MARINE ENGINEERS' HANDBOOK

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Marine Engineers' Handbook

Prepared by a Staff of Specialists

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IV

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PREFACE

This handbook is compiled especially for the use of design engineers and technical students. Operating engineers, it is hoped, will find it useful also for its presentation of design and construction details, though no attempt has been made to include instructions covering the adjustment and maintenance of marine propulsion machinery.

This book is designed by the publishers to be a successor to "Marine Engineers' Handbook," edited by the late Commander Frank W. Sterling, published in 1920, and available until recently. In attempting a revision to provide material representative of current practice, it was found that, because of virtually complete obsolescence, an entirely new book instead of a revision was in order. Commander Sterling's book was the Bible of a generation of marine engineers; it is hoped that this volume as a successor may be equally well received.

The book retains practically as originally presented the section on Reciprocating Steam Engines but all other sections on special marine engineering topics are completely new. Much material that is of pertinent value to the marine engineer has been reprinted from the fourth edition of the "Mechanical Engineers' Handbook" with the kind permission and full cooperation of Professor Lionel S. Marks.

It is the sincere belief of the editor that the book is up to date and that it incorporates all the sound and proved marine engineering practice that has been developed rapidly within the past several years. The editor has been fortunate in having the aid as contributors of many men who have been closely associated with important current developments in this field of engineering and takes this opportunity of thanking them for their prompt and painstaking work.

In so extensive a compilation there will undoubtedly be inadvertent mistakes and omissions. The editor will be thankful if comments and constructive suggestions are communicated to him through the publishers.

J. M. LABBERTON.

NEW YORK, N. Y.,
September, 1945.

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SYMBOLS AND ABBREVIATIONS

For symbols of chemical elements, see p. 530; for electric and magnetic symbols, see pp. 1688 and 1701; for abbreviations applying to metric weights and measures, see p. 72.

Pairs of parentheses, brackets, etc., are frequently used in this work to indicate corresponding values. For example, the statement that "the cost per kw of a 30,000-kw plant is $53; of a 15,000-kw plant, $62; and of an 8000-kw plant, $72," is condensed as follows: The cost per kw of a 30,000 (15,000) [8000]-kw plant is $53 (62) [72].

A.A.R.	Assoc. of American Railroads
abs	absolute
a.c.	aerodynamic center
a-c	alternating current
A.I.E.E.	American Inst. of Electrical Engineers
A.I.M.E.	Am. Inst. of Mining Engineers
air hp	air horsepower
A.M.A.	Automobile Manufacturers' Assoc.
Am. Iron & Steel Inst.	Am. Iron and Steel Inst.
amp	amperes
antilog	*antilogarithm of*
A.P.I.	Am. Petroleum Inst.
A.R.E.A.	Am. Rlwy. Eng. Assoc.
A.S.A.	Am. Standards Assoc.
A.S.C.E.	Am. Soc. of Civil Engineers
A.S.H. & V.E.	Am. Soc. of Heating & Ventilating Engineers
A.S.M.	Am. Soc. for Metals
A.S.M.E.	Am. Soc. of Mechanical Engineers
A.S.R.E.	Am. Soc. of Refrigerating Engineers
A.S.S.T.	Am. Soc. for Steel Treating
A.S.T.M.	Am. Soc. for Testing Materials
atm	atmosphere
avdp	avoirdupois
avg	average
A.W.P.A.	Am. Wood Preservation Assoc.
A.W.S.	Am. Welding Soc.
A.W.W.A.	Am. Water Works Assoc.
bar	barometer
B. & S.	Brown & Sharpe (gage)
bbl	barrels
Bé	Baumé (degrees)
B.G.	Birmingham gage (hoop and sheet)
bhp	brake horsepower
Birm.	Birmingham
B.M.	Board measure
bmep	*brake mean effective pressure*
B. of M.	Bureau of Mines
B. of S.	Bureau of Standards
bp	boiling point
Btu	British thermal units
bu	bushels
Bull.	Bulletin
B.W.G.	Birmingham wire gage
C	degrees centigrade
cal	calories
cc	cubic centimeters
c to c	center to center
C.F.	centrifugal force
cfh	cubic feet per hour
cfm	cubic feet per minute
cfs	cubic feet per second
cg	center of gravity
cgs	centimeter-gram-second
C.I.	cast iron

cir	circular
C.M.	circular mils
cm	centimeters
C.N.	cetane number
coef	coefficient
c of g	center of gravity
col	column
colog	cologarithm of
const	constant
cos	cosine of
$\cos^{-1}$	arc whose cosine is, inverse cosine of
cosh	hyperbolic cosine of
$\cosh^{-1}$	inverse hyperbolic cosine of
ctn	cotangent of
$\cot^{-1}$	arc whose cotangent is (see $\cos^{-1}$)
coth	hyperbolic cotangent of
$\coth^{-1}$	inverse hyperbolic cotangent of
covers	coversed sine of
cp	candle power, circular pitch, center of pressure
csc	cosecant of
$\csc^{-1}$	arc whose cosecant is (see $\cos^{-1}$)
csch	hyperbolic cosecant of
csch^{-1}	inverse hyperbolic cosecant of
cu	cubic
cyl	cylinder
db	decibel
d-c	direct current
def	definition
deg	degrees
diam	diameter
d.p.	diametral pitch, double pole
e	base of Napierian logarithmic system ($= 2.7182+$)
eff	efficiency
ehp	effective horsepower
elec	electric
elong	elongation
emf	electromotive force
eq	equation
evap	evaporation
exp	exponential function of
exsec	exterior secant of
ext	external
F	degrees Fahrenheit
fhp	friction horsepower
F.O.B.	free on board (cars)
fpm	feet per minute
fps	feet per second
fps	foot-pound-second (system)
F.S.	factor of safety
F.S.B.	Federal Specifications Board
ft	feet
ft-lb	foot-pounds
g	acceleration due to gravity
g	grams
gal	gallons
g-cal	gram-calories
gd	Gudermannian of
G.E.	General Electric Co.
G.M.	General Motors Co.
gpm	gallons per minute
gps	gallons per second
hhv	high heat value
hor. (horiz.)	horizontal
hp	horsepower
h-p	high-pressure
hp-hr	horsepower-hour
hr	hours
I.C.E.	Inst. of Civil Engineers
I.C.T.	International Critical Tables
i.d.	inside diameter
ihp	indicated horsepower
I.M.E.	Inst. of Mechanical Engineers
imep	indicated mean effective pressure
Imp	Imperial
in.	inches
in.-lb	inch-pounds
int	internal
Int. Soc. Test. Mat.	International Soc. for Testing Materials
i-p	intermediate-pressure
isoth	isothermal
K	degree Kelvin (centigrade abs)
kB	kilo Btu (1000 Btu)
kg	kilograms
kg-cal	kilogram-calories
kg-m	kilogram-meters
kips	thousands of pounds per sq in.
km	kilometers

kva	kilovolt-amperes
kw	kilowatts
kwhr	kilowatt-hours
l	liters
lb	pounds
lim.	limit
lin	linear
loc. cit.	work already cited
log	common logarithm of
log	*logarithm*
$\log_e$	Napierian logarithm of
$\log_{10}$	common logarithm of
l-p	low-pressure
m	meters
M	thousand
max	maximum
Mb	thousand Btu
Mcf	thousand cubic feet
mep	mean effective pressure
mhc	mean horizontal candles
min	minutes, minimum
mip	mean indicated pressure
ml	millilamberts
mlhc	mean lower hemispherical candles
mm	millimeters
mmf	magnetomotive force
mp	melting point
mph	miles per hour
msc	mean spherical candles
M.S.S.	Manufacturers Standardization Soc. of the Valve and Fittings Industry
N	number (in mathematical tables)
N.A.C.A.	National Advisory Committee on Aeronautics
nat.	natural
Nat. Dist. Htg. Assn.	National District Heating Assoc.
N.E.M.A.	National Electrical Manufacturers Assoc.
N.B.S.	National Bureau of Standards
No. (Nos.)	number(s)
O.D.	outside diameter (pipes)
O.H.	open-hearth (steel)
O.N.	octane number
op. cit.	work already cited
oz	ounces
p. (pp.)	page (pages)
p.f.	power factor
ppm	parts per million
press	pressure
Proc.	Proceedings
psi	lb per sq in.
pt	point, pint
qt	quarts
q.v.	which see
R	deg Rankine (Fahrenheit abs)
R	Reynolds number
rad	*radius*
rev	revolutions
rms	square root of mean square
rpm	revolutions per minute
rps	revolutions per second
ry.	railway
S.A.E.	Soc. of Automotive Engineers
sat	saturated
sec	seconds
sec	secant of
$\sec^{-1}$	arc whose secant is (see $\cos^{-1}$)
sech	hyperbolic secant of
sech^{-1}	inverse hyperbolic secant of
segm	segment
shp	shaft horsepower
sin	sine of
$\sin^{-1}$	arc whose sine is (see $\cos^{-1}$)
sinh	hyperbolic sine of
$\sinh^{-1}$	inverse hyperbolic sine of
sp	specific
specif	specification
sp gr	specific gravity
sp ht	*specific heat*
sq	square
SSF	see Saybolt Furol
SSU	see Saybolt Universal
std	standard
S.W.G.	Standard (British) wire gage
tan	tangent of
$\tan^{-1}$	arc whose tangent is (see $\cos^{-1}$)
tanh	hyperbolic tangent of
$\tanh^{-1}$	inverse hyperbolic tangent of
temp	temperature
thp	thrust horsepower
Trans.	Transactions
ult	ultimate

U.S.S.	United Stated Standard	W.&M.	Washburn & Moen wire gage
U.S.S.G.	U.S. Standard Gage	w.g.	water gage
vel	velocity	W.I.	wrought iron
vers	versed sine of	wt	weight
vert	vertical	yd	yards
vol	volume	yr	year(s)
vs.	versus		

ABBREVIATIONS OF TITLES OF PERIODICAL PUBLICATIONS

Am. Engr. American Engineer (New York).

Am. Mach. American Machinist (New York).

Br. A. R. C.—R. & M. Reports and Memoranda of the British Aeronautical Research Committee.

Bull. Am. Ry. Eng. Assn. Bulletin of the American Railway Engineering Association (Chicago).

Chem. and Met. Eng. Chemical and Metallurgical Engineering (New York).

El. Wld. Electrical World (New York).

Engg. Engineering (London).

Eng. News-Rec. Engineering News-Record (New York).

Engr. The Engineer (London).

Forschung. Forschung (also Forschungsarbeiten) auf dem Gebiete des Ingenieurwesens (Berlin).

Htg. and Vent. Heating and Ventilating (New York).

Ind. & Eng. Chem. Industrial and Engineering Chemistry (Easton, Pa.)

Jour. Am. Chem. Soc. Journal of the American Chemical Society (Easton, Pa.).

Jour. Am. Soc. Nav. Engrs. Journal of the American Society of Naval Engineers (Washington, D. C.).

Jour. Inst. E. E. Journal of the Institution of Electrical Engineers (London).

Jour. Inst. of Metals. Journal of the Institute of Metals (London).

Jour. Iron and Steel Inst. Journal of the Iron and Steel Institute (London).

Jour. N. E. Water Works Assn. Journal of the New England Water Works Association (Boston).

Jour. S. A. E. Journal of the Society of Automotive Engineers (New York).

Machy. Machinery (New York).

Mech. Eng. Mechanical Engineering (The Journal of the American Society of Mechanical Engineers, New York).

N.A.C.A.—T.M. Technical Memoranda of the National Advisory Committee for Aeronautics (Washington, D. C.).

N.A.C.A.—T.N. Technical Notes of the National Advisory Committee for Aeronautics (Washington, D. C.).

N.A.C.A.—T.R. Technical Reports of the National Advisory Committee for Aeronautics (Washington, D. C.).

Phil. Mag. Philosophical Magazine (London).

Proc. A. A. R. Proceedings of the Association of American Railways (New York).

Proc. Am. Ry. Eng. Assn. Proceedings of the American Railway Engineering Association (Chicago).

Proc. Am. Ry. M. M. Assn. Proceedings of the American Railway Master Mechanics' Association (Chicago).

Proc. Am. Soc. Test. Mat. (*A. S. T. M.*). Proceedings of the American Society for Testing Materials (Philadelphia).

Proc. Inst. Auto. Eng. Proceedings of the Institution of Automobile Engineers (London).

Proc. Inst. C. E. Proceedings of the Institution of Civil Engineers (London).

Proc. Inst. M. E. Proceedings of the Institution of Mechanical Engineers (London).

Proc. M. C. B. Assn. Proceedings of the Master Car Builders' Association.

Proc. Roy. Soc. Proceedings of the Royal Society (London).

Refrig. Eng. Refrigerating Engineering (New York).

Ry. Age. Railway Age (New York).

Schweiz. Bauzeitung. Schweizerische Bauzeitung (Zürich).

Trans. A. I. E. E. Transactions of the American Institute of Electrical Engineers (New York).

Trans. A. S. M. E. Transactions of the American Society of Mechanical Engineers (New York).

Trans. Inst. Naval Architects. Transactions of the Institution of Naval Architects (London).

Trans. S. A. E. Transactions of the Society of Automotive Engineers (New York).

Zeit. f. Inst. Zeitschrift für Instrumentenkunde (Berlin).

Zeit. Ver. deutsch. Ing. (*Z. V. d. I.*). Zeitschrift des Vereines deutscher Ingenieure (Berlin).

MATHEMATICAL SIGNS AND SYMBOLS

$+$ plus (sign of addition)
$+$ positive
$-$ minus (sign of subtraction)
$-$ negative
$\pm$ ($\mp$) plus or minus (minus or plus)
$\times$ times, by (multiplication sign)
$\cdot$ multiplied by
$\div$ sign of division
$/$ divided by
$:$ ratio sign, divided by, is to
$::$ equals, as (proportion)
$<$ less than
$>$ greater than
$=$ equals
$\equiv$ identical with
$\approx$ approximately equals
$\leq$ equal to or less than
$\geq$ equal to or greater than
$\mp$ $\neq$ not equal to
$\doteq$ approaches
$\propto$ varies as
∞ infinity
$\sqrt{\ }$ square root of
$\sqrt[3]{\ }$ cube root of
$\therefore$ therefore
$\|$ parallel to
$(\,)\ [\,]\ \{\,\}$ parentheses, brackets and braces; quantities enclosed by them to be taken together in multiplying, dividing, etc.
$\overline{AB}$ length of line from A to B
π pi, $= 3.14159+$
μ microns $= 0.001$ mm
$\mu\mu$ micromillimeters $= 0.001\,\mu$
$^\circ$ degrees
$'$ minutes
$''$ seconds
$\angle$ angle
dx differential of x
Δ (delta) difference
Δx increment of x
$\partial u/\partial x$ partial derivative of u with respect to x
$\int$ integral of
$\int_a^b$ integral of, between limits a and b
Σ (sigma) summation of
$f(x)$, $F(x)$ functions of x
$4!$ factorial $4 = 1 \times 2 \times 3 \times 4$
$|x|$ absolute value of x

In the writing out of formulas the practice described in the first paragraph of p. 112 is followed throughout this book.

SECTION 1

MATHEMATICAL TABLES

AND

WEIGHTS AND MEASURES

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

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MATHEMATICAL TABLES

By E. V. HUNTINGTON

WEIGHTS AND MEASURES

By H. W. BEARCE

SQUARES OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00	1.000	1.002	1.004	1.006	1.008	1.010	1.012	1.014	1.016	1.018	2
1	1.020	1.022	1.024	1.026	1.028	1.030	1.032	1.034	1.036	1.038	
2	1.040	1.042	1.044	1.047	1.049	1.051	1.053	1.055	1.057	1.059	
3	1.061	1.063	1.065	1.067	1.069	1.071	1.073	1.075	1.077	1.080	
4	1.082	1.084	1.086	1.088	1.090	1.092	1.094	1.096	1.098	1.100	
1.05	1.102	1.105	1.107	1.109	1.111	1.113	1.115	1.117	1.119	1.121	
6	1.124	1.126	1.128	1.130	1.132	1.134	1.136	1.138	1.141	1.143	
7	1.145	1.147	1.149	1.151	1.153	1.156	1.158	1.160	1.162	1.164	
8	1.166	1.169	1.171	1.173	1.175	1.177	1.179	1.182	1.184	1.186	
9	1.188	1.190	1.192	1.195	1.197	1.199	1.201	1.203	1.206	1.208	
1.10	1.210	1.212	1.214	1.217	1.219	1.221	1.223	1.225	1.228	1.230	
1	1.232	1.234	1.237	1.239	1.241	1.243	1.245	1.248	1.250	1.252	
2	1.254	1.257	1.259	1.261	1.263	1.266	1.268	1.270	1.272	1.275	
3	1.277	1.279	1.281	1.284	1.286	1.288	1.290	1.293	1.295	1.297	
4	1.300	1.302	1.304	1.306	1.309	1.311	1.313	1.316	1.318	1.320	
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6	1.346	1.348	1.350	1.353	1.355	1.357	1.360	1.362	1.364	1.367	
7	1.369	1.371	1.374	1.376	1.378	1.381	1.383	1.385	1.388	1.390	
8	1.392	1.395	1.397	1.399	1.402	1.404	1.407	1.409	1.411	1.414	
9	1.416	1.418	1.421	1.423	1.426	1.428	1.430	1.433	1.435	1.436	
1.20	1.440	1.442	1.445	1.447	1.450	1.452	1.454	1.457	1.459	1.462	
1	1.464	1.467	1.469	1.471	1.474	1.476	1.479	1.481	1.484	1.486	
2	1.488	1.491	1.493	1.496	1.498	1.501	1.503	1.506	1.508	1.510	
3	1.513	1.515	1.518	1.520	1.523	1.525	1.528	1.530	1.533	1.535	
4	1.538	1.540	1.543	1.545	1.548	1.550	1.553	1.555	1.558	1.560	
1.25	1.562	1.565	1.568	1.570	1.573	1.575	1.578	1.580	1.583	1.585	3
6	1.588	1.590	1.593	1.595	1.598	1.600	1.603	1.605	1.608	1.610	
7	1.613	1.615	1.618	1.621	1.623	1.626	1.628	1.631	1.633	1.636	
8	1.638	1.641	1.644	1.646	1.649	1.651	1.654	1.656	1.659	1.662	
9	1.664	1.667	1.669	1.672	1.674	1.677	1.680	1.682	1.685	1.687	
1.30	1.690	1.693	1.695	1.698	1.700	1.703	1.706	1.708	1.711	1.713	
1	1.716	1.719	1.721	1.724	1.727	1.729	1.732	1.734	1.737	1.740	
2	1.742	1.745	1.748	1.750	1.753	1.756	1.758	1.761	1.764	1.766	
3	1.769	1.772	1.774	1.777	1.780	1.782	1.785	1.788	1.790	1.793	
4	1.796	1.798	1.801	1.804	1.806	1.809	1.812	1.814	1.817	1.820	
1.35	1.822	1.825	1.828	1.831	1.833	1.836	1.839	1.841	1.844	1.847	
6	1.850	1.852	1.855	1.858	1.860	1.863	1.866	1.869	1.871	1.874	
7	1.877	1.880	1.882	1.885	1.888	1.891	1.893	1.896	1.899	1.902	
8	1.904	1.907	1.910	1.913	1.915	1.918	1.921	1.924	1.927	1.929	
9	1.932	1.935	1.938	1.940	1.943	1.946	1.949	1.952	1.954	1.957	
1.40	1.960	1.963	1.986	1.968	1.971	1.974	1.977	1.980	1.982	1.985	
1	1.988	1.991	1.994	1.997	1.999	2.002	2.005	2.008	2.011	2.014	
2	2.016	2.019	2.022	2.025	2.028	2.031	2.033	2.036	2.039	2.042	
3	2.045	2.048	2.051	2.053	2.056	2.059	2.062	2.065	2.068	2.071	
4	2.074	2.076	2.079	2.082	2.085	2.088	2.091	2.094	2.097	2.100	
1.45	2.102	2.105	2.108	2.111	2.114	2.117	2.120	2.123	2.126	2.129	
6	2.132	2.135	2.137	2.140	2.143	2.146	2.149	2.152	2.155	2.158	
7	2.161	2.164	2.167	2.170	2.173	2.176	2.179	2.182	2.184	2.187	
8	2.190	2.193	2.196	2.199	2.202	2.205	2.208	2.211	2.214	2.217	
9	2.220	2.223	2.226	2.229	2.232	2.235	2.238	2.241	2.244	2.247	

Moving the decimal point ONE place in N requires moving it TWO places in body of table (see p. 6).

SQUARES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.50	2.250	2.253	2.256	2.259	2.262	2.265	2.268	2.271	2.274	2.277	3
1	2.280	2.283	2.286	2.289	2.292	2.295	2.298	2.301	2.304	2.307	
2	2.310	2.313	2.316	2.320	2.323	2.326	2.329	2.332	2.335	2.338	
3	2.341	2.344	2.347	2.350	2.353	2.356	2.359	2.362	2.365	2.369	
4	2.372	2.375	2.378	2.381	2.384	2.387	2.390	2.393	2.396	2.399	
1.55	2.402	2.406	2.409	2.412	2.415	2.418	2.421	2.424	2.427	2.430	
6	2.434	2.437	2.440	2.443	2.446	2.449	2.452	2.455	2.459	2.462	
7	2.465	2.468	2.471	2.474	2.477	2.481	2.484	2.487	2.490	2.493	
8	2.496	2.500	2.503	2.506	2.509	2.512	2.515	2.519	2.522	2.525	
9	2.528	2.531	2.534	2.538	2.541	2.544	2.547	2.550	2.554	2.557	
1.60	2.560	2.563	2.566	2.570	2.573	2.576	2.579	2.582	2.586	2.589	
1	2.592	2.595	2.599	2.602	2.605	2.608	2.611	2.615	2.618	2.621	
2	2.624	2.628	2.631	2.634	2.637	2.641	2.644	2.647	2.650	2.654	
3	2.657	2.660	2.663	2.667	2.670	2.673	2.676	2.680	2.683	2.686	
4	2.690	2.693	2.696	2.699	2.703	2.706	2.709	2.713	2.716	2.719	
1.65	2.722	2.726	2.729	2.732	2.736	2.739	2.742	2.746	2.749	2.752	
6	2.756	2.759	2.762	2.766	2.769	2.772	2.776	2.779	2.782	2.786	
7	2.789	2.792	2.796	2.799	2.802	2.806	2.809	2.812	2.816	2.819	
8	2.822	2.826	2.829	2.832	2.836	2.839	2.843	2.846	2.849	2.853	
9	2.856	2.859	2.863	2.866	2.870	2.873	2.876	2.880	2.883	2.887	
1.70	2.890	2.893	2.897	2.900	2.904	2.907	2.910	2.914	2.917	2.921	
1	2.924	2.928	2.931	2.934	2.938	2.941	2.945	2.948	2.952	2.955	
2	2.958	2.962	2.965	2.969	2.972	2.976	2.979	2.983	2.986	2.989	
3	2.993	2.996	3.000	3.003	3.007	3.010	3.014	3.017	3.021	3.024	
4	3.028	3.031	3.035	3.038	3.042	3.045	3.049	3.052	3.056	3.059	
1.75	3.062	3.066	3.070	3.073	3.077	3.080	3.084	3.087	3.091	3.094	4
6	3.098	3.101	3.105	3.108	3.112	3.115	3.119	3.122	3.126	3.129	
7	3.133	3.136	3.140	3.144	3.147	3.151	3.154	3.158	3.161	3.165	
8	3.168	3.172	3.176	3.179	3.183	3.186	3.190	3.193	3.197	3.201	
9	3.204	3.208	3.211	3.215	3.218	3.222	3.226	3.229	3.233	3.236	
1.80	3.240	3.244	3.247	3.251	3.254	3.258	3.262	3.265	3.269	3.272	
1	3.276	3.280	3.283	3.287	3.291	3.294	3.298	3.301	3.305	3.309	
2	3.312	3.316	3.320	3.323	3.327	3.331	3.334	3.338	3.342	3.345	
3	3.349	3.353	3.356	3.360	3.364	3.367	3.371	3.375	3.378	3.382	
4	3.386	3.389	3.393	3.397	3.400	3.404	3.408	3.411	3.415	3.419	
1.85	3.422	3.426	3.430	3.434	3.437	3.441	3.445	3.448	3.452	3.456	
6	3.460	3.463	3.467	3.471	3.474	3.478	3.482	3.486	3.489	3.493	
7	3.497	3.501	3.504	3.508	3.512	3.516	3.519	3.523	3.527	3.531	
8	3.534	3.538	3.542	3.546	3.549	3.553	3.557	3.561	3.565	3.568	
9	3.572	3.576	3.580	3.583	3.587	3.591	3.595	3.599	3.602	3.606	
1.90	3.610	3.614	3.618	3.621	3.625	3.629	3.633	3.637	3.640	3.644	
1	3.648	3.652	3.656	3.660	3.663	3.667	3.671	3.675	3.679	3.683	
2	3.686	3.690	3.694	3.698	3.702	3.706	3.709	3.713	3.717	3.721	
3	3.725	3.729	3.733	3.736	3.740	3.744	3.748	3.752	3.756	3.760	
4	3.764	3.767	3.771	3.775	3.779	3.783	3.787	3.791	3.795	3.799	
1.95	3.802	3.806	3.810	3.814	3.818	3.822	3.826	3.830	3.834	3.838	
6	3.842	3.846	3.849	3.853	3.857	3.861	3.865	3.869	3.873	3.877	
7	3.881	3.885	3.889	3.893	3.897	3.901	3.905	3.909	3.912	3.916	
8	3.920	3.924	3.928	3.932	3.936	3.940	3.944	3.948	3.952	3.956	
9	3.960	3.964	3.968	3.972	3.976	3.980	3.984	3.988	3.992	3.996	

$\pi^2 = 9.86960$ · $1/\pi^2 = 0.101321$ $e^2 = 7.38906$

SQUARES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. dif.
2.00	4.000	4.004	4.008	4.012	4.016	4.020	4.024	4.028	4.032	4.036	4
1	4.040	4.044	4.048	4.052	4.056	4.060	4.064	4.068	4.072	4.076	
2	4.080	4.084	4.088	4.093	4.097	4.101	4.105	4.109	4.113	4.117	
3	4.121	4.125	4.129	4.133	4.137	4.141	4.145	4.149	4.153	4.158	
4	4.162	4.166	4.170	4.174	4.178	4.182	4.186	4.190	4.194	4.198	
2.05	4.202	4.207	4.211	4.215	4.219	4.223	4.227	4.231	4.235	4.239	
6	4.244	4.248	4.252	4.256	4.260	4.264	4.268	4.272	4.277	4.281	
7	4.285	4.289	4.293	4.297	4.301	4.306	4.310	4.314	4.318	4.322	
8	4.326	4.331	4.335	4.339	4.343	4.347	4.351	4.356	4.360	4.364	
9	4.368	4.372	4.376	4.381	4.385	4.389	4.393	4.397	4.402	4.406	
2.10	4.410	4.414	4.418	4.423	4.427	4.431	4.435	4.439	4.444	4.448	
1	4.452	4.456	4.461	4.465	4.469	4.473	4.477	4.482	4.486	4.490	
2	4.494	4.499	4.503	4.507	4.511	4.516	4.520	4.524	4.528	4.533	
3	4.537	4.541	4.545	4.550	4.554	4.558	4.562	4.567	4.571	4.575	
4	4.580	4.584	4.588	4.592	4.597	4.601	4.605	4.610	4.614	4.618	
2.15	4.622	4.627	4.631	4.635	4.640	4.644	4.648	4.653	4.657	4.661	
6	4.666	4.670	4.674	4.679	4.683	4.687	4.692	4.696	4.700	4.705	
7	4.709	4.713	4.718	4.722	4.726	4.731	4.735	4.739	4.744	4.748	
8	4.752	4.757	4.761	4.765	4.770	4.774	4.779	4.783	4.787	4.792	
9	4.796	4.800	4.805	4.809	4.814	4.818	4.822	4.827	4.831	4.836	
2.20	4.840	4.844	4.849	4.853	4.858	4.862	4.866	4.871	4.875	4.880	
1	4.884	4.889	4.893	4.897	4.902	4.906	4.911	4.915	4.920	4.924	
2	4.928	4.933	4.937	4.942	4.946	4.951	4.955	4.960	4.964	4.968	
3	4.973	4.977	4.982	4.986	4.991	4.995	5.000	5.004	5.009	5.013	
4	5.018	5.022	5.027	5.031	5.036	5.040	5.045	5.049	5.054	5.058	
2.25	5.062	5.067	5.072	5.076	5.081	5.085	5.090	5.094	5.099	5.103	5
6	5.108	5.112	5.117	5.121	5.126	5.130	5.135	5.139	5.144	5.148	
7	5.153	5.157	5.162	5.167	5.171	5.176	5.180	5.185	5.189	5.194	
8	5.198	5.203	5.208	5.212	5.217	5.221	5.226	5.230	5.235	5.240	
9	5.244	5.249	5.253	5.258	5.262	5.267	5.272	5.276	5.281	5.285	
2.30	5.290	5.295	5.299	5.304	5.308	5.313	5.318	5.322	5.327	5.331	
1	5.336	5.341	5.345	5.350	5.355	5.359	5.364	5.368	5.373	5.378	
2	5.382	5.387	5.392	5.396	5.401	5.406	5.410	5.415	5.420	5.424	
3	5.429	5.434	5.438	5.443	5.448	5.452	5.457	5.462	5.466	5.471	
4	5.476	5.480	5.485	5.490	5.494	5.499	5.504	5.508	5.513	5.518	
2.35	5.522	5.527	5.532	5.537	5.541	5.546	5.551	5.555	5.560	5.565	
6	5.570	5.574	5.579	5.584	5.588	5.593	5.598	5.603	5.607	5.612	
7	5.617	5.622	5.626	5.631	5.636	5.641	5.645	5.650	5.655	5.660	
8	5.664	5.669	5.674	5.679	5.683	5.688	5.693	5.698	5.703	5.707	
9	5.712	5.717	5.722	5.726	5.731	5.736	5.741	5.746	5.750	5.755	
2.40	5.760	5.765	5.770	5.774	5.779	5.784	5.789	5.794	5.798	5.803	
1	5.808	5.813	5.818	5.823	5.827	5.832	5.837	5.842	5.847	5.852	
2	5.856	5.861	5.866	5.871	5.876	5.881	5.885	5.890	5.895	5.900	
3	5.905	5.910	5.915	5.919	5.924	5.929	5.934	5.939	5.944	5.949	
4	5.954	5.958	5.963	5.968	5.973	5.978	5.983	5.988	5.993	5.998	
2.45	6.002	6.007	6.012	6.017	6.022	6.027	6.032	6.037	6.042	6.047	
6	6.052	6.057	6.061	6.066	6.071	6.076	6.081	6.086	6.091	6.096	
7	6.101	6.106	6.111	6.116	6.121	6.126	6.131	6.136	6.140	6.145	
8	6.150	6.155	6.160	6.165	6.170	6.175	6.180	6.185	6.190	6.195	
9	6.200	6.205	6.210	6.215	6.220	6.225	6.230	6.235	6.240	6.245	

Moving the decimal point ONE place in N requires moving it TWO places in body of table (see p. 6).

SQUARES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
2.50	6.250	6.255	6.260	6.265	6.270	6.275	6.280	6.285	6.290	6.295	5
1	6.300	6.305	6.310	6.315	6.320	6.325	6.330	6.335	6.340	6.345	
2	6.350	6.355	6.360	6.366	6.371	6.376	6.381	6.386	6.391	6.396	
3	6.401	6.406	6.411	6.416	6.421	6.426	6.431	6.436	6.441	6.447	
4	6.452	6.457	6.462	6.467	6.472	6.477	6.482	6.487	6.492	6.497	
2.55	6.502	6.508	6.513	6.518	6.523	6.528	6.533	6.538	6.543	6.548	
6	6.554	6.559	6.564	6.569	6.574	6.579	6.584	6.589	6.595	6.600	
7	6.605	6.610	6.615	6.620	6.625	6.631	6.636	6.641	6.646	6.651	
8	6.656	6.662	6.667	6.672	6.677	6.682	6.687	6.693	6.698	6.703	
9	6.708	6.713	6.718	6.724	6.729	6.734	6.739	6.744	6.750	6.755	
2.60	6.760	6.765	6.770	6.776	6.781	6.786	6.791	6.796	6.802	6.807	
1	6.812	6.817	6.823	6.828	6.833	6.838	6.843	6.849	6.854	6.859	
2	6.864	6.870	6.875	6.880	6.885	6.891	6.896	6.901	6.906	6.912	
3	6.917	6.922	6.927	6.933	6.938	6.943	6.948	6.954	6.959	6.964	
4	6.970	6.975	6.980	6.985	6.991	6.996	7.001	7.007	7.012	7.017	
2.65	7.022	7.028	7.033	7.038	7.044	7.049	7.054	7.060	7.065	7.070	
6	7.076	7.081	7.086	7.092	7.097	7.102	7.108	7.113	7.118	7.124	
7	7.129	7.134	7.140	7.145	7.150	7.156	7.161	7.166	7.172	7.177	
8	7.182	7.188	7.193	7.198	7.204	7.209	7.215	7.220	7.225	7.231	
9	7.236	7.241	7.247	7.252	7.258	7.263	7.268	7.274	7.279	7.285	
2.70	7.290	7.295	7.301	7.306	7.312	7.317	7.322	7.328	7.333	7.339	
1	7.344	7.350	7.355	7.360	7.366	7.371	7.377	7.382	7.388	7.393	
2	7.398	7.404	7.409	7.415	7.420	7.426	7.431	7.437	7.442	7.447	
3	7.453	7.458	7.464	7.469	7.475	7.480	7.486	7.491	7.497	7.502	
4	7.508	7.513	7.519	7.524	7.530	7.535	7.541	7.546	7.552	7.557	
2.75	7.562	7.568	7.574	7.579	7.585	7.590	7.596	7.601	7.607	7.612	6
6	7.618	7.623	7.629	7.634	7.640	7.645	7.651	7.656	7.662	7.667	
7	7.673	7.678	7.684	7.690	7.695	7.701	7.706	7.712	7.717	7.723	
8	7.728	7.734	7.740	7.745	7.751	7.756	7.762	7.767	7.773	7.779	
9	7.784	7.790	7.795	7.801	7.806	7.812	7.818	7.823	7.829	7.834	
2.80	7.840	7.846	7.851	7.857	7.862	7.868	7.874	7.879	7.885	7.890	
1	7.896	7.902	7.907	7.913	7.919	7.924	7.930	7.935	7.941	7.947	
2	7.952	7.958	7.964	7.969	7.975	7.981	7.986	7.992	7.998	8.003	
3	8.009	8.015	8.020	8.026	8.032	8.037	8.043	8.049	8.054	8.060	
4	8.066	8.071	8.077	8.083	8.088	8.094	8.100	8.105	8.111	8.117	
2.85	8.122	8.128	8.134	8.140	8.145	8.151	8.157	8.162	8.168	8.174	
6	8.180	8.185	8.191	8.197	8.202	8.208	8.214	8.220	8.225	8.231	
7	8.237	8.243	8.248	8.254	8.260	8.266	8.271	8.277	8.283	8.289	
8	8.294	8.300	8.306	8.312	8.317	8.323	8.329	8.335	8.341	8.346	
9	8.352	8.358	8.364	8.369	8.375	8.381	8.387	8.393	8.398	8.404	
2.90	8.410	8.416	8.422	8.427	8.433	8.439	8.445	8.451	8.456	8.462	
1	8.468	8.474	8.480	8.486	8.491	8.497	8.503	8.509	8.515	8.521	
2	8.526	8.532	8.538	8.544	8.550	8.556	8.561	8.567	8.573	8.579	
3	8.585	8.591	8.597	8.602	8.608	8.614	8.620	8.626	8.632	8.638	
4	8.644	8.649	8.655	8.661	8.667	8.673	8.679	8.685	8.691	8.697	
2.95	8.702	8.708	8.714	8.720	8.726	8.732	8.738	8.744	8.750	8.756	
6	8.762	8.768	8.773	8.779	8.785	8.791	8.797	8.803	8.809	8.815	
7	8.821	8.827	8.833	8.839	8.845	8.851	8.857	8.863	8.868	8.874	
8	8.880	8.886	8.892	8.898	8.904	8.910	8.916	8.922	8.928	8.934	
9	8.940	8.946	8.952	8.958	8.964	8.970	8.976	8.982	8.988	8.994	

$\pi^2 = 9.86960 \quad 1/\pi^2 = 0.101321 \quad e^2 = 7.38906$

SQUARES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
3.00	9.000	9.006	9.012	9.018	9.024	9.030	9.036	9.042	9.048	9.054	6
1	9.060	9.066	9.072	9.078	9.084	9.090	9.096	9.102	9.108	9.114	
2	9.120	9.126	9.132	9.139	9.145	9.151	9.157	9.163	9.169	9.175	
3	9.181	9.187	9.193	9.199	9.205	9.211	9.217	9.223	9.229	9.236	
4	9.242	9.248	9.254	9.260	9.266	9.272	9.278	9.284	9.290	9.296	
3.05	9.302	9.309	9.315	9.321	9.327	9.333	9.339	9.345	9.351	9.357	
6	9.364	9.370	9.376	9.382	9.388	9.394	9.400	9.406	9.413	9.419	
7	9.425	9.431	9.437	9.443	9.449	9.456	9.462	9.468	9.474	9.480	
8	9.486	9.493	9.499	9.505	9.511	9.517	9.523	9.530	9.536	9.542	
9	9.548	9.554	9.560	9.567	9.573	9.579	9.585	9.591	9.598	9.604	
3.10	9.610	9.616	9.622	9.629	9.635	9.641	9.647	9.653	9.660	9.666	
1	9.672	9.678	9.685	9.691	9.697	9.703	9.709	9.716	9.722	9.728	
2	9.734	9.741	9.747	9.753	9.759	9.766	9.772	9.778	9.784	9.791	
3	9.797	9.803	9.809	9.816	9.822	9.828	9.834	9.841	9.847	9.853	
4	9.860	9.866	9.872	9.878	9.885	9.891	9.897	9.904	9.910	9.916	
3.15	9.922	9.929	9.935	9.941	9.948	9.954	9.960	9.967	9.973	9.979	
6	9.986	9.992	9.998	10.005							6
3.1							9.99	10.05	10.11	10.18	6
2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	
3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	7
4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	
6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	
7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	8
8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	
9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	
1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	
2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	
3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	9
4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	
6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	
7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	10
8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	
9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	

$\pi^2 = 9.86960 \quad (\pi/2)^2 = 2.46740 \quad 1/\pi^2 = 0.101321$

Explanation of Table of Squares (pp. 2–7).

This table gives the value of N^2 for values of N from 1 to 10, correct to four figures (Interpolated values may be in error by 1 in the fourth figure).

To find the square of a number N outside the range from 1 to 10, note that moving the decimal point one place in column N is equivalent to moving it two places *in the body of the table. For example:*

$(3.217)^2 = 10.35; \quad (0.03217)^2 = 0.001035; \quad (3217)^2 = 10350000$

This table can also be used inversely, to give square roots.

SQUARES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	10
1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	
2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	
3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05	11
4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	
6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	
7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	
8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	12
9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	
1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32	
2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	
3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83	13
4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	
6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	
7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	
8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47	14
9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27	
1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70	
2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14	
3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	15
4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10	
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61	
6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	
7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68	
8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25	16
9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45	
1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08	
2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	
3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39	17
4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	
6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52	
7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	
8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03	18
9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82	
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63	
1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	
2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30	
3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17	19
4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06	
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	
6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	
7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84	
8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	20
9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	
10.0	100.0										

Moving the decimal point ONE place in N requires moving it TWO places in body of table (see p. 6).

CUBES OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. dif.
1.00	1.000	1.003	1.006	1.009	1.012	1.015	1.018	1.021	1.024	1.027	3
1	1.030	1.033	1.036	1.040	1.043	1.046	1.049	1.052	1.055	1.058	
2	1.061	1.064	1.067	1.071	1.074	1.077	1.080	1.083	1.086	1.090	
3	1.093	1.096	1.099	1.102	1.106	1.109	1.112	1.115	1.118	1.122	
4	1.125	1.128	1.131	1.135	1.138	1.141	1.144	1.148	1.151	1.154	
1.05	1.158	1.161	1.164	1.168	1.171	1.174	1.178	1.181	1.184	1.188	
6	1.191	1.194	1.198	1.201	1.205	1.208	1.211	1.215	1.218	1.222	
7	1.225	1.228	1.232	1.235	1.239	1.242	1.246	1.249	1.253	1.256	
8	1.260	1.263	1.267	1.270	1.274	1.277	1.281	1.284	1.288	1.291	4
9	1.295	1.299	1.302	1.306	1.309	1.313	1.317	1.320	1.324	1.327	
1.10	1.331	1.335	1.338	1.342	1.346	1.349	1.353	1.357	1.360	1.364	
1	1.368	1.371	1.375	1.379	1.382	1.386	1.390	1.394	1.397	1.401	
2	1.405	1.409	1.412	1.416	1.420	1.424	1.428	1.431	1.435	1.439	
3	1.443	1.447	1.451	1.454	1.458	1.462	1.466	1.470	1.474	1.478	
4	1.482	1.485	1.489	1.493	1.497	1.501	1.505	1.509	1.513	1.517	
1.15	1.521	1.525	1.529	1.533	1.537	1.541	1.545	1.549	1.553	1.557	
6	1.561	1.565	1.569	1.573	1.577	1.581	1.585	1.589	1.593	1.598	
7	1.602	1.606	1.610	1.614	1.618	1.622	1.626	1.631	1.635	1.639	
8	1.643	1.647	1.651	1.656	1.660	1.664	1.668	1.672	1.677	1.681	
9	1.685	1.689	1.694	1.698	1.702	1.706	1.711	1.715	1.719	1.724	
1.20	1.728	1.732	1.737	1.741	1.745	1.750	1.754	1.758	1.763	1.767	
1	1.772	1.776	1.780	1.785	1.789	1.794	1.798	1.802	1.807	1.811	
2	1.816	1.820	1.825	1.829	1.834	1.838	1.843	1.847	1.852	1.856	
3	1.861	1.865	1.870	1.875	1.879	1.884	1.888	1.893	1.897	1.902	
4	1.907	1.911	1.916	1.920	1.925	1.930	1.934	1.939	1.944	1.948	5
1.25	1.953	1.958	1.963	1.967	1.972	1.977	1.981	1.986	1.991	1.996	
6	2.000	2.005	2.010	2.015	2.019	2.024	2.029	2.034	2.039	2.044	
7	2.048	2.053	2.058	2.063	2.068	2.073	2.078	2.082	2.087	2.092	
8	2.097	2.102	2.107	2.112	2.117	2.122	2.127	2.132	2.137	2.142	
9	2.147	2.152	2.157	2.162	2.167	2.172	2.177	2.182	2.187	2.192	
1.30	2.197	2.202	2.207	2.212	2.217	2.222	2.228	2.233	2.238	2.243	
1	2.248	2.253	2.258	2.264	2.269	2.274	2.279	2.284	2.290	2.295	
2	2.300	2.305	2.310	2.316	2.321	2.326	2.331	2.337	2.342	2.347	
3	2.353	2.358	2.363	2.369	2.374	2.379	2.385	2.390	2.395	2.401	
4	2.406	2.411	2.417	2.422	2.428	2.433	2.439	2.444	2.449	2.455	
1.35	2.460	2.466	2.471	2.477	2.482	2.488	2.493	2.499	2.504	2.510	6
6	2.515	2.521	2.527	2.532	2.538	2.543	2.549	2.554	2.560	2.566	
7	2.571	2.577	2.583	2.588	2.594	2.600	2.605	2.611	2.617	2.622	
8	2.628	2.634	2.640	2.645	2.651	2.657	2.663	2.668	2.674	2.680	
9	2.686	2.691	2.697	2.703	2.709	2.715	2.721	2.726	2.732	2.738	
1.40	2.744	2.750	2.756	2.762	2.768	2.774	2.779	2.785	2.791	2.797	
1	2.803	2.809	2.815	2.821	2.827	2.833	2.839	2.845	2.851	2.857	
2	2.863	2.869	2.875	2.881	2.888	2.894	2.900	2.906	2.912	2.918	
3	2.924	2.930	2.936	2.943	2.949	2.955	2.961	2.967	2.974	2.980	
4	2.986	2.992	2.998	3.005	3.011	3.017	3.023	3.030	3.036	3.042	
1.45	3.049	3.055	3.061	3.068	3.074	3.080	3.087	3.093	3.099	3.106	
6	3.112	3.119	3.125	3.131	3.138	3.144	3.151	3.157	3.164	3.170	
7	3.177	3.183	3.190	3.196	3.203	3.209	3.216	3.222	3.229	3.235	
8	3.242	3.248	3.255	3.262	3.268	3.275	3.281	3.288	3.295	3.301	7
9	3.308	3.315	3.321	3.328	3.335	3.341	3.348	3.355	3.362	3.368	

Moving the decimal point ONE place in *N* requires moving it THREE places in body of table (see p. 10).

CUBES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avr. diff.
1.50	3.375	3.382	3.389	3.395	3.402	3.409	3.416	3.422	3.429	3.436	7
1	3.443	3.450	3.457	3.464	3.470	3.477	3.484	3.491	3.498	3.505	
2	3.512	3.519	3.526	3.533	3.540	3.547	3.554	3.561	3.568	3.575	
3	3.582	3.589	3.596	3.603	3.610	3.617	3.624	3.631	3.638	3.645	
4	3.652	3.659	3.667	3.674	3.681	3.688	3.695	3.702	3.709	3.717	
1.55	3.724	3.731	3.738	3.746	3.753	3.760	3.767	3.775	3.782	3.789	
6	3.796	3.804	3.811	3.818	3.826	3.833	3.840	3.848	3.855	3.863	
7	3.870	3.877	3.885	3.892	3.900	3.907	3.914	3.922	3.929	3.937	
8	3.944	3.952	3.959	3.967	3.974	3.982	3.989	3.997	4.005	4.012	8
9	4.020	4.027	4.035	4.042	4.050	4.058	4.065	4.073	4.081	4.088	
1.60	4.096	4.104	4.111	4.119	4.127	4.135	4.142	4.150	4.158	4.166	
1	4.173	4.181	4.189	4.197	4.204	4.212	4.220	4.228	4.236	4.244	
2	4.252	4.259	4.267	4.275	4.283	4.291	4.299	4.307	4.315	4.323	
3	4.331	4.339	4.347	4.355	4.363	4.371	4.379	4.387	4.395	4.403	
4	4.411	4.419	4.427	4.435	4.443	4.451	4.460	4.468	4.476	4.484	
1.65	4.492	4.500	4.508	4.517	4.525	4.533	4.541	4.550	4.558	4.566	
6	4.574	4.583	4.591	4.599	4.607	4.616	4.624	4.632	4.641	4.649	
7	4.657	4.666	4.674	4.683	4.691	4.699	4.708	4.716	4.725	4.733	
8	4.742	4.750	4.759	4.767	4.776	4.784	4.793	4.801	4.810	4.818	
9	4.827	4.835	4.844	4.853	4.861	4.870	4.878	4.887	4.896	4.904	9
1.70	4.913	4.922	4.930	4.939	4.948	4.956	4.965	4.974	4.983	4.991	
1	5.000	5.009	5.018	5.027	5.035	5.044	5.053	5.062	5.071	5.080	
2	5.088	5.097	5.106	5.115	5.124	5.133	5.142	5.151	5.160	5.169	
3	5.178	5.187	5.196	5.205	5.214	5.223	5.232	5.241	5.250	5.259	
4	5.268	5.277	5.286	5.295	5.304	5.314	5.323	5.332	5.341	5.350	
1.75	5.359	5.369	5.378	5.387	5.396	5.405	5.415	5.424	5.433	5.442	
6	5.452	5.461	5.470	5.480	5.489	5.498	5.508	5.517	5.526	5.536	
7	5.545	5.555	5.564	5.573	5.583	5.592	5.602	5.611	5.621	5.630	10
8	5.640	5.649	5.659	5.668	5.678	5.687	5.697	5.707	5.716	5.726	
9	5.735	5.745	5.755	5.764	5.774	5.784	5.793	5.803	5.813	5.822	
1.80	5.832	5.842	5.851	5.861	5.871	5.881	5.891	5.900	5.910	5.920	
1	5.930	5.940	5.949	5.959	5.969	5.979	5.989	5.999	6.009	6.019	
2	6.029	6.039	6.048	6.058	6.068	6.078	6.088	6.098	6.108	6.118	
3	6.128	6.139	6.149	6.159	6.169	6.179	6.189	6.199	6.209	6.219	
4	6.230	6.240	6.250	6.260	6.270	6.280	6.291	6.301	6.311	6.321	
1.85	6.332	6.342	6.352	6.362	6.373	6.383	6.393	6.404	6.414	6.424	
6	6.435	6.445	6.456	6.466	6.476	6.487	6.497	6.508	6.518	6.529	
7	6.539	6.550	6.560	6.571	6.581	6.592	6.602	6.613	6.623	6.634	11
8	6.645	6.655	6.666	6.677	6.687	6.698	6.708	6.719	6.730	6.741	
9	6.751	6.762	6.773	6.783	6.794	6.805	6.816	6.827	6.837	6.848	
1.90	6.859	6.870	6.881	6.892	6.902	6.913	6.924	6.935	6.946	6.957	
1	6.968	6.979	6.990	7.001	7.012	7.023	7.034	7.045	7.056	7.067	
2	7.078	7.089	7.100	7.111	7.122	7.133	7.144	7.156	7.167	7.178	
3	7.189	7.200	7.211	7.223	7.234	7.245	7.256	7.268	7.279	7.290	
4	7.301	7.313	7.324	7.335	7.347	7.358	7.369	7.381	7.392	7.403	
1.95	7.415	7.426	7.438	7.449	7.461	7.472	7.484	7.495	7.507	7.518	12
6	7.530	7.541	7.553	7.564	7.576	7.587	7.599	7.610	7.622	7.634	
7	7.645	7.657	7.669	7.680	7.692	7.704	7.715	7.727	7.739	7.751	
8	7.762	7.774	7.786	7.798	7.810	7.821	7.833	7.845	7.857	7.869	
9	7.881	7.892	7.904	7.916	7.928	7.940	7.952	7.964	7.976	7.988	

$\pi^3 = 31.0063 \qquad 1/\pi^3 = 0.0322515+$

CUBES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
2.00	8.000	8.012	8.024	8.036	8.048	8.060	8.072	8.084	8.096	8.108	12
1	8.121	8.133	8.145	8.157	8.169	8.181	8.194	8.206	8.218	8.230	
2	8.242	8.255	8.267	8.279	9.291	8.304	8.316	8.328	8.341	8.353	
3	8.365	8.378	8.390	8.403	8.415	8.427	8.440	8.452	8.465	8.477	
4	8.490	8.502	8.515	8.527	8.540	8.552	8.565	8.577	8.590	8.603	
2.05	8.615	8.628	8.640	8.653	8.666	8.678	8.691	8.704	8.716	8.729	13
6	8.742	8.755	8.767	8.780	8.793	8.806	8.818	8.831	8.844	8.857	
7	8.870	8.883	8.895	8.908	8.921	8.934	8.947	8.960	8.973	8.986	
8	8.999	9.012	9.025	9.038	9.051	9.064	9.077	9.090	9.103	9.116	
9	9.129	9.142	9.156	9.169	9.182	9.195	9.208	9.221	9.235	9.248	
2.10	9.261	9.274	9.287	9.301	9.314	9.327	9.341	9.354	9.367	9.381	
1	9.394	9.407	9.421	9.434	9.447	9.461	9.474	9.488	9.501	9.515	
2	9.528	9.542	9.555	9.569	9.582	9.596	9.609	9.623	9.636	9.650	14
3	9.664	9.677	9.691	9.704	9.718	9.732	9.745	9.759	9.773	9.787	
4	9.800	9.814	9.828	9.842	9.855	9.869	9.883	9.897	9.911	9.925	
2.15	9.938	9.952	9.966	9.980	9.994	10.008					14
2.1						9.94	10.08	10.22	10.36	10.50	14
2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01	15
3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65	16
4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44	18
2.5	15.62	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37	20
6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47	21
7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72	23
8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14	24
9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73	26
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50	28
1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46	30
2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61	32
3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96	34
4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51	36
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	39
6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	40
7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	42
8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	44
9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	47
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	49
1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	52
2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	54
3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	58
4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	59
4.5	91.12	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	62
6	97.34	97.97	98.61	99.25	99.90	100.54					64
6						100.5	101.2	101.8	102.5	103.2	7
7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9	7
8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9	7
9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3	7

Explanation of Table of Cubes (pp. 8–11).

This table gives the value of N^3 for values of N from 1 to 10, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the cube of a number N outside the range from 1 to 10, note that moving the decimal point one place in column N is equivalent to moving it three places in the body of the table. For example:

$(4.852)^3 = 114.2$; $(0.4852)^3 = 0.1142$; $(485.2)^3 = 114200000$

This table may also be used inversely, **to give cube roots.**

CUBES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9	8
1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8	
2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0	
3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6	9
4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5	
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7	
6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2	10
7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1	
8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3	
9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9	
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	11
1	227.0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2	
2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9	12
3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9	
4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4	
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2	13
6	287.5	288.8	290.1	291.4	292.8	294.1	295.4	296.7	298.1	299.4	
7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0	14
8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1	
9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5	
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4	15
1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7	
2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4	16
3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6	
4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2	17
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2	
6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8	18
7	456.5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7	
8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2	
9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1	19
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5	
1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4	20
2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569.7	
3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6	21
4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0	
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8	22
6	636.1	638.3	640.5	642.7	645.0	647.2	649.5	651.7	654.0	656.2	
7	658.5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2	23
8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6	24
9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6	
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1	25
1	753.6	756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2	
2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8	26
3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9	
4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7	27
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0	
6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9	28
7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3	
8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4	29
9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0	
10.0	1000.0										

$\pi^3 = 31.0063$ $\qquad 1/\pi^3 = 0.0322515+$

Moving the decimal point ONE place in *N* requires moving it THREE places in body of table (see p. 10).

SQUARE ROOTS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044	5
1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	
2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136	4
3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179	
4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261	
6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300	
7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338	
8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375	
9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446	
1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	3
2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	
3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	
4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578	
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	
6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640	
7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670	
8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700	
9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729	
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758	
1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786	
2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	
3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841	
4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868	
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895	
6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921	
7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	
8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972	
9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997	
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022	
1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047	2
2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	
3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095	
4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119	
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142	
6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	
7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189	
8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211	
9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234	

$\sqrt{\pi} = 1.77245+$ $\quad 1/\sqrt{\pi} = 0.56419 \quad \sqrt{\pi/2} = 1.25331 \quad \sqrt{e} = 1.64872$

Explanation of Table of Square Roots (pp. 12–15).

This table gives the values of $\sqrt{N}$ for values of N from 1 to 100, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the square root of a number N outside the range from 1 to 100, divide the digits of the number into blocks of two (beginning with the decimal point), and note that moving the decimal point two places in N is equivalent to *moving it one place* in the square root of N. For example:

$$\sqrt{2.718} = 1.648; \quad \sqrt{271.8} = 16.48; \quad \sqrt{0.0002718} = 0.01648;$$

$$\sqrt{27.18} = 5.213; \quad \sqrt{2718} = 52.13; \quad \sqrt{0.002718} = 0.05213.$$

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256	2
1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278	
2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300	
3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322	
4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343	
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	
6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385	
7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	
8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427	
9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447	
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	
1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	
2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	
3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	
4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567	
6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587	
7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	
8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625	
9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663	
1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	
2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	
3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718	
4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737	
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755	
6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	
7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	
8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809	
9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827	
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844	
1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862	
2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	
3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897	
4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914	
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	
6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	
7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965	
8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982	
9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998	
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015	
1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032	
2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048	
3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064	
4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097	
6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113	
7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	
8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145	
9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161	

Moving the decimal point TWO places in *N* requires moving it ONE place in body of table (see p. 12).

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
10.	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302	16
1.	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450	15
2.	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592	14
3.	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728	
4.	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860	13
15.	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987	
6.	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111	12
7.	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231	
8.	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347	
9.	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461	11
20.	4.472	4 483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572	
1.	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680	
2.	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785	
3.	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	10
4.	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990	
25.	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089	
6	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	
7.	5.196	5.206	5.215	5.225	5.235	5 244	5.254	5.263	5.273	5.282	
8.	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	9
9.	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468	
30.	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559	
1.	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648	
2.	5.657	5.666	5.675	5.683	5.692	5.201	5.710	5.718	5.727	5.736	
3.	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822	
4.	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908	8
35.	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992	
6.	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075	
7.	6.083	6.091	6.099	6.107	6 116	6.124	6.132	6.140	6.148	6.156	
8.	6.164	6.173	6.181	6.189	6.192	6.205	6.213	6.221	6.229	6.237	
9.	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317	
40.	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395	
1.	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473	
2.	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550	
3.	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626	
4.	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	
45.	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775	7
6.	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	
7.	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921	
8.	6.928	6.935	6.943	6.958	6.957	6.964	6.971	6.979	6.986	6.993	
9.	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064	

SQUARE ROOTS OF CERTAIN FRACTIONS

N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$
1/2	0.7071	3/5	0.7746	4/7	0.7559	1/9	0.3333	5/12	0.6455	9/16	0.7500
1/3	0.5774	4/5	0.8944	5/7	0.8452	2/9	0.4714	7/12	0.7638	11/16	0.8292
2/3	0.8165	1/6	0.4082	6/7	0.9258	4/9	0.6667	11/12	0.9574	13/16	0.9014
1/4	0.5000	5/6	0.9129	1/8	0.3536	5/9	0.7454	1/16	0.2500	15/16	0.9682
3/4	0.8660	1/7	0.3780	3/8	0.6124	7/9	0.8819	3/16	0.4330	1/32	0.1768
1/5	0.4472	2/7	0.5345	5/8	0.7906	8/9	0.9428	5/16	0.5590	1/64	0.1250
2/5	0.6325	3/7	0.6547	7/8	0.9354	1/12	0.2887	7/16	0.6614	1/50	0.1414

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
50.	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134	7
1.	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204	
2.	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273	
3.	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342	
4.	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	
55.	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477	
6.	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543	
7.	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609	
8.	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675	
9.	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740	6
60.	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804	
1.	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868	
2.	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931	
3.	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994	
4.	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056	
65.	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118	
6.	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179	
7.	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240	
8.	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301	
9.	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361	
70.	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420	
1.	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479	
2.	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538	
3.	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597	
4.	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654	
75.	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712	
6.	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769	
7.	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826	
8.	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883	
9.	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939	
80.	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994	
1.	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050	
2.	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105	5
3.	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160	
4.	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214	
85.	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268	
6.	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322	
7.	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375	
8.	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429	
9.	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482	
90.	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534	
1.	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586	
2.	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638	
3.	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690	
4.	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742	
95.	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793	
6.	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844	
7.	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894	
8.	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945	
9.	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995	

$\sqrt{\pi} = 1.77245+$ $\quad 1/\sqrt{\pi} = 0.56419$ $\quad \sqrt{\pi/2} = 1.25331$ $\quad \sqrt{e} = 1.64872$

Moving the decimal point TWO places in *N* requires moving it ONE place in body of table (see p. 12).

CUBE ROOTS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	1.000	1.003	1.007	1.010	1.013	1.016	1.020	1.023	1.026	1.029	3
1	1.032	1.035	1.038	1.042	1.045	1.048	1.051	1.054	1.057	1.060	
2	1.063	1.066	1.069	1.071	1.074	1.077	1.080	1.083	1.086	1.089	
3	1.091	1.094	1.097	1.100	1.102	1.105	1.108	1.111	1.113	1.116	
4	1.119	1.121	1.124	1.127	1.129	1.132	1.134	1.137	1.140	1.142	
1.5	1.145	1.147	1.150	1.152	1.155	1.157	1.160	1.162	1.165	1.167	2
6	1.170	1.172	1.174	1.177	1.179	1.182	1.184	1.186	1.189	1.191	
7	1.193	1.196	1.198	1.200	1.203	1.205	1.207	1.210	1.212	1.214	
8	1.216	1.219	1.221	1.223	1.225	1.228	1.230	1.232	1.234	1.236	
9	1.239	1.241	1.243	1.245	1.247	1.249	1.251	1.254	1.256	1.258	
2.0	1.260	1.262	1.264	1.266	1.268	1.270	1.272	1.274	1.277	1.279	
1	1.281	1.283	1.285	1.287	1.289	1.291	1.293	1.295	1.297	1.299	
2	1.301	1.303	1.305	1.306	1.308	1.310	1.312	1.314	1.316	1.318	
3	1.320	1.322	1.324	1.326	1.328	1.330	1.331	1.333	1.335	1.337	
4	1.339	1.341	1.343	1.344	1.346	1.348	1.350	1.352	1.354	1.355	
2.5	1.357	1.359	1.361	1.363	1.364	1.366	1.368	1.370	1.372	1.373	
6	1.375	1.377	1.379	1.380	1.382	1.384	1.386	1.387	1.389	1.391	
7	1.392	1.394	1.396	1.398	1.399	1.401	1.403	1.404	1.406	1.408	
8	1.409	1.411	1.413	1.414	1.416	1.418	1.419	1.421	1.423	1.424	
9	1.426	1.428	1.429	1.431	1.433	1.434	1.436	1.437	1.439	1.441	
3.0	1.442	1.444	1.445	1.447	1.449	1.450	1.452	1.453	1.455	1.457	
1	1.458	1.460	1.461	1.463	1.464	1.466	1.467	1.469	1.471	1.472	
2	1.474	1.475	1.477	1.478	1.480	1.481	1.483	1.484	1.486	1.487	
3	1.489	1.490	1.492	1.493	1.493	1.496	1.498	1.499	1.501	1.502	
4	1.504	1.505	1.507	1.508	1.510	1.511	1.512	1.514	1.515	1.517	
3.5	1.518	1.520	1.521	1.523	1.524	1.525	1.527	1.528	1.530	1.531	
6	1.533	1.534	1.535	1.537	1.538	1.540	1.541	1.542	1.544	1.545	1
7	1.547	1.548	1.549	1.551	1.552	1.554	1.555	1.556	1.558	1.559	
8	1.560	1.562	1.563	1.565	1.566	1.567	1.569	1.570	1.571	1.573	
9	1.574	1.575	1.577	1.578	1.579	1.581	1.582	1.583	1.585	1.586	
4.0	1.587	1.589	1.590	1.591	1.593	1.594	1.595	1.597	1.598	1.599	
1	1.601	1.602	1.603	1.604	1.606	1.607	1.608	1.610	1.611	1.612	
2	1.613	1.615	1.616	1.617	1.619	1.620	1.621	1.622	1.624	1.625	
3	1.626	1.627	1.679	1.630	1.631	1.632	1.634	1.635	1.636	1.637	
4	1.639	1.640	1.641	1.642	1.644	1.645	1.646	1.647	1.649	1.650	
4.5	1.651	1.652	1.653	1.655	1.656	1.657	1.658	1.659	1.661	1.662	
6	1.663	1.664	1.666	1.667	1.668	1.669	1.670	1.671	1.673	1.674	
7	1.675	1.676	1.677	1.679	1.680	1.681	1.682	1.683	1.685	1.686	
8	1.687	1.688	1.689	1.690	1.692	1.693	1.694	1.695	1.696	1.697	
9	1.698	1.700	1.701	1.702	1.703	1.704	1.705	1.707	1.708	1.709	

$\sqrt[3]{\pi} = 1.46459 \qquad 1/\sqrt[3]{\pi} = 0.682784$

Explanation of Table of Cube Roots (pp. 16–21).

This table gives the values of $\sqrt[3]{N}$ for all values of N from 1 to 1000, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the cube root of a number N outside the range from 1 to 1000, divide the digits of the number into blocks of three (beginning with the decimal point), and note that moving the decimal point three places in column N is equivalent to moving it one place in the cube root of N. For example:

$$\sqrt[3]{2.718} = 1.396; \quad \sqrt[3]{2718} = 13.96; \quad \sqrt[3]{0.000002718} = 0.01396.$$
$$\sqrt[3]{27.18} = 3.007; \quad \sqrt[3]{27180} = 30.07; \quad \sqrt[3]{0.00002718} = 0.03007.$$
$$\sqrt[3]{271.8} = 6.477; \quad \sqrt[3]{271800} = 64.77; \quad \sqrt[3]{0.0002718} = 0.06477.$$

CUBE ROOTS *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	1.710	1.711	1.712	1.713	1.715	1.716	1.717	1.718	1.719	1.720	1
1	1.721	1.722	1.724	1.725	1.726	1.727	1.728	1.729	1.730	1.731	
2	1.732	1.734	1.735	1.736	1.737	1.738	1.739	1.740	1.741	1.742	
3	1.744	1.745	1.746	1.747	1.748	1.749	1.750	1.751	1.752	1.753	
4	1.754	1.755	1.757	1.758	1.759	1.760	1.761	1.762	1.763	1.764	
5.5	1.765	1.766	1.767	1.768	1.769	1.771	1.772	1.773	1.774	1.775	
6	1.776	1.777	1.778	1.779	1.780	1.781	1.782	1.783	1.784	1.785	
7	1.786	1.787	1.788	1.789	1.790	1.792	1.793	1.794	1.795	1.796	
8	1.797	1.798	1.799	1.800	1.801	1.802	1.803	1.804	1.805	1.806	
9	1.807	1.808	1.809	1.810	1.811	1.812	1.813	1.814	1.815	1.816	
6.0	1.817	1.818	1.819	1.820	1.821	1.822	1.823	1.824	1.825	1.826	
1	1.827	1.828	1.829	1.830	1.831	1.832	1.833	1.834	1.835	1.836	
2	1.837	1.838	1.839	1.840	1.841	1.842	1.843	1.844	1.845	1.846	
3	1.847	1.848	1.849	1.850	1.851	1.852	1.853	1.854	1.855	1.856	
4	1.857	1.858	1.859	1.860	1.860	1.861	1.862	1.863	1.864	1.865	
6.5	1.866	1.867	1.868	1.869	1.870	1.871	1.872	1.873	1.874	1.875	
6	1.876	1.877	1.878	1.879	1.880	1.881	1.881	1.882	1.883	1.884	
7	1.885	1.886	1.887	1.888	1.889	1.890	1.891	1.892	1.893	1.894	
8	1.895	1.895	1.896	1.897	1.898	1.899	1.900	1.901	1.902	1.903	
9	1.904	1.905	1.906	1.907	1.907	1.908	1.909	1.910	1.911	1.912	
7.0	1.913	1.914	1.915	1.916	1.917	1.917	1.918	1.919	1.920	1.921	
1	1.922	1.923	1.924	1.925	1.926	1.926	1.927	1.928	1.929	1.930	
2	1.931	1.932	1.933	1.934	1.935	1.935	1.936	1.937	1.938	1.939	
3	1.940	1.941	1.942	1.943	1.943	1.944	1.945	1.946	1.947	1.948	
4	1.949	1.950	1.950	1.951	1.952	1.953	1.954	1.955	1.956	1.957	
7.5	1.957	1.958	1.959	1.960	1.961	1.962	1.963	1.964	1.964	1.965	
6	1.966	1.967	1.968	1.969	1.970	1.970	1.971	1.972	1.973	1.974	
7	1.975	1.976	1.976	1.977	1.978	1.979	1.980	1.981	1.981	1.982	
8	1.983	1.984	1.985	1.986	1.987	1.987	1.988	1.989	1.990	1.991	
9	1.992	1.992	1.993	1.994	1.995	1.996	1.997	1.997	1.998	1.999	
8.0	2.000	2.001	2.002	2.002	2.003	2.004	2.005	2.006	2.007	2.007	
1	2.008	2.009	2.010	2.011	2.012	2.012	2.013	2.014	2.015	2.016	
2	2.017	2.017	2.018	2.019	2.020	2.021	2.021	2.022	2.023	2.024	
3	2.025	2.026	2.026	2.027	2.028	2.029	2.030	2.030	2.031	2.032	
4	2.033	2.034	2.034	2.035	2.036	2.037	2.038	2.038	2.039	2.040	
8.5	2.041	2.042	2.042	2.043	2.044	2.045	2.046	2.046	2.047	2.048	
6	2.049	2.050	2.050	2.051	2.052	2.053	2.054	2.054	2.055	2.056	
7	2.057	2.057	2.058	2.059	2.060	2.061	2.061	2.062	2.063	2.064	
8	2.065	2.065	2.066	2.067	2.068	2.068	2.069	2.070	2.071	2.072	
9	2.072	2.073	2.074	2.075	2.075	2.076	2.077	2.078	2.079	2.079	
9.0	2.080	2.081	2.082	2.082	2.083	2.084	2.085	2.085	2.086	2.087	
1	2.088	2.089	2.089	2.090	2.091	2.092	2.092	2.093	2.094	2.095	
2	2.095	2.096	2.097	2.098	2.098	2.099	2.100	2.101	2.101	2.102	
3	2.103	2.104	2.104	2.105	2.106	2.107	2.107	2.108	2.109	2.110	
4	2.110	2.111	2.112	2.113	2.113	2.114	2.115	2.116	2.116	2.117	
9.5	2.118	2.119	2.119	2.120	2.121	2.122	2.122	2.123	2.124	2.125	
6	2.125	2.126	2.127	2.128	2.128	2.129	2.130	2.130	2.131	2.132	
7	2.133	2.133	2.134	2.135	2.136	2.136	2.137	2.138	2.139	2.139	
8	2.140	2.141	2.141	2.142	2.143	2.144	2.144	2.145	2.146	2.147	
9	2.147	2.148	2.149	2.149	2.150	2.151	2.152	2.152	2.153	2.154	

Moving the decimal point THREE places in *N* requires moving it ONE place in body of table (see p. 16).

CUBE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
10.	2.154	2.162	2.169	2.176	2.183	2.190	2.197	2.204	2.210	2.217	7
1.	2.224	2.231	2.237	2.244	2.251	2.257	2.264	2.270	2.277	2.283	6
2.	2.289	2.296	2.302	2.308	2.315	2.321	2.327	2.333	2.339	2.345	
3.	2.351	2.357	2.363	2.369	2.375	2.381	2.387	2.393	2.399	2.404	
4.	2.410	2.416	2.422	2.427	2.433	2.438	2.444	2.450	2.455	2.461	
15.	2.466	2.472	2.477	2.483	2.488	2.493	2.499	2.504	2.509	2.515	5
6.	2.520	2.525	2.530	2.535	2.541	2.546	2.551	2.556	2.561	2.566	
7.	2.571	2.576	2.581	2.586	2.591	2.596	2.601	2.606	2.611	2.616	
8.	2.621	2.626	2.630	2.635	2.640	2.645	2.650	2.654	2.659	2.664	
9.	2.668	2.673	2.678	2.682	2.687	2.692	2.696	2.701	2.705	2.710	
20.	2.714	2.719	2.723	2.728	2.732	2.737	2.741	2.746	2.750	2.755	4
1.	2.759	2.763	2.768	2.772	2.776	2.781	2.785	2.789	2.794	2.798	
2.	2.802	2.806	2.811	2.815	2.819	2.823	2.827	2.831	2.836	2.840	
3.	2.844	2.848	2.852	2.856	2.860	2.864	2.868	2.872	2.876	2.880	
4.	2.884	2.888	2.892	2.896	2.900	2.904	2.908	2.912	2.916	2.920	
25.	2.924	2.928	2.932	2.936	2.940	2.943	2.947	2.951	2.955	2.959	
6.	2.962	2.966	2.970	2.974	2.978	2.981	2.985	2.989	2.993	2.996	
7.	3.000	3.004	3.007	3.011	3.015	3.018	3.022	3.026	3.029	3.033	
8.	3.037	3.040	3.044	3.047	3.051	3.055	3.058	3.062	3.065	3.069	
9.	3.072	3.076	3.079	3.083	3.086	3.090	3.093	3.097	3.100	3.104	
30.	3.107	3.111	3.114	3.118	3.121	3.124	3.128	3.131	3.135	3.138	3
1.	3.141	3.145	3.148	3.151	3.155	3.158	3.162	3.165	3.168	3.171	
2.	3.175	3.178	3.181	3.185	3.188	3.191	3.195	3.198	3.201	3.204	
3.	3.208	3.211	3.214	3.217	3.220	3.224	3.227	3.230	3.233	3.236	
4.	3.240	3.243	3.246	3.249	3.252	3.255	3.259	3.262	3.265	3.268	
35.	3.271	3.274	3.277	3.280	3.283	3.287	3.290	3.293	3.296	3.299	
6.	3.302	3.305	3.308	3.311	3.314	3.317	3.320	3.323	3.326	3.329	
7.	3.332	3.335	3.338	3.341	3.344	3.347	3.350	3.353	3.356	3.359	
8.	3.362	3.365	3.368	3.371	3.374	3.377	3.380	3.382	3.385	3.388	
9.	3.391	3.394	3.397	3.400	3.403	3.406	3.409	3.411	3.414	3.417	
40.	3.420	3.423	3.426	3.428	3.431	3.434	3.437	3.440	3.443	3.445	
1.	3.448	3.451	3.454	3.457	3.459	3.462	3.465	3.468	3.471	3.473	
2.	3.476	3.479	3.482	3.484	3.487	3.490	3.493	3.495	3.498	3.501	
3.	3.503	3.506	3.509	3.512	3.514	3.517	3.520	3.522	3.525	3.528	
4.	3.530	3.533	3.536	3.538	3.541	3.544	3.546	3.549	3.552	3.554	
45.	3.557	3.560	3.562	3.565	3.567	3.570	3.573	3.575	3.578	3.580	
6.	3.583	3.586	3.588	3.591	3.593	3.596	3.599	3.601	3.604	3.606	
7.	3.609	3.611	3.614	3.616	3.619	3.622	3.624	3.627	3.629	3.632	
8.	3.634	3.637	3.639	3.642	3.644	3.647	3.649	3.652	3.654	3.657	2
9.	3.659	3.662	3.664	3.667	3.669	3.672	3.674	3.677	3.679	3.682	

CUBE ROOTS OF CERTAIN FRACTIONS

N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$
1/2	.7937	3/5	.8434	4/7	.8298	1/9	.4807	5/12	.7469	9/16	.8255
1/3	.6934	4/5	.9283	5/7	.8939	2/9	.6057	7/12	.8355	11/16	.8826
2/3	.8736	1/6	.5503	6/7	.9499	4/9	.7631	11/12	.9714	13/16	.9331
1/4	.6300	5/6	.9410	1/8	.5000	5/9	.8221	1/16	.3969	15/16	.9787
3/4	.9086	1/7	.5228	3/8	.7211	7/9	.9196	3/16	.5724	1/32	.3150
1/5	.5848	2/7	.6586	5/8	.8550	8/9	.9615	5/16	.6786	1/64	.2500
2/5	.7368	3/7	.7539	7/8	.9565	1/12	.4368	7/16	.7591	1/50	.2714

CUBE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
50.	3.684	3.686	3.689	3.691	3.694	3.696	3.699	3.701	3.704	3.706	2
1.	3.708	3.711	3.713	3.716	3.718	3.721	3.723	3.725	3.728	3.730	
2.	3.733	3.735	3.737	3.740	3.742	3.744	3.747	3.749	3.752	3.754	
3.	3.756	3.759	3.761	3.763	3.766	3.768	3.770	3.773	3.775	3.777	
4.	3.780	3.782	3.784	3.787	3.789	3.791	3.794	3.796	3.798	3.801	
55.	3.803	3.805	3.808	3.810	3.812	3.814	3.817	3.819	3.821	3.824	
6.	3.826	3.828	3.830	3.833	3.835	3.837	3.839	3.842	3.844	3.846	
7.	3.849	3.851	3.853	3.855	3.857	3.860	3.862	3.864	3.866	3.869	
8.	3.871	3.873	3.875	3.878	3.880	3.882	3.884	3.886	3.889	3.891	
9.	3.893	3.895	3.897	3.900	3.902	3.904	3.906	3.908	3.911	3.913	
60.	3.915	3.917	3.919	3.921	3.924	3.926	3.928	3.930	3.932	3.934	
1.	3.936	3.939	3.941	3.943	3.945	3.947	3.949	3.951	3.954	3.956	
2.	2.958	3.960	3.962	3.964	3.966	3.968	3.971	3.973	3.975	3.977	
3.	2.979	3.981	3.983	3.985	3.987	3.990	3.992	3.994	3.996	3.998	
4.	4.000	4.002	4.004	4.006	4.008	4.010	4.012	4.015	4.017	4.019	
65.	4.021	4.023	4.025	4.027	4.029	4.031	4.033	4.035	4.037	4.039	
6.	4.041	4.043	4.045	4.047	4.049	4.051	4.053	4.055	4.058	4.060	
7.	4.062	4.064	4.066	4.068	4.070	4.072	4.074	4.076	4.078	4.080	
8.	4.082	4.084	4.086	4.088	4.090	4.092	4.094	4.096	4.098	4.100	
9.	4.102	3.104	4.106	4.108	4.109	4.111	4.113	4.115	4.117	4.119	
70.	4.121	4.123	4.125	4.127	4.129	4.131	4.133	4.135	4.137	4.139	
1.	4.141	4.143	4.145	4.147	4.149	4.151	4.152	4.154	4.156	4.158	
2.	4.160	4.162	4.164	4.166	4.168	4.170	4.172	4.174	4.176	4.177	
3.	4.179	4.181	4.183	4.185	4.187	4.189	4.191	4.193	4.195	4.196	
4.	4.198	4.200	4.202	4.204	4.206	4.208	4.210	4.212	4.213	4.215	
75.	4.217	4.219	4.221	4.223	4.225	4.227	4.228	4.230	4.232	4.234	
6.	4.236	4.238	4.240	4.241	4.243	4.245	4.247	4.249	4.251	4.252	
7.	4.254	4.256	4.258	4.260	4.262	4.264	4.265	4.267	4.269	4.271	
8.	4.273	4.274	4.276	4.278	4.280	4.282	4.284	4.285	4.287	4.289	
9.	4.291	4.293	4.294	4.296	4.298	4.300	4.302	4.303	4.305	4.307	
80.	4.309	4.311	4.312	4.314	4.316	4.318	4.320	4.321	4.323	4.325	
1.	4.327	4.329	4.330	4.332	4.334	4.336	4.337	4.339	4.341	4.343	
2.	4.344	4.346	4.348	4.350	4.352	4.353	4.355	4.357	4.359	4.360	
3.	4.362	4.364	4.366	4.367	4.369	4.371	4.373	4.374	4.376	4.378	
4.	4.380	4.381	4.383	4.385	4.386	4.388	4.390	4.392	4.393	4.395	
85.	4.397	4.399	4.400	4.402	4.404	4.405	4.407	4.409	4.411	4.412	
6.	4.414	4.416	4.417	4.419	4.421	4.423	4.424	4.426	4.428	4.429	
7.	4.431	4.433	4.434	4.436	4.438	4.440	4.441	4.443	4.445	4.446	
8.	4.448	4.450	4.451	4.453	4.455	4.456	4.458	4.460	4.461	4.463	
9.	4.465	4.466	4.468	4.470	4.471	4.473	4.475	4.476	4.478	4.480	
90.	4.481	4.483	4.485	4.486	4.488	4.490	4.491	4.493	4.495	4.496	
1.	4.498	4.500	4.501	4.503	4.505	4.506	4.508	4.509	4.511	4.513	
2.	4.514	4.516	4.518	4.519	4.521	4.523	4.524	4.526	4.527	4.529	
3.	4.531	4.532	4.534	4.536	4.537	4.539	4.540	4.542	4.544	4.545	
4.	4.547	4.548	4.550	4.552	4.553	4.555	4.556	4.558	4.560	4.561	
95.	4.563	4.565	4.566	4.568	4.569	4.571	4.572	4.574	4.576	4.577	
6.	4.579	4.580	4.582	4.584	4.585	4.587	4.588	4.590	4.592	4.593	
7.	4.595	4.596	4.598	4.599	4.601	4.603	4.604	4.606	4.607	4.609	
8.	4.610	4.612	4.614	4.615	4.617	4.618	4.620	4.621	4.623	4.625	
9.	4.626	4.628	4.629	4.631	4.632	4.634	4.635	4.637	4.638	4.640	

Moving the decimal point THREE places in *N* requires moving it ONE place in body of table (see p. 16).

CUBE ROOTS (*continued*)

Cube roots of numbers from 100. to 499.

N	0.	1.	2.	3.	4.	5.	6.	7	8.	9.	Avg. diff.
10	4.642	4.657	4.672	4.688	4.703	4.718	4.733	4.747	4.762	4.777	15
1	4.791	4.806	4.820	4.835	4.849	4.863	4.877	4.891	4.905	4.919	14
2	4.932	4.946	4.960	4.973	4.987	5.000	5.013	5.027	5.040	5.053	13
3	5.066	5.079	5.092	5.104	5.117	5.130	5.143	5.155	5.168	5.180	
4	5.192	5.205	5.217	5.229	5.241	5.254	5.266	5.278	5.290	5.301	12
15	5.313	5.325	5.337	5.348	5.360	5.372	5.383	5.395	5.406	5.418	
6	5.429	5.440	5.451	5.463	5.474	5.485	5.496	5.507	5.518	5.529	11
7	5.540	5.550	5.561	5.572	5.583	5.593	5.604	5.615	5.625	5.636	
8	5.646	5.657	5.667	5.677	5.688	5.698	5.708	5.718	5.729	5.739	10
9	5.749	5.759	5.769	5.779	5.789	5.799	5.809	5.819	5.828	5.838	
20	5.848	5.858	5.867	5.877	5.887	5.896	5.906	5.915	5.925	5.934	
1	5.944	5.953	5.963	5.972	5.981	5.991	6.000	6.009	6.018	6.028	9
2	6.037	6.046	6.055	6.064	6.073	6.082	6.091	6.100	6.109	6.118	
3	6.127	6.136	6.145	6.153	6.162	6.171	6.180	6.188	6.197	6.206	
4	6.214	6.223	6.232	6.240	6.249	6.257	6.266	6.274	6.283	6.291	
25	6.300	6.308	6.316	6.325	6.333	6.341	6.350	6.358	6.366	6.374	8
6	6.383	6.391	6.399	6.407	6.415	6.423	6.431	6.439	6.447	6.455	
7	6.463	6.471	6.479	6.487	6.495	6.503	6.511	6.519	6.527	6.534	
8	6.542	6.550	6.558	6.565	6.573	6.581	6.589	6.596	6.604	6.611	
9	6.619	6.627	6.634	6.642	6.649	6.657	6.664	6.672	6.679	6.687	
30	6.694	6.702	6.709	6.717	6.724	6.731	6.739	6.746	6.753	6.761	7
1	6.768	6.775	6.782	6.790	6.797	6.804	6.811	6.818	6.826	6.833	
2	6.840	6.847	6.854	6.861	6.868	6.875	6.882	6.889	6.896	6.903	
3	6.910	6.917	6.924	6.931	6.938	6.945	6.952	6.959	6.966	6.973	
4	6.980	6.986	6.993	7.000	7.007	7.014	7.020	7.027	7.034	7.041	
35	7.047	7.054	7.061	7.067	7.074	7.081	7.087	7.094	7.101	7.107	
6	7.114	7.120	7.127	7.133	7.140	7.147	7.153	7.160	7.166	7.173	6
7	7.179	7.186	7.192	7.198	7.205	7.211	7.218	7.224	7.230	7.237	
8	7.243	7.250	7.256	7.262	7.268	7.275	7.281	7.287	7.294	7.300	
9	7.306	7.312	7.319	7.325	7.331	7.337	7.343	7.350	7.356	7.362	
40	7.368	7.374	7.380	7.386	7.393	7.399	7.405	7.411	7.417	7.423	
1	7.429	7.435	7.441	7.447	7.453	7.459	7.465	7.471	7.477	7.483	
2	7.489	7.495	7.501	7.507	7.513	7.518	7.524	7.530	7.536	7.542	
3	7.548	7.554	7.560	7.565	7.571	7.577	7.583	7.589	7.594	7.600	
4	7.606	7.612	7.617	7.623	7.629	7.635	7.640	7.646	7.652	7.657	
45	7.663	7.669	7.674	7.680	7.686	7.691	7.697	7.703	7.708	7.714	5
6	7.719	7.725	7.731	7.736	7.742	7.747	7.753	7.758	7.764	7.769	
7	7.775	7.780	7.786	7.791	7.797	7.802	7.808	7.813	7.819	7.824	
8	7.830	7.835	7.841	7.846	7.851	7.857	7.862	7.868	7.873	7.878	
9	7.884	7.889	7.894	7.900	7.905	7.910	7.916	7.921	7.926	7.932	

AUXILIARY TABLE OF TWO-THIRDS POWERS AND THREE-HALVES POWERS (see pp. 22–23)

(To assist in locating the decimal point)

N	$N^{2/3}\left(=\sqrt[3]{N^2}\right)$	$N^{3/2}\left(=\sqrt{N^3}\right)$
.0001	.002154	.000001
.001	.01	.00003162
.01	.0464	.001
.1	.2154	.03162278
1.	1.	1.
10.	4.64	31.62278
100.	21.54	1000.
1000.	100.	31622.78
10000.	464.16	1000000.

For complete table of three-halves powers, see pp. 22–23. That table, used inversely, provides a complete table of two-thirds powers.

CUBE ROOTS *(continued)*

Cube roots of numbers from 500. to 1000.

N	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Avg. diff.
50	7.937	7.942	7.945	7.953	7.958	7.963	7.969	7.974	7.979	7.984	5
1	7.990	7.995	8.000	8.005	8.010	8.816	8.021	8.026	8.031	8.036	
2	8.041	8.047	8.052	8.057	8.062	8.067	8.072	8.077	8.082	8.088	
3	8.093	8.098	8.103	8.108	8.113	8.118	8.123	8.128	8.133	8.138	
4	8.143	8.148	8.153	8.158	8.163	8.168	8.173	8.178	8.183	8.188	
55	8.193	8.198	8.203	8.208	8.213	8.218	8.223	8.228	8.233	8.238	
6	8.243	8.247	8.252	8.257	8.262	8.267	8.272	8.277	8.282	8.286	
7	8.291	8.296	8.301	8.306	8.311	8.316	8.320	8.325	8.330	8.335	
8	8.340	8.344	8.349	8.354	8.359	8.363	8.368	8.373	8.378	8.382	
9	8.387	8.392	8.397	8.401	8.406	8.411	8.416	8.420	8.425	8.430	
60	8.434	8.439	8.444	8.448	8.453	8.458	8.462	8.467	8.472	8.476	
1	8.481	8.486	8.490	8.495	8.499	8.504	8.509	8.513	8.518	8.522	
2	8.527	8.532	8.536	8.541	8.545	8.550	8.554	8.559	8.564	8.568	
3	8.573	8.577	8.582	8.586	8.591	8.595	8.600	8.604	8.609	8.613	4
4	8.618	8.622	8.627	8.631	8.636	8.640	8.645	8.649	8.653	8.658	
65	8.662	8.667	8.671	8.676	8.680	8.685	8.689	8.693	8.698	8.702	
6	8.707	8.711	8.715	8.720	8.724	8.729	8.733	8.737	8.742	8.746	
7	8.750	8.755	8.759	8.763	8.768	8.772	8.776	8.781	8.785	8.789	
8	8.794	8.798	8.802	8.807	8.811	8.815	8.819	8.824	8.828	8.832	
9	8.837	8.841	8.845	8.849	8.854	8.858	8.862	8.866	8.871	8.875	
70	8.879	8.883	8.887	8.892	8.896	8.900	8.904	8.909	8.913	8.917	
1	8.921	8.925	8.929	8.934	8.938	8.942	8.946	8.950	8.955	8.959	
2	8.963	8.967	8.971	8.975	8.979	8.984	8.988	8.992	8.996	9.000	
3	9.004	9.008	9.012	9.016	9.021	9.025	9.029	9.033	9.037	9.041	
4	9.045	9.049	9.053	9.057	9.061	9.065	9.069	9.073	9.078	9.082	
75	9.086	9.090	9.094	9.098	9.102	9.106	9.110	9.114	9.118	9.122	
6	9.126	9.130	9.134	9.138	9.142	9.146	9.150	9.154	9.158	9.162	
7	9.166	9.170	9.174	9.178	9.182	9.185	9.189	9.193	9.197	9.201	
8	9.205	9.209	9.213	9.217	9.221	9.225	9.229	9.233	9.237	9.240	
9	9.244	9.248	9.252	9.256	9.260	9.264	9.268	9.272	9.275	9.279	
80	9.283	9.287	9.291	9.295	9.299	9.302	9.306	9.310	9.314	9.318	
1	9.322	9.326	9.329	9.333	9.337	9.341	9.345	9.348	9.352	9.356	
2	9.360	9.364	9.368	9.371	9.375	9.379	9.383	9.386	9.390	9.394	
3	9.398	9.402	9.405	9.409	9.413	9.417	9.420	9.424	9.428	9.432	
4	9.435	9.439	9.443	9.447	9.458	9.454	9.458	9.462	9.465	9.469	
85	9.473	9.476	9.480	9.484	9.488	9.491	9.495	9.499	9.502	9.506	
6	9.510	9.513	9.517	9.521	9.524	9.528	9.532	9.535	9.539	9.543	
7	9.546	9.550	9.554	9.557	9.561	9.565	9.568	9.572	9.576	9.579	
8	9.583	9.586	9.590	9.594	9.597	9.601	9.605	9.608	9.612	9.615	
9	9.619	9.623	9.626	9.638	9.633	9.637	9.641	9.644	9.648	9.651	
90	9.655	9.658	9.662	9.666	9.669	9.673	9.676	9.680	9.683	9.687	
1	9.691	9.694	9.698	9.701	9.705	9.708	9.712	9.715	9.719	9.722	
2	9.726	9.729	9.733	9.736	9.740	9.743	9.747	9.750	9.754	9.758	
3	9.761	9.764	9.768	9.771	9.775	9.778	9.782	9.785	9.789	9.792	
4	9.796	9.799	9.803	9.806	9.810	9.813	9.817	9.820	9.824	9.827	
95	9.830	9.834	9.837	9.841	9.844	9.848	9.851	9.855	9.858	9.861	
6	9.865	9.868	9.872	9.875	9.879	9.882	9.885	9.889	9.892	9.896	
7	9.899	9.902	9.906	9.909	9.913	9.916	9.919	9.923	9.926	9.930	
8	9.933	9.936	9.940	9.943	9.946	9.950	9.953	9.956	9.960	9.963	
9	9.967	9.970	9.973	9.977	9.980	9.983	9.987	9.990	9.993	9.997	
100	10.00										

Moving the decimal point **THREE** places in *N* requires moving it ONE place in body of table (see p. 16).

THREE-HALVES POWERS OF NUMBERS (see also p. 20)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.	1.000	1.154	1.315	1.482	1.657	1.837	2.024	2.217	2.415	2.619	183
2.	2.828	3.043	3.263	3.488	3.218	3.953	4.192	4.437	4.685	4.939	237
3.	5.196	5.458	5.724	5.995	6.269	6.548	6.831	7.117	7.408	7.702	280
4.	8.000	8.302	8.607	8.917	9.230	9.546	9.866	10.190			313
4.								10.19	10.52	10.85	33
5.	11.18	11.52	11.86	12.20	12.55	12.90	13.25	13.61	13.97	14.33	35
6.	14.70	15.07	15.44	15.81	16.19	16.57	16.96	17.34	17.73	18.12	38
7.	18.52	18.92	19.32	19.72	20.13	20.54	20.95	21.37	21.78	22.20	41
8.	22.63	23.05	23.48	23.91	24.35	24.78	25.22	25.66	26.11	26.55	44
9.	27.00	27.45	27.90	28.36	28.82	29.28	29.74	30.21	30.68	31.15	46
10.	31.62	32.10	32.58	33.06	33.54	34.02	34.51	35.00	35.49	35.99	49
1.	36.48	36.98	37.48	37.99	38.49	39.00	39.51	40.02	40.53	41.05	51
2.	41.57	42.09	42.61	43.14	43.66	44.19	44.73	45.26	45.79	46.33	53
3.	46.87	47.41	47.96	48.50	49.05	49.60	50.15	50.71	51.26	51.82	55
4.	52.38	52.95	53.51	54.08	54.64	55.21	55.79	56.36	56.94	57.51	57
15.	58.09	58.68	59.26	59.85	60.43	61.02	61.62	62.21	62.80	63.40	59
6.	64.00	64.60	65.20	65.81	66.41	67.02	67.63	68.25	68.86	69.48	61
7.	70.09	70.71	71.33	71.96	72.58	73.21	73.84	74.47	75.10	75.73	63
8.	76.37	77.00	77.64	78.28	78.93	79.57	80.22	80.87	81.51	82.17	65
9.	82.82	83.47	84.13	84.79	85.45	86.11	86.77	87.44	88.10	88.77	66
20.	89.44	90.11	90.79	91.46	92.14	92.82	93.50	94.18	94.86	95.55	68
1.	96.23	96.92	97.61	98.30	99.00	99.69	100.38				69
1.							100.4	101.1	101.8	102.5	7
2.	103.2	103.9	104.6	105.3	106.0	106.7	107.4	108.2	108.9	109.6	7
3.	110.3	111.0	111.7	112.5	113.2	113.9	114.6	115.4	116.1	116.8	7
4.	117.6	118.3	119.0	119.8	120.5	121.3	122.0	122.8	123.5	124.3	7
25.	125.0	125.8	126.5	177.3	128.0	128.8	129.5	130.3	131.0	131.8	8
6.	132.6	133.3	134.1	134.9	135.6	136.4	137.2	138.0	138.7	139.5	8
7.	140.3	141.1	141.9	142.6	143.4	144.2	145.0	145.8	146.6	147.4	8
8.	148.2	149.0	149.8	150.5	151.3	152.1	153.0	153.8	154.6	155.4	8
9.	156.2	157.0	157.8	158.6	159.4	160.2	161.9	161.9	162.7	163.5	8
30.	164.3	165.1	166.0	166.8	167.6	168.4	169.3	170.1	170.9	171.8	8
1.	172.6	173.4	174.3	175.1	176.0	176.8	177.6	178.5	179.3	180.2	8
2.	181.0	181.9	182.7	183.6	184.4	185.3	186.1	187.0	187.9	188.7	9
3.	189.6	190.4	191.3	192.2	193.0	193.9	194.8	195.6	196.5	197.4	9
4.	198.3	199.1	200.0	200.9	201.8	202.6	203.5	204.4	205.3	206.2	9
35.	207.1	208.0	208.8	209.7	210.6	211.5	212.4	213.3	214.2	215.1	9
6.	216.0	216.9	217.8	218.7	219.6	220.5	221.4	222.3	223.2	224.2	9
7.	225.1	226.0	226.9	227.8	228.7	229.6	230.6	231.5	232.4	233.3	9
8.	234.2	235.2	236.1	237.0	238.0	238.9	239.8	240.8	241.7	242.6	9
9.	243.6	244.5	245.4	246.4	247.3	248.3	249.2	250.1	251.1	252.0	9
40.	253.0	253.9	254.9	255.8	256.8	257.7	258.7	259.7	260.6	261.6	10
1.	262.5	263.5	264.5	265.4	266.4	267.3	268.3	269.3	270.2	271.2	10
2.	272.2	273.2	274.1	275.1	276.1	277.1	278.0	279.0	280.0	281.0	10
3.	282.0	283.0	283.9	284.9	285.9	286.9	287.9	288.9	289.9	290.9	10
4.	291.9	292.9	293.9	294.9	295.9	296.9	297.9	298.9	299.9	300.9	10
45.	301.9	302.9	303.9	304.9	305.9	306.9	307.9	308.9	310.0	311.0	10
6.	312.0	313.0	314.0	315.0	316.1	317.1	318.1	319.1	320.2	321.2	10
7.	322.2	323.2	324.3	325.3	326.3	327.4	328.4	329.4	330.5	331.5	10
8.	332.6	333.6	334.6	335.7	336.7	337.8	338.8	339.9	340.9	342.0	10
9.	343.0	344.0	345.1	346.2	347.2	348.3	349.3	350.4	351.4	352.5	11

This table gives $N^{3/2}$ from $N = 1$ to $N = 100$. Moving the decimal point TWO places in N requires moving it THREE places in body of table. Thus:

$(7.23)^{3/2} = 19.44$; $(723.)^{3/2} = 19440$; $(0.0723)^{3/2} = 0.01944$

$(72.3)^{3/2} = 614.8$; $(7230.)^{3/2} = 614800$; $(0.723)^{3/2} = 0.6148$

Used inversely, table gives $M^{2/3}$ from $M = 1$ to $M = 1000$. Thus: $(0.6148)^{2/3} = 0.7230$.

THREE-HALVES POWERS (*continued*) (See also p. 20)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
50.	353.6	354.6	355.7	356.7	357.8	358.9	359.9	361.0	362.1	363.1	11
1.	364.2	365.3	366.4	367.4	368.5	369.6	370.7	371.7	372.8	373.9	11
2.	375.0	376.1	377.1	378.2	379.3	380.4	381.5	382.6	383.7	384.8	11
3	385.8	386.9	388.0	389.1	390.2	391.3	392.4	393.5	394.6	395.7	11
4.	396.8	397.9	399.0	400.1	401.2	402.3	403.4	404.6	405.7	406.8	11
55.	407.9	409.0	410.1	411.2	412.3	413.5	414.6	415.7	416.8	418.0	11
6.	419.1	420.2	421.3	422.4	423.6	424.7	425.8	426.9	428.1	429.2	11
7.	430.3	431.5	432.6	433.7	434.9	436.0	437.1	438.3	439.4	440.6	11
8.	441.7	442.9	444.0	445.1	446.3	447.4	448.6	449.7	450.9	452.0	11
9.	453.2	454.3	455.5	456.6	457.8	459.0	460.1	461.3	462.4	463.6	12
60.	464.8	465.9	467.1	468.2	469.4	470.6	471.7	472.9	474.1	475.3	12
1.	476.4	477.6	478.7	479.9	481.1	482.3	483.5	484.6	485.8	487.0	12
2.	488.2	489.4	490.6	491.7	492.9	494.1	495.3	496.5	497.7	498.9	12
3.	500.0	501.2	502.4	503.6	504.8	506.0	507.2	508.4	509.6	510.8	12
4.	512.0	513.2	514.4	515.6	516.8	518.0	519.2	520.4	521.6	522.8	12
65.	524.0	525.3	526.5	527.7	528.9	530.1	531.3	532.5	533.8	535.0	12
6.	536.2	537.4	538.6	539.8	541.1	542.3	543.5	544.7	546.0	547.2	12
7.	548.4	549.6	550.9	552.1	553.3	554.6	555.8	557.0	558.3	559.5	12
8.	560.7	562.0	563.2	564.5	565.7	566.9	568.2	569.4	570.7	571.9	12
9.	573.2	574.4	575.7	576.9	578.1	579.4	580.6	581.9	583.2	584.4	13
70.	585.7	586.9	588.2	589.4	590.7	591.9	593.2	594.5	595.7	597.0	13
1.	598.3	599.5	600.8	602.1	603.3	604.6	605.9	607.1	608.4	609.7	13
2.	610.9	612.2	613.5	614.8	616.0	617.3	618.6	619.9	621.2	622.4	13
3.	623.7	625.0	626.3	627.6	628.8	630.1	631.4	632.7	634.0	635.3	13
4.	636.6	637.9	639.2	640.4	641.7	643.0	644.3	645.6	646.9	648.2	13
75.	649.5	650.8	652.1	653.4	654.7	656.0	657.3	658.6	659.9	661.2	13
6.	662.6	663.9	665.2	666.5	667.8	669.1	670.4	671.7	673.0	674.4	13
7.	675.7	677.0	678.3	679.6	680.9	682.3	683.6	684.9	686.2	687.6	13
8.	688.9	690.2	691.5	692.9	694.2	695.5	696.8	698.2	699.5	700.8	13
9.	702.2	703.5	704.8	706.2	707.5	708.8	710.2	711.5	712.9	714.2	13
80.	715.5	716.9	718.2	719.6	720.9	722.3	723.6	725.0	726.3	727.7	13
1.	729.0	730.4	731.7	733.1	734.4	735.8	737.1	738.5	739.8	741.2	14
2.	742.5	743.9	745.3	746.6	748.0	749.3	750.7	752.1	753.4	754.8	14
3.	756.2	757.5	758.9	760.3	761.6	763.0	764.4	765.8	767.1	768.5	14
4.	769.9	771.2	772.6	774.0	775.4	776.8	778.1	779.5	780.9	782.3	14
85.	783.7	785.0	786.4	787.8	789.2	790.6	792.0	793.4	794.8	796.1	14
6.	797.5	798.9	800.3	801.7	803.1	804.5	805.9	807.3	808.7	810.1	14
7.	811.5	812.9	814.3	815.7	817.1	818.5	819.9	821.3	822.7	824.1	14
8.	825.5	826.9	828.3	829.7	831.1	832.6	834.0	835.4	836.8	838.2	14
9.	839.6	841.0	842.5	843.9	845.3	846.7	848.1	849.5	851.0	852.4	14
90.	853.8	855.2	856.7	858.1	859.5	860.9	862.4	863.8	865.2	866.7	14
1.	868.1	869.5	870.9	872.4	873.8	875.2	876.7	878.1	879.6	881.0	14
2.	882.4	883.9	885.3	886.8	888.2	889.6	891.1	892.5	894.0	895.4	14
3.	896.9	898.3	899.8	901.2	902.7	904.1	905.6	907.0	908.5	909.9	15
4.	911.4	912.8	914.3	915.7	917.2	918.6	920.1	921.6	923.0	924.5	15
95.	925.9	927.4	928.9	930.3	931.8	933.3	934.7	936.2	937.7	939.1	15
6.	940.6	942.1	943.5	945.0	946.5	948.0	949.4	950.9	952.4	953.9	15
7.	955.3	956.8	958.3	959.8	961.3	962.7	964.2	965.7	967.2	968.7	15
8.	970.2	971.6	973.1	974.6	976.1	977.6	979.1	980.6	982.1	983.5	15
9.	985.0	986.5	988.0	989.5	991.0	992.5	994.0	995.5	997.0	998.5	15
100.	1000.0										

Moving the decimal point TWO places in *N* requires moving it THREE places in body of table (see also auxiliary table on p. 20).

RECIPROCALS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00		.9990	.9980	.9970	.9960	.9950	.9940	.9930	.9921	.9911	−10
1	.9901	.9891	.9881	.9872	.9862	.9852	.9843	.9833	.9823	.9814	
2	.9804	.9794	.9785	.9775	.9766	.9756	.9747	.9737	.9728	.9718	
3	.9709	.9699	.9690	.9681	.9671	.9662	.9653	.9643	.9634	.9625	−9
4	.9615	.9606	.9597	.9588	.9579	.9569	.9560	.9551	.9542	.9533	
1.05	.9524	.9515	.9506	.9497	.9488	.9479	.9470	.9461	.9452	.9443	
6	.9434	.9425	.9416	.9407	.9398	.9390	.9381	.9372	.9363	.9355	
7	.9346	.9337	.9328	.9320	.9311	.9302	.9294	.9285	.9276	.9268	
8	.9259	.9251	.9242	.9234	.9225	.9217	.9208	.9200	.9191	.9183	−8
9	.9174	.9166	.9158	.9149	.9141	.9132	.9124	.9116	.9107	.9099	
1.10	.9091	.9083	.9074	.9066	.9058	.9050	.9042	.9033	.9025	.9017	
1	.9009	.9001	.8993	.8985	.8977	.8969	.8961	.8953	.8945	.8937	
2	.8929	.8921	.8913	.8905	.8897	.8889	.8881	.8873	.8865	.8857	
3	.8850	.8842	.8834	.8826	.8818	.8811	.8803	.8795	.8787	.8780	
4	.8772	.8764	.8757	.8749	.8741	.8734	.8726	.8718	.8711	.8703	
1.15	.8696	.8688	.8681	.8673	.8666	.8658	.8651	.8643	.8636	.8628	
6	.8621	.8613	.8606	.8598	.8591	.8584	.8576	.8569	.8562	.8554	−7
7	.8547	.8540	.8532	.8525	.8518	.8511	.8503	.8496	.8489	.8482	
8	.8475	.8467	.8460	.8453	.8446	.8439	.8432	.8425	.8418	.8410	
9	.8403	.8396	.8389	.8382	.8375	.8368	.8361	.8354	.8347	.8340	
1.20	.8333	.8326	.8319	.8313	.8306	.8299	.8292	.8285	.8278	.8271	
1	.8264	.8258	.8251	.8244	.8257	.8230	.8224	.8217	.8210	.8203	
2	.8197	.8190	.8183	.8177	.8170	.8163	.8157	.8150	.8143	.8137	
3	.8130	.8123	.8117	.8110	.8104	.8097	.8091	.8084	.8078	.8071	−6
4	.8065	.8058	.8052	.8045	.8039	.8032	.8026	.8019	.8013	.8006	
1.25	.8000	.7994	.7987	.7981	.7974	.7968	.7962	.7955	.7949	.7943	
6	.7937	.7930	.7924	.7918	.7911	.7905	.7899	.7893	.7886	.7880	
7	.7874	.7868	.7862	.7855	.7849	.7843	.7837	.7831	.7825	.7819	
8	.7812	.7806	.7800	.7794	.7788	.7782	.7776	.7770	.7764	.7758	
9	.7752	.7746	.7740	.7734	.7728	.7722	.7716	.7710	.7704	.7698	
1.30	.7692	.7686	.7680	.7675	.7669	.7663	.7657	.7651	.7645	.7639	
1	.7634	.7628	.7622	.7616	.7610	.7605	.7599	.7593	.7587	.7582	
2	.7576	.7570	.7564	.7559	.7553	.7547	.7541	.7536	.7530	.7524	
3	.7519	.7513	.7508	.7502	.7496	.7491	.7485	.7479	.7474	.7468	
4	.7463	.7457	.7452	.7446	.7440	.7435	.7429	.7424	.7418	.7413	
1.35	.7407	.7402	.7396	.7391	.7386	.7380	.7375	.7369	.7364	.7358	−5
6	.7353	.7348	.7342	.7337	.7331	.7326	.7321	.7315	.7310	.7305	
7	.7299	.7294	.7289	.7283	.7278	.7273	.7267	.7262	.7257	.7252	
8	.7246	.7241	.7236	.7231	.7225	.7220	.7215	.7210	.7205	.7199	
9	.7194	.7189	.7184	.7179	.7174	.7168	.7163	.7158	.7153	.7148	
1.40	.7143	.7138	.7133	.7128	.7123	.7117	.7112	.7107	.7102	.7097	
1	.7092	.7087	.7082	.7077	.7072	.7067	.7062	.7057	.7052	.7047	
2	.7042	.7037	.7032	.7027	.7022	.7018	.7013	.7008	.7003	.6998	
3	.6993	.6988	.6983	.6978	.6974	.6969	.6964	.6959	.6954	.6949	
4	.6944	.6940	.6935	.6930	.6925	.6920	.6916	.6911	.6906	.6901	
1.45	.6897	.6892	.6887	.6882	.6878	.6873	.6868	.6863	.6859	.6854	
6	.6849	.6845	.6840	.6835	.6831	.6826	.6821	.6817	.6812	.6807	
7	.6803	.6798	.6793	.6789	.6784	.6780	.6775	.6770	.6766	.6761	
8	.6757	.6752	.6748	.6743	.6739	.6734	.6729	.6725	.6720	.6716	
9	.6711	.6707	.6702	.6698	.6693	.6689	.6684	.6680	.6676	.6671	

$1/\pi = 0.318310 \qquad 1/e = 0.367879$

Moving the decimal point in either direction in N requires moving it in the OPPOSITE direction in body of table (see p. 26).

RECIPROCALS *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.50	.6667	.6662	.6658	.6653	.6649	.6645	.6640	.6636	.6631	.6627	−4
1	.6623	.6618	.6614	.6609	.6605	.6601	.6596	.6592	.6588	.6583	
2	.6579	.6575	.6570	.6566	.6562	.6557	.6553	.6549	.6545	.6540	
3	.6536	.6532	.6527	.6523	.6519	.6515	.6510	.6506	.6502	.6498	
4	.6494	.6489	.6485	.6481	.6477	.6472	.6468	.6464	.6460	.6456	
1.55	.6452	.6447	.6443	.6439	.6435	.6431	.6427	.6423	.6418	.6414	
6	.6410	.6406	.6402	.6398	.6394	.6390	.6386	.6382	.6378	.6373	
7	.6369	.6365	.6361	.6357	.6353	.6349	.6345	.6341	.6337	.6333	
8	.6329	.6325	.6321	.6317	.6313	.6309	.6305	.6301	.6297	.6293	
9	.6289	.6285	.6281	.6277	.6274	.6270	.6266	.6262	.6258	.6254	
1.60	.6250	.6246	.6242	.6238	.6234	.6231	.6227	.6223	.6219	.6215	
1	.6211	.6207	.6203	.6200	.6196	.6192	.6188	.6184	.6180	.6177	
2	.6173	.6169	.6165	.6161	.6158	.6154	.6150	.6146	.6143	.6139	
3	.6135	.6131	.6127	.6124	.6120	.6116	.6112	.6109	.6105	.6101	
4	.6098	.6094	.6090	.6086	.6083	.6079	.6075	.6072	.6068	.6064	
1.65	.6061	.6057	.6053	.6050	.6046	.6042	.6039	.6035	.6031	.6028	
6	.6024	.6020	.6017	.6013	.6010	.6006	.6002	.5999	.5995	.5992	
7	.5988	.5984	.5981	.5977	.5974	.5970	.5967	.5963	.5959	.5956	
8	.5952	.5949	.5945	.5942	.5938	.5935	.5931	.5928	.5924	.5921	
9	.5917	.5914	.5910	.5907	.5903	.5900	.5896	.5893	.5889	.5886	
1.70	.5882	.5879	.5875	.5872	.5869	.5865	.5862	.5858	.5855	.5851	−3
1	.5848	.5845	.5841	.5838	.5834	.5831	.5828	.5824	.5821	.5817	
2	.5814	.5811	.5807	.5804	.5800	.5797	.5794	.5790	.5787	.5784	
3	.5780	.5777	.5774	.5770	.5767	.5764	.5760	.5757	.5754	.5750	
4	.5747	.5744	.5741	.5737	.5734	.5731	.5727	.5724	.5721	.5718	
1.75	.5714	.5711	.5708	.5705	.5701	.5698	.5695	.5692	.5688	.5685	·
6	.5682	.5679	.5675	.5672	.5669	.5666	.5663	.5659	.5656	.5653	
7	.5650	.5647	.5643	.5640	.5637	.5634	.5631	.5627	.5624	.5621	
8	.5618	.5615	.5612	.5609	.5605	.5602	.5599	.5596	.5593	.5590	
9	.5587	.5583	.5580	.5577	.5574	.5571	.5568	.5565	.5562	.5559	
1.80	.5556	.5552	.5549	.5546	.5543	.5540	.5537	.5534	.5531	.5528	
1	.5525	.5522	.5519	.5516	.5513	.5510	.5507	.5504	.5501	.5498	
2	.5495	.5491	.5488	.5485	.5482	.5479	.5476	.5473	.5470	.5467	
3	.5464	.5461	.5459	.5456	.5453	.5450	.5447	.5444	.5441	.5438	
4	.5435	.5432	.5429	.5426	.5423	.5420	.5417	.5414	.5411	.5408	
1.85	.5405	.5402	.5400	.5397	.5394	.5391	.5388	.5385	.5382	.5379	·
6	.5376	.5373	.5371	.5368	.5365	.5362	.5359	.5356	.5353	.5350	
7	.5348	.5345	.5342	.5339	.5336	.5333	.5330	.5328	.5325	.5322	
8	.5319	.5316	.5313	.5311	.5308	.5305	.5302	.5299	.5297	.5294	
9	.5291	.5288	.5285	.5283	.5280	.5277	.5274	.5271	.5269	.5266	
1.90	.5263	.5260	.5258	.5255	.5252	.5249	.5247	.5244	.5241	.5238	
1	.5236	.5233	.5230	.5227	.5225	.5222	.5219	.5216	.5214	.5211	
2	.5208	.5206	.5203	.5200	.5198	.5195	.5192	.5189	.5187	.5184	
3	.5181	.5179	.5176	.5173	.5171	.5168	.5165	.5163	.5160	.5157	
4	.5155	.5152	.5149	.5147	.5144	.5141	.5139	.5136	.5133	.5131	
1.95	.5128	.5126	.5123	.5120	.5118	.5115	.5112	.5110	.5107	.5105	
6	.5102	.5099	.5097	.5094	.5092	.5089	.5086	.5084	.5081	.5079	
7	.5076	.5074	.5071	.5068	.5066	.5063	.5061	.5058	.5056	.5053	−2
8	.5051	.5048	.5045	.5043	.5040	.5038	.5035	.5033	.5030	.5028	
9	.5025	.5023	.5020	.5018	.5015	.5013	.5010	.5008	.5005	.5003	

Moving the decimal point in either direction in N requires moving it in the OPPOSITE direction in body of table (see p. 20).

RECIPROCALS *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. dif.
2.0	.5000	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785	− 24
1	.4762	.4739	.4717	.4695	.4673	.4651	.4630	.4608	.4587	.4566	− 21
2	.4545	.4525	.4505	.4484	.4464	.4444	.4425	.4405	.4386	.4367	− 20
3	.4348	.4329	.4310	.4292	.4274	.4255	.4237	.4219	.4202	.4184	− 18
4	.4167	.4149	.4132	.4115	.4098	.4082	.4065	.4049	.4032	.4016	− 17
2.5	.4000	.3984	.3968	.3953	.3937	.3922	.3906	.3891	.3876	.3861	− 15
6	.3846	.3831	.3817	.3802	.3788	.3774	.3759	.3745	.3731	.3717	− 14
7	.3704	.3690	.3676	.3663	.3650	.3636	.3623	.3610	.3597	.3584	− 13
8	.3571	.3559	.3546	.3534	.3521	.3509	.3497	.3484	.3472	.3460	− 12
9	.3448	.3436	.3425	.3413	.3401	.3390	.3378	.3367	.3356	.3344	− 12
3.0	.3333	.3322	.3311	.3300	.3289	.3279	.3268	.3257	.3247	.3236	− 11
1	.3226	.3215	.3205	.3195	.3185	.3175	.3165	.3155	.3145	.3135	− 10
2	.3125	.3115	.3106	.3096	.3086	.3077	.3067	.3058	.3049	.3040	− 10
3	.3030	.3021	.3012	.3003	.2994	.2985	.2976	.2967	.2959	.2950	− 9
4	.2941	.2933	.2924	.2915	.2907	.2899	.2890	.2882	.2874	.2865	− 8
3.5	.2857	.2849	.2841	.2833	.2825	.2817	.2809	.2801	.2793	.2786	− 8
6	.2778	.2770	.2762	.2755	.2747	.2740	.2732	.2725	.2717	.2710	− 8
7	.2703	.2695	.2688	.2681	.2674	.2667	.2660	.2653	.2646	.2639	− 7
8	.2632	.2625	.2618	.2611	.2604	.2597	.2591	.2584	.2577	.2571	− 7
9	.2564	.2558	.2551	.2545	.2538	.2532	.2525	.2519	.2513	.2506	− 6
4.0	.2500	.2494	.2488	.2481	.2475	.2469	.2463	.2457	.2451	.2445	− 6
1	.2439	.2433	.2427	.2421	.2415	.2410	.2404	.2398	.2392	.2387	− 6
2	.2381	.2375	.2370	.2364	.2358	.2353	.2347	.2342	.2336	.2331	− 6
3	.2326	.2320	.2315	.2309	.2304	.2299	.2294	.2288	.2283	.2278	− 5
4	.2273	.2268	.2262	.2257	.2252	.2247	.2242	.2237	.2232	.2227	− 5
4.5	.2222	.2217	.2212	.2208	.2203	.2198	.2193	.2188	.2183	.2179	− 5
6	.2174	.2169	.2165	.2160	.2155	.2151	.2146	.2141	.2137	.2132	− 5
7	.2128	.2123	.2119	.2114	.2110	.2105	.2101	.2096	.2092	.2088	− 4
8	.2083	.2079	.2075	.2070	.2066	.2062	.2058	.2053	.2049	.2045	− 4
9	.2041	.2037	.2033	.2028	.2024	.2020	.2016	.2012	.2008	.2004	− 4

$1/\pi = 0.318310 \qquad 1/e = 0.367879$

Explanation of Table of Reciprocals (pp. 24–27).

This table gives the values of $1/N$ for values of N from 1 to 10, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the reciprocal of a number N outside the range from 1 to 10, note that moving the decimal point any number of places in either direction in column N is equivalent to moving it the same number of places in the opposite direction in the body of the table. For example:

$$\frac{1}{3.217} = 0.3109; \quad \frac{1}{3217.} = 0.000\,3109; \quad \frac{1}{0.003217} = 310.9$$

RECIPROCALS *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	.2000	.1996	.1992	.1988	.1984	.1980	.1976	.1972	.1969	.1965	4
.1	.1961	.1957	.1953	.1949	.1946	.1942	.1938	.1934	.1931	.1927	
.2	.1923	.1919	.1916	.1912	.1908	.1905	.1901	.1898	.1894	.1890	
.3	.1887	.1883	.1880	.1876	.1873	.1869	.1866	.1862	.1859	.1855	
.4	.1852	.1848	.1845	.1842	.1838	.1835	.1832	.1828	.1825	.1821	3
5.5	.1818	.1815	.1812	.1808	.1805	.1802	.1799	.1795	.1792	.1789	
.6	.1786	.1783	.1779	.1776	.1773	.1770	.1767	.1764	.1761	.1757	
.7	.1754	.1751	.1748	.1745	.1742	.1739	.1736	.1733	.1730	.1727	
.8	.1724	.1721	.1718	.1715	.1712	.1709	.1706	.1704	.1701	.1698	
.9	.1695	.1692	.1689	.1686	.1684	.1681	.1678	.1675	.1672	.1669	
6.0	.1667	.1664	.1661	.1658	.1656	.1653	.1650	.1647	.1645	.1642	
.1	.1639	.1637	.1634	.1631	.1629	.1626	.1623	.1621	.1618	.1616	
.2	.1613	.1610	.1608	.1605	.1603	.1600	.1597	.1595	.1592	.1590	
.3	.1587	.1585	.1582	.1580	.1577	.1575	.1572	.1570	.1567	.1565	2
.4	.1563	.1560	.1558	.1555	.1553	.1550	.1548	.1546	.1543	.1541	
6.5	.1538	.1536	.1534	.1531	.1529	.1527	.1524	.1522	.1520	.1517	
.6	.1515	.1513	.1511	.1508	.1506	.1504	.1502	.1499	.1497	.1495	
.7	.1493	.1490	.1488	.1486	.1484	.1481	.1479	.1477	.1475	.1473	
.8	.1471	.1468	.1466	.1464	.1462	.1460	.1458	.1456	.1453	.1451	
.9	.1449	.1447	.1445	.1443	.1441	.1439	.1437	.1435	.1433	.1431	
7.0	.1429	.1427	.1425	.1422	.1420	.1418	.1416	.1414	.1412	.1410	
.1	.1408	.1406	.1404	.1403	.1401	.1399	.1397	.1395	.1393	.1391	
.2	.1389	.1387	.1385	.1383	.1381	.1379	.1377	.1376	.1374	.1372	
.3	.1370	.1368	.1366	.1364	.1362	.1361	.1359	.1357	.1355	.1353	
.4	.1351	.1350	.1348	.1346	.1344	.1342	.1340	.1339	.1337	.1335	
7.5	.1333	.1332	.1330	.1328	.1326	.1325	.1323	.1321	.1319	.1318	
.6	.1316	.1314	.1312	.1311	.1309	.1307	.1305	.1304	.1302	.1300	
.7	.1299	.1297	.1295	.1294	.1292	.1290	.1289	.1287	.1285	.1284	
.8	.1282	.1280	.1279	.1277	.1276	.1274	.1272	.1271	.1269	.1267	
.9	.1266	.1264	.1263	.1261	.1259	.1258	.1256	.1255	.1253	.1252	
8.0	.1250	.1248	.1247	.1245	.1244	.1242	.1241	.1239	.1238	.1236	
.1	.1235	.1233	.1232	.1230	.1229	.1227	.1225	.1224	.1222	.1221	
.2	.1220	.1218	.1217	.1215	.1214	.1212	.1211	.1209	.1208	.1206	
.3	.1205	.1203	.1202	.1200	.1199	.1198	.1196	.1195	.1193	.1192	
.4	.1190	.1189	.1188	.1186	.1185	.1183	.1182	.1181	.1179	.1178	1
8.5	.1176	.1175	.1174	.1172	.1171	.1170	.1168	.1167	.1165	.1164	
.6	.1163	.1161	.1160	.1159	.1157	.1156	.1155	.1153	.1152	.1151	
.7	.1149	.1148	.1147	.1145	.1144	.1143	.1142	.1140	.1139	.1138	
.8	.1136	.1135	.1134	.1133	.1131	.1130	.1129	.1127	.1126	.1125	
.9	.1124	.1122	.1121	.1120	.1119	.1117	.1116	.1115	.1114	.1112	
9.0	.1111	.1110	.1109	.1107	.1106	.1105	.1104	.1103	.1101	.1100	
.1	.1099	.1098	.1096	.1095	.1094	.1093	.1092	.1091	.1089	.1088	
.2	.1087	.1086	.1085	.1083	.1082	.1081	.1080	.1079	.1078	.1076	
.3	.1075	.1074	.1073	.1072	.1071	.1070	.1068	.1067	.1066	.1065	
.4	.1064	.1063	.1062	.1060	.1059	.1058	.1057	.1056	.1055	.1054	
9.5	.1053	.1052	.1050	.1049	.1048	.1047	.1046	.1045	.1044	.1043	
.6	.1042	.1041	.1040	.1038	.1037	.1036	.1035	.1034	.1033	.1032	
.7	.1031	.1030	.1029	.1028	.1027	.1026	.1025	.1024	.1022	.1021	
.8	.1020	.1019	.1018	.1017	.1016	.1015	.1014	.1013	.1012	.1011	
.9	.1010	.1009	.1008	.1007	.1006	.1005	.1004	.1003	.1002	.1001	

Moving the decimal point in either direction in *N* requires moving it in the OPPOSITE direction in body of table (see p. 20).

CIRCUMFERENCES OF CIRCLES BY HUNDREDTHS

(For circumferences by eighths, see p. 32)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	3.142	3.173	3.204	3.236	3.267	3.299	3.330	3.362	3.393	3.424	31
.1	3.456	3.487	3.519	3.550	3.581	3.613	3.644	3.676	3.707	3.738	
.2	3.770	3.801	3.833	3.864	3.896	3.927	3.958	3.990	4.021	4.053	
.3	4.084	4.115	4.147	4.178	4.210	4.241	4.273	4.304	4.335	4.367	
.4	4.398	4.430	4.461	4.492	4.524	4.555	4.587	4.618	4.650	4.681	
1.5	4.712	4.744	4.775	4.807	4.838	4.869	4.901	4.932	4.964	4.995	
.6	5.027	5.058	5.089	5.121	5.152	5.184	5.215	5.246	5.278	5.309	
.7	5.341	5.372	5.404	5.435	5.466	5.498	5.529	5.561	5.592	5.623	
.8	5.655	5.686	5.718	5.749	5.781	5.812	5.843	5.875	5.906	5.938	
.9	5.969	6.000	6.032	6.063	6.095	6.126	6.158	6.189	6.220	6.252	
2.0	6.283	6.315	6.346	6.377	6.409	6.440	6.472	6.503	6.535	6.566	
.1	6.597	6.629	6.660	6.692	6.723	6.754	6.786	6.817	6.849	6.880	
.2	6.912	6.943	6.974	7.006	7.037	7.069	7.100	7.131	7.163	7.194	
.3	7.226	7.257	7.288	7.320	7.351	7.383	7.414	7.446	7.477	7.508	
.4	7.540	7.571	7.603	7.634	7.665	7.697	7.728	7.760	7.791	7.823	
2.5	7.854	7.885	7.917	7.948	7.980	8.011	8.042	8.074	8.105	8.137	
.6	8.168	8.200	8.231	8.262	8.294	8.325	8.357	8.388	8.419	8.451	
.7	8.482	8.514	8.545	8.577	8.608	8.639	8.671	8.702	8.734	8.765	
.8	8.796	8.828	8.859	8.891	8.922	8.954	8.985	9.016	9.048	9.079	
.9	9.111	9.142	9.173	9.205	9.236	9.268	9.299	9.331	9.362	9.393	
3.0	9.425	9.456	9.488	9.519	9.550	9.582	9.613	9.645	9.676	9.708	
.1	9.739	9.770	9.802	9.833	9.865	9.896	9.927	9.959	9.990	10.022	31
1										10.02	3
.2	10.05	10.08	10.12	10.15	10.18	10.21	10.24	10.27	10.30	10.34	
.3	10.37	10.40	10.43	10.46	10.49	10.52	10.56	10.59	10.62	10.65	
.4	10.68	10.71	10.74	10.78	10.81	10.84	10.87	10.90	10.93	10.96	
3.5	11.00	11.03	11.06	11.09	11.12	11.15	11.18	11.22	11.25	11.28	
.6	11.31	11.34	11.37	11.40	11.44	11.47	11.50	11.53	11.56	11.59	
.7	11.62	11.66	11.69	11.72	11.75	11.78	11.81	11.84	11.88	11.91	
.8	11.94	11.97	12.00	12.03	12.06	12.10	12.13	12.16	12.19	12.22	
.9	12.25	12.28	12.32	12.35	12.38	12.41	12.44	12.47	12.50	12.53	
4.0	12.57	12.60	12.63	12.66	12.69	12.72	12.75	12.79	12.82	12.85	
.1	12.88	12.91	12.94	12.97	13.01	13.04	13.07	13.10	13.13	13.16	
.2	13.19	13.23	13.26	13.29	13.32	13.35	13.38	13.41	13.45	13.48	
.3	13.51	13.54	13.57	13.60	13.63	13.67	13.70	13.73	13.76	13.79	
.4	13.82	13.85	13.89	13.92	13.95	13.98	14.01	14.04	14.07	14.11	
4.5	14.14	14.17	14.20	14.23	14.26	14.29	14.33	14.36	14.39	14.42	
.6	14.45	14.48	14.51	14.55	14.58	14.61	14.64	14.67	14.70	14.73	
.7	14.77	14.80	14.83	14.86	14.89	14.92	14.95	14.99	15.02	15.05	
.8	15.08	15.11	15.14	15.17	15.21	15.24	15.27	15.30	15.33	15.36	
.9	15.39	15.43	15.46	15.49	15.52	15.52	15.58	15.61	15.65	15.68	

Explanation of Table of Circumferences (pp. 28–29)

This table gives the product of π times any number D from 1 to 10; that is, it is a table of multiples of π. (D = diameter.)

Moving the decimal point **one** place in column D is equivalent to moving it **one** place in the body of the table.

$$\text{Circumference} = \pi \times \text{diam.} = 3.141593 \times \text{diam.}$$

Conversely,

$$\textit{Diameter} = \frac{1}{\pi} \times \textit{circumf.} = 0.31831 \times \textit{circumf.}$$

CIRCUMFERENCES BY HUNDREDTHS (*continued*)

D	·0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	15.71	15.74	15.77	15.80	15.83	15.87	15.90	15.93	15.96	15.99	3
.1	16.02	16.05	16.08	16.12	16.15	16.18	16.21	16.24	16.27	16.30	
.2	16.34	16.37	16.40	16.43	16.46	16.49	16.52	16.56	16.59	16.62	
.3	16.65	16.68	16.71	16.74	16.78	16.81	16.84	16.87	16.90	16.93	
.4	16.96	17.00	17.03	17.06	17.09	17.12	17.15	17.18	17.22	17.25	
5.5	17.28	17.31	17.34	17.37	17.40	17.44	17.47	17.50	17.53	17.56	
.6	17.59	17.62	17.66	17.69	17.72	17.75	17.78	17.81	17.84	17.88	
.7	17.91	17.94	17.97	18.00	18.03	18.06	18.10	18.13	18.16	18.19	
.8	18.22	18.25	18.28	18.32	18.35	18.38	18.41	18.44	18.47	18.50	
.9	18.54	18.57	18.60	18.63	18.66	18.69	18.72	18.76	18.79	18.82	
6.0	18.85	18.88	18.91	18.94	18.98	19.01	19.04	19.07	19.10	19.13	
.1	19.16	19.20	19.23	19.26	19.29	19.32	19.35	19.38	19.42	19.45	
.2	19.48	19.51	19.54	19.57	19.60	19.63	19.67	19.70	19.73	19.76	
.3	19.79	19.82	19.85	19.89	19.92	19.95	19.98	20.01	20.04	20.07	
.4	20.11	20.14	20.17	20.20	20.23	20.26	20.29	20.33	20.36	20.39	
6.5	20.42	20.45	20.48	20.51	20.55	20.58	20.61	20.64	20.67	20.70	
.6	20.73	20.77	20.80	20.83	20.86	20.89	20.92	20.95	20.99	21.02	
.7	21.05	21.08	21.11	21.14	21.17	21.21	21.24	21.27	21.30	21.33	
.8	21.36	21.39	21.43	21.46	21.49	21.52	21.55	21.58	21.61	21.65	
.9	21.68	21.71	21.74	21.77	21.80	21.83	21.87	21.90	21.93	21.96	
7.0	21.99	22.02	22.05	22.09	22.12	22.15	22.18	22.21	22.24	22.27	
.1	22.31	22.34	22.37	22.40	22.43	22.46	22.49	22.53	22.56	22.59	
.2	22.62	22.65	22.68	22.71	22.75	22.78	22.81	22.84	22.87	22.90	
.3	22.93	22.97	23.00	23.03	23.06	23.09	23.12	23.15	23.18	23.22	
.4	23.25	23.28	23.31	23.34	23.37	23.40	23.44	23.47	23.50	23.53	
7.5	23.56	23.59	23.62	23.66	23.69	23.72	23.75	23.78	23.81	23.84	
.6	23.88	23.91	23.94	23.97	24.00	24.03	24.06	24.10	24.13	24.16	
.7	24.19	24.22	24.25	24.28	24.32	24.35	24.38	24.41	24.44	24.47	
.8	24.50	24.54	24.57	24.60	24.63	24.66	24.69	24.72	24.76	24.79	
.9	24.82	24.85	24.88	24.91	24.94	24.98	25.01	25.04	25.07	25.10	
8.0	25.13	25.16	25.20	25.23	25.26	25.29	25.32	25.35	25.38	25.42	
.1	25.45	25.48	25.51	24.54	25.57	25.60	25.64	25.67	25.70	25.73	
.2	25.76	25.79	25.82	25.86	25.89	25.92	25.95	25.98	26.01	26.04	
.3	26.08	26.11	26.14	26.17	26.20	26.23	26.26	26.30	26.33	26.36	
.4	26.39	26.42	26.45	26.48	26.52	26.55	26.58	26.61	26.64	26.67	
8.5	26.70	26.73	26.77	26.80	26.83	26.86	26.89	26.92	26.95	26.99	
.6	27.02	27.05	27.08	27.11	27.14	27.17	27.21	27.24	27.27	27.30	
.7	27.33	27.36	27.39	27.43	27.46	27.49	27.52	27.55	27.58	27.61	
.8	27.65	27.68	27.71	27.74	27.77	77.80	27.83	27.87	27.90	27.93	
.9	27.96	27.99	28.02	28.05	28.09	28.12	28.15	28.18	28.21	28.24	
9.0	28.27	28.31	28.34	28.37	28.40	28.43	28.46	28.49	28.53	28.56	
.1	28.59	28.62	28.65	28.68	28.71	28.75	28.78	28.81	28.84	28.87	
.2	28.90	28.93	28.97	29.00	29.03	29.06	29.09	29.12	29.15	29.19	
.3	29.22	29.25	29.28	29.31	29.34	29.37	29.41	29.44	29.47	29.50	
.4	29.53	29.56	29.59	29.63	29.66	29.69	29.72	29.75	29.78	29.81	
9.5	29.85	29.88	29.91	29.94	29.97	30.00	30.03	30.07	30.10	30.13	
.6	30.16	30.19	30.22	30.25	30.28	30.32	30.35	30.38	30.41	30.44	
.7	30.47	30.50	30.54	30.57	30.60	30.63	30.66	30.69	30.72	30.76	
.8	30.79	30.82	30.85	30.88	30.91	30.94	30.98	31.01	31.04	31.07	
.9	31.10	31.13	31.16	31.20	31.23	31.26	31.29	31.32	31.35	31.38	
10.0	31.42										

Moving the decimal point ONE place in *D* requires moving it ONE place in body of table (see p. 28).

AREAS OF CIRCLES BY HUNDREDTHS

(For areas by eighths, see p. 32)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933	16
.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112	18
.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307	20
.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517	21
.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744	23
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986	24
.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243	26
.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516	27
.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806	29
.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110	31
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431	32
.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767	34
.2	3.801	3.836	3.871	3.906	3.941	3.976	4.011	4.047	4.083	4.119	35
.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486	37
.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870	38
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269	40
.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683	42
.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114	43
.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560	45
.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022	46
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499	48
.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992	49
.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501	51
.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026	53
.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566	54
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.010			56
								10.01	10.07	10.12	6
.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69	6
.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28	
.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88	
.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50	
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14	7
.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79	
.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45	
.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14	
.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83	
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55	
.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28	
.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02	
.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78	8
.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56	

Explanation of Table of Areas of Circles (pp. 30–31)

Moving the decimal point one place in column D is equivalent to moving it two places in the body of the table. (D = diameter.)

$$\text{Area of circle} = \frac{\pi}{4} \times (\text{diam.}^2) = 0.785398 \times (\text{diam.}^2)$$

Conversely,

$$\text{Diam.} = \sqrt{\frac{4}{\pi}} \times \sqrt{\text{area}} = 1.128379 \times \sqrt{\text{area}}$$

AREAS OF CIRCLES BY HUNDREDTHS (*continued*)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	19.63	19.71	19.79	19.67	19.95	20.03	20.11	20.19	20.27	20.35	8
.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16	
.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98	
.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82	
.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67	9
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54	
.6	24.63	24.72	24.61	24.89	24.98	25.07	25.16	25.25	25.34	25.43	
.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33	
.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25	
.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.07	28.18	
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13	10
.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09	
.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07	
.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.67	31.97	32.07	
.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08	
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11	
.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15	
.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21	11
.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28	
.9	37.39	37.50	37.61	37.22	37.83	37.94	38.05	38.16	38.26	38.37	
7.0	38.48	38.59	38.70	38.62	38.93	39.04	39.15	39.26	39.37	39.48	
.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60	
.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74	
.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89	12
.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06	
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25	
.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45	
.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66	
.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89	
.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14	
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40	13
.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68	
.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98	
.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29	
.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61	
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95	
.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31	14
.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.69	
.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07	
.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48	
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90	
.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33	15
.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78	
.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25	
.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73	
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23	
.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75	
.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28	
.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82	
.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38	16

Moving the decimal point ONE place in D requires moving it TWO places in body of table (see p. 30).

CIRCUMFERENCES AND AREAS OF CIRCLES BY EIGHTHS, ETC.

(For tenths, see p. 28)

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
			7/8	2.749	.6013	4	12.57	12.57	9	28.27	63.62
1/64	.04909	.00019	57/64	2.798	.6230	1/16	12.76	12.96	1/8	28.67	65.40
1/32	.09817	.00077	29/32	2.847	.6450	1/8	12.96	13.36	1/4	29.06	67.20
3/64	.1473	.00173	59/64	2.896	.6675	3/16	13.16	13.77	3/8	29.45	69.03
1/16	.1963	.00307	15/16	2.945	.6903	1/4	13.35	14.19	1/2	29.85	70.88
5/64	.2454	.00479	61/64	2.994	.7135	5/16	13.55	14.61	5/8	30.24	72.76
3/32	.2945	.00690	31/32	3.043	.7371	3/8	13.74	15.03	3/4	30.63	74.66
7/64	.3436	.00940	63/64	3.093	.7610	7/16	13.94	15.47	7/8	31.02	76.59
1/8	.3927	.01227	1	3.142	.7854	1/2	14.14	15.90	10	31.42	78.54
9/64	.4418	.01553	1/16	3.338	.8866	9/16	14.33	16.35	1/8	31.81	80.52
5/32	.4909	.01917	1/8	3.534	.9940	5/8	14.53	16.80	1/4	32.20	82.52
11/64	.5400	.02320	3/16	3.731	1.108	11/16	14.73	17.26	3/8	32.59	84.54
3/16	.5890	.02761	1/4	3.927	1.227	3/4	14.92	17.72	1/2	32.99	86.59
13/64	.6381	.03241	5/16	4.123	1.353	13/16	15.12	18.19	5/8	33.38	88.66
7/32	.6872	.03758	3/8	4.320	1.485	7/8	15.32	18.67	3/4	33.77	90.76
15/64	.7363	.04314	7/16	4.516	1.623	15/16	15.51	19.15	7/8	34.16	92.89
1/4	.7854	.04909	1/2	4.712	1.767	5	15.71	19.63	11	34.56	95.03
17/64	.8345	.05542	9/16	4.909	1.917	1/16	15.90	20.13	1/8	34.95	97.21
9/32	.8836	.06213	5/8	5.105	2.074	1/8	16.10	20.63	1/4	35.34	99.40
19/64	.9327	.06922	11/16	5.301	2.237	3/16	16.30	21.14	3/8	35.74	101.6
5/16	.9817	.07670	3/4	5.498	2.405	1/4	16.49	21.65	1/2	36.13	103.9
21/64	1.031	.08456	13/16	5.694	2.580	5/16	16.69	22.17	5/8	36.52	106.1
11/32	1.080	.09281	7/8	5.890	2.761	3/8	16.89	22.69	3/4	36.91	108.4
23/64	1.129	.1014	15/16	6.087	2.948	7/16	17.08	23.22	7/8	37.31	110.8
3/8	1.178	.1104	2	6.283	3.142	1/2	17.28	23.76	12	37.70	113.1
25/64	1.227	.1198	1/16	6.480	3.341	9/16	17.48	24.30	1/8	38.09	115.5
13/32	1.276	.1296	1/8	6.676	3.547	5/8	17.67	24.85	1/4	38.48	117.9
27/64	1.325	.1398	3/16	6.872	3.758	11/16	17.87	25.41	3/8	38.88	120.3
7/16	1.374	.1503	1/4	7.069	3.976	3/4	18.06	25.97	1/2	39.27	122.7
29/64	1.424	.1613	5/16	7.265	4.200	13/16	18.26	26.53	5/8	39.66	125.2
15/32	1.473	.1726	3/8	7.461	4.430	7/8	18.46	27.11	3/4	40.06	127.7
31/64	1.522	.1843	7/16	7.658	4.666	15/16	18.65	27.69	7/8	40.45	130.2
1/2	1.571	.1963	1/2	7.854	4.909	6	18.85	28.27	13	40.84	132.7
33/64	1.620	.2088	9/16	8.050	5.157	1/8	19.24	29.46	1/8	41.23	135.3
17/32	1.669	.2217	5/8	8.247	5.412	1/4	19.63	30.68	1/4	41.63	137.9
35/64	1.718	.2349	11/16	8.443	5.673	3/8	20.03	31.92	3/8	42.02	140.5
9/16	1.767	.2485	3/4	8.639	5.940	1/2	20.42	33.18	1/2	42.41	143.1
37/64	1.816	.2625	13/16	8.836	6.213	5/8	20.81	34.47	5/8	42.80	145.8
19/32	1.865	.2769	7/8	9.032	6.492	3/4	21.21	35.78	3/4	43.20	148.5
39/64	1.914	.2916	15/16	9.228	6.777	7/8	21.60	37.12	7/8	43.59	151.2
5/8	1.963	.3068	3	9.425	7.069	7	21.99	38.48	14	43.98	153.9
41/64	2.013	.3223	1/16	9.621	7.366	1/8	22.38	39.87	1/8	44.37	156.7
21/32	2.062	.3382	1/8	9.817	7.670	1/4	22.78	41.28	1/4	44.77	159.5
43/64	2.111	.3545	3/16	10.01	7.980	3/8	23.17	42.72	3/8	45.16	162.3
11/16	2.160	.3712	1/4	10.21	8.296	1/2	23.56	44.18	1/2	45.55	165.1
45/64	2.209	.3883	5/16	10.41	8.618	5/8	23.95	45.66	5/8	45.95	168.0
23/32	2.258	.4057	3/8	10.60	8.946	3/4	24.35	47.17	3/4	46.34	170.9
47/64	2.307	.4236	7/16	10.80	9.281	7/8	24.74	48.71	7/8	46.73	173.8
3/4	2.356	.4418	1/2	11.00	9.621	8	25.13	50.27	15	47.12	176.7
49/64	2.405	.4604	9/16	11.19	9.968	1/8	25.53	51.85	1/8	47.52	179.7
25/32	2.454	.4794	5/8	11.39	10.32	1/4	25.92	53.46	1/4	47.91	182.7
51/64	2.503	.4987	11/16	11.58	10.68	3/8	26.31	55.09	3/8	48.30	185.7
13/16	2.553	.5185	3/4	11.78	11.04	1/2	26.70	56.75	1/2	48.69	188.7
53/64	2.602	.5386	13/16	11.98	11.42	5/8	27.10	58.43	5/8	49.09	191.7
27/32	2.651	.5591	7/8	12.17	11.79	3/4	27.49	60.13	3/4	49.48	194.8
55/64	2.700	.5800	15/16	12.37	12.18	7/8	27.88	61.86	7/8	49.87	197.9

CIRCUMFERENCES AND AREAS BY EIGHTHS *(continued)*

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
16	50.27	201.1	19 1/2	61.26	298.6	23	72.26	415.5	29	91.11	660.5
1/8	50.66	204.2	5/8	61.65	302.5	1/8	72.65	420.0	1/4	91.89	672.0
1/4	51.05	207.4	3/4	62.05	306.4	1/4	73.04	424.6	1/2	92.68	683.5
3/8	51.44	210.6	7/8	62.44	310.2	3/8	73.43	429.1	3/4	93.46	695.1
1/2	51.84	213.8	20	62.83	314.2	1/2	73.83	433.7	30	94.25	706.9
5/8	52.23	217.1	1/8	63.22	318.1	5/8	74.22	438.4	1/4	95.03	718.7
3/4	52.62	220.4	1/4	63.62	322.1	3/4	74.61	443.0	1/2	95.82	730.6
7/8	53.01	223.7	3/8	64.01	326.1	7/8	75.01	447.7	3/4	96.60	742.6
17	53.41	227.0	1/2	64.40	330.1	24	75.40	452.4	31	97.39	754.8
1/8	53.80	230.3	5/8	64.80	334.1	1/4	76.18	461.9	1/4	98.17	767.0
1/4	54.19	233.7	3/4	65.19	338.2	1/2	76.97	471.4	1/2	98.96	779.3
3/8	54.59	237.1	7/8	65.58	342.2	3/4	77.75	481.1	3/4	99.75	791.7
1/2	54.98	240.5	21	65.97	346.4	25	78.54	490.9	32	100.5	804.2
5/8	55.37	244.0	1/8	66.37	350.5	1/4	79.33	500.7	1/4	101.3	816.9
3/4	55.76	247.4	1/4	66.76	354.7	1/2	80.11	510.7	1/2	102.1	829.6
7/8	56.16	250.9	3/8	67.15	358.8	3/4	80.90	520.8	3/4	102.9	842.4
18	56.55	254.5	1/2	67.54	363.1	26	81.68	530.9	33	103.7	855.3
1/8	56.94	258.0	5/8	67.94	367.3	1/4	82.47	541.2	1/4	104.5	868.3
1/4	57.33	261.6	3/4	68.33	371.5	1/2	83.25	551.5	1/2	105.2	881.4
3/8	57.73	265.2	7/8	68.72	375.8	3/4	84.04	562.0	3/4	106.0	894.6
1/2	58.12	268.8	22	69.12	380.1	27	84.82	572.6	34	106.8	907.9
5/8	58.51	272.4	1/8	69.51	384.5	1/4	85.61	583.2	1/4	107.6	921.3
3/4	58.90	276.1	1/4	69.90	388.8	1/2	86.39	594.0	1/2	108.4	934.8
7/8	59.30	279.8	3/8	70.29	393.2	3/4	87.18	604.8	3/4	109.2	948.4
19	59.69	283.5	1/2	70.69	397.6	28	87.96	615.8	35	110.0	962.1
1/8	60.08	287.3	5/8	71.08	402.0	1/4	88.75	626.8	1/4	110.7	975.9
1/4	60.48	291.0	3/4	71.47	406.5	1/2	89.54	637.9	1/2	111.5	989.8
3/8	60.87	294.8	7/8	71.86	411.0	3/4	90.32	649.2	3/4	112.3	1003.8

AREAS OF CIRCLES. Diameters in Feet and Inches, Areas in Square Feet

Feet	Inches 0	1	2	3	4	5	6	7	8	9	10	11
0	.0000	.0055	.0218	.0491	.0873	.1364	.1963	.2673	.3491	.4418	.5454	.6600
1	.7854	.9218	1.069	1.227	1.396	1.576	1.767	1.969	2.182	2.405	2.640	2.885
2	3.142	3.409	3.687	3.976	4.276	4.587	4.909	5.241	5.585	5.940	6.305	6.681
3	7.069	7.467	7.876	8.296	8.727	9.168	9.621	10.08	10.56	11.04	11.54	12.05
4	12.57	13.10	13.64	14.19	14.75	15.32	15.90	16.50	17.10	17.72	18.35	18.99
5	19.63	20.29	20.97	21.65	22.34	23.04	23.76	24.48	25.22	25.97	26.73	27.49
6	28.27	29.07	29.87	30.68	31.50	32.34	33.18	34.04	34.91	35.78	36.67	37.57
7	38.48	39.41	40.34	41.28	42.24	43.20	44.18	45.17	46.16	47.17	48.19	49.22
8	50.27	51.32	52.38	53.46	54.54	55.64	56.75	57.86	58.99	60.13	61.28	62.44
9	63.62	64.80	66.00	67.20	68.42	69.64	70.88	72.13	73.39	74.66	75.94	77.24
10	78.54	79.85	81.18	82.52	83.86	85.22	86.59	87.97	89.36	90.76	92.18	93.60
11	95.03	96.48	97.93	99.40	100.9	102.4	103.9	105.4	106.9	108.4	110.0	111.5
12	113.1	114.7	116.3	117.9	119.5	121.1	122.7	124.4	126.0	127.7	129.4	131.0
13	132.7	134.4	136.2	137.9	139.6	141.4	143.1	144.9	146.7	148.5	150.3	152.1
14	153.9	155.8	157.6	159.5	161.4	163.2	165.1	167.0	168.9	170.9	172.8	174.8

If given diameter is not found in this table, reduce diameter to feet and decimals of a foot by aid of the following auxiliary table, and then find area from pp. 30–31.

From Inches and Fractions of an Inch to Decimals of a Foot

Inches	1	2	3	4	5	6	7	8	9	10	11
Feet	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167
Inches	1/8	1/4	3/8	1/2	5/8	3/4	7/8				
Feet	.0104	.0208	.0313	.0417	.0521	.0625	.0729				

Example. 5 ft 7 3/8 in. = 5.0 + 0.5833 + 0.0313 = 5.6146 ft

SEGMENTS OF CIRCLES, GIVEN h/c

Given: h = height; c = chord. (For explanation of this table, see p. 38)

$\frac{h}{c}$	$\frac{\text{Diam.}}{c}$	Diff.	$\frac{\text{Arc}}{c}$	Diff.	$\frac{\text{Area}}{h \times c}$	Diff.	Central angle, v	Diff.	$\frac{h}{\text{Diam.}}$	Diff.
.00			1.000	0	.6667	0	0.00°	458	.0000	4
1	25.010	12490	1.000	1	.6667	2	4.58	458	.0004	12
2	12.520	*4157	1.001	1	.6669	2	9.16	457	.0016	20
3	8.363	*2073	1.002	2	.6671	4	13.73	457	.0036	28
4	6.290	*1240	1.004	3	.6675	5	18.30	454	.0064	35
.05	5.050	*823	1.007	3	.6680	6	22.84°	453	.0099	43
6	4.227	*586	1.010	3	.6686	7	27.37	451	.0142	50
7	3.641	*436	1.013	4	.6693	8	31.88	448	.0192	58
8	3.205	*337	1.017	4	.6701	9	36.36	446	.0250	64
9	2.868	*268	1.021	5	.6710	10	40.82	442	.0314	71
.10	2.600	*217	1.026	6	.6720	11	45.24°	439	.0385	77
1	2.383	*180	1.032	6	.6731	12	49.63	435	.0462	83
2	2.203	*150	1.038	6	.6743	13	53.98	432	.0545	88
3	2.053	*127	1.044	7	.6756	14	58.30	427	.0633	94
4	1.926	*109	1.051	8	.6770	15	62.57	423	.0727	99
.15	1.817	*94	1.059	8	.6785	16	66.80°	418	.0826	103
6	1.723	*82	1.067	8	.6801	17	70.98	413	.0929	107
7	1.641	*72	1.075	9	.6818	18	75.11	409	.1036	111
8	1.569	*63	1.084	10	.6836	19	79.20	403	.1147	116
9	1.506	56	1.094	9	.6855	20	83.23	399	.1263	116
.20	1.450	50	1.103	11	.6875	21	87.21°	392	.1379	120
1	1.400	44	1.114	10	.6896	22	91.13	387	.1499	123
2	1.356	39	1.124	12	.6918	23	95.00	381	.1622	124
3	1.317	35	1.136	11	.6941	24	98.81	375	.1746	127
4	1.282	32	1.147	12	.6965	24	102.56	370	.1873	127
.25	1.250	28	1.159	12	.6989	25	106.26°	364	.2000	128
6	1.222	26	1.171	13	.7014	27	109.90	358	.2128	130
7	1.196	23	1.184	13	.7041	27	113.48	352	.2258	129
8	1.173	21	1.197	14	.7068	28	117.00	345	.2387	130
9	1.152	19	1.211	14	.7096	29	120.45	341	.2517	130
.30	1.133	17	1.225	14	.7125	29	123.86°	334	.2647	130
1	1.116	15	1.239	15	.7154	31	127.20	328	.2777	129
2	1.101	13	1.254	15	.7185	31	130.48	322	.2906	128
3	1.088	13	1.269	15	.7216	32	133.70	316	.3034	128
4	1.075	11	1.284	16	.7248	32	136.86	311	.3162	127
.35	1.064	10	1.300	16	.7280	34	139.97°	305	.3289	125
6	1.054	8	1.316	16	.7314	34	143.02	299	.3414	124
7	1.046	8	1.332	17	.7348	35	146.01	293	.3538	123
8	1.038	7	1.349	17	.7383	36	148.94	288	.3661	122
9	1.031	6	1.366	17	.7419	36	151.82	282	.3783	119
.40	1.025	5	1.383	18	.7455	37	154.64°	277	.3902	119
1	1.020	5	1.401	18	.7492	38	157.41	271	.4021	116
2	1.015	4	1.419	18	.7530	38	160.12	266	.4137	115
3	1.011	3	1.437	18	.7568	39	162.78	261	.4252	112
4	1.008	2	1.455	19	.7607	40	165.39	256	.4364	111
.45	1.006	3	1.474	19	.7647	40	167.95°	251	.4475	109
6	1.003	1	1.493	19	.7687	41	170.46	245	.4584	107
7	1.002	1	1.512	19	.7728	41	172.91	241	.4691	105
8	1.001	1	1.531	20	.7769	42	175.32	237	.4796	103
9	1.000	0	1.551	20	.7811	43	177.69	231	.4899	101
.50	1.000		1.571		.7854		180.00°		.5000	

* Interpolation may be inaccurate at these points.

SEGMENTS OF CIRCLES, GIVEN h/D

Given: h = height; D = diameter of circle. (For explanation of this table, see p. 38)

$\frac{h}{D}$	$\frac{\text{Arc}}{D}$	Diff.	$\frac{\text{Area}}{D^2}$	Diff.	Central angle, v	Diff.	$\frac{\text{Chord}}{D}$	Diff.	$\frac{\text{Arc}}{\text{Circumf.}}$	Diff.	$\frac{\text{Area}}{\text{Circle}}$	Diff.
.00	0.000	2003	.0000	13	0.00°	2296	.0000	*1990	.0000	*638	.0000	17
1	.2003	*835	.0013	24	22.96	*956	.1990	*810	.0638	*265	.0017	31
2	.2838	*644	.0037	32	32.52	*738	.2800	*612	.0903	*205	.0049	39
3	.3482	*545	.0069	36	39.90	*625	.3412	*507	.1108	*174	.0087	47
4	.4027	*483	.0105	42	46.15	*553	.3919	*440	.1282	*154	.0134	53
.05	.4510	*439	.0147	45	51.68°	*504	.4359	*391	.1436	*139	.0187	58
6	.4949	*406	.0192	50	56.72	*465	.4750	*353	.1575	*130	.0245	63
7	.5355	*380	.0242	52	61.37	*435	.5103	*323	.1705	121	.0308	67
8	.5735	*359	.0294	56	65.72	*411	.5426	*298	.1826	114	.0375	71
9	.6094	*341	.0350	59	69.83	*391	.5724	*276	.1940	108	.0446	74
.10	.6435	*326	.0409	61	73.74°	*374	.6000	*258	.2048	104	.0520	78
1	.6761	*314	.0470	64	77.48	*359	.6258	*241	.2152	100	.0598	82
2	.7075	*302	.0534	66	81.07	*347	.6499	*227	.2232	96	.0680	84
3	.7377	*293	.0600	68	84.54	*335	.6726	*214	.2348	93	.0764	87
4	.7670	*284	.0668	71	87.69	*326	.6940	*201	.2441	91	.0851	90
.15	.7954	276	.0739	72	91.15°	316	.7141	*191	.2532	88	.0941	92
6	.8230	270	.0811	74	94.31	309	.7332	*181	.2620	86	.1033	94
7	.8500	263	.0885	76	97.40	302	.7513	*171	.2706	83	.1127	97
8	.8763	258	.0961	78	100.42	295	.7684	162	.2789	82	.1224	99
9	.9021	252	.1039	79	103.37	289	.7846	154	.2871	81	.1323	101
.20	0.9273	248	.1118	81	106.26°	284	.8000	146	.2952	79	.1424	103
1	0.9521	243	.1199	82	109.10	279	.8146	139	.3031	77	.1527	104
2	0.9764	240	.1281	84	111.89	274	.8285	132	.3108	76	.1631	107
3	1.0004	235	.1365	84	114.63	271	.8417	125	.3184	75	.1738	108
4	1.0239	233	.1449	86	117.34	266	.8542	118	.3259	74	.1846	109
.25	1.0472	229	.1535	88	120.00°	263	.8660	113	.3333	73	.1955	111
6	1.0701	227	.1623	88	122.63	260	.8773	106	.3406	72	.2066	112
7	1.0928	224	.1711	89	125.23	256	.8879	101	.3478	72	.2178	114
8	1.1152	222	.1800	90	127.79	254	.8980	95	.3550	70	.2292	115
9	1.1374	219	.1890	92	130.33	251	.9075	90	.3620	70	.2407	116
.30	1.1593	217	.1982	92	132.84°	249	.9165	85	.3690	69	.2523	117
1	1.1810	215	.2074	93	135.33	247	.9250	80	.3759	69	.2640	119
2	1.2025	214	.2167	93	137.80	245	.9330	74	.3828	68	.2759	119
3	1.2239	212	.2260	95	140.25	242	.9404	70	.3896	67	.2878	120
4	1.2451	210	.2355	95	142.67	241	.9474	65	.3963	67	.2998	121
.35	1.2661	209	.2450	96	145.08°	240	.9539	61	.4030	67	.3119	122
6	1.2870	208	.2546	96	147.48	238	.9600	56	.4097	66	.3241	123
7	1.3078	206	.2642	97	149.86	237	.9656	52	.4163	66	.3364	123
8	1.3284	206	.2739	97	152.23	235	.9708	47	.4229	65	.3487	124
9	1.3490	204	.2836	98	154.58	235	.9755	43	.4294	65	.3611	124
.40	1.3694	204	.2934	98	156.93°	233	.9798	39	.4359	65	.3735	125
1	1.3898	203	.3032	98	159.26	233	.9837	34	.4424	65	.3860	126
2	1.4101	202	.3130	99	161.59	231	.9871	31	.4489	64	.3986	126
3	1.4303	202	.3229	99	163.90	232	.9902	26	.4553	64	.4112	126
4	1.4505	201	.3328	100	166.22	230	.9928	22	.4617	64	.4238	126
.45	1.4706	201	.3428	99	168.52°	230	.9950	18	.4681	64	.4364	127
6	1.4907	201	.3527	100	170.82	230	.9968	14	.4745	64	.4491	127
7	1.5108	200	.3627	100	173.12	229	.9982	10	.4809	64	.4618	127
8	1.5308	200	.3727	100	175.41	230	.9992	6	.4873	63	.4745	128
9	1.5508	200	.3827	100	177.71	229	.9998	2	.4936	64	.4873	127
.50	1.5708		.3927		180.00°		1.0000		.5000		.5000	

* Interpolation may be inaccurate at these points.

VOLUMES OF SPHERES BY HUNDREDTHS

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	.5236	.5395	.5556	.5722	.5890	.6061	.6236	.6414	.6596	.6781	173
.1	.6969	.7161	.7356	.7555	.7757	.7963	.8173	.8386	.8603	.8823	208
.2	.9048	.9276	.9508	.9743	.9983	1.0227					236
.2						1.023	1.047	1.073	1.098	1.124	25
.3	1.150	1.177	1.204	1.232	1.260	1.288	1.317	1.346	1.376	1.406	29
.4	1.437	1.468	1.499	1.531	1.563	1.596	1.630	1.663	1.697	1.732	33
1.5	1.767	1.803	1.839	1.875	1.912	1.950	1.988	2.026	2.065	2.105	38
.6	2.145	2.185	2.226	2.268	2.310	2.352	2.395	2.439	2.483	2.527	43
.7	2.572	2.618	2.664	2.711	2.758	2.806	2.855	2.903	2.953	3.003	48
.8	3.054	3.105	3.157	3.209	3.262	3.315	3.369	3.424	3.479	3.535	54
.9	3.591	3.648	3.706	3.764	3.823	3.882	3.942	4.003	4.064	4.126	60
2.0	4.189	4.252	4.316	4.380	4.445	4.511	4.577	4.644	4.712	4.780	66
.1	4.849	4.919	4.989	5.060	5.131	5.204	5.277	5.350	5.425	5.500	73
.2	5.575	5.652	5.729	5.806	5.885	5.964	6.044	6.125	6.206	6.288	80
.3	6.371	6.454	6.538	6.623	6.709	6.795	6.882	6.970	7.059	7.148	87
.4	7.238	7.329	7.421	7.513	7.606	7.700	7.795	7.890	7.986	8.083	94
2.5	8.181	8.280	8.379	8.478	8.580	8.682	8.785	8.888	8.992	9.097	102
.6	9.203	9.309	9.417	9.525	9.634	9.744	9.855	9.966	10.079		110
.6									10.08	10.19	11
.7	10.31	10.42	10.54	10.65	10.77	10.89	11.01	11.13	11.25	11.37	12
.8	11.49	11.62	11.74	11.87	11.99	12.12	12.25	12.38	12.51	12.64	13
.9	12.77	12.90	13.04	13.17	13.31	13.44	13.58	13.72	13.86	14.00	14
3.0	14.14	14.28	14.42	14.57	14.71	14.86	15.00	15.15	15.30	15.45	15
.1	15.60	15.75	15.90	16.06	16.21	16.37	16.52	16.68	16.84	17.00	16
.2	17.16	17.32	17.48	17.64	17.81	17.97	18.14	18.31	18.48	18.65	17
.3	18.82	18.99	19.16	19.33	19.51	19.68	19.86	20.04	20.22	20.40	18
.4	20.58	20.76	20.94	21.13	21.31	21.50	21.69	21.88	22.07	22.26	19
3.5	22.45	22.64	22.84	23.03	23.23	23.43	23.62	23.82	24.02	24.23	20
.6	24.43	24.63	24.84	25.04	25.25	25.46	25.67	25.88	26.09	26.31	21
.7	26.52	26.74	26.95	27.17	27.39	27.61	27.83	28.06	28.28	28.50	22
.8	28.73	28.96	29.19	29.42	29.65	29.88	30.11	30.35	30.58	30.82	23
.9	31.06	31.30	31.54	31.78	32.02	32.27	32.52	32.76	33.01	33.26	25
4.0	33.51	33.76	34.02	34.27	34.53	34.78	35.04	35.30	35.56	35.82	26
.1	36.09	36.35	36.62	36.88	37.15	37.42	37.69	37.97	38.24	38.52	27
.2	38.79	39.07	39.35	39.63	39.91	40.19	40.48	40.76	41.05	41.34	28
.3	41.63	41.92	42.21	42.51	42.80	43.10	43.40	43.70	44.00	44.30	30
.4	44.60	44.91	45.21	45.52	45.83	46.14	46.45	46.77	47.08	47.40	31
4.5	47.71	48.03	48.35	48.67	49.00	49.32	49.65	49.97	50.30	50.63	33
.6	50.97	51.30	51.63	51.97	52.31	52.65	52.99	53.33	53.67	54.02	34
.7	54.36	54.71	55.06	55.41	55.76	56.12	56.47	56.83	57.19	57.54	35
.8	57.91	58.27	58.63	59.00	59.37	59.73	60.10	60.48	60.85	61.22	37
.9	61.60	61.98	62.36	62.74	63.12	63.51	63.89	64.28	64.67	65.06	38

Explanation of Table of Volumes of Spheres (pp. 36–37)

Moving the decimal point one place in column D is equivalent to moving it three places in the body of the table. (D = diameter.)

$$\text{Volume of sphere} = \frac{\pi}{6} \times (\text{diam.}^3) = 0.523599 \times (\text{diam.}^3)$$

Conversely,

$$\text{Diam.} = \sqrt[3]{\frac{6}{\pi}} \times \sqrt[3]{\text{volume}} = 1.240701 \times \sqrt[3]{\text{volume}}$$

VOLUMES OF SPHERES (*continued*)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	65.45	65.84	66.24	66.64	67.03	67.43	67.83	68.24	68.64	69.05	40
.1	69.46	69.87	70.28	70.69	71.10	71.52	71.94	72.36	72.78	73.20	42
.2	73.62	74.05	74.47	74.90	75.33	75.77	76.20	76.64	77.07	77.51	43
.3	77.95	78.39	78.84	79.28	79.73	80.18	30.63	81.08	81.54	81.99	45
.4	82.45	82.91	83.37	83.83	84.29	84.76	85.23	85.70	86.17	86.64	47
5.5	87.11	87.59	88.07	88.55	89.03	89.51	90.00	90.48	90.97	91.49	48
.6	91.95	92.45	92.94	93.44	93.94	94.44	94.94	95.44	95.95	96.46	50
.7	96.97	97.48	97.99	98.51	99.02	99.54	100.05				52
.7							100.1	100.6	101.1	101.6	5
.8	102.2	102.7	103.2	103.8	104.3	104.8	105.4	105.9	106.4	107.0	5
.9	107.5	108.1	108.6	109.2	109.7	110.3	110.9	111.4	112.0	112.5	6
6.0	113.1	113.7	114.2	114.8	115.4	115.9	116.5	117.1	117.1	118.3	6
.1	118.8	119.4	120.0	120.6	121.2	121.8	122.4	123.0	123.0	124.2	
.2	124.8	125.4	126.0	126.6	127.2	127.8	128.4	129.1	129.7	130.3	
.3	130.9	131.5	132.2	132.8	133.4	134.1	134.7	135.3	136.0	136.6	
.4	137.3	137.9	138.5	139.2	139.8	140.5	141.2	141.8	142.5	143.1	7
6.5	143.8	144.5	145.1	145.8	146.5	147.1	147.8	148.5	149.2	149.8	
.6	150.5	151.2	151.9	152.6	153.3	154.0	154.7	155.4	156.1	156.8	
.7	157.5	158.2	158.9	159.6	160.3	161.0	161.7	162.5	163.2	163.9	
.8	164.6	165.4	166.1	166.8	167.6	168.3	169.0	169.8	170.5	171.3	
.9	172.0	172.8	173.5	174.3	175.0	175.8	176.5	177.3	178.1	178.8	8
7.0	179.6	180.4	181.1	181.9	182.7	183.5	184.3	185.0	185.8	186.6	
.1	187.4	188.2	189.0	189.8	190.6	191.4	192.2	193.0	193.8	194.6	
.2	195.4	196.2	197.1	197.9	198.7	199.5	200.4	201.2	202.0	202.9	
.3	203.7	204.5	205.4	206.2	207.1	207.9	208.8	209.6	210.5	211.3	
.4	212.2	213.0	213.9	214.8	215.6	216.5	217.4	218.3	219.1	220.0	9
7.5	220.9	221.8	222.7	223.6	224.4	225.3	226.2	227.1	228.0	228.9	
.6	229.8	230.8	231.7	232.6	233.5	234.4	235.3	236.3	237.2	238.1	
.7	239.0	240.0	240.9	241.8	242.8	243.7	244.7	245.6	246.6	247.5	
.8	248.5	249.4	250.4	251.4	252.3	253.3	254.3	255.2	256.2	257.2	10
.9	258.2	259.1	260.1	261.1	262.1	263.1	264.1	265.1	266.1	267.1	
8.0	268.1	269.1	270.1	271.1	272.1	273.1	274.2	225.2	276.2	277.2	
.1	278.3	279.3	280.3	281.4	282.4	283.4	284.5	285.5	286.6	287.6	11
.2	288.7	289.8	290.8	291.9	292.9	294.0	295.1	296.2	297.2	298.3	
.3	299.4	300.5	301.6	302.6	303.7	304.8	305.9	307.0	308.1	309.2	
.4	310.3	311.4	312.6	313.7	314.8	315.9	317.0	318.2	319.3	320.4	
8.5	321.6	322.7	323.8	325.0	326.1	327.3	328.4	329.6	330.7	331.9	
.6	333.0	334.2	335.4	336.5	337.7	338.9	340.1	341.2	342.4	343.6	
.7	344.8	346.0	347.2	348.4	349.6	350.8	352.0	353.2	354.4	355.6	12
.8	356.8	358.0	359.3	360.5	361.7	362.9	364.2	365.4	366.6	367.9	
.9	369.1	370.4	371.6	372.9	374.1	375.4	376.6	377.9	379.2	380.4	13
9.0	381.7	383.0	384.3	385.5	386.8	388.1	389.4	390.7	392.0	393.3	
.1	394.6	395.9	397.2	398.5	399.8	401.1	402.4	403.7	405.1	406.4	
.2	407.7	409.1	410.4	411.7	413.1	414.4	415.7	417.1	418.4	419.8	
.3	421.2	422.5	423.9	425.2	426.6	428.0	429.4	430.7	432.1	433.5	14
.4	434.9	436.3	437.7	439.1	440.5	441.9	443.3	444.7	446.1	447.5	
9.5	448.9	450.3	451.8	453.2	454.6	456.0	457.5	458.9	460.4	461.8	
.6	463.2	464.7	466.1	467.6	469.1	470.5	472.0	473.5	474.9	476.4	
.7	477.9	479.4	480.8	482.3	483.8	485.3	486.8	488.3	489.8	491.3	15
.8	492.8	494.3	495.8	497.3	498.9	500.4	501.9	503.4	505.0	506.5	
.9	508.0	509.6	511.1	512.7	514.2	515.8	517.3	518.9	520.5	522.0	16
10.0	523.6										

Moving the decimal point ONE place in *D* requires moving it THREE places in body of table (see p. 35).

SEGMENTS OF SPHERES

(h = height of segment; D = diam. of sphere)

$\frac{h}{D}$	$\frac{\text{Vol. segm.}}{D^3}$	Diff.	$\frac{\text{Vol. segm.}}{\text{Vol. sphere}}$	Diff.
0.00	0.0000	2	0.0000	3
1	0.0002	4	0.0003	9
2	0.0006	8	0.0012	14
3	0.0014	10	0.0026	21
4	0.0024	14	0.0047	26
0.05	0.0038	16	0.0073	31
6	0.0054	19	0.0104	36
7	0.0073	22	0.0140	42
8	0.0095	25	0.0182	46
9	0.0120	27	0.0228	52
0.10	0.0147	29	0.0280	56
1	0.0176	32	0.0336	61
2	0.0208	34	0.0397	66
3	0.0242	37	0.0463	70
4	0.0279	39	0.0533	74
0.15	0.0318	41	0.0607	79
6	0.0359	44	0.0686	83
7	0.0403	45	0.0769	86
8	0.0448	47	0.0855	91
9	0.0495	50	0.0946	94
0.20	0.0545	51	0.1040	98
1	0.0596	53	0.1138	101
2	0.0649	55	0.1239	105
3	0.0704	56	0.1344	108
4	0.0760	58	0.1452	110
0.25	0.0818	60	0.1562	114
6	0.0878	61	0.1676	117
7	0.0939	63	0.1793	120
8	0.1002	64	0.1913	122
9	0.1066	65	0.2035	125
0.30	0.1131	67	0.2160	127
1	0.1198	67	0.2287	130
2	0.1265	69	0.2417	131
3	0.1334	70	0.2548	134
4	0.1404	71	0.2682	135
0.35	0.1475	72	0.2817	138
6	0.1547	73	0.2955	139
7	0.1620	74	0.3094	141
8	0.1694	74	0.3235	142
9	0.1768	75	0.3377	143
0.40	0.1843	76	0.3520	145
1	0.1919	76	0.3665	145
2	0.1995	77	0.3810	147
3	0.2072	77	0.3957	147
4	0.2149	78	0.4104	148
0.45	0.2227	78	0.4252	149
6	0.2305	78	0.4401	150
7	0.2383	78	0.4551	149
8	0.2461	78	0.4700	150
9	0.2539	79	0.4850	150
0.50	0.2618		0.5000	

NOTE. Vol. segm. = $\frac{1}{6}\pi h^2 (3D-2h)$.

Explanation of Table on this page

Given, h = height of segment,
D = diam. of sphere.

To find the volume of the segment, form the ratio h/D and find from the table the value of (vol./D^3); then, by a simple multiplication,

vol. segment = $D^3 \times$ (vol./D^3).

The table gives also the ratio of the volume of the segment to the entire volume of the sphere.

NOTE. Area of zone = $\pi \times h \times D$. (Use Table of Multiples of π, p. 28)

Explanation of Table on p. 34

Given, h = height of segment,
c = chord.

To find the diam. of the circle, the length of arc, or the area of the segment, form the ratio h/c, and find from the table the value of (diam./c), (arc/c), or (area/hc); then, by a simple multiplication,

diam. = $c \times$ (diam./c),
arc = $c \times$ (arc/c),
area = $h \times c \times$ (area/hc).

The table gives also the angle subtended at the center, and the ratio of h to D. See p. 106.

Explanation of Table on p. 35

Given, h = height of segment,
D = diam. of circle.

To find the chord, the length of arc, or the area of the segment, form the ratio h/D, and find from the table the value of (chord/D), (arc/D), or (area/D^2); then, by a simple multiplication,

chord = $D \times$ (chord/D),
arc = $D \times$ (arc/D),
area = $D^2 \times$ (area/D^2).

The table gives also the angle subtended at the center, the ratio of the arc of the segment to the whole circumference, and the ratio of the area of the segment to the area of the whole circle. See p. 106.

REGULAR POLYGONS

n = number of sides;
v = $360°/n$ = angle subtended at the center by one side;
a = length of one side $= R\left(2 \sin \frac{v}{2}\right) = r\left(2 \tan \frac{v}{2}\right)$;
R = radius of circumscribed circle $= a\left(\frac{1}{2} \csc \frac{v}{2}\right) = r\left(\sec \frac{v}{2}\right)$;
r = radius of inscribed circle $= R\left(\cos \frac{v}{2}\right) = a\left(\frac{1}{2} \cot \frac{v}{2}\right)$;
Area $= a^2\left(\frac{1}{4} n \cot \frac{v}{2}\right) = R^2\left(\frac{1}{2} n \sin v\right) = r^2\left(n \tan \frac{v}{2}\right)$.

n	v	$\frac{\text{Area}}{a^2}$	$\frac{\text{Area}}{R^2}$	$\frac{\text{Area}}{r^2}$	$\frac{R}{a}$	$\frac{R}{r}$	$\frac{a}{R}$	$\frac{a}{r}$	$\frac{r}{R}$	$\frac{r}{a}$
3	120°	0.4330	1.299	5.196	0.5774	2.000	1.732	3.464	0.5000	0.2887
4	90°	1.000	2.000	4.000	0.7071	1.414	1.414	2.000	0.7071	0.5000
5	72°	1.721	2.378	3.633	0.8507	1.236	1.176	1.453	0.8090	0.6882
6	60°	2.598	2.598	3.464	1.0000	1.155	1.000	1.155	0.8660	0.8660
7	51°.43	3.634	2.736	3.371	1.152	1.110	0.8678	0.9631	0.9010	1.038
8	45°	4.828	2.828	3.314	1.307	1.082	0.7654	0.8284	0.9239	1.207
9	40°	6.182	2.893	3.276	1.462	1.064	0.6840	0.7279	0.9397	1.374
10	36°	7.694	2.939	3.249	1.618	1.052	0.6180	0.6498	0.9511	1.539
12	30°	11.20	3.000	3.215	1.932	1.035	0.5176	0.5359	0.9659	1.866
15	24°	17.64	3.051	3.188	2.405	1.022	0.4158	0.4251	0.9781	2.352
16	22°.50	20.11	3.062	3.183	2.563	1.020	0.3902	0.3978	0.9808	2.514
20	18°	31.57	3.090	3.168	3.196	1.013	0.3129	0.3168	0.9877	3.157
24	15°	45.58	3.106	3.160	3.831	1.009	0.2611	0.2633	0.9914	3.798
32	11°.25	81.23	3.121	3.152	5.101	1.005	0.1960	0.1970	0.9952	5.077
48	7°.50	183.1	3.133	3.146	7.645	1.002	0.1309	0.1311	0.9979	7.629
64	5°.625	325.7	3.137	3.144	10.19	1.001	0.0981	0.0983	0.9988	10.18

BINOMIAL COEFFICIENTS

(For table giving binomial coefficients for fractional values of n, see p. 110).

$(n)_0 = 1$; $(n)_1 = n$; $(n)_2 = \frac{n(n-1)}{1 \times 2}$; $(n)_3 = \frac{n(n-1)(n-2)}{1 \times 2 \times 3}$; etc.; in general,

$(n)_r = \frac{n(n-1)(n-2)\ .\ .\ .\ (n-[r-1])}{1 \times 2 \times 3\ .\ .\ .\ \times r}$. Another notation: $\binom{n}{r} = (n)_r$.

n	$(n)_0$	$(n)_1$	$(n)_2$	$(n)_3$	$(n)_4$	$(n)_5$	$(n)_6$	$(n)_7$	$(n)_8$	$(n)_9$	$(n)_{10}$	$(n)_{11}$	$(n)_{12}$	$(n)_{13}$
1	1	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
2	1	2	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
3	1	3	3	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	1	4	6	4	1	.....	.....	.....	.....	.....	.....	.....	.....	.....
5	1	5	10	10	5	1	.....	.....	.....	.....	.....	.....	.....	.....
6	1	6	15	20	15	6	1	.....	.....	.....	.....	.....	.....	.....
7	1	7	21	35	35	21	7	1	.....	.....	.....	.....	.....	.....
8	1	8	28	56	70	56	28	8	1	.....	.....	.....	.....	.....
9	1	9	36	84	126	126	84	36	9	1	.....	.....	.....	.....
10	1	10	45	120	210	252	210	120	45	10	1	.....	.....	.....
11	1	11	55	165	330	462	462	330	165	55	11	1	.....	.....
12	1	12	66	220	495	792	924	792	495	220	66	12	1	.....
13	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105

For $n = 14$, $(n)_{14} = 1$; for $n = 15$, $(n)_{14} = 15$, and $(n)_{15} = 1$.

COMMON LOGARITHMS (*special table*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00	0.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	4
1.01	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	
1.02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	
1.03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	
1.04	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	
1.05	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	
1.06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	
1.07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	
1.08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	
1.09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	
1.10	0.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	
1.11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	
1.12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	
1.13	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	
1.14	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	
1.15	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	
1.16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	
1.17	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	
1.18	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	
1.19	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	
1.20	0.0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	
1.21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	
1.22	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	
1.23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	
1.24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	
1.25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	3
1.26	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	
1.27	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	
1.28	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	
1.29	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	
1.30	0.1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	
1.31	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	
1.32	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	
1.33	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	
1.34	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	
1.35	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	
1.36	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	
1.37	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	
1.38	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	
1.39	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	
1.40	0.1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	
1.41	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	
1.42	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	
1.43	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	
1.44	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	
1.45	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	
1.46	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	
1.47	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	
1.48	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	
1.49	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	

Moving the decimal point n places to the right [or left] in the number requires adding $+n$ [or $-n$] in the body of the table (see p. 42).

COMMON LOGARITHMS (*special table, continued*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. dif.
1.50	0.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	3
1.51	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	
1.52	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	
1.53	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	
1.54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	
1.55	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	
1.56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	
1.57	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	
1.58	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	
1.59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	
1.60	0.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	
1.61	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	
1.62	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	
1.63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	
1.64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	
1.65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	
1.66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	
1.67	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	
1.68	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	
1.69	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	
1.70	0.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	
1.71	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	
1.72	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	
1.73	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	
1.74	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2
1.75	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	
1.76	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	
1.77	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	
1.78	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	
1.79	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	
1.80	0.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	
1.81	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	
1.82	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	
1.83	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	
1.84	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	
1.85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	
1.86	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	
1.87	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	
1.88	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	
1.89	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	
1.90	0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	
1.91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	
1.92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	
1.93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	
1.94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	
1.95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	
1.96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	
1.97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	
1.98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	
1.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	

COMMON LOGARITHMS (*special table*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00	0.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	4
1.01	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	
1.02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	
1.03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	
1.04	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	
1.05	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	
1.06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	
1.07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	
1.08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	
1.09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	
1.10	0.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	
1.11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	
1.12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	
1.13	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	
1.14	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	
1.15	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	
1.16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	
1.17	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	
1.18	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	
1.19	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	
1.20	0.0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	
1.21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	
1.22	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	
1.23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	
1.24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	
1.25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	3
1.26	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	
1.27	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	
1.28	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	
1.29	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	
1.30	0.1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	
1.31	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	
1.32	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	
1.33	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	
1.34	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	
1.35	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	
1.36	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	
1.37	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	
1.38	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	
1.39	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	
1.40	0.1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	
1.41	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	
1.42	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	
1.43	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	
1.44	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	
1.45	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	
1.46	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	
1.47	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	
1.48	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	
1.49	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	

Moving the decimal point n places to the right [or left] in the number requires adding $+n$ [or $-n$] in the body of the table (see p. 42).

COMMON LOGARITHMS (*special table, continued*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.50	0.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	3
1.51	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	
1.52	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	
1.53	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	
1.54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	
1.55	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	
1.56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	
1.57	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	
1.58	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	
1.59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	
1.60	0.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	
1.61	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	
1.62	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	
1.63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	
1.64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	
1.65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	
1.66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	
1.67	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	
1.68	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	
1.69	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	
1.70	0.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	
1.71	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	
1.72	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	
1.73	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	
1.74	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2
1.75	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	
1.76	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	
1.77	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	
1.78	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	
1.79	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	
1.80	0.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	
1.81	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	
1.82	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	
1.83	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	
1.84	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	
1.85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	
1.86	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	
1.87	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	
1.88	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	
1.89	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	
1.90	0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	
1.91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	
1.92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	
1.93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	
1.94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	
1.95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	
1.96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	
1.97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	
1.98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	
1.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	

COMMON LOGARITHMS

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	See pages 40–41
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	17
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	10
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9

$\log \pi = 0.4971$ $\log \pi/2 = 0.1961$ $\log \pi^2 = 0.9943$ $\log \sqrt{\pi} = 0.2486$
$\log e = 0.4343$ $\log (0.4343) = 0.6378 - 1$

These two pages give the common logarithms of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right [or left] in the number is equivalent to adding n [or $-n$] to the logarithm. Thus, $\log 0.017453 = 0.2419 - 2$, which may also be written $\bar{2}.2419$ or $8.2419 - 10$. See p. 91. Graphs, p. 174.

$$\log (ab) = \log a + \log b \qquad \log (a^N) = N \log a$$

$$\log \left(\frac{a}{b}\right) = \log a - \log b \qquad \log \left(\sqrt[N]{a}\right) = \frac{1}{N} \log a$$

COMMON LOGARITHMS (*continued*)

Number	0	1	2	3	4	5	6	7	8	9
5.0	0.6990	6998	7007	7016	7024	7033	7042	7050	7059	706
5.1	7076	7084	7093	7101	7110	7118	7126	7135	7143	715
5.2	7160	7168	7177	7185	7193	7202	7210	7218	7226	723
5.3	7243	7251	7259	7267	7275	7284	7292	7300	7308	731
5.4	7324	7332	7340	7348	7356	7364	7372	7380	7388	739
5.5	7404	7412	7419	7427	7435	7443	7451	7459	7466	747
5.6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
5.7	7559	7566	7574	7582	7589	7597	7604	7612	7619	762
5.8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
5.9	7709	7716	7723	7731	7738	7745	7752	7760	7767	777
6.0	0.7782	7789	7796	7803	7810	7818	7825	7832	7839	784
6.1	7853	7860	7868	7875	7882	7889	7896	7903	7910	791
6.2	7924	7931	7938	7945	7952	7959	7966	7973	7980	798
6.3	7993	8000	8007	8014	8021	8028	8035	8041	8048	805
6.4	8062	8069	8075	8082	8089	8096	8102	8109	8116	812
6.5	8129	8136	8142	8149	8156	8162	8169	8176	8182	818
6.6	8195	8202	8209	8215	8222	8228	8235	8241	8248	825
6.7	8261	8267	8274	8280	8287	8293	8299	8306	8312	831
6.8	8325	8331	8338	8344	8351	8357	8363	8370	8376	838
6.9	8388	8395	8401	8407	8414	8420	8426	8432	8439	844
7.0	0.8451	8457	8463	8470	8476	8482	8488	8494	8500	850
7.1	8513	8519	8525	8531	8537	8543	8549	8555	8561	856
7.2	8573	8579	8585	8591	8597	8603	8609	8615	8621	862
7.3	8633	8639	8645	8651	8657	8663	8669	8675	8681	868
7.4	8692	8698	8704	8710	8716	8722	8727	8733	8739	874
7.5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
7.6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
7.7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
7.8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
7.9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
8.0	0.9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
8.1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
8.2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
8.3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
8.4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
8.5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
8.6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
8.7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
8.8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
8.9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
9.0	0.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
9.1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
9.2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
9.3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
9.4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
9.5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
9.6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
9.7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
9.8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
9.9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

DEGREES AND MINUTES EXPRESSED IN RADIANS (See also p. 69)

Degrees						Hundredths				Minutes	
1°	.0175	61°	1.0647	121°	2.1118	0°.01	.0002	0°.51	.0089	1′	.0003
2	.0349	2	1.0821	2	2.1293	2	.0003	2	.0091	2′	.0006
3	.0524	3	1.0996	3	2.1468	3	.0005	3	.0093	3′	.0009
4	.0698	4	1.1170	4	2.1642	4	.0007	4	.0094	4′	.0012
5°	.0873	65°	1.1345	125°	2.1817	.05	.0009	.55	.0096	5′	.0015
6	.1047	6	1.1519	6	2.1991	6	.0010	6	.0098	6′	.0017
7	.1222	7	1.1694	7	2.2166	7	.0012	7	.0099	7′	.0020
8	.1396	8	1.1868	8	2.2340	8	.0014	8	.0101	8′	.0023
9	.1571	9	1.2043	9	2.2515	9	.0016	9	.0103	9′	.0026
10°	.1745	70°	1.2217	130°	2.2689	0°.10	.0017	0°.60	.0105	10′	.0029
1	.1920	1	1.2392	1	2.2864	1	.0019	1	.0106	11′	.0032
2	.2094	2	1.2566	2	2.3038	2	.0021	2	.0108	12′	.0035
3	.2269	3	1.2741	3	2.3213	3	.0023	3	.0110	13′	.0038
4	.2443	4	1.2915	4	2.3387	4	.0024	4	.0112	14′	.0041
15°	.2618	75°	1.3098	135°	2.3562	.15	.0026	.65	.0113	15′	.0044
6	.2793	6	1.3265	6	2.3736	6	.0028	6	.0115	16′	.0047
7	.2967	7	1.3439	7	2.3911	7	.0030	7	.0117	17′	.0049
8	.3142	8	1.3614	8	2.4086	8	.0031	8	.0119	18′	.0052
9	.3316	9	1.3788	9	2.4260	9	.0033	9	.0120	19′	.0055
20°	.3491	80°	1.3963	140°	2.4435	0°.20	.0035	0°.70	.0122	20′	.0058
1	.3665	1	1.4137	1	2.4609	1	.0037	1	.0124	21′	.0061
2	.3840	2	1.4312	2	2.4784	2	.0038	2	.0126	22′	.0064
3	.4014	3	1.4486	3	2.4958	3	.0040	3	.0127	23′	.0067
4	.4189	4	1.4661	4	2.5133	4	.0042	4	.0129	24′	.0070
25°	.4363	85°	1.4835	145°	2.5307	.25	.0044	.75	.0131	25′	.0073
6	.4538	6	1.5010	6	2.5482	6	.0045	6	.0133	26′	.0076
7	.4712	7	1.5184	7	2.5656	7	.0047	7	.0134	27′	.0079
8	.4887	8	1.5359	8	2.5831	8	.0049	8	.0136	28′	.0081
9	.5061	9	1.5533	9	2.6005	9	.0051	9	.0138	29′	.0084
30°	.5236	90°	1.5708	150°	2.6180	0°.30	.0052	0°.80	.0140	30′	.0087
1	.5411	1	1.5882	1	2.6354	1	.0054	1	.0141	31′	.0090
2	.5585	2	1.6057	2	2.6529	2	.0056	2	.0143	32′	.0093
3	.5760	3	1.6232	3	2.6704	3	.0058	3	.0145	33′	.0096
4	.5934	4	1.6406	4	2.6878	4	.0059	4	.0147	34′	.0099
35°	.6109	95°	1.6581	155°	2.7053	.35	.0061	.85	.0148	35′	.0102
6	.6283	6	1.6755	6	2.7227	6	.0063	6	.0150	36′	.0105
7	.6458	7	1.6938	7	2.7402	7	.0065	7	.0152	37′	.0108
8	.6632	8	1.7104	8	2.7576	8	.0066	8	.0154	38′	.0111
9	.6807	9	1.7279	9	2.7751	9	.0068	9	.0155	39′	.0113
40°	.6981	100°	1.7453	160°	2.7925	0°.40	.0070	0°.90	.0157	40′	.0116
1	.7156	1	1.7628	1	2.8100	1	.0072	1	.0159	41′	.0119
2	.7330	2	1.7802	2	2.8274	2	.0073	2	.0161	42′	.0122
3	.7505	3	1.7977	3	2.8449	3	.0075	3	.0162	43′	.0125
4	.7679	4	1.8151	4	2.8623	4	.0077	4	.0164	44′	.0128
45°	.7854	105°	1.8326	165°	2.8798	.45	.0079	.95	.0166	45′	.0131
6	.8029	6	1.8500	6	2.8972	6	.0080	6	.0168	46′	.0134
7	.8203	7	1.8675	7	2.9147	7	.0082	7	.0169	47′	.0137
8	.8378	8	1.8858	8	2.9322	8	.0084	8	.0171	48′	.0140
9	.8552	9	1.9024	9	2.9496	9	.0086	9	.0173	49′	.0143
50°	.8727	110°	1.9199	170°	2.9671	0°.50	.0087	1°.00	.0175	50′	.0145
1	.8901	1	1.9373	1	2.9845					51′	.0148
2	.9076	2	1.9548	2	3.0020					52′	.0151
3	.9250	3	1.9722	3	3.0194					53′	.0154
4	.9425	4	1.9897	4	3.0369					54′	.0157
55°	.9599	115°	2.0071	175°	3.0543					55′	.0160
6	.9774	6	2.0246	6	3.0718					56′	.0163
7	.9948	7	2.0420	7	3.0892					57′	.0166
8	1.0123	8	2.0595	8	3.1067					58′	.0169
9	1.0297	9	2.0769	9	3.1241					59′	.0172
60°	1.0472	120°	2.0944	180°	3.1416					60′	.0175

Arc 1° = 0.0174533 Arc 1′ = 0.000290888 Arc 1″ = 0.00000484814
1 radian = 57°.295780 = 57° 17′.7468 = 57° 17′ 44″.806

RADIANS EXPRESSED IN DEGREES

0.01	0°.57	.64	36°.67	1.27	72°.77	1.90	108°.86	2.53	144°.96
2	1°.15	.65	37°.24	8	73°.34	1	109°.43	4	145°.53
3	1°.72	6	37°.82	9	73°.91	2	110°.01	2.55	146°.10
4	2°.29	7	38°.39	1.30	74°.48	3	110°.58	6	146°.68
.05	2°.86	8	38°.96	1	75°.06	4	111°.15	7	147°.25
6	3°.44	9	39°.53	2	75°.63	1.95	111°.73	8	147°.82
7	4°.01	.70	40°.11	3	76°.20	6	112°.30	9	148°.40
8	4°.58	1	40°.68	4	76°.78	7	112°.87	2.60	148°.97
9	5°.16	2	41°.25	1.35	77°.35	8	113°.45	1	149°.54
.10	5°.73	3	41°.83	6	77°.92	9	114°.02	2	150°.11
1	6°.30	4	42°.40	7	78°.50	2.00	114°.59	3	150°.69
2	6°.88	.75	42°.97	8	79°.07	1	115°.16	4	151°.26
3	7°.45	6	43°.54	9	79°.64	2	115°.74	2.65	151°.83
4	8°.02	7	44°.12	1.40	80°.21	3	116°.31	6	152°.41
.15	8°.59	8	44°.69	1	80°.79	4	116°.88	7	152°.98
6	9°.17	9	45°.26	2	81°.36	2.05	117°.46	8	153°.55
7	9°.74	.80	45°.84	3	81°.93	6	118°.03	9	154°.13
8	10°.31	1	46°.41	4	82°.51	7	118°.60	2.70	154°.70
9	10°.89	2	46°.98	1.45	83°.08	8	119°.18	1	155°.27
.20	11°.46	3	47°.56	6	83°.65	9	119°.75	2	155°.84
1	12°.03	4	48°.13	7	84°.22	2.10	120°.32	3	156°.42
2	12°.61	.85	48°.70	8	84°.80	1	120°.89	4	156°.99
3	13°.18	6	49°.27	9	85°.37	2	121°.47	2.75	157°.56
4	13°.75	7	49°.85	1.50	85°.94	3	122°.04	6	158°.14
.25	14°.32	8	50°.42	1	86°.52	4	122°.61	7	158°.71
6	14°.90	9	50°.99	2	87°.09	2.15	123°.19	8	159°.28
7	15°.47	.90	51°.57	3	87°.66	6	123°.76	9	159°.86
8	16°.04	1	52°.14	4	88°.24	7	124°.33	2.80	160°.43
9	16°.62	2	52°.71	1.55	88°.81	8	124°.90	1	161°.00
.30	17°.19	3	53°.29	6	89°.38	9	125°.48	2	161°.57
1	17°.76	4	53°.86	7	89°.95	2.20	126°.05	3	162°.15
2	18°.33	.95	54°.43	8	90°.53	1	126°.62	4	162°.72
3	18°.91	6	55°.00	9	91°.10	2	127°.20	2.85	163°.29
4	19°.48	7	55°.58	1.60	91°.67	3	127°.77	6	163°.87
.35	20°.05	8	56°.15	1	92°.25	4	128°.34	7	164°.44
6	20°.63	9	56°.72	2	92°.82	2.25	128°.92	8	165°.01
7	21°.20	1.00	57°.30	3	93°.39	6	129°.49	9	165°.58
8	21°.77	1	57°.87	4	93°.97	7	130°.06	2.90	166°.16
9	22°.35	2	58°.44	1.65	94°.54	8	130°.63	1	166°.73
.40	22°.92	3	59°.01	6	95°.11	9	131°.21	2	167°.30
1	23°.49	4	59°.59	7	95°.68	2.30	131°.78	3	167°.88
2	24°.06	1.05	60°.16	8	96°.26	1	132°.35	4	168°.45
3	24°.64	6	60°.73	9	96°.83	2	132°.93	2.95	169°.02
4	25°.21	7	61°.31	1.70	97°.40	3	133°.50	6	169°.60
.45	25°.78	8	61°.88	1	97°.98	4	134°.07	7	170°.17
6	26°.36	9	62°.45	2	98°.55	2.35	134°.65	8	170°.74
7	26°.93	1.10	63°.03	3	99°.12	6	135°.22	9	171°.31
8	27°.50	1	63°.60	4	99°.69	7	135°.79	3.00	171°.89
9	28°.07	2	64°.17	1.75	100°.27	8	136°.36	1	172°.46
.50	28°.65	3	64°.74	6	100°.84	9	136°.94	2	173°.03
1	29°.22	4	65°.32	7	101°.41	2.40	137°.51	3	173°.61
2	29°.79	1.15	65°.89	8	101°.99	1	138°.08	4	174°.18
3	30°.37	6	66°.46	9	102°.56	2	138°.66	3.05	174°.75
4	30°.94	7	67°.04	1.80	103°.13	3	139°.23	6	175°.33
.55	31°.51	8	67°.61	1	103°.71	4	139°.80	7	175°.90
6	32°.09	9	68°.18	2	104°.28	2.45	140°.37	8	176°.47
7	32°.66	1.20	68°.75	3	104°.85	6	140°.95	9	177°.04
8	33°.23	1	69°.33	4	105°.42	7	141°.52	3.10	177°.62
9	33°.80	2	69°.90	1.85	106°.00	8	142°.09	1	178°.19
.60	34°.38	3	70°.47	6	106°.57	9	142°.67	2	178°.76
1	34°.95	4	71°.05	7	107°.14	2.50	143°.24	3	179°.34
2	35°.52	1.25	71°.62	8	107°.72	1	143°.81	4	179°.91
3	36°.10	6	72°.19	9	108°.29	2	144°.39	3.15	180°.48

Interpolation

.0002	0°.01
04	.02
06	.03
08	.05
.0010	0°.06
12	.07
14	.08
16	.09
18	.10
.0020	0°.11
22	.13
24	.14
26	.15
28	.16
.0030	0°.17
32	.18
34	.19
36	.21
38	.22
.0040	0°.23
42	.24
44	.25
46	.26
48	.28
.0050	0°.29
52	.30
54	.31
56	.32
58	.33
.0060	0°.34
62	.36
64	.37
66	.38
68	.39
.0070	0°.40
72	.41
74	.42
76	.44
78	.45
.0080	0°.46
82	.47
84	.48
86	.49
88	.50
.0090	0°.52
92	.53
94	.54
96	.55
98	.56

Multiples of π

1	3.1416	180°
2	6.2832	360°
3	9.4248	540°
4	12.5664	720°
5	15.7080	900°
6	18.8496	1080°
7	21.9911	1260°
8	25.1327	1440°
9	28.2743	1620°
10	31.4159	1800°

NATURAL SINES AND COSINES

Natural Sines at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. dif.
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87	17
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86	17
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0.0872	85	17
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84	17
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83	17
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82	17
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	1564	81	17
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	0.1736	80°	17
10°	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79	17
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78	17
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77	17
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76	17
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	0.2588	75	17
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74	17
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73	17
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72	17
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71	17
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	0.3420	70°	16
20°	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69	16
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68	16
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3907	67	16
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	4067	66	16
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	0.4226	65	16
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64	16
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63	16
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62	16
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61	15
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	0.5000	60°	15
30°	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59	15
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58	15
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57	15
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56	15
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	0.5736	55	14
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54	14
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53	14
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52	14
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51	14
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	0.6428	50°	13
40°	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49	13
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48	13
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47	13
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46	13
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	0.7071	45°	12
45°	0.7071												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

(For graphs, see p. 174.)

Natural Cosines

NATURAL SINES AND COSINES (*continued*)

Natural Sines at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. dif.
											0.7071	45°	
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193	44	12
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314	43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431	42	12
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547	41	12
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	0.7660	40°	11
50°	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771	39	11
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880	38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986	37	11
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090	36	10
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	0.8192	35	10
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8771	8281	8290	34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387	33	10
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480	32	9
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572	31	9
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	0.8660	30°	9
60°	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746	29	9
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829	28	8
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910	27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988	26	8
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	0.9063	25	7
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135	24	7
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205	23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272	22	7
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336	21	6
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	0.9397	20°	6
70°	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455	19	6
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511	18	6
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563	17	5
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613	16	5
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0.9659	15	5
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703	14	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744	13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781	12	4
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816	11	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0.9848	10°	3
80°	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877	9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903	8	3
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925	7	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	9945	6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0.9962	5	2
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976	4	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986	3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994	2	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0.9998	1	0
89	0.9998	9999	9999	9999	9999	0000	0000	0000	0000	0000	1.0000	0°	0
90°	1.0000												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural Cosines

NATURAL TANGENTS AND COTANGENTS

Natural Tangents at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524	87	17
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699	86	18
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0.0875	85	18
5	0.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051	84	18
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228	83	18
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405	82	18
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584	81	18
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	0.1763	80°	18
10°	0.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944	79	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126	78	18
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309	77	18
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493	76	18
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	0.2679	75	19
15	0.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	2867	74	19
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057	73	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249	72	19
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443	71	19
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	0.3640	70°	20
20°	0.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839	69	20
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040	68	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245	67	21
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452	66	21
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	0.4663	65	21
25	0.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4877	64	21
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095	63	22
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317	62	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543	61	23
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	0.5774	60°	23
30°	0.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6009	59	24
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249	58	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494	57	25
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745	56	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	0.7002	55	26
35	0.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265	54	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536	53	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813	52	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098	51	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	0.8391	50°	29
40°	0.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693	49	30
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	9004	48	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9325	47	32
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	0.9657	46	33
44	0.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1.0000	45°	34
45°	1.0000												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

(For graphs, see p. 174.) **Natural Cotangents**

NATURAL TANGENTS AND COTANGENTS (*continued*)

Natural Tangents at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.0000	45°	
45°	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	0355	44	35
46	0355	0392	0428	0464	0501	0530	0575	0612	.0649	0686	0724	43	37
47	0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	1106	42	38
48	1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	1504	41	40
49	1504	1544	1585	1626	1667	1709	1750	1792	1833	1875	1.1918	40°	41
50°	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	2349	39	43
51	2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	2799	38	45
52	2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	3270	37	47
53	3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	3764	36	49
54	3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	1.4281	35	52
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	4826	34	55
56	4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	5399	33	57
57	5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	6003	32	60
58	6003	6066	6128	6191	6255	6319	6383.	6447	6512	6577	6643	31	64
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	1.7321	30°	67
60°	1.732	1.739	1.746	1.753	1.760	1.767	1.775	1.782	1.789	1.797	1.804	29	7
61	1.804	1.811	1.819	1.827	1.834	1.842	1.849	1.857	1.865	1.873	1.881	28	8
62	1.881	1.889	1.897	1.905	1.913	1.921	1.929	1.937	1.946	1.954	1.963	27	8
63	1.963	1.971	1.980	1.988	1.997	2.006	2.014	2.023	2.032	2.041	2.050	26	9
64	2.050	2.059	2.069	2.078	2.087	2.097	2.106	2.116	2.125	2.135	2.145	25	9
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	2.246	24	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	2.356	23	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	2.475	22	12
68	2.475	2.488	2 500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	2.605	21	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	2.747	20°	14
70°	2.747	2.762	2.778	2.793	2.888	2.824	2.840	2.856	2.872	2.888	2.904	19	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.860	3.078	18	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.251	3.271	17	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	3.487	16	22
74	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	3.732	15	24
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	4.011	14	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	4.331	13	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	4.705	12	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	5.145	11	44
79	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	5.671	10°	53
80°	5.671	5.730	5.789	5.850	5.912	5.976	6.041	6.107	6.174	6.243	6.314	9	
81	6.314	6.386	6.460	6.535	6.612	6.691	6.772	6 855	6.940	7.026	7.115	8	
82	7.115	7.207	7.300	7.396	7.495	7.596	7.700	7.806	7.916	8.028	8.144	7	
83	8.144	8.264	8.386	8.513	8.643	8.777	8.915	9.058	9.205	9.357	9.514	6	
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	11.43	5	
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	14.30	4	
86	14.30	14.67	15.06	15.46	15.90	16.35	16.83	17.34	17.89	18.46	19.08	3	
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	28.64	2	
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	57.29	1	
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural Cotangents

NATURAL SECANTS AND COSECANTS

Natural Secants at intervals of 0°.1, or 6′.

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.0000	90°	
0°	1.0000	0000	0000	0000	0000	0000	0001	0001	0001	0001	0002	89	0
1	0002	0002	0002	0003	0003	0003	0004	0004	0005	0006	0006	88	0
2	0006	0007	0007	0008	0009	0010	0010	0011	0012	0013	0014	87	1
3	0014	0015	0016	0017	0018	0019	0020	0021	0022	0023	0024	86	1
4	0024	0026	0027	0028	0030	0031	0032	0034	0035	0037	1.0038	85	1
5	1.0038	0040	0041	0043	0045	0046	0048	0050	0051	0053	0055	84	2
6	0055	0057	0059	0061	0063	0065	0067	0069	0071	0073	0075	83	2
7	0075	0077	0079	0082	0084	0086	0089	0091	0093	0096	0098	82	2
8	0098	0101	0103	0106	0108	0111	0114	0116	0119	0122	0125	81	3
9	0125	0127	0130	0133	0136	0139	0142	0145	0148	0151	1.0154	80°	3
10°	1.0154	0157	0161	0164	0167	0170	0174	0177	0180	0184	0187	79	3
11	0187	0191	0194	0198	0201	0205	0209	0212	0216	0220	0223	78	4
12	0223	0227	0231	0235	0239	0243	0247	0251	0255	0259	0263	77	4
13	0263	0267	0271	0276	0280	0284	0288	0293	1297	0302	0306	76	4
14	0306	0311	0315	0320	0324	0329	0334	0338	0343	0348	1.0353	75	5
15	1.0353	0358	0363	0367	0372	0377	0382	0388	0393	0398	0403	74	5
16	0403	0408	0413	0419	0424	0429	0435	0440	0446	0451	0457	73	5
17	0457	0463	0468	0474	0480	0485	0491	0497	0503	0509	0515	72	6
18	0515	0521	0527	0533	0539	0545	0551	0557	0564	0570	0576	71	6
19	0576	0583	0589	0595	0602	0608	0615	0622	0628	0635	1.0642	70°	7
20°	1.0642	0649	0655	0662	0669	0676	0683	0690	0697	0704	0711	69	7
21	0711	0719	0726	0733	0740	0748	0255	0763	0770	0778	0785	68	7
22	0785	0793	0801	0808	0816	0824	0832	0840	0848	0856	0864	67	8
23	0864	0872	0880	0888	0896	0904	0913	0921	0929	0938	0946	66	8
24	0946	0955	0963	0972	0981	0989	0998	1007	1016	1025	1.1034	65	9
25	1.1034	1043	1052	1061	1070	1079	1089	1098	1107	1117	1126	64	9
26	1126	1136	1145	1155	1164	1174	1184	1194	1203	1213	1123	63	10
27	1223	1233	1243	1253	1264	1274	1284	1294	1305	1315	1326	62	10
28	1326	1336	1347	1357	1368	1379	1390	1401	1412	1423	1434	61	11
29	1434	1445	1456	1467	1478	1490	1501	1512	1524	1535	1.1547	60°	11
30°	1.1547	1559	1570	1582	1594	1606	1618	1630	1642	1654	1666	59	12
31	1666	1679	1691	1703	1716	1728	1741	1753	1766	1779	1792	58	13
32	1792	1805	1818	1831	1844	1857	1870	1883	1897	1910	1924	57	13
33	1924	1937	1951	1964	1978	1992	2006	2020	2034	2048	2062	56	14
34	2062	2076	2091	2105	2120	2134	2149	2163	2178	2193	1.2208	55	15
35	1.2208	2223	2238	2253	2268	2283	2299	2314	2329	2345	2361	54	15
36	2361	2376	2392	2408	2424	2440	2456	2472	2489	2505	2521	53	16
37	2521	2538	2554	2571	2588	2605	2622	2639	2656	2673	2690	52	17
38	2690	2708	2725	2742	2760	2778	2796	2813	2831	2849	2868	51	18
39	2868	2886	2904	2923	2941	2960	2978	2997	3016	3035	1.3054	60°	19
40°	1.3054	3073	3093	3112	3131	3151	3171	3190	3210	3230	3250	49	20
41	3250	3270	3291	3311	3331	3352	3373	3393	3414	3435	3456	48	21
42	3456	3478	3499	3520	3542	3563	3585	3607	3629	3651	3673	47	22
43	3673	3696	3718	3741	3763	3786	3809	3832	3855	3878	3902	46	23
44	3902	3925	2949	3972	3996	4020	4044	4069	4093	4118	1.4142	45°	24
45°	1.4142												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

(For graphs, see p. 174.)

Natural Cosecants

NATURAL SECANTS AND COSECANTS *(continued)*
Natural Secants at intervals of 0°.1, or 6′.

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff
											1.4142	45°	
45°	1.4142	4167	4192	4217	4242	4267	4293	4318	4344	4370	4396	44	25
46	4396	4422	4448	4474	4501	4527	4554	4581	4608	4635	4663	43	27
47	4663	4690	4718	4746	4774	4802	4830	4859	4887	4916	4945	42	28
48	4945	4974	5003	5032	5062	5092	5121	5151	5182	5212	5243	41	30
49	5243	5273	5304	5335	5366	5398	5429	5461	5493	5525	1.5557	40°	31
50°	1.5557	5590	5622	5655	5688	5721	5755	5788	5822	5856	5890	39	33
51	5890	5925	5959	5994	6029	6064	6099	6135	6171	6207	6243	38	35
52	6243	6279	6316	6353	6390	6427	6464	6502	6540	6578	6616	37	37
53	6616	6655	6694	6733	6772	6812	6852	6892	6932	6972	7013	36	40
54	7013	7054	7095	7137	7179	7221	7263	7305	7348	7391	1.7434	35	42
55	1.7434	7478	7522	7566	7610	7655	7700	7745	7791	7837	7883	34	45
56	7883	7929	7976	8023	8070	8118	8166	8214	8263	8312	8361	33	48
57	8361	8410	8460	8510	8561	8612	8663	8714	8766	8818	8871	32	51
58	8871	8924	8977	9031	9084	9139	9194	9249	9304	9360	1.9416	31	54
59	1.9416	9473	9530	9587	9645	9703	9762	9821	9880	9940	2.0000	30°	58
60°	2.000	2.006	2.012	2.018	2.025	2.031	2.037	2.043	2.050	2.056	2.063	29	6
61	2.063	2.069	2.076	2.082	2.089	2.096	2.103	2.109	2.116	2.123	2.130	28	7
62	2.130	2.137	2.144	2.151	2.158	2.166	2.173	2.180	2.188	2.195	2.203	27	7
63	2.203	2.210	2.218	2.226	2.233	2.241	2.249	2.257	2.265	2.273	2.281	26	8
64	2.281	2.289	2.298	2.306	2.314	2.323	2.331	2.340	2.349	2.357	2.366	25	8
65	2.366	2.375	2.384	2.393	2.402	2.411	2.421	2.430	2.439	2.449	2.459	24	9
66	2.459	2.468	2.478	2.488	2.498	2.508	2.518	2.528	2.538	2.549	2.559	23	10
67	2.559	2.570	2.581	2.591	2.602	2.613	2.624	2.635	2.647	2.658	2.669	22	11
68	2.669	2.681	2.693	2.705	2.716	2.729	2.741	2.753	2.765	2.778	2.790	21	12
69	2.790	2.803	2.816	2.829	2.842	2.855	2.869	2.882	2.896	2.910	2.924	20°	13
70°	2.924	2.938	2.952	2.967	2.981	2.996	3.011	3.026	3.041	3.056	3.072	19	15
71	3.072	3.087	3.103	3.119	3.135	3.152	3.168	3.185	3.202	3.219	3.236	18	16
72	3.236	3.254	3.271	3.289	3.307	3.326	3.344	3.363	3.382	3.401	3.420	17	18
73	3.420	3.440	3.460	3.480	3.500	3.521	3.542	3.563	3.584	3.606	3.628	16	21
74	3.628	3.650	3.673	3.695	3.719	3.742	3.766	3.790	3.814	3.839	3.864	15	24
75	3.864	3.889	3.915	3.941	3.967	3.994	4.021	4.049	4.077	4.105	4.134	14	27
76	4.134	4.163	4.192	4.222	4.253	4.284	4.315	4.347	4.379	4.412	4.445	13	31
77	4.445	4.479	4.514	4.549	4.584	4.620	4.657	4.694	4.732	4.771	4.810	12	36
78	4.810	4.850	4.890	4.931	4.973	5.016	5.059	5.103	5.148	5.194	5.241	11	43
79	5.241	5.288	5.337	5.386	5.436	5.487	5.540	5.593	5.647	5.702	5.759	10°	52
80°	5.759	5.816	5.875	5.935	5.996	6.059	6.123	6.188	6.255	6.323	6.392	9	
81	6.392	6.464	6.537	6.611	6.687	6.765	6.845	6.927	7.011	7.097	7.185	8	
82	7.185	7.276	7.368	7.463	7.561	7.661	7.764	7.870	7.979	8.091	8.206	7	
83	8.206	8.324	8.446	8.571	8.700	8.834	8.971	9.113	9.259	9.411	9.567	6	
84	9.567	9.728	9.895	10.07	10.25	10.43	10.63	10.83	11.03	11.25	11.47	5	
85	11.47	11.71	11.95	12.20	12.47	12.75	13.03	13.34	13.65	13.99	14.34	4	
86	14.34	14.70	15.09	15.50	15.93	16.38	16.86	17.37	17.91	18.49	19.11	3	
87	19.11	19.77	20.47	21.23	22.04	22.93	23.88	24.92	26.05	27.29	28.65	2	
88	28.65	30.16	31.84	33.71	35.81	38.20	40.93	44.08	47.75	52.09	57.30	1	
89	57.30	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural Cosecants

TRIGONOMETRIC FUNCTIONS (at intervals of 10′)

Annex −10 in columns marked*. (For 0.°1 intervals, see pp. 46–51)

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
0° 00′	0.0000	.0000	∞	1.0000	0.0000	.0000	∞	∞	∞	1.5708	90° 00′
10	0.0029	.0029	7.4637	1.0000	.0000	.0029	7.4637	343.77	2.5363	1.5679	50
20	0.0058	.0058	.7648	1.0000	.0000	.0058	.7648	171.89	.2352	1.5650	40
30	0.0087	.0087	.9408	1.0000	.0000	.0087	.9409	114.59	.0591	1.5621	30
40	0.0116	.0116	8.0658	.9999	.0000	.0116	8.0658	85.940	1.9342	1.5592	20
50	0.0145	.0145	.1627	.9999	.0000	.0145	.1627	68.750	.8373	1.5563	10
1° 00′	0.0175	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	1.5533	89° 00′
10	0.0204	.0204	.3088	.9998	.9999	.0204	.3089	49.104	.6911	1.5504	50
20	0.0233	.0233	.3668	.9997	.9999	.0233	.3669	42.964	.6331	1.5475	40
30	0.0262	.0262	.4179	.9997	.9999	.0262	.4181	38.188	.5819	1.5446	30
40	0.0291	.0291	.4637	.9996	.9998	.0291	.4638	34.368	.5362	1.5417	20
50	0.0320	.0320	.5050	.9995	.9998	.0320	.5053	31.242	.4947	1.5388	10
2° 00′	0.0349	.0349	8.5428	.9994	9.9997	.0349	8.5431	28.636	1.4569	1.5359	88° 00′
10	0.0378	.0378	.5776	.9993	.9997	.0378	.5779	26.432	.4221	1.5330	50
20	0.0407	.0407	.6097	.9992	.9996	.0407	.6101	24.542	.3899	1.5301	40
30	0.0436	.0436	.6397	.9990	.9996	.0437	.6401	22.904	.3599	1.5272	30
40	0.0465	.0465	.6677	.9989	.9995	.0466	.6682	21.470	.3318	1.5243	20
50	0.0495	.0494	.6940	.9988	.9995	.0495	.6945	20.206	.3055	1.5213	10
3° 00′	0.0524	.0523	8.7188	.9986	9.9994	.0524	8.7194	19.081	1.2806	1.5184	87° 00′
10	0.0553	.0552	.7423	.9985	.9993	.0553	.7429	18.075	.2571	1.5155	50
20	0.0582	.0581	.7645	.9983	.9993	.0582	.7652	17.169	.2348	1.5126	40
30	0.0611	.0610	.7857	.9981	.9992	.0612	.7865	16.350	.2135	1.5097	30
40	0.0640	.0640	.8059	.9980	.9991	.0641	.8067	15.605	.1933	1.5068	20
50	0.0669	.0669	.8251	.9978	.9990	.0670	.8261	14.924	.1739	1.5039	10
4° 00′	0.0698	.0698	8.8436	.9976	9.9989	.0699	8.8446	14.301	1.1554	1.5010	86° 00′
10	0.0727	.0727	.8613	.9974	.9989	.0729	.8624	13.727	.1376	1.4981	50
20	0.0756	.0756	.8783	.9971	.9988	.0758	.8795	13.197	.1205	1.4952	40
30	0.0785	.0785	.8946	.9969	.9987	.0787	.8960	12.706	.1040	1.4923	30
40	0.0814	.0814	.9104	.9967	.9986	.0816	.9118	12.251	.0882	1.4893	20
50	0.0844	.0843	.9256	.9964	.9985	.0846	.9272	11.826	.0728	1.4864	10
5° 00′	0.0873	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	1.4835	85° 00′
10	0.0902	.0901	.9545	.9959	.9982	.0904	.9563	11.059	.0437	1.4806	50
20	0.0931	.0929	.9682	.9957	.9981	.0934	.9701	10.712	.0299	1.4777	40
30	0.0960	.0958	.9816	.9954	.9980	.0963	.9836	10.385	.0164	1.4748	30
40	0.0989	.0987	.9945	.9951	.9979	.0992	.9966	10.078	.0034	1.4719	20
50	0.1018	.1016	9.0070	.9948	.9977	.1022	9.0093	9.7882	0.9907	1.4690	10
6° 00′	0.1047	.1045	9.0192	.9945	9.9976	.1051	9.0216	9.5144	0.9784	1.4661	84° 00′
10	0.1076	.1074	.0311	.9942	.9975	.1080	.0336	9.2553	.9664	1.4632	50
20	0.1105	.1103	.0426	.9939	.9973	.1110	.0453	9.0098	.9547	1.4603	40
30	0.1134	.1132	.0539	.9936	.9972	.1139	.0567	8.7769	.9433	1.4574	30
40	0.1164	.1161	.0648	.9932	.9971	.1169	.0678	8.5555	.9322	1.4544	20
50	0.1193	.1190	.0755	.9929	.9969	.1198	.0786	8.3450	.9214	1.4515	10
7° 00′	0.1222	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	1.4486	83° 00′
10	0.1251	.1248	.0961	.9922	.9966	.1257	.0995	7.9530	.9005	1.4457	50
20	0.1280	.1276	.1060	.9918	.9964	.1287	.1096	7.7704	.8904	1.4428	40
30	0.1309	.1305	.1157	.9914	.9963	.1317	.1194	7.5958	.8806	1.4399	30
40	0.1338	.1334	.1252	.9911	.9961	.1346	.1291	7.4287	.8709	1.4370	20
50	0.1367	.1363	.1345	.9907	.9959	.1376	.1385	7.2687	.8615	1.4341	10
8° 00′	0.1396	.1392	9.1436	.9903	9.9958	.1405	9.1478	7.1154	0.8522	1.4312	82° 00′
10	0.1425	.1421	.1525	.9899	.9956	.1435	.1569	6.9682	.8431	1.4283	50
20	0.1454	.1449	.1612	.9894	.9954	.1465	.1658	6.8269	.8342	1.4254	40
30	0.1484	.1478	.1697	.9890	.9952	.1495	.1745	6.6912	.8255	1.4224	30
40	0.1513	.1507	.1781	.9886	.9950	.1524	.1831	6.5606	.8169	1.4195	20
50	0.1542	.1536	.1863	.9881	.9948	.1554	.1915	6.4348	.8085	1.4166	10
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS (*continued*)

Annex −10 in columns marked*. (For 0.°1 intervals, see pp. 46–51)

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
10	0.1600	.1593	.2022	.9872	.9944	.1614	.2078	6.1970	.7922	1.4108	50
20	0.1629	.1622	.2100	.9868	.9942	.1644	.2158	6.0844	.7842	1.4079	40
30	0.1658	.1650	.2176	.9863	.9940	.1673	.2236	5.9758	.7764	1.4050	30
40	0.1687	.1679	.2251	.9858	.9938	.1703	.2313	5.8708	.7687	1.4021	20
50	0.1716	.1708	.2324	.9853	.9936	.1733	.2389	5.7694	.7611	1.3992	10
10° 00′	0.1745	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	1.3963	80° 00′
10	0.1774	.1765	.2468	.9843	.9931	.1793	.2536	5.5764	.7464	1.3934	50
20	0.1804	.1794	.2538	.9838	.9929	.1823	.2609	5.4845	.7391	1.3904	40
30	0.1833	.1822	.2606	.9833	.9927	.1853	.2680	5.3955	.7320	1.3875	30
40	0.1862	.1851	.2674	.9827	.9924	.1883	.2750	5.3093	.7250	1.3846	20
50	0.1891	.1880	.2740	.9822	.9922	.1914	.2819	5.2257	.7181	1.3817	10
11° 00′	0.1920	.1908	9.2806	.9816	9.9919	.1944	9.2887	5.1446	0.7113	1.3788	79° 00′
10	0.1949	.1937	.2870	.9811	.9917	.1974	.2953	5.0658	.7047	1.3759	50
20	0.1978	.1965	.2934	.9805	.9914	.2004	.3020	4.9894	.6980	1.3730	40
30	0.2007	.1994	.2997	.9799	.9912	.2035	.3085	4.9152	.6915	1.3701	30
40	0.2036	.2022	.3058	.9793	.9909	.2065	.3149	4.8430	.6851	1.3672	20
50	0.2065	.2051	.3119	.9787	.9907	.2095	.3212	4.7729	.6788	1.3643	10
12° 00′	0.2094	.2079	9.3179	.9781	9.9904	.2126	9.3275	4.7046	0.6725	1.3614	78° 00′
10	0.2123	.2108	.3238	.9775	.9901	.2156	.3336	4.6382	.6664	1.3584	50
20	0.2153	.2136	.3296	.9769	.9899	.2186	.3397	4.5736	.6603	1.3555	40
30	0.2182	.2164	.3353	.9763	.9896	.2217	.3458	4.5107	.6542	1.3526	30
40	0.2211	.2193	.3410	.9757	.9893	.2247	.3517	4.4494	.6483	1.3497	20
50	0.2240	.2221	.3466	.9750	.9890	.2278	.3576	4.3897	.6424	1.3468	10
13° 00′	0.2269	.2250	9.3521	.9744	9.9887	.2309	9.3634	4.3315	0.6366	1.3439	77° 00′
10	0.2298	.2278	.3575	.9737	.9884	.2339	.3691	4.2747	.6309	1.3410	50
20	0.2327	.2306	.3629	.9730	.9881	.2370	.3748	4.2193	.6252	1.3381	40
30	0.2356	.2334	.3682	.9724	.9878	.2401	.3804	4.1653	.6196	1.3352	30
40	0.2385	.2363	.3734	.9717	.9875	.2432	.3859	4.1126	.6141	1.3323	20
50	0.2414	.2391	.3786	.9710	.9872	.2462	.3914	4.0611	.6086	1.3294	10
14° 00′	0.2443	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	1.3265	76° 00′
10	0.2473	.2447	.3887	.9696	.9866	.2524	.4021	3.9617	.5979	1.3235	50
20	0.2502	.2476	.3937	.9689	.9863	.2555	.4074	3.9136	.5926	1.3206	40
30	0.2531	.2504	.3986	.9681	.9859	.2586	.4127	3.8667	.5873	1.3177	30
40	0.2560	.2532	.4035	.9674	.9856	.2617	.4178	3.8208	.5822	1.3148	20
50	0.2589	.2560	.4083	.9667	.9853	.2648	.4230	3.7760	.5770	1.3119	10
15° 00′	0.2618	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.5719	1.3090	75° 00′
10	0.2647	.2616	.4177	.9652	.9846	.2711	.4331	3.6891	.5669	1.3061	50
20	0.2676	.2644	.4223	.9644	.9843	.2742	.4381	3.6470	.5619	1.3032	40
30	0.2705	.2672	.4269	.9636	.9839	.2773	.4430	3.6059	.5570	1.3003	30
40	0.2734	.2700	.4314	.9628	.9836	.2805	.4479	3.5656	.5521	1.2974	20
50	0.2763	.2728	.4359	.9621	.9832	.2836	.4527	3.5261	.5473	1.2945	10
16° 00′	0.2793	.2756	9.4403	.9613	9.9828	.2867	9.4575	3.4874	0.5425	1.2915	74° 00′
10	0.2822	.2784	.4447	.9605	.9825	.2899	.4622	3.4495	.5378	1.2886	50
20	0.2851	.2812	.4491	.9596	.9821	.2931	.4669	3.4124	.5331	1.2857	40
30	0.2880	.2840	.4533	.9588	.9817	.2962	.4716	3.3759	.5284	1.2828	30
40	0.2909	.2868	.4576	.9580	.9814	.2994	.4762	3.3402	.5238	1.2799	20
50	0.2938	.2896	.4618	.9572	.9810	.3026	.4808	3.3052	.5192	1.2770	10
17° 00′	0.2967	.2924	9.4659	.9563	9.9806	.3057	9.4853	3.2709	0.5147	1.2741	73° 00′
10	0.2996	.2952	.4700	.9555	.9802	.3089	.4898	3.2371	.5102	1.2712	50
20	0.3025	.2979	.4741	.9546	.9798	.3121	.4943	3.2041	.5057	1.2683	40
30	0.3054	.3007	.4781	.9537	.9794	.3153	.4987	3.1716	.5013	1.2654	30
40	0.3083	.3035	.4821	.9528	.9790	.3185	.5031	3.1397	.4969	1.2625	20
50	0.3113	.3062	.4861	.9520	.9786	.3217	.5075	3.1084	.4925	1.2595	10
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS (continued)

Annex −10 in columns marked*. (For 0.°1 intervals, see pp. 46–51)

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
10	0.3171	.3118	.4939	.9502	.9778	.3281	.5161	3.0475	.4839	1.2537	50
20	0.3200	.3145	.4977	.9492	.9774	.3314	.5203	3.0178	.4797	1.2508	40
30	0.3229	.3173	.5015	.9483	.9770	.3346	.5245	2.9887	.4755	1.2479	30
40	0.3258	.3201	.5052	.9474	.9765	.3378	.5287	2.9600	.4713	1.2450	20
50	0.3287	.3228	.5090	.9465	.9761	.3411	.5329	2.9319	.4671	1.2421	10
19° 00′	0.3316	.3256	9.5126	.9455	9.9757	.3443	9.5370	2.9042	0.4630	1.2392	71° 00′
10	0.3345	.3283	.5163	.9446	.9752	.3476	.5411	2.8770	.4589	1.2363	50
20	0.3374	.3311	.5199	.9436	.9748	.3508	.5451	2.8502	.4549	1.2334	40
30	0.3403	.3338	.5235	.9426	.9743	.3541	.5491	2.8239	.4509	1.2305	30
40	0.3432	.3365	.5270	.9417	.9739	.3574	.5531	2.7980	.4469	1.2275	20
50	0.3462	.3393	.5306	.9407	.9734	.3607	.5571	2.7725	.4429	1.2246	10
20° 00′	0.3491	.3420	9.5341	.9397	9.9730	.3640	9.5611	2.7475	0.4389	1.2217	70° 00′
10	0.3520	.3448	.5375	.9387	.9725	.3673	.5650	2.7228	.4350	1.2188	50
20	0.3549	.3475	.5409	.9377	.9721	.3706	.5689	2.6985	.4311	1.2159	40
30	0.3578	.3502	.5443	.9367	.9716	.3739	.5727	2.6746	.4273	1.2130	30
40	0.3607	.3529	.5477	.9356	.9711	.3772	.5766	2.6511	.4234	1.2101	20
50	0.3636	.3557	.5510	.9346	.9706	.3805	.5804	2.6279	.4196	1.2072	10
21° 00′	0.3665	.3584	9.5543	.9336	9.9702	.3839	9.5842	2.6051	0.4158	1.2043	69° 00′
10	0.3694	.3611	.5576	.9325	.9697	.3872	.5879	2.5826	.4121	1.2014	50
20	0.3723	.3638	.5609	.9315	.9692	.3906	.5917	2.5605	.4083	1.1985	40
30	0.3752	.3665	.5641	.9304	.9687	.3939	.5954	2.5386	.4046	1.1956	30
40	0.3782	.3692	.5673	.9293	.9682	.3973	.5991	2.5172	.4009	1.1926	20
50	0.3811	.3719	.5704	.9283	.9677	.4006	.6028	2.4960	.3972	1.1897	10
22° 00′	0.3840	.3746	9.5736	.9272	9.9672	.4040	9.6064	2.4751	0.3936	1.1868	68° 00′
10	0.3869	.3773	.5767	.9261	.9667	.4074	.6100	2.4545	.3900	1.1839	50
20	0.3898	.3800	.5798	.9250	.9661	.4108	.6136	2.4342	.3864	1.1810	40
30	0.3927	.3827	.5828	.9239	.9656	.4142	.6172	2.4142	.3828	1.1781	30
40	0.3956	.3854	.5859	.9228	.9651	.4176	.6208	2.3945	.3792	1.1752	20
50	0.3985	.3881	.5889	.9216	.9646	.4210	.6243	2.3750	.3757	1.1723	10
23° 00′	0.4014	.3907	9.5919	.9205	9.9640	.4245	9.6279	2.3559	0.3721	1.1694	67° 00′
10	0.4043	.3934	.5948	.9194	.9635	.4279	.6314	2.3369	.3686	1.1665	50
20	0.4072	.3961	.5978	.9182	.9629	.4314	.6348	2.3183	.3652	1.1636	40
30	0.4102	.3987	.6007	.9171	.9624	.4348	.6383	2.2998	.3617	1.1606	30
30	0.4131	.4014	.6036	.9159	.9618	.4383	.6417	2.2817	.3583	1.1577	20
50	0.4160	.4041	.6065	.9147	.9613	.4417	.6452	2.2637	.3548	1.1548	10
24° 00′	0.4189	.4067	9.6093	.9135	9.9607	.4452	9.6486	2.2460	0.3514	1.1519	66° 00′
10	0.4218	.4094	.6121	.9124	.9602	.4487	.6520	2.2286	.3480	1.1490	50
20	0.4247	.4120	.6149	.9112	.9596	.4522	.6553	2.2113	.3447	1.1461	40
30	0.4276	.4147	.6177	.9100	.9590	.4557	.6567	2.1943	.3413	1.1432	30
40	0.4305	.4173	.6205	.9088	.9584	.4592	.6620	2.1775	.3380	1.1403	20
50	0.4334	.4200	.6232	.9075	.9579	.4628	.6654	2.1609	.3346	1.1374	10
25° 00′	0.4363	.4226	9.6259	.9063	9.9573	.4663	9.6687	2.1445	0.3313	1.1345	65° 00′
10	0.4392	.4253	.6286	.9051	.9567	.4699	.6720	2.1283	.3280	1.1316	50
20	0.4422	.4279	.6313	.9038	.9561	.4734	.6752	2.1123	.3248	1.1286	40
30	0.4451	.4305	.6340	.9026	.9555	.4770	.6785	2.0965	.3215	1.1257	30
40	0.4480	.4331	.6366	.9013	.9549	.4806	.6817	2.0809	.3183	1.1228	20
50	0.4509	.4358	.6392	.9001	.9543	.4841	.6850	2.0655	.3150	1.1199	10
26° 00′	0.4538	.4384	9.6418	.8988	9.9537	.4877	9.6882	2.0503	0.3118	1.1170	64° 00′
10	0.4567	.4410	.6444	.8975	.9530	.4913	.6914	2.0353	.3086	1.1141	50
20	0.4596	.4436	.6470	.8962	.9524	.4950	.6946	2.0204	.3054	1.1112	40
30	0.4625	.4462	.6495	.8949	.9518	.4986	.6977	2.0057	.3023	1.1083	30
40	0.4654	.4488	.6521	.8936	.9512	.5022	.7009	1.9912	.2991	1.1054	20
50	0.4683	.4514	.6546	.8923	.9505	.5059	.7040	1.9768	.2960	1.1025	10
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	63° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS *(continued)*

Annex −10 in columns marked*. (For 0°.1 intervals, see pp. 46–51)

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	63° 00′
10	0.4741	.4566	.6595	.8897	.9492	.5132	.7103	1.9486	.2897	1.0966	50
20	0.4771	.4592	.6620	.8884	.9486	.5169	.7134	1.9347	.2866	1.0937	40
30	0.4800	.4617	.6644	.8870	.9479	.5206	.7165	1.9210	.2835	1.0908	30
40	0.4829	.4643	.6668	.8857	.9473	.5243	.7196	1.9074	.2804	1.0879	20
50	0.4858	.4669	.6692	.8843	.9466	.5280	.7226	1.8940	.2774	1.0850	10
28° 00′	0.4887	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	1.0821	62° 00′
10	0.4916	.4720	.6740	.8816	.9453	.5354	.7287	1.8676	.2713	1.0792	50
20	0.4945	.4746	.6763	.8802	.9446	.5392	.7317	1.8546	.2683	1.0763	40
30	0.4974	.4772	.6787	.8788	.9439	.5430	.7348	1.8418	.2652	1.0734	30
40	0.5003	.4797	.6810	.8774	.9432	.5467	.7378	1.8291	.2622	1.0705	20
50	0.5032	.4823	.6833	.8760	.9425	.5505	.7408	1.8165	.2592	1.0676	10
29° 00′	0.5061	.4848	9.6856	.8746	9.9418	.5543	9.7438	1.8040	0.2562	1.0647	61° 00′
10	0.5091	.4874	.6878	.8732	.9411	.5581	.7467	1.7917	.2533	1.0617	50
20	0.5120	.4899	.6901	.8718	.9404	.5619	.7497	1.7796	.2503	1.0588	40
30	0.5149	.4924	.6923	.8704	.9397	.5658	.7526	1.7675	.2474	1.0559	30
40	0.5178	.4950	.6946	.8689	.9390	.5696	.7556	1.7556	.2444	1.0530	20
50	0.5207	.4975	.6968	.8675	.9383	.5735	.7585	1.7437	.2415	1.0501	10
30° 00′	0.5236	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	1.0472	60° 00′
10	0.5265	.5025	.7012	.8646	.9368	.5812	.7644	1.7205	.2356	1.0443	50
20	0.5294	.5050	.7033	.8631	.9361	.5851	.7673	1.7090	.2327	1.0414	40
30	0.5323	.5075	.7055	.8616	.9353	.5890	.7701	1.6977	.2299	1.0385	30
40	0.5352	.5100	.7076	.8601	.9346	.5930	.7730	1.6864	.2270	1.0356	20
50	0.5381	.5125	.7097	.8587	.9338	.5969	.7759	1.6753	.2241	1.0327	10
31° 00′	0.5411	.5150	9.7118	.8572	9.9331	.6009	9.7788	1.6643	0.2212	1.0297	59° 00′
10	0.5440	.5175	.7139	.8557	.9323	.6048	.7816	1.6534	.2184	1.0268	50
20	0.5469	.5200	.7160	.8542	.9315	.6088	.7845	1.6426	.2155	1.0239	40
30	0.5498	.5225	.7181	.8526	.9308	.6128	.7873	1.6319	.2127	1.0210	30
40	0.5527	.5250	.7201	.8511	.9300	.6168	.7902	1.6212	.2098	1.0181	20
50	0.5556	.5275	.7222	.8496	.9292	.6208	.7930	1.6107	.2070	1.0152	10
32° 00′	0.5585	.5299	9.7242	.8480	9.9284	.6249	9.7958	1.6003	0.2042	1.0123	58° 00′
10	0.5614	.5324	.7262	.8465	.9276	.6289	.7986	1.5900	.2014	1.0094	50
20	0.5643	.5348	.7282	.8450	.9268	.6330	.8014	1.5798	.1986	1.0065	40
30	0.5672	.5373	.7302	.8434	.9260	.6371	.8042	1.5697	.1958	1.0036	30
40	0.5701	.5398	.7322	.8418	.9252	.6412	.8070	1.5597	.1930	1.0007	20
50	0.5730	.5422	.7342	.8403	.9244	.6453	.8097	1.5497	.1903	0.9977	10
33° 00′	0.5760	.5446	9.7361	.8387	9.9236	.6494	9.8125	1.5399	0.1875	0.9948	57° 00′
10	0.5789	.5471	.7380	.8371	.9228	.6536	.8153	1.5301	.1847	0.9919	50
20	0.5818	.5495	.7400	.8355	.9219	.6577	.8180	1.5204	.1820	0.9890	40
30	0.5847	.5519	.7419	.8339	.9211	.6619	.8208	1.5108	.1792	0.9861	30
40	0.5876	.5544	.7438	.8323	.9203	.6661	.8235	1.5013	.1765	0.9832	20
50	0.5905	.5568	.7457	.8307	.9194	.6703	.8263	1.4919	.1737	0.9803	10
34° 00′	0.5934	.5592	9.7476	.8290	9.9186	.6745	9.8290	1.4826	0.1710	0.9774	56° 00′
10	0.5963	.5616	.7494	.8274	.9177	.6787	.8317	1.4733	.1683	0.9445	50
20	0.5992	.5640	.7513	.8258	.9169	.6830	.8344	1.4641	.1656	0.9716	40
30	0.6021	.5664	.7531	.8241	.9160	.6873	.8371	1.4550	.1629	0.9687	30
40	0.6050	.5688	.7550	.8225	.9151	.6916	.8398	1.4460	.1602	0.9657	20
50	0.6080	.5712	.7568	.8208	.9142	.6959	.8425	1.4370	.1575	0.9628	10
35° 00′	0.6109	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	0.9599	55° 00′
10	0.6138	.5760	.7604	.8175	.9125	.7046	.8479	1.4193	.1521	0.9570	50
20	0.6167	.5783	.7622	.8158	.9116	.7089	.8506	1.4106	.1494	0.9541	40
30	0.6196	.5807	.7640	.8141	.9107	.7133	.8533	1.4019	.1467	0.9512	30
40	0.6225	.5831	.7657	.8124	.9098	.7177	.8559	1.3934	.1441	0.9483	20
50	0.6254	.5854	.7675	.8107	.9089	.7221	.8586	1.3848	.1414	0.9454	10
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS *(continued)*

Annex −10 in columns marked*. (For 0°.1 intervals, see pp. 46–51)

Degrees	Radians	Sines Nat.	Sines Log.*	Cosines Nat.	Cosines Log.*	Tangents Nat.	Tangents Log.*	Cotangents Nat.	Cotangents Log.		
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
10	0.6312	.5901	.7710	.8073	.9070	.7310	.8639	1.3680	.1361	0.9396	50
20	0.6341	.5925	.7727	.8056	.9061	.7355	.8666	1.3597	.1334	0.9367	40
30	0.6370	.5948	.7744	.8039	.9052	.7400	.8692	1.3514	.1308	0.9338	30
40	0.6400	.5972	.7761	.8021	.9042	.7445	.8718	1.3432	.1282	0.9308	20
50	0.6429	.5995	.7778	.8004	.9033	.7490	.8745	1.3351	.1255	0.9279	10
37° 00′	0.6458	.6018	9.7795	.7986	9.9023	.7536	9.8771	1.3270	0.1229	0.9250	53° 00′
10	0.6487	.6041	.7811	.7969	.9014	.7581	.8797	1.3190	.1203	0.9221	50
20	0.6516	.6065	.7828	.7951	.9004	.7627	.8824	1.3111	.1176	0.9192	40
30	0.6545	.6088	.7844	.7934	.8995	.7673	.8850	1.3032	.1150	0.9163	30
40	0.6574	.6111	.7861	.7916	.8985	.7720	.8876	1.2954	.1124	0.9134	20
50	0.6603	.6134	.7877	.7898	.8975	.7766	.8902	1.2876	.1098	0.9105	10
38° 00′	0.6632	.6157	9.7893	.7880	9.8965	.7813	9.8928	1.2799	0.1072	0.9076	52° 00′
10	0.6661	.6180	.7910	.7862	.8955	.7860	.8954	1.2723	.1046	0.9047	50
20	0.6690	.6202	.7926	.7844	.8945	.7907	.8980	1.2647	.1020	0.9018	40
30	0.6720	.6225	.7941	.7826	.8935	.7954	.9006	1.2572	.0994	0.8988	30
40	0.6749	.6248	.7957	.7808	.8925	.8002	.9032	1.2497	.0968	0.8959	20
50	0.6778	.6271	.7973	.7790	.8915	.8050	.9058	1.2423	.0942	0.8930	10
39° 00′	0.6807	.6293	9.7989	.7771	9.8905	.8098	9.9084	1.2349	0.0916	0.8901	51° 00′
10	0.6836	.6316	.8004	.7753	.8895	.8146	.9110	1.2276	.0890	0.8872	50
20	0.6865	.6338	.8020	.7735	.8884	.8195	.9135	1.2203	.0865	0.8843	40
30	0.6894	.6361	.8035	.7716	.8874	.8243	.9161	1.2131	.0839	0.8814	30
40	0.6923	.6383	.8050	.7698	.8864	.8292	.9187	1.2059	.0813	0.8785	20
50	0.6952	.6406	.8066	.7679	.8853	.8342	.9212	1.1988	.0788	0.8756	10
40° 00′	0.6981	.6428	9.8081	.7660	9.8843	.8391	9.9238	1.1918	0.0762	0.8727	50° 00′
10	0.7010	.6450	.8096	.7642	.8832	.8441	.9264	1.1847	.0736	0.8698	50
20	0.7039	.6472	.8111	.7623	.8821	.8491	.9289	1.1778	.0711	0.8668	40
30	0.7069	.6494	.8125	.7604	.8810	.8541	.9315	1.1708	.0685	0.8639	30
40	0.7098	.6517	.8140	.7585	.8800	.8591	.9341	1.1640	.0659	0.8610	20
50	0.7127	.6539	.8155	.7566	.8789	.8642	.9366	1.1571	.0634	0.8581	10
41° 00′	0.7156	.6561	9.8169	.7547	9.8778	.8693	9.9392	1.1504	0.0608	0.8552	49° 00′
10	0.7185	.6583	.8184	.7528	.8767	.8744	.9417	1.1436	.0583	0.8523	50
20	0.7214	.6604	.8198	.7509	.8756	.8796	.9443	1.1369	.0557	0.8494	40
30	0.7243	.6626	.8213	.7490	.8745	.8847	.9468	1.1303	.0532	0.8465	30
40	0.7272	.6648	.8227	.7470	.8733	.8899	.9494	1.1237	.0506	0.8436	20
50	0.7301	.6670	.8241	.7451	.8722	.8952	.9519	1.1171	.0481	0.8407	10
42° 00′	0.7330	.6691	9.8255	.7431	9.8711	.9004	9.9544	1.1106	0.0456	0.8378	48° 00′
10	0.7359	.6713	.8269	.7412	.8699	.9057	.9570	1.1041	.0430	0.8348	50
20	0.7389	.6734	.8283	.7392	.8688	.9110	.9595	1.0977	.0405	0.8319	40
30	0.7418	.6756	.8297	.7373	.8676	.9163	.9621	1.0913	0,379	0.8290	30
40	0.7447	.6777	.8311	.7353	.8665	.9217	.9646	1.0850	.0354	0.8261	20
50	0.7476	.6799	.8324	.7333	.8653	.9271	.9671	1.0786	.0329	0.8232	10
43° 00′	0.7505	.6820	9.8338	.7314	9.8641	.9325	9.9697	1.0724	0.0303	0.8203	47° 00′
10	0.7534	.6641	.8351	.7294	.8629	.9380	.9722	1.0661	.0278	0.8174	50
20	0.7563	.6862	.8365	.7274	.8618	.9435	.9747	1.0599	.0253	0.8145	40
30	0.7592	.6884	.8378	.7254	.8606	.9490	.9772	1.0538	.0228	0.8116	30
40	0.7621	.6905	.8391	.7234	.8594	.9545	.9798	1.0477	.0202	0.8087	20
50	0.7650	.6926	.8405	.7214	.8582	.9601	.9823	1.0416	.0177	0.8058	10
44° 00′	0.7679	.6947	9.8418	.7193	9.8569	.9657	9.9848	1.0355	0.0152	0.8029	46° 00′
10	0.7709	.6967	.8431	.7173	.8557	.9713	.9874	1.0295	.0126	0.7999	50
20	0.7738	.6988	.8444	.7153	.8545	.9770	.9899	1.0235	.0101	0.7970	40
30	0.7767	.7009	.8457	.7133	.8532	.9827	.9924	1.0176	.0076	0.7941	30
40	0.7796	.7030	.8469	.7112	.8520	.9884	.9949	1.0117	.0051	0.7912	20
50	0.7825	.7050	.8482	.7092	.8507	.9942	.9975	1.0058	.0025	0.7883	10
45° 00′	0.7854	.7071	9.8495	.7071	9.8495	1.0000	0.0000	1.0000	0.0000	0.7854	45° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

EXPONENTIALS [e^n and e^{-n}]

n	e^n	Diff.	n	e^n	Diff.	n	e^n	n	e^{-n}	Diff.	n	e^{-n}	n	e^{-n}
0.00	1.000	10	0.50	1.649	16	1.0	2.718*	0.00	1.000	−10	0.50	.607	1.0	.368*
.01	1.010	10	.51	1.665	17	.1	3.004	.01	0.990	−10	.51	.600	.1	.333
.02	1.020	10	.52	1.682	17	.2	3.320	.02	.980	−10	.52	.595	.2	.301
.03	1.030	11	.53	1.699	17	.3	3.669	.03	.970	−9	.53	.589	.3	.273
.04	1.041	10	.54	1.716	17	.4	4.055	.04	.961	−10	.54	.583	.4	.247
0.05	1.051	11	0.55	1.733	18	1.5	4.482	0.05	.951	−9	0.55	.577	1.5	.223
.06	1.062	11	.56	1.751	17	.6	4.953	.06	.942	−10	.56	.571	.6	.202
.07	1.073	10	.57	1.768	18	.7	5.474	.07	.932	−9	.57	.566	.7	.183
.08	1.083	11	.58	1.786	18	.8	6.050	.08	.923	−9	.58	.560	.8	.165
.09	1.094	11	.59	1.804	18	.9	6.686	.09	.914	−9	.59	.554	.9	.150
0.10	1.105	11	0.60	1.822	18	2.0	7.389	0.10	.905	−9	0.60	.549	2.0	.135
.11	1.116	11	.61	1.840	19	.1	8.166	.11	.896	−9	.61	.543	.1	.122
.12	1.127	12	.62	1.859	19	.2	9.025	.12	.887	−9	.62	.538	.2	.111
.13	1.139	11	.63	1.878	18	.3	9.974	.13	.878	−9	.63	.533	.3	.100
.14	1.150	12	.64	1.896	20	.4	11.02	.14	.869	−8	.64	.527	.4	.0907
0.15	1.162	12	0.65	1.916	19	2.5	12.18	0.15	.861	−9	0.65	.522	2.5	.0821
.16	1.174	11	.66	1.935	19	.6	13.46	.16	.852	−8	.66	.517	.6	.0743
.17	1.185	12	.67	1.954	20	.7	14.88	.17	.844	−9	.67	.512	.7	.0672
.18	1.197	12	.68	1.974	20	.8	16.44	.18	.835	−8	.68	.507	.8	.0608
.19	1.209	12	.69	1.994	20	.9	18.17	.19	.827	−8	.69	.502	.9	.0550
0.20	1.221	13	0.70	2.014	20	3.0	20.09	0.20	.819	−8	0.70	.497	3.0	.0498
.21	1.234	12	.71	2.034	20	.1	22.20	.21	.811	−8	.71	.492	.1	.0450
.22	1.246	13	.72	2.054	21	.2	24.53	.22	.803	−8	.72	.487	.2	.0408
.23	1.259	12	.73	2.075	21	.3	27.11	.23	.795	−8	.73	.482	.3	.0369
.24	1.271	13	.74	2.096	21	.4	29.96	.24	.787	−8	.74	.477	.4	.0334
0.25	1.284	13	0.75	2.117	21	3.5	33.12	0.25	.779	−8	0.75	.472	3.5	.0302
.26	1.297	13	.76	2.138	22	.6	36.60	.26	.771	−8	.76	.468	.6	.0273
.27	1.310	13	.77	2.160	21	.7	40.45	.27	.763	−7	.77	.463	.7	.0247
.28	1.323	13	.78	2.181	22	.8	44.70	.28	.756	−0	.78	.458	.8	.0224
2.9	1.336	14	.79	2.203	23	.9	49.40	.29	.748	−7	.79	.454	.9	.0202
0.30	1.350	13	0.80	2.226	22	4.0	54.60	0.30	.741	−8	0.80	.449	4.0	.0183
.31	1.363	14	.81	2.248	22	.1	60.34	.31	.733	−7	.81	.445	.1	.0166
.32	1.377	14	.82	2.270	23	.2	66.69	.32	.726	−7	.82	.440	.2	.0150
.33	1.391	14	.83	2.293	23	.3	73.70	.33	.719	−7	.83	.436	.3	.0136
.34	1.405	14	.84	2.316	24	.4	81.45	.34	.712	−7	.84	.432	.4	.0123
0.35	1.419	14	0.85	2.340	23	4.5	90.02	0.35	.705	−7	0.85	.427	4.5	.0111
.36	1.433	15	.86	2.363	24			.36	.698	−7	.86	.423		
.37	1.448	14	.87	2.387	24	5.0	148.4	.37	.691	−7	.87	.419	6.0	.00674
.38	1.462	15	.88	2.411	24	6.0	403.4	.38	.684	−7	.88	.415	6.0	.00248
.39	1.477	15	.89	2.435	25	7.0	1097.	.39	.677	−7	.89	.411	7.0	.000912
0.40	1.492	15	0.90	2.460	24	8.0	2981.	0.40	.670	−6	0.90	.407	8.0	.000335
.41	1.507	15	.91	2.484	25	9.0	8103.	.41	.664	−7	.91	.403	9.0	.000123
.42	1.522	15	.92	2.509	26	10.0	22026.	.42	.657	−6	.92	.399	10.0	.000045
.43	1.537	16	.93	2.535	25	$\pi/2$	4.810	.43	.651	−7	.93	.395	$\pi/2$	.208
.44	1.553	15	.94	2.560	26	$2\pi/2$	23.14	.44	.644	−6	.94	.391	$2\pi/2$	.0432
0.45	1.568	16	0.95	2.586	26	$3\pi/2$	111.3	0.45	.638	−7	0.95	.387	$3\pi/2$	.00898
.46	1.584	16	.96	2.612	26	$4\pi/2$	535.5	.46	.631	−6	.96	.383	$4\pi/2$	.00187
.47	1.600	16	.97	2.638	26	$5\pi/2$	2576.	.47	.625	−6	.97	.379	$5\pi/2$	.000388
.48	1.616	16	.98	2.664	27	$6\pi/2$	12392.	.48	.619	−6	.98	.375	$6\pi/2$	.000081
.49	1.632	17	.99	2.691	27	$7\pi/2$	59610.	.49	.613	−6	.99	.372	$7\pi/2$	.000017
0.50	1.649		1.00	2.718		$8\pi/2$	286751.	0.50	0.607		1.00	.368	$8\pi/2$	.000003

* NOTE: Do not interpolate in this column.

$e = 2.71828$; $1/e = 0.367879$ $\log_{10}e = 0.4343$ $1/(0.4343) = 2.3026$

$\log_{10}(0.4343) = \bar{1}.6378$ $\log_{10}(e^n) = n(0.4343)$

For table of multiples of 0.4343, see p. 62. Graphs, p. 174.

HYPERBOLIC LOGARITHMS

These two pages give the natural (hyperbolic, or Napierian) logarithms ($\log_e$) of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right [or left] in the number is equivalent to adding n times 2.3026 [or n times $\bar{3}.6974$] to the logarithm. Base $e = 2.71828+$

n	n (2.3026)	n (0.6974–3)
1	2.3026	0.6974-3
2	4.6052	0.3948-5
3	6.9078	0.0922-7
4	9.2103	0.7897-10
5	11.5129	0.4871-12
6	13.8155	0.1845-14
7	16.1181	0.8819-17
8	18.4207	0.5793-19
9	20.7233	0.2767-21

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.0000	0100	0198	0296	0392	0488	0583	0677	0770	0862	95
1.1	0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	87
1.2	1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	80
1.3	2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	74
1.4	3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	69
1.5	0.4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	65
1.6	4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	61
1.7	5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	57
1.8	5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	54
1.9	6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	51
2.0	0.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	49
2.1	7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	47
2.2	7885	7930	7975	8020	8066	8109	8154	8198	8242	8286	44
2.3	8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	43
2.4	8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	41
2.5	0.9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	39
2.6	9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	38
2.7	0.9933	9969	*0006	*0043	*0080	*0116	*0152	*0188	*0225	*0260	36
2.8	1.0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	35
2.9	0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	34
3.0	1.0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	33
3.1	1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	32
3.2	1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	31
3.3	1939	1969	2000	2030	2060	2090	2119	2149	2179	2208	30
3.4	2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	29
3.5	1.2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	28
3.6	2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	27
3.7	3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	27
3.8	3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	26
3.9	3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	25
4.0	1.3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	25
4.1	4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	24
4.2	4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	23
4.3	4586	4609	4633	4656	4629	4702	4725	4748	4770	4793	23
4.4	4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	22
4.5	1.5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	22
4.6	5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	21
4.7	5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	21
4.8	5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	20
4.9	5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	20

$\log_e x = (2.3026) \log_{10} x$ $\log_{10} x = (0.4343) \log_e x$

where $2.3026 = \log_e 10$ and $0.4343 = \log_{10} e$ (see p. 62) For graphs, see p. 174.

HYPERBOLIC LOGARITHMS (*continued*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	1.6094	6114	6134	6154	6174	6194	6214	6233	6253	6273	20
5.1	6292	6312	6332	6351	6371	6390	6409	6429	6448	6467	19
5.2	6487	6506	6525	6544	6563	6582	6601	6620	6639	6658	19
5.3	6677	6696	6715	6734	6752	6771	6790	6808	6827	6845	18
5.4	6864	6882	6901	6919	6938	6956	6974	6993	7011	7029	18
5.5	1.7047	7066	7084	7102	7120	7138	7156	7174	7192	7210	18
5.6	7228	7246	7263	7281	7299	7317	7334	7352	7370	7387	18
5.7	7405	7422	7440	7457	7475	7492	7509	7527	7544	7561	17
5.8	7579	7596	7613	7630	7647	7664	7681	7699	7716	7733	17
5.9	7750	7766	7783	7800	7817	7834	7851	7867	7884	7901	17
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066	16
6.1	8083	8099	8116	8132	8148	8165	8181	8197	8213	8229	16
6.2	8245	8262	8228	8294	8310	8326	8342	8358	8374	8390	16
6.3	8405	8421	8437	8453	8469	8485	8500	8516	8532	8547	16
6.4	8563	8579	8594	8610	8625	8641	8656	8672	8687	8703	15
6.5	1.8718	8733	8749	8764	8779	8795	8810	8825	8840	8856	15
6.6	8871	8886	8901	8916	8931	8946	8961	8976	8991	9006	15
6.7	9021	9036	9051	9066	9081	9095	9110	9125	9140	9155	15
6.8	9169	9184	9199	9213	9228	9242	9257	9272	9286	9301	15
6.9	9315	9330	9344	9359	9373	9387	9402	9416	9430	9445	14
7.0	1.9459	9473	9488	9502	9516	9530	9544	9559	9573	9587	14
7.1	9601	9615	9629	9643	9657	9671	9685	9699	9713	9727	14
7.2	9741	9755	9769	9782	9796	9810	9824	9838	9851	9865	14
7.3	1.9879	9892	9906	9920	9933	9947	9961	9974	9988	*0001	13
7.4	2.0015	0028	0042	0055	0069	0082	0096	0109	0122	0136	13
7.5	2.0149	0162	0176	0189	0202	0215	0279	0242	0255	0268	13
7.6	0281	0295	0308	0321	0334	0347	0360	0373	0386	0399	13
7.7	0412	0425	0438	0451	0464	0477	0490	0503	0516	0528	13
7.8	0541	0554	0567	0580	0592	0605	0618	0631	0643	0656	13
7.9	0669	0681	0694	0707	0719	0732	0744	0757	0769	0782	12
8.0	2.0794	0807	0819	0832	0844	0857	0869	0882	0894	0906	12
8.1	0919	0931	0943	0956	0968	0980	0992	1005	1017	1029	12
8.2	1041	1054	1066	1078	1090	1102	1114	1126	1138	1150	12
8.3	1163	1175	1187	1199	1211	1223	1235	1247	1258	1270	12
8.4	1282	1294	1306	1318	1330	1342	1353	1365	1377	1389	12
8.5	2.1401	1412	1424	1436	1448	1459	1471	1483	1494	1506	12
8.6	1518	1529	1541	1552	1564	1576	1587	1599	1610	1622	12
8.7	1633	1645	1656	1668	1679	1691	1702	1713	1725	1736	11
8.8	1748	1759	1770	1782	1793	1804	1815	1827	1838	1849	11
8.9	1861	1872	1883	1894	1905	1917	1928	1939	1950	1961	11
9.0	2.1972	1983	1994	2006	2017	2028	2039	2050	2061	2072	11
9.1	2083	2094	2105	2116	2127	2138	2148	2159	2170	2181	11
9.2	2192	2203	2214	2225	2235	2246	2257	2268	2279	2289	11
9.3	2300	2311	2322	2332	2343	2354	2364	2375	2386	2396	11
9.4	2407	2418	2428	2439	2450	2460	2471	2481	2492	2502	11
9.5	2.2513	2523	2534	2544	2555	2565	2576	2586	2597	2607	10
9.6	2618	2628	2638	2649	2659	2670	2680	2690	2701	2711	10
9.7	2721	2732	2742	2752	2262	2773	2783	2793	2803	2814	10
9.8	2824	2834	2844	2854	2865	2875	2885	2895	2905	2915	10
9.9	2925	2935	2946	2956	2966	2976	2986	2996	3006	3016	10
10.0	2.3026										

Moving the decimal point **n places to the right [or left]** in the number requires adding *n* times 2.3026 [or *n* times **(0.6974–3)] in the body of the table.** See auxiliary table of multiples on top of the preceding page.

HYPERBOLIC SINES [$\sinh x = \frac{1}{2}(\epsilon^x - \epsilon^{-x})$]

x	0	1	2	3	4	5	6	7	8	9	Avg. diff.
0.0	.0000	.0100	.0200	.0300	.0400	.0500	.0600	.0701	.0801	.0901	100
1	.1002	.1102	.1203	.1304	.1405	.1506	.1607	.1708	.1810	.1911	101
2	.2013	.2115	.2218	.2320	.2423	.2526	.2629	.2733	.2837	.2941	103
3	.3045	.3150	.3255	.3360	.3466	.3572	.3678	.3785	.3892	.4000	106
4	.4108	.4216	.4325	.4434	.4543	.4653	.4764	.4875	.4986	.5098	110
0.5	.5211	.5324	.5438	.5552	.5666	.5782	.5897	.6014	.6131	.6248	116
6	.6367	.6485	.6605	.6725	.6646	.6967	.7090	.7213	.7336	.7461	122
7	.7586	.7712	.7838	.7966	.8094	.8223	.8353	.8484	.8615	.8748	130
8	.8881	.9015	.9150	.9286	.9423	.9561	.9700	.9840	.9981	1.012	138
9	1.027	1.041	1.055	1.070	1.085	1.099	1.114	1.129	1.145	1.160	15
1.0	1.175	1.191	1.206	1.222	1.238	1.254	1.270	1.286	1.303	1.319	16
1	1.336	1.352	1.369	1.386	1.403	1.421	1.438	1.456	1.474	1.491	17
2	1.509	1.528	1.546	1.564	1.583	1.602	1.621	1.640	1.659	1.679	19
3	1.698	1.718	1.738	1.758	1.779	1.799	1.820	1.841	1.862	1.883	21
4	1.904	1.926	1.948	1.970	1.992	2.014	2.037	2.060	2.083	2.106	22
1.5	2.129	2.153	2.177	2.201	2.225	2.250	2.274	2.299	2.324	2.350	25
6	2.376	2.401	2.428	2.454	2.481	2.507	2.535	2.562	2.590	2.617	27
7	2.646	2.674	2.703	2.732	2.761	2.790	2.820	2.850	2.881	2.911	30
8	2.942	2.973	3.005	3.037	3.069	3.101	3.134	3.167	3.200	3.234	33
9	3.268	3.303	3.337	3.372	3.408	3.443	3.479	3.516	3.552	3.589	36
2.0	3.627	3.665	3.703	3.741	3.780	3.820	3.859	3.899	3.940	3.981	39
1	4.022	4.064	4.106	4.148	4.191	4.234	4.278	4.322	4.367	4.412	44
2	4.457	4.503	4.549	4.596	4.643	4.691	4.739	4.788	4.837	4.887	48
3	4.937	4.988	5.039	5.090	5.142	5.195	5.248	5.302	5.356	5.411	53
4	5.466	5.522	5.578	5.635	5.693	5.751	5.810	5.869	5.929	5.989	58
2.5	6.050	6.112	6.174	6.237	6.300	6.365	6.429	6.495	6.561	6.627	64
6	6.695	6.763	6.831	6.901	6.971	7.042	7.113	7.185	7.258	7.332	71
7	7.406	7.481	7.557	7.634	7.711	7.789	7.868	7.948	8.028	8.110	79
8	8.192	8.275	8.359	8.443	8.529	8.615	8.702	8.790	8.879	8.969	87
9	9.060	9.151	9.244	9.337	9.431	9.527	9.623	9.720	9.819	9.918	96
3.0	10.02	10.12	10.22	10.32	10.43	10.53	10.64	10.75	10.86	10.97	11
1	11.08	11.19	11.30	11.42	11.53	11.65	11.76	11.88	12.00	12.12	12
2	12.25	12.37	12.49	12.62	12.75	12.88	13.01	13.14	13.27	13.40	13
3	13.54	13.67	13.81	13.95	14.09	14.23	14.38	14.52	14.67	14.82	14
4	14.97	15.12	15.27	15.42	15.58	15.73	15.89	16.05	16.21	16.38	16
3.5	16.54	16.71	16.88	17.05	17.22	17.39	17.57	17.74	17.92	18.10	17
6	18.29	18.47	18.66	18.84	19.03	19.22	19.42	19.61	19.81	20.01	19
7	20.21	20.41	20.62	20.83	21.04	21.25	21.46	21.68	21.90	22.12	21
8	22.34	22.56	22.79	23.02	23.25	23.49	23.72	23.96	24.20	24.45	24
9	24.69	24.94	25.19	25.44	25.70	25.96	26.22	26.48	26.75	27.02	26
4.0	27.29	27.56	27.84	28.12	28.40	28.69	28.98	29.27	29.56	29.86	29
1	30.16	30.47	30.77	31.08	31.39	31.71	32.03	32.35	32.68	33.00	32
2	33.34	33.67	34.01	34.35	34.70	35.05	35.40	35.75	36.11	36.48	35
3	36.84	37.21	37.59	37.97	38.35	38.73	39.12	39.52	39.91	40.31	39
4	40.72	41.13	41.54	41.96	42.38	42.81	43.24	43.67	44.11	44.56	43
4.5	45.00	45.46	45.91	46.37	46.84	47.31	47.79	48.27	48.75	49.24	47
6	49.74	50.24	50.74	51.25	51.77	52.29	52.81	53.34	53.88	54.42	52
7	54.97	55.52	56.08	56.64	57.21	57.79	58.37	58.96	59.55	60.15	58
8	60.75	61.36	61.98	62.60	63.23	63.87	64.51	65.16	65.81	67.47	64
9	67.14	67.82	68.50	69.19	69.88	70.58	71.29	72.01	72.73	73.46	71
5.0	74.20										

If $x > 5$, $\sinh x = \frac{1}{2}(\epsilon^x)$ and $\log_{10} \sinh x = (0.4343)x + 0.6990 - 1$, correct to four significant figures. For table of multiples of 0.4343, see p. 62. Graphs, p. 174.

HYPERBOLIC COSINES [$\cosh x = \frac{1}{2}(e^x + e^{-x})$]

x	0	1	2	3	4	5	6	7	8	9	Avg. diff.
0.0	1.000	1.000	1.000	1.000	1.001	1.001	1.002	1.002	1.003	1.004	1
1	1.005	1.006	1.007	1.008	1.010	1.011	1.013	1.014	1.016	1.018	2
2	1.020	1.022	1.024	1.027	1.029	1.031	1.034	1.037	1.039	1.042	3
3	1.045	1.048	1.052	1.055	1.058	1.062	1.066	1.069	1.073	1.077	4
4	1.081	1.085	1.090	1.094	1.098	1.103	1.108	1.112	1.117	1.122	5
0.5	1.128	1.133	1.138	1.144	1.149	1.155	1.161	1.167	1.173	1.179	6
6	1.185	1.192	1.198	1.205	1.212	1.219	1.226	1.233	1.240	1.248	7
7	1.255	1.263	1.271	1.278	1.287	1.295	1.303	1.311	1.320	1.329	8
8	1.337	1.346	1.355	1.365	1.374	1.384	1.393	1.403	1.413	1.423	10
9	1.433	1.443	1.454	1.465	1.475	1.486	1.497	1.509	1.520	1.531	11
1.0	1.543	1.555	1.567	1.579	1.591	1.604	1.616	1.629	1.642	1.655	13
1	1.669	1.682	1.696	1.709	1.723	1.737	1.752	1.766	1.781	1.796	14
2	1.811	1.826	1.841	1.857	1.872	1.888	1.905	1.921	1.937	1.954	16
3	1.971	1.988	2.005	2.023	2.040	2.058	2.076	2.095	2.113	2.132	18
4	2.151	2.170	2.189	2.209	2.229	2.249	2.269	2.290	2.310	2.331	20
1.5	2.352	2.374	2.395	2.417	2.439	2.462	2.484	2.507	2.530	2.554	23
6	2.577	2.601	2.625	2.650	2.675	2.700	2.725	2.750	2.776	2.802	25
7	2.828	2.855	2.882	2.909	2.936	2.964	2.992	3.021	3.049	3.078	28
8	3.107	3.137	3.167	3.197	3.228	3.259	3.290	3.321	3.353	3.385	31
9	3.418	3.451	3.484	3.517	3.551	3.585	3.620	3.655	3.690	3.726	34
2.0	3.762	3.799	3.835	3.873	3.910	3.948	3.987	4.026	4.065	4.104	38
1	4.144	4.185	4.226	4.267	4.309	4.351	4.393	4.436	4.480	4.524	42
2	4.568	4.613	4.658	4.704	4.750	4.797	4.844	4.891	4.939	4.988	47
3	5.037	5.087	5.137	5.188	5.239	5.290	5.343	5.395	5.449	5.503	52
4	5.557	5.612	5.667	5.723	5.780	5.837	5.895	5.954	6.013	6.072	58
2.5	6.132	6.193	6.255	6.317	6.379	6.443	6.507	6.571	6.636	6.702	64
6	6.769	6.836	6.904	6.973	7.042	7.112	7.183	7.255	7.327	7.400	70
7	7.473	7.548	7.623	7.699	7.776	7.853	7.932	8.011	8.091	8.171	78
8	8.253	8.335	8.418	8.502	8.587	8.673	8.759	8.847	8.935	9.024	86
9	9.115	9.206	9.298	9.391	9.484	9.579	9.675	9.772	9.869	9.968	95
3.0	10.07	10.17	10.27	10.37	10.48	10.58	10.69	10.79	10.90	11.01	11
1	11.12	11.23	11.35	11.46	11.57	11.69	11.81	11.92	12.04	12.16	12
2	12.29	12.41	12.53	12.66	12.79	12.91	13.04	13.17	13.31	13.44	13
3	13.57	13.71	13.85	13.99	14.13	14.27	14.41	14.56	14.70	14.85	14
4	15.00	15.15	15.30	15.45	15.61	15.77	15.92	16.08	16.25	16.41	16
3.5	16.57	16.74	16.91	17.08	17.25	17.42	17.60	17.77	17.95	18.13	17
6	18.31	18.50	18.68	18.87	19.06	19.25	19.44	19.64	19.84	20.03	19
7	20.24	20.44	20.64	20.85	21.06	21.27	21.49	21.70	21.92	22.14	21
8	22.36	22.59	22.81	23.04	23.27	23.51	23.74	23.98	24.22	24.47	23
9	24.71	24.96	25.21	25.46	25.72	25.98	26.24	26.50	26.77	27.04	26
4.0	27.31	27.58	27.86	28.14	28.42	28.71	29.00	29.29	29.58	29.88	29
1	30.18	30.48	30.79	31.10	31.41	31.72	32.04	32.37	32.69	33.02	32
2	33.35	33.69	34.02	34.37	34.71	35.06	35.41	35.77	36.13	36.49	35
3	36.86	37.23	37.60	37.98	38.36	38.75	39.13	39.53	39.93	40.33	39
4	40.73	41.14	41.55	41.97	42.39	42.82	43.25	43.68	44.12	44.57	43
4.5	45.01	45.47	45.92	46.38	46.85	47.32	47.80	48.28	48.76	49.25	47
6	49.75	50.25	50.75	51.26	51.78	52.30	52.82	57.35	57.89	54.43	52
7	54.98	55.53	56.09	56.65	57.22	57.80	58.38	58.96	59.56	60.15	58
8	60.76	61.37	61.99	62.61	63.24	63.87	64.52	65.16	65.82	66.48	64
9	67.15	67.82	68.50	69.19	69.89	70.59	71.30	72.02	72.74	73.47	71
5.0	74.21										

If $x > 5$, $\cosh x = \frac{1}{2}(e^x)$ and $\log_{10} \cosh x = (0.4343)x + 0.6990 - 1$, correct to four significant figures. For table of multiples of 0.4343, see p. 62. Graphs, p. 174.

HYPERBOLIC TANGENTS [$\tanh x = (e^x - e^{-x})/(e^x + e^{-x}) = \sinh x/\cosh x$]

x	.0	1	2	3	.4	.5	6	7	8	9	Avg. diff.
0.0	.0000	.0100	.0200	.0300	.0400	.0500	.0599	.0699	.0798	.0898	100
1	.0997	.1096	.1194	.1293	.1391	.1489	.1587	.1684	.1781	.1878	98
2	.1974	.2070	.2165	.2260	.2355	.2449	.2543	.2636	.2729	.2821	94
3	.2913	.3004	.3095	.3185	.3275	.3364	.3452	.3540	.3627	.3714	89
4	.3800	.3885	.3969	.4053	.4137	.4219	.4301	.4382	.4462	.4542	82
0.5	.4621	.4700	.4777	.4854	.4930	.5005	.5080	.5154	.5227	.5299	75
6	.5370	.5441	.5511	.5581	.5649	.5717	.5784	.5850	.5915	.5980	67
7	.6044	.6107	.6169	.6231	.6291	.6352	.6411	.6469	.6527	.6584	60
8	.6640	.6696	.6751	.6805	.6858	.6911	.6963	.7014	.7064	.7114	52
9	.7163	.7211	.7259	.7306	.7352	.7398	.7443	.7487	.7531	.7574	45
1.0	.7616	.7658	.7699	.7739	.7779	.7818	.7857	.7895	.7932	.7969	39
1	.8005	.8041	.8076	.8110	.8144	.8178	.8210	.8243	.8275	.8306	33
2	.8337	.8367	.8397	.8426	.8455	.8483	.8511	.8538	.8565	.8591	28
3	.8617	.8643	.8668	.8693	.8717	.8741	.8764	.8787	.8810	.8832	24
4	.8854	.8875	.8896	.8917	.8937	.8957	.8977	.8996	.9015	.9033	20
1.5	.9052	.9069	.9087	.9104	.9121	.9138	.9154	.9170	.9186	.9202	17
6	.9217	.9232	.9246	.9261	.9275	.9289	.9302	.9316	.9329	.9342	14
7	.9354	.9367	.9379	.9391	.9402	.9414	.9425	.9436	.9447	.9458	11
8	.9468	.9478	.9488	.9498	.9508	.9518	.9527	.9536	.9545	.9554	9
9	.9562	.9571	.9579	.9587	.9595	.9603	.9611	.9619	.9626	.9633	8
2.0	.9640	.9647	.9654	.9661	.9668	.9674	.9680	.9687	.9693	.9699	6
1	.9705	.9710	.9716	.9722	.9727	.9732	.9738	.9743	.9748	.9753	5
2	.9757	.9762	.9767	.9771	.9776	.9780	.9785	.9789	.9793	.9797	4
3	.9801	.9805	.9809	.9812	.9816	.9820	.9823	.9827	.9830	.9834	4
4	.9837	.9840	.9843	.9846	.9849	.9852	.9855	.9858	.9861	.9863	3
2.5	.9866	.9869	.9871	.9874	.9876	.9879	.9881	.9884	.9886	.9888	2
6	.9890	.9892	.9895	.9897	.9899	.9901	.9903	.9905	.9906	.9908	2
7	.9910	.9912	.9914	.9915	.9917	.9919	.9920	.9922	.9923	.9925	2
8	.9926	.9928	.9929	.9931	.9932	.9933	.9935	.9936	.9937	.9938	1
2.9	.9940	.9941	.9942	.9943	.9944	.9945	.9946	.9947	.9949	.9950	1
3.	.9951	.9959	.9967	.9973	.9978	.9982	.9985	.9988	.9990	.9992	4
4.	.9993	.9995	.9996	.9996	.9997	.9998	.9998	.9998	.9999	.9999	1
5.	.9999										

If $x > 5$, $\tanh x = 1.0000$ to four decimal places. Graphs, p. 174.

MULTIPLES OF 0.4343 ($0.43429448 = \log_{10} e$)

x	0	1	2	3	4	5	6	7	8	9
0.	0.0000	0.0434	0.0869	0.1303	0.1737	0.2171	0.2606	0.3040	0.3474	0.3909
1.	0.4343	0.4777	0.5212	0.5646	0.6080	0.6514	0.6949	0.7383	0.7817	0.8252
2.	0.8686	0.9120	0.9554	0.9989	1.0423	1.0857	1.1292	1.1726	1.2160	1.2595
3.	1.3029	1.3463	1.3897	1.4332	1.4766	1.5200	1.5635	1.6069	1.6503	1.6937
4.	1.7372	1.7806	1.8240	1.8675	1.9109	1.9543	1.9978	2.0412	2.0846	2.1280
5.	2.1715	2.2149	2.2583	2.3018	2.3452	2.3886	2.4320	2.4755	2.5189	2.5623
6.	2.6058	2.6492	2.6926	2.7361	2.7795	2.8229	2.8663	2.9098	2.9532	2.9966
7.	3.0401	3.0835	3.1269	3.1703	3.2138	3.2572	3.3006	3.3441	3.3875	3.4309
8.	3.4744	3.5178	3.5612	3.6046	3.6481	3.6915	3.7349	3.7784	3.8218	3.8652
9.	3.9087	3.9521	3.9955	4.0389	4.0824	4.1258	4.1692	4.2127	4.2561	4.2995

MULTIPLES OF 2.3026 ($2.3025851 = 1/0.4343$)

x	.0	1	2	3	.4	5	6	7	8	9
0.	0.0000	0.2303	0.4605	0.6908	0.9210	1.1513	1.3816	1.6118	1.8421	2.0723
1.	2.3026	2.5328	2.7631	2.9934	3.2236	3.4539	3.6841	3.9144	4.1447	4.3749
2.	4.6052	4.8354	5.0657	5.2959	5.5262	5.7565	5.9867	6.2170	6.4472	6.6775
3.	6.9078	7.1380	7.3683	7.5985	7.8288	8.0590	8.2893	8.5196	8.7498	8.9801
4.	9.2103	9.4406	9.6709	9.9011	10.131	10.362	10.592	10.822	11.052	11.283
5.	11.513	11.743	11.973	12.204	12.434	12.664	12.894	13.125	13.355	13.585
6.	13.816	14.046	14.276	14.506	14.737	14.967	15.197	15.427	15.658	15.888
7.	16.118	16.348	16.579	16.809	17.039	17.269	17.500	17.730	17.960	18.190
8.	18.421	18.651	18.881	19.111	19.342	19.572	19.802	20.032	20.263	20.493
9.	20.723	20.954	21.184	21.414	21.644	21.875	22.105	22.335	22.565	22.796

STANDARD DISTRIBUTION OF RESIDUALS (p. 121)

a = any positive quantity;
y = the number of residuals which are numerically $< a$;
r = the probable error of a single observation;
n = number of observations.

$\frac{a}{r}$	$\frac{y}{n}$	Diff.
0.0	.000	54
1	.054	53
2	.107	53
3	.160	53
4	.213	51
0.5	.264	50
6	.314	49
7	.363	48
8	.411	45
9	.456	44
1.0	.500	42
1	.542	40
2	.582	37
3	.619	36
4	.655	33
1.5	.688	31
6	.719	29
7	.748	27
8	.775	25
9	.800	23
2.0	.823	20
1	.843	19
2	.862	17
3	.879	16
4	.895	13
2.5	.908	13
6	.921	10
7	.931	10
8	.941	9
9	.950	7
3.0	.957	6
1	.963	6
2	.969	5
3	.974	4
4	.978	4
3.5	.982	3
6	.985	2
7	.987	3
8	.990	1
9	.991	2
4.0	.993	6
5.0	.999	

FACTORS FOR COMPUTING PROBABLE ERROR (p. 121)

n	Bessel		Peters	
	$\frac{0.6745}{\sqrt{(n-1)}}$	$\frac{0.6745}{\sqrt{n(n-1)}}$	$\frac{0.8453}{\sqrt{n(n-1)}}$	$\frac{0.8453}{n\sqrt{n-1}}$
2	.6745	.4769	.5978	.4227
3	.4769	.2754	.3451	.1993
4	.3894	.1947	.2440	.1220
5	.3372	.1508	.1890	.0845
6	.3016	.1231	.1543	.0630
7	.2754	.1041	.1304	.0493
8	.2549	.0901	.1130	.0399
9	.2385	.0795	.0996	.0332
10	.2248	.0711	.0891	.0282
11	.2133	.0643	.0806	.0243
12	.2034	.0587	.0736	.0212
13	.1947	.0540	.0677	.0188
14	.1871	.0500	.0627	.0167
15	.1803	.0465	.0583	.0151
16	.1742	.0435	.0546	.0136
17	.1686	.0409	.0513	.0124
18	.1636	.0386	.0483	.0114
19	.1590	.0365	.0457	.0105
20	.1547	.0346	.0434	.0097
21	.1508	.0329	.0412	.0090
22	.1472	.0314	.0393	.0084
23	.1438	.0300	.0376	.0078
24	.1406	.0287	.0360	.0073
25	.1377	.0275	.0345	.0069
26	.1349	.0265	.0332	.0065
27	.1323	.0255	.0319	.0061
28	.1298	.0245	.0307	.0058
29	.1275	.0237	.0297	.0055
30	.1252	.0229	.0287	.0052
31	.1231	.0221	.0277	.0050
32	.1211	.0214	.0268	.0047
33	.1192	.0208	.0260	.0045
34	.1174	.0201	.0252	.0043
35	.1157	.0196	.0245	.0041
36	.1140	.0190	.0238	.0040
37	.1124	.0185	.0232	.0038
38	.1109	.0180	.0225	.0037
39	.1094	.0175	.0220	.0035
40	.1080	.0171	.0214	.0034
45	.1017	.0152	.0190	.0028
50	.0964	.0136	.0171	.0024
55	.0918	.0124	.0155	.0021
60	.0878	.0113	.0142	.0018
65	.0843	.0105	.0131	.0016
70	.0812	.0097	.0122	.0015
75	.0784	.0091	.0113	.0013
80	.0759	.0085	.0106	.0012
85	.0736	.0080	.0100	.0011
90	.0715	.0075	.0094	.0010
95	.0696	.0071	.0089	.0009
100	.0678	.0068	.0085	.0008

COMPOUND INTEREST. AMOUNT OF A GIVEN PRINCIPAL

The amount A at the end of n years of a given principal P placed at compound interest to-day is $A = P \times x$ or $A = P \times y$ or $A = P \times z$, according as the interest (at the rate of r per cent. per annum) is compounded annually, semi-annually, or quarterly; the factor x or y or z being taken from the following tables.

Values of x. (Interest compounded annually; $A = P \times x$.)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	1.1449
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	1.2250
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.1925	1.2155	1.2625	1.3108
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	1.4026
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	1.5007
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	1.6058
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	1.7182
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.4861	1.5513	1.6895	1.8385
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	1.9672
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983	2.1049
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7959	2.0122	2.2522
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	2.4098
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609	2.5785
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966	2.7590
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	2.9522
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	3.1588
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	3.3799
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	3.6165
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	3.8697
25	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	3.3864	4.2919	5.4274
30	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	4.3219	5.7435	7.6123
40	2.2080	2.6851	3.2620	3.9593	4.8010	5.8164	7.0400	10.286	14.974
50	2.6916	3.4371	4.3839	5.5849	7.1067	9.0326	11.467	18.420	29.457
60	3.2810	4.3998	5.8916	7.8781	10.520	14.027	16.679	32.988	57.946

This table is computed from the formula $x = [1 + (r/100)]^n$.

Values of y. (Interest compounded semi-annually; $A = P \times y$.)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0201	1.0252	1.0302	1.0353	1.0404	1.0455	1.0506	1.0609	1.0712
2	1.0406	1.0509	1.0614	1.0719	1.0824	1.0931	1.1038	1.1255	1.1475
3	1.0615	1.0774	1.0934	1.1097	1.1262	1.1428	1.1597	1.1941	1.2293
4	1.0829	1.1045	1.1265	1.1489	1.1717	1.1948	1.2184	1.2668	1.3168
5	1.1046	1.1323	1.1605	1.1894	1.2190	1.2492	1.2801	1.3439	1.4106
6	1.1268	1.1608	1.1956	1.2314	1.2682	1.3060	1.3449	1.4258	1.5111
7	1.1495	1.1900	1.2318	1.2749	1.3195	1.3655	1.4130	1.5126	1.6187
8	1.1726	1.2199	1.2690	1.3199	1.3728	1.4276	1.4845	1.6047	1.7340
9	1.1961	1.2506	1.3073	1.3665	1.4282	1.4926	1.5597	1.7024	1.8575
10	1.2202	1.2820	1.3469	1.4148	1.4859	1.5605	1.6386	1.8061	1.9898
11	1.2447	1.3143	1.3876	1.4647	1.5460	1.6315	1.7216	1.9161	2.1315
12	1.2697	1.3474	1.4295	1.5164	1.6084	1.7058	1.8087	2.0328	2.2833
13	1.2953	1.3812	1.4727	1.5700	1.6734	1.7834	1.9003	2.1566	2.4460
14	1.3213	1.4160	1.5172	1.6254	1.7410	1.8645	1.9965	2.2879	2.6202
15	1.3478	1.4516	1.5631	1.6828	1.8114	1.9494	2.0976	2.4273	2.8068
16	1.3749	1.4881	1.6103	1.7422	1.8845	2.0381	2.2038	2.5751	3.0067
17	1.4026	1.5256	1.6590	1.8037	1.9607	2.1308	2.3153	2.7319	3.2209
18	1.4308	1.5639	1.7091	1.8674	2.0399	2.2278	2.4325	2.8983	3.4503
19	1.4595	1.6033	1.7608	1.9333	2.1223	2.3292	2.5557	3.0748	3.6960
20	1.4889	1.6436	1.8140	2.0016	2.2080	2.4352	2.6851	3.2620	3.9593
25	1.6446	1.8610	2.1052	2.3808	2.6916	3.0420	3.4371	4.3839	5.5849
30	1.8167	2.1072	2.4432	2.8318	3.2810	3.8001	4.3998	5.8916	7.8781
40	2.2167	2.7015	3.2907	4.0064	4.8754	5.9301	7.2096	10.641	15.676
50	2.7048	3.4634	4.4320	5.6682	7.2446	9.2540	11.814	19.219	31.191
60	3.3004	4.4402	5.9693	6.0192	10.765	14.441	19.358	34.711	62.064

Formula: $y = [1 + (r/200)]^{2n}$.

Values of z. (Interest compounded quarterly; $A = P \times z$; see opposite page)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0202	1.0252	1.0303	1.0355	1.0406	1.0458	1.0509	1.0614	1.0719
2	1.0407	1.0511	1.0616	1.0722	1.0829	1.0936	1.1045	1.1265	1.1489
3	1.0617	1.0776	1.0938	1.1102	1.1268	1.1437	1.1608	1.1956	1.2314
4	1.0831	1.1048	1.1270	1.1496	1.1726	1.1960	1.2199	1.2690	1.3199
5	1.1049	1.1327	1.1612	1.1903	1.2202	1.2503	1.2820	1.3469	1.4148
6	1.1272	1.1613	1.1964	1.2326	1.2697	1.3060	1.3474	1.4295	1.5164
7	1.1499	1.1906	1.2327	1.2763	1.3213	1.3679	1.4160	1.5172	1.6254
8	1.1730	1.2206	1.2701	1.3215	1.3749	1.4305	1.4881	1.6103	1.7422
9	1.1967	1.2514	1.3086	1.3684	1.4308	1.4959	1.5639	1.7091	1.8674
10	1.2208	1.2830	1.3483	1.4169	1.4889	1.5644	1.6436	1.8140	2.0016
11	1.2454	1.3154	1.3893	1.4672	1.5493	1.6360	1.7274	1.9253	2.1454
12	1.2705	1.3486	1.4314	1.5192	1.6122	1.7103	1.8154	2.0435	2.2996
13	1.2961	1.3826	1.4748	1.5731	1.6777	1.7891	1.9078	2.1689	2.4648
14	1.3222	1.4175	1.5196	1.6288	1.7458	1.8710	2.0050	2.3020	2.6420
15	1.3489	1.4533	1.5657	1.6866	1.8167	1.9566	2.1072	2.4432	2.8318
16	1.3760	1.4900	1.6132	1.7464	1.8905	2.0462	2.2145	2.5931	3.0353
17	1.4038	1.5276	1.6621	1.8083	1.9672	2.1398	2.3274	2.7523	3.2534
18	1.4320	1.5661	1.7126	1.8725	2.0471	2.2378	2.4459	2.9212	3.4872
19	1.4609	1.6056	1.7645	1.9389	2.1302	2.3402	2.5705	3.1004	3.7378
20	1.4903	1.6462	1.8180	2.0076	2.2167	2.4473	2.7015	3.2907	4.0064
25	1.6467	1.8646	2.1111	2.3898	2.7048	3.0609	3.4634	4.4320	5.6682
30	1.8194	2.1121	2.4514	2.8446	3.3004	3.8285	4.4402	5.9693	8.0192
40	2.2211	2.7098	3.3053	4.0306	4.9138	5.9892	7.2980	10.828	16.051
50	2.7115	3.4768	4.4567	5.7110	7.3160	9.3693	11.995	19.643	32.128
60	3.3102	4.4608	6.0092	8.0919	10.893	14.657	19.715	35.633	64.307

Formula: $z = [1 + (r/400)]^{4n}$.

AMOUNT OF AN ANNUITY

The amount S accumulated at the end of n years by a given annual payment Y set aside at the end of each year is $S = Y \times v$, where the factor v is to be taken from the following table. (Interest at r per cent. per annum, compounded annually.)

Values of v

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0200	2.0250	2.0300	2.0350	2.0400	2.0450	2.0500	2.0600	2.0700
3	3.0604	3.0756	3.0909	3.1062	3.1216	3.1370	3.1525	3.1836	3.2149
4	4.1216	4.1525	4.1836	4.2149	4.2465	4.2782	4.3101	4.3746	4.4399
5	5.2040	5.2563	5.3091	5.3625	5.4163	5.4707	5.5256	5.6371	5.7507
6	6.3081	6.3877	6.4684	6.5502	6.6330	6.7169	6.8019	6.9753	7.1533
7	7.4343	7.5474	7.6625	7.7794	7.8983	8.0192	8.1420	8.3938	8.6540
8	8.5830	8.7361	8.8923	9.0517	9.2142	9.3800	9.5491	9.8975	10.260
9	9.7546	9.9545	10.159	10.368	10.583	10.802	11.027	11.491	11.978
10	10.950	11.203	11.464	11.731	12.006	12.288	12.578	13.181	13.816
11	12.169	12.483	12.808	13.142	13.486	13.841	14.207	14.972	15.784
12	13.412	13.796	14.192	14.602	15.026	15.464	15.917	16.870	17.888
13	14.680	15.140	15.618	16.113	16.627	17.160	17.713	18.882	20.141
14	15.974	16.519	17.086	17.627	18.292	18.932	19.599	21.015	22.550
15	17.293	17.932	18.599	19.296	20.024	20.784	21.579	23.276	25.129
16	18.639	19.380	20.157	20.971	21.825	22.719	23.657	25.673	27.888
17	20.012	20.865	21.762	22.705	23.698	24.742	25.840	28.213	30.840
18	21.412	22.386	23.414	24.500	25.645	26.855	28.132	30.906	33.999
19	22.841	23.946	25.117	26.357	27.671	29.064	30.539	33.760	37.379
20	24.297	25.545	26.870	28.280	29.778	31.371	33.066	36.786	40.995
25	32.030	34.158	36.459	38.950	41.646	44.565	47.727	54.865	63.249
30	40.568	43.903	47.575	51.623	56.085	61.007	66.439	79.058	94.461
40	60.402	67.403	75.401	84.550	95.026	107.03	120.80	154.76	199.64
50	84.579	97.484	112.80	131.00	152.67	178.50	209.35	290.34	406.53
60	114.05	135.99	163.05	196.52	237.99	289.50	353.58	533.13	813.52

Formula: $v = \{[1 + (r/100)]^n - 1\} \div (r/100) = (z - 1) \div (r/100)$.

PRINCIPAL WHICH WILL AMOUNT TO A GIVEN SUM

The principal P, which, if placed at compound interest to-day, will amount to a given sum A at the end of n years is $P = A \times x'$ or $P = A \times y'$ or $P = A \times z'$, according as the interest (*at the rate of r per cent. per annum*) is compounded annually, semi-annually, or quarterly; the factor x' or y' or z' being taken from the following tables.

Values of x'. (Interest compounded annually; $P = A \times x'$)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	.98039	.97561	.97087	.96618	.96154	.95694	.95238	.94340	.93458
2	.96117	.95181	.94260	.93351	.92456	.91573	.90703	.89000	.87344
3	.94232	.92860	.91514	.90194	.88900	.87630	.86384	.83962	.81630
4	.92385	.90595	.88849	.87144	.85480	.83856	.82270	.79209	.76290
5	.90573	.88385	.86261	.84197	.82193	.80245	.78353	.74726	.71299
6	.88797	.86230	.83748	.81350	.79031	.76790	.74622	.70496	.66634
7	.87056	.84127	.81309	.78599	.75992	.73483	.71068	.66506	.62275
8	.85349	.82075	.78941	.75941	.73069	.70319	.67684	.62741	.58201
9	.83676	.80073	.76642	.73373	.70259	.67290	.64461	.59190	.54393
10	.82035	.78120	.74409	.70892	.67556	.64393	.61391	.55839	.50835
11	.80426	.76214	.72242	.68495	.64958	.61620	.58468	.52679	.47509
12	.78849	.74356	.70138	.66178	.62460	.58966	.55684	.49697	.44401
13	.77303	.72542	.68095	.63940	.60057	.56427	.53032	.46884	.41496
14	.75788	.70773	.66112	.61778	.57748	.53997	.50507	.44230	.38783
15	.74301	.69047	.64186	.59689	.55526	.51672	.48102	.41727	.36245
16	.72845	.67362	.62317	.57671	.53391	.49447	.45811	.39365	.33873
17	.71416	.65720	.60502	.55720	.51337	.47318	.43630	.37136	.31657
18	.70016	.64117	.58739	.53836	.49363	.45280	.41552	.35034	.29586
19	.68643	.62553	.57029	.52016	.47464	.43330	.39573	.33051	.27651
20	.67297	.61027	.55368	.50257	.45639	.41464	.37689	.31180	.25842
25	.60953	.53939	.47761	.42315	.37512	.33273	.29530	.23300	.18425
30	.55207	.47674	.41199	.35628	.30832	.26700	.23138	.17411	.13137
40	.45289	.37243	.30656	.25257	.20829	.17193	.14205	.09722	.06678
50	.37153	.29094	.22811	.17905	.14071	.11071	.08720	.05429	.03395
60	.30478	.22728	.16973	.12693	.09506	.07129	.05354	.03031	.01726

Formula: $x' = [1 + (r/100)]^{-n} = 1/x$.

Values of y'. (Interest compounded semi-annually; $P = A \times y'$)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	.98030	.97546	.97066	.96590	.96117	.95647	.95181	.94260	.93351
2	.96098	.95152	.94218	.93296	.92385	.91484	.90595	.88849	.87144
3	.94205	.92817	.91454	.90114	.88797	.87502	.86230	.83748	.81350
4	.92348	.90540	.88771	.87041	.85349	.83694	.82075	.78941	.75941
5	.90529	.88318	.86167	.84073	.82035	.80051	.78120	.74409	.70892
6	.88745	.86151	.83639	.81206	.78849	.76567	.74356	.70138	.66178
7	.86996	.84037	.81185	.78436	.75788	.73234	.70773	.66112	.61778
8	.85282	.81975	.78803	.75762	.72845	.70047	.67362	.62317	.57671
9	.83602	.79963	.76491	.73178	.70016	.66998	.64117	.58739	.53836
10	.81954	.78001	.74247	.70682	.67297	.64082	.61027	.55368	.50257
11	.80340	.76087	.72069	.68272	.64684	.61292	.58086	.52189	.46915
12	.78757	.74220	.69954	.65944	.62172	.58625	.55288	.49193	.43796
13	.77205	.72398	.67902	.63695	.59758	.56073	.52623	.46369	.40884
14	.75684	.70622	.65910	.61523	.57437	.53632	.50088	.43708	.38165
15	.74192	.68889	.63976	.59425	.55207	.51298	.47674	.41199	.35628
16	.72730	.67198	.62099	.57398	.53063	.49085	.45377	.38834	.33259
17	.71297	.65549	.60277	.55441	.51003	.46930	.43191	.36604	.31048
18	.69892	.63941	.58509	.53550	.49022	.44887	.41109	.34503	.28983
19	.68515	.62372	.56792	.51724	.47119	.42933	.39128	.32523	.27056
20	.67165	.60841	.55126	.49960	.45289	.41065	.37243	.30656	.25257
25	.60804	.53734	.47500	.42003	.37153	.32873	.29094	.22811	.17905
30	.55045	.47457	.40930	.35313	.30478	.26315	.22728	.16973	.12693
40	.45112	.37017	.30389	.24960	.20511	.16863	.13870	.09398	.06379
50	.36971	.28873	.22563	.17642	.13803	.10806	.08465	.05203	.03206
60	.30299	.22521	.16752	.12470	.09289	.06925	.05166	.02881	.01611

Formula: $y' = [1 + (r/200)]^{-2n} = 1/y$.

Values of z'. (Interest compounded quarterly; $P = A \times z'$; see opposite page)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	.98025	.97539	.97055	.96575	.96098	.95624	.95152	.94218	.93296
2	.96089	.95138	.94198	.93268	.92348	.91439	.90540	.88771	.87041
3	.94191	.92796	.91424	.90074	.88745	.87437	.86151	.83639	.81206
4	.92330	.90512	.88732	.86989	.85282	.83611	.81975	.78803	.75762
5	.90506	.88284	.86119	.84010	.81954	.79952	.78001	.74247	.70682
6	.88719	.86111	.83583	.81132	.78757	.76453	.74220	.69954	.65944
7	.86966	.83991	.81122	.78354	.75684	.73107	.70622	.65910	.61523
8	.85248	.81924	.78733	.75670	.72730	.69908	.67198	.62099	.57390
9	.83564	.79908	.76415	.73079	.69892	.66849	.63941	.58509	.53550
10	.81914	.77941	.74165	.70576	.67165	.63923	.60841	.55126	.49960
11	.80296	.76022	.71981	.68159	.64545	.61126	.57892	.51939	.46611
12	.78710	.74151	.69861	.65825	.62026	.58451	.55086	.48936	.43486
13	.77155	.72326	.67804	.63570	.59606	.55693	.52415	.46107	.40570
14	.75631	.70546	.65803	.61393	.57280	.53447	.49874	.43441	.37851
15	.74137	.68809	.63870	.59291	.55045	.51108	.47457	.40930	.35313
16	.72673	.67115	.61989	.57260	.52897	.48871	.45156	.38563	.32946
17	.71237	.65464	.60164	.55299	.50833	.46733	.42967	.36334	.30737
18	.69830	.63852	.58392	.53405	.48850	.44687	.40884	.34233	.28676
19	.68451	.62281	.56673	.51576	.46944	.42732	.38903	.32254	.26754
20	.67099	.60748	.55004	.49810	.45112	.40862	.37017	.30389	.24960
25	.60729	.53630	.47369	.41845	.36971	.32670	.28873	.22563	.17642
30	.54963	.47347	.40794	.35154	.30299	.26120	.22521	.16752	.12470
40	.45023	.36903	.30255	.24810	.20351	.16697	.13702	.09235	.06230
50	.36880	.28762	.22438	.17510	.13669	.10673	.08337	.05091	.03113
60	.30210	.22417	.16641	.12358	.09161	.06823	.05072	.02806	.01555

Formula: $z' = [1 + (r/400)]^{-4n} = 1/z$.

ANNUITY WHICH WILL AMOUNT TO A GIVEN SUM (SINKING FUND)

The annual payment, Y, which, if set aside at the end of each year, will amount with accumulated interest to a given sum S at the end of n years is $Y = S \times v'$, where the factor v' is given below. (Interest at r per cent. per annum, compounded annually.)

Values of v'

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
2	.49505	.49383	.49261	.49140	.49020	.48900	.48780	.48544	.48309
3	.32675	.32514	.32353	.32193	.32035	.31877	.31721	.31411	.31105
4	.24262	.24082	.23903	.23725	.23549	.23374	.23201	.22859	.22523
5	.19216	.19025	.18835	.18648	.18463	.18279	.18097	.17740	.17389
6	.15853	.15655	.15460	.15267	.15076	.14888	.14702	.14336	.13980
7	.13451	.13250	.13051	.12854	.12661	.12470	.12282	.11914	.11555
8	.11651	.11447	.11246	.11048	.10853	.10661	.10472	.10104	.09747
9	.10252	.10046	.09843	.09645	.09449	.09257	.09069	.08702	.08349
10	.09133	.08926	.08723	.08524	.08329	.08138	.07950	.07587	.07238
11	.08218	.08011	.07808	.07609	.07415	.07225	.07039	.06679	.06336
12	.07456	.07249	.07046	.06848	.06655	.06467	.06283	.05928	.05590
13	.06812	.06805	.06403	.06206	.06014	.05828	.05646	.05296	.04965
14	.06260	.06054	.05853	.05657	.05467	.05282	.05102	.04758	.04434
15	.05783	.05577	.05377	.05183	.04994	.04811	.04634	.04296	.03979
16	.05365	.05160	.04961	.04768	.04582	.04402	.04227	.03895	.03586
17	.04997	.04793	.04595	.04404	.04220	.04042	.03870	.03544	.03243
18	.04670	.04467	.04271	.04082	.03899	.03724	.03555	.03236	.02941
19	.04378	.04176	.03981	.03794	.03614	.03441	.03275	.02962	.02675
20	.04116	.03915	.03722	.03536	.03358	.03188	.03024	.02718	.02439
25	.03122	.02928	.02743	.02567	.02401	.02244	.02095	.01823	.01581
30	.02465	.02278	.02102	.01937	.01783	.01639	.01505	.01265	.01059
40	.01656	.01484	.01326	.01183	.01052	.00934	.00828	.00646	.00467
50	.01182	.01026	.00887	.00763	.00655	.00560	.00478	.00344	.00238
60	.00877	.00735	.00613	.00509	.00420	.00345	.00283	.00188	.00121

Formula: $v' = (r/100) \div \{[1 + (r/100)]^n - 1\} = 1/s$.

PRESENT WORTH OF AN ANNUITY

The capital C, which, if placed at interest to-day, will provide for a given annual payment Y for a term of n years before it is exhausted is $C = Y \times w$, where the factor w is given below. (Interest at r per cent. per annum, compounded annually.)

Values of w

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	0.9804	0.9756	0.9709	0.9662	0.9615	0.9569	0.9524	0.9434	0.9346
2	1.9416	1.9274	1.9135	1.8997	1.8861	1.8727	1.8594	1.8334	1.8080
3	2.8839	2.8560	2.8286	2.8016	2.7751	2.7490	2.7232	2.6730	2.6243
4	3.8077	3.7620	3.7171	3.6731	3.6299	3.5875	3.5460	3.4651	3.3872
5	4.7135	4.6458	4.5797	4.5151	4.4518	4.3900	4.3295	4.2124	4.1002
6	5.6014	5.5081	5.4172	5.3286	5.2421	5.1579	5.0757	4.9173	4.7665
7	6.4720	6.3494	6.2303	6.1145	6.0021	5.8927	5.7864	5.5824	5.3893
8	7.3255	7.1701	7.0197	6.8740	6.7327	6.5959	6.4632	6.2098	5.9713
9	8.1622	7.9709	7.7861	7.6077	7.4353	7.2688	7.1078	6.8017	6.5152
10	8.9826	8.7521	8.5302	8.3166	8.1109	7.9127	7.7217	7.3601	7.0236
11	9.7868	9.5142	9.2526	9.0016	8.7605	8.5289	8.3064	7.8869	7.4987
12	10.575	10.258	9.9540	9.6633	9.3851	9.1186	8.8633	8.3838	7.9427
13	11.348	10.983	10.635	10.303	9.9856	9.6829	9.3936	8.8527	8.3577
14	12.106	11.691	11.296	10.921	10.563	10.223	9.8986	9.2950	8.7455
15	12.849	12.381	11.938	11.517	11.118	10.740	10.380	9.7122	9.1079
16	13.578	13.055	12.561	12.094	11.652	11.234	10.838	10.106	9.4466
17	14.292	13.712	13.166	12.651	12.166	11.707	11.274	10.477	9.7632
18	14.992	14.353	13.754	13.190	12.659	12.160	11.690	10.828	10.059
19	15.678	14.979	14.324	13.710	13.134	12.593	12.085	11.158	10.336
20	16.351	15.589	14.877	14.212	13.590	13.008	12.462	11.470	10.594
25	19.523	18.424	17.413	16.482	15.622	14.828	14.094	12.783	11.654
30	22.396	20.930	19.600	18.392	17.292	16.289	15.372	13.765	12.409
40	27.355	25.103	23.115	21.355	19.793	18.402	17.159	15.046	13.332
50	31.424	28.362	25.730	25.456	21.482	19.762	18.256	15.762	13.801
60	34.761	30.909	27.676	24.945	22.623	20.638	18.929	16.161	14.039

Formula: $w = [1 - [1 + (r/100)]^{-n}] \div [r/100] = a/x.$

ANNUITY PROVIDED FOR BY A GIVEN CAPITAL

The annual payment Y provided for for a term of n years by a given capital C placed at interest to-day is $Y = C \times w'$. (Interest at r per cent. per annum, compounded annually; the fund supposed to be exhausted at the end of the term.)

Values of w'

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
2	.51505	.51883	.52261	.52640	.53020	.53400	.53780	.54544	.55309
3	.34675	.35014	.35353	.35693	.36035	.36377	.36721	.37411	.38105
4	.26262	.26582	.26903	.27225	.27549	.27874	.28201	.28859	.29523
5	.21216	.21525	.21835	.22148	.22463	.22779	.23097	.23740	.24389
6	.17853	.18155	.18460	.18767	.19076	.19388	.19702	.20336	.20980
7	.15451	.15750	.16051	.16354	.16661	.16970	.17282	.17914	.18555
8	.13651	.13947	.14246	.14548	.14853	.15161	.15472	.16104	.16747
9	.12252	.12546	.12843	.13145	.13449	.13757	.14069	.14702	.15349
10	.11133	.11426	.11723	.12024	.12329	.12638	.12950	.13587	.14238
11	.10218	.10511	.10808	.11109	.11415	.11725	.12039	.12679	.13336
12	.09456	.09749	.10046	.10348	.10655	.10967	.11283	.11928	.12590
13	.08812	.09105	.09403	.09706	.10014	.10328	.10646	.11296	.11965
14	.08260	.08554	.08853	.09157	.09467	.09782	.10102	.10758	.11434
15	.07783	.08077	.08377	.08683	.08994	.09311	.09634	.10296	.10979
16	.07365	.07660	.07961	.08268	.08582	.08902	.09227	.09895	.10586
17	.06997	.07293	.07595	.07904	.08220	.08542	.08870	.09544	.10243
18	.06670	.06967	.07271	.07582	.07899	.08224	.08555	.09236	.09941
19	.06378	.06676	.06981	.07294	.07614	.07941	.08275	.08962	.09675
20	.06116	.06415	.06722	.07036	.07358	.07658	.08024	.08718	.09439
25	.05122	.05428	.05743	.06067	.06401	.06744	.07095	.07823	.08581
30	.04465	.04778	.05102	.05437	.05783	.06139	.06505	.07265	.08059
40	.03656	.03984	.04326	.04683	.05052	.05434	.05828	.06646	.07467
50	.03182	.03526	.03887	.04263	.04655	.05060	.05478	.06344	.07238
60	.02877	.03235	.03613	.04009	.04420	.04845	.05283	.06188	.07121

Formula: $w' = [r/100] \div [1 - [1 + (r/100)]^{-n}] = 1/w = v' + (r/100).$

DECIMAL EQUIVALENTS

From minutes and seconds into decimal parts of a degree

0'	0°.0000	0"	0°.0000
1	.0167	1	.0003
2	.0333	2	.0006
3	.05	3	.0008
4	.0667	4	.0011
5'	.0833	5"	.0014
6	.10	6	.0017
7	.1167	7	.0019
8	.1333	8	.0022
9	.15	9	.0025
10'	0°.1667	10"	0°.0028
1	.1833	1	.0031
2	.20	2	.0033
3	.2167	3	.0036
4	.2333	4	.0039
15'	.25	15"	.0042
6	.2667	6	.0044
7	.2833	7	.0047
8	.30	8	.005
9	.3167	9	.0053
20'	0°.3333	20"	0°.0056
1	.35	1	.0058
2	.3667	2	.0061
3	.3833	3	.0064
4	.40	4	.0067
25'	.4167	25"	.0069
6	.4333	6	.0072
7	.45	7	.0075
8	.4667	8	.0078
9	.4833	9	.0081
30'	0°.50	30"	0°.0083
1	.5167	1	.0086
2	.5333	2	.0089
3	.55	3	.0092
4	.5667	4	.0094
35'	.5833	35"	.0097
6	.60	6	.01
7	.6167	7	.0103
8	.6333	8	.0106
9	.65	9	.0108
40'	0°.6667	40"	0°.0111
1	.6833	1	.0114
2	.70	2	.0117
3	.7167	3	.0119
4	.7333	4	.0122
45'	.75	45"	.0125
6	.7667	6	.0128
7	.7833	7	.0131
8	.80	8	.0133
9	.8167	9	.0136
50'	0°.8333	50"	0°.0139
1	.85	1	.0142
2	.8667	2	.0144
3	.8833	3	.0147
4	.90	4	.015
55'	.9167	55"	.0153
6	.9333	6	.0156
7	.95	7	.0158
8	.9667	8	.0161
9	.9833	9	.0164
60'	1.00	60"	0°.0167

From decimal parts of a degree into minutes and seconds (exact values)

0°.00	0'	0°.50	30'
1	0' 36"	1	30' 36"
2	1' 12"	2	31' 12"
3	1' 48"	3	31' 48"
4	2' 24"	4	32' 24"
0°.05	3'	0°.55	33'
6	3' 36"	6	33' 36"
7	4' 12"	7	34' 12"
8	4' 48"	8	34' 48"
9	5' 24"	9	35' 24"
0°.10	6'	0°.60	36'
1	6' 36"	1	36' 36"
2	7' 12"	2	37' 12"
3	7' 48"	3	37' 48"
4	8' 24"	4	38' 24"
0°.15	9'	0°.65	39'
6	9' 36"	6	39' 36"
7	10' 12"	7	40' 12"
8	10' 48"	8	40' 48"
9	11' 24"	9	41' 24"
0°.20	12'	0°.70	42'
1	12' 36"	1	42' 36"
2	13' 12"	2	43' 12"
3	13' 48"	3	43' 48"
4	14' 24"	4	44' 24"
0°.25	15'	0°.75	45'
6	15' 36"	6	45' 36"
7	16' 12"	7	46' 12"
8	16' 48"	8	46' 48"
9	17' 24"	9	47' 24"
0°.30	18'	0°.80	48'
1	18' 36"	1	48' 36"
2	19' 12"	2	49' 12"
3	19' 48"	3	49' 48"
4	20' 24"	4	50' 24"
0°.35	21'	0°.85	51'
6	21' 36"	6	51' 36"
7	22' 12"	7	52' 12"
8	22' 48"	8	52' 48"
9	23' 24"	9	53' 24"
0°.40	24'	0°.90	54'
1	24' 36"	1	54' 36"
2	25' 12"	2	55' 12"
3	25' 48"	3	55' 48"
4	26' 24"	4	56' 24"
0°.45	27'	0°.95	57'
6	27' 36"	6	57' 36"
7	28' 12"	7	58' 12"
8	28' 48"	8	58' 48"
9	29' 24"	9	59' 24"
0°.50	30'	1°.00	60'

0°.000	0".0
1	3".6
2	7".2
3	10".8
4	14".4
0°.005	18"
6	21".6
7	25".2
8	28".8
9	32".4
0°.010	36"

Common fractions				
8 ths	10 ths	32 nds	64 ths	Exact decimal values
			1	.01 5625
		1	2	.03 125
			3	.04 6875
	1	2	4	.06 25
			5	.07 8125
		3	6	.09 375
			7	.10 9375
1	2	4	8	.12 5
			9	.14 0625
		5	10	.15 625
			11	.17 1875
	3	6	12	.18 75
			13	.20 3125
		7	14	.21 875
			15	.23 4375
2	4	8	16	.25
			17	.26 5625
		9	18	.28 125
			19	.29 6875
	5	10	20	.31 25
			21	.32 8125
		11	22	.34 375
			23	.35 9375
3	6	12	24	.37 5
			25	.39 0625
		13	26	.40 625
			27	.42 1875
	7	14	28	.43 75
			29	.45 3125
		15	30	.46 875
			31	.48 4375
4	8	16	32	.50
			33	.51 5625
		17	34	.53 125
			35	.54 6875
	9	18	36	.56 25
			37	.57 8125
		19	38	.59 375
			39	.60 9375
5	10	20	40	.62 5
			41	.64 0625
		21	42	.65 625
			43	.67 1875
	11	22	44	.68 75
			45	.70 3125
		23	46	.71 875
			47	.73 4375
6	12	24	48	.75
			49	.76 5625
		25	50	.78 125
			51	.79 6875
	13	26	52	.81 25
			53	.82 8125
		27	54	.84 375
			55	.85 9375
7	14	28	56	.87 5
			57	.89 0625
		29	58	.90 625
			59	.92 1875
	15	30	60	.93 75
			61	.95 3125
		31	62	.96 875
			63	.98 4375

WEIGHTS AND MEASURES

REVISED BY

H. W. BEARCE

(Originally prepared by Louis A. Fischer)

In the United States the measures of weight and length commonly employed are identical for practical purposes with the corresponding English units, but the capacity measures differ from those now in use in the British Empire, the U. S. gallon being defined as 231 cu in. and the bushel as 2150.42 cu in., whereas the corresponding British Imperial units are, respectively, 277.418 cu in., and 2219.344 cu in. (1 Imp gal = 1.2 U. S. gal, approx; 1 Imp bu = 1.03 U. S. bu, approx).

The metric system of weights and measures was legalized and its use made permissive in the United States by an Act of Congress, passed in 1866. In 1872, by the concurrent action of the principal governments of the world, it was agreed to establish an International Bureau of Weights and Measures near Paris. The convention was held, and the treaty signed in 1875. It was ratified by the United States in 1878.

Prior to 1893, the British Imperial yard was regarded as the real standard of the United States. In 1893, the Office of Weights and Measures (now Bureau of Standards) by executive order fixed the value of the United States yard in terms of the international meter, according to the ratio: one yard = 3600/3937 meters. At the same time, the pound was fixed in terms of the international kilogram, according to the relation: one pound = 453.5924277 grams.

U. S. Customary Weights and Measures

Measures of Length

12 inches	= 1 foot
3 feet	= 1 yard
5½ yards = 16½ feet	= 1 rod, pole, or perch
40 poles = 220 yards	= 1 furlong
8 furlongs = 1,760 yards = 5,280 feet	= 1 mile
3 miles	= 1 league
4 inches	= 1 hand
9 inches	= 1 span

Nautical Units

6,080.20 feet	= 1 nautical mile
6 feet	= 1 fathom
120 fathoms	= 1 cable length
1 nautical mile per hr	= 1 knot

Surveyor's or Gunter's Measure

7.92 inches	= 1 link
100 links = 66 ft = 4 rods	= 1 chain
80 chains	= 1 mile
33⅓ inches	= 1 vara (Texas)

Measures of Area

144 square inches	= 1 square foot
9 square feet	= 1 square yard
30¼ square yards	= 1 square rod, pole, or perch
160 square rods = 10 square chains = 43,560 sq ft = 5,645 sq varas (Texas)	= 1 acre
640 acres = 1 square mile	= 1 "section" of U. S. Govt. surveyed land
1 circular inch = area of circle 1 inch in diameter	= 0.7854 sq in.
1 square inch	= 1.2732 cir in.
1 circular mil	= area of circle 0.001 in. in diam
1,000,000 cir mils	= 1 cir in.

Measures of Volume

1,728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
1 cord of wood	= 128 cu ft
1 perch of masonry	= 16½ to 25 cu ft

U. S. Customary Weights and Measures—(*continued*)

Measures of Volume

Liquid or Fluid Measure

4 gills	= 1 pint
2 pints	= 1 quart
4 quarts	= 1 gallon
7.4805 gallons	= 1 cubic foot

(There is no standard liquid barrel; by trade custom, 1 bbl of petroleum oil, unrefined = 42 gal)

Apothecaries' Liquid Measure

60 minims = 1 liquid dram or drachm
8 drams = 1 liquid ounce
16 ounces = 1 pint

Water Measure

The **miner's inch** is the quantity of water that will pass through an orifice 1 sq in. in cross section under a head of 4 to 6½ in., as fixed by statutes, and varies from 1/40 cu ft to 1/50 cu ft per sec. The units now most in use are 1 cu ft per sec and 1 gal per sec, the U. S. Reclamation Service employing the former. See p. 257.

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

1 std bbl for fruits and vegetables = 7056 cu. in. or 105 dry qt, struck measure

Shipping Measure

1 Register ton = 100 cu ft
1 U. S. shipping ton = 40 cu ft = 32.14 U. S. bu *or* 31.14 Imp bu
1 British shipping ton = 42 cu ft = 32.70 Imp bu or 33.75 U. S. bu

Board Measure

1 board foot = 144 cu in. = volume of board 1 ft sq and 1 in. thick.

The international log rule, based upon ¼ in. kerf, is expressed by the formula

$$X = 0.904762\,(0.22D^2 - 0.71D)$$

where X is the number of board feet in a 4 ft section of a log and D is the top diam in in. In computing the number of board feet in a log the taper is taken at ½ in. per 4 ft linear, and separate computation is made for each 4 ft section.

Weights
(The grain is the same in all systems)

Avoirdupois Weight

16 drams = 437.5 grains	= 1 ounce
16 ounces = 7,000 grains	= 1 pound
100 pounds	= 1 cental
2,000 pounds	= 1 short ton
2,240 pounds	= 1 long ton
1 std lime bbl, small	= 180 lb net
1 std lime bbl, large	= 280 lb net

Also (in Great Britain):

14 pounds	= 1 stone
2 stone = 28 lb	= 1 quarter
4 quarters = 112 lb	= 1 hundredweight (cwt)
20 hundredweight	= 1 long ton

Troy Weight

24 grains	= 1 pennyweight (dwt)
20 pennyweights = 480 grains	= 1 ounce
12 ounces = 5,760 grains	= 1 pound

1 Assay Ton = 29,167 milligrams, or as many milligrams as there are troy ounces in a ton of 2,000 lb avoirdupois. Consequently, the number of milligrams of precious metal yielded by an assay ton of ore gives directly the number of troy ounces that would be obtained from a ton of 2,000 lb avoirdupois.

Apothecaries' Weight

20 grains	= 1 scruple ℈
3 scruples = 60 grains	= 1 dram ʒ
8 drams	= 1 ounce ℥
12 ounces = 5,760 grains	= 1 pound

Weight for Precious Stones

1 carat = 200 milligrams
(Used by almost all important nations.)

Circular Measure

60 seconds	= 1 minute
60 minutes	= 1 degree
90 degrees	= 1 quadrant
360 degrees	= circumference
57.2957795 degrees (= 57° 17′44.806″)	= 1 radian (or angle having arc of length equal to radius)

METRIC SYSTEM

The fundamental units of the metric system are the meter (the unit of length) and the kilogram (the unit of mass). The unit of volume, the cubic decimeter (which was also designated the liter), and the unit of mass, the kilogram, were originally derived from the meter. The kilogram and the meter are now defined independently, and the liter is defined as the volume of a kilogram of water at the temperature of its maximum density, 4 C,

and under a pressure of 76 cm of mercury. The liter is slightly greater than the cubic decimeter, and according to the best information, 1 liter = 1.000027 cubic decimeters.

The U. S. customary lengths, areas, and cubic measures derived from the international meter are based on the relation 1 meter equals 39.37 inches (exactly), or 1 yard equals 0.9144018 meter.

The U. S. customary weights derived from the international kilogram are based on the value 1 avoirdupois lb = 453.5924277 grams. The value of the troy lb is based on the same relation and also the equivalent 5,760/7,000 avoirdupois lb equals 1 troy lb.

Metric Measures

Length			Area		
Unit	Symbol	Value in meters	Unit	Symbol	Value in sq. meters
Micron	μ	0.000001	...	...	...
Millimeter	mm	0.001	Sq millimeter	mm^2	0.000001
Centimeter	cm	0.01	Sq centimeter	cm^2	0.0001
Decimeter	dm	0.1	Sq decimeter	dm^2	0.01
Meter (unit)	m	1.0	Sq meter (centiare)	m^2	1.0
Dekameter	dkm	10.0	Sq dekameter (are)	a	100.0
Hectometer	hm	100.0	Hectare	ha	10,000.0
Kilometer	km	1,000.0	Sq kilometer	km^2	1,000,000.0
Myriameter	Mm	10,000.0	...	...	...
Megameter	...	1,000,000.0	...	...	...

Volume			Cubic measure		
Unit	Symbol	Value in liters	Unit	Symbol	Value in cubic meters
Milliliter	ml	0.001	Cubic kilometer	km^3	10^9
Liter (unit)	l	1.0	Cubic hectometer	hm^3	10^6
Kiloliter	kl	1,000.0	Cubic dekameter	dkm^3	10^3
Also					
Centiliter	cl	0.01	Cubic meter	m^3	1
Deciliter	dl	0.1	Cubic decimeter	dm^3	10^{-3}
Dekaliter	dkl	10.0	Cubic centimeter	cm^3	10^{-6}
Hectoliter	hl	100.0	Cubic millimeter	mm^3	10^{-9}
...	...	...	Cubic micron	μ^3	10^{-18}

Mass					
Unit	Symbol	Value in grams	Unit	Symbol	Value in grams
Microgram	γ	0.000001	Dekagram	dkg	10.0
Milligram	mg	0.001	Hectogram	hg	100.0
Centigram	cg	0.01	Kilogram	kg	1,000.0
Decigram	dg	0.1	Myriagram	Mg	10,000.0
Gram (unit)	g	1.0	Quintal	q	100,000.0
...	...	...	Ton	t	1,000,000.0

The unit of **force** defined dynamically is the dyne (p. 73); the megadyne is one million dynes. The unit of force defined by gravity is the kilogram force = 980×10^3 dynes approximately. A subsidiary unit is the milligram (gram) [ton] force which is equal to 10^{-6} (10^{-3}) [10^3] kilogram force.

SYSTEMS OF UNITS

The principal units of interest to mechanical engineers can all be derived from the three fundamental units of **force, length,** and **time.** These three fundamental units may be chosen at pleasure; each such choice gives rise to a "system" of units. The following table gives the units of the four "systems" most often met with in the literature.

The precise definitions of the units of force in these systems are as follows. In these definitions the "standard pound body" and the "standard kilogram body" refer to two material standards of mass, carefully preserved at London and Paris, respectively (the U. S. pound is derived from the kilogram); the "standard locality" means sea level, 45 deg latitude, or, more strictly, any locality in which the acceleration due to gravity has the value 980.665 cm per sec² = 32.1740 ft per sec², which may be called the standard acceleration.

The pound force is the force required to support the standard pound body against gravity, *in vacuo*, in the standard locality; or, it is the force which, if applied to the standard pound body, supposed free to move, would give that body the "standard acceleration." The word "pound" is used for the unit of both force and mass and consequently is ambiguous. To avoid uncertainty it is desirable to call the units "pound force" and "pound mass," respectively.

The kilogram force is the force required to support the standard kilogram against gravity, *in vacuo*, in the standard locality; or, it is the force which, if applied to the standard kilogram body, supposed free to move, would give that body the "standard acceleration." The word "kilogram" is used for the unit of both force and mass and consequently is ambiguous. To avoid uncertainty it is desirable to call the units "kilogram force" and "kilogram mass," respectively.

The poundal is the force which, if applied to the standard pound body, would give that body an acceleration of 1 ft per sec²; that is, 1 poundal = 1/32.1740 lb force.

The dyne is the force which, if applied to the standard gram body, would give that body an acceleration of 1 cm per sec²; that is, 1 dyne = 1/980.665 of a gram force.

Systems of Units

Name of unit	Dimensions of units in terms of F, L, T	British "gravitational" system, or "foot-pound-second" system	Metric "gravitational" system, or "kilogram-meter-second" system	Metric "absolute" system, or "C. G. S." system	British "absolute" system (little used)
Force	F	1 lb	1 kg	1 dyne	1 poundal
Length	L	1 ft	1 m	1 cm	1 ft
Time	T	1 sec	1 sec	1 sec	1 sec
Velocity	L/T	1 ft per sec	1 m per sec	1 cm per sec	1 ft per sec
Acceleration	L/T^2	1 ft per sec²	1 m per sec²	1 cm per sec²	1 ft per sec²
Pressure	F/L^2	1 lb per ft²	1 kg per m²	1 dyne per cm²	1 pdl per ft²
Impulse or momentum	FT	1 lb-sec	1 kg-sec	1 dyne-sec	1 pdl-sec
Work or energy	FL	1 ft-lb	1 kg-m	1 dyne-cm = 1 "erg"	1 ft-pdl
Power	FL/T	1 ft-lb per sec	1 kg-m per sec	1 dyne-cm per sec	1 ft-pdl per sec
Mass	$F/(L/T^2)$	1 lb per (ft per sec²) = 1 "slug."	1 kg per (m per sec²) = 1 "metric" slug."	1 dyne per (cm per sec²) = 1 gram mass.	1 pdl per (ft per sec²) = 1 pound mass.

NOTE. The "slug" (also called the "geepound," or the "engineer's unit of mass"), the "metric slug," and the "poundal" are never used in practice.

Other common units are as follows:

Work: 1 joule (absolute) = 10^7 ergs = 10,000,000 dyne-cm
1 kilowatt-hour (international) = 3,600,000 joules (international) = 3,600.864 $\times 10^{10}$ dyne-cm = 3,600,864 joules (absolute)

Power: 1 horsepower = 550 ft-lb per sec
1 poncelet = 100 kg-m per sec
1 cheval-vapeur = 75 kg-m per sec = 1 metric horsepower = 1 Pferde Starke
1 watt (international) = 1 joule (international) per sec = 1.00024 $\times 10^7$ dyne-cm per sec = 1.00024 joules (absolute) per sec
1 kilowatt = 1,000 watts = 10^{10} dyne-cm per sec

A new horsepower of 550.220 ft-lb per sec, or 746 watts, has been proposed, but has not been accepted by mechanical engineers.

The *weight* of a body (in a given locality) means a *force*, namely, the force, required to support the body against gravity (in that locality). When no particular locality

is specified, the standard locality may be assumed. Thus, the "standard weight" of the pound body is 1 lb force; the "standard weight" of the kilogram body is 1 kg force.

Heat Units. The units of heat commonly used are (1) the quantity of heat required to raise the temperature of 1 gram of water 1 C and (2) the quantity of heat required to raise the temperature of 1 pound of water 1 F. These units are (1) the **calorie** and (2) the **British thermal unit** or Btu. Work done in recent years on the International Steam Tables has led to the definition of the **IT calorie** and of the Btu in terms of other units. These definitions are

$$1 \text{ IT calorie} = \tfrac{1}{860} \text{ international watt-hour}$$
$$1 \text{ Btu} = 251.996 \text{ IT calories}$$

These units have been used on the following pages. The **kilocalorie**, some-

Force Equivalents

Dynes × 10⁶	Kilograms	Pounds	Poundals
1	1.020 0.00848	2.248 0.3518	72.33 1.85933
0.9807 $\bar{1}$.99149	1	2.205 0.34334	70.93 1.85084
0.4448 $\bar{1}$.64819	0.4536 $\bar{1}$.65667	1	32.17 1.50750
0.01383 $\bar{2}$.14067	0.01410 $\bar{2}$.14916	0.03108 $\bar{2}$.49249	1

CONVERSION TABLES

Length Equivalents

Centimeters	Inches	Feet	Yards	Meters	Chains	Kilometers	Miles
1	0.3937 $\bar{1}$.59517	0.03281 $\bar{2}$.51598	0.01094 $\bar{2}$.03886	0.01 $\bar{2}$.00000	0.0_44971 $\bar{4}$.69644	10^{-5} $\bar{5}$.00000	0.0_56214 $\bar{6}$.79335
2.540 0.40483	1	0.08333 $\bar{2}$.92082	0.02778 $\bar{2}$.44370	0.0254 $\bar{2}$.40483	0.001263 $\bar{3}$.10127	0.0_4254 $\bar{5}$.40483	0.0_41578 $\bar{5}$.19818
30.48 1.48402	12 1.07918	1	0.3333 $\bar{1}$.52288	0.3048 $\bar{1}$.48402	0.01515 $\bar{2}$.18046	0.0_33048 $\bar{4}$.48402	0.0_31894 $\bar{4}$.27736
91.44 1.96114	36 1.55630	3 0.47712	1	0.9144 $\bar{1}$.96114	0.04545 $\bar{2}$.65758	0.0_39144 $\bar{4}$.96114	0.0_35682 $\bar{4}$.75449
100 2.00000	39.37 1.59517	3.281 0.51598	1.0936 0.03886	1	0.04971 $\bar{2}$.69644	0.001 $\bar{3}$.00000	0.0_36214 $\bar{4}$.79335
2012 3.30356	792 2.89873	66 1.81954	22 1.34242	20.12 1.30356	1	0.02012 $\bar{2}$.30356	0.0125 $\bar{2}$.09691
100000 5.00000	39370 4.59517	3281 3.51598	1093.6 3.03886	1000 3.00000	49.71 1.69644	1	0.6214 $\bar{1}$.79335
160935 5.20665	63360 4.80182	5280 3.72263	1760 3.24551	1609 3.20665	80 1.90309	1.609 0.20665	1

"For the use of these tables see notes at bottom of p. 76."

Conversion of Lengths*

	Inches to millimeters	Millimeters to inches	Feet to meters	Meters to feet	Yards to meters	Meters to yards	Miles to kilometers	Kilometers to miles
1	25.40	0.03937	0.3048	3.281	0.9144	1.094	1.609	0.6214
2	50.80	0.07874	0.6096	6.562	1.829	2.187	3.219	1.243
3	76.20	0.1181	0.9144	9.842	2.743	3.281	4.828	1.864
4	101.60	0.1575	1.219	13.12	3.658	4.374	6.437	2.485
5	127.00	0.1968	1.524	16.40	4.572	5.468	8.047	3.107
6	152.40	0.2362	1.829	19.68	5.486	6.562	9.656	3.728
7	177.80	0.2756	2.134	22.97	6.401	7.655	11.27	4.350
8	203.20	0.3150	2.438	26.25	7.315	8.749	12.87	4.971
9	228.60	0.3543	2.743	29.53	8.230	9.842	14.48	5.592

* Example: 1 in. = 25.40 mm.

times called the kilogram calorie or large calorie, is equal to 1,000 calories and is used in engineering work in metric countries. The calorie is sometimes called the small calorie or gram calorie.

The mean calorie (0 to 100 C) is about 1.001 IT calorie, and the corresponding mean Btu is approximately 779 ft-lb.

Mechanical Equivalent of Heat. The values now accepted as the work equivalents of the heat units are 778.2 ft-lb for the Btu and 4.187 absolute joules for the IT calorie.

Conversion of Lengths: Inches and Millimeters

Common fractions of an inch to millimeters (From 1/64 to 1 in.)

64ths	Millimeters	64ths	Millimeters	64ths	Millimeters	64ths	Millimeters	64ths	Millimeters	64ths	Millimeters
1	0.397	13	5.159	25	9.922	37	14.684	49	19.447	57	22.622
2	0.794	14	5.556	26	10.319	38	15.081	50	19.844	58	23.019
3	1.191	15	5.953	27	10.716	39	15.478	51	20.241	59	23.416
4	1.588	16	6.350	28	11.113	40	15.875	52	20.638	60	23.813
5	1.984	17	6.747	29	11.509	41	16.272	53	21.034	61	24.209
6	2.381	18	7.144	30	11.906	42	16.669	54	21.431	62	24.606
7	2.778	19	7.541	31	12.303	43	17.066	55	21.828	63	25.003
8	3.175	20	7.938	32	12.700	44	17.463	56	22.225	64	25.400
9	3.572	21	8.334	33	13.097	45	17.859				
10	3.969	22	8.731	34	13.494	46	18.256				
11	4.366	23	9.128	35	13.891	47	18.653				
12	4.763	24	9.525	36	14.288	48	19.050				

Decimals of an inch to millimeters. (From 0.01 in. to 0.09 in.)

	0	1	2	3	4	5	6	7	8	9
.0		0.254	0.508	0.762	1.016	1.270	1.524	1.778	2.032	2.286
.1	2.540	2.794	3.048	3.302	3.556	3.810	4.064	4.318	4.572	4.826
.2	5.080	5.334	5.588	5.842	6.096	6.350	6.604	6.858	7.112	7.366
.3	7.620	7.874	8.128	8.382	8.636	8.890	9.144	9.398	9.652	9.906
.4	10.160	10.414	10.668	10.922	11.176	11.430	11.684	11.938	12.192	12.446
.5	12.700	12.954	13.208	13.462	13.716	13.970	14.224	14.478	14.732	14.986
.6	15.240	15.494	15.748	16.002	16.256	16.510	16.764	17.018	17.272	17.526
.7	17.780	18.034	18.288	18.542	18.796	19.050	19.304	19.558	19.812	20.066
.8	20.320	20.574	20.828	21.082	21.336	21.590	21.844	22.098	22.352	22.606
.9	22.860	23.114	23.368	23.622	23.876	24.130	24.384	24.638	24.892	25.146

Millimeters to decimals of an inch. (From 1 to 99 mm)

	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0		0.0394	0.0787	0.1181	0.1575	0.1968	0.2362	0.2756	0.3150	0.3543
1	0.3937	0.4331	0.4724	0.5118	0.5512	0.5986	0.6299	0.6693	0.7087	0.7480
2	0.7874	0.8268	0.8661	0.9055	0.9449	0.9842	1.0236	1.0630	1.1024	1.1417
3	1.1811	1.2205	1.2598	1.2992	1.3386	1.3780	1.4173	1.4567	1.4961	1.5354
4	1.5748	1.6142	1.6535	1.6929	1.7323	1.7716	1.8110	1.8504	1.8898	1.9291
5	1.9685	2.0079	2.0472	2.0866	2.1260	2.1654	2.2047	2.2441	2.2835	2.3228
6	2.3622	2.4016	2.4409	2.4803	2.5197	2.5590	2.5984	2.6378	2.6772	2.7165
7	2.7559	2.7953	2.8346	2.8740	2.9134	2.9528	2.9921	3.0315	3.0709	3.1102
8	3.1496	3.1890	3.2283	3.2677	3.3071	3.3464	3.3858	3.4252	3.4646	3.5039
9	3.5433	3.5827	3.6220	3.6614	3.7008	3.7402	3.7795	3.8189	3.8583	3.8976

Area Equivalents

(For conversion table see p. 77)

Square meters	Square inches	Square feet	Square yards	Square rods	Square chains	Roods	Acres	Square miles or sections
1	**1550** 3.19033	**10.76** 1.03197	**1.196** 0.07773	**0.0395** $\bar{2}.59699$	**0.002471** $\bar{3}.39288$	**0.0_39884** $\bar{3}.99494$	**0.0_32471** $\bar{4}.39288$	**0.0_63861** $\bar{7}.58670$
0.0_36452 $\bar{4}.80967$	**1**	**0.006944** $\bar{3}.84164$	**0.0_37716** $\bar{4}.88740$	**0.0_42551** $\bar{5}.40667$	**0.0_61594** $\bar{6}.20255$	**0.0_66377** $\bar{7}.80401$	**0.0_61594** $\bar{7}.20255$	**0.0_92491** $\overline{10}.39637$
0.09290 $\bar{2}.96803$	**144** 2.15836	**1**	**0.1111** $\bar{1}.04570$	**0.003673** $\bar{3}.56503$	**0.0_32296** $\bar{4}.36091$	**0.0_49184** $\bar{5}.96207$	**0.0_42296** $\bar{4}.36091$	**0.0_73587** $\bar{8}.55473$
0.8361 $\bar{1}.92227$	**1296** 3.11260	**9** 0.95424	**1**	**0.03306** $\bar{2}.51927$	**0.002066** $\bar{3}.31515$	**0.0_38264** $\bar{4}.01721$	**0.0002066** $\bar{4}.31515$	**0.0_63228** $\bar{7}.50898$
25.29 1.40300	**39204** 4.59333	**272.25** 2.43497	**30.25** 1.48072	**1**	**0.0625** $\bar{2}.79588$	**0.02500** $\bar{2}.39794$	**0.00625** $\bar{3}.79588$	**0.0_59766** $\bar{6}.98970$
404.7 2.60712	**627264** 5.79745	**4356** 3.63909	**484.** 2.68484	**16** 1.20412	**1**	**0.4** $\bar{1}.60206$	**0.1** $\bar{1}.00000$	**0.0001562** $\bar{4}.19382$
1012 3.00506	**1568160** 6.19539	**10890** 4.03703	**1210** 3.08278	**40** 1.60206	**2.5** 0.39794	**1**	**0.25** $\bar{1}.39794$	**0.0_33906** $\bar{4}.59176$
4047 3.60712	**6272640** 6.79745	**43560** 4.63909	**4840** 3.68484	**160** 2.20412	**10** 1.00000	**4** 0.60206	**1**	**0.001562** $\bar{3}.19382$
2589998 6.41330		**27878400** 7.44527	**3097600** 6.49103	**102400** 5.01030	**6400** 3.80618	**2560** 3.40824	**640** 2.80618	**1**

(1 hectare = 100 ares = 10,000 centiares or square meters)

Volume and Capacity Equivalents

(For conversion table see p. 77)

Cubic inches	Cubic feet	Cubic yards	U. S. Apothecary liquid ounces	U. S. quarts		U. S. gallons		Bushels U. S.	Liters (l)
				Liquid	Dry	Liquid	Dry		
1	**0.0_35787** $\bar{4}.76246$	**0.0_42143** $\bar{5}.33100$	**0.5541** $\bar{1}.74360$	**0.01732** $\bar{2}.23845$	**0.01488** $\bar{2}.17263$	**0.0_24329** $\bar{3}.63639$	**0.0_23720** $\bar{3}.57057$	**0.0_34650** $\bar{4}.66748$	**0.01639** $\bar{2}.21450$
1728 3.23754	**1**	**0.03704** $\bar{2}.56864$	**957.5** 2.98114	**29.92** 1.47599	**25.71** 1.41017	**7.481** 0.87393	**6.429** 0.80811	**0.8036** $\bar{1}.90502$	**28.32** 1.45205
46656 4.66891	**27** 1.43136	**1**	**25853** 4.41251	**807.9** 2.90736	**694.3** 2.84153	**202.0** 2.30530	**173.6** 2.23948	**21.70** 1.33638	**764.6** 2.88341
1.805 0.25640	**0.001044** $\bar{3}.01886$	**0.0_43868** $\bar{5}.58749$	**1**	**0.03125** $\bar{2}.49485$	**0.02686** $\bar{2}.42903$	**0.007813** $\bar{3}.89279$	**0.006714** $\bar{3}.82697$	**0.0_38392** $\bar{4}.92388$	**0.02957** $\bar{2}.47091$
57.75 1.76155	**0.03342** $\bar{2}.52401$	**0.001238** $\bar{3}.09264$	**32** 1.50515	**1**	**0.8594** $\bar{1}.93418$	**0.25** $\bar{1}.39794$	**0.2148** $\bar{1}.33212$	**0.02686** $\bar{2}.42903$	**0.9464** $\bar{1}.97606$
67.20 1.82737	**0.03889** $\bar{2}.58983$	**0.001440** $\bar{3}.15847$	**37.24** 1.57097	**1.164** 0.06582	**1**	**0.2909** $\bar{1}.46376$	**0.25** $\bar{1}.39794$	**0.03125** $\bar{2}.49485$	**1.101** 0.04188
231 2.36361	**0.1337** $\bar{1}.12607$	**0.004951** $\bar{3}.69470$	**128** 2.10721	**4** 0.60206	**3.437** 0.53624	**1**	**0.8594** $\bar{1}.93418$	**0.1074** $\bar{1}.03109$	**3.785** 0.57812
268.8 2.42943	**0.1556** $\bar{1}.19189$	**0.005761** $\bar{3}.76053$	**148.9** 2.17303	**4.655** 0.66788	**4** 0.60206	**1.164** 0.06582	**1**	**0.125** $\bar{1}.09691$	**4.405** 0.64394
2150 3.33252	**1.244** 0.09498	**0.04609** $\bar{2}.66362$	**1192** 3.07612	**37.24** 1.57097	**32** 1.50515	**9.309** 0.96891	**8** 0.90309	**1**	**35.24** 1.54703
61.02 1.78550	**0.03531** $\bar{2}.54795$	**0.001308** $\bar{3}.11659$	**33.01** 1.52909	**1.057** 0.02394	**0.9081** $\bar{1}.95812$	**0.2642** $\bar{1}.42188$	**0.2270** $\bar{1}.35606$	**0.02838** $\bar{2}.45297$	**1**

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below. In some cases the equivalents have been rounded off, while the logarithm corresponds to the equivalent carried to a greater number of decimal places.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Conversion of Areas*

	Sq in. to sq cm	Sq cm to sq in.	Sq ft to sq m	Sq m to sq ft	Sq yd to sq m	Sq m to sq yd	Acres to hectares	Hectares to acres	Sq mi to sq km	Sq km to sq mi
1	6.452	0.1550	0.0929	10.76	0.8361	1.196	0.4047	2.471	2.590	0.3861
2	12.90	0.3100	0.1858	21.53	1.672	2.392	0.8094	4.942	5.180	0.7722
3	19.35	0.4650	0.2787	32.29	2.508	3.588	1.214	7.413	7.770	1.158
4	25.81	0.6200	0.3716	43.06	3.345	4.784	1.619	9.884	10.360	1.544
5	32.26	0.7750	0.4645	53.82	4.181	5.980	2.023	12.355	12.950	1.931
6	38.71	0.9300	0.5574	64.58	5.017	7.176	2.428	14.826	15.540	2.317
7	45.16	1.085	0.6503	75.35	5.853	8.372	2.833	17.297	18.130	2.703
8	51.61	1.240	0.7432	86.11	6.689	9.568	3.237	19.768	20.720	3.089
9	58.06	1.395	0.8361	96.87	7.525	10.764	3.642	22.239	23.310	3.475

Conversion of Volumes or Cubic Measure*

	Cu in. to cu cm	Cu cm to cu in.	Cu ft to cu m	Cu m to cu ft	Cu yd. to Cu. m	Cu m to cu yd	Gallons to cu ft	Cu ft to gallons
1	16.39	0.06102	0.02832	35.31	0.7646	1.308	0.1337	7.481
2	32.77	0.1220	0.05663	70.63	1.529	2.616	0.2674	14.96
3	49.16	0.1831	0.08495	105.9	2.294	3.924	0.4010	22.44
4	65.55	0.2441	0.1133	141.3	3.058	5.232	0.5347	29.92
5	81.94	0.3051	0.1416	176.6	3.823	6.540	0.6684	37.40
6	98.32	0.3661	0.1699	211.9	4.587	7.848	0.8021	44.88
7	114.7	0.4272	0.1982	247.2	5.352	9.156	0.9358	52.36
8	131.1	0.4882	0.2265	282.5	6.116	10.46	1.069	59.84
9	147.5	0.5492	0.2549	317.8	6.881	11.77	1.203	67.32

Conversion of Volumes or Capacities*

	Liquid ounces to cu cm	Cu cm to liquid ounces	Pints to liters	Liters to pints	Quarts to liters	Liters to quarts	Gallons to liters	Liters to gallons	Bushels to hectoliters	Hectoliters to bushels
1	29.57	0.03381	0.4732	2.113	0.9463	1.057	3.785	0.2642	0.3524	2.838
2	59.15	0.06763	0.9463	4.227	1.893	2.113	7.571	0.5284	0.7048	5.676
3	88.72	0.1014	1.420	6.340	2.839	3.170	11.36	0.7925	1.057	8.513
4	118.3	0.1353	1.893	8.454	3.785	4.227	15.14	1.057	1.410	11.35
5	147.9	0.1691	2.366	10.57	4.732	5.284	18.93	1.321	1.762	14.19
6	177.4	0.2029	2.839	12.68	5.678	6.340	22.71	1.585	2.114	17.03
7	207.0	0.2367	3.312	14.79	6.624	7.397	26.50	1.849	2.467	19.86
8	236.6	0.2705	3.785	16.91	7.571	8.454	30.28	2.113	2.819	22.70
9	266.2	0.3043	4.259	19.02	8.517	9.510	34.07	2.378	3.171	25.54

Conversion of Masses*

	Grains to grams	Grams to grains	Ounces (avoir.) to grams	Grams to ounces (avoir.)	Pounds (avoir.) to kilograms	Kilograms to pounds (avoir.)	Short tons (2,000 lb) to metric tons	Metric tons (1,000 kg) to short tons	Long tons (2,240 lb) to metric tons	Metric tons to long tons
1	0.06480	15.43	28.35	0.03527	0.4536	2.205	0.907	1.102	1.016	0.984
2	0.1296	30.86	56.70	0.07055	0.9072	4.409	1.814	2.205	2.032	1.968
3	0.1944	46.30	85.05	0.1058	1.361	6.614	2.722	3.307	3.048	2.953
4	0.2592	61.73	113.40	0.1411	1.814	8.818	3.629	4.409	4.064	3.937
5	0.3240	77.16	141.75	0.1764	2.268	11.02	4.536	5.512	5.080	4.921
6	0.3888	92.59	170.10	0.2116	2.722	13.23	5.443	6.614	6.096	5.905
7	0.4536	108.03	198.45	0.2469	3.175	15.43	6.350	7.716	7.112	6.889
8	0.5184	123.46	226.80	0.2822	3.629	17.64	7.257	8.818	8.128	7.874
9	0.5832	138.89	255.15	0.3175	4.082	19.84	8.165	9.921	9.144	8.858

Example: 1 sq in. = 6.452 sq cm.

Velocity Equivalents

(For conversion table see p. 80)

Centimeters per sec.	Meters per sec.	Meters per min.	Kilometers per hour	Feet per sec.	Feet per min.	Miles per hour	Knots
1	0.01	0.6 $\bar{1}$.77815	0.036 $\bar{2}$.55630	0.03281 $\bar{2}$.51598	1.9685 0.29414	0.02237 $\bar{2}$.34965	0.01943 $\bar{2}$.28836
100 2.00000	**1**	60 1.77815	3.6 0.55630	3.281 0.51598	196.85 2.29414	2.237 0.34965	1.943 0.28836
1.667 0.22184	0.01667 $\bar{2}$.22184	**1**	0.06 $\bar{2}$.77815	0.05468 $\bar{2}$.73783	3.281 0.51598	0.03728 $\bar{2}$.57150	0.03238 $\bar{2}$.51022
27.78 1.44370	0.2778 $\bar{1}$.44370	16.67 1.22184	**1**	0.9113 $\bar{1}$.95968	54.68 1.73783	0.6214 $\bar{1}$.79335	0.53960 $\bar{1}$.73207
30.48 1.48402	0.3048 $\bar{1}$.48402	18.29 1.26217	1.097 0.04032	**1**	60 1.77815	0.6818 $\bar{1}$.83367	0.59209 $\bar{1}$.77238
0.5080 $\bar{1}$.70586	0.005080 $\bar{3}$.70586	0.3048 $\bar{1}$.48402	0.01829 $\bar{2}$.26217	0.01667 $\bar{2}$.22185	**1**	0.01136 $\bar{2}$.05553	0.00987 $\bar{3}$.99423
44.70 1.65035	0.4470 $\bar{1}$.65035	26.82 1.42850	1.609 0.20670	1.467 0.16633	88 1.94448	**1**	0.86839 $\bar{1}$.93871
51.479 1.71163	0.51479 $\bar{1}$.71163	30.887 1.48978	1.8532 0.26703	1.68894 0.22761	101.337 2.00577	1.15155 0.06128	**1**

Mass Equivalents

(For conversion table see p. 77)

Kilograms	Grains	Ounces		Pounds		Tons		
		Troy and apoth.	Avoirdupois	Troy and apoth.	Avoirdupois	Short	Long	Metric
1	15432 4.18843	32.15 1.50719	35.27 1.54745	2.6792 0.42801	2.205 0.34333	$0.0_2 1102$ $\bar{3}$.04230	$0.0_3 9842$ $\bar{4}$.99309	0.001 $\bar{3}$.00000
$0.0_4 6480$ $\bar{5}$.81157	**1**	$0.0_2 2083$ $\bar{3}$.31876	$0.0_2 2286$ $\bar{3}$.35002	$0.0_3 1736$ $\bar{4}$.23958	$0.0_3 1429$ $\bar{4}$.15490	$0.0_7 7143$ $\bar{8}$.85387	$0.0_7 6378$ $\bar{8}$.80465	$0.0_7 6480$ $\bar{8}$.81157
0.03110 $\bar{2}$.49281	480 2.68124	**1**	1.09714 0.04026	0.08333 $\bar{2}$.92082	0.06857 $\bar{2}$.83614	$0.0_4 3429$ $\bar{5}$.53511	$0.0_4 3061$ $\bar{5}$.48590	$0.0_4 3110$ $\bar{5}$.49281
0.02835 $\bar{2}$.45255	437.5 2.64098	0.9115 $\bar{1}$.95974	**1**	0.07595 $\bar{2}$.88056	0.0625 $\bar{2}$.79588	$0.0_4 3125$ $\bar{5}$.49485	$0.0_4 2790$ $\bar{5}$.44563	$0.0_4 2835$ $\bar{5}$.45255
0.3732 $\bar{1}$.57199	5760 3.76042	12 1.07918	13.17 1.11944	**1**	0.8229 $\bar{1}$.91532	$0.0_3 4114$ $\bar{4}$.61429	$0.0_3 3673$ $\bar{4}$.56508	$0.0_3 3732$ $\bar{4}$.57199
0.4536 $\bar{1}$.65667	7000 3.84510	14.58 1.16386	16 1.20412	1.215 0.08468	**1**	0.0005 $\bar{4}$.69897	$0.0_3 4464$ $\bar{4}$.64975	$0.0_3 4536$ $\bar{4}$.65667
907.2 2.95770	140_6 7.14613	29167 4.46489	320_3 4.50515	2431 3.38571	2000 3.30103	**1**	0.8929 $\bar{1}$.95078	0.9072 $\bar{1}$.95770
1016 3.00691	15680_4 7.19535	32667 4.51411	35840 4.55437	2722 3.43492	2240 3.35025	1.12 0.04922	**1**	1.016 0.00691
1000 3.00000	15432356 7.18843	32151 4.50719	35274 4.54745	2679 3.42801	2205 3.34333	1.102 0.04230	0.9842 $\bar{1}$.99309	**1**

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below. In some cases the equivalents have been rounded off, while the logarithm corresponds to the equivalent carried to a greater number of decimal places.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Pressure Equivalents

(For conversion table see p. 80)

Megabars or megadynes per sq cm	Kilograms per sq cm (metric atmospheres)	Pounds per sq in.	Short tons per sq ft	Atmospheres	Columns of mercury at temperature 0 C and g = 980.665 cm per sec²		Columns of water at temperature 15 C and g = 980.665 cm per sec²		
					Meters	Inches	Meters	Inches	Feet
1	1.0197 0.00848	14.50 1.16148	1.044 0.01882	0.9869 $\bar{1}.99427$	0.7501 $\bar{1}.87510$	29.53 1.47025	10.21 1.00886	401.8 2.60402	33.49 1.52485
0.9807 $\bar{1}.99152$	1	14.22 1.15300	1.024 0.01034	0.9678 $\bar{1}.98579$	0.7356 $\bar{1}.86662$	28.96 1.46177	10.01 1.00038	394.1 2.59556	32.84 1.51636
0.06895 $\bar{2}.83852$	0.07031 $\bar{2}.84700$	1	0.072 $\bar{2}.85733$	0.06805 $\bar{2}.83280$	0.05171 $\bar{2}.71360$	2.036 0.30876	0.7037 $\bar{1}.84738$	27.70 1.44254	2.309 0.36336
0.9576 $\bar{1}.98119$	0.9765 $\bar{1}.98966$	13.89 1.14267	1	0.9451 $\bar{1}.97547$	0.7183 $\bar{1}.85629$	28.28 1.45143	9.774 0.99006	384.8 2.58521	32.07 1.50604
1.0133 0.00573	1.0332 0.01420	14.70 1.16722	1.058 0.02453	1	0.76 $\bar{1}.88081$	29.92 1.47598	10.34 1.01459	407.2 2.60976	33.93 1.53058
1.3332 0.12490	1.3595 0.13338	19.34 1.28640	1.392 0.14373	1.316 0.11919	1	39.37 1.59517	13.61 1.13378	535.7 2.72894	44.64 1.64976
0.03386 $\bar{2}.52975$	0.03453 $\bar{2}.53823$	0.4912 $\bar{1}.69124$	0.03536 $\bar{2}.54857$	0.03342 $\bar{2}.52402$	0.02540 $\bar{2}.40484$	1	0.3456 $\bar{1}.53861$	13.61 1.13378	1.134 0.05460
0.09798 $\bar{2}.99114$	0.09991 $\bar{2}.99962$	1.421 0.15262	0.1023 $\bar{1}.00990$	0.09668 $\bar{2}.98535$	0.07349 $\bar{2}.86622$	2.893 0.46139	1	39.37 1.59517	3.281 0.51598
0.002489 $\bar{3}.39598$	0.002536 $\bar{3}.40446$	0.03609 $\bar{2}.55745$	0.002599 $\bar{3}.41479$	0.002456 $\bar{3}.39024$	0.001867 $\bar{3}.27106$	0.07349 $\bar{2}.86622$	0.02540 $\bar{2}.40484$	1	0.08333 $\bar{2}.92082$
0.02986 $\bar{2}.47516$	0.03045 $\bar{2}.48364$	0.4331 $\bar{1}.63063$	0.03119 $\bar{2}.49397$	0.02947 $\bar{2}.46942$	0.02240 $\bar{2}.35024$	0.8819 $\bar{1}.94540$	0.3048 $\bar{1}.48402$	12 1.07918	1

Energy or Work Equivalents

(For conversion table see p. 80)

Joules	Kilogram-meters	Foot-pounds	Kilo-watt-hours	Metric horse-power-hours	Horse-power-hours	Liter-atmos-pheres	Kilo-calories	British thermal units
1	0.10197 $\bar{1}.00848$	0.7376 $\bar{1}.86780$	0.0_62777 $\bar{7}.44359$	0.0_63777 $\bar{7}.57711$	0.0_63725 $\bar{7}.57113$	0.009869 $\bar{3}.99427$	0.0_32388 $\bar{4}.37800$	0.0_39478 $\bar{4}.97670$
9.80665 0.9915207	1	7.233 0.85932	0.0_52723 $\bar{6}.43511$	0.0_537037 $\bar{6}.56863$	0.0_53653 $\bar{6}.56265$	0.09678 $\bar{2}.98579$	0.002342 $\bar{3}.36961$	0.009294 $\bar{3}.96822$
1.356 0.13220	0.1383 $\bar{1}.14068$	1	0.0_63765 $\bar{7}.57580$	0.0_651206 $\bar{7}.70932$	0.0_650505 $\bar{7}.70333$	0.01338 $\bar{2}.12647$	0.0_33238 $\bar{4}.51029$	0.001285 $\bar{3}.10890$
3.601×10^6 6.55641	3.672×10^5 5.56489	2.656×10^6 6.42420	1	1.3599 0.13352	1.341 0.12743	35537 4.55068	860.0 2.93450	3413 3.53310
2.648×10^6 6.42288	270000 5.43136	1.9529×10^6 6.29068	0.7353 $\bar{1}.86648$	1	0.9863 $\bar{1}.99401$	26131 4.41715	632.4 2.80098	2509 3.39958
2.6845×10^6 6.42887	2.7375×10^5 5.43735	1.98×10^6 6.29667	0.7455 $\bar{1}.87246$	1.0139 0.00598	1	26493 4.42314	641.1 2.80696	2544 3.40557
101.33 2.00573	10.333 1.01421	74.74 1.87353	0.0_42814 $\bar{5}.44932$	0.0_43827 $\bar{5}.58284$	0.0_43775 $\bar{5}.57686$	1	0.02420 $\bar{2}.38382$	0.09603 $\bar{2}.98243$
4187 3.62191	427.0 2.63039	3088 3.48971	0.001163 $\bar{3}.06550$	0.001581 $\bar{3}.19902$	0.001560 $\bar{3}.19304$	41.32 1.61618	1	3.968 0.59861
1055 3.02300	107.6 2.03178	778.2 2.89110	0.0_32930 $\bar{4}.46690$	0.0_33985 $\bar{4}.60042$	0.0_33930 $\bar{4}.59444$	10.41 1.01757	0.25200 $\bar{1}.40139$	1

For the use of these tables, see notes at bottom of p. 78.

Linear and Angular Velocity Conversion Factors

	Cm per sec to feet per min	Feet per min to cm per sec	Cm per sec to miles per hour	Miles per hour to cm per sec	Feet per sec to miles per hour	Miles per hour to feet per sec	Radians per sec to rev per min	Rev per min to radians per sec
1	1.97	0.508	0.0224	44.7	0.682	1.47	9.55	0.1047
2	3.94	1.016	0.0447	89.41	1.364	2.93	19.10	0.2094
3	5.91	1.524	0.0671	134.1	2.045	4.40	28.65	0.3142
4	7.87	2.032	0.0895	170.8	2.727	5.87	38.20	0.4189
5	9.84	2.540	0.1118	223.5	3.409	7.33	47.75	0.5236
6	11.81	3.048	0.1342	268.2	4.091	8.80	57.30	0.6283
7	13.78	3.556	0.1566	312.9	4.773	10.27	66.84	0.7330
8	15.75	4.064	0.1790	357.6	5.455	11.73	76.39	0.8378
9	17.72	4.572	0.2013	402.3	6.136	13.20	85.94	0.9425

Conversion of Pressures*

	Pounds per sq in. to kilograms per sq cm	Kilograms per sq cm to pounds per sq in.	Atmospheres to pounds per sq in.	Pounds per sq in. to atmospheres	Atmospheres to kilograms per sq cm	Kilograms per sq cm to atmospheres
1	0.0703	14.22	14.70	0.0680	1.033	0.9678
2	0.1406	28.45	29.39	0.1361	2.066	1.936
3	0.2109	42.67	44.09	0.2041	3.100	2.904
4	0.2812	56.89	58.70	0.2722	4.133	3.871
5	0.3515	71.12	73.48	0.3402	5.166	4.839
6	0.4218	85.34	88.18	0.4083	6.199	5.807
7	0.4921	99.56	102.9	0.4763	7.233	6.775
8	0.5625	113.0	117.6	0.5444	8.266	7.743
9	0.6328	128.0	132.3	0.6124	9.299	8.711

Conversion of Energy, Work, Heat*

	Ft-lb to kilogram-meters	Kilogram-meters to ft-lb	Ft-lb to Btu	Btu to ft-lb	Kilogram-meters to kilocalories	Kilocalories to kilogram-meters	Joules to calories	Calories to joules
1	0.1383	7.233	0.001285	778.2	0.002342	427.0	0.2388	4.18
2	0.2765	14.47	0.002570	1,556.	0.004684	853.9	0.4777	8.37
3	0.4148	21.70	0.003855	2,335.	0.007026	1,281.	0.7165	12.56
4	0.5530	28.93	0.005140	3,113.	0.009369	1,708.	0.9553	16.75
5	0.6913	36.16	0.006425	3,891.	0.01171	2,135.	1.194	20.94
6	0.8295	43.40	0.007710	4,669.	0.01405	2,562.	1.433	25.12
7	0.9678	50.63	0.008995	5,448.	0.01639	2,989.	1.672	29.31
8	1.106	57.86	0.01028	6,226.	0.01874	3,416.	1.911	33.50
9	1.244	65.10	0.01156	7,004	0.02108	3,843.	2.149	37.68

Conversion of Power*

	Horsepower to kilowatts	Kilowatts to horsepower	Metric horsepower to kilowatts	Kilowatts to metric horsepower	Horsepower to metric horsepower	Metric horsepower to horsepower
1	0.7455	1.341	0.7353	1.360	1.014	0.9863
2	1.491	2.683	1.471	2.720	2.028	1.973
3	2.237	4.024	2.206	4.080	3.042	2.959
4	2.982	5.365	2.941	5.440	4.055	3.945
5	3.728	6.707	3.677	6.800	5.069	4.932
6	4.473	8.048	4.412	0.160	6.083	5.918
7	5.219	9.389	5.147	9.520	7.097	6.904
8	5.964	10.73	5.883	10.88	8.111	7.891
9	6.710	12.07	6.618	12.24	9.125	8.877

* Example: 1 lb per sq in. = 0.0703 kg per sq cm.

BOILERS AND FUELS

BY

DAVID M. SCHOENFELD and GEORGE P. HAYNES

STEAM PRESSURES AND TEMPERATURES

For merchant vessels using geared turbines or turboelectric propulsion the most popular design conditions for the steam-generating units seem to be 450 psi gage and 750 F; but, with tendencies toward higher pressures and temperatures, an increasing number of vessels are being built or are in operation using 600 psi gage and 850 F steam conditions. This is also the standard for combat vessels of the Navy.

One merchant ship has been built under U.S. Maritime Commission sponsorship utilizing 1,200 psi gage and 750 F with provision for reheating the steam.* A number of other vessels, in which the boilers will generate steam at 1,450 psi gage and 750 F with reheating provisions, are under construction for private interests.

The gains that may be achieved with pressures up to 1,450 psi gage and temperatures up to 950 F figure as follows:

Pressure, psi gage	Temperature, deg F	Approximate fuel reduction, %
450	750	0
600	850	6
900	900	10
1,450	950	13
1,450	750 (1 steam reheat)	9
	(2 steam reheat)	11
1,450	750 (1 gas reheat)	11

High-pressure boilers are smaller for a given shaft horsepower, and the initial cost is only slightly greater than for lower pressures. They introduce no particular problem in boiler design, although care should be taken to assure ample circulation in the boiler and waterwalls where headroom is restricted. High steam temperature requires special consideration of such matters as the location and size of the superheater, the design of the furnace so as to produce the required gas temperatures at the superheater, and the selection of materials for and the design of the superheater elements and the steam piping and valves. Moreover, in order to protect the turbine from excessive temperature, it is necessary to employ closer control of the superheat where high steam temperatures are involved.

Where engines are used instead of turbines for propulsion, as in the case of the vast number of Liberty ships constructed during the war, lower steam pressures and temperatures are usually selected. Pressures generally are not in excess of 300 psi gage; usually in the range of 200 to 275 psi gage. Tem-

* See *Trans. Soc. Nav. Arch. and Mar. Eng.*, **49**, 356–379, 1941; **50**, 379–387, 1942.

erally by magnetic examination of other welds; then by annealing the entire drum to relieve the stresses built up during welding.

Forced-circulation boilers depend upon mechanical means (pumps) to provide or assist circulation, rather than differences in density of water and steam, produced thermally. Wide flexibility of design results, permitting disposition of heating surfaces for most effective heat absorption, compactness of arrangement, and a saving of steel and weight especially in boilers designed to operate at high pressures of 1,200 psi and above. They have had wider

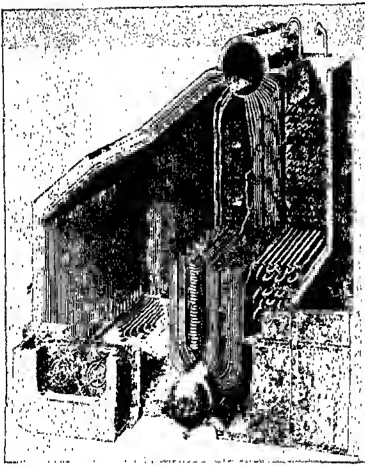

FIG. 10.—Foster Wheeler D type two-drum bent-tube boiler with vertical tube bank.

application in foreign ships than in this country. Here the applications have been mostly of an experimental nature in naval vessels. Nevertheless, with a definite trend toward higher steam pressures and temperatures, forced-circulation boilers particularly in the larger sizes required for highly powered ships are sure to receive increasing attention.

Some forced-circulation boilers are of the "once-through" type such as the Benson, Sulzer, and Besler. Such boilers have no drums. The water is forced by a high-pressure feed pump into one or more tubes (also known as circuits) and steam is delivered at the outlets. Steam demand requires such sensitive response of the fuel supply and feed pump that automatic controls are invariably used. Any impurities in the feed water are either deposited

peratures as high as 640 F have been employed but more commonly they are in the range of 450 to 550 F.

For steam conditions of 450 psi gage and 750 F and higher pressures and temperatures, it is usual to design the steam-generating units for 87 to 88 percent efficiency. To reach such efficiencies requires the use of economizers or air heaters, or a combination of the two, to reduce the temperature of the flue gases leaving the boiler. The choice between economizers or air heaters, or a combination, is generally dictated by the number of stages of feed heating selected, and this selection in turn is affected by economic and operating considerations.

For two stages of feed heating, the temperature of the boiler feed is usually about 240 F, for three stages about 315 to 320 F, and for higher pressures, as high as 385 to 420 F, depending on the number of stages and pressures selected for bleeding. With a feed-water temperature of 240 F, economizers may be used to reach 87 or 88 percent efficiency, but generally an air heater becomes too bulky to fit into the design. With a feedwater temperature of 315 to 320 F, an economizer becomes too bulky but an air heater may be used satisfactorily for these efficiencies. A combination of both economizer and air heater is used when the feed-water temperature is 385 to 420 F. These general statements of course are predicated on the provision of enough boiler surface to reduce the temperature of the flue gases to satisfactory design values. The proportions of heating surface to be incorporated in the furnace, boiler, superheater, economizer, and air heater will vary with the steam pressure and temperature, and with the ideas of different designers and operators.

The effect of economizer and air-heater surface is to increase the efficiency of the steam-generating unit by from 5 to 10 percent, the amount increasing with higher steam pressure.

BOILER TYPES

Scotch Boilers. In the earlier days of steam-propelled vessels, various kinds and designs of boilers were employed. However, owing to structural weakness, insufficient capacity, poor efficiency, or excessive weight, these unsuitable types were gradually discarded as requirements became more severe until, at present, with a few isolated exceptions, there are only two types in general use: the Scotch and the watertube, with the latter predominating. Figure 1 illustrates a Scotch marine boiler with an air preheater and fire-tube superheater.

The Scotch boiler came into use nearly a hundred years ago in Caledonian shipyards; hence the name. For many years it was the most favored type with the lower steam pressures then prevailing. It gave place to the water-tube type only when improved designs of greater capacity and efficiency, less weight, and the ability to operate at higher pressures were developed. However, under certain conditions and for certain purposes it still is regarded favorably. Its strong features are large steam space, ease of maintenance and repair, ability to use feed water of doubtful quality, and adaptability to practically any fuel. Its more undesirable limitations are excessive weight, sluggish circulation, limited pressure, relatively low capacity, and low efficiency. As usually constructed, such boilers for marine service vary from approximately 10 to 18 ft in diameter and from 10 ft 6 in. to 12 ft long. They are equipped with from two to four corrugated furnaces—the customary number being three—and with from one to four combustion chambers.

in the zone where transition from water to steam takes place or are carried over into the turbine and deposited on the blading. To avoid overheating of the zone where such deposits occur this section is usually located in a region of relatively low gas temperature and where no radiant heat will be received.

A variation of the foregoing is the type known as "spill-over," such as the "Steamotive." It is similar, but about 10 percent more water is forced through the circuits than will be transformed into steam. The mixture is discharged into a separating drum, whence the steam passes to the superheater

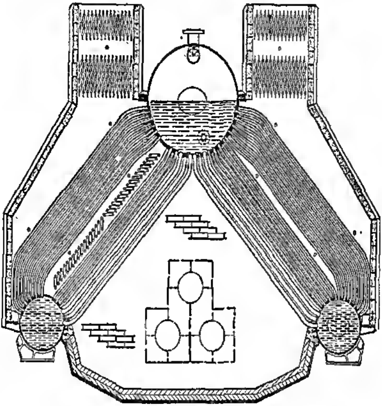

Fig. 11.—Combustion Engineering type V3M three-drum bent-tube boiler with superheater and economizers.

and the residual spill-over water is withdrawn through a pressure-reducing resistance and feed heater back to the feed-pump suction.

Other forced-circulation boilers are of the "assisted" circulation type in which a pump receives water from a drum and forces it through the tubes or circuits, the mixture of steam and water from the tubes being discharged back into the drum. Here separation occurs, the steam passing to the superheater, the water to the pump for recirculation. In the La Mont type, orifices are placed at the entrance to each tube circuit so that each circuit receives a quantity of water in proportion to the amount of steam generated in it. The amount of water circulated varies from three to eight times the quantity of steam generated. The circulating pump operates at a differential pressure head of 30 to 40 psi and requires power equivalent to 0.3 to 0.5 percent of the boiler output.

A group of tubes extends from each combustion chamber to the front head. The front head has openings to accommodate the furnaces, and holes for the tubes, access, and inspection. Flat surfaces are supported by through stays or by bridge or dog stays.

Furnaces are cylindrical shells with circumferential corrugations to afford strength against collapse and to provide for expansion. If oil fuel is used, burners are applied by means of a suitable furnace front behind which

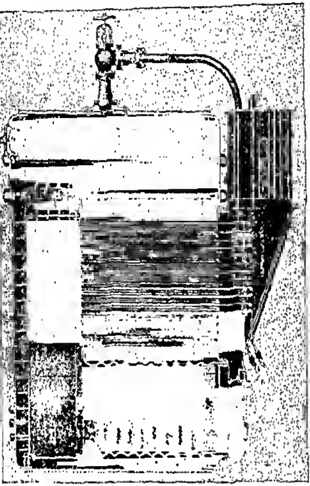

FIG. 1.—Scotch marine boiler with air heater and fire-tube superheater arranged for oil firing.

refractory is placed. With coal, grates are placed approximately on the horizontal center of the furnace. To direct air through the fuel bed a wall is erected at the rear of the grate. This wall closes the segmental area below the grate and also projects above the grate. Fronts include fire and ash doors, also connections to receive air from the forced-draft system and dampers to restrict flow when the fire door is open.

Customary practice is to provide each furnace with a separate combustion chamber. The purpose of the combustion chamber is to allow combustion to be completed and to distribute the flue gases to the tubes.

The Velox boiler is also of the assisted-circulation type but differs in that combustion is carried out under pressure usually varying from 20 to 35 psi. High combustion rates are attained, and high velocities of the gases over the heating surface permit high rates of heat absorption. The pressure of the gases leaving the heating surfaces operates a gas turbine driving a compressor

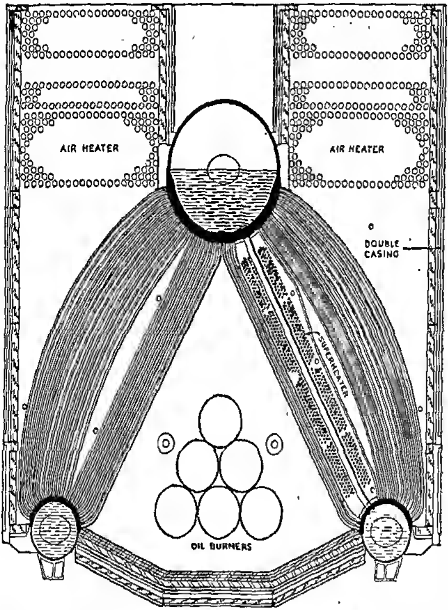

FIG. 12.—Babcock and Wilcox express type three-drum bent-tube boiler with superheater and air heaters.

which furnishes the air for combustion. The pressure at which steam is generated is optional.

In the Loeffler forced-circulation boiler, steam is generated by bubbling superheated steam through water in a drum located outside of the boiler setting. This steam is forced by a steam pump through tubes arranged both

The largest amount of heating surface is that in the tubes which are arranged in groups between the combustion chambers and the front head. Usually they are approximately 8 ft long and the customary sizes range from 2½ to 3½ in. in diameter. Tubes are of two kinds: plain and stay. Plain tubes have relatively thin walls and are held in position and made tight in their respective holes in the sheets by expanding or by rolling and beading.

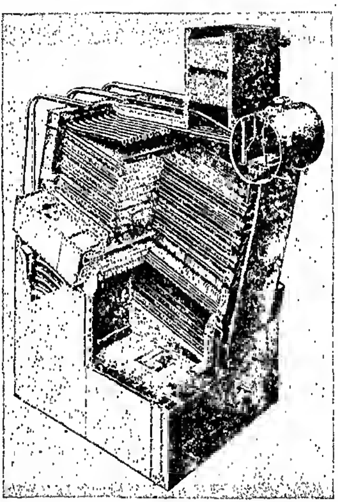

FIG. 2.—Combustion Engineering type SM sectional-header boiler as installed in Victory ships.

Stay tubes are thick walled, the ends of which usually are thickened to compensate for threading. Such tubes are used for staying tube sheets into which they are screwed and expanded. The customary ratio of plain to stay tubes is 4:1 or 5:1.

The Scotch boiler lends itself readily to the application of tubular air heaters in the uptake immediately above the boiler tubes.

Superheaters of the waste-gas or fire-tube type may be applied to new or existing boilers. Waste-heat superheaters consist of inlet and outlet headers connected by elements and are located in a horizontal position immediately

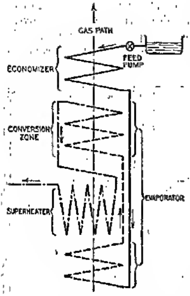

FIG. 13.—Schematic arrangement of once-through forced-circulation boiler, such as the Benson, Sulzer, and Besler types.

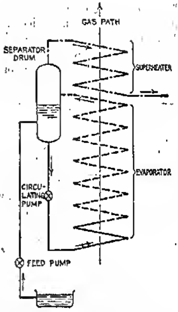

FIG. 14.—Schematic arrangement of controlled forced-circulation boiler with circulating pump—the La Mont type.

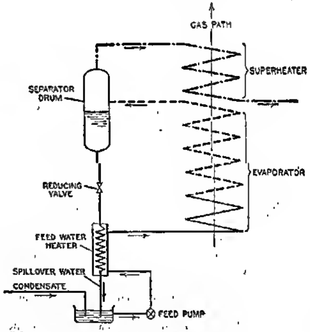

FIG. 15.—Schematic arrangement of spill-over type of forced-circulation boiler, such as the "Steamotive."

under the air heater where they are traversed by gases after they have left the boiler. Resulting superheat is of low degree. For higher superheat fire-tube superheaters are employed. These consist of one or more pairs of headers placed in the uptake and attached to the boiler head. They are connected by elements, each of which is arranged with several loops that extend into the boiler tubes.

Watertube Boilers. Current steam-generating practice in the United States employs almost exclusively watertube boilers. Fire-tube boilers or Scotch boilers are now rarely used. The watertube boilers are of two principal designs: (1) the straight-tube, header type, shown in Figs. 2 to 4, and

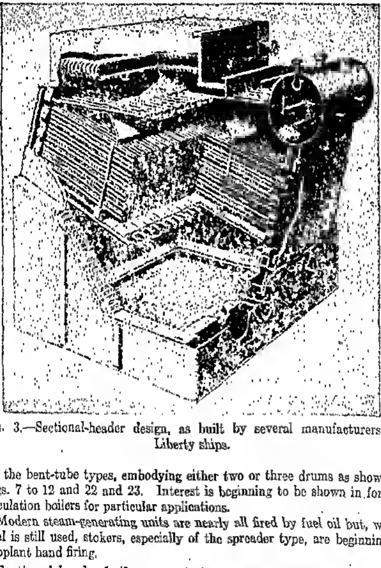

FIG. 3.—Sectional-header design, as built by several manufacturers for Liberty ships.

(2) the bent-tube types, embodying either two or three drums as shown in Figs. 7 to 12 and 22 and 23. Interest is beginning to be shown in forced-circulation boilers for particular applications.

Modern steam-generating units are nearly all fired by fuel oil but, where coal is still used, stokers, especially of the spreader type, are beginning to supplant hand firing.

Sectional-header boilers are suitable for steam pressures up to about 750 psi. They are available in sizes up to about 60,000 to 75,000 lb steam per hr. Tube sizes used vary from 4 to 1 in. O.D. When 4 and 2 in. O.D. tubes are used, the boiler is provided with baffles or flame plates to direct the flue gases in three passes over the bank of tubes. When 1¼ or 1 in. O.D.

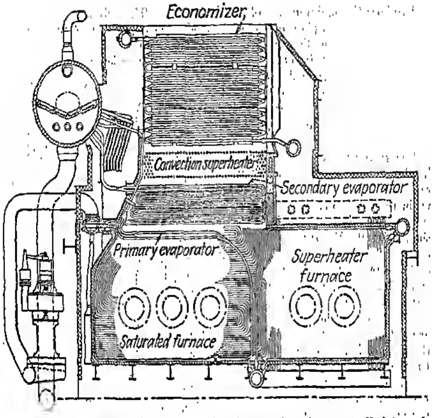

Fig. 16.—Cross section of recent Combustion Engineering controlled forced-circulation boiler, operating on principle shown in Fig. 14.

as wall cooling for the furnace and as a convection section where it is superheated. Superheating is completed in the convection section from which one portion goes to the prime mover, the remainder passing back to the drum where it generates saturated steam. Solids are precipitated in the drum out of the zones where heat absorption takes place. This type of boiler is seldom built for pressures less than 1,700 to 1,900 psi on account of the huge volumes of steam that the pump would have to handle at low pressures with consequent high power requirement. These several types of boilers are represented diagrammatically by Figs. 13 to 17.

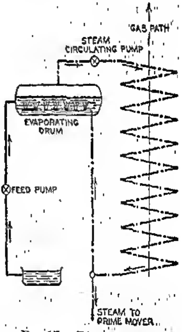

Fig. 17.—Diagram of forced-circulation boiler with circulating steam pump—Loeffler design.

FURNACES

Furnaces may be entirely of refractory construction, or water-cooled surface in the form of waterwalls may be applied to one or more walls. Even the floor of the furnace may be partly or completely water-cooled. Waterwalls permit high rates of heat release

tubes are used no baffles are employed and the gases make only a single pass over the tube bank.

The headers are generally rectangular in cross section and sinuous, so that lanes may be avoided through which the flue gases might by-pass or short-circuit the tubes, and the tubes are arranged in groups for accessibility for inspection, cleaning, or replacement through the handholes in the headers.

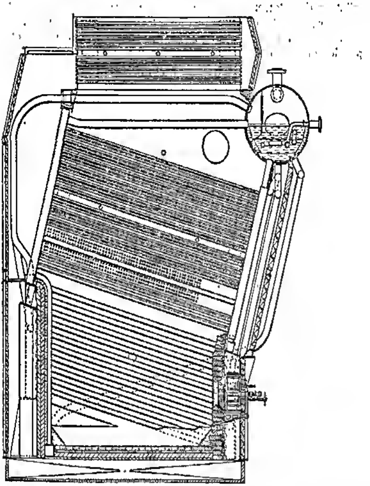

FIG. 4.—Babcock and Wilcox design of sectional-header marine boiler.

Hand holes are secured in place by arches, nuts, and washers and are sealed by gasketed seats. Details of a typical header with its handholes and parts are shown in Fig. 5.

Common practice provides under a single handhole, one 4 in. O.D. tube, or groups of either four 2 in. O.D. tubes, nine $1\frac{1}{4}$ in. O.D. tubes, or fourteen 1 in. O.D. tubes. Different combinations of these groupings may be used in the boiler as may be found desirable. Several widely used arrangements are illustrated in Fig. 6.

The location of the superheater is determined by the temperature of the flue gases required to secure the desired degree of superheat. A commonly used arrangement is indicated in Figs. 2 and 4. This is known as an "inter-

and reduce the maintenance of the brickwork. The tubes comprising the waterwalls may be plain or have extended surfaces. When plain tubes are used, they are generally arranged substantially tangent to each other. Such tubes are usually 2 in. O.D. When tubes with extended surface are used, they are usually spaced approximately two diameters from each other, and a portion is covered by a refractory material such as plastic chrome ore.

The furnace under a sectional-header boiler may be entirely of refractory construction, or waterwalls may be applied to the two side walls and the rear wall. On the side walls, the tubes are commonly disposed at the same slope as those in the boiler bank and are expanded into headers arranged vertically at the front and the rear of the boiler. The front headers are fed by downcomer tubes, and the rear headers are relieved by riser tubes connected to the steam and water drum. Vertical side waterwalls have been used. In the rear, vertical waterwall tubes are used, extending from a transverse header at the floor of the furnace upward to the uptake headers of the boiler. Downcomer tubes from the steam and water drum feed the lower header.

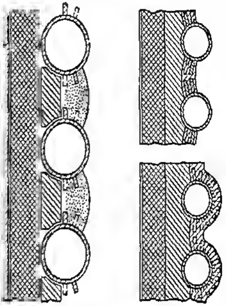

FIG. 18.—Typical details of waterwalls of sectional header boiler furnaces—left, Combustion Engineering; right, Babcock and Wilcox.

Furnaces of two-drum bent-tube boilers are always provided with water-cooled surface on the side walls and roof. The rear wall is often similarly water-cooled and occasionally even the front or burner wall. For the side

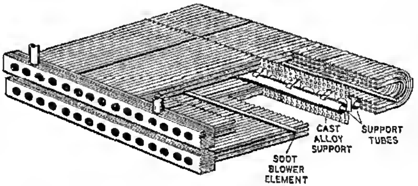

FIG. 19.—Hair-pin type of superheater as applied to a sectional-header boiler.

wall and roof a header is provided at the base of the side wall parallel to the water drum. This header is fed either by large diameter tubes, 4 or $4\frac{1}{2}$ in. O.D. extending from the steam and water drum downward to the outer ends of the header, or alternatively by a number of smaller tubes extending from

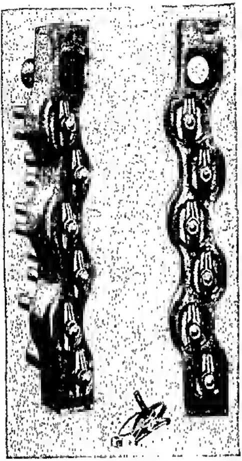

FIG. 5.—Typical header and handhole fittings.

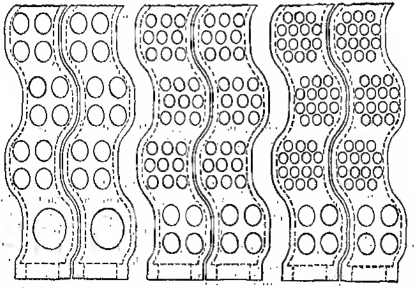

FIG. 6.—Variety of tube arrangements in headers.

the water drum across the floor of the furnace and enclosed in the refractory. The latter arrangement provides a partly water-cooled floor and assures good distribution of water all along the sidewall header. The tubes of the sidewall and roof discharge directly into the steam and water drum. When water cooling is applied to the rear or front wall, headers are provided at the floor and at the roof. The lower header is fed by a number of small tubes from the outer drum. Risers connect the upper header to the steam and water drums.

Various waterwall constructions are indicated in Figs. 2, 4, 7, 8, 10, and 12; details are shown in Fig. 18.

The rates of heat release in furnaces vary. In merchant practice the range is generally between 50,000 and 125,000 Btu/(cu ft)(hr) at normal power. The choice depends largely on the designer and the operator. Rates far in excess of these figures characterize practice in naval boilers.

SUPERHEATERS AND DESUPERHEATERS

Superheaters are either of the "hairpin" or the forged-return-bend types as illustrated in Figs. 19 and 20. The arrangement of the superheaters in the different types of boiler is evident from the typical examples shown, the design being dependent on the type of boiler, degree of superheat, fuel characteristics, type of service, etc. The desuperheater provides steam at lower temperature for auxiliaries.

For steam temperatures up to about 750 F, carbon-steel tubing has proved generally satisfactory for superheater tubes. For higher temperatures, of 800 to 850 F, carbon-molybdenum or low-chromium alloy steels are commonly used. For still higher temperatures alloy steels containing chromium and nickel are employed. The superheaters are usually designed so that only those portions subjected to these higher temperatures are constructed of these more expensive materials.

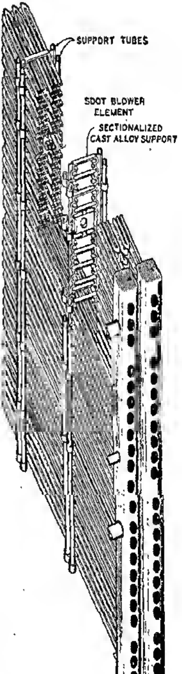

FIG. 20.—Forged return-bend superheater as applied to a two-drum bent-tube boiler.

Desuperheaters should always be fitted on boilers in which the superheaters are located in a zone where the temperature of the flue gases may be

deck" superheater. Space for a superheater is obtained by omitting from the tube bank a suitable number of tubes. Sometimes a different arrangement known as an "overdeck" or "doghouse" superheater is used as in Fig. 3.

Bent-tube boilers are generally of the two-drum design as in Figs. 7 to 10. The three-drum or so-called Yarrow type of boiler, shown in Figs. 11 and 12,

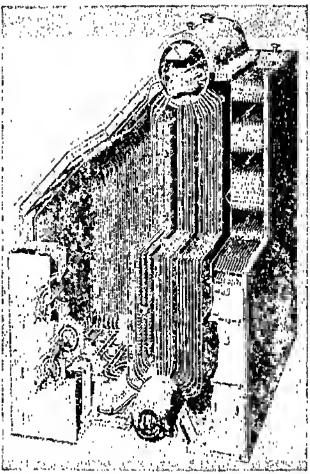

Fig. 7.—Combustion Engineering type V2M two-drum bent-tube boiler with vertical tube bank.

seems to have lost its former popularity, probably owing to its high cost and the complications involved in applying superheaters and economizers or air heaters, as well as in the uptakes. The tube banks of two-drum boilers may be arranged either vertically or inclined as shown by Figs. 7 to 10 which also indicate the customary arrangement of superheaters. Although two bent-tube boilers are sometimes arranged in a single boiler casing, the tendency in general favors each boiler in its individual casing. Both gastight single or double casings are available. Many all-welded casings have supplanted flanged casings previously used.

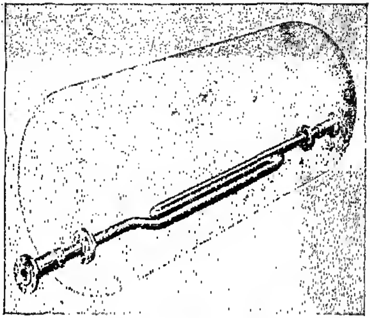

FIG. 21.—Desuperheater in boiler drum.

FIG. 22.—Foster Wheeler bent-tube boiler with separately fired superheater.

Tube sizes for these boilers are almost always 2 and $1\frac{1}{4}$ in. O.D. in merchant vessels, the former in the front bank, the latter in the rear bank. The corresponding naval practice uses $1\frac{1}{2}$ and 1 in. O.D.

The vertically arranged tube bank generally requires the use of baffles to direct the flue gases over the bank to secure proper heat absorption. Baffles are seldom required when the inclined tube bank is used.

Bent-tube boilers, especially the two-drum type, have achieved considerable popularity in recent years. Compared to sectional-header boilers, their weight and cost are relatively low. However, such boilers are inherently

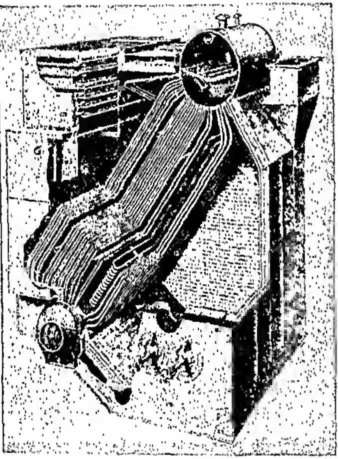

FIG. 8.—Combustion Engineering type V2M two-drum bent-tube boiler, with inclined tube bank.

susceptible to slower repair or cleaning since they must be cooled before the drum can be entered for access to the tubes. Cooling requires considerable time to prevent damage to the pressure parts and setting. On the other hand, access to the tubes of a sectional-header boiler can be had through the handholes in the headers in a much shorter time.

The design of bent-tube boilers, especially for high rates of evaporation per square foot of heating surface, requires the use of downcomer tubes located outside the path of the hot gases. These connect the steam and water drums and are usually located outside the boiler setting. The size of downcomers usually varies between $3\frac{1}{4}$ and $4\frac{1}{2}$ in. O.D. to reduce the number

expected to exceed 800 F. Otherwise, damage to superheaters is liable to occur under certain operating conditions, where the amount of steam required for auxiliaries is large and the amount of superheated steam is smaller by comparison. Under these circumstances, when the auxiliary steam is taken from the drum, not enough steam will pass through the superheater and it will overheat and deteriorate. Desuperheaters as commonly used are of the convection type located in the steam and water drum and arranged for with-

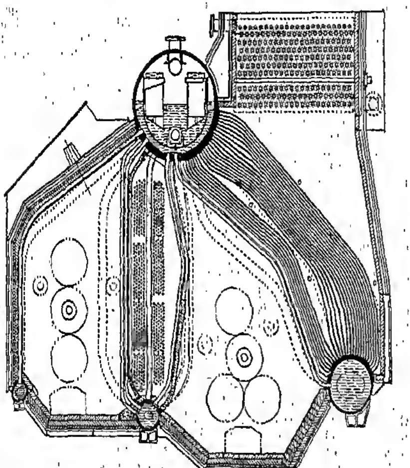

FIG. 23.—Babcock and Wilcox bent-tube boiler with separately fired superheater.

drawal through the manhole for repair or replacement (see Fig. 21). Desuperheaters are sometimes called **attemperators.**

Separately fired superheaters to provide constant superheat temperature over a wide range of steam output are available where the additional expense of such an arrangement may be justified. Such designs are shown in Figs. 22 and 23. The separately fired superheater is of the radiant type and is located in an auxiliary furnace adjacent to the main boiler furnace. The burners in the superheater furnace are fired at a rate suitable to maintain the desired total steam temperature. The flue gases pass from the superheater furnace into the main furnace through the common wall. The main furnace is fired at a rate suitable to maintain the required steam pressure and output.

required. The area in the downcomers is determined by several factors, principally the rate of heat input to the boiler and the distance between the steam and water drums.

Downcomer tubes are always used to supply the water to waterwalls. The *location of downcomer tubes with respect to swash plates in steam drums* should receive careful consideration.

Sectional-header boilers may be installed with the drums arranged either fore and aft in the vessel, or athwartship. The former is to be preferred

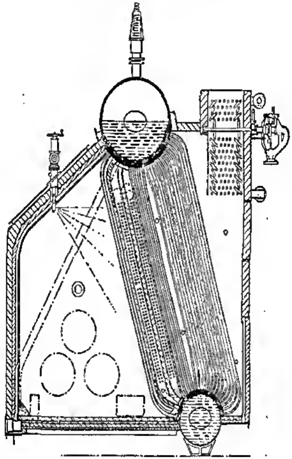

FIG. 9.—Babcock and Wilcox two-drum bent-tube boiler with inclined tube bank.

whenever possible since the motion of the ship in a seaway causes less disturbance of the water in the drums. Bent-tube boilers are always installed with the drums in a fore-and-aft direction.

Present practice no longer uses riveted drums, all drums now being manufactured by electric welding. This progressive step produces lighter, stronger drums with savings in time and expense. Furthermore, such drums are not subject to the phenomenon known as "caustic embrittlement" (arising out of improper workmanship and incorrect feed-water treatment). The welding operation is followed by an X-ray examination of all welded seams and gen-

Superheater supports are subjected to higher temperatures than the superheater tubes. To provide long life and to resist oxidation and deformation, it is customary to make them of high-chromium and nickel alloys, usually castings.

ECONOMIZERS

Economizers are used to increase the temperature of feed water before it enters the boilers, by absorbing heat from the flue gases leaving the boiler. Especially at high steam pressures, the flue gases leave the boiler at temperatures too high to escape up the stack without considerable loss in efficiency. This situation cannot be avoided since the temperature difference between the gases and the steam or saturated water is too low economically to allow use of additional boiler heating surface. However, the feed water is at a lower temperature than the water in the boiler and thus permits enough temperature difference with respect to the flue gases to allow using an economizer.

Economizers consist of bundles of tubes arranged horizontally and connected to inlet and outlet headers to form a number of parallel paths for the water. The tubes forming such continuous circuits may be joined by U-bends, by flanged bends, or expanded into short forged junction boxes. Provision is made for inspection and cleaning. The tubes may be plain or have extended surface in the form of rings, gills, fins, or pegs manufactured so as to be integral with the tube or substantially so for heat-transfer purposes. The tube sizes most commonly employed are $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{1}{2}$, and 2 in., measured on the outside diameter.

They are commonly fitted with soot blowers to clean the deposits from the gas side. The inside is cleaned by the same type of tube cleaner as is used in the boiler tubes. The widespread use of deaerating feed-water heaters and chemical treatment of boiler feed water has removed the corrosion troubles and attendant leakages previously experienced with economizers before the introduction of these advances in technique.

Boilers with economizers are shown in Figs. 2, 7 to 11, 22, and 23.

AIR HEATERS

Air heaters are generally used when multiplestage steam extraction from the turbine is employed for feed-water heating. Under such conditions the temperature of the feed water is so high that an economizer alone will not reduce the temperature of the flue gases sufficiently to attain the desired boiler efficiency. Further reduction in gas temperature is then achieved by absorbing the heat from the gases in the air required for combustion of the fuel. In many instances the economizer may be omitted altogether, only an air heater being employed to reduce the gases to a suitable temperature to meet the required efficiency.

Air heaters are nearly always of the tubular type, consisting of bundles of tubes disposed horizontally and expanded at the ends into vertical tube sheets. As commonly arranged, the hot flue gases pass upward over the outer surface of the tubes and the air makes two passes through the tubes in a substantially countercurrent direction. The most popular tube sizes are $1\frac{1}{2}$ and 2 in. O.D. Soot blowers and steam lances are used to keep the outer surfaces of the tubes clean.

Where boilers fitted with air heaters may be operated for extended periods at low rates of evaporation, as in port, poor combustion conditions together with the water vapor and sulfuric acid in the flue gases result in sticky deposits

Power Equivalents

(For conversion table see p. 80)

Horse-power	Kilo-watts	Metric horse-power	Ponce-lets	Kg-m per sec	Ft-lb per sec	Kilo-calories per sec	Btu per sec
1	**0.7455** $\bar{1}.87246$	**1.014** 0.00599	**0.7604** $\bar{1}.88105$	**76.04** 1.88105	**550** 2.74036	**0.1781** $\bar{1}.25066$	**0.7067** $\bar{1}.84926$
1.341 0.12743	**1**	**1.360** 0.13343	**1.020** 0.00848	**102.0** 2.00848	**737.7** 2.86790	**0.2389** $\bar{1}.37820$	**0.9480** $\bar{1}.97680$
0.9863 $\bar{1}.99402$	**0.7353** $\bar{1}.86648$	**1**	**0.75** $\bar{1}.87506$	**75** 1.87506	**542.5** 2.73438	**0.1757** $\bar{1}.24467$	**0.6971** $\bar{1}.84328$
1.315 0.11896	**0.9804** $\bar{1}.99142$	**1.333** 0.12493	**1**	**100** 2.00000	**723.3** 2.85932	**0.2342** $\bar{1}.36961$	**0.9294** $\bar{1}.96822$
0.01315 $\bar{2}.11896$	**0.009804** $\bar{3}.99142$	**0.01333** $\bar{2}.12493$	**0.01** $\bar{2}.00000$	**1**	**7.233** 0.85932	**0.002342** $\bar{3}.36961$	**0.009294** $\bar{3}.96822$
0.00182 $\bar{3}.25946$	**0.001355** $\bar{3}.13210$	**0.00184** $\bar{3}.26562$	**0.00138** $\bar{3}.14067$	**0.1383** $\bar{1}.14067$	**1**	**0.0_33238** $\bar{4}.51020$	**0.001285** $\bar{3}.10890$
5.615 0.74934	**4.186** 0.62180	**5.693** 0.75533	**4.270** 0.63039	**427.0** 2.63039	**3088** 3.48971	**1**	**3.968** 0.59861
1.415 0.15074	**1.055** 0.02320	**1.435** 0.15672	**1.076** 0.03178	**107.6** 2.03178	**778.2** 2.89110	**0.2520** $\bar{1}.40138$	**1**

Density Equivalents and Conversion Factors

Equivalents					Conversion factors*				
Grams per cu cm	Lb per cu in.	Lb per cu ft	Short tons (2,000 lb) per cu yd	Lb per U. S. gal		Grams per cu cm to lb per cu ft	Lb per cu ft to grams per cu cm	Grams per cu cm to short tons per cu yd	Short tons per cu yd to grams per cu cm
1	**0.03613** $\bar{2}.55787$	**62.43** 1.79539	**0.8428** $\bar{1}.92572$	**8.345** 0.92143	1 2	62.43 124.90	0.01602 0.03204	0.8428 1.6860	1.186 2.373
27.68 1.44217	**1**	**1728** 3.23754	**23.33** 1.36792	**231** 2.36361	3 4	187.30 249.70	0.04806 0.06407	2.5280 3.3710	3.600 4.746
0.01602 $\bar{2}.20466$	**0.0_35787** $\bar{4}.76245$	**1**	**0.0135** $\bar{2}.13033$	**0.1337** $\bar{1}.12613$	5 6	312.40 374.60	0.08009 0.09611	4.2140 5.0570	5.933 7.119
1.186 0.07428	**0.04286** $\bar{2}.63205$	**74.07** 1.86964	**1**	**9.902** 0.99572	7 8	437.00 499.40	0.11210 0.12820	5.9000 6.7420	8.306 9.492
0.1198 $\bar{1}.07855$	**0.004329** $\bar{3}.63639$	**7.481** 0.87396	**0.1010** $\bar{1}.00432$	**1**	9 10	561.90 624.30	0.14420 0.16020	7.5850 8.4280	10.680 11.870

* Example: 1 gm per cu cm = 62.43 lb per cu ft.

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below. In some cases the equivalents have been rounded off, although the logarithm corresponds to the equivalent carried to a greater number of decimal places.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Thermal Conductivity

Calories per sec per sq cm per cm per deg C	Watts per sq cm per cm per deg C	Calories per hr per sq cm per cm per deg C	Btu per hr per sq ft per ft per deg F	Btu per day per sq ft per in. per deg F
1	4.186	3,600	241.9	69,670
0.2389	1	860	57.79	16,643
0.0002778	0.001163	1	0.0672	19.35
0.004134	0.01730	14.88	1	288
0.00001435	0.00006008	0.05167	0.00347	1

Thermal Conductance

Calories per sec per sq cm per deg C	Watts per sq cm per deg C	Calories per hr per sq cm per deg C	Btu per hr per sq ft per deg F	Btu per day per sq ft per deg F
1	4.186	3,600	7,373	176,962
0.2359	1	860	1,761	42,274
0.0002778	0.001163	1	2.048	49.16
0.0001356	0.0005677	0.4882	1	24
0.000005651	0.00002366	0.02034	0.04167	1

Heat Flow

Calories per sec per sq cm	Watts per sq cm	Calories per hr per sq cm	Btu per hr per sq ft	Btu per day per sq ft
1	4.186	3,600	13,272	318,531
0.2389	1	860	3,171	76,094
0.0002778	0.001163	1	3.687	88.48
0.00007535	0.0003154	0.2712	1	24
0.000003139	0.00001314	0.01130	0.04167	1

TIME

Kinds of Time. Three kinds of time are recognized by astronomers, *viz.*, sidereal, apparent solar, and mean solar time. The **sidereal day** is the interval between two consecutive *transits* of some fixed celestial object across any given meridian, or it is the interval required by the earth to make one complete revolution on its axis. This interval is constant, but it is inconvenient as a time unit because the noon of the sidereal day occurs at all hours of the day and night. The **apparent solar day** is the interval between two consecutive transits of the sun across any given meridian. On account of the variable distance between the sun and earth, the variable speed of the earth in its orbit, the effect of the moon, etc., this interval is not constant and consequently cannot be kept by any simple mechanism, such as clocks or watches. To overcome the objection noted above, the **mean solar day** was devised. The mean solar day is the length of the average apparent solar day. Like the sidereal day it is constant, and like the apparent solar day its noon always occurs at approximately the same time of day. By international agreement, beginning Jan. 1, 1925, the astronomical day, like the civil day, is from midnight to midnight. The hours of the astronomical day run from 0 to 24, and the hours of the civil day usually run from 0 to 12 A.M. and 0 to 12 P.M. In some countries the hours of the civil day also run from 0 to 24.

The Year. There are three different kinds of year used, the sidereal, the tropical, and the anomalistic. **The sidereal year** is the time taken by the earth to complete one revolution around the sun from a given star to the same star again. Its length is 365 days, 6 hours, 9 minutes, and 9 seconds. The **tropical year** is the time included between two successive passages of the

of combustible material on the outer surface of the air-heater tubes. These deposits are usually heavier on the tubes where the flue gases leave the air heater and where the temperature of the entering air is lowest. When the boilers are operated at higher rating, the combustible matter deposited on the tube is likely to become ignited and may result in complete destruction of the air heaters. Therefore, care should be exercised to clean air heaters after extended periods of boiler operation at lower rates.

Boilers having air heaters are shown in Figs. 4 and 12.

COMBUSTION CONTROL

Automatic combustion-control equipment is intended to supply fuel and air to the boilers in proportion to the steam demand, to regulate the draft, and to divide the steam load properly among the boilers. Since fluctuations in steam demand are immediately reflected in corresponding fluctuations in the pressure in the main steam header, it is usual to employ a master controller, responsive to such pressure fluctuations, which sets up and transmits impulses to the various actuating elements controlling the supply of fuel and air to the boilers. Provision is made so that manual control of the boilers may be resorted to if desired.

Such control assures proper proportioning of air and fuel in following load fluctuations with improvement in efficiency and avoidance of smoke. The actuating medium is usually air, although some systems utilize oil or electricity.

CODES AND RULES

The design of pressure parts of boilers for installation in merchant tonnage is governed in the interests of safety by various codes and rules, such as "Marine Engineering Regulations and Material Specifications" of the Merchant Marine Inspection Division of the U.S. Coast Guard, "Rules for the Classification and Construction of Steel Vessels" of the American Bureau of Shipping, "Rules and Regulations" of Lloyd's Register, of Shipping, and occasionally by the "Rules for the Construction of Power Boilers" of the American Society of Mechanical Engineers. These regulations also set forth minimum clearances to be observed in arranging the boilers in the fireroom space. Boilers for use in combat ships are designed in accordance with the provisions of the "General Specifications for Machinery" of the Bureau of Ships of the Navy Department.

Care should be exercised in the use of these regulations to be assured that the latest amendments are at hand.

CAPACITY AND RATING

When steam boilers were employed principally to drive steam engines, it was convenient to rate them in horsepower. Tests at the Centennial Exposition in 1876 determined that an average engine required approximately 30 lb steam per hr to produce 1 hp, hence it was recommended that a boiler horsepower be defined as the evaporation of 30 lb water per hr at 70 psi gage from feed water at 100 F. This unit of capacity was later changed to an equivalent evaporation of 34.5 lb per hr "from and at 212 F." Based on the latest steam table values, this represents 33,475 Btu per hr. The term

from the storage bunkers by a transfer pump and the oil is heated by a series of steam grids at various levels which cause any water to precipitate, 24 hr usually being allowed for this process. The settling tanks are located so that the suctions from them are above or nearly level with the fuel-oil service pumps which deliver to preheaters, thence to the burners. Suction and discharge strainers are provided. Each tank should have a capacity such as to operate the vessel at full power for 30 hr.

Atomizers. A burner for fuel oil consists essentially of two parts: the atomizer for converting the stream of solid fuel oil into finely divided particles, and the air register for effectively introducing among the finely divided

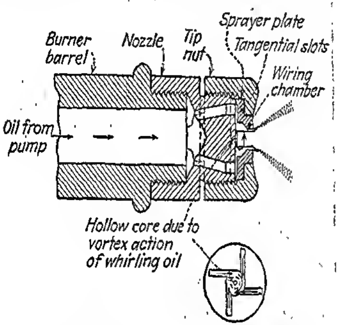

FIG. 28.—Straight pressure or mechanical atomizer.

particles of fuel oil the air required for combustion. Several means may be employed. There are four types of atomizers in common use as follows:

1. Straight pressure or mechanical atomizer (Fig. 28).
2. Straight pressure atomizer with return flow of fuel oil—wide range atomizer (Fig. 30).
3. Steam atomizer (Fig. 32).
4. Rotary-cup (spinning-cup) atomizer (Fig. 34).

The atomizing principle used in the mechanical atomizer, shown in Fig. 28, consists of delivery of several streams of fuel oil at high velocity through passages discharging tangentially into a small cylindrical chamber that verges into a cone with a discharge orifice at its apex through which the oil is sprayed into the boiler furnace. To obtain the high velocity required to produce a suitably fine spray, pressures of 75 to 300 psi are used, and the fuel oil is heated to a temperature where its viscosity will be 150 SSU. The centrifugal and axial forces set up in the chamber result in the production of a hollow conical spray. The designer can achieve considerable latitude in regard to spray angle and fineness of atomization by changing the shape and dimensions of the chamber, the number and dimensions of the tangential passages, the ratio of their total area to the area of the outlet orifice, etc.

"boiler horsepower" is now obsolete and is employed only rarely in reference to certain conventional boilers of small capacity.

The factor of evaporation is equal to the enthalpy (heat content) of 1 lb steam at the stated conditions of pressure and total temperature less the enthalpy of the liquid at feed temperature, divided by 970.3 (the latent heat of evaporation at 212 F).

At one time it was customary to rate boilers on the basis of a certain number of square feet of heating surface per boiler horsepower (10 sq ft was customary for stationary boilers), but as boiler designs and firing methods improved and as water-cooled furnaces, superheaters, economizers, and air heaters came into use, it became apparent that boilers could develop several times the capacity based on such a concept. As a result, this practice of rating fell into disuse, because it is meaningless. In the modern steam-generating unit the heat absorbed by the boiler becomes proportionately a smaller part of the total as waterwalls, superheater, economizer, and air heater are incorporated into the design, and as higher steam pressures are used.

A modern boiler, when producing its proportionate share of the steam quantity required to develop the rated shaft horsepower of the propulsion equipment at the normal speed of the ship, is considered to be operating at its normal or full power rating. Other rates of steam output are customarily stated in terms of this nominal capacity.

EFFICIENCY

The efficiency of a boiler or steam-generating unit is the ratio of the heat absorbed by the steam produced to the heat represented by complete combustion of the fuel fired. It is customary in American practice to use the high heat value of the fuel. European practice employs the low heat value. Since it is well-nigh impossible to determine or segregate the actual efficiency of a boiler only, it is usual in commercial practice to include boiler and furnace efficiency together.

The efficiency of a steam-generating unit may be found by accurate determination of the weight of steam produced or feed water used and of the fuel fired, or by careful measurements of the heat losses and a heat balance.*

With a knowledge of the composition of the fuel being burned, the composition of the products of combustion, *i.e.*, the flue gases, provides a check on how well combustion of the fuel is taking place. The analysis of the flue gases to determine their composition is customarily carried out in an Orsat apparatus. This consists essentially of a burette and three pipettes. The burette is graduated for measuring the sample of gas and the quantities of the gaseous constituents removed in the pipettes. The three pipettes contain solutions of caustic soda or caustic potash to absorb CO_2, cuprous chloride to absorb CO, and pyrogallol to absorb O_2.

Thus the Orsat apparatus provides the percentages by volume of CO_2, CO, and O_2 in the flue gases. The N_2 is assumed to be the balance. Any SO_2 appears as CO_2 but for fuels used aboard ship this slight discrepancy may be disregarded for all practical purposes.

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* For complete details of efficiency tests the reader is referred to the Test Code for Power Boilers of the A.S.M.E. or the *Trans. Amer. Soc. Naval Engrs.*, 172–176, 1942; 53; 724–734, 1941; 54.

The capacity for any given orifice size varies closely with the square root of the supply pressure. With the pressure range noted, the operating range of

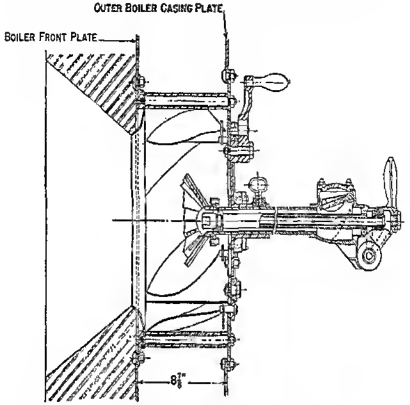


FIG. 29.—Assembly of Todd "Hex-Press" register with "Vee-Cee" atomizer.

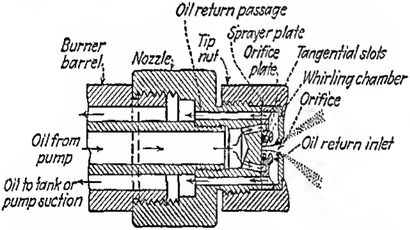


FIG. 30.—Return-flow atomizer, pressure type—Todd Combustion Equipment.

this type of atomizer is limited to a quantity ratio of 1:2. All the oil entering the chamber is discharged into the furnace.

These values provide a measure of the completeness of combustion. Satisfactory combustion of fuel oil can be obtained with 15 to 20 percent excess air and of coal on stokers with 35 to 40 percent excess air. Black smoke is ordinarily due to low excess air but may be produced with high excess air due to improper adjustment of burners and air leakage. With black smoke the Orsat analysis of the gases will usually show high CO_2, low O_2, and the presence of CO if the burners are improperly adjusted. If there is air leakage, the Orsat analysis will then show low CO_2, high O_2, and some CO. White smoke is an indication of too much excess air. Under these conditions the Orsat analysis will show low CO_2, high O_2, and no CO. With the correct quantity of air for the fuel and proper burner adjustment the stack emission should be a faint brown haze and the flue gases should contain no CO.

With the composition of the fuel and the flue gas known, it is possible to calculate various heat losses by the following relations. Sample calculations are included based upon the average analysis for a fuel oil (p. 1006).

Example. Assumed flue gas analysis, percent by volume (from Orsat):

CO_2	13.5
CO	0.2
O_2	3.4
N_2	82.9
	100.0

t_g = temperature, deg F, of flue gases leaving steam-generating unit; say 345 F
t_a = temperature, deg F, of air supply to steam-generating unit; say 100 F.

1. Weight of dry flue gases, lb per lb fuel:

$$W_g = \frac{11CO_2 + 8O_2 + 7(CO + N_2)}{3(CO_2 + CO)} \text{ (carbon burned, lb per lb fuel)}$$

$$W_g = \frac{11(13.5) + 8(3.4) + 7(0.2 + 82.9)}{3(13.5 + 0.2)} \text{ (0.864)} = 15.9 \text{ lb dry gas per lb oil}$$

2. Weight of water from combustion of hydrogen in fuel, lb per lb fuel:

$$W_h = 9 \times \text{(hydrogen in fuel, lb per lb fuel)}$$
$$9(0.107) = 0.963 \text{ lb water per lb oil}$$

3. Dry air, lb per lb fuel $= W_g + W_h - (1 - \text{refuse, lb per lb fuel})$

$$W_a = 15.90 + 0.96 - 1 = 15.86 \text{ lb dry air per lb fuel}$$

4. Weight of water vapor in air for combustion, lb per lb fuel:

$W_v = W_a \times$ (water vapor in air, lb per lb air) at 100 F dry bulb and 37 percent relative humidity, read from psychometric chart, water vapor in air = 0.015 lb per lb air

$$W_v = 15.86 \times (0.015) = 0.24 \text{ lb water vapor in air per lb fuel}$$

5. Weight of wet products of combustion, lb per lb fuel:

$$W_p = W_g + W_h + W_v$$
$$W_p = 15.9 + 0.96 + 0.24 = 17.1$$

Mechanical atomizers with return flow, illustrated in Fig. 30, have been developed to secure an increase in operating range. This is accomplished

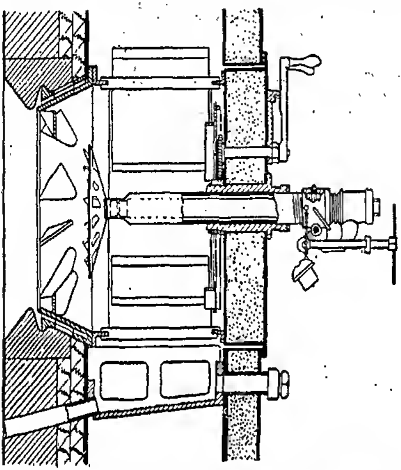


Fig. 31.—Babcock and Wilcox "Decagon" burner and register.

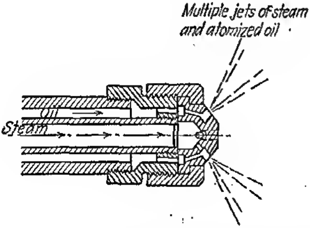


Fig. 32.—Steam atomizer.

either by supplying the whirling chamber with a quantity of fuel oil greater than that to be discharged and returning the excess to the supply pump suction or by varying the area of the tangential passages into the whirling

6. Excess air, percent above that required for complete combustion (an item not directly required for the heat balance, but of interest as indicating quality of operation):

$$X = 100\frac{(O_2 - 0.5CO)}{0.264N_2 - O_2 + 0.5CO}$$

$$X = 100\frac{[3.4 - 0.5(0.2)]}{0.264(82.9) - 3.4 + 0.5(0.2)} = 18 \text{ percent excess air}$$

Heat losses	Btu per lb fuel	Per cent
1. Loss due to sensible heat in dry flue gases: Loss $= 0.24W_g(t_g - t_a)$ $= 0.24 \times 15.9(345 - 100)$ Loss $= 935$ Btu per lb fuel or $\frac{935 \times 100}{18{,}500} = 5.05\%$	935	5.05
2. Loss due to heat in water formed by combustion of hydrogen in fuel: Loss $= W_h(1089 + 0.46t_g - t_a)$ for $t_g < 575$ F or $= W_h(1066 + 0.5t_g - t_a)$ for $t_g > 575$ F $= 0.963[1089 + 0.46(345) - 100]$ $= 0.963(1148)$ Loss $= 1110$ Btu per lb fuel, or $\frac{1110 \times 100}{18{,}500} = 6.00\%$	1110	6.00
3. Loss due to evaporating and superheating moisture contained in the fuel (may be neglected for oil): Loss $= (W_m$, lb moisture per lb fuel$)(1089 + 0.46t_g - t_a)$ for $t_g < 575$ F $= (W_m$, lb moisture per lb fuel$)(1066 + 0.5t_g - t_a)$ for $t_g > 575$ F Loss $= 0.003 \times 1148 = 3.45$ Btu per lb fuel; neglect		
4. Loss due to superheating water vapor in air for combustion: Loss $= 0.47W_v(t_g - t_a)$ $= 0.47 \times 0.24(345 - 100)$ $= 28$ Btu per lb fuel or $\frac{28 \times 100}{18{,}500} = 0.15\%$	28	0.15
5. Loss due to incomplete combustion of carbon, *i.e.*, burning carbon to CO instead of CO_2: Loss $=$ (carbon burned per lb fuel) $\left(\frac{CO}{CO_2 + CO}\right)$ 10,160 $= 0.864\left(\frac{0.2}{13.5 + 0.2}\right)$ 10,160 $= 128$ Btu per lb fuel, or $\frac{128 \times 100}{18{,}500} = 0.7\%$ (This loss is avoidable, especially with oil firing, and should be eliminated by proper firing conditions)	128	0.7
6. Loss due to unconsumed carbon in refuse (negligible for oil): Loss $=$ (refuse, lb per lb fuel $-$ ash, lb per lb fuel) 14,600		
7. Losses due to radiation, and to unburned hydrocarbons, usually known as unaccounted for loss. These can only be determined by subtracting from the heat input in the fuel the sum of the calculated losses above and the heat absorbed by the steam		

TYPICAL HEAT BALANCES

Typical efficiency curves of modern boilers for merchant and naval combat vessels are shown in Fig. 25. Typical heat balances for oil-fired boilers in merchant and naval vessels are shown in the table on page 1002.

chamber by an adjustable sleeve or plug. By either method the object is to maintain high enough velocity through the tangential passages for a good degree of atomization. With the return-flow principle, the range can be increased to as high as 1:10, depending upon furnace conditions and the maximum rate of firing. In the movable-plug design, the range can be increased to 1:4.

As the name implies, the steam atomizer employs steam as the atomizing agent. This steam atomizer is usually constructed so that a number of steam jets can be directed to impinge on an equal number of small streams of solid oil as shown in Fig. 32. The impingement breaks up the oil to

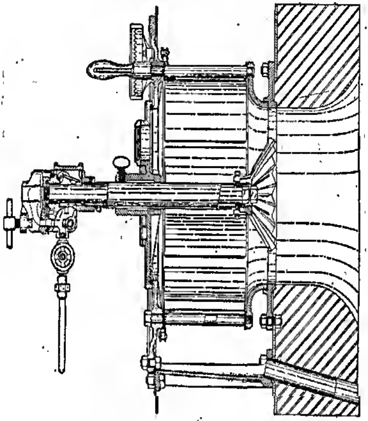


Fig. 33.—Peabody type ML burner and register.

produce an effect similar to the hollow cone-shaped spray obtained with the two types of straight pressure atomizers. In this type of atomizer, it is necessary to preheat the oil to a temperature high enough to reduce the viscosity to 300 SSU. The oil-supply pressure may be varied from 5 to 300 psi. The steam pressure relative to the oil pressure is maintained at a constant differential of about 10 psi by use of a differential valve. A flow ratio of 1:10 may be realized. There has been no widespread application of this type of atomizer on board ship. Many operators object to the steam loss, especially where it must be made up by evaporation.

The rotary-cup atomizer consists primarily of a cup shaped like the frustum of a cone and rotating at about 3,400 rpm. Oil at 300 SSU is delivered into

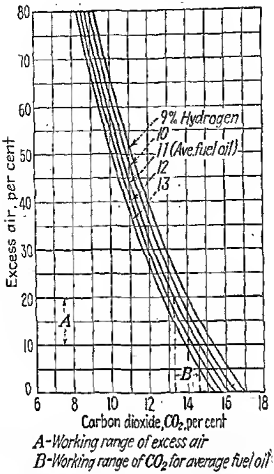


FIG. 24.—Relation between CO_2 in flue gas and excess air for fuel oils of various hydrogen content.

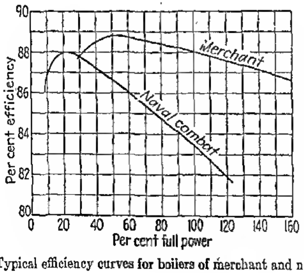


FIG. 25.—Typical efficiency curves for boilers of merchant and naval combat vessels.

the rear of the cup and is centrifuged off the leaving edge as a thin film normal to the axis of the cup. Here it is intercepted by a blast of primary air flowing

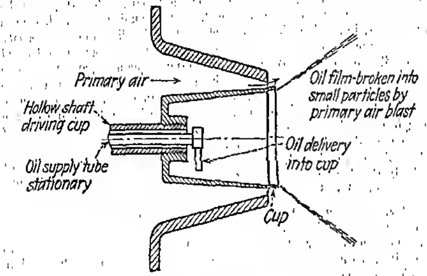


FIG. 34.—Rotary, spinning-cup atomizer.

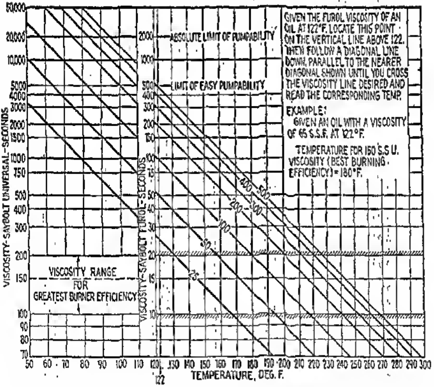


FIG. 35.—Chart for determining proper oil temperature to give efficient atomization.

at high velocity parallel to the axis of the cup. This air blast breaks up the film of oil, deflecting it into a hollow cone-shaped spray. Figure 34 shows the arrangement of a rotary-cup atomizer.

	Merchant service, %	Naval service, %
Loss due to:		
Moisture from air	0.15	0.20
Moisture from hydrogen in fuel	6.00	6.25
Sensible heat in dry gas	5.05	8.30
Radiated and unaccounted for	1.80	1.00
Total losses	13.00	15.75
Efficiency	87.00	84.25
Total	100.00	100.00

HEAT TRANSMISSION AND FLOW RESISTANCE

Heat Transmission. Heat from the products of combustion is transferred to the absorbing surfaces of the boiler unit by radiation and by convection. Luminous radiation accounts for most of the heat absorbed by the surface in the furnace. In the surface beyond the furnace most of the heat is absorbed by convection.

Furnace Heat Absorption. Some of the more important factors affecting the rate of heat absorption in a furnace are the type of fuel, the method of burning the fuel, the rate of heat input to the furnace, the furnace construction, and the condition of the absorbing surfaces.

The chemical analysis of the fuel determines the combustible constituents, the thermal energy available, and the composition of the products of combustion including water vapor. The method of burning the fuel, which is dependent upon the fuel's physical properties, influences the amount of excess air required for complete combustion. The amount of heat available is dependent primarily upon the firing rate but is affected also by the excess of combustion air and its temperature. All these factors affect the mean temperature of the luminous radiating gases which influences to a great extent the rate of heat absorption.

The radiating length of the luminous furnace gases is dependent upon the size and shape of the furnace. The proportion of radiant-heat-absorbing surface to refractory surface is an important factor influencing the rate of furnace heat absorption. Clean surfaces absorb more heat than slagged or dirty surfaces.

Rate of Radiant-heat Absorption. A correlation of actual data obtained with various furnace designs burning different fuels under varied combustion conditions allows an estimation of the rate of radiant heat transmission in a furnace under specified conditions. This rate may be expressed as follows:

$$Y = \frac{X}{1 + \frac{A}{K}(X)^n}$$

where Y = 1000 Btu absorbed per hr per sq ft effective projected radiant heating surface

X = 1000 Btu available per hr per sq ft effective projected radiant heating surface

A = lb combustion air per lb fuel

For the efficient combustion of the oil spray produced by any of the atomizers just described, it is necessary to employ an air register in conjunction with the atomizer. Air registers are designed so that they may be adapted to the type of draft under which the boiler unit is operated, such as forced draft, induced draft, or natural draft. The direction and velocity of the air supply are controlled by the register. The atomizer is so located that it will project the finely divided particles of oil along a path that will be intercepted by the air stream in such manner as to produce an intimate mixture of air and oil. The rapidity of combustion is affected by furnace temperature and dimensions, quality of atomization, velocity of air flow through the register, and temperature of the combustion air. Views of atomizers in air registers are shown in Figs. 29, 31, and 33.

A number of atomizers and registers, or oil burners, are applied to a furnace to obtain the desired quantity of oil with proper atomization and without requiring excessive air pressure.

Viscosity. On installations using straight-pressure or return-flow atomizers it is important that the operator know the viscosity of the oil as this determines the temperature to which it should be preheated in order to reduce its viscosity sufficiently to obtain proper atomization. The graph in Fig. 35 shows the effect of temperature on viscosity and, as there are several methods of calibrating viscosity, an approximate relation between the various methods is given in the accompanying table.

Approximate Relation between Engler Degrees, Saybolt and Redwood Seconds, at the Same Temperature
(From tests in the laboratories of the Texas Company)

Engler deg	Saybolt Universal sec	Saybolt Furol sec	Redwood Standard sec
2.5	83	13.9	74
2.75	92	14.5	81
3	101	15.2	88
3.25	110	15.9	96
3.5	118	16.5	104
3.75	126	17.2	112
4	135	18	119
4.25	144	18.8	127
4.5	152	19.5	134
4.75	160	20.3	142
5	169	21	150
5.5	186	22.5	165
6	203	24	181
6.5	220	25.6	196
7	237	27.2	211
7.5	253	28.7	225
8	270	30.3	240

DRAFT

Forced Draft. The closed fireroom into which the forced-draft blower discharges directly and which is entered through an air lock is not used in modern ships. This arrangement is now superseded by an open fireroom

K and n = constants depending primarily upon type of fuel, method of burning the fuel, and size, shape, and construction of furnace. For marine boiler furnaces fired with fuel oil, the value of K is about 100 and of the exponent n about 0.45

The approximate trend of this relation for an average marine boiler furnace when burning fuel oil is shown by Fig. 26. With the amount of available heat known, the absorption rate may be determined from this curve. By deducting the quantity of heat absorbed from that available, the amount remaining in the combustion gases may be ascertained. From this value and the weight of gases the sensible heat in the gases, and from it the gas temperature leaving the furnace, may be calculated.

Convection-heat Transfer. To determine the heat absorbed by the surfaces beyond the furnace as well as the gas temperature drops in each it is necessary to evaluate the convection heat-transfer coefficients existing in the boiler banks, the superheater, and the heat-recovery equipment. These coefficients depend principally upon the gas velocity, the gas temperature, and the size and arrangements of the tubes. The transfer rate coefficient may be determined from an expression having the following form:

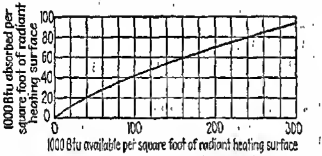


FIG. 26.—Typical curve showing ratio of heat absorbed to heat available in oil-fired furnace.

$$R = \frac{CKG^n}{D^{(1-n)}}$$

where R = heat-transfer coefficient, Btu/(hr)(sq ft)(deg F) logarithmic mean temperature difference
C = a constant
K = a variable depending the physical properties of the gas. These physical properties include the thermal conductivity, the absolute viscosity, and the specific heat
G = mass flow of the gas, lb per hr per sq ft minimum free area normal to the gas flow
D = tube diameter, ft
n = an exponent varying from 0.5 to 0.8 depending the type of flow and tube arrangement

Gas Temperature Drop. The amount of heat absorbed may be expressed by the following relation:

$$Q = R \times S \times \theta_m$$

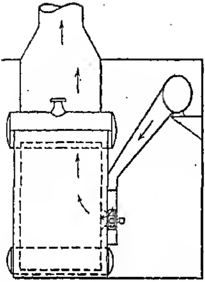


FIG. 36.—Forced draft direct from blower to double casing enclosing air registers.

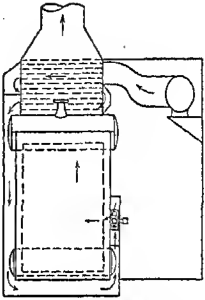


FIG. 37.—Same as Fig. 36 but with air heater.

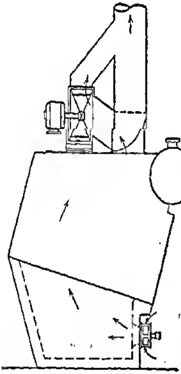


FIG. 38.—Air for combustion supplied by induced draft.

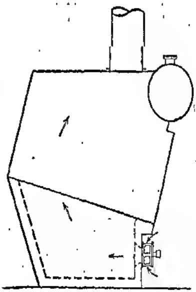


FIG. 39.—Air for combustion supplied by natural draft.

where Q = heat absorbed, Btu per hr
R = heat-transfer coefficient, Btu/(hr)(sq ft)(deg F)
S = effective heat-absorbing surface, sq ft
θ_m = logarithmic mean temperature difference, deg F

The quantity of heat absorbed is also equal to the product of the pounds of products of combustion per hour, W_g, the mean specific heat at constant pressure, C_p, and the difference between the temperature of the gases entering and leaving the heat absorbing surface, $T_1 - T_2$. The equation above, therefore, may be rewritten as follows:

$$T_1 - T_2 = \frac{R \times S \times \theta_m}{W_g \times C_p}$$

For a marine oil-fired boiler unit similar to that shown in Fig. 7, using both an economizer and an air heater to recover heat from the combustion gases, the gas temperatures throughout the unit are shown in Fig. 27. For detail information on rates of heat transfer, reference should be made to pp. 384 to 411.

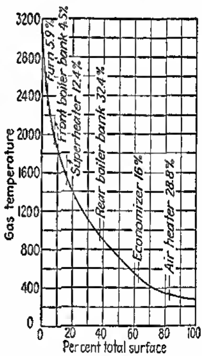


FIG. 27.—Typical curve showing distribution of heat-absorbing surface.

Fluid Flow Resistances. Resistance to flow is caused by the friction between the flowing fluid and the bounding surfaces, by change of direction such as bends and turns, and by enlargements and reductions in cross-sectional area through which flow is taking place. Frictional losses for flow inside of a tube, pipe, or duct are determined experimentally and are found to depend primarily upon the length, diameter, and velocity. The other losses are found to depend primarily upon velocity.

Steam- and Water-pressure Losses. To determine the pressure loss of water flowing through an economizer or steam through a superheater, it is necessary to determine the frictional losses. Usually it is possible to express the entrance, exit, and bend losses as equal to a certain number of tube diameters or equivalent to a fixed number of feet of straight tubing. With this method the pressure loss may be calculated as follows:

$$\Delta p = \frac{kfLvG^2}{d}$$

where Δp = pressure loss, psi
k = constant
f = friction factor (dimensionless) depending upon Reynolds number
L = equivalent length of tubing, ft

with a modern ventilation system. The forced-draft blower discharges directly to a wind box surrounding the burners on the front of the boiler as in Fig. 36. This may be varied, depending upon the design of the boiler, so that the wind box also surrounds the furnace or even the entire boiler as in Fig. 37. In other words, the furnace portion of the boiler setting, or the entire boiler, may be double cased, with wind-box air pressure in the casing. Such designs preclude the escape of flue gases into the fireroom space. Such boilers operate entirely under forced draft. This arrangement is widely predominating in both merchant and naval tonnage.

Induced draft (Fig. 38). In this arrangement the stack induction is augmented by a blower interposed between the boiler outlet and the stack. The blower exhausts the combustion gas from the boiler and discharges into the base of the stack. The entire system is always under a negative pressure. It is not frequently used in present-day practice because of the large blower capacity required and mechanical difficulties due to temperature of the stack gases.

Natural draft (Fig. 39). In this arrangement the stack induction, created by the difference in density between the column of hot gas and the outside atmosphere, is used to draw air through the burner air registers into the furnace. Changes in wind direction and velocity, changes in differential between stack gas and outside atmospheric temperature, and changes in firing rate all affect this natural draft arrangement, and the maximum firing rate available is always limited to the stack capacity. Owing to the greatly increased capacity at which present-day boilers are operated this type has practically disappeared.

COAL FIRING

Some special coastal and harbor craft on the East Coast nevertheless use coal, generally anthracite since it is cleaner and does not smoke. Many foreign vessels also operate on coal. If supplied at East Coast ports, low- and high-volatile bituminous coals from Pennsylvania, West Virginia, Virginia, and Eastern Kentucky are used. These are high-grade coals, the usual proximate analyses of which are as follows:

	Low-volatile bituminous	High-volatile bituminous	Anthracite
Moisture, %	2.6	3.0	5.5
Ash, %	5.9	6.2	10.5
Fixed carbon, %	73.7	57.0	80.0
Volatile matter, %	17.8	33.8	4.0
Sulfur, %	0.9	1.3	0.7
Heating value, Btu per lb (as received)	14,300	13,800	11,200

On the Great Lakes nearly all ships burn coal. Here coal enjoys a price differential over fuel oil owing to the expensive transportation charges for the latter. Furthermore, many of these ships transport coal or are owned by interests closely allied with the coal-producing industry.

With coal as fuel, two analyses are commonly used for practical purposes; the proximate and the ultimate, as well as the heating value. The proximate analysis generally consists of the relative amounts of (1) water or moisture, (2) volatile matter including gases and vapors, (3) fixed carbon, (4) ash or

v = specific volume of the fluid, cu ft per lb
G = mass flow of the fluid, lb per hr per sq ft of flow area
d = inside diameter of tube or pipe, in.

The water-pressure loss at normal power through the economizer of the boiler unit similar to that shown in Fig. 7 is about 5 psi, whereas the steam-pressure loss through the superheater shown is approximately 20 psi.

Resistance to Flow of Air and Gas. To obtain the frictional resistance of air or gas flowing through heating surfaces such as air-heater tubes or through ducts, casings, etc., the foregoing formula may be used with a consistent system of units. Usually the frictional loss for the straight portion of pipe, tube, or duct is used for the length (L), and the losses due to bends and to changes in cross-sectional area are calculated separately. These latter losses may be expressed as a fractional portion of a velocity head.

In calculating the resistance to flow over the banks of tubes that form the different convection heating surfaces of the boiler unit, the losses due to changes in flow area and direction are the most important. For practical purposes these are considered as one loss. This loss may be determined as follows:

$$\Delta p = \frac{kNTG^2}{10^{12}}$$

where Δp = air-pressure loss or draft loss, wg.
k = a constant depending on tube arrangement and type of heating surface, and ranges between 1.5 and 4.5, approximately
N = number of restrictions that air or gas must pass over
T = absolute temperature, deg R
G = mass flow of air or gas, lb per hr per sq ft of minimum free area normal to the gas flow

The air resistance through the burner registers is determined from standard curves based on actual test results with each specific type of burner. The correct evaluation of this loss is important since it accounts for as much as 50 percent of the total air and gas resistance.

For the boiler unit similar to that shown in Fig. 7, the air and gas resistances at normal power in inches of water are as follows:

Air resistance through air heater	1.0
Air resistance through duct from air-heater outlet to burners	0.2
Air resistance through burner registers	1.3
Gas resistance through boiler and superheater	0.3
Gas resistance through economizer	1.2
Gas resistance through air heater	1.2
Total air and gas resistance	5.2

For detail information on flow of fluids reference should be made to pp. 244 to 281 and 353 to 360.

FUELS

Steam-propelled ships generate steam by burning fuel oil or coal. Naturally, to secure the utmost in economy of operation careful consideration must be given to efficient combustion in the restricted furnaces available aboard ship. Unless this is done, losses of appreciable magnitude even extending beyond the furnaces will result.

mineral impurities. The sum of the percentages of each of these items totals 100 per cent for the coal in the "as received" state, *i.e.*, as sampled. The percentage of sulfur present is generally reported separately.

The ultimate analysis of coal is made up of the percentages of carbon, hydrogen, sulfur, oxygen, nitrogen, and ash, based on the dry coal, the sum of which is equal to 100 per cent. If based on the "as received" sample, the hydrogen and oxygen of the moisture are included in the hydrogen and oxygen of the dry sample. The ash content is the same as in the proximate analysis. Reference should be made to A.S.T.M. for the standard laboratory methods for determining the proximate and ultimate analyses of coal.

An ultimate analysis may be approximated from the proximate analysis of a coal.

In proximate analysis, the combustible = volatile matter + fixed carbon = coal as received − moisture and ash. Let V, H, C, and N be, respectively, the percentage by weight of volatile matter, hydrogen, volatile carbon, and nitrogen in the combustible. Then $H = V\{[7.35/(V + 10)] - 0.013\}$, which is accurate for American coals to about ±2 percent, approximately, by the following formulas:

$$C = 0.9(V - 10) \text{ for semianthracite}$$
$$C = 0.9(V - 14) \text{ for bituminous and semibituminous}$$

With anthracite, the standard methods of analysis usually show total carbon less than fixed carbon, because the "fixed carbon" contains other elements also.

Sulfur in coal directly increases the value of V; values of C calculated as above will be too high, approximately, by the sulfur content of the combustible.

The nitrogen (coming off in the volatile matter) may be calculated with an accuracy of ±0.5 percent by the formulas:

$$N = 0.07V, \text{ for anthracite and semianthracite}$$
$$N = 2.10 - 0.012V, \text{ for bituminous and lignite}$$

The heating value of fuel, also known as the calorific value, is the amount of heat developed by complete combustion of a definite quantity in an atmosphere of oxygen in a bomb calorimeter under a standard set of conditions. For complete details reference should be made to A.S.M.E. Power Test Codes. The value thus obtained is the high heating value at constant volume and is generally used in American practice. In European practice the low heating value is more generally employed. This is derived from the high heating value by deducting the heat of vaporization of the moisture in the fuel including the moisture formed when hydrogen is burned.

The high heating value of coal may be determined approximately by computation by Dulong's formula:

$$\text{Btu per lb} = 14{,}544C + 62{,}028\left(H - \frac{O}{8}\right) + 4050S$$

This formula employs the high heating values of the constituents found in the ultimate analysis; thus C, H, O, and S are the weight fractions from that source. The heating value of coal may be indicated on different bases, such as "as received," "dry," or "ash and moisture free."

The choice of either fuel oil or coal for merchant ships is practically entirely one of economics and availability. With oil, vessels may be fueled in much shorter time than with coal, thus reducing the time in port required for this function. This is important where ships operate on short-time turn-around, such as passenger ships. It also is becoming of increasing importance to merchant ships as improvements in cargo-handling facilities are receiving greater attention. Furthermore, fueling operations and the transfer of the fuel oil from the point of storage to the ship may be performed mechanically instead of manually. These operations are cleaner than bunkering coal. Fuel oil may be stored in the double bottoms of the ships whereas the bunkers required for coal storage encroach on cargo or other valuable space. Fuel oil poses no ash disposal problem. Mechanical handling equipment for coal and ash may be used but it is expensive and somewhat bulky. Finally, since fuel oil weighs less per unit of heating value, less weight of oil need be carried for the same operating radius; also less operating personnel is required than with coal where manually fired and where trimming must be done by hand.

These advantages in favor of fuel oil act to offset the price advantage of coal that generally prevails at East Coast ports. At Gulf Coast and West Coast ports fuel oil is seldom competitive with coal owing to the distance coal must be transported as compared to the oil.

Fuel Oil. In view of the foregoing nearly all oceangoing American tonnage propelled by steam uses fuel oil. Out of the processes of refining crude petroleum for the purpose of obtaining its most valuable and volatile constituents, residues of somewhat varying characteristics remain. Often these residues are of such a nature that lighter distillates are necessarily blended with them to obtain a product with sufficient fluidity for practical handling. Such residual oils commonly bear the name of "fuel oil" or bunker-C oil.

An average analysis of commercial fuel oil is

Carbon, %	86.4
Hydrogen, %	10.7
Sulfur, %	1.5
Nitrogen plus oxygen, %	1.0
Water, %	0.3
Sediment, %	0.1
Btu per lb	18,500
Flash point, deg F	175 F
Fire point, deg F	225 F
Specific gravity	0.975

Tanks. Delivery of fuel oil to a vessel may be from a shore station or from a barge—usually the latter. The number and arrangement of bunkers will depend upon the designer's desire to use space without interfering with cargo storage and machinery spaces. However, the oil filling system should be arranged with overflows so that while bunkering it will be necessary for the operator to observe the level in only one or two tanks, either by direct soundings or by a gage.

Storage tanks are provided with steam heating coils to reduce the oil viscosity to a point where it is readily pumpable, and the condensate from these coils is trapped. Before reentering the boiler feed system this condensate should be inspected for the presence of oil resulting from leakage.

It is customary to provide two deep "settling" tanks to permit separating out any water that may be entrained with the oil. These are filled with oil

Hand Firing. Although stokers for firing coal aboard ship have been available for years and numerous installations have been made with an increasing tendency in this respect, nevertheless most coal-burning marine boilers are hand fired. This condition prevails, even though this method of firing is the least efficient and the hard manual labor involved makes it unattractive to skilled personnel.

The method employed in hand firing depends on the kind of coal used. With anthracite and other low-volatile coals, it is usual to spread the fresh fuel evenly over the fire; with high-volatile coal either alternate firing or the coking method is employed. The former involves adding "green" coal to only half the fuel bed at a time. This permits the incandescent fuel on the other half to burn the volatiles liberated from the fresh coal. Noncaking coals are best burned in this manner. The latter method, which is well suited to caking coals, involves placing the fresh coal at the front of the fuel bed and permitting it to coke. It is then pushed back over the entire surface.

Spreader Stokers. The spreader stoker has gained popularity principally for vessels on the Great Lakes and on inland waterways, where most of the coal-burning ships in this country operate. Applications have been made to water-tube boilers of both the sectional-header and bent-tube types and to Scotch boilers. A typical arrangement is shown in Fig. 40. It projects sized coal of about ¼ to ¾ in., together with the fines, through the front wall of the furnace usually by means of rotating paddles. Sometimes jets of air or steam are used. If the coal received is larger, crushers are installed to reduce the size so it may be properly fed and burned. Most of the fines are burned in suspension. The larger pieces of coal fall to the grate and are burned there. The grate may be either of the stationary or dumping type. The distributing mechanism is usually divided into units, each consisting of a hopper, a feeding and a distributing arrangement, and the drive. The drive may be individual or from a jackshaft. Some designs utilize alternate rows of right- and left-hand distributor blades to obtain more nearly equal distribution of the coal. The speed of rotation of the blades determines the distance to which the coal is spread. Air is supplied both under the grate and over the fire.

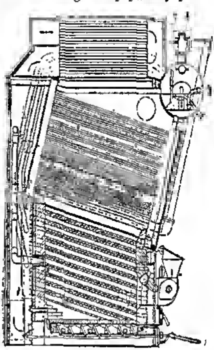


Fig. 40.—Sectional-header boiler equipped with spreader stoker.

High capacities are attainable with such stokers but, when forced, excessive quantities of cinder containing unconsumed carbon pass out of the furnace. In some installations provision is made for collecting this "carry-over" and returning it to the furnace, with resulting reduction of the loss from this source.

vernal equinox by the sun, and since the equinox moves westward 50."2 of arc a year, the tropical year is shorter by 20'23" in time than the sidereal year. As the seasons depend upon the earth's position with respect to the equinox, the tropical year is the year of civil reckoning. The **anomalistic year** is the interval between two successive passages of the perihelion, *viz.*, the time of the earth's nearest approach to the sun. The anomalistic year is only used in special calculations in astronomy.

The Calendar. The month depended originally upon the changes of the moon. The Mohammedan nations still use a lunar calendar with years of 12 lunar months, which contain 354 or 356 days in a specified cycle. According to their method of reckoning, the same month falls in different seasons and their calendars gain 1 year on ours about every 33 years. The **Julian calendar** (established 45 B.C. and now seldom used except in astronomy) discards all consideration of the moon and adopts 365¼ days as the true length of the year. The **Gregorian calendar**, now used in most of the civilized world, was adopted in Catholic countries of Europe in 1582 and in Great Britain and her colonies Jan. 1, 1752. For several years prior to the adoption of the Gregorian calendar the calendar year had begun on Mar. 25. When the change was made in Great Britain and her colonies, the year 1751 contained no January or February and no Mar. 1 to 24 inclusive. The calendar year 1751 was, therefore, short of a full year by 83 days. The calendar year 1752 was also 11 days short of a full year as the dates Sept. 3 to 13 inclusive, in that year, were dropped to correct the 11-day error that had accumulated during the use of the Julian calendar. The average length of the Gregorian calendar year is $365\frac{1}{4} - \frac{3}{400}$ days, or 365.2425 days. This is equivalent to 365 days, 5 hours, 49 minutes, 12 seconds. The length of the tropical year is 365.2422 days, or 365 days, 5 hours, 48 minutes, 46 seconds. Thus the Gregorian calendar year is longer than the tropical year by 0.0003 day, or 26 seconds. This difference amounts to 1 day in slightly more than 3,300 years, and can properly be neglected.

Standard Time. Prior to 1883, each city of the United States had its own time, which was determined by the time of passage of the sun across the local meridian. A system of standard time has been used since its first adoption by the railroads in 1883 but was first legalized on Mar. 19, 1918 when *Congress directed the Interstate Commerce Commission to establish limits* of the standard time zones. The United States, which extends from 65 to 125 deg west longitude, is divided into four zones each of 15 deg of longitude. The first or **Eastern zone** includes all territory between the Atlantic coast and an irregular line drawn from the United States-Canadian boundary just south of Drummond Island, through the Straits of Mackinac and the center of Lake Michigan, along the southern border of Michigan and the western border of Ohio, through the western part of North Carolina, and through central Georgia to the Gulf of Mexico at Apalachicola Bay, Fla. The time of this zone is that of the 75 deg meridian, which is 5 hr slower than Greenwich time. The second or **Central zone** includes all territory between the line mentioned and an irregular line drawn from the United States-Canadian boundary at the boundary line between North Dakota and Montana, along the western and southern borders of North Dakota to the Missouri River, through Phillipsburg, Kans., along the western boundary of Oklahoma and Texas to the Rio Grande River, and to the Mexican border. The time is that of the 90 deg meridian. The third or **Mountain zone** includes all territory between the last-named line and an irregular line

drawn from the United States-Canadian boundary at the northwest corner of Montana along the boundary between Montana and Idaho, the Salmon River, the western and southern borders of Idaho, through Salt Lake City, Utah, and Arizona to the United States-Mexican boundary on the Colorado River. The time is that of the 105 deg meridian. The fourth or **Pacific zone** includes all territory west of the last-named line to the Pacific coast except Alaska. The time is that of the 120 deg meridian. A fifth or **Alaska zone** includes Alaska only, the time being that of the 150 deg meridian. Standard time is uniform in each of these zones, and the time in one zone (except Alaska) differs by exactly 1 hr from the zone next to it. Four different times are used in Alaska: Pacific, 120°; Yukon, 135°; Alaska, 150°; and Western Alaska, 165°. In cities situated on the borderline of two zones, the standard time of the easterly zone is commonly used; in such cities when the time is given, it should be specified as Eastern, Central, etc. The *system of standard time has been adopted in almost all civilized countries* and now is used by ships on the high seas.

TERRESTRIAL GRAVITY

Standard acceleration of gravity is $g^0 = 980.665$ cm per sec per sec, or 32.1740 ft per sec per sec. This value g^0 is assumed to be the value of g at sea level and latitude 45 deg.

Acceleration of Gravity

(U. S. Coast and Geodetic Survey, 1912)

Latitude, deg	g		g/g^0	Latitude, deg	g		g/g^0
	Cm/sec²	Ft/sec²			Cm/sec²	Ft/sec²	
0	978.0	32.086	0.9973	50	981.1	32.187	1.0004
10	978.2	32.093	0.9975	60	981.9	32.215	1.0013
20	978.6	32.108	0.9979	70	982.6	32.238	1.0020
30	979.3	32.130	0.9986	80	983.1	32.253	1.0024
40	980.2	32.158	0.9995	90	983.2	32.258	1.0026

Correction for altitude above sea level: −0.3 cm per sec² for each 1,000 meters; −0.003 ft per sec² for each 1,000 ft.

SPECIFIC GRAVITY AND DENSITY

The **specific gravity of a solid or liquid** is the ratio of the mass of the body to the mass of an equal volume of water at some standard temperature. At the present time a temperature of 4 C (39 F) is commonly used by physicists, but the engineer uses 60 F. The specific gravity of **gases** is usually expressed in terms of hydrogen or air.

The **density** of a body is its mass per unit volume. If the gram is used as the unit of mass and the milliliter as the unit of volume, the figures representing the density are the same as the specific gravity of the body referred to water at 4 C as unity. The customary unit is pounds per cubic foot.

The specific gravity of liquids is usually measured by means of an hydrometer (see p. 250). Special arbitrary hydrometer scales are used in various

Since spreader stokers require generous furnace volume to secure proper length of flame travel and are sensitive to fuel and air control, they should be carefully regulated to avoid smoke. Air or steam jets directed across the furnace from the two front corners are effective in reducing smoke. Heat releases ranging from 20,000 to 35,000 Btu/(cu ft)(hr) are preferred for best results, but rates as high as 60,000 Btu/(cu ft)(hr) have been used successfully.

Colloidal Fuel. From time to time much emphasis seems to be placed on the use of colloidal fuel for firing boilers aboard ship. Colloidal fuel is a physical mixture of oil and coal. Usually, good grades of coal are selected. The coal must be pulverized exceedingly fine, so that 95 percent will pass through a 200-mesh screen. Even so, there is a tendency for the coal to settle out of the oil. Considerable work of an experimental nature with colloidal fuel has been carried out both aboard ship and ashore, but no widespread adoption has occurred. The abrasive character of the coal seems to have a detrimental effect on the pumps, pipe lines, valves, and burners. Furthermore, the ash deposited in the furnace and on the boiler surfaces enchances an already aggravating problem in the use of present fuel oils in maintaining furnace refractories and keeping the heating surfaces clean.

FEED-WATER AND BOILER-WATER TREATMENT

Feed water is the water fed to the boiler. Boiler water is that contained in the boiler. The latter, particularly in boilers operated at high rates of evaporation, contains many times the solids concentration of the former.

Feed water may be treated fresh water or condensate. Usually it is both, the former being known as "make-up." Although in naval practice it is considered essential to distill all make-up from sea water in order to be independent of shore water supplies, most vessels in merchant service depend on bunkering reasonably good fresh water in port. This is more economical and saves fuel. The troubles to be guarded against in the use of fresh water are scale, corrosion, foaming, and priming. The hard-scale-forming minerals are compounds of calcium, magnesium, and silica. Dissolved oxygen and acid water produce corrosion. Oil, grease, and certain organic matter are generally considered to contribute to corrosion as well as to priming and foaming.

The trend toward higher steam pressures and higher rates of evaporation in modern steam-generating units has introduced many new problems in feed-water and boiler-water treatment. Solutions of all these problems have been worked out, but it cannot be emphasized too strongly that the selection of the proper treatment should be placed in the hands of a competent chemist who specializes in this work.

Theory of Water Treatment. Waters may contain salts exhibiting two distinct solubility characteristics. Some show increasing solubility with increase of temperature; others show decreasing solubility with increase of temperature. The latter tend to become concentrated in excess of their saturation points in those parts of the boiler where evaporation is most rapid, *i.e.*, where the boiler surfaces are hottest, and deposit on such surfaces as crystals. The theory of water treatment is concerned with changing the solubility characteristics of such salts so as to increase their solubility with increasing temperature, or with changing them to other salts which will form sludges without tendency to adhere to the boiler surfaces and which will settle out in quiescent parts of the boiler where they may be blown off.

pressure cylinder is used, the value of C is the ratio of the combined volumes of the low-pressure cylinders to that of the high-pressure cylinder. Following are proposed values of R for maximum thermal efficiency in compound condensing engines with initial pressure P, pounds per square inch absolute.

P..........	90	120	150	180	200	220	250	275
R..........	10 to 13	11 to 14	13 to 17	16 to 21	19 to 23	22 to 27	25 to 32	28 to 36

P will range from 115 to 265, preferably not under 165 for condensing engines. Values of p, non-condensing, are from 15 to 17; condensing, 1 or 2, but preferably 1. Half the tabulated values of R may be used for compound non-condensing engines. Generally, in such engines, p_c is between 18 and 25 lb, increasing as P increases. The use of jackets warrants high values for R. High values are indicated when fuel is costly or the load is steady.

Clearances in compound engines are about the same, proportionately, as in simple engines. Usually $c_l > c_h$.

Diagram factors for compound engines expanding R times are about the same as those for simple engines with the $\sqrt{R}$ ratio of expansion (see p. 1023). High compression and excessive clearance reduce the value of f less than they do in simple engines. Superheat less than 150 F may be regarded as equivalent to jackets. Excessive terminal drop in the high-pressure cylinder (p. 1028) reduces the value of f. The use of a reheater between the cylinders is assumed; its omission may reduce the value of f by 0.05.

Size of Cylinder. Given f, p_{mh}, p_{ml},

$$H_i = (LNA_l f/16{,}500)[(p_{mh}/C) + p_{ml}] \qquad (12)$$

$$A_l = (H_i \times 16{,}500)/LNf[(p_{mh}/C) + p_{ml}] \qquad (13)$$

$$A_h = A_l/C$$

where A_h and A_l denote the net piston areas. (The strokes of the pistons are usually equal, although other obvious methods of arrangement are possible.) The quantity $[(p_{mh}/C) + p_{ml}]$ is the **mep referred to the low-pressure cylinder** ("equivalent mep"). This important design constant is usually between 23 and 31 lb in power-station practice. In Europe, it is about $1.2 + 0.05P$ for best thermal efficiency, increasing to $1.2 + 0.12P$ for best commercial efficiency, in condensing engines. The outputs of the two cylinders will have the ratio

$$(H_h/H_l) = (p_{mh}/Cp_{ml}) \qquad (14)$$

Note also that $2LN = S$. Values for L, N, and S are as given on p. 1024. Common values of p_{ml} in stationary practice are between 8 and 15; with the usual division of work and cylinder ratio, the corresponding values of $(p_{mh}/C + p_{ml})$ are from 24 to 32.

The receiver volume is usually 1.5 to 2 times the high-pressure cylinder volume. Large receivers make the engine sluggish in following load changes with fixed low-pressure cutoff.

When the receiver delivers part of its steam for heating or process work, the action is as suggested by the diagrams *habg*, *gmnef* (Fig. 3), and the mean effective pressure of the low-pressure cylinder becomes

$$p_{ml} = \frac{P_e}{C}(1 - x)\left(1 + \log_e \frac{C}{1 - x}\right) - p \qquad (15)$$

where x is the proportion of steam supplied to the high-pressure cylinder which is drawn off for heating. Equations (12) and (13) should then be used

Chemical Treatment. The chemicals commonly employed are soda ash, sodium phosphate, and, under certain conditions, colloidal compounds in combination with the other chemicals. Because of the rapid hydrolysis of soda ash in the boiler water, sodium phosphate is generally preferred to prevent scale and control alkalinity. This substance changes the calcium and magnesium carbonates and sulphates to the corresponding phosphates which precipitate as sludge. Phosphates do not form hard adherent scale and, if deposited, may be flushed out by a stream of water.

The phosphate may be used in the form of monosodium phosphate, disodium phosphate, trisodium phosphate, or sodium metaphosphate, depending on the alkalinity in the feed water. If the feed water contains excess alkalinity, monosodium phosphate is generally used to lower it. Monosodium phosphate has a tendency to deposit in the piping and equipment before reaching the boiler. Where its use is indicated, sodium metaphosphate may be used to avoid this trouble. It produces the same effect in the boiler, *viz.*, it breaks down into a monosodium phosphate, but the rate at which this reaction proceeds is slow enough so that it does not take place before reaching the boiler. Usually it is necessary to employ more than one form to secure excess phosphate for control of scale formation and excess alkali for control of alkalinity.

Condensate that contains only the small amount of impurities present from condenser leakage is usually treated within the boiler to neutralize any acids and to raise the pH value. Treatment with sodium phosphate renders harmless any calcium and magnesium compounds. Phosphate, except the metaphosphate, is always supplied directly to the boiler through a separate line. If added to the feed water, it causes deposits in the feed pump and feed piping.

Corrosion. Boiler corrosion is of three kinds: that due to acid, to oxygen, and to hot spots in areas of active heating surface.

Corrosion is generally controlled by maintaining the alkalinity of the boiler water above a pH value of 9.5 but not in excess of 11.0.

The water may be acid because of acids dissolved in it, or pure water may act as a weak acid. Treated water is usually alkaline but condensate, which is nearly pure water, usually requires treatment to render it alkaline. Phosphate fed into the boiler also affords certain protection against corrosion.

pH Value. This is an arbitrary symbol adopted to express the degree of acidity or alkalinity of a solution. It is the logarithm of the reciprocal of the hydrogen-ion concentration, in gram mols per liter at 71.6 F. A pH of 7 represents a neutral solution; lower values represent acidity, higher values alkalinity. Laboratory investigations have indicated that the minimum solubility of iron in distilled water occurs at a pH of 9.5.

Corrosion due to oxygen dissolved in the feed water is especially active at high pressures and temperatures. As little as 0.01 cc per liter of dissolved oxygen in the water of high-pressure boilers has been found to cause corrosion. Complete deaeration and chemical control are vital in high-pressure boiler operation.

Deaerating feed-water heaters have become an essential part of most modern high-pressure propulsion plants. Removal of oxygen by sodium sulfite is not advised as a substitute for deaeration and should be used only to remove the traces remaining from incomplete deaeration.

Compounds formerly were added to the feed water in an endeavor to remedy improper water conditions in boilers. These compounds were mix-

in design. A simple engine should be used if a large amount of low-pressure waste steam can be utilized.

Superheated Steam in Compound Engines. With superheat exceeding 150 F, Eqs. (10) and (11) are invalid. The curve abc (Fig. 3) may be represented by pv^n = const, the value of n being that given on p. 1025 for a simple engine.

Governing Compound Engines: High-pressure Terminal Drop. In early (Woolf) compound engines, without a receiver, the arrangement was necessarily either tandem or opposed and the low-pressure cylinder admitted steam throughout the full stroke. Present-day engines may have the pistons out of phase, the receiver and pipes storing steam between the cylinders; and the low-pressure cylinder works expansively while the division of work between the cylinders is controlled by variation of receiver pressure.

If an engine with any definite amount of terminal drop (*i.e.*, excess of terminal expansion pressure in the high-pressure cylinder above the receiver pressure) at normal load has a fixed point of low-pressure cutoff, increase of load will increase the receiver pressure and the proportion of work done by the low-pressure cylinder without changing the drop.

With variable low-pressure cutoff a later cutoff lowers the receiver pressure and increases the drop, without any appreciable influence on the output of the whole engine. The output of the low-pressure cylinder is decreased because of the reduced receiver pressure, in spite of its later point of cutoff. The output of the whole engine is increased by delaying high-pressure cutoff. The division of work between the cylinders may then be kept equal by also delaying low-pressure cutoff; but this will cause increased drop.

Whether any drop should be permitted at normal load is a controverted subject (see B. C. Ball, *Trans. A.S.M.E.*, **21**, p. 1002). There should be none in an engine intended for frequent overload conditions.

Factors beyond the Cylinder

Mechanical Efficiency. The power lost in friction, H_f, may be regarded as constant (see p. 1037), so that the brake horsepower, H_b, is $H_i - H_f$ at all loads. The mechanical efficiency at full load is

$$M = H_b/H_i = (H_i - H_f)/H_i$$

H_i being taken at full load. As the load increases, the **mechanical efficiency** steadily increases. At the Q proportion of full rated load, it is

$$M_1 = (Q + M - 1)/Q$$

The **commercial efficiency** of the engine is determined jointly by its thermal and mechanical efficiencies, and the most economical ratio of expansion is commercially somewhat less than that which gives highest thermal efficiency. From a standpoint of capacity alone, moreover, the maximum practicable value of R is determined by the expression

$$\frac{P}{R} - p = (1 - M)\left[\frac{P(1 + \log_e R)}{R} - p\right] \tag{16}$$

Values of M range from 0.82 to 0.97, tending to vary inversely with the initial pressure, the number of cylinders, and number of journals, and directly with the size of engine and the ratio of mean effective to maximum pressure. These variations are obscured by the factors of workmanship and lubrication; values for standard types of engine are variable within the above limits.

tures of common chemicals which the user confidently accepted as a cure-all for boiler-water troubles. With the development of modern water-treating theory and practice it has been found that there is no one compound universally applicable to all boiler conditions unless the source of the make-up and operating conditions are similar and closely controlled. Such a condition exists in the case of naval boilers. The U.S. Navy has carried out considerable research and as a result has developed a compound composed of 47 per cent anhydrous disodium phosphate, 44 per cent soda ash, and 9 per cent starch. This compound is used to adjust boiler-water alkalinity. It should be noted, however, that this compound was developed to meet the special conditions of naval boiler operation in which make-up is distilled from sea water.

It has been found that feed-water contamination from the types of tank coatings used in the bunker tanks may be more serious than condenser leakage. Serious attention should be given to the selection of coatings for this purpose.

The arrangement of the feed system and boilers should contain provision for sampling lines to provide feed-water and boiler-water samples that will be truly representative. Such sampling connections should have cooling coils so that no vapor or oxygen will be lost from the sample by flash. Sample lines of small diameter are preferred so that they may be thoroughly flushed without too much waste before securing the sample.

Test outfits have been developed for the use of the operating personnel in checking quality of the feed water and boiler water and for controlling the treatment. Such a kit should be provided aboard every ship. Nevertheless, periodic checks should be made from time to time by a qualified feed-water chemist.

Priming is the propulsion of water into the steam drum by extremely rapid boiling. It is spasmodic, in contrast to steady carry-over, and results in the entrainment of considerable moisture with the steam. It is usually caused by rapid pressure drop and sudden changes in water level, such as accompany rapidly fluctuating loads where the boiler drum is too small and hence the steam storage space or steam-disengaging surface is too limited.

BOILER CLEANING

Removal of Internal Deposits. Scale formation on the internal surfaces of boilers, waterfalls, superheaters, and economizers interferes with effective heat absorption of these surfaces. In boilers, waterwalls, and superheaters, especially where the rates of heat absorption are high, this condition is quite apt to result in overheated, bagged, or ruptured tubes, with resultant loss of the unit in outage and the expense of repairs. Such scale deposits are generally removed by rotating cutters or steel brushes driven by motor, air-, steam-, or water-powered. These devices, sometimes termed tube turbines, are designed to pass the bends customarily employed in the design of natural circulation boilers.

Methods of cleaning boilers chemically have been developed and have the possibility of saving considerable time compared to the mechanical tube cleaners. Chemical cleaning should be done only under supervision of reputable concerns which have personnel experienced in the work.

The method to be used will be governed by the composition of the scale. Some scales can be effectively removed by boiling out with a strong alkaline

STEAM-ENGINE ECONOMY

Variables Affecting Economy

Initial Pressure. The efficiency varies directly with the initial pressure. With *slight expansion, the variation is unimportant. At high ratios of expansion, high initial pressures have marked influence on efficiency.* The **ideal economies** of Tables 2 to 8, computed for cycles with incomplete expansion, illustrate these points. The results of varying initial pressure with superheated steam are shown in Tables 4 to 7. In condensing engines with superheat, high pressure is less important than in non-condensing engines.

Table 2. Ideal Engine Economies
Steam Initially Dry. Back Pressure, 2 lb abs

Ratio of expansion	Absolute steam pressure, lb per sq in.														
	70	100	160	215	315	70	100	160	215	315	70	100	160	215	315
	Btu per ihp-min					Thermodynamic efficiency, percent					Steam consumption, lb per ihp-hr				
5	249	241	230	...	228	17.0	17.6	18.2	18.3	18.5	13.8	13.2	12.5	....	12.4
7	229	221	213	208	205	18.5	19.2	19.9	20.1	20.5	12.6	12.1	11.6	11.3	11.2
10	213	205	197	192	188	19.9	20.8	21.6	22.1	22.5	11.8	11.2	10.7	10.4	10.2
15	202	191	181	176	172	21.0	22.2	23.4	24.0	24.5	11.2	10.5	9.9	9.6	9.3
20	199	185	173	168	163	21.5	22.9	24.2	25.2	25.8	10.9	10.2	9.5	9.1	8.8
25	198	182	169	163	157	21.8	23.3	24.8	26.0	26.8	10.8	10.0	9.3	8.9	8.5
30	197	180	167	160	154	21.9	23.5	25.3	26.5	27.4	10.5	9.8	9.1	8.7	8.3
40	...	...	165	156	149	....	....	25.8	27.2	28.3	....	....	9.0	8.5	8.1
50	...	...	164	155	147	....	....	26.0	27.6	28.9	....	....	8.9	8.4	7.9

Table 3. Ideal Engine Economies
Steam Initially Dry. Back Pressure, 16 lb abs

Ratio of expansion	Absolute steam pressure, lb per sq in.														
	70	100	160	215	315	70	100	160	215	315	70	100	160	215	315
	Btu per ihp-min					Thermodynamic efficiency, percent					Steam consumption, lb per ihp-hr				
2	423	390	329	304	326	9.8	10.9	12.4	13.4	13.0	26.4	23.2	20.0	18.5	19.1
3	395	338	297	279	272	10.8	12.5	14.2	15.2	15.5	23.8	20.4	17.7	16.6	16.0
4	390	321	277	260	246	10.9	13.2	15.4	16.3	17.2	23.0	19.4	16.4	15.3	14.4
5	...	316	265	246	232	....	13.4	16.1	17.2	18.3	....	19.0	15.7	14.5	13.6
6	...	...	258	237	224	....	....	16.4	17.8	19.0	....	....	15.3	14.0	13.1
7	...	...	256	232	216	....	....	16.6	18.3	19.7	....	....	15.2	13.7	12.6
10	...	...	...	228	206	....	....	....	18.6	20.6	....	....	....	13.5	12.1

High Steam Pressures. Higher efficiency of the steam-engine cycle can be obtained by increasing the temperature range. With fixed back pressure, higher steam pressures and temperatures are necessary. The practical limit of steam temperature today is about 750 to 800 F. A lubricating oil, having a flash point of about 650 F, will satisfactorily lubricate cylinders operating with steam at 800 F. Steam pressures up to 1,400 to 1,500 lb per sq in.

solution. Others require an acid solution containing an inhibitor to prevent active corrosion of the steel.

Acid cleaning requires use of a portable circulating pump and careful attention to venting of fumes to avoid toxic atmospheres in the fireroom.

Chemical cleaning is worthy of consideration as a method of cleaning boiler units installed with large economizers and superheaters which have a great number of tubes, usually with short bends.

Removal of External Deposits. Whether steam-generating units are fired by fuel oil or coal, the flue gases contain soot and other solid impurities present in the fuel. These materials are deposited in the furnace and on the outer surfaces of boiler tubes, superheater tubes, and economizer and air-heater tubes. They will also deposit on any ledges, although a properly designed boiler unit will have none of these. In zones of intense heat these deposits may even fuse to the surfaces. If not removed, such deposits, because of their definite insulating properties, will seriously interfere with *the transfer of heat* and *reduce the efficiency. If they become excessive,* the flue-gas passages are choked and require excessive draft and higher fan power. Removal of these deposits may be accomplished by a steam or air hand lance or by mechanical soot blowers. The latter are preferable.

A hand lance consists of a pipe of suitable length and diameter so it will pass between the tubes with one end flattened or nozzle-shaped and fitted with a length of hose to connect to a supply of steam or air. Inserted through openings provided in the setting, the force of the steam or air jet loosens the deposits, the fine particles being carried by the flue gases up the stack, the coarse ones falling into the furnace. This method is uncertain, dirty, tedious, sometimes hazardous, and, if steam is used, wasteful.

Such deficiencies are overcome by the use of mechanical soot blowers. As a result nearly all marine boilers now are so equipped. Mechanical soot blowers consist essentially of tubes, usually 1½ or 2 in. O.D., fitted with nozzles, and extending through the setting walls, usually at right angles to the axes of the tubes to be cleaned. Steam issuing from the nozzles dislodges the deposits. Effectively to direct the jets to clean the maximum area of heating surface, provision is made for rotating the tubes or elements as they are generally called. Such rotation is accomplished by a chain running over a sprocket wheel connected to the element by a system of gears, levers, and cams. The levers and cams serve to open the valve through which steam is supplied and to regulate the blowing arc when less than full revolution is desired. Usually a number of such units is required to clean the different parts of the boiler unit. The material of the elements is selected to withstand the temperature of the zone in which it is located.

Although considerably more economical of steam than a hand lance, mechanical steam soot blowers do cause a loss of fresh water, which may be considerable especially in large ships. To avoid such loss there are available mechanical soot blowers employing compressed air instead of steam, and many boilers are now being fitted with them.

CARE OF BOILERS OUT OF SERVICE

Boilers to be held out of service should be carefully handled and closely watched in order to minimize any tendency for corrosion of the pressure parts. A boiler to be held out of service for a period longer than 24 hr. should be laid up by either the wet or the dry method.

are in use. Single-expansion engines are used, provided the back pressure is as high as 85 to 100 lb per sq in. Such engines operate with mep of 600 to 800 lb per sq in. It is interesting to note that, under these conditions, the engines become much smaller and can be built with lighter frames and smaller bearings than equivalent engines operating with low steam pressures. The engine efficiency of high-pressure engines is as high as that of engines operating with customary low steam pressures.

Table 4. Ideal Engine Economies
Steam Superheated 150 F. Back Pressure, 2 lb abs

Ratio of expansion	Absolute steam pressure, lb per sq in.														
	70	130	215	315	415	70	130	215	315	415	70	130	215	315	415
	Btu per ihp-min					Thermodynamic efficiency, percent					Steam consumption, lb per ihp-hr				
5	234	220	...	209	206	18.3	19.2	....	20.3	20.6	12.0	11.3	....	10.4	10.2
7	216	203	196	192	188	19.6	20.9	21.6	22.1	22.6	11.2	10.4	9.8	9.5	9.3
10	203	188	182	176	174	20.9	22.5	23.4	24.1	24.5	10.5	9.6	9.1	8.7	8.6
15	195	177	169	163	160	21.7	24.	25.2	26.8	26.5	10.1	9.0	8.5	8.1	7.9
20	191	171	162	156	153	22.2	24.8	26.2	27.1	27.8	10.0	8.7	8.1	7.8	7.6
25	190	168	158	151	148	22.5	25.3	27.0	28.0	28.6	9.9	8.6	7.9	7.5	7.3
30	188	167	157	148	144	22.7	25.7	27.0	28.6	29.4	9.8	8.5	7.9	7.4	7.1
40	...	166	156	145	139	....	25.9	27.2	29.4	30.2	....	8.5	7.8	7.2	6.9

Table 5. Ideal Engine Economies
Steam Superheated 150 F. Back Pressure, 16 lb abs

Ratio of expansion	Absolute steam pressure, lb per sq in.														
	130	215	315	415	515	130	215	315	415	515	130	215	315	415	515
	Btu per ihp-min					Thermodynamic efficiency, percent					Steam consumption, lb per ihp-hr				
2	296	269	285	282	276	12.9	14.9	14.9	15.9	15.3	18.1	16.5	15.3	15.0	14.6
3	265	236	247	248	235	14.6	16.1	17.1	17.6	18.0	16.1	14.6	13.3	12.8	12.4
4	252	222	229	219	215	15.4	17.4	18.6	19.3	19.7	15.1	13.4	12.3	11.7	11.4
5	250	215	216	208	202	15.7	18.3	19.7	20.4	21.0	15.0	12.6	11.6	11.1	10.7
6	...	212	209	200	194	....	18.8	20.4	21.3	21.8	....	12.3	11.2	10.6	10.3
7	...	210	203	194	188	....	19.1	20.9	21.8	22.5	....	12.1	10.9	10.3	10.0
8	...	...	200	190	184	....	....	21.3	22.2	23.1	....	....	10.7	10.2	9.7
10	...	...	...	185	178	....	....	....	22.9	23.9	....	....	....	9.9	9.4

A comparison of the Rankine cycle efficiencies (see p. 346) of engines using steam at a temperature of 600 F but with various steam pressures is given in Table 8 (Cramer, *Trans. A.S.M.E.*, 1915).

An incidental advantage of using high-pressure steam is that it is then available for superheating the steam passing through the intermediate receivers. Tests by Schmidt of a 150 hp quadruple-expansion engine using steam at 794 lb per sq in abs at 815 F, with 28.6 in. vacuum (*Z.V.d.I.*, June

The **wet lay-up method** is used when the boiler is to be kept available for service on short notice. The alkalinity of the boiler water should be built up to a minimum of 20 grains per gal. As the boiler is being cooled, the drain and vent valves on the superheater and the desuperheater are opened to drain any condensate. All valves on the main and auxiliary steam lines are secured. When all pressure is off the boiler, the drain valves on the superheater and desuperheater are secured, but the vent valves are left open. The boiler, superheater, desuperheater, and economizer are then slowly filled with water, adding enough compound to produce an alkalinity of 20 grains per gal. Make-up water should be deaerated if possible. The unit is filled until water issues from the vent valves which are then secured.

A hydrostatic pressure of 50 to 75 psi gage is then built up by carefully cracking the feed valve. Then the vent valves on the boiler, superheater, desuperheater, and economizer are bled to assure expelling any entrapped air. The feed valve is now secured and 25 to 50 psi gage pressure allowed to remain on the boiler. Care should be exercised so that leaking feed or steam valves do not build up higher pressure in the boiler.

The **dry lay-up method** is used when the boiler is to be kept out of service for an indefinite period. The boiler is drained completely, the manhole covers opened and enough handhole plates removed from the boiler and waterwall headers to be sure the water sides are completely dry. It may be found necessary to place coke jacks with coke or wood fires in the furnace to drive off all moisture, especially in damp climates.

Then wide shallow pans of quicklime are placed in the boiler drums, the manhole covers and handhole caps replaced, and all drain and vent valves secured.

The fire sides of the boiler, superheater, economizer, and air heater are carefully cleaned while the unit is prepared for a long lay-up. Any accumulations of soot or dirt should be removed, otherwise moisture from the air will be absorbed and cause external corrosion. All air openings should be closed and the stack cover put in place to prevent circulation of air so far as possible.

25, 1921), and with superheating in the receivers to 568, 536, and 436 F, respectively, show a total steam consumption of 5.12 lb per ihp-hr, a thermal efficiency of 31.1 percent, and an "engine efficiency" (p. 346) for the engine

Table 6. Ideal Engine Economies

Steam Superheated 300 F. Back Pressure, 2 lb abs

Ratio of expansion	Absolute steam pressure, lb per sq in.														
	70	130	215	315	415	70	130	215	315	415	70	130	215	315	415
	Btu per ihp-min					Thermodynamic efficiency, percent					Steam consumption, lb per ihp-hr				
5	214	202	...	192	189	19.8	20.9	....	22.0	23.3	10.4	9.7	...	9.0	8.8
7	202	189	181	177	175	21.0	22.5	23.3	24.9	25.2	9.8	9.0	8.6	8.3	8.1
10	192	177	169	166	163	22.1	24.0	25.1	25.6	26.2	9.3	8.5	8.0	7.8	7.6
15	186	167	158	155	151	23.1	25.3	26.7	27.4	28.0	9.0	8.1	7.5	7.3	7.0
20	182	163	154	148	145	23.9	26.0	27.6	28.7	29.3	9.0	7.8	7.3	6.9	6.7
25	180	161	152	144	140	24.6	26.6	28.3	29.5	30.2	8.9	7.7	7.1	6.7	6.5
30	178	160	151	142	137	25.0	27.0	28.7	30.0	30.8	8.8	7.6	7.0	6.6	6.4
40	...	159	150	139	134	....	27.4	29.3	30.6	31.6	....	7.5	6.9	6.5	6.3

Table 7. Ideal Engine Economies

Steam Superheated 300 F. Back Pressure, 16 lb abs

Ratio of expansion	Absolute steam pressure, lb per sq in.														
	130	215	315	415	515	130	215	315	415	515	130	215	315	415	515
	Btu per ihp-min					Thermodynamic efficiency, percent					Steam consumption, lb per ihp-hr				
2	295	264	265	256	250	14.4	15.5	16.0	16.5	17.0	15.3	13.8	13.3	12.8	12.4
3	265	237	227	218	214	16.1	17.9	18.7	19.4	19.9	13.7	12.1	11.4	10.9	10.6
4	253	222	210	202	196	16.8	19.1	20.2	20.9	21.5	13.1	11.4	10.5	10.1	9.8
5	250	215	201	194	185	17.0	19.7	21.2	22.0	22.9	12.9	11.0	10.1	9.6	9.2
6	...	212	196	187	179	....	20.0	21.7	22.8	23.6	....	10.8	9.8	9.3	8.9
7	...	211	191	184	174	....	20.2	22.2	23.2	23.2	....	10.7	9.6	9.1	8.7
8	...	...	189	181	171	....	....	22.4	23.7	24.8	....	....	9.5	8.9	8.5
10	...	...	...	176	167	....	....	....	24.2	25.4	....	....	....	8.7	8.3

Table 8. Rankine Cycle Efficiencies with High Steam Pressures

Temperature of Steam, 600 F

Steam pressure, lb per sq in.	Superheat, F	Rankine cycle efficiency		Increase of efficiency, percent, compared with 200 lb pressure	
		With ½ lb absolute back pressure	With 14.7 lb absolute back pressure	With ½ lb absolute back pressure	With 14.7 lb absolute back pressure
200	218	0.329	0.197	0	0
400	155	0.361	0.243	9.5	23.4
600	113	0.373	0.260	13.5	32.0
1574	0	0.403	0.306	22.6	55.2

THE STEAM ENGINE

BY W. D. ENNIS

Revised by W. Turnwald

From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

REFERENCES: Ewing, "The Steam Engine and Other Heat Engines," Cambridge University Press. Ripper, "Steam Engine Theory and Practice," Longmans. Heck, "The Steam Engine and Turbine," Van Nostrand. Fernald and Orrok, "Engineering of Power Plants," McGraw-Hill. Gutermuth, "Die Dampfmaschinen," 4 vol., Springer. Allen, "Uniflow, Back Pressure, and Steam Extraction Engines," Pitman.

WORK AND DIMENSIONS OF THE STEAM ENGINE

Simple Engines Using Saturated Steam

Ideal Hyperbolic Diagram. Figure 1 shows the typical indicator diagram *abcdef*. Nomenclature as to valve action is shown at the points *a*, *c*, *d*, and *f*. Nomenclature as to behavior of steam is designated on the intervening lines. The **idealized diagram** on which design is based is *ghjkl*, which differs from the actual diagram in showing no clearance or compression, in extending between the throttle pressure P and atmospheric or condenser pressure p, in showing sharp corners at h, j, and k, and in following the law pv = constant between h and j.

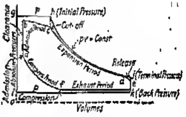


FIG. 1.—Typical Steam-engine Indicator Diagram.

Note the nomenclature adopted for pressures at h, j, and k. The volume ratio $v_j \div v_h = R$ is called the **ratio of expansion**.

The average ordinate or **mean effective pressure** of the diagram *ghjkl* is, in lb per sq in.,

$$p_m = P\frac{(1 + \log_e R)}{R} - p \tag{1}$$

in which P and p are taken in lb per sq in. abs. Table 1 gives the ratio p_m/P for zero back pressure ($p = 0$) for various values of R.

Table 1. Ratio of Mean Effective Pressure to Initial Pressure for Various Ratios of Expansion (Zero Back Pressure)

Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)	Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)	Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)	Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)
1.67	0.907	3.33	0.662	6.67	0.435	18	0.216
1.82	0.874	3.64	0.631	7.00	0.421	20	0.200
2.00	0.847	4.00	0.597	8.00	0.385	22	0.186
2.22	0.810	4.44	0.561	10.00	0.330	24	0.174
2.50	0.766	5.00	0.522	12.00	0.290	26	0.164
2.67	0.744	5.71	0.481	14.00	0.260	28	0.155
2.86	0.717	6.00	0.465	16.00	0.236	30	0.147

Pressures and Expansion Ratios [for Eq. (1)]. In ordinary practice, P ranges from 75 to 400; maximum value up to 1,800. Common values for simple engines are from 95 to 115. For non-condensing engines exhausting to the atmosphere at sea level, p is between 15 and 17. For condensing

of 81.7 percent, and for the individual cylinders of 91, 79.8, 78.6, and 80 percent, respectively. The heat rate was 136.6 Btu per ihp-min.

Figure 4 gives typical experimental results. The "total steam" lines illustrate the **Willans law,** that at fixed cutoff and variable initial pressure the steam consumed per hour is $a + bh$, where h = ihp and a and b are constants. Simple non-condensing engines, with initial pressures not much over 100 lb, use 24 to 28 lb of steam per ihp-hr.

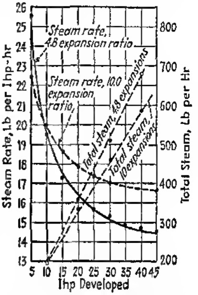


FIG. 4.—Diagram Illustrating Willans' Law of Steam Consumption.

Initial Dryness. Variation of moisture content at the throttle has no appreciable influence on economy. Carpenter and Marks (*Trans. A.S.M.E.*, 15) have shown that the **introduction of water** in proportions from 1 to 42 percent is practically **without influence on the dry steam consumption.** The water remains inert, neither helping nor hindering.

Superheat. Tables 4 to 7 show that **superheat increases efficiency,** and that ideally the increase of efficiency is nearly proportional to the amount of superheat. In practice, superheat is justified by its **influence on cylinder condensation** rather than on thermodynamic grounds. Cylinder condensation is practically eliminated when the steam is kept dry at the point of cutoff. This requires ordinarily about 150 F of superheat. The gain by superheating is progressive, probably up to that point at which the exhaust becomes superheated. **Factors favoring superheat** are slow speed, high ratios of expansion, high fuel cost, and steady load.

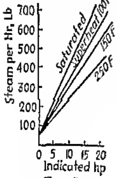


FIG. 5.

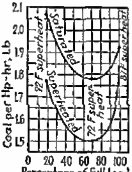


FIG. 6.

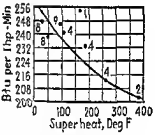


FIG. 7.—Increase in Economy of Compound Condensing Engines with Increasing Superheat.

With constant initial pressure and constant superheat, the total hourly steam consumption varies directly with the output (see Fig. 5). Experiments on H.M.S. "Brittania" show, moreover (Fig. 6) that the variation with load in over-all efficiency of plant is about the same with saturated as with superheated steam.

engines, p is between 1 and $2\frac{1}{2}$. The value 2 should be used in design. If the exhaust steam from an engine is used for heating buildings or in manufacturing processes, p may have any value from 15 upward.

The value of R is commonly around 4 in simple engines. It should increase as P increases and as p decreases, being usually between 3 and 5. It should be higher in jacketed than in unjacketed engines. The efficiency of the engine depends largely on the value chosen for R (see pp. 1034, 1035; also see under "real" and "apparent" ratios of expansion, p. 1025). High values of R will be adopted for an engine to be used where fuel is costly or the load steady. The overload capacity is similarly influenced: low values of R lead to high mep —and hence to large output from a cylinder of given size—but also to low overload capacities. In general European practice, cutoff is fixed so as to give $p_m = 18 + 0.2P$.

Upper Limits of Cutoff for Maximum Over-all Efficiency and Least Steam Consumption (Hrabak)

Throttle pressure, lb per sq in. abs	Non-condensing		Condensing		
	Simple		Simple		Compound
	Slide valve	Expansion valve	Unjacketed	Jacketed	
60	0.53–0.42	0.39–0.31	0.20–0.14	0.15–0.10	
75	0.46–0.32	0.33–0.27	0.17–0.13	0.13–0.09	0.10–0.08
90	0.40–0.28	0.28–0.23	0.15–0.125	0.11–0.08	0.09–0.07
120	0.34–0.25	0.22–0.19	0.14–0.12	0.09–0.07	0.08–0.06
150	0.29–0.20	0.19–0.17	....	....	0.07–0.05

The higher values are for operation with maximum commercial efficiency, the lower values for minimum steam consumption. The higher values given are the upper limits (latest desirable cutoff) and apply with such conditions as small engines, cheap fuel, and intermittent operation; with large engines, high-priced fuel, and continuous operation, the cutoff for maximum commercial efficiency should be made earlier.

Mean Effective Pressures Realized. The values of p_m, obtained from (1), are multiplied by a **diagram factor** f always less than 1.0, to obtain the mep likely to be realized in the actual engine. The value of f is always between 0.45 and 0.95 and usually between 0.80 and 0.95. Values given by Seaton are

Type of Engine	f
Independent cutoff, cylinder jacketed	0.90
Single valve, automatic cutoff, cylinder jacketed	0.86 to 0.88
Single valve, automatic cutoff, without jacket	0.77 to 0.81
Unjacketed throttling engines of small size and high speed	0.58 to 0.71

The following summarizes the principal influences which determine the value of f:

Influence	Effect on the Value of f
Governing by throttling as compared with cutoff regulation	Decrease 0.10 to 0.25
Jackets	Increase 0.05 to 0.15
Very early cutoff (prior to $\frac{1}{5}$ stroke)	Decrease 0.025 to 0.125
High speed (above 225 rpm)	Decrease 0.025 to 0.10
Excessive clearance (over 5 percent)	Decrease up to 0.08
Abnormally small ports and passages	Decrease 0.025 to 0.10
Interrelated valve movements (as with a single valve)	Decrease 0.025 to 0.175

Table 9. Performance of Engines Using Superheated Steam

Number of records	Size of engine, ihp	Initial absolute pressure, lb per sq in.		Superheat, deg, F		Steam consumption, lb per ihp-hr			Btu per ihp-min			References
		Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Average	Minimum	Maximum	Average	
					SIMPLE NON-CONDENSING							
2	32–300	80	91	4	41	25.6	40.0	32.8	442	668	555	
5	100–300	120	175	80	130	21.3	30.0	25.4	363	513	440	17
2	60–800	165	165	100	100	19.7	21.3	20.5	343	371	357	11
2	1,000–1,400	340	455	100	225	15.4	17.1	16.3	286	302	294	14
1	3,000	290	290	200	200	......	......	15.4	......	......	284	15
					SIMPLE CONDENSING							
5	300–600	68	94	16	59	23.6	29.8	26.2	361	408	374	2
2	60–800	165	165	100	100	14.5	15.3	14.9	284	300	292	13
1	1,500	290	290	200	200	......	......	12.6	......	......	262	16
					COMPOUND NON-CONDENSING							
1	1,000	151	151	168	168	......	......	15.4	......	......	283	3
9	175–750	160	175	110	130	20.9	26.5	22.3	368	413	390	12
1	1550	368	368	144	144	......	......	12.8	......	......	238	11
5	2,500–7,000	365	440	100	250	11.0	15.4	12.2	212	272	246	14
					COMPOUND CONDENSING							
8	650–1,100	115	165	12	45	11.9	15.1	13.2	220	283	247	4
8	400–10,000	135	190	9	86	9.0	14.0	12.7	214	259	238	[illegible]
9	130–1,500	103	191	50	132	11.3	15.1	13.0	219	291	246	[illegible]
4	1,000–1,800	123	147	71	139	11.7	13.1	12.3	230	255	241	7
1	200	154	154	167	167	......	......	12.6	......	......	254	[illegible]
4	400–1,000	147	150	176	195	10.4	11.9	11.3	208	260	233	[illegible]
4	400–1,200	142	150	216	310	9.7	10.8	10.3	205	219	213	[illegible]
2	400	132	157	375	398	9.0	9.6	9.3	198	208	203	[illegible]
					TRIPLE CONDENSING							
1	1,000	166	166	39	39	......	......	12.7	......	......	240	10
3	3,000	203	211	13	223	9.0	11.3	9.9	......	......	......	8
2	900, 3,000	170	213	110	213	9.6	9.7	9.6	187	196	192	11
2	1,127–1,129	213	215	118	129	9.0	9.1	9.0	181	184	182	17

REFERENCES: [1] Barrus, "Engine Tests," 1901 (Nos. 10, 21). [2] Barrus, *op. cit.* (Nos. 4, 8, 10, 16). [3] Sulzer power plant engine. [4] Barrus, *op. cit.* (Nos. 43, 48, 50, 51). [5] Marks, *Trans. A.S.M.E.*, **25**, p. 443; Denton, *Trans. A.S.M.E.*, **25**, p. 882; Stott, *Jour. A.S.M.E.*, Mar., 1910; Barrus, *Eng. Rec.*, **2**, 1902, p. 436. [6] Josse, "Neuere Kraftanlagen," 1911. [7] Sulzer engines tested by Schröter and Weber. [8] Sulzer engines. [9] Jacobus, *Trans. A.S.M.E.*, **25**, p. 264; Longridge, *The Engineer*, 1, 1905, p. 546. [10] Barrus, *op. cit.* (No. 59). [11] *Zeit. Ver. deut. Ing.*, 1900, p. 606. [12] Solvay Process Co. engines. [13] *Trans. A.S.M.E.*, **40**, p. 664. [14] Hopewell atmospheric nitrogen plant guarantees. [15] *Power*, **59**, p. 710. [16] *Power*, **66**, p. 291. [17] *Trans. A.S.M.E.*, **47**, p. 1295.

The tests quoted in Table 9 show, in general, an increase in economy with increasing superheat. This is illustrated (for compound condensing engines) in Fig. 7. High superheat (to a temperature of 600 F) causes a saving of about 20 percent in simple engines, 16 percent in compounds, and 8 percent in triples (see Table 10). The simple engine with superheated steam is as good as the compound engine with saturated steam, and superheating may be regarded as **a substitute for compounding.** The compound engine with superheated steam excels the triple using saturated steam.

Back Pressure. Unnecessary back pressure causes absolute loss. Comparison of experimental results from condensing and non-condensing engines illustrates the effect of changing the back pressure from 1 or 2 to 15 or 16 lb.

Cylinder Dimensions. For a double-acting engine

$$H_i = \frac{fp_m AS}{33{,}000} = \frac{fp_m ALN}{16{,}500} = fp_m K \qquad (2)$$

$$A = \frac{33{,}000 \times H_i}{fp_m S} = \frac{16{,}500 \times H_i}{fp_m LN} \qquad (3)$$

in which H_i = indicated horsepower to be expected from the engine; A = average effective area of the piston, sq in.; L = stroke of piston, ft; N = rpm; $S = 2LN$ = piston speed, fpm; $K = ALN/16{,}500 = AS/33{,}000$, a constant for a given engine. In an ordinary double-acting engine, A is the cross-sectional area of the cylinder *minus* half the cross-sectional area of the piston rod. When a tail rod is used, the deduction is the whole cross-sectional area of the rod.

Piston Speed, Revolutions per Minute. Piston speeds range about as follows: for horizontal single-cylinder engines, 600–1,100; for vertical multi-cylinder engines, 800–1,200; for air compressors, 500–750; and for locomotives about 1,500 fpm. The higher the value of S, the greater is the power realized from an engine of given size; or the less is the weight, size, and cost of an engine of given power. With very high values of S, the question of port size and arrangement becomes especially important. Maximum values of S usually accompany maximum values of L.

The value of N is chiefly limited by the type of valve and gear. It may be fixed by the mode of application of the power. Releasing-gear engines seldom run over 100 rpm. Rotary-valve engines, without releasing gear, may work up to N = 240. Ordinary slide- and piston-valve engines may run as fast as 350 rpm; poppet-valve engines to 225 rpm for horizontal and 450 rpm for vertical engines. Large engines with releasing gear almost always run between 80 and 100 rpm. Small engines generally employ maximum values of N for the type of valve gear used. At 24 in. stroke, the speed with any type of valve gear seldom exceeds 200 rpm.

Strokes of engines running around 100 rpm vary from 1½ to 3 times the piston diameter, the ratio being less with larger-size engines. In engines running above 150 rpm, the stroke is usually about equal to the diameter. Pumping engines, which must run at low piston speeds, have long strokes, and even then the value of N is abnormally low. Long strokes favor low clearances.

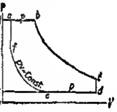


FIG. 2.—Modified Hyperbolic Diagram.

Modified Hyperbolic Diagram. In Fig. 2, the ideal diagram *abcdef* includes the influences of clearance and compression, and thus more closely approximates the indicator card than the diagram of Fig. 1. The mep is

$$p_m = \frac{P}{R}(1 + c - cR) + \frac{P}{R}(1 + c)\log_e R - p[1 + c(1 - R_c)] - pR_c c \log_e R_c \quad (4)$$

in which c = proportion of clearance = $v_a/(v_c - v_a)$, R_c = ratio of compression = v_e/v_f. When Eq. (4) is used for p_m, corresponding values of f are usually between 0.90 and 1.0.

Clearance depends upon the size of the engine, its ratio of diameter to stroke, and the type and location of valves. The **clearance volume** includes all spaces between the piston and the valve faces, when the former is at the (adjacent) end of its stroke. The linear clearance (distance from piston to cylinder head) may account for only a small part of the total clearance volume. The clearance volumes at the two ends of the cylinder are usually unequal. Designers **aim to keep clearance low.** This is more important in simple engines than in compounds, and is most important with high ratios of expan-

Such a comparison is made in **Table 11.** It shows that the condensing engine saves 26 percent over the non-condensing engine in simple four-valve types and 35 percent in releasing-gear compounds (both with saturated steam). The automatic engines profit less when running condensing, partly on account of their high compression. The simple condensing engine is more efficient than the ordinary compound non-condensing engine. With superheat, the condensing engine is more efficient than the non-condensing by 20 to 30 percent for simple types and by 23 percent for compounds.

Table 10. Comparative Results of Steam-engine Tests: Saturated vs. Superheated Steam

Type of engine	Initial pressure, (absolute) lb per sq in.	Superheat, deg F	Steam rate, lb per ihp-hr	Btu consumed per ihp-min	Remarks and references
Simple non-condensing	150–180	0	19.0–22.3	380–446	Figures represent best current practice
	150–180	260	13.4–16.1	300–353	
Simple condensing	120–150	0	16.3–17.5	306–334	
	120–150	260	10.0–11.6	226–254	
Compound condensing	145	0	11.98	225	32 expansions, Denton, *Stevens Ins Ind.* Jan., 1905
	145	307	0.99	192	
	165	0	....	213–246	Schröter engine
	165	250	....	155–223	Schröter engine
	149–151	0	13.0–14.1	(800-hp engine)	Sulzer power-station engines tested by Schröter and Weber
	150	101	11.3		
	144–145	0	14.1–14.5	(1,000-hp engine)	
	147	130	11.7		
	160	0	13.0	250	Rice and Sergent engine; Jacobus, *Trans. A.S.M.E.*, **25**, p. 204
	157	375	9.5	208	
	120–180	0	12.3–16.8	246–333	Figures represent current practice
	120–180	160	10.7–13.4	226–294	
	120–180	260	9.4–11.2	213–246	
Triple condensing	218	0	11.0	217	*Zeit. Ver. deut. Ing.* 1900, p. 606
	213	213	9.6	196	
	174	0	10.8	195	Holly pumping engine, Louisville, 1909
	170	110	9.7	187	
	180–225	0	11.4–13.4	226–253	Figures represent current practice
	180–225	160	10.0–11.2	213–239	
	180–225	260	8.9–10.0	200–220	

Altitude correspondingly affects engine efficiency and capacity, since atmospheric pressure varies about 1 lb per 2,000 ft. of elevation. In non-condensing engines, the mean effective pressure varies about ¾ lb for simples and nearly 1 lb for compounds, for this elevation. With condensing engines the variation is so slight as to be negligible.

Ratio of Expansion. Tables 2 to 7 show that ideally the efficiency varies **directly with the ratio of expansion,** and that a given amount of increase in cylinder volume pays better when the ratio is previously low than when it is high. Complete expansion (continuing until the terminal pressure is that

sion. In the low-pressure cylinders of compounds, clearances are relatively high, on account of the large ports.

Clearance Data	Percentage of Piston Displacement
Extreme range	1 to 15
Range with	
Flat slide valves at the side of the cylinder	5 to 10
Piston valves at the side of the cylinder	7 to 15
Corliss valves	2 to 8
Poppet valves, mean for 1000 hp (Lenz)	4
Flat or piston valves in the cylinder heads	2 to 7
High-speed engine } comparative values	8 to 12
Slow-speed engine } comparative values	4 to 6

"Real" and "Apparent" Ratios of Expansion. The ratio $(v_c - v_a)/(v_b - v_a)$ (Fig. 2) is the "apparent" ratio of expansion. Following the notation of Eq. (4), its value is

$$R_a = R/(1 + c - Rc) \tag{5}$$

and

$$R = (R_a c + R_a)/(R_a c + 1) \tag{6}$$

Engines are usually designed by assuming R_a and c, and then finding R from Eq. (6).

Compression has for an **extreme limit** the condition $p_f = p_a$ (Fig. 2). Where Eq. (3) is used in design, compression is regarded only as influencing the value of f. These values are low for high-speed engines partly because such engines use excessive compression. They are low for single-valve engines because such engines have high compression at light loads. Practical limits of value of p_f may be taken at $0.7(P - p) + p$ for high-speed and $0.1(P - p) + p$ for low-speed engines.

Simple Engines Using Superheated Steam

The ideal diagram of Fig. 2 represents the action of a simple engine using highly superheated steam, excepting that the expansion curve bc no longer approximates the hyperbolic. Its equation is pv^n = constant, where n has a value depending on the amount of superheat and the ratio of expansion. The value of n is about 1.10 (1.17) for 250 (400) deg F superheat. **Diagram factors** will be higher than for saturated steam though throttling, excessive speed, large clearance, restricted ports and passages, or interrelated valve movements may cause variations. **Jackets** are not used. Ratios of expansion may be 40 percent higher than with saturated steam, without reduction of diagram factor. **Piston speeds may be higher** without causing undue port friction. Corliss valves are rarely if ever used with high superheat: the poppet or piston valve may be employed.

With slight superheat, Eq. (1) is used for determining p_m, and the value of f will be governed by the considerations of p. 1023, superheating being here regarded as equivalent to jacketing.

Compound Engines

Preliminary Diagram. In Fig. 3, let the ideal diagram $habg$ correspond with $ghjkl$ of Fig. 1. The diagrams differ in that with the former the terminal pressure equals the back pressure. There is no "terminal drop" (jk, Fig. 1). In Fig. 3, the line bg represents the flow of steam, without fall of pressure, from the "high-pressure" cylinder to the **"receiver."** The latter in turn delivers

Table 11. Performance of Condensing and Non-condensing Steam Engines

(Mostly from Barrus, "Engine Tests," 1901)

Type of engine	Non-condensing			Condensing			Difference in steam rate, per cent
	Size, ihp	Initial absolute pressure, lb per sq in.	Lb steam per ihp-hr[1]	Size, ihp	Initial absolute pressure, lb per sq in.	Lb steam per ihp-hr[1]	
Simple, *four-valve*....	*50–500*	*80–117*	*29.0(13)*	*200–600*	*67–144*	*21.5(9)*	*26*
Compound, automatic cutoff............	50–350	125–182	23.6(7)	90–350	120–145	19.6(7)	17
Compound, four-valve	100	144–150	21.9(2)	300–900	110–166	14.2(18)	35
Simple, slight superheat.[3]............	32–300	80–91	32.8	300–600	68–94	26.2	20
Simple, 260° superheat.[3]............	......	150–180	13.4–16.1	....	120–150	10.0–11.6	27
Compound, superheated steam.[2].....	1000	151	15.4	130–10,000	103–191	9.3–13.2	23

[1] Figures in parentheses in this column denote number of records. [2] See Table 9. [3] See Table 10.

in the exhaust pipe) leads to excessive cylinder condensation, so that the ratio is standardized between 3 and 5 for simple engines and between 6 and 36 for multiple-expansion engines—multiple expansion itself serving to mitigate the influence of cylinder condensation. **Maximum economy** is obtained with relatively high ratios of expansion where jackets, reheaters, or superheat are used, and where the initial pressure is high or the engine *runs condensing*. *Low ratios may* be employed, regardless of thermal efficiency, to secure low first cost.

Most stationary engines are governed by varying the ratio of expansion, so that the change in economy with load illustrates the effect of ratios of expansion on efficiency. The curves of Fig. 8 show this. The flatter curves represent the type of engine best adapted for running economically over a wide range of loads. (For the shape of such curves with *superheated* steam, see p. 1032.) The effect of engine friction on desirable ratio of expansion is discussed on p. 1028.

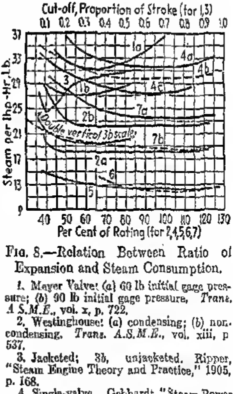


FIG. 8.—Relation Between Ratio of Expansion and Steam Consumption.

1. Mayer Valve: (*a*) 60 lb initial gage pressure; (*b*) 90 lb initial gage pressure. *Trans. A.S.M.E.*, vol. x, p. 722.

2. Westinghouse: (*a*) condensing; (*b*) non-condensing. *Trans. A.S.M.E.*, vol. xiii, p. 537.

3. Jacketed; 3*b*, unjacketed. Ripper, "Steam Engine Theory and Practice," 1905, p. 168.

4. Single-valve. Gebhardt, "Steam Power Plant Engineering," 1908, p. 262.

5. 5500-hp Westinghouse Engine, N. Y. Edison Co.

6. 700-hp Compound, *Trans. A.S.M.E.*, vol. xxiv, p. 1274.

7. (*a*) Non-condensing; (*b*) condensing. *Proc., Inst. C.E.*, vol. cxiv.

Cylinder Condensation. The walls of a steam cylinder are alternately heated and cooled during each revolution. The cold walls cause condensation

steam to a "low-pressure" cylinder, the action of which is represented by the diagram *gbcef*. The two cylinders constitute a compound engine.

To avoid noticeable fluctuations of pressure along *gb*, the receiver must be large, ½ to 1½ times the size of the high-pressure cylinder; the former ratio being used for tandem engines, the latter for cross-compounds in which the cranks are usually 90 deg apart.

Maximum stresses are greater in tandem engines than in cross-compounds, and close regulation of speed is more difficult. On the other hand, the tandem costs less and occupies less space.

If this ideal diagram correctly represented the action, then a single simple cylinder of the same size as the low-pressure cylinder of the compound, working between the same extreme pressure limits and with the same ratio of expansion $R = v_c/v_a$, would give the diagram *hacef*. The simple cylinder would give the same power, then, as the compound engine.

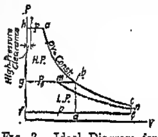


FIG. 3.—Ideal Diagram for Compound Engine.

Basis for Design. The diagram *hacef* of Fig. 3 and the output of the whole engine are determined when P, p, and $R(=v_c/v_a)$ are given. *The proportion of* work done by each cylinder depends upon the receiver pressure, P_o. The designer may proceed in any one of five ways:

1. The receiver pressure may be assumed.

2. The temperature ranges may be made the same in both cylinders. This is probably the best basis with high initial pressures. Then $T_b = (T_a + T_c)/2$, where T_b, T_a, and T_c are (for saturated steam) the temperatures corresponding, respectively, with the pressures P_o, P, and p, from which T_b and P_o may be found.

3. The cylinders may develop the same power, in which case

$$\log_e P_o = \tfrac{1}{2}\left(\log_e R + \frac{Rp}{P} - 1\right) - \log_e \frac{R}{P} \qquad (7)$$

4. The total maximum piston pressures may be equalized, when

$$P_o = P^2/(P + P_oR - pR) \qquad (8)$$

5. The ratio of volumes of the cylinders may be assumed, as $C = v_c/v_b$;

$$P_o = CP/R \qquad (9)$$

Mean Effective Pressures. Having determined P_o by any one of these five methods, and having noted the corresponding value of $C = RP_o/P$, the **approximate mean effective pressures** are, based on Fig. 3,

$$\text{high-pressure, } p_{mh} = [P/R_h] \log_e R_h = [PC/R] \log_e R_h \qquad (10)$$

$$\text{low-pressure, } p_{ml} = [(P_o/R_l)(1 + \log_e R_l)] - p = [(P/R)(1 + \log_e C)] - p \qquad (11)$$

where R_h = ratio of expansion in high-pressure cylinder $= v_b/v_a$;
R_l = ratio of expansion in low-pressure cylinder $= v_c/v_b = C$;
other symbols being as in Eq. (1).

The value of C is usually 2.5 to 3.5 in non-condensing and 4 to 6 in condensing engines. It should vary directly with R. If it is made too great, the engine will, though probably economical of steam, be costly to build and deficient in overload capacity.

At a fixed ratio of expansion R, the output of the engine is independent of the size of the high-pressure cylinder. Exceptionally high efficiencies have been obtained with value of C around 6 or 7 (see p. 1037). When no account of having reached the limit of size (about 100 in. diam), more than one low-

of steam during admission and possibly during the early part of expansion, the heat transfer being at the rate of 20 to 25 Btu per sq ft per deg of temperature difference per minute. This is followed by an evaporation of the steam during late expansion or exhaust.

See Callendar and Nicolson, *Proc. Inst. C.E.*, 131, pp. 147–268; Duchesne, *Rêvue de Mécanique*, 19, pp. 1–40; Mellanby, *Proc. Inst. Engrs. and Shipbuilders of Scotland*, 1911; Barraclough and Marks, *Min. Proc. Inst. C.E.*, 120, ii.

Since most of the condensation occurs during admission, the dryness at cutoff is a measure of economy. This may be ascertained by comparison of the indicator diagram with the experimentally determined steam consumption. The dryness at cutoff and the economy are directly influenced by the size of the engine, its speed and, chiefly, by its ratio of expansion. Figure 9 shows a typical relation between cutoff dryness and ratio of expansion. Up to a certain point, the thermodynamic gain associated with high expansive ratios more than offsets initial condensation.

Jackets. *The steam jacket, by raising the temperature of the walls,* reduces heat transfer. The jacket should be supplied with steam on its way from the throttle to the cylinder. The circulation must be free, and air pockets must be avoided. Superheat is a better device than jacketing if high

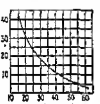


FIG. 9.—Variation of Initial Condensation with the Expansion Ratio.

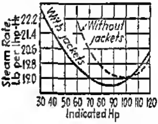


FIG. 10.—Saving Due to Jacketing.

thermal efficiency is sought. Jackets should not be needed when superheat is used.

In nearly all tests reported, the jacket shows some net gain; but unfavorable results may not be published. From 0 to 30 percent (usually 0 to 10 percent) saving in total steam consumption may be effected, the jacket meanwhile consuming 7 to 16 percent (usually 7 to 10 percent) of the total steam. *Average in 10 compounds, 10.2 percent; in 5 triples, 5.8 percent. The lower* percentages occur when superheated steam is used in the cylinders. See p. 1037 as to the omission of low-pressure jackets when reheaters are used. Figure 10 shows results of a test by Carpenter, the saving by jacketing varying directly with the ratio of expansion and becoming negative at very low ratios.

Multiple expansion *permits a high ratio of expansion without excessive* cylinder condensation, and therefore increases efficiency. Roughly, the compound uses 20 to 30 percent less steam than the simple engine. The triple may use 10 to 15 percent less than the compound. These savings (particularly those of the compound over the simple engine) are due partly to the higher initial pressure and lower leakage and clearance losses usually associated with multiple expansion.

The use of superheated steam and the uniflow engine have superseded compounding but it is sometimes resorted to as a means to reduce piston loads.

Triple-expansion engines for marine applications are economical and fairly well balanced. Average cylinder ratios are 1:2.7:8. The size of the high-

trades and industries. The most common of these are the A.P.I. and Baumé. The A.P.I. (American Petroleum Institute) scale is approved by the American Petroleum Institute, the A.S.T.M., the U. S. Bureau of Mines, and the National Bureau of Standards and is recommended for exclusive use in the United States petroleum industry, superseding the Baumé scale for liquids lighter than water. The relation between A.P.I. degrees and specific gravity is expressed by the following equation:

$$\text{Degrees A.P.I.} = \frac{141.5}{\text{sp gr } 60°/60°\text{F}} - 131.5$$

For the salinometer (salometer), see p. 2167. The specific gravities corresponding to the indications of the Baumé hydrometer are given in the following tables.

Specific Gravities at $\frac{60}{60}$ F Corresponding to Degrees A. P. I. and Weights per U. S. Gallon at 60 F.

[Calculated from the formula, specific gravity $= \dfrac{141.5}{131.5 + \text{Deg. A. P. I.}}$]

Degrees A. P. I.	Specific gravity	Lb. per U. S. gallon	Degrees A. P. I.	Specific gravity	Lb. per U. S. gallon	Degrees A. P. I.	Specific gravity	Lb. per U. S. gallon	Degrees A. P. I.	Specific gravity	Lb. per U. S. gallon
10	1.0000	8.328	33	0.8602	7.163	56	0.7547	6.283	79	0.6722	5.595
11	0.9930	8.270	34	0.8550	7.119	57	0.7507	6.249	80	0.6690	5.568
12	0.9861	8.212	35	0.8498	7.076	58	0.7467	6.216	81	0.6659	5.542
13	0.9792	8.155	36	0.8448	7.034	59	0.7428	6.184	82	0.6628	5.516
14	0.9725	8.099	37	0.8398	6.993	60	0.7389	6.151	83	0.6597	5.491
15	0.9659	8.044	38	0.8348	6.951	61	0.7351	6.119	84	0.6566	5.465
16	0.9593	7.989	39	0.8299	6.910	62	0.7313	6.087	85	0.6536	5.440
17	0.9529	7.935	40	0.8251	6.870	63	0.7275	6.056	86	0.6506	5.415
18	0.9465	7.882	41	0.8203	6.830	64	0.7238	6.025	87	0.6476	5.390
19	0.9402	7.830	42	0.8155	6.790	65	0.7201	5.994	88	0.6446	5.365
20	0.9340	7.778	43	0.8109	6.752	66	0.7165	5.964	89	0.6417	5.341
21	0.9279	7.727	44	0.8063	6.713	67	0.7128	5.934	90	0.6388	5.316
22	0.9218	7.676	45	0.8017	6.675	68	0.7093	5.904	91	0.6360	5.293
23	0.9159	7.627	46	0.7972	6.637	69	0.7057	5.874	92	0.6331	5.269
24	0.9100	7.578	47	0.7927	6.600	70	0.7022	5.845	93	0.6303	5.246
25	0.9042	7.529	48	0.7883	6.563	71	0.6988	5.817	94	0.6275	5.222
26	0.8984	7.481	49	0.7839	6.526	72	0.6953	5.788	95	0.6247	5.199
27	0.8927	7.434	50	0.7796	6.490	73	0.6919	5.759	96	0.6220	5.176
28	0.8871	7.387	51	0.7753	6.455	74	0.6886	5.731	97	0.6193	5.154
29	0.8816	7.341	52	0.7711	6.420	75	0.6852	5.703	98	0.6166	5.131
30	0.8762	7.296	53	0.7669	6.385	76	0.6819	5.676	99	0.6139	5.109
31	0.8708	7.251	54	0.7628	6.350	77	0.6787	5.649	100	0.6112	5.086
32	0.8654	7.206	55	0.7587	6.316	78	0.6754	5.622			

The weights in this table are weights in air at 60 F with humidity 50 percent and pressure 760 mm.

Specific Gravities at $\frac{60}{60}$ F Corresponding to Degrees Baumé

for Liquids Lighter than Water and Weights per U. S. Gallon at 60 F

[Calculated from the formula, specific gravity $\frac{60}{60}$ F $= \frac{140}{130 + \text{Deg Baumé}}$]

Degrees Baumé	Specific gravity	Lb per gallon	Degrees Baumé	Specific gravity	Lb per gallon	Degrees Baumé	Specific gravity	Lb per gallon	Degrees Baumé	Specific gravity	Lb per gallon
10.0	1.0000	8.328	33.0	0.8589	7.152	55.0	0.7568	6.300	78.0	0.6731	5.602
11.0	0.9929	8.269	34.0	0.8537	7.108	56.0	0.7527	6.266	79.0	0.6699	5.576
12.0	0.9859	8.211	35.0	0.8485	7.065	57.0	0.7487	6.233	80.0	0.6667	5.549
13.0	0.9790	8.153	36.0	0.8434	7.022	58.0	0.7447	6.199	81.0	0.6635	5.522
14.0	0.9722	8.096	37.0	0.8383	6.980	59.0	0.7407	6.166	82.	0.6604	5.497
15.0	0.9655	8.041	38.0	0.8333	6.939	60.0	0.7368	6.134	83.0	0.6573	5.471
16.0	0.9589	7.986	39.0	0.8284	6.898	61.0	0.7330	6.102	84.0	0.6542	5.445
17.0	0.9524	7.931	40.0	0.8235	6.057	62.0	0.7292	6.070	85.0	0.6512	5.420
18.0	0.9459	7.877	41.0	0.8187	6.817	63.0	0.7254	6.038	86.0	0.6482	5.395
19.0	0.9396	7.825	42.0	0.0140	6.777	64.0	0.7216	6.007	87.0	0.6452	5.370
20.0	0.9333	7.772	43.0	0.8092	6.738	65.0	0.7179	5.976	88.0	0.6422	5.345
21.0	0.9272	7.721	44.0	0.8046	6.699	66.0	0.7143	5.946	89.0	0.6393	5.320
22.0	0.9211	7.670	45.0	0.8000	6.661	67.0	0.7107	5.916	90.0	0.6364	5.296
23.0	0.9150	7.620	46.0	0.7955	6.623	68.0	0.7071	5.886	91.0	0.6335	5.272
24.0	0.9091	7.570	47.0	0.7910	6.586	69.0	0.7035	5.856	92.0	0.6306	5.248
25.0	0.9032	7.522	48.0	0.7865	6.548	70.0	0.7000	5.827	93.0	0.6278	5.225
26.0	0.8974	7.473	49.0	0.7821	6.511	71.0	0.6965	5.798	94.0	0.6250	5.201
27.0	0.8917	7.425	50.0	0.7778	6.476	72.	0.6931	5.769	95.0	0.6222	5.178
28.0	0.8861	7.378	51.0	0.7735	6.440	73.0	0.6897	5.741	96.0	0.6195	5.155
29.0	0.8805	7.332	52.0	0.7692	6.404	74.0	0.6863	5.712	97.0	0.6167	5.132
30.0	0.8750	7.286	53.0	0.7650	6.369	75.	0.6829	5.685	98.0	0.6140	5.110
31.0	0.8696	7.241	54.0	0.7609	6.334	76.0	0.6796	5.657	99.0	0.6114	5.088
32.0	0.8642	7.196				77.0	0.6763	5.629	100.0	0.6087	5.066

Specific Gravities at $\frac{60}{60}$ F Corresponding to Degrees Baumé

for Liquids Heavier than Water

[Calculated from the formula, specific gravity $\frac{60}{60}$ F $= \frac{145}{145 - \text{Deg Baumé}}$]

Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity
0	1.0000	12	1.0902	24	1.1983	36	1.3303	48	1.4948	60	1.7059
1	1.0069	13	1.0985	25	1.2083	37	1.3426	49	1.5104	61	1.7262
2	1.0140	14	1.1069	26	1.2185	38	1.3551	50	1.5263	62	1.7470
3	1.0211	15	1.1154	27	1.2288	39	1.3679	51	1.5426	63	1.7683
4	1.0284	16	1.1240	28	1.2393	40	1.3810	52	1.5591	64	1.7901
5	1.0357	17	1.1328	29	1.2500	41	1.3942	53	1.5761	65	1.8125
6	1.0432	18	1.1417	30	1.2609	42	1.4078	54	1.5934	66	1.8354
7	1.0507	19	1.1508	31	1.2719	43	1.4216	55	1.6111	67	1.8590
8	1.0584	20	1.1600	32	1.2832	44	1.4356	56	1.6292	68	1.8831
9	1.0662	21	1.1694	33	1.2946	45	1.4500	57	1.6477	69	1.9079
10	1.0741	22	1.1789	34	1.3063	46	1.4646	58	1.6667	70	1.9333
11	1.0821	23	1.1885	35	1.3182	47	1.4796	59	1.6860	......	....

Mohs Scale of Hardness

1. Talc. 2. Gypsum. 3. Calc spar. 4. Fluor spar. 5. Apatite. 6. Feldspar. 7. Quartz. 8. Topaz. 9. Sapphire. 10. Diamond.

pressure cylinder determines the overload capacity. Engines with small high-pressure cylinders are very limited in this respect. Clearance volume of high-pressure cylinders has little influence on economy. The reverse is true of low-pressure clearance.

Table 12. Average Steam Rates (Lb per Ihp-hr) with Saturated Steam
(From Stanwood, Barrus, *Proc. Inst. C.E.*, 93, Willans, Clark, *Trans. A.S.M.E.*, 21, etc. Compare with Table 9)

Type of engine	Non-condensing			Condensing		
	Single valve	Double valve	Four valve	Single valve	Double valve	Four valve
Simple	33	30	28	27	42	21
Compound	24	23	22	20	16	14
Triple	..	..	18½	..	..	12

High-ratio Compounds. The best triples have only slightly surpassed the best compounds. With high cylinder ratio, 6 or 7 to 1 (p. 1026), about 30 expansions, and 150 to 200 lb initial pressure, jacketed compound engines have repeatedly consumed about 12 lb of steam of ihp-hr. (*Trans. A.S.M.E.*, **13**, p. 647; **19**, pp. 155, 167, 189; *Power*, July, 1904, p. 424; Barrus, "Engine Tests," No. 47).

Reheaters. The reheater does for the low-pressure cylinders of a multiple-expansion engine just what superheat does for a single cylinder (*Trans. A.S.M.E.*, **25**, 482–492). Adequate reheat involves superheating the steam in the receivers by from 30 to 100 F. With the latter amount, Marks has shown (*op. cit.*) that the efficiency of the (compound) engine may be increased 6 to 8 percent. Good reheating makes low-pressure jackets unnecessary. With moderately superheated steam in both cylinders, the steam consumption per ihp-hr is about constant for the range from ½ load to 1¼ load.

Speed and Size. Cylinder condensation is reduced by high rotative speed, but the efficiency is not usually increased thereby. Along with high speed comes a limitation of choice of valve gear and (usually) imperfect distribution of steam. Slight variations in speed are practically without effect on the economy.

Small engines are usually wasteful. In sizes from 2 to 5 hp, non-condensing, a compound engine used 42 lb of steam per ihp-hr.; simple engines, 78 to 89 lb (*Engineering*, June 27, 1890). Thermal efficiency is not improved with large size; some of the best records have been made by units of not much over 100 hp capacity.

Clearance and Compression. Clearance is a necessary evil and should be kept as small as possible. The heat losses to the walls increase as the clearance increases. This is one of the reasons why small engines (with relatively large clearances) are wasteful and compound engines (which usually have small clearances) are economical. Klemperer gives (*Z. Ver. deut. Ing.*, **1**, 1905, p. 797), for a 7 by 18 in. Corliss engine, non-condensing, with 4½ percent clearance, 25.6 lb steam per ihp-hr; with 15.2 percent, 28.5 lb. A moderate amount of compression (varying with the speed) is justified on mechanical grounds.

Friction. The laws of friction indicate that the power lost in friction should increase somewhat with the load in either a variable-cutoff or a variable-speed engine. Conditions of workmanship and lubrication make it difficult to confirm this by actual tests of engines (see p. 1028), and the friction loss is usually regarded as constant for all loads—an assumption which

moved *to the left* a *relative* distance equal to its lap, *l*. The *absolute* movement of the auxiliary necessary to produce this relative displacement depends on the amount of movement of the main valve meanwhile taking place. The lap (and point of cutoff) may be varied by hand, by turning the valve rod; or the latter may be under the control of a governor which shifts the eccentric so as to change the angle of advance of the auxiliary. The double-piston valve gear made by the Ingersoll-Rand Co. has a separate cut-off valve.

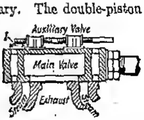
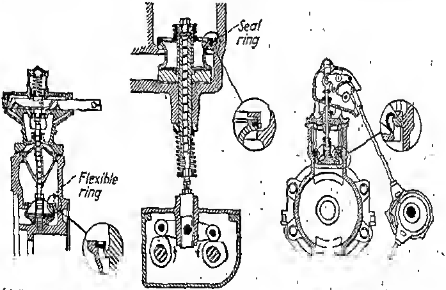


Fig. 6.—Meyer Cutoff Valve.

The **Rider valve**, like the Meyer, incorporates an auxiliary cutoff member, but the lap is changed either by rotating or transversely moving the auxiliary. In the former case, the expansion valve is of the piston type, working in a hole bored lengthwise in the main valve. From this passage, the steam ports are cut obliquely, as are also the lap ends of the auxiliary. When transverse displacement is practiced, the auxiliary is flat and of trapezoidal outline.

Poppet valves have vertical spindles, and usually lift to open. There are commonly four independent valves per cylinder. The exhaust valves are always below the steam valves. The seats are usually double and are nearly balanced, a sufficient unbalanced pressure being provided to insure tightness when closed. It is usually attempted to equalize the two steam-passage areas provided. Figure 7 shows some forms.



(a) Flexible poppet valve with seats machined in the cylinder head (b) Resilient poppet valve with double cam shaft gear (c) Rigid poppet valve in cage

Fig. 7.—Poppet Valves and Gears.

In some types of engine, the valves are operated direct from eccentrics, the minimum number of the latter being two, one each for steam and exhaust valves. More commonly, a pair of miter gears connects the engine shaft with a shaft, from which the valves are driven by eccentrics and cams.

Revolving cams are used to large extent with multicylinder uniflow engines; they are suitable for high rotative speeds. Two forms of such gears have come into use. One form employs a single cam, which is made helical on the closing side and is shifted in direction of its axis to vary the cutoff, and the other type (Fig. 7*b*) has two cams on

underlies the determination of mechanical efficiency by taking a "no-load" or "friction" indicator diagram.

The mechanical efficiency in important engines is usually between 0.9 and 0.96 at full load. The highest record seems to be nearly 0.98. Values below 0.85 are exceptional. The steam rate and thermal efficiency may be referred to brake output by introducing the mechanical efficiency as a factor.

The greater part of engine friction occurs at main bearings. Friction of piston rings and stuffing boxes comes next in importance, and the latter item may be considerably varied in operation. Unbalanced slide valves add greatly to friction losses.

Rotary engines, since the advent of the turbine, have ceased to be even desirable. Of the thousands which have been invented and forgotten, practically all manifested excessive leakage due to the substitution of line contact of wearing parts for the surface contact of an ordinary piston. Even those which are tight at first soon develop leakage.

Uniflow Engines

The typical uniflow engine (Fig. 11) has central exhaust ports which are uncovered by the piston near the end of its stroke. To accomplish this,

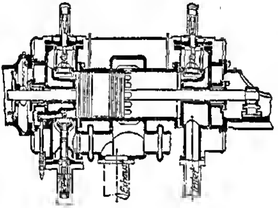


Fig. 11.—Uniflow Engine.

the piston is made of unusual length. Steam enters the cylinder usually through double-beat poppet valves (with flexible upper seat to ensure tightness under varying temperature) or through high-lift single-beat poppet valves, and, after expansion, exhausts through the central ports. The cooling of the cylinder walls and the clearance surfaces by the flow of exhaust steam past them, which is characteristic of the normal alternating- or counter-flow engine, is thereby avoided and the resulting cylinder condensation is greatly reduced. By steam-jacketing the cylinder head, the cylinder condensation loss is still further diminished and this effect is increased by compression up to the initial steam pressure. The exhaust ports close usually at 8 to 10 percent of the return stroke. The clearance in condensing engine is usually 2 to 3 percent or even less; the resulting small clearance surface is a further factor in reducing cylinder condensation.

The permissible expansion ratio in the normal steam engine is limited by the cylinder condensation loss. This limitation does not exist in the uniflow engine which can expand steam from 200 lb per sq in. or higher to a vacuum of 28 in. with an economy which is equal to or greater than that of multicylinder engines of normal type. By expansion in a single cylinder, the

parallel shafts. One cam is retarded and advanced with respect to the other, and the combined motion of the two cams is transmitted to the valve. Cam gears of this type are suitable for any desired cutoff range and for early cutoff give valve lifts that are considerably larger than can be obtained with eccentric gears.

Valve Gears

Rotary Gear. The Corliss valve is skeleton cylindrical, located transversely. It rocks back and forth through a small arc. Separate steam and exhaust valves are provided at each end of the cylinder. Some forms are shown in Fig. 8. The **typical Corliss valve gear** is shown in Fig. 9. If releasing cutoff is to be effective at cutoff later than half-stroke, separate eccentrics are used for steam and exhaust valves. The **diameters of such cylindrical valves** range from 4 to 5 times the maximum port opening. The **length of ports** is about equal to the bore of the cylinder.

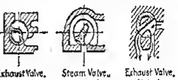


FIG. 8.—Corliss Valves.

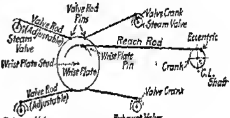


FIG. 9.—Typical Corliss Valve Gear.

Valves are usually located at the ends of the cylinder, the steam valve above and the exhaust valve below. The Corliss gear has small travel and slight friction. It gives particularly small clearances, though designs differ somewhat in this respect. Live steam and exhaust steam never pass through the same port. The valve is not suitable for use with high steam temperatures.

A fundamental feature of a Corliss engine is the **releasing gear.** In Fig. 10, the crank A is loosely mounted on the valve stem, and the grab hook H is loose on a pin fixed in A. The spring S presses the grab hook against the knock-off cam C, the position of which is controlled by the governor. C rocks freely on the valve stem, but the lever B is keyed to the stem.

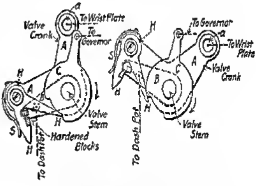


FIG. 10.—Releasing Gear of Corliss Engine.

In the left half of Fig. 10, the valve is about to rotate clockwise. This rotation is produced by the train A, H, B. Simultaneously, the rod running from B to the dashpot rises. As the motion proceeds, the upper of the hardened-steel blocks slides to the right, disengaging from the lower block—as indicated in the right half of Fig. 10. A vacuum dashpot then pulls down on B by means of the vertical rod, closing the valve by a quick counterclockwise rotation. The point of disengagement is determined by the position of the cam C.

transfer losses (pressure drops) between cylinders are avoided and the engine is much simplified and cheapened. Leakage losses are less and the back pressure is diminished because the exhaust ports have an area several times greater than the usual passage past the exhaust valve.

For **non-condensing operation,** some modification of the uniflow engine is necessary to avoid excessively high compression pressures. Generally, there is some departure from the strict uniflow principle. This is either in the form of an auxiliary exhaust valve in the cylinder head or additional clearance volume (pockets connected with the cylinder through an automatically or hand-operated relief valve). Condensing engines also have to be fitted with one or other of these devices which will come into action automatically in case the vacuum fails. The clearance for 100 (200) lb initial pressure is about 20 (12) percent with non-condensing operation. Auxiliary exhaust valves delay the beginning of compression and give a higher mep for a given ratio of

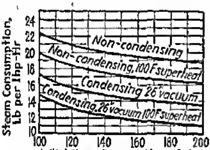


FIG. 12.—Steam Consumption of 400 Hp Uniflow Engine.

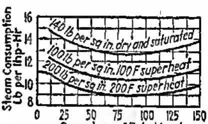


FIG. 13.—Steam Consumption of Uniflow Engine with 27½ in. Vacuum.

expansion than is obtainable with increased clearance volume but probably with some small decrease in economy.

The **steam rates** of uniflow engines are low as compared with other engines. The curves of Fig. 12 for a 400 hp engine may be taken as typical of the results obtained by the best American builders of these engines and should be compared with the performances of the usual type engine as given in Tables 9 and 12. The best results may be 5 to 10 percent better than those shown. The economy does not improve much for larger sizes; for 50 hp, the economy is about 10 percent less than for 400 hp. A notable characteristic is the **small variation in economy with load;** this is shown in Fig. 13. The normal early cutoff permits a considerable overload, usually to 175 percent of rating. Tests with high superheat have yielded the following results in condensing engines:

Observer	Engine	Steam	Lb steam per ihp-hr
Stumpf	24½ by 39 in.	140 lb, 376 F	12.25
Stumpf		120 lb, 490 F	10.85
Stumpf		162 lb, 518 F	9.9
Burmeister and Wain	115 to 220 hp	140 lb; 667 F	9.06 to 9.68
Lentz	100 hp	235 lb, 923 F	6.52
Lentz	100 hp	461 lb, 1018 F	5.67

Mechanical efficiencies are high, ranging from 92 to 96 percent. These engines are now being built in sizes up to 10,000 ihp per cylinder peak load capacity, or 20,000 ihp for twin engines.

Ordinary releasing gears do not work well at speeds above 100 rpm. Corliss valves without disengagement, operated by one fixed exhaust eccentric and a steam eccentric under control of the shaft governor, are a feature of the so-called "four-valve" engine.

Reversing Gears

An engine may be arranged to run in either direction at will, by (1) interchanging steam and exhaust ports; (2) using two alternative eccentrics, with optional engaging grabs; (3) shifting the eccentric back by an angle exceeding 180 deg + the angle of advance; (4) using a reversing gear.

The **reversing gear** is **virtually equivalent to a shifting eccentric**, but with greater flexibility of control, since it changes both position and travel of the valve. In addition, the linkage may correct inequality of distribution due to angularity of connecting rod. It does not actually move the eccentric, but its effect is the same as that due to a movement of the extremity of the valve circle from c along some such path as cp, Fig. 5. This path being determined, the steam distribution may be examined as in Fig. 5. A reversing gear may be used to give gradual variation of cutoff.

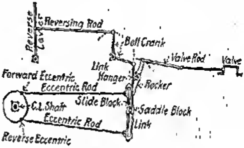


Fig. 11.—Stephenson Link Reversing Gear.

In the **Stephenson link** (Fig. 11), the eccentrics are treated as equivalent to two short cranks, driving the eccentric rods, which are hinged to the link. The radius of curvature of the link is equal to the length of eccentric rods. The valve is moved by the link by means of the slide block, rocker, and valve rod. The link may be raised or lowered by the reversing lever, reversing rod, bell crank, and link hanger. When the link is in its lowest position, the valve derives its motion from the upper eccentric rod entirely, the lower rod serving merely to rock the link about the slide-block pin as a fulcrum. In its uppermost position, the link and valve are effectively moved by the lower eccentric rod alone. At these positions, the valve travel is a maximum. A mid-height on the link, the travel is a minimum, and the motion is determined by both eccentric rods. The movement of the link from one extreme position to the other thus gradually changes the angle of advance and the travel, and both may be fixed temporarily at any desired value between the extreme limits. To minimize slip at the slide-block bearing, the slide block should be immediately under the saddle block at the usual running adjustment.

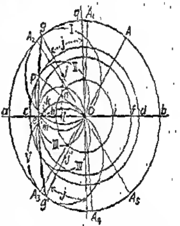


Fig. 12.—Diagram of Stephenson Link Motion.

Figure 11 shows a link motion with "open" rods. Sometimes the rods are "crossed"; i.e., when the crank is at left dead center and the eccentrics are between it and the link, the projections of the rods cross each other. (In either construction, the rods appear

Binary Vapor Engines

Steam is an unsatisfactory working substance because of (1) the limitations which it imposes on the operating temperature range of the engine, (2) the difficulty of maintaining the high vacuum corresponding to low exhaust temperatures, and (3) the excessive volume of a reciprocating engine required at such low pressures. By using a vapor with a higher boiling temperature, the maximum temperature of the cycle can be increased without corresponding increase of the vapor pressure.

A **steam-sulphur dioxide system** has been used in reciprocating engines. This extends the temperature range at the lower limit and avoids the necessity of maintaining a high steam vacuum. As worked out by Josso, the steam expands to about 3 lb abs (142 F) and on condensation generates sulphur dioxide vapor at about 160 lb per sq in. This vapor expands down to about 50 lb per sq in. (70 F) before it is condensed. To reduce steam to the same temperature would require the maintenance of a vacuum of 29.3 in., which is impracticable in a reciprocating engine. The sulphur dioxide engine is of small bulk as a result of its high pressure. Added as a third cylinder to a compound steam engine, it has increased the power output and thermal efficiency about 50 percent. This system has not proved commercially successful.

Mercury vapor and steam make a binary system which can be used for extending the upper temperature limit and is so used in certain turbines but is not practicable for a reciprocating engine.

Regenerative Cycle: Quadruple Engines. By using the receivers as successive feed-water heaters, remarkably high thermal efficiencies have been reached in certain quadruple engines. The cycle of such engines is described in *Trans. A.S.M.E.*, **21**, p. 181; **28**, No. 2, p. 221. The Wildwood pumping engine, 712 hp, 200 lb pressure, partly jacketed, used 186 Btu per ihp-min. A 1,000 hp Nordberg compressor with 257 lb steam pressure used 169 Btu. The steam rate does not measure the efficiency in this type of cycle. These results were reached with saturated steam. A similar practice is common with steam turbines (see p. 1258).

Engine efficiency is the term used for the ratio of the heat input per ihp in the ideal engine using the Rankine cycle (p. 346) to the heat input per ihp of the actual engine. It was formerly called the **efficiency ratio** (sometimes the Rankine efficiency). Its average values are about as follows, expressed in percent. Simple engines, single-valve, non-condensing 61, condensing 41. Simple engine, four-valve, non-condensing 70, condensing, 51. Compound, non-condensing 72, condensing 69. Triple, condensing 73. Superheat increases these values. For the Schmidt high-pressure engine (see p. 1030), it is 81.7. Its value is higher for the h-p cylinder of a compound than for the l-p cylinder.

Present Status. The steam engine is used extensively in sizes up to about 1,000 kw for producing power and for driving compressors; it is also used for locomotives and to a diminishing degree in the marine field. Stationary engines when horizontal are usually single-cylinder. Vertical engines of 3, 4, and 5 cylinders are extensively used and operate at higher rotative speeds with the advantage of cheaper generators and small floor space; they are generally of the uniflow type.

Compound engines are built for two-stage air or gas compressors. Triple-expansion engines are built for ships. Aggregate turbine installations passed those of steam engines in 1924.

crossed at certain crank positions.) With crossed rods, the supply of steam may be cut off at mid-position of the link. With open rods, there is positive lead at all positions.

Figure 12, following the construction of Fig. 6, shows the successive equivalent valve circles I, II, III, IV, V, as the link is shifted from "forward" to "reverse" position. The "equivalent eccentric path" is $gpeg'$. In progressing from "full" to "mid" gear, travel and port opening decrease and lead increases. The crank positions at cutoff are A, A_1, and A_2. Proceeding from mid gear to full reverse, cutoff occurs successively at A_2, A_4, A_5 *(rotation now counterclockwise)*.

With constant lead, the path $gpeg'$ would be a straight vertical line. In the Stephenson link, it is a parabola, closely coinciding with a circular arc drawn through g, g', and e, such that $oe = og\,(\cos goa \pm m \sin goa)$, where m is the ratio of link length to rod length. (The + sign is for open rods, the − sign for crossed rods.) Lead at full gear, with open rods, is usually $\frac{1}{16}$ to $\frac{3}{16}$ in.

Radial gears employ a guiding arm in place of a link. In most of them, the lead is constant, and the *path $gpeg'$ (Fig. 12) is a straight vertical line.*

The **Hackworth gear** (Fig. 13) has the eccentric rod EQ terminating at Q in a slide block. This block moves in a straight slot, the position of which is adjustable. The pin P moves in an ellipse, the proportions of which are determined by the position of the slide-block slot. The gear gives good steam distribution with sharp cutoff, and is compact.

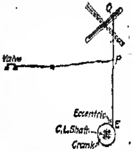


FIG. 13.—Hackworth Valve Gear.

In the **Joy gear** (Fig. 14), the valve is moved partly from the connecting rod by the linkage a, b, c, but the movement is modified by the position at which the movable fulcrum d of the radius bar e is set. There is no eccentric. A slotted guide (as in the Hackworth gear) may replace e. The Joy gear gives quick cutoff, well-balanced and moderate compression.

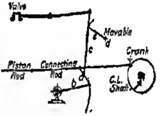


FIG. 14.—Joy Valve Gear.

The **Walschaerts gear** (Fig. 15) also combines two methods of driving: one from the crosshead, through e, f, g, h, and one from the eccentric (90 deg out of phase with the crank) through the eccentric rod, link, and radius rod attached at g. The slide block by which the radius rod is moved from the link is vertically adjustable. When the slide block is in line with the link fulcrum, the valve is moved by the crosshead alone. The link radius is equal to the length of the radius rod. If, when the piston is on dead center, the center of the link arc coincides with g, the lead will be constant. Without the crosshead linkage, there would be no lap or lead or expansion. The effect of this linkage is to increase the displacements of the valve. Minimum travel is determined by analyzing this linkage as it operates when the slide block is under the link fulcrum. Maximum travel occurs when the slide block is at one of the ends of the link. Unless the prescribed proportions are followed, the lead will vary about as it does in the Stephenson gear. The Walschaerts gear as applied almost universally to locomotives uses a supplementary

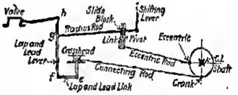


FIG. 15.—Walschaerts Valve Gear.

The types most built (on the basis of aggregate horsepower) are the four-valve and the uniflow: the former, usually, for non-condensing operation. Single-valve throttling engines of small size are built only for unimportant plants.

Table 13. Historical Development of the Reciprocating Steam Engine

Engine	Ratio of expansion	Boiler pressure, lb per sq in.	Type	Lb steam per ihp-hr
Cornish pumps, 1840	1½ to 3½	45	Simple condensing	16½ to 24
Marine, 1850–1890	6	60 to 80	Simple or compound, condensing	19½
Marine, 1850–1890	14	150	Triple condensing	15
Marine, 1850–1890	15	210	Quadruple condensing	13½
Leavitt at Lawrence, 1872	16	120	Compound (Woolf) condensing, 12 rpm	16½*
Corliss at Pawtucket, 1878	16	120	Compound condensing	13¾*
Various power-plant engines	..	...	Triple condensing	12½
Rockwood and Greene	26	150	Compound condensing 7:1 cyl ratio	12⅚*
Rice and Sargent	33	150	Compound condensing	12⅒*
Van den Kerchove	32	130	Compound condensing	12*
Westinghouse	29	185	Compound condensing	12
Leavitt, Snow, Allis, pumps	25 to 33	175	Triple condensing	11.05 to 11.26*
Allis pump	..	85	Triple condensing	10.33†
Nordberg pump	..	257	Quadruple condensing, regenerative	(*A*)
Stumpf, uniflow	..	140	Simple condensing	13.64
Van den Kerchove	..	130	Compound condensing	8.99‡
Stumpf, uniflow	..	190	Simple condensing	9.06§
Josse (binary vapor)	..	...	Triple binary	(*B*)
Locomobile	..	220	Compound condensing	8.25‖
Stumpf, uniflow	..	461	Condensing	5.67(*C*)
Schmidt (p. 1030)	..	794	Condensing	5.12(*D*)
Nordberg	..	368	Compound non condensing	12.8(*E*)

Mostly quoted by Denton, *Stevens Institute Indicator*, Jan., 1905; all engines used saturated steam except the last six. * Engines jacketed. † 196 Btu per ihp-min (*A*) 169 Btu per ihp-min. ‡ 192 Btu per ihp-min (steam superheated 307 F). § Steam superheated to 667 F. (*B*) 167 Btu per ihp-min (steam superheated). ‖ Steam superheated to 660 F. (*C*) 144 Btu per ihp-min (steam superheated to 1018 F). (*D*) 136.6 Btu per ihp-min (steam superheated to 815 F). (*E*) Engine efficiency based on ihp, 79.4 percent (steam temperature 581 F).

Relative Efficiency of Steam Engines and Turbines. The efficiency of the steam engine, compared with the turbine, depends chiefly on the size of unit and operating pressure. Leakage of steam around the blades is high in small turbines, and this loss is greater at higher pressures. On the other hand, higher initial temperatures are used on turbines, in the United States, and the turbine can utilize higher vacuum. In general, large condensing turbines require less steam than large condensing engines, but small non-condensing engines use less steam than turbines of the same size and pressure range. Table 14 gives water rates for engines and turbines. The figures given are the average for a number of installations and are based on steam temperatures of 450 F for small engines, and 550 F for the larger ones; the

outside crank in place of an eccentric. The mechanism is all outside, and occupies little width. Cutoff and lead may be equalized for the two ends.

Engine Parts

Bearings (see p. 863). Modern engines are usually equipped with pressure lubrication. Bearing pressures for high-grade side-crank engines average as follows: Crankpin, 1,300; crosshead pin, 1,700; main bearing, 300; outboard bearing, 150 lb per sq in.

Crank and crosshead pins are made large in diameter and short in length to avoid deflection. Connecting rod ends are shaped for stiffness. Wedge adjustment of bearings is usually omitted.

Main bearings of the quarter box type are still used with side crank frames. The tendency is toward bearings in halves, with the split at 45 deg.

Frames are usually made of cast iron. Sometimes the metal is held in compression by means of through tie rods. Welded frames are also used, with cast-iron crosshead guides bolted in place. With pressure lubrication, attention must be paid to oil tightness and quick collection and removal of oil. Proper ventilation of the crankcase is essential to retain oil mist.

Crosshead guides of horizontal engines are bored and the end face machined at the same setting, to ensure alignment. With high-temperature steam, the cylinder must be insulated from the frame either by restricting the contact area or by interposing distance pieces; alignment between cylinder and guide under these conditions must be obtained by other means. Flat crosshead guides with slipper-type crossheads are preferable as this type of crosshead can operate with smaller running clearance and is therefore better suited for high-speed engines.

Cylinders (see p. 845) are usually made of a semisteel mixture occasionally with the addition of nickel. Highly superheated steam requires careful design to eliminate temperature stresses. Poppet-valve cylinders for such service usually have separate steam and exhaust connection for every valve. Piston-valve cylinders, with inside admission valves, have a semiflexible valve housing with the exhaust passages attached separately.

Valves. Unbalanced slide valves are found only on small engines and direct-acting pumps. Piston valves are universally used on locomotives, either single or double ported. Surviving also is a form of double piston valve, the equivalent of the Meyer valve. Piston valves, in most cases, operate in separate cast-iron bushings and have snap rings for tightness.

Corliss valves are found in a variety of forms. Steam valves are usually double ported (see Fig. 8). They are used with superheated steam up to about 100 F superheat, in which case special oil feeds for the rubbing surfaces of the valve are provided. Non-releasing Corliss gears employ a wrist-plate motion, to reduce the valve travel during periods of unbalance. Such gears require powerful governors of the flywheel or oil-relay type.

Poppet valves are in general use with high-grade engines. They are usually made double-ported, although single-beat valves are used as exhaust valves with uniflow engines. Double-beat poppet valves have two seats in two planes. The distance of the two seats should be held as small as possible unless a resilient type of valve (Fig. 7) can be used. They can be made nearly balanced, at the expense of tightness.

Rigid cast-iron or cast-steel valves seat in cages made of the same material as the valve. If seats are machined either in the cylinder or the cylinder head, resilient valves are used to compensate for uneven expansion. Resiliency is accomplished either by flexibility of the valve itself, in which case a certain degree of unbalance must be provided, or by a built-up construction which requires an additional seal between the valve parts. Steam tightness of poppet valves is as much a matter of design as of workmanship. Valve stems of poppet valves are sealed in long cast-iron bushings with annular grooves to obtain a labyrinth effect; the slight leakage is piped off. Absence of friction and wear, except in cases of misalignment, are advantages of this type of seal.

Customary values of steam velocity past valves are: Steam valves, 200 to 300; exhaust valves, 120 to 180 fps. Exhaust port areas of uniflow engines, with the engine piston acting as a piston valve, are considerably larger than can be obtained with separate exhaust valves. Such engines therefore can utilize lower condenser pressures than engines of the counterflow type.

corresponding temperatures for turbines are from 550 to 725 F. The water rates for a condensing machine are based on 27 in. vacuum for engines and 28½ in. vacuum for turbines.

Table 14. Water Rates for Steam Engines and Turbines
(Lb steam per bhp)

Bhp		100		1,000		5,000	
Throttle pressure, lb gage	Back pressure	Engine	Turbine	Engine	Turbine	Engine	Turbine
250	Condensing	16	15	13	10	12.5	9.5
250	10 lb gage	20	27.5	17.5	21	17	18
400	Condensing	12.5	12	12	9.5	11.5	8.5
400	10 lb gage	16.5	22.5	15.5	17	15	15

While the turbine has largely replaced the reciprocating engine as a prime mover, the latter is still used to advantage:

1. Where extreme flexibility of speed control is required, and particularly where reversal of the direction of rotation is necessary, as in locomotives and rolling mill engines.

2. On small non-condensing machines operating at high initial steam pressures, and on machines operating over wide ranges of load and where low steam consumption is essential.

3. For direct drive of reciprocating machinery, such as compressors and pumps.

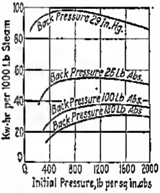


FIG. 14.—Effect of Initial Pressure on Power Obtainable from Steam at 725 F, for Various Back Pressures.

High Back-pressure Engines. For units of small capacity, the reciprocating engine ordinarily requires less steam per unit of output than a turbine. This advantage is marked in the case of high-pressure non-condensing machines and particularly so when operating against high back-pressure.

Figure 14 shows the kwhr obtainable (in engines of from 600 to 3,500 kw capacity) from 1,000 lb of steam per hr at various initial pressures, at 725 F, and with various back pressures (see Ryan, "Higher Steam Pressures in Industrial Plants," *Trans. A.S.M.E.*, 47, pp. 779–797). A comparison with the performance of a steam turbine—1,500 kw capacity—with initial pressure 400 lb abs, back pressure 180 lb abs, steam temperatures 650 F for the engine, 725 F for the turbine, is as follows:

Approximate Water Rates with High Back Pressure of Steam Engine and Steam Turbine

Load, kw	500	1000	1500
Engine water rate, lb per kwhr	73	56	53
Turbine water rate, lb per kwhr	75	69	68

The cost of an engine generator is greater than that of a turbine generator, it occupies more space, and the exhaust is contaminated with oil. Furthermore, the external heat losses are greater, so that the over-all thermal efficiency

Governors (see p. 859). Close regulation requires heavy flywheels, since the inherent friction of a governor necessitates a change of speed before the governor can come into action. A heavy wheel reduces angular acceleration to a point where overregulating and hunting is eliminated.

For a-c generator drives, a regulation of 4 to 6 percent is needed to maintain load distribution, if two or more engines run in parallel. Closer regulation may make load distribution indefinite.

Governors of engines driving a-c generators have attachments to vary the speed plus or minus 5 percent for synchronizing purposes. A small electric motor built into the wheel is used with flywheel governors, changing the tension either of the main governor spring or of an auxiliary spring. Lay-shaft governors, built with two weights for balance, employ a push rod in the hollow bored lay shaft to actuate the speed-changing device.

Variable-speed engines are usually equipped with variable-speed governors which maintain practically uniform regulation over the speed range. In some cases, indirect governing is employed, particularly with speed ratios of 1 to 4 or more.

The natural frequency of oscillation of the governor must differ sufficiently from the frequency of engine impulses to avoid conditions of resonance.

Piston rings (see p. 847) are made of semisteel and are narrow, concentric, and split. The joint is secured by dowels or spring-loaded plugs. Piston-ring joints should not be permitted to travel across ports in the cylinder wall in uniflow engines. Rings have small wall pressure and do not travel into the counterbore of the cylinder.

With high-pressure and high-temperature steam, piston-ring wear is sometimes a problem. Pure steam and the correct relative hardness of ring and cylinder surfaces are essential.

Piston and Piston Rods (see p. 846). Horizontal engines of the slow-speed type are built with self-supported pistons. Bull-ring construction offers a means to compensate for wear of cylinder and piston. Engines operated with high-pressure and high-temperatures steam should have floating pistons (tail rod) with piston rings alone making contact with the cylinder wall. Deflection of the piston rod, on large engines, is compensated for by cambering the rod.

when the exhaust steam is utilized as process steam is less than that of a turbine.

The **highest steam pressure** used for generation of power in the United States is 1840 lb at 820 F. Two boilers designed for that pressure are installed at the Lockland, Ohio, plant of the Philip Carey Mfg. Co. Power is generated in two 6,060 hp, five-cylinder, three-crank triple-expansion vertical engines, direct connected to 3,700 kw a-c generators. Steam is supplied to the throttle at 1,400 lb gage pressure and 800 F total temperature. Steam passes through two single-acting high-pressure cylinders, two single-acting intermediate-pressure cylinders, and one double-acting low-pressure cylinder, exhausting at 60 lb gage. Plant designed by W. E. S. Dyer.

Sulzer Brothers build 850 kw single-cylinder engines with steam pressures of 1,350 to 1,500 lb gage, temperatures 700 to 800 F, and back pressures 85 to 225 lb gage (*Z. Ver. deut. Ing.*, **81**, 1938). The cylinder dimensions are 10 × 23.5 in., the speed 250 rpm, and the mep at full rate 670 lb. The maximum piston load is 85,000 lb, which is less than the load on standard engines with 150 lb gage pressure operating at 150 rpm.

Steam Engines for Compressor and Pump Drives. The reciprocating engine is specially adapted for driving compressors and pumps, because of the higher efficiency of the driven unit, as compared with centrifugal machinery, as well as of the lower steam consumption of the prime mover.

The following tabulation compares the duty obtained from a 15 million gal cross-compound pumping engine with that of a turbine-driven centrifugal pump of the same capacity. Both pumps were built by the same maker and installed at the same plant. The advantage of the reciprocating unit is particularly marked at partial loads.

Duty in Million Ft-lb per Million Btu

Capacity, million gal daily	6	9	12	15
Horizontal cross-compound engine	141	143	146	147
Turbocentrifugal pump	...	...	114	124

There are in use in this country, in chemical plants, steam engines driving compressors and pumps, which operate with pressures of about 400 lb gage and with steam temperatures of 600 to 650 F. Some of these engines have back pressures as high as 225 lb, necessitating large clearance volumes to control compression.

VALVES AND VALVE GEARING

REFERENCES: Peabody, "Valve Gears for Steam Engines," Wiley. Zeuner, "Treatise on Valve Gears," Spon. Spangler, "Valve Gears," Wiley. Dalby, "Valves and Valve Gear Mechanisms," Longmans. Ewing, "The Steam Engine," Cambridge University Press.

Valves and Valve Diagrams

Action of the Valve. In Fig. 1, the **slide valve**, shown in central position, admits steam from the steam chest A through the steam ports a, a', and allows exhaust steam to flow from the cylinder through a, a', to the exhaust port b. **Steam laps** (or "laps") are represented by de and nm; **exhaust laps** are represented by fg and jk. (Either or both may be negative. A negative exhaust lap must be less than the adjoining steam lap.) **Admission** occurs through a when the valve is displaced to the right by the lap de, or through a' when it is displaced to the left by the lap mn. **Maximum port opening**, usually less than the **port width** ef which is designed for the passage of exhaust steam, is attained (on the left side of the piston) when the valve is

RECIPROCATING MARINE STEAM ENGINES

BY

WAYNE T. DIMM

REFERENCES: Bragg, "The Design of Marine Engines and Auxiliaries," Van Nostrand. Seaton, "Manual of Marine Engineering," Van Nostrand. Bauer and Robertson, "Marine Engines and Boilers," Henley. Dyson, "Practical Marine Engineering," International Marine Engineering Press. Furman, "Valves and Valve Gears," Wiley.

The theory of steam engines has been repeatedly written up, but theory without definite rules to follow is of little value to the average practical man. The intent throughout this section has been to avoid theory except where necessary to demonstrate the reason for a given practice.

American marine engine practice has been followed where possible, but where this does not cover the desired ground reference is made to European practice.

CLASSIFICATION AND DEVELOPMENT

Marine Engines Are Classified. (1) According to the arrangement of the working parts as (a) horizontal, (b) inclined, (c) vertical. (2) According to the number of expansions as (a) simple, with one or more cylinders, (b) compound, with two or more cylinders, (c) triple, with three or more cylinders, (d) quadruple, with four or more cylinders. (3) According to the type of propulsion as (a) screw, (b) side-wheel paddle, beam or inclined, (c) stern-wheel paddle, horizontal or inclined. (4) According to the service of the vessel as (a) merchant and (b) naval.

Even though several of the earlier types of marine engines had vertical cylinders, the power was transmitted to the paddle-wheel shaft through a long horizontal beam and a vertical connecting rod and these engines are spoken of as horizontal engines. Inclined engines are used in paddle steamers either of the side-wheel or stern-wheel type. The modern marine screw engine of today is of the vertical type.

Simple engines may be either condensing or noncondensing and may have two or more cylinders working on the same crankshaft. If noncondensing, they are usually spoken of as "high-pressure" engines. Compound engines were originally built with two cylinders, but as the power increased it became necessary for the purpose of construction to use two low-pressure cylinders thus making three-cylinder compound engines. Triple-expansion engines are often made with two low-pressure cylinders to secure better balance and more uniform turning moment. In very large triple-expansion engines, one low-pressure cylinder would be prohibitive on account of its size. Quadruple-expansion engines are not often built with more than four cylinders, but there have been noteworthy installations of large quadruple expansion engines having six cylinders among which may be mentioned the American Line S.S. "St. Paul," and the Hamburg-American S.S. "Deutschland," which have six-cylinder, four-crank quadruple engines with two high-pressure cylinders over the two low-pressure cylinders.

Screw engines operate at much higher revolutions than engines driving paddle wheels and hence differ considerably from paddle-wheel engines owing to the difference in length of stroke.

Merchant and naval engines differ in that no special effort is made in merchant vessels to save weight or space while in naval vessels this is of the utmost importance.

As at present, except in certain types of paddle steamers, horizontal and inclined engines are of historical interest only, this section deals chiefly with the modern, vertical, direct-acting screw engine.

Historical Development of Marine Engines. The earliest marine engines were horizontal, and for a number of years this type was used in war vessels. In these earlier war vessels, which were without armor, protection for the machinery was obtained by placing it below the water line. This fact led to the development of the many curious

displaced its maximum distance to the right; or (on the right side of the piston) when the valve is displaced its maximum distance to the left. **Cutoff** occurs when the valve is displaced as for admission, but is traveling in the opposite direction to that which produces admission. **Release** occurs through a when the valve is displaced by the amount fg of exhaust lap to the left; or through a' when it is displaced by the exhaust lap jk to the right. **Compression** occurs at the same displacements, the directions of valve movement being opposite to those for release. Admission, cutoff, release and compression are described as **critical events.**

Valve and Piston Positions. Usually, the valve will have moved a greater distance than de when the piston has reached the left-hand end of its stroke. The excess of movement over de is called the **lead.** It *is the amount of opening of the steam port when the piston is on the (adjacent) dead center.* There may also be (and usually is) **exhaust lead,** *defined as the* amount of opening of the port on the exhaust side when the piston is at the (more remote) dead center.

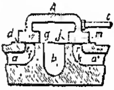


Fig. 1.—Slide Valve.

The valve is moved by an eccentric, equivalent to a crank, the eccentric rod being of great length in proportion to the eccentric throw. Displacements of the valve are therefore regarded as equal to horizontal displacements of the eccentric center, and the **travel** of the valve is equal to **twice the eccentric radius.** If a rocker is used, it may be necessary to write "proportional" for "equal" in these two statements.

With neither laps nor lead, the valve would be central (*i.e.*, in mid-travel) when the piston was at dead center: the two would be 90 deg out of phase. Steam would flow into the cylinder througbout one stroke, and out of it throughout the succeeding stroke. Lap permits cutoff and expansive working. With lap, a 90 deg phase difference between piston and valve would delay admission, and to avoid this disadvantage the phase angle is increased, the valve being displaced from its central position by an amount equal to (lap + lead) when the piston is on dead center. The increase of phase angle is called the **angle of advance**, j. The total phase angle is 90 deg $+ j$.

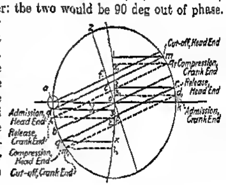


Fig. 2.—Reuleaux Valve Diagram.

Choice of a Valve Diagram. For most purposes, the Reuleaux diagram offers the most convenient method of solving problems connected with a simple eccentric valve gear. The Zeuner diagram is best for Meyer gear, or other double eccentric gears.

The Reuleaux Valve Diagram. (Also called Mueller or Sweet diagram.) In Fig. 2, describe about the center o a circle of horizontal diameter aok, representing the valve travel. Through o draw bo, making the angle $boa = j$; and oz, perpendicular to bo. Lay off oc = lap, op = exhaust lap. Draw af, dcm, qpr parallel to bo. Then the lead is cf, and the four critical events occur at piston-crank positions denoted by the points d, m, r, q. In interpreting these, the large circle is to be regarded as that described by the piston crank, rotating clockwise. The laps are those for the left-hand side of the

and ingenious types of horizontal engines, among which were the **trunk engine** (Fig. 1), the **return connecting-rod engine** (Fig. 2), and the **vibrating or side-lever engine** (Fig. 3).

When the screw propeller first came into use, gearing was used between the slow-turning engine and the propeller shaft to speed up the revolutions in order to obtain proper propeller efficiency.

As the power of marine engines increased, with increase in steam pressure, it soon became evident that marine engines must be vertical in order to be economical in space.

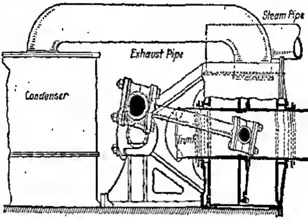


Fig. 1.—Trunk engine.

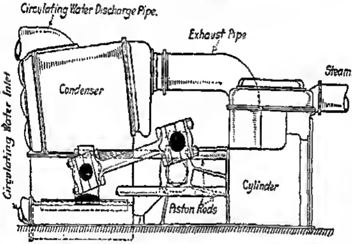


Fig. 2.—Return connecting-rod engine.

Vertical engines were used in merchant vessels long before they were adopted for naval vessels and it was not until the advent of armor which afforded them protection and of high rotative speeds which enabled short strokes to be used that they came into use for all types of naval vessels.

The development of the marine engine is covered briefly by R. H. Thurston in his "Manual of the Steam Engine," in which he says: "The steady rise in steam pressures during the 19th century is best illustrated by naval steam-engineering. In the time of Watt and up to about 1840, the usual pressure in the low-pressure side-wheel engines of that period was from 4 to 7 lb and the rude flue-boilers then in use were of the simplest and weakest forms. By the middle of the century the fire-tubular boiler had come into

valve (*de* and *fg*, Fig. 1) and the crank positions found are those for the critical events referred to that side of the piston which is controlled by the left-hand side of the valve.

On *ka* produced, find the center for an arc *ox*, described through *o*, with radius representing the length of connecting rod to the same scale as *ak* now represents the stroke of the piston. For an engine running "over," this center will be to the left of the diagram. For one running "under," it will be to the right. Then horizontal lines *qx*, *d*3, *m*1, *r*2, represent by their lengths the displacements of the piston from its central position at the critical points, again to the same scale as *ak* represents the piston stroke.

The dotted lines show the construction for the other side of the valve and the other end of the piston. On account of angularity of the connecting rod, the steam distribution is different on the two sides of the piston. Cutoff may be equalized by using different steam laps on the two sides of the valve, but this would lead to inequality of lead. The distance *cz* (Fig. 2) represents the maximum port opening (scale *oc* = lap). Note that (lap + lead) > **exhaust lap**, if release is to occur prior to the end of the stroke.

Examples. 1. Given travel, admission, release, and cutoff. Draw the circle (Fig. 2). Fix points *d* and *m*. Draw *dm*, determining **angular advance.** Fix *r* and draw *rq* parallel to *dm*, determining *q*. Draw *pz* perpendicular to *dm* and *qr*. Then *oc* = **lap**, *op* = **exhaust lap**. Through *a* draw *af* parallel to *dm*. Then *of* = **lead**, *cz* = **maximum port opening.**

2. Given travel, lead, angle of advance, and release. Draw the circle. About *a* draw a circle of radius = lead. Tangent to this circle, draw *dem* making the angle *j* with the horizontal, determining *d*, *m*. Fix *r*, and draw *rq* parallel to *dm*, determining *q*. **Laps** and **maximum port opening** are found as in Case (1).

The Zeuner Diagram. In Fig. 3, draw *jl* horizontally, *AB* vertically, intersecting at *o*; *aok* making the angle *j* with the vertical, and the large circle about *o* of diameter representing the valve travel. On the diameters *ok*, *oa*, describe the small circles. For any crank position 2, the valve displacement is *o*3, determined by drawing the vector *o*2. About *o*, describe arcs *bc*, *de* of radii equal to lap and exhaust lap, respectively. Through the intersecting points *b*, *c*, *e*, *d*, draw radii from *o*, determining

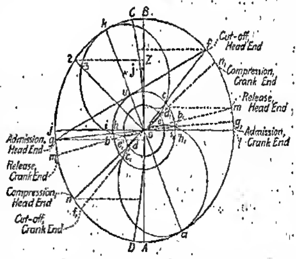


FIG. 3.—Zeuner Valve Diagram.

g, *f*, *m*, and *n* as the critical piston-crank positions. The distance *ih* is the lead, and *uk* is the maximum **port opening.** Piston positions are found as in Fig. 2, by drawing the circular arc *CD* through *o*, and drawing horizontal lines from *g*, *f*, *m*, and *n* to this arc. This construction gives the piston positions for the left-hand (head end) side of the piston in an engine running "over," rotation in Fig. 3 being clockwise. The positions for the crank end are worked out by dotted lines.

quite common use, and pressures had risen to double those above stated. Between 1850 and 1860, the customary pressures in new engines had become 20 to 25 lb, and the introduction of the surface-condenser removing the principal difficulty, the later rise in pressure was rapid and has never ceased. The use of the surface-condenser, by reducing the loss due to the deposition of the calcium sulphate contained in sea water produced a gain of 15 or 20 percent. The type of boiler next made was the cylindrical, Scotch

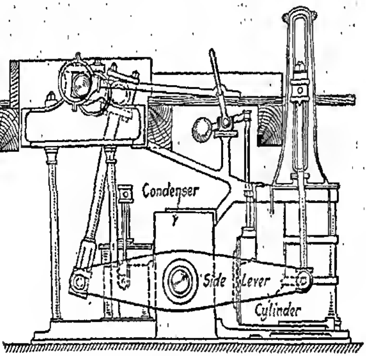


FIG. 3.—Side-lever engine.

form with large flues serving as furnaces and the gases returned through tubes, both flues and tubes enclosed in one cylindrical shell, and the compound engine introduced, the pressures rising rapidly to 60 or 75 lb, these changes resulting in a further economy of 30 or even 40 percent in engines designed during the decade 1860–1870. The next ten years carried pressures for compound engines up to 90 and 120 lb and the triple-

Table 1. Historical Development of the Marine Engine

Date	Type of boiler	Steam pressure, lb gage	Type of engine	Piston speed, fpm
1820	Flue	4-7	Paddle	175
1840	Tubular	10-15		
1850	Flat-sided	20-25	Screw	300
1860	....	30-35	Surface condensing	
1870	Cylindrical	60	Compound	450
1880	....	100		
1885	Water tube	130	Triple	600
1887	....	155		
1890	....	200	Quadruple	700
1900 to date (Merchant)	....	250	Quadruple	900
1900 to date (Naval)	....	300	Four-cylinder triple	1,000

The Zeuner diagram gives valve displacements and port openings for all crank positions in a satisfactory way, but is less accurate than the **Reuleaux diagram** for determining the crank positions at which the critical events occur.

Example. Given cutoff, angle of advance, and port opening. Draw (in Fig. 3) AB, jl, and aok, and the circle on jl, of any convenient radius. Fix f, and draw of. Draw fg perpendicular to aok, determining g. On ok as diameter, describe the small circle, fixing c. About o, describe the arc cb through c, determining h and u. Let p be the required port opening. Then lead $= p\frac{ih}{uk}$; travel $= p\frac{jl}{uk}$; lap $= p\frac{oc}{uk}$.

Crank angles and piston positions for various ratios of connecting-rod length to length of stroke are given on pp. 1006, 1007.

Piston valves may be analyzed as in Figs. 2 and 3, but with "inside edge" distribution the displacement of the valve is (Fig. 1) to the left for admission to the left-hand side of the piston.

FIG. 4.—Multiported Valve.

Multiported valves reduce friction and clearance. Figure 4 shows one form. With a large number of ports, this becomes the **gridiron valve.**

Limits of the Plain Valve. Early cutoff is *impracticable with the plain slide valve.* The point of admission being fixed, hastening of cutoff involves larger lap, earlier release and compression, and either increased travel or reduced maximum port opening. Release can then be delayed by increase of exhaust lap, but this makes compression still earlier.

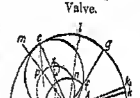


FIG. 5.—Zeuner Diagram for Slide Valve Operated by Shifting Eccentric.

The Shifting Eccentric. In Fig. 5, let ab, cod, ef, and iA be the elements of a Zeuner diagram fixing the four critical events at h, g, k, and j. Let it be required to fix cutoff at l, without change in admission. Draw ol, and om bisecting the angle hol. Find a center, on om, for a circle epn, which shall pass through o and n, the point of intersection of ol and ef. The required conditions are met when epn becomes the valve circle; *i.e.*, when the angle of advance is increased by com and the travel is reduced from cd to $2 \times op$. The symmetrically placed circle on oB now fixes release at k_2 and compression at j_2, the latter event being considerably hastened. Maximum port opening is decreased, but the distribution is superior to that in a plain slide-valve gear.

The **change in throw and angle of eccentric** is accomplished in various ways such as by using a pivoted arm on the flywheel, under governor control. The action may be described by plotting the "path of the eccentric center" cp (Fig. 5). If this path is *straight* and *vertical*, the lead will be constant. If it is *convex* toward o, the travel will be relatively great at late cutoff, and port openings small at early cutoff. Equalization of steam distribution is facilitated when the path is *concave* toward o.

Separate Cutoff Valves. Some of the disadvantages of the plain valve are overcome by employing a second auxiliary mechanism, driven from an independent eccentric, for control of cutoff only, the point of cutoff being varied by hand or by the governor. In the **Meyer valve** (Fig. 6), cutoff occurs (on the left-hand side of the piston) when the auxiliary valve has

expansion engine, coming into use, 1875–1880, the pressure has risen one-fourth or one-third more, this type giving a gain of 15 to 20 percent over the earlier compound engines."

A brief outline of the development of the marine engine is given in Table 1.

Steam Pressure. The earliest marine engines followed land practice and worked with a steam pressure of 4 or 5 lb in a single cylinder with a cut-off of 0.70 to 0.85. Increase in steam pressure was slow, and it was not until about 1880 that 100 lb was reached. The rise in steam pressure was limited by the construction of the boilers on the one hand and by the type of the engine on the other hand. Flat surface or box boilers could not carry pressures over about 30 lb. With the advent of cylindrical boilers (about 1860) the obstacle to the increase in steam pressure from the boiler side was removed. The limit of pressure for single-cylinder engines in the early days was about 50 lb. As compound-, triple-, and quadruple-expansion engines came into use, pressures rose until those in use at the present time were reached. The curve, Fig. 4, shows the increase in steam pressure graphically. Except in radical or freak designs, among which may be mentioned the steam yacht "Arrow," which had a triple-expansion engine built for 350 to 400 lb working pressure, boiler pressures for reciprocating engines have not exceeded 300 lb and this pressure is not likely to be exceeded unless means are found to make use of greater total expansion.

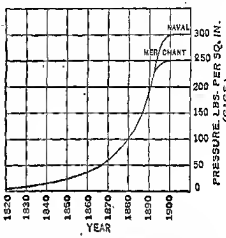


Fig. 4.—Rise in steam pressure.

Pressure Drop. There is always a drop in pressure between the boilers and the throttle valve at the engine, the amount depending on the quality and velocity of the steam, the length of the steam pipe, and the number of bends and valves in the line. For properly designed piping, this drop should not exceed from 5 to 8 lb for merchant work, and from 15 to 20 lb for naval work, the larger drop in naval work being accounted for by the fact that the distance from the boilers to the engines is usually greater, and, in order to reduce pipe sizes and save weight, velocities are higher.

There is also a drop between the main steam pipe and the cylinder through the throttle valve, the engine valves, and the ports. In good designs, this drop should not exceed from 10 to 15 lb in either merchant or naval work.

Pressures in Cylinders and Valve Chests. For the purpose of designing, it is necessary to know the maximum pressure in each cylinder, and the pressures in the valve chests. The maximum pressure in each cylinder occurs at the beginning of the stroke and corresponds to the initial pressure during admission. With slide valves and with piston valves taking outside steam, the pressure on the valve-chest covers corresponds to the initial pressures in the cylinders. For piston valves taking inside steam the pressure on the valve-chest covers corresponds to the pressure of the exhaust from the cylinder

SECTION 2

MATHEMATICS

BY

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

CONTENTS

MATHEMATICS

BY

EDWARD V. HUNTINGTON

ARITHMETIC

NUMERICAL COMPUTATION

Number of Significant Figures. In any engineering computation, the data are ordinarily the result of measurement and are correct only to a limited number of significant figures. Each of the numbers 3.840 and 0.003840 is said to be given "correct to four figures"; the true value lies in the first case between 3.8395 and 3.8405; in the second case, between 0.0038395 and 0.0038405. The **absolute error** is less than 0.001 in the first case, and less than 0.000001 in the second; but the **relative error** is the same in both cases, namely, an error of less than "one part in 3840."

If a number is written as 384,000, the reader is left in doubt whether the number of correct significant figures is 3, 4, 5, or 6. This doubt can be removed by writing the number as 3.84×10^5, or 3.840×10^6, or 3.8400×10^4, or 3.84000×10^5.

In any numerical computation, the possible or desirable degree of accuracy should be decided on and the computation should then be so arranged that the required number of significant figures, and no more, is secured. Carrying out the work to a larger number of places than is justified by the data is to be avoided, (1) because the form of the results leads to an erroneous impression of their accuracy, and (2) because time and labor are wasted in superfluous computation. The labor of working with six-place tables is nearly three times as great as that with four-place tables. In computations involving several steps, it is desirable to retain one extra figure until just before the final result is reached, in order to protect the last figure against the possible cumulative effect of small tabular errors. **In discarding superfluous figures,** if the first discarded figure is 5 or more, increase the preceding figure by 1. Thus, 3.14159, written correct to four figures, is 3.142; correct to three figures, 3.14. Again, 6.1297, correct to four figures, is 6.130.

Addition. In adding numbers, note that a doubtful final figure in any one number will render doubtful the whole column in which the figure lies; hence all figures to the right of that column are superfluous and contribute nothing to the accuracy of the result.

```
  0.2056x
  2.572xx
 14.25xxx
576.1xxxx
---------
593.1
```

Subtraction. The "Austrian" or "shop" method is recommended. The mental process is as follows, the figures here printed in boldface type being the only ones written down:

(3 plus how many is 12?) 3 plus **9** is 12; 1 to carry.	14752
(7 plus how many is 15?) 7 plus **8** is 15; 1 to carry.	8463
5 plus **2** is 7. 8 plus **6** is 14.	6289

or the initial pressure in the next cylinder. It is usual to express the initial pressure in the cylinders and the pressure in the valve chests as a percentage of the steam pressure. Table 2 gives the value of this factor for different types of engines, the factors being based on the steam pressure at the engine.

Table 2. Pressures in Cylinders and Valve Chests

P = Boiler Pressure (Gage); $p = P -$ drop from Boiler to Engine

Type of engine	Compound		Triple			Quadruple			
Boiler pressure	125 to 175		175 to 200			200 to 250			
Cylinder	H-p	L-p	H-p	L-p	L-p	H-p	1st I-p	2d I-p	L-p
Initial pressure in cylinders	$0.85p$ to $0.9p$	$0.15p$	$0.9p$ to $0.95p$	$0.3p$ to $0.35p$	$0.03p$ to $0.04p$	$0.9p$ to $0.95p$	$0.35p$ to $0.4p$	$0.1p$ to $0.2p$	$0.01p$ to $0.02p$
Pressure on valve-chest covers for piston valves with outside steam and for slide valves	$0.85p$ to $0.9p$	$0.15p$	$0.9p$ to $0.95p$	$0.3p$ to $0.35p$	$0.03p$ to $0.04p$	$0.9p$ to $0.95p$	$0.35p$ to $0.4p$	$0.1p$ to $0.2p$	$0.01p$ to $0.02p$
Pressure on valve-chest covers for piston valves with inside steam	$0.15p$	3 to 4 lb abs	$0.3p$ to $0.35p$	$0.03p$ to $0.04p$	3 to 4 lb abs	$0.35p$ to $0.4p$	$0.1p$ to $0.2p$	$0.01p$ to $0.02p$	3 to 4 lb abs

Expansion. The number of times the steam admitted to the high-pressure cylinder of a multiple expansion engine up to the point of cut-off, increases in volume up to the point of release in the low-pressure cylinder, is called the **ratio of expansion** or the number of expansions. If there were no clearance, the expansion in the high-pressure cylinder would be the ratio of the total volume swept through by the piston during one stroke, to the volume of the cylinder at the point of cut-off, or $r = 1/b$, r being the expansion in the cylinder and b the cut-off in percent of the stroke. But **clearance** must always be taken into account and its effect is to reduce the expansion in the high-pressure cylinder and hence throughout the engine. The expansion in the high-pressure cylinder, considering clearance, is $r = (1 + c) \div (b + c)$, where c is the clearance volume expressed as a percent of the volume swept through by the piston. The total expansion in a multiple-expansion engine is the expansion in the high-pressure cylinder multiplied by the ratio of the areas of the low-pressure and high-pressure cylinders. This is on the assumption that the point of release in the low-pressure cylinder is at the end of the stroke, which, for all practical purposes, is sufficiently accurate.

Example. Cut-off in the high-pressure cylinder is 0.75, the clearance volume at each end of the high-pressure cylinder is 0.12, and the ratio of the areas of the low-pressure and high-pressure cylinders, 8; the expansion in the high-pressure cylinder, if clearance is disregarded, is $r = 1/0.75 = 1.33$, and the total expansion $1.33 \times 8 = 10.6$. With clearance taken into consideration, $r = (1 + 0.12) \div (0.75 + 0.12) = 1.29$, and the total expansion is $1.29 \times 8 = 10.3$. The reduction in the total expansion of the engine due to this clearance is therefore 3 percent.

As the number of expansions greatly affects the economy of the engine it is of importance, when designing an engine, that the proper expansion be

early cutoff in the high-pressure cylinder increases the total expansion, improves the economy of the engine, and affects the size of the engine for a given horsepower. The cut off in each cylinder after the high-pressure depends on the ratio of its volume to the preceding one. In order to obtain a uniform turning moment, it is desirable to distribute the power equally on the cranks. This distribution may be obtained approximately by varying the cutoffs in the cylinders after the high-pressure by adjusting the valve gear. In a triple-expansion engine, if the cut off in the intermediate-pressure cylinder is increased the back pressure in the high-pressure cylinder is decreased and more power will be developed in that cylinder and less in the intermediate-pressure cylinder. Similarly increasing the cut off in the low-pressure cylinder will reduce the back pressure in the intermediate-pressure and more work will be done in the intermediate-pressure cylinder and less in the low-pressure. The *cutoff usually used in the high-pressure cylinder of different types of engines* is given in Table 3.

Data and Performance of Marine Engines. Table 7 giving the sizes, proportions, and performances of a number of marine engines for vessels of various types will prove useful for reference. Owing to the scarcity of steam consumption data this table lists the engines of several old vessels on which steam consumption tests were made.

Example. Determine the size of a three-cylinder triple-expansion engine for a large cargo steamer to develop 5,000 ihp with 200 lb steam pressure at the boilers and 27 in. vacuum in the condenser. From Table 5 we will select the rpm as 75 and the piston speed as 750 for an engine of this class and ihp. The length of stroke will therefore be $\frac{750 \times 12}{2 \times 75} = 60$ in. The next step is to determine the proper M.R.P. to use in calculating the area of the low-pressure cylinder. Assume a drop of 10 lb from boilers to engine and 15 lb from throttle valve to cylinder. The absolute initial pressure in the cylinder, P_1, will therefore be 190 lb. Assume a back pressure of 4 lb abs. From Table 3 the ratio of the areas of the low-pressure and the high-pressure cylinders for this pressure may be taken as 8.5 and the cutoff in the high-pressure cylinder at full power as 0.75. Table 4 indicates that the clearance in the high-pressure cylinder may be 12 percent. Expansion in high-pressure cylinder $= \frac{1 + 0.12}{0.75 + 0.12} = 1.29$ and total expansion $r = 1.29 \times 8.5 = 11$. Theoretical mep, $P_m = 190 \frac{(1 + \log_e 11)}{11} - 4$. From Fig. 10 the value of $\frac{(1 + \log_e 11)}{11} = 0.31$. Therefore $P_m = 190 \times 0.31 - 4 = 55$. From Table 6 select a card factor of 0.58 as it is well in determining the size of an engine to allow some margin by assuming a safe card factor. M.R.P. then $= 55 \times 0.58 = 31.9$. Area low-pressure cylinder $= \frac{5{,}000 \times 33{,}000}{750 \times 31.9} = 6{,}900$ sq in. and diam $= 94$ in. Area high-pressure cylinder $= 6{,}900 \div 8.5 = 810$ sq in. and diam $= 32$ in. If the diameter of the intermediate-pressure cylinder is taken 54 in., the ratios of successive cylinders will be 2.85 and 3.03.

PROPORTIONING OF ENGINE PARTS

Throughout this section the details of the engine are proportioned by the calculation of pressures, stresses, and factors of safety as is done by the practical designer rather than by rules and empirical formulas. As reciprocating engines are no longer being designed for the fighting ships of the Navy, the

selected to suit the other conditions. The total expansion to use in design depends upon the boiler pressure, the cut-off in the high-pressure cylinder, and the ratio of the areas of the low-pressure and high-pressure cylinders. The total expansion in the best present-day practice in marine engines is given in Table 3. The **total expansion is approximately** absolute initial pressure in the high-pressure cylinder ÷ 22, 17.5, and 16 for compound-, triple-, and quadruple-expansion engines.

Table 3. Total Expansions for Different Types of Marine Engines

Type of engine	Boiler pressure, lb gage	Cut-off in h-p cylinder, percent	Ratio of l-p to h-p cylinder	Total expansion
Compound	125–175	0.65–0.75	4–5	5.5–7.5
Triple (merchant)	175–200	0.70–0.77	7.5–8.5	9.5–11
Quadruple	200–250	0.65–0.72	9–12	12–15

Back Pressure. The term "back pressure" is usually applied to the pressure of the exhaust from the low-pressure cylinder, although of course there is back pressure in the other cylinders due to the exhausting of the steam into the next cylinder. It is of importance to keep the back pressure in the low-pressure cylinder as low as possible as this pressure acts on the piston in opposition to the steam pressure. In order to keep the back pressure low, care should be taken in the design of the low-pressure cylinder to make the exhaust passages and exhaust pipes of ample area and free from obstructions and abrupt turns. The back pressure in a well-designed low-pressure cylinder **should not exceed 3 to 4 lb abs.** It may be found from the indicator card and is usually about 2 lb more than the pressure in the condenser as shown by the vacuum gage.

Vacuum. In recent years, considerable attention has been paid to the design of condensing apparatus in order to obtain a higher vacuum. Formerly it was the belief among many chief engineers that there was no gain in economy in working with a vacuum over 24 or 25 in. (5 or 6 in. Hg abs), with the accompanying low temperature of air-pump discharge water, as in many merchant steamers there is not enough exhaust steam from auxiliaries properly to heat the feed water. A high vacuum may reduce the steam used by the main engine, but when the total heat used is taken into consideration, in case of insufficient auxiliary exhaust steam to heat the feed water, the fuel consumed may actually increase. There is a limit to the vacuum to which existing engines, designed without special attention being paid to exhaust ports and passages, will respond, and any increase in vacuum over about 25 in. may actually prove a loss owing to the increase in power required for air and circulating pumps.

Clearance. **Volumetric clearance** is the volume enclosed between the piston and the valve face at the beginning of the stroke and is expressed as a percentage of piston displacement. This percentage varies with the size of the engine, the ratio of diameter to stroke, the location of the valves, and the size and length of ports. The percentage of clearance is greater in a fast running engine, as the ports are made larger to avoid excessive steam velocities. The **actual clearance volumes** of an engine are found by measuring the amount of water or other liquid required to fill the clearance spaces.

Table 7. Data and Performances of Marine Engines

Vessel	Type vessel	Date	Type engine	Diam cylinders, in.	Stroke
Virginia..........	River steamer	....	3-cyl. triple	2(17-27-43)	24
Iona:............	....	....	3-cyl. triple	21.9-34-57	39
Vespasian.........	Freight	1909	3-cyl. triple	22¼-35-59	42
Brookline.........	....	....	3-cyl. triple	23-25-57	36
Mariana..........	Freight	1915	3-cyl. triple	21½-36-63	42
Meteor...........	....	....	3-cyl. triple	29.4-44-70.1	48
Topila...........	Oil tanker	1913	3-cyl. triple	24½-41¼-72	48
Madison..........	Freight & passenger	1911	3-cyl. triple	26½-44-74	54
Neches...........	Freight	1914	3-cyl. triple	29-49-84	54
Minnesota........	Freight & passenger	....	3-cyl. triple	2(29-51-89)	57
El Sol...........	Freight	1910	3-cyl. triple	34½-57-96	60
Idalia............	Yacht	1909	4-cyl. triple	11½-19-22 11/16-22 11/16	18
Florida...........	Bay steamer	....	4-cyl. triple	24½-40-47-47	42
Southland........	Bay steamer	1909	4-cyl. triple	26-41-50-50	42
Birmingham.......	U. S. cruiser	1909	4-cyl. triple	28¼-46-62-62	36
Wilhelmina.......	Freight & passenger	1909	4-cyl. triple	35-58-69-69	60
South Carolina....	U. S. battleship	1909	4-cyl. triple	2(32-52-72-72)	48
Delaware.........	U. S. battleship	1909	4-cyl. triple	2(38½-57-76-76)	48
James C. Wallace..	Freight	1905	Quadruple	18½-28½-43½-66	42
Huron............	Freight	1914	Quadruple	19½-28¼-41-60	42
J. D. Rockefeller...	Oil tanker	1914	Quadruple	24-35-51-75	
Pres. Grant.......	River steamer	....	Quadruple	2(25-36-52-75)	51 54
Manoa............	Freight & passenger	1913	Quadruple	27-39-58-87	54
Saxonia..........	Freight & passenger	1901	Quadruple	2(29-41½-59-84)	54
Mongolia.........	Freight & passenger	1904	Quadruple	2(30-43-63-89)	60
Cap Finisterre	Freight & passenger	....	Quadruple	2(30-44-62-90)	55
Geo. Washington..	Freight & passenger	1908	Quadruple	2(28.2-56.6-79.9-112.2)	66.9
Germania.........	Tug	....	Compound	13-24	18
Rush.............	Revenue cutter	....	Compound	24-38	27
Fusi Yama........	....	....	Compound	27.4-50 3	33
Colchester........	....	....	Compound	30-57	36
Ville De Douvres..	....	....	Compound	50.1-97.1	72
Gallatin..........	Revenue cutter	....	Single	34.1	30
Tivoli............	River steamer	....	Beam	38	102
Lancaster.........	River steamer	....	Beam	48	132
Smithfield........	River steamer	....	Beam	51	96
Adirondack.......	River steamer	....	Beam	18	144

Clearance volume may also be calculated from the engine drawings with a fair degree of accuracy. **Linear or piston clearance** (see p. 1077) is the distance between the piston and the cylinder at each end and may account for only a small part of the volumetric clearance. The steam in the clearance space does no work except during expansion and, as it is not practicable to carry compression completely to admission pressure, a certain amount of *fresh steam must be supplied each stroke to fill the clearance space; clearance* therefore **increases steam consumption.** Clearance also **reduces the ratio of expansion** (see p. 1057) and consequently **reduces the work** done per pound of steam.

Although clearance should be kept as small as possible, a certain amount is necessary at each end of the cylinder to provide space for the water which may be condensed when starting the engine, or which may be carried over from the boilers. By keeping the valves close to the cylinders and making the ports horizontal the values given under "short ports" in Table 4 may generally be realized.

Table 4. Volumetric Clearances in Percent of Piston Displacement

	Short ports	Medium length ports	Very long ports
H-p cylinders with piston valves	10–12	16–18	22–26
I-p cylinders with piston valves	10–12	12–14	18–22
L-p cylinders with piston valves	10–12	12–14	16–18
L-p cylinders with slide valves	6–8	8–10	12–14

EFFICIENCY

Thermal

The transformation of heat energy into mechanical energy or work is accomplished by a series of processes called a cycle, during which the working fluid passes through successive states of pressure, volume, and temperature, which, after completion, leaves it in its initial state. The efficiency of a cycle is the ratio of the heat energy converted into useful work to the heat energy supplied.

The **Carnot cycle** as shown in Fig. 5 consists of two isothermal lines *ab* and *cd* representing the application and rejection of heat at constant temperature and two adiabatic lines *bc* and *da* representing expansion and compression without transmission of heat to or from the fluid. It is impracticable to construct an engine to operate on this cycle, but it is important as being the most efficient cycle. The efficiency of the Carnot cycle, $E_c = (T_h - T_c) \div T_h$, where T_h and T_c are the absolute temperatures of the steam at admission and exhaust, respectively.

The **Rankine or ideal cycle** more nearly approaches the cycle of events in an actual engine. An engine operating on this cycle, Fig. 6, would receive vapor at constant pressure (*ab*), expand adiabatically to exhaust pressure (*bc*), exhaust against constant back pressure (*cd*), and compress adiabatically to admission pressure (*da*). In such an engine the exhaust valve would not open until expansion is complete, the walls would be nonconducting, there would be no friction, no leakage past the valves or the piston, and no drop in pressure through the throttle, admission, and exhaust valves. The efficiency

Table 7. Data and Performances of Marine Engines—(*continued*)

Ratio $\frac{l\text{-}p}{h\text{-}p}$	Cut-off h-p cyl	Pressure At boiler	Pressure At engine	Super-heat, deg F	Ihp one engine	Rpm	Piston speed, fpm	Re-ferred mep	Vacu-um	Steam per ihp-hr
6.4	.....	200	.....	.....	1,250	190	760	37.8		
6.78	.....	...	165	.....	615	61.1	397	21.0	23.5	13.4
7.03	0.710	149	.....	.....	1,007	70.1	490	24.7	25.5	17.2
6.15	.....	...	145	.....	1,137	93	558	26.3	21.7	17.0
8.58	0.752	174	164	.....	1,291	84	588	32.3	25.5	
5.68	.....	...	145	.....	1,994	71.8	575	29.8	24.4	15.0
8.64	0.652	201	195	.....	2,320	73	531	32.2	23.1	
7.80	0.765	...	194	.....	3,815	91	819	35.8	25	
8.39	0.587	197	185	.....	4,102	89	801	30.5	27	
9.42	.....	250	240	.....	5,970	89.5	850	37.3	23	
7.74	0.75	195	190	.....	5,541	75	750	33.8	26	
7.8	.....	...	{190	105	{572.3	194	583	36.0	25.5	18.3
			{203		{502.2	193.1	579	35.4	25.2	15.5
7.35	.....	175	.....	.....	2,582	105.4	738	38.2		
7.41	0.815	175	173	.....	3,547	115	805	37.1	24.5	
9.62	.....	...	246	.....	7,200	150	900	43.8	27.9	15.3
7.76	0.763	...	190	.....	5,559	74	740	33.15	25.7	
10.15	0.795	285	275	47.5	8,826	121.3	970	37.04	26.24	
7.8	.....	300	286	61.6	14,289	128.4	1,025	50.9	26.31	13.38
12.75	{0.574	238	234	86.9	{1,559	79	553	27.2	24.5	
	{0.574	237	235		{1,741	80.3	563	29.9	24.6	
9.47	.....	208	.....	.....	1,731	84.9	594	34	21.2	
9.75	0.625	219	214	.....	2,614	71.5	608	32.1	28.7	12.8
9.00	.....	215	.....	.....	4,250	86	774	41		
0.4	0.624	220	212	.....	3,959	80	720	30.5	26.2	
8.4	.....	199	192	.....	4,550	78	702	39	25.35	13.47
8.82	.....	205	200	.....	5,200	85	850	32.5	26	
9.00	.....	213	.....	.....	5,650	82	750	39.1		
8.62	.....	213	.....	.....	10,000	83	926	36.1		
3.41	.....	...	104	.....	256	170	510	36.6	25.5	
2.51	.....	...	69.1	.....	266.5	71	319	24.3	26.5	18.4
3.88	.....	...	56.8	.....	371	55.6	306	20.1	25.1	21.2
3.61	.....	...	80.5	.....	1,022	86	516	25.6	24.8	21.7
3.75	.....	...	105.8	.....	2,977	36	432	30.7	20.3	29.8
.....	.....	...	65.4	.....	260.5	51	306	36.9	25.1	22.0
.....	.....	60	59	.....	826	37.0	629	38.2		
.....	.....	55	.....	.....	810	20¼	446	33.2		
.....	0.358	60	.....	.....	703	28.9	462	24.8	.....	21.35
.....	.....	55	.....	.....	4,000	26	623			

of the Rankine cycle, $E_r = q_r \div q$, where q_r is heat converted into work by this cycle (adiabatic or isentropic heat drop) and q is heat supplied.

In the actual engine however the above conditions are not realized and the actual efficiency is much less than the ideal. The **actual thermal efficiency,** $E_a = q_a \div q$, where q_a is heat converted into work by the actual engine.

It is usual to compare the actual efficiency of an engine with the efficiency of the Rankine or ideal engine. This is called the cylinder efficiency or efficiency ratio, $E_r \div E_a = q_a \div q_r$, and is the ratio of the heat converted into work in the actual engine to the heat which an ideal engine (Rankine cycle) operating with the same admission and exhaust pressures, could transform into work.

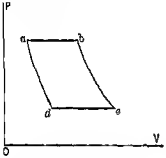


FIG. 5.—Carnot cycle for steam.

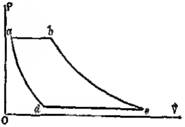


FIG. 6.—Rankine cycle.

All the above efficiencies are based on 1 lb steam, and the heat supplied to the steam, q, is the heat supplied by the boiler and the feed-water heater above the heat of the liquid at exhaust pressure.

Example. The steam consumption of a marine engine at full power is 14 lb per ihp-hr. The pressure in the main steam pipe is 200 lb gage with 50 deg superheat and the vacuum is 27 in. Find (a) Carnot efficiency, (b) Rankine or ideal efficiency, (c) actual efficiency, and (d) efficiency ratio.

(a) Absolute temperature of steam at 200 lb gage and 50 deg superheat, $T_h = 438 + 460 = 898$ deg. Absolute temperature of steam at 27 in. vacuum, $T_e = 115 + 460 = 575$ deg F.

$$\text{Carnot efficiency} = \frac{898 - 575}{898} = 0.36$$

(b) The adiabatic heat drop from 200 lb gage and 50 deg superheat to 27 in. vacuum is best found from an entropy chart.

		Btu
Heat content of steam at 200 lb gage and 50 deg superheat	=	1232
Heat content of steam at 27 in. and same entropy	=	901
Isentropic heat drop, q_r	=	331
Heat content of steam at 200 lb gage and 50 deg superheat	=	1232
Heat content of water at 27 in. vacuum (115–32 deg)	=	83
Heat supplied, q	=	1149

$$\text{Rankine or ideal efficiency} = \frac{331}{1149} = 0.29$$

$$\text{(c) Btu per hp-hr} = \frac{33{,}000 \times 60}{778} = 2545$$

proportions given here are for engines of merchant vessels or of naval auxiliary vessels, which, when using reciprocating engines, follow merchant practice.

Stresses in marine engines are kept low, and large **factors of safety** are used to make the engines safe against the strains caused by rolling and pitching. Furthermore, low stresses and massive parts guard against breakdowns at sea with their accompanying dangers and delays.

Indicated and Effective Thrust. It is usual in designing thrust bearings for reciprocating engines to base the stresses and pressures on the indicated thrust and the proportions of thrust bearings in this section are on this basis. The actual or effective thrust may be found by multiplying the indicated thrust by 0.55 which is the combined efficiency of engine and propeller, assuming the efficiency of the propeller 65 per cent and the mechanical efficiency of the engine, including shaft bearings and attached pumps, 85 per cent.

Destructive Loads for Columns. The sizes of piston rods, connecting *rods, and other parts acting as struts are determined by treating them as* columns. This involves the ratio of length (L) to radius of gyration (ρ), and the destructive load for the material of which the part is made. The column calculation may be greatly simplified by the use of Table 8, giving the destructive loads for various values of $L \div \rho$ for different classes of material. These values are based on the results of an extensive series of experiments on iron and steel struts made by the Pencoyd Iron Works and published in their handbook. The steel used in merchant engines averages about 70,000 lb ultimate tensile strength, and this value is used in the safe stresses and factors of safety given in this section. These figures should be corrected if steel of different ultimate tensile strength is used.

Table 8. Destructive Loads for Iron and Steel Struts
(Adapted from Pencoyd's Handbook)
L = Length of column in in. ρ = Radius of gyration in in.

$\frac{L}{\rho}$	Wrought iron or soft steel of about 46,000 to 55,000 lb tensile strength			Medium steel of about 70,000 lb tensile strength			Hard steel of about 100,000 lb tensile strength		
	Fixed ends	Hinged ends	Round ends	Fixed ends	Hinged ends	Round ends	Fixed ends	Hinged ends	Round ends
20	46,000	46,000	44,000	70,000	70,000	66,900	100,000	100,000	95,600
30	43,000	43,000	40,250	51,000	51,000	47,700	74,000	74,000	69,300
40	40,000	40,000	36,500	46,000	46,000	41,900	62,000	62,000	56,600
50	38,000	38,000	33,500	44,000	44,000	38,800	60,000	60,000	52,900
60	36,000	36,000	30,500	42,000	42,000	35,600	58,000	58,000	49,100
70	34,000	33,750	27,750	40,000	39,700	32,600	55,000	55,100	45,300
80	32,000	31,500	25,000	38,000	37,400	29,700	53,000	52,200	41,400
90	31,000	29,750	22,750	36,100	34,700	26,500	49,900	47,800	36,600
100	30,000	28,000	20,500	34,200	31,900	23,400	46,800	43,700	32,000
110	29,000	26,150	18,500	33,100	29,600	21,100	44,700	40,400	28,500
120	28,000	24,300	16,500	31,900	27,700	18,800	42,600	36,900	25,100
130	26,750	22,650	14,650	30,100	25,500	16,500	39,400	33,500	21,600
140	25,500	21,000	12,800	28,200	23,200	14,200	36,300	29,900	18,200
150	24,250	18,750	11,150	26,800	20,700	12,300	34,200	26,500	15,700
160	23,000	16,500	9,500	25,300	18,100	10,400	32,200	23,100	13,300
170	21,500	14,650	8,500	23,400	15,900	9,240	29,800	20,300	11,800
180	20,000	12,800	7,500	21,400	13,700	8,030	27,400	17,500	10,300
190	18,750	11,800	6,750	19,400	12,200	6,990	25,100	15,800	9,060
200	17,500	10,800	6,000	17,900	11,000	6,120	22,900	14,100	7,860

Actual heat turned into work, per lb steam, $q_a = \frac{2545}{14} = 182$ Btu

Actual efficiency $= \frac{182}{1149} = 0.16$

(*d*) Efficiency ratio $= \frac{0.16}{0.29} = \frac{182}{331} = 0.55$

Aside from improvement in mechanical features the efficiency of an engine may be increased by (1) increasing the initial steam pressure; (2) reducing the back pressure; (3) reducing condensation and reevaporation in the cylinder by (*a*) superheating, (*b*) steam jacketing, (*c*) compounding.

The effect of increasing initial pressure and reducing back pressure is illustrated by the curves in Fig. 7. It will be noted that a large increase in initial pressure is required to produce an effect equal to a small reduction in back pressure.

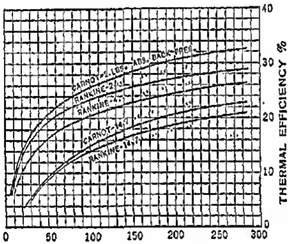


FIG. 7.—Carnot and Rankine efficiency with dry saturated steam.

Cylinder Condensation and Reevaporation. The following treatment of this subject is from Barton's "Naval Engines and Machinery," U.S. Naval Institute: Excepting the loss of heat to the condenser in the exhaust steam, initial condensation in the cylinders is the most serious loss connected with the use of steam as a working medium. During admission, expansion, and exhaust, there is a complex interchange of heat between the cylinder walls and the steam, greatly increasing the steam consumption and reducing the efficiency. Steam at admission comes in contact with the metallic surfaces of the cylinder walls which have been chilled by contact with the cool steam of expansion and exhaust of the previous stroke. Condensation immediately commences and continues until the temperature of the cylinder walls approaches that of the steam. Since the throttle is still open, fresh steam is supplied to take the place of the steam liquefied, so that at the point of cut-off, much more steam has been supplied than corresponds to the volume of the admission space. After cut-off the deposited moisture and water begin to reevaporate as soon as the temperature of the steam falls below that of the cylinder walls, and the weight of the steam present in the cylinder as steam

Section modulus is required in the calculation of bending stresses and, as the sections of the parts of engines figured as beams are usually complicated, the method of finding the section modulus is presented.

The moment of inertia and section modulus of a compound section may be obtained by treating it as a combination of elementary sections of which the properties are already known or can be found. The first step is to find the center of gravity, which, in the case of a symmetrical section, is at the center of the figure. For an unsymmetrical section the center of gravity is determined by multiplying the areas of the component parts by the distances of their centers of gravity from an assumed axis, and dividing the sum of these products by the sum of the areas, which gives the distance of the center of gravity of the whole section from the assumed axis. The moment of inertia of the compound section about an axis through its center of gravity is the sum of the moments of inertia of each component part about an axis through its own center of gravity parallel to the axis of the compound section, plus the sum of the products of each area by the square of the distance of its center of gravity from the axis through the center of gravity of the compound section. The section moduli for tension and compression are obtained by dividing the moment of inertia of the whole section by the distances from the neutral axis

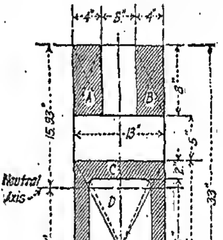


FIG. 11.

Table 9. Method of Finding Section Modulus of a Compound Section

Element	Width (b)	Height (h)	Area (A)	Distance c.g. of element from HK (l)	Moment ($A \times l$)	Moment of inertia about its own axis I_x	Distance from c.g. of element to neutral axis ($l - d = D$)	AD^2
A	4	8	32	29.0	928	171	11.93	4,560
B	4	8	32	29.0	928	171	11.93	4,560
C	13	20	260	10.0	2,600	8,665	7.07	13,000
D	9	15	−135	10.5	−1,417	−2,535	6.57	−5,825
E	4	3	−12	1.5	−18	−9	15.57	−2,910
Total	..	..	177	....	3,021	6,463	.....	13,385

d = Distance of center of gravity above assumed axis HK = $3021 \div 177 = 17.07$ in. I_x of rectangular section $= Ah^2 \div 12$. Total moment of inertia I of section $= I_x + AD^2 = 6463 + 13,385 = 19,848$. Y_t = distance to extreme fiber in tension = 17.07 in. Y_c = distance to extreme fiber in compression = 15.93 in. Z_t = section modulus (tension) $= I \div Y_t = 1163$. Z_c = section modulus (compression) $= I \div Y_c = 1245$.

gradually increases toward the end of the stroke. This heat transferred to the steam by reevaporation during expansion and before exhaust does a small amount of work on the piston, but the heat transferred to the steam during exhaust goes directly to the condenser thus increasing the already large exhaust loss. In general in a single cylinder, condensation and loss from reevaporation are increased by an increase in the range of temperature between admission and exhaust.

Superheat. The practical advantages of superheated steam are primarily due to the reduction of the losses caused by cylinder condensation. If the steam is kept dry to the point of cut-off, these losses are practically eliminated. This would probably require higher superheat than is otherwise desirable in marine work. In early days with the very low boiler pressures in use, considerable gain in economy resulted from the use of superheated steam as a large quantity of heat could be added without increasing the temperature to a dangerous extent. As boiler pressures rose to about 60 lb, the total temperature became too high for the construction and materials then in use and superheat was discontinued on the ground that the high temperature dried up the unguents, destroyed the packing in the stuffing boxes (metallic packing was unknown), and caused excessive wear in the cylinders. Later on, however, the use of superheat in marine engines was revived and during the past 20 years there have been many successful installations using moderate and high superheat in both merchant and naval vessels. There is about 1 per cent increase in economy for each 10 deg F of superheat.

As steam at high temperature has a chemically corrosive effect on all brass and copper besides causing other troubles, the present tendency with reciprocating marine engines is to use moderate superheat (not exceeding 100 deg F) to ensure dry or slightly superheated steam at admission.

With saturated steam the cylinder walls are lubricated by the condensed steam and by the oil entering the cylinder from the use of oil swabs on the rods, but with superheated steam oil lubrication must be supplied and this oil must be removed from the feed water before it enters the boilers. On account of the high temperature special metallic packing is required at least for the piston rods and valve stems of the high-pressure cylinder.

Steam Jackets. By constructing the cylinder with jacket spaces and filling these spaces with live steam, cylinder condensation may be reduced. The jacket steam, to be effective, must be kept in circulation, and this causes a loss by condensation in the jackets which may entirely offset the gain produced by the prevention of condensation within the cylinders. Tests made to determine the economy of steam jackets have usually indicated some gain. However, it is doubtful if in everyday practice there is much economy in the use of steam jackets on account of general defects in the system and lack of attention by the engineers. Also in large engines the thickness of the walls interferes with the rapid interchange of heat. But even if there be no economy in the use of steam jackets, the jacket provides a method of warming the cylinders before starting up. At present, steam jackets are very seldom used in merchant engines and, when used, only those cylinders are jacketed which are fitted with liners which provide the jacket spaces. In naval work it is the practice to jacket all cylinders. In order to prevent excessive condensation in the jackets, jacket steam for intermediate- and low-pressure cylinders should be reduced to a pressure corresponding to the temperature of admission steam within the cylinders.

of the extreme fibers on the tension and compression sides. Table 9 gives the calculations for the moment of inertia and section moduli for the section shown in Fig. 11.

Unbalanced Load. The unbalanced load (*P*) on the high-pressure piston is taken as the basis for proportioning and calculating the strength of the piston rods, crossheads, connecting rods, and housings and for calculating the pressure on the crosshead pins, crosshead slippers, crankpins, and main bearings. The unbalanced pressure is the difference in the absolute pressure on the opposite sides of the piston at the time of admission and is found from the indicator card. The unbalanced load then is the area of the cylinder multiplied by the unbalanced pressure. The unbalanced loads in the

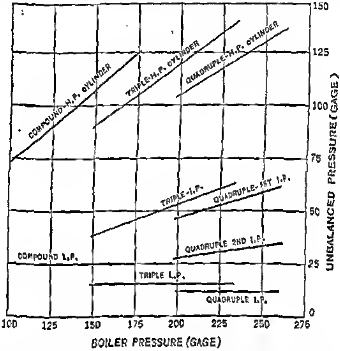


Fig. 12.—Unbalanced steam pressures in marine engines.

different cylinders vary with the ratios of the cylinders and the valve setting, the load on the intermediate-pressure piston of a triple expansion and on the 2d intermediate-piston of a quadruple expansion engine being higher than the load on the high-pressure piston, but for all practical purposes the high-pressure unbalanced load may be used for all cylinders. The unbalanced pressure in all cylinders except the low-pressure vary with the boiler pressure, and the curves in Fig. 12, taken from the indicator cards from a number of engines, show the unbalanced pressures in the cylinders for different boiler pressures. The piston rods, crossheads, and connecting rods are usually made the same size for all cylinders. For the purpose of calculating the strength of these parts for the low-pressure cylinders of four-cylinder triple-expansion engines, it is advisable to assume the unbalanced load in these cylinders as 60 per cent of the unbalanced load in the high-pressure cylinder as the actual unbalanced load often runs higher than the theoretical 50 per cent.

Compounding. To obtain the full benefits of high pressures and large ratios of expansion, it is necessary that the expansion be carried out in two or more cylinders in order to reduce cylinder condensation by reducing the range of temperature in each. Furthermore, the steam reevaporated during the exhaust stroke in one cylinder is available for expansion in the next instead of passing directly to the condenser. The usual pressure ranges for compound-, triple-, and quadruple-expansion engines are given in Table 3.

Mechanical

The **mechanical efficiency** of an engine is the ratio of the horsepower developed in the cylinders, ihp, to the horsepower delivered to the shaft, shp, as determined by the use of torsion meters. The difference between the two is the **frictional loss.** The mechanical efficiency of marine engines varies from 0.90 to 0.93 at full power to 0.94 to 0.95 at low powers. Bauer, in "Marine Engines and Boilers," cites several marine engines in which the mechanical efficiency runs from 0.91 for a 1,640 ihp engine to 0.935 for one of 4,500 ihp. Archibald Denny in a paper read before the Institution of Naval Architects, March, 1907, gives a curve of mechanical efficiency of a certain marine engine which shows values of about 0.925 at 1,325 ihp, 0.937 at 400 ihp and 0.947 at 230 ihp. The mechanical efficiency of **engines with attached pumps** will be at least 5 percent lower than the above figures.

Steam Consumption. It is usual to rate the economy of marine engines in terms of steam consumption per ihp-hr. In general, the steam consumption per ihp will be lower for engines with high ratios of expansion if properly proportioned as to ratios of cylinders, cut-off, clearance, etc., than for engines with low ratios of expansion. Other considerations which improve economy are superheating, lubrication, good workmanship and alignment, and proper care and operation. Very few tests of steam-consumption of reciprocating merchant engines have been made and published but Table 7 gives the steam consumption of several merchant and naval engines of different types.

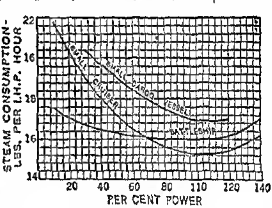


FIG. 8.—Characteristic reciprocating-engine steam-consumption curves.

The steam consumption per ihp **increases as the power decreases.** It is also usually **higher at overload than at designed full power.** These characteristics are shown by the curves in Fig. 8, in which steam consumption per ihp is plotted against per cent of power, calling designed full power 100 per cent. The steam consumption per ihp of merchant engines, which are proportioned to meet the conditions given in Table 3, exclusive of auxiliaries, is approximately as follows: compound 500 to 1,000 ihp, 18 to 22 lb; triple 1,000 to 2,000 ihp, 16 to 17 lb; 3,000 to 6,000 ihp, 15 to 16 lb; quadruple 2,000 to 4,000 ihp, 12.5 to 13.5 lb. The use of superheat should reduce these values about 1 per cent for each 10 deg.

Cylinders

Cylinder Liners and Barrels. The cylinder barrel, bottom, and valve chest are cast in one piece which makes a complicated casting. For this reason a good quality of close-grained cast iron that will run easily must be used. This quality of cast iron does not resist the wear of the piston rings as well as will a harder iron but hardness must be sacrificed to ensure a sound casting. On account of the high pressure and temperature in the high-pressure cylinder it is more liable to fracture and wear than are the other cylinders and so it is customary to fit this cylinder with a close-grained cast-iron liner as hard as can be conveniently machined. In merchant practice, it is not usual to fit liners in low-pressure cylinders, but they are often fitted in intermediate-pressure cylinders. Some owners, however, prefer to have liners in all cylinders to lessen the cost of renewal in case of fracture or serious wear of the cylinder bore.

Figure 13 shows a **h-p cylinder** with a **piston-valve chest** and with straight and direct ports. This gives a much smaller **volumetric clearance** than is possible with the long curved ports formerly used. It will be noted that the top and bottom of the valve chest extend above and below the cylinder, but this is less objectionable than large clearances. The space between the liner and the cylinder is made small to reduce to a minimum the volume for steam in case the joints between the liner and the cylinder leak. For examining the valve laps without removing the top valve-chest cover, inspection holes fitted with covers are provided on the side of the valve chest in line with the top and bottom steam ports.

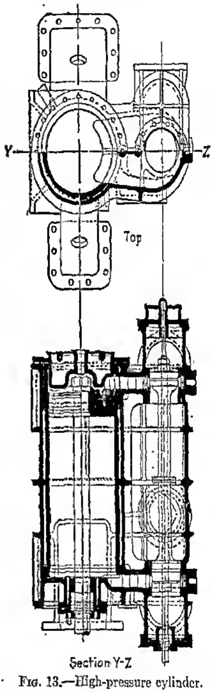


FIG. 13.—High-pressure cylinder.

Figure 14 shows a **low-pressure cylinder** from another engine with a **double-ported slide-valve chest** and with the ports as short and direct as is possible with this type of valve. The volumetric clearance is small and the slide valve permits the top of the valve chest to come in line with the top of the cylinder. A manhole in the bottom allows access.

DETERMINING SIZE OF THE ENGINE

Size of Low-pressure Cylinder. As the low-pressure cylinder must be of sufficient size to pass all the steam used by the engine at terminal pressure it is logical, in determining the size of an engine for a given horsepower, that the diameter of the low-pressure cylinder be determined first. The length of the stroke, revolutions or piston speed (P.S.), and the mep referred to the low-pressure cylinder (M.R.P.) must first be selected. Then area low-pressure cylinder in sq in. $= \frac{\text{ihp} \times 33{,}000}{\text{P.S.} \times \text{M.R.P.}}$. The diameters of the high-pressure and low-pressure cylinders are determined by the ratios of their areas to the area of the low-pressure cylinder.

Length of stroke is chosen to suit the size of the engine and the conditions under which it is to operate and is ordinarily 0.6 to 0.7 of the diam of the low-pressure cylinder. When there are two low-power cylinders, this factor refers to the diam of one equivalent cylinder.

Revolutions per minute (rpm) also vary with the size of the engine and the class of service. Table 5 gives the revolutions and piston speed generally adopted for reciprocating engines for vessels of various types. Piston speed (P.S.) in ft per min $= \frac{\text{rpm} \times \text{stroke} \times 2}{12}$

Table 5. Revolutions and Piston Speed of Reciprocating Engines

Type of vessel	Rpm	Piston speed, fpm
Tugboats and other small craft	100–200	400–550
Screw river and bay steamers	100–150	500–700
Large passenger steamers	75–100	700–1,000
Large cargo steamers	70–85	550–750
Small cargo steamers	65–85	500–700
Beam paddle-wheel engines	15–35	400–700
Inclined paddle-wheel engines	35–60	400–700

Mean Referred Pressure (M.R.P.). In a multiple-cylinder engine the mean referred pressure is the mep that would be required if the work of all the cylinders was done in the low-pressure cylinder. $\text{M.R.P.} = \frac{\text{ihp} \times 33{,}000}{\text{area l-p} \times \text{P.S}}$. The area of the piston rod is not deducted. The M.R.P. is always calculated when the power of an engine is determined from indicator cards as a means of comparison with other engines, and it is also used in determining the size of the low-pressure cylinder of a multiple expansion engine. The designer in determining the M.R.P. must properly coordinate the factors upon which the M.R.P. depends, *viz.*; the cutoff in the high-pressure cylinder, the initial steam pressure, the back pressure, and the ratio of the areas of the low-pressure and high-pressure cylinders. The initial pressure in the high-pressure cylinder depends upon the drop in pressure between the boilers and the cylinder as discussed on p. 1056. Back pressure in the low-pressure cylinder has also been discussed on p. 1058. The M.R.P. to suit the given conditions is found by calculating the theoretical mep and

multiplying this by the **card factor.** The mean referred pressures for engines of different types are given in Table 7.

Theoretical Mean Effective Pressure. This is the mean pressure of admission and expansion as shown by the theoretical diagram in Fig. 9. This diagram represents the pressure and volume of the steam admitted to the high-pressure cylinder, the expansion through all the cylinders, the volume of the steam at release, and the theoretical pressure at exhaust. It is usual to assume that the steam expands in accordance with the law pv = constant in which case the expansion curve is hyperbolic and the theoretical mep,

$$P_{m_t} = P_1 \frac{(1 + \log_e r)}{r} - p_b$$

in which P_1 is absolute initial pressure, r is total expansion, and p_b absolute back pressure. To simplify the calculation the values of $\frac{1 + \log_e r}{r}$ for different values of r may be taken from the curve in Fig. 10. In calculating P_m it is advisable to use a drop of about 10 lb between the boilers and the engine and a drop of about 15 lb from the throttle valve to the cylinder for the ordinary arrangement of machinery and piping.

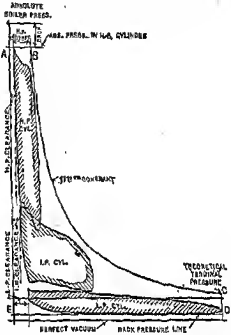


Fig. 9.—Combined actual and theoretical indicator diagram for a triple-expansion engine.

With these assumed drops the absolute initial pressure, P_1, will be 10 lb less than the boiler pressure by gage. Ordinarily a back pressure of 4 lb abs may be assumed. The value of r is calculated as shown on p. 1057.

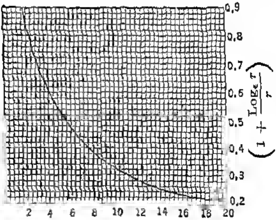


Fig. 10.

Card Factor. The ratio of the areas of the actual indicator cards to the theoretical combined card $ABCDE$ as shown in Fig. 9 or the ratio of the actual mean referred pressure to the theoretical mep (M.R.P. $\div P_m$) is called the "card factor." This factor represents the losses due to cylinder condensation, drop in pressure through ports and passages, and all other losses between the heat available in the steam and the

Thickness of Liners and Barrels. In determining the thickness of liners and of cylinders without liners, allowance is always made for reboring. The following formula, in which P = boiler pressure, D = diam in in., f = allowable fiber stress, and c = a constant for reboring, is used in determining the thickness of high-pressure cylinders and liners: thickness $= (P \times D) \div 2f + c$. The constant c is usually taken as $\frac{1}{8}$ in. and the stress is kept low; 1,500 lb for small and 2,500 lb for large high-pressure cylinders is good practice. In determining the thickness of cylinder barrels in which liners are fitted, the same formula and the same stress are used but no allowance is made for reboring; in this case D = the actual cylinder-barrel diameter. The thicknesses of different sizes of high-pressure cylinder liners and of cylinders fitted with liners for a steam pressure of 200 lb are given in Table 10. For pressures above or below 200 lb the thickness should be increased or decreased accordingly. It is usual to make the thickness of cylinders and liners for the other cylinders the same as the high-pressure. In designing cylinders much depends on the judgment of the designer in giving due consideration to factors such as the minimum thickness of a large cylinder to ensure a sound casting, the ribbing of flat surfaces, and the avoidance of lumps of metal which may cause cracks or spongy spots in the casting. Table 11 gives the thickness of low-pressure cylinders with and without liners.

Table 10. Thickness of H-p Cylinders Fitted with Liners
For 200 Lb Steam Pressure

Diam of bore	Thickness cylinder	Thickness liner
15	$1\frac{1}{16}$	$1\frac{3}{16}$
20	$1\frac{1}{8}$	$1\frac{1}{4}$
25	$1\frac{1}{4}$	$1\frac{3}{8}$
30	$1\frac{3}{8}$	$1\frac{1}{2}$
35	$1\frac{1}{2}$	$1\frac{5}{8}$
40	$1\frac{5}{8}$	$1\frac{3}{4}$

Table 11. Thickness of L-p Cylinders

Diam of bore	If liner fitted	If no liner fitted
40	1	$1\frac{1}{8}$
50	1	$1\frac{3}{16}$
60	$1\frac{1}{8}$	$1\frac{5}{16}$
70	$1\frac{1}{8}$	$1\frac{3}{8}$
80	$1\frac{1}{4}$	$1\frac{1}{2}$
90	$1\frac{3}{8}$	$1\frac{5}{8}$
100	$1\frac{1}{2}$	$1\frac{3}{4}$
110	$1\frac{5}{8}$	$1\frac{7}{8}$

Thickness of Cylinder Walls. Representing the thickness of metal in the cylinder without liners by t, the thickness of the different walls of the cylinder may be taken as follows: thickness of metal in bottom $= t$; in steam ports and valve chest $= 0.85t$; in slide-valve face $= 1.3t$; in cylinder feet $= t$ to $1.1t$; in ribs in cover or bottom $= t$.

The cylinder bottom should be strongly ribbed and a good connection made to the cylinder feet as the unbalanced load is transmitted through the

heat actually converted into work in the cylinders. This factor varies with the number of cylinders in which expansion takes place, and various other considerations. . For the usual ratios of cylinders and total expansions the values in Table 6 may be used for saturated steam. Methods of calculating

Table 6. Ratio of Actual to Theoretical Mean Referred Pressure (Card Factor)

Type of engine	Card factor
Single-cylinder, l-p paddle engines	0.68–0.72
Two-cylinder, compound paddle engines	0.62–0.66
Two-cylinder, small compound screw engines	0.58–0.66
Three-cylinder, triple-expansion merchant engines	0.58–0.63
Four-cylinder, triple-expansion merchant engines	0.57–0.62
Four-cylinder, quadruple-expansion merchant engines	0.60–0.64

card factors differ in that P_1 is sometimes taken as the boiler pressure and sometimes the back pressure and clearance are neglected. With superheat the specific volume of the steam is increased and larger cylinders are required for the same power. It therefore follows that smaller card factors must be used. Data from actual engines show that the card factor is reduced approximately 1 per cent for each 10 deg of superheat. The use of steam jackets increases the card factor.

Ratios of Cylinders. The ratio of the areas of the low-pressure and high-pressure cylinders of a multiple-expansion engine must be taken into consideration in connection aith the total ratio of expansion. The ratios for different boiler pressures and ratios of expansion for well-proportioned marino enginos are given in Table 3. The ratios of the areas of successive cylinders, in connection with the cutoff in each cylinder, affect the power distribution in the different cylinders. In practice it is usual to determine the sizes of the intermediate cylinders more or less arbitrarily and proportion the cutoffs to suit the ratios. In triple-expansion engines the ratio of the areas of successive cylinders is approximately $\sqrt{\text{area l.p.} \div \text{area h.p.}}$ and in quadruple engines approximately $\sqrt[3]{\text{area l.p.} \div \text{area h.p.}}$ However, it is usual to increase somewhat the ratios of the areas of successive cylinders, toward the low-pressure cylinder, approximately as follows:

	$\frac{\text{1st i-p}}{\text{h-p}}$	$\frac{\text{2d i-p}}{\text{1st i-p}}$	$\frac{\text{L-p}}{\text{2d i-p}}$	$\frac{\text{L-p}}{\text{H-p}}$
Triple	2.65–2.8		2.85–3.0	7.5–8.5
Quadruple	2–2.2	2.1–2.3	2.2–2.4	9–12

The ratios of cylinders for engines of different types are given in Table 7.

Cutoff. The cutoff in the high-pressure cylinder regulates the total horsepower of the engine, and the cutoff in the other cylinders regulates the distribution of power in the different cylinders. Decreasing the high-pressure cutoff decreases the total power as less steam is admitted per stroke. Similarly increasing the high-pressure cutoff increases the total power. An

feet to the engine framing. Sometimes for very large cylinders or when jackets are used, the bottom is made double. In large cylinders a manhole is fitted in the bottom for access as shown in Fig. 14.

Water Testing of Cylinders. After the cylinder castings are machined, they should be tested by water pressure for strength and soundness as follows: high-pressure cylinders to 1.5 times the boiler pressure, intermediate-pressure cylinders to 1.5 times the initial pressures given in Table 2, and low-pressure cylinders to 25 lb.

Steam ports are made as **wide** as practicable to keep the **height** a minimum and thereby reduce the valve travel. The area is determined by the steam velocities given in Table 13. The steam ports are made as **short and direct** as practicable to keep the volumetric clearance small and to reduce resistance and condensation. Abrupt turns and projections in the ports should be avoided.

Cylinder Bolting. Cylinder cover studs vary from 1⅛ in. for a 15-in. cylinder to 1¾ in. for a 40-in. high-pressure cylinder. Valve-chest cover

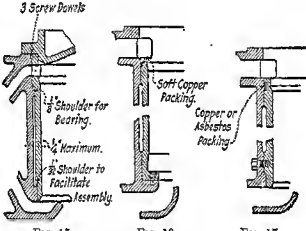


FIG. 15. FIG. 16. FIG. 17.
Methods of securing cylinder liners.

studs vary from 1 to 1½ in. for the same size cylinders. One size of studs is used for all cylinder covers and one size for all valve-chest covers in the same engine. **Pitch of studs** in diameters to ensure tight joints should not exceed: 3 to 4 for high-pressure, 4 to 5 for intermediate-pressure, and 5 to 6 for low-pressure cylinders. The stress in the studs should be based on the initial pressures in the cylinders and valve chests given in Table 2. Stresses should not exceed 3,000 lb for small and 5,500 lb for large studs. **The flanges for cylinder and valve-chest covers** should be 2.6 to 2.8 times the diameter of the studs in width and 1.4 to 1.8 times the diameter of the studs in thickness. For **bolting in the feet** see p. 1083.

Securing Cylinder Liners. It is very difficult to make joints between the cylinder and the liner which will remain steamtight after the engine is in commission. It is therefore important that the space between the cylinder and the liner, which may be filled with steam, be reduced to a minimum. Figure 15 shows a liner supported by a shoulder turned near the upper end. The liner extends to the top of the cylinder and is held in place by the cylinder

This method is especially useful when it is desired to subtract from a given number the sum of several other numbers.

7 plus 1 is 8; plus 5 is 13; plus 9 is 22; 2 to carry.	14752
5 plus 0 is 5; plus 2 is 7; plus 8 is 15; 1 to carry.	3125
3 plus 1 is 4; plus 1 is 5; plus 2 is 7.	101
5 plus 3 is 8; plus 6 is 14.	5237
	6289

The use of a wavy line to indicate subtraction is also recommended, as it will minimize the danger of adding when subtraction is intended.

Multiplication. In long examples in multiplication, the arrangement of work here illustrated is recommended, since it facilitates the abbreviation of the work by the omission, in practice, of all the figures on the right of the vertical line.

```
 4956
 8372
39648|
 1486|8
  346|92
    9|912
41492|xxx
```

The **position of the decimal point** should be determined by reference to the first, or left-hand, figures of the numbers, rather than by "pointing off" so-and-so many places from the right-hand end. For the right-hand figures of a number are the least important ones, and in many cases are entirely unknown (especially when the slide rule or a computing machine is used). The mental process for determining the decimal point is as follows:

(*a*) If the multiplier is a number like 3.1416, with only one figure preceding the decimal point, think of this number as "a little over 3"; then the product must be "a little over three times the number which is being multiplied"; and this gives the position of the decimal point at once, by inspection.

(*b*) If the multiplier is a number like 3141.6 [or 0.000 003 141 6], think of this number as "about 3, with the point moved three places to the right" [or "about 3, with the point moved six places to the left"]; then think what the answer would be if the multiplier were simply "about 3," and shift the decimal point accordingly.

Multiplication Tables. Crelle's large volume (G. Reimer, Berlin) gives the product of every three-figure number by every three-figure number; Peters's (G. Reimer, Berlin), of every four-figure number by every two-figure number. The smaller table of H. Zimmerman (Wm. Ernst, Berlin) gives the product of every three-figure number by every two-figure number. Tsuneta Yano's Table (published by the First Mutual Life Ins. Co., Tokyo, Japan) gives the same information as Crelle's Table in small book form.

Division. In long division, where the numbers are given only approximately, the work can be much abbreviated without loss of accuracy by "cutting off" one figure of the divisor at each step, instead of "bringing down" a doubtful zero in the dividend. Thus, $3.1416 \div 2.3026 = 1.3644$.

```
23026)31416(1
      23026
 2303) 8390(3
       6909
  230) 1481(6
       1380
   23)  101(4
         92
    2)    9(4
```

To determine the **position of the decimal point** in a problem of fractional division, shift the point (mentally) in both numerator and denominator (the same number of places in each) until the denominator is a number in the "standard form," that is, a number with only one figure preceding the decimal point. (This will not change the value of the fraction.) Then estimate the approximate magnitude of the quotient by inspection. Thus:

$$\frac{0.2718}{3141.6} = \frac{0.000\ 2718}{3.1416} = \text{"about } 0.000\ 09\text{"} = 0.000\ 08652;$$

$$\frac{31.416}{0.002718} = \frac{31{,}416}{2.718} = \text{"about } 10{,}000\text{"} = 11{,}558.$$

Reciprocals. The reciprocal of N is $1/N$. Instead of dividing by a long number N, it is often better to multiply by the reciprocal of N. The table of reciprocals on pp. 24–27 gives the reciprocal of any number, correct to four figures. Barlow's Table (Spon & Chamberlain, New York) gives the reciprocal of every four-figure number correct to seven figures (but without facilities for interpolation). The reciprocals of numbers having more than four figures are best found by the use of a large table of logarithms.

Reciprocals of $1 \pm x$ when x is Small.

$1/(1+x) = 1 - x + [\text{error} < x^2$, if x is between 0 and 1],
$\quad = 1 - x + x^2 - [\text{error} < x^3$, if x is between 0 and 1].
$1/(1-x) = 1 + x + [\text{error} < x^2 + 2x^3$, if x is between 0 and ½],
$\quad = 1 + x + x^2 + [\text{error} < x^3 + 2x^4$, if x is between 0 and ½].

NOTE. $1/(a \pm b) = (1/a)[1/(1 \pm x)]$, where $x = b/a$.

Notation by Powers of 10. All questions concerning the position of the decimal point are readily answered if each number is expressed in the "standard form," that is, as the product of two factors, one of which is a number with only one figure preceding the decimal point, while the other is a positive or negative power of 10. Thus, 3.1416×10^3 means 3.1416 with the point moved three places to the right, that is, 3141.6. Again, 3.1416×10^{-6} means 3.1416 with the point moved six places to the left, that is, 0.000 003 1416. This notation by powers of 10 should always be used in dealing with very large or very small numbers. Among electrical engineers its use is very general, even for numbers of moderate size.

Square Root. (a) If four figures of the root are sufficient, take the answer directly from the table of square roots, pp. 12–15. (b) To obtain a root of six or seven figures from the table, use the formula: $\sqrt{N} = a + [(N - a^2)/2a]$ (approx.), where a is the nearest value of $\sqrt{N}$ obtainable from the table, with three or four ciphers annexed. Here a^2 must be found exactly, by direct multiplication, so that at least three significant figures of the difference $N - a^2$ shall be known correctly; but this done, the division of $N - a^2$ by $2a$ should be carried to only three figures (logarithms or slide rule may be used).

NOTE. The simplest way to obtain any root of a seven-figure number correct to seven figures is to use a seven-place table of logarithms, if such a table is at hand.

Square Roots of $1 \pm x$ when x is Small.

$(1+x)^{1/2} = 1 + \tfrac{1}{2}x - [\text{error less than } \tfrac{1}{8}x^2 \text{ if } 0 < x < 1]$
$\quad = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + [\text{error} < \tfrac{1}{16}x^3 \text{ if } 0 < x < 1]$

$(1-x)^{1/2} = 1 - \tfrac{1}{2}x - [\text{error} < \tfrac{1}{8}x^2 + \tfrac{1}{8}x^3 \text{ if } 0 < x < \tfrac{1}{2}]$
$\quad = 1 - \tfrac{1}{2}x - \tfrac{1}{8}x^2 - [\text{error} < \tfrac{1}{16}x^3 + \tfrac{1}{16}x^4 \text{ if } 0 < x < \tfrac{1}{2}]$

NOTE. $\sqrt{a+b} = \sqrt{a}\,(1+x)^{1/2}$, where $x = b/a$.

Cube Root. (a) If four figures of the root are sufficient, take the answer directly from the table of cube roots, pp. 16–21. (b) To obtain a root of six or seven figures from the table, use the formula: $\sqrt[3]{N} = a + [(N - a^3)/3a^2]$ (approx), where a is the nearest value of $\sqrt[3]{N}$ obtainable from the table, with three or four ciphers annexed. Here a^3 must be found correct to seven or eight figures, by direct multiplication, so that at least three significant figures of the difference $N - a^3$ shall be known; but this done, the division of $N - a^3$ by $3a^2$ should be carried to only three or four figures (logarithms or the slide rule may be used).

cover and expands downward. Three screw dowels should be fitted at the top to keep it from working loose. On account of the small space between the liner and the cylinder the liner must be finished on the outside. Figure 16 shows a liner with a flange on the lower end through which tag bolts secure it to the cylinder bottom. At the top is a dovetailed recess into which is calked a ring of soft copper. A modification of this type is shown in Fig. 17 which reduces the volumetric clearance. All these methods allow the liner to expand and contract freely.

Counterbore. To prevent shoulders from being worn in its working surface by the piston rings, the liner or cylinder is counterbored about $\frac{1}{8}$ in. larger in diameter at each end as shown in Figs. 13 and 14, the length of the wearing surface being such that the piston rings will either come flush or overrun a short distance at each end, say 0 to $\frac{1}{8}$ in. at the bottom and $\frac{1}{8}$ to $\frac{3}{8}$ in. at the top. On the other hand the length of the counterbore must be limited so that, when striking clearances of the piston are taken, the piston rings will not entirely overrun and stick on the shoulders.

Table 12

Stroke, in.	Piston clearances, in.	
	Top	Bottom
15–21	$\frac{1}{4}$	$\frac{3}{8}$
22–31	$\frac{1}{4}$	$\frac{1}{2}$
32–41	$\frac{3}{8}$	$\frac{1}{2}$
42–48	$\frac{3}{8}$	$\frac{5}{8}$
49–54	$\frac{1}{2}$	$\frac{5}{8}$
55–66	$\frac{1}{2}$	$\frac{3}{4}$

Piston Clearances. Clearance must be allowed between the piston and the cylinder at each end. This clearance is usually made larger at the bottom than at the top as the wear of the bearings in time will lower the piston. The clearances allowable should be based on the length of stroke rather than on cylinder diameter as the clearance is made the same in all cylinders of the same engine. Clearances should not be less than given in Table 12 unless the pistons and cylinder ends are machined. The cylinder-cover joints will slightly increase the top clearances given.

Cylinder Relief or Safety Valves. Relief valves are provided on top and bottom of all cylinders to prevent fracture of the cylinder parts in case excessive pressure is built up or water hammer occurs. The diameters of relief valves for the different cylinders bear the following relations (to the nearest half inch) to the diameters of the cylinders: high-pressure, 0.10 to 0.115; intermediate-pressure, 0.065 to 0.085; low-pressure 0.045 to 0.06.

Safety valves are usually fitted to the receivers and are the same size as or slightly less than the cylinder relief valve for the corresponding cylinder. Relief and safety valves for high-pressure cylinders are set at 1.10 to 1.15 times the boiler pressure, for intermediate-pressure cylinders at 1.25 to 1.5 times the initial pressures in the cylinders as given in Table 2, and for low-pressure cylinders of triple- and quadruple-expansion engines at about 30 lb.

A cylinder relief valve of the type generally used in shown in Fig. 18. The casing prevents the scalding of attendants by the escaping water, the exit

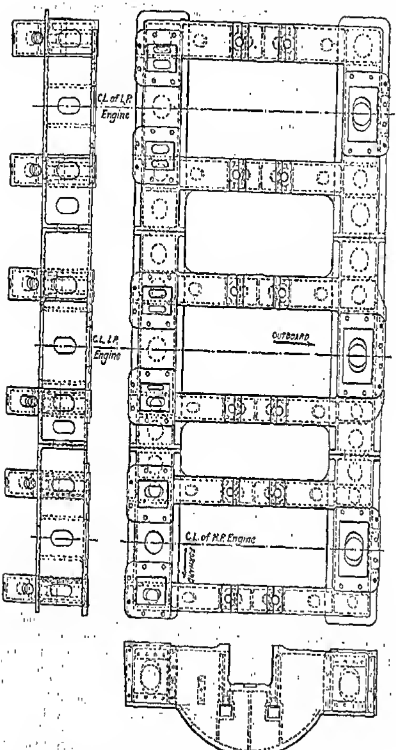


FIG. 28.—Bedplate with dropped bottom.

being on each side near the bottom. If it is desired to reduce volumetric clearance to a minimum, the type of relief valve shown in Fig. 19 should be used. The seat of this valve is fitted in the inner wall of the cover and the exit is through the cored passage (a) to a pocket in the cylinder cover. This pocket is drained by a pipe with its flange on the same level as the relief-valve flange.

Cylinder Drains. Drain cocks or valves are fitted at the lowest point in each cylinder, and valve chest and gear should be provided for operating them from the working platform. The drains are piped to the hot well or condenser. The sizes of cylinder drains, to the nearest quarter inch, for the different cylinders are: high-pressure, 0.04 to 0.055; intermediate-pressure,

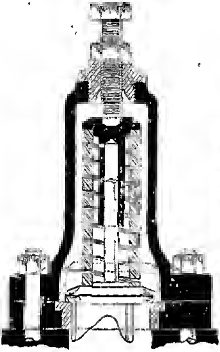


Fig. 18.

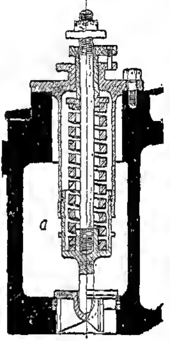


Fig. 19.

Relief valve.

0.03 to 0.04; low-pressure, 0.02 to 0.03 times the diameter of the cylinders. The valve-chest drains are usually made ¼ or ½ in. smaller than the corresponding cylinder drains.

By-pass Valves. For admitting live steam to the intermediate-pressure or low-pressure cylinders for starting the ending easily or for starting the engine in case the high-pressure crank should be on its dead center, by-pass valves are fitted. Steam is taken from the main steam pipe or from the inlet side of the throttle valve. Two-inch to 3-in. valves are used for engines of 1,000 to 3,000 ihp, and 3- to 4-in. for larger engines.

Indicator connections of ¾ or 1 in. for indicator pipes are provided at each end of the cylinder, care being taken that the connections are not near the cylinder ports so that the inrushing steam during admission will

exceed 1,200 lb. The **over-all dimensions** of bedplates and the distance of the side girders and bottom from the center of the shaft, to ensure sufficient clearance from the path of the connecting rod, are determined when laying down the engine. The **depth of the side girders** is from 1.5 to 2 and the width from 1.25 to 1.6 times the diameter of the shaft. The **width of the cross girders** is 2 to 4 in. less than the lengths of the main bearings which they carry. The **thickness of metal in bedplates** for an 8-in. crankshaft is about 1 in. on top and bottom and $\frac{3}{4}$ in. on the sides. For an 18-in. shaft the corresponding figures are $1\frac{1}{2}$ and $1\frac{1}{4}$ in. The **thickness of flanges** for the holding-down bolts should be 1.4 to 1.8 times the thickness of the

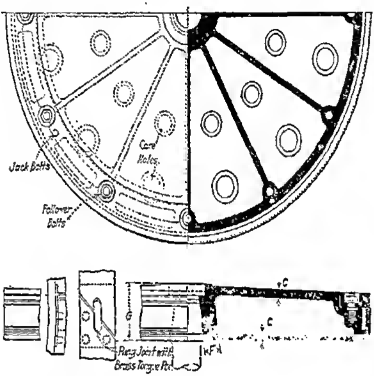


FIG. 29.—Cast-iron piston.

adjoining metal and the width about 2.75 times the diameter of the bolt. **To facilitate casting and handling,** bedplates are often made in two or more sections bolted together. If the main condenser is built in as part of the engine framing, it may rest on top of the bedplate or extend to the foundation, in which case the bedplate is bolted to it. Bedplates are secured to the foundations by bolts passing through cast-iron or steel liners of thickness to suit the alignment. The **size and number of holding-down bolts** must be sufficient to keep the engine in alignment when the vessel is rolling. The diameters run from $1\frac{1}{8}$ in. for a 1,500- to $1\frac{1}{2}$ in. for a 7,500 ihp engine; pitched 8 or 9 diameters. A certain number should be fitted bolts or all of them should be a close fit. Sometimes cast-steel chocks, riveted to the structural foundation, are fitted at each corner of the bedplate in lieu of fitted bolts.

not affect the pressure recorded by the indicator. The three-way cock to which the indicator is connected should be located near the middle of the cylinder. Valves should be fitted close to the cylinder at top and bottom so the pipes can be closed off when the indicators are not in use.

Lagging. All cylinders and valve chests are lagged all over with 1½- to 2-in. magnesia block or plaster and covered over with galvanized or planished steel secured to the lagging flanges by screws.

Stuffing Boxes. The piston-rod stuffing box is usually a separate casting bolted to bottom of cylinder (Figs. 13 and 14). The opening thus provided permits the use of a heavy bar in boring the cylinder. The stuffing box should be designed to accommodate any of the standard makes of metallic packing to allow change in the type of packing. The **diameter of**

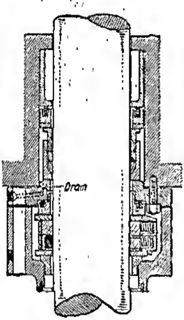


Fig. 20.

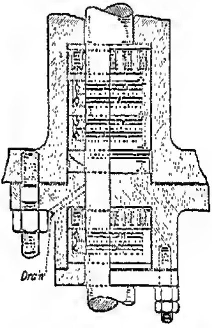


Fig. 21.

Types of metallic packing.

stuffing box for metallic packing is approximately diameter of rod plus 2 in. for 4-in. rods to diameter plus 3 in. for 9-in. rods. The **depth of box** varies with the design of packing. The space required for soft packing is about the same as for metallic. The **stuffing box for a piston-valve** stem is usually cast with the bottom valve chest cover (Fig. 13) and **for a slide-valve** stem with the valve-chest bottom (Fig. 14).

Metallic Packing. There are many types in use but all are essentially the same in principle, consisting of one or more systems of ring segments held securely against the rod by pressure induced by axial, radial, or circumferential springs. Figure 20 illustrates a typical example of double metallic packing used **for high pressure.** White metal packing rings of the inner set are held against the moving rod by axial springs and conical seatings in a retaining cup, and those for the outer set by a combination of radial and axial springs without the use of the conical seating. **For lower pressure,** the outer set is often made a duplicate of the inner set. Figure 21 is a **double packing** in

Reciprocating Parts

Pistons may be of **cast iron of box section** (Fig. 29) or of **cast steel conical in form** (Fig. 30). Either form combines lightness and strength. As the relative weights of the different pistons greatly affect the balance of the engine, the high-pressure piston is often made of cast iron almost or entirely solid, and the low-pressure piston of cast steel as light as consistent with the necessary strength. The curves in Figs. 31 and 32 give **proportions of cast-iron and cast-steel pistons.** If the high-pressure piston is of cast

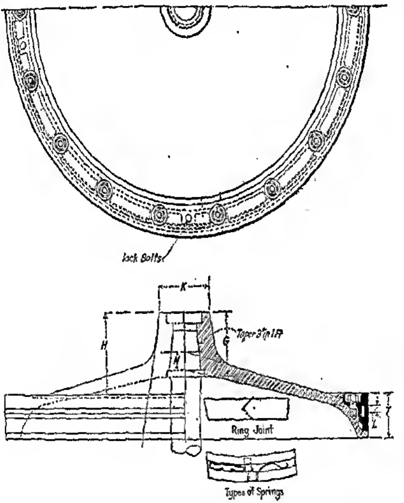


Fig. 30.—Cast-steel piston.

steel, it should be much thicker than indicated by the curves to increase the weight. It is usual to make the depth of face and depth of hub the same for all cylinders. In hollow cast-iron pistons the top, bottom, and ribs are generally made the same thickness, and the number of follower or junk ring-bolts corresponds to the number of ribs. For solid cast-iron and for cast-steel pistons the pitch of the bolts in diameters is 6 for high-pressure, 9 for intermediate pressure, and 12 for low-pressure pistons. Bolts and nuts must be securely locked against backing out and should not project above the follower face. To prevent bolts from rusting fast in the piston, brass plugs are some-

which the elements of the inner and outer sets are interchangeable. The packing rings are held against the moving rod by axial and circumferential springs. When double metallic packing is used, the chamber for holding the outer set is a separate casting bolted on the outside of the stuffing box. For high-pressure cylinders using **superheated steam** or saturated steam above 190 lb, the packing rings must be of a special composition.

A drain must always be fitted to take off the leakage or condensation which would otherwise cause an objectionable drip from the stuffing box; these drains are piped to the condenser, except the low-pressure piston rod drain which is generally carried into the low-pressure receiver to prevent air being drawn into the condenser. A separator or collector is sometimes fitted to prevent cylinder oil from being drawn into the condenser. A commercial brand of soft packing is sometimes used for the low-pressure piston rods and valve stems. When metallic packing is used **in a new engine**, it is very important that the steam ports and passages be thoroughly cleaned, preferably by pickling; otherwise sand and grit will enter the stuffing boxes and wear the packing. Sometimes the stuffing boxes are filled with soft packing until the grit has all worked out and then the metallic packing is installed. In **designing stuffing boxes** for metallic packing, it is advisable to prepare tentative sketches, giving fixed dimensions and limits, and to submit these to packing manufacturers for recommendations and criticisms in order to use a standard size. For **materials for metallic packings**, see p. 898.

Main Steam, Receiver, and Exhaust Pipes and Passages. Steam velocities are based on piston displacement. Velocity in fpm = area of cylinder × piston speed ÷ area of pipe or passage; piston speed is in feet per minute and areas in sq in. To avoid drop in pressure the areas of pipes and passages should be such that velocities will not exceed those given in Table 13 but velocities should be kept lower if possible.

Table 13. Velocities of Steam in Pipes and Passages

	H-p	First I-p	Second I-p	L-p
Main steam pipe	5,500			
Steam to first i-p cylinder	.....	6,500		
Steam to second i-p cylinder	....	. .	7,500	
Steam to l-p cylinder	....	.....	.....	8,500
Exhaust to condenser	. . .	.	. . .	6,000
Steam through valve liner at maximum port opening	5,000	6,500	8,000	9,000
Steam or exhaust through ports of cylinder	5,000	6,500	7,500	8,500
Exhaust through total area of valve liner	4,000	5,000	6,500	8,000

On account of the increased **resistance through the throttle valve** it should be ½ to 1 in. larger in diameter than the main steam pipe. **Velocities through cylinder ports** should be fairly high to reduce condensation and permit of small clearance volumes, but the **exhaust through the valve liners** should be low on account of the increased resistance. Instead of receiver pipes the exhaust from one cylinder is often led to the next through a passage cast with the cylinders. For long receiver pipes expansion joints must be fitted unless expansion can be taken care of by pipe bends (see p. 942). The exhaust pipe from the low-pressure cylinder to the condenser should be of ample area and as short and direct as possible as it is important that excessive

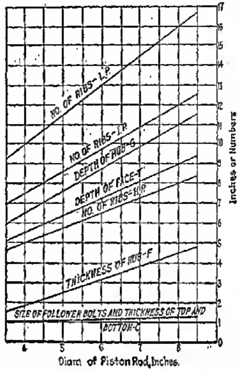


Fig. 31.—Proportions of cast-iron pistons (letters refer to Fig. 29).

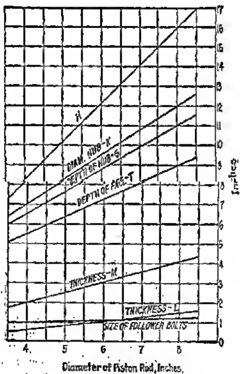


Fig. 32.—Proportions of cast-steel pistons (letters refer to Fig. 30).

back pressure be avoided. Receiver and exhaust pipes may be cast iron, steel, or copper.

Cylinder covers in merchant vessels are of cast iron with a **single wall** with deep radial ribs on the outside unless it is desired to use steam jackets in which case **double walls** are used. Double walls may also be used for covers of very large cylinders. As it is difficult to calculate the stresses in

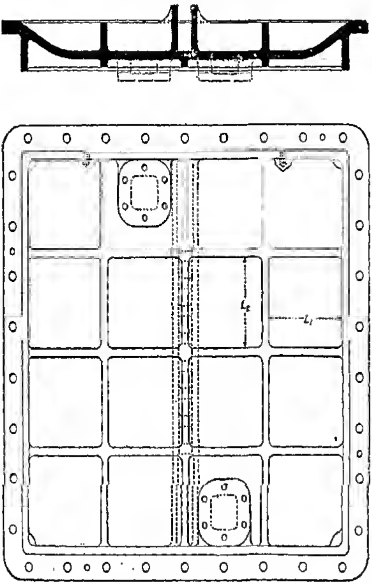


Fig. 22.—Slide-valve chest cover.

cylinder covers, it is usual to proportion them from successful practice. **Thickness of metal in cover** is about 0.85t for small cylinders and t for large cylinders, where t is the thickness of cylinder without liner. The **ribs** are the same thickness or sometimes slightly heavier. Cylinder covers are conical to follow the shape of the piston and to obtain greater strength. They should be well ribbed to avoid too large flat surfaces between ribs. The width of the steam joint is about 2.8 $\times$ d and the thickness of bolting

times fitted in the piston into which the bolts are screwed. Piston castings up to 35 in. diameter should be turned about $\frac{1}{32}$ in. less than the diameter of the cylinder bore and larger pistons about $\frac{1}{16}$ in. The top and bottom walls of a cast-iron piston must be of sufficient strength to withstand the shock due to water collecting in the cylinder. The strength of a cast-steel piston should be calculated by considering half the piston as a cantilever loaded at the center of pressure and breaking through the center line. The cross section is divided into a number of triangles or rectangles as shown in Fig. 33 and the section modulus found by the method on p. 1071. Stress $= \frac{P \times A \times r}{2 \times 2 \times Z}$ in which P is pressure per sq in., A is total area of piston in sq. in., r is radius to center of pressure $= 0.424 \times$ radius of piston, and Z is modulus of half section. Stress figured by this method should not exceed 3,000 psi. It is usual to use a pressure of 30 lb in the calculation for low-pressure pistons.

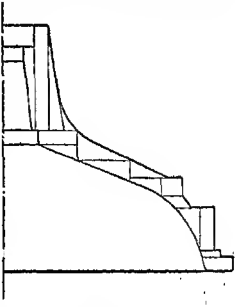


FIG. 33.

Piston Rings. No parts of the reciprocating engine require more care in their design than the piston rings, the function of which is to prevent leakage

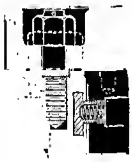


FIG. 34.

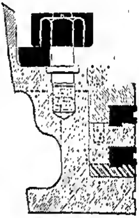


FIG. 35.

Types of Piston packing.

of steam past the piston by exerting a uniform pressure all around without excessive friction. To accomplish this was a relatively simple matter when low-pressure wet steam was used, but with high-pressure and superheated steam the duties of the piston rings, especially for high-pressure cylinders, have become much more exacting. The unequal expansion of the piston and

flange about 1.35 $\times d$ where d is diameter of the studs. In large cylinders a **manhole** is fitted in the cover (Fig. 14) so that the piston and cylinder may be examined without lifting the cover. A **false cover** of cast iron or checkered steel plate is fitted on top of the ribs and the space between ribs is filled with magnesia. If a **tail rod** is used, the cylinder cover is fitted with a stuffing box or a closed cast-iron casing in which the tail rod works.

Valve-chest covers for piston-valve chests are of the same construction as cylinder covers. They contain the balance piston cylinder, when fitted, as shown in Fig. 14. Valve-chest covers for slide valves are rectangular in shape and on account of the large area must be heavily ribbed as shown in Fig. 22. The **width of the steam joint** should be about 2.6 $\times d$ and the **thickness of the bolting flange** about 1.25 $\times d$ where d is the diameter of the studs. The **thickness of metal in cover** is usually the same as the cylinder walls. Slide valves are used only on cylinders where the pressure is low and therefore the design of this cover is not so difficult as if it carried full-pressure steam.

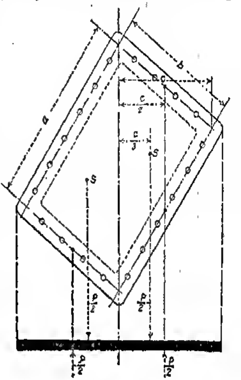


Fig. 23.

It is well however to calculate the **stress in the cover** by the following method given by Bauer: Experiments have shown that square or rectangular plates, when subjected to a uniform strain under hydraulic pressure, split across the diagonal and should therefore be calculated for a bending stress along that line. Load on cover (see Fig. 23), $P = a \times b \times p$ where p is test pressure psi and a and b are dimensions of cover to center line of bolts. Half of the load may be considered as acting at the center of gravity S of half the cover; then the bending moment about the diagonal is $\frac{P}{2} \times \frac{c}{3}$. This load is counteracted by the resultant of two forces which act along the sides a and b. Then the moment about the diagonal $= M_b = \frac{P}{2} \times \frac{c}{2} - \frac{P}{2} \times \frac{c}{3} = \frac{Pc}{12}$. Expressing c in terms of a and b and $P = a \times b \times p$, then $M_b = \frac{pa^2b^2}{12a^2 + b^2}$. Stress $= \frac{Mb}{Z}$ where Z is modulus of the diagonal section found by the method given on p. 1071. Stresses calculated by this method

the cylinder walls and the necessity of providing for wear render some means of adjustment necessary.

Ordinary **concentric rings** of square or rectangular section, cut and sprung into position, do not exert a uniform pressure but bear much harder near the ends. This has a tendency to wear the cylinder bore oval. To overcome this and at the same time to improve the steam joint by making the pressure of the rings uniform all around the cylinder wall, springs are often fitted behind the rings as shown in Figs. 30 and 34. Rings of this type should not be used as **floating snap rings** unless they are restrained by clamping of the follower or by **nonelastic bull rings,** which do not bear against the cylinder walls (Figs. 30, 35 to 37). Holes are tapped into the bull ring so that it may be removed with the rings without drawing the piston.

Eccentric or **ramsbottom rings,** thicker in the middle than at the ends, must be used for all snap rings. To prevent steam leaking past them, the rings are usually dowelled together so that the joints will come on diametrically opposite sides of the piston. In addition a brass tongue piece is some-

FIG. 36.

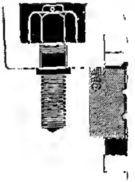


FIG. 37.

Types of piston packing.

times fitted as shown in Fig. 29. Piston rings are always made of hard cast iron to resist wear. Figure 37 shows a very satisfactory type of piston packing for high pressures in which the rings are so fitted that when the follower bolts are loosened the rings will expand and fill the cylinder bore and when the bolts are tightened the rings will be pinched in position. This arrangement gives the effect of an accurately fitted solid piston with provision for adjustment. Figures 35 and 36 show other satisfactory types of high-pressure packing rings. In these the rings float, but the amount is limited by shoulders on the bearing rings. Figures 30 and 34 show satisfactory types of intermediate-pressure and low-pressure piston rings, and Fig. 29 a very satisfactory low-pressure packing ring. There are a number of patented piston packings of various designs in use.

Piston Rods. The **length** is found when laying down the engine and is usually about $1\frac{1}{2}$ times the stroke. The rods for all cylinders are made **interchangeable,** if possible. The **diameter** is determined from the unbalanced load, P, treating the rod as a column with round ends. The stress in the body of the rod $= P \div$ area and is usually about 2,000 psi. The

may safely run as high as 6,000 psi. If a guide for the slide valve is cast with the cover and is a strength member, it should be considered in calculating the section modulus. The pressure is considered as acting to the center line of the bolts.

Strength of Flat Plates. In designing cylinders and cylinder and valve chest covers it is often necessary to calculate the strength of flat sections of metal between ribs. A number of formulas for flat plates have been published, but rarely is the same formula satisfactory for both thick and thin plates. In cylinder parts the flat plates are fixed around the edges and uniformly loaded, and it is usual to calculate them for the test rather than for the working pressure. Probably the simplest and most satisfactory method of figuring flat plates is by equating deflections, although the strength of a flat plate can be determined only approximately. If the plate to be figured is not rectangular, an equivalent rectangle should be assumed. Total load, W = pressure per sq in. $\times$ area. Part of the total load, W_1, is carried by the edge L_2 (Fig. 22) and part of the load, W_2, is carried by the edge L_1. Equating the deflections at the middle of the rectangular plate $(W_1 \div W_2) = (L_2^4 \div L_1^4)$. Therefore $W_1 = WL_2^4 \div (L_1^4 + L_2^4)$ and $W_2 = WL_1^4 \div (L_2^4 + L_1^4)$. Stress due to $W_1 = \dfrac{6W_1L_1}{12L_2t^2}$ and stress due to $W_2 = \dfrac{6W_2L_2}{12L_1t^2}$, where t is thickness. Stress figured by this method should not exceed 2,000 psi for cast iron, based on test pressure, but it will usually be much less than this.

Framing and Beds

Framing of merchant engines usually consists of cast-iron housings of box section either straight or inverted Y (Fig. 24). In small engines, and in larger engines where it is desired to keep the front of the engine open, forged-steel front columns are used with straight or **Y-box housings** carrying the crosshead guides in the back. In engines of intermediate and large sizes **straight box housings** are generally used in front and Y-housings in the back, each housing carrying a crosshead guide. In very large engines four cast-iron or cast-steel housings may be used for each cylinder each carrying a guide. In **naval and yacht engines**, where saving of weight is important, the framing is made up of **forged-steel columns** and braces. Housings are subjected to direct tensile and compressive stress from the unbalanced load, to direct compressive stress from the weight of the cylinders, which may be greatly augmented by rolling of the vessel, and to bending stress from the crosshead pressure on the guides. As the stresses are always low, housings must be proportioned for rigidity rather than strength to avoid deflection. The bending stress from the crosshead is ordinarily only about 200 psi and need not be computed. The direct stress at the top of the housing from the unbalanced load and the weight of the cylinders should not exceed 800 psi. The **thickness of metal in housings** is about 1 in. for a 20-in. and $1\frac{3}{8}$ in. for a 35-in. cylinder. Back housings should have one-fifth to one-third more sectional area than front housings. The thickness of the metal should be increased at top and bottom to provide better attachment for the flanges. The **thickness of flanges** is about 1.4 times the diameter of the bolts. **Number and diameter of bolts** securing the housings to the cylinders and bedplates are determined from the unbalanced load, P. Stresses should be kept low to ensure rigidity, about 2,400 lb for the upper and 1,800 lb for the lower bolts being good practice. Diameters average about $1\frac{1}{2}$ in. for engines

factor of safety, which is usually about 20, is found by dividing the destructive load per square inch, from Pencoyd's tables given on p. 1070, by the stress in the rod. These destructive loads are based on $L \div \rho$, where L is the length of the column and ρ, the least radius of gyration, both in inches. $\rho = D \div 4$ for solid rods and $\sqrt{D^2 + d^2} \div 4$ for hollow rods in which D is diameter of rod and d is diameter of hole. The stresses are kept low and the factors of safety high, to permit turning down if the rods should be scored by the

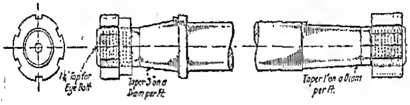


FIG. 38.—Piston rod.

metallic packing. The piston rod is tapered about 3 in. per ft on diameter in the piston and 1 in. in the crosshead (Fig. 38). The large taper facilitates the removal of the piston. A collar at the end of the taper against which the piston rests prevents this large taper from splitting the piston. The **diameter of the thread** at each end is about 0.7 to 0.8 times the diameter of the rod and is proportioned from the unbalanced load, P, for a stress of 3,500 to 4,000 lb. Four threads per inch are generally used for all sizes. The **diameter of tail rod,** if fitted, is about 0.7 times the diameter of the piston rod, or as large as the threads of the piston-rod nut will permit.

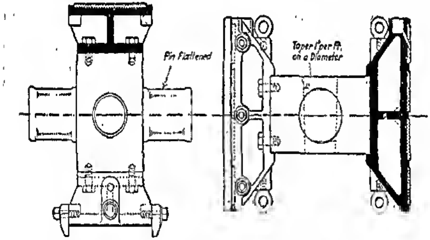


FIG. 39.—Central-block type of crosshead.

Crossheads. In merchant engines the **central-block type** of crosshead with two pins is generally used (Fig. 39). The **depth of the central block** is 1.6 to 1.8 and the **length** 1.5 to 1.8 times the diameter of the piston rod. The **width,** when one slipper is used, is about the same as the depth; with two slippers it is made much wider to keep the slippers clear of the connecting rod. The central block should be figured for strength as a beam supported at the middle of the pins and loaded by the unbalanced load on the piston rod P. Stress $= PL \div 4R$ where L is length between middle of pins and R is resistance of the section $= bd^2 \div 6$; b is width less the average width of the

with a 20-in. and $2\frac{1}{4}$ in. for a 35-in. high-pressure cylinder, the lower bolts usually being made somewhat larger than the upper. Forged-steel front columns are proportioned from the unbalanced load, P, considering them as columns fixed at the ends by the method given under piston rods (p. 1092). The full unbalanced load, P, should be used with a factor of safety of about 25 to allow for the weight of the cylinders and the increased load due to rolling of the vessel. If the main condenser is built in as a part of the engine framing, the back housings rest on top of the condenser.

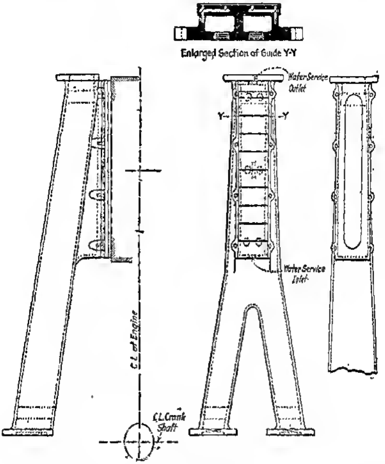


Fig. 24.—Cast-iron housings.

Crosshead Guides. Small engines usually have guides of the bar type (Fig. 25). This type is favored for tugboat and other engines which do a lot of backing as the surface is the same for backing as for going ahead. The guide is generally cast iron of box section for water circulation, but when used on engines with a stroke as long as 42 in. cast or forged steel should be used to ensure rigidity. The strength of the bar should be computed as a beam supported at the ends and loaded in the middle by the load on the crosshead slippers, P_2 (see Fig. 44). Stress for cast iron should not exceed 2,750 psi. Engines of small and intermediate size often have guides of the single type (Fig. 26). The ahead guide, which is hollow for water circulation, is

hole for the piston rod and d is depth. Stress may safely run as high as 3,000 lb. The **single-pin type** of crosshead, which usually consists of a bearing box on the end of the piston rod, is also used (Fig. 40). In this type the pin is shrunk into the fork of the connecting rod. Stresses in the bolts

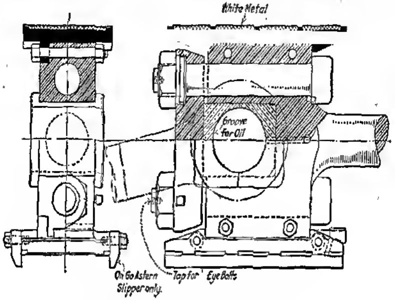


Fig. 40.—Single-pin type of crosshead.

and caps, due to the unbalanced load P, should be calculated as shown on p. 1100. The same stresses are allowable.

Crosshead Pins. As the crosshead-pin bearings are oscillating instead of rotating bearings, higher pressures per inch are allowable than on the crankshaft and crankpin bearings. The pressure is produced by the load on the

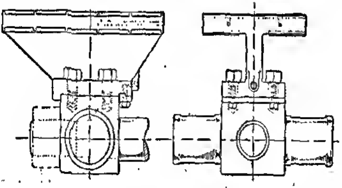


Fig. 41.

connecting rod P_1 (see Fig. 44), and pressures of 500 to 600 psi are allowable. The pins must be proportioned for strength as well as area. The **diameter of the pins** of the type shown in Fig. 39 is 1.1 to 1.2 times the diam of the piston rod and the length 1.1 to 1.25 times their diam. The pin is figured as a cantilever loaded at the middle of its length by one-half the load on the connecting rod, P_1. Stress $= P_1l \div 2 \times 2R$ where l is length of pin and

bolted to the back housing, the same bolts also securing the backing guides. The load on these bolts $= \frac{P_1(a + b)}{2}$ in which a and b are as shown in Fig. 26.

Intermediate and large engines usually have **guides on both front and back housings.** The guides are usually separate castings, cored for water circulation and bolted to the housings (Fig. 24). Very large engines with four housings to each cylinder have a guide on each housing. The **surface of the guides** is proportioned to suit the area of the crosshead slippers, and the **length** is such that the slippers overrun about 1 in. at each end to prevent wearing of shoulders. The bar and single-slipper types of guides require no adjustment to allow for expansion of the cylinder; this is required with guides on both housings, the adjustment being made in the latter type, to prevent crossheads from pounding after the engine has been running for some time. The working surfaces of the guides are not lined with white metal but usually have grooves to retain the oil.

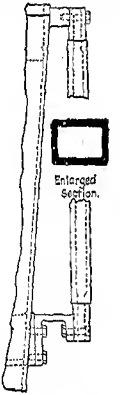


Fig. 25.

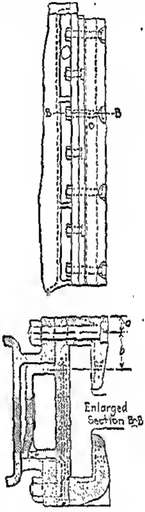


Fig. 26.

Types of crosshead guides.

Bedplates for merchant engines are cast iron of box section with either flat or dropped bottom as shown in Figs. 27 and 28. The **dropped-bottom type** is preferable as the housings are shorter and the shaft lower while ample space is allowed for building rigid foundation girders under the bed-

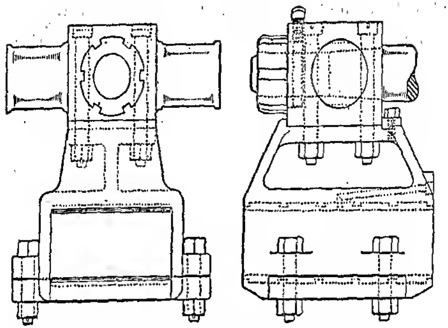


FIG. 42.

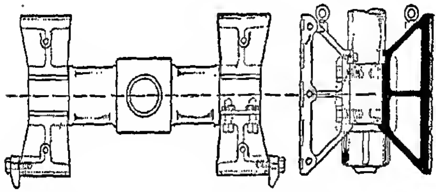


FIG. 43.

R is resistance to bending $= d^3 \div 10.2$. Crosshead pins are flattened on the sides to facilitate the distribution of oil from top to bottom.

In the single-pin type (Fig. 40) diameter of the pin is 1.2 *to 1.5 times the diameter of the piston rod and the length* about 1.4 to 1.6 times its diameter. The pin is figured as a beam fixed at the ends and loaded at the middle of its length by the load on the connecting rod, P_1. Stress $= P_1l \div 8 \times R$, where l is length of pin and R is resistance to bending $= d^3 \div 10.2$. Stresses in pins of either type may safely run as high as 3,500 psi.

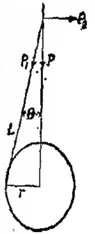


FIG. 44.

Crosshead Slippers. *Crossheads with two slippers* (Fig. 39) are generally used in intermediate and large engines. For smaller engines one slipper of the type shown in Fig. 41 or the box type (Fig. 42) are used. For very large engines with four housings to each cylinder four slippers are used (Fig. 43). *Slippers are usually of cast iron but some-* times of cast steel and lined with white metal. When one

plates. Bedplates are proportioned for mass and rigidity rather than for strength, but it is usual to calculate the stress in the cross girders as beams

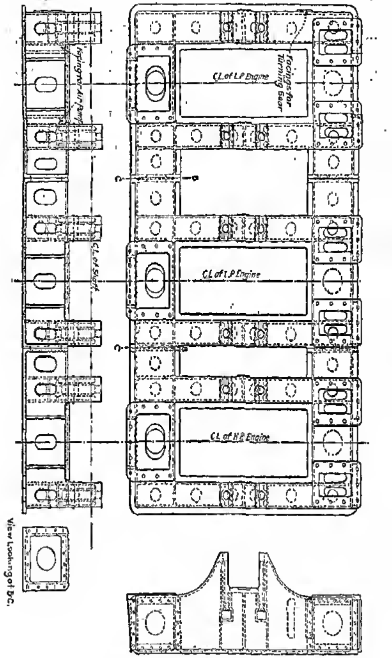


FIG. 27.—Bedplate with flat bottom.

supported at the middle of the side girders and loaded in the middle by the unbalanced load, P. Stress $= Pl \div 2 \times 4R$ where R is the modulus of section found by the method on p. 1071. To ensure rigidity, stress should not

slipper is used, the bolts securing the slipper to the central block must be proportioned to carry the load when backing. When two or four slippers are used the ahead or astern slippers must be fitted with gibs on each side as

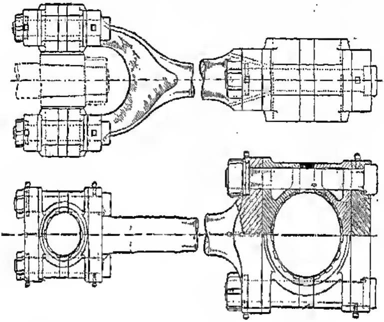


Fig. 46.

Connecting rods.

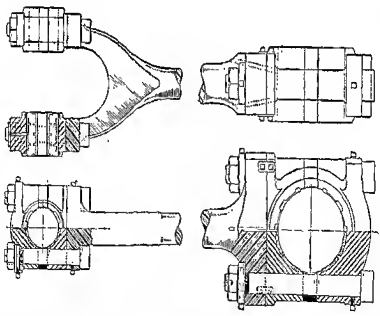


Fig. 45.

shown in Figs. 39, 40, and 43 to keep the crosshead in alignment. To facilitate the removal of the slippers keys are fitted in the upper side of the central block as shown in Figs. 41 to 43.

NOTE. The simplest way to obtain any root of a seven-figure number correct to seven figures is to use a seven-place table of logarithms, if such a table is at hand.

Cube Roots of $1 \pm x$ when x is Small.

$$(1+x)^{1/3} = 1 + \tfrac{1}{3}x - [\text{error} < \tfrac{1}{9}x^2 \text{ if } 0 < x < 1].$$
$$= 1 + \tfrac{1}{3}x - \tfrac{1}{9}x^2 + [\text{error} < \tfrac{1}{16}x^3 \text{ if } 0 < x < 1]$$
$$(1-x)^{1/3} = 1 - \tfrac{1}{3}x - [\text{error} < \tfrac{1}{9}x^2 + \tfrac{1}{10}x^3 \text{ if } 0 < x < \tfrac{1}{2}]$$
$$= 1 - \tfrac{1}{3}x - \tfrac{1}{9}x^2 - [\text{error} < \tfrac{1}{16}x^3 + \tfrac{1}{16}x^4 \text{ if } 0 < x < \tfrac{1}{2}]$$

NOTE. $\sqrt[3]{a+b} = \sqrt[3]{a}(1+x)^{1/3}$, where $x = b/a$.

LOGARITHMS

Tables of Logarithms. The use of a table of logarithms greatly reduces the labor of multiplication, division, raising to powers, and extracting roots. The table on pp. 42–43 is carried out to four significant figures, and the following explanations should be sufficient to permit the use of the table readily, even by one without previous experience. For algebraic theory, see p. 113.

If more than four-figure accuracy is required, recourse must be had to a larger table. Five-place tables are available in great variety; the Macmillan Tables, 1913 or Albrecht, "Logarithmisch-Trigonometrische Tafeln," are perhaps as convenient as any. If more than five figures are required, use Bremiker's six-place table, or proceed at once to a seven-place table: Schrön (Vieweg & Sohn, Braunschweig); Bruhns; Vega-Bremiker. If extreme accuracy is required, use the eight-place table by Bauschinger and Peters (Engelmann, Leipzig). Logarithmic paper, see p. 170.

To Find the Logarithm of Any Given (Positive) Number.

(*a*) WHEN THE GIVEN NUMBER IS BETWEEN 1 AND 10.

An inspection of the table on pp. 42–43 shows that as the number increases from 1 to 9.99. . . the logarithm of that number increases continuously from 0 to 0.999. . . For example, log 2.97 = 0.4728; log 2.98 = 0.4742.

If the given number contains four significant figures, it is necessary to interpolate between the tabulated values, as follows:

To find log 2.973, notice that this number is 3/10 of the way from 2.97 to 2.98; hence its logarithm will be (approximately) 3/10 of the way from 0.4728 to 0.4742. The difference here is 14 units, and 3/10 of this difference is 4 (to the nearest unit); hence, by adding this 4 to 4728, log 2.973 = 0.4732. This process of interpolating should be performed mentally; the step of finding the tabular difference will be facilitated by a glance at the last column on the right, which gives, for each line of the table, the average of the differences along that line.

Again, to find log 4.098: From table, log 4.09 = 0.6117; adding 8/10 of the difference (11), or about 9, gives: log 4.098 = 0.6126. Or better, since 8/10 of the way forward is equal to 2/10 of the way back, find in table log 4.10 = 0.6128, and *subtract* 2/10 of 11, or 2, giving log 4.098 = 0.6126. It should be noted that any interpolated value may be in error by 1 in the last place.

If the given number contains more than four significant figures, it should be cut down to four figures (see p. 88), since the later figures will not affect the result in four-place computations.

(*b*) WHEN THE GIVEN NUMBER IS LESS THAN 1 OR MORE THAN 10, it is simply necessary to notice that every such number can be regarded as obtainable from some number between 1 and 10 by merely shifting the decimal point (see p. 90); and that according to the rule at the foot of the table, moving the decimal point n places to the right [or left] in the number-column is equivalent to adding n [or $-n$] to the logarithm in the body of the table.

For example, to find log 2973. Here $2973 = 2.973 \times 10^3$ (*i.e.*, 2.973 with the decimal point moved 3 places to the right). From the table, log 2.973 = 0.4732. Hence, log 2973 = 0.4732 + 3, which may be written as 3.4732.

Again, to find log 0.0002973. Here $0.0002973 = 2.973 \times 10^{-4}$ (*i.e.*, 2.973 with the decimal point moved 4 places to the left). From the table, log 2.973 = 0.4732. Hence, log 0.0002973 = 0.4732 − 4. (This may be written as $\bar{4}.4732$, if desired, and is equal of course, to − 3.5268; this latter form, however, is not convenient in practice.)

It is thus evident that the logarithm of every positive number may be regarded as consisting of two parts: a decimal fraction, which is always positive (or zero); and a whole number, which may be positive, negative, or zero. The fractional part is called the **mantissa**, and is found from the table; the whole-number part is called the **characteristic**, and is determined by inspection.

To Find the Number Corresponding to a Given Logarithm.

(*a*) When the given logarithm is a positive decimal fraction (characteristic zero), simply reverse the process for finding the logarithm of a number between 1 and 10.

For example, given log $N = 0.4732$; to find N. In the body of the table it is seen that 0.4732 lies a little beyond 0.4728; hence N must lie a little beyond 2.97. By taking differences it is found that 4728 is in fact $\frac{4}{14}$ of the way from 0.4728 to the next higher logarithm; therefore N must be $\frac{4}{14}$ of the way from 2.97 to the next higher number. But $\frac{4}{14}$ of 1 is 0.3 (to the nearest tenth), hence $N = 2.973$.

Again, given log $N = 0.6126$; to find N. Here, 0.6126 is $\frac{9}{11}$ of the way from 0.6117 to the next higher logarithm; therefore N must be $\frac{9}{11}$ of the way from 4.09 to the next higher number. But $\frac{9}{11}$ of 1 is 0.8 (to the nearest tenth), hence $N = 4.098$.

(*b*) When the given logarithm has any given value (characteristic not zero), proceed as follows: First, be sure the given logarithm is in the "standard form," that is, a positive decimal fraction (mantissa) plus a positive or negative whole number (characteristic). For example, if log N is originally given in the form log $N = -3.5268$, this must first be reduced to the (equivalent) form log $N = 0.4732 - 4$ (or $\bar{4}.4732$), before entering the table. Having the logarithm given in the standard form, suppose for the moment that the characteristic is zero, and find in the table the number corresponding to the given mantissa; then move the decimal point to the right or left according as the value of the characteristic is positive or negative.

For example, given log $N = 0.4732 + 3$; to find N. From the table, the number corresponding to 0.4732 is 2.973. The characteristic (+ 3) directs that the decimal point be moved 3 places to the right; hence $N = 2.973 \times 10^3 = 2973$.

Again, given log $N = 0.4732 - 4$; to find N. From the table, the number corresponding to 0.4732 is 2.973. The characteristic (− 4) indicates that the decimal point is to be moved 4 places to the left; hence $N = 2.973 \times 10^{-4} = 0.0002973$.

The number corresponding to a given logarithm is called its **antilogarithm**. Thus, if log 2973 = 0.4732 + 3, then 2973 = antilog (0.4732 + 3).

Note 1. In most tables of logarithms the decimal point is omitted, the tables being in fact not tables of logarithms, but tables of mantissas. This omission is of no consequence to the experienced computer but is often perplexing to one who makes only occasional use of such tables.

Note 2. Many computers prefer to write negative characteristics in the form of some positive number minus some multiple of 10; thus, 0.4732 − 4 = 6.4732 − 10; 0.4732 − 13 = 7.4732 − 20; etc.

Fundamental Properties of Logarithms. The usefulness of logarithms in computation depends on the following properties:

$$(1)\ \log (ab) = \log a + \log b; \quad (3)\ \log (a^n) = n \log a;$$
$$(2)\ \log (a/b) = \log a - \log b; \quad (4)\ \log \sqrt[n]{a} = (1/n) \log a;$$
$$(5)\ \log 10^n = n$$

It is to be noted also that log 1 = 0, log 10 = 1, and log $(1/n) = -\log n$.

The pressure of the slipper on the guide, P_2, is the component of the load on the connecting rod, P_1, perpendicular to the guide as shown in Fig. 44; $P_2 = P_1 \sin\theta$ or $P \tan\theta$. The surface of the slipper should be proportioned for a pressure of about 40 psi. When single slippers of the type shown in Fig. 41 are used the backing surface is 65 to 75 percent of the ahead surface.

Connecting Rods. The usual type used in merchant engines is shown in Fig. 45. The crank-end and crosshead-end boxes and caps are lined with white metal and are usually of cast steel, although sometimes of brass. Caps are generally cast in one piece, but cast-iron or brass boxes and forged-steel binders are sometimes used as shown in Fig. 46. Setscrews are fitted in the crankpin cap and in the upper end of the connecting rod to keep the bolts from falling out when the nuts are removed, enabling caps and bolts to be handled together. For intermediate and large engines there should be two oil holes to the crankpin with pipes led to the oil boxes on top of the crosshead-pin bearings. If the single-pin type of crosshead is used, the fork of the connecting rod is made with solid eyes and the pin shrunk in. The length between centers of connecting rods for main engines, is usually 2.25 times the stroke, but this ratio is often made 2.5 for small engines and 2 for large engines. The diameter at the upper end is 0.90 to 0.95 times the diameter of the piston rod, and the diameter at the lower end about 1.25 times the diameter of the upper end. The rod must always be figured for strength as a column with hinged ends. This is done by the method explained on p. 1092, except that the load is taken as $P_1 = P \sec.\ \theta$, where P is the unbalanced load and θ the angle whose sine is equal to the crank radius ÷ length of rod (Fig. 44). In figuring the factor of safety the stress and radius of gyration must be taken at the middle of the rod. The factor of safety is usually about 20. The method of figuring bolts and caps is the same as explained on p. 1101, except that P_1 is used instead of P. The same stresses are allowable. The thickness over flats at the fork end is about ½ in. more than the diam of the rod. The fork should be figured for stress by assuming an average rectangular section at a point A-A (Fig. 47) and considering the total stress equal to the sum of the direct tensile or compressive stress and the stress due to bending. Total stress $= \dfrac{P_1 \times \sin 30°}{2b \times d} + \dfrac{6P_1 \times \cos 30° \times c}{2b \times d^2}$ which may be reduced to the form $\dfrac{P_1}{4bd^2}(d + 6a)$. The stress figured by this method may safely be 5,000 to 6,000 lb.

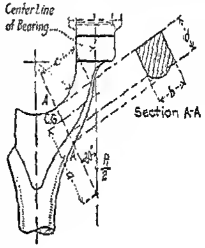


FIG. 47.

Bearings

White metal is cast in the bearing boxes at least ¼ in. thicker than required by the finished bearing to allow for hammering in. After the bear-

In built-up shafts the shafts and crankpins are shrunk into the webs. Square or round keys are always fitted in the shafts and sometimes in the pins.

Table 15. Proportions of Crank Webs for Built-up Shafts

Diameter bore for crankshaft	Diam shaft + 1½ in.
Diameter bore for crankpin	Diam pin + 1 in.
Metal around crankshaft	0.5 × diam shaft
Metal around crankpin	0.4 × diam pin
Thickness of webs if sections of shaft are interchangeable	0.68 × diam shaft
Thickness of webs if sections of shaft not interchangeable	Forward cylinder = 0.52 × diam shaft After cylinder = 0.73 × diam shaft.

Crankpins. It is usual to make the crankpin the same diameter as the crankshaft, although it is sometimes made larger to reduce the pressure, if it is desired to shorten the pin, or to reduce the stress, if a long pin is required to keep the pressure low. **Ratios of length to diameter** of pin vary in good practice from 0.95 to 1.0 for triple- and quadruple-expansion engines and from 1.0 to 1.1 for compound engines.

The pressure on the crankpin is figured from the thrust of the connecting rod, P_1, produced by the unbalanced pressure on the piston (see p. 1094). In computing the pressure the actual length of bearing surface of the connecting rod is used which is about ¼ in. less than the length of the pin. **Allowable pressures** per square inch are 300 to 325 lb for triple- and quadruple-expansion engines and 400 to 425 lb for compound engines. Some designers give consideration to the heat produced by the friction of the crankpin in proportioning its size, but unless the design is out of range of ordinary practice this is unnecessary.

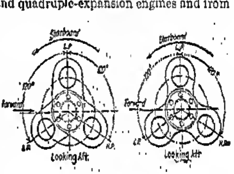


FIG. 60.

The **after crankpin**, through which the power from the forward cylinders all passes, should always be figured for stress. This pin is subjected to a bending moment produced by the torque from the forward cylinders applied at the forward end, and also to a bending moment from the torque produced by the after cylinder, applied at the middle of its length. These bending moments are combined and the resulting bending moment used in figuring the stress as follows:

Force from forward cylinders applied at forward end of pin,

$$F_1 = \frac{\text{ihp of forward cylinders} \times 33{,}000 \times 12}{2 \times \pi \times r \times \text{rpm}} \times C$$

Force from after cylinder applied at middle of pin,

$$F_2 = \frac{\text{ihp of after cylinder} \times 33{,}000 \times 12}{2 \times S \times \text{rpm}} \times 1.1$$

ing is bored out, it is scraped preferably to its own journal. The metal is held in place by dovetails $\frac{1}{4}$ to $\frac{1}{2}$ in. deep, arranged in a manner similar to that shown in *Fig. 48.* The **total thickness** of white metal should be approximately as follows: $\frac{9}{16}$ in. for 10-in. to $\frac{3}{4}$ in. for 20-in. main bearings and crankpins; $\frac{1}{2}$ in. for 7- to 15-in. and $\frac{5}{8}$ in. for 16- to 22-in. steady bearings; $\frac{3}{8}$ in. for 5-in. to $\frac{5}{8}$. in for 10-in. crosshead pins; $\frac{9}{16}$ to $\frac{3}{4}$ in. for thrust shoes and about $\frac{1}{2}$ in. for crosshead slippers and other flat surfaces.

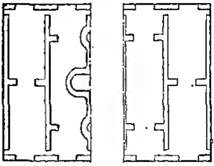


Fig. 48.—Dovetails securing white metal in bearings.

Main Bearings. Two bearings to each cylinder or crank are usual although, in order to shorten the engine and save weight, one bearing is often used between two cranks, in which case it is made longer than ordinarily required, or the bearing may be in two parts covered by the same cap, with a short distance between them. The **bottom boxes** may be cast steel, brass, or cast iron, lined with white metal. In naval, and sometimes in merchant engines, a separate cylindrical brass casting lined with white metal is used

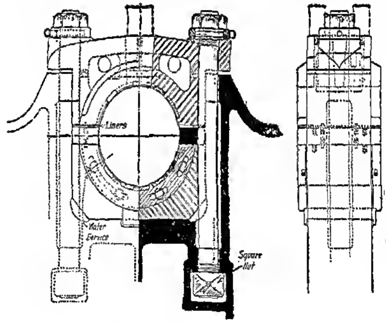


Fig. 49.

as shown in Fig. 49. This permits the bottom bearing to be rolled out for examination or repairs without taking up the crankshaft as is necessary with bearings of the type shown in Fig. 50. The **caps** are made either in one piece of cast steel or cast iron lined with white metal as shown in Fig. 49,

where r = radius of crank, in.

S = stroke, in.

C = ratio to max. to mean twisting moment as given in Table 14.

Let L = the length of crank pin in in., then maximum bending moment on after pin, $B_m = F_1 \times L + F_2 \times \frac{L}{2}$. Stress in pin $= B_m \div R_b$ where R_b = resistance of the pin to bending $= D^3 \div 10.2$ for solid pins and $(D^4 - d^4) \div 10.2D$ for hollow pins in which D = diameter of pin and d = diameter of hole. Stress in the crankpin computed by this method should not exceed 5,000 psi for merchant engines.

Thrust Shafts. The **diameter** is determined by the Classification Society rules, but it should be figured for stress due to combined bending and twisting, the bending moment being produced by its own weight between centers of the end bearings. The maximum twisting moment, T, will correspond to the value given under crankshafts. Maximum bending moment $= (W \times L) \div 8$, where W is the weight and L is length of shaft between centers of bearings in inches. The equivalent twisting moment and stress may then be found, as shown under crankshaft (p. 1106). Stresses of 5,000 to 6,000 psi are allowable in merchant work. For **number of thrust collars** see Thrust Bearings, p. 1101. In order to be stiff and not deflect the **diameter of the thrust, collars** should not exceed 1.45 to 1.65 times the diam of the shaft. With collars of these proportions pressures per square inch of 65 lb for the smaller ratio to 40 lb for the larger ratio may be carried safely. For the same reason the **thickness of collars** should not be less than 0.6 to 0.5 times the height. To provide sufficient space for stiff thrust shoes the **space between collars** should be 2.3 to 2.6 times the thickness of the collars for cast-iron and 1.4 to 1.6 for cast-steel shoes.

Line Shafts. The **diameter** is determined by the Classification Society rules but it should be figured for stress by the method given under Thrust Shaft. If the stress exceeds 5,000 psi for small or 6,000 lb for large shafts, the length between bearings should be reduced. In any event the **over-all unsupported length of line shafts** should not exceed 20 ft to avoid deflection.

Propeller Shafts. The **diameter** is determined by the Classification Society rules but the stresses due to combined twisting and bending should always be calculated. The maximum twisting moment is the same as given under crankshafts and the bending moment is produced by the weight of the shaft. The bending moment $= (2W \times L) \div 8$, in which W = weight of shaft between bearings, including brass sleeves, if fitted, and L = length in in. between centers of bearings. The factor 2, according to Seaton, is added for rough weather. Stress = equivalent twisting moment ÷ resistance to twisting. The stress in the after or overhung end of the shaft due to combined twisting and bending should also be computed. The bending moment $= W_1 \times L_1$, where W_1 = weight of propeller and shaft aft of the bearing, and L_1 = distance in in. from center of propeller to 3 in. inside of bearing. Stress = equivalent twisting moment ÷ resistance to twisting. (The brass sleeve should not be included in figuring the resistance.) Stresses in propeller shafts between bearings or in the overhung end should not exceed 5,000 psi for small and 6,000 lb for large shafts. Propeller shafts are fitted with brass sleeves in the stern-tube or strut bearings, the thickness of which should not

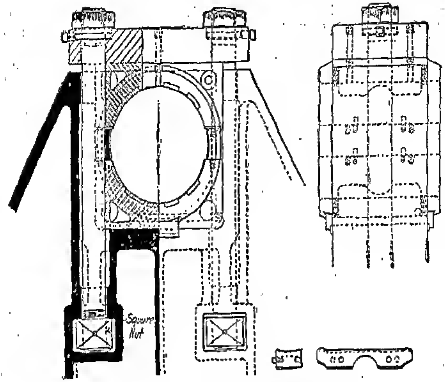


FIG. 50.—Types of main bearings.

or consist of a box of cast steel, cast iron, or brass held in place by a forged-steel keeper as shown in Fig. 50. Another type of main bearing is shown in Fig. 51. In this type the cap has lugs fitted over the bedplate to strengthen the bedplate against side thrust from the shaft. The distance pieces between the bottom brass and the cap prevent the bottom brass from nipping the shaft in case the bearing heats. Unless bearings are unusually long, two **bolts** are sufficient to secure the cap. The caps are fitted with **oil boxes** and a **handhole** with cover for the purpose of feeling the shaft and for applying cold water directly to the shaft in case the bearing warms up. The bottom boxes and sometimes the caps are cored for water circulation except in small engines.

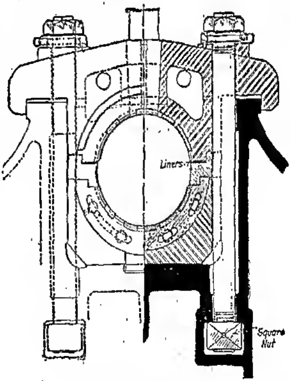


FIG. 51.—Main bearing.

be less than $\frac{3}{4}$ in. for all sizes of shafts up to 18 in., and 1 in. for shafts above 18 in. diam.

It is usual to cover the shaft between bearings with **brass sleeves**, the thickness of which may be about one-half that of the bearing sleeves. When brass sleeves are made in several sections the joints between the sections are scarfed leaving about $\frac{1}{8}$ in. open space for burning in or calking full of soft copper as shown in Fig. 61. In twin-screw vessels where struts are used and the propeller shafts are accessible, the shafts between bearing sleeves are often protected by wrapping with marlin or painting with anticorrosive paint.

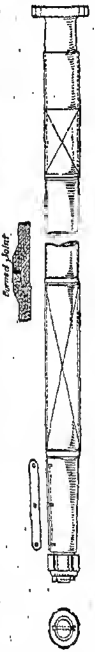


FIG. 61.—Propeller shaft.

Flanged Shaft Couplings. The diameter of the coupling flanges on crank, thrust, line, and propeller shafts is about 1.8 to 2 times diameter of shaft and the thickness about 0.23 to 0.28 times diameter of shaft but the couplings **should be proportioned from the size of bolts required.** The bolts must be of a size to safely resist the shear produced by the maximum twisting moment, T.

$$\text{Stress} = T \div n \times a \times r$$

where n = number of bolts
a = area of each bolt at face of coupling
r = radius of pitch circle

Allowable stress is 5,000 to 6,000 lb. **Six bolts are used unless** a greater number is required to avoid excessive size. Coupling bolts are tapered as shown in Fig. 62 except in interchangeable crankshafts in which case they must be straight as shown in Fig. 63. If diameter of bolt at face of coupling = A, then proportions of couplings and bolts when six bolts are used will be as follows: Thickness of coupling, $B = 1.2A$; distance from outside of shaft to pitch circle, $D = 0.9A$; distance to outside of coupling, C = from $2\frac{1}{8}$ in. for a 2-in. bolt to $3\frac{3}{8}$ in. for a $3\frac{1}{2}$-in. bolt; diameter of thread, $F = 0.75A$; depth of nut, $G = 0.75A$; depth of head for straight bolts, M = 0.7 to $0.65A$; size of head cross corners, $N = 1.38A$; taper for tapered bolts = $\frac{3}{4}$ in. per ft on diameter.

Clamp Shaft Couplings. When the propeller shaft is too long to be withdrawn inboard or if it is desired for other reasons to withdraw it outboard, a removable coupling of the clamp type of cast or forged steel as shown in Fig. 64 is used. The two fore-and-aft keys in each shaft transmit the torque and should be figured for shear from the maximum twisting moment as shown under Propeller Shafts, p. 1108. The diameter of coupling is from 1.7 to 2.0 times the diameter of shaft. The length must be sufficient to keep the key stresses within the allowable limit and is usually from 3 to 4 times the

The maximum pressure on the main bearings is produced by the unbalanced load and occurs twice each revolution, once upward and once downward, and is usually taken as the basis of figuring the pressure for purposes of design. The allowable pressure per square inch from the unbalanced load taken by two bearings runs from 125 to 175 lb. The ratio of length to diameter of main bearings is 1.0 to 1.15. The unbalanced load is not a true measure of the load on the bearings, but as it operates in the nature of a blow tending to destroy the oil film and bring the shaft and bearing metal into direct contact, it is always well to figure pressures in this way in addition to any other method used.

The true load on the main bearings is a function of the power passing through them and increases from forward to aft, as the power from the for-

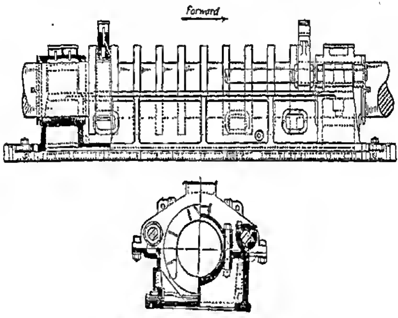
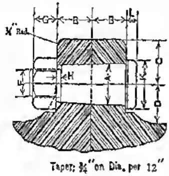


Fig. 52.—Thrust bearing, horseshoe type.

ward cylinders exerts a turning force on the after crank webs, which is transmitted to the bearings. Higher pressure per square inch is allowable on the after bearings as the turning force transmitted through them is more uniform and, therefore, is more evenly distributed around the bearing. In designing, however, it is usual to proportion the bearings on the average load and make all bearings the same length except the extreme forward bearing, which, having no power passing through it, frequently is made 10 per cent shorter than the intermediate bearings. For a similar reason it is desirable to make the extreme after bearing, through which all the power passes, 10 percent longer than the intermediate bearings. The average load on each bearing, from the power, is $\frac{\text{ihp} \times 33{,}000}{P.S. \times \text{no. bearings}}$ The allowable pressure per square inch from mean load runs from 80 to 120 lb. In the bearing pressures given above consideration has not been given to the weight and inertia of the crankshaft and reciprocating parts, or the centrifugal force. The pressure due to weight amounts to an additional 40 to 60 psi, but the effects of inertia and

diameter of shaft. The bolts should be as close to the shaft as possible and large enough to cause the coupling halves to grip the shaft tightly. The bolts are sometimes heated through the openings in the sides of the coupling while tightening up, in which case there will be a stress due to contraction in addition to the initial tension. The following formula adapted from Bauer gives the total area of all bolts on both sides at bottom of threads: Area $= C \times l \times (D - d) \div 2$, in which D, d, and l = outside diameter, bore, and length of coupling, respectively, and C is a constant = 0.25 to 0.35. When going astern, the propeller thrust is carried by projections on the inside of the coupling which fit into grooves turned in the shafts. These projections and the metal between the grooves and the ends of the shafts should be figured



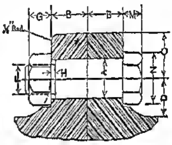


Fig. 62. Fig. 63.
Proportions of shaft couplings and bolts.

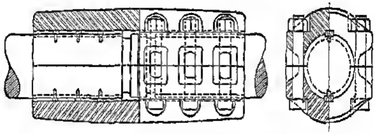


Fig. 64.—Clamp shaft coupling.

for shear and crushing due to the astern propeller thrust. Astern thrust in pounds $= \dfrac{\text{astern ihp} \times 33{,}000}{V \times 101.3}$ where V = astern speed of the ship in knots. The astern ihp may be taken as 80 per cent of the ahead ihp and V as 60 per cent of the ahead full speed.

Shaft Brake. In twin-screw and sometimes in single-screw vessels a brake similar to that shown in Fig. 65 is fitted to a shaft coupling aft of the thrust bearing to keep the shaft from turning while making repairs, or, in twin-screw vessels, to keep one shaft from turning while under way with the other engine.

Valve Gear

The two general classes of valve gears which permit of reversal are (1) link motion with eccentrics and (2) radial gears with or without eccentrics.

centrifugal force are small. Some designers, in determining allowable bearing pressures, take into account the product of the pressure and the rubbing speed of the bearings, but for merchant marine engines this is not considered necessary unless the engines are of a size and speed beyond the range of ordinary practice.

Main bearing caps and bolts must be proportioned for the maximum unbalanced load, P. The tensile stress in the bolts $= P \div (2n \times a)$ where n is number of bolts in one bearing and a is area at bottom of threads. The stress in the bolts, on account of the intermittent and suddenly applied load should not exceed 3,500 lb. The cap is figured as a beam loaded in the middle and supported at the ends. Stress $= Pl \div (8 \times 2R)$ for caps cast in a single piece, where l is distance between bolts, and R is section modulus which is determined by the method given on p. 1071. The section modulus is always figured under the white metal and the handhole and oil hole taken out. If the cap consists of a box with a keeper, it is considered that the separate box has the effect of making the load on the keeper a mean between concen-

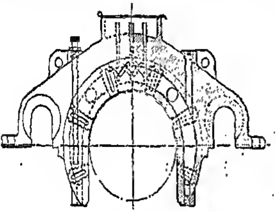


Fig. 53.

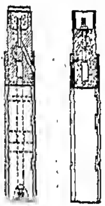


Fig. 54.

Thrust shoes.

trated and distributed load, and the load is assumed as all being taken by the keeper and stress $= Pl \div (10 \times 2R)$. On account of the intermittent and suddenly applied load and to secure stiffness, the stresses in caps should not exceed 2,500 psi for cast steel, 1,500 for cast iron, and 3,500 for a forged-steel keeper.

Thrust Bearings. The **horseshoe type** of thrust bearing (Fig. 52) is generally used with reciprocating engines as the nuts on the side rods provide means of adjustment. The **number of shoes and their area** are determined from the indicated thrust T, which $= \dfrac{\text{ihp} \times 33{,}000}{\text{rpm} \times P \times \%}$ or $\dfrac{\text{ihp} \times 33{,}000}{S \times 101.3}$ where P is pitch of propeller in feet, % is percent slip, and S is speed of the vessel in knots. For **diameter of thrust collars** see p. 1108. In this type of bearing the effective area of the thrust shoe is only about 70 percent of the annular area of the thrust collar. The **pressure on the shoes** is usually 40 to 60 psi, but with very stiff thrust collars and shoes pressure as high as 65 lb may be carried safely. The thrust shoes (Fig. 53) are cored for water circulation and are usually of cast-iron, although sometimes of cast steel, and are lined with white metal on both faces. The **thickness of the**

In America **Stephenson link motion** with two eccentrics is the most common type of valve gear used but **radial gears of the Marshall type,** with one eccentric, and the **Joy type,** worked by links from the connecting rod, are frequently used. With Stephenson link motion the valves are in the fore-and-aft center line of the cylinders although sometimes the high-pressure valve is placed on the side of the cylinder and the valve motion transmitted from the links through a rocker arm With the Marshall and Joy types the valves are on the side of the cylinders and hence the fore-and-aft length of the engine is less. A valve gear to be perfect would have a motion that would instantly open the valve wide at the beginning of the stroke and instantly close it at the point of cutoff without wire drawing; it would also instantly open it wide to exhaust at the end of the stroke and instantly close it when exhaust is completed. To more nearly attain this motion, **double-ported and trick valves** are often introduced. This motion is more nearly approached by radial valve gears than by link motion. With radial valve gears the lead is constant or nearly constant for all cutoffs. This is considered a very desirable feature by some designers.

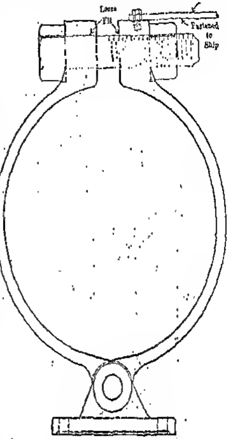


Fig. 65.—Shaft brake.

Stephenson Link Motion. In this type of valve gear there are two eccentrics keyed to the crankshaft, each operating an eccentric rod connected to a common link as shown in Fig. 106. The positions of the eccentrics are such that one eccentric rod operates the valve for going ahead and the other for going astern, the common link acting as the means for changing from one to the other. The valve stem is connected to the link block which slides on the link and changes the positions of the eccentric rods relative to the valve stem. When the block is in mid-position, there is practically no motion imparted to the valve and the engine stops. When it is in extreme position on the **go-ahead** eccentric rod the link is in **full gear** and **linked up** when the link block is in any position between extreme and mid-positions. Reversal is accomplished by moving the links by the suspension rods so that the sliding block is changed from the go-ahead to the **go-astern** eccentric rod which reverses the motion of the valve. Each cylinder may be **linked up independently** by means of the screw in the slotted reverse arm thus moving the links to bring the sliding block into the desired intermediate position. When linked up both eccentrics act on the valve and the motion is equivalent to that of an eccentric with less eccentricity and greater angle of advance

thrust shoe, including the babbitt, should be 2.3 to 2.6 times the thickness of the thrust collar for cast iron and 1.4 to 1.6 for cast steel. Sometimes instead of the customary dovetailed babbitted surface, a loose babbitt or bronze face is secured to the "go-astern" side of the thrust shoe by screws as shown in Fig. 54. Liners may then be inserted behind the face to take up wear. In calculating the **stress in the thrust shoe** it is considered as a beam supported at the centers of the side rods, the load being taken as a mean between concentrated and distributed. Bending moment $= TL \div 10n$, where L = distance between side rods in in. and n = no. of shoes. The modulus of the section at the center is found in accordance with the method given on p. 1071. Stress = bending moment ÷ section modulus. In order to avoid deflection, which would cause unequal bearing on the surface, stresses in thrust shoes should be kept low, good practice being not over 1,500 psi for cast iron and not over 3,000 lb for cast steel. The tensile **stress in the side rods** is calculated by dividing the total indicated thrust by the area of the

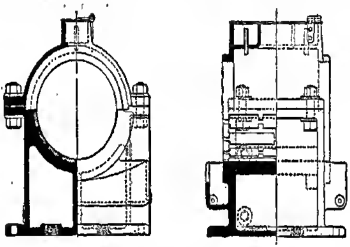


FIG. 55.—Line-shaft steady bearing.

rods at the bottom of the thread. A stress of 5,000 to 6,000 psi is allowable for either steel or bronze rods. **Steady bearings** of the same design as the line-shaft steady bearings are fitted at each end of the thrust bearing bedplate as shown in Fig. 52. The length of these bearings is 1.0 to 1.1 × diam of shaft. Sometimes the cap is lined with white metal but this is unnecessary. A stuffing box is fitted around the lower half of the shaft to retain the oil in the thrust-bearing bedplate. A pipe coil for water circulation for cooling the oil is fitted in the bottom of the thrust-bearing bedplate.

The **all-round type** of thrust bearing is occasionally used. To permit of adjustment, the "go-ahead" and "go-astern" bearings are separate, although both are fitted on a common bedplate. In this type the bearing surface covers the entire annulus of the collar, and as the collars on the shaft, owing to their form, are less liable to deflect, they may be made of larger diameter, necessitating the use of fewer shoes to maintain the same pressure per square inch. These bearings, however, lack the adjustability of the horseshoe type and are now seldom used in marine work.

Sole Plate. The thrust bearing is bolted down, through elliptical holes, on a cast-iron sole plate which is riveted to the hull foundation. Adjusting

resulting in shorter valve travel and a change in lead, cutoff, release, and compression. The change in these functions is shown by the valve diagram.

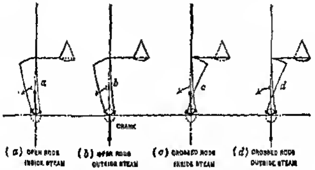


FIG. 66.—Open and crossed eccentric rods.

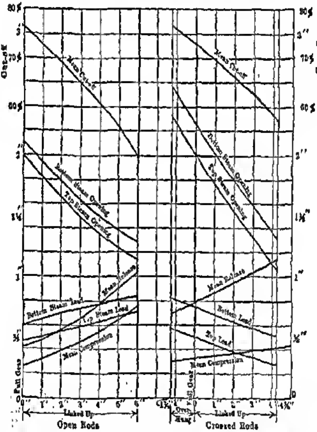


FIG. 67.—Changes in valve functions for open and crossed rods.

The go-astern eccentric is set at the same angle relative to the crank as the go-ahead eccentric but on the opposite side of the crank.

Open and Crossed Rods. With the eccentricity toward the links, the eccentric rods are open when arranged as shown at *a* and *b*, and crossed when arranged as shown at *c* and *d* (Fig. 66). In full gear the valve motion

wedges are fitted at each end between the bearing flanges and projections on the sole plate. It is very important that the foundation under the sole plate be strong and rigid to avoid deflection. Sometimes to ensure rigidity it is preferred to cast the sole plate with V-shaped legs on the forward end for bolting to the main engine bedplate. For merchant vessels of low power it is satisfactory to omit the sole plate and to bolt the thrust bearing to the hull foundation on liners through which the bolts pass, chocks being riveted forward and aft of the bearing and liners driven up hard against the bearing flanges to lock the whole solid after bearing is adjusted. Low stresses (not over 4,000 psi in single shear) should be allowed in the rivets securing the sole plate and in the holding-down bolts securing the bearing.

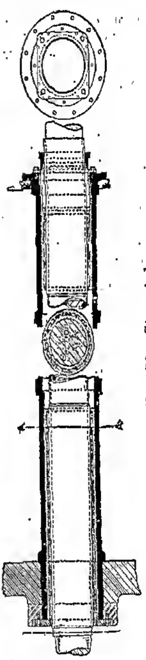


FIG. 56.—Stern tube.

Line-shaft Steady or Spring Bearings. It is customary to make steady bearings the same diameter as the shaft, it being unnecessary to increase the shaft in the bearing even if it is desired to leave the rest of the shaft rough-turned; only one bearing is generally used to each section of shaft, but with large shafts a bearing is sometimes fitted at each end of each shaft to facilitate lining up and handling. The **lengths of steady bearings** vary from 1.25 to 1.5 diameters and the **pressures per square inch of projected area** due to the weight of the shaft run from 35 to 50 lb. If the distance between bearings is too great, the shaft may deflect due to its weight and cause heating of the bearings. With only one bearing per shaft a **safe length between center of bearings** in feet $= c\sqrt{d}$ in which d = diameter of shaft in in. and c = 5 to 5.5. Figure 55 shows a typical steady bearing. The **pedestal** is of cast iron lined with white metal, and provided with an oil receptacle. The **cap**, which is usually merely a shell almost touching the shaft at each end to keep out dirt, has wick-feed oil boxes and a **handhole** for feeling the shaft. The bearings are sometimes piped for water circulation. Caps are secured by four **bolts**, the diameters of which vary from 1 in. for an 8-in. bearing to $1\frac{1}{2}$ in. for an 18-in. bearing. For the same size bearings the **holding-down bolts** vary from 1 to $1\frac{1}{4}$ in. diameter, pitched about 6 diameters. Ring oiling is sometimes used.

Stern tubes in merchant work are of cast iron and in one piece unless very long (Fig. 56). The **thickness of the casting** in the stern frame and between bearings varies from $1\frac{1}{2}$ in. for a 10-in. shaft to $2\frac{1}{2}$ in. for a 20-in. shaft. The **minimum length of the after bearing** is 4 times the diameter of propeller shaft under the bearing sleeve according to Lloyd's rules, but this

is the same for open as for crossed rods but, when the gear is linked up, the effect on the valve functions is materially different. The curves in Fig. 67 show that with open rods a greater range of cutoff may be obtained with a smaller reduction of port opening than with crossed rods. As the gear with open rods is linked up, the lead increases while with crossed rods it decreases and may become negative. Linking up with crossed rods decreases the valve travel very rapidly with consequent reduction of both steam and exhaust port openings and therefore crossed rods are not as well adapted for expansive working as open rods. With crossed rods the links are thrown toward the back of the engine when in ahead gear, which leaves the front of the engine open for oiling and inspection. Also the suspension rods are much longer which reduces the slotting action of the link block. Most marine engines are fitted with open rods.

Slip of the Link Block. In laying out a valve gear the slip or slotting action of the link block on the links should be given consideration. Although this slip has no practical effect on the action of the valve, the length of the suspension rods and the point of suspension should be chosen to reduce this action as much as possible on account of wear. The nearer the link block to the point of suspension the smaller will be the slip. If the links are suspended directly over the position of the link block when in full gear ahead, the slip may be considerable when the engine is reversed but this is of little importance if it is a minimum for the usual working position of the links. When the probable location of the reverse arm has been determined, the path of the point of suspension for one complete revolution should be plotted for full gear ahead and astern and maximum linked-up position ahead and astern and clearances carefully checked. By examination of these loops, examples of which are shown in Fig. 68, it may be possible to choose a position of the reverse arm which will reduce the slotting action.

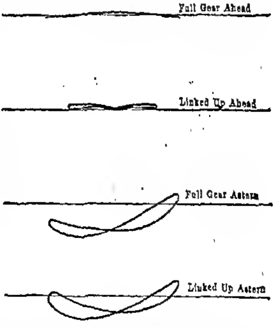


FIG. 68.—Slip of the link block.

Marshall Radial Valve Gear. This type of marine-engine valve gear as usually constructed in America has the valve rod connected to the eccentric rod outside of the point of attachment of the guide link while European practice is to make the connection between the guide link and the eccentric. The arrangement of this gear for each cylinder consists of one eccentric mounted on the crankshaft, giving motion to a short and massive eccentric rod A (Fig. 69) which is guided at a by the link B and is connected at b to the valve rod D. The guide link B is supported at d by the bell crank

sometimes is made longer. **The length of the forward bearing** depends on the length of the stern tube, but the minimum length is 1½ times the diameter of shaft. **The pressure due to weight** of the shaft, sleeve, and propeller should not exceed 30 psi for the after bearing and 20 lb for the forward bearing. In all cases a steady bearing is fitted on the line shaft as close as possible to the inboard coupling. The stern-tube bearings are usually of lignum vitae contained in brass bushings, made in halves, the joints being in the horizontal plane slightly inclined fore and aft to facilitate withdrawal. **The thickness of the wood,** which always bears on the end of the grain, runs from 1 in. for a 10-in. shaft to 1¼ in. for a 20-in. shaft. **The thickness of the brass bushing** under the wood for the same size shafts is about ½ and ¾ in. The stern tube is held in place in the stern frame by a cast- or forged-steel nut in which there are holes or slots for a spanner wrench for tightening up. Four threads per inch are used for nuts of all sizes. The thickness of the nut should be about 3¼ in. for a 10-in. shaft and 6½ in. for a 20-in. shaft. At the forward end of the stern tube is fitted a **stuffing box** of the usual type for soft packing. Very often a bronze or cast-iron bushing is substituted for the forward lignum vitae bearing.

No lubricant other than water is required for stern tubes with lignum vitae bearings. There should be a pipe connection to one of the salt-water pressure pumps in the engine room for washing out the stern tube and also a connection for draining the space between the forward and after bearings. Sometimes the after bearing is babbitted and the forward bearing bronze bushed, in which case a protecting stuffing box must be fitted between the propeller and the after end of the stern tube to keep out sand and dirt. With these stern tubes oil is used for lubrication. There are several patented stern tubes and protective stuffing boxes on the market.

Strut Bearings. In twin-screw vessels in which the stern tube is not carried back to the propeller a separate bearing is carried in the strut for taking the weight of the shaft and propeller. The length of the bearing, the thicknesses of the wood and the brass bushing, and the pressure per square inch correspond to the figures given for the after stern-tube bearing.

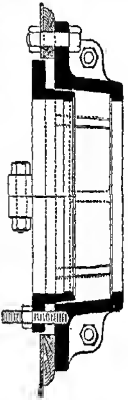


FIG. 57.—Bulkhead stuffing box.

Lignum vitae is a greenish-brown, hard, heavy wood of specific gravity, 1.10. It is used for stern-tube and strut bearings, and makes an excellent low-pressure bearing when **lubricated by** the free circulation of water. Lignum vitae **for use in bearings** should be generally straight, sound, and well seasoned; it **should be free from** injurious shakes, wormholes, excessive sap, large or unsound **knots**, and other injurious defects. Ordinary season checks, small amounts of sap, slight heart and ring shakes are not generally considered defects.

Bulkhead stuffing boxes of cast iron are fitted where line shafting passes through a water-tight bulkhead (Fig. 57). The box and gland are in halves.

lever C having its axis at c and connected by the rod F to the reverse arm G. The valve rod D may be connected to the valve stem direct or through a rocker arm. The gear is linked up by shortening the valve travel which is done by means of the adjustment block in the bell-crank lever C. The movement of the point b is an ellipse as shown inclined to the horizontal in one direction for ahead motion and in the opposite direction for astern motion, the change being effected by the reversing engine through the reversing rod F. The angle between the eccentric and the crank is either 0 or 180 deg according to whether the valve is driven direct or through a rocker arm or whether it takes steam inside or outside. Lead is constant, and a sharp cut-off and good steam distribution are obtained but to the curvature at the path of the point a, the top and bottom cut-offs are unequal. The large number of joints subject to wear results in excessive lost motion and frequent adjustments must be made. The oval-valve diagram is used for this type of gear and it is usually worked out by means of a simple model.

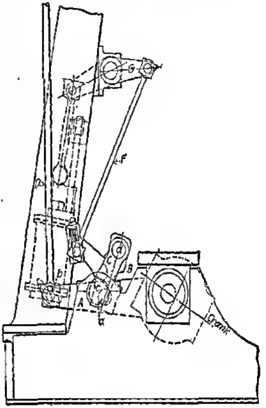


Fig. 69.—Marshall radial valve gear.

Joy Radial Gear. In this type of valve gear no eccentrics are used, the movement being taken from the connecting rod by a series of links. By utilizing the reciprocating and vibrating actions of the rod a movement results which gives almost equal cutoff for both sides of the piston for all points of expansion, less compression at short cutoff, and much quicker action at the points of admission and cutoff than is given by Stephenson link motion. Lead is constant. The vibrating link A (Fig. 70) is connected to a point on the connecting rod and to the radius rod B. The movement of the point a is an ellipse and of the point b an arc of a circle as shown. Another lever C with a fulcrum at d is connected at c to a point on the vibrating lever and at e to the valve link E which actuates the valve stem G. The fulcrum d is connected to the reverse lever D or to a link block sliding in a slotted link. The movement of the point e is an ellipse, the inclination of which from the horizontal, as effected by the position of the reverse lever D, or slotted link, governs the degree of expansion. The engine is reversed by inclining the axis of this ellipse the other way by means of the reverse lever through the link F.

The stuffing box is carried on a portable plate in halves large enough to ship the shafts and bearings. Soft packing is used.

Shafting

Arrangement. The location of the after end of the shafting is determined by the diameter and the depth of immersion of the propeller. In single-screw vessels a clearance of 4 to 6 in. must be allowed between the tips of the propeller and the stern frame shoe. In twin-screw vessels ample clearance between the tips of the propellers and the hull must be allowed (see p. 1425). The height of the **forward end of the shafting** is determined by the depth necessary for a strong and rigid foundation under the main engine bedplate. Shafting is kept parallel with the base line of the vessel except where the inner bottom or the water tanks under the engine makes this impossible. In twin-screw vessels it is often impossible owing to the arrangement of the machinery, to keep the shafting parallel in the horizontal plane.

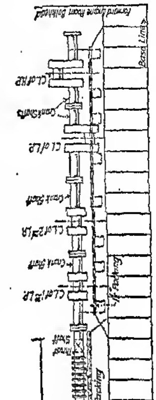


FIG. 58.—Arrangement of shafting.

The shaft tunnel is fitted its entire length with a platform from which the bearings may be attended. The forward end of the shaft tunnel is isolated from the engine room by a sliding watertight door operated from above. Overhead in the tunnel are located eyebolts or other lifting gear for handling the propeller and thrust shafts and the stern-tube bearings. The propeller shaft is drawn inboard for examination unless its length is prohibitive. A water service pipe is carried through the tunnel with a connection to each bearing and to the stern-tube stuffing box. This pipe is usually led so it can be used as a handrail. Figure 58 shows the arrangement of shafting for a single-screw vessel with the machinery located in the stern.

Crankshafts for merchant engines are usually of the "built-up" type, *i.e.*, the pieces of shaft, webs, and pins are forged separately and shrunk together. For naval engines, yachts, and other engines where saving of weight and space is important, crankshafts are "solid-forged" and usually hollow. In order that one spare section of crankshaft may serve for any crank, the sections are often made interchangeable and sometimes interchangeable and reversible, *i.e.*, one spare section will serve for any crank with either end forward. The over-all dimensions of cylinders and

For this type of valve gear the **oval-valve diagram** is used and it is best worked out by means of a simple model. The defects of the Joy gear are the large number of joints subject to wear, resulting in excessive lost motion, and the difficulty of inspecting and taking care of the crosshead and crankpin owing to the obstruction of the moving parts while the engine is running.

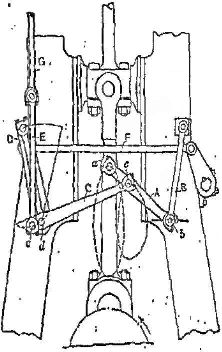


FIG. 70.—Joy radial valve gear.

Action of the Valve. The successful operation of the engine depends upon the proper design and setting of the valves. The action of the valve is the same whether piston or flat slide valves are used, and the elementary principles apply to both. In mid-position, the valve not only covers the steam ports, but overlaps the edges of the ports, as shown in Fig. 71. The amount of overlap a, on the steam side is the **steam lap** and on the exhaust side b, the **exhaust lap.** The exhaust lap b is **positive** if overlapping the edge of the port and **negative** when not covering the port. The other elements are c, **steam port width;** d, **maximum steam port opening;** e, **exhaust port width;** f, **minimum exhaust port opening;** and g, **half travel.** The complete cycle of distribution of steam in the cylinder consists of admission, expansion, release, and compression. Admission begins through c, when the outer edge of the valve has traveled a distance equal to the steam lap a and uncovered the edge of the steam port (**point of admission**), and ends when the valve has reached this point again, traveling in the opposite direction (**point of cutoff**). Expansion then takes place until the inner edge of the valve has uncovered the edge of the steam port to exhaust (**point of release**). Exhaust continues until the valve closes the port traveling in the opposite direction (**point of compression**).

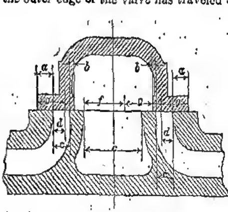


FIG. 71.—Elementary slide valve.

From this point compression takes place until the valve is again opened to admission. The maximum steam port opening d is usually less than the width of port c, and is reached when the valve has traveled its maximum distance from mid-position. The amount which the steam port is open when the piston is at the end of the stroke is the **lead,** or steam lead. Similarly the amount which the exhaust port is open is the exhaust lead, commonly

bedplate are increased if the sections of crankshaft are made interchangeable, or interchangeable and reversible. This question is discussed on p. 1153.

The diameter of the crankshaft for a merchant engine is determined from the rules of the Classification Society under which the vessel is constructed, but the designer should always calculate the stress in the after end of the shaft produced by the combined bending and twisting moment. Maximum twisting moment in inch-pounds, $T = \frac{63{,}000 \times \text{ihp}}{\text{rpm}} \times C$, where C = ratio of the maximum to the mean twisting moment. This ratio varies with the number of cranks, as shown in Table 14. The bending moment is produced by the twisting moment, which, for all practical purposes, may be taken the same as T, considered at the middle of the crankpin. Then maximum bending moment, $B = T \times L \div 8 \times r$, where L = distance between edges of bearings of after crank in in. + 1 in. and r = radius of crank in in. Then equivalent maximum twisting moment, $T_e = B + B\sqrt{1 + (T \div B)^2}$.

Stress in shaft = $T_e \div R_t$, where R_t is resistance of the shaft to twisting $= D^3 \div 5.1$ for solid shafts and $(D^4 - d^4) \div 5.1D$ for hollow shafts, in which D = diameter of shaft and d = diameter of hole. Stress should not exceed 6,000 psi although it is often less than this, due to the desire of owners to have shafts larger than required by the Classification Society rules.

Table 14. Ratio of Maximun to Mean Twisting Moment

Type of Engine	Ratio for Average Cut-off of 70 Percent in H-p Cylinders
One-crank	2.00
Two-crank	1.50
Three-crank	1.33
Four-crank	1.25

The effect of inertia is not considered in the above ratios. According to Bragg, inertia may increase the ratio for three-crank engines as much as 18 per cent, while for engines with cranks at 180 and 90 deg, the inertia effects are nearly balanced.

Crank webs for merchant engines are usually of the type shown in Fig. 59, with straight sides, although they are sometimes made as shown in Fig. 60

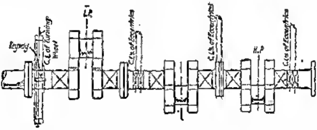


Fig. 59.—Crankshaft.

to save weight; the weight saved, however, does not justify the extra labor involved. Crank webs are subjected to a bending stress in the direction of rotation due to the maximum twisting moment and also to a bending stress in a fore-and-aft direction owing to the unbalanced load on the crankpin. These stresses, however, are small and need not be computed, if the webs are proportioned by Table 15.

called release. To bring the edge of the valve into proper position relative to the edge of the steam port when the piston is at the end of the stroke, it is necessary to move the eccentric through an angle which will move the valve from its central position a distance equal to the steam lap plus the steam lead. This angle is called the **angle of advance.**

Valve Functions. If there were neither lap nor lead, the valve would be central each time the piston reached the end of the stroke and steam would enter the cylinder throughout one stroke and be exhausted throughout the next. There would be no expansion and no compression. The use of lap makes cutoff and expansion possible. It is customary to make the top steam lap slightly larger than the bottom lap to lessen the difference between the top and bottom cutoffs. Steam laps of 1 to 2½ in. are commonly used in marine engines. Positive **exhaust lap** retards the period of release and makes compression earlier while negative exhaust lap has the opposite effect. The amount of lap, whether positive or negative, is determined by the exhaust opening which should be wide open at the end of the stroke. Exhaust lap, if negative, should always be less than the steam lap to avoid a passage for steam from one end of the cylinder to the other at the moments of admission and cutoff. Frequently exhaust lap is zero but it may be as much as ± 1 in. **Steam lead** is generally settled according to the judgment of the designer. The inertia of the moving parts relative to the area of the piston should be taken into consideration in determining lead as ordinarily compression is not sufficient to overcome the inertia of the moving parts by the time the piston reaches the end of the stroke. Lead therefore helps to cushion the piston and should be greater for large than for small cylinders. It may vary from ⅛ in. in small high-pressure cylinders to 1½ in. in large low-pressure cylinders. On account of the angularity of the connecting rod the bottom lead is always made more than the top lead. Top lap plus lead must always equal bottom lap plus lead. As linking up increases the lead with open rods and decreases it with crossed rods, as shown in Fig. 67, normal lead is made greater with crossed rods. **Lead with Marshall, Joy, and other radial valve gears** is constant for all grades of expansion. The **cutoff** for equal power in each cylinder of a multiple-expansion engine depends on the relative volumes of the cylinders as discussed on p. 1066. With Stephenson link motion the bottom cutoff is from 5 to 9 percent less than the top cut off due to the angularity of the connecting rod. **Release**

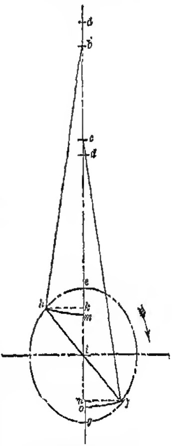


FIG. 72.—Effect of angularity of the connecting rod.

To Multiply by Logarithms. Find from the table the log of each factor, and add; the result will be the log of the product. Then find the product itself from the table.

EXAMPLE. To find $x = (4.098)(0.0002973)(72.1)$	$\log 4.098 = 0.6126$
	$\log 0.0002973 = 0.4732 - 4$
Answer: $x = 8.784 \times 10^{-2}$	$\log 72.1 = \underline{0.8579 + 1}$
$= 0.08784$	$\log x = 1.9437 - 3 = 0.9437 - 2.$

To Divide by Logarithms. First Method: Find from the table the log of the numerator and the log of the denominator, and subtract the second from the first; the result will be the logarithm of the quotient. Then find the quotient itself from the table.

EXAMPLE. To find $x = \dfrac{4.098}{0.0002973}$	$\log 4.098 = 0.6126$
	$\log 0.0002973 = \underline{0.4732 - 4}$
Answer: $x = 1.378 \times 10^4 = 13780$	$\log x = 0.1394 + 4$

In order to avoid negative mantissas in cases where a larger mantissa would have to be subtracted from a smaller, modify the upper logarithm by adding and subtracting 1.

EXAMPLE. To find $x = \dfrac{0.0291}{63.4}$	$\log 0.0291 = 0.4639 - 2 = 1.4639 - 3$
	$\log 63.4 = 0.8021 + 1 = \underline{0.8021 + 1}$
Answer: $x = 4.590 \times 10^{-4}$ $= 0.0004590$	$\log x = 0.6618 - 4$

But if the logarithms are written with the characteristics in front, and the "shop method" of subtraction is used (see p. 88), then no such special device is here required. Thus:

$$\log 0.0291 = \bar{2}.4639$$
$$\log 63.4 = \underline{1.8021}$$
$$\log x = \bar{4}.6618$$

To Divide by Logarithms. Second Method: Instead of subtracting the log of a number, it is often convenient to add the **cologarithm** of that number; the colog of N being defined by: $\text{colog } N = \log(1/N) = -\log N$.

To find the colog of a number, write the log. of the number in the standard form, and subtract it from $1.0000 - 1$, as in the following examples:

$$\begin{array}{rl} & 1.0000 - 1 \\ \log 69.5 = & \underline{0.8420 + 1} \\ \text{colog } 69.5 = & 0.1580 - 2 \end{array} \qquad \begin{array}{rl} & 1.0000 - 1 \\ \log 0.0002973 = & \underline{0.4732 - 4} \\ \text{colog } 0.0002973 = & 0.5268 + 3 \end{array}$$

This subtraction should be performed mentally. Thus, to subtract the mantissa, subtract each digit from 9 until the last non-zero digit is arrived at, and subtract this from 10; to subtract the characteristic, follow the regular rule of algebra ("reverse the sign and add"). Hence, if the logarithm itself is already written down, or can be read off from the table without interpolation, the cologarithm can be written down at once, by inspection. The use of cologarithms is not essential in logarithmic computation, but it often facilitates a compact arrangement of the work, especially in cases where the denominator of a fraction is itself the product of two or more factors.

To Find the nth Power of a Number by Logarithms. Find from the table the log of the number, and multiply it by n; the result will be the logarithm of the nth power of that number. Then find the power itself from the tables.

EXAMPLE 1. Find $x = (0.0291)^3$	$\log 0.0291 = 0.4639 - 2$
Answer: $x = 2.464 \times 10^{-5}$	$\underline{\quad 3}$
$= 0.00002464$	$\log x = 1.3917 - 6 = 0.3917 - 5$

EXAMPLE 2. Find $x = (0.0291)^{1.41}$ $\log 0.0291 = 0.4639 - 2 = -1.5361$

Answer: $x = 6.825 \times 10^{-3}$

$= 0.006825$

$$\begin{array}{r} -1.5361 \\ 1.41 \\ \hline 15361 \\ 61444 \\ 15361 \\ \hline \end{array}$$

$$\log x = -2.1659 = 0.8341 - 3$$

To Find the nth Root of a Number by Logarithms. Find from the table tho log of the number, and divide it by n; the result will be the log of the nth root of that number. Then find the root itself from the table.

EXAMPLE. Find $x = \sqrt[3]{4.098}$ $\log 4.098 = 0.6126$

Answer: $x = 1.600$ $\log x = 0.2042$

In order to avoid fractional characteristics, if the characteristic is not divisible by n, make it so divisible by adding and subtracting a suitable number before dividing.

EXAMPLE. Find $x = \sqrt[3]{0.0004590}$ $\log 0.0004590 = 0.6618 - 4$

$3)\underline{2.6618 - 6}$

Answer: $x = 7.714 \times 10^{-2}$

$= 0.07714$ $\log x = 0.8873 - 2$

But if the characteristic is positive, it is simpler to write it in front of the mantissa, and then divide directly.

THE SLIDE RULE

The slide rule is an indispensable aid in all problems in multiplication, division, proportion, squares, square roots, etc., in which a limited degree of accuracy is sufficient. The ordinary 10 in. Mannheim rule (see below) costs \$3 to \$4.50 and gives three significant figures correctly; the 20 in. rule (\$12.50) gives from three to four figures; the Fuller spiral rule (\$30) or the Thacher cylindrical rule (\$35) gives from four to five figures. For many problems the slide rule gives results more rapidly than a table of logarithms; it requires, however, more care in placing the decimal point in the answer. In all work with the slide rule, the position of the decimal point should be determined by inspection (see p. 89), only the sequence of digits being obtained from the instrument itself. Rapidity in the use of the instrument depends mainly on the skill with which the eye can estimate the values of the various divisions on the scale; expertness in this respect comes only with practice. The following explanations should be sufficient to permit the use of the ordinary slide rule successfully without previous experience and without knowledge of logarithms.

Multiplication and Division with a (Theoretical) Complete Logarithmic Scale. Consider a *complete* logarithmic scale (D, Fig. 1), assumed to extend indefinitely in both directions, only the main section, from 1 to 10, however, being usually available. Note that the divisions within the several sections are identical, except that the numeral attached to each division of any one section is ten times the numeral attached to the corresponding division in the preceding section. [The distances laid off from 1 are proportional to the logarithms of the corresponding numbers, the distance from 1 to 10 being taken as unity.] Consider also a duplicate scale C, numbered from 1 to 10, and arranged to slide along the fixed scale D as in the figures. By means of such a scale D and slide C any two numbers between 1 and 10 (and hence any two numbers whatever, with proper attention to the decimal point) can be multiplied or divided, as in the following examples.

(sometimes called exhaust lead) should be early enough to give a free passage for exhaust to avoid wire drawing; 7 to 16 percent is the range for marine engines. **Compression** should not be carried beyond the pressure of admission; it varies from about 6 percent in high-pressure cylinders to about 12 percent in low-pressure cylinders. The **angle of advance** is the lap angle plus **lead angle** and may range from 33 deg for high-pressure to 45 deg for low-pressure cylinders. With valves taking outside steam the eccentric precedes the crank 90 deg plus angle of advance, and with valves taking inside steam the eccentric follows the crank 90 deg minus angle of advance. Although the **angularity of the connecting rod** affects the position of the piston with reference to the crank, the length of the eccentric rod with reference to the throw of the eccentric is so great that the displacement of the valve is considered equivalent to the vertical travel of the center of the eccentric and the valve travel equal to twice the radius of the eccentric.

Valve Travel. It is important that the travel of the valve be kept as short as practicable to reduce frictional loss. The travel of a piston valve may be reduced by increasing the diameter and of a slide valve by making it double ported, or in case of very large cylinders with long strokes, triple ported. The valve travel of all cylinders is kept the same if possible but it is often found, when working out the valve diagram, that it is better to increase the travel of the low-pressure valve. The travel is usually 2 to 3 times the width of the steam port.

Angularity of the Connecting Rod. Inspection of Fig. 72 will show that, for the same angular position of the crank relative to the beginning of the stroke, the piston moves a much greater distance on the down stroke than on the up stroke. If Fig. 72 represents the positions of the piston and crank at cutoff, it is evident that with a connecting rod of infinite length, the piston would have traveled equal distances and ac and db would be equal to en and gk and the cutoffs would be equal; but the angularity of the connecting rod increases the cutoff on the down stroke and reduces it on the up stroke making $ac = eo$ and $db = gm$. Actually a movement of the crank of 120 deg, with a connecting rod length equal to 4 times the crank, corresponds to a cutoff or piston travel of 80 percent on the down stroke and 70 percent on the upstroke. To partly overcome this difference, the top steam lap is made larger than the bottom. However, this difference with Stephenson link motion can never be reduced below about 5 percent.

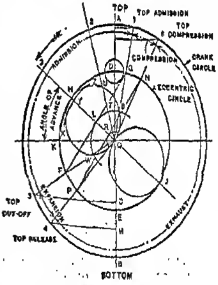


FIG. 73.—Construction of Zeuner valve diagram.

Zeuner Valve Diagram. In laying down a valve diagram the valve travel, cutoff, steam lead, and exhaust release are assumed and the other functions determined by trial. The Zeuner valve diagram is generally used for Stephenson link motion and is constructed as follows:

Lovekin Assistant Cylinders. In this type of balance cylinder (Fig. 85) the admission of steam to the underside of the piston is controlled by the movement of the piston past a steam port. The steam is taken directly from the chest and the action is automatic. Its purpose is to balance the inertia force of the valve gear on the down stroke as well as the weight, by

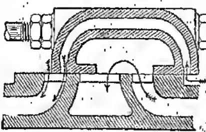


FIG. 83.—Trick slide valve.

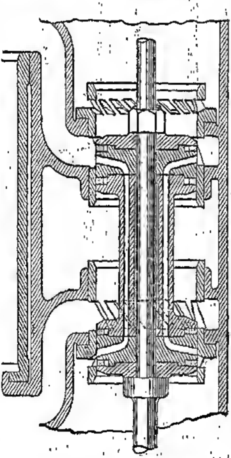


FIG. 84.—Trick piston valve.

compression of steam on the underside of the piston to a pressure higher than that of the valve chest. In cases where the inertia force is much greater than the weight of the valve gear, steam is also admitted on top of the piston to provide compression on the up stroke as well as the down stroke. In this case the steam required is only that necessary to replace the steam which condenses as there are no exhaust ports.

In Fig. 73 let the line AB be the direction of travel of the valve. With O as a center describe the circle DKE with radius equal to the eccentricity (full size) and also the crankpin circle $A3B$ reduced to a convenient scale. Assume A top and B bottom of stroke. Offset AC from A equal to the top cut-off and draw the corresponding crank position $O3$, using the length of the connecting rod reduced to scale, as a radius. With D as a center draw a circle with radius equal to the top steam lead and from F a line FG tangent to this circle. Bisect this line with a normal HJ through O. A circle HSO with HO as diameter will represent the movement of the valve from the central posi-

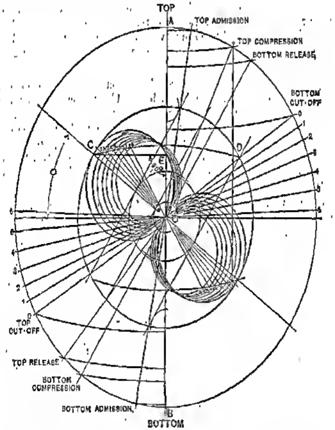


FIG. 74.—Zeuner valve diagram for open eccentric rods.

tion. The angle HOK is the **angle of advance**, LO the **steam lap**, and LH the **maximum steam opening** which occurs at crank position $O2$. The **steam opening for any other crank position** $O5$ is TU. Draw $O1$ through G which passes through the intersection of the steam lap circle WLS and the valve circle HSO. The line $O3$ also passes through the intersection of these circles. The points S and W, therefore, represent the **opening and closing of the valve**. Complete the construction for the bottom of the stroke in a similar manner. Offset BM from B equal to the top exhaust release and draw the crank position $O4$. Also draw $O6$ making angle $6O2$ equal to angle $4O2$. The point 6 on the crank circle is the crank position when **top compression begins**. With a straight line connect points N and P and, with O as a center,

Valve Load. A number of formulas are in use for obtaining the loads on the valves in order to proportion bearing surfaces and the strength of the various parts of the valve gear. Some formulas take into account the weight and inertia of the valve gear and others the friction of the valves. Any formula must be more or less empirical, and it is well to simplify the calculation by basing the load on the friction of the valves only and keeping the pressures and stresses low enough to cover all other contingencies. The pressures and stresses mentioned under valve gear details have been found by

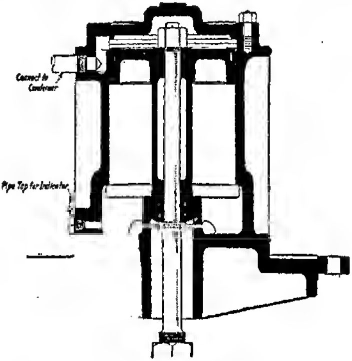


FIG. 85.—Lovekin assistant cylinder.

successful practice to be satisfactory and are based on valve loads found by the following formulas:

$$\text{Valve load in lb} = L = c \times b \times l \times p \text{ for slide valves}$$
$$= c \times \pi d \times l_1 \times p \text{ for piston valves}$$

where c = coefficient of friction
b and l = width and length of slide valve
d and l_1 = outside diameter and length of top + bottom piston valve rings
p = unbalanced absolute pressure between the steam and exhaust sides of the valve

The constant c should be taken 0.15 for intermediate-pressure and low-pressure slide valves and 0.22 for high-pressure and intermediate-pressure piston valves which values are high enough to cover almost dry surfaces, a condition which is liable to occur. For low-pressure cylinders, p should be taken not less than 30 lb for compound, 25 lb for triple, and 20 lb for quad-

draw a circle tangent to NP. The radius OR of this circle is the exhaust lap. If this circle lies within circle HO, the lap is negative and, if within circle OJ, the lap is positive. When the width of the port is less than HL, an arc XYZ is drawn making the distance YL equal to the port opening. The steam lap, lead, release, and valve travel must be adjusted until considered satisfactory for the required cutoff. The diagram is always laid off in the same manner relative to the top of the stroke whether the valve takes steam inside or outside. A separate diagram must be drawn for each cylinder. To

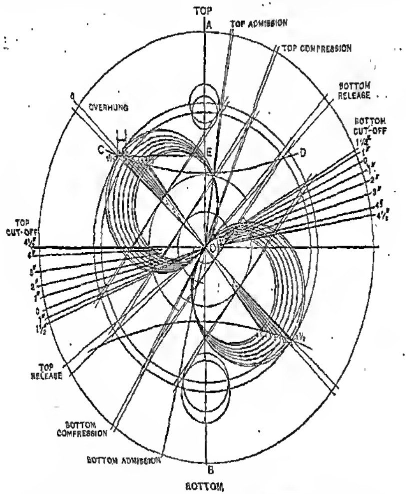


FIG. 75.—Zeuner valve diagram for crossed eccentric rods.

find the valve functions for linked up positions, see Fig. 74, which is a complete diagram for open rods. The arc CD is described through point C from a center on OB extended and with a radius equal to

$$\frac{\text{Length of eccentric rod} \times 2 \times EC \text{ (Fig. 74)}}{2 \times \text{distance between eccentric rod pins on link}}$$

Considering the length of the arc CD as equivalent to the length of the link, lay off proportionally from E the points 1, 2, 3, etc., to represent linking up of 1, 2, 3, etc., in., and describe circles with $O1$, $O2$, $O3$, etc., as diameters. From these circles the points of admission, cutoff, release, and compression,

ruple engines; for other cylinders p may be taken from Table 2. For slide valves the total area of the back of the valve is used, neglecting the effect of the relief ring, if any, as the valve gear must be proportioned to operate the valve should the relief ring fail to work. The load for piston valves depends on the fit of the valve rings. With tight rings and dry surfaces the load may exceed that given by the formula. The valve loads for all cylinders should be determined and, unless the loads are materially different, the valve gear proportioned from the largest load and the parts made the same throughout the engine. Often, however, the valve load for the low-pressure cylinder is much greater than for the other cylinders, and it is not unusual to increase the size of the parts for this cylinder, The following details of valve gear cover Stephenson link motion, but the details of other types may be designed in the same manner by using the valve load L as a basis.

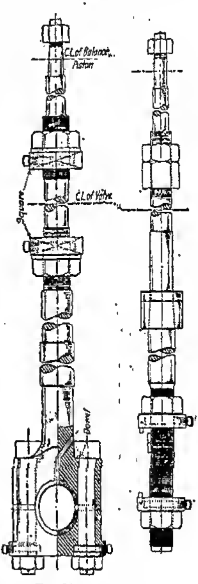


FIG. 86. FIG. 87.
Valve stems.

Valve Stems. When one valve is used, the valve stem is fitted with a **cap and adjustable brass bushing** at the lower end (Fig. 86) which attaches to the link block. When there are two valves, the stem (Fig. 87) is **screwed into a crosshead**, which connects to the link block. Piston valves are secured to the stem by a collar and washer below and lock nuts above the valve. **Adjustment of the valve along the stem** is made by changing the thickness of the washer. **To provide for wear** of a slide valve and to allow the steam pressure to hold it firmly against its seat, the hole through which the stem passes is made oval. Nuts above and below the valve are locked against a pipe distance piece over the stem or in some other manner so that the valve is free on the stem. The diameter of the valve stem below the valve is determined by the method given under Piston Rods, p. 1092, using the valve load L and a factor of safety of about 30. The stress at the bottom of the thread in the valve should not exceed 3,000 lb. The cap and bolts are proportioned in a manner similar to connecting rod caps and bolts using the same allowable stresses. The stems, collars, nuts, caps, and bolts are forged steel.

Valve-stem Crossheads. When two piston valves are used a crosshead, usually of cast steel, of a design similar to Fig. 89 is used. The lower end of the crosshead carries the bushing for attaching to the link block.

corresponding to the linked-up positions, may be found in the same manner as for full-gear condition first described. **For crossed rods** the arc CD is laid off from a center on OA extended instead of on OB as shown in Fig. 75 which is a complete diagram for crossed rods.

In some cases the links are arranged for carrying the cutoff beyond the point of attachment of the eccentric rods thus increasing the valve travel and decreasing the angle of advance; this may be represented on the diagram by a proportionate extension of the equivalent link line CD (Fig. 75) beyond the point C and is generally called **overhung gear.** The difference between open and crossed rods can best be understood by comparing the valve functions for different cutoffs as shown by the curves in Fig. 67, which are drawn for valve gears with open and crossed rods suitable for the same engine. For crossed rods in this case it was found advisable to overhang the links $1\frac{1}{4}$ in. to get the benefit of larger valve travel and the proper range of cutoff without reducing too much the steam opening and steam lead when **linked up.** The large steam opening and steam lead at maximum linked-out position necessary to obtain good running conditions in the maximum linked-up position may be noted.

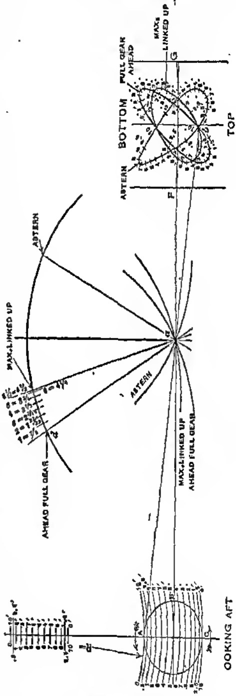


FIG. 76.—Construction of oval-valve diagram for Marshall gear.

The oval-valve diagram is frequently used in connection with radial valve gears. Its construction for Marshall valve gear is as follows:

The path of the lower valve rod pin b, during one revolution, is plotted by laying off to a convenient scale or by use of a simple model, the locations of the

Valve-stem Guides. The valve stem must be guided at the lower end to prevent the side strain due to the angularity of the eccentric rods or the action of the links, from bending the stem or throwing a pressure on the stuffing box. For a **single valve** this guide may be a separate casting or cast with the lower valve chest cover as shown in Fig. 88. The bearing surface will be ample if the diameter in the guide is made ¼ to ½ in. more than the diameter of the stem and the length 3 times the diameter. The guide is lined with brass and fitted with a removable cap. **When a crosshead is used,** the guide is cast iron or steel as shown in Fig. 89 with adjustable brass gibs fitted in the crosshead. As one valve is liable to work much stiffer than the other and throw continual pressure on the guides, it is advisable to make the bearing surface almost double that used for a single valve. The upper end of the valve stem is guided by the balance piston or by a bronze bushing in a cast-iron hood.

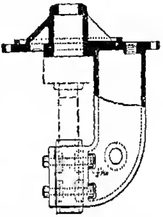


Fig. 88.—Valve-stem guide.

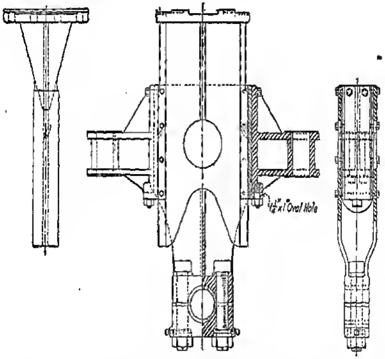


Fig. 89.—Valve-stem crosshead and guide.

Links. The double-bar type (Fig. 90) is generally used. The mean radius is the distance between the center of the eccentric and the center of the link. The links are forged steel machined all over and held together at the ends by bolts through cast-iron distance pieces. The four forged-steel eccen-

centers of the eccentric rod, the suspension pin d on the bell crank and the attachment of the swinging link B to the eccentric rod at a, Fig. 69. On the eccentric circle ABC (Fig. 76) with a radius =

$$\frac{2 \times \text{eccentric} \times \text{length of connecting rod}}{\text{stroke}},$$

arcs are drawn corresponding to each tenth position of the piston. The intersections of these arcs with the eccentric circle marked 1, 2, 3, etc., for the down stroke and 1′, 2′, 3′, etc., for the upstroke, are used in plotting the path of the lower valve rod pin, which is done for full gear ahead, maximum linked up, and astern positions as shown. Points on the path of the valve rod pin are marked to correspond to the piston positions. In Fig. 77, draw line DE representing the stroke of the piston to a convenient scale and divide into tenths. On the left side of line DE plot the positions of the valve for the down stroke of the piston which are found by scaling off the vertical distances of the points on the path of the valve rod pin from the arc FG, the radius of which is the length of the valve rod. On the right side of DE plot the positions for the up stroke. A line through these points will describe an oval figure representing the positions of the valve for the corresponding positions of the piston. Draw the different lap lines for steam and exhaust parallel to DE, the left side representing the down stroke and the right side the up stroke. The intersections of the lap lines and the oval represent the **points of admission, cutoff, exhaust release, and compression.** If the exhaust lap is positive, the lap line is drawn on the opposite side of its steam lap and, if negative, on the same side. In the diagram shown in Fig. 77, the top exhaust lap is zero. A separate diagram must be drawn for each cylinder. The diagram shows that increasing the steam lap decreases the lead and the cutoff, and increasing the exhaust lap makes release later and compression earlier. Owing to the angularity of the connecting rod and the curvature of the path of the point a, Fig. 69, the oval is not a true ellipse but one end is fuller than the other which shows that the top and bottom cutoffs are unequal. To partly overcome this the laps are made unequal. As lead is constant for all grades of expansions ovals drawn for linked-up positions will all pass through the same top and bottom points of lead.

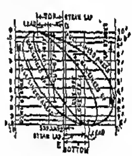


FIG. 77.—Oval-valve diagram for Marshall gear.

Valve Diagram Data. When the valve diagrams have been drawn for each cylinder the data are tabulated on the drawing for easy reference as shown in Table 16. It is also well to plot on the drawing, curves similar to Fig. 78 showing the changes in the valve functions for different link positions so that the cutoff, release, compression, etc., may be easily read for any position of the links. Table 16 and Fig. 78 are from the valve diagram of an actual engine.

Flat Slide Valves. On cylinders in which the pressure will not cause excessive wear, it is customary to use a slide valve. If the cylinder is large, this valve may be double-ported as shown in Fig. 79 or even triple-ported to reduce the travel and to give quick-closing ports. The faces of the valve and seat are sometimes scraped to a true surface but usually, if these faces are

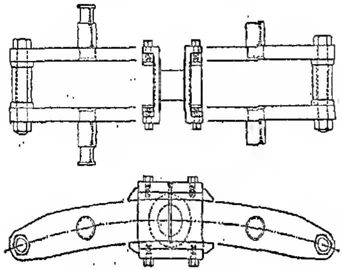


FIG. 90.—Links and link block.

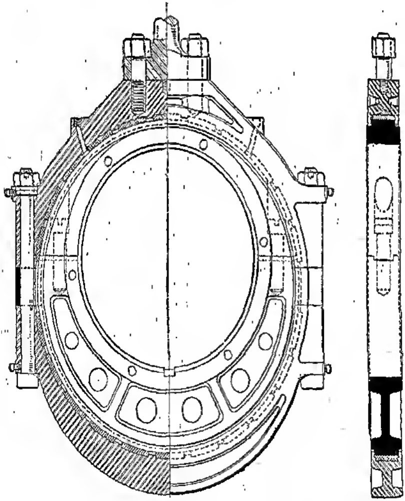


FIG. 91.—Eccentric and eccentric straps.

planed at right angles, they will soon wear to a good surface. To reduce friction, grooves are often cut in the bearing surface of the valve or seat so steam may enter and act as a lubricant. Care must be taken that the valve has sufficient bearing surface in order to keep the pressure per square inch within proper limits. If the ports are very wide, one or more central bars must be used. Even though the pressure per square inch on a slide valve may

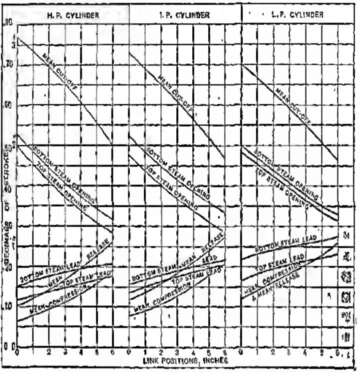


Fig. 78.—Changes in valve functions for different link positions.

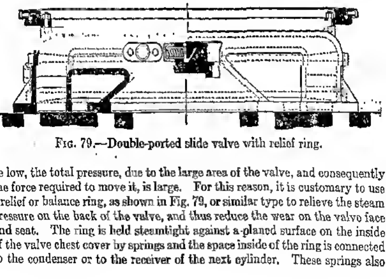


Fig. 79.—Double-ported slide valve with relief ring.

be low, the total pressure, due to the large area of the valve, and consequently the force required to move it, is large. For this reason, it is customary to use a relief or balance ring, as shown in Fig. 79, or similar type to relieve the steam pressure on the back of the valve, and thus reduce the wear on the valve face and seat. The ring is held steamtight against a planed surface on the inside of the valve chest cover by springs and the space inside of the ring is connected to the condenser or to the receiver of the next cylinder. These springs also

tric rod pins, two of which are extended to form the suspension rod pins, are riveted through the link bars. The suspension rods, for merchant engines, are generally attached to the go-ahead eccentric rod pins to reduce the slotting action of the link block in ahead gear. The **depth of the bars** is made about three times the thickness and the **distance between eccentric rod pins** six to seven times the eccentricity. The **eccentric rod pins are proportioned** for a bearing pressure of 600 to 800 psi. The **area of the suspension rod pins** is about three-fourths that of the eccentric rod pins. Each **link bar is proportioned** as a beam, supported at the eccentric rod pins, to carry half the valve load, L, in the middle; 5,000 to 5,000 lb stress may be allowed.

Link Blocks. Figure 90 shows a forged-steel link block of the usual type. The pin for the attachment of the valve stem is proportioned to carry the valve load with a pressure of 600 to 950 psi. The **length** is made equal to the diameter. Brass faces or gibs, proportioned for a pressure of 250 to 350 psi, are fitted between the link block and the links.

Eccentric Rods. The **diameter** of the eccentric rod is determined in the same manner as a connecting rod using hinged ends, the valve load, L, a length equal to the distance between the centers of the eccentric strap and the pins on the links, and a factor of safety of 18 to 22. The **diameter of the lower end** is made about 1.3 times that of the upper end. The rods are forged steel fitted at the upper end with adjustable brass bushings secured by a forged steel or brass cap. The fork of the ahead rod is usually made symmetrical and the astern rod offset (see Fig. 107). The **fork should be proportioned** as explained under connecting rods, allowing a stress of about 4,000 lb. The lower end of the rod is secured to the eccentric strap by two bolts which must be proportioned to carry the valve load. The length of the eccentric rod varies from 8 to 15 times the valve travel.

Eccentrics. The usual type of eccentric (Fig. 91) is of cast iron although the smaller part is sometimes forged or cast steel. The parts are held together by bolts about 1 in. for a 12-in. to 2-in. for a 80-in. eccentric. The keyway, which is always in the large part, is often made wider than the key and side keys fitted so that small changes in the angle of advance may be made. **To secure the eccentric against moving sideways** a setscrew is fitted, or the ahead and astern eccentrics are bolted together. The joint between the parts is stepped to take side strain off the bolts. The **width of the eccentric** is determined from the valve load, L, the projected area being such that the pressure per square inch is 60 to 140 lb.

Eccentric Straps. Figure 91 shows a cast-steel eccentric strap lined with white metal of the type generally used. A lip at each side of the bearing surface keeps the strap in place. The **strap should be sufficiently stiff** not to pull away from the eccentric where bolted to the rod or cause excessive friction by closing in at the sides. The bolts connecting the top and bottom must safely carry the load on the valve gear. As spare eccentric straps are usually carried, the top and bottom parts are often made from the same pattern to reduce the number of spare parts (see Fig. 107). The lower part of the strap is proportioned by considering it as a beam supported at the centers of the bolts and loaded in the middle with the valve load, L, allowing a stress of not over 4,000 lb for cast steel.

Reversing Gear

For small engines reversing is accomplished by a hand lever attached directly to the reverse shaft or by some hand-operated gear, either worm and

Table 16. Valve Diagram Data
(See Fig. 78)

Engine 24½-41½-72 × 48 in. stroke Ihp = 2,200. Rpm = 70 Piston speed = 560
Stephenson link motion—open rods Length of connecting rod 9 ft 0 in.

	H-p		I-p		L-p	
No. and size of valves	One 11″ at top 10½″ at bottom,		One 22 1/16″ at top 22″ at bottom,		One double-ported slide valve,	
Side of which steam is taken	inside		outside		outside	
Angle of advance	34°-5′		37°-20′		40°-40′	
Eccentricity and valve travel	3½-7		3½-7		3½-7	
	Top	Bottom	Top	Bottom	Top	Bottom
Data for full gear						
Steam lap, in.	1 1/8	1 3/8	1 9/16	1 1/2	1 19/32	1 53/64
Exhaust lap, in.	−1/2	0	−1/2	+1/16	−5/16	+5/16
Steam lead, in.	15/32	19/32	9/16	5/8	11/16	7/8
Cutoff, in.	38 1/8	35 1/8	36 3/4	32 13/16	35 5/16	32 3/16
Cutoff in dec. of stroke	0.794	0.731	0.7656	0.683	0.735	0.670
Mean cutoff in dec. of stroke	0.7622		0.7243		0.7025	
Exhaust release, in.	5 1/16	5	5 7/8	5 9/16	5 7/8	5 3/8
Exhaust release in dec. of stroke	0.1055	0.1041	0.1224	0.1160	0.1225	0.112
Compression, in.	2 7/8	3 5/16	3 5/8	4 3/16	5 3/8	5 7/8
Compression in dec. of stroke	0.0599	0.069	0.0755	0.0872	0.1120	0.1225
Width of port in liner, in.	3	3	3 1/4	3 1/4	2 × 2 1/2	2 × 2 1/2
Steam opening in liner, in.	2	2 1/8	1 15/16	2	2 × 1 29/32	2 × 2 3/32
Exhaust opening in liner	Full port		Full port		Full port	
Area of steam opening in liner, sq in.	53.8	54.0	103.0	106.0	250.0	274.0
Area of exhaust opening in liner, sq in.	80.62	76.12	172.25	172.25	327.5	327.5
Area of port in cylinder, sq in.	58.1	58.1	140.0	140.0	327.5	327.5
Velocity through steam opening in liner	4900	4620	7360	7010	9120	8260
Velocity through exhaust opening in liner	3275	3275	4400	4310	6960	6910
Velocity through port in cylinder	4540	4295	5410	5300	6960	6910
Data for 6-in. linked-up position						
Cutoff in dec. of stroke	0.555	0.463	0.522	0.423	0.5025	0.430
Mean cutoff in dec. of stroke	0.509		0.4725		0.466	
Steam opening, in.	15/32	19/32	1 3/16	1 1/8	1 1/4	1 7/16
Exhaust opening, in.	Full port	2 21/32	Full port	2 1/16	Full port	Full port

wheel, or screw. For large engines, steam reversing gear is fitted. The "all-round" type is almost universally used in European practice, and the "direct-acting" type in American practice.

The all-round type (Fig. 92) consists of a small one- or two-cylinder engine driving a worm and worm wheel. The worm wheel is connected

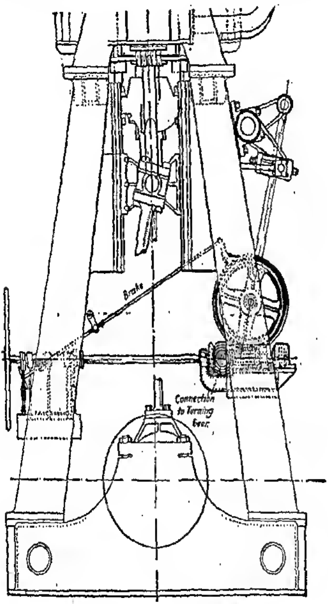


FIG. 92.—All-round reversing gear.

to an arm on the reverse shaft by a connecting rod and a crankshaft or a connecting rod and a pin in the rim of the wheel. The engine is usually fitted with a flywheel of large diameter with a light smooth rim readily accessible for use as a handwheel for starting the reversing engine when on dead center, or in making slight movements of the valves. The flywheel also forms an emergency means of starting the valve gear in case it should

serve to reseat the valve in case it is lifted off its seat by water in the cylinder or any other cause.

Slide-valve False Face. A hard close-grained cast-iron false face or liner is generally fitted to the cylinder as a working surface for a slide valve, as shown in Fig. 80. This liner varies from about 1 to 2 in. thick depending on the size of the engine and is firmly secured to the cylinder by a number of bronze screws set about ⅛ in. below the surface to allow for wear.

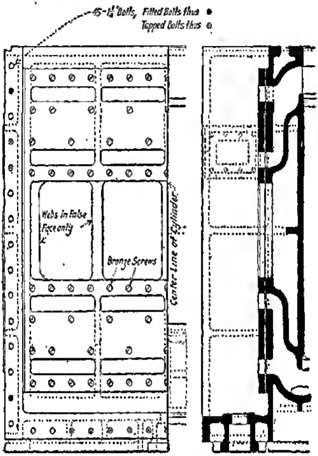


Fig. 80.—Slide-valve false face.

Piston Valves. It is customary to use a piston valve on the high-pressure cylinder as the wear of a slide valve is excessive under high pressure. The volumetric clearance is more than with a slide valve, owing to the steam passages extending around the valve but the use of straight ports reduces this difference. A piston valve, if properly fitted, works with little friction but is not as steamtight as a slide valve and, as it cannot lift from its seat if dirt or an obstruction enters the port, it is liable to stick and throw severe strains on the valve gear. For the same reason water in the cylinder is unable to escape. Piston valves take **inside steam** when the steam enters the cylinder past the inner edges of the valve rings and **outside steam** when the steam

stick. The all-round type of gear is also useful in warming-up as the reversing engine may be kept running and continuously reversing the valves. It is less liable to be damaged by careless handling, and the engine may also be arranged to operate the turning gear. The valve gear for the reversing engine is sometimes fitted with a "follow-up" gear which stops the reversing engine when the gear of the main engine has reached the desired position. An indicator is usually fitted so that the valve position may be readily seen. Sometimes a brake is fitted so that the engine may be stopped quickly and held at any point.

The "direct-acting" type (Figs. 93 and 106) consists of a cylinder with a connecting rod directly connected to the reverse shaft arm, one stroke of the piston moving the reverse shaft from ahead to astern position. The valve gear of this engine is operated by a hand lever fitted with a quadrant for securing it. A follow-up gear stops the reversing engine when the gear of the main engine has reached the desired position. The piston should be provided with stops at each end of the stroke consisting of springs or of bosses on the piston which strike the cylinder head. The valve of the reversing engine is usually of the *D* slide type. A V-notch is often cut in the steam lap at each end of the valve (Fig. 93) which makes the steam balance maintained by the follow-up gear more sensitive and the movement of the piston more easily controlled by slight movements of the valve. To provide a cushioning effect, the reversing gear of a large installation sometimes has an oil or water cylinder in tandem with the steam cylinder with a valve also in tandem, so that any movement of the steam valve and piston causes a corresponding movement of the hydraulic valve and piston allowing the fluid to flow from one end of the hydraulic cylinder to the other. The fluid may be throttled, so as to regulate the speed of the movement. Sometimes the hydraulic cylinder is also fitted with an auxiliary hand pump which may be used as a hand reversing gear or as an emergency gear in case the main engine valve gear sticks.

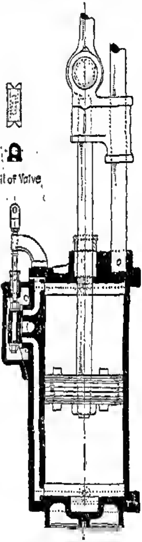


FIG. 93.—Direct-acting reversing engine.

The size of reversing engine is determined more or less empirically but, with constants taken from practice, the following formula, adapted from Bauer, gives good results:

$$d_s^2 = \frac{1 \times t}{p \times n} \times c$$

enters past the outer edges. The general type of piston valve is shown in Fig. 81. Large piston valves are often made with cast-steel ends riveted to a central section of steel tubing, as shown in Fig. 82. The rings are sometimes held against the valve liner by springs but, on account of excessive wear, it is better to use solid rings or, to provide for adjustment, to have the rings cut and the ends fastened together by screws or bolts as shown in Fig. 82. The diameter of the rings may then be increased by the use of liners. On intermediate-pressure cylinders, either piston or slide valves are used, according to the idea of the designer. To avoid excessive distances from the center of the cylinder, two piston valves may be used instead of one as discussed on p. 1153.

Piston Valve Liners. A hard close-grained cast-iron liner is usually fitted in the valve chest as a working surface for a piston valve as shown in Fig. 81. The thickness varies from about ⅞ to 1¼ in., depending on the size of the engine. The bridges in the steam port (see Fig. 81) are arranged diagonally to ensure equal wear on the valve rings and are usually about 25 per cent of the circumference of the valve. The openings in the valve liners are accurately machined. The ends of the liners are counterbored to allow the valve rings to overrun a short distance at each end.

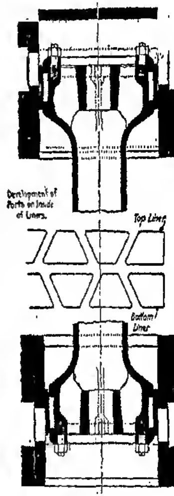


FIG. 81.—Cast-iron piston valve.

Expansion Valves. With the ordinary types of valve gear and the usual valve laps a variation of cutoff of 15 to 25 per cent may be obtained. If a greater range or an earlier cutoff is desired, a separate expansion or cutoff valve, consisting of an auxiliary valve, working on top of or within the main valve and driven by a separate eccentric, is fitted. This valve is arranged so that the cutoff desired may be obtained by changing its position by means of a screw. To obtain a wide range the main valve is usually designed for a very late cutoff so the auxiliary valve can effect only an earlier cutoff. The other functions of the main valve are not changed. Although seldom fitted at the present time, there is a large number of engines with these valves still in service.

Trick Valves. To obtain a quicker and fuller opening of the steam port, trick valves are sometimes used. Figure 83 shows a trick slide valve and Fig. 84 a trick piston valve. Passages are cored through the valve and so arranged that their entrances just overlap the end of the valve seat or liner when the valve is at the end of its stroke. As the valve uncovers the steam port at one end, the cored passages admit steam to the same port from the other end of the valve. The width of the trick ports is such that they are completely open to steam at the same time as the steam port. Exhaust takes place only through the inner passage of the main valve in the same manner as valves of the ordinary type. The valve diagram for a trick valve is drawn in the same manner as for an ordinary valve keeping in mind that the steam opening is the sum of the openings of the main and the trick valves.

where d and s = diam. and stroke of reversing engine
l = maximum load on one main engine valve (see Valve Load)
t = half travel of the valves
p = boiler press. (gage)
n = revolutions of reversing engine required to reverse main engine (= 1 for direct-acting type)
c = constant

All dimensions are in inches. If a double-cylinder all-round engine is used, d^2 is the sum of the squares of the two cylinders. For three-cylinder triple-expansion engines with cranks at 120 deg, Bauer's values of c, modified to agree with the above formula, are 7 for direct-acting, and 17 for all-round type; and for four-cylinder triple- and quadruple-expansion engines with cranks at 90 deg, 10 for direct-acting, and 23 for all-round type. However, reversing engines are often made of such size that the constants are from 1½ to 2 times these values.

For the direct-acting type the diameter of the reversing cylinder may be taken roughly as 0.15 to 0.23 times the diameter of the low-pressure cylinder for all types of engines and the stroke 2 to 3 times the valve travel. For the all-round type it is usual to make the stroke about equal to the cylinder diameter.

Reversing Gear Load. The various parts of the reversing gear are proportioned from the **reversing-engine load, $P = Ap$,** where A = area of reversing-engine cylinder and p = steam pressure, which for purpose of design is taken as boiler press. As this maximum load comes on the gear only in case the valves stick, stresses figured by this method may run much higher than ordinary working stresses. For simplicity and in view of the margin of safety in using boiler press, the angularity of the reversing-engine connecting rod and the suspension rods is neglected.

Reversing-engine Piston Rod, Connecting Rod, and Crosshead. The piston rod is forged steel tapered at each end and secured to the piston and the crosshead by nuts. The **crosshead** (Fig. 106) is of cast steel or composition and carries the forged-steel connecting rod pin and the crosshead guide shoe which is composition bushed if the crosshead is of steel. The **connecting rod** (Fig. 106) is a steel forging with a solid brass bushing at the crosshead end and a split brass bushing with steel cap at the other end. The piston rod and connecting rod are proportioned by the method given under main engine piston and connecting rods, using a factor of safety of 8 to 10. The direct tension in the piston rod at the base of the threads may run as high as 10,000 lb. The stress in the crosshead pin is figured in the same manner as a main crosshead pin of the same type, a stress of about 5,000 lb being allowable. The bearing pressure on the crosshead pin may be as high as 3,500 psi. The **length of the connecting rod** is usually about 1½ to 2 times the stroke of the reversing engine.

Reverse Shaft. The reverse shaft (Fig. 106) is forged steel and supported by bearings bolted to the engine housings or to the cylinders. The stress in the shaft is figured from the equivalent twisting moment. Usually at one or both ends of the shaft a reversing arm is located outside of the bearing, so that it is overhung. The **reversing-engine arm** should be located near the middle of the shaft to keep torsional stresses low, and near a bearing to reduce bending. The equivalent twisting moment is figured at each reversing arm to determine the maximum value between bearings and overhung. The stresses are then found by dividing the largest equivalent twisting moment

Balance Cylinders and Pistons. To relieve the eccentrics of a large portion of their load, a balance cylinder with its piston attached to an extension of the valve stem is often fitted in the upper valve chest cover of heavy piston or flat slide valves as shown in Figs. 106 and 107. The underside of the piston is in communication with the valve chest or receiver and the upper side connected to the condenser or a steam chest of lower pressure.

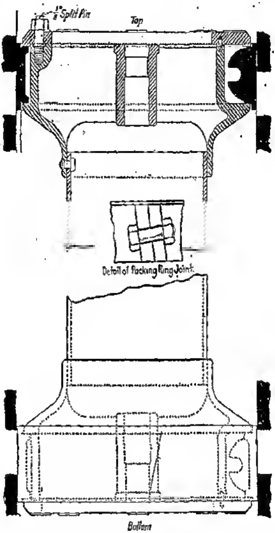


Fig. 82.—Steel-piston valve.

The area of the piston is made such as to just support the weight of the valve and gear so that the valve load is made up entirely of friction. In case of the low-pressure valve, where the low steam pressure in the chest would necessitate a very large balance piston, the underside of the piston is sometimes connected to the intermediate-pressure instead of to the low-pressure chest or receiver. The pistons are solid cast iron packed with several small cast-iron rings. The balance piston acts as a guide for the upper end of the valve stem.

for each condition by the resistance. If T = twisting moment and B = bending moment, the equivalent twisting moment $= B + B\sqrt{1 + (T \div B)^2}$. The resistance to twisting $= D^3/5.1$ where D is the shaft diameter. The maximum twisting moment occurs at the reversing-engine arm, and $= P \times L$, where L = length of the arm. Usually the maximum equivalent twisting moment between bearings will be at the reversing-engine arm. If there is also a reversing arm between the same bearings, the bending moment used is the resultant of the components of the bending moments. For reversing arms at the ends of the shaft, the twisting moment is the product of the load on the reversing arm times the length of the arm. For reversing arms between the ends and the reversing-engine arm, the twisting moment will be the sum of the twisting moments of all the arms between the end of the shaft

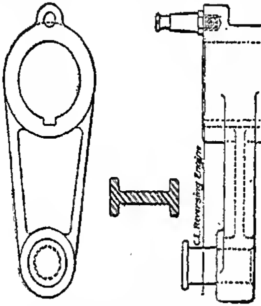


Fig. 94.—Reversing-engine arm.

and that point. Stresses in reverse shafts figured by this method may safely run from 8,000 to 10,000 lb.

The **reversing-engine arm** (Fig. 94), usually of cast steel of I section, is figured for bending stress near the hub due to the reversing-engine load P. If b = distance from center of the pin to the section taken and R = resistance of the section, stress $= \frac{P \times b}{R}$. This stress may safely be 10,000 lb. The pin in the reversing-engine arm is proportioned for a bearing pressure of about 3,000 lb due to the load P, and also for bending stress. If l = length of pin and d = diameter in the arm, stress $= \frac{P \times l \times 10.2}{2 \times d^3}$. The allowable stress may run as high as 18,000 lb. Diameter of reversing-engine arm boss = 1.6 times diameter of the shaft and the length equal to shaft diameter. The lengths of the reversing-engine arm and the reversing arms are proportioned approximately so that

To Multiply 4 by 6. In Fig. 1, starting with point 1 of the fixed scale, run the eye along from 1 to 4; then set the 1 of the slide opposite this point 4, and *run the eye forward along the slide from* 1 *to* 6; the point thus reached on the fixed scale is 24, which is equal to 4 × 6. This process gives the distance from 1 to 4 *plus* the distance from 1 to 6, and is, in fact, a mechanical method of adding the logarithms of these numbers; hence the result is the product of the numbers. Conversely,

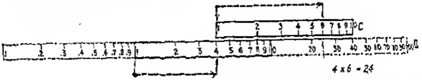


Fig. 1.

To Divide 4 by 6. In Fig. 2, starting with the point 1 of the fixed scale, run the eye along from 1 to 4; then set the 6 of the slide opposite the point 4, and *run the eye backward along the slide from* 6 *to* 1; the point thus reached on the fixed scale is 0.667, which is equal to 4 ÷ 6. This process gives the distance from 1 to 4 *minus* the distance from 1 to 6; and is, in fact, a mechanical method of subtracting the logarithms of these numbers; hence the result is their quotient.

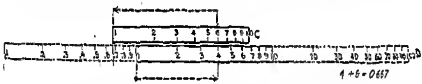


Fig. 2.

Multiplication and Division, Using Only a Single Section of the Scale. If only the main section of scale *D* is available (as is usually the case in practice), the result of multiplication may fall beyond the scale, as it does in Fig. 1. In such cases *divide the first factor by* 10 *before beginning to multiply;* this will bring the result within the scale, without affecting the sequence of digits.

For example, to multiply 4 by 6. Having found that the setting shown in Fig. 1 is not successful, *reset the slide as in Fig.* 3, *with* 10 *instead of* 1 *opposite* 4; run the eye backward along the slide from 10 to 1, thus reaching the (unrecorded) point corresponding to 4 ÷ 10; then, continuing from this point, run the eye forward along the slide from 1 to 6, as before; the point finally reached on the main scale is 2.4, which has the same sequence of digits as the required value 24. After a little practice, this preliminary step of dividing by 10 will be performed almost intuitively. Whether or not this step is necessary in any given case can be determined only by trial.

The **general rule for multiplication** may be stated as follows, if preferred: To find the product of two factors, find one factor on the fixed scale; opposite this, set (tentatively) point 1 of the slide; on the slide find the second factor, and opposite this read the product on the main scale, if possible. If the product falls beyond the scale, begin over again, using point 10 of the slide instead of point 1.

In division also, the result may fall beyond the main section of the scale, as it does in Fig. 2. In such cases, it suffices merely to *multiply the result by* 10 in order to bring it within the scale; this will not affect the sequence of digits.

For example, to divide 4 by 6, set the slide as in Fig. 4, and follow out mentally the steps indicated by the arrows. It will be noticed that the supplementary step of multiplying by 10 is performed by simply running the eye along the slide from 1 to 10 without resetting the slide; for this reason, division on the slide rule is slightly easier than multiplication.

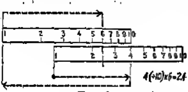


FIG. 3.

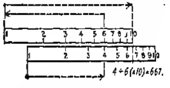


FIG. 4.

The Ordinary Mannheim Slide Rule has four scales, A, B, C, D, as shown in Fig. 5. Scales C and D are essentially the same as the C and D scales described above, and the principle just explained shows how they are used in multiplication and division. The fact that the D scale covers only the main section from 1 to 10 (all decimal points being omitted) is practically no restriction on the scope of the scale, as is seen in the preceding examples. A runner is provided, so that intermediate positions reached in the course of an extended computation may be indicated temporarily on the scale without the necessity of reading off their numerical values. The best runners are those which have no side frame to obscure the numerals.

FIG. 5.

In problems involving successive multiplications and divisions, arrange the work so that multiplication and division are performed alternately.

For example, to calculate $\dfrac{a \times b \times c}{d \times e}$, divide the product $a \times b$ by d; multiply this quotient by c; and divide this product by e. Each operation will require only one shifting either of the slide (for multiplication) or of the runner (for division).

To multiply a number of different quantities by a *constant multiplier* x, set the point 1 of slide opposite x, and read, by aid of the runner, the products of x by all the quantities which do not fall beyond the scale; then reset the slide, setting 10 instead of 1 opposite x, and read the products of x by all the remaining quantities.

To divide a number of different quantities by a *constant divisor* y, first find (by the slide rule) the quotient $1 \div y$, and then use this as a constant multiplier.

Scales A and B are exactly like scales C and D, except that they cover two sections of the complete logarithmic scale, the graduations being only half as fine. Either pair of scales may be used for multiplication and division; C and D give more accurate readings, but have the disadvantage that in the case of multiplication the slide must often be shifted to the other end in order to keep the result on the scale—an inconvenience which is not present when the less accurate scales A and B are employed.

By the use of both pairs of scales, problems in squares and square roots may be readily solved; for every number on A, except for the decimal point, is the square of the number directly below it on D (use the runner).

$$\frac{\text{Distance bet. ecc. rod pins on links}}{\text{Length of reversing arm}} = \frac{\text{length of reversing arm}}{\text{length of reversing-engine arm}}$$

The follow-up gear of the reversing engine is often attached to a pin in the boss of the reversing engine arm, as shown in Fig. 94.

The **reversing arms** (Fig. 95) are usually of cast steel of I section and should be placed near a bearing to reduce bending moments in the shaft. Each reversing arm is fitted with a screw working in a crosshead having a pin at each end, to which the suspension rods are attached so that the linkage of any cylinder may be varied by adjusting the screw. **The slot in the reversing arm** should be approximately parallel with the suspension rods in ahead gear and perpendicular to them in astern gear so that linking up has no effect when going astern. The reversing arms are also figured for bending stress near the hub. Let Q = the load on the arm, b_1 = distance from center of crosshead to the section taken, and R_1 = resistance of the section. As the reversing arms for all cylinders are usually made the same, R_1 and b_1 will be

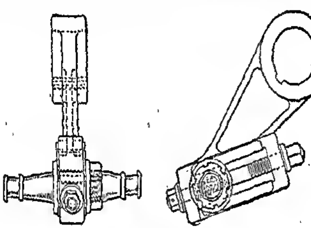


Fig. 95.—Reversing arm.

the same for all cylinders, but Q will vary in proportion to the valve loads. If L = length of reversing-engine arm from center of pin to center of shaft, L_1 = corresponding length of the reversing arm, then the **total load on all reversing arms**, $P_1 = P \times \frac{L}{L_1}$. The load P_1 is divided among the reversing arms in proportion to the valve loads, *i.e.*, the load Q on any reversing arm is equal to $P_1 \times$ the ratio of the valve load for that cylinder to the sum of the valve loads. Stresses may safely run as high as 10,000 lb. The diameter of the reversing arm boss = 1.6 times and the length 0.8 times diameter of shaft. The adjusting screw is figured for direct tension at the bottom of the threads, the load being Q and the allowable stress about 5,000 lb.

The **reversing-arm crosshead** (Fig. 95) is figured for bending stress in a section through the middle, through which the adjusting screw passes. If L' = distance between centers of suspension rod pins, B = width and D = depth of middle section, and d = diameter of hole for the adjusting screw, stress $= \frac{Q \times L \times 6}{4 \times D^2(B - d)}$ which may run to 6,000 lb. The crosshead is also figured as a cantilever at the collar, where the tapered portion joins the central

has returned to normal, the pivoted weight is returned to its original position by the springs, and the detent, D, alters the position of the pawls, PP, so that the upper pawl picks up the lever, H, and opens the throttle. Should the speed continue excessive the repeated movement of the weight causes the first pawl to remain set, and the throttle remains closed until the speed has become normal. This governor is also fitted with an emergency weight, A, which comes into use in the case of very excessive speed such as would result from losing a propeller or breaking a shaft. The action of the weight, A, causes the weight, W, to be locked which results in the throttle remaining shut until the engine stops and the throttle can be unlocked. This governor may be arranged to operate the reversing gear instead of the throttle.

Handling Gear. All gear necessary for the control and maneuvering of the engine should be located at one point and arranged for quick and easy handling under emergency conditions. The levers for operating the throttle, butterfly valve, and reversing engine are usually located together on a bracket

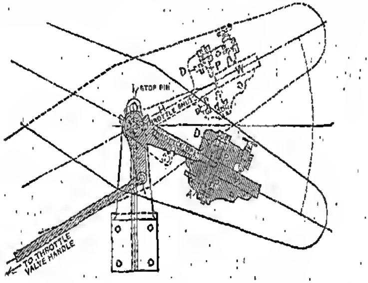


FIG. 105.—Aspinall governor.

fitted to the side of the forward or second housing at a convenient height from the engine-room floor. The receiver-steam and cylinder-drain controls are located on the same side of the engine as the throttle control but not necessarily at the same point. The butterfly valve, used for quick control of the engine when racing due to pitching of the vessel in a heavy sea, is also connected to the governor, if fitted, for automatic control under such conditions. The various water-service and lubricating-oil cocks are placed where most convenient.

Working Platform and Gratings. The engine controls are handled from the engine-room floor, but gratings should be placed wherever necessary to provide access for oiling, indicating, and making repairs on the main engine as well as for access to other equipment about the engine room. The usual arrangement consists of a grating all around the engine below the bottom of the cylinders with cross gratings between the housings for access to the crossheads and stuffing boxes. Another grating is fitted all around the engine near the top of the cylinders for use when indicating or making repairs on cylinders or pistons.

block. This section is usually circular and, if l = distance from center of pin to section taken and d = diameter of section, stress $= \frac{Q \times l \times 10.2}{2 \times d^3}$. This stress is usually higher than the stress in the central block, and runs about 8,000 lb. The pins are proportioned for a bearing pressure of about 1,000 lb.

The **suspension rod** (Fig. 96) is figured as a column with hinged ends, loaded with half the reversing-arm load. The lowest factor of safety is usually in the low-pressure rods and runs about 30 to 40. The caps and bolts are figured in the usual manner.

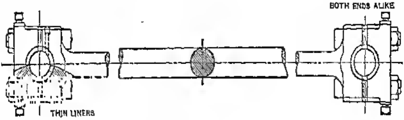


FIG. 96.—Suspension rod.

Turning Gear

Provision is always made for turning the engine for the purpose of setting the valves, making repairs, or for keeping the bearings and working parts in condition when in port by turning the engine several revolutions when considered necessary. A simple form of turning gear used on **small engines** consists of recesses in the coupling, or a notched wheel attached to the shaft to which a bar or jack may be applied. For **larger engines,** a worm wheel is fitted on the shaft or coupling and operated by a worm which may be disengaged by having the worm shaft pivoted or the worm working on a feather on the shaft thus allowing it to be moved axially out of gear when the turning gear is not in use. This worm is usually actuated by a small single- or double-cylinder engine although a ratchet lever is often used on engines of small size. If hand operation is used, the ratio of revolutions of the worm to the revolutions of the engine is about 50:1. If an engine is used, a second worm and wheel are introduced, making the ratio of gearing such that the revolutions of the turning engine required for one revolution of the main engine is from 1,200:2,000 or even more for very large engines. The turning engine operates at 200 to 400 rpm and it required 5 to 8 min to turn the main engine one revolution. The engine should be of sufficient size to turn over the main engine in port with steam pressure of 80 lb or less. A flywheel is usually fitted and the engine has a change valve, links, or shifting eccentric for reversing so the main engine may be turned in either direction. Holes in the rim of the flywheel for inserting a bar provide means for turning the engine by hand. The engine is generally located on the engine housing or bedplate, a typical arrangement being shown in Fig. 97. If an all-round reversing engine is used, it is generally also arranged for turning through a belt, chain, or shafting and a separate engine is not required.

The following formula, adapted from Bragg, may be used for determining the size of **turning engine**: $d^2s = \frac{D^2 \times S}{R} \times C$ where d and s = diameter and stroke of turning engine D and S = diameter of low-pressure cylinder

Lubrication. Merchant engines are fitted with gravity lubrication to all bearings, crosshead guides, and thrust and line-shaft bearings. Oil cups or boxes may be fitted directly on the part to be lubricated, but it is usual to provide a box or reservoir, of a capacity to last several hours, on the side of each cylinder from which pipes are led to the cups or boxes on the engine. These boxes or reservoirs have small tubes fitted with wicks leading up through the oil, which by capillary attraction and siphon effect feed the oil through a sight feed to the point to be lubricated. The size and number of wicks control the flow. Receptacles mounted on rapidly moving parts into which oil is dropped from the distributing system should be packed with curled hair held in place by a perforated cover or wires to prevent the oil being thrown out and to keep dirt from plugging the pipes. An oil box is mounted on each main bearing cap with one or more tubes feeding the bearing. A cup is fitted on each eccentric rod with a pipe leading down to the eccentric strap. For all valve gear, air pump, and reversing- and turning-engine parts, where the movement is intermittent, an oil cup of commercial make is usually fitted. The crosshead guides have oil piped to both the forward and after sides and lead by suitable drillings to the face of the guide, on which are oil-retaining grooves. At each stroke a wiper on the crosshead dips into a box fitted at the bottom of the guide and distributes on the face of the guide the oil that runs down into the cup. Oil boxes on top of the crosshead pin caps, lubricate the crosshead pins through drillings in the caps. The same boxes lubricate the crankpin through one or two pipes lead down the connecting rod. The thrust bearing is lubricated by a cup with capillary feeders on each shoe with oil channels lead to the face of each collar. The base of the thrust bearing is usually filled with oil in which the lower part of the collars is immersed. The line-shaft bearings are lubricated either by oil cups with capillary feeders mounted on the caps or by a block of grease placed in an opening in the cap which lubricates by rubbing on the shaft. The piston rods and valve stems are lubricated by oil swabs while the engine is running, the oil thus carried into the cylinder usually being all that is required for internal lubrication with saturated steam. With superheated steam, oil must be forced into the cylinder by a lubricator. The oil is distributed around the bearings by grooves cut in the white metal so arranged that oil will not leak out the ends of the bearings. The grooves are about 1/4 to 1/2 in. wide and 1/8 to 1/4 in. deep.

Water Service. Connections for circulating sea water are led to the parts of the main engine, thrust, line-shaft, and stern-tube bearings which are liable to heat. These connections may circulate water in cored passages in the part to be cooled, thereby providing indirect cooling, or the water may be allowed to flow on the part to be cooled thus providing direct cooling. The main bearings and crosshead guides are generally fitted with indirect service. Direct service through 1/2- to 3/4-in. pipes, taken off a main supply pipe, is supplied for almost every bearing, in addition to hose connections for emergency use in case of excessive local heating. These pipes are on adjustable fittings to direct the flow of water within certain limits and are fitted with valves so that they may be operated individually. The crankpins are often served by a transverse perforated pipe on each side distributing water thereby from above during a complete revolution. The thrust shoes are cored for indirect water circulation, and sometimes cooling coils are fitted in the oil chamber in the base. Direct water service is also provided by branch pipes similar to those on the main engine and so arranged that water

and stroke of main engine R = ratio of gearing C = constant. All dimensions are in inches. If a double-cylinder turning engine is used, d^2 is the sum of the squares of the two cylinders; if there are two low-pressure cylinders, D^2 is the sum of the squares of the two. For single-cylinder turning engines a safe value of the constant C is 1.75 for three-cylinder main engines and 2 for four-cylinder main engines. The constants may be reduced somewhat for

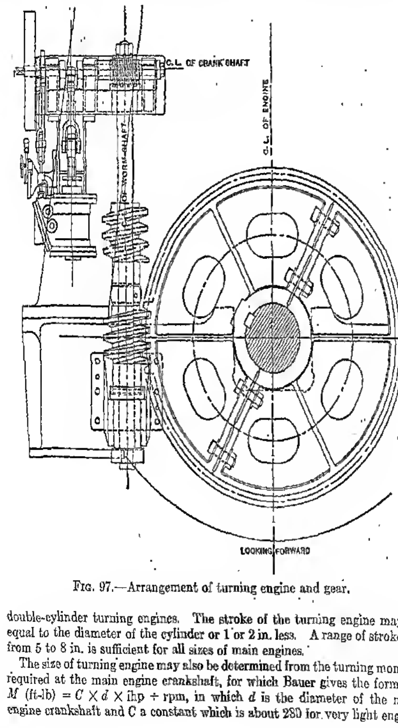


FIG. 97.—Arrangement of turning engine and gear.

double-cylinder turning engines. The stroke of the turning engine may be equal to the diameter of the cylinder or 1 or 2 in. less. A range of strokes of from 5 to 8 in. is sufficient for all sizes of main engines.

The size of turning engine may also be determined from the turning moment required at the main engine crankshaft, for which Bauer gives the formula: M (ft-lb) $= C \times d \times$ ihp $\div$ rpm, in which d is the diameter of the main engine crankshaft and C a constant which is about 280 for very light engines

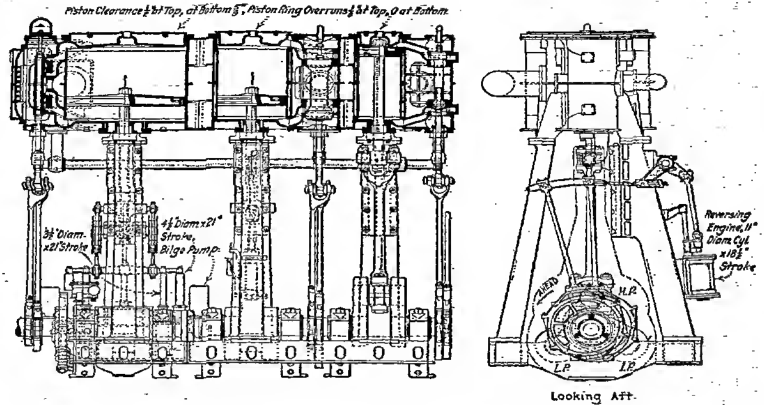


FIG. 106.—Triple-expansion engine designed and built by the Newport News Shipbuilding and Dry Dock Company.

and 560 to 670 for heavy engines. If this formula is used, the turning engine should be made large enough to give at least 1.5 times the twisting moment required, with 30 to 50 lb mep, taking into account the gear ratio and the efficiency of the gearing which will be from 13 to 15 per cent.

For merchant engines the main and auxiliary worm wheels have cast teeth and are generally of cast iron. The main and auxiliary worms are single thread and generally of cast iron although sometimes forged steel or hard bronze is used. For very large engines the auxiliary wheel may be bronze with cut teeth. The main wheel is usually split which requires that an even number of teeth be used. The pitch is made in even quarter or half inches. The following proportions of the worms and wheels may be used:

Per cent of total gear ratio in main and auxiliary gears: 75 and 25.

Pitch diam of main wheel: 4 $\times$ diam main engine crankshaft.

Pitch of teeth in main wheel: 2 in. for a 40-in. to $3\frac{1}{2}$ in. for an 80-in. wheel.

Pitch of teeth in auxiliary wheel: $1\frac{1}{2}$ to $2\frac{1}{4}$ in. for all sizes.

Pitch diam of worms: 3 $\times$ pitch.

Length of worms over teeth: 3.5 to 4.5 $\times$ pitch.

Height of teeth in wheels: 0.7 $\times$ pitch.

Height of teeth above pitch circle: 0.3 $\times$ pitch.

Thickness of teeth at pitch circle: 0.46 $\times$ pitch.

Width of face of wheels: 2 to 2.5 $\times$ pitch.

The strength of the wheel teeth, shafts, and other parts should be figured by the usual methods. The loads on the teeth may be found from the power of the turning engine, assuming a mep of 30 to 50 lb, and taking the efficiency of each worm 40 per cent, each set of thrust collars 95 per cent, and each set of bearings 95 per cent. If n_1 and n_2 = no. of teeth in the auxiliary and main worm wheels, r_1 and r_2 = radius of the auxiliary and main worm wheels, and C = 2.0 for single- and 1.5 for double-cylinder turning engines: maximum twisting moment on turning-engine crankshaft, $T = \dfrac{63{,}000 \times \text{ihp} \times C}{\text{rpm}}$.

Twisting moment on auxiliary shaft, $T_1 = T \times n_1 \times 0.40 \times 0.95 \times 0.95 = 0.36T \times n_1$.

Force on auxiliary worm wheel teeth, $F_1 = T_1 \div r_1$.

Twisting moment on main engine shaft, $T_2 = T_1 \times n_2 \times 0.40 \times 0.95 \times 0.95 = 0.36T_1 \times n_2$.

Force on main worm wheel teeth, $F_2 = T_2 \div r_2$.

Attached Pumps

On merchant ships, with engines of about 5,000 ihp and under, it is customary to drive the main air and bilge pumps through links and beams from one of the main engine crossheads, usually the low-pressure. Frequently the feed and sanitary pumps are also driven in this manner. The pumps are vertical, single-acting, and the stroke is about 45 per cent of the stroke of the main engine. The arrangement of attached pumps and gear is shown in Fig. 107.

Main Air Pumps. The bucket type of (attached) air pump in which there are foot, bucket, and head valves is shown in Fig. 98. The bucket, which is fitted with soft packing, has a depth of face of $\frac{1}{4}$ to $\frac{1}{5}$ of the diam. The pump is located so that the condensate flows from the condenser by gravity into the bottom of the pump. The down stroke of the bucket closes the foot valves and forces the trapped air and water through the bucket

may be played on each collar. The bases of the line-shaft bearings are usually piped for water circulation, while the cap may be fitted with direct service from above. The supply of water is taken from the discharge side of the main circulating or other pump, and in the case of indirect service, which is usually in use when the main engine is running, this water is returned either to some pump suction or to the bilge. The circulation for the guides is often taken from the sanitary pump in which case the water is returned to the

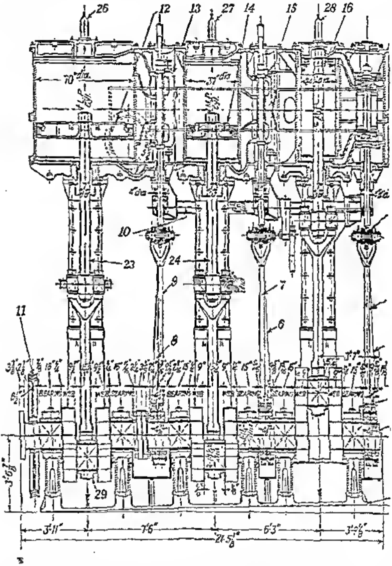


FIG. 106a.—Triple-expansion slide-valve engine used

sanitary system. The water used in direct service drains to the bilge. The water-service system for the main engine is sometimes kept separate from the system supplying the thrust bearing and line-shaft bearings on large installations, but usually one system suffices.

valves. On the up stroke the bucket valves are closed and the air and water pass through the head valves into the pump discharge. The up stroke forms a partial vacuum below the bucket which causes air and water to flow through the foot valves. The **Edwards type of (attached)** air pump in which there are only head valves (Fig. 99) is now generally used. The absence of foot and bucket valves reduces the liability to breakdown and increases the efficiency. The face of the bucket is packed by water grooves as shown and is about one-fourth of the diameter in depth. The condensate flows by gravity into the conical base of the pump and the bucket, which is also conical, on its down stroke *forces* the water through the ports to the upper side of the bucket. On the up stroke the bucket closes the ports and the trapped air and water are forced through the head valves into the pump discharge. The clearances at top and bottom of the stroke are made much smaller than for the bucket type, ¼ to ¾ in. at the top and ¾ in. at the bottom being sufficient. The angle of the bottom and the bucket is about 130 deg and that of the direction of discharge through the ports about 35 deg from the horizontal. The ports occupy about 75 per cent of the circumference of the liner and their area is 35 to 50 per cent of the area of the bucket. In this type of pump a relief valve, one-sixth to one-eighth the diameter of the pump, is always fitted at the bottom (Fig. 99) to avoid damage in starting up if an excessive amount of water has accumulated. With surface condensers the **rate of displacement of the low-pressure piston** on a double stroke **to the displacement of the air pump bucket** on a single stroke is about 28 for the bucket type and about 35 for the Edwards type. For either type the **ratio of the area through the** valves to the area of the bucket is about 0.27 and the **ratio of the area of the suction and discharge openings to the area of the bucket** is about 0.18. The air pump barrel is usually of cast iron fitted with a removable brass liner. In large sizes a manhole, which forms part of the cylinder wall, is fitted for access to the

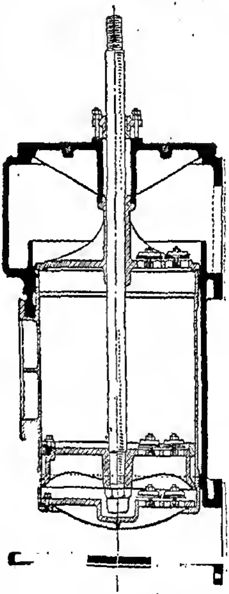
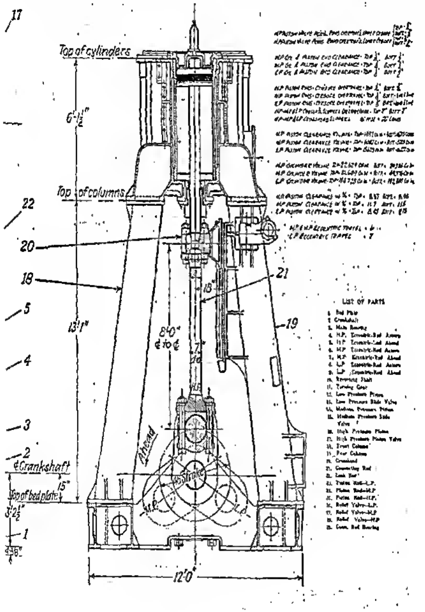


FIG. 98.—Bucket type of attached air-pump.

Gages and Gage Board. The engine-room gage board is usually located on the forward engine-room bulkhead near the engine control gear and within easy sight of the engineer handling the throttle. A clock and the following gages are fitted on the gage board: main steam or boiler steam; high-pressure chest; intermediate-pressure receiver; low-pressure receiver; vacuum.

Tools and Wrenches. The engine-room tool equipment should include a full set of mechanics' handtools such as vise, wrenches, hammers, chisels,



in Liberty ships. (*Courtesy of Nautical Gazette.*)

files, taps, dies, screwjack, chain falls, soldering copper, and ratchet drill. Also a full set of special wrenches for the main engine and large wrenches for the stern-tube nut and propeller nut.

Lifting Gear. Suitable lifting gear should be provided over the engine for lifting the cylinder covers and drawing the pistons and valves. This gear

valves as shown in Fig. 98. The bucket in small sizes is of brass and in large sizes of cast iron with a brass casing. The pump rod is either steel with a brass sleeve or solid tobin bronze. An extension attached to the upper end of the piston rod works in a guide as shown in Fig. 107. The thickness of the cast-iron barrel, bottom, and cover varies from about 7/8 in. for a 20-in. to 1 in. for a 30-in. pump and the brass liner from about 1/2 to 3/4 in. The valve decks are made about 1.2 times the thickness of the liner.

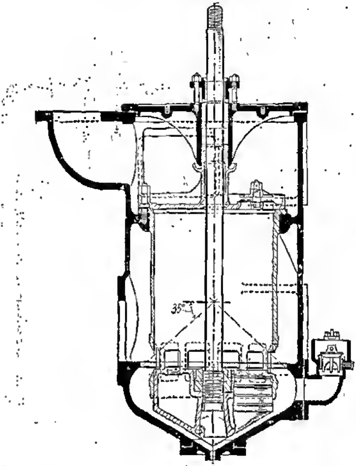


Fig. 99.—Edwards type of attached air pump.

Unless the pump discharge is taken off in such a manner that there is a head of water on the discharge valves, these valves are kept submerged by casting a rim around the valve deck or the pump cylinder as shown in Figs. 98 and 99. This head of water keeps the valves closed on the down stroke. A standpipe, open at the top, connected to and usually the same size as the discharge pipe, extends about 15 ft above the pump discharge and serves as a relief in case an abnormal amount of water enters the pump when starting up or from any other cause.

Air-pump valves are now usually made of three sheet bronze disks about 3/64 in. thick (Fig. 100). Rubber is liable to be affected by the oil which finds its way into the steam from the lubrication of the stuffing boxes. Light-

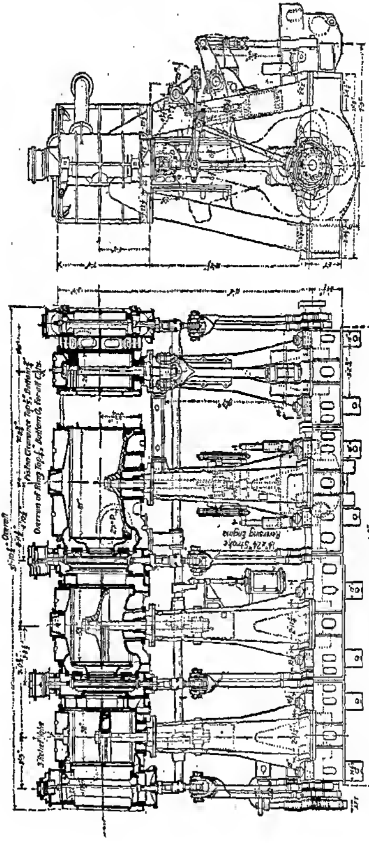


FIG. 107.—Quadruple-expansion engine designed and built by the Newport News Shipbuilding and Dry Dock Company.

ness, which is essential to good efficiency, is obtained by using brass disks. Holes in the middle and lower disks, as shown, allow the suction to act on all the disks, which assists in seating the valve and prevents the edges from curling. The valve seats, which are also brass, are in the form of grids to reduce the unsupported area of the disks. A brass guard on a stud regulates the lift of the valve which must be such that the circumferential discharge area is equal to the clear area through the seat. There may be one row of large or two rows of small valves.

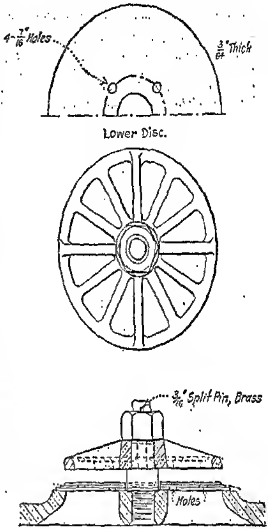


FIG. 100.—Bronze air-pump valve.

Feed pumps are of the plunger type, single-acting, and are worked from one or both ends of the air pump crosshead. The plunger works through a stuffing box fitted with metallic or soft packing. The inlet and outlet valves are carried in a valve box which is cast separate from the pump barrel to reduce the cost of replacement in case one or the other is damaged. The valves are of brass with springs to assist in seating. An air vessel and a relief valve are fitted on the pump or in the discharge piping. Feed pumps are designed for a theoretical capacity of about 2.25 times the boiler evaporation except for small engines where the ratio is increased. The evaporation may be assumed as 20 lb per ihp for compound engines, 16 to 17 lb for triple, and 15 lb for quadruple engines. Feed and other attached pumps usually have cast-iron barrels and valve boxes and brass plungers, but steel plungers are often used for feed pumps to avoid excessive wear. No definite ratios can be given for the sizes of bilge and sanitary pumps, as their capacities depend upon the type of vessel.

Attached Pump Gear. The beams are made up of two pairs of steel plates connected by a rocker shaft which works in bearings on the engine housings (Fig. 101). The link pins at each end are carried between the plates. Each link consists of two steel bars with bushings at the ends. The working parts are designed for the load on the pumps which is taken as 30 psi on the air-pump bucket and 20 psi on the plungers of the other pumps. These pressures are high enough to cover the weight of the reciprocating parts as

consists of one or more chain blocks of suitable capacity hung from travelers fitted to a strong I beam placed fore and aft above the centerline of the engine at a height sufficient to lift out the piston with rod attached. In the bottoms of the cylinders are tapped holes for eyebolts for attaching chain falls for handling the main bearing caps and other heavy parts.

Arrangement of Cylinders and Cranks. The order in which the cylinders are arranged beginning with the forward cylinder is called the order of cylinders. Different orders of cylinders is discussed under Balancing, p. 1169. The order in which the cranks pass the top center when the engine is going ahead is called the sequence of cranks. The high-pressure is commonly called the leading crank when the sequence is high-pressure, intermediate-pressure, low-pressure. Similarly the low-pressure is the leading crank when the sequence is low-pressure, intermediate-pressure, high-pressure. A right-hand engine has the working platform on the starboard side, and the cranks in passing their top position turn to the starboard. In other words, a right-hand engine turns clockwise when viewed from the after end.

Engine Designs. Figure 106 shows a triple-expansion engine in which the ports are short but not as short as if the valve chests were extended above and below the cylinders. The crankshaft is in three interchangeable and reversible sections, so that one section of spare shaft may be used for any cylinder with either end forward. The top and bottom of the eccentric straps are made from the same pattern to reduce the number of spare parts carried. Figure 107 shows a quadruple-expansion engine with the parts as short as it is possible to make them. The crankshaft is in four interchangeable but not reversible sections. The position of the links and reversing arms indicates that the engine shown in Fig. 106 has open eccentric rods and the engine in Fig. 107, crossed rods. These are representative merchant engines, the proportions and details of which are in accordance with the methods given in this section. Figure 106a shows the slide-valve reciprocating engine used in the Liberty ships during the Second World War. Figure 107a shows a uniflow engine used in the same service.

DRAFTING-ROOM METHOD OF LAYING DOWN THE ENGINE

The following method used in the design of a quadruple-expansion engine is presented by Mr. B. Meurk formerly of the Engineering Department of the Newport News Shipbuilding and Dry Dock Company:

In the engine taken as an example the crankshaft is in four interchangeable sections; piston valves are used in the high-pressure and 1st intermediate-pressure and slide valves in the 2d intermediate-pressure and low-pressure cylinders; liners are fitted in the high-pressure, 1st intermediate-pressure, and 2d intermediate-pressure cylinders. The size of cylinders, crankshaft, coupling bolts, connecting rods and bolts, crosshead, and main bearings are all determined according to the methods previously described. The best balance of a quadruple-expansion engine is obtained when the order of cylinders is high-pressure, low-pressure, 2d intermediate-pressure, and 1st intermediate-pressure and when the sequence of cranks is high-pressure, 1st. intermediate-pressure, low-pressure, and 2d intermediate-pressure, and this arrangement only is discussed.

Block out the low-pressure and 2d intermediate-pressure cylinders, valve chests, and slide valves and enough of a valve diagram (Fig. 108) to determine the stroke of the eccentrics and the areas of the passages necessary in the

well as the working pressure (or vacuum). The up stroke load is used. The pump beams are proportioned for a bending stress of about 4,500 lb at the section B (Fig. 101). If P is the total load at the pump end, A, b, and t, the width and thickness of one beam plate at B, and d the diameter of the rocker shaft in the beam plate, the stress in the beam at B $= \frac{6P \times b}{4t(b^2 - d^2)}$. The pins are proportioned for working pressures in the link bushings of about 400 lb at A and 300 lb at B. The load at C is found by multiplying the load at A by the ratio x/y. The load on the rocker shaft at B is equal to the sum of the loads at A and C. The working pressure on the rocker shaft bearings is about 150 lb. The air-pump load only is used when figuring the pump rod

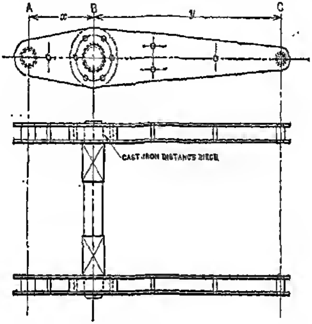


Fig. 101.—Attached pump beams.

and the bending stress in the middle of the pump crosshead. If the breadth and depth of an average section in the middle of the pump crosshead are b and h, the average diameter of the hole d, the distance between the centers of the link bearings l and the load on the pump rod P_1, the stress in the central block $= \frac{P_1 l}{4h^2(b - d)}$. The air-pump rod is figured as a column fixed at the ends, by the usual method, using a factor of safety of about 20, and also for tensile strength at the bottom of the threads, the allowable stress being about 3,500 lb. The link rods and caps are proportioned by the usual methods.

Piping and Fittings

Throttle Valves. The ability to maneuver quickly requires that the throttle valve be balanced for easy handling and arranged for convenient operation from the working platform either by a lever or a handwheel. The

cylinders and slide valves to give the proper steam and exhaust velocities. This is done to determine the shortest distance between center lines of

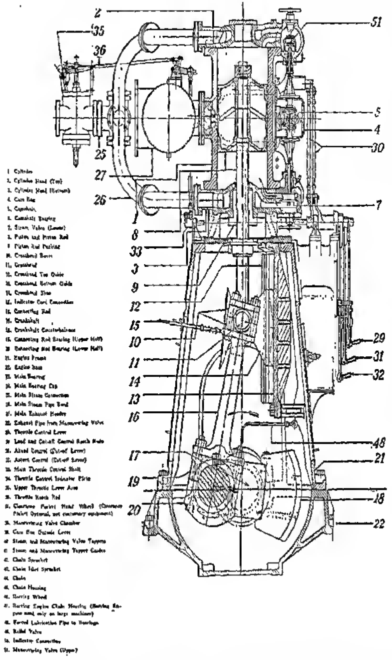


Fig. 107a.—Three-cylinder uniflow engine used

cylinders and valve stems. Lay down the crankshaft and main bearings (Fig. 108) allowing ⅛-in. clearance between the sides of the crank webs and the crankpin and main bearing brasses. The distance between main

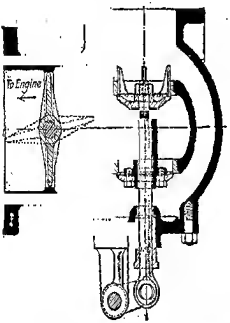


FIG. 102.—Double-beat poppet throttle valve and butterfly valve.

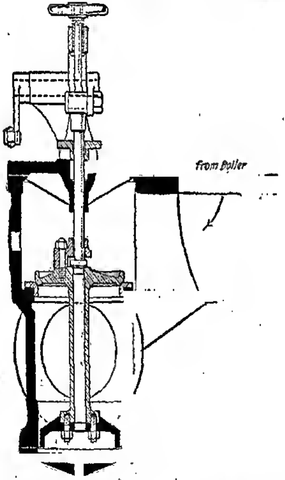
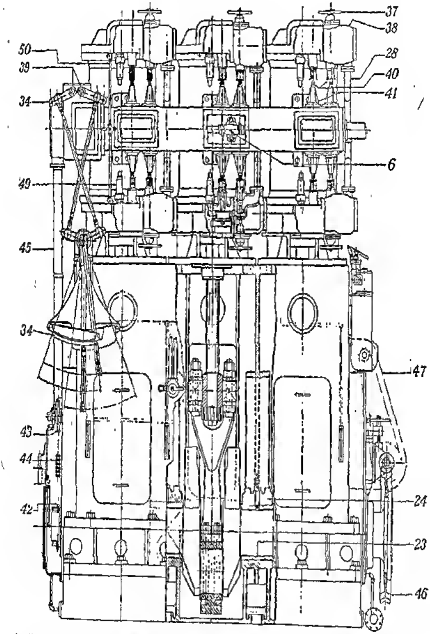


FIG. 103.—Throttle valve with balance piston and pilot valve.

bearings Nos. (2) and (3) and Nos. (6) and (7) should be made as small as possible, clearance only being allowed for the coupling bolt nuts. The distance a is settled by allowing ¼-in. clearance between the after main-bearing brasses and the 1st intermediate-pressure eccentrics and sufficient space for the coupling bolt nuts. The outline of the cylinder and valve chest covers should next be completed (Fig. 108) facing off the covers and offsetting



in Liberty ships. (*Courtesy of Nautical Gazette.*)

the ahead eccentric rod about ½ in. if necessary to close up the centers. The width of the main bearing girders is determined by allowing the brasses to overhang about 1¾ in. at the sides to allow for fitting water-service pipes.

double-seated valve, sometimes called the double-beat poppet, shown in Fig. 102, is the type most commonly used. One disk is made slightly larger than the other so that the unbalanced pressure will act to close the valve yet not be so great as to cause difficulty in opening it by hand. To reduce the uneven expansion between the valve disks and the valve body, the distance piece between the disks is generally made of the same material as the body. If the valve is operated by a hand lever, it is well to fit a gag screw with a handwheel to limit the stroke or to lock the valve tight when in port. **Large single-seated valves should be avoided** as they are slow in closing and require special balancing arrangements which are delicate and liable to give trouble. A throttle valve **fitted with balance piston and pilot valve** is shown in Fig. 103. Unless this valve is carefully designed, it will not be balanced sufficiently to operate with a hand lever. The balance piston should be on the engine side of the valve where it is less apt to chatter than if on the steam side. The **combined disk and cylindrical valve**, fitted with a pilot valve as shown in Fig. 104 (from Bauer), is easy to balance and gives good adjustment but is rather slow to open and shut. Steam leaking through the valve liner presses the disk firmly on its seat. As the spindle rises, the pilot valve is opened, the steam above the disk escapes, and the main valve is easily operated. The flow of steam is regulated by the uncovering of the openings in the valve liner.

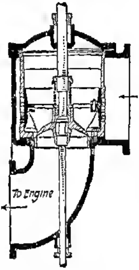


FIG. 104.

A butterfly valve (Fig. 102) is often fitted in the steam line in addition to the throttle valve for quick handling when the engine is racing. The angle of travel of the valve is generally from 60 to 90 deg. The valve is controlled by a hand lever from the working platform and is attached to the governor gear if fitted.

Governors. To prevent racing in bad weather or if the engine is suddenly relieved of its load, a governor is often fitted. **The governor may be actuated** by the pressure under the stern of the vessel or by the revolutions of the engine above a predetermined speed. In the first type a pipe is led from a chamber connected to the water at the stern to a pressure chamber at the engine fitted with a diaphragm. Any change in draft at the stern due to pitching of the vessel will vary the pressure on the diaphragm which, through suitable gear, operates the reversing engine which in turn controls the speed of the engine by moving the links. Governors of the second type may make use of the centrifugal force of revolving weights or of the force of inertia. Governors operating on the latter principle are more frequently used, one of the most popular being the **Aspinall** shown in Fig. 105. This device is carried on the air-pump beam or on a rocker arm supported on one of the housings and actuated by a link attached to the crosshead. The mechanism consists of a spring-retarded weight, *W*, whose inertia at speeds of about 5 per cent in excess of a predetermined speed is sufficient to overcome the spring and set the pawls, *PP*, so that the lower pawl on, the next up stroke, picks up the lever, *H*, which closes the throttle valve. When the speed of the engine

Before advancing further, it is well to look into different arrangements of valves and crankshaft. With the same order of cylinders, the variation in the dimensions X, Y, and Z for different arrangements of valves and crankshaft may be noted in the following table, which is made up for an engine with cylinders 26-, 38-, 55-, 81-, by 51-in. stroke.

	X	Y	Z	
Fig. 108	31 ft 10¼ in	29 ft 0¾ in	19¼ in	Crankshaft in four interchangeable sections
Fig. 109	28 ft 11¾ in	27 ft 2¼ in	2 ft 7½ in	Crankshaft in two interchangeable sections
Fig. 109	28 ft 11¾ in	27 ft 2¼ in	11½ in	Crankshaft in two interchangeable sections. Forward end of shaft cut off (one spare section only required)
Fig. 110	28 ft 11¾ in	27 ft 2¼ in	8 in	Crankshaft in two interchangeable sections

After deciding on a suitable arrangement the end view is worked out (Fig. 111). Draw the crank circle and the connecting rod on the top center. Sweep the connecting rod on the crank circle a full stroke and spot the clear-

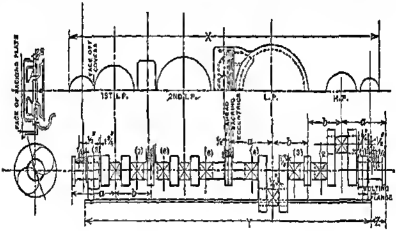


Fig. 108.

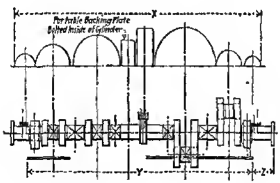


Fig. 109.

A scale of sines, tangents, and logarithms is often printed on the back of the slide. For further details concerning the use of the slide rule in various problems, see the instruction books furnished with each instrument: Wm. Cox, "Manual of the Mannheim Slide Rule"; F. A. Halsey, "Manual of the Slide Rule"; etc.

Other Types of Slide Rules. The **duplex** slide rule ($5 to $18 according to length) shows on one face the regular A, B, C, D scales, and on the other face the scales A, B', C', D (where B' and C' are the same as B and C, only numbered in the reverse order), with a runner encircling the whole scale. This arrangement makes possible the solution of more complicated problems with fewer settings of the slide, but if the rule is to be used only for simple problems, the multiplicity of scales is rather confusing. Less complicated is the **polyphase rule**, which is like a Mannheim rule with the addition of a single inverted scale, C', printed in the middle of the slide. The **log log duplex slide rule** (10 in., $8) is especially adapted for handling complex problems involving fractional powers or roots, hyperbolic logarithms, etc. A number of **circular slide rules** are on the market, the best of which are operated by a milled thumbnut, like the stem wind of a watch. The advantage of the circular rule, aside from its compact size (some models are scarcely larger than a watch), lies in the fact that the scale is endless, so that the slide never has to be reset in order to bring the result within the scale. A disadvantage is found in the necessity of reading the figures in oblique positions, or else continually turning the instrument as a whole in the hand. The **Fuller and Thacher rules** already mentioned are invaluable for problems requiring greater accuracy than can be obtained with the ordinary rules. There are also many **special slide rules**, adapted to various special types of computation, such as calculating discharge of water through pipes, horse power of engines, dimensions of lumber, stadia measurements, and electric circuits. For a description of a large variety of slide rules and other calculating apparatus, see the Catalogue of the Collection in the Science Museum, South Kensington, or Horsburgh, "Napier Tercentenary Celebration" (Royal Society of Edinburgh).

COMPUTING MACHINES

For certain purposes, computing machines have ceased to be luxuries and have become almost necessities; but they are expensive, and should be selected with reference to the special work that is to be done. The machines may be classified roughly into three groups, as follows:

Adding Machines, Non-listing. Of the machines of this kind, the most convenient in the hands of a careful operator is the well-known **Comptometer** (Felt & Tarrant Co., Chicago, Ill.; $300 to $400 according to size), or the **Burroughs non-listing** adding machine (Detroit, Mich., $175). To add a number, simply press a key in the proper column; the result appears on the dials in front of the keyboard. Multiplication as well as addition can be performed on this machine with great rapidity, and division also after a little practice. Weight, about 15 lb.

Adding and Listing Machines. The machines of this group not only add, but also print the items, totals, and subtotals. The **Burroughs** (Detroit, Mich.), the **Barrett**, and the **Wales** resemble each other in having an 81-key keyboard; the **Dalton** (Cincinnati, Ohio) and the **Sundstrand** (Rockford, Ill.) have a 10-key keyboard, admitting of operation by the touch method. On all these machines, in order to add a number, first depress the proper keys and then pull a handle (or, in the case of electrically driven machines, press a button) to record the item. The prices range from $125 to $1,100, according to size and style, new models being constantly devised for special commercial purposes.

Calculating Machines. Machines of this third group are intended primarily for multiplication and division; the types which have a keyboard can be used effectively for addition and subtraction also. They are all non-listing. The best known are the **Monroe**, the **Marchant**, and the **Millionaire**, all of which are available in either hand-driven or electrically driven models.

For graphical methods of computation, see pp. 106, 119, 170, 173–186.

Automatic calculating machines, such as those manufactured by the International Business Machines Corp., have found wide application in accounting. They are capable of a variety of simple algebraic operations; data is fed to the machines by means of perforated cards. For details see the manufacturer's bulletins, and for application to statistical methods see Baehne, "Practical Applications of the Punched Card Method" (Columbia University Press).

FINANCIAL ARITHMETIC

For the facts that are commonly required in regard to compound interest, sinking funds, etc., see the headings of the tables on pp. 64–68. More extended tables may be found in J. W. Glover's "Tables of Applied Mathematics" (Ann Arbor, Mich., 1923).

ance line, being sure that the line covers all projecting parts such as setscrews. This line should clear the bedplate and housings by about 2½ or 3 in. Next work out the conditions surrounding the upper end of the connecting rod including the crosshead, piston-rod stuffing boxes, pistons, and bottom and

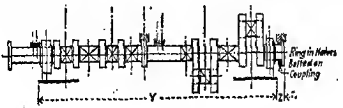


FIG. 110.

top of high-pressure and low-pressure cylinders. Sufficient space should be allowed for the piston-rod packing and gland to accommodate any type of metallic packing commonly used, as the type of packing is often changed.

The length of the eccentric rod is determined by laying down the valve in its bottom position and locating the bottom valve chest cover with its stuffing box and gland. The clearance between the gland and the guide face on the valve stem should be sufficient to allow repacking the gland with the valve in its top position. The length of the guide face should be such as to allow the valve stem facing to overrun at top and bottom in maximum linked-up gear. The valve-stem guide and the main links in their top position may then be located, allowing ample space between the links and the guide bracket for oil cups on the link block and eccentric rod. The length of eccentric rod is then the distance between the center of the links and the top of the eccentric circle drawn on the crank center.

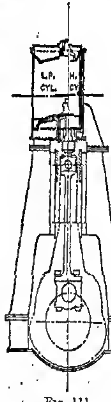


FIG. 111.

To locate the reverse shaft center, lengths and reversing-engine arm and reversing arms, angle of slot in reversing arms, and outer end of the suspension rod, it is necessary to plot the paths of the eccentric-rod pins on the links for full gear ahead and astern and maximum linked-up astern positions. The motion of the main links is determined by the suspension rod and the two eccentric rods. The point where the suspension rod attaches to the link moves in a circular arc with a radius equal to the length of the rod. The point where the astern eccentric rod is attached to the link has a compound motion due to the ahead eccentric rod and the suspension rod which causes the point to travel through a figure eight. This figure should be plotted for the link positions mentioned above and the reverse shaft, reversing arms, and outer end of suspension rod located to give the smallest possible slotting action of the link block. It is also advisable to check the clearances between the link block and the connecting pins at the ends of the links for the corresponding link positions. Figure 68 shows the

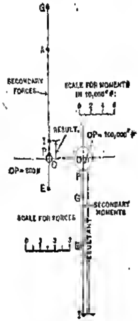


Fig. 118.—Secondary-force and moment polygons.

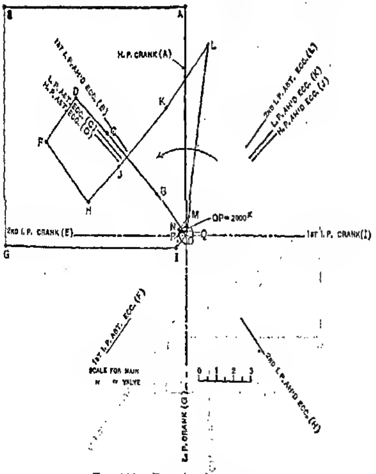


Fig. 119.—Rotating-force polygon.

slotting action of the link block for full gear ahead, linked-up ahead, full gear astern, and linked-up astern positions.

BALANCING

The following text was prepared by Mr. J. B. Woodward, Jr., of the Engineering Department of the Newport News Shipbuilding and Dry Dock Company:

The object in balancing marine engines is to avoid vibrations in the ship's structure. The tremors produced by pounding in the engine due to slack bearings or improper valve setting are also objectionable but should not be confused with the vibrations caused by the unbalanced forces. The magnitude of the unbalanced forces is dependent upon the weight and size of the moving parts and the speed of rotating of the engine, but is independent of the steam pressure. When the piston with its attached weights is being accelerated, that portion of the steam pressure necessary to accelerate it is absorbed in giving energy to the moving parts and is not transmitted to the engine framing. It follows that the pressure on the cylinder head is only partly balanced by that portion of the steam pressure upon the piston which is transmitted to the engine framing by way of the piston rod, connecting rod, and crankshaft. It is obvious then that the unbalanced forces developed when an engine is revolving are equal and opposite to the forces necessary to accelerate or retard the moving parts.

Analytical Method

The most serious unbalanced forces are those which act in a vertical direction. The horizontal forces in a vertical engine are much smaller, also the ship itself is usually stiffer in the horizontal plane. In determining forces, the moving parts are divided into two groups, reciprocating and rotating, and the cylinder and valve gear parts are treated separately, as follows:

1. Cylinder Reciprocating Parts. Pistons, piston rods, crossheads, and top half of connecting rods. The connecting rod is divided into two parts at the center of gravity, one assumed to move with the crosshead and the other to revolve with the crankpin. The location of the center of gravity of the connecting rod may be assumed with sufficient accuracy.

2. Valve Gear Reciprocating Parts. Valves, valve stems, balance pistons, links and blocks, ahead eccentric rods, one-half the astern eccentric rods, and one-half the suspension rods.

3. Cylinder Rotating Parts. Crank webs, crankpins, and bottom half of connecting rods.

4. Valve-gear Rotating Parts. Ahead and astern eccentrics and straps.

Formulas for Vertical Forces. The rotating parts of an engine give rise to a force which is constant in amount but variable in direction while the reciprocating parts set up a force constant in direction but variable in amount. In dealing with the rotating parts, only the vertical component of the force will be considered.

Assume A = length of crank arm, B = length of connecting rod, $U = B \div A$, b = length of eccentric rod, a = throw of eccentric, $u = b \div a$, l = sum of the mean of top and bottom steam laps and the mean of top and bottom steam leads, N = revolutions per sec, and θ = angle of the crank from

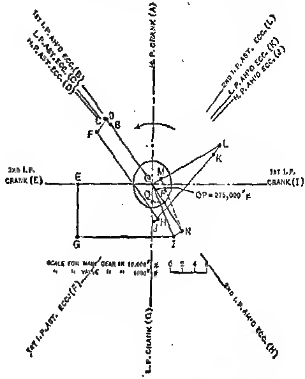


Fig. 120.—Primary-moment polygon.

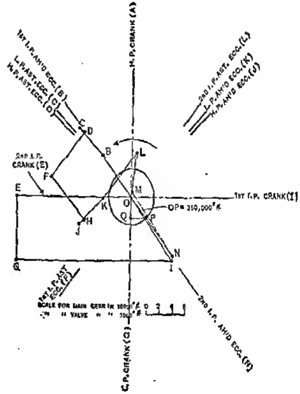


Fig. 121.—Rotating-moment polygon.

the vertical axis. All dimensions are in feet. Then for a single-cylinder engine, or for each cylinder of a multi-cylinder engine:

$$\text{Cylinder reciprocating parts: } F_1 = \frac{4\pi^2}{32.2} N^2 A W_1 \left(\cos\theta + \frac{1}{U}\cos 2\theta\right)$$

$$= 1.226 N^2 A W_1 \left(\cos\theta + \frac{1}{U}\cos 2\theta\right) \quad (1)$$

where W_1 = the weight of the parts

$$\text{Valve-gear reciprocating parts: } F_2 = 1.226 N^2 a W_2 \cos\theta \quad (2)$$

where W_2 = weight of the parts

The second term $\frac{1}{U}\cos 2\theta$ has been neglected, as the weight of the valve gear is small compared with the cylinder parts and as the ratio of eccentric rod length to eccentricity is relatively large.

$$\text{Cylinder rotating parts: } F_3 = 1.226 N^2 A W_3 \cos\theta \quad (3)$$

where W_3 = weight of the parts

$$\text{Valve-gear rotating parts: } F_4 = 1.226 N^2 l W_4 \cos\theta \quad (4)$$

where W_4 = weight of the parts

Application of Formulas. To show the application of the formulas the vertical forces will be computed for a triple-expansion engine of the usual marine type with cranks at 120 deg.

Example. Engine size is 20½-35-60 in. × 42-in. stroke with attached air and bilge pumps driven by a beam from the low-pressure crosshead. The order of cylinders from the forward end is high-pressure, intermediate-pressure, and low-pressure. Designed ihp is 1,500 at 85 rpm. As the crank webs, crankpins, and connecting rods are the same for the three cylinders, there is no unbalanced force arising from the cylinder rotating.

Table 17

Designed rps (N) (85 ÷ 60)	1.42
Length of crank arm (A), ft	1.75
Length of connecting rod (B), ft	7.50
Connecting rod ratio (U)	4.28
1.226 N^2A	4.31

Size and type valve	H-p 8-in. piston	I-p 14-in. piston	L-p double-ported slide
Valve takes steam	Inside	Inside	Outside
Throw of eccentric (a)—ft	0.33	0.33	0.27
Angle of advance	36°-38′	28°-22′	42°-30′
Sum of mean steam lap and mean steam lead (l)—ft	0.20	0.21	0.18
Weight in lb			
Piston	560	1,400	2,850
Valve-gear reciprocating parts	930	1,200	2,560
Valve-gear rotating parts	2,100	2,100	1,200

on a crank angle base (Fig. 122). The projection OQ of OP on the vertical axis represents the magnitude of the vertical force due to the reciprocating parts when the high-pressure crank is passing the top center. By revolving OP about O and finding its projection on the vertical axis for any position, the vertical force of the reciprocating parts may be found for the corresponding high-pressure crank angle. Thus, if it is desired to find the force when the high-pressure crank has moved 20 deg from the vertical position, it is only necessary to revolve OP 20 deg in the direction of rotation and obtain its projection on the vertical axis.

The secondary-force polygon (Fig. 118) is constructed in a manner similar to the primary polygon except that each weight is laid off in a position corresponding to double the angle in Fig. 115. As a rule, the virtual secondary weights of the valve gear may be neglected as is done in this case. The closing line (IO) of the polygon is multiplied by $1.226N^2A \div (B/A) = 0.79$ and laid off (OP) to the scale of forces in Fig. 122. In finding the vertical forces OP is revolved at twice the rate of the high-pressure crank and projected on the vertical axis.

The rotating-force polygon (Fig. 119) is drawn in a manner similar to the primary polygon for the reciprocating parts, and the same factor 3.54 is applied to the closing line. The horizontal forces, if desired, may readily be obtained by projecting OP on the horizontal axis.

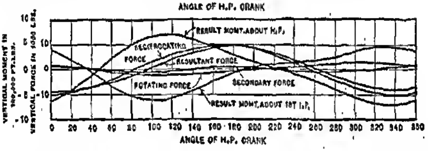


FIG. 122.—Unbalanced vertical forces and rocking moments—quadruple-expansion engine.

Moment polygons (Figs. 118, 120 and 121) are plotted in the same manner as the force polygons, using moments instead of weights and applying the same factors to the closing lines as in the force polygons.

Force and Moment Curves (Fig. 122). The reciprocating and rotating forces are plotted on a crank angle base and combined into a single resultant vertical force. The moments about the forward crank may be plotted and combined in the same manner. The resultant moment only is shown in Fig. 122. The resultant moment about the after crank is also shown.

Means of Improving Balance

In a single-crank engine, the unbalanced force of the rotating parts can be compensated for by a **counterweight** on the crank equal in weight to the rotating parts. If the counterweight be made equal to the rotating and reciprocating parts, the unbalanced force of the latter also disappears, but a new horizontal force is introduced equal to the radial acceleration of the reciprocating parts.

In a two-crank engine, revolving balance may be secured by the use of counterweights opposite each crank and, if balance of the reciprocating parts is necessary, it is probably best to use lever balance weights for each cylinder. Perfect balance may be secured only by special arrangements in which the cranks are no longer at 90 deg.

The arrangement of cylinders generally used in **three-crank triple-expansion engines** counting from the forward end is high-pressure, intermedi-

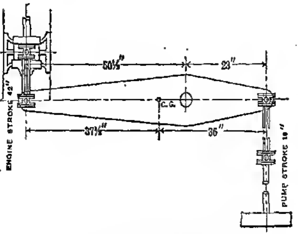


FIG. 112.—Arrangement of attached pumps.

parts. When these parts are not of the same size, the unbalanced force for each crank may be calculated by formula 3. Except for the pump lever pins on the low-pressure, crosshead (which are included with the pump gear in Table 18) the piston rods and crossheads are also alike for the three cylinders. Their resultant force for this crank arrangement is zero, and they may be omitted from the computation. The connecting rods are omitted for the same reason. The low-pressure eccentrics are on the shaft while the other eccentrics are on the shaft couplings. The data required are given in Table 17.

The attached pumps (Fig. 112) may be combined with the cylinder reciprocating parts by determining the force which would have to be applied at the low-pressure crosshead to balance the combined moment of the whole gear (Table 18). The obliquity of the links may be neglected as its effect is small.

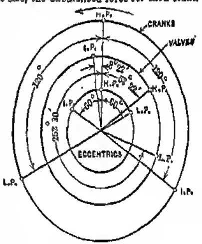


FIG. 113.—Arrangement of cranks and eccentrics—looking aft.

Table 18

Item	Weight, lb	Arm about beam axis, in.	Moment about beam axis, in.-lb
Pump plungers, bucket and rod	350	23.0	8,050
Pump crosshead	250	23.0	5,750
Pump links and link pins	230	23.0	5,290
Crosshead links and link pins	150	−50.5	−7,570
Crosshead pins	20	−50.5	−1,010
Pump beam	405	−13.0	−5,265
Resultant	1,405	+3.73	5,245

ate-pressure, and low-pressure with the cranks at 120 deg. The crankpins and webs and also the piston rods, crossheads, and connecting rods are usually made the same for all three cylinders. With such an arrangement the balance may be improved by increasing the weight of the high-pressure piston and lightening the low-pressure so as to make the weights of the reciprocating parts as nearly equal as possible. Even when the crank webs are of unequal thickness it is seldom advisable to add weight and complication to the engine by using counterweights to balance the rotating masses.

In **four-crank engines** Bauer and Robertson state that balancing is most effectually carried out when (1) the cylinders with the heaviest connecting rods are in the center and (2) the distance between the two middle cylinders is as large as possible, as compared with the distance between the two cylinders at either end of the engine. "If these conditions cannot be secured in practice, the outer connecting rods must be artificially lightened by means of counterweights and the connecting rods of the middle cylinders artificially weighted by the use of heavy pistons. In triple-expansion engines good balancing is most easily obtained by having two low-pressure cylinders, each doing purposely less work and having lighter connecting rods than the high-pressure and intermediate-pressure cylinders, outside which they are placed. An engine with four cranks, in which the forward pair of cranks is at right angles to the after pair, and the cranks of each pair are at an angle of 180 deg to each other, gives an almost perfect balance, if the distance between the two pairs of cranks is relatively large in proportion to the distance between the individual cranks constituting each pair, and when the masses of the forward pair are equal to the masses of the after pair." Four-cylinder quadruple-expansion engines should be arranged with the high-pressure and 1st intermediate-pressure cylinders on the ends and the 2d intermediate-pressure and low-pressure inside, with the attached pumps, if any, driven by a beam from the low-pressure crosshead. The low-pressure piston should be made as light as possible. The pair of cylinders on each end should be as close together as practicable, which is facilitated by placing the valves for the high-pressure and the 1st intermediate-pressure cylinders at the ends of the engine and the valves for the other two cylinders in the middle.

Revolving balance may be obtained in four-crank engines in a number of ways, simplicity and cheapness usually being the determining factors. Instead of fitting balance weights opposite each crank, two revolving counterweights are often used, one forward of the engine framing and the other aft. The turning wheel may generally be used for the attachment of one weight, and the other may be made in the form of a disk attached to the forward end of the crankshaft, extended a few inches if necessary. The amount and angular position of such weights may be readily determined from the force and moment polygons. Whether or not counterweights are used it is well to consider the advantage to be gained by drilling holes in one or more crankpins if solid, or filling one or more with lead if hollow.

Yarrow-Schlick-Tweedy System. Complete primary balance can be obtained in four-crank engines on the assumption of infinite connecting rods, thus eliminating the secondary forces from consideration. The balance is complete if (1) the sum of the moments of the horizontal forces and the sum of the moments of the vertical forces about the forward crank = 0 and (2) the sum of the moments of the horizontal forces and the sum of the moments of the vertical forces about the after crank = 0. By proper choice of crank angles and by changing weights of moving parts and cylinder spacing, in so

Table 19. Vertical Forces

H-p crank angle	Cylinder reciprocating parts						Valve gear			Total resultant
	H-p	L-p	I-p	Cylinder resultant	Attached pump	Resultant including attached pumps	2,772 × cos θ	1,578 × sin θ	Resultant	
0	2,985	−7,600	−3,730	−8,345	172	−8,173	2,772	0	2,772	−5,401
20	2,710	−8,920	−2,370	−8,580	362	−8,218	2,605	540	3,145	−5,073
40	1,950	−9,370	−272	−7,692	502	−7,190	2,125	1,012	3,137	−4,053
60	928	−9,430	2,310	−6,192	553	−5,639	1,385	1,365	2,750	−2,889
80	−109	−9,370	4,875	−4,604	502	−4,102	482	1,555	2,037	−2,065
100	−952	−8,920	6,760	−3,112	362	−2,750	−482	1,555	1,073	−1,677
120	−1,495	−7,600	7,450	−1,645	172	−1,473	−1,385	1,365	−20	−1,493
140	−1,755	−4,830	6,760	175	−20	155	−2,125	1,012	−1,113	−958
160	−1,840	−553	4,875	2,482	−176	2,306	−2,605	540	−2,065	241
180	−1,855	4,720	2,310	5,175	−276	4,899	−2,772	0	−2,772	2,127
200	−1,840	9,930	−372	7,818	−325	7,493	−2,605	−540	−3,145	4,348
220	−1,755	13,780	−2,370	9,655	−341	9,314	−2,125	−1,102	−3,237	6,077
240	−1,495	15,160	−3,730	9,935	−344	9,591	−1,385	−1,365	−2,750	6,841
260	−952	13,780	−4,380	8,448	−341	8,107	−482	−1,555	−2,037	6,070
280	109	9,930	−4,600	5,221	−325	4,896	482	−1,555	−1,073	3,823
300	928	4,720	−4,640	1,008	−276	732	1,385	−1,365	20	752
320	1,950	−553	−4,600	−3,203	−176	−3,379	2,125	−1,102	1,113	−2,266
340	2,710	−4,830	−4,830	−6,500	−20	−6,520	2,605	−540	2,065	−4,455
360	2,985	−7,600	−3,730	−8,345	172	−8,173	2,772	0	2,772	−5,401

far as the design of the engine permits, it is possible to approximate very closely the above conditions.

The work is best done graphically. The vertical moment about the forward crank is found for each cylinder and valve, using for the cylinder weights the sum of the actual weights of the cylinder reciprocating and rotating parts, and for the valve-gear weights the sum of the virtual weights of the valve-gear reciprocating and rotating parts. The horizontal moment about the forward crank is found for each cylinder and valve, using the actual weights of the cylinder rotating parts and the virtual weights of the valve-gear rotating parts. The moments about the after crank are obtained in the same manner. The four moment polygons are then plotted by laying off each moment parallel to its corresponding crank or eccentric position (see Graphical Method). The sum of the moments represented by a particular polygon is zero when the polygon is a closed figure. All the polygons may usually be made very nearly or entirely to close by properly choosing the angles, weights, and cylinder spacing. If necessary, counterweights may be used at the forward and after ends of the engine, the amount and position of such weight being determined in each case from the moment polygons. If the ratio $L \div l$, in which L is the length between center lines of the pair of cylinders at either end of the engine and l the distance between center lines of the inner pair of cylinders, is less than $\frac{5}{3}$, the angles of the two outer cranks are apt to be small, which has an injurious effect upon the maneuvering qualities and the regularity of the turning moment. If the cylinders with the heaviest moving parts are placed in the middle and $L \div l$ made about 2, the best crank angles will be obtained, the smallest being about 65 to 70 deg, and complete balance can be obtained without the use of counterweights.

A full description of the Yarrow-Schlick-Tweedy system of balancing may be found in Bauer and Robertson.

Critical Revolutions and Vibrations of Hull. It is important that the rpm of the engine at its working speed do not coincide with the natural period of vibration of the ship. In cases where heavy vibrations have been set up they have been relieved by changing the working revolutions of the engine. The number of revolutions which produces the maximum vibratory effect is given by Bauer and Robertson from the formula derived by Schlick (*Trans. of the Inst. of Naval Architects*, 1894) as follows:

$$N = K\sqrt{T \div DL^3}$$

where N = rpm

T = the moment of inertia of the midship section in foot units

D = the displacement in tons

L = the water-line length, ft

K = a constant having the following values:

For large fast ships with fine lines, $K = 1.872 \times 10^5$

For cargo vessels with full lines, $K = 1.668 \times 10^6$

Investigations by Mr. William Gatewood on an oil tank steamer, *Trans. Soc. Nav. Arch. and Mar. Engrs.*, 1915, gave results closely in agreement with the above formula.

TURNING EFFORT

While it is important that the resultant of the tangential forces acting upon the crankpins, frequently called the turning effort, be as uniform as

A weight of 104 (= 5.245 ÷ 50.5) lb applied at the crosshead end would completely balance 1,405 lb with a moment of 5.245 in.-lb. The effect of the attached pumps is then virtually to reduce the low-pressure cylinder reciprocating weight by 104 lb. It is better, however, to treat this weight separately as in Table 19 rather than to combine it with cylinder weight.

The arrangement of the cranks and valve gear is shown diagrammatically in Fig. 113. The position shown for the eccentric in each case is the mean position of the ahead and astern eccentrics.

Calling the angle of the high-pressure crank θ, the formulas become:

Cylinder Reciprocating Parts:

H-p, $$F_1 = 2{,}420\left(\cos\theta + \frac{\cos 2\theta}{4.28}\right)$$

I-p, $$F_1 = 6{,}040\left[\cos(\theta - 120^\circ) + \frac{\cos 2(\theta - 120^\circ)}{4.28}\right]$$

L-p, $$F_1 = 12{,}300\left[\cos(\theta + 120^\circ) + \frac{\cos 2(\theta + 120^\circ)}{4.28}\right]$$

Attached pumps, $$F_1 = 450\left[\cos(\theta - 60^\circ) + \frac{\cos 2(\theta - 60^\circ)}{4.28}\right]$$

Valve-gear Parts:

		H-p	I-p	L-p
Angle of valve from h-p crank	=	$306^\circ - 38'$	$8^\circ - 22'$	$252^\circ - 30'$
1.226 N^2aW_2	=	702	985	1,670
1.226 N^2W_4	=	1,035	1,090	535

Then the resultant force of the valve-gear reciprocating parts, $F_2 = 762 \cos(\theta + 306^\circ\text{-}38') + 985 \cos(\theta + 8^\circ - 22') + 1{,}670 \cos(\theta + 252^\circ\text{-}30') = (762 \cos 306^\circ\text{-}38' + 985 \cos 8^\circ\text{-}22' + 1{,}670 \cos 252^\circ\text{-}30') \cos\theta - (762 \sin 306^\circ\text{-}38' + 985 \sin 8^\circ\text{-}22' + 1{,}670$

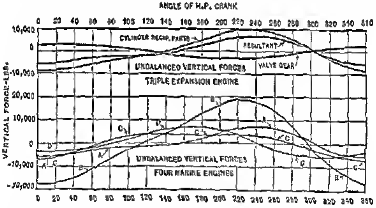


FIG. 114.—Curves of unbalanced vertical forces.

$\sin 252^\circ\text{-}30') \sin\theta = 925 \cos\theta + 2{,}060 \sin\theta$. The resultant force of the valve-gear *rotating* parts, $F_4 = 1{,}035 \cos\theta + 1{,}090 \cos(\theta + 60^\circ) + 535 \cos(\theta - 60^\circ) = 1{,}847 \cos\theta - 482 \sin\theta$.

The total force of the valve gear is then $= 2{,}772 \cos\theta + 1{,}578 \sin\theta$.

The vertical forces of the cylinder parts and the valve gear and the resultant vertical force are given in Table 19 and are shown graphically on a crank angle base in Fig. 114. *The resultant vertical forces for the four engines below* are shown in Fig. 114.

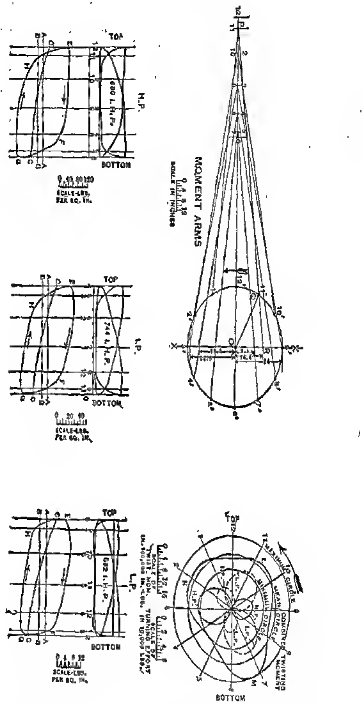


Fig. 123.—Construction of twisting-moment diagram for triple-expansion engine.

Curve	Size, in.	Rpm	Ihp	Order of cylinders	Sequence of cranks
A	20½-35-60 × 42 stroke	85	1,500	H-p, i-p, l-p	H-p, i-p, l-p
B	31-50-84 × 54 stroke	82	4,000	H-p, i-p, l-p	H-p, i-p, l-p
C	35-58-69-69 × 60 stroke	75	5,500	F-l-p, h-p, i-p, a-l-p	F-l-p, a-l-p, h-p, i-p
D	24-35-51-75 × 51 stroke	70	2,600	H-p, l-p, 2 i-p, 1 i-p	H-p, 1 i-p, l-p, 2 i-p

In all cases the cranks are equally spaced and the cylinder rotating parts are balanced in all cases except *B*, where the crank webs are of unequal thickness. In large three-cylinder triple-expansion engines (*B*), the vertical forces are necessarily great unless a counterweight is used opposite the low-pressure crank to offset the effect of the heavy piston, which however is not often done. No counterweights have been used in any of the above engines.

Rocking Moments. As the unbalanced forces of the various cylinders act at different points along the axis of the engine, unbalanced moments arise which tend to tilt, or rock, the engine in a *fore-and-aft* direction. It is convenient to refer these moments to a horizontal axis at right angles to the axis of the crankshaft and passing through the intersection of the crankshaft axis with the center line of the forward cylinder. The arms are then measured from the point of intersection to the axis of the various cylinders and valves. Where there is a valve forward of the point of intersection, its arm is considered negative. The formulas for the rocking moments may be derived from formulas (1), (2), (3), and (4) by inserting a factor in each to represent the arm about the given axis expressed in feet.

Primary and Secondary Forces and Moments. It is useful, particularly in representing the forces graphically, to separate the force due to the reciprocating members into two parts, one part varying as cos θ and called the **primary force**, and the other varying as cos 2θ and called the **secondary force.** Corresponding to the primary and secondary forces there are also primary and secondary moments. The weights for the secondary forces, as may be seen in formula (1), are only $1/U$ times the weights for the primary forces. However, it should be borne in mind that the forces or moments, thus separated for convenience into primary and secondary parts, actually appear combined as a single force or moment.

Graphical Method

The method given here is based on Rear-Admiral D. W. Taylor's paper "Balancing Marine Engines," *Trans. Soc. Nav. Arch. and Mar. Engrs.*, 9.

Polygons are drawn for the primary and for the secondary forces due to the reciprocating parts, and for the forces due to the rotating parts. The corresponding moment polygons are also drawn. In order to combine the valve gear in the same polygon with the main gear, the valve weights are reduced to virtual weights, having the travel of the piston for reciprocating weights and revolving with the crank radius for rotating weights. The reduction factor for the primary polygons is the ratio of valve travel to engine stroke ($a \div A$); and for the secondary polygons, the factor is the product of the ratio of valve travel to stroke ($a \div A$) by the ratio of the throw of the eccentric to the length of the eccentric rod ($a \div b$).

possible throughout the revolution to ensure steadiness of running and to reduce the stresses in the shafting, it is not as a rule necessary to construct the turning effort diagram, unless the design departs from usual practice, in which case it is advisable to construct a set of theoretical indicator cards and proceed as outlined below. It is sufficient for the purpose of calculating shafting stresses to know the ratio of maximum to *mean twisting moment*, which has the same value as the ratio of maximum to mean turning effort. The value of this ratio is given for various types of engines on p. 1106. **Variation in the turning effort is due to** (1) unequal distribution of power in the different cylinders, (2) variation in steam pressure on the pistons, (3) weight and inertia of the moving parts, and (4) the angularity of the connecting rod. In general **the greater the number of cranks the more uniform the turning effort.**

The turning effort on any one crank in pounds multiplied by the crank radius in inches gives the twisting moment on the shaft in pound-inches for that particular crank. The same result is more conveniently obtained if the net vertical force on the piston due to steam pressure and to inertia and dead weight of the moving parts is multiplied by OR (Fig. 123) the distance from the shaft center O to the intersection of PQ, the centerline of the connecting rod, with the axis XOX. The turning effort may be readily obtained from the twisting moment by dividing by the crank radius, or from the twisting moment diagram (Fig. 123) by applying the proper scale.

For all practical purposes the indicator cards may be assumed to be taken on the same revolution of the engine. On the crankpin circle (Fig. 123) lay off the points 1', 2', 3', 4', etc., 30 deg apart and locate the corresponding *points* 1, 2, 3, 4, etc., *in the travel of the piston.* Locate these piston positions for each set of cards, and at each position erect an ordinate. At any position of the down stroke the intersection of the ordinate between the down stroke line of the top card and down stroke line of the bottom card represents the difference in steam pressure on the top and the bottom of the piston; and in similar manner for any up stroke position. The ordinates from the base line AA to $EFGH$ show the pressure difference on the pistons in pounds per square inch for a complete revolution, ordinates above AA representing downward and ordinates below, upward pressure.

The moving parts are being accelerated during the early portion of the stroke, and their inertia must be subtracted from the steam pressure on the piston; during the later portion the parts are being retarded and their inertia must be added to the steam pressure. **The inertia force** (F) may be calculated by the formula

$$F = 1.226N^2AW\left(\cos\theta + \frac{\cos 2\theta}{U}\right),$$

where N = rps

A = length of crank arm, ft

U = ratio of connecting rod length to crank arm

θ = angle of the crank

W = weight in lb of the piston, piston rod, crosshead, and top half of connecting rod

Attached pumps should be treated as described under Balancing, p. 1160. The vertical component of the centrifugal force of the rotating parts is not considered as, when combined with the horizontal component, the resultant is a

The valve gear is here separated into (1) ahead reciprocating parts, consisting of valves, valve stems, balance pistons, link blocks, and one-half the links, suspension rods, and ahead eccentric rods; (2) astern reciprocating parts, consisting of one-half the links and astern eccentric rods; (3) ahead rotating parts, consisting of the ahead eccentrics and straps and one-half the ahead eccentric rods; and (4) astern rotating parts, consisting of astern eccentrics and straps and one-half the astern eccentric rods.

Example. The method will be applied to a quadruple-expansion engine 26-in., 38-in., 55-in., 81-in. by 51-in. stroke designed for 3,000 ihp at 70 rpm. The particulars of the engine and the data required to construct the polygons are given in Tables 20 and 21.

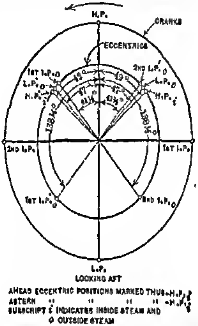


Fig. 115.—Arrangement of cranks and eccentrics.

The arrangement of cranks and eccentrics is shown in Fig. 115 and the arrangement of cylinders and valves in Fig. 116. The air, bilge, and sanitary pumps attached to the low-pressure crosshead have an upward resultant of 560 lb, which is subtracted from the low-pressure reciprocating weights.

Table 20

Designed rps (N) (70 ÷ 60):	1.167
Length of crank arm (A), ft	2.125
Length of connecting rod (B), ft	9.580
Connecting rod ratio (U)	4.500
Length of eccentric rods (b), ft	9.250
$1.226N^2A$	3.540

	H-p	1st i-p	2d i-p	L-p
Travel of valves, in	8.0	8½	8½	9.0
Primary reduction factor for valve-gear weights	0.1570	0.1670	0.1670	0.1760
Secondary reduction factor for valve-gear weights	0.0057	0.0064	0.0064	0.0071

radial force. The friction, weight, and inertia of the valve gear has little effect upon the turning effort and need not be considered.

The dead weight w, which includes all of W and in addition the bottom half of the connecting rod, the crankpin, and the eccentric portion of the webs, must be added directly to the pressure on the piston during the down stroke and subtracted during the up stroke. In Fig. 123, if BB be laid off from AA as a base to represent w per sq in. of piston area and CC laid off from BB to represent F per sq in. of piston area, then the ordinates between CC and $EFGH$ will represent the vertical pressure per square inch on the piston due to the steam and to the weight and inertia of the moving parts.

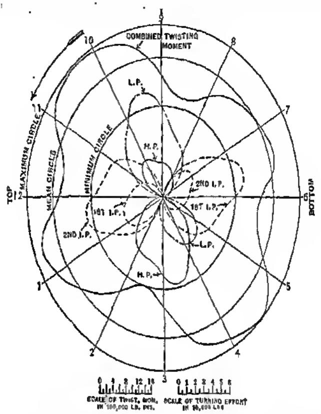


FIG. 124.—Twisting-moment diagram for quadruple-expansion engine.

In Tables 23 to 24 the data and calculations and in Fig. 123 the twisting moment diagram are shown for a 24½ in.-41½ in.-72 in. × 48 in. stroke triple-expansion engine. The length of the connecting rod is 9 ft and the rpm = 72. The twisting moment in pound-inches is plotted radially on the corresponding crank position for each crank separately, and the three smaller figures then combined into the resultant LMN. The ratio of the maximum to the mean twisting moment as shown by the figure is 1.32. The mean twisting moment is 1,800,000 lb-in. against 1,850,000 lb-in. by the formula (63,000 ihp ÷ rpm).

In Fig. 124 is shown the twisting-moment diagram for a 27-in., 39-in., 58-in., 87-in. by 54-in. stroke quadruple-expansion engine turning at 85 rpm. The ratio of maximum to mean twisting moment in this case is 1.233.

Table 21. Weights and Moments
(Moments are expressed in foot-pounds)

	Rotating				Reciprocating			
	H-p	1st i-p	2d i-p	L-p	H-p	1st i-p	2d i-p	L-p
Actual weights:								
Cylinder parts....	10,400	10,400	11,000	11,000	5,500	6,500	7,000	8,900
Valve-gear parts ahead.........	1,580	1,580	2,220	2,220	1,540	2,230	4,230	5,580
Valve-gear parts astern.........	1,600	1,600	2,250	1,250	500	500	500	500
Virtual primary weights:								
Valve-gear parts ahead.........	250	260	370	390	240	370	710	980
Valve-gear parts astern.........	250	270	380	400	80	80	80	90
Virtual secondary weights:								
Valve-gear parts ahead.........	9	10	14	16	9	14	27	40
Valve-gear parts astern.........	9	10	14	16	3	3	3	4
Moment arms about h-p:								
Cylinders........	0	23.08	16.02	7.06	0	23.08	16.02	7.06
Valves...........	−2.85	26.33	19.73	11.92	−2.85	26.33	19.73	11.92
Moments about h-p.								
Cylinder parts....	0	240,000	176,000	78,000	0	150,000	112,000	63,000
Valve-gear parts ahead.........	−700	6,800	7,300	4,700	−700	9,700	14,000	11,700
Valve-gear parts astern.........	−700	7,100	7,500	4,800	−200	2,100	1,600	1,100
Moment arms about 1st i-p:								
Cylinders........	23.08	0	7.06	16.02	23.08	0	7.06	16.02
Valves...........	25.93	−3.23	3.35	11.16	25.93	−3.23	3.35	11.16
Moments about 1st i-p:								
Cylinder parts....	240,000	0	77,500	176,000	127,000	0	49,500	142,500
Valve-gear parts ahead.........	6,500	−800	1,200	4,400	6,200	−1,200	2,400	11,000
Valve-gear parts astern.........	6,500	−900	1,300	4,500	2,100	−300	300	1,000

Primary-force Polygon (Fig. 117). From O as a center to a convenient scale construct the polygon $OAEGI$ of which each side represents by its length the weight (Table 21) of the cylinder reciprocating parts for one of the four cranks and parallels the crank position. The letter at the end of each line is the same as that marking the corresponding crank position. In similar manner construct the polygon $OBCDFHJKL$ for the valve-gear reciprocating parts, the sides being equal in length to the virtual primary weights (Table 21) of the various valve-gear reciprocating parts and parallel to the corresponding eccentric positions. On account of the relatively small weight of the valve gear it is convenient to use a larger scale for the valve-gear polygons. The closing line LO to the scale of the main polygon $OAEGI$ reduces to MO. IO and MO are combined by means of the parallelogram $OMNI$ into NO, which then represents the weight and position of an imaginary weight which revolving on the crank radius would completely balance the primary forces of the main and valve-gear reciprocating parts. To obtain the absolute value of the vertical force, ON is multiplied in length by $1.226N^2A$ ($= 3.54$) and the new line OP laid off to the scale which it is desired to use when plotting the forces

Table 22

Cylinder	H-p	I-p	L-p
Area of cylinder	471.44	1352.7	4071.5
$1.226N^2A$ ÷ cylinder area	0.0075	0.00261	0.00087
Dead weight on piston (w)	10,350	11,750	13,400
Dead weight ÷ cylinder area	22.0	8.7	3.3
Weights for inertia forces (W)	4,450	6,950	7,500

Table 23. Inertia Forces

Crank position θ deg	$\frac{\cos\theta + \frac{\cos 2\theta}{U}}{}$	Inertia force—psi cylinder		
		H-p	I-p	L-p
0	1.2222	40.5	22.1	7.9
30	0.9771	32.5	17.6	6.4
60	0.3889	13.0	7.0	2.5
90	−0.2222	− 7.4	− 4.0	−1.5
120	−0.6111	−20.4	−11.1	−4.0
150	−0.7549	−25.2	−13.7	−4.9
180	−0.7778	−25.9	−14.0	−5.1
210	−0.7549	−25.2	−13.7	−4.9
240	−0.6111	−20.4	−11.1	−4.0
270	−0.2222	− 7.4	− 4.0	−1.5
300	0.3889	13.0	7.0	2.5
330	0.9771	32.5	17.6	6.4
360	1.2222	40.5	22.1	7.9

Table 24. Twisting Moment

Crank position	Arms—Fig. 123			Pressure in psi from Fig. 123*			Twisting moment, lb-in.			
	H-p	I-p	L-p	H-p	I-p	L-p	H-p	I-p	L-p	Total
1	14.4	24.0	9.6	98	25	11.5	665,000	810,000	449,000	1,924,000
2	22.8	22.8	0.0	115	26	− 2.0	1,236,000	802,000	0	2,038,000
3	24.0	14.4	9.6	128	13	3.7	1,450,000	253,000	145,000	1,848,000
4	18.5	0.0	18.5	118	−23	5.2	1,030,000	0	391,000	1,421,000
5	9.6	14.4	24.0	40	34	6.5	181,000	662,000	635,000	1,478,000
6	0.0	22.8	22.8	56	45	8.5	0	1,385,000	788,000	2,173,000
7	9.6	24.0	14.4	66	50	9.3	298,000	1,623,000	545,000	2,466,000
8	18.5	18.5	0.0	22	46	− 4.5	628,000	1,150,000	0	1,778,000
9	24.0	9.6	14.4	80	23	8.5	906,000	298,000	498,000	1,702,000
10	22.8	0.0	22.8	70	−16	13.0	752,000	0	1,205,000	1,957,000
11	14.4	9.6	24.0	34	22	14.8	231,000	285,000	1,445,000	1,961,000
12	0.0	18.5	18.5	80	25	14.5	0	625,000	1,090,000	1,715,000

* Including steam pressure, inertia force, and dead weight.

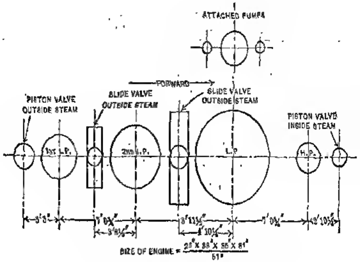


Fig. 116.—Arrangement of cylinders and valves.

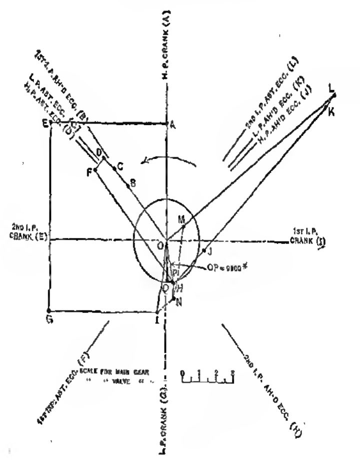


Fig. 117.—Primary-force polygon.

Revolutions for Different Percentages of Full Power. The revolutions of reciprocating engines corresponding to different indicated horsepowers may be taken from the curve in Fig. 125 which is plotted from data from a number of engines. If the revolutions for full power are known, the revolutions for other powers will agree closely with values taken from the curve.

Backing Power of Marine Engines. In reversing a reciprocating engine a much greater proportion of ahead full power can be obtained than in a turbine installation in which the backing turbine is a separate element, the power of which is kept as low as consistent with safety in handling the ship in order to reduce the turbine size. Commander S. M. Robinson in the *Jour. A.S.N.E.* for February, 1916, gives the results of the backing trials of the battleship "Delaware" when suddenly reversed from full speed ahead. At the beginning, 44 per cent of full ahead power (28,500 ihp) was developed at 40 per cent of ahead revolutions (130 rpm), which had increased to 55 per cent at 54 per cent of revolutions when the vessel stopped. The maximum power developed as the backing continued was 81 per cent at 74 per cent of revolutions. The Coast Guard cutter "Tallapoosa," when reversed, developed 53 per cent of full ahead power (1,000 ihp) at 62 per cent of ahead revolutions (129 rpm), 60 per cent at 78 per cent of revolutions and 90 per cent at 109 per cent of revolutions. The torque developed while backing often exceeds the full power ahead torque, the backing trials of the "Delaware" showing about 9 per cent excess.

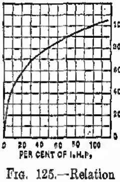


Fig. 125.—Relation of ihp and rpm.

WEIGHTS AND DIMENSIONS

Weights of Marine Engines. There are many methods, some of which are very complicated, for estimating the weights of marine engines. For most purposes, the total weight of the engine is all that is desired and that may be obtained in several ways, the most direct of which is based on ihp. The following **weights per ihp of vertical marine engines** will give a general idea of the weights of different types: compounds, 65 to 75 lb; three-cylinder triples, 85 to 110 lb; quadruples, 110 to 135 lb. However, on account of the different ratings of ihp which the same size engine may have, the total weight may be estimated much more closely if based on cylinder volumes; or, for the sake of simplicity, the sum of the squares of the cylinder diameters multiplied by the stroke, all in inches, which may be designated as Σ. The product of $\Sigma \times$ constant C (which varies with the number of cylinders and the length of stroke) closely approximates the weight of the engine. C for vertical marine engines, exclusive of attached pumps and condensers, for engines with 24 to 40 in. stroke may be taken as follows: 0.9 to 0.75 for three cylinder triples; 1.10 to 0.9 for four-cylinder triples. For engines with 40 to 66 in. stroke: 0.75 to 0.6 for three-cylinder triples; 0.9 to 0.7 for four-cyclinder triples; and 0.9 to 0.7 for quadruples. For compound engines with 20 to 30 in. stroke, C may be taken as 1 to 0.85. These constants hold for engines of usual proportions but, for very light or very heavy engines, the constants may come outside of the range given.

Center of Gravity of Engines. In computing trim and stability of a vessel the center of gravity of machinery must be known. In calculating this

ELEMENTARY GEOMETRY AND MENSURATION

GEOMETRICAL THEOREMS

(For geometrical constructions, see p. 101)

Right Triangles. $a^2 + b^2 = c^2$. (See Fig. 1). $\angle A + \angle B = 90°$. $p^2 = mn$. $a^2 = mc$. $b^2 = nc$. See also p. 105 and p. 132.

Oblique Triangles. (See also pp. 105, 134.) Sum of angles = 180°. An exterior angle = sum of the two opposite interior angles. (Fig. 1.)

The medians, joining each vertex with the middle point of the opposite side, meet in the center of gravity G (Fig. 2), which trisects each median.

The altitudes meet in a point called the **orthocenter**, O.

The perpendiculars erected at the midpoints of the sides meet in a point C, the center of the circumscribed circle. [In any triangle G, O, and C lie in line, and G is two-thirds of the way from O to C.]

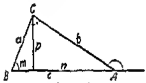


FIG. 1.

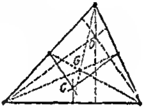


FIG. 2.

The bisectors of the angles meet in the center of the inscribed circle (Fig. 3).

The largest side of a triangle is opposite the largest angle; it is less than the sum of the other two sides, and greater than their difference

FIG. 3.

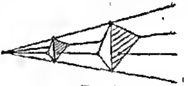


FIG. 4.

Similar Figures. Any two similar figures, in a plane or in space, can be placed in "perspective," that is, so that straight lines joining corresponding points of the two figures will pass through a common point (Fig. 4). That is, of two similar figures, one is merely an enlargement of the other. Assume that each length in one figure is k times the corresponding length in the other; then each area in the first figure is k^2 times the corresponding area in the second, and each volume in the first figure is k^3 times the corresponding volume in the second. If two lines are cut by a set of parallel lines (or parallel planes), the corresponding segments are proportional.

The Circle. (See also pp. 106, 137.) An angle inscribed in a semicircle is a right angle (Fig. 5). An angle inscribed in a circle, or an angle between a chord and a tangent, is measured by half the intercepted arc (Fig. 6). An angle formed by any two lines which meet a circle is measured by half the sum or half the difference of the intercepted arcs, according as the point of intersection of the lines lies inside (Fig. 7) or outside the circle (Fig. 8).

A tangent is perpendicular to the radius drawn to the point of contact.

If a variable line through A (Figs. 9 and 10) cuts a circle in P and Q, then

$\overline{AP} \times \overline{AQ}$ is constant; in particular, if A is an external point, $\overline{AP} \times \overline{AQ} = \overline{AT}^2$, where AT is the tangent from A.

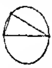


FIG. 5.

FIG. 6.

FIG. 7.

FIG. 8.

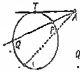


FIG. 9.

FIG. 10.

The radical axis (Fig. 11) of two circles is a straight line such that the tangents drawn from any point of this line to the two circles are of equal length. If the two circles intersect, the radical axis passes through their points of intersection. In any case, the radical axis bisects the common tangents of the two circles. The three radical axes of a set of three circles meet in a common point. (For equations, see p. 137.)

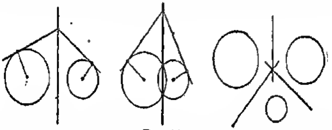


FIG. 11.

Dihedral Angles. The dihedral angle between two planes is measured by a plane angle formed by two lines, one in each plane, perpendicular to the edge (Fig. 12). (For solid angles, see p. 110.)

In a **tetrahedron,** or triangular pyramid, the four medians, joining each vertex with the center of gravity of the opposite face, meet in a point, the center of gravity of the tetrahedron; this point is ¾ of the way from any vertex to the center of gravity of the opposite face. The four perpendiculars erected at the circumcenters of the four faces meet in a point, the center of the circumscribed sphere. The four altitudes meet in a point called the orthocenter of the tetrahedron. The planes bisecting the six dihedral angles meet in a point, the center of the inscribed sphere.

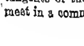


FIG. 12.

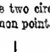


FIG. 13.

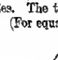


FIG. 14.

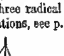


FIG. 15.

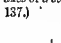


FIG. 16.

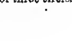


FIG. 17.

Regular Polyhedra (see also p. 110): Regular tetrahedron (Fig. 13), bounded by four equilateral triangles; cube (Fig. 14), bounded by six squares; octahedron (Fig. 15), bounded by eight equilateral triangles; dodecahedron (Fig. 16), bounded by twelve regular pentagons; icosahedron (Fig. 17), bounded by twenty equilateral triangles. Figs. 13–17 show how these solids can be made by cutting the surface out of paper and folding it together.

The Sphere. (See also p. 109.) If AB is a diameter, any plane perpendicular to AB cuts the sphere in a circle, of which A and B are called the poles. A great circle on the sphere is formed by a plane passing through the center. A spherical triangle is bounded by arcs of great circles (see p.

it is necessary to approximate the location of the center of gravity of the main engine. The **vertical center of gravity** of a vertical marine engine above the center of the crankshaft is approximately 1.75 times the length of stroke. The **horizontal center of gravity** aft of the center of the forward cylinder depends on the order of cylinders, but the following constants, in which D is the sum of the cylinder diameters, will be found sufficiently accurate: compounds with the high-pressure cylinder forward, about $0.5D$; three-cylinder triples with high-pressure cylinder forward and low-pressure aft, about $0.65D$; four-cylinder triples with the low-pressure cylinders on the ends, about $0.65D$; quadruples with the high-pressure cylinder forward, about $0.75D$.

Over-all Dimensions of Engines. In laying out an engine-room arrangement it is necessary to know approximately the over-all dimensions of the engine. The design of the engine affects these dimensions to a certain extent, but in general the length of each type of engine bears a fairly constant relation to the sum of the cylinder diameters, which may be designated as D, and the height bears a similarly constant relation to the length of stroke. The constants given in Table 25 represent a fair average for vertical marine engines

Table 25. Over-all Dimensions of Vertical Marine Engines
(D = sum of cylinder diameters)

Type of engine	Compound	Three-cylinder triple valves on center line	Three-cylinder triple valves at side	Four-cylinder triple	Quadruple
$\frac{\text{Length over cylinders}}{D}$	2.00	1.80	1.65	1.85	1.90
$\frac{\text{Length over bedplate}}{D}$	1.70	1.70	1.70	1.60	1.75
$\frac{\text{Height, center of shaft to top of cylinders}}{\text{Stroke}}$	5.25	4.75	4.75	4.75	4.75
$\frac{\text{Width of bedplate}}{\text{Stroke}}$	2.7 to 3.0 for all types				

Length of Engine Room. Proceeding along the same lines as above, there are certain ratios that can be used to approximate the length of the engine room for average conditions. When the machinery is amidships, the length of the engine room is approximately from 2.25 to 2.75 times the sum of the cylinder diameters; when in the stern, the length from the forward engine-room bulkhead to the after-peak bulkhead is approximately from 2.5 to 3.5 times the sum of the cylinder diameters for triple and quadruple engines. With compound engines the length of engine room is about five times the sum of the cylinder diameters.

MACHINING AND TOLERANCES

The allowances and limitations given below are based on good American practice for merchant marine engines.

Table 1. Nozzle Coefficient Correction for Over- or Underexpansion

	Underexpanding					Overexpanding				
d = nozzle deviation...	−40	−30	−20	−10	0	+10	+15	+20	+25	+30
Velocity coef factor....	0.964	0.977	0.989	0.997	1.00	0.997	0.993	0.984	0.965	0.933

should be the same as that entering if conditions were ideal. Actually there are friction and turbulence in this passage and the result is a loss that appears as a diminution of exit velocity and a gain in enthalpy of the steam. The bucket velocity coefficients as derived from tests are given in Table 2.

Table 2. Bucket Velocity Coefficients

Velocity relative to bucket, fps..........	200	400	600	800	1,000	1,500	2,000	2,500	3,000	4,000
Velocity coef.........	*0.953*	*0.918*	*0.888*	*0.863*	*0.841*	*0.801*	*0.774*	*0.754*	*0.737*	*0.716*

Blade velocity diagrams are shown in Figs. 4 and 5. The various velocities can be calculated as outlined previously and these diagrams drawn. The work done and efficiencies obtained will be as given in *Eqs.* (1) to (13).

IMPULSE TURBINES

The total available heat drop is divided among several stages, each stage consisting of a set of nozzles and a wheel carrying a row of buckets. The velocity ratio $\rho = u/v_1$, can be made high enough to obtain good efficiency, provided v_1 can be held down. This is possible when a relatively small amount of energy is utilized in each stage. Usually the pressure drop is so subdivided that the same amount of energy is utilized in each stage. The first wheels can then be made of the same pitch diameter. Often the first stage is made with velocity compounding so as to utilize a greater portion of the energy and thereby reduce the pressure and temperature considerably. This lowers the maximum temperature and pressure to which the casing is subjected, reduces the density of the steam through which the wheel must revolve, and makes the leakage problem easier. As the steam goes through the turbine, it expands and occupies more space; consequently the pitch diameters of the last wheels are usually made larger so as to provide more area. This also results in greater blade velocity, but this is permissible in view of the lower density of steam in these stages. In the first stages the nozzles occupy only a portion of the periphery but, as progress through the turbine is made and the steam increases in volume, more and more of the periphery is used. The approximate number of stages required can be determined as follows, assuming that the pitch diameter of the blade, energy per stage, and the velocity ratio, ρ, are held constant:

$$\left(\frac{\text{Speed for single stage}}{\text{Desired speed}}\right)^2 = \text{number of pressure stages} \qquad (25)$$

Speed for single stage is the speed that would be reached if the total energy were used in one stage with the pitch diameter and velocity ratio, ρ, as decided upon.

Interior of Cylinders. The bores of cylinders and piston-valve chests should be cylindrical within the following tolerances:

From 20 to 40 in. diam. .... 0.003–0.005 in.
Larger than 40 in. diam. .... 0.005–0.008 in.
Difference in diam at top and bottom .... 0.010 in.

Cylinder Bore and Piston Clearances. Between the bore of the cylinder and the diameter of the piston and follower the clearances in inches should not be less than the following:

Diam. of cylinder, in.	H-p	I-p	L-p
21–30	0.035	0.0275	0.025
31–40	0.045	0.035	0.030
41–50	.....	0.05	0.0375
51–60	.....	0.06	0.050
61–80	.....	.....	0.065
81–100	.....	.....	0.085

Working Fits of Piston and Piston-Valve Rings. The following allowances have been found satisfactory:

Diameter, in.	Minimum, in.	Maximum, in.
12–15	0.004	0.007
16–20	0.006	0.009
21–30	0.007	0.015
31–40	0.010	0.020

Slide Valves and Seats. Working surfaces should be true planes, free from blemish and thoroughly smooth. It is advisable to plane the working surfaces of slide valves at right angles to the direction of planing of the seats.

Piston rods and valve stems should be accurately ground or machined within the following tolerances:

Tolerances on diam. plus 0.012 in. minus 0.005 in.
Deviation from exact roundness, 0.003 in.

Bearings should be accurately machined and fitted, and special care taken to secure hardness and smoothness. The following tolerances in inches are permissible:

Diam. of shaft, in.	Plus or minus in diam.	Deviation from exact roundness
6	0.005	0.0025
10	0.006	0.003
16	0.007	0.0035
Larger	0.008	0.004

Figure 8 shows a cross section through the high-pressure element of a Westinghouse marine turbine. Steam flows from right to left. The first stage has two velocity compoundings, and the succeeding stages are simple impulse. The pitch diameter of all these stages is approximately constant.

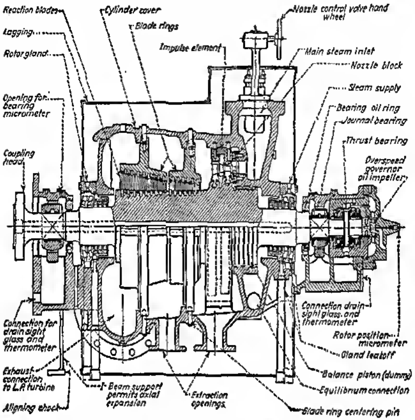


FIG. 8.—Typical cross section of a Westinghouse high-pressure turbine.

The steam exhausts from this casing into another lower pressure turbine (Fig. 9).

Rotation Loss or Windage. Experiments have shown that the rotation of the disk carrying the buckets requires energy equal to

$$L_d = \frac{143.35D^2}{\overline{V}}\left(\frac{u}{100}\right)^{2.9} \qquad \text{Btu per hr} \qquad (26)$$

and the rotation of the buckets requires energy equal to

$$L_b = \frac{638.2Dh^{1.25}}{\overline{V}}\left(\frac{u}{100}\right)^{2.8} \qquad \text{Btu per hr} \qquad (27)$$

where D = pitch diameter, ft
$\overline{V}$ = specific volume of steam, cu ft per lb
u = wheel speed at pitch diameter, fps
h = mean blade height, in.

The following clearances for **pins and journals** are considered satisfactory:

Clearance in in. for pins		Diameter, in.	Clearance in in. for journals	
Max	Min		Max	Min
0.005	0.002	5	0.005	0.0025
0.008	0.004	10	0.010	0.005
0.012	0.006	15	0.015	0.0075
0.015	0.008	20	0.020	0.010
0.018	0.010	25	0.025	0.0125

Bearing clearances in inches as follows are recommended for large engines in good alignment:

Crosshead pin brasses	0.006–0.007
Crankpin brasses	0.008–0.009
Main bearings	0.009–0.010
Eccentric straps	0.010–0.015
Crosshead slippers	0.010–0.012
Thrust shoes	0.005–0.007

Stern Bearings. Lignum-vitae must be well water-soaked before being bored out. The following clearances are permissible on **lignum-vitae bearings**:

Diam of journal, in.	Min clearance in in. for new bearings.	Wear in in. at which bearings should be renewed
8	0.050	$\frac{1}{4}$
10	0.060	$\frac{1}{4}$
12	0.075	$\frac{5}{16}$
16	0.100	$\frac{3}{8}$
20	0.125	$\frac{3}{8}$

Bearing Brasses. Sufficient clearance to permit of expansion should be allowed at the ends of the brasses where the two halves meet. Cooling water should not leak onto the bearing surface. Hollow brasses should be tested to demonstrate their ability to withstand pressure of water service.

Thrust bearings must be so fitted that all the rings perform equal duty.

Allowance for Expansion in Castings. Large iron castings subjected to steam of high temperature will expand or grow and take a permanent set. Where large cylinders are used, ample allowance for fore-and-aft expansion of the cylinders must be made in locating valve-rod guides and eccentrics.

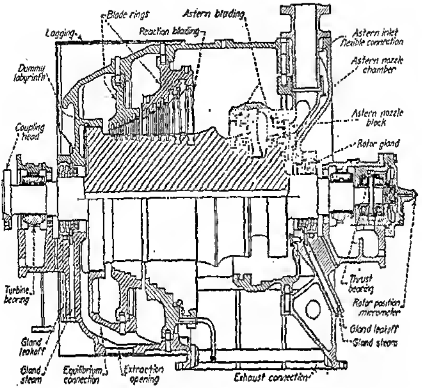


FIG. 9.—Typical cross section of a Westinghouse low-pressure turbine.

These are the losses for dry and saturated steam; for steam of other character, correct according to Table 3. For wheels above 36 in. diameter increase the losses 4 per cent above those as calculated by these formulas.

Table 3. Variation of Rotation Loss with Steam Quality

Superheat deg F, or moisture, %	100°	50°	0	2	4	6	8	10	12
Rotation loss factor	0.82	0.91	1.0	1.04	1.09	1.19	1.23	1.35	1.81

REACTION TURBINES

Referring to Fig. 5, the velocity of the steam v_1 is determined in the same manner as in the case of any nozzle except that, usually, reaction turbines are designed so that the steam leaving the moving blades of one stage enters the stationary blades of the succeeding stage at the proper angle to utilize the entering velocity. If v_2 is the leaving velocity of the preceding stage then

$$v_1 = 223.7 \sqrt{\left[\left(\frac{v_2}{223.7}\right)^2 + (h_a - h_b)\right] e_{n_s}} \text{ ft per sec} \quad (28)$$

where $h_a - h_b$ = isentropic drop through the stationary blades or nozzle
e_{n_s} = nozzle efficiency (stationary nozzle, see Figs. 5 and 10).

STEAM TURBINES

BY

J. M. LABBERTON

REFERENCES: Stodola, "Steam and Gas Turbines" (translated by L. C. Loewenstein), McGraw-Hill. Goudie "Steam Turbines," Longmans. Kearton, "Steam Turbine Theory and Practice," Pitman. Church, "Steam Turbines," McGraw-Hill.

TYPES

There are three fundamental types of turbines based on the manner in which the staging or blading makes use of the energy in the steam.

1. Impulse staging, in which the steam expands or gives up part of its thermal energy in the stationary nozzles, this thermal energy being converted to kinetic energy which in turn is converted into energy in the form of speed and torque at the shaft. In the impulse stage there is no change in pressure

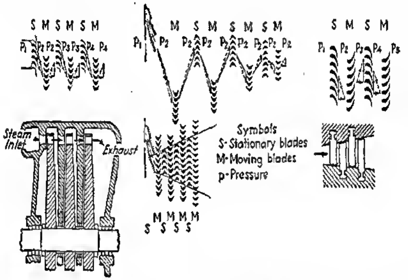


FIG. 1.—Impulse turbine with single-velocity stages. FIG. 2.—Impulse turbine with multivelocity stages. FIG. 3.—Reaction turbine.

FIGS. 1–3.—Diagrammatic illustrations of turbine elements with corresponding bucket-velocity diagrams.

in passing through the moving blade. There is a gain in thermal energy due to friction and other losses, after leaving the nozzle, known as reheat.

2. Reaction staging, in which the steam expands or gives up part of its thermal energy in the stationary nozzles and also gives up still more in the moving blades. Actually a reaction stage is a combination of impulse and reaction.

3. Velocity compound staging, in which the steam, after having part of its thermal energy converted into kinetic energy in the stationary nozzles, gives up part of this kinetic energy to one set of moving blades, then, being redirected by stationary blading or guiding devices, gives up more of the

In similar manner,

$$R_2 = 223.7 \sqrt{\left[\left(\frac{R_1}{223.7}\right)^2 + (h_{b'} - h_{b''})\right] e_{nm}} \text{ ft per sec} \quad (29)$$

where $h_{b'} - h_{b''}$ = isentropic drop through the moving blades or nozzle
e_{nm} = nozzle efficiency (moving nozzle, see Figs. 5 and 10)

TURBINE LOSSES

The losses in a steam turbine are as follows (see Fig. 11):

1. Leakage of Steam. In a single-stage impulse turbine there should be no leakage because all the steam goes through the nozzles and thereby is immediately brought to exhaust pressure. However, in multistage turbines, the steam from one stage enters a nozzle or nozzles at the exit of that stage and expands to the next lower stage. Naturally, both wheels of these two stages are carried on the same shaft. These two stages, therefore, must be separated by a wall or diaphragm, which, where it approaches the shaft, must have a packing arrangement or stuffing box because there is relative motion at that point. Therefore, there will be a tendency toward leakage at that point and steam will go through there instead of through the nozzle or nozzles, the resultant energy being lost. This loss may be as much as 5 per cent of the available energy.

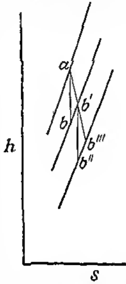


FIG. 10.—Condition of steam passing through reaction blading. b is condition leaving stationary and entering moving blades.

2. Nozzle Losses. As explained on p. 1184, there are frictional resistance and turbulence in the nozzle, resulting in a loss of 3 to 13 per cent of the available energy.

3. Blade Loss. Owing to the friction and turbulence in the blade passage there is a loss. Also there is a leaving loss due to the exit velocity of the steam from the blades. The total is 10 to 25 per cent of the available energy.

4. Windage Losses. The wheel of the turbine containing the blades rotates in an enveloping mass of dead or relatively motionless steam at the pressure and density of the steam leaving the blades. The churning of this steam requires energy, and torque at the shaft is reduced thereby. Fortunately, the largest blades and the largest diameter wheels are at the lower stages where the steam density is lower. These losses range from 2 to 7 percent of the available energy.

5. Mechanical Losses. These are due to friction of the bearings and to any load of accessories such as oil pumps and governors that may be driven from the main shaft. These losses should amount to not more than 2 per cent of the available energy.

kinetic energy to another set (or the same set) of moving blades, and so on to another and another. The velocity compounded stage is a special case of the impulse, and there is no change in pressure once the steam has left the stationary nozzles. There is the gain in thermal energy (reheat) as in the impulse stage. Simple impulse is velocity compounding with one set of moving blades instead of several or redirection.

Multistage turbines are composed of a number of stages and all three types of staging may exist or be incorporated in one turbine.

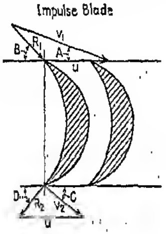


FIG. 4.

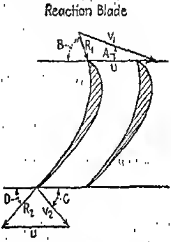


FIG. 5.

The velocity and direction of the steam through the blades are shown in Figs. 4 and 5 where the following symbols are used. All velocities are in feet per second, enthalpies in Btu per pound.

v_1 = absolute velocity of jet at blade entrance
v_2 = absolute velocity of jet at blade exit
R_1 = relative velocity of jet at blade entrance
R_2 = relative velocity of jet at blade exit
u = peripheral velocity of blade
h_1 = enthalpy at blade entrance
h_2 = enthalpy at blade exit
w = weight of steam passing, lb per sec

The work done on the blade is

$$W_b = \frac{w[(v_1^2 - v_2^2) - (R_1^2 - R_2^2)]}{50,000} \quad \text{Btu per sec} \qquad (1)$$

or

$$W_b = \frac{w}{g}(v_1 \cos A - v_2 \cos C)u \quad \text{ft-lb per sec} \qquad (2)$$

or

$$W_b = \frac{w}{g}(R_1 \cos B + R_2 \cos D)u \quad \text{ft-lb per sec} \qquad (3)$$

The force on the blade is

$$F_b = \frac{w}{g}(v_1 \cos A - v_2 \cos C) \quad \text{lb} \qquad (4)$$

or

$$F_b = \frac{w}{g}(R_1 \cos B + R_2 \cos D) \quad \text{lb} \qquad (5)$$

6. Radiation from Casing. This loss is usually so small that it can be ignored.

Turbine Efficiency. There are three types of efficiency generally considered.

1. *Thermal efficiency* is the ratio of the turbine output to the difference between the energy in the steam at the throttle and the energy in the liquid water at exhaust pressure. Figure 7 shows a temperature-entropy diagram. If h_a is the enthalpy at a, h_b the enthalpy at b, and h_c the enthalpy at c, then if P is the output at the turbine shaft in Btu, thermal efficiency is

$$E_{\text{thermal}} = \frac{P}{h_a - h_c} \qquad (30)$$

The logic behind this definition is that $h_a - h_c$ is the amount of energy that must be put into each pound of steam by the boilers. A reference to

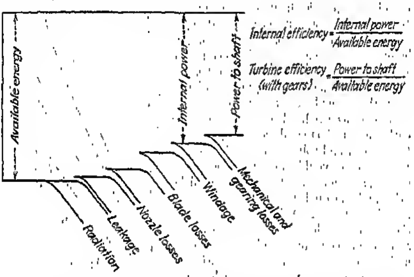


FIG. 11.

the diagram will make this clear. Assuming that the feed water enters the boilers at condition c, this must first be raised in temperature to condition d, then evaporated to condition e, and finally superheated to condition a, under which condition it enters the throttle. In short, the total amount of energy that must be supplied to the turbine per pound of steam is $h_a - h_c$.

2. *Engine efficiency* or turbine efficiency is the ratio of the turbine output to the energy available to the turbine. In Fig. 7 the energy $h_b - h_c$ all goes into the sea water used in the condenser. There is no way known to man at the present time of converting heat of condensation into mechanical power. A turbine that is 100 per cent efficient would exhaust under condition b. Therefore, the total energy available to the turbine is $h_a - h_b$. The engine efficiency or turbine efficiency, therefore, is

A comparison of engine or turbine efficiency is a much more illuminating method of judging turbines as to performance than any other. A com-

The torque in foot-pounds or force acting at a 1-ft radius where r ft is the mean radius of the row of blades is

$$\text{Torque} = F_b r = \frac{w}{g}(v_1 \cos A - v_2 \cos C)r \quad \text{ft-lb} \tag{6}$$

or,

$$\text{Torque} = \frac{w}{g}(R_1 \cos B + R_2 \cos D)r \quad \text{ft-lb} \tag{7}$$

In the case of **impulse staging only** the blade efficiency is

$$e_{ib} = \frac{2u(R_1 \cos B + R_2 \cos B)}{v_1^2} \quad \text{where } B = D \tag{8}$$

taking $\rho = \frac{u}{v_1}$ as the ratio of the blade velocity to the jet velocity

$$e_{ib} = (2\rho \cos A - 2\rho^2)\left(1 + \frac{R_2}{R_1}\right) \tag{9}$$

The optimum value for ρ or value for maximum blade efficiency is

$$\rho = \frac{\cos A}{2} \tag{10}$$

This is true regardless of the relative values of R_1 and R_2. R_2 is always less than R_1 in the case of impulse blading on account of friction loss (see p. 1190). The maximum possible blade efficiency is

$$e_{ib\,\max} = \left(\frac{\cos^2 A}{2}\right)\left(1 + \frac{R_2}{R_1}\right) \tag{11}$$

In the case of **reaction staging only** no simple and general expression for efficiency can be derived because of the varying proportions of the available energy converted to kinetic energy in the stationary nozzle and the moving blades. However, as regards the optimum velocity ratio ρ, when approximately the same amount of energy is used in both stationary and in moving blades (or nozzles)

$$\rho = \cos A \tag{12}$$

This means that, in general, the blade speed, u, must be twice as fast in the case of a reaction stage as with an impulse stage. There is no appreciable difference in efficiency.

In the case of **velocity compounding** the velocity ratio can efficiently be much lower than either impulse or reaction staging. If q is the number of compoundings or number of times the steam is redirected then

$$\rho = \frac{\cos A}{2q} \tag{13}$$

for maximum efficiency with **symmetrical and frictionless blades.** Actually, there is always some friction and, as a consequence, the velocity ratio for optimum efficiency is somewhat lower.

Comparison of Impulse and Reaction Turbines. Since there is no pressure drop across the impulse blade, there is little tendency for leakage of steam over the tip of the blade. In the first stage there should be no leak-

parison of water rates, *i.e.*, the pounds of steam per horsepower-hour, does *not* mean much unless throttle and exhaust conditions are known.

3. *Internal efficiency* is the ratio of the energy delivered to the blades to the energy available to the turbine. It corresponds to the "indicated efficiency" of a reciprocating engine. Internal horsepower is equal to power output plus bearing friction losses plus any power used to drive pumps, governors, etc. Therefore, internal efficiency is

$$E_{\text{int}} = \frac{P_{\text{int}}}{h_a - h_b} \tag{32}$$

P_{int} must be in Btu per pound of steam (see Fig. 11).

Reheat. All the internal losses that occur in any stage of a turbine, *i.e.*, those losses occurring before the net power is delivered to the blades, produce friction and turbulence in the steam and thereby reduce the final power delivered. If the negligible amount of heat that might be *transferred out through the casing is* ignored, all these losses are absorbed by the steam itself. This results in more heat in the steam at the exhaust pressure of this stage than would exist if isentropic expansion had occurred. Therefore, since the enthalpy has increased (at this pressure), the entropy and volume will also be greater. If the steam would have been wet under isentropic conditions, the steam would be drier and the quality improved under the actual *conditions. This excess of heat in* the steam at exhaust over and above the amount of heat that would have existed under conditions of isentropic expansion to the same exhaust pressure is called the reheat or RH of that stage.

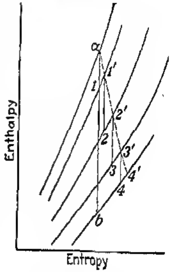


FIG. 12.

Reheat in any one stage means that less energy has been delivered to the blades in that stage and that more energy is thereby available in the next stage. In Fig. 12, suppose the steam to enter the stage under condition a. If isentropic expansion took place, it would exhaust under condition 1. However, since there are losses and since these losses reheat the steam, it actually exhausts under condition 1′, which is at the same pressure but with more heat content. The reheat RH_1 of this first stage is

$$RH_1 = h_1' - h_1$$

If this action is continued through the several stages of the turbine, the path followed by the steam will be as shown on Fig. 12 from a to 1′ to 2′, etc. From this it can readily be seen that, whereas if the whole expansion had been isentropic, the available energy would have been $h_a - h_b$, on account of the losses and reheat the actual available energy was

$$(h_a - h_1) + (h_1' - h_2) + (h_2' - h_3) + (h_3' - h_4)$$

age at all. In the other stages the predominant leakage is usually between the shaft and the diaphragm or sheet separating the stages. In reaction turbines, leakage may be seriously great if the clearance space between the blade tips and casing is an appreciable percentage of the blade length. This is true in the upper or higher pressure stages where the blades are short. In the lower stages where the blades are longer this leakage may be negligible. Tip leakage can be reduced by placing a soft metal shroud over the tips, thereby reducing the clearance to a value lower than would be tolerated for blades alone. In the event of accidental contact, the soft metal will wear away with little consequential damage.

In reaction turbines, owing to the very nature of the action, *i.e.*, pressure drop across the moving blade, it is necessary to have full peripheral entry of steam all around the wheel. The impulse stage has the advantage that stationary nozzles may occupy only a portion of the periphery permitting more liberal design and also permitting economical control by cutting nozzles in and out as the load changes. However, the efficiency increases as more of the perimeter is used. This is due to the greater windage loss of idle blades and also to the necessity of accelerating the idle or stagnant steam riding between the blades between nozzle areas or active zones.

Fairly economical control can be obtained in the case of full peripheral entry (reaction turbines) by by-passing some stages at light loads.

In view of the higher velocity ratio necessary in the case of reaction staging, the enthalpy drop per stage for a given turbine rpm, is much lower for reaction staging. Therefore, more reaction stages than impulse stages are necessary, all other things being equal—approximately twice as many.

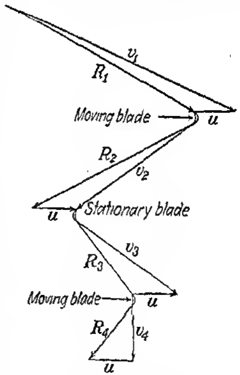


FIG. 6.—Velocity compounding (compounded twice).

Generally, the very first stage is made with velocity compounding as this permits of a still greater enthalpy drop than does simple impulse (Fig. 6). The advantage of this is that when high-pressure high-temperature throttle steam is used the high temperature can be fairly well disposed of in the nozzles of the first stage. This permits of assembling the throttle, steam chest, and first nozzle block separately or not integral with the main turbine casing and thereby obviating the necessity of subjecting the casing to the initial drastic steam conditions.

Capacity of a turbine depends upon the throttle and exhaust conditions and upon the weight of steam passed within a unit time. This latter depends upon the volume of the steam and the area of the path. Since condenser

This sum is invariably greater than $h_a - h_5$ because, as will be disclosed by an inspection of a Mollier diagram of steam, there is a continuing divergence of the constant-pressure curves as the heat content increases. In other words, these curves spread as the enthalpy and entropy increase.

Since the sum is greater than $h_a - h_b$, if the sum is divided by $h_a - h_b$, a factor is obtained by which the apparent available energy is multiplied to obtain the actual available energy. This factor is called the reheat factor or RF.

$$\text{RF} = \frac{(h_a - h_1) + (h_1' - h_2) + (h_2' - h_3) + (h_3' - h_4)}{(h_a - h_b)}$$

and this may run anywhere from unity to 1.1. In a single-stage turbine, the RF would, of course, be unity.

In any turbine the stage efficiency e_s is the ratio of energy delivered to the blades to that available in that stage. For example, in a single-stage turbine, the stage efficiency e_s is equal to the internal efficiency E_{int}. However, for a multistage turbine, the internal efficiency is greater than the stage efficiency because reheat increases the actual energy available. Therefore,

$$E_{int} = \text{RF}e_s \qquad \text{for any turbine} \qquad (33)$$

The impression should not be gained from the foregoing that, since reheat is always available in the next stage, stage efficiency is of no importance because the losses are always available for more work. It should be remembered that there is no opportunity to use any of the reheat in the last stage.

CONDITION CURVE

The path the steam takes in going through the turbine such as shown in Fig. 12 is called the condition curve. The condition curves show the character of the steam in various stages of the turbine, and this permits intelligent selection of materials for use at these points and furnishes information for extraction points for feed-water heating and gland sealing. It is best to obtain condition curves from the manufacturer as he has the most detailed and reliable information. However, when this is not available, a curve can be approximated as follows.

Suppose it is desired to determine the condition curve for a 4,000-shp turbine, receiving throttle steam at 455 psia, 740 F, and exhausting at 1.5 in. Hg abs. Plot initial point a on a Mollier diagram (Fig. 13). Assume 3½ percent pressure drop through the throttle or 16 psi. This is a constant enthalpy process, there being no gain or loss of heat; therefore, in the steam chest the pressure would be 439 psia and the condition shown as point b on the diagram. From Fig. 14 it is seen that the combined efficiency of a 4,000-shp turbine should be 73 per cent. From Fig. 3 (p. 1443), full-load gear efficiency is 96.8 per cent. $\dfrac{100 \times 0.73}{0.968} = 75.5$ per cent engine efficiency of turbine alone. From Fig. 15 it is seen that the mechanical loss for a 4,000-shp turbine is 2.25 per cent. $75.5 + 2.25 = 77.75$ per cent internal efficiency of the turbine. The isentropic drop from 455 psia, 740 F to 1.5 in. Hg abs is 480 Btu per lb (see Fig. 13 point c).

$$480 \times 0.7775 = 374 \quad \text{Btu per lb actual heat drop}$$

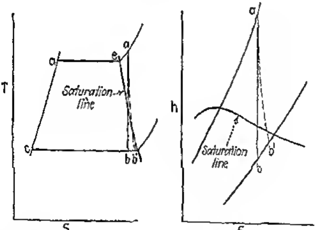
pressure in modern marine installations varies little from 1.5 in. Hg abs. the area of path or the area of annulus of the last row of blades together with throttle conditions is a measure of capacity.

General Advantage of Steam Turbines. Compared with other marine prime movers such as diesel engines or reciprocating steam engines, the turbine occupies less space, is lighter, requires less attention, has lower lubricating oil consumption, no lubricating oil in the exhaust (not a disadvantage as compared with diesel), less vibration, uniform torque, no rubbing parts except bearings, great overload capacity, great reliability, lower maintenance, and, what is perhaps most important of all considering fuel economy, the ability to use steam of higher pressure and temperature at the throttle and to exhaust to lower pressure than any reciprocating steam engine. In the sizes used for ship propulsion the efficiency of the turbine surpasses that of the reciprocating steam engine. Single units can be built in greater capacity than can any other type. The cost is much less. Turbines have the disadvantage of being inherently high-speed machines whereas a ship's propeller is inherently a slow-speed device. This necessitates the interposition of some system of speed reduction such as gearing or electric drive. Also, the turbine, requiring close clearances and relatively fine workmanship, cannot be built in so many plants as can the not so finely constructed reciprocating steam engine.

STEAM FLOW

Nozzles. The velocity of steam leaving a nozzle is

$$v = 223.7\sqrt{(h_a - h_b)e_n} \quad \text{ft per sec} \qquad (14)$$



Temperature-Entropy Diagram Mollier Diagram

FIG. 7.

where e_n = nozzle efficiency

$$e_n = \frac{h_a - h_b'}{h_a - h_b} \qquad \text{(see Fig. 7)} \qquad (15)$$

where $e_n = \dfrac{\text{actual gain in kinetic energy in actual nozzle}}{\text{calculated gain in kinetic energy in ideal nozzle}}$

Enthalpy at point a is 1381.6 Btu per lb.

$$1381.6 - 374 = 1007.6. \quad \text{Btu lb in exhaust}$$

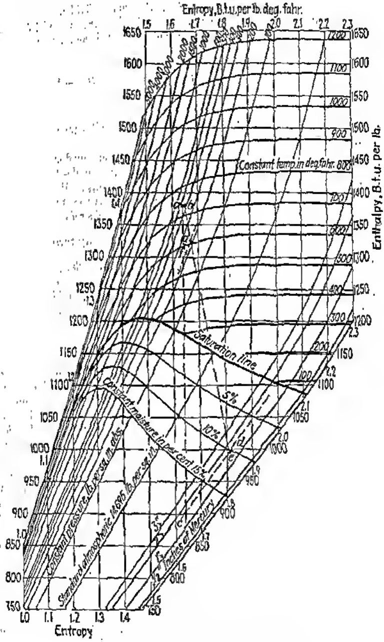


Fig. 13.—Turbine-steam condition curve.

This is the point d at 1.5 in. Hg shown on Fig. 13. In ordinary designs, the steam leaves the blades in the last stage at approximately 700 fps or with an

In well-designed nozzles c_n varies from 0.90 to 1.00 for discharge velocities not greater than 2,000 fps.
Where c_v is the velocity coefficient,

$$c_v = \frac{\text{actual discharge velocity measured by impulse or reaction}}{\text{calculated ideal velocity}}$$

$$c_n = c_v^2 \tag{16}$$

For a given weight of steam passed per second, w, the area at any point, x, in the nozzle in square feet is

$$A_x = \frac{wV_x}{v_x} \tag{17}$$

where V_x is the specific volume of the steam at point x.

If the passage areas are calculated, it will be found that for ordinary pressures at the entrance and exhausting at something less than half this pressure the areas will first decrease in magnitude, *i.e.*, the nozzle is convergent; then, as the discharge is approached, the areas increase in magnitude and the nozzle becomes divergent.

The reason for this is that, at first, the velocity increases faster than does the specific volume but, later, the specific volume increases faster than does the velocity. If the exhaust pressure is not made low enough to permit the latter situation, the nozzle will consist of only the converging portion and will be a *convergent* nozzle. Practically all loss occurs in the divergent position.

The pressure at the point where the area of the nozzle is a minimum is known as the **critical pressure**, and the corresponding velocity is the critical velocity. The ratio of the critical pressure to the initial pressure is

$$\frac{p_t}{p_a} = \left(\frac{2}{n+1}\right)^{\left(\frac{n}{n-1}\right)} \tag{18}$$

where n = exponent in the relationship

$$PV^n = \text{a constant} \qquad \text{(see p. 315)}$$

The *weight of steam*, w, in pounds per second that will flow under ideal conditions, *i.e.*, no friction, or with a discharge coefficient, C_d, of unity is

$$w = 0.668A_b\sqrt{\frac{n}{n-1}\frac{p_a}{\overline{V}_a}\left[\left(\frac{p_b}{p_a}\right)^{\frac{2}{n}} - \left(\frac{p_b}{p_a}\right)^{\frac{n+1}{n}}\right]} \tag{19}$$

where A_b = area at discharge of nozzle be it converging or diverging, sq in.
$\overline{V}_a$ = cu ft per lb steam
p_a = initial pressure, psia
p_b = exit pressure, psia

Obviously the weight of steam flowing is governed by the minimum nozzle area and, since the minimum area in a converging-diverging nozzle is at the throat or critical point, any discharge or exit pressure lower than the critical will have no effect on the weight of steam discharged. In short, the weight discharged with a discharge pressure equal to the critical is not increased

energy of approximately 10 Btu per lb which would be available in the following stage if there were one. Therefore, the condition curve should reach point e, 10 Btu below d, then jump to d. The steam does not take exactly a straight-line path in going from b to e on account of the different efficiencies of the various types of stages passed through, but it is approximately straight. As mentioned before, the first stage is usually velocity compounded twice, and approximately half of the total pressure is expended

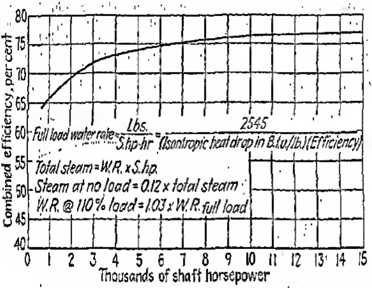


FIG. 14.—Typical combined efficiencies of marine turbines, including gears, at full load.

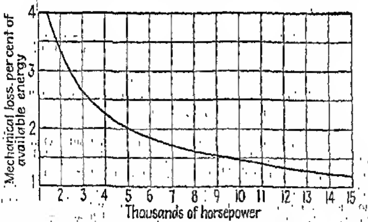


FIG. 15.—Marine-turbine mechanical losses in per cent of available energy.

in this stage. Therefore, the actual pressure in this stage would be approximately $0.50 \times 439 = 220$ psia. The isentropic drop to this pressure, b to f (see Fig. 13), would be to 1302 Btu per lb, point f. The isentropic drop is $1381.6 - 1302 = 79.6$ Btu. The efficiency for such a stage should be approximately 68 percent (see Fig. 16). Therefore, $0.68 \times 79.6 = 54.2$ Btu. $\therefore 1381.6 - 54.2 = 1327.4$ Btu per lb at 220 psia. This is point g. Draw a straight line from g to e. This is the approximate condition curve.

if the discharge pressure is lowered further. Consequently, substituting a value of $n = 1.13$, that corresponding to saturated steam, and discharge pressure equal to the critical in the foregoing equation, the result is

$$w_{sat} = 0.306A_t\sqrt{\frac{p_a}{V_a}} \qquad \text{lb per sec} \qquad (20)$$

In the same manner, if a value of $n = 1.3$, that corresponding to superheated steam, and discharge pressure equal to the critical is substituted, the result is

$$w_{supht} = 0.316A_t\sqrt{\frac{p_a}{V_a}} \qquad \text{lb per sec} \qquad (21)$$

where A_t = throat area, sq in.
p_a = initial pressure, psia
A_a = initial volume, cu ft per lb

These expressions are for steam being discharged at and below the critical pressure.

In the same manner, for steam being discharged at higher pressure than the critical,

$$w_{sat} = 1.97A_b\sqrt{\frac{p_a}{V_a}\left[\left(\frac{p_b}{p_a}\right)^{1.77} - \left(\frac{p_b}{p_a}\right)^{1.89}\right]} \qquad \text{lb per sec} \qquad (22)$$

$$w_{supht} = 1.39A_b\sqrt{\frac{p_b}{V_a}\left[\left(\frac{p_b}{p_a}\right)^{1.54} - \left(\frac{p_b}{p_a}\right)^{1.77}\right]} \qquad \text{lb per sec} \qquad (23)$$

The above weights are for steam flow under ideal conditions. Actually the steam discharge varies somewhat and the "ideal" rate must be multiplied by a discharge coefficient, C_d. C_d varies from 0.97 to 1.02 in the case of nozzles discharging at velocities of from 500 to 3,000 fps. The higher values of discharge coefficient are for wet steam said to be "supersaturated."

A nozzle that first converges and then diverges can be proportioned for only one set of initial and exhaust pressures and, at times, it is required that operation is necessary at other initial and exhaust pressures. The deviation from correct ratio for existing conditions may be expressed as

$$d = \left(\frac{r - R}{r}\right) 100 \qquad (24)$$

where R = correct exit to throat area ratio
r = actual ratio

Obviously this deviation may be positive or negative. If the exit is larger than it should be, the nozzle will overexpand the steam; if the exit is smaller, the steam will be underexpanded. The proper multiplying factor to correct for velocity is given in Table 1.

Bucket Velocity Coefficients. In impulse blading, since there is no expansion effect, the relative velocity of the steam leaving the blade passage

GENERAL DESIGN

Velocity ratio, $\rho = u/v_1$, is an important factor in the turbine efficiency. As previously mentioned, this value must be greatest to obtain maximum efficiency from a reaction stage and can be lower for simple or "one-row" impulse, and still lower for velocity compounding. The more times the velocity is compounded, the lower ρ can efficiently be. The curve (Fig. 16) shows the efficiencies to be expected at different velocity ratios for different types of blading with a nozzle angle of 20 deg. At the low-pressure end of turbines, the blade must usually be so long, sometimes nearly half the radius of the wheel, that the speeds are appreciably different from one end to the other. Since such a situation renders it difficult to satisfy all parts of the blade length with one steam velocity, the blades are made to act as impulse

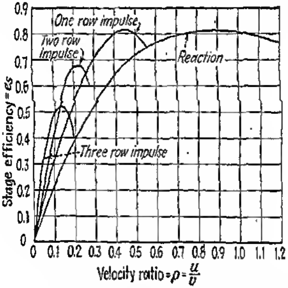


FIG. 16.—Variation of stage efficiency of turbine elements with velocity ratio—nozzle angle 20 deg.

blades at the inner end and as reaction blades at the outer end. When a high heat drop through the system (all stages) is to be contended with, it is often so difficult to reconcile blade speeds in the first stage with those in the last stage that the best solution is to divide the steam expansion into two or more separate casings. Sometimes the low-pressure casings are arranged double flow, *i.e.*, the steam enters at the center and flows toward the ends through identical sets of wheels. The high-pressure element usually is designed for higher speed than the low-pressure element. This construction lends itself well to gear drive for marine propulsion where two or more pinions of different diameter can be meshed with a single (bull) gear.

Owing to the fact that as expansion or heat drop continues below the point of saturation, liquid water appears in the steam stream. This liquid may be 15,000 times as dense as the saturated steam accompanying it in the lower stages. As a consequence, the velocity of the liquid will not be so great as that of the gas. This means that the moving blades will strike these water

134). In two polar triangles, each angle in one is the supplement of the corresponding side in the other. In two symmetrical triangles, the sides and angles of one are equal to the corresponding sides and angles of the other, but arranged in the reverse order (like right-handed and left-handed gloves).

GEOMETRICAL CONSTRUCTIONS

To Bisect a Line AB (Fig. 18). (*a*) From A and B as centers, and with equal radii, describe arcs intersecting in P and Q, and draw PQ, which will bisect AB in M.

(*b*) Lay off $AC = BD$ = approximately half of AB, and then bisect CD.

To Draw a Parallel to a Given Line l Through a Given Point A (Fig. 19). With point A as center drow an arc just touching tho line l; with any point O of the line as center, draw an oro BC with tho same rodius. Then a lino through A touching this arc will ho tho required parallel. Or, use a straight edge and triangle. Or, use a sheet of celluloid with a set of lines parallel to one edge and ahout ¼ in. apart ruled upon it.

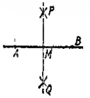


FIG. 18.

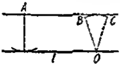


FIG. 19.

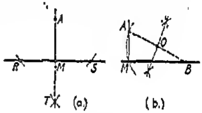


FIG. 20.

To Draw a Perpendicular to a Given Line from a Given Point A Outside the Line (Fig. 20). (*a*) With A as center, describe an arc cutting tho line in R and S, and bisect RS in M. Then M is tho foot of the perpendicular. (*b*) If A is nearly opposite one end of the line, take any point B of the line and bisect AB in O; then with O as center, and OA or OB as radius, draw an arc cutting the line in M. Or, (*c*) use a straight edge and triangle.

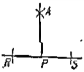


FIG. 21.

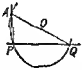


FIG. 22.

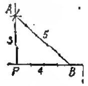


FIG. 23.

To Erect a Perpendicular to a Given Line at a Given Point P. (*a*) Lay off $PR = PS$ (Fig. 21), and with R and S as centers drow arcs intersecting at A. Then PA is the required perpendicular. (*b*) If P is near the end of the line, take any convenient point O (Fig. 22) above the line as center, and with radius OP draw an arc cutting the line in Q. Produce QO to meet the arc in A; then PA is the required perpendicular. (*c*) Lay off $PB = 4$ units of any scale (Fig. 23); from P and B as centers lay off $PA = 3$ and $BA = 5$; then APB is a right angle.

To Divide a Line AB into n Equal Parts (Fig. 24). Through A draw a line AX at any angle, and lay off n equol steps along this line. Connect the last of these divisions with B, and draw parallels through the other divi-

sions. These parallels will divide the given line into n equal parts. A similar method may be used to divide a line into parts which shall be proportional to any given numbers.

Fig. 24.

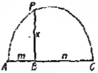


Fig. 25.

Fig. 26.

To Construct a Mean Proportional (or Geometric Mean) Between Two Lengths, m and n (Fig. 25). Lay off $AB = m$ and $BC = n$ and construct a semicircle on AC as diameter. Let the perpendicular erected at B meet the circumference at P. Then $BP = \sqrt{mn}$. (See p. 115.)

To Divide a Line AB in Extreme and Mean Ratio (the "golden section"). At one end, B, of the given line (Fig. 26), erect a perpendicular, BO, equal to half AB, and join OA. Along OA lay off $OP = OB$, and along AB lay off $AX = AP$. Then X is the required point of division; that is, $\overline{AX}^2 = AB \times BX$. Numerically, $AX = \frac{1}{2}(\sqrt{5} - 1)(AB) = 0.618(AB)$.

To Bisect an Angle AOB *(Fig. 27).* Lay off $OA = OB$. From A and B as centers, with any convenient radius, draw arcs meeting in M; then OM is the required bisector.

To draw the bisector of an angle when the vertex of the angle is not accessible (Fig. 28). Parallel to the given lines a, b, and equidistant from them, draw two lines a', b' which intersect; then bisect the angle between a' and b'.

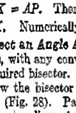


Fig. 27.

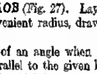


Fig. 28.

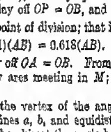


Fig. 29.

To Draw a Line Through a Given Point A and in the Direction of the Point of Intersection of Two Given Lines, when this point of intersection is inaccessible (Fig. 29). Draw any two parallel lines PQ and $P'Q'$ as in the figure; through P' draw a line parallel to PA, and through Q' draw a line parallel to QA; let these lines intersect in A', and draw the line AA'. This line AA' will (if produced) pass through the intersection of the two given lines.

To Construct, Approximately, the Length of a Circular Arc (Rankine). *In Fig. 30 draw a tangent at A. Prolong the chord BA to C, making* $AC = \frac{1}{2} AB$. With C as center, and radius CB, draw arc cutting the tangent in D. Then AD = arc AB, approximately (error about 4 min in an arc of 60 deg). Conversely, to find an arc AB on a given circle to equal a given length AD, take E one-fourth of the way from A to D, and with E as center and radius ED draw an arc cutting the circumference in B. Then arc $AB = AD$, approximately.

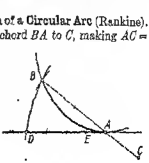


Fig. 30.

droplets if the design is such as to render smooth passage of the gas. This results in a retarding force to the blade and also a pitting of the blade. This can be minimized by shielding the inlet edges of the blades with a hard strong material such as stellite. Any amount of moisture, in the liquid state, greater than 10 per cent can be expected to cause trouble.

Rotative Speed. All other things being equal, the power of a turbine is directly proportional to its rotative speed. For this reason it is most economical from weight and cost standpoints to operate at the maximum possible low-pressure blade speeds. Blade speeds are now approaching 1,500 fps. With high speeds resulting in smaller turbines, the casing problem as regards expansion due to temperature fluctuations is eased. Also the length of time required for warming up the rotor is lessened as well as the attendant danger due to distortion from ununiform heating.

Balancing. A rotor that is of negligible length (one wheel) is in practical dynamic balance when it is in static balance (see pp. 509 and 510). When the length is of an appreciable amount, such as is the case where there are several wheels on one spindle, balancing is usually done on a balancing machine. The rotors undergoing balance must not be run at operating speed because the blading acts as fans or blowers and too much power would be involved. Dynamic balance can be obtained at low speeds provided there is sufficient force indicated to locate the point where balance weight must be added or removed. This has nothing to do with critical speed, which exists regardless of how perfectly the rotor is balanced, and is dependent on the amount of deflection of the rotor due to gravity when the rotor is at rest. Critical speed may be approximately determined by the expression

$$\text{rpm}_c = 187.7\sqrt{\frac{1}{\Delta}} \tag{34}$$

where Δ = deflection at maximum point, in.

Blading. Impulse blades, generally being in the initial stages of the turbine where the steam is superheated and thereby subject to corrosion, are usually made of corrosion-resisting material. These may either be machined from bar stock, an expensive method but justifiable when there are relatively few blades to be made, or be made from stock having a cross section similar to that desired. Long blades having change in cross section over the length are usually first forged and then finish-machined.

The material in most general use is a low-carbon stainless steel of approximately the following composition: C, 0.06 to 0.13 per cent; Cr, 11 to 13 per cent; Ni, 0.50 per cent max; Mn, 0.8 per cent max; Si, 0.50 per cent max. It should have approximately the following physical characteristics: tensile strength, 100,000 psi; minimum elongation in 2 in., 20 per cent; minimum reduction in area, 60 per cent; Brinell hardness, 200.

A chrome-nickel steel of the following characteristics is also often used: C, 0.45 per cent max; Cr, 7 per cent; Ni, 20 per cent; Mn, 0.75 per cent max; Si, 1.25 per cent max. The physical characteristics should be approximately as follows: tensile strength, 105,000 psi; max elongation in 2 in., 27 per cent; min reduction in area, 40 per cent; Brinell hardness, 200.

The common types of root fastenings are shown in Fig. 17.

Reaction blades, usually at the lower end of the turbine, are usually made of copper-zinc or copper-nickel good for temperatures not exceeding 600 F but are made of stainless steel such as is used for impulse blades when reaction

Also it should not be forgotten that the success of this device depends upon the use of metals under high stress at temperatures around 1500 F. If this problem is solved for the gas turbine, the solution can be applied to the steam turbine, which would result in great improvement in this prime mover.

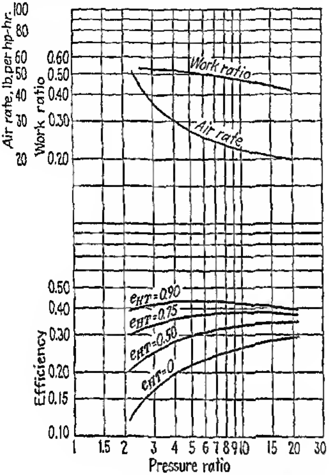


FIG. 28.—Combustion-gas turbine performance.

GOVERNING

The fundamental principles of governing are covered on p. 859. The main propulsion turbines of a ship are not governed by any device that maintains approximately constant speed as in the case of turbines driving electric generators or other auxiliaries. The load of driving the ship will prevent the propulsion turbines from overspeeding except in the event of the propeller's coming out of the water as might happen in stormy weather, or in the event of the loss of the propeller altogether. These contingencies are usually cared for by an oil-actuated overspeed device, a typical one being shown in Fig. 29. The complete assembly includes the maneuvering valves for the ahead and astern elements. Usually a guard valve is installed in series with the regular astern valve. These valves are not automatic but are hand-operated. It is good practice to place a small drain line from between the astern valve and the guard valve in order to remove the condensate, which might otherwise get into the astern blading. It is also good practice to

staging is used in the upper or higher temperature section of the turbine. Usually being long, they are sometimes made with integral spacing pieces. They may be manufactured in the same manner as are impulse blades but usually are rolled or drawn. A few standard shapes are available which are used in all designs, the design being modified to suit. The blades, both stationary and moving, are usually fastened according to the center illustration in Fig. 17. Since there is a drop in pressure across these blades, they are sometimes shrouded to permit of greater radial clearance and yet maintain low leakage.

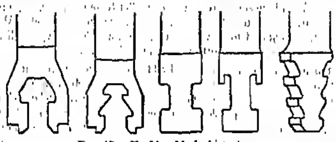


Fig. 17.—Turbine-blade fastenings.

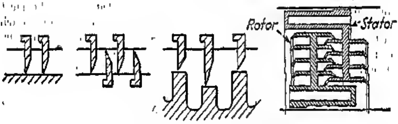


Fig. 18.—Labyrinths with radial clearance.

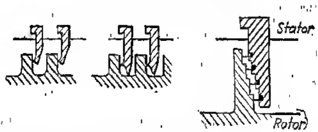


Fig. 19.—Labyrinths with axial clearance.

Leakage. To prevent steam leakage from stage to stage or to atmosphere, either carbon packings or metallic labyrinth packings are employed. In carbon packings, rings made of carbon are used in sections of 90 or 120 deg with overlapping joints at the ends and held against the shaft by means of springs. Where there is considerable difference in pressure, these rings are arranged in multiple. Labyrinths may be arranged to depend on either radial clearance (Fig. 18) or axial clearance (Fig. 19).

Dummy Pistons. In reaction turbines, since there is a pressure difference on each side of the moving blade, considerable end thrust is built up over several stages. This thrust is usually balanced by dummy pistons mounted on the rotor and running with close clearance with the casing and of the same

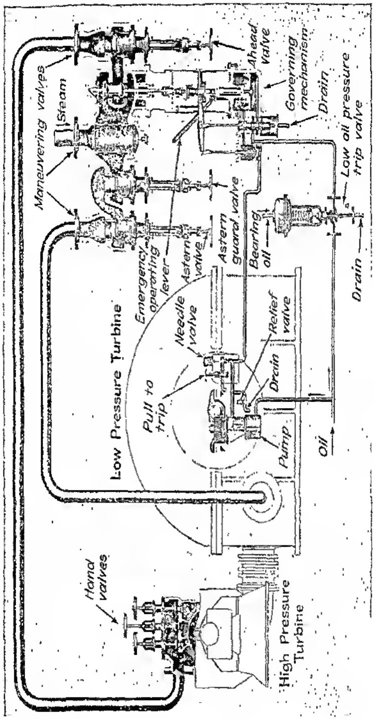


FIG. 29.—General Electric turbine governing system.

mean diameter as the steam path. This piston is then subjected to approximately the same difference of pressure as exists over the several stages so that the thrust is in the opposite direction to that due to the rows of moving blades. The result is zero thrust. The steam is led to the dummy piston by means of steam passages from front to rear, and leakage around the dummy is minimized by means of labyrinth packing (see Figs. 18 and 19).

Gland Sealing. As regards the sealing of joints where the shaft passes through the casing, the pressures run from almost throttle pressure to considerably below atmospheric. These joints are sealed by means of glands consisting of numerous rings or collars so as to provide a labyrinth (see Figs. 18 and 19). Steam condensing in this passage aids in sealing. The low-

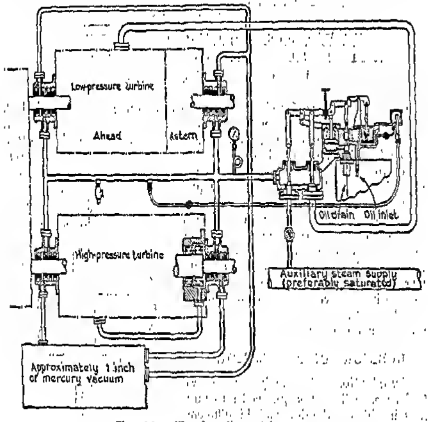


FIG. 20.—Gland sealing with steam.

pressure glands of both the straight carbon-ring and labyrinth types always have intermediate grooves that are supplied with steam under pressure higher than atmospheric. If this steam were not supplied, the air would leak into these lower stages owing to atmospheric pressure. This air would then enter the condenser and plug it up with noncondensable gas of greater amount than the air ejectors could eliminate.

However, if the intermediate grooving is supplied with steam, some of it will leak into the lower stages and be carried to the condenser, where it will condense; some of it leaking out to atmosphere. In some of the older installations, this gland leak-off steam is allowed to escape into the engine room, and the presence of this small plume of steam is an indication that the gland is properly sealed. There are two arguments against this practice:

place some hand valves at the inlet of the high-pressure turbine to permit some adjustment of the first-stage nozzle area—*i.e.*, to permit the cutting in or out of some first-stage nozzles, depending on the load.

The main propulsion-governing mechanism proper has two chief purposes: (1) to provide an upper limit to the speed of the turbines and (2) to shut off

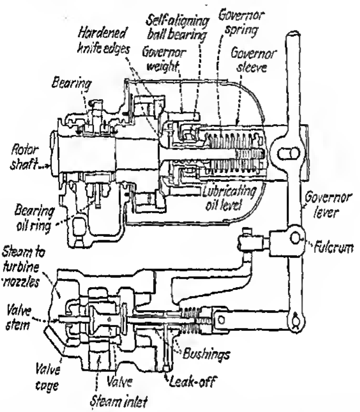


FIG. 30.—Horizontal centrifugal weight governor. This is the simplest of governors used on small turbines. It provides speed regulation suitable for pump, fan, and other mechanical drives where a single governor valve, not over 5 in. in diameter, is used. Where desired, the governor can be arranged for operation with pump or exhaust pressure control.

At three important points, friction that might impair sensitivity has been minimized in this governor. Knife edges and seats are hardened to protect against wear. Self-aligning and oil-lubricated ball bearing transmits governor movements to the valve linkage. A bushing seal around the valve stem insures freedom of valve movement.

In action, the centrifugal force of the governor weights is transmitted through knife edges and the thrust bearing to the governor lever, which operates directly on the governor valve.

A hand speed changer, adjustable while the turbine is running, can be used.

throttle steam automatically in the event of imminent failure of oil supply to bearings or gears. The operation of the system shown in Fig. 29 can be understood by studying the diagram. A positive-displacement gear-type pump is on the forward end of the low-pressure turbine shaft. It produces an oil pressure of an amount in accordance with a predetermined setting of

(1) it will fog up the engine room, impair visibility, and render conditions humid and uncomfortable; (2) it is a waste of valuable feed water.

Modern practice involves trapping this gland leak-off steam and condensing it, returning the condensate to the feed system. This will be covered more thoroughly in the section on Heat Balance, p. 1264 (see Figs. 20 and 21).

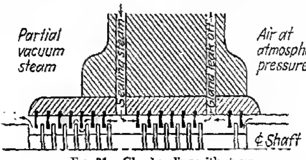


FIG. 21.—Gland sealing with steam.

The normal source for gland-sealing steam at full power is the leak-off from the high-pressure end or a tap from one of the lower stages. However, in running at reduced power, the pressure at these points may be less than atmospheric; under such circumstances, gland-sealing steam must be supplied from the exhaust line or some such reliable source. Gland-sealing steam is chargeable to the steam consumption of the turbine.

Water-sealed glands in which an impeller revolves in a pocket containing water are not in favor with some marine engineers. Complications of water supply and the hazard of a broken seal-impeller rendering the complete turbine inoperative are the reasons for this (see Fig. 22). In turbogenerator turbines, they are more acceptable.

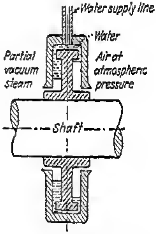


FIG. 22.—Water-sealed gland.

LUBRICATION AND BEARINGS

Lubrication is invariably of the forced-feed type. Oil is often pumped into tanks set fairly high in the ship, and this difference in head causes it to run to the turbine bearings (as well as the gearbox). Sometimes pressure from a pump is used. The bearings are of the sleeve type, babbitt-lined. The oil cools as well as lubricates the bearings and then passes through oil coolers and back to the tanks. Sensitive control devices maintain the proper level in the tanks and close the main turbine throttle in the event that the level in the tanks drops below a predetermined point.

Bearings. Small turbines may be equipped with either ball bearings or oil-ring sleeve bearings. It is considered bad practice to have both types on one turbine. Propulsion turbines usually are equipped with plain sleeve bearings lubricated by means of oil under pressure. The bearings are split and the shells made of cast iron or steel as a rule although bronze

the by-pass needle valve shown. If the speed of the turbines should exceed the desired maximum, the oil pump increases the pressure sufficiently to close the

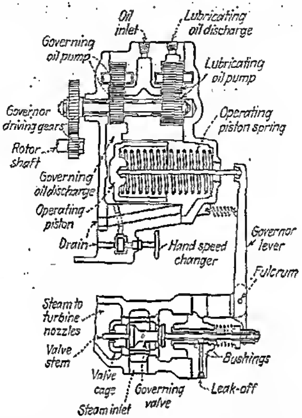


FIG. 31.—Hydraulic orifice governor. Speed adjustment over a 3:1 range is provided by this simple, positive governor for variable speed drives such as pumps, fans, and blowers. The varying pressure developed at different speeds by a gear oil pump supplies the motivating force in this governor.

The oil pressure, proportional to the square of the turbine speed, opposes a spring-loaded operating piston, which thus becomes responsive to speed changes. This piston operates the governor valve through the fulcrumed governor lever.

Increased power is provided when valve sizes exceed 5 in. in diameter, by interposing a hydraulic servomotor between the governor lever and the governor valve stem. The governor then operates only the servomotor relay, while high pressure oil supplied to the servomotor by another pump in the governor casing provides the power for operating the governor valve. The second gear pump also supplies oil under pressure to the main turbine bearings and to a reduction gear when used.

As is the case with the horizontal centrifugal governor, control by pump or exhaust pressure regulator can be used.

preemergency governor valve. In the same manner, if the lubricating-oil pressure should become low, the low-oil-pressure trip valve operates so as to close the throttle.

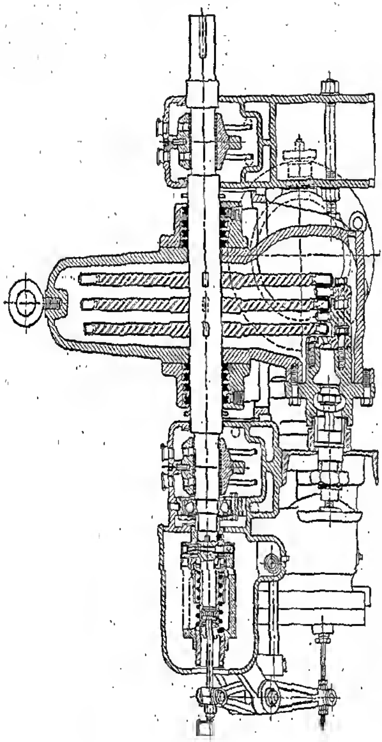


Fig. 23.—Single-stage velocity-compounded turbine. (Courtesy of Elliot Company.)

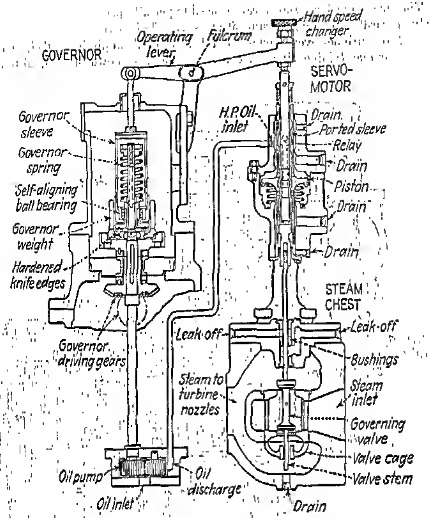


FIG. 32.—Vertical centrifugal weight governor. Powerful and positive in action, this governor, for single-valve steam chests, supplies the quality of speed regulation required for generator drive, or other machinery that must be run with small speed variation. For added power, a hydraulic servomotor is interposed between the centrifugal weight governor and the valve.

Movement of the governor weights in response to speed changes is transmitted through a ball type thrust bearing to the governor sleeve and operating lever to the servomotor relay. This relay operates within a ported sleeve integral with the operating piston, and controls the flow of high pressure oil to and from the operating cylinder. The servomotor piston is connected directly to the governor valve stem.

A gear-type oil pump at the lower end of the governor spindle supplies the high pressure oil for operating the servomotor as well as lubrication for the turbine bearings.

Either a hand or motor-operated speed changer can be used and may be adjusted while the turbine is in operation.

or brass would be better from the standpoint of anchoring the babbitt lining. Grooves are machined in the shell to help anchor the lining, and the shell is tinned and heated before the babbitt is poured. The two halves are then bolted together, and the bearing is finish-machined. Oil is furnished to the bearing at a pressure of 8 to 10 psi and a temperature of 110 to 120 F. The oil both lubricates and cools the bearing, the temperature rise of the oil being approximately 20 F. Oil seals similar to labyrinth packing are provided to prevent oil leakage or creapage along the shaft. Where high-temperature steam is used, oil leakage is a fire hazard.

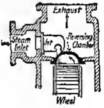


FIG. 24.—Helical-flow turbine.

Impulse turbines have some end thrust due to the axial flow of the steam through the turbine, but this is small. The dummy piston, as described previously, offsets the inherent thrust of the reaction turbine. In order to care for any remaining thrust a collar on the shaft bearing on a babbitt surface extending over the edge journal bearing is sufficient for turbines with sleeve bearings. For small turbines equipped with ball bearings, a deep-groove ball bearing or a ball-thrust bearing will suffice.

SMALL TURBINES

Small turbines for driving pumps and other auxiliary apparatus are usually made with only one pressure stage and are velocity-compounded. The most popular types at present are those shown in Figs. 23 and 24. Figure 21 shows a turbine in which the steam is redirected into moving blades twice. All through this stage the pressure is the same, but the steam continues to lose velocity as it passes through the moving wheels. Figure 22 shows a helical-flow turbine. The steam impinges tangentially on the wheel which has blades or semicircular recesses milled into its periphery, mounted around which are groups of nozzles with contiguous reversing chambers. The steam is expanded down to exhaust pressure in a single nozzle and, after striking one side of the wheel recess or blade, is reversed in direction, leaving the opposite side of the same recess or blade. This operation is repeated as many times as is necessary to reduce the kinetic energy of the steam to a relatively low value.

REHEATING

Some experimental propulsion equipment has been installed involving reheating. This is done by taking steam from the turbine at one of the lower stages, reheating it, and then returning it to the next stage. This increases the enthalpy at the given pressure and consequently increases the available energy. The steam should be removed for reheating before the saturation line is reached so that the reheating is a superheating rather than an evaporation process. Generally the steam is reheated to the original throttle temperature or thereabouts. The maximum gain is accomplished when the steam is allowed to expand and drop in temperature in the turbine until the temperature drop is approximately half the total drop that would have occurred had the steam gone to exhaust pressure in the conventional manner. Practically, it is not expedient to drop to this point because of the volume of the steam under such circumstances. Furthermore, the reheating increases this volume still further because under no circumstances should steam be

The emergency lever shown can be used to permit opening of the emergency valve regardless of the governor oil pressure. This permits operation of the propulsion turbines even though the lubricating oil supply has failed. Governors for maintaining virtually constant speed are shown in Figs. 30 to 33.

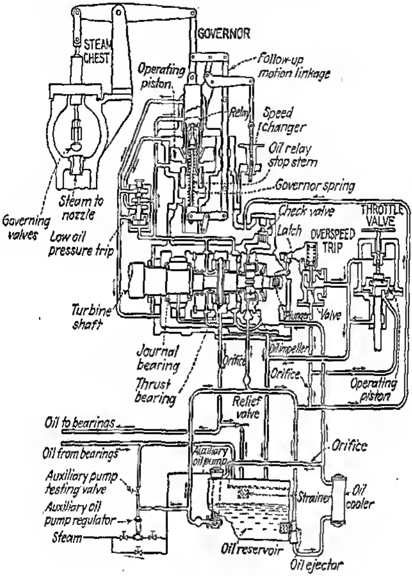


FIG. 33.—Impeller-type oil-pressure governor.

DESIGN EXAMPLE

Example. *It is desired to design one stage of an impulse turbine, the steam to enter this stage at 300 psia, 680 F; the available energy consumption or isentropic heat drop to be 30 Btu per lb steam; 50,000 lb per hr steam to be used; and the stage to run at 3,650 rpm.*

bled from the turbine for reheating unless there are a few degrees of superheat still remaining. Since the volume involved complicates an already complicated piping problem, reheating does not pay unless the throttle pressure is in excess of 1,200 lb or the temperature at throttle is approximately 1000 F. This can readily be seen by inspecting a Mollier diagram.

There are two practical ways of reheating. One is to lead the steam from the turbine into a heat exchanger, the hot side of which is receiving steam at throttle pressure and temperature from the boilers. This is known as "steam reheat" and has the advantage that the runs of large piping from turbine and return are small. The heat exchanger can be put alongside the turbine. The other method is to lead the steam from the turbine over to the boiler and through a heat exchanger across the hot side of which hot combustion gases flow. This is known as "gas reheat." This second is somewhat more economical.

In short, reheating involves relatively high throttle pressures and temperatures and increases piping complication. It has the advantage that there is a great increase in available energy and that it renders the amount of heat lost in the condenser, which is approximately the same for each pound of steam regardless of throttle conditions, a smaller percentage of the heat applied to the water as it passes through the boilers. Furthermore, a glance at a Mollier diagram will disclose that for a given throttle condition there will be less moisture in the steam at exhaust in the case of reheat than otherwise, this for a given exhaust pressure. This is a decided advantage when the turbine maintenance is considered, because moisture causes blade erosion in the lower stages. Any amount greater than 10 per cent is dangerous.

Several experimental ships, mostly with gas reheat, have been built and are operating, but no conclusive results are available yet. The problem involves mostly boiler design and piping layout. Greater economy is certainly possible with these schemes, and more research will undoubtedly be done along this line.

WATER RATES AND EFFICIENCY

The water rates of main propulsion turbines may be obtained approximately from the curve in Fig. 14. For turboelectric generators see the curve in Fig. 25. For auxiliary turbines, operating noncondensing, the turbine or "engine" efficiency can be approximated by means of the empirical expression

$$e = [10.5 + (\text{hp})^{0.46}] \left(\frac{\text{rpm}}{1{,}000}\right)^{0.43} \tag{35}$$

This holds fairly well for horsepowers from 35 to 1000. Below 35 hp the efficiency will be 5 per cent less.

ASTERN OPERATION

A steam turbine cannot be reversed unless there is some means whereby all the blades can be reversed on the wheels and all the stationary nozzles reversed. Obviously, this is extremely impractical, and no attempt is made to do it. The only alternative is to install an additional turbine with blades and nozzles arranged to obtain this torque. This is the common practice (see Figs. 9 and 34).

Reference to a Mollier diagram will disclose that with a 30 Btu per lb isentropic drop, the pressure in the stage would be 230 psia and the isentropic enthalpy 1327.7 Btu per lb. 230/300 = 0.768, therefore the nozzles would be discharging at a pressure higher than the critical and would be convergent. The efficiency of such a nozzle should be 1.00.

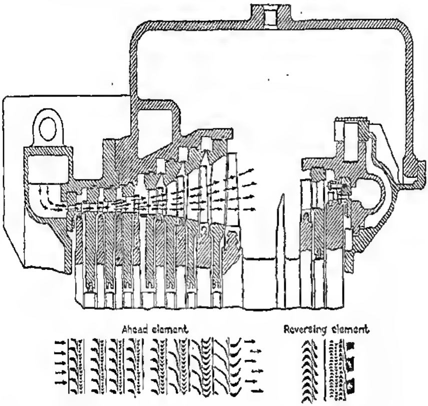


FIG. 34.—General Electric low-pressure turbine containing both impulse and reaction staging. The last two stages are reaction. Note astern turbine.

Take the discharge coefficient, C_d, as unity. Since this is superheated steam, we obtain, from Eq. (23),

$$\frac{50{,}000}{3{,}600} = 1.39A_b \sqrt{\frac{300}{2.184}[(0.768)^{1.54} - (0.768)^{1.77}]}$$

$$A_b = 4.33 \text{ sq in.}$$

where A_b = total area of all the nozzles blowing into this stage

$$v_1 = 223.7\sqrt{(h_a - h_b)e_n} = 223.7\sqrt{30} = 1{,}222 \text{ ft per sec}$$

Make the nozzle angle 20 deg. This is the angle A in Fig. 4. For maximum efficiency

$$\rho = \frac{\cos A}{2} = \frac{u}{v} = \frac{0.94}{2} = 0.47$$

$$u = 0.47 \times 1{,}222 = 575 \text{ ft per sec}$$

Required speed is 3,650 rpm. Therefore $\pi D 3{,}650 = 575 \times 60$ or $D = 3$ ft, pitch diameter of blades. Refer to Fig. 35.

Since this turbine is an additional piece of apparatus of no use in the normal operation of a ship, it is undesirable to make it any larger, heavier, or more expensive than absolutely necessary. Consequently, turbine-driven merchant ships seldom have astern power of more than 70 percent of ahead power, and some have as little as 25 percent; 40 percent is a good figure. In the case of some naval combat vessels, such as aircraft carriers where backing at certain speeds is essential to facilitate landing of planes, special considerations affect the decision as to the amount of astern power.

The astern turbine is usually in the form of several stages mounted on the low-pressure turbine shaft. The best practice is to install these reverse wheels at the exhaust or lowest pressure end. This permits the astern blading to run in steam at exhaust pressure when the ship is going ahead. This is the lowest density steam in the system and, consequently, the windage losses due to this blading are minimized. Generally, only two or three stages are

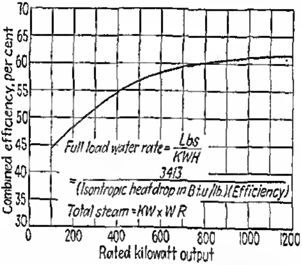


FIG. 25.—Typical combined efficiencies, including gears, of turboelectric generators at full load. Curve is for d-c generator; for alternating current, multiply by 1.02.

necessary to obtain the desired power. As in the ahead turbine, the first stage should be velocity-compounded twice, *i.e.*, the throttle steam should be brought through the nozzle and directed on one row of blades. After leaving this first row the steam should be redirected on a second row. After this the steam can be further expanded through one or two ordinary impulse stages. Reaction blading is not suited for this sort of application but has been used where tradition favors such blading.

Full-pressure and temperature steam, of course, enters the astern turbines. Consequently, with these few stages they are not very efficient. The exhaust entering the condenser is often superheated steam. This should be considered in the condenser design. Economy for astern operation is of little importance because of the short time of duration. However, unless some provision is made for ventilation steam through the ahead blading (which is seldom done), the full-power astern operation should be continued for a limited time; otherwise, the windage losses in the ahead blading may result in a detrimental temperature rise in the blading.

$$R_1 = \sqrt{u^2 + v_1^2 - 2\,uv_1 \cos A}$$

$$R_1 = \sqrt{575^2 + 1{,}222^2 - 2 \times 575 \times 1{,}222 \times 0.94} = 715 \text{ ft per sec}$$

$$R_2 = 0.875 \times 715 = 625 \text{ ft per sec (Table 2)}$$

$$\text{Cos } B = \frac{v_1 \cos A - u}{R_1} = \frac{1{,}222 \times 0.94 - 575}{715} = 0.804$$

$$B = 36.5 \text{ deg.} \qquad \text{Make} \therefore B = D.$$

The rate of doing work on the blade according to Eq. (3) is

$$W_b = \frac{W}{g}(R_1 \cos B + R_2 \cos D)u \quad \text{ft-lb per sec}$$

$$= \frac{50{,}000}{3{,}600 \times 32.2}(715 \times 0.804 + 625 \times 0.804)575$$

$$= 269{,}000 \text{ ft-lb per sec}$$

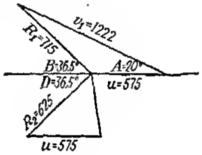


FIG. 35.

The blade efficiency is

$$e_b = \left(\frac{\cos^2 A}{2}\right)\left(1 + \frac{R_2}{R_1}\right) \tag{11}$$

$$= \frac{\overline{0.94}^2}{2} \times 1.875 = 0.83$$

The losses up to this point are nozzle losses plus blade losses; or the total efficiency to this point is 0.83 since the nozzle efficiency is considered as 1.00.

$$1 - 0.83 = 0.17 \text{ loss}$$

or

$$0.17 \times 30 = 5.1 \text{ Btu per lb}$$

Therefore, the enthalpy of the steam leaving the blades would be 1327.7 + 5.1 = 1332.8 Btu per lb at 230 psia, 228 F of superheat. Extrapolating Table 3 results in a factor of 0.60.

$$\text{Specific volume } \bar{V} = 2.727 \text{ cu ft per lb}$$

This is the character of the steam in which the wheel rotates. Use four nozzles. Total area of exit = 4.33 sq in. or 1.09 sq in. per nozzle. Make each nozzle 1.1 in square. Therefore the blades would be 1.1 in. high. If the pitch diameter is 3 ft,

$$L_d = \frac{143.35D^2}{\bar{V}}\left(\frac{u}{100}\right)^{2.9} \quad \text{Btu per hr} \tag{26}$$

$$L_d = \frac{143.35 \times \bar{3}^2}{2.727}(5.75)^{2.9} = 75{,}000 \quad \text{Btu per hr}$$

COMBUSTION-GAS TURBINE

The combustion-gas turbine is a device that uses high-temperature combustion gas at fairly high pressure as the motive fluid. When steam is used in the conventional turbine, the water must be evaporated in a boiler, but this latent heat of vaporization is not fully converted into energy in the turbine, much of it being lost in the condenser. With combustion gas, all action takes place in the superheat region, and there is no loss in a condenser. The simplest form of the cycle is shown in Fig. 26. Air is taken into a compressor at atmospheric pressure and compressed to a value of several atmospheres. Fuel is then injected and burned. The hot products of combustion then enter a turbine where expansion takes place down to atmospheric pressure. Part of the power of the turbine is used to drive the compressor, and the excess is available for ship propulsion. This is not a very efficient cycle, and it is obvious that the use of heat exchangers and reheating would give better results. Figure 27 shows a cycle that is more practical. Note that the high-pressure turbine drives only the low-pressure compressor, the useful power being delivered by the low-pressure turbine which also drives the high-pressure compressor. This is an arrangement that lends itself well to part-load operation. The efficiency of this cycle depends, to a large extent, on the amount of heat-transfer surface incorporated. On the basis of the following efficiencies and temperatures, the performance to be expected from such a cycle operating at 1500 F max is shown in Fig. 28.

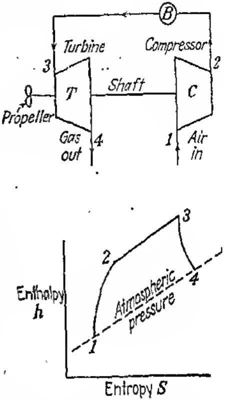


FIG. 26.—Simple combustion-gas turbine cycle.

Mechanical efficiency of turbines	0.98
Mechanical efficiency of compressors	0.98
Internal efficiency of turbines	0.90
Internal efficiency of compressors	0.85
Reheat temperature	initial turbine temperature
Inlet air temperature	70 F
Intercooler discharge temperature	90 F
Burner efficiency	0.98
Heat-transfer efficiency	$e_{ht} = \dfrac{h_5 - h_4}{h_9 - h_4}$

The air-rate and work-rate ratios are also shown in Fig. 28. Note that work-rate ratio means that if the ratio is 0.45 the plant will develop 450 useful

$$L_b = \frac{638.2Dh^{1.25}}{2{,}727}\left(\frac{u}{100}\right)^{2.8} \quad \text{Btu per hr} \tag{27}$$

$$L_b = \frac{638.2 \times 3 \times 1.1^{1.25}}{2{,}727}(5.75)^{2.8} = 125{,}000 \text{ Btu per hr}$$

Total windage loss,

$$0.60\left(\frac{75{,}000 + 125{,}000}{50{,}000}\right) = 2.4 \text{ Btu per lb}$$

Assume leakage loss as 1 percent or 0.3 Btu per lb.

$$\text{Total loss in stage} = 5.1 + 2.4 + 0.3 = 7.8 \text{ Btu per lb}$$

$$\text{Stage efficiency} = \frac{30 - 7.8}{30} = 74 \text{ per cent}$$

$$\text{Internal horsepower of this stage} = \frac{0.74 \times 30 \text{ Btu/lb} \times 50{,}000 \text{ lb/hr}}{2545 \text{ Btu/hp-hr}} = 435$$

horsepower for each 1000 hp of turbine capacity installed, the remaining 550 hp being used to drive the compressors, overcome pressure drops, etc. The higher the heat-transfer efficiency, e_{ht}, the lower the work ratio; but the difference is not great so the work ratio is shown as one approximate curve. Also the higher the heat-transfer efficiency, e_{ht}, the higher the air rate; but the difference is not great so air rate is shown as one approximate curve.

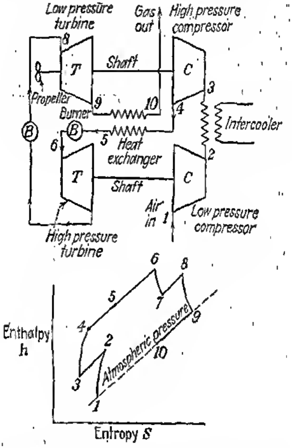


FIG. 27.—Combustion-gas turbine cycle involving heat exchangers and reheat.

From the values given it can be seen that, although the boilers, feed pumps, and other similar auxiliaries can be dispensed with if combustion-gas turbines are used, a great deal of extra heat-transfer surface must be installed as well as excess turbine capacity to say nothing of the compressors. Starting is also a problem. This can be done by means of auxiliary electric motors or diesel engines, but such a method involves units of considerable size. Therefore the practicability of such a method is questionable. Another method is to use compressed air as in the case of conventional diesel engine starting. However, it would not be necessary to have the air at more than 100 psi gage.

Astern operation presents additional difficulties. Probably electric drive is the best solution for this problem.

CONDENSATION

BY G. A. ORROK

From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

REFERENCES. Bausbrand, "Evaporating, Condensing and Cooling Apparatus," Scott, Greenwood & Co. Dalby, "Heat Transmission," *Proc. Inst. M.E.*, 1909. Orrok, *Trans. A.S.M.E.*, vols. 32 and 34. Kaula and Robinson, "Condensing Plant," Pitman. Royds, "Heat Transmission in Boilers, Condensers and Evaporators," Van Nostrand. McAdams, "Heat Transmission," McGraw-Hill. Standards of Heat Exchange Institute. Power Test Code, "Condensing Apparatus," A.S.M.E.

General. Condensation is either by direct contact between steam and water, as in the case of the jet condenser; or by surface condensation, where a wall of metal prevents the mixing of the steam and the cooling medium, as in the case of the ordinary surface condenser type. The cooling medium is generally water. Direct-contact condensers may be divided into three classes: (1) jet, (2) barometric, and (3) ejector condensers. Surface condensers may be classed as (1) water-cooled, (2) air-cooled, and (3) evaporative.

In a **jet condenser**, the exhaust steam and the cooling water enter at or near the top of the condenser head and the steam is condensed by the water falling in a fine spray, the resulting mixture of condensed steam, cooling water, and air being removed by a tail pump; or an independent pump may be used to remove the air. For high vacuum, the independent air pump is necessary. The tail pump is replaced by the tail pipe in the **barometric condenser**, and the cooling water and condensed steam flow from the bottom of the tail pipe without the aid of a pump. An independent air pump is generally used, but is not necessary with some types. The **ejector condenser** acts on the same principle as a steam ejector; the exhaust steam enters the ejector through a series of orifices and is condensed by the cooling water, resulting in the water being given a velocity high enough to discharge it against atmospheric pressure.

In a **water-cooled surface condenser**, the water flows through tubes and the steam is condensed by being brought into contact with the outside of the tubes. The steam generally enters at the top of the condenser, and the cold water first passes through the lower tubes and then through the upper tubes. This is known as the counterflow type. If the condensing water enters at the top near the steam inlet, the condenser is of the parallel-flow type. Condensers are single, two, or three pass. In **air-cooled condensers**, the heat is removed from the steam by the passage of cool air through the tubes or over plates. In **evaporative condensers**, steam is brought into contact with one side of a plate or tube and water is allowed to flow over the other side. The heat which is absorbed by the water is carried away by a stream of air passing over the surface of the water, part of the water being evaporated.

Direct-contact Condensers

Condensing Water Requirements. If t_s, t_1, and t_2 are the temperatures of the steam to be condensed, of the inlet water, and of the outlet water, respectively, deg F; h_s and h_2 the enthalpys of the steam and of the outlet water at t_2, Btu per lb; W_s and W_w the weights of steam per hr to be condensed and of water per hr needed for condensing; then the ratio R of water to steam required for condensation is given by $R = W_w/W_s = (h_s - h_2)/(t_2 - t_1)$. For turbines, $h_s - h_2$ may be taken as 950 Btu; for engines 1000 Btu. Owing

To Inscribe a Hexagon in a Circle (Fig. 31). Step around the circumference with a chord equal to the radius. Or, use a 60 deg triangle.

To Circumscribe a Hexagon About a Circle (Fig. 32). Draw a chord AB equal to the radius. Bisect the arc AB in T. Draw the tangent at T (parallel to AB), meeting OA and OB in P and Q. Then draw a circle with radius OP or OQ and inscribe in it a hexagon, one side being PQ.


Fig. 31.

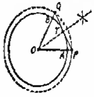


Fig. 32.

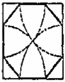


Fig. 33.

To Inscribe an Octagon in a Square (Fig. 33). From the corners as centers, and with radius equal to half the diagonal, draw four arcs, cutting the sides in eight points. The points will be the vertices of the octagon.

To Inscribe an Octagon in a Circle. Draw two perpendicular diameters, and bisect each of the quadrant arcs.

To Circumscribe an Octagon About a Circle. Draw a square about the circle, and draw the tangents to the circle at the points where the circle is cut by the diagonals of the square.

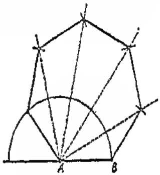


Fig. 34.

To Construct a Polygon of n Sides, One Side AB being Given (Fig. 34). With A as center and AB as radius, draw a semicircle, and divide it into n parts, of which $n - 2$ parts (counting from B) are to be used. Draw rays from A through these points of division, and complete the construction as in the figure (in which $n = 7$). Note that the center of the polygon must lie in the perpendicular bisector of each side.

To Draw a Tangent to a Circle from an external point A (Fig. 35). Bisect AC in M; with M as center and radius MC, draw arc cutting circle in P; then P is the required point of tangency.

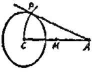


Fig. 35.

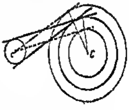


Fig. 36.

To Draw a Common Tangent to Two Given Circles (Fig. 36). Let C and c be the centers and R and r the radii ($R > r$). From C as center, draw two concentric circles with radii $R + r$ and $R - r$; draw tangents to these circles from c; then draw parallels to these lines at distance r. These parallels will be the required common tangents.

To Draw a Circle Through Three Given Points A, B, C, or to find the center of a given circular arc (Fig. 37). Draw the perpendicular bisectors of AB and BC; these will meet in the center, O.


Fig. 37.

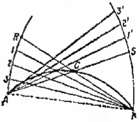


Fig. 38.

To Draw a Circular Arc Through Three Given Points When the Center is not Available (Fig. 38). With A and B as centers, and chord

AB as radius, draw arcs, cut by BC in R and by AC in S. Divide RA into n equal parts, 1, 2, 3, . . . Divide BS into the same number of equal parts, and continue these divisions at $1', 2', 3'$, . . . Connect A with $1', 2', 3'$, . . . and B with 1, 2, 3, . . . Then the points of intersection of corresponding lines will be points of the required arc. (Construction valid only when $CA = CB$.)

To Draw a Circle Through Two Given Points, A, B, and Touching a Given Line, l (Fig. 39). Let AB meet line l in C. Draw any circle through A and B, and let CT be tangent to this circle from C. Along l, lay off CP and CQ equal to CT. Then either P or Q is the required point of tangency. (Two solutions.) Note that the center of the required circle lies in the perpendicular bisector of AB.

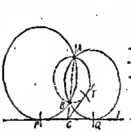


FIG. 39.

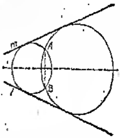


FIG. 40

To Draw a Circle Through One Given Point, A, and Touching Two Given Lines, l and m (Fig. 40). Draw the bisector of the angle between l and m, and let B be the reflection of A in this line. Then draw a circle through A and B and touching l (or m), as in preceding construction. (Two solutions.)

To Draw a Circle Touching Three Given Lines (Fig. 41). Draw the bisectors of the three angles; these will meet in the center O. (Four solutions.) The perpendiculars from O to the three lines give the points of tangency.

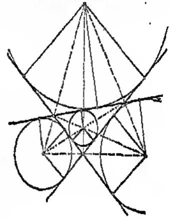


FIG. 41.

To Draw a Circle Through Two Given Points A, B, and Touching a Given Circle (Fig. 42). Draw any circle through A and B, cutting the given circle in C and D. Let AB and CD meet in E, and let ET be tangent from E to the circle just drawn. With E as center, and radius ET, draw an arc cutting the given circle in P and Q. Either P or Q is the required point of contact. (Two solutions.)

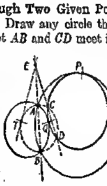


FIG. 42.

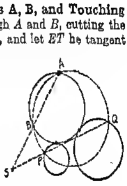


FIG. 43.

To Draw a Circle Through One Given Point, A, and Touching Two Given Circles (Fig. 43). Let S be a center of similitude for the two given circles, that is, the point of intersection of two external (or internal) common tangents. Through S draw any line cutting one circle in two points, the nearer of which shall be called P, and the other in two points, the more remote of which shall be called Q. Through A, P, Q

to the presence of air and imperfect mixing, t_h is 5 to 10 F lower than t_i. In proportioning ordinary jet and barometric condensers, W_s is the normal amount of steam to be condensed; a 50 percent overload is common at some reduction of vacuum.

Jet-condenser Design. Injection and tail pipes are usually designed for an allowable velocity of 5 fps. The steam entrance nozzle may be designed for a velocity of 300 to 600 fps. In the barometric type, the top of the tail pipe should be at least 35 ft above the hot well level. The condenser head should be as near to the exhaust flange as possible, for friction and velocity head increase rapidly with increasing vacuum. Heads of practically any shape may be used in jet and barometric types and are equally efficient if the water and steam are brought into intimate contact and the air is collected and carried away. The use of these types of condenser for power work is decreasing; in the chemical industry it is increasing, particularly the ejector type.

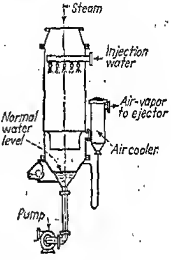


FIG. 1.—Parallel-flow Jet Condenser.

Surface Condensers

Surface Condensation. Let q be the total heat to be transmitted per hour, Btu; A the total outside surface of tubes, sq ft; W_s the steam condensed per hour, lb; W_w the condensing water required per hour, lb; $R = W_w/W_s$; U the coefficient of heat transmission, Btu per sq ft per deg F per hr; t_m the mean temperature difference of water and steam, deg F; t_s the temperature corresponding to the vacuum, deg F; t_c the hot-well temperature, deg F; t_1 and t_2 the temperature of circulating water at inlet and outlet, respectively, deg F; h_s the enthalpy, of the entering steam, Btu; h_c the enthalpy of the hot well water, Btu. Then, $q = Ut_mA = W_s(h_s - h_c)$; $W_w = W_s(h_s - h_c)/(t_2 - t_1)$; and $A = W_w(t_2 - t_1)/Ut_m = W_s(h_s - h_c)/Ut_m$. The value of U depends on the velocity of the water, the material of the tubes, the cleanliness factor, and the amount of air present. The mean temperature difference for rough calculations with small rise in temperature of the circulating water may be taken as the arithmetical mean without serious error, but for most calculations the logarithmic mean should be used. See Table 1. The water velocity in the tubes should not exceed 8 fps; and under normal condition, a velocity between 6 and 7 ft is the most economical. If the material coefficient for copper, Muntz metal, red brass, Admiralty metal, and aluminum brass is taken as unity, coefficient of 0.97 should be used

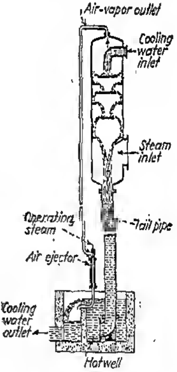


FIG. 2.—Counter-current Barometric Condenser.

Leaving temperature = 102 + 8.6 = 110.6 F.

$$\Delta_m = \frac{\Delta_{01} - \Delta_{02}}{\log_e \frac{\Delta_{01}}{\Delta_{02}}} \qquad (5)$$

Assume counter flow.

$$\Delta_m = \frac{75.4 - 8}{\log_e \frac{75.4}{8}} = 30 \text{ F}$$

Assuming that the temperature drop through the metal of the tube is negligible in so far as calculation of film temperature is concerned; also assuming that the drop through each film is approximately the same, then the film temperature should be approximately $\frac{30}{4} = 7.5$ F or 8 F higher or lower than the average water temperature or 148 − 8 = 140 F for the hot water and $\frac{102 + 110.6}{2} + 8 = 114.6$ F for the cold water.

$$h_0 = 160(1 + 0.012t_f)\frac{V_i^{0.8}}{D_i'^{0.2}} \qquad (6b)$$

$$h_i = 160(1 + 1.38)\frac{5^{0.8}}{0.527^{0.2}} = 380\frac{3.624}{0.8748} = 1{,}560$$

$$\frac{1}{UA} = \frac{1}{h_iA_i} + \frac{L}{KA_m} + \frac{1}{h_0A_0} \qquad (5a)$$

K for Admiralty metal = 64.

$$L = \frac{0.049}{12} = 0.004 \text{ ft}$$

$$\frac{1}{UA} = \frac{1}{1{,}770 \times 0.138} + \frac{0.004}{64 \times 0.153} + \frac{1}{1{,}560 \times 0.164}$$

$$\frac{1}{UA} = 0.0085$$

If A is considered as the outside area,

$$A_0 = \frac{\pi 0.625}{12} = 0.164 \text{ sq ft per ft length}$$

$$U_0 = \frac{1}{0.0085 \times 0.164} = 718 \text{ Btu/(hr) (sq ft) (deg F)}$$

$$\text{Area required} = \frac{\text{Btu per hr}}{\Delta_m U}$$

$$= \frac{3{,}400(186 - 110)}{30.0 \times 718} = 12 \text{ sq ft}$$

$$\frac{12 \text{ sq ft}}{0.164 \text{ sq ft/ft length}} = 73 \text{ ft of tubing}$$

$$\frac{3{,}400 \text{ lb/hr}}{1{,}080 \text{ lb/hr/tube}} = 2 \text{ tubes in a one-pass cooler, } \frac{73}{2} = 36.5 \text{ ft long}$$

for aluminum bronze, 0.90 for 70-30 cupronickel, and 0.84 for monel metal. The cleanliness factor varies between 0.80 and 0.90 for normally clean water, but if the water contains scale-forming ingredients a cleanliness factor of 0.70 may be assumed.

Table 1. Logarithmic Mean Temperature Difference between Steam and Condensing Water

(As given by the expression $(t_2 - t_1)/\log_e[(t_s - t_1)/(t_s - t_2)]$, where t_s is the temperature of the steam, t_1 of the injection water, and t_2 of the discharge water)

$t_2 - t_1$	$t_s - t_1$													
	15	20	25	30	35	40	45	50	55	60	65	70	75	80
5	12.3	17.4	22.4	27.4	32.4	37.4	42.4	47.5	52.5	57.5	62.5	67.5	72.5	77.5
10	9.1	14.4	19.6	24.6	29.7	34.8	39.8	44.8	49.8	54.8	59.8	64.8	69.9	74.9
15	.....	10.8	16.4	21.6	26.8	31.9	37.0	42.0	47.0	52.2	57.2	62.2	67.2	72.2
20	.....	.....	12.4	18.2	23.6	28.9	34.1	39.2	44.2	49.3	54.4	59.4	64.4	69.6
25	.....	.....	.....	13.9	19.9	25.5	30.8	36.1	41.3	46.4	51.5	56.6	61.7	66.8
30	.....	.....	.....	.....	15.4	21.6	27.4	33.1	38.1	43.3	48.5	53.7	58.7	63.8

The Heat Exchange Institute has stated the performance that may be expected from a well-designed condenser with new and clean tubes. Figure 3 shows the expected heat-transfer rates with circulation water at 70 F and for tubes of Muntz metal, Admiralty, red brass, aluminum brass, copper, or arsenical copper; with tubes of 70-30 copper-nickel, or 80-20 copper-nickel, the rates are to be reduced 10 percent. The temperature-correction curve gives correction factors for water temperatures

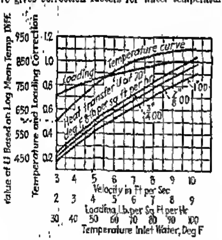


FIG. 3.—Heat-transfer Rates through Condenser Tubes (Heat Exchange Institute).

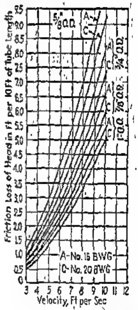


FIG. 4.—Friction Loss in Condenser Tubes (Heat Exchange Institute).

other than 70 F. The curves are for a condensation rate of 8 lb per sq ft per hr; for other rates, a correction factor is given by the load curve.

The head loss through clean condenser tubes, according to the Heat Exchange Institute, may be assumed to be as in Fig. 4. The water-box losses per pass is stated to be as follows:

Water velocity in tubes, fps	4	6	8	10
Loss of head in boxes, ft	0.63	1.13	1.72	2.4

The total condenser friction is the sum of the tube loss and the water-box losses.

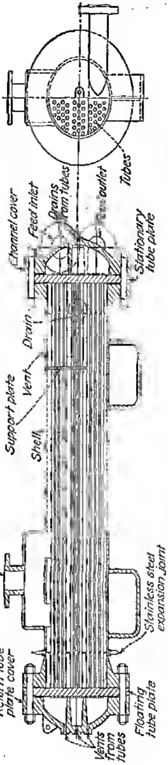


FIG. 1.—Typical section of feed-water heater. (*Courtesy of Foster Wheeler Corporation.*)

Design of Surface Condensers. (See pp. 394 and 399.) In condenser design, the given quantities usually are W_s, t_1, and the required vacuum. It is important that the place of measurement of the vacuum should be stated, and this is usually in the nozzle connecting the prime mover to the condenser. The highest vacuum will always be found at the air-pump suction, less in the body of the condenser, and the lowest vacuum in the nozzle. The vacuum inside the prime mover will be less by the velocity head necessary to give motion to the exhaust and by friction in the nozzle. The allowable velocity in a turbine nozzle is about 600 fps. The loss (or "drop") in a well-designed condenser should be less than 0.1 in. of mercury, and t_s should be taken as the temperature corresponding to this reduced vacuum. For good practice, t_2 should be 8 to 10 F lower than t_s. W_b/W_s, or the ratio of cooling water to condenser steam, usually ranges from 80 to 150 for single-pass condensers and from 50 to 90 for multipass condensers for turbine application. Single- and two-pass condensers are generally used. Single pass is used where the pumping head external to the condenser is small. Two pass is indicated for cooling tower or cooling pond installations.

Small tubes are best for the transmission of heat, but cannot be used with dirty water, so that the usual sizes are 5/8, 3/4, 7/8, and 1 in., with 7/8 in. most common. In two-pass condensers, the tube length is usually 16 to 20 ft; in single pass, 20 to 30 ft.

Surface condensers for large turbine installations have generally 0.6 to 1.5 sq ft of **condensing surface per kw**, or approximately 1/10 to 1/4 sq ft per lb of steam condensed per hr. Small turbogenerators require 2 to 4 sq ft per kw, or 1/10 to 1/8 sq ft per lb of steam per hr.

For **dry-air condensers** the coefficient of conductivity in Btu per hr per sq ft of surface per deg F difference of temperature varies from 3 in still air to 14 with air moving at a velocity of 1,500 fpm.

Materials of Construction. (See p. 632.) The materials generally used for **tubes** are Admiralty, copper, aluminum bronze, and Muntz metal. Any of these will give satisfactory service with fresh water, but with salt water (see p. 663) trouble may be experienced. Admiralty mixture (70Cu, 29Zn, 1Sn with .1 percent arsenic to prevent dezincification) aluminum bronze, and aluminum brass are generally used for salt water. Tubes are made of No. 20, No. 18, and sometimes No. 16 B. W. G. **Tube spacing** is important, as there must be room for the glands and sufficient metal between them for strength. The minimum allowable spacing or pitch of tubes is as follows (number of tubes per sq ft $= 166/s^2$, approx, s being the pitch in inches):

Diam of tube, in.	5/8	3/4	7/8	1	1 1/4
Pitch of tubes, in.	15/16	1 1/16	1 3/16	1 5/16	1 9/16
Number of tubes per sq ft	189	147	118	96	81

The **tube sheets** should be of rolled Muntz metal or Naval brass. A rolled sheet will give the best service, although cast sheets are used. Thickness should be at least 1/8 in. greater than the tube diameter but has been standardized by the Heat Exchange Institute as follows:

Area of tube sheet, sq in.	Up to 1965	1965–3739	3740–8495	8496–30791	30792 and over
Thickness, in.	7/8	1	1 1/8	1 1/4	1 1/2

Tubes are usually rolled into both inlet and outlet tube sheets, and expansion is provided for by the use of a floating head, a welded steel joint in the shell, or by bowing the tubes. The inlet tube sheet should be faced with a

Make eight pass.

$$\frac{36.5}{8} = 4.5 \text{ ft between tube sheets}$$

$$2 \times 8 = 16 \text{ tubes total}$$

$$\frac{16 \times 0.7854 \times \overline{0.625}^2}{144} = 0.0342 \text{ sq ft}$$

Cold water must run 5 fps. Average temperature is 106 F. 0.01615 cu ft per lb at this temperature.

$$\frac{30{,}000 \text{ lb/hr} \times 0.01615 \text{ cu ft/lb}}{3{,}600 \text{ sec/hr} \times 5 \text{ fps}} = 0.027 \text{ sq ft}$$

$$0.0342 + 0.027 = 0.0612 \text{ sq ft}$$

Make 16/189 = 0.0846 sq ft (see p. 1219)

Inside shell or $d = \sqrt{\frac{0.0846}{0.7854}} = 0.327$ ft or 3.93 in. Make O.D. = 4 in.

Over-all dimensions of cooler, approximately 5 ft × 4 in. containing sixteen ⅝-in. tubes arranged for eight passes.

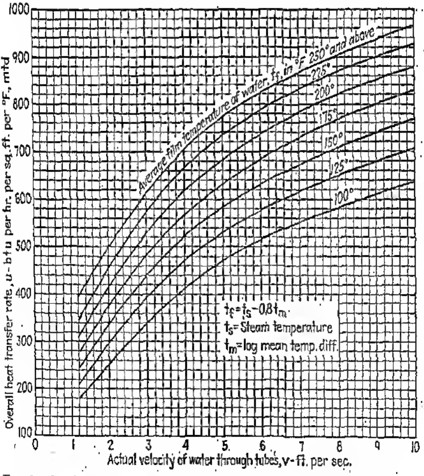


FIG. 2.—Feed-water heaters' heat-transfer rates. For 1-in. Admiralty tubes and smaller. For steel use 80 per cent of curve valve.

Table 2. Standard Condenser-tube Data[a]

Outside diam, in.	Size number, B.W.G.	Weight per ft, lb[b]	Thickness, in.	Inside diam, in.	Surface sq ft per ft length: Outside	Surface sq ft per ft length: Inside	Inside sectional area, sq in.	Velocity, fps, for 1 gpm	Capacity at velocity of 1 fps: U. S. gal per min	Capacity at velocity of 1 fps: Lb water per hr
5/8	16	0.424	0.065	0.495	0.1636	0.1296	0.1925	1.667	0.5999	300.0
	18	0.329	0.049	0.527	0.1636	0.1380	0.2181	1.472	0.6793	339.7
	20	0.241	0.035	0.555	0.1636	0.1453	0.2420	1.326	0.7542	377.1
3/4	14	0.644	0.083	0.584	0.1963	0.1528	0.2678	1.198	0.8347	417.4
	16	0.518	0.065	0.620	0.1963	0.1613	0.3019	1.063	0.9407	470.4
	18	0.400	0.049	0.652	0.1963	0.1706	0.3339	0.9611	1.041	520.5
7/8	14	0.769	0.083	0.709	0.2291	0.1856	0.3949	0.8126	1.230	615.0
	16	0.613	0.065	0.745	0.2291	0.1951	0.4360	0.7360	1.358	679.0
	18	0.472	0.049	0.777	0.2291	0.2034	0.4740	0.6770	1.477	738.5
1	14	0.887	0.083	0.834	0.2618	0.2183	0.5463	0.5874	1.702	851.0
	16	0.708	0.065	0.870	0.2618	0.2277	0.5945	0.5398	1.852	926.0
	18	0.535	0.049	0.902	0.2618	0.2361	0.6390	0.5022	1.991	995.5
1¼	14	1.13	0.083	1.084	0.3271	0.2839	0.9229	0.3477	2.877	1439
	16	0.898	0.065	1.120	0.3271	0.2932	0.9852	0.3257	3.070	1535
	18	0.675	0.049	1.152	0.3271	0.3015	1.043	0.3075	3.253	1627

[a] Prepared by T. B. Drew.
[b] In brass, specific gravity = 8.56.

radius tool to form a bell mouth; the flaring of the tube over this bell mouth improves the entrance condition to the tube. The hole in the tube sheet may have circumferential grooves to improve the holding power. Occasionally the inlet end is made tight by metallic packing which is preferred to fiber packing or paraffined corset lace. This is held in place by glands with rounded entrance. On the outlet side, metallic packing, without a ferrule, is often used.

Condenser shells are usually made of welded steel reinforced against collapsing pressure. Tubes should be supported at distances of 40 to 50 diam by supporting plates (usually of cast iron) drilled with clearance around the tubes. Water boxes should be large and designed to offer little resistance to the passage of the water and to provide uniform distribution of water among the tubes. Several holes, ¼ or 1 in. diam, should be provided in the *water partition of multipass condensers to permit draining* of the tubes and box and to vent excess air from the first to the last water pass. Where possible the steam should enter from the top and water at the bottom (countercurrent principle), but this is not essential, as parallel flow condensers give good results. The bottom of the circulating-water outlet should be above the highest point of the tube bank. If this cannot be done at the water box, *the discharge pipe should be carried up to the* same height away from the condenser.

The steam passage should be direct to the tube bank, and the nozzle should be spread so that no dead pockets may be left away from the path of the steam. The tube layout in a given condenser shell should be chosen so as to offer the least resistance to the steam penetration into the tube nest. The velocity of

Feed-water Heaters. Open or direct-contact feed-water heaters, being merely mixing devices, do not operate on a basis of heat transfer. Only in *closed, noncontact,* or surface heaters does a transfer of heat occur from the steam to the water. All heaters on a ship, other than the deaerating heater, are of this type and consist essentially of a number of straight or curved tubes connected at their ends to water boxes, or heads, enclosed in a shell, usually cylindrical in shape. The tubes are usually made of brass or copper

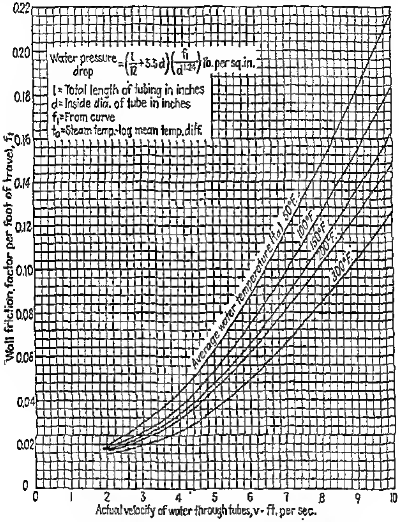
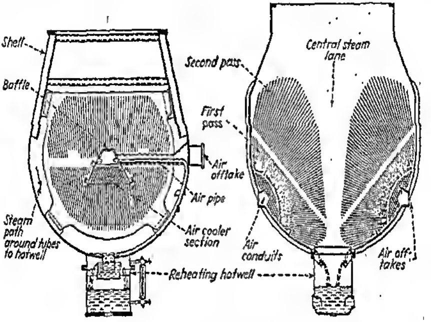


FIG. 3.—Feed-water heaters. Water friction loss.

alloy in order that they may resist corrosion. The heaters may be made single pass or multipass, usually the latter in small capacities in order that an economical shape may be obtained (see Fig. 1).

Structurally, the feed-water heater resembles the condenser, especially when the heater is to use steam below atmospheric pressure. In this case the shell is under pressure tending to cause it to collapse as in the case of a condenser. The higher pressure heaters are subjected to pressures tending

steam in the nest should be kept nearly uniform, and consequently the area of passage should decrease fast at first and slower toward the air suction. The dripping condensate reduces the heat transfer, and in general the lower tubes are the least efficient. In order to provide sufficient steam entrance area, a differential spacing of tubes is used in smaller condensers, sometimes in combination with triangular shape of the tube nest. In larger condensers, steam lanes are also provided. Radial-flow condensers with steam admission on the full circumference of the nest give good results. Eccentric radial layouts of tubes are employed in many condensers built for high vacuum. It is essential that the area of the steam passage near the suction to the air pump be reduced sufficiently to provide a fair velocity of the remaining steam-air mixture, otherwise, the part of the tube nest at the end of steam path will become air bound.



Westinghouse Elliott

FIG. 5.—Surface Condensers.

Water connections should be figured for a velocity of 6 to 8 fps and the air-pump connection should be at least twice that of the hot-well pump suction, which should be figured for about 4 fps. **Air-pump connections** should be arranged to draw the air-steam mixture evenly from the end of the steam path. In high-vacuum installations, an addition of a special air cooler either inside or outside the main shell increases the efficiency of the main tube nest and permits the reduction of the bulk and weight of condenser shells. Condensers with sealed hotwells are used to reduce the oxygen content in feed water by protecting the condensate from contact with the rich part of the air-steam mixture.

AIR PUMPS

The amount of air in feed water varies from 0.75 to 5 percent by volume at atmospheric temperature and pressure. If closed heaters are used, all the air passes through the boilers and prime movers into the condenser. If open heaters are used, the air in the feed water (at 200 F) should not be over 1 percent. When deaerators are installed, the non-condensible gases

to cause the shell to explode, and they must be designed, with this in mind, to withstand such pressures. Information for the design of tube-type feed-water heaters is given in Figs. 2 and 3.

An example of the use of this information follows:

Example. Assume that it is necessary to design a feed-water heater to heat 40,000 lb per hr water from 240 to 320 F with steam entering at 1260 Btu per lb at 100 psia. Drain to leave at saturation temperature or 328 F ⅝ in. O.D. Admiralty metal tubes to be used with a maximum tube length of 12 ft. Proceed as follows:

Enthalpy of entering steam	=	1260 Btu/lb
Enthalpy of leaving drain	=	298.4
Heat transferred per lb	=	961.6 Btu/lb

Temperature of steam = 328 F

$$\Delta_m = \frac{\Delta_{01} - \Delta_{02}}{\log_e \dfrac{\Delta_{01}}{\Delta_{02}}} = \frac{88 - 8}{\log_e {}^{88}\!/_{8}} = 33.3 \text{ F} \tag{5}$$

$$t_f = t_s - 0.8t_m = 328 - 26.6 = 301.4 \text{ F}$$

Take 8 fps velocity (Fig. 2.)

$$U = 900$$

Enthalpy of feed water entering	=	208.34 Btu/lb
Enthalpy of feed water leaving	=	290.28
Transferred	=	81.94 Btu/lb

Total heat transferred = 40,000 × 81.94 = 3,277,600 Btu/hr

$$\text{Surface area} = \frac{3,277,600}{900 \times 33.3} = 109.2 \text{ sq ft}$$

Volume of water at $\frac{240 + 320}{2} = 280$ F is 0.01726 cu ft per lb.

$$(\text{lb/tube/hr})\left(\frac{0.01726}{3600}\right) = (\text{lb/tube/hr})\ (4.8 \times 10^{-6}) = (\text{cu ft/sec/tube})$$

$$\frac{(\text{Cu ft/sec/tube})}{(\text{sq ft/tube})} = \text{ft/sec}$$

$$\frac{(\text{Lb/hr water})(\text{number of passes})}{(\text{Total number of tubes})} = \text{lb/tube/hr}$$

Therefore

$$\frac{(\text{Lb/hr water})(\text{number of passes})(4.8 \times 10^{-6})}{(\text{Total number of tubes})(\text{sq ft/tube})} = v$$

$$\text{Total number of tubes} = \frac{(\text{lb/hr water})(\text{passes})(4.8 \times 10^{-6})}{v(\text{sq ft/tube})}$$

Using No. 18 B.W.G. tube, ⅝ O.D.

$$\text{I.D.} = 0.625 - 2(0.049) = 0.527$$

$$\frac{0.7854(0.527)^2}{144} = 0.001525 \text{ sq ft/tube}$$

$$\text{Total number of tubes} = \frac{(40,000)(1)(4.8 \times 10^{-6})}{8 \times 0.001525} = 15.75$$

should be less than $\frac{1}{10}$ of 1 percent. The leakage in surface condensers is an extremely variable quantity, ranging from 1 percent in tight condensers to 25 percent or more in condensers in poor condition. The average leakage is from 5 to 10 percent of the volume of the feed water. As there is always leakage through glands, pipes, or joints, the air pump must have a displacement greater than that given by the above formula. The Heat Exchange Institute specifies that the capacity of the air-removal equipment for turbines should be not less than the following quantities; for engines these quantities are to be doubled. They are expressed in cfm of free air at 70 F.

Steam condensed, lb per hr..........	5,000	10,000	20,000	40,000	75,000	100,000
Air removal capacity, cfm...........	2.20	2.65	3.00	3.65	5.00	6.5
Steam condensed, lb per hr....	150,000	250,000	350,000	450,000	600,000 and over	
Air removal capacity, cfm.....	8.5	10.0	11.5	13.5	16.0	

The volume of free air (cfm at 70 F) liberated from 1,000 gal of injection water varies with the water temperature and may be assumed to be as follows:

Injection water temp, F...............	90	80	70	60	50	40	35
Air liberated, cu ft..................	2.18	2.40	2.64	2.93	3.30	3.75	4.0

In jet or barometric condensers, the combined circulating and air pump generally has a displacement of 3 times the volume of the injection water. If an independent air pump is used, the displacement should be twice the volume of the injection water.

The **air-removal equipment** may be a reciprocating dry-vacuum pump, a "burling-water" pump (Fig. 6), or **steam-jet air ejectors** (Fig. 7). In new installations and for replacement, the steam jet has replaced other types because it has no moving parts, is relatively free from maintenance, and is usually more economical to operate. One, two, or three stages are used depending on the vacuum desired, and two or three jets in parallel can make up each stage. Two-stage ejectors are most common.

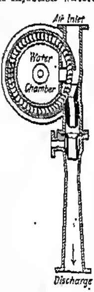


FIG. 6.—Westinghouse-Leblanc Rotary Air Pump.

Air pumps perform best when all possible vapor is condensed and removed before the air reaches the jets. The final condensing of the steam from the air-rich mixture and the cooling of the resulting saturated air-vapor mixture is accomplished either in an internal air cooler, usually a group of tubes baffled off from the main bank, or in an external air cooler. The location of the air-cooler section within the condenser, air offtake arrangement, and use of external coolers are important items of condenser design.

A single-stage ejector can be vented to atmosphere for the discharge of air and steam mixture from the jet. A two-stage ejector requires for efficient operation that first-stage steam be condensed and air-vapor mixture again cooled before passing to the second-stage jet (Figs. 8 and 9). This cooling is usually accomplished in a shell-and-tube **intercondenser** using condensate as cooling medium to recover ejector-steam heat.

An **aftercondenser** (Fig. 9) recovers heat from the final-stage steam, permits measuring air leakage, and avoids exhausting steam to atmosphere. For use in starting, when no condensate flow is available, the aftercondenser can be arranged with a separate raw-water section or a by-pass control

Call it 16 tubes for one-pass heater.

$$\text{Tube length} = \frac{\text{area of surface}}{\left(\frac{\text{number of tubes}}{\text{pass}}\right)(\text{passes})(\text{area/ft length tube})}$$

$$= \frac{109.2}{16 \times 1 \times 0.1636} = 41.8 \text{ ft for one pass}$$

For 12 ft, maximum length,

$$\frac{41.8}{12} = 3.5, \text{ call it 4 passes}$$

$$\text{Tube length} = \frac{109.2}{16 \times 4 \times 0.1636} = 10.5 \text{ ft} = \frac{l}{12}$$

Water pressure drop (see Fig. 3),

$$\left(\frac{l}{12} + 5.5d\right)\left(\frac{f_1}{d^{1.24}}\right)(\text{passes})$$

$$f_1 = 0.084 \text{ from curve}$$

$$(10.5 + 5.5 \times 0.527)\left(\frac{0.084}{0.527^{1.24}}\right)(4) = 10 \text{ psi}$$

For four passes, drop per pass in the water box at the end is 1.72 ft (see table, p. 1218).

$$\frac{4 \times 1.72}{2.3} = 3 \text{ psi}$$

$$10 + 3 = 13 \text{ psi, total drop}$$

Total number of tubes in the heater is $4 \times 16 = 64$. Taking $\frac{3}{8}$-in. clearance between tubes, the pitch for $\frac{5}{8}$-in. tubes would be $\frac{5}{8} + \frac{3}{8} = 1$ in. In any case, the number of tubes per square foot is $166/(\text{pitch})^2$ and in this case $166/1^2 = 166$ tubes per square foot.

$$\frac{64}{166} = 0.386 \text{ sq ft or } 55.5 \text{ sq in.}$$

$$\text{Diameter inside shell} = \sqrt{\frac{\text{area}}{0.7854}} = \sqrt{\frac{55.5}{0.7854}} = 8.4 \text{ in.}$$

With steam pressure at 100 psia or 85 psi gage and 1,000 psi stress allowable in the shell $t = \frac{pd}{2s} = \frac{85 \times 8.4}{2 \times 1{,}000} = 0.358$ in. would be sufficient. Make it $\frac{3}{8}$ in. The outside diameter should be $8.4 + 0.375 = 9.15$ in.

Fuel-oil Heaters. Fuel-oil heaters are, in general, very similar to closed, or noncontact, feed-water heaters in which the steam and the fluid to be heated are separated by metallic surfaces through which the heat is transferred from the steam to the fluid. There are three general types:

1. Multipass heaters, using straight or slightly curved tubes, oil inside.
2. Same as type 1 except with oil outside the tubes.
3. Heaters with the heat-transfer area consisting of flat surfaces with small sections or passages for the oil.

Type 1 is the most popular; in general, it should have not less than four passes. Type 2 is in disfavor, probably because the low velocity of the oil in the shell and the dependence on convection retard heat transfer. Type 3 is

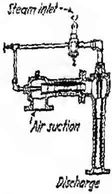


FIG. 7.—Two-stage Steam-jet Air Ejector.

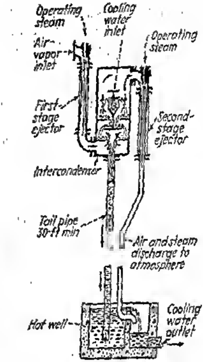
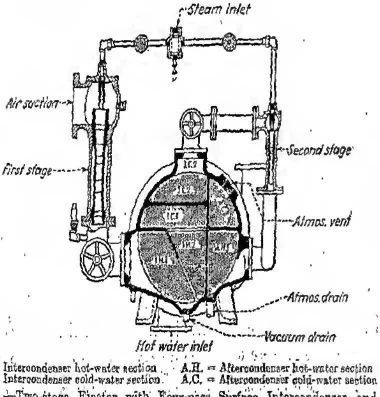


FIG. 8.—Two-stage Ejector with Direct-contact Barometric Intercondenser.



I.H. = Intercondenser hot-water section A.H. = Aftercondenser hot-water section
I.C. = Intercondenser cold-water section A.C. = Aftercondenser cold-water section

FIG. 9.—Two-stage Ejector with Four-pass Surface Intercondenser and Two-pass Aftercondenser.

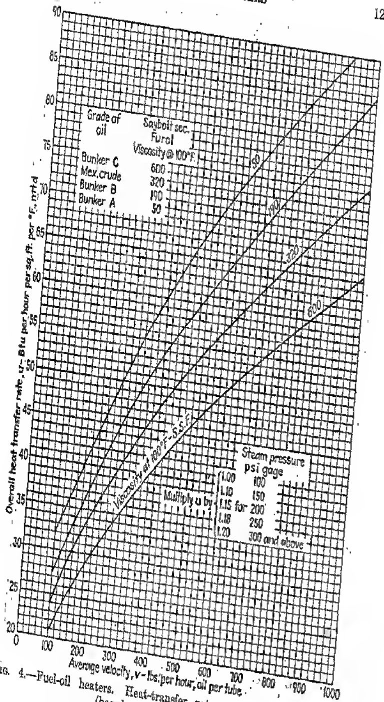


Fig. 4.—Fuel-oil heaters. Heat-transfer rates with ⅝-in. O.D. tubes (based on clean tube surface).

provided to recirculate condensate back to the condenser shell. Three-stage ejectors require two intercondensers and one aftercondenser.

Condensate from the intercondenser is usually returned to the condenser shell through a loop seal. Intercondenser steam space operates at high vacuum, 22 to 26 in., requiring a 10 to 15 ft water-leg seal.

The aftercondenser operates at atmospheric pressure. Drain can be piped to an open feed-water heater or trapped to the hot well.

Single-stage ejectors are customarily used up to 26.5 in. vacuum. For better vacuums up to 29.5, the two-stage ejectors should be used. Single-stage ejectors use about 7½ lb of live steam per lb of compressed vapor mixture at 26.5 in. vacuum. Two-stage non-condensing ejectors use 18 to 21 lb for 29 in. vacuum and 9 to 12 lb for 28 in.; the use of intercondensers reduces these quantities to 4 and 3 lb, respectively. The preceding figures apply to an average air-vapor mixture containing about 2.3 lb of steam per lb of air.

An augmenter is a device for boosting vacuum and is applied to a condenser served by a mechanical vacuum pump (Fig. 10). It consists of an ejector which may use exhaust steam of 3 to 10 lb gage pressure and an auxiliary condenser. With 5 lb operating pressure, it has been possible to augment the vacuum from 26 to 28.5 in. using 2 lb of exhaust steam per lb of air-vapor mixture compressed.

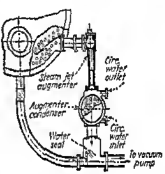


Fig. 10.—Vacuum Augmenter.

Power Required by Auxiliaries. The power required by large turbine auxiliaries varies from 1 to 3 percent, generally between 1 and 2 percent of the power of the main unit. In small installations, the power consumed varies from 2 to 10 percent. Centrifugal hot-well pumps or surface condensers generally require 0.05 to 0.10 hp for each 1,000 lb steam condensed per hour. Reciprocating air pumps require 0.10 to 0.3 hp per 1,000 lb steam per hour. Centrifugal air pumps require 0.25 to 0.5 hp per 1,000 lb steam per hour. The power required by circulating pumps varies with the design of the condensers, quantity of water pumped, and the efficiency of the pump. The pumping head may be as low as 10 ft in some condensers and as high as 35 ft or more in others, the average being about 20 ft. Pump efficiencies range from 75 to 85 percent for this service.

The steam required for auxiliaries, using moderate steam pressure, in pounds per brake horsepower per hour is as follows:

Circulating pumps, slow-speed, engine-driven	25 to 75
Circulating pumps, high-speed, engine-driven	40 to 100
Circulating pumps, turbine-driven, geared	25 to 30
Circulating pumps, turbine-driven, direct-connected	30 to 60
Air pumps, reciprocating	40 to 100
Hydraulic air pumps	40 to 55
Hot-well pumps, reciprocating	50 to 100
Hot-well pumps, centrifugal	40 to 60

The power required by jet condensers varies from 1 hp per 1,000 lb steam used per hour by the main turbine engine in the case of large units, to 3 hp per 1,000 lb steam for small units. Barometric condensers require 0.5 to 1.5 hp per 1,000 lb steam per hour.

just the opposite from type 2 as regards heat transfer, but the small passages tend to clog. There is a modification of type 1 gaining in favor owing to its low cost. Instead of being only slightly bent, the tubes have been made

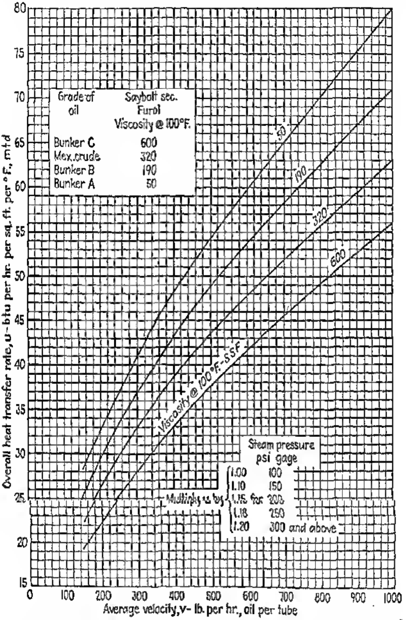


FIG. 5.—Fuel-oil heaters. Heat-transfer rates with ¾-in. O.D. tubes (based on clean tube surface).

into a coil. This minimizes expansion trouble, but the resistance to oil flow is increased. The design of type 1 only will be discussed here.

Because of the fact that the viscosity of the oil decreases so much owing to the rise in temperature, the pressure drop falls rapidly. Therefore it is

Most Economical Vacuum. No definite rule can be laid down for the most economical vacuum to be used. Each case must be considered separately on account of such factors as the cost of condensers, auxiliaries, and piping (all of which vary with temperature of inlet water), foundations, space, cost of coal, load factor, and fixed charges on main unit. For medium-size turbines a vacuum of 28½ in. and for large size 29 in. is generally considered the most economical with inlet water at 60 to 65 F. For engines, a vacuum of 26 in. is the most economical with water of ordinary temperatures. A higher vacuum will not increase engine economy.

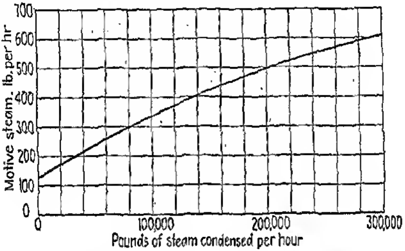


FIG. 11.—Steam Consumption of Two-stage Air Ejectors.

Salt-water Leakage. Where salt water must be used for circulating water in surface condensers, it is important that there should not be an excessive leakage into the condensed steam which is to be used for boiler feed. Glandless tubes with one floating tube sheet have been used successfully. If the hot-well water contains more than 1 percent of salt, it should not be supplied to the boilers. The method of determining this percentage is as follows:

Ten cc of the condenser water are measured out by a pipette, placed in a beaker, and two or three drops of potassium chromate solution added. A standard solution of silver nitrate is then added drop by drop from a burette, but only until the liquid in the beaker maintains a faint pink color on agitating. The volume of silver nitrate solution used is then recorded. Ten cc of the sample of circulating sea water are now measured out, 90 cc distilled water added, mixed, and 10 cc of the mixture taken. A few drops of the chromate solution are now added, and the silver nitrate added as before, the amount of the latter used being recorded. The amount of silver nitrate used for the circulating water multiplied by 10 and divided into the amount used for the condenser water and the quotient multiplied by 100 gives the percentage of sea-water leakage in the condenser water. The silver nitrate solution usually used is made up by dissolving 9.6 g silver nitrate in 2 liters of water. This is of such strength that 1 cc is equivalent to 1 mg of chlorine. Electrical-resistance recording indicators for salt-water leakage are available.

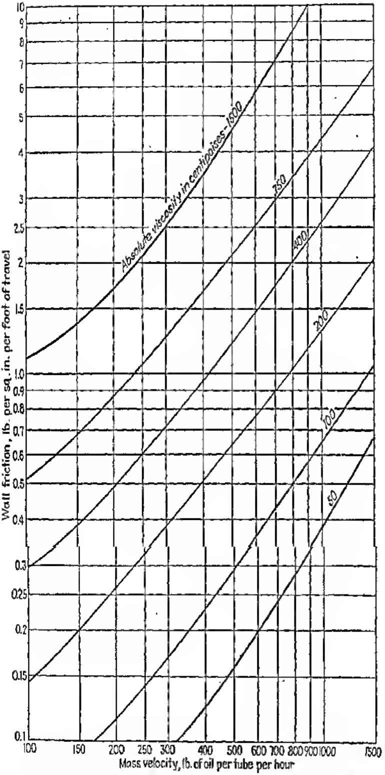


FIG. 6.—Fuel-oil friction with ⅝-in. O.D. tubing, No. 18 B.W.G.

HIGH-PRESSURE HEAT EXCHANGERS

BY

J. M. LABBERTON

General. Heat exchangers that operate at pressures higher than atmospheric present problems different from those operating at lower than atmospheric pressures, such as condensers. These high-pressure devices are used for transferring heat from steam to water, as in feed-water heaters; from water to water, as in drain coolers; from steam to oil, as in fuel-oil heaters; from oil to water, as in lubricating oil coolers.

Drain Coolers. The amount of surface required for drain coolers can be determined by using Eqs. (4), (5), (5a), and (6b), pp. 389, 391 to 393. These are generally shell and tube or double pipe devices in which there is a definite *velocity of water above the critical, i.e., turbulent flow. An example of the* use of this information follows.

Example. *Assume that it is desired to design a drain cooler to cool 3,400 lb per hr water at 186 to 110 F using 30,000 lb per hr water which enters at 102 F.*

Use 5/8 in. O.D., No. 18 B.W.G. tubing, Admiralty metal.

$$\text{I.D.} = 0.625 - 2(0.049) = 0.527$$

$$\frac{0.7854(0.527)^2}{144} = 0.001525 \text{ sq ft per tube}$$

Make velocity 5 fps.

$$0.001525 \times 5 = 0.007625 \text{ cu ft per tube per sec}$$

Average temperature of water $= \dfrac{186 + 110}{2} = 148$ F. 0.01633 cu ft per lb at this temperature.

$$\frac{0.007625 \times 3{,}600}{0.01633} = 1{,}680 \text{ lb per hr per tube}$$

Arrange so that water flow outside of tube is at approximately the same velocity as on the inside, i.e., 5 fps.

$$h_i = 160(1 + 0.012t_f)\frac{V_f^{0.8}}{D_f^{0.2}} \qquad (6b)$$

$t = 140$ F approximately. This is to be checked later.

$$h_i = 160(1 + 1.68)\frac{5^{0.8}}{0.527^{0.2}} = 430\frac{3.624}{0.8748} = 1{,}770$$

$$A_m = \frac{A_1 - A_0}{\log_e \dfrac{A_1}{A_0}} \qquad (4)$$

$$A_m = \frac{\dfrac{\pi 0.625}{12} - \dfrac{\pi 0.527}{12}}{\log_e \dfrac{0.625}{0.527}} = 0.153 \text{ sq ft per ft length}$$

$$\frac{3{,}400 \text{ lb per hr } (186 - 110)}{30{,}000 \text{ lb per hr}} = 8.6 \text{ F } \textit{rise in cold water}$$

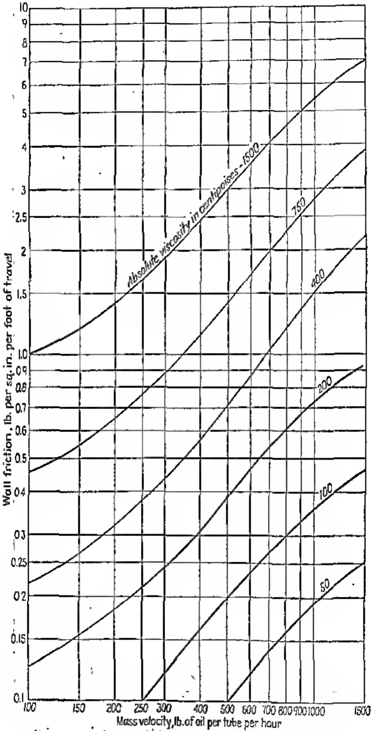


FIG. 7.—Fuel-oil friction with ¾-in. O.D. tubing, No. 16 B.W.G.

draw a circle cutting SA in B. Then draw a circle through A and B and touching one of the given circles (see preceding construction). This circle will touch the other given circle also. (Four solutions.)

To Draw an Annulus Which Shall Contain a Given Number of Equal Contiguous Circles (Fig. 44). (An annulus is a ring-shaped area enclosed between two concentric circles.) Let $R + r$ and $R - r$ be the inner and outer radii of the annulus, r being the radius of each of the n circles. Then the required relation between these quantities is given by $r = R \sin (180^\circ/n)$, or $r = (R + r)[\sin (180^\circ/n)]/[1 + \sin (180^\circ/n)]$.

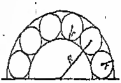


FIG 44.

For methods of constructing ellipses and other curves, see pp. 139–156.

LENGTHS AND AREAS OF PLANE FIGURES

Right Triangle (Fig. 45). $a^2 + b^2 = c^2$.
Area $= \frac{1}{2} ab = \frac{1}{2}a^2 \cot A = \frac{1}{2}b^2 \tan A = \frac{1}{4}c^2 \sin 2A$.

Equilateral Triangle (Fig. 46). Area $= \frac{1}{4}a^2\sqrt{3} = 0.43301a^2$.

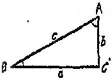


FIG. 45.

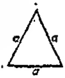


FIG. 46.

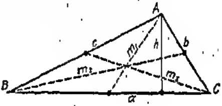


FIG. 47.

Any Triangle (Fig. 47). $s = \frac{1}{2}(a + b + c)$, $t = \frac{1}{2}(m_1 + m_2 + m_3)$

$r = \sqrt{(s - a)(s - b)(s - c)/s}$ = radius inscribed circle

$R = \frac{1}{2} a/\sin A = \frac{1}{2}b/\sin B = \frac{1}{2}c/\sin C$ = radius circumscribed circle

Area $= \frac{1}{2}$ base $\times$ altitude $= \frac{1}{2}ah = \frac{1}{2}ab \sin C = rs = abc/4R$

$= \sqrt{s(s - a)(s - b)(s - c)} = \frac{4}{3}\sqrt{t(t - m_1)(t - m_2)(t - m_3)}$

$= r^2 \cot \frac{1}{2} A \cot \frac{1}{2} B \cot \frac{1}{2} C = 2R^2 \sin A \sin B \sin C$

$= \pm\frac{1}{2}\{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\}$, where (x_1, y_1), (x_2, y_2), (x_3, y_3) are co-ordinates of vertices. See also p. 134.

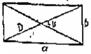


FIG. 48.

FIG. 49.

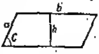


FIG. 50.

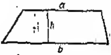


FIG. 51.

Rectangle (Fig. 48). Area $= ab = \frac{1}{2}D^2 \sin u$. [$u$ = angle between diagonals D; D.]

Rhombus (Fig. 49). Area $= a^2 \sin C = \frac{1}{2}D_1D_2$. [$C$ = angle between two adjacent sides; D_1, D_2 = diagonals.]

Parallelogram (Fig. 50). Area $= bh = ab \sin C = \frac{1}{2}D_1D_2 \sin u$. [$u$ = angle between diagonals D_1 and D_2; $D_1^2 + D_2^2 = 2(a^2 + b^2)$].

Trapezoid (Fig. 51). Area $= \frac{1}{2}(a + b)h = \frac{1}{2}D_1D_2 \sin u$. [Bases a and b are parallel; u = angle between diagonals D_1 and D_2.]

Quadrilateral Inscribed in a Circle (Fig. 52). Area $= \frac{1}{2}D_1D_2 \sin u = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \frac{1}{2}(ac+bd)\sin u$; $s = \frac{1}{2}(a+b+c+d)$.

Any Quadrilateral (Fig. 53). Area $= \frac{1}{2}D_1D_2 \sin u$.

NOTE. $a^2+b^2+c^2+d^2 = D_1^2+D_2^2+4m^2$, where m = distance between midpoints of D_1 and D_2.

Polygons. See table, p. 39.

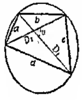


FIG. 52.

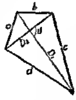


FIG. 53.

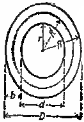


FIG. 54.

FIG. 55.

Circle. Area $= \pi r^2 = \frac{1}{2}Cr = \frac{1}{4}Cd = \frac{1}{4}\pi d^2 = 0.785398d^2$ (table, p. 30). Here r = radius, d = diam., C = circumference $= 2\pi r = \pi d$ (table, p. 28).

Annulus (Fig. 54). Area $= \pi(R^2 - r^2) = \pi(D^2 - d^2)/4 = 2\pi R'b$, where R' = mean radius $= \frac{1}{2}(R+r)$, and $b = R - r$.

Sector (Fig. 55). Area $= \frac{1}{2}rs = \pi r^2(A/360°) = \frac{1}{2}r^2$ rad A, where rad A = radian measure of angle A, and s = length of arc $= r$ rad A (table, p. 44).

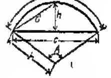


FIG. 56.

Segment (Fig. 56). Area $= \frac{1}{2}r^2$ (rad $A - \sin A$) $= \frac{1}{2}[r(s-c)+ch]$, where rad A = radian measure of angle A (table, pp. 34–35, 44). For small arcs, $s = \frac{1}{3}(8c' - c)$, where c' = chord of half the arc. (Huygens's approximation.) NOTE. $c = 2\sqrt{h(d-h)}$; $c' = \sqrt{dh}$ or $d = c'^2/h$, where d = diameter of circle; $h = r(1 - \cos\frac{1}{2}A)$, $s = 2r$ rad $\frac{1}{2}A$.

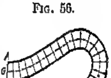


FIG. 57.

Ribbon bounded by two parallel curves (Fig. 57). If a straight line AB moves so that it is always perpendicular to the path traced by its middle point G, then the area of the ribbon or strip thus generated is equal to the length of AB times the length of the path traced by G. (It is assumed that the radius of curvature of G's path is never less than $\frac{1}{2}AB$, so that successive positions of the generating line will not intersect.)

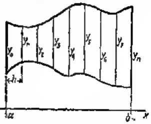


FIG. 58.

Simpson's Rule (Fig. 58). Divide the given area into n panels (where n is some even number) by means of $n+1$ parallel lines, called ordinates, drawn at constant distance h apart; and denote the lengths of these ordinates by $y_0, y_1, y_2, \ldots, y_n$. (Note that y_0 or y_n may be zero.) Then Area $= \frac{1}{3}h[(y_0 + y_n) + 4(y_1 + y_3 + y_5 \ldots) + 2(y_2 + y_4 + y_6 \ldots)]$, approx. The greater the number of divisions, the more accurate the result. Note: Taking $y = f(x)$, where x varies from $x = a$ to $x = b$, and $h = (b-a)/n$, then the

$$\text{error} = -\frac{1}{180}\frac{(b-a)^5}{n^4}f''''(X),$$

where $f''''(X)$ is the value of the fourth derivative of $f(x)$ for some (unknown) value $x = X$, between a and b.

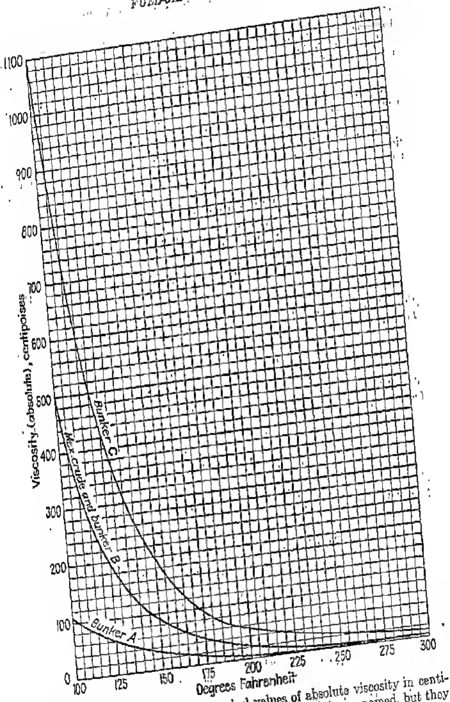


FIG. 8.—Fuel-oil viscosity. Typical values of absolute viscosity in centipoises. The values are not exact for all oils in the category named, but they are sufficiently accurate for practical determination of pressure drop in piping or heaters.

The combined flashed and generated vapors in the second-effect shell pass through the baffles and separator as in the first effect. The drains from the second-effect evaporator tube nest pass into the second-effect coil drain flash

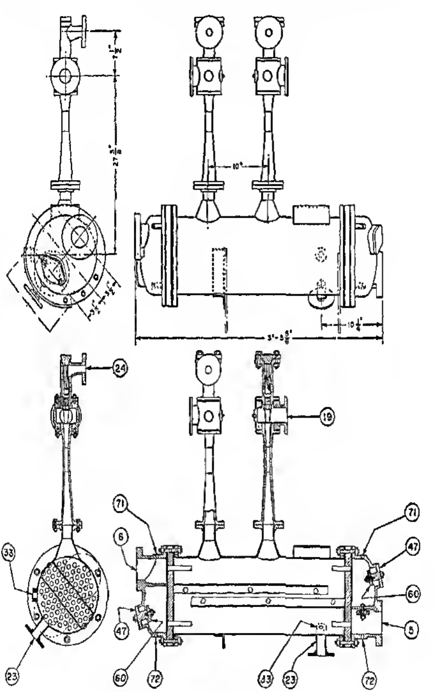


FIG. 14.—Air ejectors and after condenser for double-effect system. (*Courtesy of Foster Wheeler Corporation.*)

tank and finally join the drains from the distiller condenser. The vapor released in the second-effect coil drain flash tank, together with the vapor from the shell of the second-effect evaporator, passes into the distiller con-

good practice to have twice as many tubes per pass in the first half of the total passes as in the last half.

The curves in Figs. 4 to 8 will be used to determine the design of the fuel-oil heater. Their use is demonstrated by means of the following problem.

Example. It is desired to design a fuel-oil heater to heat 3,000 lb per hr of Bunker C oil from 100 to 230 F by means of 50 psi gage saturated steam and draining with no subcooling or at 298 F.

The specific heat of fuel oil of this character may be taken to be 0.48 over the practical temperature range with very little introduction of error. The amount of heat required is

$$(3{,}000)(230 - 100)(0.48) = 187{,}000 \text{ Btu/hr}$$

$$\frac{187{,}000 \text{ Btu/hr}}{911.6 \text{ Btu/lb}} = 205 \text{ lb/hr steam}$$

$$A_m = \frac{A_i - A_o}{\log_e \dfrac{A_i}{A_o}} = \frac{198 - 68}{\log_e \dfrac{198}{68}} = 122 \text{ F} \qquad (4)$$

Use $\frac{5}{8}$-in. O.D. tubing. Take velocity as 400 lb per tube per hr, average. $U = 41$ (see Fig. 4).

$$\text{Area} = \frac{187{,}000}{41 \times 122} = 37.4 \text{ sq ft}$$

Take eight passes. Two-thirds of surface in first four passes; one-third of surface in last four passes.

$$\tfrac{2}{3} \times 37.4 = 25 \text{ sq ft}$$

$$\frac{\text{Sq ft area}}{\text{Ft length}} = 0.1636 \text{ for } \tfrac{5}{8}\text{-in. O.D. tube}$$

If 400 lb per hr is average velocity $\frac{2}{3} \times 400 = 267$ lb per hr in first four passes. $\frac{4}{3} \times 400 = 534$ lb per hr in last four passes.

$$\text{Tube length} = l = \frac{\text{area}}{\text{number tubes} \times \dfrac{\text{area}}{\text{ft tube}}}$$

$$v = \frac{\text{lb oil}}{\text{tubes/hr}} = \frac{\text{lb oil/hr} \times \text{passes}}{\text{total tubes}}$$

$$267 = \frac{3{,}000 \times 4}{\text{total tubes}}$$

$$\text{Total tubes} = \frac{3{,}000 \times 4}{267} = 45 \text{ in first 4 passes}$$

There must be 48 to make an integral number of tubes per pass, or 6 tubes per pass. This necessitates a velocity correction. $v = \dfrac{3{,}000 \times 4}{48} = 250$ for first four passes, and 500 for last four passes.

$$U = 32 \text{ for } v = 250$$
$$U = 46 \text{ for } v = 500$$
$$U = \frac{32 + 46}{2} = 39 \text{ average}$$

denser. The distillate joins the drains from the second-effect evaporator tube nest, and both are pumped through the tubes of the distillate cooler into the test tank by the distillate condensate pump. The air ejector removes

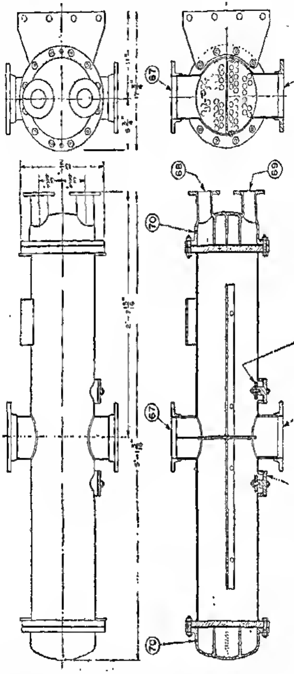


FIG. 15.—Distillate cooler for double-effect system. (*Courtesy of Foster Wheeler Corporation.*)

the noncondensible gases from the distilling condenser. The drains from the ejector after condenser join the drains from the first-effect evaporator tube nest and are returned to the ship's feed system.

$$\text{Area} = \frac{187{,}000}{39 \times 122} = 39.3$$

$$\tfrac{2}{3} \times 39.3 = 26.2 \text{ sq ft}$$

$$l = \frac{39.3}{(48 + 24)(0.1636)} = 3.33 \text{ ft}$$

Call it 3 ft 4 in.

For convenience, divide into four groups. Each pass could be worked out if desirable.

	Sq Ft
1st 2 passes 2 × 12 tubes × 0.1636 × 3.33 ft	13.1
2d 2 passes 2 × 12 tubes × 0.1636 × 3.33 ft	13.1
3d 2 passes 2 × 6 tubes × 0.1636 × 3.33 ft	6.6
4th 2 passes 2 × 6 tubes × 0.1636 × 3.33 ft	6.6
Total	39.4

If t_i = temperature of oil in and t_o = temperature of oil out, then the following expression may be used for determination of the temperature of the oil after having passed a given area:

$$t_o = t_i + t_e(1 - e^n)$$

where $n = -\dfrac{AU}{(\text{lb oil/hr})(0.48)}$

A = surface area
U = heat-transfer coefficient
t_e = temperature difference of steam and entering oil

To continue with the problem in hand. First two passes:

$$n = -\frac{(13.1)(32)}{(3{,}000)(0.48)} = -0.291$$

$$t_o = 100° + 198°(1 - e^{(-0.291)}) = 150 \text{ F}$$

Second two passes:

$$n = -0.291 \text{ (same as first two passes)}$$
$$t_o = 150 + 148(1 - e^{(-0.291)}) = 187 \text{ F}$$

Third two passes:

$$n = -\frac{(6.6)(46)}{(3{,}000)(0.48)} = -0.21$$

$$t_o = 187 + 111(1 - e^{(-0.21)}) = 208 \text{ F}$$

Fourth two passes:

$$n = -0.21$$
$$t_o = 208 + 90(1 - e^{(-0.21)}) = 225 \text{ F}$$

The latter value should be 230 F, *which is an indication of the error to be expected in this method of calculation.*

In any event, the average temperature of the oil in the first two passes is approximately 125 F. In the curve (Fig. 8) it can be seen that the absolute viscosity in centipoises is 420. In the curve (Fig. 6) it is seen that for a value of v = 250 (the velocity in the first pass) the wall friction is approximately 0.65 psi per ft travel. Pressure drop through the first two passes is therefore 0.65 ×6.66 ft = 4.35 psi

In similar manner, through the second two passes the average temperature is $\dfrac{150 + 187}{2}$ = 169 F.

Absolute viscosity = 110 centipoises
Wall friction = 0.16

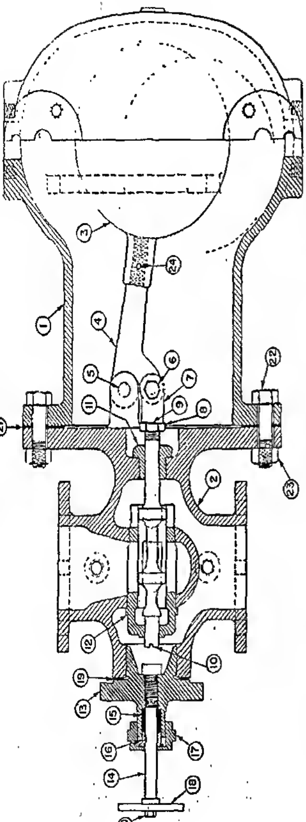


FIG. 16.—Drain regulator for double-effect system. (*Courtesy of Foster Wheeler Corporation.*)

Pressure drop through the second two passes is therefore $0.16 \times 6.66 = 1.07$ psi.

In similar manner, through the third two passes the average temperature is $\frac{187 + 208}{2} = 198$ F.

$$\text{Absolute viscosity} = 58 \text{ centipoises}$$
$$\text{Wall friction} = 0.17, \quad \text{for} \quad z = 500$$

Pressure drop through the third two passes is therefore

$$0.17 \times 6.66 = 1.15 \text{ psi}$$

In similar manner, through the fourth pass the average temperature is

$$\frac{208 + 225}{2} = 217 \text{ F}$$

$$\text{Absolute viscosity} = 40 \text{ centipoises}$$
$$\text{Wall friction} = 0.14$$

Pressure drop through fourth two passes is therefore

$$0.14 \times 6.66 = 0.94 \text{ psi}$$
$$\text{Total pressure drop is } 4.35 + 1.07 + 1.15 + 0.94 = 7.5 \text{ psi}$$

Drop through end boxes is negligible.

Oil Coolers. The amount of surface for oil coolers can be determined by calculating the water side as outlined on p. 1226 and by calculating the oil side by means of Fig. 3 on p. 394. μ in centipoises can be determined from Fig. 8 but when used in connection with Fig. 3 on p. 394 it must be multiplied by 2.42. The specific heat of oil, C_p, can be taken as 0.48.

$$K = 0.065 \text{ Btu/(hr)(sq ft)(deg F)(ft)}$$

The over-all heat transfer, U, can be determined as demonstrated on p. 392.

Evaporators. The purpose of evaporators is either to make fresh water from sea water or to evaporate the fresh water available on a ship and feed it into the feed system as vapor thus retaining almost all the heat that has been put into it. Small ships which do not use much fresh water usually are equipped with "single-effect" evaporators. Such systems evaporate the sea water in one shell, and the vapor therefrom is either used directly in the feed system or condensed and stored. Large ships and naval vessels which require a large amount of fresh water are equipped with "double-effect" or "triple-effect" evaporators. In such systems, the vapor from the first evaporator is led into a second evaporator and the heat therefrom used to evaporate still more water. If the vapor from the second evaporator is used to vaporize water in a third evaporator then it is said to be "triple-effect." It should be noted that the water is *not* double- nor triple-distilled.

Figure 9 shows a "single-effect" evaporator such as would be used on a small ship. This evaporator will produce 52,800 lb fresh water from sea water in 24 hr. Evaporation takes place at 10 psi gage, and the motive steam pressure may vary from 100 to 220 psi gage.

Feed water from the main condenser circulating water discharge is generally used for making fresh water. It enters the shell through the feed controller located on the side of the shell. The level of the water is maintained constant by this device. Approximately one-third of the water is

FEED SYSTEMS

BY

J. M. LABBERTON

General. The path traveled by the steam (or water) in any steam power plant is from boiler to turbine (or engine), thence to the condenser, and back to the boiler. This simple circuit requires a pump to force the water from the condenser to the boiler because of the higher pressure in the boiler. This is shown in Fig. 1.

That part of the circuit which handles water and which is necessary to feed this water to the boiler so that the boiler will have sufficient—no more nor less—material from which to make the required amount of steam is known as the feed system.

It is obvious that the only conditions under which the layout shown in Fig. 1 will operate successfully are absolutely unvarying load and no leakage.

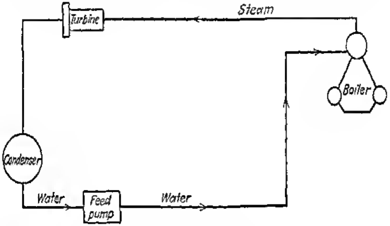


Fig. 1.—Basic feed system.

If the load is decreased, the feed pump will scavenge all the water out of the condenser and put too much in the boiler, filling it completely so that there will be no space for steam and, there being no elastic or compressible gas, the noncompressible water will burst the boiler or will flow into the turbine and destroy the blades.

If the load is increased, the condenser will fill with water and the boiler will run dry and then explode when water does reach it.

The effect of leakage is obvious.

Therefore, the scheme, as it stands, is impracticable, but it has two great advantages. All the heat in the condenser water is saved and available. The system is completely closed, and there is no possibility of the boiler feed water becoming contaminated with oxygen which would cause boiler-tube corrosion, assuming that there is no leakage of air into the system at the condenser, the pressure of which is below that of atmosphere. We can, however, render this system feasible by complicating it slightly with additional apparatus. Figure 2 shows the layout with the necessary items added.

These consist of a condensate pump, a surge or feed tank, a boiler-water-level regulating valve, and a means for adding water that may be lost by

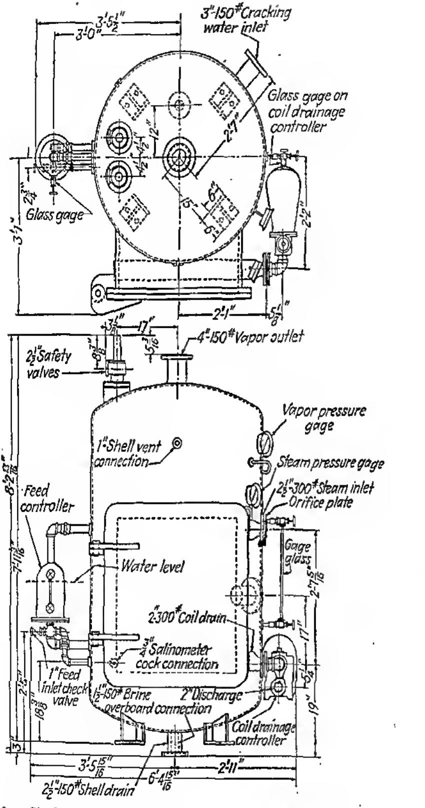


FIG. 9a.—Single-effect evaporator—elevation and plan. (*Courtesy of Foster Wheeler Corporation.*)

leakage. This latter water to be added is known as make-up feed. The boiler water level is controlled here by the regulator, and the variations of the water in the circuit are compensated for changes in the amount of water in the surge tank.

It will also be noticed that a separate circuit for steam to drive auxiliaries is included. These auxiliaries, of course, include the turbines for driving the condensate and main feed pumps, or the auxiliaries may consist of a turboelectric generator which furnishes power for the motors that drive the pumps.

In the case of the scheme shown in Fig. 1 it could be supposed that the feed pump was driven by an extension on the main turbine shaft.

The scheme shown in Fig. 2 is practicable. In fact, many ships are on the high seas, as you read this, operating with just such a feed system. It will be noted that the make-up water is shown being fed into the condenser. This is done so that the air ejectors, included among the auxiliaries, may have a chance to remove some of the air brought in by the raw feed water. Another

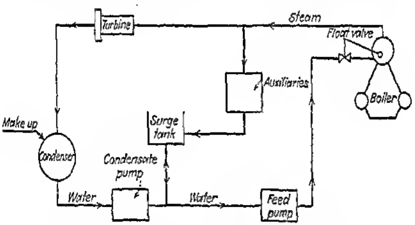


Fig. 2.—Practical open-feed system.

reason is that no pump or pressure above atmospheric is required to inject make-up feed into the system, the lower pressure of the condenser permitting atmospheric pressure to force the water in. Often, however, the make-up feed is put into the open surge tank.

This open surge tank places this system in the class of "open" feed systems. It is the weak spot in this system because oxygen in the air will surely dissolve in this water, and this corrosive solution will be carried into the boiler.

Deaeration. Attempts to obviate this trouble have resulted in the evolution of various types of "closed" feed systems. Obviously, since changes in load must be experienced, a surge tank or its equivalent should be incorporated in the system. No air should enter this surge tank directly from the atmosphere. Since in various sections of the feed system pressures range from condenser pressure to slightly above full boiler pressure, we have quite a choice of pressures at which to operate the surge tank. If it is placed in any part of the system at atmospheric pressure or below, it must be closed to atmosphere. However, it must also be vented at the top otherwise any air that might get into it, through leakage or in any other manner, would soon become bound at the top and eventually reduce the capacity of the tank as well as dissolve in the water and nullify the effects sought.

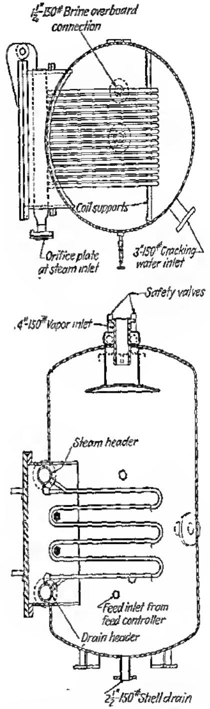


FIG. 9b.—Single-effect evaporator—horizontal section and sectional elevation. (*Courtesy of Foster Wheeler Corporation.*)

The only way to vent such a tank would be into a portion of the system at lower pressure. Naturally, the choice settles on the condenser, and this is often done. Figure 3 shows a popular general type of closed feed system incorporating a surge tank operating at lower than atmospheric pressure and vented to the condenser. Naturally, better elimination of air would be accomplished if the water in the surge tank were boiling. Since the pressure in the tank is low, boiling will take place at low temperatures and the heat can be supplied by means of low-pressure steam. In the case of Fig. 3, this steam is part of the exhaust of auxiliary turbines which are designed to operate at rather high exhaust pressure, say 10 psi gage. All the heat in this exhaust steam is not lost but passes on to the boiler in the feed water.

There is one portion of it, however, which is lost and that is one of the bad features of this system. Since the water in the surge tank is caused to boil, water vapor as well as air passes over to the condenser. The water vapor is

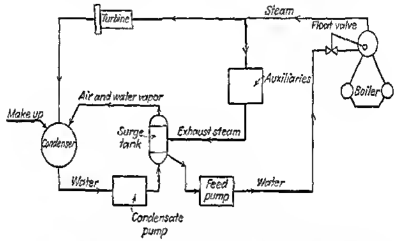


FIG. 3.—Closed-feed system with deaerating surge tank below atmospheric pressure.

condensed, and the air passes out through the air ejectors. However, in the condensation of the water vapor, all the latent heat of vaporization, approximately 1000 Btu per lb, goes out into the sea with the condenser circulating water and is lost forever. The object in the boiling is to get rid of the air and entrain it with vapor passing over. The more the vapor, the better the air elimination but, by the same token, the greater the loss.

Another undesirable feature is that the resistance to air and vapor flow between surge tank and condenser should be kept low. This means a very large pipe (or conduit) or an extremely short run or both. Often this is inconvenient.

If the surge tank is placed in any part of the feed system at higher than atmospheric pressure, the tank can be vented to atmosphere or to any pressure lower. The choice of venting points is practically limited to the atmosphere or to the condenser. If to the condenser, there is no point in going to a pressure higher than atmosphere for the surge tank. If to the atmosphere, there is no point in going to any pressure except one slightly higher than atmospheric.

converted into vapor which passes up through the baffled vapor outlet into the condenser. The remaining water is discharged overboard as concentrated brine.

Descaling, otherwise known as "temperature cracking," should be performed every 24 hr. This consists in shutting off the steam and filling the shell with cold sea water. Then steam is suddenly admitted, and the rapid resultant expansion causes any scale to crack and fall off.

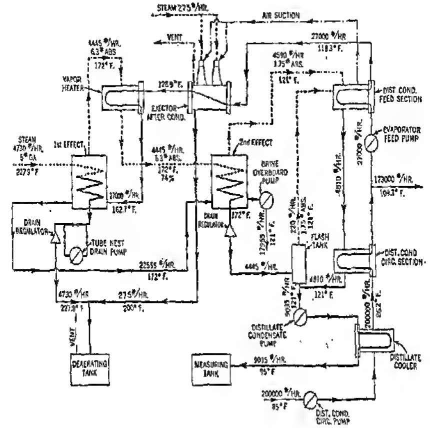


FIG. 10.—Double-effect evaporator. Heat-flow diagram under clean-tube conditions—26,000 gal per 24 hr. (Courtesy of Foster Wheeler Corporation.)

Access to the coils is gained by means of the door shown.

Figures 10 to 16 show the elements and flow cycle of a double-effect evaporator capable of 167,000 lb fresh water in 24 hr.

Feed water from the sea chest is pumped into the shell of the distillate cooler by the distilling condenser circulating pump. It then passes through the tubes of the distilling condenser condensing section where it absorbs heat from the vapor in the shell. After passing through the condensing section, about 85 percent of the feed water is pumped overboard. The remainder is picked up by the evaporator feed-pump suction and is further heated by being fed to the distiller condenser feed section, the air ejector after condenser, the first-effect vapor heater, in that order, and finally entering the shell of the first-effect evaporator. Part of the feed water will flash when it flows from the first-effect evaporator shell into the second-effect evaporator shell because of the low pressures. Brine, from the second-effect evaporator shell, is pumped overboard by the brine overboard discharge

A pressure slightly higher than atmospheric is the one usually chosen when venting to the atmosphere (see Fig. 4).

For the reasons mentioned before, it is well to have the atmospheric surge tank boiling. However, if water vapor boils off, not only is the heat lost as in the case of the surge tank venting to the condenser, but the valuable water is lost and the engine room is filled with opaque fog and vapor which is both annoying and dangerous.

The problem then resolves itself into getting rid of the air yet retaining the vapor. This is done by having this mixture of air and water vapor go through what is called a "vent condenser." This is a nest of cooling tubes, the surface temperature of which is considerably lower than 212 F. This cooling is usually accomplished by running the feed water, which is on its way to the deaerating surge tank, first through the coils of this vent condenser. Naturally, the water vapor will be condensed and fall back into the surge tank while the air, which is noncondensable under such circumstances, will

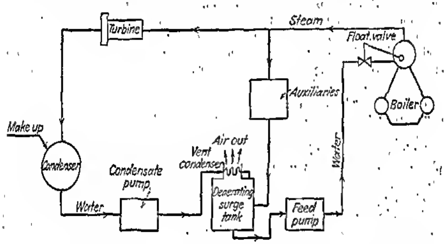


FIG. 4.—Closed-feed system with deaerating surge tank above atmospheric pressure.

pass on out. Usually, since it is not quite possible to obtain a perfect balance to the point where all the water vapor is condensed and all the air passes out of its own volition, a duct or conduit is led from the vent condenser to some point outside the machinery space and connected with the ventilation exhaust or some similar system. Of course, the air is not dry, so the feed system does lose a small amount of water in this manner.

It should be borne in mind that all of the latent heat of vaporization of the water boiled off with the air in the surge tank is saved aside from that small amount of water vapor leaving the vent condenser with the air. This heat is transferred to the feed water entering the surge tank. Consequently, considering the vent condenser as an integral part of the surge tank assembly, there is a mere circulation of heat within this assembly and none is lost. This conception greatly simplifies calculation of the heat balance of the complete system.

Pumps. Reference to Fig. 1 will disclose that only one pump is necessary. The suction of this pump will be at lower than condenser pressure whereas the discharge is somewhat greater than boiler drum pressure; however, reference to Fig. 2 will disclose two pumps in the feed line. The necessity

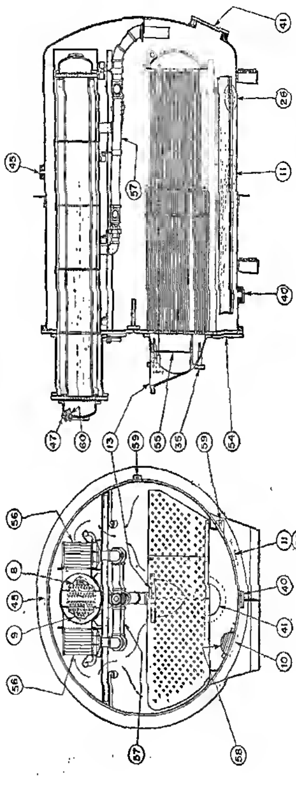


FIG. 11.—First-effect evaporator of double-effect system. (*Courtesy of Foster Wheeler Corporation.*)

for the additional or "condensate pump" is clear when it is realized that the open surge tank has atmospheric pressure on it and that this pressure would force all of the water back into the condenser. A similar condition exists in the case of Fig. 3 where the surge tank, though not at atmospheric pressure, is still at higher than condenser pressure. The same is true of Fig. 4.

In other words, in cases where turbine exhaust is at lower than atmospheric pressure, at least two pumps are necessary in the feed line in any practical feed system. Sometimes an additional or "booster" pump is desirable as will be discussed later.

In general, there should be at least one condensate pump for each condenser except sometimes in the case of auxiliary turbogenerator condensers. Since it must have a suction pressure approaching vacuum, it is invariably operated under submerged conditions and must be placed underneath the condenser it serves. On board ship this is sometimes difficult to do because the bottom of the condenser should also be placed low, it being under the main propulsion turbines, so as to keep the center of gravity of the total machinery weight low for stability reasons. In general, also, condensate pumps are driven by an electric motor of simple and rugged construction such as that of a squirrel-cage induction motor if a-c power is available on the ship, otherwise by a d-c shunt motor. The discharge pressure may be anywhere from 20 to 75 psi gage.

The feed booster pumps, when used, take water at a suction pressure about equal to that of condensate pump discharge. They discharge at from 50 to 100 psi gage.

Main feed pumps take water at a suction pressure of anywhere from atmospheric to 75 psi gage and discharge at somewhat higher than boiler-drum pressure.

This may be a proper place to mention that, if the suction pressure of the main feed pump is anywhere near the boiling point, corresponding to temperature and pressure of the water entering, boiling or vaporization is likely to occur and the pump will "cavitate" or vapor-lock. A margin of pressure should be assured here. To state this in another way, if the feed-water temperature entering the pump is high enough and its pressure is suddenly reduced, it will flash into steam. This steam has the same pressure, of course, and occupies the same space that water would have occupied. Its density, however, is relatively low and, when it finds its way into the passages at the entrance of the pump impeller, the centrifugal force, which is the basis of the discharge pressure, decreases greatly and the pump operates erratically if at all.

The principal purpose of a feed booster pump is to ensure a sufficiently high suction pressure on the main feed pump. For this reason, surge tanks at atmospheric pressure, whether deaerating or not, are usually placed at as high a point as practicable to ensure pressure at the main feed-pump intake.

Sometimes the main feed pump takes water from the last feed heater, if there are several stages of feed-water heating, and discharges directly into the boiler. Under such circumstances the feed water would be quite hot, considerably above 212 F and, in that event, a booster feed pump is almost essential so that no point will be at a pressure so low as to permit vaporization or steam formation in the feed line.

The actual specification as to capacity and power required to drive will be taken up in more detail on p. 1282.

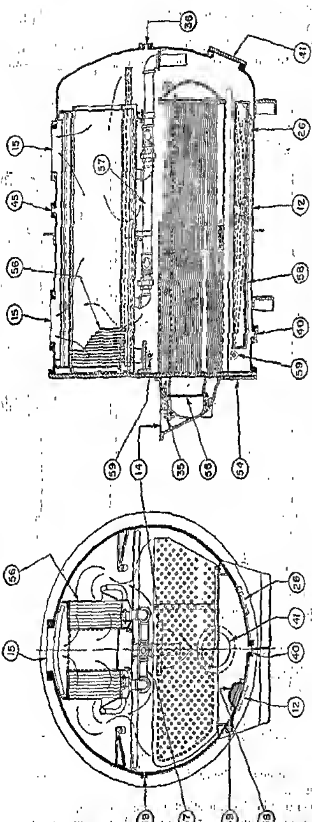


FIG. 12.—Second-effect evaporator of double-effect system. (Courtesy of Foster Wheeler Corporation.)

Heating. The main condenser in marine work is generally designed for operation at 1.5 in. Hg abs pressure. This pressure is somewhat higher than is common in the best stationary or land plants, but it should be remembered that space and weight on board ship are at a great premium. Consequently the design is a compromise with all the factors having influence on the design according to their importance, as is the case in all good engineering.

In the case of land installations, the condensers can be so built that there is very little subcooling of the condensate below the temperature corresponding to condenser pressure and, consequently, less heat is required for feed-water heating.

In the case of marine condensers, cramped space results in an unavoidable amount of subcooling. A reasonable figure to use as a basis for calculation and to specify is 5 F.

The saturation temperature corresponding to 1.5 in. Hg abs is 92 F; 5 F below this is 87 F; therefore it may be assumed that the lowest temperature of the feed water is 87 F and that the feed water must be raised to the final feed-water temperature from this value of 87 F. The value to be specified for final temperature will be discussed shortly.

Naturally, all savings possible should be made concerning heat in the system. Any heat lost or thrown away must be supplied by fuel.

There are many drains around a ship which are hotter than 87 F. These come from air-ejector condensers, gland leak-off condensers, condensate from lower stage feed heaters, etc. The temperature of this mixture, which can be collected in a drain tank, might be from 175 to 200 F. If a collecting tank is used, it should be vented either to atmosphere or to one of the condensers, otherwise it will soon become airbound and drains will not flow in.

These drains, which, of course, consist of good clean, distilled water, can then be led to the condenser but should be taken via a heat exchanger generally referred to a "drain cooler." On one side of the surface would be the cool feed water from the condenser entering at about 87 F and on the other side would be the relatively hot drains entering at about 175 to 200 F. This passage through the drain cooler might raise the feed water 5 to 20 F in temperature.

However, it might be desirable to pass the feed water through the air-ejector condensers first.

Air ejectors in marine service are assembled close to the condenser shell and are usually in two stages. The first-stage condenser will absorb all of the heat of the ejector motive steam plus about two-thirds of the heat of the water vapor carried over from the main condenser less the heat of the drains leaving. The second-stage condenser will absorb all of the heat of the ejector motive steam plus the other third of the heat of the water vapor carried over from the main condenser (and continued to be carried over to the second stage) less the heat of the drains leaving.

At full power, not more than 5 or 6 F will be picked up by the feed water in passing through the intercooler or aftercooler, nor more than 10 or 12 F in passing through the combination. Consequently feed water at 110 F could still well utilize the heat available in a drain cooler.

In general, passing the feed water through the air-ejector system first is to be preferred. It is conducive to better air-ejector action when the intercoolers and aftercoolers are colder.

Up to the present time, the discussion has been on the basis of one main propulsion turbine exhausting to one condenser. In the case of a *four-screw*

pump in sufficient quantity to maintain a maximum of 1½ thirty-seconds concentration.

The steam passes through a reducing valve, reducing the pressure to 5 psi gage or less, to the first-effect evaporator tube nest. The drains from the

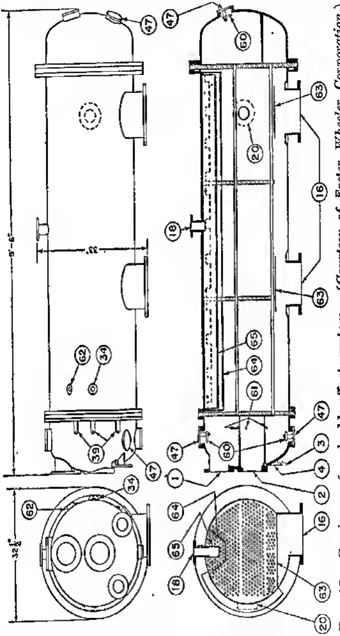


FIG. 13.—Condenser for double-effect system. (*Courtesy of Foster Wheeler Corporation.*)

tube nest are returned to the feed system of the ship by the first-effect tube nest drain pump.

Vapor, generated in the first-effect evaporator shell, rises through a system of baffle plates and a separator to the vapor heater tubes in the upper half of the shell. This vapor then passes into the second-effect evaporator tube nest.

ship there would be four sets of turbines, condensers, and air ejectors. They should then be considered as groups, each group acting as a unit of one turbine, and so on.

However, there will also be on the ship electric turbogenerators. These will operate in groups or, at times, there may be only one turbogenerator operating, with the main propulsion secured or shut down as when in port. In that event the turbogenerators are a little system within themselves and should be so considered. Therefore, it is common practice to run the condensate from the turbogenerators or, as some of the older marine people call them, the "dynamo condensers" through its own air-ejector system and then join it with the main or feed at a point just beyond the main aftercooler.

Often a connection is made between the exhaust of the turbogenerators and the main condensers. Then, when at sea and the main condensers are in operation, the turbogenerator condensers can be secured together with condensate pumps and air ejectors, and the turbogenerator turbine exhaust fed into the main condensers. This is an economical scheme but should be considered carefully before being adopted because of complication, weight, and space of piping.

From this point on to the boilers, there are two basic methods of applying heat to the feed water. One is to use exhaust steam from the auxiliaries plus the heat from such high-pressure drains as the steam heating system, fuel-oil heating system, etc. The other is to use steam bled from the main turbines plus the heat from the high-pressure drains. In the latter case, the auxiliaries would be driven by electric power, and the electric generator turbines would exhaust to condensers. Consequently there would be no auxiliary exhaust steam available for feed heating.

The former method will be considered first. It has already been mentioned and discussed to some extent. It is hardly practical to use more than one exhaust pressure on board ship because, if several exhaust pressures are used, there must be several exhaust-steam lines. Too much complication in the attempt to gain economy in marine steam installations is not justified and, since reliability and simplicity go hand in hand to some extent, some economy must be sacrificed to obtain a reasonable degree of simplicity and low cost.

Therefore, assuming that there will be but one exhaust pressure, the value of this pressure can be determined when the value of the feed-water temperature at the entrance to the boiler (or economizer) has been chosen. The saturation temperature of the exhaust steam used for feed-water heating should be approximately 25 F greater than that of the feed-water final temperature.

If a deaerating surge tank at slightly more than condenser pressure is used, such as shown in Fig. 3, the saturation or boiling temperature in this tank will be quite low, very close to 100 F. Consequently, if this is the only place where heat is applied to the feed water, a very low feed-water temperature will be the result or practically no feed-water heating at all. This is uneconomical and unsatisfactory because little or no use could be made of the exhaust steam from the auxiliary turbines. Therefore, under such circumstances, it would be better to put a feed heater of the shell and tube or surface type just ahead of the main feed pump or just after it.

If it is placed just ahead of the main feed pump, it will probably be necessary to install a feed booster pump between the surge tank and the feed heater so as to ensure high enough suction pressure at the main feed pump to avoid vaporization or cavitation at this point, as mentioned before.

Ellipse (Fig. 59; see also p. 140). Area of ellipse $= \pi ab$. Area of shaded segment $= xy + ab \sin^{-1}(x/a)$. Length of perimeter of ellipse $= \pi(a + b)K$, where $K = [1 + \frac{1}{4}m^2 + \frac{1}{64}m^4 + \frac{1}{256}m^6 + \ldots]$, $m = (a - b)/(a + b)$.

For $m =$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$K =$	1.002	1.010	1.023	1.040	1.064	1.092	1.127	1.168	1.216	1.273

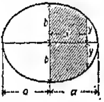


FIG. 59.

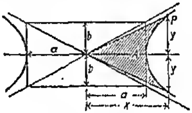


FIG. 60.

Hyperbola (Fig. 60; see also p. 144). In any hyperbola, shaded area $A = ab \log_e\left(\frac{x}{a} + \frac{y}{b}\right)$. In an equilateral hyperbola ($a = b$), area $A = a^2 \sinh^{-1}(y/a) = a^2 \cosh^{-1}(x/a)$. For tables of hyperbolic functions, see p. 60. Here x and y are co-ordinates of point P.

Parabola (Fig. 61; see also p. 138). Shaded area $A = \frac{2}{3}ch$. In Fig. 62, length of arc $OP = s = \frac{1}{2}PT + \frac{1}{2}p \log_e \cot \frac{1}{2}u$. Here c = any chord; p = semi-latus rectum; PT = tangent at P. Note: $OT = OM = x$.

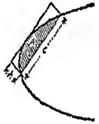


FIG. 61.

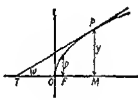


FIG. 62.

Other Curves. For lengths and areas, see pp. 147–156.

SURFACES AND VOLUMES OF SOLIDS

Regular Prism (Fig. 63). Volume $= \frac{1}{2}nrah = Bh$. Lateral area $= nah = Ph$. Here n = number of sides; B = area of base; P = perimeter of base.

Right Circular Cylinder (Fig. 64). Volume $= \pi r^2 h = Bh$. Lateral area $= 2\pi rh = Ph$. Here B = area of base; P = perimeter of base.

FIG. 63.

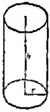


FIG. 64.

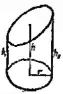


FIG. 65.

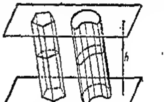


FIG. 66.

Truncated Right Circular Cylinder (Fig. 65). Volume $= \pi r^2 h = Bh$. Lateral area $= 2\pi rh = Ph$. Here h = mean height $= \frac{1}{2}(h_1 + h_2)$; B = area of base; P = perimeter of base.

Any Prism or Cylinder (Fig. 66). Volume $= Bh = Nl$. Lateral area $= Ql$. Here l = length of an element or lateral edge; B = area of base; N = area of normal section; Q = perimeter of normal section.

Any Truncated Prism or Cylinder (Fig. 67). Volume $= Nl$. Lateral area $= Qk$. Here l = distance between centers of gravity of areas of the two bases; k = distance between centers of gravity of perimeters of the two bases; N = area of normal section; Q = perimeter of normal section. For a truncated triangular prism with lateral edges a,b,c, $l = k = \frac{1}{3}(a + b + c)$. Note: l and k will always be parallel to the elements.

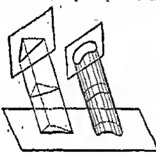


Fig. 67.


Fig. 68.

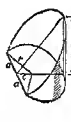


Fig. 69.

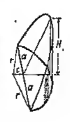


Fig. 70.

Special Ungula of a right circular cylinder. (Fig. 68.). Volume $= \frac{2}{3}r^2H$. Lateral area $= 2rH$. r = radius. (Upper surface is a semi-ellipse.)

Any Ungula of a right circular cylinder. (Figs. 69 and 70.) Volume $= H(\frac{2}{3}a^3 \pm cB)/(r \pm c) = H[a(r^2 - \frac{1}{3}a^2) \pm r^2 c \operatorname{rad} u]/(r \pm c)$. Lateral area $= H(2ra \pm cs)/(r \pm c) = 2rH(a \pm c \operatorname{rad} u)/(r \pm c)$. If base is greater (less) than a semicircle, use $+$ $(-)$ sign. r = radius of base; B = area of base; s = arc of base; u = half the angle subtended by arc s at center; rad u = radian measure of angle u (see table, p. 44).

Hollow Cylinder (right and circular). Volume $= \pi h(R^2 - r^2) = \pi hb(D - b) = \pi hb(d + b) = \pi hbD' = \pi hb\ (R + r)$. Here h = altitude; $r,R(d,D)$ = inner and outer radii (diameters); b = thickness $= R - r$; D' = mean diam $= \frac{1}{2}(d + D) = D - b = d + b$.


Fig. 71.

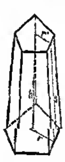


Fig. 72.

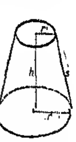


Fig. 73.

Regular Pyramid (Fig. 71). Volume $= \frac{1}{3}$ altitude $\times$ area of base $= \frac{1}{6}hran$. Lateral area $= \frac{1}{2}$ slant height $\times$ perimeter of base $= \frac{1}{2}san$. Here r = radius of inscribed circle; a = side (of regular polygon); n = number of sides; $s = \sqrt{r^2 + h^2}$. Vertex of pyramid directly above center of base.

Right Circular Cone. Volume $= \frac{1}{3}\pi r^2 h$. Lateral area $= \pi rs$. Here r = radius of base; h = altitude; s = slant height $= \sqrt{r^2 + h^2}$.

Frustum of Regular Pyramid (Fig. 72).

Volume $= \frac{1}{6}hran[1 + (a'/a) + (a'/a)^2]$.

Lateral area = slant height $\times$ half sum of perimeters of bases = slant height $\times$ perimeter of mid-section $= \frac{1}{2}sn(r + r')$. Here r,r' = radii

If it is placed just after toe main feed pump, the feed heater is at full boiler-drum pressure, slightly higher, in fact; this, though practicable, is obviously undesirable. Many ships, however, are in operation with feed heaters so placed.

With this system there is a wide choice of feed temperatures available, as high as 250 or 300 F with an exhaust pressure of possibly 75 psi gage. This, however, is unusual and the feed temperature is generally chosen at approximately 215 or 220 F with an exhaust pressure of 10 psi gage, approximately.

If a feed system incorporating an atmospheric deaerating surge tank is chosen, the feed temperature, in the case of single-stage heating such as is now being discussed, is fairly definitely fixed.

Water boils at 212 F at atmospheric pressure. Therefore the actual temperature in the tank must be higher owing to the higher pressure in the tank. Under no circumstances should it be less than 213 F, for then no deaerating effect would exist.

As mentioned before, as the main feed pump would take suction from this tank and discharge to the boilers, the tank should be placed as high as possible.

It should be borne in mind that, in the case of feed-heating systems using exhaust steam from auxiliaries for feed heating, once the feed temperature is decided upon, this fixes the exhaust pressure of the auxiliaries. This means that just enough exhaust steam, through very narrow limits, should be available for this feed heating and no more. If there is too little steam, the feed temperature will not be brought up to the proper point—considering deaeration, among other things. If there is too much exhaust steam, there will be a wasteful boiling off of water in the case of a deaerating surge tank and, in the case of a shell and tube feed heater, the exhaust steam will simply not condense and there will be a rise in exhaust pressure which will adversely affect auxiliary turbine performance.

Therefore, the heat balance must be carefully worked out to ensure just enough turbine-driven auxiliaries, the others being electrically driven, to furnish the proper amount of steam.

If conditions of variable power rates exist caused by running at various speeds under different circumstances, such as are encountered in the case of naval vessels, the problem is considerably complicated. Under such circumstances, the only practical solution, from an economical viewpoint, is to select auxiliaries so that there will be just enough steam under the most adverse condition and then feed the excess back into the main propulsion turbines at one of the lower stages under the other conditions.

This is bad from a turbine standpoint for the reason that at various loads on the turbines, various pressures occur at a given stage, and there would therefore have to be several feedback points on the turbine for a given exhaust pressure. Another argument against it is that the largest, most expensive, most difficult to design from a mechanical stress standpoint, part of the turbine is the low-pressure end. And, with this steam injected at low pressures, this undesirable situation is amplified.

The other basic method of heating feed water, i.e., by the use of steam bled from the turbines plus the heat from the high-pressure drains, is the more economical method and should be used whenever practically possible.

The fundamental nature of this economy can be readily understood when it is realized that the largest loss in the power plant is due to the heat given up in the exhaust. This exhaust is at approximately 110 F, and all the latent

involved. Saturated steam at higher temperatures is less deleterious than superheated steam. With the latter, the valve seats deteriorate rapidly.

HEAT BALANCE

Suppose that it is desired to design or specify the propulsion machinery for a ship having a displacement of 17,600 tons and requiring 8,500 shp; 100 kw electrical power is required for purposes other than auxiliary equipment for propulsion; and 2,000 lb per hr desuperheated steam is required for evaporators and reciprocating pumps. The electrical power for purposes other than propulsion is usually specified by the naval architect, whereas the steam consumption for the evaporators and small pumps is calculated by the marine engineer.

Assuming that 800 F total temperature of steam is satisfactory, reference to the Mollier diagram (Fig. 2) will disclose that, without reheat, the optimum pressure to use is not far from 600 psia; *i.e.*, with an exhaust pressure of 1.5 in. Hg abs.

It may appear that this temperature and pressure are arrived at rather quickly and without much investigation, but actually there is not much argument to it. Certainly the highest temperature should be used commensurate with the heat-withstanding qualities of the materials available. This is obvious from the Mollier diagram. Also, the proper pressure is obvious from the diagram aside from the argument for a higher pressure to reduce pipe size and taking into account moisture in the exhaust steam which must be a limited amount (see p. 1266).

Taking into account the pressure and temperature drop from boiler-superheater outlet to turbine throttle, the boiler-superheater-outlet condition should be approximately 615 psia at 810 F. The drop through the superheater of the boiler should be approximately 25 psi; therefore, the drum pressure should be 640 psia.

Some desuperheated steam will be necessary. If the drop through the desuperheater is considered negligible and 50 F of superheat remains (an average condition), the condition of the steam at the desuperheater outlet would be 615 psia, 539 F with an enthalpy of 1245.4 Btu per lb.

Using the method outlined on p. 1193, a condition curve of steam through the turbine is obtained (see Fig. 2).

Propulsion Steam Turbine Consumption. The combined efficiency with gears of an 8,500-shp unit should be approximately 76 percent (see Fig. 14, p. 1195). Therefore the water rate (W.R.) should be

$$\text{W.R.} = \frac{2{,}545}{(\text{isentropic drop})e_{\text{turbine}}}$$

The isentropic drop from 600 psia, 800 F, to 1.5 in. Hg is 509.7 Btu per lb, therefore,

$$\text{W.R.} = \frac{2{,}545}{(509.7)(0.76)} = 6.58 \text{ lb/shp-hr}$$

$$\text{Total steam} = 8{,}500 \times 6.58 = 56{,}000 \text{ lb/hr, no bleeding}$$

Turbogenerator Steam Consumption. Turbogenerators should be installed with sufficient capacity to obtain approximately 80 percent of full load when the ship is operating at full power. The reason for this is to

heat of vaporization of the steam being condensed in the condenser is thrown away in warming up sea water.

As was discussed in the case of using exhaust from the auxiliary turbines, practically all of the heat in the steam fed to auxiliary turbines is recovered in heating feed water. Instead of sea water carrying it away, feed water carries it to the boiler.

By bleeding steam from the main propulsion turbines, an attempt is made to recover part of the heat fed in, which is ordinarily lost in exhaust (see Fig. 5).

Obviously, if steam is bled from the turbines at a given pressure, the maximum temperature to which feed water can be heated is the saturation temperature corresponding to that pressure. Also, assuming for the sake of simplicity, a shell and tube type of heater with bled steam condensing in the shell, the maximum amount of steam that will flow out of the turbine and be

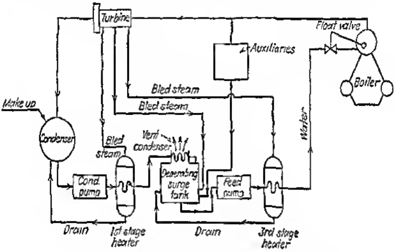


Fig. 5.—Regenerative feed-water heating system.

condensed is that amount sufficient to heat the feed water to the saturation temperature.

Naturally, the greatest economy will occur when the maximum amount of steam is diverted from the heat-wasting condenser and, as seen above, this amount is limited.

Assume that a certain amount of steam has been condensed in the condenser and assume that, at a pressure point only slightly higher than condenser pressure, steam is bled and used for heating this condensate from the condenser up to a temperature corresponding to the bled pressure. Assume further that the condensed steam that was bled is mixed with the water from the condenser, all now at the same temperature.

This bled steam has been completely useful. Its available energy has been applied to the turbine, and a certain percentage of mechanical energy, depending upon the efficiency of the turbine and gearing, has been transmitted to the propeller. The remaining heat has been utilized for a purpose for which otherwise fuel would have had to be supplied. True, the heat involved has been small because it was possible to raise the temperature of the feed water only slightly, but the gain was definite and positive.

permit the adding of more electrical load (after the ship has been in service) in the way of ventilating motors, etc., without the necessity of starting up another generator.

Assume, as a basis for preliminary calculation, that 175 kw will be necessary for auxiliary equipment for propulsion. This will be verified later.

$175 + 100 = 275$ kw operating load. Install two 350-kw d-c generators. One to operate at 275 kw. The combined efficiency of a 350-kw turbogenerator should be approximately 53.5 percent (see Fig. 25, p. 1204). Therefore the water rate should be

$$\text{W.R.} = \frac{3{,}413}{(\text{isentropic drop})e_{\text{turbogenerator}}}$$

Using the same throttle-steam conditions and exhaust pressure as in the case of the main turbines, the isentropic drop would be 509.7 Btu per lb. Therefore,

$$\text{W.R.} = \frac{3{,}413}{(509.7)(0.535)} = 12.55 \text{ lb/kwhr}$$

at full load. Taking the efficiency at 80 percent load to be 95 percent as good, the fractional-load water rate should be $12.55/0.95 = 13.2$ lb/kwhr. The total steam consumption should, therefore, be

$$275 \text{ kw} \times 13.2 \text{ lb/kwhr} = 3{,}630 \text{ lb/hr}$$

Air-ejector Steam Consumption. Refer to curve, Fig. 11, p. 1225. In the case of regenerative feed-water heating with three stages, approximately 90 percent of the throttle steam, no bleeding, goes to the condenser. Therefore, the steam to the main condenser would be approximately

$0.90 \times 56{,}000 = 50{,}400$ lb per hr (To be checked later)
240 lb per hr steam to main ejectors
140 lb per hr steam to turbogenerator ejectors

Feed-water Temperature. As demonstrated on p. 1252, the temperature of the feed water in the deaerating feed heater corresponds to the pressure in that heater. It is not necessary that the pressure be greater than sufficient to drive the air out. Probably 10 psi gage or 25 psia is a fair value to use in calculations. This would correspond to a temperature of 240 F, the temperature of the feed water leaving the deaerating heater.

Since the turbine exhaust temperature is 92 F (corresponding to 1.5 in. Hg with no subcooling) and since the saturation temperature at 600 psia is 486 F, $486 - 92 = 394$ F is the maximum possible rise in feed water through the three heaters. Referring again to p. 1263, it is seen that the optimum rise should be approximately

$$40 + 8 \times 3 = 64\% \text{ of the maximum rise}$$

or

$$0.64 \times 394 \text{ F} = 252 \text{ F, actual rise}$$
$$252 + 92 = 344 \text{ F}$$

call it 345 F for the final feed-water temperature. This corresponds to 126 psia. To care for some temperature and pressure drop the steam should be bled from the turbine at approximately 20 psi higher pressure or 145 psia.

Now, suppose that more steam is bled at a point slightly higher and the process repeated. A further small saving would result. Suppose this process to be repeated and repeated at small increments of pressure until the maximum bleeding pressure possible is reached. This maximum pressure is the throttle pressure of the turbine.

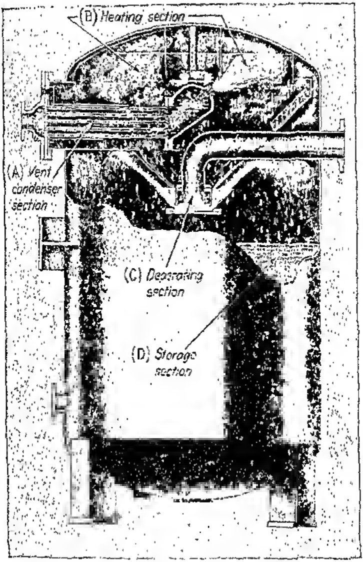


FIG. 6.—Deaerating feed-water heater and vent condenser. (*Courtesy of Elliott Company.*)

If an infinite number of bleeding pressures and heaters is used, each one to bring the feed-water temperature up to the saturation temperature at that point, the maximum economy will be realized.

This can be readily seen to be true when viewed from the standpoint that under such conditions the maximum amount of steam is permitted to pass through the turbine and only the latent heat of vaporization of the bled steam is required for feed heating, yet all the steam that it is possible to use for this purpose has been used.

If there is 5 F subcooling in the main condenser, this being a fair figure to use, the actual temperature of the condensate leaving the condenser should be approximately $92 - 5 = 87$ F.

In going from the condenser to the deaerating feed heater at 240 F there is a temperature rise of $240 - 87 = 153$ F. Assume that half of this is accomplished in the first-stage heater, then

$$153/2 + 87 = 163.5 \text{ F or } 164 \text{ F}$$

should be the temperature of the feed water leaving the first-stage heater. As this corresponds to 5.25 psia, the bleeding pressure should be made 10 psia to care for the drops. The bleed pressures at the turbine as well as the feed-water temperatures and enthalpies leaving the various heaters are shown on the accompanying table. It is assumed that loss of enthalpy in going from the turbine to the heater in each case is negligible.

Stage	Feed temp, deg F	Enthalpy Btu per lb	Bleed pressure, psia
1st	164	132	10
2d	240	208	35
3d	345	316	145

The major part of the condensate going through the first-stage heater is made up of main turbine and turbogenerator steam (see Fig. 1). Therefore, using an estimate of the amount of steam to be bled for first-stage, heating can be obtained.

50,400 lb/hr, main turbines
3,630 lb/hr, turbogenerators
54,030 lb/hr

Enthalpy of condensate leaving the two condensers at 87 F is 55 Btu per lb.

$$54{,}030 \text{ lb/hr } (132 - 55) = 4{,}160{,}000 \text{ Btu/hr to be supplied.}$$

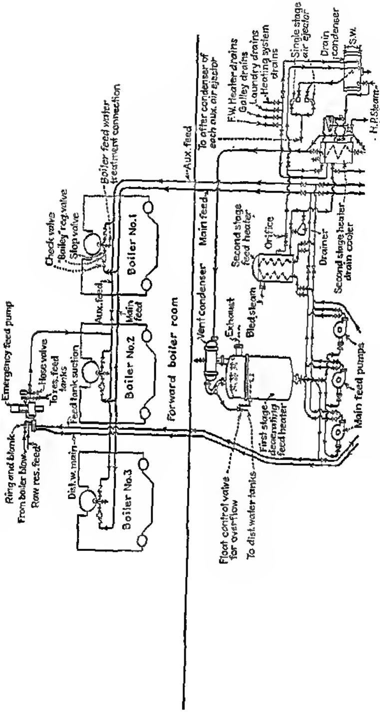
As previously mentioned, the air ejectors should use 380 lb per hr and, if the single-stage gland leak-off ejector takes 70 lb per hr, which is a reasonable figure, the total air-ejector steam would be $380 + 70 = 450$ lb per hr. Each pound of this steam is good for approximately 1000 Btu per lb. The total is, therefore, good for $450 \times 1000 = 450{,}000$ Btu per hr.

$$4{,}160{,}000 - 450{,}000 = 3{,}710{,}000 \text{ Btu/hr}$$

Referring to the condition curve of the turbine (Fig. 2), it is seen that the enthalpy of the steam bled for the first-stage heater at 10 psia is 1123 Btu per lb. Assume that the drain leaves at an enthalpy corresponding to 5 psia or 130 Btu per lb, then the heat transferred will be $1123 - 130 = 993$ Btu per lb.

$$\frac{3{,}710{,}000 \text{ Btu/hr}}{993 \text{ Btu/lb}} = 3{,}740 \text{ lb/hr}$$

to be bled for the first-stage heater.



Ring and blank
From boiler blow
Raw res. feed
Emergency feed pump
Hose valve
To res. feed tanks
Feed tank suction
Dist. w. main
Boiler No. 3
Boiler No. 2
Boiler No. 1
Aux. feed
Main feed
Check valve
"Bailey" reg. valve
Stop valve
Boiler feed water treatment connection
Forward boiler room
Aux. feed
Main feed
To after condenser of each aux. air ejector
Vent condenser
Exhaust
Bled steam
Second stage feed heater
Orifice
Drainer
Second stage heater drain cooler
F.W. Heater drains
Galley drains
Laundry drains
Heating system drains
Single stage air ejector
Drain condenser
S.W.
H.P. Steam drains
Float control valve for overflow
To dist. water tanks
First stage deaerating feed heater
Main feed pumps

FIG. 7.

All feed water goes through the deaerating feed heater, and the additional feed from the various auxiliaries averages up approximately to an enthalpy corresponding to the temperature of the feed leaving the heater; consequently, in raising from 164 to 240 F there is required

$$(54{,}030 \text{ lb/hr})(208 - 132) = 4{,}110{,}000 \text{ Btu/hr}$$

Referring to the condition curve of the turbine (Fig. 2), it is seen that the enthalpy of the steam bled for the deaerating feed heater at 35 psia is 1195 Btu per lb. This steam leaves as feed water at 208 Btu per lb. Therefore there is a transfer of 1195 − 208 = 987 Btu per lb.

$$\frac{4{,}110{,}000 \text{ Btu/hr}}{987 \text{ Btu/lb}} = 4{,}170 \text{ lb hr}$$

to be bled for the deaerating heater or second stage.

Ordinarily, in this class of ship, the steam for all purposes is approximately 30 or 40 percent more than the main turbine steam to condenser. Assume it is 40 percent.

$$1.4 \times 50{,}400 = 70{,}500 \text{ lb/hr}$$

This leaves the deaerating heater at 208 Btu per lb and is raised to 316 Btu per lb in the third-stage heater. Referring to the condition curve of the turbine (Fig. 2), it is seen that the enthalpy of the steam bled for the third-stage heater at 145 psia is 1295 Btu per lb. Assume that the drain leaves at an enthalpy corresponding to 130 psia or 319 Btu per lb. Then the heat transferred will be 1295 − 319 = 976 Btu/lb.

$$\frac{(70{,}500 \text{ lb/hr})(316 - 208)}{976 \text{ Btu/lb}} = 7{,}780 \text{ lb/hr}$$

to be bled for the third-stage heater.

With these estimated values of bled steam, turbine throttle steam can be calculated.

Extraction or Bleeding. Figure 3 shows the "condition curve" of a turbine on a Mollier diagram. The point a is the throttle condition, and the path abc is the isentropic path, or the path followed under conditions of 100 percent internal efficiency of the turbine. Actually, the path will be $ab'c''$. The available energy is, of course, $h_a - h_c$. But if steam is extracted or bled from the turbine at the pressure indicated by the pressure curve bb', then the available energy remaining in this bled steam would be $h_{b'} - h_{c'}$, and this is lost in so far as turbine work output is concerned.

If $w_{b'}$ = weight of steam bled at b', w_t = weight of throttle steam required to perform the remaining work, $w_{b'}$ would have done if it had not been bled. Then

$$w_{b'}(h_{b'} - h_{c'}) = w_t(h_a - h_c)$$

This is on the basis that the internal efficiency of the complete turbine is equal to the internal efficiency of that section from b' to c''. This, of course, is not strictly true, owing to the reheat differing (see p. 1192), but it is sufficiently close for all practical purposes. The extra work and calculation in

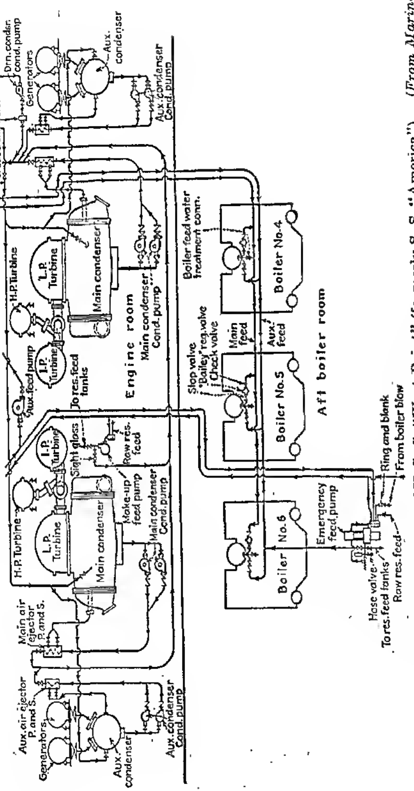


FIG. 7.—Feed and condensate system of U. S. S. "West Point" (formerly S. S. "America"). (*From Marine Engineering and Shipping Review.*)

correction for this small difference are not justified. Therefore,

$$\frac{h_{b'} - h_{c'}}{h_a - h_c} w_{b'} = w_t$$

or the steam to be added at the throttle, when $w_{b'}$ is bled, in order to maintain the same power output.

Induction or Feeding Back. If, on the other hand, there is excess steam of lower than throttle pressure that must be disposed of and the condition of this steam is as indicated by b' and if

$w_{b'}$ = weight of steam feed into turbine under condition b'

w_t = weight of throttle steam required to perform the work that $w_{b'}$ amount of steam would perform

then $$w_{b'}(h_{b'} - h_{c'}) = w_t(h_a - h_c)$$

therefore, $$\frac{h_{b'} - h_{c'}}{h_a - h_c} w_{b'} = w_t$$

or the steam saved at the throttle when maintaining the same power output

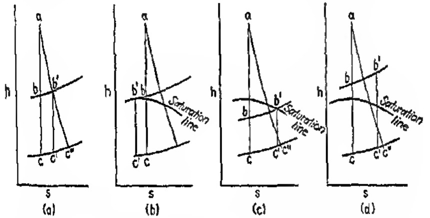


FIG. 3.—Extraction (bleeding) and induction (feeding back) of steam from and into turbines.

In either extraction or induction the term $(h_{b'} - h_{c'})/(h_a - h_c)$ is known as the **replacement factor.**

It should be carefully noted that when steam is bled from a turbine it may come out superheated, saturated, or wet, depending on the stage from which it is bled. However, when steam is inducted, it is usually saturated and may be superheated but should never be wet. In any event, it is highly improbable that the inducted steam will be of exactly the same character as that in the stage being entered. In Fig. 3*b*, *c*, and *d*, note the various circumstances that may exist in inducting or feeding back at a given pressure. Note that the condition in a stage may be superior or inferior to the condition of the steam coming in, but that, in any event, the isentropic drop from b', properly located, must be used as demonstrated above.

Steam to the Main Condenser. Referring to the condition curve (Fig. 2), the replacement factor for the first bleed point or that for the first-

To obtain a better conception, the two extremes and a medium point of bleeding will be considered.

Suppose that steam is bled from the turbine at throttle pressure and at this point only. This steam is used to heat feed water to the temperature corresponding to this pressure. At this point, the bled steam contains considerable available energy which could be converted to mechanical energy. This high-grade steam is used to do, to a large extent, a low-grade job, *i.e.*, to bring low-temperature feed water up to a high temperature when a low-pressure, low-grade steam could bring the temperature part of the way up just as well. This low-grade steam is what is left after the high-grade steam has been used to obtain some mechanical energy.

Obviously, this is not so economical as in the case of an infinite number of bleed points and heaters.

Suppose that steam is bled from the turbine at exhaust pressure. Of course, that is merely conventional operation and all the latent heat of vaporization of all the steam is lost in the condenser and that is not at all economical. It is what we are trying to avoid.

Suppose that steam is bled from the turbine at a point midway between throttle and exhaust. This is certainly better than bleeding at throttle pressure because some of the available energy of the bled steam has been converted to mechanical energy, and it is certainly better than no bleeding at all because some of the latent heat of vaporization has been salvaged. But it is not so economical as using an infinite number of heaters.

Therefore, it follows that the more the bleed points or stages of heating, the better the economy.

Naturally, it is impossible to install an infinite number of stages of heating on board ship. It would be impossible to have more stages of heating than there are pressure stages in the turbine and this is far too many for practical purposes. The maximum number that is, at present, acceptable to most ship operators is three, and it has been generally found that equal increments of feed-water temperature are best for economy. But this cannot always be exactly realized in marine installations because the construction of the turbines influence bleeding pressures and the middle-stage temperature is fixed by the deaerating surge tank.

The foregoing discussion has made no mention of losses, but there are, of course, losses. In the first place, since an infinite number of heaters is not used there is some loss due to using relatively high-temperature steam to bring relatively low-temperature water up to approximately the steam temperature. Also, there is a drop in pressure between the turbine and the heaters as well as a differential in temperature between steam and water in shell and tube heaters. In the second place, there is loss of heat due to heat transfer to the outside atmosphere from the various heaters and piping.

There are, on the other hand, various attendant advantages which are very important. (1) The turbine blading design becomes more rugged owing to the fact that the greater amount of steam passes through the upper, smaller, high-pressure stages. (2) The size and cost of the turbine exhaust shell become less for a given capacity. (3) The size and cost of the condenser decrease.

The best feed-water temperature has been the subject of considerable study. Obviously, the greater the number of heaters, the greater the optimum feed-water temperature. As already demonstrated, for an infinite number of heaters, the optimum temperature is the saturation temperature of the throttle steam, or the highest temperature it is possible to reach.

stage heater would be

$$\frac{h_{b'} - h_{e'}}{h_s - h_c} = \frac{1{,}123 - 965}{509.7} = 0.311$$

For the second bleed point or second-stage heater, the replacement factor would be

$$\frac{h_{b'} - h_{e'}}{h_s - h_c} = \frac{1{,}195 - 948}{509.7} = 0.485$$

For the third bleed point or third-stage heater, the replacement factor would be

$$\frac{h_{b'} - h_{e'}}{h_s - h_c} = \frac{1{,}295 - 926}{509.7} = 0.725$$

A table can now be made like the accompanying.

Heater	Bled steam, lb per hr	Replacement factor	Steam added, lb per hr
1st stage	3,740	0.311	1,162
2d stage	4,170	0.485	2,020
3d stage	7,780	0.725	5,650
Total bled	15,690	To be added	8,832

Throttle steam = 56,000 + 8832 = 64,832 lb/hr
To condenser = 64,832 − 15,690 = 49,142 lb/hr

(This is within 2 percent of the estimated condensate used to estimate amount of bleeding. This is close enough for the preliminary work now being done.)

A preliminary steam summary can now be made

PRELIMINARY STEAM SUMMARY

	Lb per hr	
Main propulsion turbines	64,832	
Turbogenerators	3,630	
Total superheated steam		68,462
Evaporators and reciprocating pumps	2,000	
Air ejectors	450	
Total desuperheated steam		2,450
Total steam, all purposes		70,912

This is sufficiently close to the 70,500 lb per hr used to estimate the bleeding of the turbine.

The heat balance will now be calculated. Refer to the steam flow diagram (Fig. 1) and start at the turbogenerator.

From the turbogenerator condenser, assume 10 lb per hr gland leak-off steam from all turbine bearings.

When operating at 8,500 shp, have all drains from the atmospheric drain tank enter the drain cooler.

Of course, the quantity or value really under consideration is the temperature rise because the bled steam really raises the temperature above that of condensate temperature. An empirical expression which may be used to obtain a first estimate of the optimum temperature is as follows:

Temperature rise of feed water in percent of maximum possible rise
$$= 40 + 8 \text{ (number of heaters)}$$

When the final temperature has been determined in this manner, it may be well to work out a couple of temperatures on either side to see if there is any change in economy. However, the efficiency curve at the optimum point is quite flat and there can be considerable change in temperature with very little change in economy.

The amount to be bled and the effect on turbine throttle steam value are also taken up on p. 1272.

It is not always practical to use this most economical scheme. Obviously, for certain bleeding pressures, it is set for a certain load on the turbines, normally the designed cruising speed of the ship. If the load on the turbines is changed to any extent, the pressures in the various stages will change and the corresponding temperatures of the bled steam to the various heaters will be entirely different. This will result in the system's being out of adjustment and a loss of economy.

It is not practical to have a multiplicity of bleeding points on a turbine and to change from one to the other quickly.

In the case of most naval vessels, maneuverability is important, and this involves wide changes in load. In such cases it may be best to have most auxiliary units steam-driven and have one bleed point on the turbine, this at auxiliary exhaust pressure; then steam can either be bled from or fed back in at this point when there is not enough or too much auxiliary exhaust for feed heating. In other words, economy must sometimes be sacrificed for flexibility.

In the case of destroyers, the cruising speed at which the ship operates more than 90 percent of the time requires power less than 10 percent of that required at full speed. In such case, it might be best to have a special cruising turbine and use bled steam for feed heating, driving the small cruising auxiliaries electrically; then for powers above this, steam-driven auxiliaries with their attendant flexibility could be used and their exhaust used for feed heating. In short, there would be two systems on one ship, one of less than 10 percent the capacity of the other.

"Carry-over," or water in the air removed by air ejectors, can generally be taken as 0.00002 *times the steam condensed. Also it can be assumed that* 70 percent of this carry-over is condensed in the intercooler and the other 30 percent in the aftercooler.

$$0.00002 \times 3{,}630 = 0.0726 \text{ lb/hr (negligible)}$$
$$\text{Turbogenerator intercooler drain} = 70 \text{ lb/hr}$$

Total condensate from turbogenerator condenser = 3,630 + 70 = 3,700 lb/hr. At 1.5 in. Hg,

$$\text{Temperature} = 92 - 5 = 87 \text{ F}$$
$$\text{Enthalpy} = 55 \text{ Btu/lb}$$

Total heat leaving turbogenerator condenser = 3,700 × 55 Btu/lb
= 203,500 Btu/hr

Gland Leak-off Condenser. Assume drains at 209 F or 177 Btu per lb, 10 lb per hr steam at atmospheric pressure.

10 lb/hr × 1150 Btu/lb	11,500 Btu/hr
10 lb/hr × 177 Btu/lb	1,770
Heat transferred	9,730 Btu/hr
Heat leaving turbogenerator	203,500
Total heat leaving condenser	213,230 Btu/hr

$$\frac{213{,}230 \text{ Btu/hr}}{3{,}700 \text{ lb/hr}} = 57.7 \text{ Btu/lb or } 90 \text{ F}$$

Turbogenerator Intercooler. The temperature of the leaving drains is approximately 121 F or at 89 Btu per lb.

70 lb/hr × 1245.4 Btu/lb	87,200 Btu/hr
70 lb/hr × 89 Btu/lb	6,230
Heat transferred	80,970 Btu/hr
In condensate	213,230
In condensate leaving	294,200 Btu/hr

Drain to turbogenerator condenser.

Turbogenerator Aftercooler. The temperature of the leaving drains is approximately 209 F or at 177 Btu per lb. There are two ejectors blowing into this condenser: the gland leak-off ejector, and the second stage of the turbogenerator ejection system, each at 70 lb per hr.

70 lb/hr × 1245.4 Btu/lb	87,200 Btu/hr
70 lb/hr × 1245.4 Btu/lb	87,200
Entering	174,400 Btu/hr
140 lb/hr × 177 Btu/lb	24,800
Transferred	149,600 Btu/hr
In condensate	294,200
In condensate leaving	443,800 Btu/hr

$$\frac{443{,}800 \text{ Btu/hr}}{3{,}700 \text{ lb/hr}} = 120 \text{ Btu/lb or } 152 \text{ F}$$

Drain to atmospheric drain tank.

Atmospheric Drain Tank. "Carry-over" from main condenser,

$$0.00002 \times 49{,}142 \text{ lb/hr} = 0.98 \text{ lb/hr}$$

HEAT BALANCE AND SIZES OF AUXILIARIES

BY

J. M. LABBERTON

STEAM PRESSURE AND SUPERHEAT

Steam pressure and steam temperature should be chosen to obtain the greatest amount of available energy from steam, the available energy being defined as that energy useful to the turbine or isentropic heat drop. Reference to the typical steam Mollier diagram (Fig. 2) will disclose that the available energy increases as the temperature increases, throttle pressure and exhaust pressure being held constant. However, there is a limit to the temperature due to the materials available for use in the upper stages of the turbine. At present this limit is practically about 1000 F, but much developmental work is being done, especially in view of the gas turbine (see p. 1205), and it is probable that the practical temperature will be raised considerably in the near future. The Mollier diagram, on the other hand, discloses that, although the available energy does increase as the pressure increases, this increase does not amount to much after 600 psia is passed, provided temperature and exhaust pressure are held constant. Consequently there is little to be gained by increasing the pressure above 600 psia if available energy is the only consideration.

Since steam volume decreases almost directly with pressure increase, temperature being held constant, there is some argument involving the size of high-pressure piping in favor of a pressure higher than 600 psia. The smaller this piping, temperature being held constant, the easier is the problem of pipe stresses. The maximum pressure without reheat should probably be 850 to 900 psia.

In the event that reheating of the steam, after it has passed through a portion of the turbine (see p. 1202), is contemplated, then the pressure should be increased to not less than 1,200 psia, otherwise the volumes of steam involved during reheating will be so large as to be impractical.

As to exhaust pressure, when space, weight, and economy are considered, 1.5 in. Hg abs seems to be the optimum pressure at the present time. This permits a condenser of good reasonable design.

FEED SYSTEM

As demonstrated on pp. 1258 to 1263, there is little argument in favor of any feed system other than the regenerative system, except in cases where a ship must be run continuously at several widely different speeds, as in the case of naval vessels.

A steam-flow diagram, such as that shown in Fig. 1, is a practical one and can be modified to include more or less stages of feed-water heating or to eliminate or add drain coolers, etc., as may be desirable from an economical or simplification standpoint. Calculations will show how much is to be saved at various points by the units under consideration.

It will be noted that the diagram includes a "contaminated" water system. This permits a small separate system to take care of water that might become contaminated with oil or other substances injurious to the boilers or other parts of the main system. As shown, this small system feeds the fuel-oil

Call it 1 lb per hr. Although this is divided 70 and 30 percent in the inter- and aftercoolers, in order to avoid dealing in fractions of a pound, assume that all of this carry-over is condensed in the intercooler. Therefore, the only drain from the main aftercooler is due to the motive steam of the air ejector.

Main aftercooler............	120 lb/hr × 177 Btu/lb = 21,240 Btu/hr
Turbogenerator aftercooler...	140 lb/hr × 177 Btu/lb = 24,780
Gland leak-off..............	10 lb/hr × 177 Btu/lb = 1,770
	270 lb/hr 47,790 Btu/hr

Drain Cooler. From first-stage feed-water heater at 5 psia (assuming pressure drop in coming from bleed point on turbine), 162 F or 130 Btu per lb.

3740 lb/hr × 130 Btu/lb = 486,000 Btu/hr
270 lb/hr × 177 Btu/lb = 47,790
4010 lb/hr 533,790 Btu/hr

$$\frac{533{,}790 \text{ Btu/hr}}{4{,}010 \text{ lb/hr}} = 133 \text{ Btu/lb or } 185 \text{ F entering.}$$

These drain coolers are usually designed with sufficient surface to result in a leaving drain temperature of 110 F or 78 Btu per lb.

$$133 - 78 = 65 \text{ Btu/lb heat transferred}$$
$$\text{Total transfer} = 4010 \text{ lb/hr} \times 65 \text{ Btu/lb} = 260{,}500 \text{ Btu/hr}$$

This transferred heat will be applied to the combined condensates of the turbogenerator condenser and main condenser. Calculations for the latter follow.

From Main Condenser.

	Lb per Hr
Main propulsion turbines..................................	49,142
Gland leak-off..	−10
Drain cooler..	4,010
Main intercooler..	121
Through air ejectors..	−1
Entering main condensate pump..............................	53,262

Assuming some subcooling as in the case of the turbogenerator condenser, the condensate is at 55 Btu per lb.

$$\text{Heat leaving} = 53{,}262 \text{ lb/hr} \times 55 \text{ Btu/lb} = 2{,}930{,}000 \text{ Btu/hr}$$

Main Intercoolers. 121 F drain leaving, 89 Btu per lb.

120 lb/hr × 1245.4 Btu/lb........................	149,600 Btu/hr
1 lb/hr × 1102 Btu/lb..........................	1,102
121 lb/hr..	150,702 Btu/hr
121 lb/hr × 89 Btu/lb...........................	10,780
Heat transferred..............................	139,922 Btu/hr
In condensate.................................	2,930,000
In condensate leaving..........................	3,069,922 Btu/hr

$$\frac{3{,}069{,}922 \text{ Btu/hr}}{53{,}262 \text{ lb/hr}} = 57.6 \text{ Btu/lb or } 90 \text{ F}$$

Drain to main condenser.

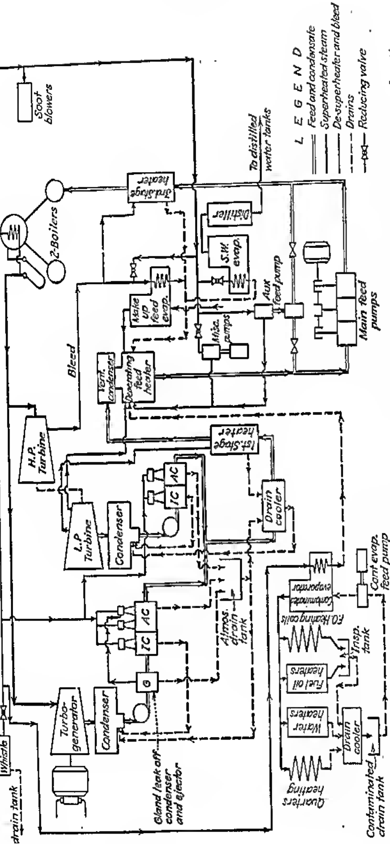


FIG. 1.—Steam and feed flow diagram. Closed regenerative system with three stages of feed-water heating.

Main Aftercooler. 209 F drain leaving, 177 Btu per lb.

120 lb/hr × 1245.4 Btu/lb	149,600 Btu/hr
120 lb/hr × 177 Btu/lb	21,240
Heat transferred	128,360 Btu/hr
In condensate	3,069,922
In condensate leaving	3,198,282 Btu/hr

$$\frac{3{,}198{,}282 \text{ Btu/hr}}{53{,}262 \text{ lb/hr}} = 60 \text{ Btu/lb or } 92 \text{ F}$$

Combination of Condensates.

53,262 lb/hr at	3,198,282 Btu/hr
3,700 lb/hr at	443,800
56,962 lb/hr at	3,642,082 Btu/hr

$$\frac{3{,}642{,}082 \text{ Btu/hr}}{56{,}962 \text{ lb/hr}} = 64 \text{ Btu/lb or } 96 \text{ F}$$

This enters the cold side of the drain cooler.

Drain Cooler.

In condensate	3,642,082 Btu/hr
Transferred in cooler	260,500
56,962 lb/hr at	3,902,582 Btu/hr

$$\frac{3{,}902{,}582 \text{ Btu/hr}}{56{,}962 \text{ lb/hr}} = 68.8 \text{ Btu/lb or } 101 \text{ F}$$

First-stage Heater.

Enthalpy of steam entering heater	1123 Btu/lb
Enthalpy of drain leaving	130
Transferred per lb	993 Btu/lb

3,740 lb/hr × 993 Btu/lb = 3,710,000 Btu/hr

In condensate	3,902,582
In condensate leaving	7,612,582 Btu/hr

$$\frac{7{,}612{,}582 \text{ Btu/hr}}{56{,}962 \text{ lb/hr}} = 134 \text{ Btu/lb or } 166 \text{ F}$$

164 F was the temperature sought for this stage.

Deaerating Feed Heater and Vent Condenser. A total of 4,170 lb per hr is to be bled from the main turbines at 1195 Btu per lb and introduced directly into the heater. A total of 7,780 lb per hr is to be bled at the highest pressure, 145 psia, and with an enthalpy of 1295 Btu per lb for use in the third-stage heater. Assume that 100 lb per hr of this is diverted to the make-up feed evaporator where it will be used to generate steam at approximately 20 psia which enters the deaerating feed heater for make-up. Such a scheme assures absolutely pure make-up water, and the heat used for vaporizing the feed water is not lost. The remaining 7,680 lb per hr will condense in the third-stage heater and return to the deaerating heater as water at 319 Btu per lb.

The fluid entering the deaerating heater will include the drain from the make-up feed evaporator or 100 lb per hr at 196 Btu per lb, assuming no subcooling.

heaters and quarters-heating system with low-pressure saturated steam. Such a system has the additional advantage over schemes that use steam, the

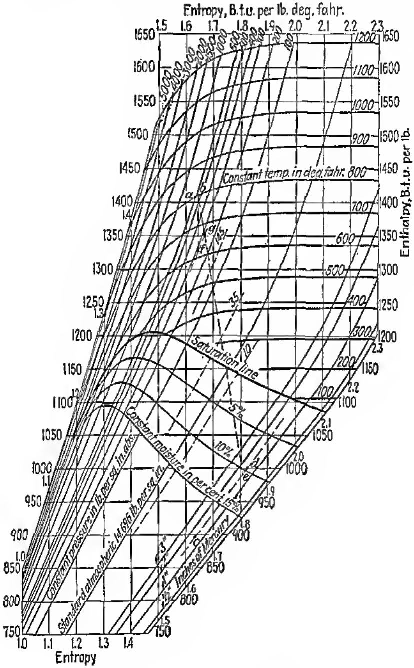


FIG. 2.—Condition curve of propulsion turbines.

pressure of which is reduced by means of reducing valves, in that there is no superheat. Superheat is bad in a quarters-heating system because of the corrosive action of superheated steam in spite of the low temperatures

The sum of the fluids entering the deaerating heater is as follows:

In condensate	56,962 lb/hr	7,612,582 Btu/hr
In bled steam	4,170 lb/hr × 1195 Btu/lb	4,980,000
Evaporators and reciprocating pumps	2,000 lb/hr × 335 Btu/lb	670,000
Make-up evaporator	100 lb/hr × 196 Btu/lb	19,600
Third-stage heater drain	7,680 lb/hr × 319 Btu/lb	2,450,000
	70,912 lb/hr	15,732,182 Btu/hr

$$\frac{15{,}732{,}182 \text{ Btu/hr}}{70{,}912 \text{ lb/hr}} = 222 \text{ Btu/lb or } 254 \text{ F}$$

This is too high a temperature at this point, and it is apparent that the bleeding from the turbine must be reduced. However, a good method is to accept this temperature for the time being and continue the calculations for the third-stage heater, then make all corrections at once.

It is well to note at this point that the total weight of feed water leaving the deaerating heater should be the same as the weight of steam, all purposes (see Preliminary Steam Summary, p. 1272), as no more water is added from this point on to the boiler. Note also that it is assumed here that the steam requiring make-up is lost from the system at the same pressure as that of the make-up steam entering the deaerating heater. This simplifies the calculations and is as justifiable an assumption as any other.

Third-stage Heater.

	Btu per Lb	Btu per Hr
Enthalpy of steam entering	1295	
Enthalpy of drain leaving	319	
Heat transferred per lb	976	
7,680 lb/hr × 976 Btu/lb		7,500,000
In condensate		15,732,182
Total in feed		23,232,182

$$\frac{23{,}232{,}182 \text{ Btu/hr}}{70{,}912 \text{ lb/hr}} = 328 \text{ Btu/lb or } 356 \text{ F}$$

This is 11 F too high or 12 Btu per lb too much.

The enthalpy leaving the second-stage heater should have been 208 Btu per lb whereas it was 222 Btu per lb or 14 Btu per lb too much. Therefore, the second stage should be decreased 14 Btu per lb and the third stage increased 2 Btu per lb.

$$\frac{70{,}912 \text{ lb/hr} \times 14 \text{ Btu/lb}}{(1195 - 208) \text{ Btu/lb}} = 1005 \text{ lb/hr reduction}$$

$4170 - 1005 = 3165$ lb/hr to be bled for second stage.

$$\frac{70{,}912 \text{ lb/hr} \times 2 \text{ Btu/lb}}{976 \text{ Btu/lb}} = 145 \text{ lb/hr increase}$$

$7780 + 145 = 7925$ lb/hr to be bled for third stage.

Corrected Steam Summary and Heat Balance. Make a table for the new bleedings similar to the one on p. 1272.

of inscribed circles; $s = \sqrt{(r - r')^2 + h^2}$; a,a' = sides of lower and upper bases; n = number of sides.

Frustum of Right Circular Cone (Fig. 73). Volume = $\frac{1}{3}\pi r^2 h[1 + (r'/r) + (r'/r)^2] = \frac{1}{3}\pi h(r^2 + rr' + r'^2) = \frac{1}{4}\pi h[(r + r')^2 + \frac{1}{3}(r - r')^2]$. Lateral area $= \pi s(r + r')$; $s = \sqrt{(r - r')^2 + h^2}$.

Any Pyramid or Cone. Volume $= \frac{1}{3}Bh$. B = area of base; h = perpendicular distance from vertex to plane in which base lies.

Any Pyramidal or Conical Frustum (Fig. 74). Volume $= \frac{1}{3}h(B + \sqrt{BB'} + B') = \frac{1}{3}hB[1 + (P'/P) + (P'/P)^2]$. Here B, B' = areas of lower and upper bases; P, P' = perimeters of lower and upper bases.

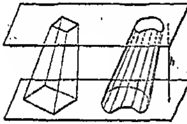


FIG. 74.

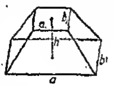


FIG. 75.

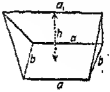


FIG. 76.

Obelisk (Frustum of a rectangular pyramid. Fig. 75).
Volume $= \frac{1}{6}h[(2a + a_1)b + (2a_1 + a)b_1] = \frac{1}{6}h[ab + (a + a_1)(b + b_1) + a_1b_1]$.

Wedge (Rectangular base; a_1 parallel to a,a and at distance h above base. Fig. 76). Volume $= \frac{1}{6}hb(2a + a_1)$.

Sphere. Volume $= V = \frac{4}{3}\pi r^3 = 4.188790r^3 = \frac{1}{6}\pi d^3 = 0.523599d^3$ (table, p. 36) = $\frac{2}{3}$ volume of circumscribed cylinder. Area $= A = 4\pi r^2$ = four great circles (table, p. 30) $= \pi d^2 = 3.14159d^2$ = lateral area of circumscribed cylinder. Here r = radius; $d = 2r$ = diameter $= \sqrt[3]{6V/\pi} = 1.24070\sqrt[3]{V} = \sqrt{A/\pi} = 0.56419\sqrt{A}$.

Hollow Sphere, or spherical shell. Volume $= \frac{4}{3}\pi(R^3 - r^3) = \frac{1}{6}\pi(D^3 - d^3) = 4\pi R_1^2 t + \frac{1}{3}\pi t^3$. Here R,r = outer and inner radii; D,d = outer and inner diameters; t = thickness $= R - r$; R_1 = mean radius $= \frac{1}{2}(R + r)$.

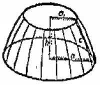


FIG. 77.

Spherical Segment of One Base. Zone (spherical "cap" of Fig. 78). Volume $= \frac{1}{6}\pi h(3a^2 + h^2) = \frac{1}{3}\pi h^2(3r - h)$ (table, p. 38). Lateral area (of zone) $= 2\pi rh = \pi(a^2 + h^2)$. Note: $a^2 = h(2r - h)$, where r = radius of sphere.

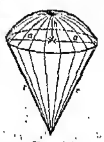


FIG. 78.

Any Spherical Segment. Zone (Fig. 77). Volume $= \frac{1}{6}\pi h(3a^2 + 3a_1^2 + h^2)$. Lateral area (zone) $= 2\pi rh$. Here r = radius of sphere. If the inscribed frustum of a cone be removed from the spherical segment, the volume remaining is $\frac{1}{6}\pi hc^2$, where c = slant height of frustum $= \sqrt{h^2 + (a - a_1)^2}$.

Spherical Sector (Fig. 78). Volume $= \frac{1}{3}r \times$ area of cap $= \frac{2}{3}\pi r^2 h$. Total area = area of cap + area of cone $= 2\pi rh + \pi ra$. Note: $a^2 = h(2r - h)$.

Spherical Wedge bounded by two plane semicircles and a lune. (Fig. 79.) Volume of wedge ÷ volume of sphere $= u/360°$. Area of lune ÷ area of sphere $= u/360°$. u = dihedral angle of the wedge.

Spherical Triangle bounded by arcs of three great circles. (Fig. 80.) Area of triangle $= \pi r^2 E/180°$ = area of octant $\times E/90°$. E = spherical excess $= 180° - (A + B + C)$, where A, B, and C are angles of the triangle. See also p. 134.

Solid Angles. Any portion of a spherical surface subtends what is called a **solid angle** at the center of the sphere. If the area of the given portion of spherical surface is equal to the square of the radius, the subtended solid angle is called a **steradian**, and this is commonly taken as the unit. The entire solid angle about the center is called a **steregon**, so that 4π steradians = 1 steregon. A so-called "solid right angle" is the solid angle subtended by a quadrantal (or trirectangular) spherical triangle, and a "spherical degree" (now little used) is a solid angle equal to $\frac{1}{90}$ of a solid right angle. Hence 720 spherical degrees = 1 steregon, or π steradians = 180 spherical degrees. If u = the angle which an element of a cone makes with its axis, then the solid angle of the cone contains $2\pi(1 - \cos u)$ steradians.

FIG. 79.

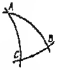


FIG. 80.

Regular Polyhedra. A = area of surface; V = volume; a = edge.

Name of solid (see p. 100)	Bounded by	A/a^2	V/a^3
Tetrahedron	4 triangles	1.7321	0.1179
Cube	6 squares	6.0000	1.0000
Octahedron	8 triangles	3.4641	0.4714
Dodecahedron	12 pentagons	20.6457	7.6631
Icosahedron	20 triangles	8.6603	2.1817

Ellipsoid (Fig. 81). Volume $= \frac{4}{3}\pi abc$, where a, b, c = semi-axes.

Spheroid (or ellipsoid of revolution). The volume of any segment made by two planes perpendicular to the axis of revolution may be found accurately by the prismoidal formula (p. 111).

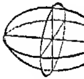


FIG. 81.

FIG. 82.

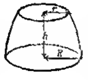


FIG. 83.

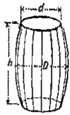


FIG. 84.

Paraboloid of Revolution (Fig. 82). Volume $= \frac{1}{2}\pi r^2 h = \frac{1}{2}$ volume of circumscribed cylinder.

Segment of Paraboloid of Revolution (Bases perpendicular to axis, Fig. 83). Volume of segment $= \frac{1}{2}\pi(R^2 + r^2)h$.

Barrels or Casks (Fig. 84). Volume $= \frac{1}{12}\pi h(2D^2 + d^2)$ approx for circular staves. Volume $= \frac{1}{15}\pi h(2D^2 + Dd + \frac{3}{4}d^2)$ exactly for parabolic staves.

Heater	Bled steam, lb per hr	Replacement factor	Steam added, lb per hr
1st stage	3,740	0.311	1,162
2d stage	3,165	0.485	1,536
3d stage	7,925	0.725	5,750
Total bled	14,830	To be added	8,448

Throttle steam = 56,000 + 8,448 = 64,448 lb/hr.
To condenser = 64,448 − 14,830 = 49,618 lb/hr

This is even closer to the estimated condensate than in the case of the preliminary calculations. A final steam summary can now be made.

Final Steam Summary

	Lb per Hr	
Main propulsion turbines	64,448	
Turbogenerators	3,630	
Total superheated steam		68,078
Evaporators and reciprocating pumps	2,000	
Air ejectors	450	
Total desuperheated steam		2,450
Total steam, all purposes		70,528

This is very close to the 70,500 lb per hr used to estimate the bleeding of the turbine.

From the steam flow diagram (Fig. 1) it can be seen that the turbogenerator steam flow and amount are unchanged. Therefore, start at the main condenser.

From Main Condenser.

	Lb per Hr	
Main propulsion turbines	49,618	
Gland leak-off	−10	
Drain cooler	4,010	
Main intercooler	121	
Through air ejectors	−1	
	53,738	
Heat leaving = 53,738 lb/hr × 55 Btu/lb	2,953,500	Btu/hr
Transferred in main intercooler	139,922	
Transferred in main aftercooler	128,360	
Total heat in feed	3,221,782	Btu/hr

$$\frac{3{,}221{,}782 \text{ Btu/hr}}{53{,}738 \text{ lb/hr}} = 60 \text{ Btu/lb or } 92 \text{ F}$$

Combination of Condensates.

Lb per Hr		Btu per Hr
53,738	at	3,221,782
3,700	at	443,800
57,438	at	3,665,582

$$\frac{3{,}665{,}582 \text{ Btu/hr}}{57{,}438 \text{ lb/hr}} = 64 \text{ Btu/lb or } 96 \text{ F}$$

be placed so that it is possible to remove the rotor without removing the frame from the bedplate. Small turbines should be placed so that the top half of the casing and the rotor may be removed without disturbing the rest of the assembly. Sufficient space should be available around boilers and condensers so that tubes can be removed and replaced without disturbing

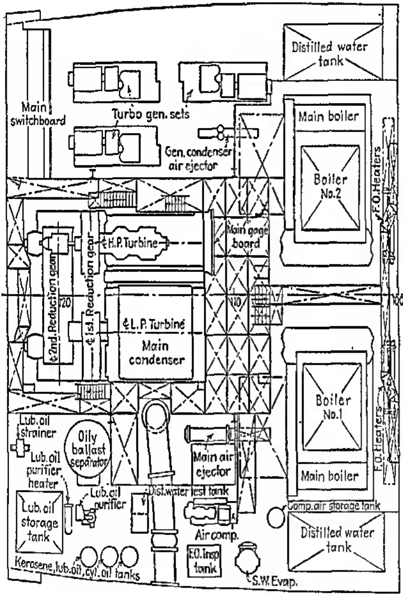


FIG. 5.—Intermediate level of S. S. "Exceller."

other apparatus. Probably the easiest way to assure these features is to make scale-size cardboard cutouts of the various motor and pump assemblies, etc., and have this cutout extended to the point where sufficient space is available to make the repairs mentioned above. Then in the preliminary stages of the layout these cutouts can be moved around and pinned in place temporarily on the layout board. These devices will, of course, show

	Btu per Hr
In condensate	3,665,582
Transferred in drain cooler	260,500
Transferred in first-stage heater	3,710,000
57,438 lb/hr at	7,636,082

$$\frac{7{,}636{,}082 \text{ Btu/hr}}{57{,}438 \text{ lb/hr}} = 133 \text{ Btu/lb or } 165 \text{ F}$$

This enters the deaerating feed heater via the vent condenser at 165 F.

Deaerating Feed Heater and Vent Condenser.

In condensate	57,438 lb/hr	7,636,082 Btu/hr
In bled steam	3,165 lb/hr × 1195 Btu/lb =	3,780,000
Evaporators and reciprocating pumps	2,000 lb/hr × 335 Btu/lb =	670,000
Make-up evaporator	100 lb/hr × 196 Btu/lb =	19,600
Third-stage heater drain	7,825 lb/hr × 319 Btu/lb =	2,500,000
	70,528 lb/hr	14,605,682 Btu/hr

$$\frac{14{,}605{,}682 \text{ Btu/hr}}{70{,}528 \text{ lb/hr}} = 207.5 \text{ Btu/lb or } 239 \text{ F}$$

As 240 F was desired at this point, 239 F is satisfactory.

Third-stage Heater.

7825 lb/hr × 976 Btu/lb	7,650,000 Btu/hr
In condensate	14,605,682
	22,255,682 Btu/hr

$$\frac{22{,}255{,}682 \text{ Btu/hr}}{70{,}528 \text{ lb/hr}} = 316 \text{ Btu/lb or } 345 \text{ F}$$

which was the final feed temperature originally sought.

To determine the fuel consumption, the boiler performance must be calculated.

Boilers.

Enthalpy steam at 615 psia, 810 F	1412.2 Btu/lb
Enthalpy feed water at 345 F	316
Heat gain	1096.2 Btu/lb
Enthalpy steam at 615 psia, 539 F	1245.4
Enthalpy feed water at 345 F	316
Heat gain	929.4 Btu/lb

Superheated 68,078 lb/hr × 1096.2 Btu/lb = 74,600,000 Btu/hr
Desuperheated 2,450 lb/hr × 929.4 Btu/lb = 2,280,000
76,880,000 Btu/hr

Assuming a boiler efficiency of 88 percent and fuel oil with a calorific value of 18,500 Btu per lb, the oil consumption should be

$$\frac{76{,}880{,}000 \text{ Btu/hr}}{0.88 \times 18{,}500 \text{ Btu/lb}} = 4710 \text{ lb/hr fuel}$$

$$\frac{4{,}710 \text{ lb/hr}}{8{,}500 \text{ shp}} = 0.555 \text{ lb fuel/shp/hr}$$

Install two boilers.

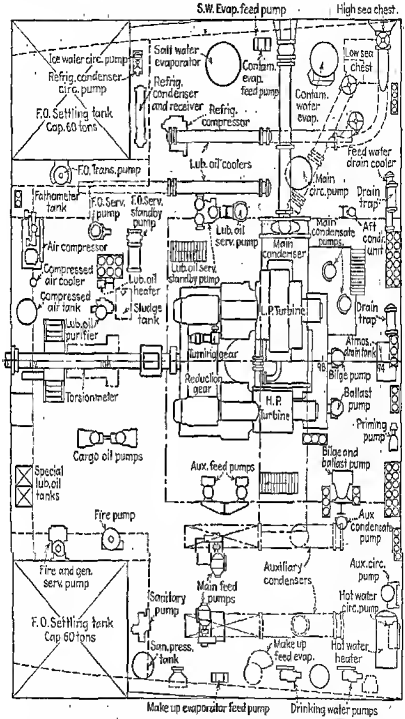


FIG. 6.—Engine-room layout of the S. S. "Joseph Lykes," a 4,000 S.H.P., C-1-B. steamship designed by the U.S. Maritime Commission. (*From Marine Engineering and Shipping Review.*)

ELECTRICAL LOAD AND SIZES OF AUXILIARIES

Fireroom **Blowers.** Assume that three fireroom blowers will be installed any one of which will be able completely to handle the requirements of each of the two boilers. Assume that the total air pressure required is to be 8 in water when 225 cu ft air per lb fuel is being supplied. This is a fair set of operating conditions for modern boilers and is probably on the conservative side.

From the heat balance,

$$\frac{4{,}710 \text{ lb/hr} \times 225 \text{ cu ft/lb}}{60 \text{ min}} = 17{,}700 \text{ cfm}$$

for 2 blowers.

$$\frac{17{,}700}{2} = 8{,}850 \text{ cfm/blower at 8 in. water}$$

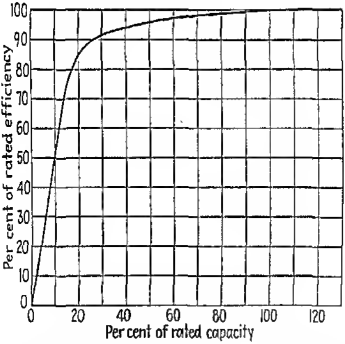


FIG. 4.—Electric motors. Approximate percentage of full load efficiency at fractional loads.

Assume 60 percent blower efficiency. This is a conservative figure, and the manufacturer of the blower may guarantee a higher efficiency.

$$\text{Hp} = \frac{0.0001573 \times \text{cfm} \times \text{in. water}}{\text{efficiency}}$$

$$= \frac{0.0001573 \times 8{,}850 \times 8}{0.60} = 18.6$$

Install a 20-hp 1,750-rpm motor. Table 13, p. 1727, shows the performance to be expected of conventional designs of d-c motors.

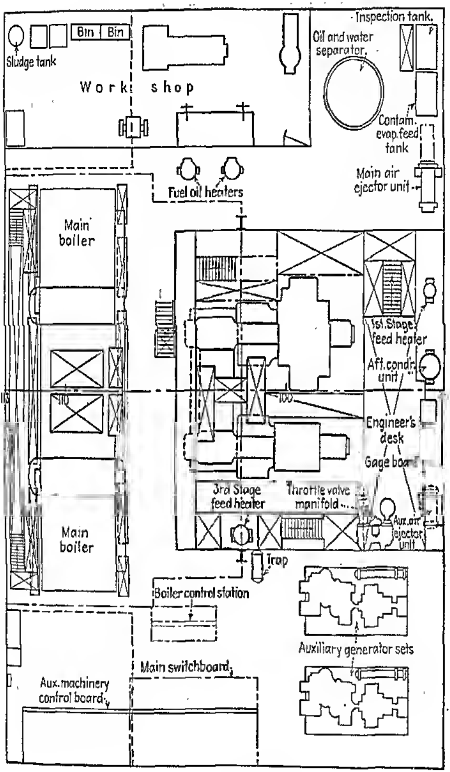


FIG. 6.—*Continued.*

18.6/20 = 94 percent of rated capacity. Efficiency at this load is 87.5 percent (see Fig. 4).

$$\text{Kw input to motor} = \frac{18.6 \times 0.746}{0.875} = 15.9$$

$$\text{Total kw} = 2 \times 15.9 = 31.8$$

Main Circulating Pump. Install one, operate one. Later, the actual water and pressure required for the condenser can be obtained from the condenser manufacturer; at the present stage a preliminary estimate must be made. Assuming 1000 Btu dissipated in the condenser for each pound of turbine exhaust and a 10 F rise in sea water passing through the condenser, which are conservative figures, the Btu per gallon on sea water can be approximated as follows. There are 8.58 lb per gal in sea water under average temperature conditions and the specific heat is 0.94.

0.94 × 8.58 × 10 F = 80.6 Btu/gal
49,618 lb/hr to condenser (see heat balance, p. 1277)

$$\frac{49{,}618 \text{ lb/hr} \times 1000 \text{ Btu/lb}}{80.6 \text{ Btu/gal} \times 60 \text{ min}} = 10{,}300 \text{ gpm}$$

which should be the normal capacity of the pump. Install with 20 percent margin, or 1.2 × 10,300 = 12,380 gpm. Call it 12,500 gpm at 10 psi total head and assume motor drive at 690 rpm. Take 10 psi as equivalent to 23 ft ahead. Make the impeller so as to have water enter each side as in Fig. 1 in the pump section. This would result in 6.250 gpm per side at 23 ft 690 rpm (see Fig. 10, p. 1606). N_s = 5,170 (see Fig. 11, p. 1607). Efficiency = 87 percent. A mix-flo type of pump.

$$\text{Hp} = \frac{\text{gpm} \times \text{psi}}{1{,}715 \times \text{efficiency}}$$

$$\text{Hp} = \frac{12{,}500 \times 10}{1{,}715 \times 0.87} = 83.8$$

Install 85-hp motor capable of speed adjustment down to 75 percent of rated speed by means of shunt field rheostat.

Operating load on the pump is 10,300 gpm or $\frac{10{,}300}{12{,}500}$ = 82.5 percent of rated capacity (see Fig. 12, p. 1608).

$$\text{Efficiency} = 0.95 \times 0.87 = 0.826$$

$$\text{Operating head is} \left(\frac{10{,}300}{12{,}500}\right)^2 (10) = 6.8 \text{ psi}$$

$$\text{Operating hp} = \frac{10{,}300 \times 6.8}{1715 \times 0.826} = 49.4$$

See Table 13, page 1727. Motor efficiency is 0.89. Motor efficiency at 49.4 hp is .87 percent (see Fig. 4).

$$\text{kw input} = \frac{0.746 \times \text{hp}}{\text{efficiency}} = \frac{0.746 \times 49.4}{0.87} = 42.4$$

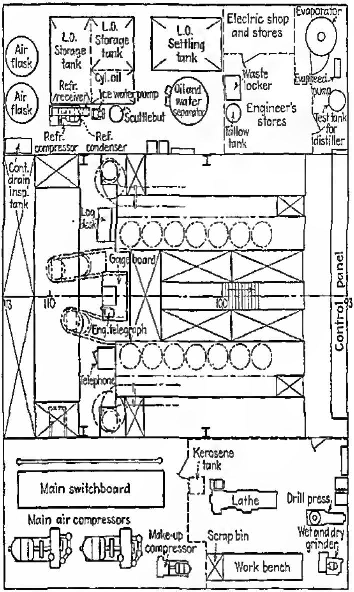


FIG. 7.—Engine-room layout of the S. S. "American Manufacturer," a 4,000 S.H.P., C-1-B. diesel ship designed by the U. S. Maritime Commission. (*From Marine Engineering and Shipping Review.*)

Main Condensate Pumps. Install two, operate one.

$$\frac{53{,}738 \text{ lb/hr}}{8.33 \text{ lb/gal} \times 60 \text{ min}} = 107.4 \text{ gpm}$$

Install with 35 percent margin.

$$1.35 \times 107.4 = 145 \text{ gpm}$$
$$75 \text{ psi total head}$$

Make two stage and drive at 1,750 rpm (see Fig. 10, p. 1606).

$$75 \text{ psi} = 172.5 \text{ ft head}$$
$$\frac{172.5}{2} = 86.25 \text{ ft head per stage}$$
$$N_s = 748 \qquad \text{(See Fig. 11, p. 1607)}$$
$$\text{Efficiency} = 58 \text{ percent}$$
$$\text{Hp} = \frac{145 \times 75}{1{,}715 \times 0.58} = 10.92$$

Install a 12-hp motor. Operating capacity of the pump is $\frac{107.4 \text{ gpm}}{145 \text{ gpm}} = 74$ percent of rated (see Fig. 12, p. 1608).

$$\text{Operating efficiency} = 0.90 \times .58 = 52.2 \text{ percent}$$
$$\text{Operating hp} = \frac{107.4 \times 75}{1{,}715 \times 0.522} = 0$$

See Table 13, page 1727. Full-load efficiency = 86 percent. Motor efficiency at 75 percent load = 85 percent (see Fig. 4).

$$\text{Kw input} = \frac{0.746 \times 9}{0.85} = 7.92$$

Auxiliary Condensate Pumps. One pump for each generator,

$$\text{Full-load water rate} = 12.55 \text{ lb/kwhr}$$
$$350 \text{ kw} \times 12.55 \text{ lb/kwhr} = 4{,}400 \text{ lb/hr}$$
$$\frac{4{,}400 \text{ lb/hr}}{8.33 \text{ lb/gal} \times 60 \text{ min}} = 8.8 \text{ gpm normal}$$

Install with 35 percent margin.

$$1.35 \times 8.8 = 11.9$$

Call it 12 gpm at 75 psi total head. Make two stage and drive at 1,750 rpm.

$$75 \text{ psi} = 172.5 \text{ ft load}$$
$$172.5$$

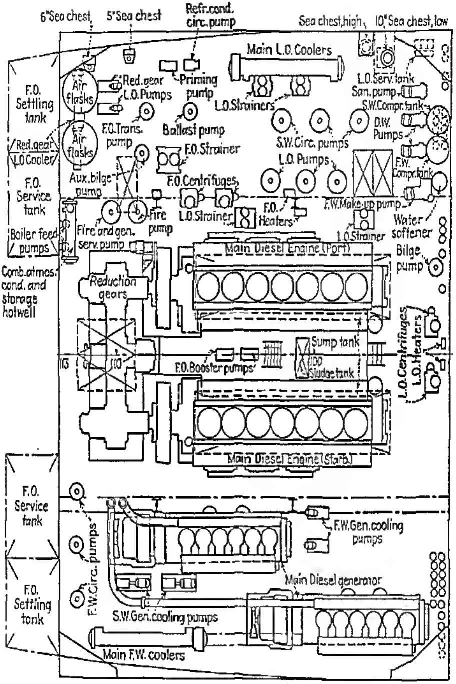


FIG. 7.—*Continued.*

Refer to Fig. 11, p. 1607. This value of specific speed is off the curve, but extrapolation indicates an efficiency of approximately 30 percent.

$$\text{Hp} = \frac{12 \times 75}{1{,}715 \times 0.30} = 1.75$$

Install 2-hp motor. Operating capacity of the pump is $8.8/12 = 73.5$ percent of rated (see Fig. 12, p. 1608). Operating efficiency, $0.90 \times 0.30 = 27$ percent.

$$\text{Operating hp} = \frac{8.8 \times 75}{1{,}715 \times 0.27} = 1.425$$

See Table 13, p. 1727 Motor efficiency, 78 percent. At 1,425 hp, motor efficiency $= 77.5$ percent (see Fig. 4)

$$\text{Kw input} = \frac{0.746 \times 1.425}{0.775} = 1.37$$

Auxiliary Circulating Pumps. One pump for each generator. Assume 1100 Btu per lb dissipated in condenser because of the lower efficiency of the turbogenerator turbine.

$$\frac{4{,}400 \text{ lb/hr} \times 1100 \text{ Btu/lb}}{80.6 \text{ Btu/gal} \times 60 \text{ min}} = 1{,}000 \text{ gpm}$$

Install with 20 percent margin. $1.2 \times 1{,}000 = 1{,}200$ *gpm at 10 psi total* head, 23 ft. Make this small pump of the centrifugal type, single stage, operating at 1,750 rpm (see Fig. 11, p. 1607). Efficiency $= 75$ percent.

$$\text{Hp} = \frac{1{,}200 \times 10}{1{,}715 \times 0.75} = 9.35$$

Install 10-hp motor. 86 percent rated efficiency (see Table 13, p. 1727).

$$\text{Operating pressure} = \left(\frac{1{,}000}{1{,}200}\right)^2 (10) = 7$$

$$\text{Pump operating efficiency} = 0.95 \times 0.75 = 71 \text{ percent}$$

$$\textit{Operating hp} = \frac{1{,}000 \times 7}{1{,}715 \times 0.71} = 5.75$$

Operating motor efficiency $= 83.5$ percent (see Fig. 4)

$$\text{Kw input} = \frac{0.746 \times 5.75}{0.835} = 5.15$$

Main Feed Pumps. *Two installed, two operating.* *70,528 lb per hr* total steam all of which passes through the main feed pump. Weight of water at this temperature is 7.9 lb per gal.

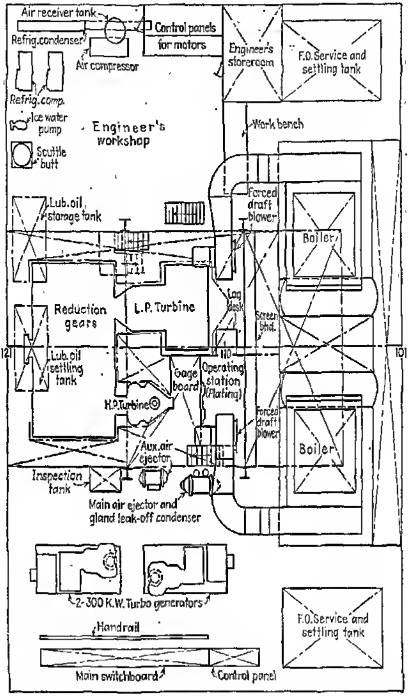


Fig. 8.—Engine-room layout of the S. S. "Hawaiian Planter," an 8,500 S.H.P., C-3 steamship designed by the U. S. Maritime Commission. (*From Marine Engineering and Shipping Review.*)

Boiler drum pressure	640 psia
5% line drop	32 psi
Check valve	50 psi
Economizer	16 psi
3-stage heater	20 psi
Discharge pressure	758 psia
Assume suction pressure	20 psia
Total head	738 psi

$$\frac{70{,}528 \text{ lb/hr}}{7.9 \text{ lb/gal} \times 60 \text{ min} \times 2 \text{ pumps}} = 74.5 \text{ gpm}$$

Install with 25 percent margin.

$$1.25 \times 74.5 = 93 \text{ gpm per pump}$$

Install 100-gpm pumps. Investigation will show that for this small capacity so many stages would be needed to obtain a centrifugal pump with reasonable efficiency that it would be better to use a motor-driver reciprocating pump (see Fig. 32, p. 1627). Use with double reduction gearing.

$$\text{Water hp} = \frac{100 \times 738}{1{,}715} = 43$$

$$\text{Efficiency} = 82 - 5 = 77 \text{ percent}$$

$$\text{Hp input} = \frac{43}{0.77} = 56$$

Install 60-hp motor. 89 percent rated efficiency (see Table 13, p. 1727).

$$\textit{Operating hp} = \frac{74.5 \times 738}{1{,}715 \times 0.77} = 41.6$$

Motor efficiency at 41.6 hp is 87 percent (see Fig. 4).

$$\text{Total kw input} = \frac{2 \times 0.740 \times 41.6}{0.87} = 71.5$$

Fuel-oil Service Pumps. Two installed, one operating. Fuel oil weighs 8.1 lb per gal.

$$\frac{4{,}710 \text{ lb/hr fuel}}{8.1 \text{ lb/gal} \times 60 \text{ min}} = 9.7 \text{ gpm operating}$$

Install with 30 per cent margin.

$$1.3 \times 9.7 = 12.6. \quad \text{Call it } 12.5 \text{ gpm.}$$

Fuel-oil service pumps, regardless of the horsepower or steam pressure of the ship, are usually installed capable of 400 psi total head. This is on account of the atomizing pressure required at the burners (see Fig. 36, p. 1630). Rated efficiency = 62 percent.

$$\text{Hp} = \frac{12.5 \times 400}{1{,}715 \times 0.62} = 4.7$$

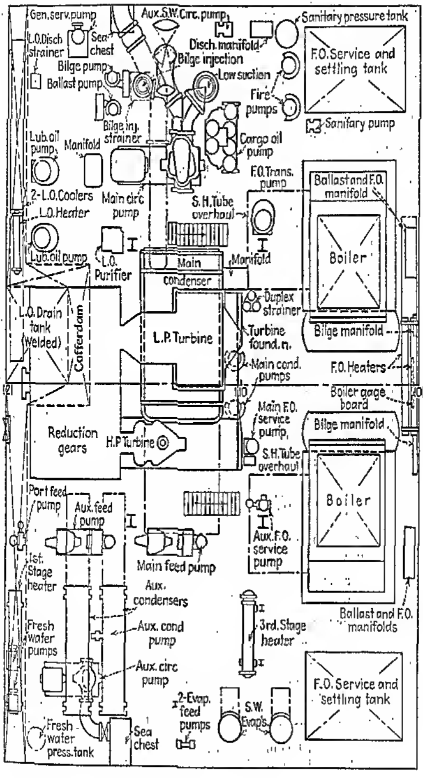


FIG. 8.—*Continued.*

Install 5-hp motor. Rated motor efficiency = 80 percent (see Table 13, p. 1727).

$$\text{Operating capacity of pump} = \frac{9.7}{12.5} = 77.5 \text{ percent}$$

$$\text{Operating pump efficiency} = 1.00 \times 0.62 = 62 \text{ percent}$$

$$\text{Operating hp} = \frac{9.7 \times 400}{1{,}715 \times 0.62} = 3.65$$

Operating motor efficiency 78.5 percent (see Fig. 4).

$$\text{Total kw} = \frac{3.65 \times 0.746}{0.785} = 3.47$$

Lubricating-oil Service Pumps. Two installed, one operating at full capacity. Obtain preliminary capacity from empirical formula.

$$\begin{aligned} \text{Gpm} &= 36 + \sqrt{(7.15)(\text{shp}) + 1{,}300} \\ &= 36 + \sqrt{7.15 \times 8{,}500 + 1{,}300} = 286 \end{aligned}$$

Call it 300 gpm at 50 psi total head (see Fig. 35, p. 1629). Rated efficiency = 71 percent.

$$\text{Hp} = \frac{300 \times 50}{1{,}715 \times 0.71} = 12.3$$

Install 15-hp motor, 87 percent efficiency. At 12.3 hp the efficiency should be approximately the same as at rated capacity.

$$\text{Total kw} = \frac{12.3 \times 0.746}{0.87} = 10.55$$

Total Electrical Load. The total kilowatt load due to these pumps and fans is as follows:

Fireroom blower	31.8
Main circulating pump	42.4
Main condensate pump	7.9
Auxiliary condensate pumps	1.4
Auxiliary circulating pumps	5.2
Main feed pumps	71.5
Fuel-oil service pump	3.5
Lubricating-oil service pump	10.6
	174.3

This compares favorably with the estimate of 175 kw made to cover straight propulsion load. The remainder of the electrical load consisting of bilge pumps, sanitary water pumps, ventilating fans, galley, laundry, etc., and of such items as are not controlled by the propulsion equipment was assumed to be 100 kw. This information should be furnished by the naval architect. Total operating load, 275 kw.

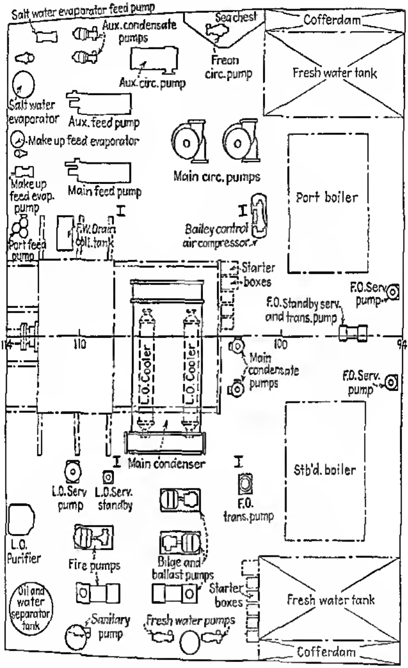


FIG. 9.—Engine-room layout of the S. S. "Robin Locksey," a 6,300 S.H.P. steamship. (*From Marine Engineering and Shipping Review.*)

LAYOUT

General. There are a few fundamental principles that should be followed in machinery layout. The rest is common sense and ingenuity.

1. Apparatus should be accessible for operation, inspection, and repair.
2. Apparatus should be compactly arranged so as to occupy as little space as necessary so as to free space for revenue purposes.

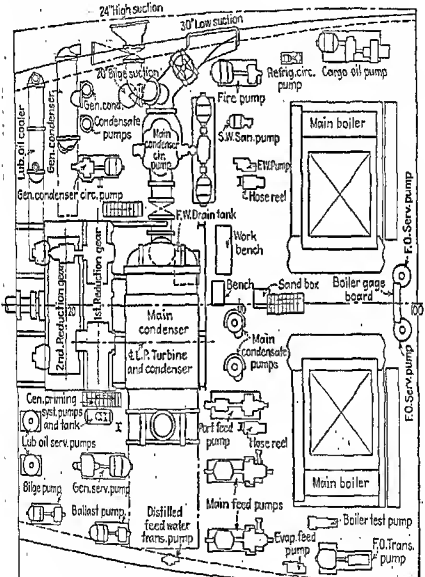


FIG. 5.—Lower level of S. S. "Exceller."

3. Certain types of apparatus should occupy certain relative positions, or they will render the plant inoperative.
4. Apparatus should be so disposed as to assure trim of the ship.

Requirements 1 and 2 are opposed to each other and, as in all successful engineering, the best compromise should be reached. Electric motors should

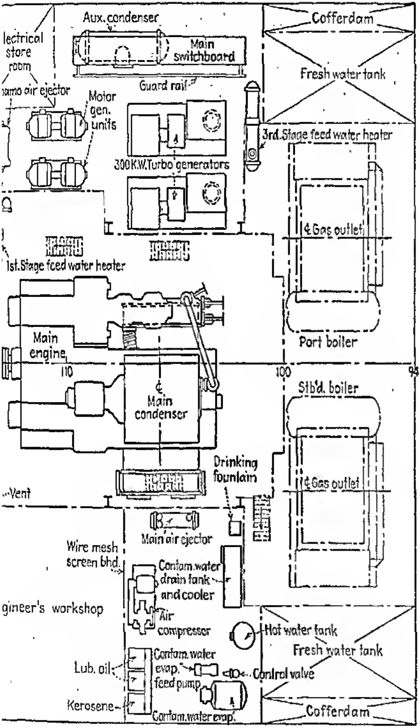


FIG. 9.—*Continued.*

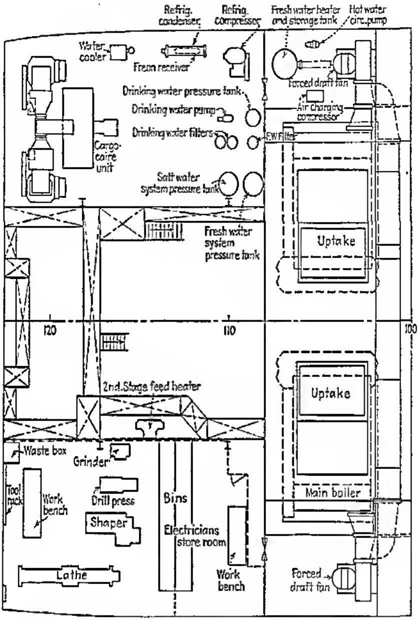


Fig. 5.—Engine-room layout (upper level) of the S. S. "Exceller," a 7,500 S.H.P. steamship. *(From Marine Engineering and Shipping Review.)*

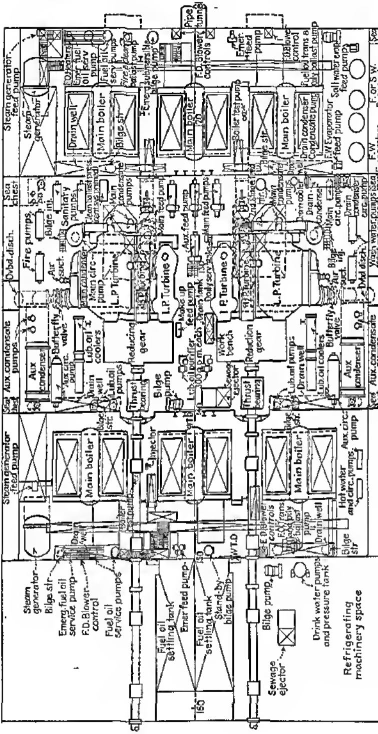


FIG. 10a.—Engine-room layout of the U. S. S. "West Point" (formerly S. S. "America"), a 34,000 S.H.P. steamship. (*From Marine Engineering and Shipping Review.*)

For a standing cask, partially full, compute contents by the prismoidal formula, p. 111. Roughly, the number of gallons, G, in a cask is given by $G = 0.0034n^2h$, where n = number of inches in the mean diameter, or $\frac{1}{2}(D + d)$, and h = number of inches in the height.

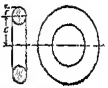


FIG. 85.

Torus, or Anchor Ring (Fig. 85). Volume $= 2\pi^2cr^2$. Area $= 4\pi^2cr$ (Proof by theorems of Pappus).

Theorems of Pappus. 1. Assume that a plane figure, area A, revolves about an axis in its plane but not cutting it; and let s = length of circular arc traced by its center of gravity. Then volume of the solid generated by A is $V = As$. For a complete revolution, $V = 2\pi rA$, where r = distance from axis to center of gravity of A.

2. Assume that a plane curve, length l, revolves about an axis in its plane but not cutting it; and let s = length of circular arc traced by its center of gravity. Then area of the surface generated by l is $S = ls$. For a complete revolution, $S = 2\pi rl$, where r = distance from axis to center of gravity of l.

NOTE. If V_1 or S_1 about any axis is known, then V_2 or S_2 about any parallel axis can be readily computed when the distance between the axes is known.

Generalized Theorems of Pappus. Consider any curved path of length s. If (1) a plane figure, area A [or (2) a plane curve, length l] moves so that its center of gravity slides along this curved path (Fig. 86), while the plane of A [or l] remains always perpendicular to the path, then (1) the volume generated by A is $V = As$ [and (2) the area generated by l is $S = ls$]. The path is assumed to curve so gradually that successive positions of A [or l] will not intersect.

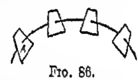


FIG. 86.

The Prismoidal Formula (Fig. 87). Volume $= \frac{1}{6}h(A + B + 4M)$, where h = altitude, A and B = areas of bases and M = area of a plane section midway between the bases. This formula is exactly true for any solid lying between two parallel planes and such that the area of a section at distance x from one of these planes is expressible as a polynomial of not higher than the third degree in x. It is approximately true for many other solids.

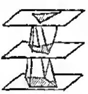


FIG. 87.

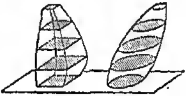


FIG. 88.

Simpson's Rule may be applied to finding volumes, if the ordinates y_1, y_2, be interpreted as the areas of plane sections, at constant distance h apart (p. 106).

Cavalieri's Theorem. Assume two solids to have their bases in the same plane. If the plane section of one solid at every distance x above the base is equal in area to the plane section of the other solid at the same distance x above the base, then the volumes of the two solids will be equal. See Fig. 88.

ALGEBRA

FORMAL ALGEBRA

Notation. The main points of separation in a simple algebraic expression are the $+$ and $-$ signs. Thus, $a + b \times c - d \div x + y$ is to be interpreted as $a + (b \times c) - (d \div x) + y$. In other words, the range of operation of the symbols $\times$ and $\div$ extends only so far as the next $+$ or $-$ sign. As between the signs $\times$ and $\div$ themselves, $a \div b \times c$ means, properly speaking, $a \div (b \times c)$; that is, the $\div$ sign is the stronger separative; but this rule is not always strictly followed, and in order to avoid ambiguity it is better to use the parentheses.

The range of influence of exponents and radical signs extends only over the next adjacent quantity. Thus, $2ax^3$ means $2a(x^3)$, and $\sqrt{2}ax$ means $(\sqrt{2})(ax)$. Instead of $\sqrt{2}ax$, it is safer, however, to write $\sqrt{2}\cdot ax$, or, better, $ax\sqrt{2}$.

Any expression within parentheses is to be treated as a single quantity. A horizontal bar serves the same purpose as parentheses.

The notation $a \cdot b$, or simply ab, means $a \times b$; and $a:b$, or a/b, means $a \div b$.

The symbol $|a|$ means the "absolute value of a," regardless of sign; thus, $|-2| = |+2| = 2$.

The symbol $n!$ (where n is a whole number) is read: "n factorial," and means the product of the natural numbers from 1 to n, inclusive. Thus $1! = 1$; $2! = 1 \times 2$; $3! = 1 \times 2 \times 3$; $4! = 1 \times 2 \times 3 \times 4$; etc.

The symbol $\neq$ or $\not\doteq$ means "not equal to"; $\pm$ means "plus or minus."

The symbol $\approx$ is sometimes used for "approximately equal to."

Addition and Subtraction. $a + b = b + a$.

$(a + b) + c = a + (b + c)$. $a - (-b) = a + b$. $a - a = 0$.

$a + (x - y + z) = a + x - y + z$. $a - (x - y + z) = a - x + y - z$.

A minus sign preceding a parenthesis operates to reverse the sign of every term within, when the parentheses are removed.

Multiplication and Simple Factoring. $ab = ba$. $(ab)c = a(bc)$. $a(b + c) = ab + ac$. $a(b - c) = ab - ac$. Also, $a \times (-b) = -ab$, and $(-a) \times (-b) = ab$; "unlike signs give minus; like signs give plus."

$(a + b)(a - b) = a^2 - b^2$.

$(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$.

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$; etc.

(See table of binomial coefficients, p. 39; also p. 114.)

$a^2 - b^2 = (a - b)(a + b)$, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$.

$a^n + b^n$ is factorable by $a + b$ only when n is odd; thus,

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$,

$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$; etc.

The following transformation is sometimes useful:

$$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2\right]$$

Fractions. If m is not zero, $\dfrac{ma + mb + mc}{mx + my} = \dfrac{a + b + c}{x + y}$; that is, both numerator and denominator of a fraction may be multiplied or divided

clearances on the plan view only; further check must be made for vertical clearances.

As to requirement 3, the deaerating feed heaters should be placed at as high a point as possible in order to obtain sufficient head on the feed pumps. If this cannot be done, booster pumps will be necessary. For the same reasons, the feed pumps must be placed at as low a point as possible. Bilge pumps should, of course, be placed at as low a point as possible. Condensers should always be placed directly under the turbines they serve and should be

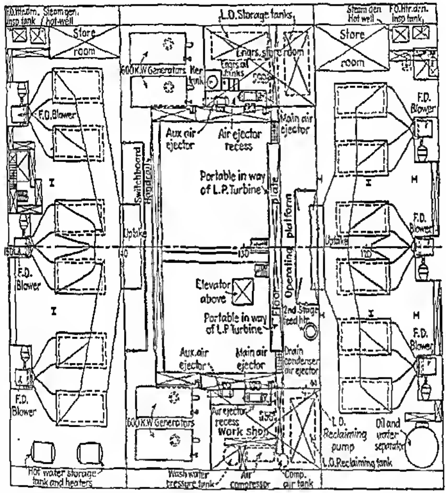


FIG. 10*b*.—Upper level of the U. S. S. "West Point" (formerly S. S. "America"). (*From Marine Engineering and Shipping Review.*)

supported by the turbines. The main propulsion turbines and gears should be so disposed as to have the shortest runs of propulsion shafting and to clear other shafting. The shafting should have as little angle with the center line of the ship as is practical. In the case of a single-screw ship, the propulsion shafting line should coincide with the center line of the ship. A slight angle of elevation is permissible so as to permit the turbines to be above the condenser yet have as great a submergence of the propeller as practical. The tip of the propeller blade must not go below keel.

As to requirement 4, ordinary symmetry of arrangement of the heavier items such as turbines, gears, and boilers will almost solve the problem. A

turbine may be shut down from the deck if desired. This feature is required by the U.S. Coast Guard as a remote means for securing the main turbine if it is necessary to abandon ship and no one is in the engine room to secure the turbine.

Diesel-engine and High-speed Reciprocating-engine Lubricating-oil Service System. Lubricating-oil service systems installed for diesel engines are usually of the pressure type, similar to that installed for a steam turbine-driven vessel. They do differ, however, from the turbine installation in that they require larger quantities of oil at higher pressures at the oil inlet connection than a turbine installation of the same shaft horsepower. In order to maintain constant oil pressure at the engine, the oil delivery is controlled by adjusting the speed of the service pump, which should be fitted for about 50 percent speed control. This wide speed control is necessary since for a constant oil pressure most engines require much more oil at reduced speeds than when operating at full speed. In a diesel installation the oil becomes contaminated with fine carbon particles in a very short time, and these are usually removed by installing a by-pass type of oil filter. These filters employ fuller's earth, or cellulose material, as a filtering medium.

Oil service systems for high-speed reciprocating engines employing forced oil feed are very similar to those installed for steam turbine-driven vessels employing the pressure-type oil system, except that the oil pressure required at the engine is relatively high and the quantity of oil required is much less than for turbines of the same shaft horsepower.

The oil storage, transfer, and purifying systems are the same for diesel and high-speed reciprocating engines except that on diesel installations the oil storage tank must be large to allow for the relatively large amount of oil consumed by the engine.

Piping Materials. The lubricating-oil piping materials, installed on most vessels that have been built to the U.S. Maritime Commission requirements, have used standard-weight black steel pipe with all valves, flanges, and fittings either forged or cast steel. All take-down joints are generally flanged along with all valves 2 in. and above in size. All small sizes of valves and fittings are either of screwed or socket-welded design, with the screwed design of valves seal-welded to the pipe. All fittings 2 in. in size and larger are generally of butt-welded design with a few cast-steel flanged fittings being used where necessary. With the foregoing arrangement, a tight system is assured since all joints are either of welded or flanged design. The use of an all-steel system is an advantage since it offers the maximum protection against a fire hazard due to the high melting point of the steel as compared with copper or brass.

The oil-piping material installed on all new U.S. Navy constructions employs the use of thin-wall steel tubing with all valves, flanges, and fittings either forged or cast steel. All joints are either welded or flanged similar to merchant practice. In the past, however, the Navy used copper tubing with composition valves and fittings, and many of these vessels are still in use today with these materials.

Extreme precaution should be taken at all times from the fabrication of all the parts to the completion of the system, in order to ensure that all foreign matter is removed and has no opportunity to reenter the system. After all piping has been fabricated and fitted into the vessel, it should be disassembled and thoroughly cleaned by sandblasting. The use of pickling in lieu of sandblasting is not generally satisfactory since the pickling will not

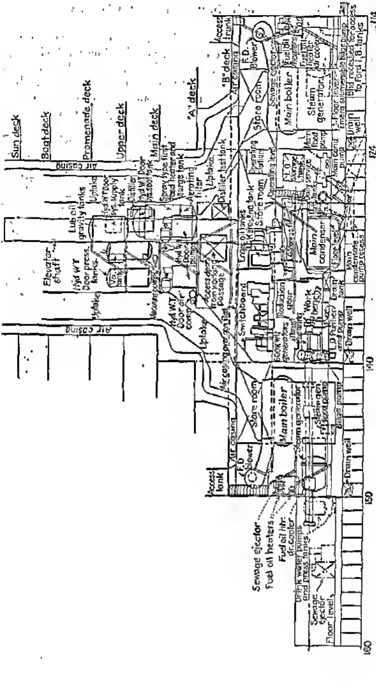


FIG. 10a.—Longitudinal elevation of machinery arrangement in the U. S. S. "West Point" (formerly S. S. "America"). (*From Marine Engineering and Shipping Review.*)

remove the lacquer or varnish usually placed on the steel pipe by the manufacturer. After sandblasting, all of the pipe should be reassembled and flushed with a light grade of flushing oil for a period of about 24 to 48 hr. During this operation, the oil should not pass through the turbine and gears for foreign matter may lodge in the bearings and cause trouble at some later date.

Capacity Service Pumps. It is difficult to determine the actual quantity of oil required to be delivered by the lubricating-oil service pumps, since it varies, depending upon the type of unit involved, the individual ideas of the manufacturer, the shaft horsepower, and the propeller revolutions. As an example of the effect the individual designs of the manufacturer have upon the lubricating-oil requirements it may be noted that for 8,500 shp geared turbine units installed on the U.S. Maritime Commission Victory ships and supplied by three of the leading manufacturers and all operating with the same propeller rpm and steam conditions, the individual oil requirements varied from 230 gpm as a minimum to 310 gpm as a maximum. Therefore, no definite rule can be stated as to the size of pumps required. The accompanying table, however, gives the size required for recent merchant vessels. Pages 1284 and 1633 show a method of estimating capacity required.

Type drive	Normal shp	Propeller rpm	Installed lubricating-oil pump capacity	Installed lubricating-oil pump, T.D.H., psi
Geared turbine	4,000	90	175	50
Geared turbine	6,000	92	250	47
Geared turbine	8,500	85	325	48
Diesel direct	1,700	160	500	50
Diesel direct	6,000	92	375	50

Duplex Strainers. A lubricating-oil duplex strainer is in effect two strainers built into one body, complete with inlet and outlet valves and designed so that one strainer may be in service while the other is available for cleaning. These strainers are of two basic designs. One is the plug type, which employs two plug cock valves for controlling the flow of the oil and the other employs a sliding gate valve. In both designs it is most important that there be some foolproof manner provided to prevent the stoppage of flow when shifting from one compartment to the other at cleaning time. In the case of the plug-type strainer, this feature is obtained by merely interlocking the inlet and outlet plug valves so that they may both work together. With the design of strainer shown in Figs. 5 and 6, an interlocking chain drive is provided between the sliding gate valves. The chain must have sufficient slack so that each individual sliding gate valve may be seated tight by the use of its handwheel. It is also desirable to have an indicator with both the plug and sliding gate valve type of strainer to enable the operator to tell at a glance which compartment is in service and which may be opened for inspection and cleaning. To open the strainer for inspection, generally all that is required is to loosen two or four bolts on the strainer cover; then the cover and strainer basket may be removed.

The materials of the body of a lubricating-oil duplex strainer are generally cast steel, fabricated steel, or a combination of the two. It is best, however, to make the body containing the valve parts, of solid steel casting. Cast

table of moments due to all the apparatus about the centerline of the ship and another table of moments about a line generally somewhat aft of amidships depending upon hull weight distribution should be made up. The moments tending to roll the ship starboard could be called positive and those tending to roll the ship port could be called negative. All these moments should add up to zero. It is a simple matter to shift apparatus, other than

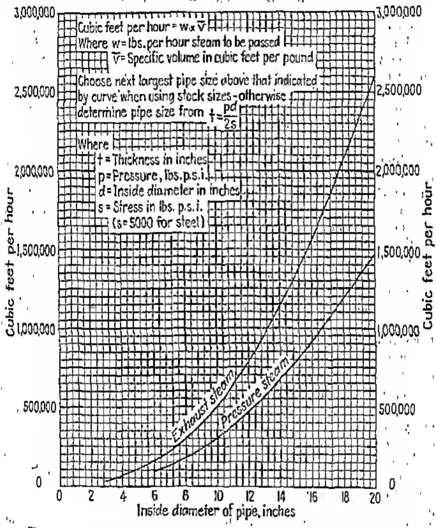


FIG. 11.—Approximate pipe sizes for pressure and exhaust steam.

that covered under requirement 3, so as to obtain a total of zero. In a similar manner the moments about the line aft of amidships should add up to zero. With these tables of moments available any proposal concerning the addition or elimination of apparatus can be disposed of intelligently. Some typical layouts are shown in Figs. 5 to 10.

Figure 11 shows a curve from which the sizes of piping may be determined for both pressure and exhaust steam.

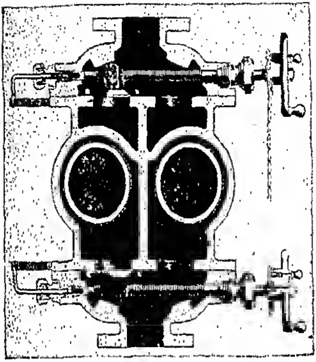


FIG. 5.—Lubricating-oil duplex strainer—sliding-valve design (see Fig. 6). (*Courtesy of Elliott Company.*)

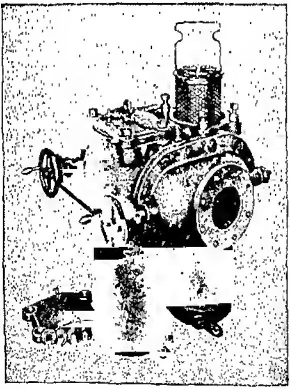


FIG. 6.—Lubricating-oil duplex strainer—sliding-valve design (see Fig. 5). (*Courtesy of Elliott Company.*)

FUEL- AND LUBRICATING-OIL SYSTEMS

BY

ROBERT P. GIBLON

LUBRICATING-OIL SYSTEM

One of the most important parts of the whole marine power plant is the lubricating-oil system. This system may be of a very simple design, such as used with most steam reciprocating engines, where hand lubrication methods with the aid of small tubes and wick feeds are used, or it may be of a more complex design, such as the pressure feed type, as used on turbine, diesel, and some designs of high-speed steam reciprocating engines. The system to be discussed in the next few pages is the pressure feed type, particularly with reference to its use on geared turbine-driven merchant vessels.

Principal Divisions. The lubricating-oil system for a geared turbine-driven vessel is divided into three divisions, the principal one being the **lubricating-oil service system.** The functions of the service system are (1) to supply clean lubricating oil to the main turbins and gear bearings and gear meshes and (2) to remove the heat that is transmitted to the oil from the bearings and gears. The second principal division of the lubricating-oil system is the **storage, transfer, and purifying system,** whose principal functions are (1) to provide storage space for oil that is consumed by the main turbines, turbogenerators, line-shaft bearings, and auxiliary machinery and (2) to provide a method of purifying the oil used by the main turbine and turbogenerator. The third division of the lubricating-oil system is the **turbine governor system,** which operates to protect the turbine from overspeeding and also to shut down the unit immediately should the lubricating-oil service system fail.

Service System. The simplest lubricating-oil system is the pressure type, which is shown by Fig. 1. A system similar to the one shown is used on many merchant vessels. In this system a motor-driven rotary service pump takes suction through a macomb strainer from the turbine and gear sump tank, discharges the oil through a lubricating-oil cooler, a duplex magnetic strainer, and delivers it under pressure directly to the main turbine bearings and gears. The proper distribution of oil to the gears, bearings, and other parts requiring lubrication is accomplished by means of a system of piping, manifolds, and spray nozzles, which are built into the turbine and gear unit. A system of drain piping, also built into the gear unit, carries the oil away from the bearings and gears, back to the oil sump tank. In parallel with the motor-driven service pump is a steam-driven lubricating-oil service stand-by pump, which is installed so that it will operate immediately in event of the failure of the main motor-driven pump and thus prevent damage to the main turbines and gears owing to lack of lubrication. To keep the stand-by pump available for immediate service, it is essential that it be kept operating at an idling speed at all times when the system is in use so that the steam end will remain hot and properly drained. Steam is supplied to the pump for this condition through an orifice fitted in a by-pass around the stand-by pump governor valve. If the service pump should fail, the drop in oil discharge pressure will immediately open the governor valve in the steam supply line and place the stand-by pump in operation. The pump will then deliver

iron may be used, however, if the strainer is to meet only the current 1944 requirements of the American Bureau of Shipping and the U.S. Coast Guard and not the requirements of *Senate Report* 184. The valves are generally bronze with steel stems and handwheels. The strainer baskets are usually bronze and, in the case of a discharge strainer, are lined with bronze or monel wire cloth.

The friction drop and free hole area are two important points to be considered in selecting the size of strainer desired. It is generally considered good practice to have a free hole area through the strainer basket of at least six times the area of the inlet pipe connection, with the velocity of the liquid in the pipe not exceeding approximately 5 fps if the strainer is installed in the suction of a pump, or 6 fps if the strainer is in the discharge side of the pump. To maintain the friction drop as low as possible, it is essential that all ports and passages be as large as possible so that the velocity, and thus friction drop, is maintained at a minimum.

The size of perforations in a strainer depends upon its location and purpose in the system. Strainers located in the suction line to service pumps are used to protect the pump only; therefore, the perforations need to be $\frac{1}{16}$ in. diameter. Small diameter perforations are not recommended for this service since they will present too much resistance to the flow of oil and thus may reduce the capacity of the service pump. Strainers located in the discharge side of the service pump are used to protect the bearings and gears of the engine or turbine from all foreign solid substances. The baskets of these strainers are generally formed of $\frac{1}{16}$ in. thickness plate fitted with $\frac{3}{8}$ in. diameter holes. This basket is then lined with a wire cloth having about 80 wires of 0.004 in. diameter to the inch. The use of any finer mesh is not recommended since it has a tendency to wear very easily.

One of the duplex strainers installed in a lubricating-oil service system should be fitted with magnets effectively clamped in the basket which will remove iron or steel filings, welding beads, or other ferrous material. If bright metal chips are deposited on these magnets, it is a good indication that trouble is developing within the unit being served.

Purifying Lubricating Oil. The entrance of water along with small particles of dirt, rust, and scale into the lubricating-oil system cannot be prevented. These foreign substances, along with solids, formed as a result of decomposition of the oil, should be removed frequently from the lubricating oil. There are three prime methods by which these foreign substances may be removed from oil on shipboard: by settling, by filtering, or by the use of a centrifugal purifier. Generally, a combination of two of the methods or all three are used since each has some feature which the other does not possess.

The use of gravity settling in a tank is the simplest method of purification to employ, but it does not function well at sea owing to the motion of the ship and the relatively long time required to accomplish the settling. The use of filters to purify oil is satisfactory when it is desired to remove only foreign solid substances, but it is not designed for the removal of water from oil. A centrifugal purifier has the advantage over the use of a filter or settling tank, since it will remove both solid foreign substances and liquids in a relatively short time. A filter does, however, tend to remove very fine substances more effectively.

Centrifugal Purifiers. The manner in which separation of two liquids takes place in a centrifugal oil purifier may be easily understood by examining the

enough oil to prevent any damage to the main turbine bearings and gears. To operate the system at its normal pressure conditions, however, the tension of the governor spring should be readjusted by means of a readily accessible handwheel provided on the governor valve.

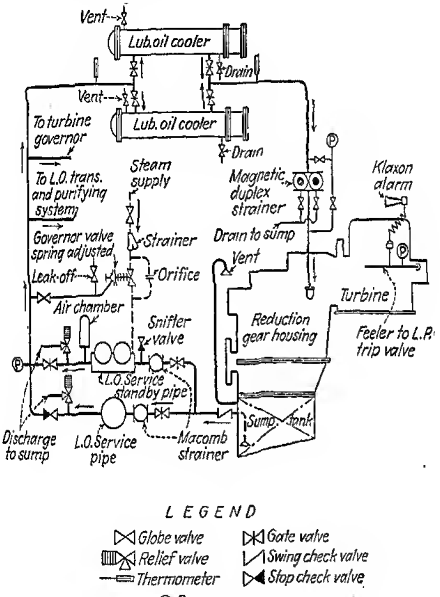


Fig. 1.—Lubricating-oil service system—pressure type.

The gravity-type system, shown by Fig. 2, is similar to the pressure type, except that gravity tanks are fitted in the piping system between the lubricating oil cooler and the turbine and gears. These tanks are generally located about 30 ft above the turbine in order to assure the required pressure at the turbine and gear oil inlet connection. An orifice valve is generally fitted immediately before the inlet connection to allow close adjustment of the oil pressure and flow. The service pump is adjusted to pump oil in excess of that

simplest of all separating devices, the settling tank. Generally speaking, any two liquids that have different specific gravities and are not soluble in each other can be separated by gravity, but they can be separated much more thoroughly and in a fraction of the time by correctly applied centrifugal force.

The primary advantage of a centrifugal purifier over a gravity tank is that it makes use of a force many thousands of times stronger than gravity. When applied to the best advantage, as in the purifying member of either of the designs of purifier shown (see Figs. 7 and 8), centrifugal force is so much more intense than gravity that it will often accomplish work that gravity separation alone would not be able to do. A cubic inch of oil at the bottom of a water-filled tank has a buoyant force of a fraction of an ounce acting on it tending to make it rise to the top. If the same cubic inch of oil were in the purifying member of a separator, the buoyant force would be a good many pounds, enabling separation to be effected in a much shorter time. Moreover, it sometimes happens, as in the case of an emulsion, that a heavy liquid is distributed through a lighter liquid in such very fine droplets that the use of force of gravity cannot cause perceptible movement. Under the influence of centrifugal force, these fine droplets will move and separate out. In addition to separating water from oil, the centrifugal purifier will also remove solid substances. A portion of the solid substances separated is generally removed from the machine along with the water; however, a portion of the solid substances remains in the purifying member and must be removed by hand cleaning methods.

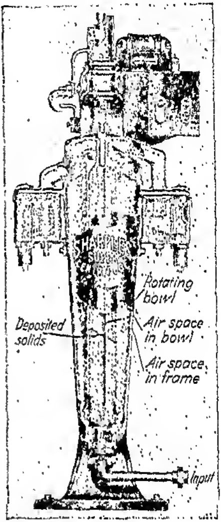


FIG. 7.—Sharples centrifugal oil purifier.

Types of Centrifugal Purifiers. Two common designs of marine lubricating-oil purifiers are shown in Figs. 7 and 8, one of which is the Sharples design and the other the DeLaval design. Each, when installed on shipboard, is fitted with suction and discharge pumps which are capable of lifting oil from one reservoir and discharging it to another. In the Sharples design an additional motor is required to drive the transfer pumps (motor and transfer pumps not shown). On the DeLaval design shown, the attached transfer pumps and centrifuge are all driven by one motor. In the two designs of centrifuges shown, the same principle is employed to accomplish the separating, but the design of the units is quite different, for the Sharples design

required by the turbines and gears, and this excess oil runs through the overflow fitted in the gravity tank and then by a sight glass to the sump tank.

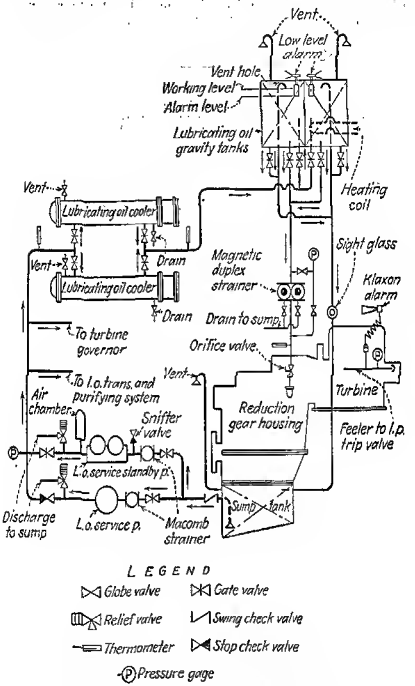


FIG. 2.—Gravity-type lubricating-oil system.

The gravity tanks are generally sized such that there will be about a 3- to 4-min reserve supply that may be used by the main unit after the service pump

employs the use of "tubular-type" bowl, while the DeLaval uses the "disk-type" bowl. The tubular-type bowl is relatively small in diameter, which permits operation at a high rotative speed, which in turn develops high centrifugal force. Also, with the bowl designed with a length many times the depth of the liquid layer, the distance the oil has to travel is many times the settling distance. This allows for a relatively long settling time. In addition, the bowl is provided on the inside with a simple three-wing device,

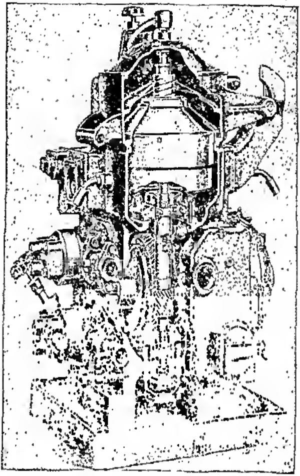


Fig. 8.—De Laval centrifugal oil purifier.

whose function is to keep the liquid up to the speed of the bowl without slipping. The disk-type bowl (see Figs. 8 and 9) is characterized by a relatively larger diameter and by a series of disks that separate the liquid into thin layers and thus creates shallow settling distances. It is general knowledge that in gravity settling, shallow vessels work more rapidly and effectively than deep vessels, since with deep vessels, the particles to be separated have a greater distance to travel before they reach the bottom of the vessel; therefore, more time is required. In the bowl of the centrifuge, the same conditions exist, and both designs tend to employ this principle. In the

ceases to deliver oil. With this reserve supply on hand, sufficient time is usually available to start the steam stand-by pump and prevent damage to the bearings. Gravity tanks are sometimes arranged with a water drain-off connection so that the tank not in use may be used for gravity settling if desired.

In both system designs, it is most important that consideration be given to the location of the lubricating-oil service pumps so that their suction lift is not greater than that for which they are designed, especially when the vessel is not on even keel. The suction pipe in the sump tank should be bell-mouthed and located where it will have not less than ½ ft oil over the open end when the vessel is rolling about 45 deg. The suction piping should be free from bends and of large diameter to assure minimum drop. The macomb strainer indicated in the suction line to the motor-driven service pump is sometimes replaced by a strainer built into the pump suction, which reduces the friction drop in the piping.

On both designs indicated, two lubricating-oil coolers have been shown, since the installation of two units is the most common commercial practice. On many installations, however, only one cooler with an oil by-pass is provided.

The location chosen for the magnetic duplex strainers is a very important item. These strainers should be placed as close to the turbine and gear inlet connection as possible for it should be remembered that the turbine and gears are being protected from foreign matter and not the gravity tanks, oil coolers, or interconnecting piping.

The pressure and gravity systems have certain advantages and disadvantages, and it is a matter of much discussion as to which system is the better to install on a particular design of vessel.

The principal advantages of the gravity system over the pressure system are as follows: (1) In the event of failure of the service pump, about 3 to 4 min are available to restore the flow of oil by the use of the stand-by pump before damage may be done to the main unit. In the pressure system, however, one depends entirely upon the automatic starting of the stand-by pump, which is not considered very dependable by some designers and operators. (2) Close regulation of the oil quantity delivered by the service pump is not required since all excess oil flows through the overflow fitted in the gravity tank. (3) The sight glass provided in the overflow gives a visible indication that sufficient oil is being supplied to the main turbine unit.

The principal advantages of the pressure system over the gravity system are as follows: (1) The pressure system lends itself well to a vessel that has low headroom in the engine room, such as a destroyer or other naval vessel. (2) The fire hazard is greatly reduced since 3,000 to 4,000 gal less of lubricating oil are carried in the engine room. This is particularly an advantage in wartime. (3) The weight of the piping, tanks, etc., for a pressure system is about 50 percent less than that of the gravity system. (4) The installation of a pressure system is almost a requisite for lubrication of high-speed steam reciprocating or diesel engines owing to the relatively high pressure required at the bearings. (5) The size of the lubricating-oil coolers and pumps is smaller than in the gravity system.

Storage, Transfer, and Purifying Systems. The principal tanks installed in the lubricating-oil system are a storage tank, a settling tank, a sump tank, and, in the case of a gravity system, one or two gravity tanks. The storage tank is generally sized sufficiently large to hold one complete fresh charge of lubricating oil for the service system, plus the

Sharples design the bowl is belt-driven from the motor and is suspended by a flexible spindle from a bearing at the top of the housing; in the DeLaval design the bowl is ordinarily gear-driven from below, with the spindle itself rigid but the upper bearing flexibly mounted. The feed to the tubular-type bowl is through a feed nozzle at the bottom, whereas the disk-type bowl is ordinarily fed from the top through a center tube, by which the liquid is directed toward a series of holes punched through the disk stack and then flows upward through these holes. Separated light liquid moves toward the center to discharge into one of the covers, and the separated heavier liquid moves

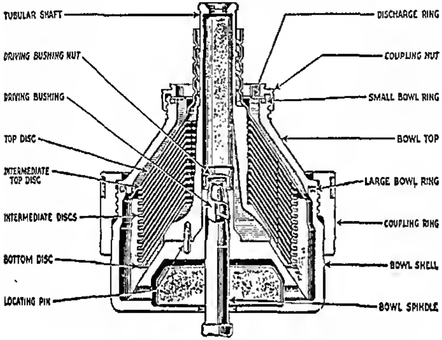


FIG. 9.—Bowl for De Laval centrifugal purifier.

toward the outside, being diverted from the bowl into the other cover (see Fig. 9).

FUEL-OIL SYSTEM

The duty of the fuel-oil service system is to deliver a required quantity of fuel oil at the desired pressure and temperature conditions, to the burners of a boiler in the case of a steam-propelled vessel, or to the injectors of the engine in the case of a diesel-propelled vessel. The system must also be fitted with the necessary filters, strainers, purifiers, etc., so that the oil is delivered in the required purity.

Service System for Steam-propelled Vessels. The system to be discussed is the type that is installed on many steam-propelled merchant vessels that use heavy-grade bunker oil as fuel, since this is the common design. Figure 10 shows a typical fuel-oil service system. In this system, oil may be drawn from either of the two fuel-oil settling tanks or direct from the inner bottom or deep bunker oil tanks. Each settling tank generally has sufficient capacity for about a 28-hr fuel supply for the vessel when it is operating at normal power. For normal operation of the system, oil is pumped by means of a fuel-oil transfer pump (not shown in sketch) from the

oil that will be consumed by the engines during the voyage. The settling and sump tanks generally are sized sufficiently large to take all the oil in the service system. The gravity tanks, when used, usually have a capacity of about 3- to 4-min supply of oil for the normal requirements of the service system.

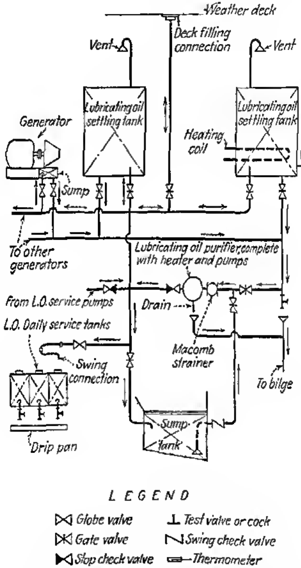


FIG. 3.—Lubricating-oil storage, transfer, and purifying system.

Figure 3 shows a typical piping arrangement for a lubricating-oil storage, transfer, and purifying system. The storage system is designed so that oil from deck may be led to the storage tank where it may be run by gravity to the main or turbogenerator sump tanks. The transfer part of the system is designed so that oil may be pumped from the main sump tank by means of the lubricating-oil purifier transfer pumps and/or the stand-by lubricating-

inner bottom and/or deep fuel-oil storage tanks to the fuel-oil settling tank that is empty. After the tank is filled, it is heated to about 100 to 115 F by means of the tank heating coil provided. The exact temperature to which the oil should be heated depends upon its viscosity characteristics. The oil is then left in the tank for about one day, and water and sediment are allowed

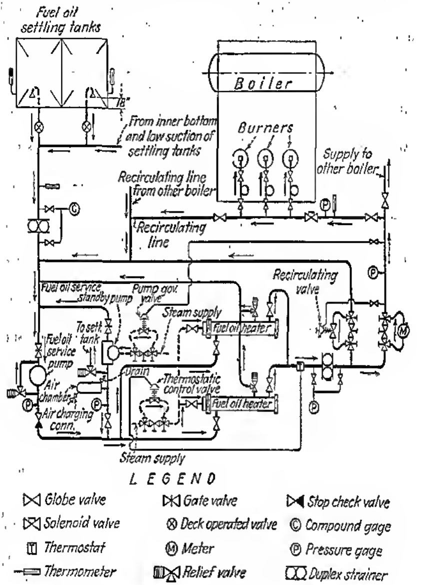


FIG. 10.—Fuel-oil service system for steamships.

to settle. While settling is taking place in the one tank, the service pump will be taking suction from the other tank. When the tank on which the service pump is operating becomes nearly empty, the suction to the full tank is opened and the suction to the empty tank is closed. The empty tank is then filled as described above. For most economical operation, the fuel-oil

oil service pump. These pumps are fitted so that they can discharge either to the storage or settling tanks or to deck. If the turbogenerator sump tanks are to be emptied, the lubricating-oil transfer pumps attached to the purifier may be used or the oil may be run by gravity to the main turbine sump tank.

The purification system is arranged so that either "continuous" or "batch" purification methods may be employed. To accomplish this, a centrifugal lubricating-oil purifier with attached suction and discharge pumps, a lubricating-oil heater, and a lubricating-oil settling tank fitted with a heating coil are generally provided. For continuous purifications, suction is taken from the main turbine sump by means of the purifier suction pump, and oil is discharged through the lubricating-oil heater to the centrifuge where it is purified. The discharge pump takes suction from the purifier and discharges purified oil back to the sump tank. The same operation may be performed for the oil in the turbogenerator sump tanks if desired. The lubricating-oil purifier is generally of sufficient capacity so that it need operate only about 8 hr per day while the vessel is in operation.

For batch purification, suction is taken from the main turbine sump by means of either the lubricating-oil service stand-by pump or the lubricating-oil purifier transfer pumps, and discharged to the lubricating-oil settling tank. Fresh oil may then be run from the storage tank to the sump tank if desired. The oil in the settling tank may then be heated to approximately 180 deg by means of the steam coil provided, and the water will settle from the oil. When the settling has been completed, the water may be drawn off by means of the drain-off valve provided, and then the oil may be run to the centrifuge and purified. The purified oil may then be discharged to the main sump or storage tank depending on whether or not a new charge is desired in the service system. Purification by the centrifuge after settling is not required since the settling tank alone will purify the oil very well.

Purification by the batch method has one serious limitation, however, for it cannot be used unless the main turbines are shut down for the period when the sump tank is empty. The batch method, therefore, is the common one to use when the vessel is in port and the continuous method is the common one to use when at sea.

Turbine Governor System. The turbine governor system is designed primarily to protect the turbines, rotors, and other parts from damage due to overspeeding or from damage to the bearings and gears due to an insufficient supply of lubricating oil. Figure 4 shows a typical governor system. The design shown is similar to that used on many of the Victory ships built for the U.S. Maritime Commission. The sketch shown employs a Westinghouse Electric and Manufacturing Company design of throttle valve manifold complete with its attached oil-operated governing mechanism.

The overspeed protection is obtained by the use of the governor mechanism in the manifold, which is actuated by a small oil pump attached to the turbine shaft. This pump may be of the centrifugal or gear type. In the manifold is fitted a small spring-loaded piston upon which the discharge pressure from the pump acts. This piston moves an oil pilot valve which in turn regulates the oil flow to the large spring-loaded oil piston used to move the steam throttle valve. The oil used for operating the main piston is obtained from the discharge line from the main lubricating-oil service pumps. Thus, if the turbine should overspeed beyond a predetermined figure, the increased oil discharge pressure from the turbine pump will actuate the oil pilot valve. This in turn will dump the oil that holds the main piston in the normal

service pumps are generally of two designs, one being a motor-driven positive-displacement pump and the other a steam reciprocating duplex or simplex pump. The fuel-oil pumps are usually sized to have a capacity at least 10 percent in excess of the maximum oil required to be pumped. The motor-driven service pump is generally fitted for about a 25 to 50 percent speed control with the control rheostat located at the boiler operating station.

Oil from the settling tank is pumped by means of the service pumps through the suction main, a coarse-mesh duplex suction strainer, and is discharged into the service main. The discharge pressure required depends upon the quality of oil to be burned and the type of burner used. The discharge pressure is usually from 125 to 350 lb gage. The oil, after entering the discharge line, is pumped through a fuel-oil heater, a fine-mesh duplex strainer, a meter, and then to the burners.

To control the supply pressure to the burner or to the combustion control valve, when a motor-driven service pump is used, a back-pressure valve is fitted. This back-pressure valve should be located so that all oil pumped will pass through the fuel-oil heaters and so that it may be easily adjusted by the operating personnel. The accessibility of the valve is especially important if an automatic combustion control is not provided and hand operation is required.

In most cases, the steam reciprocating pump is fitted with a steam governor valve that is actuated by the oil discharge pressure. In order to smooth out the pump pulsations created by the reciprocating pump, a large air chamber must be fitted. This chamber should have a capacity of about 4 or 5 cu ft and should be fitted with an air charging and a drain connection.

Both service pumps must be fitted with relief valves and, in the case of the steam reciprocating pump, this valve must be installed so that its discharge will be connected to the settling tank. This is necessary since a reciprocating pump can build up a very high discharge pressure, if both the suction and the discharge valves on the liquid end are closed and the steam pump piston should move, owing to small steam leakages. Many fuel-oil service pumps have been damaged owing to improper connection of the relief valve. Each fuel-oil heater should be fitted with a relief valve which will protect the unit from excessive pressure due to expansion of the oil when both the inlet and the outlet valves are secured and steam is accidentally applied to the unit.

If good combustion is desired, accurate control of the fuel-oil temperature is required. This is imperative since small changes in temperature produce a relatively large change in the quantity of oil burned. This is especially true if an automatic combustion control is fitted since its successful operation depends upon a known quantity of oil being burned for a predetermined oil pressure.

The solenoid valve shown on the sketch is installed if a motor-driven fan is fitted to cut off the supply of oil to the burners in the event of loss of air supply to the boiler. This valve is interlocked electrically with the forced- or induced-draft motor fan, so that if it stops the solenoid valve will close. This valve should also be fitted with a manual quick-closing feature. The installation of this valve is a great help in preventing furnace explosions. If the fan is steam-driven, the solenoid valve should be replaced by a quick-closing valve.

The location and arrangement of the fuel-oil suction piping should be given careful consideration when designing a fuel-oil system for it is necessary that the pressure drop be kept to a minimum. The fuel-oil service pumps

operating position to the sump tank. The main oil piston will then move down owing to the spring loading pressure and close the throttle valve.

For testing the overspeed mechanism, it is not necessary to overspeed the turbine, since a simulated overspeed condition will be obtained if the test valve installed in the governor oil pump return line is opened. Opening this valve creates an increased differential pressure on the oil pilot valve piston since it reduces the back pressure on the oil return line, thereby simulating increased supply pressure or overspeeding.

To protect the turbine and gear bearings and the gear meshes from an insufficient supply of lubricating oil, the governing mechanism is also designed

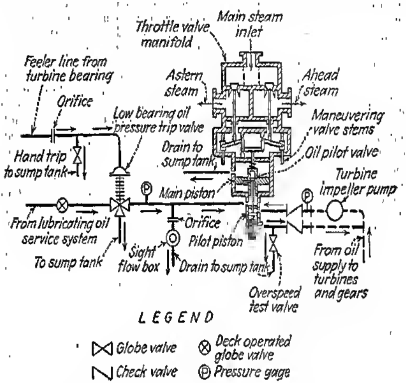


FIG. 4.—Westinghouse turbine governor system.

to cut off the steam to the turbine in the event that (1) the lubricating-oil service pump stops, (2) the oil pump discharge pressure drops below approximately 15 to 20 psi gage, or (3) the bearing oil pressure at the turbine is insufficient. These features are obtained since the main oil piston which operates the steam throttle valve is held open by the oil discharged from the service pump (see Figs. 1 and 2). Thus, if the oil discharge pressure at the pump drops, the spring will move down the main oil piston and cut off the steam supply. The three-way trip valve is installed to cut off the oil supply to the governor in the event that the oil pressure at the turbine bearings drops below a predetermined figure (usually from 5 to 7 psi gage). The operation of both the three-way trip valve and the governor valve may be tested by opening the hand trip valve, which will simulate a low bearing oil pressure.

The deck-operated valve indicated in the sketch is provided so that the

should be located so that they have the minimum static suction lift and so that the suction piping will be as short as possible. All piping should be of ample size, and all valves should be of the gate type. This is a particularly important item if wide-range burners are employed which return hot oil to the pump suction.

The system design shown in the sketch is satisfactory for the use of straight mechanical-pressure atomizer-type burners or wide-range burners not requiring the use of return oil. If it is desired to install a system employing the wide-range burners requiring return oil, the arrangement shown should be modified to install the necessary piping to return the excess oil from the burners back to the pump suction. The fuel-oil meter should be relocated to the suction side of the service pump so that it measures only the oil burned. To arrange the system shown for automatic combustion control, it is merely necessary to add an additional control valve in the oil supply line to each boiler. Several designs of combustion control are in use today on commercial vessels, the most common of which are manufactured by the Bailey Meter Company, Mason-Neilan Regulator Company, General Regulator Company, and Hagan Corporation.

Service System for Diesel-propelled Vessels. The fuel-oil service system for a diesel-propelled vessel is quite different from that used on an oil-burning steam vessel since all oil delivered to the engine must be run through centrifugal purifiers and filters. On a diesel vessel, two daily service tanks are installed, in addition to the two settling tanks. Oil is settled in the settling tanks the same as on a steam-propelled vessel, except that instead of pumping the oil from the settling tanks direct to the engine, the oil is delivered from the settling tank to a centrifugal oil purifier where it is purified and returned to the oil service tanks. The design of centrifuge employed is the same as that employed for lubricating oils. For a detailed description see p. 1311. Oil is then pumped from the service tank by the fuel-oil service pump similar to the system installed on a steam-propelled vessel, except the discharge pressure of the service pump is relatively low, being only from 30 to 50 lb gage. All oil delivered by the service pump must be pumped through a metal edge strainer and a filter, for all oil delivered to the engine must be of the utmost purity. The oil that is delivered by the service system is then handled by the engine injector where it is metered and injected into the cylinders at a high pressure. The oil pressure to the engine is maintained by the use of a back-pressure valve located in the fuel-oil header at the engine. All oil passing through the back-pressure valve is returned to the pump suction.

Piping Materials. The oil piping materials, installed in the suction piping lines to the fuel-oil service pumps on most vessels that have been built for the U.S. Maritime Commission and to meet *Senate Report* 184 requirements, have employed standard-weight black steel pipe with all valves and fittings either forged or cast steel. All take-down joints should be flanged along with all valves 2 in. and above in size. All small sizes of valves and fittings are either of screwed or socket-welded design with the screwed design of valve seal-welded to the pipe. All fittings 2 in. in size and larger are generally of butt-welded design with a few steel-flanged fittings being used where necessary. With the design of system described above, a tight system is assured since all joints are either of welded or flanged design. If the vessels being constructed do not have to meet the requirements of *Senate Report* 184, then cast-iron, bronze, and malleable-iron valves and fittings may be installed in the suction piping. It is best, however, to make all cast-iron valves and

by any quantity different from zero, without altering the value of the fraction.

To add two fractions, reduce each to a common denominator, and add the numerators: $\frac{a}{b}+\frac{x}{y}=\frac{ay}{by}+\frac{bx}{by}=\frac{ay+bx}{by}$.

To multiply two fractions: $\frac{a}{b}\times\frac{x}{y}=\frac{ax}{by}$; $\frac{a}{b}\times x=\frac{a}{b}\times\frac{x}{1}=\frac{ax}{b}$.

To divide one fraction by another, invert the divisor and multiply:

$$\frac{a}{b}\div\frac{x}{y}=\frac{a}{b}\times\frac{y}{x}=\frac{ay}{bx};\quad \frac{a}{b}\div x=\frac{a}{b}\times\frac{1}{x}=\frac{a}{bx}$$

Ratio and Proportion. The notation $a:b::c:d$, which is now passing out of use, is read: "a is to b as c is to d," and means simply $(a/b) = (c/d)$, or $ad = bc$. a and d are called the "extremes," b and c the "means," and d the "fourth proportional" to a, b, and c. The "mean proportional" between two numbers is the square root of their product; also called the "geometric mean" of the numbers (p. 115). If $a/b = c/d$, then $(a+b)/b = (c+d)/d$, and $(a-b)/b = (c-d)/d$; whence also, $(a+b)/(a-b) = (c+d)/(c-d)$. If $a/x = b/y = c/z = \ldots = r$, then

$$(a+b+c+\ldots)/(x+y+z+\ldots)=r$$

Variation. The notation $x \propto y$ is read: "x varies directly as y," or "x is directly proportional to y," and means $x = ky$, where k is some constant. To determine the constant k, it is sufficient to know any pair of values, as x_1 and y_1, which belong together; then $x_1 = ky_1$, and hence $x/x_1 = y/y_1$, or $x = (x_1/y_1)y$. The expression "x varies inversely as y," or "x is inversely proportional to y," means that x is proportional to $1/y$, or $x = k/y$.

Exponents. $a^{m+n} = a^m a^n$. $a^{m-n} = a^m/a^n$. $a^0 = 1$ (if $a \neq 0$). $a^{-m} = 1/a^m$. $(a^m)^n = a^{mn}$. $a^{1/n} = \sqrt[n]{a}$. Thus: $a^{1/2} = \sqrt{a}$, and $a^{1/3} = \sqrt[3]{a}$. $a^{m/n} = \sqrt[n]{a^m}$. Thus: $a^{2/3} = \sqrt[3]{a^2}$ and $a^{3/2} = \sqrt{a^3}$. $(\sqrt[n]{a})^n = a$. $(ab)^n = a^n b^n$. $(a/b)^n = a^n/b^n$. $(-a)^n = a^n$ if n is even. $(-a)^n = -a^n$ if n is odd. If n is positive and increases indefinitely, a^n becomes infinite if $a > 1$, and approaches 0 if $a < 1$ (a being always positive). Graphs, p. 174; series, p. 160.

Radicals. Except in the simple cases of square root and cube root, radical signs should always be replaced by fractional exponents: $\sqrt[n]{a} = a^{1/n}$. $(\sqrt[n]{a})^n = (a^{1/n})^n = a$. If n is odd, $\sqrt[n]{-a} = -\sqrt[n]{a}$; but if n is even, $\sqrt[n]{-a}$ is imaginary. Every positive number a has two square roots, one positive and the other negative; but the notation $\sqrt{a}$ always means the positive root; thus, $\sqrt{9} = 3$; $-\sqrt{9} = -3$. If the denominator of a fraction is of the form $\sqrt{a} \pm \sqrt{b}$, it is possible to "rationalize the denominator" by multiplying both numerator and denominator by $\sqrt{a} \mp \sqrt{b}$. Thus:

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\frac{a+b+2\sqrt{ab}}{a-b}$$

Logarithms. (For the use of logarithms in numerical computation, see p. 91.) The logarithm of a (positive) number N is the exponent of that power to which the base (10 or e) must be raised to produce N. Thus, $x = \log_{10} N$ means that $10^x = N$, and $x = \log_e N$ means that $e^x = N$. Logarithms to base 10 are called **common, denary,** or **Briggsian** logarithms. For table of 4-place common logarithms see pp. 40–43.

Logarithms to base e are called **hyperbolic, natural,** or **Napierian** logarithms. Here $e = 1 + 1 + 1/2! + 1/3! + 1/4! + \ldots = 2.7182818284\,59\ldots$ For table of 4-place hyperbolic logarithms see pp. 58, 59.

If the subscript 10 or e is omitted, the base must be inferred from the context, the base 10 being used in numerical computation, and the base e in theoretical work. In either system,

$$\log (ab) = \log a + \log b \qquad \log (a^n) = n \log a \qquad \log 0 = -\infty$$
$$\log (a/b) = \log a - \log b \qquad \log (\sqrt[n]{a}) = (1/n) \log a \qquad \log 1 = 0$$
$$\log (1/n) = -\log n \qquad \log (\text{base}) = 1 \qquad \log \infty = \infty$$

The two systems are related as follows:

$$\log_{10} e = M = 0.4342944819\ldots; \quad \log_e 10 = 1/M = 2.3025850930\ldots$$
$$\log_{10} x = 0.4343 \log_e x; \qquad \log_e x = 2.3026 \log_{10} x.$$

For tables of multiples of M and $1/M$, see p. 62. For graphs of the logarithmic and exponential functions, see p. 174; series, p. 160.

The Binomial Theorem. (For table of binomial coefficients, see p. 39 and p. 116.)

Let $(n)_1 = n$, $(n)_2 = \dfrac{n(n-1)}{1 \times 2}$, $(n)_3 = \dfrac{n(n-1)(n-2)}{1 \times 2 \times 3}$,

$$(n)_4 = \frac{n(n-1)(n-2)(n-3)}{1 \times 2 \times 3 \times 4}, \ldots$$

Then, for any value of n, provided $|x| < 1$,

$$(1 + x)^n = 1 + (n)_1 x + (n)_2 x^2 + (n)_3 x^3 + (n)_4 x^4 + \ldots$$

(If n is a positive integer, the series breaks off with the term in x^n, and is valid without restrictions on x, see p. 112.)

The most useful **special cases** are the following:

$$\sqrt{1+x} = (1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \ldots \quad (|x| < 1)$$

$$\sqrt[3]{1+x} = (1+x)^{1/3} = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \frac{10}{243}x^4 + \ldots \quad (|x| < 1)$$

$$\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots \quad (|x| < 1)$$

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \ldots \quad (|x| < 1)$$

$$\frac{1}{\sqrt[3]{1+x}} = (1+x)^{-1/3} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4 - \ldots \quad (|x| < 1)$$

$$\sqrt{(1+x)^3} = (1+x)^{3/2} = 1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3 + \frac{3}{128}x^4 - \ldots \quad (|x| < 1)$$

$$\frac{1}{\sqrt{(1+x)^3}} = (1+x)^{-3/2} = 1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{35}{16}x^3 + \frac{315}{128}x^4 - \ldots \quad (|x| < 1)$$

with corresponding formulæ for $\sqrt{1-x}$, etc., obtained by reversing the signs of the odd powers of x. Also, provided $|b| < |a|$:

$$(a+b)^n = a^n \left(1 + \frac{b}{a}\right)^n = a^n + (n)_1 a^{n-1} b + (n)_2 a^{n-2} b^2 + (n)_3 a^{n-3} b^3 + \ldots$$

where $(n)_1$, $(n)_2$, etc., have the values given above.

Arithmetical Progression. In an arithmetical progression, a; $a + d$; $a + 2d$; $a + 3d$; . . ., each term is obtained from the preceding term by adding a constant, called the constant difference, d. If n is the number of terms, the last term is $l = a + (n-1)d$; the "average" term is $\frac{1}{2}(a + l)$;

fittings of the flanged design and also to limit the maximum size of the bronze and malleable valves and fittings to about $1\frac{1}{2}$ in. in size.

The fuel-oil service piping installed in the discharge of the fuel-oil service pump should be extra-heavyweight black steel pipe with all valves, flanges, and fittings of a heavyweight forged or cast-steel design. All take-down joints should be flanged along with all valves and fittings $1\frac{1}{2}$ in. in size and larger. All small sizes of valves and fittings are either of screwed or socket-welded design with the screwed design of valve seal-welded.

The fuel-oil suction piping material installed on all new U.S. Navy construction employs thin-wall steel tubing with all valves, flanges, and fittings either forged or cast steel. All joints are either welded or flanged, and no

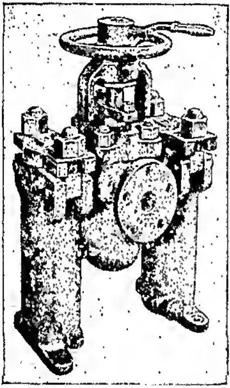


FIG. 11.—Plug-type duplex strainer for fuel oil. (*Courtesy of Elliott Company.*)

screwed joints are permitted. The oil service discharge piping material is the same except heavier weight tubing, flanges, valves, and fittings are required.

Strainers. The strainers installed in a fuel-oil service system should be of the duplex design so that the flow of oil will not be interrupted when cleaning of a strainer basket is required. This is most important since the interrupting of the flow of oil to the burners of a boiler in a steam-propelled vessel may result in a furnace explosion.

The basic design of a fuel-oil duplex strainer is the same as for a lubricating-oil duplex strainer, of which a detailed description is given on p. 1308. The outline and cross-sectional drawings (Figs. 11 and 12) show a typical plug-cock design of strainer, which is the most common in use for fuel-oil service systems.

The material of the bodies of the duplex strainers are cast steel, but in the case of the suction strainer cast iron may be used if the merchant vessel is not

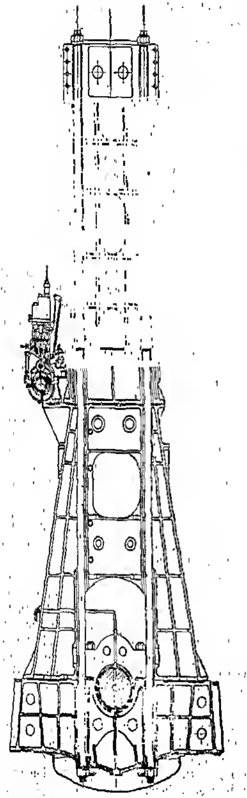


FIG. 7a.—M.A.N. two-cycle double-acting engine.

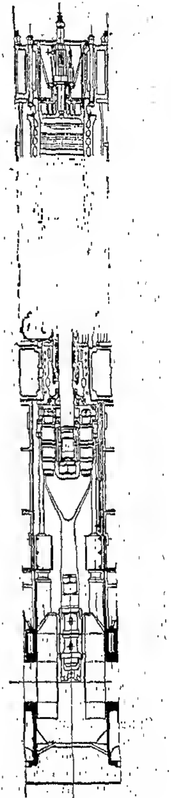


FIG. 7b.—M.A.N. two-cycle double-acting engine.

designed to the rules set forth in *Senate Report* 184. The valve or plugs are generally bronze or stainless steel. The strainer baskets are usually monel with $\frac{1}{16}$ in. diameter perforations for the suction strainers and $\frac{3}{8}$ in. diameter perforations with No. 30 wire cloth for the discharge strainer. The free

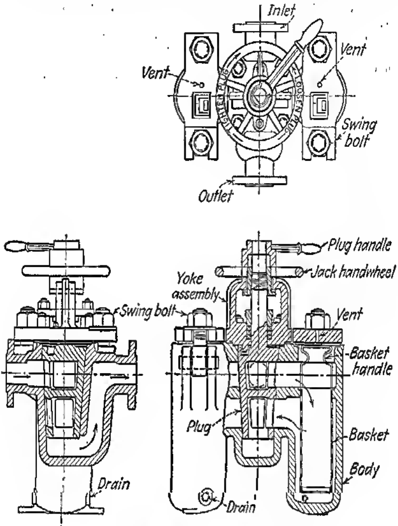


FIG. 12.—Fuel-oil strainer—duplex plug-cock type.

hole area through the strainer basket should be at least six times the area of the inlet pipe connection with the velocity of the liquid in the pipe not exceeding approximately 1.5 fps for a suction strainer and 4 fps for a discharge strainer.

part of the piston except in the case of crankcase scavenging engines, which will be explained in a later section.

Double-acting Engine. Just as in the case of the steam engine, in which the cylinder is closed at both ends and steam is admitted to first one side of the piston and then the other, it has been found possible to close both ends of the diesel cylinder and utilize both sides of the piston to produce power strokes. The conditions of operation are, however, more difficult because the piston rod must operate in a cylinder filled with gas that attains a temperature of more than 3000 deg and must pass through a stuffing box that will prevent leakage of gas at 500 to 700 lb pressure.

The necessity for this stuffing box and the fact that the presence of the piston rod interferes with the distribution of fuel in the lower combustion chamber and requires the use of two or more injection valves in the lower head, add complication to the head. Despite these difficulties the double-acting engine has proved highly successful and is widely used for large power installations.

The first successful double-acting engines operated on the four-cycle principle, but the necessity for air inlet and exhaust valves in both ends of the cylinder, with the accompanying complicated valve gear, made this type of engine very cumbersome and it was eventually abandoned in favor of the much simpler two-cycle design.

The cylinder of the two-cycle double-acting engine, as shown in Fig. 7, is essentially two single-acting cylinders placed end to end. There are two sets of scavenging and exhaust ports in the middle of the cylinder, one set of which is uncovered by the piston on its downward stroke, the other being uncovered on the up stroke. The piston is likewise, in its essentials, two single-acting pistons joined to form one long piston which is attached to a piston rod. The cyclic processes in the cylinder are the same as those described for the single-acting engine, but they occur alternately in the opposite ends of the cylinder so that the piston receives a power impulse on every stroke.

The absence of valves other than the injection valves makes the two-cycle double-acting engine comparatively simple. The starting air valve which is essential for starting the engine is generally used only in the upper cylinder head for the sake of simplicity. The double-acting engine is essentially a high engine, and in general its use in marine service is confined to power plants in which a high concentration of power per unit is desired. At one time it appeared that the double-acting two-cycle engine would entirely displace the single-acting type for powering large ships, but the development of better methods of scavenging, multiple-effect gear drives, and supercharging has enabled the single-acting engine to give it strong competition. At present its use is confined to large ships with the exception of a special design of double-acting two-cycle engine that finds considerable application in submarine service.

Opposed-piston Engine. A development that arose from the desire to obtain more power from a single cylinder than is possible by the use of the single-acting principle is that of the opposed-piston engine. As may be seen from the schematic diagram in Fig. 8, the cylinder of this type of engine is simply a cylindrical trunk, open at both ends, in which there are two pistons. The lower piston is connected to the crankpin in the conventional way through the medium of a piston rod, crosshead, and connecting rod. The upper

INTERNAL-COMBUSTION ENGINES

BY

LOUIS R. FORD

GENERAL

Diesel engine is a term commonly used to designate any engine in which atmospheric air is compressed to a pressure that will produce a temperature high enough to ignite the fuel that is discharged into the cylinder at the end of the compression stroke. The resulting combustion of the fuel produces a highly heated gas, the expansive force of which is utilized to perform work.

This definition is not strictly accurate, since the designation diesel was originally intended to apply to a type of engine invented by Rudolf Diesel to utilize a special thermal cycle conceived by him. Subsequent development of this engine resulted in modifications of the processes executed within it that involved variations of the cycle. In time the name became associated more with the kind of fuel used than with the cycle involved. As a result, in present-day practice the terms oil engine and diesel engine are practically synonymous although, as will be explained later, an oil engine is not necessarily a diesel engine.

METHOD OF OPERATION

In the usual diesel engine the cylinder is charged with atmospheric air while the piston is at the bottom of its stroke, and this air is compressed to a pressure of approximately 500 lb by the piston on its return stroke, this compression raising the temperature of the air to about 1000 F. At the end of the compression stroke, fuel oil in a highly atomized state is injected into this air and is ignited by its high temperature. The resultant burning of the oil raises the temperature of the gases produced to about 3000 F and increases the pressure accordingly. The piston in the meantime is moving downward on its power stroke under the influence of this pressure, and theoretically the resulting increase in volume occurs without rise in pressure of the gases during the combustion period. After combustion is completed, expansion of the gas behind the moving piston continues with a constantly decreasing pressure until the piston nears the end of its stroke, when an outlet from the cylinder is opened and the gases escape to the atmosphere. The cylinder is again charged with air and the cycle of operation repeated. An engine operating on the diesel cycle may, however, use fuel other than oil but no other fuel is used in marine service.

HEAT CYCLES USED IN OIL ENGINES

There are three thermal cycles that have a direct relation to oil-engine operation: the Otto cycle, the diesel cycle, and the dual cycle. In each of these there are five phases: (1) the cylinder is charged with air, (2) the air is compressed, (3) the fuel is injected and burned, (4) the resulting gas is expanded, (5) the burned gas is exhausted from the cylinder.

Otto Cycle. The Otto cycle, so named after the engineer who devised this system of operation, consists of five phases as follows: first, the cylinder is charged with air, then the air is compressed, and at the end of compression

piston has its piston rod connected to a crossbeam which has a side rod attached to each of its outer ends. These side rods are connected to two cranks, one on each side of the crank to which the lower piston is attached, through the medium of crossheads and connecting rods. Near the bottom end of the cylinder is a row of scavenging ports, and in a corresponding location at the top of the cylinder is a row of exhaust ports.

The action of the engine is as follows: Starting with the cylinder charged with air and both pistons at the outer ends of their strokes, *i.e.*, at opposite ends of the cylinder, the two pistons move inward and the air in the cylinder is compressed between them. At their point of nearest approach in the middle of the cylinder, compression is complete and fuel is injected into the combustion space between the two pistons. Pressure of the gas, produced by the resulting combustion, forces the pistons apart and they move out toward the ends of the cylinder, the lower piston pushing downward on its crank and the upper piston pulling upward on the two side cranks. As they approach the end of their travel, the upper piston uncovers the exhaust ports and shortly afterward the lower piston uncovers the scavenging ports. The scavenging air entering through the lower ports blows straight through the cylinder and out through the upper ports. This constitutes what is known as **uniflow scavenging.**

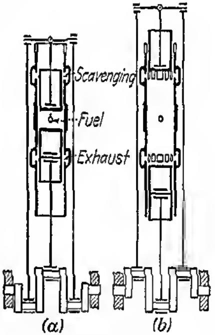


FIG. 8.—Schematic diagram of opposed-piston engine.

An apparent disadvantage of this type of construction is the fact that there are three cranks between two main bearings, which would seem to involve the possibility of crankshaft trouble. As a matter of fact, very little trouble has ever been experienced with such crankshafts and the opposed-piston engine is highly successful in service. Although a few small engines of this type have been built, this method of construction is usually associated in marine service with large, slow-speed engines. Since each cylinder of the engine is, in effect, two single-acting cylinders, it is readily seen that this construction enables a large amount of power to be concentrated in a short engine. The over-all height, however, is greater than that of the corresponding engine of the single-piston type.

A variation from the structural arrangement described in the foregoing is found in the opposed-piston engine using two crankshafts, one below the cylinders and one above. The lower piston is connected to the lower crankshaft, and the upper piston is connected in the same way to the upper crankshaft, the two shafts being connected by gears so that their rotation is synchronized. This arrangement eliminates the necessity for side rods and permits making the engine even shorter. The use of engines of this type is at present confined to submarine propulsion and auxiliary services, but they will undoubtedly be available for commercial service after the war.

the fuel is injected and burns during a period of time when the cylinder volume is unchanged, with the result that the pressure created by the heat of the gases is added to the pressure of compression. On the PV diagram, as shown in Fig. 1, the combustion line is vertical. At the end of the combustion period, expansion begins and the gases are expanded down to the point of release, after which the phase of exhaust occurs. As will be seen from the diagram, the use of this cycle involves combustion at constant volume. Accordingly, the pressure to which the air may be compressed in an oil engine is limited by the maximum allowable pressure in the engine cylinder and some external agency is required for igniting the fuel. This is the cycle on which the ordinary gasoline engine operates.

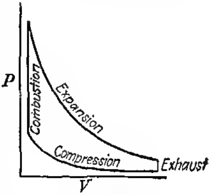


FIG. 1.—Otto cycle.

Diesel Cycle. The principal difference between the diesel cycle and the Otto cycle is, that, although the air-compression phase occurs in the former as in the latter, it is carried to a higher pressure and the fuel is injected at a graduated rate so that combustion occurs while the cylinder volume is increasing. Accordingly, the combustion pressure does not rise above the compression pressure and is represented on the diagram in Fig. 2 by a horizontal line. At the end of the combustion period, expansion occurs as in the Otto cycle and is followed by the exhaust phase at the end of the expansion phase. The combustion occurring in the manner described is known as **combustion at constant pressure.**

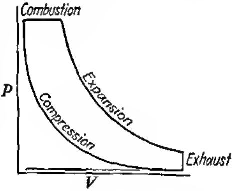


FIG. 2.—Diesel cycle.

An important difference between these two cycles lies in the way in which the combustion occurs. In the diesel cycle, the compression can be carried to any desired pressure, thereby increasing the temperature of the air enough to cause autoignition when the fuel is injected.

It should be noted that the theoretical diesel cycle, as described, differs somewhat from the practical cycle usually carried out in the actual diesel engine. This is shown in the diagrams in Fig. 3, which are indicator diagrams, taken from operating engines. Here it will be noted that the combustion line instead of being horizontal is curved upward. Although it is possible to adjust a diesel engine so that it will produce a combustion line that conforms closely to the shape of the theoretical diagram, it is found in practice that better efficiency is obtained if the combustion line has this upward-curving characteristic.

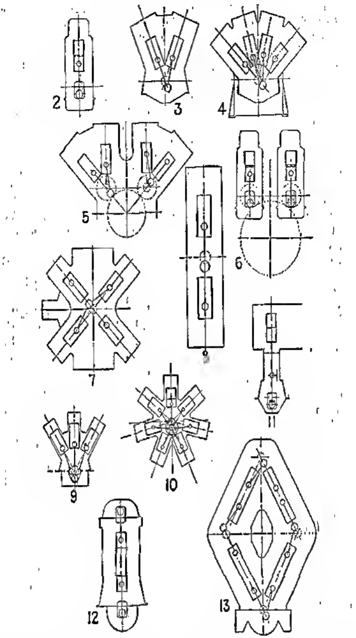
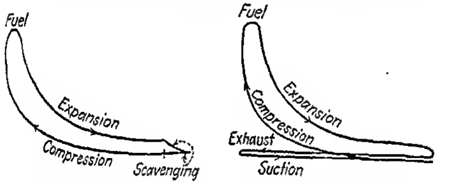


FIG. 9.—Special cylinder arrangements: 2, two-cycle cylinder with uniflow scavenging through valves in head; 3, V arrangement built in 6, 8, 12, and 16 cylinders; 4, porcupine type with two V's, producing a 32-cylinder unit; 5, double V with pinion between each pair of four-throw crankshafts; 6, quad unit of 24 cylinders; 7, pancake arrangement of four cylinders in same plane; 8, opposed cylinders produce a narrow unit; 9, W arrangement with two link rods on master connecting rod; 10, aircraft radial engine; 11, double-acting engine of well-known type; 12, two-crankshaft arrangement with opposed pistons; 13, diamond arrangement of four opposed piston engines.

Dual Cycle. Further departure from the theoretical cycles is found in operating engines, especially those operating at high speed. The high-speed engine with smaller cylinders requires such a small fuel charge at each injection that graduated injection becomes very difficult. Also, certain inherent characteristics of combustion, which will be discussed later, make it practically impossible to burn the fuel at a rate corresponding to the increase of cylinder volume. As a result, we get a condition in the cylinder that is in effect a combination of Otto-cycle and diesel-cycle operation. The combustion line in this case will show a partial rise corresponding to combustion at constant volume, followed by a period of combustion at constant pressure. When this condition exists, the engine is said to be operating on a dual cycle. Practically all small high-speed diesels operate on this dual cycle.

Mechanical Cycles. The cycle discussed in the foregoing section are designated as thermal cycles and are not to be confused with mechanical cycles, of which there are two involved in diesel-engine operation. The thermal cycles represent heat processes, and such a cycle may be considered as a preliminary plan of engine operation. Mechanical cycles on the other



Two-cycle engine Four-cycle engine
FIG. 3.—Practical diesel cycle. Shapes of actual indicator cards.

hand are descriptive of the mechanical arrangements that are used for carrying out the heat cycles.

Two-stroke Cycle. In an engine operating on the two-stroke cycle, the cylinder is charged with air and the piston makes an upward stroke to compress the air. At the end of this compression period, fuel injection and combustion occur and the piston moves downward on its power stroke. When about 80 to 85 percent of the stroke has been completed, exhaust ports in the lower wall of the cylinder are uncovered or exhaust valves in the cylinder head are opened, depending on the construction of the engine, and the gases escape to the atmosphere, the pressure in the cylinder falling to atmospheric. Further movement of the piston uncovers scavenging ports in the side of the cylinder opposite the exhaust ports and air under a pressure of 1 to 5 lb blows into the cylinder, pushes out the remaining gases, and fills the cylinder with a fresh charge of air, after which the cycle is repeated.

It is thus seen that all the phases in the cycle of operations, namely, charging with air, injection and combustion of fuel, expansion, and exhaust occur during two strokes of the piston. From this we get the designation **two-stroke cycle.** In general practice this term is shortened to two-cycle and engines operating in this manner are known as two-cycle engines.

CYLINDER ARRANGEMENTS

Cylinders in Line. The conventional cylinder arrangement and the one most widely used is that in which all the cylinders are placed in line above the crankshaft and in a vertical position. Numerous combinations are found, including the 1-, 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 12-, and 16-cylinder combinations, but for marine propulsion it is not customary to use fewer than six cylinders for four-cycle and five for two-cycle engines, except in the case of opposed-piston engines or engines connected to the propeller shaft through reverse gears. Single-cylinder and two-cylinder engines are used only for auxiliary service, such as generator or pump drive.

V-type Engine. The trend toward greater concentration of power and more compactness of engines has found diesel designers taking a leaf from the book of designers of automotive and other types of gasoline engines as far as cylinder arrangement is concerned. A number of cylinder arrangements departing from the in-line practice have been developed, but the one most widely used is the V-type engine. Two lines of cylinders are placed in an inclined position diverging at the top and connected to a single crankshaft located at the point of intersection of the center lines. This arrangement permits the use of a very rigid type of frame and a high concentration of power within a given over-all space.

Originally designed for railway service, where limitations on space are severe, the V-type engine found ready acceptance in submarine service where the space limitations are still more severe. This type has also found wide application in certain classes of commercial marine service, particularly for electric drive.

Miscellaneous Arrangements. In the search for ways of concentrating power in a small space, several different cylinder arrangements have been designed. These are illustrated in the sketch in Fig. 9 which is self-explanatory. Aside from the V arrangement, the special arrangements shown have found very little use outside of naval service where they have been applied to the drive of special types of vessels, or in cases where some very special conditions call for a departure from conventional arrangements.

FUEL SYSTEMS

What may be considered the heart of the diesel engine is the fuel system, by means of which the fuel oil is delivered in metered quantities in an atomized state into the cylinders of the engine at the proper time. Starting at the bunkers or tanks in which the fuel oil is stored, the oil is pumped from there by a transfer pump and delivered to daily service tanks which are elevated above the engine. In most installations it is customary to interpose cleaning equipment between the supply pump and the service tanks, this cleaning equipment consisting of centrifuges or some type of filter. From the service tank, the fuel flows by gravity to the fuel-injection pumps, which perform the double function of metering the charge for each injection into the cylinders and forcing that charge into the cylinders.

At this point we find a difference in the early diesel engines and those of modern design. Originally all diesel engines utilized compressed air for injecting the fuel into the cylinders. Later development of mechanical or airless injection methods resulted in the abandonment of air injection in new marine installations. All marine diesel engines built today are equipped

Mention was made in a preceding paragraph of two methods of exhausting; one through ports in the cylinder wall and the other through valves in the cylinder head. In the former case, as may be seen in Fig. 4, the ports are near the bottom of the cylinder wall with the exhaust ports on one side and the scavenging ports on the other, the exhaust ports being higher than the scavenging ports. This latter arrangement is necessary in order that the exhaust ports will be uncovered and release the gases before the scavenging ports are opened. The scavenging ports are in communication with an air receiver, which is kept charged with air by means of a scavenging air pump either driven directly from the crankshaft of the engine or operated independently by means of an electric motor. The scavenging ports are shaped in such a way as to direct the entering air upward toward the top of the cylinder in order to clear the gases out of the upper end of the cylinder; in addition they are sometimes given an angular position so that the entering air will follow a helical path.

It will be seen that with this arrangement the exhaust ports must necessarily be uncovered before the scavenging ports, while the piston is making its downward stroke, and on the upward stroke the scavenging ports are covered by the piston before the exhaust ports have been covered, thus leaving the cylinder open to the exhaust receiver for a short period during the compression stroke, thereby permitting the entry of some burned gas. This condition is prevented in some two-stroke engines by the use of two rows of scavenging ports, one above the other, with automatic valves in the air receiver to close the upper row of ports or, in one design, both rows. On the down stroke of the piston the upper row of scavenging ports is uncovered before the exhaust ports, but the pressure of the gases in the cylinder holds the valves on their seats, and none of the gas can escape through the scavenging ports. As the piston proceeds downward and uncovers the exhaust ports and later the lower row of scavenging ports, exhaust and scavenging occur as previously described but, as soon as the pressure in the cylinder falls below the pressure in the air receiver, the automatic valves open and additional scavenging air enters through the upper row of ports. On the up stroke the piston closes the lower row of scavenging ports, but air continues to blow into the cylinder through the upper row and scavenging continues. When the moving piston covers the exhaust ports, the upper row of scavenging ports is still open and a slight supercharge is given to the cylinder, owing to the pressure of air in the air receiver. After further movement of the piston covers the upper row of scavenging ports, the cycle continues as before. A two-cycle engine with two rows of scavenging ports controlled by automatic valves is shown in Fig. 4.

In the crankcase scavenged engine the compressor is eliminated by using the undersides of the pistons to compress the air in the closed crankcase and providing passages from the crankcase to the scavenging ports.

Four-stroke Cycle. An engine operating on the four-stroke cycle requires air inlet and exhaust valves in the cylinder head. Starting with the piston at the bottom of the cylinder, the cylinder charged with air, and all valves closed, the operation is as follows: The piston moves upward on the compression stroke, at the end of which fuel is injected, combustion occurs, and the piston moves downward on the expansion stroke. Near the bottom of the stroke the exhaust valve in the cylinder head opens and, as the piston moves upward on its next stroke, the burned gases are pushed out of the cylinder. After the piston passes top dead center, the exhaust valve closes, the inlet

for mechanical fuel injection, but there are still many of the old, air-injection engines in service.

Air Injection. In this type of engine the fuel pump meters each charge of fuel and delivers it into the body of the injection valve located in the cylinder head. The body of this valve is also connected to a high-pressure air system so that it is at all times filled with air at about 900 lb pressure.

After the fuel charge is delivered into the valve, the valve gear operates to open the valve at the proper time in the cycle, and the high-pressure air in the valve blows the fuel violently into the cylinder. In the bottom of the valve body is an atomizer, usually consisting of some form of labyrinthine passage through which the oil is blown, thus causing it to be broken into fine particles and ejected from the valve body into the cylinder in the form of fog.

It would be perfectly feasible to operate the fuel-injection pump by an independent motor but, since its action must be accurately synchronized with the movements of the engine piston so that the fuel charge will be delivered at the proper time, it is obvious that the best method of operating it is through some sort of mechanism driven by the crankshaft of the engine. For this reason the fuel pump is always mounted on the engine where it can be operated by cams and gears driven from the shaft. The fuel pump may accordingly be considered a part of the engine itself.

Mechanical Injection. The mechanical injection of fuel as now used on all new marine engines differs from the earlier air injection principally by reason of the fact that no air is used in connection with the injection or atomization of the fuel. Instead, the fuel is forced into the cylinder by the fuel pump under high pressure and, by forcing it through minute orifices in the injection nozzle, this high pressure is converted into velocity, which causes the fuel to be blown into the cylinder in a fine mist similar to that produced by the air-injection valve.

There are two types of mechanical-injection systems in use. One is known as the common rail system and the other the jerk pump system, the latter being the more widely used. In the common rail system a supply pump delivers the fuel to a pressure pump, the discharge from which need have no definite time relation to engine rotation. This pressure pump maintains a pressure of 3,000 to 9,000 lb in a header or common rail, to which each of the injection valves is connected. The bodies of the injection valves—except some modified types—are filled at all times with this high-pressure oil and, when the valve-operating gear causes a valve to open, the oil is projected into the cylinder. The amount of fuel thus introduced into the cylinder depends upon the length of time the valve is held open. Provision is made in the valve-operating gear for regulating this duration of opening.

In the jerk pump system each fuel pump is connected directly to its corresponding injection valve or built into it. Although the line between the pump and the valve is at all times filled with oil, it is not under pressure except during the period when injection occurs. The fuel pump is of the constant-stroke type, and the same amount of oil is drawn into the pump barrel on each successive stroke. As soon as the plunger starts its discharge stroke, oil is forced out of the other end of the pipe connecting to the injection valve.

The injection valve is simply an accurately made spring-loaded check valve held on its seat by the tension of its spring. When the pressure, owing to the oil's being forced into the valve by the pump, is high enough to overcome the tension of the spring, the valve lifts and oil is forced through fine

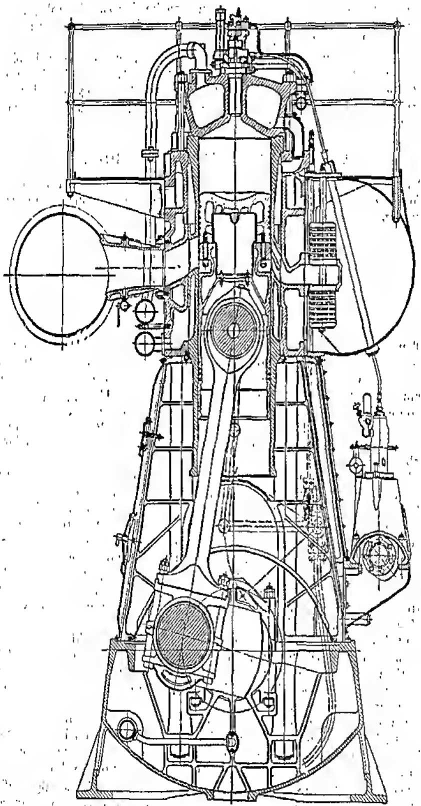


FIG. 4.—Nordberg two-cycle trunk-piston engine.

orifices in the injection valve oozzle and is projected into the combustion space in the form of a mist.

The amout of fuel that is thus forced into the cylinder must, of course, be varied to suit operating conditions. This variation is obtained by establishing, at a variable and controllable point in the stroke, a connection between the pump barrel and the suction line. This is done either by a spill valve operated by a linkage attached to the plunger or by a helical groove in the plunger in connection with a port in the plunger barrel. In this latter case, when the edge of this groove io the plunger uncovers the port, the pressure in the pump barrel is released. The helical shape of the groove makes it possible to vary this time of uncovering the port by rotating the plunger by means of a control mechanism leadiog to the control station.

A separate pump is provided for each cylinder on the engine, and all the pumps may be assembled in a single pump unit having a common suction tank, or each pump may be an independent unit mounted on the engine adjacent to the cylinder that it serves. The pump plungers are in all cases operated by cams upon a camshaft which is driven by gearing from the crankshaft of the engine.

COOLING SYSTEMS

During the process of combustion of fuel in the cylinder of the diesel engine the momentary temperature of the resultant gas reaches a point higher than the melting point of the metal of the cylinder. Some of the metal surfaces enclosing this high-temperature gas have relative motion and rubbing contact and must, therefore, be lubricated. Cooling of the engine cylinders is accordingly necessary, not only to keep the temperature of the metal below the melting point but also to keep the temperature low enough to permit maintaining a film of lubricating oil on the rubbing surfaces.

Water is the universally used cooling medium. Provision of an adequate and properly arranged path for the cooling water through the engine is a functioo of the engine designer, but it is of equal importance that pumps of the right type and capacity be selected and the associated piping be correctly arranged when the engine is installed io a ship.

In the early days of marioe diesel engineering it was the universal practice to use the flotation water for cooling, because of its ready availability and the simplicity of its application. All that was required was a pump to draw in water through a sea suction, force it through the cooling system, and discharge it overboard. Later, improved engineering practice eliminated the use of flotation water, but on some of the older vessels it is still used. Also for operation in localities such as the Great Lakes, where the water is fresh and soft, this is common practice.

There are three things that make it inadvisable to use water drawn from overboard for cooling: First, sea water, eveo when it is perfectly clean, contains dissolved magnesium, sodium, and lime which settle out in the form of hard scale if the temperature of the water rises beyond a certain point. This scale forms on the surfaces of the cylioder jackets and seriously interferes with the necessary transmission of heat.

Second, in harbors where the water is shallow and usually contaminated with sewage the bad effect of salt water is added to by the heavy silt content of the water, causing the jackets to fill up with mud unless cleaned frequently. In fresh-water rivers the scale-forming salts may be absent, but the silt content is usually very high and mud formatioo in the jackets is rapid.

valve opens and, as the piston continues on its downward stroke, a fresh charge of air is drawn into the cylinder. Near the bottom dead center the inlet valve closes and the cycle begins again.

It is thus seen that for the execution of this cycle, four strokes of the piston are required, hence the designation **four-stroke cycle.** As in the case of the two-cycle engine, this term is usually shortened to four-cycle, and engines operating in this manner are designated as four-cycle engines. A typical four-cycle engine is shown in Fig. 5.

CLASSIFICATION OF DIESEL ENGINES

All diesel engines fall into one of two type classifications, either two-cycle or four-cycle, but a further subdivision according to structural arrangement may be made as follows:

$$\text{2-cycle}\left\{\begin{array}{l}\text{Trunk-piston type}\\ \text{Crosshead type}\\ \text{Single-acting type}\\ \text{Double-acting type}\end{array}\right\}\text{—4-cycle}$$

Opposed-piston type—2-cycle only

Trunk-piston Engine. This type, examples of which are shown in Figs. 4 and 5, gets its name from the fact that the piston is made in the form of a cylindrical trunk, closed at the upper end and open at the lower, and having a length equal to about twice the diameter. The upper part of the piston contains the rings, and the walls are of thickened section to enable it to carry the pressure load and transmit heat rapidly. Below the head is an extension or skirt, which may or may not be made as a separate piece bolted to the head.

About midway of the piston are cast bosses, bored to receive a piston pin for attaching the piston to the connecting rod, which serves as the connecting link between the piston and the crankshaft and converts the reciprocating motion of the piston into rotary motion at the shaft. An alternate method of attaching the connecting rod to the piston utilizes a separate piston-pin carrier which is bolted inside the piston at its upper end. Another method is to attach the piston pin directly to flanges inside the piston by long studs which pass through the flat ends of the pin. These alternate methods eliminate the heavy bosses and the holes through the sides of the piston and permit making it a symmetrical, unbroken cylindrical casting.

In this trunk-piston form of construction the piston, in addition to forming a closure for the cylinder, serves in lieu of a crosshead for transmitting the side thrust of the connection rod. In the two-cycle engine the piston skirt also serves to cover the exhaust and scavenging ports when the piston is in the upper part of its stroke.

Another feature of the trunk-piston engine is that the lower end of the cylinder is open to the crankcase, and means must be provided for preventing oil vapor from the crankcase being drawn up into the cylinder. This generally takes the form of scraper rings in the lower end of the skirt.

Crosshead Engine. A type of construction that is generally confined to large engines is shown in Fig. 6. Here it may be seen that the connection between the piston and the crankshaft is through the medium of a piston rod, a crosshead, and a connecting rod. The piston rod, to which the upper end

The third troublesome factor is the highly corrosive action of harbor waters containing sewage and by-products from manufacturing plants. Fresh river water may also be corrosive, owing to the character of the land through which it flows.

Closed-system Cooling. The foregoing considerations led to the universal adoption of the closed-system method of cooling, except in such localities as the Great Lakes, previously mentioned. In this type of system fresh, soft water is circulated through the engine jackets, then passed through a cooler, and returned to the pump for recirculation. The fresh water is used over and over, and the only use made of flotation water is to pass it through the heat exchanger for cooling the jacket water.

Engine System. The cooling system may best be considered in two divisions: that part of it forming a part of the engine structure, and the part forming the circulatory system exterior to the engine. The focal point of the system lies in the metal surrounding the combustion space, where the most rapid transfer of heat must occur. The amount of heat transferred at this point and the rate of heat flow required are the factors that determine the pump capacity and arrangement of the circulatory system.

The usual arrangement found on the engine calls for a fore-and-aft header located near the lower ends of the cylinders, with a branch connecting to each cylinder jacket near its bottom end. Water flowing into each jacket rises to the top and passes into the cooling space in the cylinder head, either through holes in the top of the jacket that register with corresponding holes in the head, or through external jumper pipes connecting the jacket and the head. Since the area around the fuel-injection valve requires intensive cooling and in many engine designs is the most obstructed part of the head, it is customary to arrange nozzles or baffles in the head to direct and accelerate the flow of water around the fuel-valve housing or provide a separate water line to the fuel-valve jackets.

In the case of larger engines in which the exhaust valves must be cooled, a connection from the cylinder head allows the water to flow from the cylinder-head jacket into a jacket built into the exhaust-valve cage, from where it leaves through an outlet pipe that usually connects with a jacket surrounding the exhaust manifold. From the exhaust manifold it leads back to the systems or is discharged overboard, depending on whether the closed or open system is used. It should be noted that in some cases the exhaust manifold is not provided with a water jacket, in which case the connection to the exhaust manifold is omitted. In smaller engines that do not require cooling of the exhaust valves the discharge from the cylinder usually goes direct to the exhaust manifold or to the system, as the case may be. If the air compressor is attached to the engine, a branch from the main water manifold supplies water to the jackets on the compressor. Likewise, in the case of exhaust turbo-charged engines, the jacket of the exhaust turbine is connected to the cooling system.

Piston Cooling. Although the piston-cooling arrangements of modern engines do not form a part of the water circulation system, it is in order to mention it here because it is a part of the engine-cooling system. In smaller engines except those of high output the heat absorbed by the piston from the gases in the cylinder is conducted to the cylinder walls and is carried away by the jacket-cooling water, but in most engines having pistons of approximately 15 in. or more in diameter, the heat transfer by this method is not rapid enough to prevent overheating of pistons, and a direct means of cooling

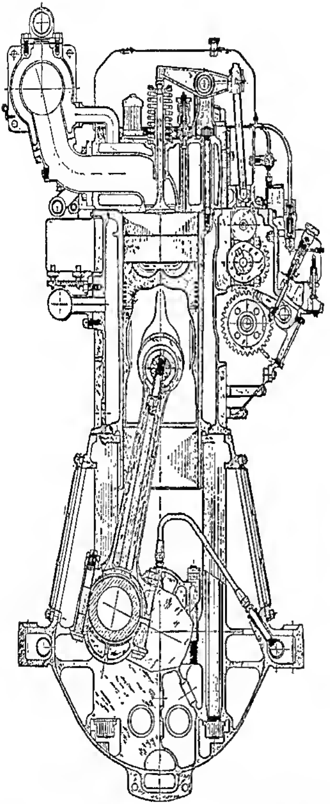


Fig. 5.—Cooper-Bessemer four-cycle trunk-piston engine.

the pistons must be provided. It should be noted, however, that there are some engines operating with uncooled pistons as much as 18 in. in diameter. In the early diesel engines water was used for piston cooling, but this practice has been abandoned in favor of the use of oil as the cooling medium. The most convenient source of oil for this purpose is the lubricating system, and in all except very large engines the oil for cooling the pistons is taken from this system. The manner in which it is done will be explained in connection with the discussion of lubrication. In very large engines it is customary to make the piston-cooling system entirely separate from the lubricating system, with its own pumps and circulatory system.

Circulatory System. In closed-system cooling the circulatory system consists of a fresh-water storage tank, a circulating pump which draws the water from this tank and discharges it to the engine, a heat exchanger through which the water is passed before reaching the engine, and a raw-water pump which pumps flotation water through the heat exchanger for cooling the fresh water. The course of the cooling water is from the storage tank into the circulating pump, thence through the heat exchanger to the engine jackets, then back to the storage tank. In installations where the total amount of water in the system is large, the storage tank may be eliminated and the water returning from the engine jackets may go directly to the pump. An elevated expansion tank located at any convenient point above the engine serves to ensure the system's being filled with water at all times.

It is common practice to insert a lubricating-oil cooler in the raw-water circuit so that the same water that cools the fresh jacket water may be used to cool the lubricating oil. There are heat exchangers available that combine the lubricating-oil cooler and water cooler in a single shell with suitable connections.

Of late years there has been a growing tendency to connect to engines of smaller sizes all the auxiliaries that are needed to keep it in operation, so that when the vessel is under way it will not be necessary to operate independent auxiliaries. In this case the circulating pumps for fresh water and raw water are driven from the crankshaft of the engine, and in the cases of small engines the heat exchanger may be mounted on the engine frame. In most cases it is customary to use centrifugal pumps for circulating the water; when they are attached to the engine, a type is used that will operate in either direction so that reversal of the engine does not interfere with the flow of water. When attached pumps are used, an independently driven unit can be used to circulate jacket water before starting and after stopping the engine. This is known as a "before-and-after pump," and its principal value is in preventing overheating of the engine after it is stopped. This pump is often combined with a lubricating-oil circulating pump, in a single unit.

The heat exchangers used for cooling the water are usually of the tubular type, but there are several different designs of tube arrangements. A commonly used one is made up of a nest of small tubes expanded in headers and inserted into a cylindrical shell. The raw water passes through the tubes, and the water to be cooled fills the shell and surrounds the exterior of the tubes, with baffles arranged in the shell to cause the water to flow back and forth across the tubes. Another type of heat exchanger used on small engines utilizes a flat type of tube, but the cooling action is the same.

Cooling-water Requirements. The amount of cooling water required and hence the capacity of the pumping equipment are governed by the amount of heat to be transferred from the burning fuel in the cylinder to the

of the piston is attached, is fastened at its lower end to the crosshead, which is supported in guides bolted to the engine frame. In an alternate design the piston rod is omitted, and the crosshead is attached directly to the bottom of the piston. A crosshead pin serves to connect the top end of the connecting rod to the crosshead, the bottom end being attached to the crankpin.

Since with this arrangement all of *the side thrust of the connecting rod is* transmitted by the crosshead, the piston is subjected only to the vertical thrust due to gas pressure, and there is no need for the long piston skirt in so far as side thrust is concerned. It is needed, however, in the two-cycle engine to cover the exhaust and scavenging ports. The absence of ports in the four-cycle cylinder makes the skirt unnecessary with crosshead construction and such engines are characterized by a short piston in the form of a closed cylindrical box, bolted to the top end of the piston rod.

Use of the crosshead construction with a piston rod permits the cylinder to be isolated from the crankcase by means of a horizontal diaphragm, the piston rod operating through a packed stuffing box in the diaphrogm.

It may be seen that the introduction of a crosshead and piston rod increases the height of the engine and, where headroom is a factor, trunk-piston construction is to be preferred. In the small and medium sizes, the trunk-piston engine has a lighter specific weight and is cheaper to build. For large engines the crosshead construction is preferable because the piston is simpler, the cylinder may be isolated from the crankcase, and the rate of cylinder liner wear is generally less. Consequently the two types of construction are generally identified, respectively, with the small and medium-sized engines in one case and large engines in the other.

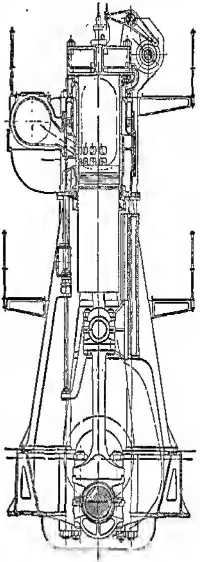


FIG. 6.—H.O.R. crosshead-type engine.

Single-acting Engine. In the engines thus far shown, all the processes involved in the thermal cycles used are carried out on one side of the piston. Consequently, the piston receives a power impulse only once in every two strokes in the two-cycle and once in every four strokes in the four-cycle. All such engines are designated single-acting engines, and the vast majority of diesel engines thus far built are of this type. No use is made of the lower

water in the jackets. It has been found that, in the average engine, 25 to 30 percent of the total heat in the fuel will be transferred to the cooling water, and for purposes of estimating capacity requirements the higher figure is customarily used. When using a fuel with a heat value of 18,500 Btu at the rate of 0.4 lb per hp per hr, the amount of heat to be carried away by the cooling water will be 18,500 × 0.4 × 0.30 = 2,220 Btu/(hp)(hr).

The controlling factor in determining the amount of water that must be circulated in order to carry away this heat is the temperature rise desired. It is inadvisable to have too great a difference between the temperature of the water entering the jackets and that of the water leaving the jackets, as this sets up severe temperature stresses in the metal of the cylinder liner. This temperature difference should not be more than 15 to 25 deg with an outlet temperature of between 120 and 160 deg. As a general rule it is conducive to engine efficiency to operate with an outlet temperature as high as is consistent with maintaining cylinder lubrication. Where the high outlet temperature is used, it may be necessary to regulate the temperature of the inlet water if the temperature difference is to be kept right. In general it is better to maintain a large volume of flow with the inlet at a high temperature than to use a small flow of very cold water.

The amount of water in gallons per minute required for cooling the engine may be determined by the following formula:

$$gpm = \frac{hhv \times W \times h \times P}{(t_2 - t_1) \times 500}$$

where hhv = heat content of fuel per lb
W = lb fuel used per hp per hr
h = percent of heat absorbed by cooling water
P = bhp
t_1 = temperature of inlet water
t_2 = temperature of outlet water

Example. Take an engine of 800 bhp, which uses 0.4 lb fuel per hp per hr, the heat content of the fuel being 18,500 Btu. If a temperature rise of 25 deg is desired, which will be $(t_1 - t_2)$, then the gallons per minute will be

$$\frac{18{,}500 \times 0.4 \times 0.30 \times 800}{25 \times 500} = 242 \text{ gpm}$$

The foregoing formula may or may not agree with the engine manufacturer's recommendation. In such cases, the latter should be followed.

Pump Capacity. It is general practice to provide circulating pump capacity considerably in excess of the amount required for cooling as designated by calculations such as this, but there is no universally accepted rule followed in this connection. Investigation by the author of a number of ships in service equipped with different makes of engines showed that the circulating-water-pump capacities varied from 5 to 9 gal per hp per hr. There is a tendency in modern practice to use larger amounts of the order of 15 to 20 gal per hp per hr in order to maintain a small temperature drop.

Cooler Capacity. There will be found a similar variation in practice in regard to the capacity ratings of different makes of heat exchangers. In determining capacity, the amount of heat to be taken away by the water is

and the sum of the n terms is n times the average term, or $S = \frac{1}{2}n(a + l)$. The **arithmetical mean** between a and b is $(a + b)/2$.

Geometrical Progression. In a geometrical progression, a; ar; ar^2; ar^3; . . ., each term is obtained from the preceding term by multiplying by a constant, called the **constant ratio, r.** The nth term is ar^{n-1}. The sum of the first n terms is $S = a(r^n - 1)/(r - 1) = a(1 - r^n)/(1 - r)$. If r is a positive or negative fraction, that is, if $-1 < r < +1$, then r^n will approach zero as n increases, and the sum of n terms will approach $a/(1 - r)$ as a limit. The **geometric mean** between a and b is $\sqrt{ab}$; also called the **mean proportional** between a and b (p. 113; construction, p. 102).

The **harmonic mean** between a and b is $2ab/(a + b)$.

Summation of Certain Series by Second and Third Differences. Let $a_1, a_2, a_3, \ldots a_n$ be any series of n numbers, as in the first column of the adjoining scheme. By subtracting each number from the next following, form the column of "first differences," and by repeating this process, form the columns of second, third, etc., differences. If the kth differences are all equal, so that subsequent differences are all zero, the original series is called an arithmetical series of the kth order. In this special case the series can be summed as follows: Denote the numbers which stand at the head of the successive columns of differences by $D', D'', D''', \ldots$ Then the nth term of the series is a_n, and the sum of the first n terms is S_n, where

Numbers	1st diff.	2nd diff.	3rd diff.
−64			
	37		
−27		−18	
	19		6
−8		−12	
	7		6
−1		−6	
	1		6
0		0	
	1		6
1		6	
	7		.
8		.	
	.		
.			

$$a_n = a_1 + (n - 1)D' + \frac{(n-1)(n-2)}{1 \times 2}D'' + \frac{(n-1)(n-2)(n-3)}{1 \times 2 \times 3}D''' + \ldots$$

$$S_n = na_1 + \frac{n(n-1)}{1 \times 2}D' + \frac{n(n-1)(n-2)}{1 \times 2 \times 3}D'' + \frac{n(n-1)(n-2)(n-3)}{1 \times 2 \times 3 \times 4}D''' + \ldots$$

If the series is, for example, of the third order, each of these formulæ will stop with the term involving D'''; and only a few terms of the series are required for the computation of the D's. (Differentials, p. 159.)

Sum of the Squares or Cubes of the First n Natural Numbers.

$$1 + 2 + 3 + \ldots + (n - 1) + n = \tfrac{1}{2}n(n + 1)$$
$$1^2 + 2^2 + 3^2 + \ldots + (n - 1)^2 + n^2 = \tfrac{1}{6}n(n + 1)(2n + 1)$$
$$1^3 + 2^3 + 3^3 + \ldots + (n - 1)^3 + n^3 = [\tfrac{1}{2}n(n + 1)]^2$$

Formula for Interpolation by Second Differences. In any ordinary table giving a quantity y as a function of a variable x, let it be required to find the value of y corresponding to a value of x which is not given directly in the table, but which lies between two tabulated values, as x_1 and x_2. If $x = x_1 + md$, where $d = x_2 - x_1$ = the constant interval between two successive x's, and m is some proper fraction, then the corresponding value of y will be given by the formula

$$y = y_1 + mD' + \frac{m(m-1)}{1 \times 2}D'' + \frac{m(m-1)(m-2)}{1 \times 2 \times 3}D''' + \ldots$$

where $D', D'', D''', \ldots$ are the first, second, third, . . . differences in the

series of y's which begins with y_1 (see above), provided the function is of such a nature that the differences of higher orders become negligibly small.

The coefficients of D', D'', D''', . . . in the formula are the binomial coefficients for fractional values of m (see following table). The several terms of the formula (with careful attention to sign) are the successive corrections which must be added to y_1; the sum of these corrections should be rounded out to the nearest unit of the last significant place before adding. If $D' < 4$, the term involving D'', and later terms, can be neglected; the formula then reduces to $y = y_1 + mD'$, which is the familiar formula for ordinary, or "linear," interpolation. If $D''' < 8$ (or $D'''' < 12$, or $D''''' < 16$), the term involving D''' (or D'''', or D''''') can be neglected.

Binomial Coefficients for Fractional Values of m

m	$(m)_2$	$(m)_3$	$(m)_4$	$(m)_5$
0.0	− 0.0000	0.0000	− 0.0000	0.0000
0.1	− 0.0450	0.0285	− 0.0207	0.0161
0.2	− 0.0800	0.0480	− 0.0336	0.0255
0.3	− 0.1050	0.0595	− 0.0402	0.0297
0.4	− 0.1200	0.0640	− 0.0416	0.0300
0.5	− 0.1250	0.0625	− 0.0391	0.0273
0.6	− 0.1200	0.0560	− 0.0336	0.0228
0.7	− 0.1050	0.0455	− 0.0262	0.0173
0.8	− 0.0800	0.0320	− 0.0176	0.0113
0.9	− 0.0450	0.0165	− 0.0087	0.0054

Here $(m)_2 = \dfrac{m(m-1)}{1 \times 2}$, $(m)_3 = \dfrac{m(m-1)(m-2)}{1 \times 2 \times 3}$, $(m)_4 = \dfrac{m(m-1)(m-2)(m-3)}{1 \times 2 \times 3 \times 4}$, etc. Compare p. 39.

Permutations. The number of possible permutations or arrangements of n different elements is $1 \times 2 \times 3 \times \ldots \times n = n!$ (read: "n factorial").

If among the n elements there are p equal ones of one sort, q equal ones of another sort, r equal ones of a third sort, etc., then the number of possible permutations is $(n!)/(p! \times q! \times r! \times \ldots)$, where $p + q + r + \ldots = n$.

Combinations. The number of possible combinations or groups of n elements taken r at a time (without repetition of any element within any one group), is $[n(n-1)(n-2)(n-3) \ldots (n-r+1)]/(r!) = (n)_r$. (See table of binomial coefficients, p. 39.) If repetitions are allowed, so that a group, for example, may contain as many as r equal elements, then the number of combinations of n elements taken r at a time is $(m)_r$, where $m = n + r - 1$. Note: $(n)_1 + (n)_2 + \ldots + (n)_n = 2^n - 1$.

SOLUTION OF EQUATIONS IN ONE UNKNOWN QUANTITY

Roots of an Equation. An equation containing a single variable x will in general be true for some values of x and false for other values. Any value of x for which the equation is true is called a **root** of the equation. To "solve" an equation means to find all its roots. Any root of an equation, when substituted therein for x, will "satisfy" the equation. An equation which is true for all values of x, like $(x+1)^2 = x^2 + 2x + 1$, is called an **identity** [often written $(x+1)^2 \equiv x^2 + 2x + 1$].

Types of Equations.

(a) Algebraic Equations:

of the first degree (linear), *e.g.*, $2x + 6 = 0$ (root: $x = -3$);
of the second degree (quadratic), *e.g.*, $x^2 - 2x - 3 = 0$ (roots: $-1, 3$);
of the third degree (cubic), *e.g.*, $x^3 - 6x^2 + 5x + 12 = 0$ (roots: $-1, 3, 4$).

usually assumed to be 3000 Btu/(hp)(hr), although this is higher than the amount actually dissipated. The rate at which the heat is transferred will vary with the velocity of water through the cooler. With a velocity of 7 to 5 fps, an average rate of heat transfer of 275 Btu/(sq ft)(hr)(deg of mean temperature difference) is obtained. The amount of raw water used will be an amount equal to or exceeding by three times the amount of water to be cooled when using higher temperature differentials such as 25 to 30 deg. A formula that has been developed for calculating the amount of surface required for the heat exchanger is as follows:

$$\text{Surface, sq ft} = \frac{Q}{t_m \times C \times F}$$

where Q = heat injection, Btu per hr

t_m = log mean temperature difference between raw water and jacket water

U = a factor representing over-all rate of heat transfer

F = a foulage factor, 100 percent for clean tubes to 85 percent for foul tubes

The amount of raw water required in gallons per minute will be

$$\text{Gpm} = \frac{Q}{(t_2 - t_1) \times 500}$$

where t_1 = inlet temperature of raw water

t_2 = outlet temperature of raw water

LUBRICATING SYSTEM

There is probably no other factor in diesel-engine operation that has been given more study by designers than lubrication. This has resulted in the development of a standard arrangement of the lubricating system. The action of the lubricating oil and the chemistry involved in the production of oils suitable for lubrication, under the special conditions that exist in diesel engines in different types of service, are still the subject of widespread study and experiment. The application of lubricant to the rubbing surfaces in the engine is accomplished through the medium of two separate systems: one for the cylinders and one for the bearings.

Cylinder-lubricating System. The lubrication of the cylinder liner surface upon which the piston rings must travel and which is repeatedly exposed to the flaming gas in the cylinder presents a special problem requiring a method of application that makes it necessary to use equipment for this purpose separate from the equipment used in the lubrication of the rest of the engine. The oil must be applied in very small quantities, at frequent intervals, and must be distributed around the circumference of the liner so that it can be spread over the entire surface by the piston rings during their travel.

The mechanism used consists of a number of small pumps assembled in groups of 2 to 24 in units known as mechanical lubricators. Each of these small pumps is connected by means of small-diameter tubing to a fitting on the outside of the cylinder that passes through the jacket and connects with a hole in the wall of the liner, the number of such fittings and the correspond-

reverse direction with air, the camshaft remains stationary until the lost motion in the coupling is taken up, thus changing the angular position of the cams relative to the crankshaft.

Gear Reversing. When small engines of about 200 hp or less are used for propulsion, the need for reversing mechanisms is eliminated by the use of reverse gears, by means of which the direction of propeller rotation is reversed without stopping the engine or changing its direction of rotation. This arrangement permits the use of electric starting and eliminates the air compressor and its associated tanks and piping.

The reverse-gear unit is placed between the crankshaft and propeller shaft and comprises two trains of gears and clutches operated by a single lever. One train has a crankshaft gear and a propeller-shaft gear, with one intermediate gear, thus causing the propeller shaft to turn in the same direction as the crankshaft. The other has two intermediate gears, thus giving the propeller shaft rotation opposite to that of the crankshaft. Movement of the control lever in one direction engages the clutch connected to one train, while movement in the opposite direction engages the other train. Placing the lever in mid-position disengages both clutches, and the engine continues to run while the propeller shaft remains stationary.

In another type of gear the gear train is used only in the reverse direction. When running ahead, the entire unit is locked together and acts as a solid connection between crankshaft and propeller shaft. For use in driving a propeller with a high-speed engine, the reverse gear is combined with a reduction gear, the combined unit forming an integral part of the engine structure.

SUPERCHARGING

The first marine application of supercharging, in the form of an electrically driven blower that discharged air into the intake manifolds of two engines, gave results that led to the development of present-day methods and greatly widened the field of the four-cycle engine. The basic purpose of supercharging is to get more air into the engine cylinder during each charging period than is possible by natural aspiration, in order that more fuel may be burned, a higher mep obtained, and more power produced with a cylinder of given dimensions. The procedure has now been developed to the point where supercharging equipment is an integral part of the design of some makes of engines and is optional with many others.

Three methods of supercharging have been developed commercially: (1) the use of the bottom ends of the cylinders as air compressors that discharge the air into a receiver cast around the cylinders; (2) the use of a rotary or displacement type of blower driven by gears or chains from the engine crankshaft; (3) the use of the exhaust gases from the engine to drive a turbine connected to a blower that discharges air into the inlet manifold of the engine.

This last is the most widely used and appears to be in a fair way toward acceptance as the universally standard method. It is possible, however, that experimental work now being carried on will develop some radically new concepts of supercharging. An advantage of the exhaust-turbo method is that it operates on free power that would otherwise be wasted, yet does not rob the exhaust gas of its heat and prevent its use in heat-recovery apparatus.

Effect of Supercharging. Since the pressure at the end of compression, with any given compression ratio, depends upon the pressure at the beginning of compression, it follows that if air at 4 to 6 lb pressure is forced into a

ing number of pumps required for each cylinder varying from four to eight. The mechanical lubricator assembly is arranged so that the pumps are all connected to a common suction chamber which is kept filled with oil.

All the pumps are operated by cams driven by a shaft connected through gearing to some moving part of the engine. The amount of oil discharged by each pump can be regulated, but the discharge is at the rate of only a few drops per minute. The lubricators are always situated below the cylinder they serve, and the tubing connecting them with the inlet fittings on the cylinder is always filled with a solid column of oil, so that each time a pump discharges a drop of oil into its tubing a corresponding drop is forced out of the other end into the cylinder. The rate of feed is a matter that must be determined by experience, but in general it will vary between 7 to 15 drops per minute from each pump. The number of points at which oil is discharged into the cylinder is also a matter to be determined by cut-and-try methods. In general, the objective sought is to have them spaced so that the oil spreading sideways on the cylinder wall from each feed will be able to meet the oil from the adjacent feeds.

Bearing Lubrication System. The bearings on all present-day diesel engines—except crankcase-scavenged types—are lubricated by means of a pressure system in which oil is continuously circulated and used over and over. The principal elements in the system as shown in Fig. 10 are a storage tank from which make-up oil is fed

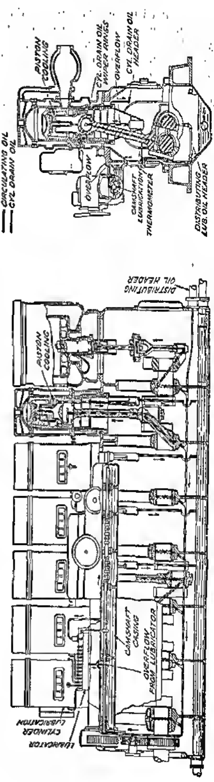


FIG. 10.—Typical lubricating system as used in Fairbanks-Morse engine.

cylinder that is designed for operation with atmospheric aspiration, the resulting compression pressure will be above normal. Consequently the supercharged engine may have a lower compression ratio than the unsupercharged engine and, if the supercharged engine is operated without the supercharger, the compression pressure will be below normal.

Reference to the diagram in Fig. 12, which shows an indicator card from a supercharged cylinder imposed on one from an unsupercharged cylinder, will show the effects obtained by supercharging. Since the greater weight of air in the cylinder permits the burning of more fuel, the combustion line will be higher; thus the mep will be greater although the maximum pressure will be the same, provided the compression ratio is properly adjusted. This means that the power obtained from the cylinder will be increased without any change in the dimensions of the engine. The practical application of this is to make possible the use of a smaller engine for a given propulsion power or provide more power for a given ship.

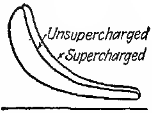


Fig. 12.—Indicator diagram showing effects of supercharging.

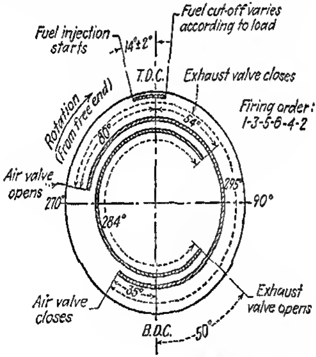


Fig. 13.—Valve diagram of Alco supercharged engine showing valve overlap.

The fact that more oil is burned in the cylinder does not, as might be expected, create a higher temperature in the cylinder or a higher temperature in the exhaust. Although more oil is burned, the ratio of oil to air is the same, owing to the presence of the additional air. Also the overlap of opening of inlet and exhaust valves allows air to blow through the combustion

into the system as required; a sump tank, which forms the reservoir from and to which the oil is circulated; a circulating pump, which forces the oil under pressure through the circulatory system on the engine; a heat exchanger for cooling the oil after it has passed through the engine; and some form of filter for cleaning the oil.

Circulation through the engine occurs as follows: The oil from the circulating pump is discharged into a fore-and-aft header which has a connection with each main bearing on the engine. These connections are sometimes made through the bearing cap and sometimes through the bottom half of the bearing, while the header may be a pipe or it may be a passage cored in the bedplate. The oil spreads out in the main bearings and, by way of a circumferential groove in each bearing that registers with a hole in the shaft, flows into the crankpin bearings. From the crankpin bearing it flows into a groove in the bearing shell, then flows upward through an axial passage in the connecting rod to the piston pin bearing. At each bearing some of the oil spreads out through the bearing, flows out of the end, and finally falls into the crankcase and thence to the sump.

Engines that require piston cooling have a connection between the crosshead or piston-pin bearing and the piston, arranged so that the oil flows from the piston-pin bearing up into the piston head, out through a connection in the piston, and thence into the crankcase.

There are a number of different forms of connections between the crosshead or wrist-pin bearings and the piston, but the function of all of them is the same. In some small engines the oil is jetted against the underside of the piston through a nozzle on top of the piston-pin bearing. The heat exchanger for cooling the oil is usually placed between the circulating pump and the distribution header on the engine, and is connected to the same raw-water pump that cools the jacket water. Current Navy practice, followed in some commercial installations, is to pass the jacket water through the oil-heat exchanger.

A necessary feature of an engine lubricated in this manner is a closed crankcase. With the oil being forced into the system under pressure and flowing out of each bearing, it is obvious that the rapidly moving cranks and connecting rods will dash the oil around violently and keep the crankcase filled with a heavy fog of oil. All openings in the crankcase must have oil-tight closures, otherwise there will be a serious loss of oil out of the system. As a matter of fact, the lack of tightness of the crankcase is a very common cause of loss of oil and resulting excessive oil consumption by the engine. It will also be evident that, since the cylinder of the trunk-piston type of engine is open to the crankcase at its lower end, there will be a large amount of oil thrown up onto the walls of the cylinder liner. This is the reason for the use of the scraper rings mentioned in connection with pistons.

The use of pressure lubrication has in modern engines been extended to include not only the principal bearings mentioned in the foregoing but also all the secondary bearings such as those for the camshaft. In many engines every moving part that requires lubrication is served by branches from the pressure system, and all hand oiling is eliminated.

Cleaning Lubricating Oil. The lubricating oil in its rapid passage through the engine is subjected to a number of deteriorating influences that cause the formation of carbon, gums, etc., in it. It also picks up any dirt or particles of metal that may be present in the engine. For this reason it is essential that a means be provided for cleaning the oil. There are two

space during the latter part of the exhaust period, thus scavenging the cylinder, cooling the piston and cylinder wall, cooling the gases, and taking away in the exhaust gases some of the heat that would otherwise be taken up by the cooling water. The manner in which the valve-timing overlap is produced is shown in the timing diagram of a supercharged four-cycle engine shown in Fig. 13.

The changes in operating conditions resulting from supercharging may be summed up as follows: The pressure at start of compression is higher, the compression pressure is the same, the mean pressure is higher, the exhaust pressure is higher, the temperature of combustion is the same, the mean temperature is higher, the exhaust gas temperature is lower, and less heat is carried away in the cooling water although the total heat is greater owing to higher hp output.

Exhaust Turbocharging. The elements in the exhaust turbocharging system are a gas-driven turbine, usually mounted on the engine, and a multiple-exhaust manifold for over three cylinders. The manifold must be arranged so that the exhaust valve of one cylinder is not opening when the inlet and exhaust valves of another cylinder are open.

A four-cylinder engine will require two pipes to the turbine arranged so that the cylinders on the same crank-phase angle are in one pipe. This would mean that, with the standard crank arrangement, cylinders 1 and 4 would have one pipe and numbers 2 and 3 the other pipe.

On six-cylinder engines with the standard arrangement of cranks, cylinders 1, 2, and 3 would have a common pipe and cylinders 4, 5, and 6 a common pipe.

An eight-cylinder engine with standard crank arrangements requires four pipes to the turbine. This is necessary because otherwise the scavenging effect of supercharging would be entirely lost. A centrifugal blower is mounted on the same shaft with the turbine and an air-inlet manifold connected to the discharge of the blower and to the air-inlet passage of the cylinder.

Multiple exhaust manifolds are used in order to take advantage of the fluctuating pressure of the gas, which varies from 2 to 7 lb. If all cylinders exhaust into a single manifold, the maximum and minimum pressures tend to neutralize each other, and a uniform pressure will exist in the manifold equal to or higher than that in the air-inlet manifold. With multiple manifolds the condition of fluctuating pressure can be maintained and the period of scavenging previously referred to, when the inlet and exhaust valves are both open, can be made to coincide with the time of lowest pressure in the exhaust passage. By this means positive scavenging and charging of the cylinder are obtained even though the average pressure of the gas in the exhaust manifold is about 5 lb and the pressure of the air in the inlet manifold may be as little as 4 lb. This will be understood by reference to Fig. 14.

It has been stated that the heat carried off by the cooling water is less in the supercharged engine. It should be noted that this applies only when sufficient overlap has been given to the exhaust and inlet valves to produce a pronounced scavenging effect. If an engine with normal valve setting is supercharged, it will be found that the heat carried off by the cooling water will increase directly with the horsepower, and the cooling water capacity must be increased accordingly.

At the present time about 500 hp seems to be the low limit on size of engine to which exhaust turbocharging can be economically applied. Engines below this size utilize built-in blowers driven by gears or chains, the blowers usually

general types of equipment used for this purpose: the centrifuge and the filter. At present there are only two types of centrifuge in use, and the principle on which they operate is the same. The dirty oil is run into a rapidly rotating bowl and, under the influence of centrifugal force, the foreign matter contained in the oil is separated and thrown outward against the walls of the bowl, and the clean oil flows out of the top of the bowl. This method of cleaning is very successful in removing all foreign matter held in suspension, but it will not remove collodial carbon or correct acidity (see p. 1310).

The other method of cleaning involves the use of filters, of which there are many different types, although the basic principle of all is the same. This involves forcing the oil through passages so minute that the suspended impurities cannot get through with the oil. These minute passages are provided by various types of material that are used for filtering. These include cotton waste, cloth bags, cellulose, and fuller's earth. Other types use metal or paper disks and other arrangements that are assembled in such a way as to provide minute passages between the disks for the flow of oil and the interception of foreign matter.

Various arrangements of the filtering equipment are used, and the methods of operation include batch cleaning and continuous by-pass cleaning. Theoretically, the best procedure would be to pass all of the oil through filters on its way to the engine, but this would involve filtering equipment of very great capacity. In the batch method of cleaning, all or a large part of the oil in the system is removed periodically for cleaning and replaced with new oil. After cleaning, the used oil is stored in the clean oil tank to be used as replacement oil. By this method the engine operates for a time with very clean oil, but as time goes on the oil gets progressively dirtier and may become extremely so before the next cleaning period.

The method most commonly used is continuously to by-pass a part of the oil through the filters and back to the system. Thus, the engine is always operating on a mixture of clean and dirty oil.

The amount of oil that must be circulated in the system is an indeterminate quantity, and there seems to be no generally accepted ratio of oil-pump capacity to engine power. Consequently wide variations in the quantity of oil circulated are found between different installations. As an example, it is found that in one 2,000-hp installation the oil-circulating pump has a capacity of 50 gpm, while in another installation of 950 hp a pump capacity of 91 gpm is provided. This is a situation that need not cause concern because it has little effect on design, layout, or operation of the plant. The oil in a forced-feed system circulates continuously in the system, regardless of what storage-tank capacity may be installed. Once filled, a system that holds 1,000 gal requires no more reserve storage capacity than one that holds 500 gal.

Lubricating-oil Consumption. The amount of oil circulated has little relation to oil consumed by the engine. Theoretically the only lubricating oil required by the engine should be that fed into the cylinders, and the oil circulated in the bearing lubrication system should last indefinitely. Actually there is some oil lost out of the system by being drawn up into the cylinders and burned, and considerably more is lost by leakage. In some designs this leakage is prevented by maintaining slightly less than atmospheric pressure in the crankcase.

Oil consumption of the engine is the amount used for cylinder lubrication, plus the amount of bearing oil burned or lost by leakage. It may be expressed

being of the Roots type, preferably with helical vanes. Development work is being done with small turbochargers, and it is expected that in the future it will be possible to apply them to much smaller engines.

An arrangement that is used in some recently built large diesel tugs employs independent supercharging blowers driven by electric motors. In order to prevent waste of power by the blower when the engine is operating at reduced

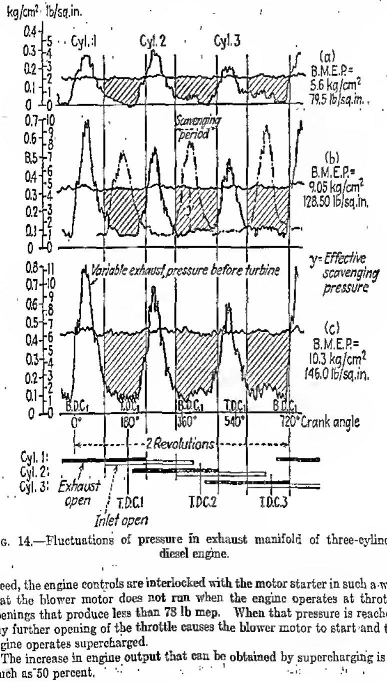


FIG. 14.—Fluctuations of pressure in exhaust manifold of three-cylinder diesel engine.

speed, the engine controls are interlocked with the motor starter in such a way that the blower motor does not run when the engine operates at throttle openings that produce less than 78 lb mep. When that pressure is reached, any further opening of the throttle causes the blower motor to start and the engine operates supercharged.

The increase in engine output that can be obtained by supercharging is as much as 50 percent.

in several ways, but the simplest and most easily visualized method is to state it in terms of rated horsepower-hours per gallon of oil. On this basis it will be found that the consumption will cover a wide range, from 1,500 to 10,000 hp-hr per gal. In general, the trunk-piston engine will use more than the crosshead engine, the high-speed engine will use more than the slow-speed engine, and the two-cycle engine will use more than the four-cycle engine. Aside from differences due to engine type, the condition of the engine and the quality of attendance will have a marked influence on consumption.

AIR SYSTEM

In addition to the air compression that is performed in the working cylinders of the engine and constitutes a phase in the diesel cycle, compression of air is required for the purpose of starting the engine, for the scavenging of the two-cycle engine and, in the case of the air-injection engine, for injecting fuel into the cylinders. Air systems may then be discussed under three divisions: injection system, scavenging system, and starting system.

Injection System. As explained previously, there are still a number of air-injection engines in service, and the air compressor constitutes a very important part of them. As pointed out, the fuel is injected into the cylinders of such an engine by air that must be at a pressure sufficiently high to project the fuel vigorously into a body of air in the combustion chamber that is compressed to about 500 lb. To do this the injection air must be compressed to about 900 to 1,200 lb.

Perfectly satisfactory operation can be obtained if the air compressor is independently driven, either by an electric motor or an auxiliary diesel engine, but by building the compressor into the engine and driving it from an extra crank on the engine shaft, operation is made much more convenient. In this case the compressor runs only when the engine runs and changes speed as the engine speed changes. Consequently it became standard practice to build the compressor into the engine and provide a separate independently driven compressor as a stand-by and for starting air. On some of the older ships an independent compressor was used to give the air its first stage of compression, the remaining stages being carried out by the attached compressor. The air may be taken from the engine room, from the outside, or from both. If all of it is taken from the engine room, the engine-room temperature may at times be reduced too much for the crew's comfort. Provision is frequently made to regulate the amount taken from either source by a butterfly valve in each intake pipe.

In order to reduce the amount of work required to compress the air, it is customary to carry out the compression in three stages and pass the air through coolers after each stage of compression. Pressures in the three stages are approximately in the ratio of 60: 240: 900 lb. The compressor is driven from a crank on the forward end of the crankshaft, and the piston is made as a single unit of three different diameters, with cylinder diameters to match. The cylinder bores are usually arranged so that the second-stage section is on the bottom, next to the crankcase. This prevents oil fog from the crankcase being drawn into the cylinder, as would be the case if the first stage were on the bottom, and also gives a better distribution of load.

Air from the first-stage cylinder passes from the discharge valves into an intercooler and, after being cooled, is drawn into the second-stage cylinder. After the second compression it is passed through a second intercooler and into the third-stage cylinder. After, this third compression it is passed

METHODS OF APPLICATION

Two general methods of application of the diesel engine to marine propulsion are in use: (1) direct drive, with one or more propellers driven by engines coupled directly to the propeller shafts, and (2) indirect drive, with one or more engines driving the propeller shaft through reduction gears or through the medium of electric motors supplied with current from generators driven by the engines.

A further variation in the methods of application is found in the number of screws used. In the early days of diesel propulsion, the use of twin screws was considered necessary in order to permit the use of propellers that could operate at the high speeds at which diesel engines were run and to provide a factor of safety by having power still available even if one engine should break down. Development of slower speed engines and improved propeller design in time eliminated the first condition and the demonstration of reliability in operation equal or superior to other types of propulsion machinery made multiple engines unnecessary from a safety standpoint.

In general it may be said that where weight and space are not controlling factors, direct drive is simpler and in most cases cheaper but, if space is limited and saving of weight is important, indirect drive is preferable.

Direct Drive. In considering direct-driven propellers there are several generalizations that apply. For the average vessel the rpm should be in inverse ratio to the horsepower, also the higher the speed for which a vessel is designed the higher the revolutions that can be used with a given power. Where for any reason a comparatively small propeller must be used, as on boats operating in shallow rivers or canals, the propeller speed will usually be near enough to the designed engine speed to call for direct drive.

With direct drive the usual arrangement is to locate the engine approximately amidship unless the type of service or the kind of cargo carried, such as in oil tankers, makes it more advantageous to place it in the extreme stern. Aft of the engine is a thrust bearing which prevents the linear thrust from the propeller from reaching the engine. In most large installations the thrust bearing is independent of the engine and mounted on a separate foundation, but in many of the smaller sizes it is attached to the bedplate and forms a part of the engine assembly. The length of intermediate shaft between the thrust bearing and the propeller shaft will of course vary with the location of the engine; in the case of tankers it disappears altogether, the propeller shaft being connected directly to the thrust bearing.

GEAR DRIVES

Although propulsion by direct drive has proved thoroughly satisfactory from the operating standpoint, there has been a distinct reversal of trend in regard to engine rotative speed and a growing acceptance of higher speed; but, in order to realize the advantages of reduced cost, weight, and space associated with the high-speed engine, indirect drive of the propeller must be resorted to. Two methods of indirect drive are in use: one utilizes reduction gears between the engine and the propeller shaft and the other uses the engine to drive a generator which supplies current to a propulsion motor on the propeller shaft.

Gear Arrangements. The main elements in a reduction gear are a large gear, known as a bull wheel, mounted on the propeller shaft; one or two

through an aftercooler and thence to the storage bottles and the injection valves.

The elements in the system are the compressor, the storage bottles, and the high-pressure piping leading to the injection valves. The capacity of the compressor is determined by the volume of the first-stage cylinder and is expressed in terms of cubic feet of free air per minute. In practice there is a wide variation among different builders in regard to capacity. It has been found that this variation in existing installations is from 0.2 to 0.35 *cu ft free air per min per bhp.*

The air bottles in which the air is stored are usually two in number and only one is used at a time, the other being shut off and kept in reserve. The one bottle that is in use serves as a surge tank to prevent fluctuations of pressure at the injection valves. The capacity of the air bottles should be based on the stroke volume of the engine cylinders. In practice, it is found that this capacity varies from fourteen times the stroke volume for small engines to one to two times the stroke volume for engines with cylinders 22 in. or more in diameter.

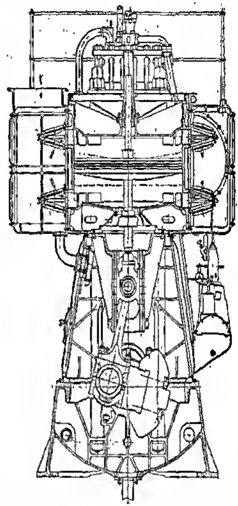


Fig. 11.—Engine-driven double-acting scavenging pump as used on Nordberg engine.

Scavenging Air System. As previously explained, the operation of the two-cycle engine requires the use of low-pressure air to blow the burned gases out of the cylinders. The compressor used to supply this air may be either a reciprocating pump or a rotary blower and is of light construction, owing to the fact that the pressure required is between 1 and 3 lb only. It should have a capacity sufficient to give a ratio of scavenging air volume to power cylinder volume of about 1.5 :1, and never more than 2: 1.

When the reciprocating type of scavenging pump is used, it is built into the engine, as shown in Fig. 11, is driven by an extra crank on the crankshaft, and is usually double-acting. The air is discharged into a receiver which in most cases consists of a fore-and-aft manifold bolted to the cylinders. In some cases a part of the engine frame or housing is closed in to form a receiver.

pinion gears, to which the engine or engines are connected; and the necessary bearings and lubrication equipment. These elements are assembled as a complete unit within its own casing. The ratio of reduction is usually about 1:2 and never exceeds 1:6. With a unit of this sort one, two, or four engines may be used. Three-engine operation is never resorted to except when one engine of a four-engine installation may be shut down temporarily.

For a one-engine installation the pinion may be in either the vertical or the horizontal plane passing through the center of the bull-wheel shaft. With the pinion in the vertical plane, the engine shaft must be at a height above the center line of the propeller shaft equal to the radius of the bull wheel, plus the radius of the pinion. It is obvious that this arrangement is suitable only for an engine of low over-all height. With the pinion in the horizontal plane, the engine may be placed low in the ship, but it will be displaced to one side of the centerline and its weight must be compensated for by placing auxiliary machinery on the other side.

Two-engine and four-engine installations permit symmetrical arrangement of engines and gears. Two pinions are used in each case, with one engine connected to each or two engines connected to each, according to whether two of four engines are used. In the two-engine installation the gear set may be located either aft or forward of the engines. In the latter case the propeller shaft leads aft between the two engines. This is an arrangement well suited to tankers or other bulk-cargo ships in which the engine room is located in the extreme stern. With the four-engine arrangement, the engines are located at the four corners of a rectangle with the gear set in the center. The schematic arrangement of a two-engine gear drive is shown in Fig. 15.

Gear Forms. A factor that contributed more than any other to the success of reduction gears was the development of methods for accurately cutting herringbone teeth. The helical form of tooth with the gradual engagement of each succeeding pair of teeth produces noiseless operation and also creates an axial thrust tending to jam the teeth together. In the double-helical or herringbone form the thrust acts in equal and opposite directions. Consequently most reduction gears are made with either continuous or interrupted herringbone teeth. In the former each tooth is in the form of a closed V, while in the latter the V is open at the point.

Although the thrust due to tooth action is thus automatically eliminated, provision must be made to prevent the thrust of the propeller from reaching the gears. To accomplish this a thrust bearing is mounted on the propeller shaft, and it is customary to build this thrust bearing into the gear set as a part of the complete unit.

In order to prevent any possibility of binding or unequal wear of the gear teeth, the pinion shaft is given a small amount of float so that the teeth can center themselves. In some cases a certain amount of flexibility is given to the pinion shaft by making it in the form of a quill shaft. Complete closure of the casing that houses the gears permits the use of pressure lubrication of the gears and bearings. It is customary to provide the gear set with its own circulating pump, cooler, filter, and necessary piping.

Couplings. Although some early reduction-gear installations were made with rigid connections between the engines and the pinion gears, all present-day installations are made with some form of coupling that will prevent the transmission of harmful vibration to the gears. The first form of vibration absorber was a quill-shaft drive. This was followed by several different types of mechanical flexible couplings, until the advent of the hydraulic coupling

Of late years there has been a decided trend toward the use of rotary blowers as scavenging air pumps. These blowers are of the Roots type and may be mounted on the engine and driven by gears, V-belts, or chains from the crankshaft, or they may be arranged for independent drive by electric motors. When attached blowers are used, some type of flexible or fluid coupling should be used in the drive in order to prevent damage by synchronous vibration or by inertia forces due to stopping and starting.

Crankcase Scavenging. In some types of engines the bottom ends of the pistons are used as scavenging pumps. The crankcase is made airtight and provided with suction valves. Each piston on the up stroke creates a partial vacuum in the crankcase and air is drawn in. On the downstroke this air is slightly compressed and, when the piston uncovers the scavenging ports, the air blows into the cylinder through a passage connecting the crankcase with the ports. With this arrangement the ratio of scavenging volume to power-cylinder volume can never be more than 1:1, and the mep in the cylinders is necessarily much less than when a higher scavenging ratio is used.

Starting Air System. Although all small engines and some fairly large ones used in connection with electric drive are started by electric motors, the method almost universally used for starting engines of more than 200 hp is by admitting air at 250 to 400 lb pressure to the cylinders through timed valves. Air for this purpose is provided by a compressor which discharges the air to tanks located at any convenient point in the engine room and connected to the starting valves in the cylinder heads.

This compressor is similar in construction to the injection-air compressor previously discussed, except that it is of two-stage construction because of the lower pressure to be handled. In some cases the compressor is built into the engine but driven by gears, chains, or V-belts; but in most installations, especially of large engines, it is independently operated, usually by an electric motor but in some cases by a separate auxiliary engine. Since the maintenance of an adequate supply of starting air is vitally essential, the air compressors should be installed in duplicate. If an attached compressor is used, a small independent unit should be provided; if no attached compressor is used, two and sometimes three units are installed.

In order to eliminate the possibility of the compressor's not being started in time during extended periods of maneuvering, automatic controls should be installed that will start and stop the compressors in accordance with pressure changes in the air tanks.

An excellent arrangement used on some U.S. Maritime Commission motorships is to provide three motor-driven compressors, two of which are of 48 cfm capacity each at 390 lb pressure, while the third is a make-up unit of 10 cfm capacity. Automatic controls are arranged so that when the pressure in the air tanks falls to 375 lb the small compressor starts; if the pressure falls to 350 lb one large compressor will start and the make-up compressor will stop; if the pressure falls to 300 lb, the second large compressor will start. All compressors will stop when the pressure in the tank reached 390 lb.

Air Starting Methods. Although every air starting system uses timed valves in the cylinder heads for admitting air into the cylinders at the right point in the cycle, there are several different methods in use for operating these valves. In every case the air main leading from the tanks is connected to each starting valve, but air is turned into these lines only during the times when the valves are in operation.

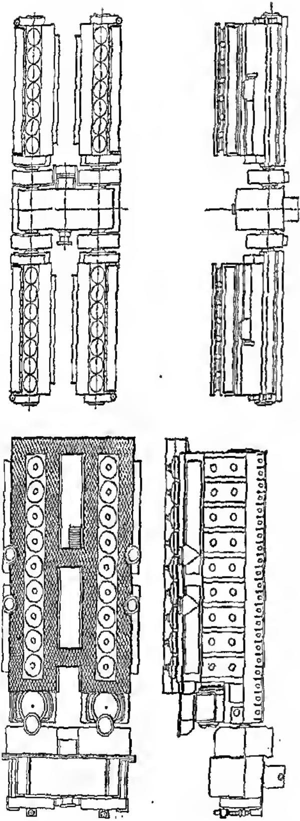


FIG. 15.—Typical arrangements of two-engine and four-engine gear drives. (*From D.E.M.A. Marine Diesel Standards.*)

In one arrangement the valves are operated by cams and levers, the levers being mounted on eccentric fulcrum bearings. Normally the rollers are held up clear of the cams but, when the eccentric bearings are rotated by the control gear, which at the same time operates to open the valve that admits air to the lines leading to the cylinders, the rollers drop onto the cams and the valves open and close in accordance with their timing under the influence of the cams and valve levers.

In another arrangement the starting valves in the heads are simply spring-loaded valves that open when air pressure in the valve body exceeds the spring pressure. Air is admitted to the valves at the right times by cam-operated control valves in a control unit at the operating station. In other designs the starting valve is operated by a piston in a cylinder incorporated in the valve body. Air is admitted to this cylinder through a small pipe line leading to a timed pilot valve in the control unit at the operating station. In every case the maneuvering gear is arranged so that air from the tanks is automatically turned on when the starting controls are moved to the start position, and cut off when they are moved to the run or stop positions.

Starting Air Capacity. The classification societies have established a requirement that diesel-powered vessels must have starting air-storage capacity sufficient for 12 consecutive starts of each engine. This is not an accurately measurable quantity because there are highly variable factors, such as temperature of the cylinders, tightness of rings, and skill of the operator, which have an influence on the amount of air required for each start.

As a starting point for a more definite evaluation, the amount of air-storage capacity should be related to the swept volume of the power cylinders. A ratio that is practical and will meet the requirement mentioned above is a storage capacity equal to 35 times the swept volume of one cylinder. With this as a basis the operating factors that have an influence must be considered. A large ship that normally executes but a few maneuvers, as when entering or leaving port, will require a much lower capacity ratio than a tug, which may often require a large number of starts when maneuvering in close quarters. Also a nonreversing engine used in connection with electric drive or reverse gears will require much less air capacity then a direct-connected reversing engine. In general the requirements for each different class of service are best determined on the basis of experience.

Air-compressor Capacity. The air-compressor capacity is also an indefinite quantity that has a relation to type of service. The controlling factor should be the length of time decided upon for the compressor to be able to fill the air tanks. The compressor capacity may then be determined by the following formula:

$$\text{cfm} = \frac{V \times p_t}{p_a T}$$

where V = volume of tanks, cu ft

p_t = pressure (absolute) to which tanks are to be charged

T = time allowed for charging, min

p_a = atmospheric pressure

Power Required for Compressors. The engine power absorbed by attached compressors, or the motor power required for independent drive, will vary with the pressure, number of stages, and free air capacity. The

and later the electromagnetic coupling. It is now common practice to use one or the other of these two latter types in reduction-gear-drives.

The advantage of both of these types of coupling lies in the fact that there is no mechanical connection between the engine and the gears it drives, and not only are the gears protected against engine vibration but also the engine is protected against sudden shocks due to the propeller's striking an obstruction.

Hydraulic Coupling. The principal elements in the hydraulic coupling, as shown in Fig. 16, are the impeller and the runner, which are two radially

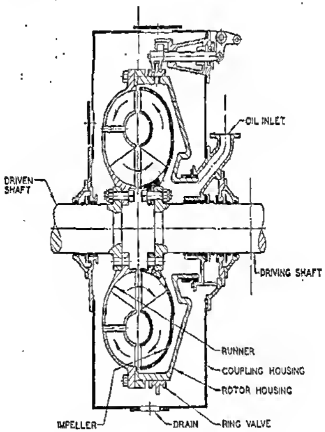


Fig. 16.—Section through hydraulic coupling.

vaned members with a concentric semicircular trough in each. When the two members are placed face to face, the outer shells and inner troughs form two concentric rings with an annular space between them. Both the runner and the impeller have an unequal number of radial vanes.

The impeller is mounted on the engine shaft and the runner on the pinion shaft, facing but not quite touching. A housing bolted to the runner encloses the back of the impeller and, by fitting closely around the driving shaft, forms a container for the working fluid, a mineral oil of 180 to 200 Saybolt viscosity. The entire unit is enclosed within an outer casing that forms a part of the fixed structure and is made oiltight by labyrinth packing where the shafts go through it.

horsepower theoretically required to compress 1 cu ft air under various conditions is shown in Table 1. Since the values given are for an assumed compression efficiency of 100 percent, whereas the actual efficiency is about 65 percent, these values should be multiplied by 1.54.

REVERSING SYSTEMS

Every engine connected to a propeller shaft should be provided with a means of reversing the direction of engine rotation or have a reverse gear interposed between the engine and the propeller shaft, in order to give the necessary backing motion to the propeller. It has been explained how the engine may be started by either air or electric motor but, if the engine must be started in either direction, only air starting is used. The basic requirement for reversing is that before starting the engine the valve-timing mechanism must be positioned to give the valves the correct timing for the desired direction of rotation. Since the valves are always operated by cams that have a definite angular relation to the crankshaft, the change of timing for changing the direction of rotation is accomplished by changing this angular relation.

Four-cycle Reversing. The method generally used for reversing the four-cycle engine is to provide two identical sets of cams on the camshaft, one set positioned to time the valves for ahead running and the other for astern. With the engine running in the ahead direction and the cam rollers resting on the ahead cams, reversal is accomplished by stopping the engine, shifting the camshaft in the direction of its length to bring the astern cams under the rollers, then starting the engine with air, after which rotation continues in the opposite direction in accordance with the changed timing of the valves.

Shifting of the camshaft is accomplished by one of several different methods, the most commonly used being by means of an air- or hydraulically-operated piston linked to the camshaft. This operation must be preceded by lifting the cam rollers off the cams, followed by lowering them onto the cams again after the shaft is moved, unless the cams are made with beveled edges that will permit the rollers to ride up on them.

The mechanisms involved are so linked together that the operations of raising the rollers, shifting the shaft, and lowering the rollers are executed by the movement of a single lever or handwheel at the control station. In some designs the starting air control and fuel control are also connected to this lever or handwheel, and the entire sequence of operations required for stopping, reversing, starting, and fuel admission is thus carried out by movement of the one lever or handwheel.

Two-cycle Reversing. In the case of the port-scavenged two-cycle engine the only timing that has to be changed for reversal is that of the fuel-injection pumps and air starting valves. If, however, exhaust valves are used, the timing of these also may have to be changed. In most two-cycle designs of the first type the air starting controls are separate from those for the fuel pumps, although they may be operated by the same control lever.

In this case it is necessary only to rotate the camshaft relative to the crankshaft about 30 deg, and the same cams used for ahead running are then in a position for astern timing. This relative rotation may be obtained by using an air-operated piston to slide a helical gear or turn one of the gears in the camshaft gear train, while the engine is stationary, enough to give the desired camshaft movement, or a lost-motion coupling may be used in the camshaft drive. In this latter case, when the engine is stopped and then started in the

An inlet pipe mounted on the casing, with a sliding fit on the inner housing, serves for admitting oil to the housing. Ports in the periphery of the housing, covered by a ring valve, serve for emptying the coupling rapidly when desired. This ring valve is interlocked with the inlet valve so that when one is open the other is closed.

With the impeller and runner assembled face to face, the vanes in the two form a continuous spiral passage around the central concentric ring. If the impeller is rotated with the coupling full of oil, centrifugal force causes the oil to flow radially outward, as shown by the arrows in Fig. 16, across the gap into the runner, then inward toward the lower part of the runner, and across the bottom gap into the impeller. Since the passage formed by the relative positions of the vanes is spiral, this movement causes the oil to travel continuously in a spiral path around the circumference of the coupling.

The kinetic energy generated as the oil flows outward and gains rotational velocity about the axis of the coupling, is released when the oil turns inward in the runner, and flows toward the axis of the coupling and its velocity is decreased. This energy is the force that causes the runner to turn. The torque input is equal to the torque output, and the power lost is represented by the slip between the impeller and the runner. Thus the efficiency of the coupling is the ratio between the speeds of the driving and the driven shafts.

In order to maintain a condition of difference in centrifugal force in the two members that will cause the oil to circulate as described, the coupling for any given power and speed must be selected so that there will be a difference of 3 percent between the speeds of the impeller and runner.

In addition to transmitting the engine power to the gears without vibration or torque fluctuation, the hydraulic coupling permits quick and easy disconnection of one engine without stopping the others, simply by dumping the oil. This is done by means of the peripheral ports and ring valve previously mentioned. If the ring valve is moved by the controls to uncover the ports, the inlet valve is closed and the centrifugal force throws the oil out through the ports, thus bringing the driven member to rest even though the driver continues to turn. On installations where it is not necessary to dump the oil in order to disconnect the engine while running, another type of hydraulic coupling, known as the "traction" coupling, is popular since it is cheap to purchase and install. This coupling has exactly the same driving members as the dump coupling but does not have the filling or dumping features with the coolers, pumps, and piping. The coupling must be filled through a plug with the engine at rest and dumped in the same manner if it is desired to disconnect the engine. It is calculated to have enough cooling surface itself to keep the oil within operating limits.

Electromagnetic Coupling. The electromagnetic coupling, or electric coupling, as it is commonly called, offers another method of driving the reduction gears without any mechanical connection between the engine shaft and the pinion shaft. In construction it resembles an induction motor, having an inner member or armature mounted on the engine shaft and an outer member, carrying salient field poles, mounted on the pinion shaft. The inner member is positioned inside the circle of field poles so that there is an air gap of 0.2 to 0.4 in. between its periphery and the faces of the field poles, as shown schematically in Fig. 17. The inner member has a squirrel-cage winding similar to an induction-motor armature. When the field of the outer member is excited by direct current through collector rings and brushes, a strong magnetic flux is set up which causes the inner member to rotate with the field at the speed of the field, less the slip, which amounts to 1.3 to 1.5 per cent.

Table 1. Power Required to Drive Compressors

Gage pressure	Ratio of compression	Weight of 1 cu ft at 62 F	Single stage: Mep	Single stage: Final temp, deg F	Single stage: Horsepower to compress 1 cu ft free air per min	Two stage: Ratio of compression in each stage	Two stage: Final temp in each stage, deg F	Two stage: Horsepower, to compress of 1 cu ft free air per min	Three stage: Ratio of compression in each stage	Three stage: Final temp in each stage, deg F	Three stage: Horsepower to compress 1 cu ft free air per min
5	1.34	.1020	4.50	108	.0197						
10	1.68	.1279	8.30	144	.0362						
15	2.02	.1537	11.51	177	.0045						
20	2.36	.1796	14.40	207	.0628						
25	2.70	.2055	17.00	235	.0742						
30	3.04	.2313	19.40	259	.0845						
35	3.58	.2572	21.65	280	.0944						
40	3.72	.2831	23.60	303	.1030						
45	4.06	.3090	25.50	321	.1112						
50	4.40	.5548	27.50	341	.1195	2.10	180	.1065			
55	4.74	.3607	29.10	358	.1268	2.17	189	.1123			
60	5.08	.3866	30.75	373	.1339	2.25	196	.1184			
65	5.42	.4124	32.30	392	.1408	2.33	200	.1235			
70	5.76	.4383	53.80	405	.1472	2.40	207	.1286			
75	6.10	.4642	55.18	420	.1532	2.47	214	.1329			
80	6.44	.4901	36.55	434	.1590	2 54	222	.1372			
85	6.78	.5159	37.90	447	.1650	2.60	227	.1410			
90	7.12	.5418	39.10	461	.1705	2.67	233	.1462			
95	7.46	.5676	40.35	473	.1758	2.73	238	.1500			
100	7.80	.5935	41.65	485	.1812	2.79	242	.1542			
105	8.14	.6194	42.30	497	.1841	2.85	246	.1578			
110	8.48	.6453	43.75	508	.1908	2.90	251	.1615			
115	8.82	.6712	45.16	519	.1965	2 99	256	.1648			
120	9.16	.6971	46.00	530	.2008	3.02	259	.1681			
125	9.50	.7230	47.05	540	.2045	3.08	262	.1710			
130	9.84	.7488	47.80	550	.2085	3.14	266	.1740			
135	10.18	.7747	48.85	560	.2135	3 19	269	.1775			
140	10.52	.8005	49.90	569	.2176	5.24	272	.1810			
145	10.86	.8264	51.00	578	.2220	3.79	276	.1837			
150	11.20	.8522	51.70	587	.2255	3.35	280	.1865			
200	14.6	1.1110	.....	...	.....	3.82	308	.2110	2.44	215	.1965
250	18.0	1.3697	.....	...	.....	4.24	332	.2315	2.62	226	.2140
300	21.4	1.6285	.....	...	.....	4.63	353	.2490	2.78	241	.2295
350	24.8	1.8872	.....	...	.....	4 98	370	.2640	2 92	251	.2418
400	28.2	2.1459	.....	...	.....	5.31	386	.2770	3.04	259	.2535
450	31.6	2.4048	.....	...	.....	5.61	399	.2895	3.16	267	.2630
500	35.0	2.6634	.....	...	.....	5.91	412	.2915	3.27	275	.2730
550	38.4	2.9221	.....	...	.....	....	...	.....	3.37	281	.2830
600	41.8	3.1810	.....	...	.....	....	...	.....	3.47	287	.2910
650	45.2	3.4395	.....	...	.....	....	...	.....	3 56	292	.2960
700	48.6	3.6982	.....	...	.....	....	...	.....	3.64	297	.3025
750	52.0	3.9570	.....	...	.....	...	...	.....	3.73	302	.3090
800	55 4	4.2155	.....	...	.....	....	...	.....	3.80	307	.3150
850	58.8	4.4745	.....	...	.....	....	...	.....	3.83	312	.3210
900	62.2	4.7330	.....	...	.....	....	...	.....	3.96	316	.3260
950	65.6	4.9920	.....	...	.....	....	...	.....	4.03	320	.3315
1000	69.0	5.2510	.....	...	.....	....	...	.....	4.10	324	.3360

The power required for excitation amounts to about 1 percent of the power transmitted through the coupling and the maximum torque that can be transmitted by the coupling is about 200 percent of normal. If the load on the gears and the propeller shaft for any reason exceeds this amount, the coupling will fall out of step and the load on the engine will be reduced. Also, the torque transmitted has no relation to the speed of rotation of the unit as a whole but depends upon the relative speeds of the two parts of the coupling; hence the coupling can transmit the full torque exerted by the engine even when the engine is operating at slow speed.

As in the case of the hydraulic coupling, there is no mechanical connection between the engine shaft and the pinion shaft; therefore pulsations of torque and critical vibrations do not reach the gears, and sudden shocks caused by the propeller striking an obstruction do not reach the engine crankshaft. Also, any one engine can be disconnected instantaneously from the gear set simply by opening a switch in the excitation circuit.

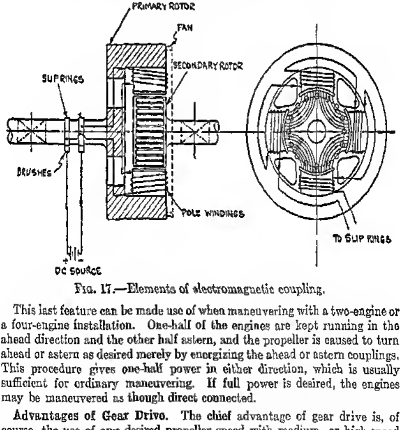


FIG. 17.—Elements of electromagnetic coupling.

This last feature can be made use of when maneuvering with a two-engine or a four-engine installation. One-half of the engines are kept running in the ahead direction and the other half astern, and the propeller is caused to turn ahead or astern as desired merely by energizing the ahead or astern couplings. This procedure gives one-half power in either direction, which is usually sufficient for ordinary maneuvering. If full power is desired, the engines may be maneuvered as though direct connected.

Advantages of Gear Drive. The chief advantage of gear drive is, of course, the use of any desired propeller speed with medium- or high-speed engines. There are incidental advantages such as less headroom required, and the facility with which repairs or examination of one engine at a time can be carried out at sea without stopping the ship. Also, the use of smaller and lighter engines reduces the size of parts to be handled and provides greater flexibility. The chief disadvantage of gear drive is the greater cost of the installation as compared with a direct-drive plant of the same power, except when very low propeller speeds are used.

(*b*) Transcendental Equations:
exponential equations, *e.g.*, $2^x = 32$ (root: $x = 5$); $2^x = -32$ (no root); trigonometric equations, *e.g.*, $10 \sin x - \sin 3x = 4$ (roots: 30°, 150°).

Legitimate Operations on Equations. An equation which is true for a particular value of x will remain true for that value of x after any one of the following operations is performed:

Adding any quantity to both sides; subtracting any quantity from both sides; transposing any term from one side to the other, provided its sign be changed; multiplying or dividing both sides by any quantity which is not zero; changing the signs of all the terms; raising both sides to any positive integral power; extracting any odd root of both sides; extracting any even root of both sides, provided the $\pm$ sign is used; taking the logarithms of both sides (both sides being positive); taking the sin, cos, tan, etc., of both sides.

Notice, however, that the new equation obtained by some of these operations may possess "additional roots" which did not belong to the original equation. This occurs especially when both sides are squared; thus, $x = -2$ has only one root, namely, -2; but $x^2 = 4$, obtained by squaring, has not only the root -2 but also another root, $+2$.

Equations of the First Degree (Linear Equations). Solution: Collect all the terms involving x on one side of the equation, thus: $ax = b$, where a and b are known numbers. Then divide through by the coefficient of x, obtaining $x = b/a$ as the root.

Equations of the Second Degree (Quadratic Equations). Solution: Throw the equation into the standard form $ax^2 + bx + c = 0$. Then the two roots are:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad\qquad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The roots are real-and-distinct, coincident, or imaginary, according as $b^2 - 4ac$ is positive, zero, or negative. The sum of the roots is $x_1 + x_2 = -b/a$; the product of the roots is $x_1x_2 = c/a$.

GRAPHICAL SOLUTION. Write the equation in the form $x^2 = px + q$, and plot the parabola $y_1 = x^2$, and the straight line $y_2 = px + q$. The abscissæ of the points of intersection will be the roots of the equation. If the line does not cut the parabola, the roots are imaginary.

Equations of the Third Degree with Term in x^2 Absent. Solution: After dividing through by the coefficient of x^3, any equation of this type can be written $x^3 = Ax + B$. Let $p = A/3$ and $q = B/2$. The general solution is as follows:

Case 1. $q^2 - p^3$ positive. One root is real, namely

$$x_1 = \sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}}$$

the other two roots are imaginary.

Case 2. $q^2 - p^3 =$ zero. Three roots real, but two of them equal.

$$x_1 = 2\sqrt[3]{q},\ x_2 = -\sqrt[3]{q},\ x_3 = -\sqrt[3]{q}$$

Case 3. $q^2 - p^3$ negative. All three roots real and distinct. Determine an angle u between 0 and 180°, such that $\cos u = q/(p\sqrt{p})$. Then

$$x_1 = 2\sqrt{p}\cos(u/3),\ x_2 = 2\sqrt{p}\cos(u/3 + 120°),\ x_3 = 2\sqrt{p}\cos(u/3 + 240°).$$

GRAPHICAL SOLUTION. Plot the curve $y_1 = x^3$, and the straight line $y_2 = Ax + B$. The abscissæ of the points of intersection will be the roots of the equation.

Equations of the Third Degree (General Case). Solution: The general cubic equation, after dividing through by the coefficient of the highest

power, may be written $x^3 + ax^2 + bx + c = 0$. To get rid of the term in x^2, let $x = x_1 - a/3$. The equation then becomes $x_1{}^3 = Ax_1 + B$, where $A = 3(a/3)^2 - b$, and $B = -2(a/3)^3 + b(a/3) - c$. Solve this equation for x_1, by the method above, and then find x itself from $x = x_1 - (a/3)$.

GRAPHICAL SOLUTION. Without getting rid of the term in x^2, write the equation in the form $x^3 = -a[x + (b/2a)]^2 + [a(b/2a)^2 - c]$, and solve by the graphical method.

General Properties of Algebraic Equations. An algebraic equation of the nth degree in x is an equation of the type

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$$

where the a's are any given numbers (a_0 not zero), the expression on the left being called a **polynomial** of the nth degree in x. Such an equation will, in general, have n roots; but some of these n roots may be equal, and some may be imaginary. **Imaginary roots** always occur in pairs.

If the equation is written in the form: (a polynomial in x) $= 0$, then (1) *if a is a root of the equation, $x - a$ is a factor of the polynomial; (2) if the* polynomial can be factored in the form $(x - p)(x - q)(x - r) \dots = 0$, each of the quantities $p, q, r, \dots$ is a root of the equation; (3) if x is very large (either positive or negative), the higher powers of x are the most important; (4) if x is very small, the higher powers may be neglected.

Short Method of Substitution in a Polynomial. To find the value of $4x^4 - 14x^3 + 23x - 26$ when $x = 3$, for example, first arrange the terms in order of descending powers of x, and write the detached coefficients, with their signs, in a row, taking care to supply a zero coefficient for any missing term, including the constant term. Then, beginning at the left, bring down the first coefficient; multiply this by 3, and add to the second coefficient; multiply this result by 3 again, and add to the third coefficient; and so on. The final result, -11, is the value of the polynomial when $x = 3$.

4	−14	0	23	−26	(3
	12	−6	−18	15	
4	−2	−6	5	−11	

Short Method of Dividing a Polynomial by x − a. The device just explained gives not only the value of the polynomial when $x = 3$, but also the result of dividing the polynomial by $x - 3$. Thus, in the case illustrated, the quotient is $4x^3 - 2x^2 - 6x + 5$ and the remainder is -11. That is, $4x^4 - 14x^3 + 0x^2 + 23x - 26 = (x - 3)(4x^3 - 2x^2 - 6x + 5) - 11$.

Exponential Equations. To solve an equation of the form $a^x = b$, take the logarithms of both sides: $x \log a = \log b$, whence $x = (\log b)/(\log a)$. For example, if $3^x = 0.4$, $x = \log 0.4/\log 3 = (0.6021 - 1)/0.4771 = -0.3979/0.4771 = -0.8340$. Notice that the complete logarithm must be taken, not merely the mantissa.

Trigonometric Equations. (1) To solve $a \cos x + b \sin x = c$, where a and b are positive: Find the acute angle u for which $\tan u = b/a$, and the angle v (between 0 and 180°) for which $\cos v = c/\sqrt{a^2 + b^2}$. Then $x_1 = u + v$ and $x_2 = u - v$ are roots of the equation.

(2) To solve $a \cos x - b \sin x = c$, where a and b are positive: Find u and v as above. Then $x_1 = -(u + v)$ and $x_2 = -(u - v)$ are roots of the equation.

General Method of Solution by Trial and Error. This method is applicable to a numerical equation of any form, and can be carried out to any desired degree of approximation. It is especially useful when a first approximation to a root is already known. Write the equation in the form

ELECTRIC DRIVE

From the standpoint of machinery arrangement, the diesel-electric is the most flexible of all drives. It permits dividing the power plant into as many units as desired, and the propulsion motor is the only principal element in the plant that is limited to a fixed location by its function. The generating sets that furnish power for the motor may be located at any point in the ship that is most convenient or desirable and may be placed remote from the propulsion motor. Also, the controls may be located at any desired point. Its principal disadvantage is its loss in efficiency due to losses in generators and motors, amounting to approximately 15 per cent.

The essential elements in such a plant include one or more diesel-driven generators, a propulsion motor, a source of current for excitation, and control equipment. The engine driving the generator is required to run in only one direction and, therefore, needs no reversing gear.

It can be designed to run at any desired speed, thus permitting the use of lightweight engines. Earlier practice called for constant-speed engines but, in present-day installations, it is common practice to arrange the engine for variable-speed operation. In this case the engine is operated at about half speed and all propeller speed variations are obtained by varying the voltage of the propulsion motor field, up to half speed, at which point all resistance is out of the field circuit. Beyond that point, further increases of speed are obtained by increasing the engine speed.

Generators. The generators are shunt-wound, separately excited d-c machines, with the exciter armatures mounted on the extended shaft of the generators or mounted above the main generators and driven by belts. Since the exciter must deliver constant voltage while the engine speed varies, a voltage regulator is required. Usually the exciter has a capacity in excess of the power required for excitation, in order that it may supply current for operating auxiliaries.

Propulsion Motor. The propulsion motor is a separately excited, shunt-wound machine and, when attached directly to the propeller shaft, must necessarily operate at slow speed. This calls for a comparatively large motor and, in order to reduce the diameter, it is customary to use a double-armature motor which is in effect two motors placed side by side. In later installations the use of large, heavy motors is avoided by the placing of reduction gears between the motors and the propeller shaft. This permits the use of high-speed motors of small size.

Controls. For control of the propulsion-motor speed by the variable voltage method, with the engine running at constant speed, the armatures of the motor and main generators are connected in series and their fields are independently connected to the excitation circuit. The motor field is excited at a constant value, and its speed is varied by varying the intensity of the generator field. Increasing the generator field increases the generator voltage impressed on the motor. Since the motor field is constant, the speed of the motor will vary directly as the impressed voltage. In types of service in which it is desirable to enable the propeller to absorb full engine power while turning at slow speed, as in tug operation, the amount of torque developed by the motor can be adjusted by the use of a rheostat in the motor field circuit. Reversal of the motor is accomplished by reversing the direction of the generator excitation current. The resulting reversal of generator voltage causes the motor to reverse its direction of rotation.

Available Fuels. The purchaser who buys diesel fuel in large quantities can usually obtain oil to any desired specification, but the transient vessel or the small consumer that bunkers from oil company stock in any given port must take whatever is available.

Table 3. Conversion Equivalents for Fuel Oil

Deg Baumé	Deg A.P.I.	Sp gr	Viscosity SSU at 60 F	Lb per gal	Lb per bbl	Bbl per ton
10	10.00	1.00	....	8.328		
11	11.01	0.9930	....	8.270		
12	12.02	0.9861	....	8.212		
13	13.03	0.9792	....	8.155		
14	14.04	0.9725	....	8.009	340.3	5.877
15	15.05	0.9659	....	8.044	338.2	5.957
16	16.06	0.9593	14,000	7.989	335.7	5.957
17	17.08	0.9529	6 500	7.935	333.3	6.001
18	18.09	0.9465	3,200	7.882	331.2	6.039
19	19.10	0.9402	1,700	7.830	329.1	6.078
20	20.11	0.9340	950	7.778	326.6	6.123
21	21.12	0.9279	575	7.727	324.5	6.163
22	22.13	0.9218	370	7.676	322.4	6.203
23	23.14	0.9159	240	7.627	320.3	6.243
24	24.15	0.9100	165	7.578	318.2	6.285
25	25.16	0.9042	125	7.529	316.1	6.326
26	26.17	0.8984	94	7.481	314.0	6.369
27	27.18	0.8927	75	7.434	312.3	6.405
28	28.19	0.8871	63	7.387	310.2	6.448
29	29.20	0.8816	55	7.341	308.5	6.484
30	30.21	0.8762	49	7.296	306.4	6.528
31	31.22	0.8008	45	7.251		
32	32.24	0.8654	42	7.206		
33	33.25	0 8602	40	7.163		
34	34.26	0.8550	38	7.119		
35	35.27	0.8498	36.5	7.076		
36	36.28	0.8448	35	7.034		
37	37.29	0 8398	34	6.993		
38	38.30	0.8348	33	6.951		

Posted prices for diesel fuel in New York Harbor apply to a fuel of 30 A.P.I. gravity or over, carbon content less than 1 percent, and a viscosity of about 40 SSU at 100 F. This is a fuel suitable for high-speed engines but not needed for slow-speed units. A grade of oil known as heavy marine diesel fuel can be obtained for 20 to 25 cents per barrel less that has a gravity of about 20 A.P.I., viscosity 100 SSU at 100 F carbon about 1 percent, and a cetane number as high as 40.

Effect of Fuel Price. Roughly, the cost of fuel amounts to about 50 percent of the operating cost of the average cargo ship and is thus the item that offers the most attractive target for efforts at saving. Aside from all other considerations, such as increased cargo capacity due to the smaller weight of fuel that must be carried, the diesel engine would offer a large saving in the cost of fuel, as compared with the equivalent steam plant if both used the same kind of fuel. Although there is a definite trend toward the use of boiler

Alternating-current Drive. Although most diesel-electric drives utilize direct current, there is a growing trend toward the use of alternating current, because of the greater efficiency of the a-c plant and the lower cost, lower weight, and simplicity of the equipment. The disadvantages of alternating drive are that it is similar to a fixed-ratio gear drive. For instance, if two generator units are driving a single propulsion motor at rated full speed and for any reason it becomes necessary to stop one generator, the other generating unit will automatically have to slow down to approximately 63 per cent of its top rating in order not to be overloaded. This complies with the law that the horsepower varies as the cube of the speed. With the d-c drive this would not be necessary, for by changing the field current the remaining generating unit could be operated at its full capacity without any injury to itself and thus bring the propeller up to 79 per cent of full speed. The principal elements in the a-c system are one or more salient pole generators and a synchronous motor on each propeller shaft.

The general principle of operation is to start the engines, bring them up to idling speed, then synchronize the generators, and close the reversing switch for the direction desired. The motor will then operate as an induction motor with its fields short-circuited; when it comes up to the same synchronous speed as the engines, the motor field current is applied and the motor brought into step with the generators.

With the generators and motor in step, any increases of speed may be obtained by increasing the speed of the engines. Below the idling speed further speed reductions are obtained by operating the motor as an induction motor with voltage low enough to produce slip. When maneuvering ahead and astern, the required torque is obtained by overexciting the generator fields. A greater range of speed changes can be obtained if the motor is provided with two sets of poles.

ECONOMICS OF THE DIESEL ENGINE

Considered on the basis of its ability to convert energy released from the fuel in the form of heat into power available at the crankshaft for performing work, the diesel engine is the most efficient prime mover yet produced. Between this initial conversion of energy and the final book balance between the cost of producing power and the income derived from the application of this power, there are several variable factors that should be taken into account. This is particularly true in marine service, and it is impossible to establish any one set of figures that can be given blanket application for comparing the economy of diesel-propulsion machinery with other types used in marine service. There are, however, certain basic factors that can be evaluated for use in establishing a figure for over-all economy under any given conditions of operation.

Fuel Consumption. Fuel consumption of marine diesel engines is sometimes stated in terms of pounds of fuel consumed per indicated horsepower per hour, because the only power that can be measured after the engine is installed in the ship is that determined from indicator cards. The only power the shipowner is interested in, however, is that available at the shaft that can be used for income-producing work. The fuel consumption, then, should be expressed in terms of pounds of fuel per brake horsepower per hour. The terms brake horsepower and shaft horsepower are used synonymously, but the former is usually preferred because the economy of the engine is determined when it is run in the shop and the load applied by some form of brake, usually

Table 4. Approximate Cetane Number from Viscosity and Gravity of Fuel

(Moore and Kaye)[a]

Viscosity, SSU, at 100 F	A.P.I. gravity																
	18	20	22	24	26	27	28	29	30	31	32	33	34	35	36	37	38
34	21	25	29	33	37	39	40	42	44	46	47	49	51	52	54	55	57
35	22	26	30	34	38	40	42	44	46	48	49	51	53	54	56	57	59
36	23	27	31	35	39	41	43	45	47	49	51	52	54	56	58	59	61
37	24	28	32	36	40	42	44	46	48	50	52	53	55	57	59	60	62
38	24	29	33	37	41	43	45	47	49	51	53	54	56	58	60	61	63
39	25	29	33	37	41	43	45	47	49	51	53	55	57	59	61	62	64
40	25	30	34	38	42	44	46	48	50	52	54	56	58	60	62	63	65
42	26	30	35	39	43	45	47	49	51	53	55	57	59	61	63	64	66
44	26	31	35	40	44	46	48	50	52	54	56	58	60	62	64	65	67
46	27	32	36	41	45	47	49	51	53	55	57	59	61	63	65	66	68
48	27	32	36	41	45	47	49	51	53	55	57	59	61	63	65	67	68
50	28	33	37	42	46	48	50	52	54	56	58	60	62	64	66	68	69
60	29	34	39	43	47	50	52	54	56	58	60	62	64	66	68	69	70
80	31	35	40	45	50	52	54	56	58	60	63	65	67	69	71	72	
100	32	37	42	47	52	54	56	58	60	62	65	67	69	71			
150	34	39	44	49	54	56	58	60	63	65	67	69	72				
200	35	40	45	50	55	58	60	62	65	67	69	71					
300	36	41	47	52	57	60	62	65	67	69	72						
400	37	42	48	53	59	61	64	66	68	71							
500	38	43	49	55	60	63	65	67	70								

[a] Courtesy of the Texas Company.

Table 5. Conversion of Diesel Index to Cetane Number

Diesel Index	Cetane No.
0	18
5	20
10	24
15	28
20	30
25	34
30	37
35	40
40	43
45	46
50	50
55	53
60	56
65	59
70	62
75	65
80	68
85	71
90	75
95	78
100	81

a water brake. If a more precise definition of shaft horsepower is taken to mean only the power delivered to the propeller, the friction in thrust bearings and line-shaft bearings must be deducted. Also a deduction must be made for losses in diesel-electric drives and for slip of coupling in gear drives. The amount of brake horsepower that can be obtained from a given quantity of fuel is a true measure of the engine's economy.

The amount of fuel consumed per brake horsepower will vary between narrow limits with different engines, but it should never be more than 0.4 lb per bhp-hr at normal full load for the engine without attached auxiliaries. Many engines use less than this. The records of some engines in service show a normal consumption of 0.334 lb per bhp-hr. An interesting characteristic of the diesel engine is the small difference in fuel consumption of different sized engines. The graph of full-load consumption for units of various sizes, from 5 to 15,000 hp, is practically a straight line.

Lubricating-oil Consumption. Far more variable than fuel consumption is the amount of lubricating oil used by the engine. As explained on p. 1337, the oil applied to the cylinder walls is not recoverable but is burned in operation. There is no way of determining definitely, while the engine is running, whether the amount of lubricating oil being fed into the cylinders is enough, too little, or too much. Therefore, the lubricator feed must be adjusted on the basis of past experience or judgment and is most often likely to be too much. In general it may be assumed that the two-cycle engine will require more oil than the four-cycle for cylinder lubrication.

There will also be wide variations in the amounts of oil used in the bearing lubricating system. In a large crosshead type engine in which the cylinders are separated from the crankcase by a diaphragm, there will be less oil lost by being drawn up into the cylinders and burned than will be the case with a trunk-piston engine, especially if the latter is of the high-speed type. In both engines the amount of oil lost by leakage will depend upon the degree of tightness of crankcase doors and joints throughout the system and will accordingly vary with the quality of attendance the engine receives and to some extent upon the provisions made by the designer for oil tightness.

These variable conditions account for the variation of oil consumption mentioned. Expressed in terms of the number of horsepower-hours produced per gallon of lubricating oil used, the consumption may vary from about 1 gal per 1,000 hp-hr in small, high-speed engines to 1 gal per 10,000 hp-hr in large, slow-speed units. Since most engines in service are of small size, it will be found that a fair average consumption by existing engines is somewhere between 1 gal per 1,500 hp-hr and 1 gal per 2,000 hp-hr. If small high-speed engines are excluded, the average consumption will be 1 gal per 3,000 to 5,000 hp-hr.

Effect of Wear. The variations noted and their causes apply to engines in normally good operating condition. Further variations will be introduced by the condition of the engine, particularly in regard to piston ring and liner wear. Such wear in a trunk-piston engine will usually result in an increase in the amount of crankcase oil drawn up into the cylinders and burned. Also the resulting blow-by of gases into the crankcase causes a large amount of oil vapor to be blown out through the crankcase breather pipe.

Over-all Economy. The fact that the diesel engine is inherently the most efficient prime mover yet devised is not in itself a guarantee that the diesel-propulsion plant will be the most economical under all conditions. Numerous other factors, some within the plant and others outside but related

oil for diesel fuel, most diesel engines use a refined fuel costing considerably more than boiler oil.

This difference in cost has the effect of reducing the comparative net economy of the diesel plant, and the greater the difference between the costs of the two fuels, the greater this reduction will be. Fuel prices vary in different ports. At time of writing, standard diesel fuel costs 60 percent more than boiler fuel in New York Harbor but in Los Angeles harbor the difference is only 30 percent. It is obvious that in the case of two ships, one a steamer and the other a motorship, both with machinery plants of the same total horsepower, the higher this percentage difference is, the nearer the steamer will approach the motorship in cost of power. The point at which the cost difference disappears can be determined by a simple calculation.

Assuming two ships of 4,000 bhp, one using Bunker C fuel and the other diesel fuel, and basing their respective fuel consumptions on reports of the service performance of efficient ships of their types, the steamer will consume 13,020 Btu per shp per hr and the motorship will consume 7030 Btu. The other data may be tabulated as follows:

	Steamer	Motorship
Gravity of fuel, A.P.I.	11	29
Pounds per bbl	343	308
Heat content, Btu per lb	18,500	19,000
Price per bbl	$1.10	$1.45
Btu per bbl	6,345,500	5,852,000
Cost per million Btu, cents	17.2	24.8
Btu per shp	13,020	7,030
Total Btu per hr	52,080,000	28,120,000
Cost of fuel per hr	$8.95	$6.97

If the cost of fuel for the steamer is $8.95 per hour, the motorship can pay as much as 895/18.2 = 31.8 cents per million Btu, or $1.86 per barrel, before its hourly fuel cost will equal that of the steamer. On a Btu basis this shows that the motorship can pay $\frac{31.8 - 17.2}{17.2} = 85$ percent more for its fuel than does the steamer without its fuel bill exceeding that of the steamer. This relation will hold true in any port in the world, regardless of relative fuel prices, as long as the relative consumptions are as stated.

It is obvious from the foregoing discussion that the difference in costs of fuel used for steamers and for diesel-powered ships penalizes the latter to a degree that becomes greater as the disparity in prices increases until, when the 85 percent point is reached, the two are on an equal footing in regard to fuel cost. This indicates that it is very much to the advantage of the diesel engine to avoid the use of high-priced special fuel and burn the same type of oil that is burned under boilers.

Use of Heavy Fuel. The economic penalty imposed on the motorship by the use of high-grade diesel fuel has created an increasing tendency toward the use of the heavier grades commonly used as boiler fuel and loosely classified as Bunker C. This does not identify any one grade of oil since it includes those varying in gravity from 10 to 17 deg or more A.P.I. and having Saybolt viscosities from 2,000 to 4,000 at 100 F. Such oils usually contain considerable abrasive matter that tends to increase the rate of wear of liners and

to its operation, have an influence on its economic balance. The over-all economy of the entire propelling plant is the matter in which the ship operator is most interested. Among the factors that enter into it, the principal ones are fuel oil and lubricating-oil consumption of main and auxiliary engines and cost of maintenance.

These determine the variable costs that must be added to such fixed charges as interest on investment, depreciation charges, insurance, and crew cost, in determining the final cost per unit of power delivered to the propeller. The rate of fuel consumption may prove to be less important than the kind of fuel consumed. The ability of an engine to burn a heavy fuel that is cheaper than high-grade diesel fuel may spell the difference between profit and loss.

Numerous factors enter into the cost of maintenance, the first being the quality of design. Assuming that the engine is correctly operated, the rate of wear is strongly influenced by this design factor, as is also the frequency of parts breakage. Accessibility is an important factor in maintenance cost and should be given consideration in the original design and the installation plan. The most accessible engine may be made extremely difficult to keep in good operating condition if no consideration is given to accessibility in designing the installation arrangement in the ship.

The items mentioned in the foregoing are merely indications of the wide scope that would have to be covered if any detailed analysis of over-all economy is to be made. A more specific means of favorably affecting over-all economy is by utilization of that part of the heat content of the fuel that is discharged to the atmosphere in the exhaust without doing useful work.

WASTE-HEAT RECOVERY

The heat balance of the average diesel engine shows that about 30 percent of the total heat supplied is carried away in the exhaust and accordingly wasted. The amount of this waste heat can be indicated by assuming a 1,000-hp engine to use 0.4 lb fuel per hp per hr, and the fuel to have a heating valve of 18,500 Btu. Then if 30 percent of the heat is carried away in the exhaust, the total amount lost will be $H = 1{,}000 \times 0.4 \times 18{,}500 \times 0.30 = 2{,}220{,}000$ Btu per hr. Although not all of this heat is recoverable, enough of it can be recovered by means of an exhaust-gas boiler to increase appreciably the over-all economy of the plant.

The amount of heat that theoretically can be recovered in such a boiler is the difference between the total heat in the gases at the temperature at which they leave the engine and their total heat at the temperature of the steam at the pressure carried in the boiler. It is not practicable, however, to effect a transfer of heat that will reduce the gases to the same temperature as the steam. About the best that can be done is to reduce the gas temperature to about 75 deg higher than that of the steam in the boiler. It follows, then, that the higher the steam pressure carried in the boiler the less the amount of heat that can be recovered from the gases.

Amount of Heat Recoverable. To estimate the amount of heat that can be recovered, the pressure desired, the boiler efficiency, and the temperature of the gas leaving the engine must be known. Usually the efficiency of the boiler is not accurately known, but a figure can be assumed for the difference in temperature between the steam and the gas leaving the boiler, such as the 75 deg previously mentioned.

rings, a condition aggravated by the fact that, if the oil is not properly conditioned before injection into the cylinder, the resulting reactions interfere with lubrication.

It has been definitely established that engines with large- or medium-bore cylinders, with correspondingly large combustion-chamber volume, experience no trouble in burning heavy oil satisfactorily, but as the cylinder bore becomes smaller and the revolutions higher more difficulty is experienced. This has led to the conclusion in some quarters that on oceangoing ships two different grades of fuel should be carried, one for the main engines and one for the auxiliary engines. Other informed opinion holds that this is not necessary because engines of the sizes used for auxiliary service in motorships of average size can operate on heavy fuel if the two prime requisites—cleaning of the fuel and heating to the proper degree—are provided.

Liner Wear. The principal deterrent to the use of heavy fuels has been the resulting increase in rate of liner wear. Renewal of liners is the most expensive item in maintenance costs and, if it is required too often, may nullify the gains resulting from the use of cheap fuel. On the other hand the renewal frequency, even though it is greater than when standard diesel fuel is used, may be considerably overbalanced by the saving in fuel cost resulting from the use of the cheaper oil. The fuel bill with standard fuel may in two years amount to as much as the first cost of the engines and, if this bill can be reduced by even a small percentage, it will more than pay for the increased cost due to more frequent liner renewal.

Liner wear resulting from abrasive impurities can be dismissed as unnecessary because the necessity for cleaning is well known and the equipment available for that purpose is readily available. Experiments reported by Broeze and Gravesteyn indicate that although carbon and ash content have some influence on rate of wear, the sulfur content is the principal factor, with incomplete combustion second to it. These experimenters reported that when the sulfur content is more than 1 percent the formation of sulfur trioxide causes corrosion in the combustion space that creates the effect of liner wear.

Incomplete combustion is preventable by strict observance of the requirements for correct heating, adequate compression pressure, correct cylinder temperature, and correct timing of injection. The most promising method of preventing the corrosion due to sulfur is by the use of chromium-plated liners. The investigators referred to found that with a very low grade of fuel, containing high percentages of sulfur, carbon, and ash, chromium-plated liners showed a rate of wear only one-tenth of that of unplated liners, using the same fuel.

Fuel Conditioning. The heating of oil to reduce its viscosity is the first step in the conditioning of heavy fuel. An initial heating in the bunkers may or may not be necessary to make it fluid enough to be handled by the transfer pump, but it is desirable to heat the oil before it is passed through the centrifuge for cleaning. After cleaning, further heating is required to reduce the viscosity enough for the fuel-injection pumps to be able to handle it. Maintenance at an even temperature is facilitated if the fuel-supply system is arranged for continuous circulation of oil through the heaters. A viscosity that permits ready passage of the oil through the fuel-injection pumps is also low enough for atomization by the injection valves.

To prevent a fall in temperature and rise in viscosity during the passage of the oil from the pumps to the injection valves, each connecting tube should

Example. Consider the case where the gas leaves the engine at a temperature of 700 deg and steam at a pressure of 100 lb is desired. Since the temperature of the steam is 337 F, the temperature of the gas leaving the boiler will be 337 + 75 = 412 F and the temperature drop will be 700 − 412 = 288 F. If the air temperature is assumed to be 70 deg, the percent of heat recovered will be 288/700 − 70 = 45.7 percent. In the case of the 1,000-hp engine cited previously the heat recovered will be 2,220,000 × 0.457 = 1,014,540 Btu per hr.

The amount of heat required to produce 1 lb steam at 100 lb pressure, assuming a feed-water temperature of 140 deg, will be 880 + 337 − 140 = 1077 Btu, and the amount of steam per hour that could be produced by the 1,000-hp engine would be 1,014,540/1077.4 = 942 lb. This amount represents a rate of production of 0.942 lb per bhp per hr.

A more detailed method of calculation is presented by Gregson, an example of which is that for a four-cycle engine consuming 0.4 lb fuel per bhp per hr and having an exhaust gas temperature of 740 F. Steam is generated at a pressure of 160 lb, and the gas leaving the boiler has a temperature of 370 + 70 = 440 F. The excess air is taken as 125 percent, corresponding to a CO_2 content of 600 percent in the gas, and the boiler efficiency is 95 percent.

Btu recovered per bhp per hr = weight of gas per hr × temperature drop × specific heat × 0.95. With a specific heat of 0.24 this becomes

$$\text{Btu (hr) (bhp)} = 0.4 \times 32.84 \times 300 \times 0.24 \times 0.95 = 900$$
$$\text{Steam per hr per bhp} = {}^{900}\!/_{960} = 0.93 \text{ lb}$$

Exhaust-gas Boiler. Since there is no radiant-heat effect in the exhaust-gas boiler and the heat flow is by conduction only, the heating surface should be large and the gas velocity high, with the flow path arranged to break up the gas stream. The types of boilers in use include those with horizontal tubes of hairpin shape; those using thimble tubes projecting from a cylindrical shell and closed at their outer ends; the tubular type, similar to a fire-tube boiler; the bent-tube type, in which the tubes are arranged as flat coils laid one on top of the other; and the type in which a water jacket surrounding the exhaust muffler is used to produce steam.

Steam from the exhaust-gas boiler is, of course, available only when the engine is in operation, and the rate of evaporation will decrease when the engine is operated at reduced speeds. For these reasons the greatest benefit from heat reclamation is obtained on voyages long enough to call for continuous operation of the engine for extended periods.

The greatest economy is obtained when the steam produced is used to operate the auxiliaries ordinarily required at sea, such as the steering engine, oil and water pumps, and lighting generator. A most desirable arrangement is to have the auxiliaries driven by electric motors and supply them with current from a steam-turbine-driven generator, operated by steam from the exhaust-gas boiler, with a diesel-driven generator as a stand-by. They can then be operated with current from the turbogenerator or the diesel generator as desired.

By arranging the exhaust-gas boiler for oil firing it can be used in port or its rate of evaporation can be maintained at a maximum during periods of slow-speed operation at sea. This latter possibility exists only if the boiler is designed with a separate combustion chamber isolated from the gas-inlet chamber. Such a boiler is in most cases simply two separate boilers with a common steam drum or steam space.

An arrangement that has been used with success in some recent tankers utilizes an exhaust-gas boiler of this type, in connection with automatic controls. Normally the b

be paralleled by a small steam pipe and the two wrapped together and insulated. As stated before, the temperature of the oil should be high enough to lower its viscosity to about 200 SSU.

To ensure complete combustion, the compression pressure should be raised to about 25 lb higher than when operating with light fuel. Also the jacket-water temperature should be maintained as high as is consistent with good lubrication and freedom from piston seizure. No specific temperature for the jacket water can be stated, since it will vary with different engines and can be determined only by trial in each case. The ignition lag is usually greater and the rate of combustion slower with heavy grades of oil. For this reason the time of injection should be advanced, the amount of advance to be determined by trial.

VIBRATION AND NOISE SUPPRESSION

The vibration occurring in any reciprocating engine has its origin in the movement of the reciprocating parts and the force applied periodically to the piston to accelerate these parts. This force can be calculated by the fundamental formula:

$$F = \frac{12WV^2(\cos d + r \cos 2d/l)}{Agr}$$

where W = total weight of reciprocating parts
V = velocity of crank, fps
d = angle between crank and center line through cylinder
l = length of connecting rods
A = area of piston, sq in.
g = acceleration due to gravity
r = radius of crank

If the values of F obtained by this formula are plotted against crank angle, the result will be a sine curve with an amplitude of 180 deg between maximum plus valves and maximum minus valves, as shown by curve a in Fig. 18. If the expression outside the parenthesis and the one inside it are calculated separately and plotted, the two curves b and c will be obtained, the first representing the influence of the reciprocating masses considered concentrated at the crankpin and the second the influence of the angularity of the connecting rod. These forces are the first and second harmonics, and it will be noted that the second has twice the frequency of the first and much less amplitude. By accurate calculation of the centrifugal force, succeeding harmonics of increasing frequency and decreasing amplitude can be plotted, as shown by curve d which is the fourth harmonic. Harmonics above the fourth order have little influence on vibration and may be neglected.

Cylinder Combinations. Since the forces causing vibration, as described in the foregoing, occur in each cylinder and the phase relation of these different sets of forces depends on relative crank positions, it is evident that the number of cylinders in an engine and the crank settings have an important influence on vibration. In a three-cylinder engine with cranks 120 deg apart, the first, second, and fourth harmonics are completely balanced, but there are couples in the vertical and horizontal planes that cause vibration. In the six-cylinder four-cycle engine, which is in effect two three-cylinder engines, these couples will act in opposite directions and cancel each other

steam being generated is insufficient to meet the demands of the auxiliaries, the pressure falls and at a predetermined pressure the controls cut in the oil burner. When the pressure rises to a given point, the burner cuts out.

AUXILIARIES

In order to keep the diesel engine in operation and supply such services as lubrication and cooling of the engine, certain auxiliary equipment is required. Also, diesel generators must be provided for supplying power to operate deck machinery, such as steering engine, anchor windlass, capstans, and deck winches; also to supply lighting current, domestic water, sanitary water, and water for fire protection, as well as power for operating bilge pumps and refrigerating and ventilating equipment. Some of this auxiliary equipment must be in operation at all times while the propulsion engine is running, and some is required to be operated only intermittently.

Independent Auxiliaries. It is universal practice to use electric-motor-driven auxiliaries in all cases, except when they are attached directly to the engine, and in all large ships the current for their operation is provided by diesel-driven generator sets. The amount of generating capacity required will vary widely according to the type of ship. In the case of a cargo ship the controlling factor will be the amount of power required for operating cargo winches; in the passenger ship it will be the power required for the hotel load, *i.e.*, service to passengers, that will control.

This total power should be divided between two or more diesel-generator sets for the sake of economy in operation, owing to the fact that only a fraction of the total power is required most of the time. A cargo ship or tanker while in port will require a large amount of power to operate winches or cargo pumps, and all the generators must be in operation; at sea, one will usually carry all the load. The auxiliary diesels are thus enabled always to operate at somewhere near their rated load if the total maximum power is divided between several units.

The auxiliaries required to keep the engine in operation are the lubricating-oil-circulating pump, the fresh-water-circulating pump, the raw-water-circulating pump, and sometimes a fuel-booster pump with coolers in the oil and water circuits. If the open type of cooling system is used, the raw-water pump and water cooler are dispensed with. The water pumps are usually of the centrifugal type, and the lubricating-oil pump of the rotary or gear type. As a matter of precaution, both are usually installed in duplicate.

A form of diesel unit widely used on small diesel vessels is the so-called auxiliary set, which combines in one unit all the principal auxiliaries other than the pumps mentioned in the foregoing. It consists of a small diesel engine, a generator, air compressor, general service pump, and the oil and water pumps for the engine, all mounted on a common base. Connection to the engine is made by clutches so that any one or more of the units in the set may be operated at the same time.

Engine-driven Auxiliaries. Some accessory equipment such as fuel injection pumps, cylinder lubricators, and, in air-injection engines, the air compressor, have always been built into the engine structure and driven by the engine. Present-day design of engines below approximately 1,500 hp extends this practice to other auxiliaries so that in some types of vessels no diesel generator need be in operation when the main engine is running. There is no engineering reason why this should not be done in all cases, but

if cranks 1 and 6 are in phase and if 2 and 5 and 3 and 4 are likewise paired. The six-cylinder engine is thus inherently balanced.

Torsional Vibration. Vibration becomes synchronous when the vibrating impulses applied to any part have a frequency that coincides with the natural frequency of that part. The speed at which this condition is set up in an engine is known as the **critical speed**, and it becomes most serious when synchronous torsional vibration is set up in the crankshaft. Torsional vibration is the vibration set up in a shaft when the torque applied to the shaft causes it to twist and the stress thus set up causes it to untwist. A single impulse applied in this way will be followed by alternate twisting and untwisting with decreasing amplitude until the internal friction of the shaft gradually brings it to rest. If a series of such impulses is applied at a frequency corresponding to the natural frequency of the shaft, a condition of synchronous

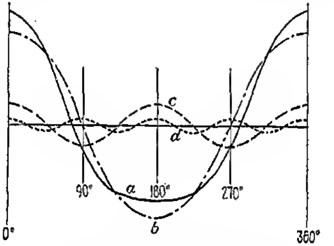


FIG. 18.—Inertia forces causing vibration.

torsional vibration is established and, unless the energy input is absorbed by damping in some way, the shaft will eventually fail.

This condition will exist whenever the natural frequency of the shaft divided by the engine speed is a whole number or a half number for the four-cycle engine and a whole number for the two-cycle engine. Accordingly, an engine may have more than one critical speed, yet may operate very smoothly at all other speeds. By methods of analysis (see p. 1437) the natural frequency of the mass elastic system of a given engine, and the relative locations of the applied forces and the forces resisting oscillation, can be calculated. In this way it can be determined whether the critical speeds of the engine lie within the range of speeds at which it will normally operate. If they do, the natural frequency of the shaft may be changed by increasing the diameter of the shaft, decreasing the weight of the flywheel, or shortening the shaft.

If a critical speed remains after all possible changes have been made, the tachometer for indicating engine speed should be marked with a red line at the critical speed and operation at this speed avoided.

A method of controlling torsional vibration which is coming into general use utilizes a vibration damper on the shaft to absorb the energy of vibration

in large ships where the power required by auxiliaries is large, it would require extra power to be built into the main engine.

There is a growing tendency in small vessels to include the water and oil pumps in the engine structure and drive them by means of gearing from the crankshaft. When the generator is also engine-driven, chains or V-belts are utilized for the drive. When this is done, stand-by auxiliaries driven from an independent source of power are provided.

An arrangement that has been practically standardized in tug practice is to have a diesel-driven generator and a complete set of motor-driven auxiliaries in addition to the built-in equipment of the main engine. When getting under way or during maneuvers that require starting and stopping of the main engine over a considerable time, the diesel generator is kept running to operate the starting air compressor, steering engine, electric lights, etc. After the engine is started and during subsequent maneuvering, water and oil circulation are taken care of by the attached pumps and, after the boat is in the fairway, the electrical load is shifted to the attached generator and the diesel generator is shut down.

In some cases occasionally used auxiliaries, such as the bilge pump and fire pump which require considerable power, are connected by chain drives to the intermediate or propeller shaft, with clutches interposed, so that they may be operated whenever desired merely by engaging the clutches. Where there are several of these units, they may be driven from a jackshaft, each with a separate chain drive and a single chain used to drive the jackshaft.

Battery Systems. Auxiliary arrangements such as have been described for smaller vessels, in which a generator driven from the main engine is used, generally include a storage battery, with "floating system" connections. The battery comprises enough cells to give the same voltage as the generator and is connected across the main power bus. If the engine is stopped or is slowed down enough to reduce the generator voltage below normal, current from the battery flows into the line and the battery takes the load. When the engine is speeded up to the point where the generator voltage is again normal, the generator again takes the load and current flow back into the battery to bring it up to a normal state of charge. Also, the battery helps to carry the load if a sudden, short demand is made for power beyond the capacity of the generator.

Although attached generators are designed to have a flat voltage characteristic over a fairly wide range of speed, it is customary to provide a voltage regulator, usually of the carbon-pile type, in connection with such generators. This regulator acts to maintain a constant voltage whenever the engine is operated at varying speeds.

FUELS

Although under special circumstances diesel engines have been operated with a variety of fuels, normal operation requires the use of a fuel refined from crude petroleum and obtained after the gasoline, kerosene, and other light products are removed from the crude. Wide variations in the quality of fuel are found, but in general the small, high-speed engines are confined to the use of lighter oil of comparatively high gravity, while the large engines that have greater combustion-chamber volume, more time for combustion, and a smaller ratio of exposed-surface-to-air volume in the combustion chamber can burn oils of low gravities, corresponding to the oil burned under

by friction. In one form it comprises a flywheel held between friction disks. When the angular velocity of the shaft decreases or reverses, the flywheel continues to move at the original velocity and the friction dampens the vibration. Another form uses hydraulic friction as the damping force, and a third utilizes spring-controlled rotating weights. Any change in velocity causes the radius of rotation of the weights to change, thus varying the moment of inertia and causing the natural frequency of the shaft to change.

Resilient Mounting. After all vibration that might be harmful to the engine has been removed or neutralized, there will always be a certain amount of residual vibration which, although harmless to the engine structure, may sometimes cause unpleasant vibration in the vessel's hull or superstructure, or rattling of pipes and small parts. A method in common use for eliminating these effects is applicable to auxiliary engines or generator sets used for electric drive but has not as yet had any extensive application to propulsion engines. It uses resilient mounting under the engine so that vibration will be confined to the engine and will not be transmitted through the foundation to the hull.

Rubber mountings, arranged so that there is no metal-to-metal contact between the foundation and the engine bed, or holding-down bolts, have been used successfully with small engines. Similar mountings of cork have been used. The most effective isolation of vibration is obtained with steel-spring mountings. These are made up in the form of complete units that are inserted under the bedplate in place of the usual chocks and arranged so that the tension of the springs can be adjusted to suit the particular engine conditions. In its latest form this type of mounting is designed to prevent dislocation of the engine by shock resulting from gunfire, bombs, etc.

The application of resilient mounting to propulsion engines must take account of the fact that the engine is connected to a rigidly restrained propeller shaft. Also the engine is heavy and its center of gravity relatively high, resulting in an overturning moment when the vessel rolls in a seaway. In the few installations that have been made in connection with propulsion engines a flexible coupling is inserted between the crankshaft and propeller shaft to allow freedom of movement in excess of any movement of the engine and the engine bed is chocked fore and aft to prevent end movement.

Complete isolation of engine vibration requires that no rigid pipe connection be made to the engine. All piping for oil or water should be connected by means of flexible metal or rubber tubing. In addition to the flexible metal connection between the exhaust manifold and muffler, the muffler should also be resiliently mounted.

Noise Suppression. Since sound is the result of air waves produced by vibration, the matter of noise suppression should be considered in connection with vibration. From the first inception of the diesel engine the need for silencing or muffling the sound of the exhaust has been recognized. Early hit-or-miss methods have been succeeded in later years by development of exhaust silencers based on scientific study of the action of sound waves.

It has been found that the most effective damping of exhaust noise is obtained by varying the cross sections of the channels through which the gases flow. This has led to various designs of silencers, which are divided into separate chambers in which the desired change of cross section is obtained by perforated pipes, pipes with several branches, or partitions with holes for the gas passage. Effective silencing can be produced by throttling, whereby the pressure is converted to kinetic energy, after which the kinetic

steam boilers. Certain characteristics of fuel oil that have a relation to its value for use in diesel engines may be briefly explained as follows:

Gravity. The gravity of an oil is expressed as specific gravity, Baumé gravity, or A.P.I. gravity. The last has been established as a standard of the American Petroleum Institute and is now the most widely used in American diesel-engine practice. It is expressed in degrees, as determined by the following formula:

$$\text{Deg A.P.I. 60 F} = \frac{141.5}{\text{specific gravity at 60 F}} - 131.5$$

By this formula the gravity of water is 10, that of liquids heavier than water is less than 10, and that of liquids lighter than water is more than 10. *Although gravity of itself is not an index to the quality of the oil, it serves* as an indicator of the suitability of an oil for use as diesel fuel. In general, the higher the A.P.I. gravity, the less trouble will be experienced in burning the oil.

Heat Content. The heat content of an oil, which determines the amount of work that may be obtained from it, is expressed as the number of Btu per pound, as determined by burning a sample in a calorimeter and measuring the heat liberated. Since this measurement includes the latent heat that is released by condensation of the vapor formed by the combustion of the hydrogen in the oil, it represents some heat that can never be utilized in a working engine, because the gas resulting from combustion leaves the cylinder at a temperature far above the condensation temperature (see p. 366).

Flash Point. The temperature of the oil at which an open flame applied near the surface of the oil will cause the vapor from the oil to ignite is the flash point. Although it has no relation to the performance of the fuel in the engine, it is important in relation to safety in handling, storing, and transporting. In general a flash point of 150 F or more indicates that the oil is a safe one.

Pour Point. Another characteristic that is of importance in connection with the handling and storing the oil is its pour point, the lowest temperature at which it will flow. For marine use, a pour point of 15 F is satisfactory.

Viscosity. The fluidity of an oil, in terms of viscosity, has an important relation to its suitability for pumping and atomization. There are several different kinds of viscosimeters in use, with correspondingly different scales, but the one most widely used in the United States is the Standard Saybolt Universal, known as the SSU. The viscosity of an oil can be reduced by heating, but the temperature required to produce a given viscosity will vary with oils of different origins. In general it may be said that any oil fluid enough to be handled by the injection pump will atomize sufficiently at the injection nozzle, and any oil with a viscosity of 200 SSU will meet this requirement. If the viscosity is higher than this, the oil must be heated to a temperature which will reduce the viscosity to 200 SSU.

Carbon Content. At one time the carbon content of an oil was considered a highly important factor in relation to the combustion characteristics. As determined by the Conradson method, it is the percentage by weight of carbon remaining after a sample of the oil has been evaporated over a burner. It will vary from 0.5 to 10 percent in different oils. At present there is considerable difference of opinion as to the influence of carbon

energy is converted into heat by turbulence and internal friction, but this results in loss of engine output if appreciable back pressure is produced. One way in which this principle may be applied effectively is by the use of an exhaust turbine. The turbine acts as a throttle that produces power and also dampens the noise without producing harmful back pressure.

The annoyance caused by sparks issuing with the exhaust gases, as well as the danger in the case of vessels carrying inflammable cargo or working around oil docks, etc., led to the development of spark-arresting exhaust silencers. The use of water in the silencer for quenching is undesirable, and the method commonly used in modern silencers is to utilize centrifugal force to extract the spark-creating solids from the exhaust gas. The gas passage is arranged to give the gas stream a whirling motion, thus causing the solids to be thrown out and caught in a collector, from where they may be removed at intervals.

The principles of exhaust-gas silencing have been extended to apply to the air inlet on the four-cycle engine and the scavenging pump inlet on the two-cycle. This is particularly desirable in the case of high-speed two-cycle engines using rotary blowers with high-intake velocities. The general principles of noise suppression in the exhaust gases apply here with certain structural differences due to the tonal differences in the exhaust gas and inflowing air.

In considering the noises originating in the moving parts of an engine, it may be stated as a generalization that as engine speeds become higher such noise increases and its tonal quality becomes more annoying. With the modern trend toward high speeds the matter of general noise suppression must be given increasing attention. In such special installations as submarine boat machinery it has become a matter of major importance because of its mental and physical effects on personnel. Much progress has been made in the direction of sound absorption and elimination of sound reflections from bulkheads and other resonant surfaces.

Indicative of the methods used in suppressing noise by insulation is the procedure that proved effective in the case of a large, seagoing tug. The main engine is mounted on Micarta blocks instead of steel chocks; the auxiliary generator sets are mounted on spring-type dampers and each is enclosed in an acoustic cabinet; the bulkheads, ceiling, and gratings are sheathed with acoustic paneling; the undersides of engine-room ladder treads and floor plates are lined with insulating material; and the feet of the ladders rest on rubber pads. The result in this case is the elimination of resonance and reflected noise. The engine noises have a flat tone, and the absence of magnification permits conversation in the engine room in normal tones without effort.

MANUFACTURERS' STANDARDS

The conditions under which marine diesel engines operate vary widely and there are many factors that have a bearing on types and sizes of accessory equipment; also there are many optional variables that enter into the design of such engines. For these reasons it is not possible to set up rigid standards that will apply to all installations. A limited amount of standardization has been adopted by the engine builders who are members of the Diesel Engine Manufacturers Association. Some of the standards are presented here in brief form, to indicate their nature and are not necessarily presented in the exact language of the code of the association.

on combustion, but the Conradson carbon content is always included in fuel-oil specifications. As a generalization it may be stated that for satisfactory operation the smaller the engine the smaller should be the Conradson carbon content.

Sulfur. The bad effects of sulfur in the fuel were considered in earlier days important enough to require the sulfur content to be held to a fraction of 1 percent, but it is now recognized that little harm can come from it. About the only way in which it can be harmful is by the sulfur dioxide formed during combustion combining with the water in the exhaust gas, if the latter falls below the condensation temperature, attacking the metal of the exhaust lines and muffler. Recent experiments, to be mentioned in another section, indicate that there may be a relation between sulfur content and rate of cylinder liner wear. The sulfur content of present-day fuel may vary from 0.5 to 4 percent.

Water and Sediment. The water and sediment content, sometimes expressed as BS&W, for bottom sediment and water, is likely to be highly variable because an oil that leaves the refinery entirely free from water and sediment is subject to contamination in shipping and handling. A considerable amount of water may be harmless if it is dispersed throughout the fuel, but if it enters the fuel system in slugs it may cause misfiring. Sediment is harmful to the injection pumps and nozzles. Fuel specifications usually require the BS&W to be less than 1 percent, but in view of the ever-present possibility of contamination, all fuel should be filtered or centrifuged before being used in the engine.

Ignition Lag. The injection process that occurs after the fuel is injected takes place in four stages, as follows: ignition lag, ignition and combustion, controlled combustion during the injection period, and afterburning. The most important phase is ignition lag, which is the time that elapses between the start of injection and the ignition of the first particles of fuel; by measuring it, the so-called **ignition quality** of the fuel is determined.

If the ignition lag is too great, injection is completed before the initial ignition occurs; when it does occur, the whole charge of fuel is inflamed, so that combustion occurs with explosive violence, the engine knocks, the exhaust is smoky, and carbon is deposited in the cylinders. On the other hand, if the lag is too short, it lengthens the time of combustion, causes increased heat losses to cooling water and exhaust, and prevents thorough mixing of fuel and air. These things are of prime importance with high-speed small-bore engines but of less importance with large, slow-speed engines.

Ignition quality is expressed in several ways, the simplest of which is the diesel index; the expression for which is

$$\text{Diesel index} = \frac{\text{aniline number} \times \text{A.P.I. gravity}}{100}$$

The aniline number is the lowest temperature at which freshly distilled aniline will mix with the oil to be tested. The mixture is heated until the solution is clear, then allowed to cool. The temperature at which it becomes turbid is the aniline number. The diesel index does not give a very accurate indication of the ignition quality of oils from mixed crudes.

A more accurate method of expressing ignition quality is by cetane number, obtained by comparing the fuel with a standard reference fuel composed of

Ratings. The standard rating is the net brake horsepower that the engine will deliver continuously with atmospheric temperature not over 90 F, and barometric pressure not less than 29 in. Hg. The rating must be such that the engine will operate on the test stand with an overload of 10 percent for 2 hr, consecutively.

Brake Mean Pressure. Because of the many variables in engine design and construction, no standard can be established for bmep that would have general application.

Net Brake Horsepower. The standard net brake horsepower is measured by a dynamometer at the engine coupling, without correction for power required by auxiliaries which may or may not be driven by the engine. When comparing proposals the buyer should consider any variations in attached auxiliaries in their relation to power, fuel consumption, weight, and additional auxiliary power required.

Horsepower and Fuel-consumption Guarantee. Precise determination of power output after installation in a vessel is rarely possible. For this reason tests for fulfillment of horsepower and fuel consumption guarantees are limited to the engine builder's test stand. Fuel-consumption guarantees are made in terms of fractions of a pound per net brake horsepower per hour at rated power and speed. Such guarantees are based on fuel having a high heat value of 19,350 Btu per lb.

For nonsupercharged engines the power and fuel consumption guarantees are contingent on the following conditions:

1. Intake air temperature between 40 and 90 F.
2. Barometric pressure of intake air between 29 and 30 in. Hg.
3. Fuel oil to conform to manufacturer's specifications for the type of engine under test.

All fuel-consumption guarantees are subject to a tolerance of 5 percent, owing to variations of intake-air density caused by atmospheric temperature and barometric pressure, and to variations in low-heat value of fuels having the same high-heat value. Fuel-consumption guarantees are not made at other than rated speeds and load except for constant-speed engines for electric drive. The association has not yet established a basis for fuel-consumption guarantees for supercharged engines.

Critical Speeds. *Although reasonably accurate predetermination of* critical speeds can be made as far as the entire propulsion assembly as a rotating system is concerned, the uncertainties of propeller and water flow to propeller make it impracticable to guarantee the exact location and amplitude of torsional vibration.

Equipment. To suit the variable requirements of different types of engines the association has established a minimum list of equipment to be supplied with every marine engine with certain additions to be applied according to whether the engine is (1) reverse-geared, (2) direct-connected, reversible, (3) reduction-geared, direct reversible, or (4) the main engine for electric drive or the auxiliary engine for electric power.

Additional equipment is supplied as required by the purchaser.

Spare Parts. Most ships are classified with one or more of the classification societies, and the spare parts supplied with engines for such ships conform to the societies' requirements. For unclassified vessels it is standard practice of the association to furnish certain minimum spares, which are specified with the bid.

cetane and alpha-methyl-naphthalene. The former ignites very easily and is given an arbitrary scale number of 100, while the latter is very hard to ignite and is given a rating of zero. The procedure is to determine in a special test engine, known as the CFR engine, the angle of crankshaft rotation between the time injection begins and the time ignition occurs, when the fuel being tested is injected. The standard reference fuel is then tested until a mixture of cetane and alpha-methyl-naphthalene is found, the delay angle of which will be the same as that of the test fuel. The percentage of cetane in this mixture will then be the cetane number of the fuel tested.

Ash. The solid matter remaining after a sample of the fuel has been burned in a test dish is called ash and is expressed as a percentage by weight of the fuel burned. In general it includes the abrasive impurities that may cause scoring of the liners and rings and should be as low as possible, the usual figure being 0.1 percent.

Fuel Classification. Some years ago it was thought that a specification could be developed that would produce a single fuel suitable for all diesel engines. Subsequent extensive research definitely established the fact that no one standard fuel can be suitable for all engines. The only way in which standardization can be accomplished is by providing several different fuels on the basis of high-speed, medium-speed, and slow-speed engines. Research work to date is by no means conclusive but the American Society for Testing Materials has established a tentative classification of fuel oils as shown in Table 2.

Table 2. A.S.T.M. Proposed Diesel Fuel Classification

Characteristic	Grade				
	1-D	3-D	4-D	5-D	6-D
Flash point, deg F, min[a]	115 or legal	150	150	150	150
Water and sediment, % by volume, max	0.05	0.1	0.6	1.0	2.0
Viscosity, SSU at 100 F	Min[b] 32 Max 50	32 70	 250		
Viscosity, SSU at 122 F, max	......	......	......	100	300
Carbon residue, % by wt, max	0.2	0.5	3.0	6.0	
Ash, % by wt, max	0.02	0.02	0.04	0.08	
Pour point, deg F, max[c]	35	35	35		
Sulfur, % by weight, max[d]	1.5	1.5	2.0	2.0	
Cetane no., min	50	40	30		
Diesel index, min	45	30	20		
Viscosity-gravity No., max	0.86	0.89	0.91		
Boiling point-gravity No; max	188	195	200		

[a] As stated or as required by local fire regulations, Underwriters, or state laws.
[b] For viscosities below 35 SSU at 100 F, other methods may be used and results converted.
[c] Lower pour points may be specified when required by local conditions, though less than 0 F should not be necessary.
[d] Not ordinarily of consequence as regards combustion.

Fuel-oil Tables. The values given in the tables that follow should be considered as approximations only, but they are close enough to the true values to serve as a guide to the general relation between the properties of any given oil that may be encountered in service.

Wrenches and Tools. All necessary special tools and wrenches that cannot be purchased readily in the open market are supplied as standard engine equipment, but no duplication of tools is made for two or more engines in the same engine room. *Other* tools and wrenches of commercial types may be supplied by agreement between buyer and seller.

Engine Classification. It is the general practice of members of the association to construct all engines to comply with the rules of Lloyd's and the American Bureau of Shipping, but inspection and certification by the societies are optional with the purchaser and at his own expense.

Fuel Specifications. *Members of the association will recommend some* one of the A.S.T.M. grades of diesel fuel (tabulated in a preceding section) for each of their engines and favor the buying of fuel by A.S.T.M. designation.

Foundations. Foundations are not a part of the engine or inclusive in the engine contract, but the engine manufacturers will cooperate with naval architects and shipbuilders by suggesting changes of design that may be helpful in obtaining suitable foundations.

Torsional Calculations. Members of the association assume responsibility for the accuracy of torsional vibration data and calculation of critical speeds, but they have no control over the hydraulic conditions under which the propellers will operate. Hence they cannot guarantee final results.

DESIGN CONSIDERATIONS

Since several factors, such as number of cylinders, bore and stroke, mep, and piston speed, must be taken into account when designing a diesel engine and their relation is such that any change in one causes a changes in one or more of the others, it is customary to choose one of these factors as the basis for the design and adjust the others to suit. A commonly used starting point for design is the selection of the piston speed desired.

Piston Speed. In the early days of diesel-engine design piston speed, expressed in feet per minute by multiplying the rpm by twice the piston stroke in feet, usually fell within the range 600 to 700 fpm. In succeeding years there has been a continued increase in piston speed so that some engines built today have a piston speed of 2,000 fpm. The divergence of opinion among designers in regard to piston speed is shown by Table 6 which is the result

Table 6. *Variation of Piston Speed and Piston Stroke*

Stroke range, in.	Piston speed range, fpm	No. of engines checked
	Four-cycle engines	
4–10	600–2,000	80
10–15	660–1,800	48
15–20	600–1,550	38
20–32	900–1,200	19
	Two-cycle engines	
4–10	600–1,670	9
10–15	600–1,440	14
15–20	735–1,000	8
20–36	1,000–1,275	7

$f(x) = 0$, where $f(x)$ means any function of x, and plot the curve $y = f(x)$ for a sufficient number of values of x to obtain a general idea of the shape of the curve. Then pick out the regions in which the curve appears to cross the axis of x, and plot the curve more accurately in each of these regions. Thus, by successive approximations, plotting the important parts of the curve on a larger and larger scale, determine as accurately as necessary the points where the curve crosses the axis—that is, the values of x which make $f(x)$ equal to zero.

Thus, suppose that $f(x) = 3.0$ when $x = 2.6$ and -5.0 when $x = 2.7$ (see Fig. 1). Then the curve must cross the axis somewhere between $x = 2.6$ and $x = 2.7$; and since it will not vary greatly from a straight line between those points, it is seen that it must

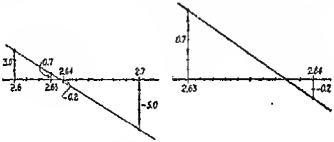


FIG. 1.

cross near 2.64. Suppose the value of $f(x)$ when computed for $x = 2.64$, is -0.2, and when computed for $x = 2.63$ is $+0.7$; then the root lies between $x = 2.63$ and 2.64. Plotting this section on the larger scale, it is seen that the next guess should be about 2.638; and so on.

Instead of writing the original equation with all the terms on the left-hand side, it is often better to divide the expression into two parts, say $f_1(x)$ and $f_2(x)$, writing the equation in the form $f_1(x) = f_2(x)$. If then the two curves $y_1 = f_1(x)$ and $y_2 = f_2(x)$ be plotted separately, on the same diagram, the value of x corresponding to their point of intersection will be the desired root.

SOLUTION OF SIMULTANEOUS EQUATIONS

The Meaning of a System of Simultaneous Equations. To solve a system of n simultaneous equations in n unknowns, means to find all the sets of values of the unknowns (if any) which, when substituted in the given equations, will satisfy all the equations at the same time. If a system of equations has no solution, the equations are "inconsistent"; if it has an infinite number of solutions, the equations are "not all independent."

Simultaneous Equations of the First Degree in Two Unknowns.

$$\begin{array}{l|c|c|}
 & \multicolumn{2}{c}{\text{Factors}} \\
(1)\ a_1x + b_1y = c_1 & b_2 & -a_2 \\
(2)\ a_2x + b_2y = c_2 & -b_1 & a_1 \\
\hline
\end{array}$$

$$(a_1b_2 - a_2b_1)x = b_2c_1 - b_1c_2 \qquad \therefore x = (b_2c_1 - b_1c_2)/(a_1b_2 - a_2b_1)$$
$$(a_1b_2 - a_2b_1)y = a_1c_2 - a_2c_1 \qquad \therefore y = (a_1c_2 - a_2c_1)/(a_1b_2 - a_2b_1)$$

Here (1) is multiplied by b_2, (2) by $-b_1$, and the products added so as to eliminate y; again, (1) is multiplied by $-a_2$, (2) by a_1, and the products added so as to eliminate x. (The process is most conveniently performed as follows: Write the multipliers, as b_2 and $-b_1$, at the right of the equations; multiply the first term of each equation by its proper multiplier and add; then multiply the second term of each equation by its proper multiplier, and add; and so on. This is simpler than the common practice of multiplying out each equation separately before adding.) If $a_1b_2 - a_2b_1 = 0$, the equations have no solution when $c_1 \neq c_2$, and an infinite number of solutions when

$c_1 = c_2$. The following **special solution** is possible when the sum and difference of the two unknowns are given:

$$\begin{array}{lll} & \text{Let } x + y = m & (1) \\ & \text{and } x - y = n & (2) \\ (1) + (2): & 2x = m + n & \therefore x = \tfrac{1}{2}(m + n) \\ (1) - (2): & 2y = m - n & \therefore y = \tfrac{1}{2}(m - n) \end{array}$$

Simultaneous Equations of the Second Degree in Two Unknowns.
(a) When the product of the unknowns, and their sum or difference, are given:

	$x + y = 5$ (1)	$x - y = 3$ (1)
	$xy = 4$ (2)	$xy = 4$ (2)
Squaring (1),	$x^2 + 2xy + y^2 = 25$	$x^2 - 2xy + y^2 = 9$
From (2),	$-4xy = -16$	$4xy = 16$
Adding,	$x^2 - 2xy + y^2 = 9$	$x^2 + 2xy + y^2 = 25$
Hence,	$x - y = 3$ or -3	$x + y = 5$ or -5
But	$x + y = 5$ or 5	$x - y = 3$ or 3
Therefore	$\begin{array}{c} x = 4 \\ y = 1 \end{array}$ or $\begin{array}{c} x = 1 \\ y = 4 \end{array}$	$\begin{array}{c} x = 4 \\ y = 1 \end{array}$ or $\begin{array}{c} x = -1 \\ y = -4 \end{array}$

(b) When the product and the sum of the squares are given:

$$\begin{array}{lrl} & xy = 5 \ (1) & \sqrt{(4)}: x + y = 6 \text{ or } \ 6 \text{ or } -6 \text{ or } -6 \\ & x^2 + y^2 = 26 \ (2) & \sqrt{(5)}: x - y = 4 \text{ or } -4 \text{ or } \ 4 \text{ or } -4 \\ \text{From (1),} & 2xy = 10 \ (3) & \begin{array}{l} \therefore x = 5 \\ \therefore y = 1 \end{array} \text{ or } \begin{array}{r} 1 \\ 5 \end{array} \text{ or } \begin{array}{r} -1 \\ -5 \end{array} \text{ or } \begin{array}{r} -5 \\ -1 \end{array} \\ (2)+(3): & x^2 + 2xy + y^2 = 36 \ (4) & \\ (2)-(3): & x^2 - 2xy + y^2 = 16 \ (5) & \end{array}$$

(c) When the sum or difference, and the sum of the squares, are given:

	$x + y = 5$ (1)		$x - y = 3$ (1)
	$x^2 + y^2 = 17$ (2)		$x^2 + y^2 = 17$ (2)
$(1)^2$:	$x^2 + 2xy + y^2 = 25$	$(1)^2$:	$x^2 - 2xy + y^2 = 9$
(2) :	$x^2 + y^2 = 17$	(2) :	$x^2 + y^2 = 17$
$(1)^2 - (2)$:	$2xy = 8$	$(1)^2 - (2)$:	$-2xy = -8$
	$xy = 4$		$xy = 4$
Then proceed as in case (*a*), above.		Then proceed as in case (*a*), above.	

(d) When one equation is of the first degree and the other of the second, as $ax + by = c$, and $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$: Solve the first equation for y in terms of x, and substitute in the second. This will give a quadratic equation in x. Solve this quadratic for the two values of x, and for each of these values of x find the corresponding value of y by substituting in the equation of the first degree.

Simultaneous Equations of the First Degree in n Unknowns. For example:

		Factors		
(*a*)	$2x - y + 3z + 5w = 29$	3	1	2
(*b*)	$5x + 2y - 2z + 3w = 15$	-5		
(*c*)	$3x - 4y + 7z - w = 12$		5	
(*d*)	$4x + 3y - 5z + 2w = 3$			-5
(*e*)	$-19x - 13y + 19z = 12$	-2	-31	
(*f*)	$17x - 21y + 38z = 89$	1		
(*g*)	$-16x - 17y + 31z = 43$		19	
(*h*)	$55x + 5y = 65$	16		
(*i*)	$285x + 80y = 445$	-1		

of a check of 185 different makes of engines. This table shows the range of speeds over different ranges of stroke. It is interesting to note that, as the stroke becomes greater, the range of piston speeds becomes less, also the range of piston speeds of two-cycle engines is narrower than that of four-cycle. The tabulated values were obtained from existing engines but there is no reason to assume that future development will not produce higher speeds.

Importance of Piston Speed. The principal reason for the heretofore general acceptance of piston speed as the basis of design is that the rate of wear of liners and piston rings is directly proportional to it although some authorities claim that other factors may be of greater importance. This takes no account of the influence of rotational speed on inertia and centrifugal forces, the two factors that have an influence on the stresses set up in the engine, the balancing, and noise in operation.

The formula for inertia force of reciprocation parts is (see p. 855)

$$F = 0.0000284N^2Wr(\cos a + C \cos 2a)$$

where N = rpm
W = weight of reciprocating parts
r = crank radius, in.

$$C = \frac{r}{l}$$

l = length of connecting rod, in.
a = crank angle

The formula for centrifugal force due to rotating parts is

$$F = \frac{WV^2}{gR}$$

where W = weight of two-thirds of connecting rod
V = velocity of crankpin
g = acceleration due to gravity
r = radius of crank, in.

It may be seen that the velocity of the crankpin enters into inertia and centrifugal forces; thus the rate of rotation is an important factor.

Piston speed alone takes no account of the variations in rates of rotation in different engines since a long-stroke engine turning at slow speed may have the same piston speed as a short-stroke engine turning at high speed. It appears then that rpm is not a satisfactory basis on which to rate an engine as high-speed or slow-speed.

A more definite classification of engines as low-, medium-, and high-speed on the basis of both rotative speed and piston speed is proposed by Prof. V. L. Maleev, who suggests a speed characteristic, designated as C_s, which is

$$C_s = \frac{NC}{100{,}000}$$

where N = rpm
C = piston speed, fpm

Substituting in this formula the expression for piston speed $C = Nl/6$, in which l is the length of stroke in inches, it becomes

$$C_s = \frac{N^2l}{600{,}000}$$

M_t is no longer nearly constant. In order to determine the resisting or stability moment at a specific large angle of inclination, the problem becomes one of determining the shift of the center of buoyancy and the measurement of the normal distance between F_b and F_w, the resisting or stability moment being the force couple.

It is to be noted that the stability moment at small angles is dependent upon the value of I_t, V, and the location of B and G, and that for large angles the stability moment is dependent upon the location of G and the magnitude of the shift of B. The shift of B is dependent directly upon the hull form.

By inspection of Fig. 2 it is apparent that the condition of positive stability requires that the point G be below the metacenter, M. In the event that the point G is above the metacenter, the tendency of the ship will be to roll to an angle of inclination at which equilibrium will result. Such is the condition of negative stability and, if it is sufficiently large, capsizing may result. In the event that the point G and the metacenter are coincident, a condition of indifference will result.

It is particularly pertinent to note that the presence of free liquid surface in the ship will cause a reduction in the $\overline{GM}$ or stability of the ship. It may be readily shown that the following relationship exists:

$$\overline{gm} = \frac{i}{V} p \tag{14}$$

where $\overline{gm}$ = reduction in $\overline{GM}$, ft
- i = moment of inertia, ft^4 of the free surface about a fore-and-aft-axis through its center of gravity
- V = displacement of the whole ship, cu ft
- p = ratio of the density of the free surface liquid to the sea water displaced by the ship

Therefore, the actual stability is $\overline{GM} - \overline{gm}$. Attention is invited to the seeming paradox that the reduction in stability, $\overline{gm}$, is wholly independent of the quantity of the free liquid.

Inclining Experiment. Calculation of the $\overline{GM}$ necessitates location of the center of gravity, G, of the ship. Its location by analytical means is a laborious program and, owing to the magnitude of the work, is apt to be in error. The location of the center of gravity may be easily determined relative to the metacenter by experiment. For small angles of inclination, the following relationship will directly determine the $\overline{GM}$:

$$\overline{GM} = \frac{M}{\tan\theta \times W} \tag{15}$$

where $\overline{GM}$ = metacentric height, ft
- M = a heeling moment, ft-tons
- θ = a small angle of heel, deg
- W = weight of ship, tons

The heeling moment, M, may be induced by moving a weight, w, athwartship a distance of d ft. The tan θ of Eq. (15) may be determined by use of a plumb-line pendulum. Letting l denote the length of the pendulum

Engines with the same C_s may be considered as having the same speed, regardless of variations in their rpm. Maleev classifies all engines with a characteristic below 3 as slow-speed, those with a characteristic between 3 and 9 as medium-speed, from 9 to 27 high-speed, and above 27 super high-speed.

It is interesting to note that there is no fixed relation between speed characteristic and stroke-bore ratio. About as many low-speed engines are found with a small stroke-bore ratio as there are high-speed engines with long stroke. In general the stroke-bore ratio of existing engines varies between 1.00 and 1.9, with an average value between 1.3 and 1.35.

Numbers of Cylinders. Most marine diesel engines are built with vertical in-line cylinders, with the V arrangement becoming more common where space and weight considerations are important. The number of cylinders will vary from 1 to 16, with the largest number found in the 6-cylinder group because of the better balance, as noted in a previous section. Since the weight of an engine per horsepower decreases as the number of cylinders is increased the 8-, 10-, 12-, and 16-cylinder arrangements are used where low specific weights are desirable. Also they offer a means of increasing the engine power without increasing the height and width.

General Procedure. Although a detailed analysis of the step-by-step procedure in designing a new engine cannot be presented here, the procedure can be outlined briefly. The starting point is the selection of the principal characteristics such as the speed characteristic previously explained, the stroke-bore ratio L/D, and the mep P. This latter value and the compression ratio R are sufficiently standardized in practice to permit their selection on the basis of previous experience, without calculation. The number of cylinders will be determined by the factors mentioned previously.

From Maleev's method of analysis, the basis of computation for determining the revolutions and cylinder diameter is the standard formula for horsepower, $hp = PLAN/33{,}000$, but to make it more workable it is modified by expressing L, A, and N in terms of D as $L/D = Q$. The revolutions will then be

$$N = 774.6\sqrt{\frac{C_s}{QD}}$$

and for the two-cycle single-acting cylinder

$$hp = 0.001534PD^{2.5}\sqrt{QC_s}$$

from which the cylinder diameter will be

$$D = \left(\frac{651.5\text{ hp}}{P\sqrt{QC_s}}\right)^{0.4}$$

For the four-cycle engine with half as many working strokes

$$D = \left(\frac{1{,}303\text{ hp}}{P\sqrt{QC_s}}\right)^{0.4}$$

Dimensions. After the principal characteristics have been determined, the dimensions of the various parts are calculated on the basis of a theoretical indicator diagram, from which the stresses due to gas pressure, pressure acting on the piston, inertia forces, etc., can be determined.

in feet and a denote the transverse movement of the pendulum, Eq. (15) may be expressed as follows:

$$\overline{GM} = \frac{w \times d}{\dfrac{a \times W}{l}} \tag{16}$$

The plumb line should be swung in a hatchway to afford maximum length. It is customary to dampen the action of the pendulum by immersing the plumb bob in a bucket of oil. Needless to say several plumb lines should be used and judiciously located forward and aft, the results averaged and checked. Care should be taken to perform the experiment under conditions of slackened mooring lines, calm water, and light winds in order to eliminate unknown errors. The condition of the ship should be carefully noted in regard to ullage of all tanks, foreign weights, and the $\overline{GM}$ should be corrected for free liquid surfaces in accordance with Eq. (14). It is to be remembered that the $\overline{GM}$ derived from the inclining experiment represents the stability at the time of the experiment and that any subsequent changes, additions, or removal of weights must be accounted for.

Effect of Shifting of Weights. A very useful principle, often used in stability calculations, for determining the **shift of the center of gravity** of a weight system is as follows: Assume the notation, w = weight shifted; W = total weight of the whole system; d = distance through which w has been moved; c.g. = center of gravity of the whole system.

$$\text{Shift of c.g.} = \frac{w \times d}{W} \tag{17}$$

The direction of the shift will be parallel to the movement of w.

Example. A weight of 80 tons is vertically raised 30 ft from the hold to the main deck of a ship of 10,000 tons displacement. The shift of center of gravity, in this case the rise of center of gravity, will be $\frac{80 \times 30}{10{,}000}$, or 0.24 ft. In event the weight is moved horizontally 20 ft, the center of gravity will shift horizontally aft $\frac{80 \times 20}{10{,}000}$, or 0.16 ft. This principle may be used for determining the shift of the center of gravity caused by adding additional weight by first assuming that the added weight is placed aboard the ship at the center of gravity of the ship, then assuming the weight is moved to its assigned location and calculating the shift of the center of gravity in a manner similar to the above example. The removal of a cargo weight may be handled likewise, but in reverse order.

Transverse Dynamic Stability. For any particular angle of heel the amount of work required to heel the ship to that angle is called the dynamic stability for the particular inclination. Thus the dynamic stability, at any angle θ, equals $\int_0^\theta D \times GZ\, d\theta$. In other words, the area under a **curve of stability or righting moments** plotted against an angle of heel of θ is the dynamic stability. Such a curve can be constructed and the area measured. The character of the curve of righting moments is usually such that the righting moment is proportional to the angle of heel up to an inclination

SECTION 9

HULL AND PROPULSION

BY

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whereby the deck edge becomes immersed. This angle of inclination is usually the inflection point of the curve. For ships of positive stability, the slope of the curve is positive to the inflection point.

Period of Roll. The period required for a ship to roll from the vertical to the port, returning past the vertical to the starboard, and thence returning to the vertical is termed the period of roll.

For angles of roll up to and approaching the inflection point of the curve of stability moments, and assuming simple harmonic oscillation, the period of roll may be determined with reasonable accuracy by the following formula:

$$T = \frac{1.108P}{\sqrt{\overline{GM}}} \tag{18}$$

where T = period as defined, sec

$\overline{GM}$ = transverse metacentric height, ft

P = radius of gyration, ft, of the mass of the ship about a fore-and-aft axis through its center of gravity. An average value of 0.4 times the beam may be used for most merchant vessels.

From an inspection of Eq. (18) it will be noted that an increase in $\overline{GM}$ will shorten and that an increase of P will lengthen the period of roll.

Forces Due to Dynamic Effect of Rolling. Rolling induces forces upon various parts of a ship due to the acceleration of the roll and the inertia effect of the mass of the various parts. The maximum velocity will occur at the vertical position and diminish to zero at the extremities of the roll. Therefore, the acceleration will be maximum at the extremities, and the maximum dynamic effect will occur at the maximum angle of roll. The tangential force may be determined as follows:

$$F = \left(\frac{W \times d}{46.7T^2}\right)\theta \tag{19}$$

where F = tangential force, lb

θ = maximum angle of roll, deg

T = period of complete roll, sec

W = weight of ship part being considered, lb

d = distance from the center of roll (see Center of Flotation, p. 1385) to center of gravity of the ship part

Longitudinal Stability. Analytically, the problem of longitudinal stability is similar to that for transverse stability except that the longitudinal metacentric radius is expressed by the following relationship:

$$\overline{BM}_L = \frac{I_L}{V} \tag{20}$$

where $\overline{BM}_L$ = longitudinal metacentric radius, ft (height of longitudinal metacenter above the center of buoyancy)

I_L = moment of inertia of the water plane of flotation about a transverse axis through the center of flotation

V = displacement of vessel, cu ft

BOWES DRIVE

Staff Contribution
Based on data from the Elliott Company

It is apparent that I_L will be considerably larger than I_T in the case of transverse stability. It follows that the longitudinal metacentric height, $\overline{GM}_L$, will be many times larger than the transverse metacentric height. The longitudinal metacentric height is of interest principally in determining changes in trim.

Trim. The difference between the draft at the bow and at the stern is termed the trim. **Change in trim** is simply the rotation of the ship about a transverse axis through the center of flotation whereby opposite changes in draft occur at the bow and the stern.

Movement, addition, or removal of weights forward or aft of the center of flotation will induce a change in trim in accordance with the following relationship:

$$\text{Change in trim} = \frac{L \times w \times a}{D \times \overline{GM}_L}\ \text{ft} \tag{21}$$

where L = length of water line, ft
w = weight shifted, tons
a = distance weight is shifted in a fore or aft movement, ft
D = displacement of ship, tons
$\overline{GM}_L$ = longitudinal metacentric height, ft

The **trimming moment** is equal to $w \times a$; therefore,

$$\text{Moment to trim 1 in.} = \frac{D \times \overline{GM}_L}{12 \times L} \quad \text{ft-tons} \tag{22}$$

and the total change in trim at bow and stern in inches is equal to the trimming moment divided by the moment to trim 1 in. Allocation of the change in draft at the bow and the stern is proportional about the center of flotation as follows:

$$\begin{matrix}\text{Change of draft}\\ \text{at stern}\end{matrix} = \left(\frac{\begin{matrix}\text{distance from}\\ \text{after perpendicular}\\ \text{to center of flotation}\end{matrix}}{\text{water line length}}\right) \times \text{change in trim} \tag{23}$$

$$\begin{matrix}\text{Change of draft}\\ \text{at bow}\end{matrix} = \left(\frac{\begin{matrix}\text{distance from}\\ \text{forward perpendicular}\\ \text{to center of flotation}\end{matrix}}{\text{water line length}}\right) \times \text{change in trim} \tag{24}$$

For changes in trim caused by the removal of weights, it is necessary to make an imaginary assumption that the weight is first moved over the center of flotation of the ship and the change in trim is calculated. Then assuming direct removal of the weight, the change of draft due to lightening of the ship may be directly calculated and the final draft ascertained. The effect of the addition of weights may be handled similarly in the reverse order.

NAVAL ARCHITECTURE

BY

E. BRYCE McCROHAN, Jr.

REFERENCES: Taylor, "Speed and Power of Ships," Ransdell, Inc., Rossell and Chapman, "Principles of Naval Architecture," Vol. II, Soc. Nav. Arch. and Mar. Engrs. Marks, "Mechanical Engineers' Handbook," McGraw-Hill. Sterling, "Marine Engineers' Handbook," McGraw-Hill. *Trans. Soc. Nav. Arch. and Mar. Engrs.*

GENERAL

The characteristics of the final completed design of a ship are the result of many compromises. A naval architect defines the character of a ship to suit many conflicting requirements. To be successful, it must be designed to suit the requirements of the service for which it is intended and must conform to the rules and regulations as applicable. The hull form and the dimensions of a ship are the primary characteristics upon which it is built. It is the duty of the naval architect to determine the primary characteristics of a ship and to coordinate them with the requirements of the marine engineer. The primary function of the marine engineer is to design and arrange the power plant of the ship and to coordinate the same to suit the overall requirements. It is the purpose of this section to provide the marine engineer with knowledge necessary to make preliminary estimates of hull characteristics.

It is necessary that the marine engineer be fully cognizant of the basic fundamentals that may influence the design and also be aware of the source of the data from which he must work. The character of the hull is of prime importance to his work.

The character and form of the hull are defined by the architect on the **line draft** plan, which is a graphical, geometrical representation of the hull form. It usually consists of three geometrical projections, as follows:

1. A **profile** or **sheer plan** which shows the outboard profile and the shape of the hull in elevation. The profile is usually constructed on a base line or base plane from which all dimensions are taken upward. Water planes are represented as water lines on the profile and are spaced at convenient and regular intervals parallel to the base line. Buttock lines are represented on the profile usually as curved lines and represent the trace on the hull surface of the intersection of planes parallel to the medial centerline of the ship, spaced at regular, convenient intervals, outboard and normal to the base plane.

2. The **half-breadth plan** indicates the shape and contour of half water planes in plan view as shown on the profile plan. The projections of the buttock lines are represented as straight lines spaced as stated above and parallel to the centerline plane. The vertical projection of the deck edge is also indicated.

3. The **body plan** consists of two half end-view transverse elevations. To the right hand is usually indicated the fore body viewed from the bow and to the left the after body viewed from the stern which represents the trace of transverse planes as they intersect the boundary of the hull form. The water

SHIP RESISTANCE AND POWERING

Water Resistance. The movement of a ship through the water is induced by the application of a driving force or thrust exerted upon it. In the case of steam or motor ships the driving force or thrust is supplied by mechanisms, such as screw propellers or paddle wheels, driven by the ship's propulsion machinery. The driving force in a sailing vessel is derived directly from the reaction of the wind force in the sails; in the case of towed barges, etc., the driving force is simply the towrope pull. In all cases the movement of the ship through the water creates various reactionary or retardant forces equal and opposite to the driving force. These reactionary or retardant water forces are termed **ship resistance** and may be expressed as follows:

$$R_t = R_f + R_r \tag{25}$$

where R_t = total bare-hull resistance, lb
R_f = frictional or skin resistance, lb
R_r = residual resistance, lb

These are described as follows:

1. **Frictional resistance** is the force required to overcome eddying water close about the wetted surface caused by the roughness of the surface and the ship's movement through the water.

2. **Residual resistance** is the difference between the total resistance and the frictional resistance and may be said to comprise the following:

a. **Wave-making resistance** or the absorption of energy resulting principally in the creation of wave systems set up in the water by the variations of the water pressure induced by the flow of water from bow to stern around and about the hull.

b. **Eddy-making, streamline or form resistance** resulting from the absorption of energy in setting up eddies of water at the after portion of the hull, owing to failure of the water to flow smoothly in streamline fashion about the hull. In most cases this resistance is relatively small for vessels of normal ship forms driven at ship speeds. It is usually neglected without serious error.

Frictional or Skin Resistance. The frictional resistance may be closely estimated by the following formula:

$$R_f = f \times A \times V^n \tag{26}$$

where R_f = frictional resistance, lb
f = a friction constant which varies with the length and roughness of the wetted surface
A = area of wetted surface, sq ft [Eq. (10)]
V = speed of vessel through the water, knots
n = an exponent

Froude established values of the friction constant for various lengths of hulls as given in Table 1 where the value of the exponent, $n = 1.825$. Thus for Froude's values of the friction constant, Eq. (26) is $R_f = f \times A \times V^{1.825}$.

Tideman established values of the friction constant and the exponent n of Eq. (26) for various lengths of hulls and characters of the wetted surface as given in Table 2.

planes are represented as parallel lines spaced as shown on the profile plan and parallel to the base plane. Similarly, the buttock lines are represented as vertical lines parallel to the centerline plane. Usually the deck edge is curved in both the half-breadth and profile plan and, therefore, shows as a curved line on the body plan.

The bounding surface of the hull form as shown on the line draft is defined as the molded form. Its dimensions are defined as molded dimensions.

PRINCIPAL DIMENSIONS OF THE HULL

The hull dimensions are as follows:

Length over-all is the length between the foremost to the aftermost part of the ship.

Length between perpendiculars is the distance between two vertical lines erected normal to the base plane. The forward perpendicular is located at the forward edge of the stem and passes through the designed water line at which the vessel is expected to float. The after perpendicular is usually located on single-screw vessels at the after edge of the rudder stock. On multiscrew vessels it usually passes through the after edge of the sternpost or the aftermost part of the designed water line. The location of the after perpendicular is subject to some variation, depending more or less on the type of the stern. For purposes of calculation it is a distinct advantage to locate the after perpendicular at the aftermost part of the designed water line.

Beam (molded) is the extreme breadth of the hull form as shown on the line draft.

Beam (extreme) is the extreme over-all breadth of the ship measured to the outside of the hull structure.

Depth (molded) is the vertical distance measured amidships, from the top of the keel plate to the top of the weather deck beams at the deck edge amidships.

Draft is the greatest vertical distance from the bottom of the keel to the water plane at which the ship is floating.

Displacement is the amount of water displaced by the immersed part of the vessel in either cubic dimensions of volume or the weight of the water so displaced. By Archimedes' principle the ship will displace a weight of water equal to the weight of the ship and everything on board. The density of sea water averages about 64 lb per cu ft (1 long ton, 2,240 lb, equals 35 cu ft); hence, the displacement in sea water is measured by the immersed volume at the water line divided by 35. In fresh water the corresponding divisor is usually taken as 36.

Tonnage is an arbitrary measure of the capacity of the entire enclosed portion of the ship including deckhouses and enclosures, assuming that 100 cu ft represents 1 ton. **Gross tonnage** refers to the total enclosed capacity, and **net tonnage** refers to enclosed capacity after deducting the space that is necessary to operate the ship, such as engine rooms, fuel, crew, and similar spaces. In general, net tonnage refers to the space available for earning capacity. Methods of measurement and taking of dimensions are defined by law and vary for the various maritime nations.

Hull Coefficients of Form. Assuming the following notation: L = length of water line, ft; B = beam, ft (usually molded beam); H = draft, ft; D = displacement in tons of sea water (2,240 lb per ton); V = volume of sea water displaced, cu ft; A = area of water plane, sq ft; M = area of amidship transverse section below the water line, sq ft, then

Table 1.[a] Froude's Friction Constants for Ships in Salt Water

Length, ft	f	Length, ft	f	Length, ft	f
5	0.012585	23	0.010361	140	0.009085
6	0.012345	24	0.010311	160	0.009046
7	0.012126	25	0.010269	180	0.009016
8	0.011932	26	0.010224	200	0.008992
9	0.011751	27	0.010182	250	0.008943
10	0.011579	28	0.010139	300	0.008902
11	0.011423	29	0.010103	350	0.008867
12	0.011282	30	0.010068	400	0.008832
13	0.011151	35	0.009908	450	0.008802
14	0.011033	40	0.009791	500	0.008776
15	0.010925	45	0.009691	550	0.008750
16	0.010829	50	0.009607	600	0.008726
17	0.010742	60	0.009475	700	0.008680
18	0.010661	70	0.009382	800	0.008639
19	0.010596	80	0.009309	900	0.008608
20	0.010524	90	0.009252	1000	0.008574
21	0.010468	100	0.009207	1100	0.008548
22	0.010413	120	0.009135		

For fresh-water values of f, multiply by ratio 62.4/64.0.

[a] From Taylor, "Speed and Power of Ships," Ransdell, Inc., and Rossell and Chapman, "Principles of Naval Architecture," Vol. II, Soc. of Nav. Arch. and Mar. Engrs.

Table 2.[a] Tideman Friction Constants for Ships in Salt Water

Length of ship, ft	Iron bottom clean and well painted		Copper- or zinc-sheathed: Sheathing smooth and in good condition		Copper- or zinc-sheathed: Sheathing rough and in bad condition	
	f	n	f	n	f	n
10	0.01124	1.8530	0.01000	1.9175	0.01400	1.8700
20	0.01057	1.8484	0.00990	1.9000	0.01350	1.8610
30	0.01018	1.8440	0.00983	1.8650	0.01310	1.8530
50	0.00991	1.8357	0.00976	1.8300	0.01250	1.8430
100	0.00970	1.8290	0.00966	1.8270	0.01200	1.8430
150	0.00957	1.8290	0.00953	1.8270	0.01183	1.8430
200	0.00944	1.8290	0.00943	1.8270	0.01170	1.8430
250	0.00933	1.8290	0.00936	1.8270	0.01160	1.8430
300	0.00923	1.8290	0.00930	1.8270	0.01152	1.8430
350	0.00916	1.8290	0.00927	1.8270	0.01145	1.8430
400	0.00910	1.8290	0.00926	1.8270	0.01140	1.8430
450	0.00906	1.8290	0.00926	1.8270	0.01137	1.8430
500	0.00904	1.8290	0.00926	1.8270	0.01136	1.8430

For fresh-water values of f, multiply by ratio 62.4/64.0

[a] From Taylor, "Speed and Power of Ships," Ransdell, Inc., and Rossell and Chapman, "Principles of Naval Architecture," Vol. II, Soc. of Nav. Arch. and Mar. Engrs.

$$C_b = \text{block coefficient of fineness} = \frac{V}{L \times B \times H} = \frac{35D}{L \times B \times H} \tag{1}$$

$$C_w = \text{water plane coefficient} = \frac{A}{L \times B} \tag{2}$$

$$C_m = \text{midship section coefficient} = \frac{M}{B \times H} \tag{3}$$

$$C_p = \text{longitudinal prismatic coefficient} = \frac{V}{M \times L} = \frac{35D}{M \times L} \tag{4}$$

$$C_q = \text{vertical prismatic coefficient} = \frac{V}{A \times H} = \frac{35D}{A \times H} \tag{5}$$

The following relationships exist between the hull coefficients of form:

$$C_p = \frac{C_b}{C_m}; \qquad C_q = \frac{C_b}{C_w} \qquad \frac{C_p}{C_q} = \frac{C_w}{C_m}; \qquad C_b = \sqrt{C_p \times C_q \times C_m \times C_w}$$

The values of hull coefficients that may be expected range about as follows:

	C_b	C_w	C_m	C_p	C_q
Freight steamers	0.65–0.80	0.75–0.85	0.85–0.95	0.75–0.85	0.80–0.95
Passenger liners	0.55–0.65	0.65–0.75	0.80–0.90	0.60–0.70	0.80–0.90
Battleships	0.60–0.65	0.70–0.75	0.85–0.92	0.65–0.70	0.80–0.90
Cruisers	0.50–0.55	0.60–0.65	0.80–0.90	0.55–0.60	0.75–0.85
Torpedo craft, yachts, etc.	0.35–0.55	0.55–0.60	0.75–0.85	0.50–0.60	0.65–0.80

Ratios. Various ratios of the dimensions of the immersed portion of the hull are used for comparative purposes and are as follows:

$$\text{Length-beam ratio} = \frac{L}{B} \tag{6}$$

$$\text{Length-draft ratio} = \frac{L}{H} \tag{7}$$

$$\text{Beam-draft ratio} = \frac{B}{H} \tag{8}$$

$$\text{Displacement—length ratio} = \frac{D}{\left(\frac{L}{100}\right)^3} \tag{9}$$

Curve of Sectional Areas. The curve of sectional areas is drawn to a convenient scale upon a base line as the abscissa equal in length to the water line and with the ordinates representing the cross-sectional areas of the immersed portion of the hull. The curve graphically represents the fore-and-aft distribution of the displacement. The shape of the curve has great influence on the resistance of the ship. The area under the curve to the base line represents the displacement V of the vessel, and the center of gravity represents the longitudinal position of the center of buoyancy (see p. 1385).

It is of interest to note that in some cases where barnacles and other marine growths increase the roughness of the wetted surface, the frictional resistance may be increased to values as much as 100 percent greater than that for smooth and clean bottoms. In extreme cases increases up to 300 percent have been noted.

Residual Resistance. Neglecting eddy and streamline resistance and assuming that the residual resistance is comprised principally of wave resistance, it may be said that the wave resistance of geometrically similar ships or models is proportional to their displacements when their speeds are proportional to the square root of their lengths. Such speeds are termed **corresponding speeds** and comparisons may be made when the ratio $V/\sqrt{L}$ (where V = speed, knots; L = length of water line, ft) of the ships or models being compared is the same. The ratio $V/\sqrt{L}$ is the speed-length ratio. The foregoing is known as **Froude's law of comparison** and is often expressed in many other ways.

"Geometrically similar" refers to ships or models whose underbodies have similar hull coefficients of form, dimension ratios, and whose hulls are geometrically similar with like curves of sectional areas.

Thus a system of estimating the residual resistance of a ship by comparison with that measured from existing ships or from the results of model tests is indicated by the law of comparison.

Methods of Estimating Resistance. By the use of the **frictional resistance** relationship and **Froude's law of comparison,** three methods are in common use for estimating the **total bare hull resistance** as follows:

1. *By Model Tests.* A reduced-scale model of the hull in question is constructed. The model is properly trimmed and ballasted to a draft as required, then towed at a proper **corresponding speed** in a ship-model testing tank and the **total resistance** is measured. Knowing the speed of the model, the area of the wetted surface, the frictional constant, and the exponent n of Eq. (26) from Tables 1 or 2, the frictional resistance of the model may be readily estimated. It is to be noted that the wetted surface of the model, when smoothed and in consideration of the scale, is closely similar to that of a clean and painted ship's wetted surface. By use of Eq. (25), the **residual resistance** of the model at a corresponding speed may be calculated. By the **law of comparison,** the residual resistance of the ship will be that of the model increased by the ratio of the displacement of the ship to the displacement of the model. For practical reasons, the water of testing tanks is usually fresh. Therefore, the ratio of displacements of ship to model must be multiplied by the ratio 64/62.4 (the ratio of densities of salt to fresh water). Again by the use of Eq. (26), the **frictional resistance** of the ship may be estimated. With the ship values of the residual and frictional resistance, Eq. (25) will give the **total bare-hull resistance.**

2. *Similar Ships.* Very often ships are designed by using an existing known hull of good repute as the parent or model and fashioning a new form from the known hull as a **parent design.** Where hulls are closely similar, excellent estimates of resistance may be made by using the parent hull as a model in a fashion similar to the method indicated for model tests in the preceding paragraph. Where vessels are dissimilar and the dissimilarity is such that its effect on the resistance may be estimated, very good results may be obtained.

3. *Taylor's Standard Series.* The late Rear Admiral D. W. Taylor, USN, by study of the results of many model tests made at the Washington Experi-

Wetted Surface. The area of the immersed portion of the hull in contact with the water is termed the wetted surface. Determination of this area is necessary in the estimation of the frictional or skin resistance. There are several more or less complex methods for estimating the wetted surface from the hull lines, expanded girths, etc., taken from the line draft. However, there are several simplified approximate methods for calculation. Taylor proposed the following formula which gives good results for ships of normal forms:

$$A = C\sqrt{D \times L} \tag{10}$$

where A = area of wetted surface, sq ft, of the hull proper not including appendages

D = displacement, tons 2,240 lb

L = length between perpendiculars or length of water line, ft

C = a coefficient dependent upon the beam-draft ratio, $\frac{B}{H}$ [Eq. (8)], and the midship section coefficient, C_m [Eq. (3)].

Taylor's contour curves of the coefficient may be selected from Fig. 1.

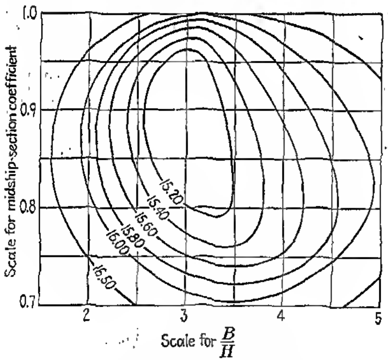


FIG. 1.—Taylor's contour curves.

For normal ship forms an average value for the coefficient of 15.6 is often satisfactory. Thus Eq. (10) is often represented as $A = 15.6\sqrt{D \cdot \times L}$. It is to be noted that the wetted surface of similar ships varies as the square of the ratio of their lengths.

Tons per Inch Immersion. The tons per inch of immersion is the weight in tons required to increase the draft by 1 in. and is expressed as follows:

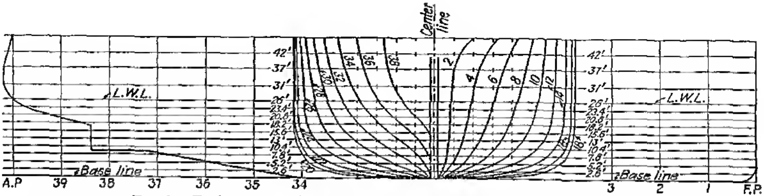


FIG. 3.—Taylor's parent-hull form—body plan and stem and stern profiles.

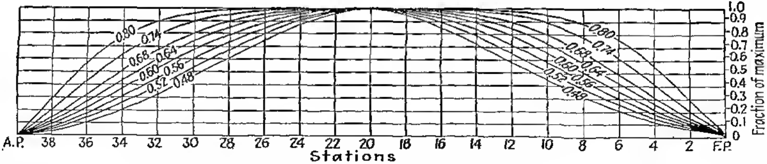


FIG. 4.—Taylor's parent-hull form—relative curves of section areas. Each curve has a corresponding longitudinal coefficient noted thereon.

$$\text{Tons per inch immersion} = \frac{\text{area of water plane}}{35 \times 12}, \quad \text{for salt water} \tag{11}$$

$$= \frac{\text{area of water plane}}{36 \times 12} \quad \text{for fresh water}$$

It is apparent that for vertical-sided hulls Eq. (11) will be exactly true and that for normal ships whose hulls are not wholly vertical sided, it will be closely true. In the latter case, the tons per inch of immersion will vary for each water line or depth of draft.

Center of Flotation. The **center of gravity** of the water plane at which the vessel is floating is the **center of flotation**. This is the **center of** rotation about which the ship rolls or pitches. **Rolling** is rotation about a fore-and-aft axis lying in the water plane. For small angles of roll it is assumed to be the fore-and-aft center line of the water plane (coincident with the centerline of the vessel). **Pitching** is rotation about a transverse axis through the center of gravity of the water plane whereby opposite changes in draft occur at the bow and stern.

Center of Buoyancy. The center of gravity of the displaced volume of the immersed portion of the hull is termed the center of buoyancy. The resultant force of all the upward buoyancy forces acts vertically through the center of buoyancy.

STABILITY

Archimedes' principle states, in part, that any buoyant body freely floating on the surface of a liquid will displace its own weight of the liquid. Assume that no external forces are applied to the body and that there are, therefore, only two forces acting: (1) The upward force of the water pressure distributed over the immersed portion of the body. (The resultant force of the summation of all buoyant forces will act upward through the center of gravity or the center of buoyancy of the displaced volume as the point of application.) (2) The weight of the body acting downward. (The resultant force of the summation of all the downward-weight forces will act downward through the center of gravity of the entire body as the point of application.)

When the body is floating in equilibrium, the two forces will act in the same vertical line and be equal in magnitude and opposite in direction. For any other condition, the body will tend to adjust its position on the surface of the liquid and seek such a position of equilibrium.

Similarly, the resultant upward-buoyancy force of the water upon a ship will act vertically upward through the center of buoyancy, and the total weight of the ship, including the weight of everything aboard, will act downward as a weight force and may be represented as a resultant force acting downward through the center of gravity of the whole ship. The condition for equilibrium requires that the weight force be equal and opposite to the force of buoyancy and lie in the same vertical line. In the event that the upward and downward forces are equal but do not lie in the same vertical line, the ship will adjust itself by changing its transverse and longitudinal position until equilibrium exists. For convenience, the problem of stability of a ship is resolved into transverse and longitudinal stability and analyzed separately as such.

Transverse Stability. Assuming that there is no change in the weight or the displacement of a ship and that a small angle of inclination, θ (from

mental Model Basin, evolved a parent hull form shown in Fig. 3 which had good resistance characteristics with hull proportions of proper stability and seagoing characteristics. A series of models were constructed with curves of areas as shown on Fig. 4 with prismatic coefficients [Eq. (4)] varying from 0.48 to 0.86 and with their displacement-length ratios [Eq. (9)] varying from 20 to 250. Two series of models were made with beam-draft ratios [Eq. (8)] of 2.25 and 3.75.

The models were towed in the testing tank at speeds giving **speed-length ratios** from 0.30 to 2.00. The **residual resistance** was calculated for these ratios. A family of curves of residual resistance in pounds per ton of displacement were drawn, with the prismatic coefficient as the abscissa and the displacement-length ratio as the ordinate, for each speed-length ratio. A series of charts were drawn for beam-draft ratios of 2.25 and 3.75.

In all cases the models had a common midship section coefficient [Eq. (3)] of 0.926. Taylor's experiments indicated that the midship coefficient could be varied between extremes of 0.70 to 1.10 with only minor variation in the residual resistance. The selection of a midship coefficient of 0.926 appeared to be the most economical, and the selection of the beam-draft ratios of 2.25 and 3.75 represented the extremes that were likely to occur in practice. Taylor also indicated that an increase in the beam-draft ratio from 2.25 to 3.75 made only a moderate increase in the resistance.

Table 3.[a] Residual Resistance per Ton (R_r/D). Taylor's Standard Series. $B/H = 2.25$

C_p	$\frac{D}{\left(\frac{L}{100}\right)^3}$	$\frac{V}{\sqrt{L}}=$ 0.60	$\frac{V}{\sqrt{L}}=$ 0.65	$\frac{V}{\sqrt{L}}=$ 0.70	$\frac{V}{\sqrt{L}}=$ 0.75	$\frac{V}{\sqrt{L}}=$ 0.80	$\frac{V}{\sqrt{L}}=$ 0.85	$\frac{V}{\sqrt{L}}=$ 0.90	$\frac{V}{\sqrt{L}}=$ 0.95	$\frac{V}{\sqrt{L}}=$ 1.00	$\frac{V}{\sqrt{L}}=$ 1.05	$\frac{V}{\sqrt{L}}=$ 1.10
0.50	50	0.41	0.50	0.69	0.85	1.10	1.20	1.40	1.65	2.00	3.15	5.25
	100	0.53	0.67	0.91	1.09	1.25	1.55	1.75	2.00	2.60	3.95	7.00
	150	0.62	0.78	0.99	1.20	1.40	1.80	2.20	2.55	3.30	4.90	8.40
	200	0.67	0.83	1.03	1.27	1.62	2.20	2.50	3.00	4.00	6.00	9.90
	250	0.71	0.86	1.07	1.29	1.96	2.30	2.90	3.60	4.90	7.50	13.00
0.60	50	0.49	0.62	0.83	1.22	1.61	2.05	2.70	3.80	5.40	6.20	6.80
	100	0.62	0.74	0.99	1.30	1.69	2.15	3.00	4.45	6.70	8.10	9.20
	150	0.68	0.83	1.05	1.34	1.71	2.20	3.20	4.85	7.60	9.00	10.60
	200	0.72	0.88	1.09	1.35	1.74	2.30	3.35	5.20	8.10	9.60	11.00
	250	0.74	0.92	1.12	1.36	1.79	2.48	3.60	5.60	8.50	10.00	11.50
0.70	50	0.80	1.02	1.32	1.83	2.34	3.35	4.70	7.30	11.50	14.30	15.00
	100	0.81	1.03	1.32	1.83	2.46	3.50	5.30	8.90	15.60	20.00	22.40
	150	0.81	1.03	1.33	1.83	2.46	3.50	5.45	9.75	17.80	24.00	28.20
	200	0.82	1.04	1.33	1.83	2.46	3.50	5.50	10.30	19.00	26.80	33.00
	250	0.83	1.06	1.34	1.83	2.50	3.65	5.50	10.60	20.00	28.90	35.70
0.80	50	1.00	1.27	2.00	3.52	6.70	9.30	10.30	13.20	19.30	25.50	28.00
	100	1.00	1.27	2.00	3.52	6.70	10.50	12.00	15.60	24.70	35.30	43.50
	150	1.00	1.27	2.00	3.52	6.70	10.90	12.50	16.80	26.70	40.20	52.50
	200	1.00	1.27	2.00	3.52	6.70	11.10	12.80	17.35	27.70	42.00	57.50
	250	1.00	1.27	2.00	3.52	6.70	11.30	13.20	17.60	28.30	41.50	

Where R_T = residual resistance, lb per ton of displacement; D = displacement tons, 2,240 lb; B = beam, ft; H = draft, ft; V = speed, knots; L = length of water line, ft; C_p = longitudinal prismatic coefficient.

[a] Condensed tables by Prof. K. S. M. Davidson from Rossell and Chapman, "Principles of Naval Architecture," vol. II, Soc. Nav. Arch. and Mar. Engrs.

0 to 5 or 10 deg) is induced by the application of an external overturning moment, M_o, it is readily discernible that the ship will roll from the normal water line, wl, to an inclined water line, w_1l_1, as shown in Fig. 2.

For small angles of θ, it may be assumed that the emerged wedge, W_1OW, will be similar to the immersed wedge, L_1OL and, since no change in displacement has been assumed, it may therefore be said that the axis of rotation will be about the point O (the fore-and-aft axis through the center of flotation). The center of buoyancy, B, will shift accordingly, to some point B_1. The resultant weight force, F_w, of the ship will act downward through the center of gravity, G, of the ship as the point of application, and the resultant upward-buoyancy force, F_b, will act vertically upward through the center of buoyancy, B_1, as the point of application. It is apparent that a force couple will result tending to resist the overturning moment, M_o. The value of this resisting couple equals $F_w \times \overline{GZ} = F_b \times \overline{GZ}$, where $\overline{GZ}$ is the normal distance between F_w and F_b. The distance $\overline{GZ}$ is termed the **righting lever.**

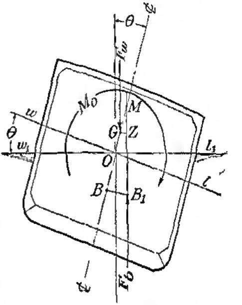


FIG. 2.—Transverse stability.

For small angles of θ the line of action of the force F_b will intersect the vertical centerline of the ship at a point M, which for small angles is very nearly constant. The point M is termed the **metacenter.** Letting $\overline{GM}$ denote the distance between the points M and G, it is apparent that $\overline{GZ} = \overline{GM} \sin \theta$ and that

$$\text{Resisting stability moment} = (\overline{GM} \times \sin \theta)F_b = (\overline{GM} \times \sin \theta)F_w. \quad (12)$$

For small angles $\overline{GM}$ is very nearly constant, and it is obvious that $\overline{GM}$ is, therefore, a measure of stability for such angles of inclination. $\overline{GM}$ is termed the **metacentric height** and, as such, is known as the stability of the ship.

The distance $\overline{BM}$ between the center of buoyancy, B, and the metacenter, M, is termed the **metacentric radius.** It can be easily proved that

$$\overline{BM} = \frac{I_t}{V} \quad (13)$$

where I_t denotes the moment of inertia of the water-line plane about a fore-and-aft axis passing through the center of flotation.

For angles of inclination larger than 5 or 10 deg, the line of action of the resultant buoyancy force, F_b, does not intersect the vertical centerline of the ship at a constant point. Therefore, for large angles of inclination the point,

This series of hulls constitutes the **Taylor Standard Series.** The complete data are available in his book, "The Speed and Power of Ships," and were intended to form a criterion and basis for design. Hull forms have been developed with somewhat better characteristics than the hull forms of his series. Little excuse exists for any hull form inferior to that of the Taylor Standard Series, the parent Taylor forms.

Since the Standard Series covers a wide range of varying hull proportions, the series may also be used for estimating the residual resistance for hulls that are reasonably similar to that of the series.

Condensed tables of the residual resistance for speed-length ratios from 0.60 to 1.10 and displacement-length ratios from 50 to 250 have been tabulated from the curves of residual resistance of the Taylor Standard Series and are given here in Tables 3 and 4. For purposes of preliminary estimates, these tables are convenient. For values within the ranges given, the residual resistance per ton of displacement may be determined by interpolation. Values also may be obtained for B/H ratios between 2.25 and 3.75 by interpolation between tables. It is to be noted that these tables are taken from curves which are somewhat irregular and that the determination of residual resistance by interpolation is subject to some error. For precise values of residual resistance it is recommended that Taylor's book be consulted.

Table 4.[a] Residual Resistance per Ton (R_r/D). Taylor's Standard Series. $B/H = 3.75$

C_p	$\frac{D}{\left(\frac{L}{100}\right)^3}$	$\frac{V}{\sqrt{L}}=$ 0.60	$\frac{V}{\sqrt{L}}=$ 0.65	$\frac{V}{\sqrt{L}}=$ 0.70	$\frac{V}{\sqrt{L}}=$ 0.75	$\frac{V}{\sqrt{L}}=$ 0.80	$\frac{V}{\sqrt{L}}=$ 0.85	$\frac{V}{\sqrt{L}}=$ 0.90	$\frac{V}{\sqrt{L}}=$ 0.95	$\frac{V}{\sqrt{L}}=$ 1.00	$\frac{V}{\sqrt{L}}=$ 1.05	$\frac{V}{\sqrt{L}}=$ 1.10
0.50	50	0.46	0.52	0.67	0.88	1.20	1.55	1.90	2.40	3.00	4.10	5.80
	100	0.72	0.93	1.16	1.46	1.85	2.40	2.90	3.45	4.30	5.70	8.50
	150	0.80	1.02	1.32	1.73	2.20	2.90	3.50	4.20	5.20	7.20	11.30
	200	0.86	1.05	1.38	1.81	2.30	3.20	3.80	4.45	5.70	8.20	13.10
	250	0.89	1.08	1.43	1.90	2.47	3.40	4.02	4.65	6.50	8.70	14.30
0.60	50	0.55	0.72	0.91	1.28	1.70	2.20	2.80	4.00	5.35	6.60	8.10
	100	0.73	0.92	1.21	1.52	1.85	2.50	3.40	4.85	7.10	9.15	11.10
	150	0.83	1.05	1.37	1.69	2.15	2.70	3.60	5.30	7.80	10.30	12.70
	200	0.88	1.13	1.48	1.88	2.30	2.85	3.80	5.50	8.00	10.80	13.70
	250	0.93	1.20	1.51	1.93	2.45	3.00	4.00	5.60	8.40	11.30	14.20
0.70	50	0.89	1.16	1.66	2.34	3.20	4.40	6.00	8.80	12.20	15.60	17.30
	100	1.00	1.35	1.81	2.52	3.30	4.60	6.60	9.80	15.70	21.20	25.20
	150	1.03	1.39	1.87	2.55	3.35	4.70	6.80	10.60	17.40	24.30	
	200	1.05	1.40	1.86	2.50	3.35	4.65	6.80	10.70	17.50		
	250	1.08	1.41	1.84	2.49	3.35	4.60	6.50	10.30	16.70		
0.80	50	1.63	2.42	3.80	5.60	7.80	10.50	13.50	17.00	22.90	28.20	32.00
	100	1.55	2.35	3.51	5.20	7.75	11.40	15.30	20.30	28.00	37.30	45.00
	150	1.31	1.92	2.77	4.20	6.30	9.70	12.90	17.00	26.00	36.30	
	200	1.27	1.77	2.52	3.60	5.50	8.40	11.00	14.80	23.70		
	250	1.32	1.94	2.50	3.65	6.20	8.20	11.00	14.90	23.70		

Where R_r = residual resistance, lb per ton of displacement; D = displacement, tons, 2,240 lb; B = beam, ft; H = draft, ft; V = speed, knots; L = length of water line, ft; C_p = longitudinal prismatic coefficient.

[a] Condensed tables by Prof. K. S. M. Davidson from Rossell and Chapman, "Principles of Naval Architecture," vol. II, Soc. Nav. Arch. and Mar. Engrs.

$$
\begin{array}{lrll}
(j) & 595x = & 595; & \therefore x = 1 \\
 & 5y = & 65 - 55x = 65 - 55 = 10; & \therefore y = 2 \\
 & 19z = & 12 + 19x + 13y = 12 + 19 + 26 = 57; & \therefore z = 3 \\
 & 2w = & 3 - 4x - 3y + 5z = 3 - 4 - 6 + 15 = 8; & \therefore w = 4
\end{array}
$$

Here w is eliminated from (a) and (b), obtaining (e); from (a) and (c), obtaining (f); and from (a) and (d), obtaining (g). Then z is eliminated from (e) and (f), obtaining (h), and from (e) and (g), obtaining (i). Then y is eliminated from (h) and (i), obtaining (j), which contains only the single variable x. Hence $x = 1$. Now substituting this value of x in either (h) or (i), y is found; substituting these values of x and y in either (e), (f), or (g), z is found; and so on. (Solution by determinants, see p. 123.)

Approximate Solution of a Set of Simultaneous Equations of the First Degree When the Number of Equations is Greater Than the Number of Unknowns. (Method of Least Squares.)

Case 1. Single Unknown Quantity. Given n equations in one unknown x; for example, n equally careful, independent measurements of some physical quantity:

$$x = x_1, x = x_2, \ldots x = x_n.$$

As the "best" value of x, take the arithmetic mean, x_0, of the several determinations, namely, $x_0 = (x_1 + x_2 + \ldots + x_n)/n$. The quantities $v_1 = x_0 - x_1$, $v_2 = x_0 - x_2$, $\ldots$ $v_n = x_0 - x_n$ are called the **residuals** of the observed values with respect to x_0, and their absolute values (that is, their numerical values without regard to sign) are denoted by $|v_1|, |v_2|, \ldots |v_n|$. [It can be shown that the sum of the squares of the residuals with respect to x_0 is smaller than the sum of the squares of the residuals with respect to any other value x'_0; hence the name of the method: "least squares."]

The quantities r and r_0, defined exactly by Bessel's formulæ:

$$r = \frac{0.6745}{\sqrt{n-1}}\sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

$$r_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

or given approximately by the simpler formulæ of Peters:

$$r = \frac{0.8453}{\sqrt{n(n-1)}}(|v_1| + |v_2| + \ldots + |v_n|)$$

$$r_0 = \frac{0.8453}{n\sqrt{n-1}}(|v_1| + |v_2| + \ldots + |v_n|)$$

are called the **probable error of a single observation** (r), and the **probable error of the mean** (r_0), for the given series of observations. Note that $r_0 = r/\sqrt{n}$. For tables of the coefficients, see p. 63. This quantity r (or r_0) is best regarded as merely a conventional means of recording the relative precision of different sets of observations. If r is small, it may be inferred that most errors of the "accidental" class have been eliminated; but it should be especially noted that the smallness of r gives no information in regard to "constant" or "systematic" errors.

A statement like "x is equal to 2.36 with a probable error of 0.02," is written: $x = 2.36 \pm 0.02$, and is usually understood to mean that the true value of x, as far as can be told, is just as likely to lie inside as outside the

To test the **distribution of residuals**, arrange the residuals in order of magnitude, without regard to sign, and count the number, y, of residuals which are numerically less than some assigned value a; divide y by n, the total number of observations, and divide a by r, the probable error of a single observation. Do this for various values of a, and compare the results with the table on p. 63, which gives the standard distribution of residuals, as found from experience from a large number of different series of observations. In particular, the number of residuals numerically less than r should be about equal to the number numerically greater than r (if n is large). If any large discrepancy appears, the series of observations should be regarded as unsatisfactory.

NOTE. The "mean square error" sometimes met with is equal to the probable error divided by 0.6745.

Case 2. Several Unknown Quantities. Assume that there have been obtained by measurement or observation n different equations of the first degree involving, say, three unknown quantities, x, y, z. There are then n simultaneous equations in three unknowns, and if $n > 3$ there will be, in general, no set of values of x, y, z which will satisfy all these n equations exactly. In such a case, the "best" set of values, x_0, y_0, z_0, may be found by the method of least squares as follows. (The process usually involves a large amount of labor; the use of a computing machine is advisable.)

Given Equations

$$a_1x + b_1y + c_1z = p_1$$
$$a_2x + b_2y + c_2z = p_2$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$a_nx + b_ny + c_nz = p_n$$

First, arrange the n given equations in the form indicated, being careful not to modify any of them by multiplication or division. (Any of the coefficients may of course be zero.)

Next, form the three "normal equations" as follows: (1) Multiply each of the given equations by the coefficient of x in that equation, and add; the result will be the first normal equation. (2) Multiply each of the given equations by the coefficient of y in that equation, and add; the result will be the second normal equation. (3) Similarly for the third. {Notation: $[aa] = a_1^2 + a_2^2 + \ . \ . \ . + a_n^2$; $[ab] = a_1b_1 + a_2b_2 + \ . \ . \ . + a_nb_n$; $[ap] = a_1p_1 + a_2p_2 + \ . \ . \ . + a_np_n$; etc.}

Normal Equations

$$[aa]x_0 + [ab]y_0 + [ac]z_0 = [ap]$$
$$[ba]x_0 + [bb]y_0 + [bc]z_0 = [bp]$$
$$[ca]x_0 + [cb]y_0 + [cc]z_0 = [cp]$$

Finally, solve the three normal equations for the three unknowns in the usual way.

The quantities $v_1 = a_1x_0 + b_1y_0 + c_1z_0 - p_1$, etc., are called the **residuals** with respect to x_0, y_0, z_0. [It can be shown that the sum of the squares of the residuals with respect to x_0, y_0, z_0 is smaller than the corresponding quantity with respect to any other set of values, x'_0, y'_0, z'_0; this relation is taken as the criterion for the "best" set of values of x, y, z.]

The probable error of a single observation is

$$r = \frac{0.6745}{\sqrt{n-m}}\sqrt{v_1^2 + v_2^2 + \ . \ . \ . + v_n^2}, \text{ or approximately,}$$

$$r = \frac{0.8453}{\sqrt{n(n-m)}}(|v_1| + |v_2| + \ . \ . \ . + |v_n|),$$

where m = the number of unknown quantities (here $m = 3$).

Example. The following uses Taylor's Standard Series as a basis for estimating the power requirements of an assumed twin-screw ship with dimensions and characteristics as follows: length on water line, $L = 480$ ft; beam, $B = 72$ ft; draft, $H = 25.5$ ft; displacement, $D = 16{,}600$ tons; midship section coefficient, $C_m = 0.963$; longitudinal prismatic coefficient, $C_p = 0.682$; block coefficient, $C_b = 0.651$; and speed, $V = 14.25$ knots. Therefore $B/H = 2.82$; $V/\sqrt{L} = 0.65$; and $D/(L/100)^3 = 150$.

Wetted Surface Calculation:

$$A = C\sqrt{D \times L} \tag{10}$$

from Fig. 1, $C = 15.5$. Therefore,

$$\text{Area of wetted surface} = A = 15.5\sqrt{16{,}600 \times 480} = 43{,}753 \text{ sq ft}$$

Frictional Resistance (R_f):

$$R_f = f \times A \times V^n \tag{26}$$

from Table 2, $f = 0.00905$ (about) and $n = 1.829$. Therefore,

$$R_f = 0.00905 \times 43{,}753 \times 14.25^{1.829} = 51{,}047 \text{ lb}$$

Residual Resistance (R_r):
From Table 3,

$$R_r \text{ per ton displacement } \begin{aligned} &= 1.03 \text{ for } C_p = 0.70 \\ &= 0.83 \text{ for } C_p = 0.60 \end{aligned} \Bigg\} \text{ at } \frac{V}{\sqrt{L}} = 0.65 \text{ and } \frac{B}{H} = 2.25$$

By interpolation, $R_r = 0.994$ for $C_p = 0.682$ of the assumed ship.
From Table 4,

$$R_r \text{ per ton displacement } \begin{aligned} &= 1.39 \text{ for } C_p = 0.70 \\ &= 1.05 \text{ for } C_p = 0.60 \end{aligned} \Bigg\} \text{ at } \frac{V}{\sqrt{L}} = 0.65 \text{ and } \frac{B}{H} = 3.75$$

By interpolation, $R_r = 1.328$ for $C_p = 0.682$ of the assumed ship. By interpolation between $R_r = 0.994$ for $B/H = 2.25$ and $R_r = 1.328$ for $B/H = 3.75$, the R_r of the assumed ship $= 1.121$ lb per ton of displacement at a $B/H = 2.82$.
Therefore the residual resistance of the assumed ship is

$$1.121 \times 16{,}600 = 18{,}609 \text{ lb}$$

Total Bare-hull Resistance (R_t):

$$R_t = R_f + R_r \tag{25}$$
$$R_t = 51{,}047 + 18{,}609 = 69{,}656 \text{ lb}$$

Total Hull Resistance (R_T):
Assuming an appendage resistance of 9 percent of the bare-hull resistance, then

$$R_T = 69{,}656 \times 1.09 = 75{,}925 \text{ lb}$$ (approximately) see p. 1399.

Effective Horsepower:

$$\text{Ehp} = \frac{R_T \times V}{325.5} \tag{28}$$

$$\text{Ehp} = \frac{75{,}925 \times 14.25}{325.5} = 3{,}324 \tag{28}$$

Assuming a propulsive coefficient [Eq. (29)] equal to 0.69, then

$$\text{Shp} = \frac{3{,}324}{0.69} = 4{,}820 \quad \text{(approximately)}$$

propeller is accelerated, and its speed relative to the hull is increased in this region. Obviously, the resistance of the hull will be increased. It is common practice to assume this increase in resistance as a loss in propeller thrust. For such an assumption, the propeller thrust will be greater than the resistance. The ratio of this increase in thrust to the resistance is termed the **thrust deduction coefficient.** Thus,

$$t = \frac{T - R}{T} \tag{35}$$

where T = the propeller thrust in accordance with the foregoing assumption
R = resistance of the ship
t = thrust deduction coefficient

The **wake and thrust deduction** are interrelated and, therefore, may not be considered as totally independent. The interrelationship may be expressed as follows:

$$E_h = \frac{1 - t}{1 - w} \tag{36}$$

where t = thrust deduction fraction
w = wake fraction
E_h = hull efficiency, as defined below

The work done on a ship is equal to the resistance times the speed of the ship through still water, and the work done on the propeller is the thrust times its speed of advance. The ratio of the work done on a ship to the work done by the propeller is termed the hull efficiency.

For purposes of preliminary propeller design, Taylor considered that an assumption of a hull efficiency equal to unity was sufficiently accurate. Such an assumption assumes that any gain from the wake is offset on the loss in propeller thrust. For such an assumption the **wake fraction** will equal the **thrust deduction coefficient.**

Examination of model and trial data indicates that for the average twin-screw ship, Taylor's assumption of a hull efficiency equal to unity is usually satisfactory. However, it appears that the hull efficiency of **single-screw** ships is more likely to range between 1.08 to 1.25. Further, it appears that the hull efficiencies of **triple- and quadruple-screw** ships are likely to range from values less than unity to unity.

Effect of Hull Form on Frictional Resistance. Equation (26) assumes that the hull form affects the frictional resistance only in so far as the form affects the area of the wetted surface and the length of the hull form affects the selection of the frictional constant. (In the case of the Tideman constants, the length of the hull form also affects the exponent n.) Such an assumption is not strictly true. However, for all practical purposes, Eq. (26) will give satisfactory results. For medium-sized vessels, the errors will be negligible. For very large vessels of high speed, the use of Eq. (26) may underestimate the total resistance as much as 10 percent. In the final analysis such vessels require special investigation.

Effect of Hull Form on Residual Resistance. The effect of hull form on the residual resistance is very complex and, to date, no simple analytical methods have been developed for purposes of estimation. In general, the length of the hull form, the prismatic coefficient, the displacement-length ratio, the shape of the curve of sectional areas, and the beam-draft ratio are particularly effective in influencing the residual resistance as follows:

1. The **length of the hull** form is particularly significant. A curve of residual resistance plotted against the speed-length ratio will follow an exponential form with certain irregularities or humps. These humps are caused by the interference of the various wave systems set up in the water at the bow, along the vessel, and at the stern. The first significant hump or rise in the residual resistance occurs at speed-length ratios between about 0.75 to 0.82. The second significant hump occurs at speed-length ratios of about 1.0 to 1.1. The third significant hump occurs at speed-length ratios of about 1.25 to 1.7. Experience has shown that for heavy cargo vessels of full form, speed-length ratios in excess of 0.75 are not economical, and that for fast passenger vessels of the liner type speed-length ratios greater than 1.0 to 1.1 are not practicable. For vessels such as cruisers, destroyers, and yachts where speed is considered paramount, speed-length ratios above 1.1 are often encountered. Such vessels have a large ratio of horsepower to displacement, and large changes in trim will be experienced at high speeds, unless special preventive modifications are made to the hull form. Such modifications are usually in the form of flat sterns, level and straight buttock lines aft, and often hard bilges incorporated with sharp knuckles or chines. All of which generally make for uneconomical hull forms at lower speed-length ratios.

2. The **prismatic coefficient** is an indication of the fineness of the hull form and is particularly significant. Low prismatic coefficients are associated with fine forms and low residual resistances. High prismatic coefficients are associated with full forms, bluff bows and sterns, and high residual resistances. Tables 3 and 4 show the effectiveness of the prismatic coefficient in the Taylor Standard Series.

3. The **curve of sectional areas** has a significant influence on the residual resistance in that, for vessels of normal form, it is closely allied to the prismatic coefficient. For the higher values of speed-length ratios and lower values of the prismatic coefficient, the curve of sectional areas is extremely significant. In general, a form of curve similar to a curve of versed sines gives the least residual resistance. Increasing the prismatic coefficient and reducing the speed-length ratio reduce the significance of the curve of sectional areas accordingly.

4. The effect of varying **displacement-length** and **beam-draft ratios** is significant to a lesser extent in their effect on residual resistance. Tables 3

PROPELLERS

BY

J. M. LABBERTON

REFERENCES: Taylor, "Speed and Power of Ships," Ransdell, Inc. Labberton, "Marine Engineering," McGraw-Hill.

Basic Coefficient and Performance. A propeller is a screw, the length of which is much less than the pitch. As a consequence, if the "thread" or blade were made continuous, it would not cover the complete circumference and the result would be a single-bladed lop-sided propeller. In order to obtain balance, this screw is divided up into two, three, or four blades. Some of the first tests made indicated that efficiency was greatly impaired by too much blade width or too long a screw, owing to the motion of the water past the surface.

In the case of any screw propeller of a given pitch working in a viscous fluid, it can be shown by dimensional analysis that the thrust developed is

$$T = \rho d^2 V_a^2 \left[f\left(\frac{nd}{V_a}\right), \frac{\gamma}{dV_a}, \frac{dg}{V_a^2} \right]$$

where T = propeller thrust, lb
d = propeller diameter, ft
n = propeller rps
V_a = speed of advance, fps
ρ = density of fluid, lb per cu ft
γ = kinetic viscosity of fluid
g = gravity acceleration

It will be noted that the second term in the parenthesis is the Reynolds number inverted and that the third term is Froude's number. For reasons that cannot be covered in the space here available it is known that these last two functions can be neglected in the case of practical propellers not in cavitation and the expression for thrust can be simplified to

$$T = \rho d^2 V_a^2 \times f\left(\frac{nd}{V_a}\right)$$

$f\left(\frac{nd}{V_a}\right)$ is a function of slip and from the above can be derived a dimensionless coefficient $B = \frac{nP^{1/2}}{\rho^{1/2} V_a^{5/2}}$ fixing the performance of any propeller regardless of size but having a given pitch ratio, number of blades, blade proportions, etc., and at a given slip but not in cavitation. However, since water is the only fluid with which we are concerned at present, we can dispense with the variable ρ. Moreover, since it is customary in propeller problems to use knots instead of feet per second and revolutions per minute instead of revolutions per second, V_a = knots and N = rpm can be used. The dimensionless coefficient then becomes a quasi-dimensionless coefficient

$$B_p = \frac{NP^{1/2}}{V_a^{5/2}}$$

and 4 again illustrate their effect on the residual resistance for the Taylor Standard Series. For speed-length ratios in excess of 0.75 to 0.82, the residual resistance rapidly increases in significance.

Resistance of Appendages. Appendages to the hull such as shaft struts, shafts, condenser scoops, plate laps, and bilge keels also induce added resistance. Taylor has indicated that the total resistance of appendages for twin-screw vessels with large bilge keels, two pairs of struts per shaft, may amount to as much as 20 percent of the bare-hull resistance, and that for single-screw vessels without bilge keels the appendage resistance may be as low as 3 to 4 percent of the bare-hull resistance. Careful streamlining of the shaft struts, bosses, etc., to the flow line of the water about the hull is necessary to avoid the high appendage resistance of the first case.

With bilge keels following the lines of flow, the additional resistance of the bilge keel is little different from that of the additional wetted surface.

Resistance in Shallow Water and Restricted Passages. The passage of vessels through restricted channels, such as canals, and through shallow water greatly increases the resistance. Ships of normal form operating in such waters experience a large increase in draft and change of trim, because the lines of water flow are forced out of their normal paths. For vessels passing through canals at moderate speeds where the canal width is about six to 10 times greater than that of the ship, the added resistance will be small; for a ratio of 2.5 or 3, the increase in resistance will be excessive, compelling a speed so low as to prohibit operation for vessels of normal form. Depths of water greater than six to ten times the draft are required for vessels of the slow-speed type; depths greater than the length of the vessel are required for high-speed types in order that the resistance may be free from the effects of shallow water. Vessels designed for operation in shallow or restricted water require special consideration.

Air Resistance. The above-water portion of a ship's hull and superstructure moving through the air also induces resistance to the forward motion of the ship. In the case of water resistance the ship advances directly into the water, but it does not always advance directly into the wind except in the case of a head wind. More often the wind would be abeam or astern, in which case it may assist the forward motion of the ship. In general, the air or wind resistance is only a small fraction of the water resistance and may often be neglected.

Taylor indicated that reasonably good estimates of air resistance may be calculated by the following formula:

$$R_a = 0.004 \frac{B^2 \times V_a^2}{2} \tag{27}$$

where R_a = air resistance, lb

B = beam of ship, ft

V_a = speed of wind, knots

This formula assumes that the projected area presented to the wind is equal to $B^2/2$. (The ship is assumed to be half the beam in height above the water.)

Effective Horsepower. The effective horsepower may be defined as the "towrope" horsepower or the power transmitted through a towrope, assuming the vessel to be towed at a given speed, and may be calculated by the following relationship:

where the subscript p indicates that the coefficient is concerned with power input to the propeller rather than the useful power output.

Therefore, if any model propeller is tested at a given slip and rpm, and if power input in horsepower and speed of advance in knots are measured, the coefficient B_p can be determined which will hold good for any size of propeller, of geometrically similar proportions. If

$$\delta = \frac{Nd}{V_a} \quad \text{and} \quad a = \frac{p}{d} \text{ the pitch ratio,}$$

then it can be shown that $1 - S = \frac{101.33}{a\delta}$ where S is slip ratio, that percentage of a propeller revolution turned which does not advance the propeller in proportion to pitch.

From the foregoing it follows that, in similar manner, the power output, U, of the propeller could be used as well as the power input, P, and a similar coefficient

$$B_u = \frac{NU^{1/2}}{V_a^{5/2}}$$

would result, the only difference being the factor of efficiency. If e_p = propeller efficiency, $e_pP = U$,

or

$$B_p\sqrt{e_p} = \frac{NU^{1/2}}{V_a^{5/2}} = B_u$$

Naturally, a plot of values of B_u is much more valuable than a plot of B_p because it is generally required to choose a propeller for a given job rather than to determine what a given propeller, already chosen, will do.

Figures 1 and 2 show plots of three- and four-bladed propeller performance as made by Rear Admiral D. W. Taylor, USN, from tests made at the Washington Navy Yard. Figure 3 shows corrections to be made when the propeller has a mean width ratio (M.W.R.) or a blade thickness fraction (B.T.F.) other than M.W.R. = 0.25 or B.T.F. = 0.05, the conditions covering Figs. 1 and 2.

It will be noticed that as the values of the B_u increase the maximum efficiency obtainable decreases and the correct pitch ratio to obtain this maximum efficiency also decreases. It will be noted that there is one best value of $\delta = \frac{Nd}{V_a}$ and one pitch ratio that should be used to obtain the highest possible efficiency for a given B_u.

A curve has been drawn on Figs. 1 and 2 through these maximum efficiency points. Mention has been made of mean width ratio and blade thickness fraction. Figure 4 shows a conventional elliptical blade with a 20 percent diameter, hub. The term conventional elliptical blade is used because that is the general shape of the blade as recommended by Dyson. The characteristics of such blades are set forth in Fig. 6 where, with a propeller radius of unity, the chords of the arcs between the middle line of the projected area and either leading or following edge of the projection at each of the

$$\text{ehp} = \frac{R_T \times V}{326} \tag{28}$$

where ehp = effective horsepower
R_T = total resistance, lb, of the ship including bare hull, appendage, and air resistance
V = speed, knots

Propulsive Coefficient. The ratio of ehp to shp is termed the propulsive coefficient, *i.e.*,

$$\frac{ehp}{shp} = \text{propulsive coefficient} \tag{29}$$

The term indicated horsepower is applied in the case of reciprocating machinery. In the case of modern steam-turbine-driven vessels or diesel electric vessels, the shaft horsepower (shp) is used. Similarly, for diesel- or gasoline-powered vessels, the brake horsepower (bhp) is used in lieu of ihp. It is to be noted that the ihp includes the necessary power to overcome the losses of the propeller, shaft (stuffing boxes and bearings), and the mechanical efficiency of the engine. The shp is the horsepower in the shaft and does not include the power losses in the reduction gears or turbine machinery. In the case of diesel or gasoline engines the power delivered by the engine at the coupling is termed the brake horsepower (bhp). It is apparent that the propulsive coefficient must be defined specifically in regard to ihp, shp, or bhp.

Propulsive coefficients ranging from 0.40 to 0.70 may be expected when referred to ihp and, when the coefficient is referred to shp, values ranging from 0.50 to 0.80 may be expected. In cases where the mechanical efficiencies are high, values in the upper half of the ranges given may be expected. However, conservative estimates are in order.

Power by Admiralty Formula. The power requirements to propel a ship are often estimated directly by assuming that the total resistance, R_T, is proportional to $D^{2/3} \times V^3$ and that the propulsive coefficient is constant. With such an assumption the following relationship can be expressed:

$$\text{hp} = \frac{D^{2/3} \times V^3}{K} \quad \text{(Admiralty formula)} \tag{30}$$

where hp = engine indicated horsepower, brake, or shaft horsepower
D = displacement, tons
V = speed, knots
K = the Admiralty coefficient.

The Admiralty coefficient includes all factors due to the influence of form, dimensions, propeller, and shaft efficiencies. In cases where the ihp is used in the relationship, it will also include the mechanical efficiency of the engine. It is extremely pertinent to note that for speeds less than about $0.75\sqrt{L}$, the Admiralty coefficient is relatively constant for a particular hull. For speeds in excess of about $0.75\sqrt{L}$, the coefficient may vary irregularly with the speed. Therefore, it is to be noted that considerable experience is required to select a proper coefficient. The following briefly furnishes guidance in selecting a coefficient.

values 0.2, 0.3, 0.4, 0.5, etc., of the radius are given for each value of projected-area ratio of a three-bladed propeller, from $PA/DA = 0.15$ to $PA/DA = 0.7$.

$$\frac{PA}{DA} = \frac{\text{projected area including all blades}}{\text{disk area}}$$

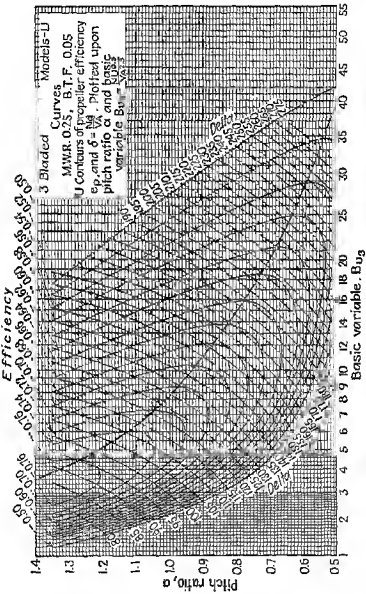


FIG. 1.—Performance of Taylor's series of three-bladed propellers. (*From Admiral D. W. Taylor, U.S.N., The Speed and Power of Ships, Ransdell, Inc.*)

For a four-bladed propeller, the conventional form corresponding to the total projected-area ratio will be for three-fourths of that ratio; for a two-bladed propeller it would be $1\frac{1}{2}$ times that ratio.

In laying down the projected-area form for any desired projected-area ratio it should be borne in mind that all projected areas are outside the $0.2R$

Admiralty Coefficient Values. For ships of large size and with fine to medium characteristics of form (in particular the prismatic coefficient) at speeds such that $V/\sqrt{L} \leqq 0.8$ to 0.9, values of $K = 250$ to 350 are found. For similar ships at higher speed when $V/\sqrt{L} \geqq 1.0$, the values of K range from 250 to 275.

With medium to full hull forms at speeds where $V/\sqrt{L} < 1.0$, K values of 220 to 250 are common.

With medium to small ships, other things being equal, the values range somewhat smaller than for large ships. With medium to small ships of average characteristics and at speeds where $V/\sqrt{L}$ may considerably exceed 1.0, K values of 150 to 200 are common.

For small craft of fine lines driven at excessively high speed and of such underwater form as to rise partly out of the water, the resistance is decreased with corresponding reduction in power and an increase in the value of K. In such cases, values of 200 to 250 and more are met.

For similar ships driven at corresponding speeds, the value K will be the same.

It will be noted from the foregoing that no exact rules can be given for selecting the value of K. Table 5 of ship data is useful and may be used in estimating the proper value for K. Care should be exercised in using ship data for selecting values of K, so that the similarity of the ships is as close as the available data permit.

By the application of **Froude's law of comparison,** extended to cover approximately all components of ship resistance, much more reliable results may be determined. The law may be restated that for similar ships at corresponding speeds the powers will be in the ratio of the products of the displacements by the speeds, or the power ratio will equal the product of the displacement ratio by the speed ratio.

Thus, given a ship having a length of 400 ft, a displacement of 7,500 tons, a speed of 16 knots, and requiring 0,300 ihp, by substitution in Eq. (30) the value of K is readily found to be 250. Then, assuming a similar ship 480 ft in length; length ratio $= 480/400 = 1.2$; corresponding speed ratio $= \sqrt{1.2} = 1.095$; corresponding speed $= 16 \times 1.095 = 17.53$; displacement ratio $= (1.2)^3 = 1.728$; displacement $= 7,500 \times 1.728 = 12,960$. Then using value of $K = 250$, displacement $= 12,960$, and speed $= 17.53$ and substituting in Eq. (30), the ihp required at the corresponding speed will be 11,900. For moderate differences in block coefficient, C_b, and in length to beam ratio, L/B, K may be taken to vary inversely as $(C_b)^{2/3}$ and $(\text{beam})^{1/3}$.

Power by Stevens Formula. A modification of the Admiralty formula known as the Stevens formula is sometimes used with fairly good results:

$$\text{hp} = \frac{D \times V^3}{C \times \sqrt{L}} \qquad \text{(Stevens formula)} \quad (31)$$

where hp = engine ihp or bhp
D = displacement, tons (2,240 lb per ton)
L = length, ft, of load water line
C = a coefficient to be selected

Values of C may be estimated from nearly similar ships. Table 5 may again be used for this purpose. Values of C are given in Fig. 5. The two curves that are plotted upon displacement show the limits through which

circle, and any variation from this in the actual blade due to variation of the hub diameter may be neglected. The lengths of the chords of the half arcs for any diameter propeller will equal the length of the chord as given in the table for each fraction of the radius and the particular projected-area ratio desired, multiplied by the radius of the propeller.

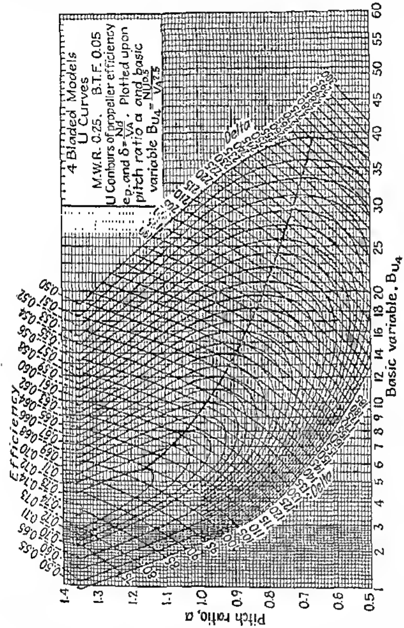


FIG. 2.—Performance of Taylor's series of four-bladed propellers. (*From Admiral D. W. Taylor, U.S.N., The Speed and Power of Ships, Ransdell, Inc.*)

Thus, suppose the diameter of the propeller to be 10 ft and the desired projected-area ratio to be 0.6. The length of the chord at the half arc at $0.7R$ will be $0.499 \times 5 = 2.495$ ft.

The greatest circular width of the projected-area form for conventional elliptical blades is at the $0.7R$ radius.

The term projected-area ratio or PA/DA is falling into disuse, probably because it does not take pitch into account, and mean width ratio is taking its place.

Table 5.

Ship	Type	Year built	Length water line,[a] ft, L	Beam, molded, ft, B	Depth, molded, ft, D_s	Load, draft, ft, H	Displacement tons, D	Deadweight capacity, tons	Tonnage, gross	Block coefficient, C_b
Normandie	P & C	1935	962	118	91.9	36.6	67,000	11,810	79,000	0.56
Queen Mary	P & C	1937	1,004	118	92.5	38.8	77,400	.....	80,700	0 59
Bremen	P & C	1930	900	102	79.4	33.9	54,750	14,390	51,656	0.625
Nieuw Amsterdam	P & C	1938	700	88	55	31.5	36,240	10,260	36,290	0.63
America	P & C	1940	690.3	93.9	91.9	32.5	35,440	14,331	27,000	0.586
Manhattan	P & C	1932	685	86	47	30.7	33,500	13,224	24,290	0.65
Patria	Di.el.P	1939	577	75.8	49.2	25.5	19,780	8,370	16,595	0.62
Mariposa	P & C	1932	628	79	....	28.2	26,140	11,408	18,020	0.65
Scharnhorst	P & C	1935	610	73.8	....	29.0	23,900	10,800	18,000	0.64
Pretoria	P & C	1938	570	72	44.5	26.5	20,104	9,750	16,660	0.65
Oslofjord	g.Di.P	1938	560	73	46	27.0	20,500	7,750	18,670	0.65
Panama	P & C	1939	486	64	38 5	26.0	14,030	6,800	10,020	0.61
Excalibur	P & C	1931	450	61.5	42.3	27.9	15,500	9,300	9,360	0.70
City of New York	P & C	1930	450	61.5	37 0	26.0	14,950	9,400	8,270	0.72
Columbia	P & C	1933	385	57 5	31 5	23.5	9,490	5,236	5,236	0.64
Acadia	P & C	1932	400	61 0	29.8	18.0	6,811	2,494	6,185	0.54
Prince Boudoin	CC	....	361	45.9	24.8	11.1	2,755	......	3,300	0.53
Red Jacket	C	1939	435	63	40.5	25.8	13,900	7,620	......	0.69
Sea Fox	C	1939	473	69 5	42.5	27 3	17,600	11,920	......	.....
Pacific Trader	Di.C	1926	420	58	28.1	26.7	13,930	9,380	6,327	0.75
Black Falcon	C	....	390	54	32	24.3	11,200	7,500	3,700	0.745
Angelina	C	1934	390	55	30.5	24.6	10,530	7,250	4,773	0.70
Arc Wear	C	1934	360	57.5	26 8	22.2[b]	9,060	7,000	......	0.70
Ramb I	Di.Fr.	1937	354	47.9	..	18.2	5,200	2,300	3,500	0.57
Quaker	C	1939	280	48.5	32 2	18.5	4,215	2,050	......	0.585
Carl D. Bradley	L	1927	615	65	33	22.5	22,000	15,600	10,030	0.86
J. W. Van Dyke	T	1938	530	70	40	29.6	23,970	16,100	11,650	0.76
Brunswick	Di.Ta.	...	469	63	36 8	26 5	17,275	13,400	8,950	0.765
Esso Bayonne	T	1938	445	66 5	34 5	28.0	17,050	13,080	7,700	0.724
Destroyer	D	1935	320	31.2	18.8	10	.....	......	......	.....
De Ruyter	Cr	1936	550	51.2	....	18	6,800	......	......	0.47
Corsair	Y	1930	280	42.7	21.7	16.2[a]	2,590	......	......	0 47
Alex. Hamilton	C.G.Cu	1937	308	41.0	....	12.7	2,350	......	......	0 51
[illegible]	C.G.Cu	....	239	42	....	12.9	1,662	......	......	0 49
Thetis	C.G.Cu	1931	160.8	23 8	....	7.7	334	......	......	0.378
Naugatuck	C.G.Cu	1939	110.0	26.4	15.0	10.5	328	......	......	0.377
Onwego	Y	1938	119.0	20.9[d]	....	5.1	157	......	......	0.435
Cleopatra	Y	1931	104 0	19.4[e]	....	4.6	103	......	......	0.396
Cossack	Y	1929	74.6	12.2[c]	....	2.8	29.3	......	......	0.396
Walter Wyman	Cu	1932	92	23.2	14.2	9.3	270	......	......	0.48
Patrol Boat	C.G.Cu	....	70	12.8	....	3.6	31	......	......	0.45

Abbreviations: P, Passenger; C, Cargo; CC, Cross channel; C.G., Coast Guard; Cr, Cruiser; Cu, Cutter; D, Destroyer; Di.C, Diesel Cargo; Di.Fr., Diesel Fruiter; Di.Ta, Diesel Tanker; L, Laker; T, Tanker; Y, Yacht

[a] M.A.N., two-stroke, single-acting.
[b] Two-stroke, 212 rpm.
[c] Sun-Doxford, two-stroke.
[d] Sulzer, two-stroke, single-acting.
[e] Fiat two-stroke, 20.5 × 32.3 in.

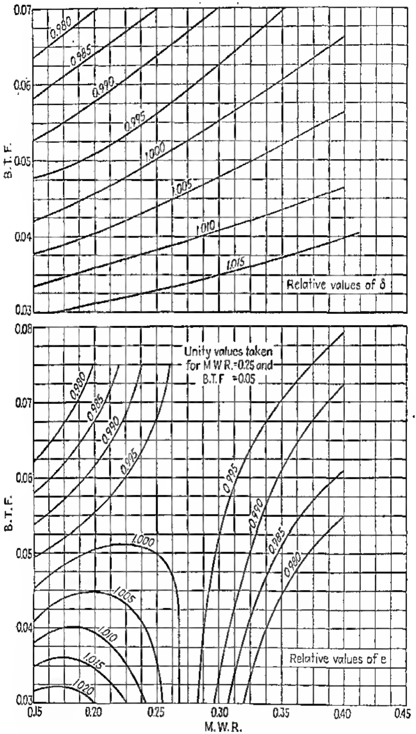


FIG. 3.—Correction curves for Figs. 1 and 2. (*From Admiral D. W. Taylor, U. S. N., The Speed and Power of Ships, Ransdell, Inc.*)

Ship Data

Normal speed, knots. V	$V/\sqrt{L}$	Propellers	Rpm	Total shp	Machinery: Engines	Boilers: Number	Boilers: Type	Boiler heating surface,[g] sq ft	Boiler press, lb per sq in., gage	Steam temp, F	Mach'y weight, lb per shp[h]	Admiralty coef
28.5	0.92	4	....	160,000	4 m.t.g.	29	3 dr.	310,000	400	660		
29.0	0.92	4	180	158,000	4 s.r.	24	4 dr.	....	400	700		
27	0.90	4	182	100,000	3 cyl. s.r.	20	3 dr.	183,500	375	707		
20.5	0.77	2	131	34,000	4 cyl. s.r.	6	4 dr.	51,000	550	740		
22	0.84	2	128	34,000	4 dr.	6	dr.	63,000	500	725		
20.3	0.78	2	125	30,000	3 cyl. s.r.	6	3 dr.	63,000	400	675	....	316
17	0.71	2	125	15,000	5-8 cyl. Di[a]							
20.5	0.82	2	125	22,000	3 cyl. s.r.	12	cr. dr.	53,520	375	650	....	353
21.0	0.85	2	130	26,000	2 m.t.g.	4	2 dr.	27,900	735	880		
18	0.76	2	125	14,200	3 cyl. s.r.	2	Benson	13,800	1135	900		
19	0.80	2	92	15,800	4-7 cyl. g. Di[b]							
16.5	....	2	90	9,000	2 cyl. d.r.	2	3 dr.	18,400	445	750	....	382
16	0.76	1	97	8,000	3 cyl. s.r.	4	cr. dr.	16,800	350	620	224	
13.5	0.64	2	100	5,400	4 cyl. Di[c]	..	....	....	....	....	519	
16	0.82	1	110.6	8,275	3 cyl. s.r.	4	cr. dr.	17,780	400	680	233	
20	1.0	2	168	9,500	2 cyl. s.r.	4	cr. dr.	29,136	375	680	248	
22	1.16	2	258	15,000	12 cyl. Di[d]	..	....	....	....	....	105	
15.5	0.75	1	92	6,000	2 cyl. d.r.	2	....	....	450	750	....	448
16.5	....	1	85	8,500	2 cyl. d.r.	2	....	....	450	750		
11.5	0.56	1	87	2,900	4 cyl. Di[e]	..	....	....	....	....	580	
13.2[k]	0.67	1	99.5	2,974	1 cyl. d.r.	3	cr. dr.	9,075	200	....	....	370
13	0.66	1	90	3,150	2 cyl. d.r.	2	cr. dr.	7,884	300	547		
12.1[k]	0.64	1	72.5	1,976[o]	Triple exp.;	3	s.e.S.	9,075	200	....	....	430
18.5[k]	0.98	2	....	5,000	9 cyl. Di[c]							
16.5	0.99	1	120	4,000	2 cyl. d.r.	2	cr. dr.	9,360	400	715		
12.0	0.48	1	105	4,500	1 m.t.g.	2	cr. dr.	11,260	325	707		
13.3	0.58	1	90	5,000	1 m.t.g.	2	cr. dr.	6,444	625	835	182	410
11.5	0.53	1	....	2,690	4 cyl. Di. el.[f]	..	....	....	....	....	....	376
13.8[k]	0.65	1	102[k]	4,100[k]	2 cyl. d.r.	2	2 dr.	4,570	400	750		
36.7[k]	2.05	2	446[k]	31,280[k]	2 cyl. s.r.	3	4 dr.	17,970	386	660	28.5	
32	1.36	2	320	66,000	2 cyl. s.r.	6	3 dr.	43,000	415	660		
16.0[k]	0.96	2	210	3,140[k]	2 m.t.g.	4	cr. dr.	13,290	300	620	....	247
20[k]	1.14	2	241	5,250[k]	2 cyl. d.r.	2	....	....	400	650	177	270
17[k]	1.10	1	163	3,350	1 m.t.g.	2	cr. dr.	6,330	265	586	216	206
16	1.27	2	450	1,340	2 Di	..	....	....	....	....	147	
12	1.14	1	240	610	2 Di.el.							
13.9	1.28	2	700	600	2-8 cyl. Di							
13	1.36	2	700	400	2-6 cyl. Di							
24.5	2.84	2	1300	1,000	2-8 cyl. m							
11.7[k]	1.22	1	180	557	2-5 cyl. Di. el.	..	....	....	....	....	500	120
30.4[k]	3.63	2	1490	1,490	4-12 cyl. V.[m]	..	....	....	....	....	9.4	

cr. dr., cross drum
cyl., cylinder
Di, Diesel
Di. el., Diesel electric
dr., drum
d.r., double-reduction geared turbine
g.Di., geared Diesel
m.t.g., main turbogenerators
s.e. S., single-ended Scotch
s.r., single-reduction geared turbine

[f] 19.5 × 24 in.
[g] Boiler heating surface = steam generating surface.
[h] See *Trans. Soc. Naval Architects & Marine Engrs.*, 1932, 1937.
[k] Trial results.
[m] Gasoline engine.
[n] Length between …
[o] Indicated horsepower.
[p] Mean draft.
[q] Beam …

Figure 4 shows the developed outline of the conventional elliptical blade above the 20 percent hub. If the area of this developed blade is divided by the distance from the hub to the tip, the result will be a mean width, a very important dimension. Mean width divided by the diameter of the propeller results in a dimensionless ratio called the mean width ratio, which is a measure of the proportionate size of the blade, its surface area, and, con-

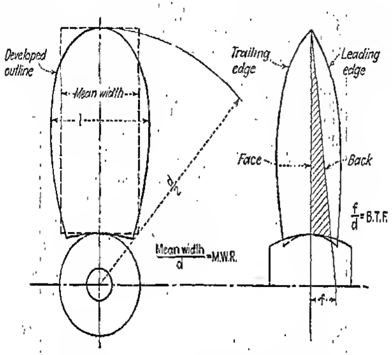


FIG. 4. FIG. 5.
Conventional elliptical blade and nomenclature.

sequently, unit surface pressure which has to do with cavitation as will be discussed later.

Therefore, referring to Fig. 4 and its notation, $\dfrac{\text{mean width}}{d} = \text{M.W.R.}$

With reference to the conventional forms

$$\frac{PA}{DA} = (0.543 - 0.1106a)n(\text{M.W.R.})$$

where a = pitch ratio, $\dfrac{p}{d}$

n = number of blades

The maximum developed width of the blade can be determined by the expression

$$l = 1.188(\text{M.W.R.})d$$

C may vary. These curves were derived from ships with a ratio of beam to draft (B/H) of about 2 and $V/\sqrt{L}$ of 0.5 to 0.6. Ships with shallower draft and higher speed-length ratios will fall near the lower curve; deeper draft ships of moderate speed-length ratios and good form will fall near the upper curve. Ships of higher speed-length ratios and with propellers making a high number of revolutions, values of C, will fall below the lower curve. It is to be noted that values of C taken from the curves of Fig. 5 are for use with the formula when hp = ihp and assuming that ihp = 2 × ehp.

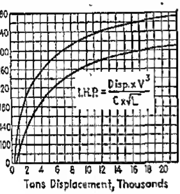


FIG. 5

Other Formulas for Estimating Power. In addition to the Admiralty formula and the Stevens formula, other and lesser known relationships are sometimes used for estimating power requirements:

$$V = \frac{C\sqrt[3]{L \times \text{hp}}}{B} \tag{32}$$

where V = speed, knots
L = length, ft, of water line
hp = horsepower
C = coefficient to be selected
B = beam, ft.

Also,

$$V = \sqrt[3]{\frac{\text{hp} \times 1{,}000}{D}} \times \sqrt{L} \times C \tag{33}$$

where V = speed, mph (5,280 ft per hr)
D = displacement, lb
L = length of water line, ft
hp = horsepower
C = a coefficient to be selected

In Eqs. (32) and (33) selection of a proper coefficient, as in the Admiralty formula, requires experience and judgment, and similarly to the Admiralty coefficient the value of C includes many factors. It is pertinent to note that Eq. (33) is in common use in the field of motorboat designing and good results may be obtained for such vessels.

Power Requirements. In estimating the power requirements, it is pertinent to note that the foregoing methods of calculating power by the use of model tests, similar ships, or Taylor's Standard Series give trial-trip speeds. In the case of the Admiralty or Stevens formula, the coefficients are often calculated on trial-trip conditions. Trial trips are usually made under ideal conditions of clean bottoms and fair weather. Service conditions are far different. Semifoul bottoms and bad weather are to be expected. The final selection of the power requirements should provide for maintenance of service speed, and an additional allowance in power should be provided. Experience has shown that increases ranging from 20 to 35 percent are in order depending upon the vessel and the conditions of the service in which the vessel will operate. Slow-speed vessels of full form are less apt to be retarded by heavy weather than vessels of fine form. Foul bottoms will usually slow down fine-lined fast vessels more than slow full-form cargo vessels.

TABLE OF HALF CHORDS, C..RADIUS = UNITY

P.A./D.A.	DECIMAL PART OF RADIUS WITH HALF CHORDS UNDER											
	.2	.3	.4	.5	.6	.7	.8	.9	.925	.95	.975	.9875
.15	.042	.064	.087	.107	.120	.128	.126	.105	.094	.080	.059	.042
.20	.055	.084	.114	.140	.159	.171	.168	.139	.125	.108	.078	.058
.25	.069	.107	.142	.173	.198	.212	.209	.174	.157	.133	.097	.069
.30	.082	.128	.170	.207	.236	.253	.250	.210	.190	.161	.120	.088
.35	.097	.150	.199	.242	.274	.295	.293	.245	.222	.192	.142	.105
.40	.109	.170	.227	.277	.316	.338	.333	.278	.251	.214	.158	.118
.45	.123	.191	.253	.308	.352	.378	.374	.313	.284	.243	.178	.127
.50	.136	.210	.279	.341	.389	.417	.415	.346	.312	.269	.195	.143
.55	.148	.230	.306	.372	.425	.459	.455	.382	.348	.293	.218	.165
.60	.159	.249	.332	.405	.463	.499	.497	.413	.377	.322	.238	.164
.65	.172	.270	.359	.437	.497	.539	.537	.445	.403	.343	.255	.200
.70	.184	.288	.382	.466	.533	.578	.575	.481	.437	.378	.280	.220

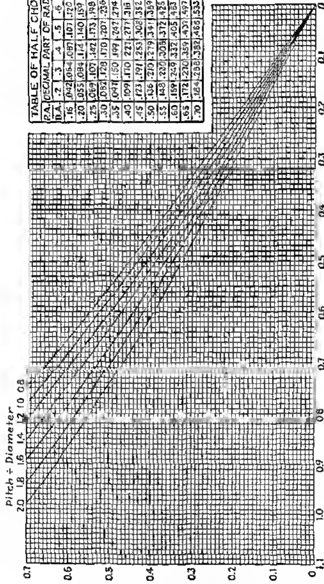


FIG. 6.—Dyson's standard forms of propellers. (*Courtesy*

WAKE AND WAKE FRACTION

The speed of a ship, the friction of the water, the roughness of the wetted surface, the hull form, and the appendages create a tendency to drag a boundary layer of water along with its movement. The water closely adjacent to the wetted surface moves with the speed of the ship, and its speed rapidly decreases toward the outward extremities of the layer. The thickness of this moving layer of water is at a minimum at the bow and increases in thickness progressively toward the stern. There, the forward motion of the water on the surface is often augmented by the stern wave. This whole effect of moving water is termed the **wake**, and its speed is maximum directly astern of the sternpost and retrogressively decreases away from this vicinity of the ship. The effect of the wake is that of the propeller doing its work advancing through a current of water at a speed different from that of the ship moving through the surrounding still water. The speed of the propeller advancing through the wake is termed the **speed of advance** and is defined in accordance with the following relationship:*

$$V_a = (1 - w)V \tag{34}$$

where V_a = speed of advance, knots
V = speed of ship through still water, knots
w = wake fraction

It is of interest to note that the propeller derives from the wake in which it is working an increased thrust resulting in a return to the vessel of a portion of the energy spent in overcoming the resistance. Since the propellers of single-screw ships are in the region of maximum wake, it follows that such ships will benefit more from this source than multiscrew ships.

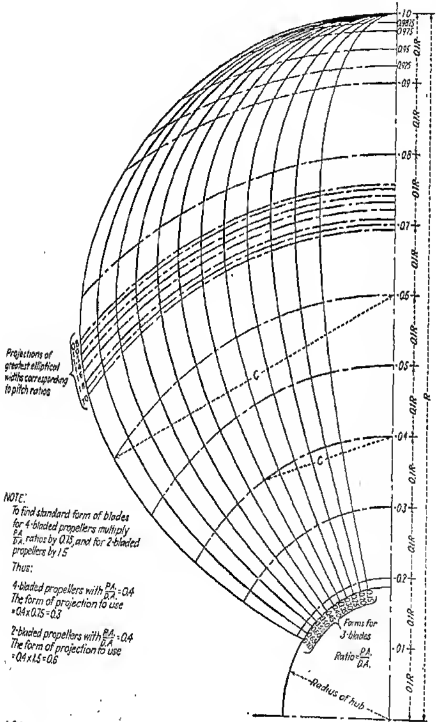
Obviously, the wake fraction is of major importance to the propeller designer. Selection of a proper wake fraction can best be made from the results of tests with models fitted with appendages and propellers. In the absence of such tests, the following derived from data published in the Taylor and *Trans. Soc. Nav. Arch. and Mar. Engrs.*, references, given at the beginning of this article, will furnish guidance for approximating the value of the wake fraction:

1. For the average single-screw ship, the wake fraction in the vicinity of the screw ranges approximately in a straight-line relationship from 0.15 for ship speeds of 20 knots to 0.35 for ship speeds of 14 knots. For ships of full form with high block coefficients, higher values may be expected.

2. For the average twin-screw ship, the wake fraction in the vicinity of the screws ranges approximately in a straight-line relationship from 0.05 for ship speeds of 30 knots to 0.20 for ship speeds of 14 knots. For ships of finer than average form and with low block coefficients, lower values of the wake fraction may be expected. For ships of light displacement and high speeds, slightly negative values are often encountered. For ships of more than average full form, the wake fraction may be expected to be greater than the values given.

3. For triple-screw ships where the center screw is aft and relatively clear of the wash of the outboard wing screws, the wake fraction for the center screw may be expected to be slightly less than that for single-screw

* Froude expressed the relationship as $V/V_a = (1 + x)$, where x = wake percentage. Thus $w = \dfrac{x}{1 + x}$. The wake fraction is not to be confused with the wake percentage.



of Simmons-Boardman Publishing Corporation.)

ships. The wake fraction for the wing screws will have values about the same as given for twin-screw ships.

4. For quadruple-screw ships, the establishment of average values of the wake fraction in the vicinity of the screws is especially difficult. In most quadruple-screw ships it is quite difficult and often impracticable to locate the outboard or inboard screws so that the inboard screws may be entirely free of the wash of the outboard screws. Therefore, it may be expected that the wake fraction of the inboard screws will be somewhat less than the values given for twin-screw ships. Since the outboard screws are usually farther away from the maximum wake, it may be expected that the wake values of the same will generally be slightly less than that of the inboard screws.

It is to be noted that there is a general relationship between the block coefficient and the wake fraction. High wake fractions are generally associated with high block coefficients, and low wake fractions are generally associated with low block coefficients.

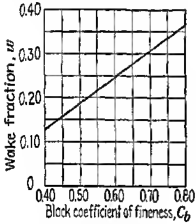


FIG. 6.—Wake fraction for single-screw ships.

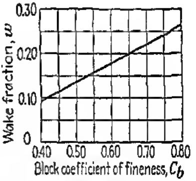


FIG. 7.—Wake fraction for twin-screw ships.

Taylor, by approximate determination from numerous trial results of full-size ships, derived wake fraction values for twin-screw ships ranging from −0.038 for ships with block coefficients of 0.50 to values of 0.200 for ships with block coefficients of 0.80. The wake fraction values of single-screw ships ranged from 0.230 for ships with block coefficients of 0.50 to values of 0.477 for ships with block coefficients of 0.90. Since low block coefficients are generally associated with fine form, it is reasonable to presume that the fine forms were driven at higher than average speeds (see Figs. 6 and 7).

The subject of the wake fraction has been investigated by many authorities, and the trends of values of the fraction are generally consistent. However, the establishment of wake fraction values by means of a relationship correlating influencing factors has not been entirely satisfactory or consistent in the determination of numerical values. In the absence of specific test data it is suggested that Figs. 6 and 7 be used.

Thrust Deduction Coefficient and Hull Efficiency. The propeller, doing its work in the stern wake, disturbs the streamlines of the water flow as they close in about the stern. The water approaching and leaving the

For values of a between 0.6 and 2.0 the following expressions hold:

$$\text{Total developed blade area} = 0.4nd^2(\text{M.W.R.})$$
$$\text{Projected area} = PA = (0.4267 - 0.0916a)nd^2(\text{M.W.R.})$$
$$\frac{PA}{\text{Developed area}} = 1.067 - 0.229a$$
$$\frac{\text{Developed area}}{\text{Disk area}} = 0.509n(\text{M.W.R.})$$

There is a limit to mean width ratio vitally affecting efficiency. The reason for increasing the blade width is to decrease the unit pressure, but naturally this presents more surface for drag friction. This loss of efficiency

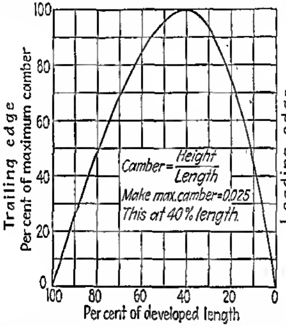


FIG. 7.—Streamline blade sections. Center-line cambers.

progresses fairly proportionately until a value of approximately 0.50 for mean width ratio is reached, and then there is a sudden great increase in the loss and the efficiency falls off very rapidly.

In Fig. 5 the face is shown as a straight line. When this straight line revolves at a constant angular velocity and moves forward at a constant linear velocity, a helicoidal surface is generated, which is the face of the propeller blade. Generally the straight line is perpendicular to the axis of rotation; sometimes, in order to clear some part of the hull, the straight line, or generatrix, is inclined. In either event, the propeller action is the same.

The propeller blade acts as a cantilever beam with a distributed load, the total load being the total pressure upon the blade. The stress then depends, for a given load, on the section modulus at the root or point where the blade joins the hub. Since this is true, the thicker the blade at the root, the less the stress or the stronger the blade or the stronger the propeller. A measure

DETERMINANTS

Determinants are used chiefly in formulating theoretical results; they are seldom of use in numerical computation.

Evaluation of Determinants:

Of the second order:

$$\begin{vmatrix} a_1 b_1 \\ a_2 b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Of the third order:

$$\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 c_2 \\ b_3 c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 c_1 \\ b_3 c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 c_1 \\ b_2 c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Of the fourth order:

$$\begin{vmatrix} a_1 b_1 c_1 d_1 \\ a_2 b_2 c_2 d_2 \\ a_3 b_3 c_3 d_3 \\ a_4 b_4 c_4 d_4 \end{vmatrix} = a_1 \begin{vmatrix} b_2 c_2 d_2 \\ b_3 c_3 d_3 \\ b_4 c_4 d_4 \end{vmatrix} - a_2 \begin{vmatrix} b_1 c_1 d_1 \\ b_3 c_3 d_3 \\ b_4 c_4 d_4 \end{vmatrix} + a_3 \begin{vmatrix} b_1 c_1 d_1 \\ b_2 c_2 d_2 \\ b_4 c_4 d_4 \end{vmatrix} - a_4 \begin{vmatrix} b_1 c_1 d_1 \\ b_2 c_2 d_2 \\ b_3 c_3 d_3 \end{vmatrix}$$

etc. In general, to evaluate a determinant of the nth order, take the elements of the first column with signs alternately plus and minus, and form the sum of the products obtained by multiplying each of these elements by its corresponding **minor**. The minor corresponding to any element a_1 is the determinant (of next lower order) obtained by striking out from the given determinant the row and column containing a_1.

Properties of Determinants.

1. The columns may be changed to rows and the rows to columns:

$$\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} = \begin{vmatrix} a_1 a_2 a_3 \\ b_1 b_2 b_3 \\ c_1 c_2 c_3 \end{vmatrix}$$

2. Interchanging two adjacent columns changes the sign of the result.
3. If two columns are equal, the determinant is zero.
4. If the elements of one column are m times the elements of another column, the determinant is zero.
5. To multiply a determinant by any number m, multiply all the elements of any one column by m.
6. $$\begin{vmatrix} a_1 + p_1 + q_1, & b_1 & c_1 \\ a_2 + p_2 + q_2, & b_2 & c_2 \\ a_3 + p_3 + q_3, & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} + \begin{vmatrix} p_1 b_1 c_1 \\ p_2 b_2 c_2 \\ p_3 b_3 c_3 \end{vmatrix} + \begin{vmatrix} q_1 b_1 c_1 \\ q_2 b_2 c_2 \\ q_3 b_3 c_3 \end{vmatrix}$$
7. $$\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} = \begin{vmatrix} a_1 + mb_1, & b_1 & c_1 \\ a_2 + mb_2, & b_2 & c_2 \\ a_3 + mb_3, & b_3 & c_3 \end{vmatrix}$$

Solution of Simultaneous Equations by Determinants.

If
$$\begin{aligned} a_1x + b_1y + c_1z &= p_1 \\ a_2x + b_2y + c_2z &= p_2 \\ a_3x + b_3y + c_3z &= p_3 \end{aligned} \quad \text{where } D = \begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} \neq 0,$$

then
$$\begin{aligned} x &= D_1/D, \\ y &= D_2/D, \text{ where } D_1 = \begin{vmatrix} p_1 b_1 c_1 \\ p_2 b_2 c_2 \\ p_3 b_3 c_3 \end{vmatrix}, \; D_2 = \begin{vmatrix} a_1 p_1 c_1 \\ a_2 p_2 c_2 \\ a_3 p_3 c_3 \end{vmatrix}, \; D_3 = \begin{vmatrix} a_1 b_1 p_1 \\ a_2 b_2 p_2 \\ a_3 b_3 p_3 \end{vmatrix} \\ z &= D_3/D, \end{aligned}$$

Similarly for a larger (or smaller) number of equations.

THE ALGEBRA OF IMAGINARY OR COMPLEX QUANTITIES

In the algebra of imaginary or complex quantities, the objects on which the operations of the algebra are performed are not numbers in any ordinary sense of the word, but are best thought of as **points in a plane** (or as **vectors** drawn from a fixed origin to these points). The "**complex plane**" is determined by three fundamental points, O, U, i, arranged as in Fig. 2 and called the **zero point**, the **unit point**, and the **imaginary unit point**, respectively. All points on the line through O and U are called **real points**—positive if on the right of O, negative if on the left. All the remaining points in the plane are called **imaginary points**—those on the line through O and i being called the **pure imaginary points**.

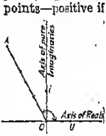


FIG. 2.

The position of any point A in the plane may be determined by the *distance* from the origin O, measured in terms of OU as the unit length, and the *angle* φ which OA makes with the positive direction of the axis of reals. The distance r is sometimes called the modulus or absolute value of the point; the angle φ is sometimes called the amplitude or argument of the point. The notation $A = (3, \angle 120°)$ means the point whose *distance*, r, is 3 times OU, and whose *angle*, φ, is 120°. The development of the algebra depends wholly on the definitions of three fundamental operations denoted by $A + B$, $A \times B$, and e^A, as follows.

Addition and Subtraction. The **sum**, $A + B$, of two points A and B is defined as the point reached by starting from A and performing a journey *equal in length and direction to the journey from O to B. That is, the vector* from O to $A + B$ is the vector sum of the vectors OA and OB. In case A and B are not in line with O, the point $A + B$ is the fourth vertex of a parallelogram of which OA and OB are the sides (Fig. 3). Conversely, if any two points A and B are given, there is a definite point X such that $A = B + X$; this point X is called the **remainder**, A minus B, and is denoted by $A - B$. The point $O - B$ is denoted for brevity by $-B$. With these definitions of $A + B$ and $A - B$, all the ordinary laws of addition and subtraction that hold in the algebra of real numbers hold also in the algebra of complex quantities. In particular, the zero point O has all the formal properties of the number zero, and is denoted by 0.

FIG. 3.

[Note: If A and B are "real" points, $A + B$ and $A - B$ will also be real.]

Repeated Addition. Multiples and Submultiples. The point $A + A + A + \ldots + A$ to n terms is called the **nth multiple of A** and is denoted by nA. The points U, $2U$, $3U$, . . . are denoted, for brevity, by 1, 2, 3, ... Conversely, if any point A, and any positive integer n are given, there is a definite point X such that $nX = A$; this point X is called the **nth submultiple of A**, and is denoted by A/n. The points $U/2$, $U/3$, . . . are denoted, for brevity, by ½, ⅓, ...

Multiplication and Division. The **product**, $A \times B$, or $A{\cdot}B$, or AB, of two points A and B is defined as the point whose angle is the sum of the angles of the given points, and whose distance is the product of the distances. (See Fig. 4.) Thus, if $A = (5, \angle 120°)$ and $B = (2, \angle 270°)$, then $AB = (10, \angle 30°)$. Conversely, if any two points A and B are given, provided B is not zero, there is a definite point X such that

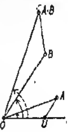


FIG. 4.

of this proportionate thickness is a measure of proportionate strength. This thickness is usually designated as the **blade thickness fraction** and is the ratio between the length of the axial intercept between the front and back lines (extended to the axis) of a central radial section of the propeller blade and the diameter of the blade (see Fig. 5). B.T.F. $= f/d$.

In times past it was usual to make the circumferential section through the blade ogival in shape or as a portion cut from a disk. Recently, owing to the marked advance in the science of fluid mechanics, it has been found that blades of "airfoil" section offer much less resistance to the flow of water past, and consequently a somewhat higher efficiency propeller is the result. This increase in the efficiency due to the adoption of airfoil section is not

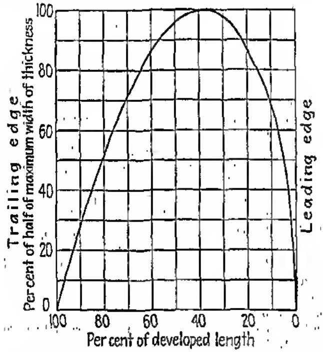


FIG. 8.—Streamline blade sections. Streamline contours about center line.

marked, being a matter of several percent. Also, it has been found that airfoil sections tend to increase cavitation, all other things being equal, because the airfoil section is thicker.

One system of streamlining is covered in Figs. 7 and 8. This is based on the conception of the blade section resembling the plan view of a fish. Assume the backbone or center line of the fish to be curved or cambered so that one side of the fish is fairly flat. This flat side would be the driving face. Having fixed the length of the section according to Fig. 6, the center-line camber is determined according to Fig. 7. The blade thickness, for strength reasons, having been fixed as discussed later, the streamline contours about the curved or cambered center line are determined according to Fig. 8.

Since propellers have been discussed up to this point on a proportionate basis, the law of comparison between a large and a small, or model, propeller will now be given.

(wake and thrust deduction must be taken into account as in the case of propellers working forward) and a given rpm. The curve shown in Fig. 12 is from an actual propeller. Note that

$$\frac{T/V^2d^2}{Q/V^2d^3} = \frac{Td}{Q} = 5.5$$

This latter relationship holds approximately for all propellers regardless of number of blades, blade shape, or pitch.

If it is more convenient, the curve can be obtained by running at one value of water speed and varying the rpm of the model propeller. Note

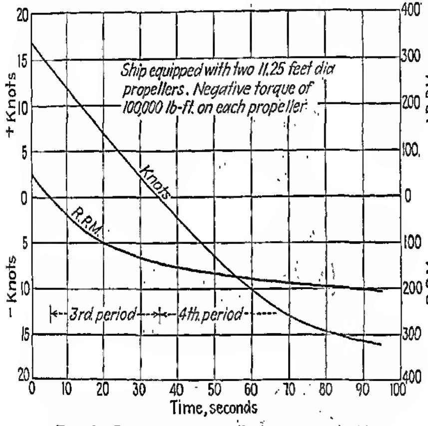


FIG. 13.—Performance of propeller in reverse or backing.

that when Nd/V is zero, the rpm is zero but thrust and torque remain. The different phases in the reversing or backing operation as listed above are indicated on the curve, Fig. 12. Phase 1 is, of course, normal forward operation of the propeller and would not appear on the reverse-operation curve. To obviate any confusion that may be due to negative torque existing during phase 1, note that if there were zero torque, the propeller would pass through this phase but not quite so quickly. The practical effect of this is that a few seconds can elapse between the time the ahead throttle is closed and the reverse is opened with no appreciable effect on the "ahead-reach" or distance it takes for the ship to come dead in the water.

Figure 13 shows the performance of an actual ship in reverse.

Kort Nozzles. The Kort nozzle is a nozzle or ring of airfoil section in which the propeller rotates. Naturally, since space is occupied by the nozzle

Law of Comparison

Diameter, ft	D	d
Rpm	N	n
Speed on advance, knots	V	v
Thrust, lb	T	t
Torque, lb-ft	Q	q
Pressure on propeller, psi	P_1	p_1
Power absorbed	P	p

λ = ratio of linear dimensions.

$$D = \lambda d \qquad Q = \lambda^4 q$$
$$T = \lambda^3 t \qquad V = v\sqrt{\lambda}$$
$$N = \frac{n}{\sqrt{\lambda}} \qquad P = \lambda p_1$$
$$P = \lambda^{3.5} p$$

These equations hold until cavitation starts.

It should be mentioned here that, in running tests to determine the performance of a small model in order that this performance may be converted to that of a propeller of large proportions, elaborate equipment is necessary if there is any likelihood that the large propeller might operate near the cavitation point. This equipment is necessary in order to obtain less than atmospheric pressure on the surface of the water in which the test is run.

Such tests are usually run in a water tunnel with a vacuum pump attached and controls so arranged that the desired pressure will be maintained on the water surface throughout the test. It is beyond the scope of this book to describe the detailed operation of such a water tunnel.

As will be brought out later, the pressure on the propeller has a great effect on the point at which cavitation or boiling will occur. As an example of this phase of model performance, consider the case of a ship that is to be equipped with a 15 ft diam propeller with 4-ft tip submergence. This is equivalent to 11.5 ft water at the hub plus atmospheric pressure of 15 psi 11.5 ft $\times$ 0.4335 = 5 psi. $P = 5 + 15 = 20$ psi abs pressure on the hub.

Suppose a 6-in. model is to be run in order to predict therefrom the performance of the 15-ft propeller.

$$\lambda = \frac{15}{0.5} = 30$$

$$p_1 = \frac{P_1}{\lambda} = \frac{20}{30} = 0.67 \text{ psi}$$

or 1.36 in. Hg abs. With a 30-in. barometer this would correspond to a negative manometer reading of 28.64 in. vacuum.

Tests that are run with full atmospheric pressure on the surface of the water are called "open-water" tests. All readings below the point at which cavitation would occur if the proper pressure were on the propeller are quite accurate. In other words, below cavitation, pressure does not appreciably affect torque and thrust readings.

This extra-large proportionate pressure which exists in the case of small propellers operating in open water is one reason why motorboat propellers can operate at such high values of rpm without cavitation difficulties. It is also the reason why self-propelled tests of ship's models in open water are not always a true measure of performance, especially at the highest powers.

or ring surrounding the propeller, a propeller of smaller diameter must be installed than would otherwise be possible. The advantage of this arrangement is that the thrust is increased at speeds below approximately 8 knots. Therefore, the device is of some advantage when applied to tugs, trawlers, or

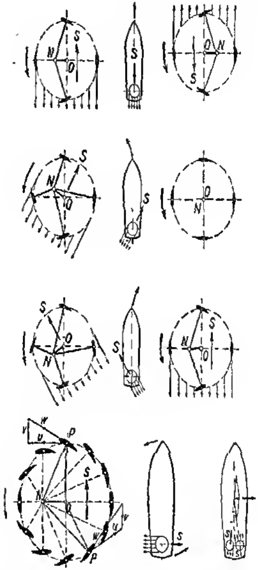


FIG. 14.—Voith-Schneider propeller action.

river towboats. Above approximately 8 knots the arrangement is less efficient than is the conventional propeller.

Voith-Schneider Propeller. The Voith-Schneider propeller consists of an assembly containing long blades of airfoil section projecting vertically downward. This assembly is so arranged that the blades project downward from the after part of the ship's bottom. There may be one or more such

All those concerned with the testing of ship's models under self-propelled conditions dream of the time when they will have a variable-pressure basin in which all factors are adjusted.

In order to choose a propeller for a given job it is necessary to have all the data for the determination of B_u. Required are N, the rpm of the propeller: U, the useful horsepower; and V_a the speed of advance of the propeller.

The useful horsepower, U, is that part of the initial shaft horsepower (shp) which is available for propulsion after the propeller losses are taken into account. Having the useful horsepower the hull effect must be considered in order to obtain the actual work on the ship, which is, of course, the effective horsepower (ehp) of the ship at that speed (see p. 1399).

The hull efficiency may be greater than 100 percent, and consequently the useful horsepower be less than the ehp. A strong wake helps or increases the hull efficiency, whereas a strong or heavy thrust deduction detracts from it.

Wake effect is specified as "wake fraction" and is the speed of the wake divided by the speed of the ship with reference to undisturbed water. Since the propeller operates in water, the speed of advance of the propeller through the water in which it finds itself will not be the same as the speed of the ship unless the wake has a speed of zero. If w = wake fraction, V = speed of ship, knots, V_a = speed of advance of propeller,

$$w = \frac{V - V_a}{V} \qquad \text{or} \qquad V_a = V(1 - w)$$

Thrust deduction is due to the propeller's drawing water past the surface or skin of the hull and thereby causing a backward drag. If the propeller were not there, this would not exist; consequently, it is not a regular part of the resistance such as would be evident or become evident in a towed test.

Thrust-deduction effect is specified as the thrust-deduction coefficient and is the ratio between the thrust deduction and the initial thrust of the propeller. If T = propeller thrust, lb, t = thrust-deduction coefficient, $(1 - t)$ = proportion of the propeller thrust useful in driving the ship,

$$T(1 - t) = \text{useful thrust}$$

$$T = \frac{326 \text{ ehp}}{(1 - t)V}$$

Hull efficiency is defined as

$$\frac{1 - t}{1 - w}$$

Therefore, it can be seen that, if the wake fraction is greater than the thrust deduction, the hull efficiency is greater than 100 percent.

$$\text{Ehp.} = U\left(\frac{1 - t}{1 - w}\right)$$

Therefore,

$$U = \frac{\text{Ehp}}{\dfrac{1 - t}{1 - w}} = e_p \text{ shp}$$

and

$$\text{Shp} = \frac{\text{ehp}}{e_p\left(\dfrac{1 - t}{1 - w}\right)}$$

From the foregoing it is obvious that, if values for ehp, w, t, V, and N are

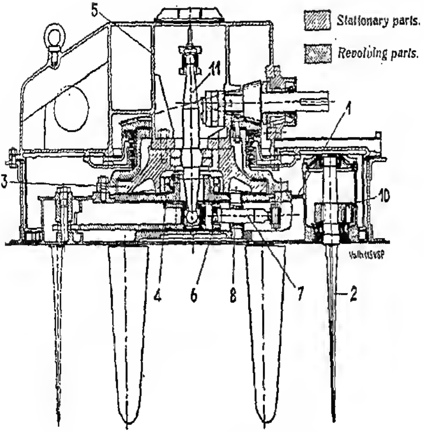


FIG. 15a.—Voith-Schneider propeller assembly.

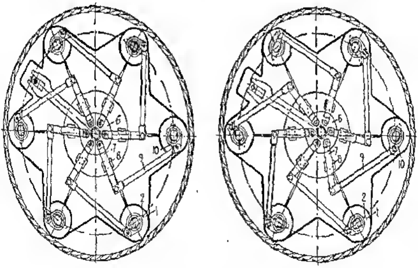


FIG. 15b.—Voith-Schneider propeller feathering motion.

values are determined from hull characteristics with the exception of N, the propeller designer should obtain them from the naval architect. N, of course, is determined by the marine engineer, and the propeller designer should obtain that value from him.

Cavitation. If the blade of a propeller of given blade area is driving a ship through water, a pressure exists on the driving face of the blade. For a given thrust on this part of the propeller the unit pressure will increase as the blade is made more narrow. At the same time, on the reverse side of the blade, the pressure is relieved; and, if the revolutions are sufficiently high and the slip sufficiently great, a partial vacuum may exist to the extent that vaporization of the water takes place and a conventional boiling of the water on the back of the blade is evident. The result of this is that the whole propeller may become enveloped in a boiling mass of water and thrust may no longer increase in the same proportion to the torque applied as was the case before this boiling started to occur. The energy put into the propeller shaft is going, to a great degree, to boiling water rather than to propelling the ship. This phenomenon is called cavitation and, from its nature, is more likely to occur under the following conditions:

1. *The Pitch Ratio Is Great.* If the pitch is small, the propeller more nearly approaches the condition of a disk rotating in the water, which would be the case were the pitch ratio zero. In this case, there would be no tendency to create a vacuum back of the blades and consequently no boiling. As the pitch is increased, this tendency to create the vacuum back of the blade grows greater with the allied tendency to boil.

2. *The Slip Is Great.* If there were no slip, the effect would be the same as that of a disk revolving in the water, as in the case of zero pitch; therefore, there could be no tendency toward a vacuum and consequently boiling.

3. *The Hub Depth Is Small.* The deeper the hub, the greater will be the pressure of water on the propeller and, consequently, the less the tendency toward a vacuum and consequent boiling.

4. *The Blade Area Is Small.* The smaller the blade area, the greater will be the unit pressure on the driving face and this greater tendency to cause vacuum on the back of the blade.

The foregoing characteristics refer to a propeller of given diameter. Since the speed of any portion of the surface for a given rpm is proportional to the diameter, it follows that the rpm at which cavitation will start is inversely proportional to the diameter.

Capt. E. F. Eggert, USN, developed a method for determining the approximate rpm at which cavitation will start. Among other factors, this method involves the thickness ratio of the blade section, or its maximum thickness divided by its width, where the maximum thickness is in the center as in the case of an ogival blade. Capt. Eggert called this ratio c. For the airfoil section where the maximum thickness is at about one-third the width from the leading edge, c is three-fourths the thickness ratio described above (see Figs. 7 and 8). Naturally, as the blade width is increased, the thickness increases; but, for all practical purposes in the case of ogival sections,

$$c = \frac{0.46 - 0.74b}{5.7}$$

where b = M.W.R.

For a given temperature the boiling pressure of water is the absolute pressure rather than the gage pressure. This can also be expressed in terms

assemblies per ship. The blades are caused to rotate about a vertical central axis; at the same time they feather or oscillate around their own axes. By changing the feathering angle, this being done from the bridge by fairly simple control devices, the thrust can be made to act in any direction, even in reverse. As a consequence, no rudder is needed. Also, this results in a highly maneuverable ship. They have not been built in powers greater than 2,500 shp. Claims for extraordinary efficiency are based on the fact that the rudder, struts, and other such appendages can be removed from the hull, thereby reducing the hull resistance. The actual efficiency of the blade arrangement is little different from that of the conventional propeller (see Figs. 14 and 15).

Example. In order to use the foregoing data and information, take the ship for which the effective horsepower was worked out on p 1397. The ehp = 3,324 (see p. 1399). Block coefficient, $C_p = 0.651$ (see p. 1383). Refer to the curve (Fig. 7) on p. 1406 and find the wake fraction, $w = 0.20$. The hull efficiency, e_h, for a twin-screw ship of conventional design should be approximately unity (see p. 1407).

$$e_h = \frac{1-t}{1-w} = 1.00 = \frac{1-t}{1-0.20}$$

$$t = 0.20$$
$$V = 14.25 \text{ knots}$$
$$H = 25.5 \text{ ft draft}$$
$$N = \text{rpm} = 100$$

$$U = \frac{\text{ehp}}{\dfrac{1-t}{1-w}} = \frac{3{,}324}{2\left(\dfrac{1-0.20}{1-0.20}\right)} = 1{,}662$$

$$U^{1/2} = 40.75$$
$$V_a = V(1-w) = 14.25(1-0.20) = 11.4$$
$$V_a^{5/2} = 439$$

$$B_u = \frac{NU^{1/2}}{V_a^{5/2}} = \frac{100 \times 40.75}{439} = 9.28$$

Since this value of B_u is such that the efficiency of either three- or four-bladed propellers will be the same (see Figs. 1 and 2), suppose a four-bladed propeller is chosen.

For maximum efficiency, $a = \frac{p}{d} = 1.00$ and $\delta = 131$

$$e_p = 0.695$$

$$d = \frac{\delta V_a}{N} = \frac{131 \times 11.4}{100} = 14.95 \text{ ft, call it 15 ft}$$

All this is on a basis of M.W.R. = 0.25 and B.T.F. = 0.05. This to be checked for cavitation.

$$\text{rpm}_c = \frac{60}{d}\sqrt{\frac{h[7.3(1+4b)]}{aS+5.7c}}$$

Since the draft, H, is 25.5 ft, the hub depth could be 17.5 ft.

$$h = 33 + 17.5 = 50.5$$
$$5.7c = 0.46 - 0.74b = 0.275$$

of feet head of water. Atmospheric pressure in terms of feet head is 33 ft. Since the absolute pressure on the propeller blades is the depth of water plus the atmospheric pressure on the surface of the water, the absolute head is

$$h = 33 + \text{depth in ft}$$

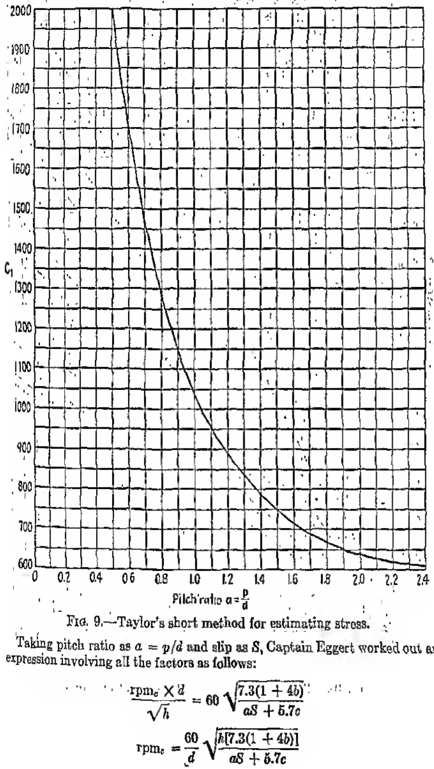


Fig. 9.—Taylor's short method for estimating stress.

Taking pitch ratio as $a = p/d$ and slip as S, Captain Eggert worked out an expression involving all the factors as follows:

$$\frac{\text{rpm}_c \times d}{\sqrt{h}} = 60\sqrt{\frac{7.3(1 + 4b)}{aS + 5.7c}}$$

$$\text{rpm}_c = \frac{60}{d}\sqrt{\frac{h[7.3(1 + 4b)]}{aS + 5.7c}}$$

$$1 - S = \frac{101.33}{a\delta} = \frac{101.33}{1.00 \times 131} = 0.774$$

$$S = 0.226$$

$$\text{rpm}_c = \frac{60}{15}\sqrt{\frac{50.5(7.3 \times 2)}{0.226 + 0.275}} = 153 \text{ which is higher than necessary.}$$

Try M.W.R. = 0.20

$$\text{rpm}_c = \frac{60}{15}\sqrt{\frac{50.5(7.3 \times 1.8)}{0.226 + 0.312}} = 140$$

This is more than enough margin for safety and there is little, if anything, to be gained in the way of efficiency by reducing the blade width further.

Check for strength (see Figs. 9 and 10)

$$c_1 = 1{,}040, \quad X = \frac{c_1 P_1}{N d^3}$$

The power per propeller will be approximately $\dfrac{3{,}324}{2 \times 0.695} = 2{,}395$

$$P_1 = \frac{2{,}395}{4} = 598$$

$$X = \frac{1{,}040 \times 598}{100 \times 15^3} = 1.84$$

$$Y = \tfrac{2}{3}\,(\text{M.W.R.})\,(\text{B.T.F.})^2 = 0.00036 \text{ for } 5{,}000 \text{ psi.}$$

$$\tfrac{2}{3}(0.20)(\text{B.T.F.})^2 = 0.00036$$

$$\text{B.T.F.} = 0.052$$

Correct for δ and e_p (see Fig. 3).

$$\delta \text{ corrected} = 131 \times 0.995 = 130$$

$$e_p \text{ corrected} = 0.695 \times 1.00 = 0.695$$

Consequently, from an efficiency standpoint it would be just as well to retain the M.W.R. = 0.25 and B.T.F. = 0.05 and there would be a gain in strength.

Check for strength.

$$X = 1.84$$

$$Y = \tfrac{2}{3}(0.25)(0.05)^2 = 0.000416$$

Stress approximately 4,000 psi

$$\text{shp} = \frac{\text{ehp}}{e_p\left(\dfrac{1-t}{1-w}\right)} = \frac{3{,}324}{2 \times 0.695} = 2{,}395 \text{ per shaft.}$$

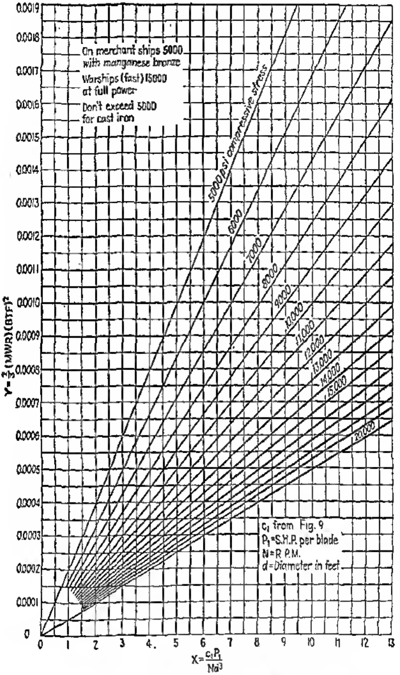


FIG. 10.—Taylor's short method for estimating stress.

PROPULSION SHAFTING

BY

AUSTIN H. CHURCH and J. M. LABBERTON

REFERENCE: Den Hartog, "Mechanical Vibrations," McGraw-Hill.

General. Shafting, being long, is invariably made up of several sections, which are fastened together inside the ship by means of conventional flanged couplings. The flanges are formed by upsetting the ends of the shafts and are, therefore, forged integrally therewith. Either parallel bolts with heads and a slight clearance between the body of the bolt and the hole may be used, or tapered bolts without heads that completely fill the hole. In the latter case the two flanges should be reamed and tapered together with a taper of ¾ in. to the foot. This makes an excellent joint but there is no interchangeability. If it is taken down, it should be reamed again.

Shear stress in the bolts should not exceed 8,500 psi for high-speed vessels where the material has an ultimate tensile strength of 120,000 psi, nor should it exceed 5,000 psi on merchant ships where the material has an ultimate tensile strength of 60,000 psi. In addition to shear stresses there are, of course, tension stresses due to any bending of the assembled shafting. The bolt sizes are usually fixed by classification society rules. Generally the bolt material has the same physical characteristics as the shafting.

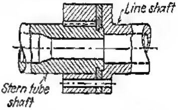


FIG. 1.—Inboard shaft coupling.

A coupling such as shown in Fig. 1 is usually fitted just inboard of the stern tube. Such a coupling will transmit ahead and astern thrust and torque. Often it is desirable to pull the stern-tube shaft out through the stern-tube

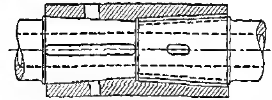


FIG. 2.—Outboard shaft coupling.

bearing in order to make repairs. This is much more convenient than taking the shaft into the hull and shaft alley and out through the engine room. It can be done with the coupling such as shown in Fig. 1. The stern-tube shaft has the same diameter all the way through. During astern operation the load is taken by a collar held in place by bolts as shown. Outside the ship where the shaft is exposed to the sea, couplings such as shown in Fig. 2 are generally used. Note that keys are used across the diameters to prevent the shaft sections from pulling out of the tapers when the ship is backing. One advantage of this type of coupling is that it offers less resistance to water flow than would a conventional flange coupling.

where rpm_c is the predicted rpm at which cavitation will start. It will be noted from this expression that high pitch ratio, slip, and diameter lower the speed at which cavitation will start and that the deeper the propeller works in the water and the wider the blade, the higher the speed that can be attained before cavitation will start.

Hub depth is the depth considered, for this is the average depth of the blades.

Strength. To calculate the blade stresses in propellers under conditions of load is a complicated and involved procedure. However, in order to obtain a quick, if not too accurate, estimate of the strength of a propeller,

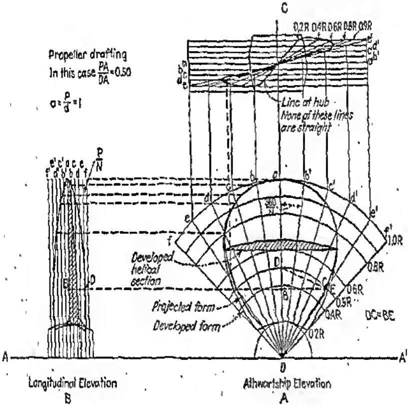


FIG. 11.—Drafting of propellers.

Admiral Taylor drew some curves through points indicating the strength of a number of propellers. By means of these curves, it is possible to determine quickly the maximum compressive stress approximately. These curves are shown in Figs. 9 and 10.

Drafting of Propellers. The object in making a drawing of a propeller is to obtain a visual idea as to how it would appear, to obtain clearance dimensions with the hull, and to furnish the patternmaker and machinist with sufficient information to manufacture the propeller properly. The drafting of propellers is fairly simple provided that the geometry is thoroughly understood. There are several methods of depicting propellers. The one that will now be demonstrated has proved satisfactory in practice.

Sometimes it is necessary to incline the blades of a propeller in order to clear the ship's rudder or other stern parts. In this event, the surface of a

Stresses. As regards the shafting proper, it must withstand combined torsional, bending, and thrust stress. Also the flexural and torsional critical speeds must be out of the range of normal operating speeds. Finally the classification society rules must be met.

Torsional stress is caused by the twisting moment. In the case of geared-turbine or electric drive the torque is fairly smooth but, in the case of reciprocating-engine drive, steam or diesel, the torque pulsates or rises and falls in

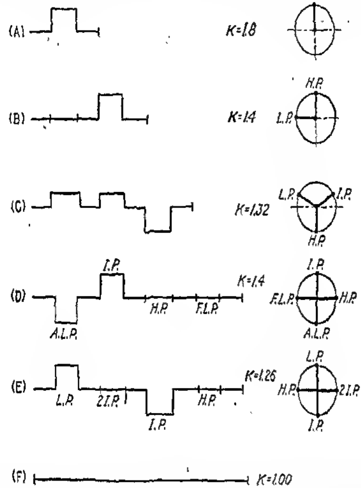


FIG. 3.—Values of K for shafting driven by reciprocating steam engines.

magnitude. Naturally the shaft must withstand the maximum stress. The twisting moment in inch-pounds is

$$M_T = \frac{\text{shp}}{\text{rpm}} \times 63.024 \times K$$

where K is a factor depending on the number of cylinders and crank arrangements of the reciprocating engine. In the case of geared-turbine or electric drive, K is unity. The value of K for various steam-engine arrangements is shown in Fig. 3. If the ihp only is known, a mechanical efficiency of 92 percent may be assumed, in which case

$$M_T = \frac{\text{ihp}}{\text{rpm}} \times 57{,}980 \times K$$

screw is generated by a straight line moving forward at a certain rate, rotating about a fixed axis, and inclined at a fixed angle to the axis of rotation. In most cases, however, the surface is generated by a straight line moving forward at a certain rate, rotating about a fixed axis, and vertical to the fixed axis. The former is called an **inclined generatrix** and the latter a **vertical generatrix**.

Propellers having a vertical generatrix will be considered first. Let R be the radius of the propeller, which in this case is unity. In any practical case the actual diameter should be used. However, it would be more convenient to continue to consider the even percentages of the radius for determining the blade sections, as will be demonstrated. Draw radial lines ob, ob', etc., forming equal angles at o (Fig. 11A).

$$N = \text{number of equal angles for a complete circle}$$
$$p = \text{pitch of propeller}$$
$$\frac{360}{N} = \text{degrees in each equal angle}$$

Draw arcs with radii of $0.2R$, $0.3R$, etc. Plot projected form of blade by means of proportions given in Fig. 6.

Projection on Longitudinal Plane (Fig. 11B). Draw axes as before. Draw vertical lines spaced p/N apart and letter a, b, c, etc. Project points of intersection of projected form with the radial lines of Fig. 11A over to Fig. 11B. The points where these projection lines intersect corresponding lettered lines are points of longitudinal projected form as shown on Fig. 11B.

Plan (Fig. 11C). Draw axis a, a_1. Draw horizontal lines spaced p/N apart, and letter b, b', etc. Project points of intersection of radial lines with arcs $0.2R$, $0.3R$, etc. (Figs. 11A to 11C). The intersections of these projection lines and the corresponding horizontal lettered lines (Fig. 11C) are points of helices, $0.2R$, $0.3R$, etc. Project points of intersection of projected form with arcs (Figs. 11A to 11C). The intersections of these projection lines and the corresponding lettered helices are points of plan projected form as shown in Fig. 11C.

Developed Form (Fig. 11A). Take any arc as $0.5R$ and draw a horizontal line BC through its intersection with the projected form. Extend this line to the longitudinal section (Fig. 11B) and obtain BD. Transfer BD to the athwartship elevation (Fig. 11A) to axis oa. Draw DC. With DC as a radius and B as a center, draw an arc intersecting BC projected. This intersection is a point on the developed form, E.

In the case of the **inclined generatrix** the procedure is exactly the same except that in the longitudinal elevation the lines a, b, c, d, etc., are inclined to the right or left, as the case may be, at the proper angle but spaced apart horizontally a distance equal to p/N. Correction must be made for this inclination in the plan view by inclining, from which $0.2R$, $0.4R$, $0.6R$, etc., are drawn.

Blade Sections. In the case of an ogival blade the section is formed by a straight line and an arc; in the case of a streamlined blade, by a series of curves. For the sake of simplicity, consider an ogival section as shown in Fig. 11A, but bear in mind that this section can be replaced by a streamlined one. In any case, the length of the blade section is

$$\text{Length} = \sqrt{(\text{circumference})^2 + (\text{pitch})^2} \times \frac{\text{angle}}{360}$$

In the case of diesel engines with direct-drive, the following values of K should be used.

$$M_T = \frac{\text{ihp}}{\text{rpm}} \times 50{,}420 \times K \quad \text{(attached auxiliaries)}$$

$$M_T = \frac{\text{ihp}}{\text{rpm}} \times 53{,}867 \times K \quad \text{(independent auxiliaries)}$$

	Cylinders	Value of K
4-cycle single-acting	1 4 6 8 10 12	17 2.3 1.6 1.38 1.24 1.15
2-cycle single-acting	2 3 4 6	3.48 2.10 1.38 1.15
4-cycle double-acting	2 3	1.38 1.15

Calculation of Stresses. The line shaft is subjected to three primary stresses which act simultaneously: (1) torsion, due to the driver f_s, (2) direct compression, due to the propeller thrust f_t, and (3) bending, due to its own weight, f_b.

A more effective use of the material for bending and torsional strength from a weight standpoint may be obtained by using a hollow shaft. For this reason propulsion shafting is often made hollow with the inside diameter about 60 percent of the outside.

The transmitted torque, M_T, in inch-pounds will induce a torsion or shear stress of magnitude

$$f_s = \frac{5.1 M_T D}{D^4 - d^4}$$

where D = outside diameter of the shaft, in.
d = inside diameter, in. If the shaft is solid, d equals zero.

The propeller thrust, T lb, may be found from the equation

$$T = \frac{326 \text{ ehp}}{(1 - t)V}$$

where ehp = effective horsepower
t = thrust-deduction coefficient (see p. 1419)
V = speed of the ship, knots

The action of this thrust may produce a column effect in the shaft, particularly if the ratio of bearing span to shaft diameter is large. This is included in the

The angle is at o between radii through the intersection of the projected form and the arc at the radius at which the blade section is to be determined. Calculate the length as outlined above, project over to the longitudinal section to obtain the blade thickness at that point, and the section is known.

Noise. Propeller noise is usually caused by the propeller tips being too close to the hull, which causes the hull to vibrate, or by the leading edges of the blade being too blunt. Propeller blade tips in general should not swing within 24 in. of the hull plating. Also, too few blades sometimes are the cause of synchronous vibrations. The greater the number of blades, the greater the frequency at given rpm (see p. 1436).

Bad propeller whine, or "singing," has been cured merely by sharpening the leading edges of the blades.

Weights. A quick approximation of propeller weight can be determined from the expression

$$\text{Weight, lb} = Kd^3(\text{M.W.R.})(\text{B.T.F.})$$

where K = 345 for three blades
K = 450 for four blades

$$\text{Radius of gyration} = 0.21d$$

where d = diameter, ft.
The above values of K are for bronze propellers. If another material is used, correction must be made for the density.

Backing of Propellers. In the case of any propeller, the thrust is a function of the torque. The exact relationship is uncertain but model-basin tests indicate that when propelling forward, the driving being done by the face of the blades,

$$\frac{Td}{Q} = 8 \text{ approx}$$

But when revolving backward, the driving being done by the backs of the blades,

$$\frac{Td}{Q} = 5.5 \text{ approx}$$

where Q = torque, lb-ft

When a ship is proceeding forward with the propeller operating normally, driving is done by the face of the propeller blades and there is slip. If the torque on the propeller is reversed, the slip first goes to zero. While going to zero, the thrust is still forward until the slip reaches zero at which instant the thrust is zero. This is the normal forward operation of a propeller.

This is followed by a second period during which the slip becomes negative and gradually increases negatively until the rpm is zero. During this period the thrust has reversed although the propeller continues to turn in the forward direction. At approximately the point of zero rpm erratic behavior of both torque and thrust may be expected. This may be due to a cavitation phenomenon but it is not certain. In any event, the propeller at this instant is dragged through the water by the ship and the effect is as if the propeller an appendage fastened firmly to the stern.

en a third period or phase of operation begins when the propeller, owing e reverse torque, starts to turn or revolve in reverse and then increases n until the ship comes to rest in the water and is traveling at zero knots.

equation

$$f_t = \frac{T}{A}\left[1 + \frac{16L^2}{25{,}000(D^2 + d^2)}\right]$$

where A = cross-sectional area of the shaft, sq in.
L = distance between the bearing centers, in.

Since the shaft is supported on many bearings, the bending action is similar to that of a uniformly loaded continuous beam, and the bending moments should be calculated on that basis. However, it is generally sufficiently accurate to treat each span as though it were a beam with fixed ends and a distributed load. The distributed load is that due to the weight of the shaft. On this latter basis, the maximum bending stress is given by the equation

$$f_b = \frac{0.849wL^2D}{D^4 - d^4}$$

where w = shaft weight per in. of length, lb

These primary stresses may be combined to obtain the resultant shear and compressive stresses by the standard formulas

$$f_t' = \tfrac{1}{2}(f_t + f_b) + \tfrac{1}{2}\sqrt{(f_t + f_b)^2 + 4f_s^2}$$
$$f_s' = \tfrac{1}{2}\sqrt{(f_t + f_b)^2 + 4f_s^2}$$

Generally there is no bearing placed aft of the propeller. In such cases this section is overhung and a cantilever action takes place. The bending stress is then due to the weight of the propeller itself plus the distributed weight of the overhung shaft. The bending moment, when the ship is lying idle in calm water, is the weight of the propeller with its nut, etc., multiplied by the shaft length from the after-strut bearing to the propeller-assembly center of gravity plus the relatively slight moment due to the stub end of the shaft weight.

When the ship is at sea under storm conditions, there will be additional inertia stresses due to the sudden accelerations and decelerations as the ship pitches and yaws. Since it is impossible to predict the exact magnitude of these stresses, it is common practice to design by doubling the bending moment due to the propeller assembly weight alone, i.e., let the moment $M_B = 2F_1L_1$ where F_1 is the impeller assembly weight and L_1 is the distance from the after-strut bearing to the center of gravity of the propeller assembly in inches.

The primary stresses, which can be combined as noted for the previous case, become

$$f_b = \frac{10.2M_BD}{D^4 - d^4}$$

$$f_t = \frac{T}{0.7854(D^2 - d^2)}$$

$$f_s = \frac{5.1M_TD}{D^4 - d^4}$$

A fourth period or final phase of operation begins when the ship starts to back or make sternway. This continues until a steady state of ship-speed astern, and astern or reverse rpm is attained.

These four phases may be summarized as follows:

1. Negative torque; positive rpm; positive knots; positive slip; positive thrust. Ending in zero slip.

2. Negative torque; positive rpm; positive knots; negative slip; negative thrust. Ending in zero rpm.

3. Negative torque; negative rpm; positive knots; negative slip; negative thrust. Ending in zero knots.

4. Negative torque; negative rpm; negative knots; negative slip; negative thrust. Ending in a fixed backing speed.

In a manner similar to that in which $B_u = \frac{NU^{1/2}}{V^{5/2}}$ was shown to hold constant for any value of Nd/V, it can be shown that T/V^2d^2 is constant as well as Q/V^2d^3 is constant. This being true, a model propeller of the proper

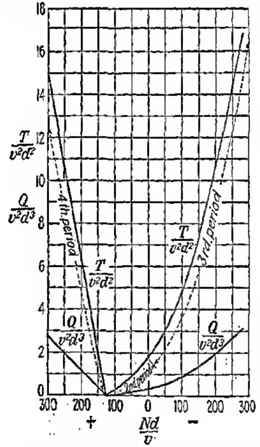


FIG. 12.—Performance of propeller in reverse or backing.

proportions can be run in reverse in a water tunnel and the data from this test plotted as shown in Fig. 12. This curve can then be used to determine the thrust and torque at a given speed of motion backward through the water

It has been observed that for some ships there is a rise and fall in the torque on the propeller shaft due to the proximity of the propeller blades to the hull of the ship. For ordinary clearances encountered in present-day design this pulsating torque amounts to approximately $7\frac{1}{2}$ percent of the average torque carried by shaft in ships having a rounded stern. For propellers operating closely behind a strut or skeg, this pulsation may amount to 15 percent of the mean torque.

Stress Concentration. So far no mention has been made of increased stresses due to stress concentration brought about by small-radii fillets at points where the shaft section changes, as, for instance, where flanges are made for couplings. Methods for allowing for this are shown on p. 420. The greater the change in diameter and the smaller the radius of the fillet, the greater the concentration of the stress at the point of the fillet, amounting to as much as five times the stress in other parts of the member.

Bearings. There must be bearings both inside the hull and outside exposed to the sea. The bearings inside are usually oil-ring-lubricated and babbit-lined such as are used on most slow-speed industrial rotatingapp aratus (see p. 873).

The bearings outside the hull are lubricated by the water that surrounds them and goes through them. The forward stern-tube bearing receives its lubricating water usually from the general service pump in the ship; however, sometimes, the packing gland is loosened and water is permitted to flow in from the sea through the bearing, this water later being discharged overboard by the bilge pumps. The bearing consists of longitudinal strips of wood (lignum vitae blocks with grain on edge) driven into grooves, rubber-bearing surfaces vulcanized to brass backing strips which are driven into the grooves, or phenolic composition blocks made up of cloth and plastic driven into the grooves. The friction losses in these bearings are negligible.

One virtue claimed for the rubber is that if sand or small stones are drawn into the bearing, the rubber will flex and the foreign matter will roll right out; in the case of wood or fabric-plastic, the stones would become embedded and score the shaft.

Thrust bearings are mounted just aft of the engine in a separate casing on reciprocating-engine drive; in the case of geared-turbine drive, they are in the gear box (see p. 1444). The horseshoe type is rarely used except on old installations, having been superseded by the Kingsbury or Michell type. Constructional details and operational characteristics of these bearings are shown on p. 875.

Torsional Vibration. After the moving masses and the interconnecting shafts of a marine drive have been tentatively designed, it is necessary to determine the natural torsional frequencies of the system. The frequencies of the disturbing impulses imposed either by the propeller (number of blades times the propeller rpm) or the driving engine should not equal the natural frequency or whole multiples of it. If this precaution is not observed, resonant conditions may develop which will cause severe strains on the system, shock loading on the gear teeth, or fatigue failures of the shafting. If resonance seems probable, the sizes of the various parts should be altered to avoid this condition.

In determining the natural frequencies of vibration, the positions of the nodes may also be located. Frequently the drive is designed to secure a **nodal drive,** *i.e.*, have a node located at the gear. This will reduce the dynamic impulses on the gear teeth due to the vibration.

$A = BX$. This point X is called the **quotient,** A divided by B, and is denoted by A/B (where $B \neq 0$). Thus, the point A/B is a point whose angle is the angle of A minus the angle of B, and whose distance is the distance of A divided by the distance of B. The point U/B ($B \neq 0$) is called the **reciprocal** of the point B, and is denoted by $1/B$. (See Fig. 5.) With these definitions of AB and A/B the elementary laws of multiplication and division that hold in the algebra of real numbers hold also in the algebra of complex quantities. In particular, the point U has all the formal properties of the number unity, and is denoted by 1.

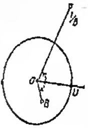


FIG. 5.

[Note: If A and B are real, AB and A/B will also be real.]

Repeated Multiplication. Powers and Roots. The point $A \times A \times A \times \ldots \times A$ to n factors is called the nth power of A and is denoted by A^n (Fig. 6). Conversely, if any point A (not 0) and any positive integer n are given, there will be n distinct points X such that $X^n = A$; each of these points is called an **nth root of A**, some one of them, usually the one with the smallest positive angle, being denoted by $\sqrt[n]{A}$ or $A^{1/n}$. Thus, the point $\sqrt[n]{A}$ is a point whose distance is the nth root of the distance of A, and whose angle is $1/n$th of the angle of A. All the nth roots of A will lie on the circumference of a circle about O as center, and will divide that circumference into n equal parts (Fig. 7). Every point A (not 0) has two square roots, three cube roots, etc. Hence the theorem "If $A^n = B^n$ then $A = B$" does not hold in this algebra, and the ordinary rules for radical signs must be applied with caution. For example, if A and B are positive reals, $\sqrt{-A}\cdot\sqrt{-B} = -\sqrt{AB}$ and not $\sqrt{(-A)(-B)}$, which would give $+\sqrt{AB}$.

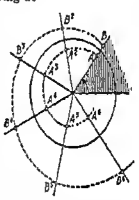


FIG. 6.

[Note: If A is real and positive, $\sqrt[n]{A}$ will be real and positive; if A is real and negative, $\sqrt[n]{A}$ will be real if n is odd and imaginary if n is even.]

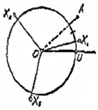


FIG. 7.

Properties of i. The *point i is the point whose distance is 1 and whose angle is 90 deg.* It follows from the definition above that **multiplying any point A by i has the effect of rotating the point through an angle of $+90°$** without changing its distance from O. In particular,

$i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i$, etc.; $i = \sqrt{-1}$, $-i = -\sqrt{-1}$; where "1" denotes not the *number* one, but the *point* U.

Similarly, multiplying any point A by -1 has the effect of rotating the point through 180 deg.

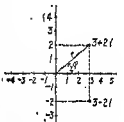


FIG. 8.

First Standard Form for a Complex Quantity (Fig. 8). Any point A can be expressed in the form $x + iy$, where x and y are real points. For example, the three cube roots of 1 are 1, $-\frac{1}{2} + \frac{1}{2}i\sqrt{3}$, and $-\frac{1}{2} - \frac{1}{2}i\sqrt{3}$.

In general, $(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2)$;
$(x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_2y_1 + x_1y_2)$;

$$\frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\,\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$

If two complex quantities are equal, their real parts must be equal, and the coefficients of their pure imaginary parts must also be equal. That is, if $x_1 + iy_1 = x_2 + iy_2$, then $x_1 = x_2$ and $y_1 = y_2$. Thus a single equation between complex quantities is equivalent to two equations between real quantities.

Conjugate Imaginaries. Two points $A = x + iy$ and $B = x - iy$ are called conjugate imaginaries. Two such points are symmetrically situated with regard to the axis of reals. The sum and product of two conjugate imaginaries will be real.

Second Standard Form for a Complex Quantity. Since $x = r\cos\varphi$ and $y = r\sin\varphi$, any point $A = x + iy$ can be expressed $A = r(\cos\varphi + i\sin\varphi)$, where r is real and positive (namely, the distance of A), and φ is real (namely the angle of A). For example, the three cube roots of 1 are 1, $\cos 120° + i\sin 120°$, and $\cos 240° + i\sin 240°$. In general,
$[r_1(\cos\varphi_1 + i\sin\varphi_1)][r_2(\cos\varphi_2 + i\sin\varphi_2)] = r_1r_2[(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1+\varphi_2)]$;
$[r(\cos\varphi + i\sin\varphi)]^n = r^n[\cos(n\varphi) + i\sin(n\varphi)]$ **(De Moivre's Theorem).**

The Exponential Function, e^A, or $\exp A$, of any point $A = x + iy$ is defined as the point whose distance is e^x and whose angle (measured in radians) is y. That is, $e^{x+iy} = e^x(\cos y + i\sin y)$. Here e^x means the ordinary exponential function of the real quantity x, where $e = 2.718$.

From this definition, the usual formal laws of exponents can be deduced: $e^Ae^B = e^{A+B}$, $(e^A)^n = e^{nA}$, $e^{-A} = 1/e^A$; $e^1 = e$, $e^0 = 1$.

The function e^A is a periodic function with a pure imaginary period $2\pi i$; that is, $e^{A \pm k2\pi i} = e^A$, where k is any positive integer.

If A is made to move along a line parallel to the axis of reals [or axis of pure imaginaries], the corresponding point e^A will move along a straight line through O [or along a circle about O as center].

Properties of $e^{i\varphi}$. The point $e^{i\varphi}$ is a point whose distance is 1 and whose angle is φ. It follows from the definitions above that **multiplying any point A by $e^{i\varphi}$ has the effect of rotating the point through an angle** φ, without changing its distance from O. In particular, $e^{i\pi} = -1$, $e^{-i\pi} = -1$; $e^{i\pi/2} = i$; $e^{-i\pi/2} = -i$; $e^{2\pi i} = 1$.

Third Standard Form for a Complex Quantity. Any point A can be expressed in the form $A = re^{i\varphi}$, where r is the distance and φ the angle of the point. For example, the three cube roots of 1 are 1, $e^{\frac{1}{3}2\pi i}$, $e^{\frac{2}{3}2\pi i}$. In general,

$$(r_1e^{i\varphi_1})(r_2e^{i\varphi_2}) = (r_1r_2)e^{i(\varphi_1+\varphi_2)};\quad (re^{i\varphi})^n = (r^n)e^{in\varphi}.$$

If $x + iy = re^{i\varphi}$, then $r = \sqrt{x^2 + y^2}$, $\sin\varphi = \dfrac{y}{r}$, $\cos\varphi = \dfrac{x}{r}$, $\tan\varphi = \dfrac{y}{x}$.

If two complex quantities are equal, their distances will be equal, and their angles will differ at most by some multiple of 2π. Thus, if $r_1e^{i\varphi_1} = r_2e^{i\varphi_2}$ then $r_1 = r_2$ and $\varphi_1 = \varphi_2$ or $\varphi_2 \pm k2\pi$. Here again a single equation between complex quantities is equivalent to two equations between real quantities.

Equivalent Systems. Many marine systems employ gears between the driver and propeller so that each may operate at its most efficient speed. Since the frequency calculation is based upon the energies stored in the masses and shafts and since these are proportional to the square of the speed of rotation, it is necessary to correct the actual values of inertia and spring scale to that of an equivalent system operating at an equivalent speed (usually that of the propeller). The equations for conversion are

$$\text{Equivalent spring scale, } k_{eq} = k_{act}\left(\frac{\text{actual speed}}{\text{equivalent speed}}\right)^2$$

$$\text{Equivalent inertia, } I_{eq} = I_{act}\left(\frac{\text{actual speed}}{\text{equivalent speed}}\right)^2$$

The inertia of a mass may be found by summing up the product of the weight and the radius of gyration squared of each of the component parts and dividing the result by g, the acceleration of gravity. These values are generally furnished by the manufacturer of the part concerned. The actual inertia of the propeller is usually increased by 20 percent to allow for the entrained water which must move with it. The inertia of the shaft is generally accounted for by taking one-third of it and considering this to act with the adjacent mass. It can be shown that this procedure is theoretically correct.

The shaft-spring scale between the masses is given by $k = \dfrac{G}{L_{eq}}\dfrac{\pi d_{eq}^4}{32}$, where G is the shearing modulus of elasticity of the material (12,000,000 for steel), L_{eq} is the equivalent length of the shaft in inches having an equivalent diameter d_{eq} in. Since the shafts are always stepped, it is convenient to refer the various lengths and diameters to one equivalent diameter (usually 1 in.) before the spring scale is found. The corresponding equivalent length, L_{eq}, is one that will give the same deflection for a given torque as the actual shaft. Thus to determine the equivalent length of shaft, $L_{eq} = L_{act}\left(\dfrac{d_{eq}}{d_{act}}\right)^4$ or if the equivalent shaft diameter is 1 in., $L_{eq} = L_{act}/d_{act}^4$. The sum of the equivalent lengths between the masses is used in the spring-scale equation given above. The equivalent length of crankshafts is given on p. 514.

Frequency Determination. If the system can be approximated by two or three masses connected by shafts, the frequencies may be found by the equations given on pp. 513 and 514. Generally such approximations are avoided for the final determination, the frequencies, nodes, and amplitudes being calculated in tabular form by the Holzer method (Holzer, "Die Berechnung der Drehschwingungen").

The Holzer method is based upon the principle that, at the natural frequency, the inertia and elastic torques of the system are in equilibrium; *i.e.*, the sum of the inertia torques of the system equals zero when it is vibrating at the natural frequency. If I_1, I_2, etc., are the mass moments of inertia of the various masses, β_1, β_2, etc., are the corresponding maximum amplitudes of these masses, and ω is the natural frequency in radians per second, then $(I_1\beta_1 + I_2\beta_2 + \text{etc.})\omega^2 = 0$. The amplitude of an end mass is assumed to be 1 radian, and the other amplitudes are found in relation to this amplitude by considering the inertia and elastic torques; thus $\beta_2 = \beta_1 - I_1\omega^2\beta_1/k_1$, where

If the helix angle is considered and the approximate profile error is known, the second method may be used.

$$F_d = \frac{0.05V(F_t + LC\cos^2\psi)\cos\psi}{0.05V + (F_t + LC\cos^2\psi)^{1/2}} + F_t$$

where C is a factor dependent upon the tooth form, materials, and is directly proportional to the error. For a steel pinion and gear with an error of 0.001 in., C is 1,600 for 14½-deg teeth, 1,660 for 20-deg full-depth teeth, and

Table 1. Values of Form Factor y Based on Virtual Number of Teeth

N_v	14½ deg, composite and involute	20 deg, full depth	20-deg stub
12	0.113	0.132	0.158
13	0.120	0.141	0.164
14	0.127	0.149	0.172
15	0.132	0.156	0.177
16	0.137	0.160	0.184
17	0.142	0.163	0.187
18	0.146	0.166	0.192
19	0.150	0.170	0.196
20	0.153	0.173	0.200
22	0.158	0.178	0.206
24	0.162	0.182	0.211
26	0.166	0.187	0.216
28	0.170	0.190	0.219
30	0.172	0.193	0.222
34	0.176	0.200	0.227
38	0.180	0.207	0.232
43	0.183	0.214	0.235
50	0.187	0.221	0.241
60	0.192	0.227	0.246
75	0.195	0.234	0.252
100	0.198	0.241	0.257
150	0.202	0.248	0.264
300	0.207	0.255	0.272
Rack	0.210	0.262	0.280

1,720 for 20-deg stub teeth. As noted above, C is directly proportional to the error. The maximum error to be expected for hobbed and ground teeth is a function of the tooth size and is approximately:

Diametral pitch........	1	2	3	4	5	6 and finer
Max error, in..........	0.0012	0.0010	0.0008	0.0007	0.0006	0.0005

To avoid surface fatigue failures, the ability of teeth to wear well should also be investigated. The limiting load for wear is

$$F_w = D_p LKQ/\cos^2\psi$$

k_1 is the torsional spring scale between masses I_1 and I_2. A frequency ω is assumed that will satisfy the above basic equation, and the various amplitudes calculated. If the assumed frequency is not correct, the torque summation will not equal zero, and another assumption must be made. Thus the solution is a trial-and-error one and can be worked best in tabular form. After the correct frequency is found, the amplitudes β may be plotted at the various masses to locate the nodes. The process is best illustrated by an example.

Example. Consider a four-cylinder marine diesel engine with a flywheel driving a propeller as shown schematically in Fig. 4*a*. The values of the inertias and spring scales of the shafts are given on the figure. A circular frequency of 24.5 radians per sec is assumed ($\omega^2 = 600$) and Table 1 is set up.

The inertias are listed in the second column and are multiplied by the constant value of ω^2, which is 600, in the third. The spring constants of the shafts are listed in the

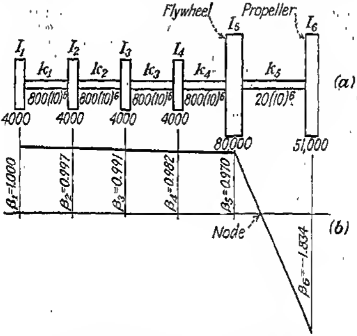


FIG. 4.—*Example of frequency determination.*

seventh column. Mass 1 is assumed to have an amplitude of 1 radian (top line of fourth column), hence the inertia torque of this mass is $I_1\omega^2\beta_1$ or $2.40(10)^6$. Dividing this by the spring constant k_1 of $800(10)^6$ gives the shaft deflection between masses 1 and 2 (top line of last column). If this value of 0.003 radian is subtracted from β_1, the value of β_2 which is 0.997 radian is obtained (second line of fourth column). The consequent inertia torque of mass 2 is $I_2\omega^2\beta_2$ or $2.39(10)^6$. The total torque acting on the shaft between masses 2 and 3 is the sum of the inertia torques of masses 1 and 2 since they act simultaneously in the same direction. This value is $4.79(10)^6$ as given in the second line of the sixth column. The deflection in the latter shaft is calculated, and the amplitude of mass 3 found. The process continues in this manner until the last mass (propeller) is reached. The algebraic sum of the inertia torques for the whole system should equal zero.

In the example, the remainder is $-0.03(10)^6$ which is negligibly small compared to the torques involved. Since this is so, the assumed frequency must be the correct one, and the first natural frequency is $24.5 \times 60/2\pi = 234$

where $Q = 2N_g/(N_g + N_p)$

N_g and N_p = numbers of teeth on gear and pinion, respectively

K = factor based upon the profile shape, material, surface hardness, and surface endurance limit (see Table 2)

If the original Brinell number of the gear is less than 180, it may be assumed that it will be increased to that value by the work-hardening effect of the pinion.

To secure satisfactory operation of the gears, the load acting on the teeth including the dynamic effects, F_d, must be less than the endurance strength, F_s, and wearing ability, F_w.

Table 2. Values of Coefficient K

Brinell No. of pinion	Brinell No. of gear	K for 14½-deg teeth	K for 20-deg teeth
150	150	30	41
200	150	43	58
250	150	58	79
200	200	58	79
250	200	76	103
300	200	96	131

Example. To check the design of the first reduction gears connected to the high-pressure turbine. Data: power 4,250 hp; pinion speed 4,960 rpm; gear speed 643 rpm; solid pinion diameter $D_p = 9.5$ in.; gear diameter $D_g = 73.33$ in.; pinion has two bearings with a 1½-in. gap between the helices; number of pinion teeth $N_p = 57$; number of gear teeth $N_g = 440$; 20-deg stub teeth; Brinell number of pinion teeth = 225, of gear teeth = 175; normal diametral pitch $P_{dn} = 7$; diametral pitch $P_d = 6$; helix angle $\psi = 31$ deg; active face width $L = 20$ in.

Factors of numbers of teeth:

Gear, $2^3 \times 5 \times 11 = 440$ teeth

Pinion, $3 \times 19 = 57$ teeth

Since there are no common factors, the numbers of teeth are satisfactory.

Lateral Deflection:

$$\frac{L + \text{gap}}{D_p} = \frac{20 + 1.5}{9.5} = 2.26 \text{ (less than 2.5, so satisfactory)}$$

Torsional Deflection:

$$t = \frac{0.013 \text{ hp } L}{nD_p^3} = \frac{0.013 \times 4{,}250 \times 20}{4{,}960 \times 9.5^3} = 0.00026 \text{ in. (less than 0.001 in., so satisfactory)}$$

Pitch Line Speed:

$$V = \frac{\pi Dn}{12} = \frac{\pi 9.5 \times 4{,}960}{12} = \frac{\pi 73.33 \times 643}{12} = 12{,}330 \text{ fpm}$$

Transmitted Load:

$$F_1 = \frac{33{,}000 \text{ hp}}{V} = \frac{33{,}000 \times 4{,}250}{12{,}330} = 11{,}370 \text{ lb}$$

cycles per min. The values of the relative amplitude β of the various masses as found in the fourth column are plotted on Fig. 4*b*. The node is located at the point of zero amplitude between masses 5 and 6. Its axial position is proportional to the amplitudes and distance between the masses.

Table 1
(Assume $\omega^2 = 600$)

Item	I	$I\omega^2$	β	$I\omega^2\beta$	$\Sigma I\omega^2\beta$	k	$\frac{1}{k}\,\Sigma I\omega^2\beta$
1	4,000	$2.4(10)^6$	1.000	$2.40(10)^6$	$2.40(10)^6$	$800(10)^6$	0.003
2	4,000	$2.4(10)^6$	0.997	$2.39(10)^6$	$4.79(10)^6$	$800(10)^6$	0.006
3	4,000	$2.4(10)^6$	0.991	$2.38(10)^6$	$7.17(10)^6$	$800(10)^6$	0.009
4	4,000	$2.4(10)^6$	0.982	$2.36(10)^6$	$9.53(10)^6$	$800(10)^6$	0.012
5	80,000	$48.0(10)^6$	0.970	$46.56(10)^6$	$56.09(10)^6$	$20(10)^6$	2.804
6	51,000	$30.6(10)^6$	-1.834	$-56.12(10)^6$	$-0.03(10)^6$		. .

If the propeller of the example just given runs at 100 rpm and has three blades, the shafting will receive 3 × 100 or 300 impulses per min from it. As the diesel is a four-cycle engine, it will give the shaft 2 impulses per revolution or 200 impulses per min. The natural frequency of 234 cycles per min is about 28 percent below the number of propeller impulses, but only about 17 percent above the number of engine impulses. It would, therefore, be advisable to reduce the mass inertias or stiffen the shafts slightly to raise the natural frequency to about 250 cycles per min in this case.

If higher frequencies are assumed, it will be found that the torque summation may again be made equal to zero. These are the higher natural modes of vibration and are distinguished by the number of nodes, each mode having an equal number of nodes. They are of decreasing severity in amplitude but should be investigated to ensure safe operation.

Branch systems having high- and low-pressure turbines connected to a common propeller shaft through reduction gears are handled in a similar manner with the added condition that both branches must have the same amplitude at the bull gear (see Den Hartog, "Mechanical Vibrations," p. 236).

Lateral Vibration. In addition to the torsional vibrations just considered, lateral vibrations or critical speeds in which the shaft center line vibrates radially may occur in the shafting. If the operating and critical speeds are close together, a resonant condition will result. Then severe dynamic forces will be transmitted through the bearings to the hull, and the shafting system is subjected to strains. Hence it should be avoided.

The basic principles are covered on pp. 514 to 520 and so are not repeated here.

The shafting between the gears or driver and the stern-tube bearing is made so stiff from a stress standpoint that there is no danger of lateral-vibration trouble there. However, from the stern-tube bearing to the propeller, there may be considerable deflection and a possibility of vibration.

The procedure for determining the critical speed of this section follows that outlined on pp. 514 and 520. The shaft weight is approximated by a series of concentrated loads along its length, and the static deflection curve of shaft center line is calculated or approximated. It should be noted that for the lowest natural frequency the portions of the shaft in adjacent spans will deflect in opposite directions. Forward of the stern-tube bearing, the

Tooth Loadings:

$$\frac{F_t}{LD_p} = \frac{11{,}370}{20 \times 9.5} = 59.8 \quad \text{(less than 60, so satisfactory)}$$

$$\frac{F_t}{L\sqrt{D_p}} = \frac{11{,}370}{20 \times 3.083} = 184 \quad \text{(less than 190 to 225, so satisfactory)}$$

K Factor:

$$\text{Speed ratio} = \frac{440}{57} = 7.71 \cdot \frac{R}{(R+1)} = \frac{7.71}{8.71} = 0.885$$

$$K = \frac{F_t}{LD_pR/(R+1)} = \frac{11{,}370}{20 \times 9.5 \times 0.885} = 67.6 \quad \text{(within safe range)}$$

Endurance Strength of Teeth:

For pinion: Virtual number of teeth, $N_v = N/\cos^3\psi = 57/0.857^3 = 90.5$. $y = 0.255$ (from Table 1). $p_n = \pi/P_{dn} = \pi/7 = 0.449$ in.

$$\begin{aligned} s_t &= 225 \times 250 = 56{,}000 \text{ psi} \\ F_t &= 0.75 s_t p_n L y \cos\psi \\ &= 0.75 \times 56{,}000 \times 0.449 \times 20 \times 0.255 \times 0.857 = 82{,}900 \text{ lb} \end{aligned}$$

For gear: Virtual number of teeth, $N_v = N/\cos^3\psi = 440/0.857^3 = 699$. $y = 0.276$ (from Table 1).

$$\begin{aligned} s_t &= 175 \times 250 = 43{,}750 \text{ psi} \\ F_t &= 0.75 s_t p_n L y \cos\psi \\ &= 0.75 \times 43{,}750 \times 0.449 \times 20 \times 0.276 \times 0.857 = 69{,}800 \text{ lb} \end{aligned}$$

Dynamic Load on Teeth:

$$F_d = \frac{78 + \sqrt{V}}{78} F_t = \frac{78 + 111}{78} \times 11{,}370 = 27{,}550 \text{ lb}$$

Assuming an error in the profile of 0.0005 in., the corresponding value of C for stub teeth is $\frac{1}{2} \times 1720 = 860$.

$$\begin{aligned} F_d &= \frac{0.05V(F_t + LC\cos^2\psi)\cos\psi}{0.05V + (F_t + LC\cos^2\psi)^{1/2}} + F_t \\ &= \frac{0.05 \times 12{,}330(11{,}370 + 20 \times 860 \times 0.857^2)0.857}{0.05 \times 12{,}330 + (11{,}370 + 20 \times 860 \times 0.857^2)} + 11{,}370 \\ &= 16{,}430 + 11{,}370 = 27{,}800 \text{ lb} \end{aligned}$$

Limiting Wear Load on Teeth:

$$Q = \frac{2N_g}{N_g + N_p} = \frac{2 \times 440}{440 + 57} = 1.77 \cdot K = 79 \qquad \text{(from Table 2)}$$

$$F_w = \frac{D_pLKQ}{\cos^2\psi} = \frac{9.5 \times 20 \times 79 \times 1.77}{0.857^2} = 36{,}200 \text{ lb}$$

Since the strength and wear loads are both greater than the dynamic load on the teeth, the pair of gears will be satisfactory.

deflection may be assumed zero; *i.e.*, the end condition at the stern-tube bearing is "fixed."

The values of the concentrated weights, W_1, W_2, etc., and the corresponding deflections y_1, y_2, etc., are substituted in Eq. (32), p. 515, which is

$$N_c = 187.7\sqrt{\frac{W_1y_1 + W_2y_2 + W_3y_3 + \text{etc.}}{W_1y_1^2 + W_2y_2^2 + W_3y_3^2 + \text{etc.}}}$$

to determine the critical speed. All deflections are considered to be positive in this substitution; *i.e.*, the sign of the deflection is neglected. For safe operation, the critical speed should be 10 to 20 percent away from the operating speed.

Hull Vibration Due to Propeller. Owing to the pulsating torque previously mentioned, there will be a pulsating thrust on the propeller shaft. If this pulsating-thrust frequency approaches or coincides with the natural frequency of the hull, a hull vibration will be set up that may break the piping and cause general annoyance. If the natural frequency of the hull can be predetermined, this condition may be avoided. In general, attempts to stiffen up the hull when this condition arises have been unsuccessful. Changing the number of blades on the propeller seems to be the most practical solution. Sometimes it is necessary to have as many as five blades on the propeller.

DECK MACHINERY

BY

L. S. McCREADY

GENERAL

Deck machinery for merchant vessels as discussed here includes windlasses, cargo winches, mooring winches, capstans, and steering engines. It should receive the most careful attention from the designer because it plays a key role in the security, maneuverability, and earning capacity of the vessel throughout her career. One of the most fertile fields for development of original design is in cargo-handling machinery. Present methods are time-tried and well proven, but little in the way of new developments has come forward for years except the change from steam to electric power.

Upon the windlass falls the duty of handling the vessel's anchors with unfailing dependability. It is located in the most exposed part of the vessel and, like other deck machinery, generally receives a minimum amount of care. Its failure at an inopportune time may cause loss of the vessel or serious delay in her schedule.

The cargo winches play a key role in the over-all efficiency of the vessel in loading and discharging cargo. Any improvement in design and reliability will be reflected in substantial savings to the owners over a period of years, owing to the increased efficiency of cargo handling and shortening of the time in port.

Capstans and mooring winches play a less important role, yet they must be carefully designed and well made because, when they are in use, the vessel is in crowded waters being docked or undocked, and failure must not occur.

The steering engine plays a critical part in the operation of the vessel from the time her voyage begins until it terminates. Countless vessels have been lost or badly damaged from failure of the steering machinery. Wartime hazards from air and underwater attack place the highest premium on rapid and positive steering. Steering machinery must receive the most critical attention in its basic design. It cannot be overemphasized that it is of the highest importance in the over-all design and future operation of the vessel.

Many general requirements must be met in the design of all deck machinery. The basic design must conform to all the rules set forth by the U.S. Coast Guard, Bureau of Marine Inspection, and the American Bureau of Shipping. The latter body publishes annually its volume, entitled "Rules for the Classification and Construction of Steel Vessels." This volume, as well as all other pertinent rules, should be assiduously studied by the designer. Before starting design of deck machinery, its requirements must be established. Various factors to consider include the following, which should be given careful attention:

1. Reliability
2. Ruggedness and simplicity
3. Accessibility of all parts
4. Economy of operation
5. Quietness of operation
6. Lubrication and maintenance
7. Weatherproofing
8. Interchangeability of parts and motors with other machinery on board the ship
9. Adequacy of spare parts list
10. Ease of repair
11. Weight
12. Cost
13. National defense features
14. Guarding of moving parts
15. Protection from sabotage

MARINE REDUCTION GEARS

BY

AUSTIN H. CHURCH

REFERENCES: Buckingham, "Manual of Gear Design," Industrial Press. Waller and Peterson, Chap. 8, in "Marine Engineering," Soc. Nav. Arch. and Mar. Engrs. Gearing, pp. 816 to 829 of this handbook.

Since the operating speeds of modern steam-turbine or diesel drives are greater than the propeller speeds, it is necessary to employ reduction gears between the driver and the propeller line shaft. Marine reduction gears

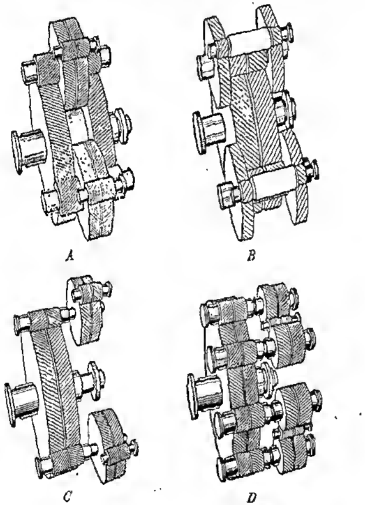


FIG. 1.—Typical marine double-reduction gear arrangements for use with cross-compound turbines. *A*, nested type; *B*, Victory ship nested type; *C*, three-case or articulated type; and *D*, lock-train type.

are always of the double-helix type. They are either single- or double-reduction, depending upon the speed ratio required. The maximum reduction ordinarily obtained with a single pair of gears is 10:1 or 12:1. When greater ratios are required a double-reduction train is used.

ANCHOR WINDLASS

Definition. A ship's windless is a power-operated device to hoist the anchors, drop them when desired, and to veer chain as required. It has a secondary function to handle lines and warps when the vessel is docking or undocking, which is done by means of drums or warping heads on extensions of one or more of its shafts.

Requirements. The windlass must be able to perform several operations:

1. Drop both anchors together or independently by using the brakes.
2. Slack out both anchors together by power.
3. Slackout either anchor by power, at will.
4. Veer chain on either anchor or both anchors.
5. Heave in both anchors together at not less than 30 ft per min.
6. Heave in either anchor at not less than 30 ft per min.
7. Warp hawsers with a maximum pull of 75 percent of hawser breaking strength.

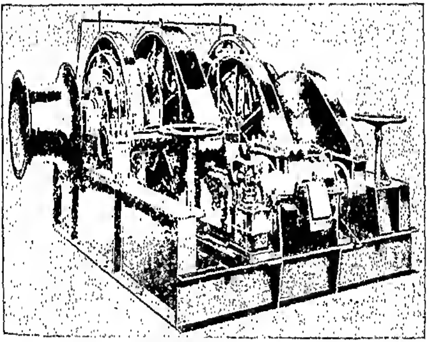


FIG. 1.—Anchor windlass.

Performance and Specifications. Specifications for the windlass are normally based upon the size of the vessel. The windlass on a typical cargo-carrying merchant vessel of 7,900 gross tons, displacing 17,000 tons, will have specifications similar to the following:

1. The windlass shall be capable of hoisting two bower anchors simultaneously from 30 fathoms of water at a chain speed of 30 ft per min. It shall be capable of hoisting one anchor and the maximum scope of 135 fathoms of chain under all service operating conditions.

2. Wildcats and warping heads shall be of cast steel. Their mountings shall not only safely withstand all service loads but also withstand stresses that may result in breaking a chain or snapping a hawser. Warping heads shall be of the smooth barrel type, without whelps.

ARRANGEMENT

There are three general methods of arranging double-reduction gears as illustrated in Fig. 1. The **nested type** shown in parts *A* and *B* is simple, compact, reliable, and has few bearings and couplings. The **articulated type** shown in part *C* has greater flexibility since quill shafts are generally used between the pairs of gears. The **lock-train** type of part *D* is more intricate but has greater load-carrying capacity, owing to the greater number of gears used. Since the points of contact of the first reduction pair are diametrically opposite, the lateral deflection of the pinion is eliminated.

MECHANICAL DETAILS

The gear casing may be cast of either iron or steel, or it may be built up of welded plates. The latter is lighter. One or more vents are provided to

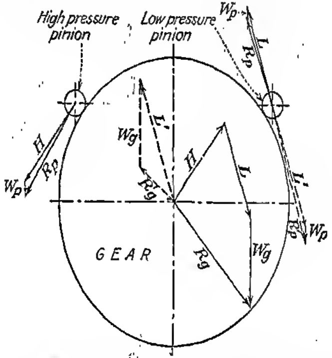


FIG. 2.—Graphical determination of bearing loads for second reduction gears driven by high- and low-pressure turbines.

remove the oil vapors from the casing. They should be located so that the vapors are removed from the engine room, *i.e.*, discharge into the exhaust ventilating system. The bearing supports should be very rigid to prevent misalignment under load.

The bearings are usually of the journal type, although on small sets roller bearings are sometimes used. The bearings are split perpendicularly to the load and usually supplied with oil under pressure. They are babbitt-lined in a cast-iron or steel shell. The bearing size is based upon a pressure of 100 to 200 psi of projected area, depending upon the rubbing speed of the journal. Since the gears and pinions have a double helix, the end thrust is balanced.

3. Each wildcat shall be fitted with a driving and locking head designed to lock in both engaged and disengaged positions. Each wildcat shall have a hand-operated brake of sufficient capacity to stop and hold the anchor and chain when "let go" under control of the brake. Chain clearers shall be furnished for each wildcat, fastened to the windlass bedplate.

4. Warping heads shall have a capacity of 29,000 lb each at 30 ft per min, and a light line speed 75 ft per min. The maximum line pull at the warping head shall not exceed 75 percent of the hawser strength estimated at 58,000 lb. The warping heads are to be driven on the main shaft ends.

5. The cast-steel common chain stoppers are to be furnished with the windlass. (The devil's claws are customarily furnished by the shipyard.)

6. The motor shall not be overloaded when hoisting one anchor. It shall not be overloaded more than 25 percent when hoisting both anchors.

7. The motor shall be at least 70 hp, compound-wound, 30 min rated after 1 hr at no load.

8. Electrical spares shall be furnished in accordance with rules of the American Bureau of Shipping, Sec. 35, Par. 72; U.S. Coast Guard, Merchant Marine Inspection Division; *Senate Report* 184, and the *A.I.E.E. Standard* 45, and the letter of the specifications.

General Design and Construction of Windlass. In windlass design, the most important considerations are

1. Reliability
2. Ruggedness and simplicity
3. Lubrication
4. Accessibility of parts
5. Weatherproofing of gears and electrical circuits
6. Ease of repair

The windlass is rather infrequently used and is in an out-of-the-way location. Hence, economy and quietness are of lesser importance. Considerable trouble will arise if the installation is not adequately weatherproofed. The windlass is in the most exposed part of the ship. In rough weather it is continually drenched by tons of breaking seas, sometimes for days on end. From subzero cold to tropical extremes, the equipment undergoes severe exposure. Inaccessible parts invariably rust after a few years' service. Out-of-the-way bearings may not receive adequate lubrication, and moisture tends to seep into the electrical circuits. The motor, brake, coil, and wiring are often on deck fully exposed and must have complete waterproofing. It cannot be overemphasized that the electrical installation must be exceptionally well weatherproofed. The bulk of all trouble developing in service arises from moisture accumulating in the controller, motor, or wiring.

The windlass is generally built up with commercial quality material from a cast-steel or fabricated welded-steel base and frame of heavy design. Machinery-steel shafting is employed, in common split, adjustable, babbitt-lined, or bronze bearings. The slow-speed shafts should have only bronze bearings. The main gear is best made of welded construction, with machine-cut teeth; pinions should be specified to be made from forged steel, and other gears may be of cast steel, all with cut teeth. All gears should be shrouded. The motor gear should be totally enclosed, running in oil. The gear housing should be fitted with a carefully designed vent to prevent climatic changes from causing condensation inside in cold weather, or expulsion of oil around the bearing in hot climates. The windlass bedplate should have an oil-retaining rim for cleanliness. The locking devices should be designed for

The transmitted force, F_t, acts tangentially to the pitch circles and equals 33,000 hp/V, where V is the pitch line speed in feet per minute and hp is the transmitted horsepower. The resultant load on the teeth acts along the pressure line and equals $F_t/\cos\phi$, where ϕ is the pressure angle.

The bearing loads are best determined graphically by adding vectorially the gear weight to the resultant tooth loads. The resultant of these forces gives the magnitude and direction of the bearing load. The bearings are split approximately perpendicular to these loads, and the bearing size is

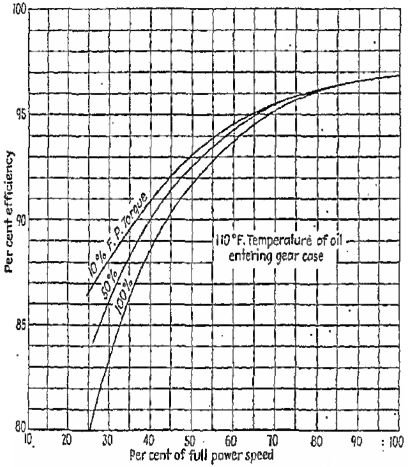


FIG. 3.—Typical efficiency curves of double reduction main propulsion gearing.

based upon their magnitude. Figure 2 illustrates the procedure. H and L are the resultant tooth loads of the high- and low-pressure pinions, respectively, W the gear or pinion weight, and R the resultant bearing load. The full lines represent ahead operation, while the dotted lines and primed letters apply to astern operation.

On large units, *high- and low-pressure turbines* operate in parallel, each having its own pinions as indicated in Fig. 1, and divide the load approximately equally for ahead operation. For astern operation, the low-pressure turbine supplies the entire power while the high-pressure pinion idles. A general rule for this condition is that the total power is 40 percent and the

extreme safety of operation, as this is normally a hazardous operation if there is any strain on the windlass.

Stresses in gearing, shafting, and other parts should not exceed 40 percent of the materials' elastic limit and maximum stresses should not exceed 80 percent of the yield point. On such materials as cast iron having no definite yield point, the working stresses should be calculated using a factor of safety of not less than 4. All forgings and steel castings should be of good commercial quality, stress-relieved. If heat-treated, the heat-treating should follow S.A.E. requirements.

When general design of the bedplate, framing, bearings, and parts of the windlass is undertaken, the proportioning is done in conformity with common machine-design practice and should present no difficulties. Welded structures are preferred to elaborate steel castings, and all parts should be robust. Although weight is not a factor to be neglected, strength and reliability are much more important and should in no wise be sacrificed to save weight. Parts should be designed for ease in disassembly. Standard tables of wrench clearances should be carefully followed in order that future maintenance work may be readily done, especially in cramped locations.

All windlasses have extensive gearing. Certain further details to follow in the design of gearing should specify gear bronze for worm wheels, and if larger than 8 in. in pitch diameter they should be made with a bronze rim on a steel center. Worms are made integrally with their shafts, heat-treated or case-hardened, depending on their size. Reduction-gear housings for worm drive should be made of welded steel or alloy cast iron, fully enclosed, and the worm shaft should have roller bearings to absorb annular and thrust loads. These gears run in oil, with suitable access plates and split joints in the housing for ready inspection and repair. Suitable plugs for filling, draining, and means for measurement of oil level must be embodied in the design. Pressure-gun lubrication fittings should be provided for all bearings not automatically oiled.

A suitable set of operating and lubrication instructions in the form of a framed chart should be provided in the specifications, to be mounted in the controller compartment, below deck.

The windlass motor is mounted on the bedplate, unless driving the windlass by an extension shaft from below decks. The coupling between the motor and reduction-gear train may be a flexible coupling of the all-metallic type.

Dimensions of windlass warping heads will vary roughly from 18 by 18 in. in a merchant ship of 10,500 tons D.W.T. to 24 by 24 in. in a vessel of 13,000 tons D.W.T. and upward. Designers should be careful that all warping heads on deck machinery are designed to be smooth, without whelps. Whelps damage rope severely when rope is surged over the head, and do not materially increase the frictional grip of the rope on the head. Although whelps damage the rope somewhat, they are effective, however, in gripping frozen mooring lines while smooth heads are not. Heads should be left unpainted. Vessels carry hawsers whose size is determined from the American Bureau of Standards rules, Table 16; they will be found to be approximately 8 to 8½ in. in circumference for the above vessels, for example. Table 1 gives the breaking strain of three-strand pure manila rope in pounds. The working strength is taken as one-fifth of the breaking strength. Four-strand rope has approximately the same tensile strength as three-strand rope. This table may be consulted in designing the required pull on windlass warping heads when rope size required on the vessel has been determined. Maximum designed pull should not exceed 75 percent of the breaking strength of the hawser.

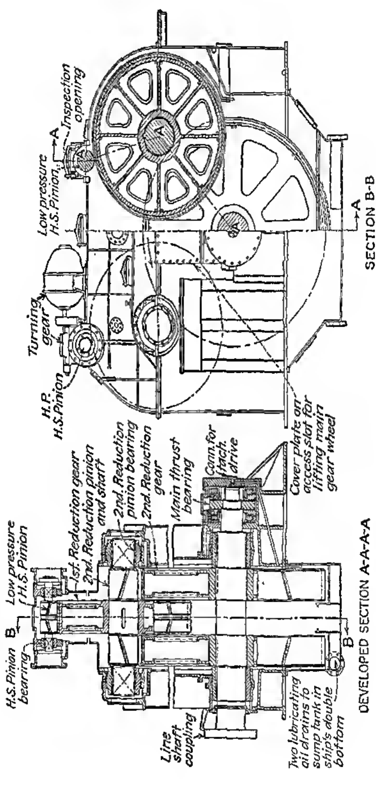


FIG. 4.—De Laval double-reduction gearing of the nested type.

Table 1. **Breaking Strength of Manila Rope**

Rope Circumference	Breaking Strain, Lb
5	22,500
5½	26,500
6	31,000
6½	36,000
7	41,000
7½	46,500
8	52,000
8½	58,000
9	64,000
9½	71,000
10	77,000
11	91,000
12	105,000

Further construction details specify cast steel for the wildcat locking head, bronze for the locking ring, and forged steel for the block keys. Brake linkage is actuated by a worm and wheel requiring not more than five turns of the handwheel from "full off" to "full on." The brake wheels have sockets in their rims for removable brake levers to use in locking and unlocking the brake bands.

Windlass motors are compounded with a heavy series field to give characteristics tending toward those of the series motor, which are ideal for hoisting service (see Fig. 2). The motor and controls are customarily designed to meet the specifications of the A.I.E.E., American Bureau of Shipping, U.S. Coast Guard, Merchant Marine Inspection Division, and *Senate Report* 184. Service on American cargo vessels is designed generally for 230 volts, direct current. Motors should be designed waterproof and provided with easy means of access for lubrication and servicing. Heavy-duty motors should have roller bearings, so designed that grease under pressure cannot be accidentally forced from the bearings into the motor housing.

Direct-current windlass motor control may well be designed so that the master-controller handle must be returned to the "off" position to reset the under-voltage relay, and the "off" position should be of sufficient width to ensure against inadvertently leaving the controller on either of the first positions. Control is the magnetic type, with combined dynamic lowering and reversing, designed to differentiate between warping duty and anchor-handling duty, with a step-back relay to limit the armature current under stalled conditions to 125 percent rated value and ensure satisfactory anchor handling and warping. The jamming or step-back relay is fitted in addition to regular overload protection. The windlass motor should have an electric brake on its shaft, like that on a cargo winch, of sufficient capacity to stop and hold the load under all service-operating conditions when hoisting or slacking out the anchors under power. All parts of the windlass should be assembled on the bedplate, except the resistors, control panel, and other control equipment located below deck.

Alternating current is being introduced rapidly in the merchant marine and Navy. Although winch duty is best served by d-c motors, windlasses may readily be designed for alternating current. In this case the general windlass design is followed as described, but the motor is a wound-rotor motor having high starting torque and low starting current.

speed 50 percent of that for ahead operation; hence the torque and forces acting on the low-pressure train are then 60 percent greater than for ahead operation, while the torque on the high-pressure train is zero.

The oil used for all the bearings and tooth lubrication is generally the same as that used for the turbines and usually has a viscosity of 400 to 600 SSU at 100 F. In addition to lubricating, the oil has the function of removing the heat generated. The hot oil is removed from the casing and passed through a heat exchanger. The cooling medium is sea water from the flushing line of the ship. After filtering, it is pumped to a gravity supply tank. The teeth are lubricated by spray nozzles located above and/or below the point of contact. Slow-speed gears are sometimes splash-lubricated by dipping into a reservoir of relatively high viscosity oil.

The **efficiency** of the unit is relatively high. The losses are due to friction between the teeth and in the bearings. Figure 3 gives typical efficiency curves for various torques and speeds and may be used for estimating purposes.

The **propeller thrust** is taken by the gear case through a thrust bearing of the Kingsbury or Michell type connected to the second reduction or "bull" gear (Fig. 4). This bearing must be able to absorb thrust in either direction for ahead and astern operation. The design pressure varies from about 250 to 400 psi.

The unit is provided with a **turning gear** (Fig. 4) attached to the high-speed pinion of the high-pressure turbine. This device is driven by a motor through a worm or reduction gear to cause the propeller shaft to make one revolution every 5 or 10 min for at least 2 hr without overheating. It is used to warm up the turbines, for inspection purposes, and to turn the gears over during long stays in port thus avoiding nonuniform corrosion of the teeth. There is also a provision for turning the gears by hand.

Flexible couplings are used to connect the turbines and pinions, and also between the first and second reduction gears in articulated trains. They are generally made with mating internal and external teeth. These teeth may be large and relatively few in number (9 to 16); or many small teeth may be used, giving a coupling of the Fast type. To secure satisfactory operation in absorbing the shock loads, oil films must be maintained between the contact surfaces.

The **pinions** are made of heat-treated forged carbon or nickel steel and generally have a Brinell number between 200 and 240. The **gears** may be made of cast iron or cast steel, the teeth being cut directly into the casting. For larger sizes the gear body may be made of cast iron or built up of welded steel plates, and a forged carbon-steel ring shrunk on the body. The Brinell number of the ring ranges from 160 to 200. The teeth are cut in the rim after the assembly of rim, body, and shaft is completed. The gear is then balanced.

The teeth are of involute form, generally of the 20-deg stub type, being hobbed and then run in with an abrasive or lapping compound. The normal diametral pitch ranges between 3 and 8, and the helix angle between 20 and 45 deg. The pressure angle in the plane of rotation ϕ differs from the normal pressure angle ϕ_n; the relationship between them being $\tan \phi_n = \tan \phi \cos \psi$, where ψ is the helix angle. The numbers of teeth in each pair of gears (and preferably in the entire train) should have no common factors that would tend to cause resonance and hence noise or vibration.

To secure an even load distribution across the face of the gears, their **deflection** must not be excessive. Since the gear is much heavier and

Power of Windlass Motor. The power of the windlass motor, of course, depends primarily upon the weight of both of the anchors and their chain. The weight of anchors required to be carried on the ship is determined from reference to the rules of the classification society, American Bureau of Shipping, or Lloyd's, depending on which rules were used in the building of the vessel. The American Bureau of Shipping rules, Sec. 24, state that the

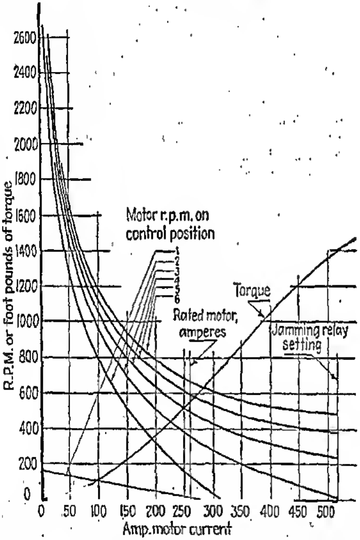


FIG. 2.—Anchor windlass motor characteristics—70 hp, 230 volts, direct current, 650 rpm. (*Courtesy of Westinghouse Electric & Manufacturing Company.*)

anchor weight is determined by the vessel's equipment tonnage. These rules further state that either ordinary or stockless anchors are permitted, but modern vessels all use the stockless pattern whose head is equal to not less than three-fifths of the total weight of the anchor. Table 16, American Bureau of Shipping rules, gives the required anchor weights and chain sizes, once the vessel's equipment tonnage is known.

Chains are led through the hawsepipes, whose friction is a more or less indeterminate factor. Practice has been to calculate the frictional efficiency

stiffer than the pinion, the usual assumption is that it does not deflect. The two kinds of deflection to be considered are the lateral and the torsional. The lateral deflection is usually limited by the empirical rules that the active face of the helices plus the gap (required to facilitate hobbing) between them or the total span divided by the pitch diameter of the pinion should not exceed 2.5 for a two-bearing pinion. The active face width of each helix divided by the pinion pitch diameter should not exceed 2 for three-bearing pinions. The torsional deflection is generally limited to 0.001 in., based upon the assumptions that the torque is removed at a constant rate across the pinion face and that the effective outside diameter of the pinion corresponds to the pitch diameter. The maximum torsional deflection in inches is given by $t = 0.013$ hp $D_pL/n(D_p^4 - d^4)$, where D_p is the pinion pitch diameter, d the inside diameter of a hollow pinion, L the width of active face in inches, n the rpm, and hp the transmitted horsepower. If the pinion is solid, this equation reduces to $t = 0.013$ hp L/nD_p^3.

TOOTH DESIGN

As it is essential that the gears give reliable operation for long periods, the tooth loads should be within safe values. Two empirical rules commonly used to determine these are $F_t/L = ADp$, where the symbols have the meanings given above, with the addition that A is a coefficient that is not greater than 95 for naval vessels, 50 to 60 for merchant ships, and about 40 for diesel drives. The second rule is $F_t/L = (190 \text{ to } 225)\sqrt{D_p}$, where the symbols have the meanings given above.

A more rational method based upon contact stresses involves a K factor as given in the equation $F_t/LD_p = KR/(R + 1)$, where R is the speed ratio (always greater than unity) and K is a factor dependent upon the service and reduction. It ranges from 65 to 110. On the Victory ships, K for the first reduction pair equals 100; for the second pair it equals 70.

A more detailed method of checking the design, based upon the strength and wearing ability of the teeth, is given in Sec. 3 of the "Manual of Gear Design" by Buckingham. The application to marine gears follows.

The load that can be transmitted for long periods of time without resulting in a fatigue failure, F_s, may be found by a modified form of the Lewis equation wherein the load is assumed to act approximately on the pitch circle, *i.e.*,

$$F_s = 0.75 s_t p_n L y \cos \psi$$

where s_t = bending endurance strength of the material and approximates 250 times the Brinell number

p_n = normal circular pitch, in.

L = active face width, in.

y = Lewis factor, taken from Table 1 and based upon the virtual number of teeth as found from $N_v = N/\cos^3 \psi$

N = actual number of teeth

ψ = helix angle

Owing to inaccuracies in the profile and spacing, the gears will suffer rapid accelerations and decelerations resulting in dynamic loads on the teeth which are frequently much greater than the load transmitted by the gears. There are two methods of calculating these dynamic loads. By the first method, $F_d = (3{,}000 + V)F_t/3{,}000$ for pitch line speeds (V) from 2,000 to 4,000 fpm; or $F_d = \left(\dfrac{78 + \sqrt{V}}{78}\right) F_t$ for pitch line speeds greater than 4,000 fpm.

of the hawsepipe at from 50 to 60 percent, the U.S. Maritime Commission not exceeding the latter figure. The over-all mechanical efficiency of the windlass itself should not be considered as being much in excess of 60 to 65 percent, although the gearing by itself is higher.

The main shaft, gearing, bearings, and other parts should be designed from a basis of transmitting maximum motor torque. Operating conditions in service are very rough indeed, however, indicating the need of generous proportioning of all parts and a factor of safety of not less than 4. Windlass hold-down bolts should have a factor of safety of 6. Knowing the weight of each anchor and 30 fathoms of chain and estimating the efficiency of the hawsepipe, windlass wildcats, gearing, and bearings, the motor horsepower can be calculated using the standard of hoisting speed of 30 fpm customarily used in all windlass design. The total weight to be hoisted is taken as that of both anchors and a total of 30 fathoms, or 180 ft of chain. Usually a generous addition to the calculated horsepower is specified to ensure ample power under adverse conditions, and actual hoisting speed may reach 40 fpm.

Spare Parts Specified. A specimen spare-parts list for the windlass installation is submitted below:

1 only	Armature
1 set	Coils, field, for motor
1 only	Coil, motor brake
1 set	Coils, contactor, one of each kind, for control panel
2 only	Assemblies, bearing, complete with inner and outer races and cage
2 only	Holders, brush, with 6 spare springs and stud insulation
2 sets	Brushes, motor
1 set	Grids, resistance, for control panel, comprising 1 grid of each size and kind
2 only	Elements, heater, for control and resistor enclosure heater
2 sets	Contacts, for controller, both fixed and moving, each size and kind
1 set	Springs, for controller contacts
1 set	Springs, brake
1 set	Linings, brake

ADDITIONAL INITIAL SUPPLIES AND TOOLS RECOMMENDED

1 only	Puller, gear, for motor brake wheel and motor and bell housing
1 only	Seater, brush
1 only	Undercutter, commutator
1 length	Wire, asbestos heatproof, for emergency-grid connections

The above list of spares is more complete than usual but will be found most worth while after the vessel is in service. A point to observe is to design the rack for any spare armature so that its shaft is vertical. If stored horizontally, a distortion of the coils may be expected to develop over a period of years.

Wiring. The windlass motor should be wired with varnished, cambric-insulated, leaded, basket-weave, bronze-armored cable, adequately protected where necessary and fitted with stuffing boxes of best design. An approved type of thermostatically controlled heater element should be provided in the enclosure for the motor and control panel to keep the equipment dry in damp weather, similar in design to heaters in winch-resistor houses.

Windlass Design on Oil-tank Vessels. Owing to the hazards natural to the carrying of volatile oil cargoes on these vessels, electrical deck machinery is not always used. On many such vessels the windlass will still be steam-driven. Its design will follow long-established practice and should present

Definition of A^B. Let $A = re^{i\varphi}$; then $A^B = \exp[(\log_e r + i\varphi)B]$.

For example, $i^i = e^{-\pi/2}$ where $i = \sqrt{-1}$.

If a is a positive real, $a^{x+iy} = a^x[\cos(y \log_e a) + i \sin(y \log_e a)]$.

Trigonometric and Hyperbolic Functions of a Complex Variable. If A is any point, then, by definition,

$$\sin A = \frac{e^{iA} - e^{-iA}}{2i}, \quad \cos A = \frac{e^{iA} + e^{-iA}}{2}, \quad \tan A = \frac{\sin A}{\cos A} \quad (\cos A \neq 0);$$

$$\sinh A = \frac{e^A - e^{-A}}{2}, \quad \cosh A = \frac{e^A + e^{-A}}{2}, \quad \tanh A = \frac{\sinh A}{\cosh A}.$$

Hence the formulæ that hold for these functions in the real case (p. 131; p. 135; p. 161) hold also for the complex case. Further:

$$\begin{array}{ll}
\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y, & \sin iy = i \sinh y; \\
\cos(x + iy) = \cos x \cosh y - i \sin x \sinh y, & \cos iy = \cosh y; \\
\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y, & \sinh iy = i \sin y; \\
\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y, & \cosh iy = \cos y;
\end{array}$$

where $\sin x$, $\sinh x$, etc., are the ordinary trigonometric and hyperbolic functions of the real variables x and y. The functions $\sin A$ and $\cos A$ are periodic with a real period 2π. The functions $\sinh A$ and $\cosh A$ are periodic with a pure imaginary period $2\pi i$.

Logarithmic and Other Inverse Functions of a Complex Variable. If any point A is given, there will be an infinite number of points X such that $e^X = A$; any one of these points may be called a logarithm of A, and be denoted by $\log A$. All the values of the logarithm of A may be obtained from any one value by adding multiples of $2\pi i$.

If $x + iy = re^{i\varphi}$, then $\log_e(x + iy) = \log_e r + i\varphi \pm k \cdot 2\pi i$.

If any point A is given, there will be an infinite number of points X such that $\sin X = A$; any one of these may be denoted by $\sin^{-1} A$. The functions $\cos^{-1} A$, $\sinh^{-1} A$, etc., are defined in a similar way.

The elementary laws of operation which hold for these functions in the algebra of reals hold also, in a general way, in the algebra of complex quantities; but caution must be used, on account of the ambiguity in the symbols $\log A$, $\sin^{-1} A$, etc., which denote many-valued functions.

Differentiation of Functions of a Complex Variable. If $w = f(z)$, the derivative of w with respect to z is defined as

$$dw/dz = \lim \{[f(z + \Delta z) - f(z)]/\Delta z\} \text{ when } \Delta z \text{ approaches } 0.$$

It can be shown that $\lim \{[\exp \Delta z - 1]/\Delta z\} = 1$; hence $d(e^z) = e^z dz$, $d(\sin z) = \cos z \, dz$, etc., so that the formulæ for differentiation here are the same as in the case of a real variable (p. 157).

NOTE. For the algebra of vector analysis, which differs in important respects from the algebra of complex quantities, see p. 186.

TRIGONOMETRY

FORMAL TRIGONOMETRY

Angles, or Rotations. An **angle** is generated by the rotation of a ray, as Or, about a fixed point O in the plane. Every angle has an **initial line** (OA) from which the rotation started (Fig. 1), and a **terminal line** (OB) where it stopped; and the counterclockwise direction of rotation is taken as positive. Since the rotating ray may revolve as often as desired, angles of any magnitude, positive or negative, may be obtained. Two angles are **congruent** if they may be superposed so that their initial lines coincide and their terminal lines coincide. That is, two congruent angles are either equal or differ by some multiple of 360 deg. Two angles are **complementary** if their sum is 90 deg.; **supplementary** if their sum is 180 deg. (The acute angles of a right-angled triangle are complementary.) If the initial line is placed so that it runs horizontally to the right, as in Fig. 2, then the angle is said to be an angle in the 1st, 2nd, 3rd, or 4th **quadrant** according as the terminal line lies across the region marked I, II, III, or IV. The angles 0, 90, 180, 270 deg are called the quadrantal angles.

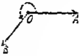


FIG. 1.

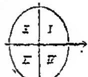


FIG. 2.

Units of Angular Measurement.

(1) SEXAGESIMAL MEASURE. (360 degrees = 1 revolution.) 1 degree = $1° = \frac{1}{90}$ of a right angle. The degree is usually divided into 60 equal parts called minutes (′), and each minute into 60 equal parts called seconds (″); while the second is subdivided decimally. But for many purposes it is more convenient to divide the degree itself into decimal parts, thus avoiding the use of minutes and seconds. (See tables, pp. 46–51.)

(2) CENTESIMAL MEASURE, used chiefly in France. (400 grades = 1 revolution.) 1 grade = $\frac{1}{100}$ of a right angle. The grade is always divided decimally, the following terms being sometimes used: 1 "centesimal minute" = $\frac{1}{100}$ of a grade; 1 "centesimal second" = $\frac{1}{100}$ of a centesimal minute. In reading Continental books it is important to notice carefully which system is employed.

(3) RADIAN, OR CIRCULAR, MEASURE. (π radians = 180 degrees.) 1 radian = the angle subtended by an arc whose length is equal to the length of the radius. The radian is constantly used in higher mathematics and in mechanics, and is always divided decimally. Table, pp. 44–45.

1 radian = $57°.30-$ = $57°.2957795131$ = $57°\,17'\,44''.806247$ = $180°/\pi$.

$1° = 0.01745\ldots$ radian = 0.01745 32925 radian.

$1' = 0.00029\,08882$ radian. $1'' = 0.00000\,48481$ radian.

(For 10-place conversion tables, see the Smithsonian Tables of Hyperbolic Functions, Washington, D. C.)

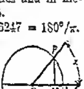


FIG. 3.

Definitions of the Trigonometric Functions. Let x be any angle whose initial line is OA and terminal line OP (see Fig. 3). Drop a perpendicular from P on OA or OA produced. In the right triangle OMP, the three sides are MP = "side opposite" O (positive if running upward); OM = "side adjacent" to O (positive if running to the right); OP = "hypothenuse" or "radius" (may always be taken as positive); and the six ratios between these sides are the principal trigonometric functions of the angle x; thus:

no unusual features. In certain tankers electric deck machinery will be used, but it must be given extremely careful design to eliminate danger from any inflammable vapors that could accumulate.

Weight Estimates. A rough estimate of windlass weight may be taken from Table 2, which is based upon the size of the stud-link anchor chain used on the vessel.

Table 2. Weight of Windlasses

Chain Size, In.	Weight, Tons
1	2.7
1¼	4.0
1½	6.0
1¾	7.0
2	9.0
2¼	12.5
2½	18.0
2¾	32.0
3	35.0

ELECTRIC CARGO WINCHES

Definition. A cargo winch is a power-driven cable-winding drum, horizontally mounted, whose shaft is extended to carry one or two winch heads or warping heads. Its principal purpose is to hoist or lower cargo, but it is frequently used for topping and lowering cargo booms, handling lines, and assisting in warping the ship in or out of her berth. Nearly all modern vessels are designed with electric cargo winches. Steam is still used on many new oil-tank vessels because of the reduced fire hazard. Winches are designed right- and left-hand, single- or double-geared.

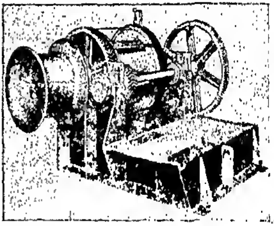


FIG. 3.—Cargo winch without motor. Note bedplate for motor.

Requirements. The cargo winch must be able to perform the following operations:

1. Pick up rated load without shock and accelerate to rated hoisting speed.
2. Decelerate the load being hoisted to a standstill without shock.
3. Lower the load under perfect control, and "lay it down" at approximately 10 percent of normal lowering speed when landing the load.
4. Stop lowering a load smoothly, without shock.
5. Stop a load automatically and smoothly in case of power failure.
6. Pick up a load from the deepest hold, hoist it, and lower it overside to water level when the ship is in light draft with one lay of hoisting wire left on the drum.
7. Have empty-hook speed double normal hoisting speed.

Performance. Drafts of general cargo up to 3 tons are handled on a single purchase or tackle by winches found in the merchant marine. Two tons on a

Layout. Depending on the size and kind of the vessel, particularly with reference to her stern design, the general layout is made. Larger vessels require two rams with four cylinders in all; smaller vessels employ one ram and two opposed cylinders. Rams actuate the tiller or rudder yoke in each case by the well-known Rapson slide, which has the advantage of transmitting an increasing torque as the rudder swings, provided a constant oil pressure is maintained in the cylinders. Other designs for large merchant or naval vessels use parallel links instead of a Rapson slide, as shown in Fig. 9. The controls for pumps and motors and the unit itself should be designed to

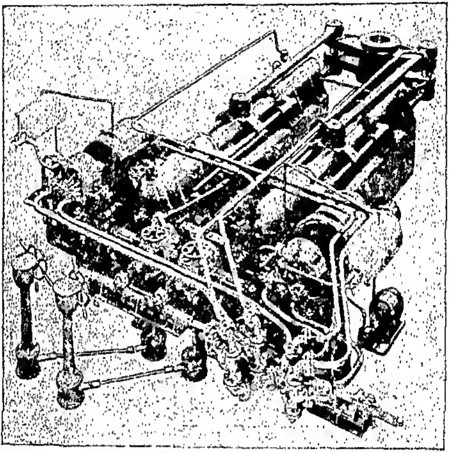


FIG. 9.—Electrohydraulic steering gear of parallel-link type as fitted to some C-3 cargo vessels. (*Courtesy of American Engineering Company.*)

minimize interference between working parts, particularly with reference to overhaul work which entails removal of various parts, both small and large. Clearance should be left for rise of the rudder stock when the vessel is laboring in heavy seas, and also for normal vertical weardown of the rudder as well as removal of the stock in shipyard when necessary. A ½ to ¾ in. allowance for weardown is common practice. A minor point not likely to be easily realized by the designer, for example, is the loss to the vessel if she cannot easily be kept on a steady, true course but must constantly be given an undue amount of corrective rudder. Such a vessel will consume noticeably more fuel, owing chiefly to the increased rudder drag but also to the increased work done by the steering engine and the longer distance sailed. A ship should be

single whip is the average maximum load hoisted at speeds from 200 to 300 ft per min; for medium-heavy lifts of 5 to 10 tons, a two-part purchase is used, with double-geared winches in low gear for the heavier loads. Heavy lifts

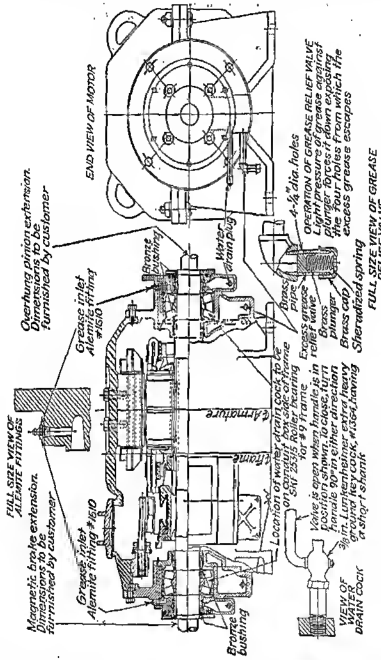


Fig. 4.—Typical winch motor. (Courtesy of General Electric Company.)

up to 30 tons, usually the limit on the ordinary cargo vessel's heavy-lift gear, are handled on a multiple purchase up to seven parts. Heavy-lift gear is served by compound-geared winches. Characteristic winch performance is presented in Table 3.

able to be steered by a good quartermaster so as to hold within $1\frac{1}{2}$ deg of her course in good weather, and not more than 3 deg in the average sea with force 3 wind.

Stresses and Loads. The loads imposed come from two sources: (1) the resistance offered by the water when the rudder is turned away from the centerline and (2) the motion of the ship in a seaway which tends to slam the rudder first one way and then the other, always with great force. The first can be calculated; the second is more or less indeterminate. The rudder is limited from turning more than 35 deg on each side, because to exceed this optimum angle gives no greater turning force but only increases the drag instead. In practice, the rudder seldom needs to move more than 10 or 15 deg from the centerline, except when maneuvering. When the vessel is sailing normally, the rudder torque seldom exceeds 10 percent of its maximum value, and the pump motors are accordingly but lightly loaded most of the time.

Torque on a rudder at positions up to approximately 15 deg will not greatly exceed 10 to 15 percent of the maximum hardover value at full speed ahead. Going astern, with 15 deg rudder angle, the torque is roughly between 30 and 40 percent full hardover astern value. The rudder, being of course greatly unbalanced in astern direction, causes greatly increased torque. Indeed, the limiting torque for design purposes is not always determined by the hardover torque at full speed ahead but, on the contrary, is determined by the hardover torque at full speed astern, which is greater, especially on twin-screw ships.

When running ahead, the steering engine is required to develop power to move the rudder to the right or left and overcome the water-pressure load on the rudder surface. The power required of the steering engine increases with the angularity of the rudder owing to the greater effective rudder area and the increase of distance of the center of pressure from the rudder stock. Thus the power for moving the rudder while the vessel is running ahead is greatest when the rudder is put hard over. The power needed by the steering engine decreases as the vessel responds to the rudder and starts to turn owing to the swinging of the ship and the increase in lateral pressure on that side of the rudder on the outside of the turning circle. The rudder torque, after the vessel responds to the rudder, is about 60 percent

Table 4

Rudder movement	Time, sec	Ram pressure	Motor, amp
Ahead maneuvering at full speed			
0-35L	13	1,000	26
35L-35R.	26	1,500	54
35R.-35L	23	1,050	36
35L-0	11	400	20
Astern maneuvering at full backing power			
0-35L	11	100	18
35L-35R.	22	450	24
35R.-35L	22	600	24
35L-0	12	700	24

Note also the steering gear tests of the S.S. "Red Jacket" page 1944.

Table 3. Cargo Winch Performance

Hook load, tons	Line pull at drum, lb	Tackle	Nominal drum line speed, ft per min
No load	400	Single	525
1½	3,760	Single	290
3	7,400	Single	225
5	6,200	2 part	235
30	14,500	7 part	105

Winches on a somewhat smaller vessel of 10,500 gross tons handle loads up to 6,640 lb at the drum at 200 ft per min on a single whip, and 11,200 lb on the hook with a two-part purchase. This vessel carries a total of 10 such winches, 5 right-hand and 5 left-hand.

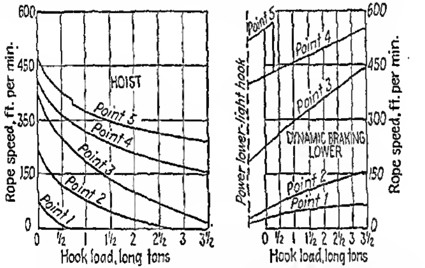


FIG. 6.—Characteristic curves of high-speed electric cargo winches. (*Courtesy of General Electric Company.*)

General Design. The essential characteristics to embody in the design of electric cargo winch installations are

1. Reliability
2. Smoothness of operation
3. Ruggedness and simplicity
4. Weatherproofing
5. Ease of lubrication and repair
6. Ventilation of resistor house or enclosure
7. Quietness
8. Adequate guarding of moving parts

Cargo winches are used intermittently. They remain idle for long periods during voyages, then are run under heavy duty while loading or discharging cargo. At sea, they are subjected to continuous and heavy drenching by salt water in tropical or freezing weather. The impact of heavy seas coming aboard strikes heavily upon the deck machinery, driving water into every part not waterproofed to the highest degree. In cold weather they are ice-coated, and dampness tends to set in in the winch resistor houses. When winches are in operation, they may likewise be running in freezing weather, which impairs their lubrication, or in extremes of tropical heat, which may

of the torque required to put the rudder hard over. It requires little or no power to return the rudder amidships when going ahead as the water pressure tends to do this. Table 4 shows how the steering engine of a large tanker was loaded during various trial-run maneuvers.

When running astern, any movement of the rudder away from the centerline is aided by the pressure of the water; in fact, the steering gear is then under negative torque and the steering engine actually retards the speed of movement. However, when putting the rudder amidships from its hardover position, it then requires the most torque when running astern.

For equal conditions of draft and speed it is obvious that the rudder torque when going astern is the greater, particularly where unbalanced rudders are used. The rudder may not be designed by the designer of the steering engine, but it is mentioned here that its type has a large part in determining the size of the steering engine required. Since balanced rudders will turn with much less power, the degree of balance and the pattern of the rudder will thus

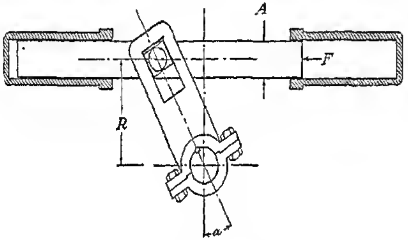


FIG. 10.—Ram arrangement.

influence the power required and hence directly influence the size of the steering engine itself.

The calculation of rudder torque from the hull and rudder dimensions is a very uncertain operation as most methods have been found to be unreliable—sometimes to the extent of 300 percent. Model tests furnish the only data on which any dependence can be placed and, for that reason, no formulas covering rudder torque are included here.

When the maximum rudder torque has been determined from model tests, the design can proceed.

A conservative figure for maximum shearing stress in the rudder stock is 5,000 psi. Commercial applications of machine steel for shafting will normally allow about 8,000 psi, but for the critical duty imposed on the rudder stock no greater than 4,500 to 5,000 psi should be allowed. The diameter of stock required can be calculated thus:

$$D = \sqrt[3]{\frac{16T}{\pi f_s}} \quad \text{or} \quad 1.72 \sqrt[3]{\frac{T}{f_s}}$$

overheat resistance grids and motors. They generally receive a minimum of care and maintenance and indeed are subjected to very severe duty.

Winch Motor Power. The power of winch motors is, of course, governed by the load to be lifted and the desired hoisting speed. In practice, however, cargo winches in the merchant service generally have 50-hp motors. Cargo lifts or drafts are measured in terms of long tons of 2,240 lb.

$$\text{Winch hp} = \frac{\text{line speed} \times \text{line pull}}{33{,}000 \times \text{winch efficiency}}$$

where line speed is in feet per minute and line pull in pounds. Winch efficiency is about 75 to 80 percent.

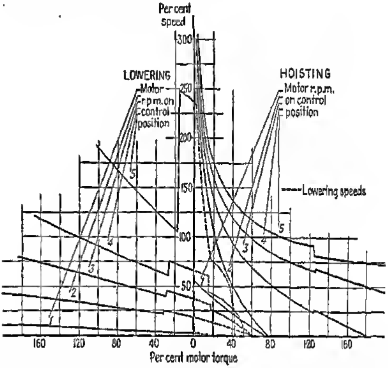


Fig. 6.—Cargo winch motor characteristics—50 hp, 230 volts, direct current, 600 rpm. (*Courtesy of Westinghouse Electric and Manufacturing Company.*)

The relationship between line pull and the load being hoisted depends upon the purchase of the tackle, for in multiple purchases much power is consumed in overcoming friction. The actual applied line pull at the winch is increased by an amount depending on the condition of the rope, the friction of the block, and the number of sheaves. A practical rule for calculating the actual pull required to hoist a load is to add 10 percent to the nominal load for every sheave over which the line passes; using this sum and the number of parts, calculate the power. This rule may be used safely in winch design.

Winch motors are customarily designed for 230 volts direct current, meeting the general requirements of the A.I.E.E. and U.S. Coast Guard, Merchant

where D = diameter of solid stock, in.
f_s = maximum allowable shearing stress
T = torque, lb-in.

The formula for determining the torque or moment the stock will withstand, becomes

$$T = \frac{\pi}{16} D^3 f_s$$

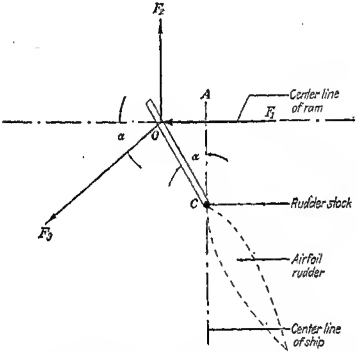


FIG. 11.—Force diagram of Rapson slide.

The working pressure in the rams can be chosen from the rated delivery pressure of the manufacturer's pump, usually 1,000 to 1,500 psi. Then the ram diameters can be calculated to give the desired torque at the rudder stock.

$$T = \frac{P \times A \times R \times E}{\cos^2 a} \qquad \text{(See Fig. 10)}$$

where T = torque on rudder stock, lb-in.
A = ram cross-sectional area, sq in.
R = normal tiller radius, in.
E = efficiency of ram, estimated at 80 percent
a = rudder angle
P = oil pressure acting on ram

NOTE: $\cos 35° = 0.819$; $\cos^2 35° = 0.671$.

Some steering installations have two rams. In such cases, of course, only half the force is produced by a single ram and the formula must be modified.

Marine Inspection Division. Winches run intermittently while handling cargo yet remain in general service many hours a day. Motors in this service heat to the same degree that they would if the winch were steadily operated at its fastest cargo cycle for 5 continuous hours. Winch motors should be of the waterproof, split-frame type, for locations on deck. Since winches nearly always operate gypsy heads as well, the motors should be compound-wound and rated at full load for ½ hr. They should have roller bearings, protected by spring-loaded relief valves from unintentional greasing to excess. Particular emphasis is laid upon accessibility of the motor and its sealing against the weather. Frequently trouble arises in service after the motor inspection plates are opened a few times because of failure of the gasketed joint to reseat absolutely waterproof.

Recent practice has been to build the entire winch and its control equipment in a single unit. The control panel, resistors, brake, and wiring are all enclosed, yet readily accessible. This arrangement saves deck space, eliminates deckhouses for winch controllers, and greatly simplifies the wiring necessary in the shipyard. Such winches on the average cargo ship have a 50-hp, 230-volt, d-c, compound motor turning 600 rpm and fitted with a vertical-type brake.

Control. Deck winches have magnetic, reversible control, very close acceleration and speed control, dynamic braking for lowering, and a magnetic brake on the motor shaft capable of controlling the load under any conditions. Small utility hoists below 15 hp may have manual control. The master controller and foot brake should be arranged for easy operation, but the master controller should not be located farther than approximately 24 in. from the hatch coaming. The effort for operating the motor brake is supplied by a helical spring which forces the shoes against the wheel when the magnet coil is deenergized. This coil is normally across the line and is rated for continuous service. When the brake is released, the pull rod pulls the shoes away from the wheel. Torque adjustments are made by changing spring compression by the spring support. The pull-rod assembly carries adjusting nuts to compensate for brake lining wear. All such fittings should be corrosion-resistant.

Construction. As in windlass design, normal stresses in gearing, shafting, etc., should not exceed 40 percent of the elastic limit of the material, and maximum stresses should not exceed 80 percent of the yield point. All parts of the winch should be strong enough to withstand stalling under maximum overload. Winches operate under severe duty, sustain shock loads, and must be ruggedly built. Material is commercial quality, following S.A.E. or A.S.T.M. specifications. The general requirements for winch gears, bearings, forgings, and castings may follow those given previously for windlass design. Winches may have spur gears, although most designers employ herringbone gears for greater smoothness and quietness. Single-geared winches have a gear ratio approximately 5:1. Compound-geared winches have a ratio of 10:1 or 15:1. The winches on passenger vessels in particular should receive most careful design to ensure quietness. Double-geared winches must have a positive steel jaw clutch, suitably enclosed. The gears must run in oil, in a casing properly vented and fitted with drain plug, filling plug, and oil-test plug for determining the oil level inside.

Winch drums vary from approximately 16 to 22 in. diam, from 18 to 30 in. in length, and are flanged. It is important to note in drum design that all the wire-rope hoisting cable must be able to be wound in not more than one

The Rapson slide has an interesting characteristic. The farther the rudder is put over, the greater the torque developed at the rudder stock by the mechanism of the slide (see Fig. 11). This figure shows the force diagram of a Rapson slide, and the abovementioned characteristic is explained as follows:

Case (*a*). Rudder amidships:

$$\text{Torque} = \text{ram force} \times \text{normal tiller radius}$$

or

$$= F_1 \times AC$$

Case (*b*). Rudder put over at angle a:

Torque = perpendicular force on tiller jaw × distance between rudder stock and force

or $= F_3 \times OC$

The relation between F_1 and F_3 and the torque is further shown thus, for any rudder angle a:

$$\text{Torque} = F_3 \times OC$$

$$F_3 = \frac{F_1}{\cos a}$$

As the rudder is put over, the effective tiller arm increases thus:

$$OC = \frac{AC}{\cos a}$$

Hence,

$$\text{Torque} = \frac{F_1}{\cos a} \times \frac{AC}{\cos a} = \frac{F_1 AC}{\cos^2 a}$$

The net effect of the increased effective tiller arm and multiplication of the force applied by the ram is an increase in rudder torque governed in amount by the square of the cosine of the rudder angle. Those steering gears employing two rams and four cylinders simply divide the total torque between two rams. The basic design, of course, remains the same.

The stroke of the ram will be sufficient to turn the rudder slightly more than 35 deg. Designers specify from 36 to 40 deg as the maximum possible movement of the tiller before the rams meet mechanical interference.

$$\text{Stroke of ram} = R \tan a$$

where stroke = in.
R = normal tiller radius
a = rudder angle from the centerline

Total stroke, from hardover to hardover, is twice this value.

To calculate the ram horsepower, it is necessary to know the maximum load and the number of seconds desired for the ram to make the stroke from

"lay" or layer on the drum. The only exception to this is the winch used for the heavy-lift gear where not more than five lays should be wound upon the drum. Drums should have a suitable clip for securing the cable, which is led through a 1-in. hole or slot at the end of the drum, either radially through the drum at the flange or else tangentially through the flange itself. They should have well-designed wire-rope guards to prevent accident to winchmen or fouling of the cable. The drum must have sufficient wire capacity to lower the hook to the bottom of the vessel's cargo hold and still have one complete turn holding on the drum. Drums should have a strong foot brake of sufficient capacity to stop and hold the loads under all conditions of operation. Winch heads for warping duty should be made of cast steel or alloy cast iron, without the projections or whelps formerly used for greater friction, and be made of a grade of material with maximum properties for resisting the abrasion incidental to long service. Drums should be made of cast steel or good-grade alloy cast iron. Bearings on the drum shaft are usually bronze-bushed. Drums are not grooved for the rope but are left smooth. The design of the winch must ensure that its acceleration and braking will not be great enough to overstrain the cable or any parts. Wire rope used on cargo winches varies generally from ½ to ¾ in. in diameter. It is nearly always specified as ungalvanized, six-strand, 19-wire improved plow-steel wire rope. The designer's factor of safety should be 5.

Winch Spares. Ample spares for the winches should be specified. Cargo ships carry from 10 to 20 winches of similar design, all working on a strenuous schedule. Winch breakdowns occur when the winches are working cargo at a time when they can least afford to be out of service. Breakdowns may occur in remote parts far from any source of extra parts. Such occurrences cause serious delay in cargo handling and entail attendant confusion and loss of earning power for the vessel. Specifications of the vessel's design should call for enough spares for the winches to be adequately serviced. Spare-parts lists should be specified from consultation with rules of the same societies previously listed under windlass design. A specimen list is submitted below. The designer may find that the owner prefers fewer spares to be included in the original contract for the vessel, desiring instead to put more spares aboard after delivery of the vessel.

1 only	Armature, for each six motors of same size
1 set	Coils, field, for each six motors of same size
1 only	Coil, motor brake, for each six motors of same size
1 set	Assemblies, bearing, complete, for each six motors of same size
1 only	Holder, brush, for each six motors of same size
1 set	Brushes, motor, for each three motors
1 set	Grids, resistance, for control panel comprising 1 grid of each size and kind for each six controllers
1 only	Element, heater, for control and resistor enclosure heater
2 sets	Contacts, for controller, both fixed and moving, for each four controllers of same design
1 set	Springs, for controller contacts, for each four controllers
1 set	Springs, brake, for each four winches
1 set	Linings, brake, for each four winches; with spare shoes for each eight winches
2 only	Fuses, for each fuse installed up to maximum of 30 spares, for each rating. Renewable links, where used, should include five renewals for each fuse and 10 percent of each size of fuse case

Further additions or deletions to the above list may be made according to owner's desires, manufacturer's recommendations, special trade-route

hardover to hardover. A minimum of 30 sec for this is allowable, and the ram should actually be able to accomplish this stroke in somewhat less time.

$$\text{Hp} = \frac{A \times P \times D}{550 \times t \times E}$$

where A = cross-sectional area of ram, sq in.
P = working pressure, psi
D = distance moved by ram, ft
t = time, sec
E = efficiency of ram, estimated at 80 percent

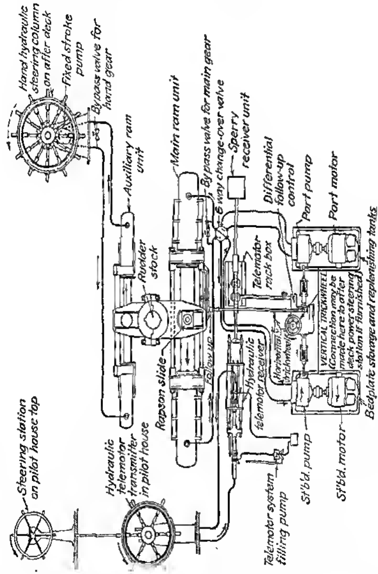


FIG. 12.—Typical arrangement of electrohydraulic steering gear. (Courtesy of American Engineering Company.)

To determine the power of the motor required, an increase is necessary to allow for the normal loss in pump efficiency. The oil pumps used on electrohydraulic systems have approximately 85 percent efficiency.

requirements, and changes in the requirements of the classification societies.

Wiring. Winches should be wired with varnished, cambric-insulated, leaded, basket-weave, bronze-armored cable, well protected by kick pipes, thimbles, and stuffing boxes on deck, and mechanical protective guards as necessary. As specified for the windlass controller, the winch houses for the controller should be provided with thermostatically controlled electric heaters to keep the panels, coils, and contacts dry in damp weather. A further source of trouble is the extreme heat reached in these enclosures when cargo is handled in the tropics. The overheating or "sagging" of resistance grids is not at all uncommon, and the ship's electrician must make repairs by jumping the grid or fitting a spare one under adverse conditions. The house should have an electric blower of from 800 to 1000 cfm capacity, or sufficient to create an air change at least once a minute in the compartment.

Practice may specify mounting several control panels in the house together, or on a common panel. Winches of unit-type design have all necessary controller parts and wiring inside their own individual casings. Exceptional provisions for waterproofing should be made in the entire installation in any event.

Weight of Cargo Winches. Electric cargo winches may vary in weight from approximately 5,800 to 10,000 lb. For general examples, an open-type, 50-hp, compound-geared, two-speed winch will weigh about 9,000 lb. One design of heavy-duty, electric, two-speed, single-drum winch used for handling extra-heavy deck loads, such as locomotives, weighs 12,000 lb.

MOORING WINCHES AND CAPSTANS

Mooring Winches

Definition. Mooring winches, sometimes called **warping winches**, are essentially cargo winches with no drum and long extensions on the main

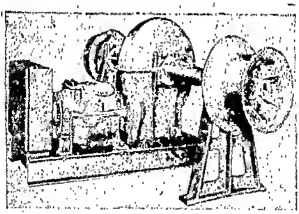


FIG. 7.—Mooring winch

shaft which carry warping heads. Also fitted near the ends of the shaft, inboard of the pedestal bearings, are smaller grooved drums for winding wire rope.

The purpose of the warping winch is primarily for warping the ship during docking and undocking. It is also occasionally used for hoisting boats and handling cargo booms. The winch is usually located well aft, so that its

$$\text{Motor hp} = \frac{\text{ram hp}}{\text{efficiency of pump}} = \frac{\text{ram hp}}{0.85}$$

Emergency Provisions. Each motor must be capable of operating the rudder when the ship is going ahead at maximum shp and full-load draft, or astern at full astern power and full-load draft. The motor should not be overloaded more than 25 percent during any maneuvering astern. The design of the selector valve should enable both pumps to be run at once when desired, since when the vessel is maneuvering this enables the rudder to be swung much more rapidly and contributes much to the maneuverability

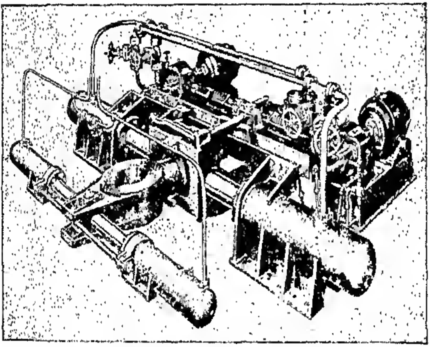


Fig. 13.—Electrohydraulic steering gear on the tanker, "Pennsylvania Sun." (*Courtesy of American Engineering Company.*)

of the ship. Those installations having duplex rams need have no separate relieving tackle or separate auxiliary gear, and one ram alone should be able to handle the rudder up to half the ship's designed full-ahead speed. In case the system becomes inoperative, there should be a hand-operated rudder brake strong enough to hold the rudder in position. On many occasions the rams will put the rudder hard over and the "hardover stops" for limiting the ram movement should be able to withstand the full load on them. These stops will be set at 35 deg but will allow overtravel to a 37- or 38-deg positive stop location. Some designers make them adjustable. Usual design limits the maximum working oil pressure during ahead or astern maneuvering to 1,500 psi. The separate oil-storage and drain tank should have a capacity at least 120 to 130 percent of the system's capacity. It is provided with its own pump and piping. The unit should have a trick wheel for each pump and control unit directly connected to its control differential.

wire-rope drums can be used for emergency steering. As the shaft turns, one drum winds the steering-gear emergency relieving tackle while the other drum unwinds on the other side. Control of the winch will thus steer the ship by power when other means fail if the steering gear is designed to permit this. A warping winch is generally preferred to a capstan if there is sufficient room. Its shaft lies in an athwartship direction.

Design. A warping winch should have a light line speed of from 75 to 90 fpm, for taking in slack lines. The pull may be taken at not over 2,500 lb under this condition. When warping, the winch should develop a pull on the lines equal to at least half the breaking strength of the hawsers and warps required to be carried on the vessel. It should be able to withstand full breaking strength of this line, without damage. Working pull is about one-third the breaking strength of the line.

The warping winch may be single- or double-geared. In general, its design follows requirements already discussed under windlasses and winches. Its outboard shaft pedestal bearings, of the sleeve type, must withstand the full pull of a breaking hawser. Warping heads are smooth. No brakes need be fitted. The control must be reversible, and its design is more or less similar to cargo winch design.

Its power may be calculated on the basis of the required warping line speed and the rated pull. The winch efficiency is about 70 to 75 percent.

$$\text{hp} = \frac{\text{line speed} \times \text{line pull}}{33{,}000 \times \text{efficiency}}$$

Capstans

Definition. A capstan is a motor-operated vertical drum used principally for warping the ship while docking or undocking. In warping ship, a line is sent ashore and made fast at the dock or pier. The capstan is then used to put sufficient strain on the line to draw the ship slowly toward the dock.

Design. A capstan must be well and carefully designed. In service, it is subjected to heavy loads created by the movement or inertia of the entire vessel. Its chief requirements are that it must accomplish the following:

1. Run in either direction.
2. Have provision for hand operation in event of power failure.
3. Have a light line speed from 70 to 80 fpm.
4. Have a loaded line speed of approximately 30 fpm.
5. Have a motor brake sufficient to control full load.
6. Develop rated pull equal to 75 percent of the breaking strength of the hawsers specified for the vessel.

Requirements. Capstan design is based upon the key requirements of reliability, ruggedness, and weatherproofing. Economy, quietness, weight, etc., are of much less importance. The capstan is not run for long periods at a time, being altogether different in this respect from cargo winches. In general, the proportioning of parts and the selection of material follow the previously discussed material given under windlass design.

Capstan power is determined from the rated line pull and rated line speed under load, in the same manner as the calculations given for determining the power of mooring winches. Motors and controls follow windlass and winch design rather closely, with the exception that a-c wound-rotor motors

Materials and Construction. The bedplate must be very strong and rigid for transmitting the heavy forces safely to the ship's framing. Generally this is of welded-steel construction. The cylinders are of cast steel fitted with bronze guide bushings and oil-resistant outside packing for the rams. The rams may be solid, hollow-forged steel with a plugged end or built up from welded steel and machined. Long service causes the packing to wear a neck in the rams at the point of their most frequent travel, when the rudder is more or less amidships. To avoid this, the ram surface in that location should be casehardened or flame-hardened before the final finish is ground. The crosshead or tiller should be of cast steel, heavily keyed to the stock. Guide blocks, guide rods, tie rods, pins, and other stressed parts should be of forged steel. All parts of the storage motion control should be

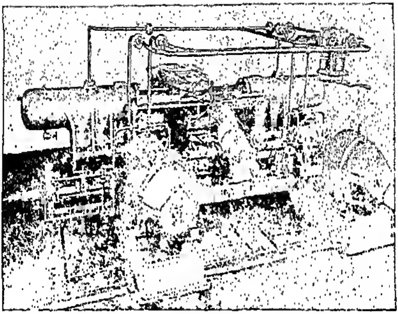


FIG. 14.—Steering gear of type fitted to some C-2 cargo vessels. (*Courtesy of American Engineering Company.*)

casehardened or heat-treated to resist wear. All heavy parts should be provided with one or more bosses tapped for oversize eyebolts for lifting during overhaul. Heavy bearings should be provided with wick-feed or automatic lubricators.

Piping. The piping in the high-pressure system must be of seamless steel, with bolted flanged joints, tested at maximum relief valve pressure of the system. The relief is set about 15 percent above working pressure. Screwed, or screwed and welded, joints are not allowed. Relief valves and vent cocks are fitted as required. High-pressure piping should be sized to permit velocity of oil flow not in excess of 18 or 20 fps. Low-pressure tubing should be of annealed copper with coned joints. Brazed joints should be avoided as they tend to produce foreign matter in the form of bits of spelter or flux, which get in the system. The oil-supply tank should be located not less than 3 or 4 ft above the highest part of the piping. All oil-tank-filling

may be used for capstans though they would not be desirable in winch work owing to the marked superiority of direct current for the latter service.

The capstan is built up from a substantial cast-steel or welded-steel base provided with exceptionally rugged means of being secured to the deck. The drum is of cast steel or alloy cast iron, smooth-barreled and driven by an extension shaft through the weather deck from the worm wheel inside the base. The worm is sometimes driven by a spur gear reduction from the motor; otherwise by direct coupling to the motor shaft. If both spur and worm gearing are embodied in the design, separate oil enclosures should be provided in a common, rigid housing because they require different grades of oil. The worm is carried on ball bearings designed for both thrust and annular loads.

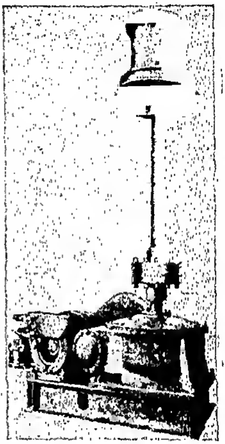


FIG. 8.—Capstan with extension shaft so that the drive may be mounted below the weather deck.

Proportioning of shafting and gearing is done on a basis of transmitting maximum motor torque and, further, resisting safely the strain brought by a breaking hawser. Capstan drive must be self-overhauling and, since worm drive is employed, the hawser's pull on the drum cannot overhaul or turn the motor under this condition unless the helix angle of worm and wheel is at least 15 deg. The vertical capstan shaft is of two-piece design and arranged with a slip joint to provide adequate lift and allow for ½ to ¾ in. deflection of the deck above the motor when the unit is working under heavy strain.

The motor, usually 230 volts, compound-wound, is 30 min rated after 1 hr at no load. It is the heavy-duty marine type, with roller bearings. Its maximum speed should not be exceeded if load is thrown off the capstan. Motor control is usually the magnetic reversing type, with a torque limiting relay positively limiting line pull on the drum of 75 percent of the warping hawser's breaking strength. No overload relay is required. In general, the motor and control equipment should provide 5-point control, automatic acceleration, adequate torque to maintain specified line pulls, and protection against stalling and low voltage.

STEERING MACHINERY

Introduction. The importance of the steering machinery has been previously mentioned. This discussion leaves out entirely all mention of various mechanical hand gears, drum and cable-controlled tillers, or small chain- or screw-type gears fitted on small ships. The only kinds of steering

openings should be designed for locking to prevent sabotage from emery or carborundum dust being thrown in the oil. The entire steering engine room should, in fact, be kept locked, if sabotage is thought possible.

Motor Selection. On hydraulic systems the commonly used motor is 230 volts, shunt-wound, dripproof, constant-speed, 1 hr rated, fitted with ball bearings. On direct-drive systems the motor is compound-wound. The duty imposed offers no problem in control and the common across-the-line starter is used. Motor-starting push buttons should be located near the trick wheels and the six-way valve controls, to minimize the time necessary for emergency changeover. The A.I.E.E. requires a spare set of bearings, a brush holder, and a set of brushes to be carried as spares for each motor.

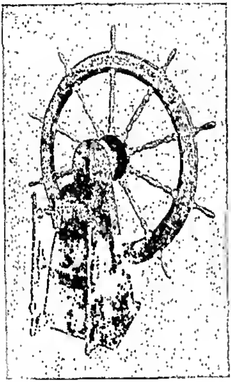


FIG. 15.—Hydraulic telemotor steering stand. (*Courtesy of American Engineering Company.*)

The designer or owner may desire more, however. 440-volt, three-phase, 60-cycle alternating current is gaining rapidly on merchant and naval vessels and a-c steering engine motors are well adapted to this service. Motors have overload relays, and a-c installations should have alarm lamps indicating open phases or reversed phase sequence.

The undervoltage release for each steering motor is located in the steering engine room and should provide automatic restarting upon restoration of the ship's voltage. Motor feeders should run to each motor from two separate cables, with transfer switches, from the main switchboard, one being on each side of the ship. The overload protection should be located on the ship's main switchboard, provided with audio and visual signals to tell when either motor loses its current supply. Each motor requires an indicator light

machinery considered suitable for oceangoing vessels are direct-drive electric gear and the more popular electrohydraulic ram-type gear. Steam steering machinery is considered obsolete in so far as progressive new design is concerned.

The electrohydraulic gear is by far the most efficient. Unlike the direct motor drive, its motor need not be stopped and reversed to reverse the rudder but may run continuously at all times, its load being varied according to the telemotor control. Its characteristics permit low headroom, relatively light weight, and many various methods of locating the rams, pumps, and controls to suit the vessel's design.

Societies governing design of steering machinery are the American Bureau of Shipping, U.S. Coast Guard, Merchant Marine Inspection Division, A.I.E.E., and also *Senate Report* 184. To these must be added the owner's special requirements and, of course, any and all special national defense features required to be embodied in the design by the Navy. The scope of steering machinery in this discussion extends from the calculation of rudder torque from the rudder shape and area as determined by the naval architect, through the entire design of the steering machinery itself, the elements of telemotor design, and in general all detail work necessary to design the installation including the steering stand and telemotor control.

Basic Requirements. Steering machinery should be designed to carry out the following functions on any oceangoing vessel over 250 ft long:

1. Move the rudder smoothly to any desired position within limits of 35 deg left or 35 deg right of the ship's centerline.
2. Move rudder from hardover to hardover through a total of 70 deg in not more than 30 sec when vessel is going ahead at a speed corresponding to maximum shp at full-load draft.
3. Move rudder from hardover to hardover through a total of 70 deg in not more than 60 sec when the ship is going astern, at a speed corresponding to maximum astern hp at full-load draft.
4. Move rudder by means of an emergency source of power from 15 deg left to 15 deg right through a total of 30 deg in not more than 60 sec when the vessel is going ahead at half the maximum designed sea speed or 7 knots, whichever is greater.
5. Have a brake on the rudder stock, or have a hand oil pump on the rams capable of producing a torque equal to one-fourth the maximum ahead torque.
6. Hold the rudder in any desired position against all stresses imposed upon the rudder during normal operation of the ship.
7. Yield under influence of extreme stresses imposed upon the rudder in heavy seas, and return automatically to the set position when the overload has passed.
8. Operate with 100 percent reliability by designing with main, auxiliary or stand-by, and emergency power.
9. Operate quietly, accurately, and economically.

General Qualifications. The steering machinery should be designed from the general consideration of the following essential characteristics:

1. Reliability
2. Strength
3. Economy
4. Accessibility of parts
5. Quietness of operation
6. Ease of maintenance and repair
7. Ease of changeover to emergency gear
8. Positiveness of telemotor control
9. Protection from sabotage

in the engine room. Audio alarms have a silencing and reset button in the engine room to be used after the alarm is noted by those on watch.

Pump Selection. The Hele-Shaw and the Waterbury pumps are most frequently used. These are the popular radial piston or axial piston designs, much used for hydraulic-pressure service. They stand up excellently under long operation, and no excess capacity need be allowed for them in design of the steering machinery as they maintain nearly constant efficiency throughout their life. They should be selected at a rating equal to 90 percent of their full stroke capacity. A flexible coupling should be provided between the motor and the pump. Those pumps of parallel-piston design need adequate provisions for recirculating the oil. Radial pumps need a motor-driven leak-off pump and tank for each pump to return the maximum possible amount of leakage oil to the replenishing tank. A 10-gal capacity will suffice for the overhead replenishing tanks. The oil for the system is a high-grade mineral oil, like turbine oil, of "heavy-medium" viscosity, although sometimes an "extra-heavy" oil may be used.

Telemotor Control. The telemotor is the control whereby the helmsman at the wheel transmits mechanical motion to the steering engine itself to set the power mechanism of the latter in operation. Formerly wire or steel shafting was used, running directly from the wheelhouse, but this is now obsolete. Only the hydraulic telemotor and the electrical control are now used. Figure 15 shows a hydraulic telemotor steering stand. Telemotors use medium-grade pure mineral oil. Of course glycerin and water can be used, but oil is preferred. The oil should have a cold pour point 40 deg below 0 F. All large modern vessels have a Sperry gyro-control unit for automatic steering, which also actuates the steering engine control when so desired. The telemotor and the Sperry unit both control the stroking of the oil pumps, hence the steering itself. All steering machinery should have suitable follow-up gear. Ships generally may be steered from three locations: pilot house, aft steering station, and steering-gear room. Some may be steered from the top wheelhouse if so constructed.

Requirements of telemotor design are

1. Reliability
2. Simplicity
3. Tightness of connections
4. Protection from loss of control by freezing
5. Absence of excessive number of connections between wheel and steering engine
6. Freedom from disturbing magnetic influences near compasses
7. Automatic equalizing when wheel is amidships

Hydraulic Telemotors. These follow conventional design throughout. Telemotor tubing requires care in installation, however. It is copper tubing, ½ to ¾ in. in diameter, led as straight as possible. It should always lead forward on a constantly rising level, avoiding all dips and excessive bends. Care should be taken to keep it clear of boiler tops, the fiddley, steam lines, etc. It should be designed for a 500-lb test pressure, although working pressure is much lower. The charging tank will generally be 7 or 8 gal; the forward replenishing tanks hold about 3 gal as part of the telemotor housing. Air cocks should be provided for venting.

Electric Telemotors. Those vessels having alternating current for auxiliary use, as well as main propulsion, are suited for a "synchro-tie" or

$$\begin{aligned}
\text{sine of } x &= \sin x = \text{opp/hyp} = MP/OP;\\
\text{cosine of } x &= \cos x = \text{adj/hyp} = OM/OP;\\
\text{tangent of } x &= \tan x = \text{opp/adj} = MP/OM;\\
\text{cotangent of } x &= \cot x = \text{adj/opp} = OM/MP;\\
\text{secant of } x &= \sec x = \text{hyp/adj} = OP/OM;\\
\text{cosecant of } x &= \csc x = \text{hyp/opp} = OP/MP.
\end{aligned}$$

The last three are best remembered as the reciprocals of the first three:

$$\cot x = 1/\tan x;\ \sec x = 1/\cos x;\ \csc x = 1/\sin x.$$

Other functions in use are the versed sine, the coversed sine, and the exterior secant:

$$\text{vers } x = 1 - \cos x;\ \text{covers } x = 1 - \sin x;\ \text{exsec } x = \sec x - 1.$$

For graphs, see p. 174; series, p. 161.

Signs of the Trigonometric Functions

If x is in quadrant	I	II	III	IV
$\sin x$ and $\csc x$ are..................	+	+	−	−
$\cos x$ and $\sec x$ are..................	+	−	−	+
$\tan x$ and $\cot x$ are..................	+	−	+	−

vers x and covers x are always positive.

Variations in the Functions as x Varies from 0 to 360 deg are shown in the accompanying table. The variations in the sine and cosine are best remembered by noting the changes in the lines MP and OM (Fig. 4) in the "unit circle" (that is, a circle with radius $= OP = 1$), as P moves around the circumference.

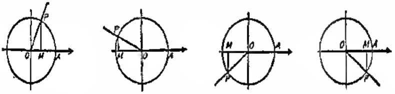


FIG. 4.

x	0° to 90°	90° to 180°	180° to 270°	270° to 360°	Values at 30°	45°	60°
sin x	+0 to +1	+1 to +0	−0 to −1	−1 to −0	½	½$\sqrt{2}$	½$\sqrt{3}$
csc x	+∞ to +1	+1 to +∞	−∞ to −1	−1 to −∞	2	$\sqrt{2}$	⅔$\sqrt{3}$
cos x	+1 to +0	−0 to −1	−1 to −0	+0 to +1	½$\sqrt{3}$	½$\sqrt{2}$	½
sec x	+1 to +∞	−∞ to −1	−1 to −∞	+∞ to +1	⅔$\sqrt{3}$	$\sqrt{2}$	2
tan x	+0 to +∞	−∞ to −0	+0 to +∞	−∞ to −0	⅓$\sqrt{3}$	1	$\sqrt{3}$
cot x	+∞ to +0	−0 to −∞	+∞ to +0	−0 to −∞	$\sqrt{3}$	1	⅓$\sqrt{3}$
vers x	+0 to +1	+1 to +2	+2 to +1	+1 to +0			
covers x	+1 to +0	+0 to +1	+1 to +2	+2 to +1			

$\sqrt{2} = 1.4142$; ½$\sqrt{2} = 0.7071$; $\sqrt{3} = 1.7321$; ½$\sqrt{3} = 0.8660$; ⅓$\sqrt{3} = 0.5774$; ⅔$\sqrt{3} = 1.1547$

Trigonometrical Tables. The tables on pp. 46–56 give the values of the principal trigonometric functions and of their logarithms, correct to four places of decimals, the angle advancing either by tenths of a degree (p. 46) or by 10 min (p. 52). These tables will be found adequate for most

computations in which an accuracy of 1 part in 1000 is sufficient. If much computing is to be done, it is advisable to use a separate volume of tables, containing more facilities for interpolation, and printed in larger type, such as the four-place tables of E. V. Huntington (Houghton Mifflin Company, Boston, Mass.), with convenient marginal tabs; the five-place tables published by Macmillan or many others; the six-place tables of Bremiker; the standard seven-place tables of Schrön, Vega, or Bruhns (angles advancing by 10 sec); or the great eight-place of Bauschinger and Peters (angles advancing at intervals of 1 sec from 0 to 90 deg). The larger tables give only the logarithms of the functions, not the natural values.

To Find Any Function of a Given Angle. (Reduction to the first quadrant.) It is often required to find the functions of any angle x from a table that includes only angles between 0 and 90 deg. If x is not already between 0 and 360 deg, first "reduce to the first revolution" by simply adding or subtracting the proper multiple of 360 deg [for any function of (x) = the same function of $(x \pm n \times 360°)$]. Next reduce to the first quadrant as follows:

If x is between		90° and 180°	180° and 270°	270° and 360°
Subtract		90° from x	180° from x	270° from x
Then	$\sin x$	$= +\cos(x-90°)$	$= -\sin(x-180°)$	$= -\cos(x-270°)$
	$\csc x$	$= +\sec(x-90°)$	$= -\csc(x-180°)$	$= -\sec(x-270°)$
	$\cos x$	$= -\sin(x-90°)$	$= -\cos(x-180°)$	$= +\sin(x-270°)$
	$\sec x$	$= -\csc(x-90°)$	$= -\sec(x-180°)$	$= +\csc(x-270°)$
	$\tan x$	$= -\cot(x-90°)$	$= +\tan(x-180°)$	$= -\cot(x-270°)$
	$\cot x$	$= -\tan(x-90°)$	$= +\cot(x-180°)$	$= -\tan(x-270°)$
	$\text{vers}\, x$	$= 1+\sin(x-90°)$	$= 1+\cos(x-180°)$	$= 1-\sin(x-270°)$
	$\text{covers}\, x$	$= 1-\cos(x-90°)$	$= 1+\sin(x-180°)$	$= 1+\cos(x-270°)$

The "reduced angle" ($x - 90°$, or $x - 180°$, or $x - 270°$) will in each case be an angle between 0° and 90°, whose functions can then be found in the table.

[NOTE. The formulæ for sine and cosine are best remembered by aid of the unit circle.]

To Find the Angle When One of Its Functions is Given. In general, there will be two angles between 0 and 360 deg corresponding to any given function. The following tabulated rules show how to find those angles.

Given	First find from the tables an *acute* angle x_0 such that	Then the required angles x_1 and x_2 will be
$\sin x = +a$	$\sin x_0 = a$	x_0 and $180° - x_0$
$\cos x = +a$	$\cos x_0 = a$	x_0 and $[360° - x_0]$
$\tan x = +a$	$\tan x_0 = a$	x_0 and $[180° + x_0]$
$\cot x = +a$	$\cot x_0 = a$	x_0 and $[180° + x_0]$
$\sin x = -a$	$\sin x_0 = a$	$[180° + x_0]$ and $[360° - x_0]$
$\cos x = -a$	$\cos x_0 = a$	$180° - x_0$ and $[180° + x_0]$
$\tan x = -a$	$\tan x_0 = a$	$180° - x_0$ and $[360° - x_0]$
$\cot x = -a$	$\cot x_0 = a$	$180° - x_0$ and $[360° - x_0]$

The angles enclosed in brackets lie outside the range 0 to 180 deg, and hence cannot occur as angles in a triangle.

For solution of trigonometric equations, see p. 118.

Selsyn telemotor. These units have both transmitter and receiver alike for interchangeability of spares, single-phase stators, and three-phase rotors, with a minimum of cables between stations. The primary runs at 440 volts, and the rotor windings are interconnected. Any movement of the transmitter rotor is duplicated exactly back aft in the steering engine-room receiver. The unit must have sufficient torque to prevent the units from pulling out of step when the helmsman turns the wheel quickly. Ten turns of the wheel will be normally required for putting the rudder hardover to hardover through

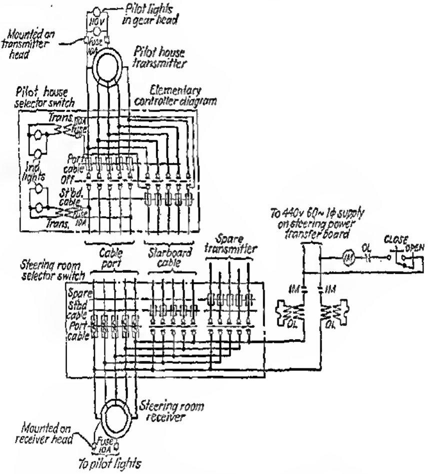


Fig. 16.—Schematic control diagram for electric telemotor.

70 deg. Figure 16 shows a specimen schematic wiring diagram for one of these units. The transmitter is located vertically in the case of each steering stand, and the receiver is located in the steering engine room, near the stroking control of the pumps. Follow-up motion from the rudder movement itself governs the system. An overload line contactor and line selector switch are necessary, there being duplicate cables run between stations. As the selector switch in the pilothouse, with its indicating lamps, is duplicated aft, the cable selection can be controlled from the steering engine room as well.

Design and Layout. The top wheelhouse steering stand, a nonmagnetic design, is connected mechanically to the wheelhouse telemotor transmitter by shaft, clutch, and gears. The aft steering station has a nonmagnetic

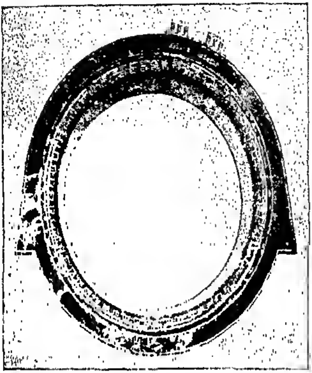


FIG. 8—Stator for synchronous marine motor. *(Courtesy of General Electric Company.)*

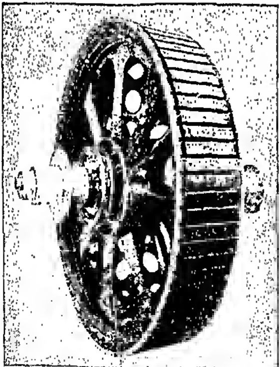


FIG. 9.—Revolving field for synchronous marine motor. *(Courtesy of General Electric Company.)*

stand also, mechanically connected to the trick stations are required by the U.S. Coast Guard, Merchant Marine Inspection Division.

The Sperry gyro-pilot system, if fitted, embodies the following: steering stand in wheelhouse; servo-power unit; transfer switches; and motor-control panel all in the steering engine room, and two d-c–a-c, 110-volt, 60-cycle, motor generator sets. The gyro pilot also has a repeater for the helmsman, a rudder-angle indicator graduated in degrees, adequate lighting provisions, and adjustments for various weather and rudder conditions. The control should be able to disconnect the system entirely, put it on automatic steering, or put it on hand steering. The rudder-angle indicator should be accurate to within ½ deg from the indicated setting. The gyro-pilot power unit should not need any means of supplementary torque amplification but should stroke either or both the main pumps directly. Limit switches should be provided to control the hardover rudder settings at nominal 35 deg settings. When steering by gyro, the hydraulic system should be completely by-passed.

Control. The steering wheel, usually between 36 to 42 in. in diameter, must turn to the right to turn the vessel's rudder and the bow to the right; conversely, it turns left to turn the ship to the left. The words **port** and **starboard** are no longer used in giving steering directions owing to the possibility of confusion of intent in emergencies, such as the collision between the "Mohawk" and the "Talisman," which resulted when the telemotor liquid froze and the emergency signals were misunderstood on the "Mohawk." The rudder indicator must have an adjustable pointer and graduations 35 deg left and right. No more than 10 or 15 turns should be necessary to put the rudder through 70 deg.

The specifications should clearly include complete operating and lubrication instructions for the entire steering system to be posted in the steering room. These should be clearly posted and should include complete directions for all conditions of operation, especially those movements required to shift over to emergency steering.

attained when the countertorque applied does not cause propeller cavitation. In reversing a synchronous motor on its induction motor winding, the opposing torque gradually brings the propeller to rest without causing undue stresses in the line shafting.

In one of the first synchronous motor applications, an experiment was made in which the generator and motor fields were removed, the main line contactors reversed, the slip rings of the generator short-circuited, and the field current then applied to the motor only. This operation was performed with the turbine-speed lever in the maneuvering position. This caused the motor to act as a generator and the generator as an impedance, thereby occasioning a dynamic braking action. A more effective method of dynamic braking is attained by temporarily inserting resistors across the motor terminals until the high peak of propeller torque caused by the ship's driving action is overcome.

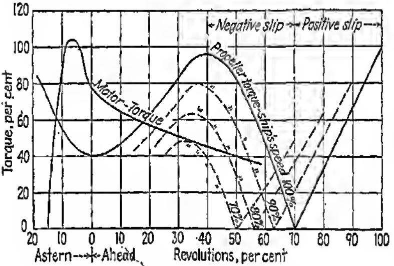


FIG. 10.—Propeller-torque curve during reversal of propeller with ship maintaining constant-ahead speed.

Power Required during Reversal. When a ship is being driven ahead and the power to the propeller is disconnected, zero torque on the propeller will take place when the propeller slip decreases to zero. The exact speed corresponding to zero torque being a function of actual slip, it will differ on vessels having different propellers and wake factors. Usually, this point corresponds to approximately 70 percent of the ahead revolutions corresponding to the ship's speed.

In applying power to reverse the propeller, the ship's coasting speed has a tendency to keep the propeller revolving in the ahead direction and, if sufficient retarding torque is applied to create a negative slip equal to the positive slip corresponding to that attained when driving the ship in the ahead direction, the torque will be approximately equal.

The percentage of the ahead revolutions at which maximum negative torque is attained when dynamically braking the forward movement of the propeller varies widely on different classes of vessels, and also on identical vessels operating at different drafts. The factors that influence the brak-

ELECTRIC SHIP PROPULSION

BY

FRANK V. SMITH

TURBINE-ELECTRIC DRIVE

Classification of System. The term "turbine-electric drive" designates a form of ship-propulsion apparatus in which a steam-turbine-driven generator is used to furnish power to a propelling motor. In installations using a-c generators and motors, propeller reversals are accomplished by a switching of electrical connections between the generator and the motor; speed changes are effected by altering the speed of the prime mover. In installations using d-c generators and motors, speed changes are made by altering the voltage, and reversals are made by changing the direction of current flow. Both of these functions are controlled by the generator field.

With few exceptions, a-c generators and motors have been used in turbine-electric-drive installations—the few d-c installations being mainly of historical interest.

Type of Electrical Apparatus Used. During the developmental stages of turbine-electric drive, induction motors of the squirrel-cage type were used. Notable examples of this type were found on the U.S.S. "Jupiter" (later renamed the "Langley")—the first large ship to be electrically propelled; the U.S. Navy battleship "New Mexico"; and the U.S. Navy airplane carriers, the "Saratoga" and "Lexington."

In modern practice, the synchronous type of a-c motor is almost universally used because of its lighter weight, higher efficiency, and unity power-factor characteristics.

The complete apparatus, considered from the standpoint of a single propelling unit, consists of

1. Steam turbine of the condensing type directly connected to a three-phase a-c generator.
2. Propelling motor of the synchronous type mechanically constructed for direct connection to the propeller shaft.
3. Control panel upon which are mounted the levers for operating the reversing contactors; generator and motor field contactors; and turbine speed control.
4. Exciter generator for furnishing direct current to the generator and motor field circuits.

Primary Purpose and Inherent Advantages. Turbine-electric drive is used primarily to effect a reduction in speed between a high-speed prime mover and a slow-speed propeller. Within certain limitations turbine-electric drive permits a satisfactory fixed speed-reduction ratio between the two for the attainment of a high over-all propulsive efficiency.

The inherent advantages are as follows:

1. The turbine generator on certain types of ships can be used as a universal power plant, thereby eliminating the duplication of powering equipment for loading and unloading operations and for special power purposes.
2. In ships having multiple generating equipments and propeller shafts, electrical connections can be provided that permit the paralleling of motors on a single generator under reduced power conditions.

ing torque during the deceleration period are (1) the stern design of the hull and (2) the tendency of the propeller to cavitate. Both of these items change with the draft and trim of the vessel.

When a ship is operated in the astern direction, the thrust column of water from the propeller flows forward against the hull of the vessel thereby partly canceling the effectiveness of the astern thrust caused by the propeller. The rate of revolutions at which normal rated propeller torque in the astern

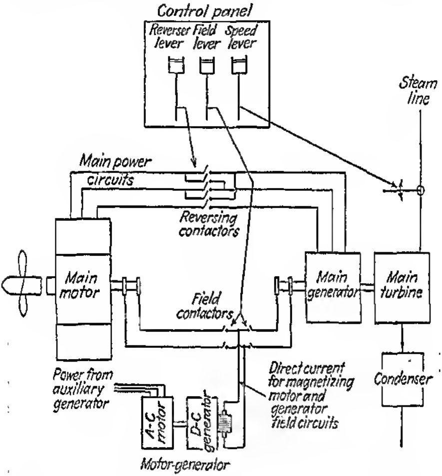


Fig. 11.—Fundamental control and power circuits for turbine-electric drive system.

direction is attained varies on different types of vessels and differs appreciably between single-screw vessels and vessels with wing screws.

A typical propeller-torque curve during reversal based on the assumption that the ship's speed remains constant at 100 percent in the forward direction is shown in Fig. 10. In practice a ship decelerates very rapidly from its top speed, and as its speed is reduced the negative torque that must be applied to the propeller to cause its reversal also decreases very rapidly. If it is assumed that propeller slip is as effective in producing torque on one side of the propeller blading as the other, then the retarding torque required to reverse the propeller is a function of the ship's speed at the time of reversal. Actually, the back of the blade is not so effective as is the face.

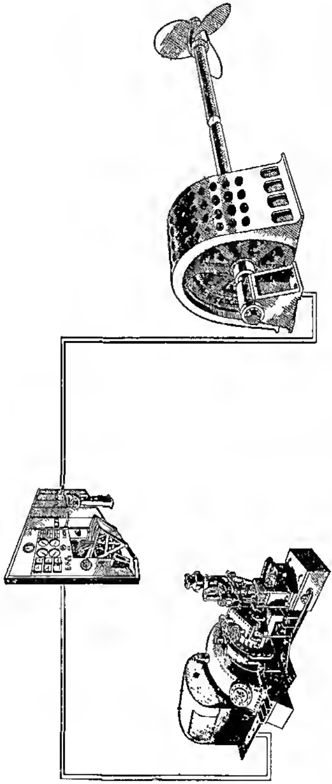


FIG. 1.—Turbine-generator set, control board, and synchronous motor, main items in marine turbine-electric drive. (*Courtesy of General Electric Company.*)

To drive a ship ahead at 100 percent speed requires 100 percent torque on the propeller. Assuming torque to vary as the square of the revolutions, approximately 81 percent torque is required at 90 percent speed; 64 percent at 80 percent speed; and 49 percent at 70 percent speed.

In reversing a propeller by means of a motor, the designer is interested in knowing the time element involved in a ship's deceleration as well as the peak of the torque curves during reversal. Very few ships have the same reversing torque characteristics, and it is general practice when ships go into commission to take extensive oscillograph records during crash reversals, and all other conditions of operation.

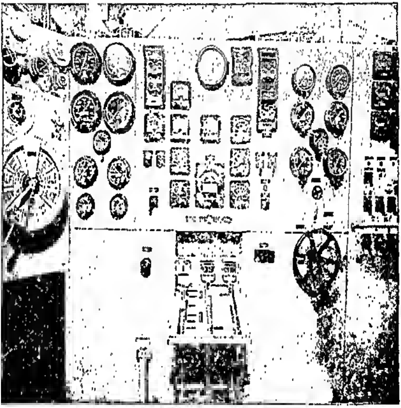


FIG. 12.—General Electric propulsion control panel on 10,000 shp tanker.

This accumulation of design information acts as a guide in considering new designs and in adequately designing the electrical characteristics of the motor to effect proper retarding effect and synchronization after the motor has attained its slip speed as an induction motor in the reverse direction.

Pulsating power loads caused by heavy seas vary widely. At one instant the propeller may emerge from the water resulting in zero torque, and at the next instant it may be submerged deeply. A complete understanding of the torque speed characteristics of the propeller under all conditions of operation is necessary before the electrical design factors can be chosen (see p. 1425 Propellers Backing or in Reverse).

Control of Turbine-electric Drive. From an operative point of view, there are four functions to be considered as follows:

3. Flexibility of power arrangement. The turbine-generator location in the ship can be made independently of the location of the propelling motor.

4. Ease and rapidity of maneuvering electrical connections can be quickly switched for propeller reversal without altering the direction of rotation of the prime mover.

5. Maximum power available for astern operation.

6. Improved relation between propeller speed and torque. Torsional vibrations that emanate from the propeller are not transmitted to the prime mover, or are reduced.

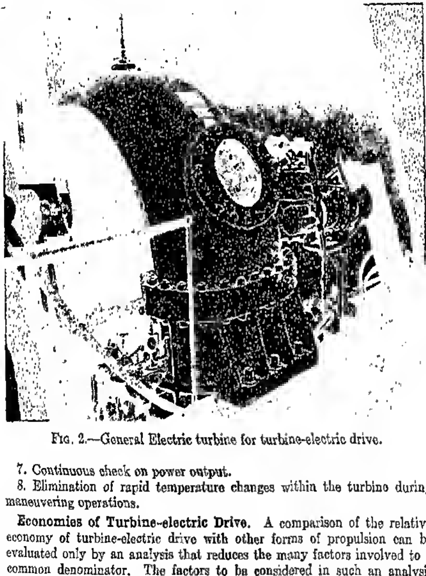


FIG. 2.—General Electric turbine for turbine-electric drive.

7. Continuous check on power output.

8. Elimination of rapid temperature changes within the turbine during maneuvering operations.

Economies of Turbine-electric Drive. A comparison of the relative economy of turbine-electric drive with other forms of propulsion can be evaluated only by an analysis that reduces the many factors involved to a common denominator. The factors to be considered in such an analysis consist of the specific type of ship, its proposed trade route, speeds between ports of call, space and weight factors, auxiliary power needs, first cost and resultant capital charges, depreciation, operating charges, and fuel costs at ports of call.

The foregoing type of analysis is made by computing a complete heat balance for a specific type of ship, taking into consideration the extraneous

1. Turbine-generator speed control.
2. Switching of power leads for propeller reversal.
3. Means for closing switches in generator and motor field circuits in proper sequence.
4. Means for varying generator and motor field current to suit the power and speed conditions.

In modern marine practice functions (1), (2), and (3) are controlled by two levers situated on the control panel. Function (4) may be either manually or automatically controlled. One lever called the reversing and field lever combines functions (2) and (3) and is supplied with three posi-

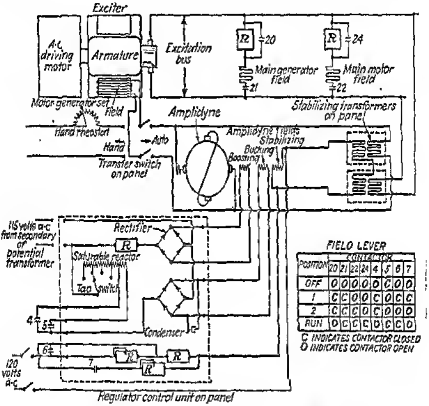


FIG. 13.—Amplidyne schematic wiring diagram.

tions in both the ahead and the astern position. Position one on this lever closes the main-line contactors for the ahead or reverse operation as desired; position 2 closes the generator field switch; and position 3 closes the motor field switch. The second lever controls the speed of the turbine, and interlocking arrangements are provided which make it necessary to return the speed lever to a predetermined maneuvering position before moving the field and reversing lever.

In the earlier turbine-electric-drive installations functions (1), (2), and (3) were controlled by separate levers. Interlocking arrangements were provided which made it necessary to open the field circuits prior to opening or closing the main-line switches.

The motor and generator field current was manually controlled by means of rheostats in the earlier installations. Power-factor meters and excitation

apparatus found necessary with each form of drive. This shows the fuel rates per shaft horsepower per hour for all purposes, which can be quickly interpreted into terms of nautical miles per ton of fuel if the propulsive efficiency is known (see p. 1267).

The fuel rate on a ship is dependent upon the design factors of the power plant, such as boiler efficiency, steam pressure and superheat, vacuum obtainable with sea temperatures existing on a given trade route, type of heat-recovery apparatus installed, type and efficiency of auxiliary apparatus, and magnitude of auxiliary power needs.

Because of the unidirectional rotation characteristics of the turbine on electric-drive ships, such turbines are especially adapted for making use of high-pressure, high-temperature steam, and also for the inclusion of steam-extraction systems for multiple-stage feed-water heating.

The transmission efficiency between the turbine and the propeller shaft is a function of the designed horsepower output of the equipment. In the smaller powers an average of 92 to 93 percent is generally obtainable, increasing to as high as 96.2 percent in the larger powers. The transmission efficiency has an almost flat characteristic over a very wide range of loads. This is due to the core and windage losses being reduced approximately in proportion to the square of the frequency.

Turbine efficiency is the function of its speed, number of stages, steam conditions, and power output. In cases where it has not been necessary to compromise any of these factors, the maximum efficiency is obtained. Whether it pays to design for minimum steam consumption, however, depends upon a correct evaluation of the many other pieces of apparatus that operate within the power cycle.

Types of Ships to Which Adapted. High-powered passenger ships and combination passenger-cargo ships of the twin-screw and quadruple-screw type offer the most advantageous field for turbine-electric drive. It is in this field that the turbine generator designers and motor-designing engineers can cooperate freely with naval architects in obtaining the most economic results. There are no design limitations to the amount of power that can be applied, and by using motors in tandem (if necessary) on a propeller shaft 50,000 hp per shaft and above can be as readily applied as motors of lesser rating.

Cargo ships and tankers lie within the natural province of turbine-electric drive provided the special features of the system so warrant, and the nominal horsepower range is 5,000 hp or above. In tankers, use can be made of the main generating equipment to drive motor-driven cargo oil pumps of large capacity, without duplication of powering equipment. Many types of self-unloading bulk-freight carriers can also make use of the main power plant in port under special conditions.

Naval vessels can best meet special maneuvering requirements by means of turbine-electric drive.

The turbine speed should be selected with reference to the generator speed after a satisfactory speed-reduction ratio between the generator and the motor speed has been selected. In installations that make use of the main propulsion generators for auxiliary purposes in port, 60-cycle current is generally preferable, which with a two-pole generator gives a resultant speed of 3,600 rpm. This does not restrict the turbine generator from operating on a variable-frequency basis when driving the propelling motor alone, but it may influence the normal designed operating speed, especially if the auxiliary power requirements in port are of sufficient magnitude.

indicators were furnished on the control panel for the operator's guidance. In the modern installations, this function is automatically controlled by means of an amplidyne generator whose power output controls the field of the exciter generator. The fields of the amplidyne are controlled by means of a voltage regulator that functions to maintain the proper relationship between the voltage and the frequency, thereby providing a sufficient margin of motor torque to keep the synchronous propelling motor in step even under pulsating load conditions.

The amplidyne generator is similar in appearance to a d-c generator except that it has one pair of brushes short-circuited. The amplification factor between the field and the armature circuit is extremely high, being in the order of 10,000: 1. The small amount of current needed for the amplidyne

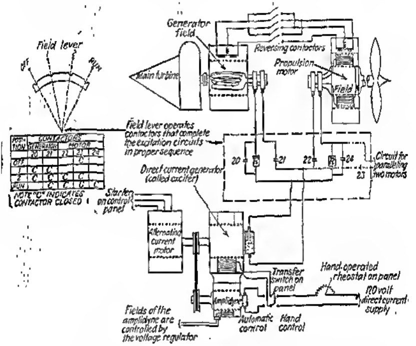


FIG. 14.—Amplidyne control showing power circuits.

fields is furnished by a small transformer from the main power circuit and converted to direct current by means of small rectifiers. The general system in use is shown in Figs. 13 to 14.

DIESEL-ELECTRIC DRIVE USING DIRECT-CURRENT MACHINERY

Classification. Diesel-electric drive is a form of propulsion in which a generator and a motor are interposed between the prime mover and propeller shaft for purposes of speed reduction and maneuverability. Direct-current generators and motors have been almost universally used in connection with diesel-electric drive, because of the variable-speed-reduction characteristic that can be obtained. This characteristic permits the operation of the diesel engine at its most advantageous speed considered from the standpoint of torque and exhaust temperatures irrespective of the pro-

Generators.—In practically all modern turbine-electric-drive installations the a-c generators are of the three-phase, two-pole, Y-wound type. The two-pole type of generator is used because it permits the highest speed-reduction ratio between the turbine generator and the propelling motor and permits the

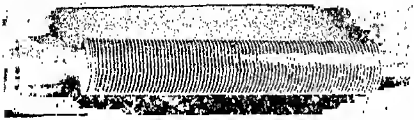


FIG. 3.—Fin tube for General Electric surface air cooler.

maximum turbine speed with the minimum number of poles on the motor. This speed-reduction ratio is a direct proportion of the number of poles on the motor to the number of poles on the alternator and is fixed, once it is selected.

The generators are usually directly connected to the turbines, forming a three-bearing set, and totally enclosed for protection against dust or oil-

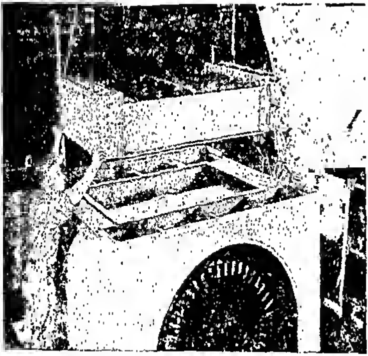


FIG. 4.—Top-mounted surface air cooler for turbine generator. (*Courtesy of General Electric Company.*)

sludge deposit on the windings. In this type of construction air coolers are provided on either the upper or the lower section of the casing, and the air is recirculated through the generator and cooler by means of a fan directly connected to the generator rotor. The coolers use circulating water as the medium for carrying away the heat.

peller speed, and the inclusion of one or a multiple number of diesel-engine-driven generators within the power loop to satisfy a specific power demand.

Type of Electrical Apparatus Used. Direct-current generators of the separately excited shunt-field type are almost universally used with diesel-electric drive in conjunction with the variable voltage system of control. In a few instances the constant-voltage system has been adopted on special types of ships but they have been few in number and are of minor importance.

The complete electrical apparatus considered from the standpoint of a single unit consists of the following:

1. Direct-current generator arranged for direct coupling to a diesel engine.
2. Direct-current motor for direct coupling to the propeller shaft. In the high-power range, two direct-drive motors mounted in tandem on a common shaft (called a double-armature motor), or two motors driving the shaft through gearing are used.
3. Exciter-generator: There are several methods of furnishing exciting current for the generator and motor field poles; among those fulfilling the majority of applications are the following:

   a. From the ship's auxiliary power bus.
   b. From an overhung type of d-c generator driven from an extension of the main generator shaft.
   c. From a motor-generator set.

   NOTE. Types (b) and (c) are adapted to main generator voltage control through control of the exciter field. This results in a system of control that uses a very small amount of current in the speed controllers but does require a separate source of excitation for the motor field.

4. Main-control panel upon which are mounted the switches, speed controllers, meters, rheostats, and instruments for making operational checks.
5. Speed controllers for mounting on the bridge. This item is optional.

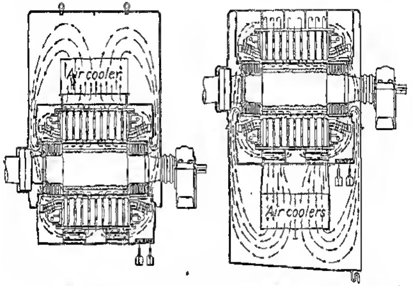
Claimed Advantages of Diesel-electric Drive.

1. Flexibility with relation to placement of powering units in ship.
2. Main propulsion units can be used for dual purposes. When the constant-voltage system is used, the units can be employed simultaneously to furnish power for propulsion and the ship's auxiliaries. When the variable-voltage system is used, units can be withdrawn from the propulsion-power loop for special power purposes at sea or used to furnish power for large auxiliary-power loads in port.
3. The installation of a multiple number of powering units permits a machine to be taken out of service at sea for repair or overhaul.
4. Rapidity and ease of control from any location desired.
5. Increase of propulsive efficiency because of independent selection of engine speed and propeller speed.
6. Adjustment of propeller speed-torque relationship for absorption of normal rated horsepower of engines under varying load conditions.
7. Low stand-by losses.
8. Elimination of excessive torque on engines during surging power loads, thereby preventing high exhaust temperatures.
9. Smooth deceleration and acceleration during reversal, without altering direction of rotation of engines.
10. Engines can be properly warmed up prior to ship's operation.

The devices that protect against electrical faults on such generators consist of unbalanced phase relays and ground relays. Each of these devices removes the field from the generator and motor in case of faults on the system.

The instruments used for observation of the operating conditions consist of ammeters, voltmeters, and temperature indicators. The temperature measurement of the stator windings is made by means of temperature detectors embedded among the windings in each phase. It is also the accepted practice to install heaters in the casing so that the temperature within it can be maintained above room temperature when the machine is stationary and thus avoid condensation on the windings.

Propelling Motors. The synchronous type of polyphase motor is in general use on modern ships. During the developmental stages of turbine-



AIR COOLER MOUNTED ON TOP OF GENERATOR AIR COOLER INSTALLED BELOW THE FLOOR

Fig. 5.—Air cooler in operation.

electric drive induction motors with pole-changing characteristics were used to attain different speed ratios between the generator and the motor, but these are now of only slight interest. The higher efficiency and lighter weight of the synchronous motor, in comparison to either the squirrel-cage or wound-rotor type of induction motor, and its unity power-factor characteristic, which greatly lessens the physical dimensions of the generator, more than offset the advantages of correction in turbine speed at low powers.

The speed-reduction ratio that can be accomplished, when the propelling motor is directly connected to the propeller shaft, is a function of its number of poles, and this in turn is restricted by the diameter of the motor that can be installed in a given ship.

The synchronous type of motor as developed for ship propulsion utilizes a heavy squirrel-cage winding for starting and reversing operations. This winding is embedded in the faces of the rotor field poles and becomes inactive after the motor is brought into synchronism. In first accelerating the motor

Operating Principle. With constant field excitation the speed of a shunt motor varies almost directly with the applied voltage, and the direction of rotation with the direction of the current through the armature. As the armature current is a function of the applied voltage and counter emf produced by the field, the torque-speed relationship of this type of motor can be closely adjusted by regulating either the line voltage or the field excitation.

The voltage produced by a generator is a function of its speed, and the magnetic intensity of its field poles. With constant field excitation, the armature voltage varies directly with the revolutions, and at a constant rate of revolutions the voltage varies almost directly with the field current. The polarity at the armature terminals depends upon the direction of current flow through the generator field coils.

In applying diesel-electric drive, the designer has two systems of control from which to choose: (1) the variable-voltage system and (2) the constant-voltage system. The choice between the two is dependent upon the use of the generator at sea.

In installations that use the main propulsion generators exclusively for propulsive power at sea, the variable-voltage system is preferable because it permits both the rate of propeller revolutions and the direction of rotation to be controlled by the generator field excitation. This does not prevent such a generator, however, from being removed from the propulsion circuit at sea for special power purposes or from being used in port as a constant potential unit. Several ingenious methods have been used in modern installations to vary the field current to the main generators efficiently with minimum loss in the field resistors. This has been accomplished by installing a small generator for furnishing excitation to the main generator or generators only and by varying the field of this smaller generator.

In the constant-voltage system of control, speed variations are obtained by means of resistors in the armature circuit, and reversals by a switching of the armature leads. This system results in an appreciable power loss if the motor is to be operated at reduced speed continuously and is therefore applicable only to ships that operate at their designed speed the greater part of the time.

Application of Variable-voltage System of Control. Because of the ease with which diesel-electric drive can be controlled through the generator field alone, the system is adaptable to bridge control, engine-room control, or from any location desired. The apparatus used to control this function consists of a variable resistor and reversing contactors, which can be easily operated by means of a single wheel or lever.

In the earlier installations it was the general practice to operate the diesel engine at a constant speed throughout the power range and to vary the generator field strength to effect speed changes. In view of the fact that a ship's power varies as the cube and greater exponents of the speed and the propeller torque approximately as the square of the revolutions, the high engine speeds are not compatible with good efficiency in the low power range.

In the modern types of installations it is general practice to operate the engines at half speed, at propeller speeds below 50 percent speed, and to effect speed changes in this range by means of the generator field excitation. At speeds above 50 percent the excitation is held constant, and the voltage that affects the motor speed is regulated by means of the diesel-engine speed. This type of control has a very beneficial effect on the engines. If a heavy surging power load is imposed on the propeller, the additional current required

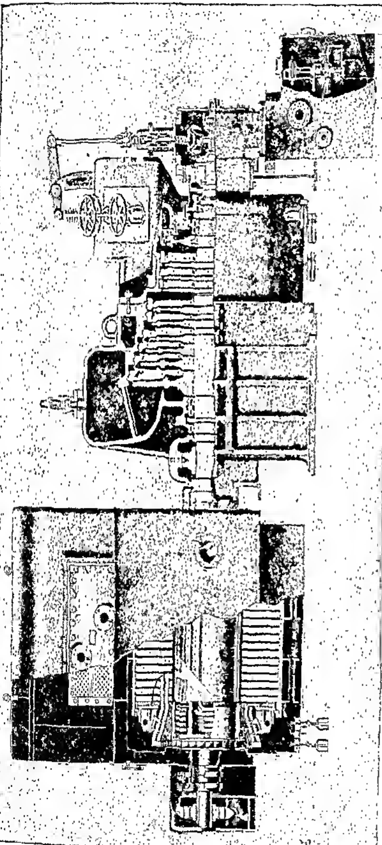


FIG. 6.—Turbine-generator set for ship propulsion. (*Courtesy of General Electric Company.*)

to meet this torque is reflected back to the engine, in which case the speed of the engine is reduced, thereby reducing the voltage and propeller revolutions. As current and torque are proportional and the torque can be controlled by the motor *field rheostat*, the system lends itself to a wide range of propeller torque-speed relationships, without imposing torques above normal on the engine. Figure 15 shows the revolutions, torque, and voltage characteristics.

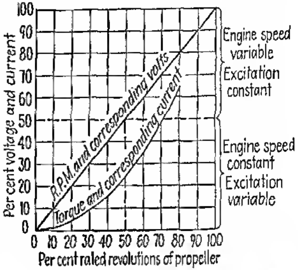


FIG. 15.—*Rpm, torque,* and voltage characteristics of d-c marine drive.

Types of Ships to Which Adapted. The development of relatively high-speed, high-powered diesel engines is continually opening up new fields for marine propulsion application when combined with an electric form of variable-speed reduction between the prime mover and the propeller shaft.

The types of ships to which diesel-electric drive is best adapted are those which require 2,500 hp or less per propeller shaft, and in which the duty and maneuvering requirements cover a wide range. Another factor that may have an equal bearing on the selection of diesel-electric drive is the ability to use the main power units for auxiliary power purposes in port, or a portion of the units at sea for special power purposes.

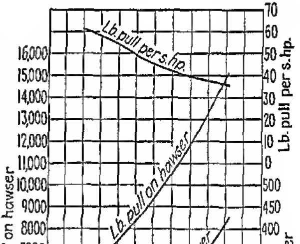


FIG. 16.—Standstill tests of tugboat "Van Dyke 1."

Towboats, seagoing tugs, tankers, small passenger vessels, cargo ships, ferries, salvage vessels, submarines, net tenders, minesweepers, tenders, repair ships, self-propelled dredges, canal boats, self-

up to its slip speed as an induction motor prior to synchronization, the system voltage is increased by temporarily increasing the generator field excitation.

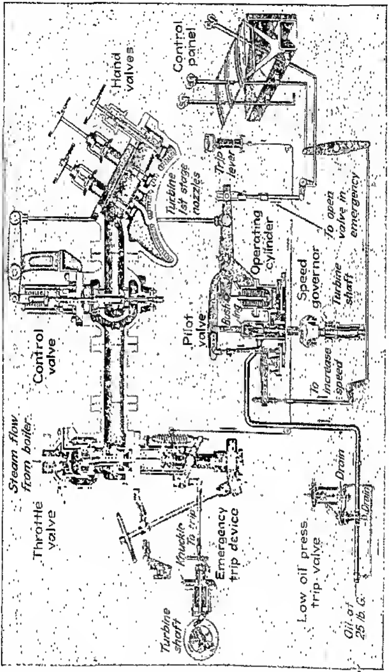


FIG. 7.—Governing system for General Electric steam-turbine ship-propulsion system.

In reversing a ship's propeller the main objective is to bring the ship to rest in the shortest length of time, rather than the propeller. This is usually

unloading bulk-freight carriers, refrigerator ships, and many special types of ships all lie within the natural province of diesel-electric-drive application.

Tugboats. The variable-speed-reduction characteristic of diesel-electric drive makes it especially adaptable to tug boats. The ability to utilize the full rated power output of the engines from standstill to full-speed running light, without imposing above-normal torque on the diesel engines, results in the maximum towing efficiency. The propeller can also be designed especially for towing, thereby resulting in an appreciable gain in propulsion efficiency.

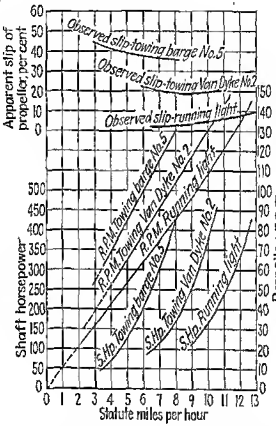


FIG. 17.—Towing tests of tugboat "Van Dyke 1."

The extremes met with in tugboat service range from the condition existing when first accelerating a tow, in which the propeller apparent slip is 100 percent and all of the power is expended in exerting a dead pull on the hawser, to the running-light condition in which all of the power is expended in propelling the boat. Knowledge concerning the torque-speed relationship of the propeller, between the two extremes of power expenditure, is of vital concern to the designer, as it enables him to predict with a degree of certainty the division of power available for towing and to calculate the value of towing efficiency.

A series of tests made on the diesel electrically propelled tugboat "Van Dyke 1," showing the complete characteristics over the entire power range,

Relations Between the Functions of a Single Angle. (See Fig. 5.)

$\sin^2 x + \cos^2 x = 1$; $\tan x = \dfrac{\sin x}{\cos x}$; $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$;

$1 + \tan^2 x = \sec^2 x = \dfrac{1}{\cos^2 x}$; $1 + \cot^2 x = \csc^2 x = \dfrac{1}{\sin^2 x}$;

$\sin x = \sqrt{1 - \cos^2 x} = \dfrac{\tan x}{\sqrt{1 + \tan^2 x}} = \dfrac{1}{\sqrt{1 + \cot^2 x}}$;

$\cos x = \sqrt{1 - \sin^2 x} = \dfrac{1}{\sqrt{1 + \tan^2 x}} = \dfrac{\cot x}{\sqrt{1 + \cot^2 x}}$.

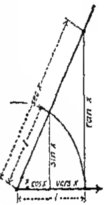


Fig. 5.

Functions of Negative Angles. $\sin(-x) = -\sin x$; $\cos(-x) = \cos x$; $\tan(-x) = -\tan x$.

Functions of the Sum and Difference of Two Angles.

$\sin(x + y) = \sin x \cos y + \cos x \sin y$;
$\cos(x + y) = \cos x \cos y - \sin x \sin y$;
$\tan(x + y) = [\tan x + \tan y]/[1 - \tan x \tan y]$;
$\cot(x + y) = [\cot x \cot y - 1]/[\cot x + \cot y]$;
$\sin(x - y) = \sin x \cos y - \cos x \sin y$;
$\cos(x - y) = \cos x \cos y + \sin x \sin y$;
$\tan(x - y) = [\tan x - \tan y]/[1 + \tan x \tan y]$;
$\cot(x - y) = [\cot x \cot y + 1]/[\cot y - \cot x]$;
$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$;
$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$;
$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$;
$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$;

$\tan x + \tan y = \dfrac{\sin(x + y)}{\cos x \cos y}$; $\cot x + \cot y = \dfrac{\sin(x + y)}{\sin x \sin y}$;

$\tan x - \tan y = \dfrac{\sin(x - y)}{\cos x \cos y}$; $\cot x - \cot y = \dfrac{\sin(y - x)}{\sin x \sin y}$;

$\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x = \sin(x + y) \sin(x - y)$;
$\cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x = \cos(x + y) \cos(x - y)$;
$\sin(45° + x) = \cos(45° - x)$; $\tan(45° + x) = \cot(45° - x)$;
$\sin(45° - x) = \cos(45° + x)$; $\tan(45° - x) = \cot(45° + x)$.

In the following transformations, a and b are supposed to be positive, $c = \sqrt{a^2 + b^2}$, A = the positive acute angle for which $\tan A = a/b$, and B = the positive acute angle for which $\tan B = b/a$:

$a \cos x + b \sin x = c \sin(A + x) = c \cos(B - x)$;
$a \cos x - b \sin x = c \sin(A - x) = c \cos(B + x)$.

Functions of Multiple Angles and Half Angles.

$\sin 2x = 2 \sin x \cos x$; $\sin x = 2 \sin \tfrac{1}{2}x \cos \tfrac{1}{2}x$;
$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$;

$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$; $\cot 2x = \dfrac{\cot^2 x - 1}{2 \cot x}$;

$\sin 3x = 3 \sin x - 4 \sin^3 x$; $\tan 3x = \dfrac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$;

$\cos 3x = 4 \cos^3 x - 3 \cos x$;

$$\sin(nx) = n \sin x \cos^{n-1} x - (n)_3 \sin^3 x \cos^{n-3} x + (n)_5 \sin^5 x \cos^{n-5} x - \ . \ . \ . ;$$

$$\cos(nx) = \cos^n x - (n)_2 \sin^2 x \cos^{n-2} x + (n)_4 \sin^4 x \cos^{n-4} x - \ . \ . \ . ,$$

where $(n)_2, (n)_3, \ . \ . \ .$ *are the binomial coefficients* (see p. 39).

$$\sin \tfrac{1}{2} x = \pm \sqrt{\tfrac{1}{2}(1 - \cos x)};\ 1 - \cos x = 2 \sin^2 \tfrac{1}{2}x;$$

$$\cos \tfrac{1}{2}x = \pm \sqrt{\tfrac{1}{2}(1 + \cos x)};\ 1 + \cos x = 2 \cos^2 \tfrac{1}{2}x;$$

$$\tan \tfrac{1}{2}x = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x};$$

$$\tan\left(\frac{x}{2} + 45°\right) = \pm \sqrt{\frac{1 + \sin x}{1 - \sin x}}.$$

Here the $+$ or $-$ sign is to be used according to the sign of the left-hand side of the equation.

Relations Between Three Angles Whose Sum is 180°.

$$\sin A + \sin B + \sin C = 4 \cos \tfrac{1}{2}A \cos \tfrac{1}{2}B \cos \tfrac{1}{2}C;$$
$$\cos A + \cos B + \cos C = 4 \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \sin \tfrac{1}{2}C + 1;$$
$$\sin A + \sin B - \sin C = 4 \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \cos \tfrac{1}{2}C;$$
$$\cos A + \cos B - \cos C = 4 \cos \tfrac{1}{2}A \cos \tfrac{1}{2}B \sin \tfrac{1}{2}C - 1;$$
$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \cos A \cos B \cos C + 2;$$
$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C;$$
$$\tan A + \tan B + \tan C = \tan A \tan B \tan C;$$
$$\cot \tfrac{1}{2}A + \cot \tfrac{1}{2}B + \cot \tfrac{1}{2}C = \cot \tfrac{1}{2}A \cot \tfrac{1}{2}B \cot \tfrac{1}{2}C;$$
$$\cot A \cot B + \cot A \cot C + \cot B \cot C = 1;$$
$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C;$$
$$\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C.$$

Inverse Trigonometric Functions. The notation $\sin^{-1}x$ (read: antisine of x, or inverse sine of x; sometimes written arc sin x) means the principal angle whose sine is x. Similarly for $\cos^{-1}x$, $\tan^{-1}x$, etc. (The principal angle means an angle between $-90°$ and $+90°$ in case of $\sin^{-1}$ and $\tan^{-1}$, and between 0° and 180° in the case of $\cos^{-1}$.) For graphs, see p. 174.

SOLUTION OF PLANE TRIANGLES

The "parts" of a plane triangle are its three sides, a, b, c, and its three angles A, B, C (A being opposite a, B opposite b, C opposite c, and $A + B + C = 180°$). A triangle is, in general, determined by any three parts (not all angles). To "solve" a triangle means to find the unknown parts from the known. The fundamental formulæ are:

Law of sines: $\dfrac{a}{b} = \dfrac{\sin A}{\sin B}$. Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.

Right Triangles. Use the definitions of the trigonometric functions, selecting for each unknown part a relation which connects that unknown with known quantities; then solve the resulting equations. Thus, in Fig. 6, if $C = 90°$, then $A + B = 90°$, $c^2 = a^2 + b^2$,

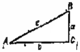


FIG. 6.

$$\sin A = a/c,\ \cos A = b/c,\ \tan A = a/b,\ \cot A = b/a.$$

If A is very small, use $\tan \tfrac{1}{2}A = \sqrt{(c - b)/(c + b)}$.

Oblique Triangles. There are four cases. It is highly desirable in all these cases to draw a sketch of the triangle approximately to scale before commencing the computation, so that any large numerical error may be readily detected.

Case 1. GIVEN TWO ANGLES (provided their sum is $<$ 180 deg), AND ONE

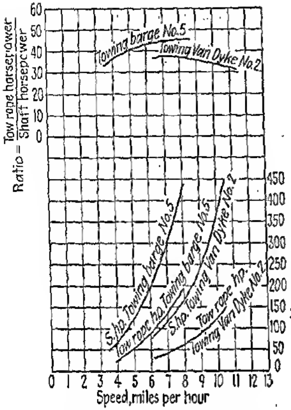


Fig. 18.—Towing tests of tugboat "Van Dyke" 1.

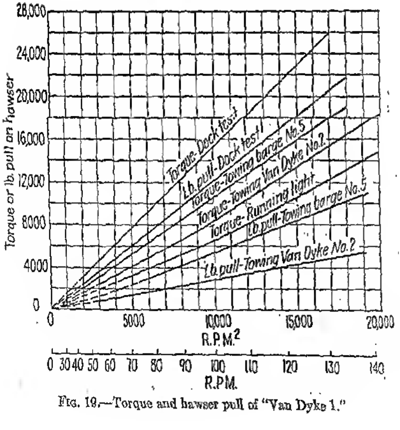


Fig. 19.—Torque and hawser pull of "Van Dyke 1."

usually present in the field of a synchronous motor. This element is provided with removable end bells which serve to make the entire unit drip-proof, and it also supports the brush rigging associated with the collector rings of the engine element.

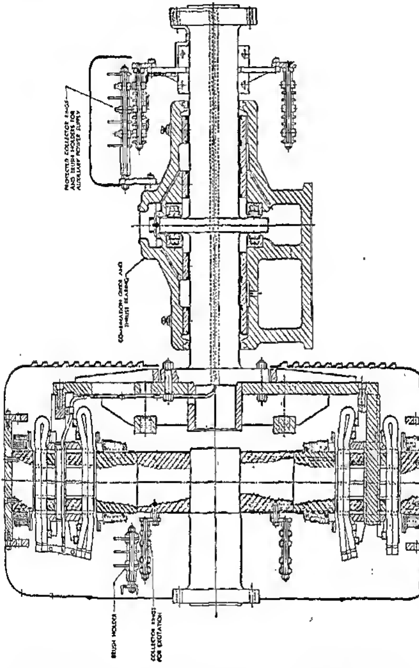


Fig. 1.—Bowes drive.

Operation. The Bowes drive is operated in the following manner: With the engine running at its idling speed, excitation is supplied to the engine element. This sets up a rotating magnetic field which moves past the winding on the inner periphery of the shaft element and generates a voltage in this winding exactly as in the case of the conventional generator. As

are shown in Figs. 16 to 20. This boat is rated 375 shp at 140 rpm running light.

Figure 16 shows the pounds pull on the hawser at standstill of the boat. With the propeller operating at 100 percent apparent slip, rated full power was expended at 107 rpm, or 76.5 percent rated revolutions.

Figure 17 shows the shaft horsepower, revolutions, and observed propeller slip, under a heavy towing condition, light towing condition, and under free-running condition. Figure 18 shows the percentage of power available for towing under light- and heavy-towing conditions. As the "Van Dyke 2," which constituted the light tow, was similar in every respect to the "Van Dyke 1," it is possible to deduce from this test the actual hull resistance or thrust required to drive this ship, and ascertain with a fair degree of accuracy the propulsive efficiency of either boat. Running free, the "Van Dyke 1" required 200 hp to obtain a speed of 10.5 mph, whereas the "Van Dyke 2" required 150 tow-rope hp to attain a similar speed. The test was made with the power disconnected and propeller free to turn on the "Van Dyke 2." Figure 19 shows the torque and pounds pull plotted against the revolutions squared. Figure 20 shows a composite curve of all tests.

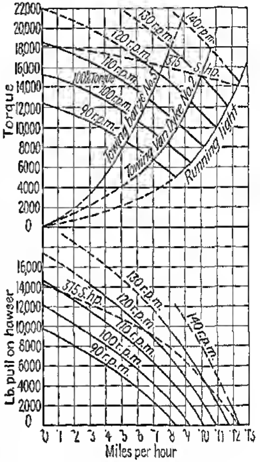


Fig. 20.—Torque, speed, and towing force curves of "Van Dyke 1."

Ships for Special Service. In this classification may be included all types of ships which because of their duty require special maneuvering qualities or ships that can use the main power plant for other purposes than propulsion. Cutters and salvage boats which may be called upon to operate near reefs or which may be pressed into towing service are especially benefited by diesel-electric drive. In installations using two or more main generating units, a switching arrangement is provided whereby a given generator can be disconnected from the propulsive-power loop while underway and diverted to other purposes.

Double-ended ferryboats offer an excellent field for diesel-electric drive as the motor driving the forward propeller may be either electrically dis-

soon as power is developed in this winding, a reaction torque is developed which causes the shaft element to start to revolve in the same direction as the engine element. (This reaction torque is also present in the conventional generator, but the armature cannot turn because it is held stationary by its supports.) The power that is being generated is fed directly into the winding on the outer periphery of the shaft element and, since this is the armature of a synchronous motor, an additional torque is developed to accelerate the shaft element. During this acceleration the field winding of the stationary element is short-circuited through an external resistance. When the shaft element is revolving at close to the synchronous speed of the drive, excitation is applied to the stationary field and the shaft element is pulled into synchronism. Since the engine element continues to run at a higher speed than the shaft element as determined by the speed ratio, there is power developed in the generator winding to operate the motor side of the drive. After the drive has been synchronized, the engine speed may be adjusted to any speed from idling to full load, and the speed of the shaft element will be varied in the same ratio.

In shutting down the Bowes drive, excitation is removed from both the engine element and the stationary element. This deenergizes the drive. If a reversal is required, the drive is deenergized, the engine brought to a standstill, reversed, and brought up to idling speed in the reversed direction. The excitation is then applied to the drive as before.

The Bowes drive is ventilated by means of an external-motor-driven fan arranged to force air into one end of the drive. This air discharges through openings in the end bell on the other side of the drive.

If desired, leads from the generator winding on the shaft element can be brought out to collector rings and a small amount of auxiliary power provided when the unit is running. It is also possible to lock the driven shaft with a brake, open the connections between the two shaft element windings and, by running the engine at the proper speed, generate power for other uses.

Control. The control for the Bowes drive consists of apparatus properly to apply excitation to the field windings in the correct sequence and also to protect the drive. The required relays, contacts, circuit breakers, field discharge, and starting resistors are supplied.

Excitation for the Bowes drive may be supplied from a constant-potential d-c generator driven by the engine or from an auxiliary power supply.

Maintenance. The Bowes drive requires that maintenance which is usually required for proper protection of the conventional a-c motor and generator. Repairs may be made when necessary in the same manner as those required by the conventional synchronous motor or generator.

In cases of emergency it is possible to bolt the engine and shaft elements together so that they act as a solid coupling. The driven shaft may then be operated at engine speed, this speed being set so that overloads on the engine do not occur.

connected or operated at a speed corresponding to zero torque when under way. This permits the power to be applied to the after propeller where it is more effective in producing a net thrust.

Design Characteristics. In all problems dealing with power application, a complete understanding of the characteristics of the driven apparatus should be obtained before starting the design of the driver. In the case of ship-propulsion machinery these items are the hull and propeller.

Motors and generators are designed for a normal power output when operating at a given speed without exceeding a definite temperature, and for temporary operation at a given maximum power without dangerous overheating. As the speed of the diesel-engine-driven generator is independent of the propeller speed, the main concern of the designer is the duty cycle of the motor.

As the most extreme conditions prevail in a tugboat, this application will be considered first. Tests have shown that a propeller when operating at 100 percent apparent slip, the condition ensuing at standstill of boat, absorbs full rated power at approximately 75 percent speed. The torque corresponding to this condition is 133 percent and, current being proportional to the torque produced, the motor must necessarily be designed for this condition. As the conditions existing during acceleration of a tow are of a temporary nature, it is necessary to consider the time element involved and the effect that this excessive current will have on the gradual overheating of the motor.

The heaviest towing condition, which is a continuous operation, dictates the design of the motor and may be considered the basic design factor. Speeds above the heavy towing condition are not of great concern if the machine has been adequately designed for the heaviest operating condition. Motors, however, must be designed for operation between definite current and voltage relationships for, if the speed of a shunt motor is suppressed by increasing the field current, the heating of the field may become a factor.

The accompanying table shows the speed, voltage, and current relationships on a typical basis, for a propelling motor designed for a towboat and operating under towing and running-free conditions.

Propeller, rpm		Volts	Amperes
Towing	Running free		
150	200	240	2,000
135	180	216	1,620
120	160	192	1,280
105	140	168	980
90	120	144	720

It will be noted from the table that the motor under the towing condition has the most strenuous duty and produces the same power output at but 75 percent of the speed attained under the light running condition. The towing condition obviously dictates the motor design.

In actual design practice the engineer has three methods at his disposal for meeting the powering requirements of a ship and in designing the motors to give the highest over-all propulsion efficiency: (1) the single-armature type

SECTION 10

HEATING, VENTILATING, AIR CONDITIONING, ILLUMINATION, SOUND AND NOISE

BY

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CONTENTS

of motor, (2) the double-armature motor, and (3) the use of two or more high-speed motors with gearing. Each of the foregoing types should be studied with relation to the weight, dimensional, and first-cost factors. For shallow craft that limit the motor diameter, the geared arrangement often proves to be the best arrangement from the standpoint of space. The double-armature type of motor is also used for similar purposes.

Ships differ with respect to adequate ventilation in the engine and motor room, and this is a subject of primary importance to the designer. If the heat losses of the engines, generators, and motors are allowed to become cumulative, thereby creating high ambient temperatures, the safe output of the electrical apparatus is limited.

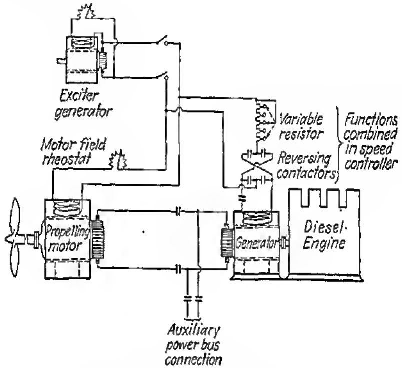


FIG. 21.—Elements of diesel-electric drive system.

In cases where open-type generators and motors are used with integral fans on the rotors, adequate engine-room ventilation must be provided to eliminate the continuous recirculation of the heated air. In installations using a relatively large amount of power, it has been the general practice to enclose the motors and furnish the cooling air by means of separately driven blowers. In the higher power installations the general trend is toward the use of air coolers that use water as the circulating medium for carrying away the heat. This prevents the deposit of dust or grease on the windings and at the same time results in an accurate means of temperature control.

A number of methods are presented to the designer for obtaining the exciting current for the fields of the generators: (1) from an exciter generator driven by the main propulsion unit, (2) from a motor-generator set, and (3) directly from the ship's auxiliary-power bus.

HEATING, VENTILATION, AND AIR CONDITIONING

BY

ABRAHAM THAELER and H. H. HOLLY

GENERAL

Purpose. The purpose of ventilation and heating is the maintenance of satisfactory standards of comfort for the passengers and crew and the establishment of proper atmospheric conditions for protection of the ship's equipment and cargo. Atmospheric conditions through which a ship passes vary more widely and more rapidly than for a fixed location ashore, yet the ship and its contents must be maintained at reasonably constant conditions throughout all weathers.

Until recent years, ventilation and heating were considered as separate problems and were so handled. In modern practice, however, they are treated as two closely allied phases of the same problem. Older ships were heated by a system of direct radiation, generally employing steam-heated radiators; and were ventilated by an entirely separate system of ventilators, ducts, and fans. Recent practice combines the two into a single system, using unheated or cooled air in warm weather and heated air in cold weather.

Marine Conditions. Maintenance of a satisfactory standard of conditions afloat is more difficult than ashore. The wide and rapid temperature fluctuations require an adaptable system; in addition, a marine installation is subjected to many inherent problems. The **watertight integrity** of all watertight decks and bulkheads must be maintained, where ducts and pipes pierce them. The **structural integrity** of strength decks and bulkheads must also be maintained. **Weather openings** are subjected to unusual and severe service. The necessity of recognizing and designing for these conditions is shown by the rules of marine classification societies such as the American Bureau of Shipping and Lloyd's Register of Shipping, which are established to ensure watertight and structural integrity of the weather deck.

Fire hazards must be recognized and reduced to a minimum. The regulations of government agencies such as the U.S. Coast Guard Marine Inspection Service, and agreements such as the rules established by the 1929 International Convention for Safety of Life at Sea and *U.S. Senate Report* 184 must be met. Adequate closures and emergency stops to prevent spread of fire; fireproof or fire-resistant insulation; isolation of inflammable or dangerous materials, etc., should be provided.

Ratproofing of ventilating systems is necessary. Owing to the nature of a ship's structure and the prevalence of rats in many ports, they are a greater problem afloat than ashore. Precautions should be taken to prevent not only establishment of rat homes in ventilating ducts but also migration of rats from one compartment to another by way of the ducts. The U.S. Public Health Service has established regulations dealing with this problem.

Weather conditions vary widely. For vessels designed to operate generally in all parts of the world, air temperatures ranging from 90 to 10 F should be considered for design. For ships in North Atlantic service during the winter, the heating system should be designed for an outside air tempera-

The control panel upon which are mounted the disconnecting switches, speed controller, motor field rheostat, and meters for observation of power output can be made quite simply as the functions are of an elementary nature. As the main power circuits are switched only when the main power circuits are deenergized, the switches are generally operated manually by means of a wheel. A speed controller is generally mounted on the panel, and

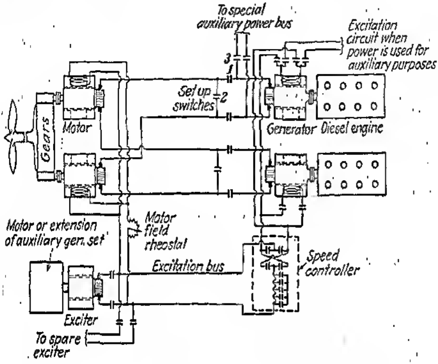


Fig. 22.—Two-generator, two-motor powering system.

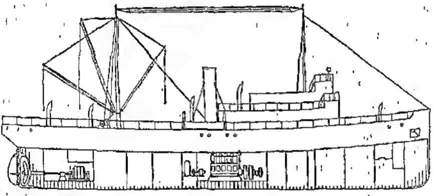


Fig. 23.—Typical method of installation of diesel-electric drive.

a transfer switch supplied for transferring the control to either the bridge or the engine room as desired.

Powering Combinations. It is the general tendency in marine practice to install two diesel-engine-driven generators per propeller shaft, and one or a multiple number of motors per shaft depending upon the power output required. This combination permits a generating unit to be disconnected

ture of 0 F. The humidity of sea air is generally high and, in passing from warm to cool weather, the moisture in the air will condense on cool surfaces and cause "sweat." For this reason, cargo holds should be provided with suitable means for ventilation, to protect the cargo from "sweat damage." Ventilating systems afloat must be able to operate under other adverse conditions, with the ship pitching and rolling; and equipment should operate satisfactorily when tilted as much as 15 deg in any direction and with the usual weather openings closed in rough weather.

Space and weight are invariably limited. This fact must be considered for ducts as well as heaters, fans, and ancillary units. As a result, higher air velocities in the ducts are acceptable, as long as the noise created thereby is not objectionable. The relatively lower ceilings of living quarters require special attention in order to obtain proper diffusion of air. The camber and sheer of ship structure require that ducts follow closely these irregular contours. As living and working quarters are frequently located adjacent to machinery spaces and heat-producing areas, their boundaries require insulation therefrom. **Drainage** of ducts and equipment should take into account the ship's forward motion as well as rolling and pitching. **Corrosion** due to salt air is an important factor, and all materials must be safeguarded. Steel should be protected by galvanizing, painting, or both. The corrosion resistance of aluminum is increased by anodizing, painting, or both.

As the trade routes and operating personnel are frequently changed, **informative labelling** and operating instructions are very important and desirable. Ventilating systems are generally labeled in accordance with the following numbering system.

Designation. Owing to the multiplicity of ventilating systems on many large ships, both merchant and naval, the following method of designation is used for numbering the fans and their attached systems. Each system is designated by a series of numbers: the first indicates the deck immediately below the fan; the second indicates the ship-frame number immediately forward of the fan; the third indicates the athwartship location of the fan relative to other fans at the same deck and frame location. For the third number, odd numbers are used for starboard and even numbers for port fans, starting from the ship's centerline

Deck numbers are as follows:

03. Third deck above weather deck
02. Second deck above weather deck
01. First deck above weather deck
1. Weather deck
2. First deck below weather deck
3. Second deck below weather deck, etc.

Thus, a system numbered 1-47-2 is supplied by a fan on the weather deck; the fan is immediately abaft ship frame 47; the fan is on the port side of the ship. The presence of the third number indicates that more than one fan is at location 1-47.

Special Navy Conditions. Naval ships involve special problems, owing to the following: (1) increased subdivision of compartments; (2) adequate ventilation of ammunition storage spaces, gun turrets, congested living and working quarters, etc.; (3) possible failure of equipment due to combat casualty; (4) fewer permissible weather openings; (5) special restrictions imposed by Navy specifications. The last-named should always be carefully reviewed prior to designing a heating and ventilating system.

from the line under reduced-power conditions, or for overhaul if occasion demands.

Figure 21 shows the elementary, main, and field circuits of a single generator and motor, and Fig. 22 a typical method of electrically connecting two generators and two propelling motors in a series loop. The set-up switches are so arranged that the power may be used for propulsion, cut out of the power loop entirely, or diverted to auxiliary uses. When the generator units are used for special power purposes, the field-excitation circuit is generally connected to the ship's d-c auxiliary-power bus.

The most satisfactory method of driving the exciter generator can be determined only after a study of the auxiliary-power needs and the preferred engine-room arrangement.

Power Limitations When Using Direct Current. Direct-current machinery is used primarily in a low voltage system; consequently the current becomes quite high when large powers are involved. The voltage of the generators generally used is 250. Commutation and the current-carrying capacity of the armature coils become limiting factors in such installations. At the present time the practical field of application lies within a power range of a few hundred horsepower per shaft, up to 2,000 shp when using two generators and two motors per shaft.

DIESEL-ELECTRIC DRIVE USING ALTERNATING CURRENT MACHINERY

Operating Principles and Purpose. In the diesel-electric-drive system of ship propulsion using a-c generators and motors, the operating characteristics are similar in some respects to those obtained with turbine-electric drive. The prime movers operate in one direction of rotation, and the propeller reversals are accomplished by switching the electrical connections between the generator and the motor.

The primary purpose of diesel-electric a-c drive is to adapt the high-speed diesel engine to ships that require powers above those normally obtainable with d-c apparatus. High powers are obtained by paralleling a multiple number of diesel-engine-driven generators on a single power bus. The power that can be applied by this method is limited only by the number of units that can be conveniently located in a ship. Individual generating units can be either connected or disconnected from the power bus as the power demands warrant while the ship is under way.

The speed-reduction ratio between the prime mover and the propeller shaft is a fixed value when the synchronous type of propelling motor is used, and propeller-speed variations are obtained by alternating the speed of the diesel engines.

The torque limitations on the engines when operating at reduced speed are deciding factors in determining the fractional number of units to use for a given power and speed. Figure 24 has been plotted on the assumption that four diesel-engine-driven generators supply the power to an individual propeller shaft at 100 percent propeller revolutions and 100 percent engine torque. It has been further assumed that the minimum safe speed of the engines is 25 percent of rated full speed, and that an engine torque of 50 percent should not be exceeded at this point under continuous operating conditions. Both of these items, however, are subject to the specific recommendations of the engine manufacturer. In this arrangement and assuming that the torque can be safely increased proportionately with the engine speed,

LIVING AND RELATED SPACES

General. In living spaces, the personal comfort of the occupants is the primary consideration. These spaces include public rooms and staterooms for officers and passengers, crew's quarters, and hospital spaces. Also included are spaces in which proper working conditions must be maintained, such as galleys, pantries, bakeries, butcher shop, laundry, radio room, chartroom, wheelhouse, and workshops. This latter group is segregated from the former because it requires special treatment owing to unusual heating and cooling conditions. In all, however, personal comfort is paramount as to both air requirements and distribution.

Miscellaneous spaces such as dry stores, linen lockers, boatswain's stores, CO_2 bottle rooms, deck lockers, gyro rooms, paint and lamp rooms, slop chest, and other infrequently occupied spaces are ventilated for protection of their contents, as well as to ensure safe conditions for personnel who enter the compartment.

Conditions. Table 1 lists desired air changes and temperature conditions for various living and related spaces.

Table 1. Air Changes

Space	Air change, min[g]		Summer max rise[f] deg F	Winter design temp deg F and type of heating
	Supply	Exhaust		
Staterooms, passenger and crew	5	..	10	70[a]
Group berthing	5	10	10	70[a]
Hospital spaces	4	4	10	75[a]
Dining and messrooms, lounges, etc.	4-5	[h]	10	70[a]
Theater and dance hall	6	[h]	10	70[a]
Barber shop	5	5	10	70[a]
Toilets and washrooms	....	4	..	70[b]
Passages	....	[h]	..	60-70
Galley and bakery	4-6	1-2	15	60[d]
Pantry	4-6	1-2	10	60[d]
Butcher shop	....	4	15	60[b]
Laundry and tailor shop	4-6	2	15	60[d]
Workshops	6-10	..	..	60[c]
Wheelhouse	5	..	..	70[b]
Chartroom	5	..	10	70[a]
Radio room	5	[h]	10	70[a]
Gyro room	[h]	[h]	15	50[c]
Dry stores	15	..	..	50[d]
Boatswain's stores and deck lockers	[e]	[e]		
Slop chest	10		10	60[d]
Lockers, clean linen	15			
Lockers, soiled linen and oilskins	....	15		
Paint and lamp rooms	[e]			
CO_2 bottle room	....	10		

[a] *Hot blast or convector.*
[b] Convector.
[c] Tempered air, or unit heater.
[d] Tempered air.
[e] Natural ventilation.
[f] Maximum difference between air in space and outside air temperature.
[g] *Based on maximum designed quantity* supplied.
[h] Quantity determined by factors other than air change.

approximately 40 percent ship's speed can be obtained with one engine, 60 percent with two engines, and 81 percent with three engines, assuming the shaft horsepower varies as the cube of the speed over the entire operating range. At the higher speeds, the exponent is greater than three.

Paralleling of Alternating-current Generators. The paralleling of an a-c generator is quite a simple process. This type of generator, when connected to the line with its field circuit externally short-circuited, has a similar characteristic to that of an induction motor. This feature permits the second and succeeding diesel-engine-driven generators to be started without

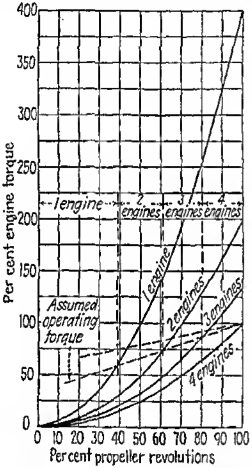


Fig. 24.—Engine torque at varying propeller revolutions.

load, brought up to an approximate speed approaching that of the generator or generators in operation, before being connected across the line. As soon as the generator has attained its slip speed as an induction motor, excitation is applied to the field which brings the generator into synchronism with those in operation. From this point on, the division in load between the generators in operation is controlled by the engine fuel-oil governing system.

Maneuvering Characteristics. During maneuvering operations a propelling motor of the synchronous type is brought up to its slip speed as an induction motor before applying excitation to its field for synchronization. The starting current during this period of operation is several times normal current and, in order to sustain the line voltage, it becomes necessary to

Air changes as given in Table 1 are based on usual minimum air requirements for breathing comfort and on air movement required for removal of odors and body cooling. They do not take into consideration heating and cooling loads and should, therefore, be modified to suit these factors.

Another closely related criterion for air supply to a living space is the minimum air requirement per occupant, accepted values for which are as follows:

	Cfm per Man
Quarters	40–50
Group berthing	30
Mess spaces	25–30

Air volumes based on these requirements will generally exceed those based on air change for a crowded berthing or mess space, while air change, heating load, or cooling load will generally be the determining factor in less densely occupied spaces.

Air supplied to living spaces should be delivered without creating objectionable drafts and as evenly distributed as possible. For private and semiprivate spaces, means for controlling the volume of air delivered should be accessible to the occupant. For public and berthing spaces the individual outlet controls should be accessible to the ship's personnel. For working spaces, air should be supplied at high velocity from terminals with directional adjustment to permit "spot cooling." Both directional and volume control should be accessible to the occupants of the space. For storerooms and similar unoccupied spaces, a continuous air change should be provided with no means of closing off the air, except by means of a fire damper.

Certain spaces that require special treatment in connection with the foregoing air changes are as follows:

Dining and messrooms should have exhaust slightly in excess of supply to prevent spread of odors to adjacent spaces. If the supply is mechanical, the total exhaust should also be mechanical. The excess of exhaust over supply in adjoining galleys or pantries can be utilized for this purpose by means of door louvers or bulkhead grilles. This may take care of the entire exhaust for small messrooms but, for large spaces, additional direct outlets are required to give better distribution and prevent drafts.

Lounges and other public rooms should have exhaust approximately equal to the supply, either direct from the space or from adjoining service rooms and passages. Smoking rooms and bars should have an excess of direct exhaust.

Hospital spaces require special care to assure uniform distribution and freedom from drafts. Individual rooms, or groups of rooms on the same passage, should be individually balanced for supply and exhaust to prevent the spread of odors. Isolation spaces should have a separate mechanical exhaust fan or, if included in a larger system, should have a separate exhaust duct run directly to the exhaust fan inlet.

Galleys, pantries, and other spaces containing heat-producing equipment should have exhaust inlets located directly over such equipment. Canopies should be provided over any equipment from which vapors or fumes are emitted. Local exhausts of this type should always be located on the far side of the equipment, so as to draw heat and fumes away from operating personnel.

Paint and lamp rooms should have but one opening, located in the deckhead or high in the bulkhead and provided with a means of closure.

overexcite the field of the generator. Because of the limited torque characteristics of a diesel engine at slow speed and because above-normal torque is required during periods of acceleration and retardation, it is not feasible to maneuver with only one of a group of engines in operation. In determining the maximum rated motor that can be safely connected to a power line without causing undue line disturbance, the rating of the generating plant must be considered. In normal practice, the generator rating is many times the rating of the motor, whereas in the problem under consideration, the total power of the generating units just equals the power of the motor. Disconnecting power units from the line further decreases the power available.

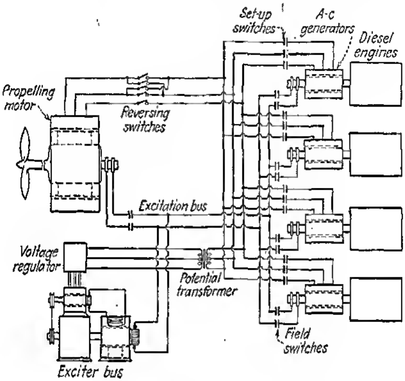


FIG. 25.—Diesel-electric a-c drive with four generating units.

In a turbine-drive ship the power is usually provided by a single large unit in which the torque capacity rapidly increases as the rate of revolutions is forcefully decreased, whereas the opposite is the case when a diesel engine is used.

In reversing a propeller a countertorque must be applied to oppose that produced by the ship's coasting speed which tends to keep the propeller rotating in the forward direction. As the propeller torque is directly reflected on the prime mover in the a-c system of propulsion, the time element involved in getting possession of the propeller will be of longer duration with diesel-electric drive than with turbine-electric drive.

A second consideration in applying diesel-electric drive with a-c generators and motors is the problem of obtaining propeller speeds below those corresponding to the lowest safe speed of the engines. The recommended

CO_2 *bottle room* exhaust should be taken from a point about 6 in. above the deck, with natural supply into the upper part of the space.

Temperatures as shown should be maintained in the living zone of all spaces. Heat, whether by hot blast or convector, should be distributed generally but with particular attention to the cold boundaries—usually outside bulkheads. Insulation should be provided on all boundaries where a high temperature difference exists, to reduce the heating requirements. Attention should be given to the location of exhaust inlets in relation to heating sources, to aid and not hinder the heating operation.

A unit heater serving a space with natural ventilation should, if possible, be located in front of the supply outlet so as to heat the incoming air.

The usual practice of heating **toilets and washrooms** by convector should be modified in cases where there are no adjacent heated spaces from which to draw natural supply. In such cases the outside air supply should be at least tempered. It is then more practical to supply all the heat with a single heater instead of a preheater and convector. In this case the heated supply air should be introduced at low velocity close to the deck, with the exhaust of equal volume taken, as usual, from high in the space. In such cases and provided the size of the space warrants it, it is good practice to use two fans with the systems interconnected. Then, in case one fan fails, the other can supply sufficient air to the entire space to prevent the freezing of piping.

The allowable temperature-rise figures are used to determine the volumes of outside air required for cooling. A definite temperature figure cannot be given since the outside air temperature is the ultimate limit that can be approached by this method of cooling. The amount of outside air introduced into the space should, however, be sufficient to keep the inside air temperature from rising above these limits. The relative amounts of positive exhaust and positive supply required are dependent on the nature of the particular space as regards equipment contained and the degree of occupancy. Inasmuch as air movement is an important factor in body comfort, this factor as well as heat removal should be given due consideration in the design of a system for cooling and, since vessels in most services operate well over half the time under cooling load conditions, its importance should be recognized.

The other method of cooling is by distributing cooled air. This is one function of **air conditioning**. Since this involves a number of other factors, this method will be considered under that heading. It may be said, however, that the temperature to be maintained depends upon the conditions under which the ship operates. In general, 70 to 80 deg effective temperature provides personal comfort in air-conditioned spaces.

Purity and humidity are considered only in living spaces unless liberated fumes, smoke, or gases are present; in which case they must be removed by exhaust. Air supplied to hospital spaces should be filtered, as should also the air supplied to other living spaces if feasible. Space limitations may make the installation of filters impractical. Filters are particularly desirable, however, on vessels in service where people live aboard ship during long stays in port.

Insect screens should always be provided for natural supply and exhaust openings to living spaces, and on mechanical supply intakes to living spaces where filters are not installed.

The problems of oxygen and CO_2 content of air are covered by the air changes recommended and need no further consideration. Warnings should

minimum speeds vary from 25 to 33⅓ percent of normal rated speed among different engine manufacturers.

In many types of ships the minimum propeller speeds obtainable by engine-speed regulation are sufficient. For ships requiring lower speeds, the same must be accomplished electrically. Two methods have been proposed to meet this problem: (1) by means of a small a-c motor attached to the end of the main motor shaft and (2) by operating the main motor on its squirrel-cage starting winding at reduced voltage, thereby increasing its slip.

The type of motor usually recommended for the first solution is the wound-rotor type of induction motor, in which speed control is obtained by means of variable external resistors. Assuming the minimum propeller speed obtainable by engine-speed control to be 25 percent, the corresponding torque is 6.25 percent, and the corresponding horsepower 1.5625 percent. The motor rating must be adequate to provide breakaway torque at the start and be designed for continuous operation at the maximum speed and power for which intended.

The second method entails the incorporation of a special type of generator excitation control with automatic change-over when the speed lever is moved below that corresponding to the lowest safe speed of the engines.

The problems associated with diesel-electric drive which concern the application engineer are (1) a satisfactory method of providing a dynamic braking action on the propeller during maneuvering operations without reflecting this action back to the engines in the form of excessive torque and (2) a satisfactory governing system for parallel operation of the powering units over a wide speed range.

be posted, however, against personnel entering an unoccupied compartment alone without first making sure that the air within is fit for breathing. Portable fan units should be supplied for blowing out tanks and other spaces not provided with installed ventilation.

Humidity control is a phase of air conditioning and will be discussed under that subject. In general, however, 45 to 65 percent relative humidity is the comfort range; the preferred value for a particular case depends on other conditions, both inside and outside.

Methods. Ventilation is the supplying and exhausting of air. Natural ventilation accomplishes this object without the use of a fan. Mechanical ventilation utilizes a fan for propelling the air either directly into or out of a space or through ductwork. A self-evident but often neglected fact is that, in order to accomplish any air change, means for both supply and exhaust must be provided. Since equal mechanical supply and exhaust are seldom provided in a given space, a balancing air quantity must be able to pass into or out of the space under all operating conditions. This may equal the full amount of mechanical air change, and, in such a case, the size and location of the openings are of great importance. Door louvers, undercut doors, or bulkhead openings are the usual means provided for this air movement. Infiltration should be neglected in all shipwork. Thus it is evident that most mechanical systems involve some natural ventilation.

Natural ventilation may be either positive supply, positive exhaust, or neutral, depending on the type of weather opening used. Cowls, porthole wind scoops, and some patented intake heads are generally used for positive supply. Torpedo-type vents, porthole exhausters, and various patented aspirator-type exhausters are used for positive exhaust. Some of the patented exhausters have an advantage over other positive ventilators, in that they do not require trimming into the wind and are practically rain- and sprayproof. Neutral openings—those having no positive action and used for either natural supply or natural exhaust—include mushrooms, goosenecks, and bulkhead openings covered with louvers or wire mesh. The disadvantages of these natural vents, particularly the positive types, are that their performance is dependent on wind conditions; unless of very large size, they handle comparatively small quantities of air against low static pressures; they cannot be operated in bad weather; and their performance cannot be well controlled. A single natural vent head of the positive type will serve a limited number of spaces, since lack of static pressure makes it necessary to keep ductwork to a minimum. When there is more than one outlet on a positive system, regulating dampers should be provided to assure proper distribution, even though it is very difficult to get a close adjustment. It is not practicable to heat natural supply air by duct heaters, owing to the constantly varying volume.

Natural ventilation in living spaces is generally supplemented by bracket fans. Unless such a fan can be located close to a natural vent opening, it will merely circulate the air but, in so doing, will add considerably to body comfort. Porthole fans may also be used for either supply or exhaust. Strictly speaking, they provide mechanical ventilation; but they are generally used in conjunction with natural ventilation, in lieu of an installed mechanical system.

Mechanical ventilation is more expensive to install and operate than natural ventilation. In all other respects, however, it is to be preferred for living-space ventilation, as it is more positive and controllable, requires fewer

BOWES DRIVE

STAFF CONTRIBUTION BASED ON DATA FROM THE ELLIOTT COMPANY

AN ELECTRICAL SPEED-REDUCING TORQUE-MULTIPLYING DEVICE

The Bowes drive is an electrical device for the reduction of speed between high-speed prime movers and driven shafts for mechanical applications where gear and coupling are ordinarily necessary. The immediate application is in ship propulsion where it will be used to reduce the speed of the prime mover, particularly the high-speed diesel engine, to the lower speed required for most efficient operation of the propeller. It may also be used for other applications where speed reducers are required.

Sizes and Types. The Bowes drive covers a range of 300 to 3,000 hp and is built in speed ratios up to 6:1. Any desired speed-reduction ratio up to this value is obtainable, either exactly or very nearly so. The drive is so constructed that the speed ratio once selected is the same for all speeds of the engine and is not adjustable. The drives in operation are of the nonreversing type; *i.e.*, in order to cause a reversal of the driven shaft it is necessary to stop and reverse the drive engine. It can also be built as a reversing drive which will drive the load in either direction without the necessity of reversing the engine.

Description. The following description is for a nonreversing Bowes drive. There are three main elements:

1. An **engine element**, which is solidly bolted to the shaft of the prime mover and which revolves at the same speed as the prime mover. This structure is very similar to the rotating field of a conventional a-c generator. It consists of a spider wheel upon which field poles are mounted. The number of poles is selected so that in conjunction with the other elements of the drive the desired speed ratio is obtained. A field winding is mounted on these poles, and leads from this winding are brought out to collector rings which are mounted on the engine side of this element. Direct current for exciting the field is supplied to this winding through brushes and the collector rings.

2. A **shaft element**, which is solidly attached to the driven shaft. This structure is larger in diameter than the engine element and is mounted concentric with it. This element contains the laminated core and windings usually associated with the armature of an a-c generator. A set of windings on the inner periphery is selected to match the poles on the engine member so that, with the engine running and the generator field poles excited, a-c voltage is generated in this winding. The terminals of this winding are connected directly to the terminals of a second winding which is located on the outside periphery of this member. This winding, since it receives power from the generator winding, becomes the armature of an a-c motor.

3. A **stationary element**, which is mounted concentric with the other two elements and on the outside. It is supported by feet bolted to the supporting foundation. This element consists of a steel ring, inside of which are field poles similar to those on the engine element but of a number selected to produce the desired speed ratio. These poles are also supplied with a field winding and, in conjunction with the winding on the outer periphery of the shaft element, complete a synchronous motor. In addition, a special starting winding is embedded in the pole faces in a manner similar to that

ventilators on the open deck, and is capable, by the use of ductwork, of reaching all parts of the vessel with air taken in through protected inlets located high in the ship. Furthermore, because it is constant and controllable, it can be used for heating and air conditioning.

Mechanical ventilation air is drawn from or discharged to the weather *through louvered openings, mushrooms, or goosenecks* located as high in the ship as possible. Supply intakes should be located so that there is no chance of drawing in gases discharged from the stack, exhaust vent openings, or open skylights. A desirable arrangement for toilet or galley exhausts is to discharge them into the outer casing surrounding the live stack. This removes the ventilator from the open deck and carries the foul air clear of the ship.

Three principal types of fan are all used for mechanical ventilation: propeller, centrifugal, and axial. Each has individual characteristics that recommend it for a particular application. Since these characteristics will be discussed under Equipment (p. 1542), only the fan applications will be mentioned here.

Propeller fans, generally exhaust, may be mounted directly on the bulkhead with no duct work on either side of the fan. Wire mesh should be installed over the bulkhead opening. If the fan is within a man's reach, a mesh guard should be installed on the motor side as well. The cover over the bulkhead opening serves as a fire closure for such systems. If ductwork is connected to the intake side of the fan, easy access must be provided for motor servicing and repair. Ductwork should be kept to a minimum and carefully designed, since the performance of a propeller fan is adversely affected by resistance to the air flow.

Centrifugal fans almost invariably are fitted with ducts connected to both intake and outlet. Large units are generally located in fan rooms, owing to the large amount of deck space they require. Small centrifugal fans may, however, be bulkhead or deckhead mounted in working or even living spaces.

Axial fans are generally located close to their weather opening, either in a fan room or on the open deck. Being of large capacity and with a relatively higher noise level, there should be a reasonable distance between a fan and the first outlet of the system. Their use for living spaces is principally on systems requiring large air volume at fairly high static pressures.

Mechanical supply ventilation to living spaces is generally handled by large systems serving a number of rooms. The systems vary in size from a small fan serving isolated compartments to a group of fans handling large sections of quarters. The arrangement of the vessel must, of course, determine the size of system to be used, but fans of from 5,000 to 8,000 cfm capacity and serving 20 to 40 outlets are commonly used. The upper limit is generally determined by the space limitations of the fan room and the size of duct that may be run. Wherever possible, main ducts should be run in passageways, with branches running outboard between transverse deck beams to the individual spaces. This gives a maximum of headroom below the ducts. Steel structure, piping, and wireways are the principal interferences to be avoided in such a layout. Each main branch should be provided with a splitter damper, and each individual branch with a splitter, regulating damper, or other type of volume control, so that the air delivery from each outlet may be controlled. Each outlet should have, in addition, a shutoff damper. Diffusers and wall registers are most commonly used as

SIDE (say a, Fig. 7). The third angle is known, since $A + B + C = 180°$. To find the remaining sides, use $b = \dfrac{a \sin B}{\sin A}$, $c = \dfrac{a \sin C}{\sin A}$. Or, drop a perpendicular from either B or C on the opposite side, and solve by right triangles.

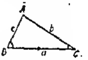


FIG. 7.

Check: $c \cos B + b \cos C = a$.

CASE 2. GIVEN TWO SIDES (say a and b), AND THE INCLUDED ANGLE (C); and suppose $a > b$. Fig. 8.

First Method: Find c from $c^2 = a^2 + b^2 - 2ab \cos C$ [or $c^2 = (a - b)^2 + 2ab \text{ vers } C$]; then find the smaller angle, B, from $\sin B = (b/c) \sin C$; and finally, find A from $A = 180° - (B + C)$. Check: $a \cos B + b \cos A = c$.

Second Method: Find $\frac{1}{2}(A - B)$ from the law of tangents:

$$\tan \tfrac{1}{2}(A - B) = [(a - b)/(a + b)] \cot \tfrac{1}{2}C,$$

and $\frac{1}{2}(A + B)$ from $\frac{1}{2}(A + B) = 90° - C/2$; hence $A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$ and $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$. Then find c from $c = a \sin C/\sin A$ or $c = b \sin C/\sin B$. Check: $a \cos B + b \cos A = c$.

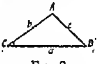


FIG. 8.

Third Method: Drop a perpendicular from A to the opposite side, and solve by right triangles.

CASE 3. GIVEN THE THREE SIDES (provided the largest is less than the sum of the other two), Fig. 9.

First Method: Find the largest angle A (which may be acute or obtuse) from $\cos A = (b^2 + c^2 - a^2)/2bc$ [or $\text{vers } A = [a^2 - (b - c)^2]/2bc$] and then find B and C (which will always be acute) from $\sin B = b \sin A/a$ and $\sin C = c \sin A/a$. Check: $A + B + C = 180°$.

Second Method: Find A, B, and C from $\tan \frac{1}{2}A = r/(s - a)$, $\tan \frac{1}{2}B = r/(s - b)$, $\tan \frac{1}{2}C = r/(s - c)$, where $s = \frac{1}{2}(a + b + c)$, and $r = \sqrt{(s - a)(s - b)(s - c)/s}$. Check: $A + B + C = 180°$.

Third Method: If only one angle, say A, is required, use

$$\sin \tfrac{1}{2}A = \sqrt{(s - b)(s - c)/bc} \quad \text{or}$$

$$\cos \tfrac{1}{2}A = \sqrt{s(s - a)/bc},$$

according as $\frac{1}{2}A$ is nearer 0° or nearer 90°.

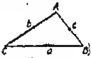


FIG. 9.

CASE 4. GIVEN TWO SIDES (say b and c) AND THE ANGLE OPPOSITE ONE OF THEM (B). This is the "ambiguous case" in which there may be two solutions, or one, or none (see Fig. 10).

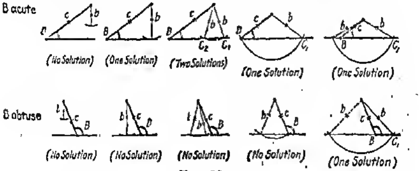


FIG. 10.

First, try to find C from $\sin C = c \sin B/b$. If $\sin C > 1$, there is no solution. If $\sin C = 1$, $C = 90°$ and the triangle is a right triangle. If $\sin C < 1$, this determines two angles C, namely, an acute angle C_1, and an obtuse angle $C_2 = 180° - C_1$. Then C_2 will yield a solution when and only when

$C_1 + B < 180°$ (see Case 1); and similarly C_2 will yield a solution when and only when $C_2 + B < 180°$ (see Case 1).

Other Properties of Triangles. (See also p. 99 and p. 105.)

Area $= \frac{1}{2}ab \sin C = \sqrt{s(s-a)(s-b)(s-c)} = rs$, where $s = \frac{1}{2}(a+b+c)$, and r = radius of inscribed circle $= \sqrt{(s-a)(s-b)(s-c)/s}$.

Radius of circumscribed circle $= R$, where

$$2R = a/\sin A = b/\sin B = c/\sin C;\ r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{abc}{4Rs}.$$

The length of the bisector of the angle C is

$$z = \frac{2\sqrt{abs(s-c)}}{a+b} = \frac{\sqrt{ab[(a+b)^2 - c^2]}}{a+b}.$$

The median from C to the middle point of c is $m = \frac{1}{2}\sqrt{2(a^2+b^2) - c^2}$.

SOLUTION OF SPHERICAL TRIANGLES

For the occasional solution of a spherical triangle the following formulas will be sufficient. For a detailed discussion, see any text-book on spherical trigonometry.

Let a, b, c be the sides of the spherical triangle, that is, portions of arcs of great circles of the sphere; and let A, B, C be the angles of the triangle, that is, the angles made by tangents drawn to the sides at their points of intersection on the sphere. The sum of the angles will always be greater than two right angles, and may be nearly six right angles. The angle $E = A + B + C - 180°$ is called the **spherical excess** of the triangle. (See also p. 100.)

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B};\quad \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C};\quad \frac{\sin c}{\sin C} = \frac{\sin a}{\sin A}.$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

with similar formulæ for $\cos b$ and $\cos c$.

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a,$$

with similar formulæ for $\cos B$ and $\cos C$. Other formulas are:

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A, \text{ etc.};$$

$$\sin A \cos b = \cos B \sin C + \sin B \cos C \cos a, \text{ etc.}$$

If $s = \frac{1}{2}(a + b + c)$ and $S = \frac{1}{2}(A + B + C)$; and if

$$\tan r = \sqrt{\sin (s-a) \sin (s-b) \sin (s-c)/\sin s}, \text{ and}$$

$$\tan R = \sqrt{(-\cos S)/[\cos (S-A) \cos (S-B) \cos (S-C)]}; \text{ then}$$

$$\tan \tfrac{1}{2}A = (\tan r)/\sin (s-a), \text{ and } \tan \tfrac{1}{2}a = \tan R \cos (S-A),$$

with similar formulæ for $\tan \frac{1}{2}B$, $\tan \frac{1}{2}C$, and for $\tan \frac{1}{2}b$, $\tan \frac{1}{2}c$. (Here r and R are the radii, on the spherical surface, of the inscribed and circumscribed circles, respectively.) If $E = A + B + C - 180°$, $\tan \frac{1}{4}E = \sqrt{\tan \frac{1}{2}s \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)}$. If E is small, then approximately, $\sin E = F/K^2$, where K = radius of sphere, and F = area of triangle, regarded as a plane triangle. In any case,

$$\frac{\text{The area of a spherical triangle}}{\text{area of a great circle}} = \frac{\text{spherical excess}}{180°}.$$

In the special case of a right spherical triangle, in which $C = 90°$,
$\cos c = \cos a \cos b = \cot A \cot B$; $\cos a = \cos A/\sin B$; $\cos b = \cos B/\sin A$; $\sin A = \sin a/\sin c$; $\cos A = \tan b/\tan c$; $\tan A = \tan a/\sin b$.

terminals in living spaces. The **punkah louver**, a high-velocity outlet with both directional and volume control, should also be mentioned since it is widely used on foreign vessels and on some American ships. Registers are generally mounted in a cut in the passage bulkhead at the end of a short branch duct, blowing across the space just below ceiling height. If the duct is within a room, the branch (which should be not less than 6 in. long) can be taken off the side, and the register mounted in the vertical ceiling furred in around the duct. A register, particularly when delivering heated or conditioned air, should blow toward an exposed bulkhead and should be designed with sufficient "throw" and dispersion to cover this area. The passage bulkhead location for a register has an additional advantage in the fact that, with mechanical supply ventilation, the exhaust is frequently through a louver set low in the passage door, thus giving the air a full circuit of the room.

Diffusers are mounted either in the ceiling or in the underside of a duct. They should be connected to the duct by a collar at least equal in length to the neck diameter of the diffuser, to prevent too much air discharge on the far side of the terminal. Some diffuser manufacturers supply adjustable grids for this purpose, but the collar is to be preferred for shipwork. It is nevertheless safe practice to have the duct serving a diffuser run toward the exposed side of the room so that any overbalance of air on the far side of the diffuser will discharge toward the point of principal heating and cooling load. The diffuser should be as centrally located as possible so as to reach all parts of the space ventilated. Even when using a combination diffuser and lighting fixture, the location should be dictated essentially by ventilation considerations. Lining up with paneling or light fixtures is desirable for appearance, but such considerations should not jeopardize the ventilation. Where drop ceilings are installed, dampers may be either part of the diffuser or a duct damper with remote control. The punkah may be either bulkhead or ceiling mounted.

Mechanical exhaust may be accomplished either by small individual fans generally located in or adjacent to the space served or by a large fan with ductwork. The latter is the preferred arrangement. Food-handling spaces should be ventilated by a separate system. An exception may be made for a small isolated food space, in which case a separate duct should be run to the fan intake. Toilet systems also should be kept separate, although lockers and other miscellaneous spaces may be incorporated. Exhaust systems are usually smaller than living space supply systems, owing to the fact that exhaust is generally drawn from small widely separated spaces. Ducts are run similar to supply ducts. Outlets should draw from the top of the space, with the exception of those located in spaces where heavy gases such as CO_2 or gasoline may be emitted. Splitter or regulating dampers should be provided for every outlet and main branch. Openings with ½-in. wire-mesh screens are generally used for exhaust. Louvers or commercial grilles should, however, be employed in dining rooms, lounges, and other spaces where appearance is a consideration. Exhaust openings should be located as close as possible to a heat source and at sufficient distance from the supply so as not to short-circuit the air. As in the case of supply, spaces having an excess of exhaust should have door louvers, undercut doors, or bulkhead openings to permit passage of air. Shutoff dampers need not be provided for exhaust terminals, except those whose operation can directly affect the heating of the space in which they are located, such as hospital, radio room, or lounge.

To solve for inlet air temperature, transpose the formula as follows:

$$t_i - 70° = \frac{4141 \text{ Btu}}{1.08 \times 224 \text{ cfm}} = 17 \text{ F}$$

$$\text{Inlet air temperature} = 70 + 17 = 87 \text{ F}$$

The **zone system of hot-blast heating** requires balancing temperature and air volume as illustrated by the following example based on 8 spaces on one reheater:

Example.

1	2	3	4	5	6	7	8	9	10
Room	Cubic	Summer air change	Room, deg F	Btu loss	Est winter air, 65%	Cfm est for 92.6°	Final winter cfm	Final summer cfm	Final air change
A	1,120	5	70°	4,141	146	170	170	260	4.3
B	980	5	70°	2,010	128	112	128	195	5.0
C	980	5	70°	3,055	128	126	126	195	5.0
D	806	5	70°	2,740	105	113	113	175	4.6
E	1,040	5	70°	3,390	135	140	140	215	4.0
F	1,330	5	70°	3,425	173	141	173	270	4.9
G	1,150	5	70°	3,940	150	162	162	250	4.6
H	1,020	4	75°	3,980	166	211	211	325	3.2
Totals......................				27,481	1,131	...	1,223	1,885	

A preliminary inlet air temperature is figured from the above totals (item *H* included as 70 F).

$$t_d = \frac{27{,}481}{1.08 \times 1{,}131} = 22.5 \text{ F}$$

$$t_i = t_r + t_d = 92.5 \text{ F}$$

Using 17.5 F as t_d for item *H* and 22.5 F as t_d for all other items, the cfm for each space is calculated by the same formula and recorded in column 7.

Since four of the eight items in column 7 are reasonably close for the temperature chosen, and two are slightly high, the cfm of items *B* and *F* will be increased to give the final values shown in columns 8, 9, and 10.

The heater for such a system will be as follows: Preheater 0 to 60 F temperature range.

$$\text{Btu} = 1.08 \times 1{,}223 \text{ cfm} \times 60 \text{ F} = 79{,}000 \text{ Btu}$$

Pounds of 35 lb steam required $= \frac{79{,}000}{924} = 86$ lb condensate per hr. Reheater 60 to 92.5 F temperature range.

$$\text{Btu} = \frac{60 \times 1{,}223 \text{ cfm} \times 32.5 \text{ F}}{55.6} = 43{,}000 \text{ Btu}$$

Pounds of 35 lb steam required $= \frac{43{,}000}{924} = 47$ lb condensate per hr.

Mechanical supply and exhaust is the generally accepted standard arrangement for living spaces. In such an arrangement the two should be balanced for any enclosed section of the ship. The smaller the group of spaces that can be balanced among themselves the better, for this cuts down the movement of large air volumes and the consequent danger of unpleasant drafts. It also ensures better distribution of the exhaust, since the spaces affected will be more equally distant from the exhaust and, therefore, more equally benefited by it. The unit may be a deck, part of a deck, or a vertical section through several decks.

It can be seen from Table 1 that the spaces having excess supply predominate in both number and size, the excess air flowing into the passages. Sometimes this extra supply air can be balanced by increasing the volume of exhaust from adjacent toilets, pantries, etc., above the specified air change. It is generally necessary, however, to provide a direct exhaust from the passage equal in volume to the excess supply. This may be by means of a separate system, usually a propeller fan, discharging directly to the weather or into the machinery casing, or by means of an outlet from an existing system. If the supply system is run at reduced capacity for winter operation, the exhaust should likewise be reduced to maintain the balance. This can be done either by reducing the speed of the exhaust fans (this should not be done with a propeller fan) or by shutting down some of the passage exhaust fans. If the fan is run at reduced speed, it is advisable to close some of the passage and other less important outlets so as to maintain the higher air change in toilets or heat-producing spaces. A slight excess of supply is desirable to prevent infiltration.

Heating on shipboard is generally accomplished by one of three methods: direct radiation, hot blast, or unit heater. Steam is by far the most widely used heating medium for shipwork, being generally available and easily handled. Hot-water heating is capable of closer control where small quantities of heat are involved, but it requires a special hot-water generator, as well as larger, and therefore heavier, heaters and piping. Electricity for heating is more easily handled and controlled than either steam or hot water but is too valuable a commodity on shipboard for general heating use. Its principal application at present is for heating isolated spaces which would otherwise require a special steam line.

Steam for heating is taken through a reducing valve from an auxiliary steam line or, on motor vessels, from the waste-heat boiler. Gage pressures ranging from 5 to 50 lb at the heater are used, 30 or 35 lb being the most widely accepted. A two-pipe system is used, with returns run to an atmospheric drain tank from which it is pumped to the deaerating heater and back into the system. All parts of the system must be pitched to drain under normal conditions of trim, drainage forward or aft being better than transverse.

The purpose of providing heat is to maintain a predetermined temperature within a space for the personal comfort of the occupants or the protection of equipment. The heat supplied must be sufficient to replace that lost by transmission through the boundaries of the spaces and to raise the temperature of incoming air to that being maintained in the space. Wherever there is a temperature difference between the air on the two sides of a partition, there is a flow of heat from the warmer to the colder side, the rate of flow depending on the conductivity of the material or materials making up the partition. For bare steel this figure is very high. It is, therefore, necessary to insulate exposed steel boundaries if the amount of heat to be supplied is to be kept

Cooling calculations to determine the quantity of outside air required for cooling to a 10 deg temperature rise can be illustrated by further reference to room *A* in Fig. 2. Assume the heat load in the room to be as follows:

Electrical equipment 1 kw input × 3413	3413
4 lights at 50 watts each = ⅕ kw × 3413	683
2 men at 200 Btu each	400
Solar heat gain (calculated above)	965
Total heat load in Btu	5461

$$\text{Required cfm} = \frac{5461 \text{ Btu}}{1.08 \times 10 \text{ F}} = 506 \text{ cfm}$$

This amount of air would be provided as mechanical exhaust, the excess over mechanical supply (506 − 260 or 246 cfm) being taken in through door louvers or bulkhead opening. If properly located over the main heat source, this amount of exhaust will undoubtedly limit the temperature rise to less than the calculated 10 deg.

Special Features. *Galley pantry and bakery* ventilation is primarily a cooling problem, although, if tempered air is not supplied, some radiation should be provided for an exposed space to keep equipment from freezing when not in use. Air should be supplied from directional terminals located at the working stations preferably about midway between heat-producing areas. Exhaust should be taken from as high as possible over all heat-producing equipment, a wire-mesh covered opening on either the top or bottom of the duct being customary. An intake velocity of about 750 fpm based on gross area of opening should be used.

Canopies should be installed over all equipment emitting large amounts of heat, steam, smoke, or odors. These include ranges, friers, steam kettles, baking or roasting ovens, griddles, dishwashers, etc. A simple and effective canopy consists of a braced sheet-metal curtain plate extending from the deckhead to a point about 6 ft 6 in. from the deck. Around steam kettles, a small gutter to catch condensation should be provided on the inside of the curtain plate. Piping, wireways, etc., should be kept out of the canopy. If this is not possible, a full hood should be built under all obstructions that might collect dirt. From 50 to 75 percent of the galley exhaust should be taken from the canopies, the balance being from over sinks, garbage grinders, etc., or from points necessary for good distribution. Grease filters and a fusible-link fire damper should be installed in the canopies over ranges, friers, and large griddles. Exhaust from a galley should average about 60 cfm per kw input for electrical equipment and about 12 cfm per gal steam-kettle capacity, and figuring approximately 66 percent usage factor for the galley as a whole.

Laundries and tailor shops should be treated in a manner similar to galleys, but canopies are seldom required.

Wheelhouse radiation calculations should include about 25 percent excess to take care of possible open windows.

The **construction** of living-space ventilation systems differs from other systems in a few features owing to the special conditions encountered.

The **magnetic circle,** *i.e.*, all space within 6 ft of a magnetic compass, should be kept clear of ductwork. Any equipment within this area should be of nonferrous material.

Cuts for ducts passing through joiner bulkheads and ceilings should be slightly oversized and finished with a collar. A flanged connection is unnecessary.

within reasonable limits and if the space is to be heated uniformly. Heat supply and insulation of living spaces are parts of the same problem, and the two should be worked out together to determine the most practical combination. The amount of outside air introduced during the heating season should be kept to the minimum ventilation requirements, to keep down the steam load. Since steam is a valuable commodity on shipboard, any heating system should be designed to keep its consumption to a minimum.

Direct radiation denotes heating by radiator, pipe coil, convector, or panel. Cast-iron radiators and bare pipe coils have largely been supplanted by the more efficient convector, although pipes hung on an exposed bulkhead are still used in places such as dry stores when a little heat is needed to prevent freezing. For a given heating area, convectors are considerably smaller and lighter than cast-iron radiators and, therefore, are preferred for most applications. They are best installed on a bulkhead, close to the heating load, with the bottom 8 to 12 in. above the deck. A cutoff valve should be provided in the supply, with a thermostatic steam trap and cutoff valve in the drain line. Most direct radiation systems are manually controlled.

Radiant or panel heating is mentioned here only as a principle that may play a part in future ship heating. The human body experiences the feeling of cold owing to loss of heat from it by radiation. The usual heating methods raise the temperature of the surrounding air and thus cut down to a comfortable point the heat loss from the body. In radiant heating, however, the surrounding surfaces are heated so that heat radiation from them cuts down the radiation heat loss from the body and thus creates a comfortable condition. This is accomplished by heating coils set inside the structure of the room boundaries. Since few living spaces on a ship have more than one or two exposed boundaries and since the ceiled-in air space between structural frames could be easily adapted for heating panels, radiant heating would seem well suited for shipwork. For any given application, however, its feasibility should be determined by weighing the possible saving in heat against the extra weight of piping that panel heating would entail.

Hot-blast heating is a combination of heating and mechanical ventilation. Under this system the heat required to maintain a specified temperature is supplied in the ventilation air by heaters installed in the ductwork. Its principal advantages are that the introduction of heated air permits a higher winter ventilation air change than is otherwise possible; it requires far less steam piping than does direct radiation; it takes up no cabin space; and it can be controlled automatically.

With hot-blast heating, the fan speed should be reduced during the winter heating period to save heat required to raise the temperature of the full amount of outside air. Between 60 and 75 percent of the total summer capacity gives a satisfactory air change and a reasonable entering air temperature for most applications. Main ducts carrying heated air should be insulated and kept as short as possible, to maintain the air temperature to the most remote outlet. Heated air ducts within the space served need not be insulated. Although some recirculation can be used with this system, it has not been widely used, owing to the extra ductwork it would require. Steam lines for blast heaters should have a stop valve in the supply, with a cutoff valve and float-operated thermostatic or inverted bucket trap on the drain.

There are several hot-blast heating arrangements in common use. The simplest is the central system, in which all the air supplied by a given fan is

Owing to limited deck heights and dropped ceilings, ducts in living spaces are generally of flat rectangular cross section. The ratio of width to depth should be kept below 2:1 where possible. If it is necessary to exceed 4:1, a dividing diaphragm should be inserted to give stiffness as well as to aid the air flow.

Insulation is mentioned here only in its relation to ventilation. The insulation of outside boundaries against heat gain and loss should be balanced against the heating and ventilation to determine the optimum amount of each. Sketches of the common types of outside boundaries—bulkhead, shell, or deck—are shown with both welded and riveted stiffening indicated.

Usual insulating materials for boundaries of living spaces are rock-wool or glass-wool insulation with marine-board sheathing. Sheet-metal sheathing instead of marine board is used in washrooms, storerooms, etc. Cork paint is frequently used in washrooms; but this is primarily to stop sweating, and its insulating value can be neglected.

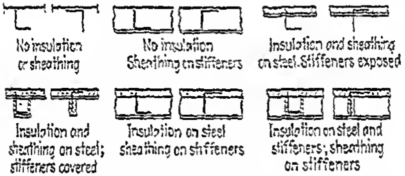


FIG. 3.—Typical outside boundaries.

Ducts carrying cooling air should be insulated wherever they pass through the machinery casing or other hot spaces. Ducts, heaters, and fans carrying heated air should be insulated except within the space they serve. An exception is a single fan and heater system in a fan room, used as an intake plenum, in which case any heat dissipated is drawn back into the system. Insulation should be blocks of mineral wool, or its equal, generally 1 in. thick, securely fastened by adhesive and metal bands, spaced not over 18 in. apart. Where ducts are exposed, a canvas cover should be sewed on over lagging. Insulation over access holes should be portable.

Noise problems are frequently very complex and so belong to the sound engineer and not to the ventilation designer. There are, however, certain rules of good design that can reduce noise in a ventilation system.

There is no general rule as to noise limits, since each location has its own desirable sound level. Although the general sound level on shipboard is comparatively high, allowing more liberty in ventilation noise, ship construction materials are better sound conductors than most building materials. As in shore work, however, noise is carried in the air stream, through the ship structure, or can be caused by the air in the terminal. Also, as in shore work, there are two remedies for excessive ventilation noises: elimination of the source and cutting down the transmission of existing noise.

The fan is the principal source of ventilation noise. The first step in preventing noise should therefore be the selection of the proper fan for a particular application. A rating of 85 decibels is acceptable for a machinery

heated to the predetermined temperature necessary to maintain the specified temperature in the space or spaces served. This may be accomplished by a single heater covering the total range or by a preheater to temper the cold outside air, and a reheater to raise the air to the required temperature. The two-heater arrangement is capable of closer control. Either room thermostats or compensated duct thermostats may be used for control. If only one heater is used, the control should be such that, when the desired temperature is reached in the space, the heater can deliver tempered air only. Such a system has the advantage of simplicity but can be used only for a group of spaces with comparable heat loss and specified air change and with outlets within a fairly limited area.

The most widely used hot-blast heating arrangement is the zone system which differs from the central system in that it consists of one preheater and several reheaters, each serving a heating zone. The thermostat-controlled preheater on the intake side of the fan delivers tempered air at a constant temperature, generally 50 to 60 F. The latter is desirable if there are spaces to receive tempered air from the system. The duct supplying such spaces is taken off directly after the fan. Other main branches each contain a reheater controlled by either room thermostat or compensated duct thermostat. Each reheater is sized to provide air at the calculated temperature required to maintain the specified condition in the spaces served. The reheater should be located so that the distance to the most remote outlet is not too great. There are frequently spaces in a given section of the ship, inside rooms for example, which require the same air change as near-by spaces but which have far less heating load. These spaces should receive all tempered air; or have two outlets, one supplying heated air and one tempered air; or should have the air change provided by exhaust with the supply of 70-deg air drawn in from the passage. The choice of method will depend on the conditions in each case. The zone arrangement gives greater flexibility than the central system, since it can serve groups of spaces requiring different entering air temperatures and also deliver tempered air. A typical zone layout is shown in Fig. 16.

The individual system differs from the zone system only in that there are separate reheaters for each individual space, controlled manually or by a room thermostat. Since the heat required from each reheater under this arrangement is small, hot-water or electric heat has advantages over steam. Individual control is the great asset of this system, but its use is not often feasible owing to difficult installation and expense of the piping and numerous heaters.

Another arrangement giving individual control is the dual-duct system by which two ducts, one carrying heated air and one tempered air, are run together into each space, where they meet in a mixing chamber at the terminal. By adjusting the terminal, the room occupant can regulate the temperature of the entering air mixture. The punkah louver is frequently used with this system. The disadvantage of the dual-duct system is the difficulty and expense of installing the two ducts throughout a ship.

Unit heaters are used in workshops and miscellaneous spaces which have no installed mechanical ventilation but which need both heat and air change at times when men are working therein. Being easily located and installed, unit heaters are well suited to take care of these special cases. They are generally steam-heated but can be electric- or hot-water-heated. Steam lines should have a stop valve in the supply and a cutoff valve and float thermostatic or inverted bucket trap preceded by a dirt trap or drain.

space fan but is far too high for a hospital system. Axial fans are in general noisier than centrifugal fans and therefore should not be used where noise is objectionable. Tip speed is a rough index of fan noise and should be kept low if possible. A limit of 8,000 to 10,000 fpm is desirable and should be exceeded only after consideration of the noise problem. Also, care should be taken to see that a large fan is running in its range of greatest efficiency since axial fans and forward-curved-blade centrifugal fans in particular become noisier if run otherwise. In cases such as engine rooms where shutting off several terminals will greatly affect the fan performance, instructions should be given to reduce the fan speed when more than about one-third of the terminals are closed. Also, the air entering a fan should be evenly supplied over the entire inlet, since uneven supply creates an unbalance that increases noise. All fans should be rigidly mounted and the surrounding structure stiffened if necessary so that they cannot vibrate. Canvas connections between centrifugal blowers and ductwork, commonly used in shore work, are not ratproof and cannot be used. Since ratproofing such a connection is not satisfactory, the preferred connection, with no metal-to-metal contact, is by means of a 1-in. rubber gasket with fiber ferrules and washers as shown in Fig. 4.

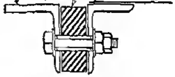


FIG. 4.—Fan connection.

Air noise in terminals can be prevented only by keeping the outlet velocity within proper limits for the type of outlet in question. In ducts, air noise can be prevented by using proper velocities and being sure that the inside surfaces of the ducts are smooth and free from projections and bad fittings.

In regard to noise carried in the air stream, it is popular fallacy that sound does not travel against the air flow. Since sound travels in air at the rate of around 65,000 fpm, the effect of the air velocity—under 3,000 fpm—is negligible. The noise level of a system should be considered not only for the space ventilated but also in the vicinity of the intake, which may be near a navigating station or sleeping space. If the source of the excessive noise cannot be eliminated, the installation of a sound-absorbing material is necessary. If the problem is in an existing installation, the system should be tested with a decibel meter with analyzer, and a sound engineer should be consulted as to the proper treatment, since sounds of different frequencies should be treated in different ways. In designing sound treatment for a new system, it should be remembered that the penetrating whine which comes from a high-speed fan is mostly a high-frequency sound which is best removed by closely spaced sound-insulation units. A general rule for the amount of insulation required is 10 to 15 diameters of duct or its equivalent. Since lining this length of duct is seldom feasible in shipwork, a sound trap is usually more practical and, for equal area, more effective, particularly to minimize high-frequency sound.

The sound-absorbing material should be noninflammable, unaffected by dampness and, if of the mineral-wool type, about 1 in. thick and held in place by wire screen or perforated sheet metal. A practical and effective sound trap can be made by using panels with material exposed on both sides, spaced on about 6 in. centers, extending the width of the trunk, and running parallel to the air stream. If space and the static pressure of the fan permit,

Air conditioning is the process of maintaining, within fixed limits, the temperature, humidity, movement, and quality of air in a space. In its purpose, it differs from straight mechanical ventilation only in the degree of treatment and control, but practically the cooling of air for temperature and humidity control introduces so many new factors that air conditioning must be regarded as a separate subject. In the following, an attempt is made, not to describe the subject thoroughly, but merely to bring out a few points in connection with its use on shipboard.

Complete air conditioning requires a large installation, the cost of which is not justified in most cases on shipboard. There are few large passenger vessels, however, which are not equipped with some form of cooling and humidity control for the public spaces for passengers and crew. How many additional spaces should be conditioned and the extent of the treatment depend on the service for which the vessel is intended. It seems likely, *however, that passenger ships in many services will have at least some* cooling in all staterooms, either for comfort or for its advertising value.

The index of air conditioning is the "effective temperature" or so-called comfort scale. This is an experimentally established scale of comfort which takes into consideration dry- and wet-bulb temperature and air movement. On this basis, different combinations of the three factors can produce the same effective temperature and the same degree of comfort. What exact values should be maintained depends on the outside conditions, since extreme contrasts are unpleasant, even though the inside effective temperature may be well within the comfort range. In other words, no one temperature is the optimum under all conditions. For example, a ship operating in the Caribbean with 95 deg temperature has an air-conditioning plant to give 80 F and 70 to 75 deg effective temperature; a ship in Red Sea trade, where 100 F and 80 percent relative humidity are met, has a plant to give 90 F and about 80 deg effective temperature; one in the Pacific with 90 deg and 75 percent relative humidity has a plant designed for 85 F and 70 to 75 deg effective temperature. Thus, although the effective temperatures generally fall between 70 and 80 deg, the plant must be chosen on the basis of the outside conditions to be encountered.

The required refrigeration can be estimated as follows: From the desired inside temperature, the expected outside temperature, and number of occupants, the heat gain in the space can be calculated (see p. 1523). The amount of outside air to be brought in and the amount of dehumidification having been decided upon (this being affected by the amount of recirculated air), the temperature and total heat of air leaving the cooling surface can be determined (see p. 375). The heat gain in the space, plus the dehumidification, plus the sensible cooling of the outside air to space temperature are the refrigeration required.

The question of purity is not so vital in average shipwork as it is in land work, owing to the relatively high purity of the air encountered by ships in most services. As a consequence and because of the importance of saving space and weight, filters may be used with installations that would warrant an air washer ashore. The proper amounts of oxygen and CO_2 in the space are assured by the constant introduction of about 25 percent of outside air.

Heating is handled by a hot-blast system, similar to straight mechanical supply ventilation; plus provision for raising the humidity as necessary by injection of steam or a water spray into the air stream.

A duct layout for air conditioning is very similar to that used with mechanical supply ventilation and heating, plus a return duct system to pick up and

the trap can be made more effective by zigzagging the panels, using 9- to 12-in. straight sections and sufficient angle to close the direct line of air flow through the trap. Panels should be removable for repairs and should be evenly spaced with their leading and trailing edges formed so as to disturb the air stream as little as possible. Another type of sound trap applicable to weather openings is obtained by lining the intake chamber or mushroom and vanes between the fan and the weather. This arrangement is simpler than the panel type but is less effective, owing to the smaller area of absorbing material.

The **fan room** is the heart of a ventilation system and, as such, its design is important to the operation of the entire unit. Although fan rooms may be of any size and contain any number of fans and other pieces of ventilating equipment, there are general features that should apply to all. They should be located close to the spaces ventilated and thus keep ducts short. The intake should be in as protected and weatherproof a location as possible, to

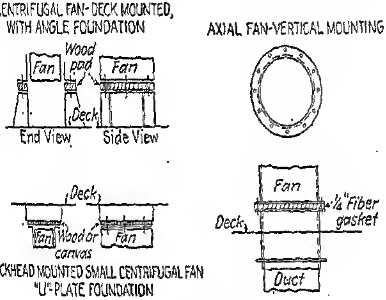


FIG. 5.—Fan foundations.

minimize chance of forced shutdown. In a fan room for living-space supply either the room intake or the fan inlet must have filters or insect screen. A single fan may be direct-connected to the weather opening. In cases of more than one fan of the same type—supply units particularly—the fan room itself is generally used as a plenum chamber. Filters should be connected to the fan inlet but, if space requires that they be connected to the room intake, there should be at least 18 in. between the weather opening and the filters to cut down the chance of water freezing on the filters. Washing and recharging equipment should be located close to filters and provided with permanent drain and water connections. If the room is used as a plenum, the gastight door to the compartment should be kept closed at all times. Preheaters should be mounted on the fan suction, with sufficient space between filters and heaters and between heater and fan intake, to assure good air distribution. Heaters should be firmly supported from either deck or deckhead with ample space allowed for piping connections. Fans should be rigidly mounted on foundations or blocks set on deck, with shafts either vertical or fore and

recirculate about 75 percent of the supply air. The remaining 25 percent must be exhausted by regular means. Ductwork should be insulated. Distribution is a feature of the utmost importance in any air-conditioning system but, with the low ceilings encountered on shipboard, it requires even greater attention. Outlet and intake velocities must be kept low to prevent

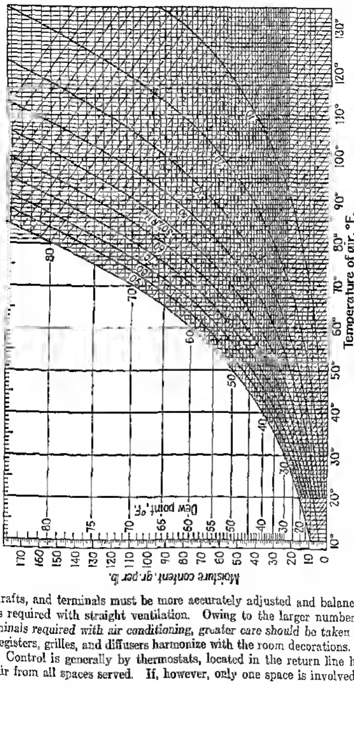


FIG. 1.—Psychrometric chart for determining the dew point and moisture content of air from wet- and dry-bulb readings or from temperature and relative humidity readings (see pp. 376–378).

drafts, and terminals must be more accurately adjusted and balanced than is required with straight ventilation. Owing to the larger number of terminals required with air conditioning, greater care should be taken to have registers, grilles, and diffusers harmonize with the room decorations.

Control is generally by thermostats, located in the return line handling air from all spaces served. If, however, only one space is involved or if a

aft, to reduce wear on bearings. Figure 5 shows some general types of fan foundations. Space must be allowed to permit easy servicing of fans and for the unshipping of the motor if this should be necessary. If it is desirable to isolate the fan for noise reduction, fiber ferrules and washers can be used in addition to the pads shown to prevent metal-to-metal contact.

Axial fans with duct connections on both sides must have a spacer ring above the fan, to permit making up the flange connection to the fan. An axial fan drawing or discharging without ductwork should have a sheet-metal bell to ease the air flow.

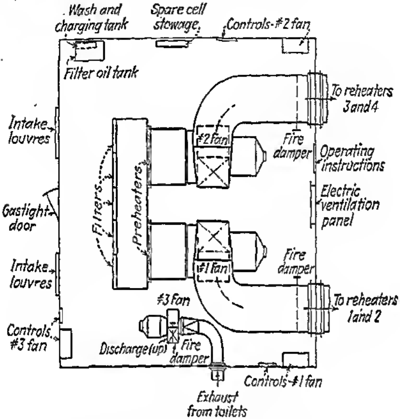


FIG. 6.—Typical fan-room layout.

Electrical controls should be located on a bulkhead close to the fans and out of the intake air stream. Compensating thermostatic bulbs must be located directly in the intake air stream so that they will register true outside air temperature. Reheaters should be located in the fan room if doing so does not make the duct run beyond the heater too long. Reheaters should always be pitched 1 in. for drainage. Fire dampers should be located in fan rooms where they are easily controlled from the one place.

A layout for a fan room containing three living-space centrifugal fans—two supply and one exhaust—is shown in Fig. 6.

MACHINERY SPACES

General. Machinery spaces include those spaces housing boilers, main and auxiliary engines and equipment, emergency generators, steering gear,

particular space requires especially close control, a room thermostat should be used. Automatic humidity controls are available, but more often manual control based on a humidity indicator is provided.

Air-conditioning equipment can be of the regular commercial type, with necessary marine features incorporated to adapt it for marine use. Freon is the most generally accepted refrigerant. Central systems, in which all the equipment is located in a machine room, are preferred. There are also, however, self-contained units in which the fan, compressor, condenser, etc., are in a self-contained cabinet and unit coolers with independent fan and coils. Certain installations, frequently those on conversions, necessitate the use of self-contained units, owing to simplicity of installation. Particularly for vessels that encounter extreme variation in cooling load, it is good practice to provide two compressor units which together have peak-load capacity, and either one of which can handle the average load, with the other as a stand-by. In all such considerations it should be remembered that air conditioning is a luxury and not a vital service, as are ventilation and heating. Economy of installation and operation is therefore of great importance.

Calculations. Heat gain or loss from any space equals the algebraic sum of the heat transferred through its various boundaries.

The heat transmitted through a boundary may be determined as follows:

$$H = A \times U \times t_d$$

where H = total heat transmitted, Btu per hr
A = area of the boundary, sq ft
U = over-all coefficient of heat transfer, Btu/(hr)/(sq ft)/(deg diff)
t_d = difference in temperature between the two sides of the boundary, $= t_1 - t_2$, in deg F

The value of U is determined by the following formula:

$$U = \frac{1}{\dfrac{1}{f_o} + \dfrac{X_1}{K_1} + \dfrac{1}{a} + \dfrac{X_2}{K_2} + \cdots + \dfrac{1}{f_i}}$$

where f_o = outside surface conductance, Btu/(hr)/(sq ft)/(deg F)
f_i = inside surface conductance, Btu/(hr)/(sq ft)/(deg F)
K = conductivity of the material, Btu/(hr)/(sq ft)/(deg F)/(in. thickness)
X = thickness of the material, in.
a = conductance of the air space between the materials, Btu/(hr)/(sq ft)/(deg F)

All of the foregoing are definite values, but they vary somewhat with the temperature at which the heat transfer takes place. Also, the surface conductances vary greatly with the velocity of the air moving over it and whether or not the surface is wetted. Table 2 lists values for typical ship boundaries as generally used for average conditions. These do not necessarily agree with the simple formula, owing to the fin effect of stiffeners and other contingent factors.

cargo-oil pumps, and special electrical equipment such as electrical storage batteries and resistors. Where the equipment is in continual use and is constantly radiating at least sufficient heat to ensure comfortable working conditions, ventilation alone is required; the chief purpose of the ventilating system in these spaces is to supply sufficient fresh air to limit the temperature to a maximum consistent with comfort. Main and auxiliary boiler and engine rooms generally are in this category. Where operation of equipment is intermittent or where the equipment is not giving off sufficient heat to ensure proper conditions, both heating and ventilation may be required. Compartments housing emergency generators and steering gear are typical spaces that should have both ventilation and heating. For spaces housing special electrical equipment such as large storage batteries, which give off objectionable vapors and which must be kept from extremes of heat and cold, both heating and ventilation should be provided. Electrical equipment is adversely affected by very moist air, particularly salt air; and electrical controls and resistors for intermittent-service auxiliaries, such as deck winches and capstans, should be protected from moisture and condensation during inoperative periods by proper heating equipment. When such auxiliaries are in use, the heat radiated therefrom must be removed by ventilation. Emergency generators are frequently located in inside rooms, without direct doors or openings to the weather. Unless proper provision is made for the supply and exhaust of ventilating air to such spaces, satisfactory operation is impossible, as the heat given off by their radiators is very considerable and the cooling medium will soon reach boiling temperature.

Cargo-oil pump rooms on tankers require adequate ventilation to remove explosive, noxious, and toxic vapors emitted by the oil. On tankers carrying light petroleum products such as gasoline or naphtha, the vapor released in cargo pump rooms, particularly in warm weather, is considerable. The vapor, being heavier than air, settles in the bilges and should be removed to reduce the fire hazard and eliminate the toxic effect on the operating crew.

Refrigerating-machinery rooms require, generally, the same consideration as other machinery spaces. Where ammonia is the refrigerant, ventilation should be adequate and positive, as ammonia is toxic and corrosive to copper and cuprous alloys. It has been largely superseded for marine use by the halide refrigerants, of which dichloro-difluoromethane, generally known as Freon, is the most common. Freon is nontoxic and noninflammable, and hence the ventilation of Freon refrigerating rooms is not so critical.

Conditions. Main machinery spaces do not generally require any heating, as the heat radiated from the machinery is sufficient to ensure comfort, even in cold weather. Ventilation, however, is most important, particularly for operation in warm water and weather. All the heat losses in the machinery spaces must be carried away by the ventilating air and, to ensure reasonable comfort for the operators, the temperature rise of the ventilating air should not exceed 30 deg F. A maximum rise of 25 deg F is preferable. With outside air at a temperature of 90 F and an air temperature rise of 25 deg, the temperature of air leaving the space will be 115 deg, which is a maximum desirable for comfort of the operators.

The system should permit regulation of the quantity of ventilating air, otherwise the spaces will be unduly cold in winter weather. If natural ventilation is used, control dampers in the supply ducts should be installed. For mechanical ventilation, speed variation of the fans of at least 25 percent

Table 2. Over-all Heat-transfer Coefficients

(For various types of boundary construction)

Construction	U
Painted steel:	
Inside bulkhead ($K = 300$)	1.00
Outside shell (12-knot wind)	1.10
Outside steel—air space—¼-in. marine board	0.45
Steel:	
2-in. rock wool—sheet-metal facing	0.20
2-in. rock wool—³⁄₁₆-in. marine board	0.19
2-in. rock wool—air space—½-in. marine board	0.15
2-in. cork, painted	0.25
2-in. glass wool—glass cloth	0.19
Steel deck:	
Painted—¾-in. magnesite covering	0.55
12-in. cork (refrigerated spaces)	0.04
Wood deck—steel, painted	0.31
¾-in. marine board bulkhead	0.48
Airport light—14 in. diam (closed)	0.50

In Table 2, the following values of conductivity and conductances were used:

Steel, conductivity	$K = 300$
Marine board, conductivity	$K = 0.86$
Rock wool, conductivity	$K = 0.27$
Cork, conductivity	$K = 0.30$
Glass wool, conductivity	$K = 0.25$
Outside surfaces, film conductance	$f_o = 6.00$
Inside surface, film conductance	$f_i = 1.65$
Air space, over ¾ in. thickness, conductance	$a = 1.10$

Heat-transfer calculations should be based on the extreme outside temperatures, in order to determine the peak heating or cooling load. These vary with the service for which the vessel is intended. The following may, however, be used as designed extreme temperatures, in degrees Fahrenheit, for general cases:

Element	Temperature, deg F	
	Winter	Summer
Outside air	0	85
Sea water	35	85
Hold, unheated	20	85
Deck:		
Wood	0	120
Steel	0	140
Vertical bulkhead, steel	0	120

In determining cooling loads, the heat from human occupants should be based on 200 Btu per man per hour, based on total occupancy; the heat from electric lighting or resistance heat = 3413 Btu per kwhr. In public spaces, the electric-lighting load is about 2 watts per sq ft deck area. These should not be considered as a source of heat when determining the heating load.

is advisable, and the outlet terminals should also have control dampers for further reduction of air supply in cold weather.

Steering engine rooms do not generally have an operator in constant attendance and do not, therefore, require so close temperature regulation. As the heat liberated is generally small, natural ventilation, either by supply or exhaust, is sufficient. However, some degree of heating is advisable, particularly if the steering gear is electric, and may be provided by electric or steam heaters, of either convector, unit heater, or radiator type.

Emergency generator rooms should have sufficient ventilation to ensure adequate cooling air for the air-cooled radiator and for combustion.

Methods. Ventilating systems may be either supply, exhaust, or a combination of supply and exhaust; and the air may be handled by either natural or mechanical means. **Natural ventilation,** by the use of supply and exhaust cowls and their attendant ducts, has been much used in the past and is fairly satisfactory if well laid out, but it requires large ducts to ensure low friction loss. Natural ventilation depends on the velocity of the air relative to the ship for its efficiency, and the supply cowls must be trimmed in the direction of the relative air velocity. Whenever the ship changes course or the wind direction changes, the position of the cowls should be changed accordingly. If there is a following wind, the ventilating effect will be very small and the space excessively hot. The prevalent trend to machinery casings of small area also restricts the possible sectional areas of vent ducts for main machinery spaces. **Mechanical ventilation,** with either supply or exhaust fans or a combination of both, ensures a positive and controllable supply of air for all operating conditions and also permits use of ducts of moderate dimensions. Owing to the larger permissible friction losses, the ducts may be more extensive and a better distribution of air obtained, resulting in a more equable temperature of the spaces.

Mechanical supply ventilation comprises one or more supply fans, taking outside air from fixed intake heads and discharging it through a duct system to the various outlets, properly located to scavenge any hot zones. The heated air rises up the machinery casing and is discharged to the outside through either a skylight or other exhaust opening.

Mechanical exhaust ventilation comprises one or more exhaust fans, collecting heated or vitiated air from various locations through a duct system and discharging it through a suitable exhaust opening to the outside air. The air is naturally supplied to the lower part of the space either through large ducts or down the machinery casing itself. This type of system is not generally used by itself.

Combined mechanical system is a combination of mechanical supply and mechanical exhaust. Exhaust inlets are properly located to withdraw vitiated air from "hot spots" or zones where considerable heat is liberated.

Of the above systems, the mechanical supply system is probably the most efficient and economical for the main machinery spaces. The ability to provide effective distribution of cool air and the utilization of the machinery casing as a natural draft "flue" for the heated air are desirable factors. In steamships, where a large supply of air is required for combustion of fuel in the boilers, the forced-draft fans frequently take their suction from the upper *machinery space, where the air is warm, and draw it down through an air* casing surrounding the stack, thus further assisting in the ventilation.

Frequently a large outer stack is fitted around the live stack, for the sake of ship appearance. If the exhaust ventilating air is permitted to rise to

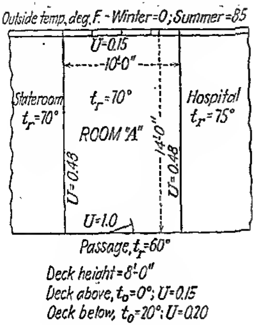


FIG. 2.—Typical space.

Figure 2 represents a typical space, for which the winter heat loss would be calculated as follows:

Boundary	Dimensions	Area, A	Heat-transfer coef U	Temp difference, t_d	Heat transmitted, $H = A \times U \times t_d$
Outboard side....	10' × 8'	80	0.15	(70 − 0) = 70	840
Forward side.....	14' × 8'	112	0.48	(70 − 75) = −5	(−) 369
Inboard side.....	10' × 8'	80	1.0	(70 − 60) = 10	800
Aft side.........	14' × 8'	112	0.48	(70 − 70) = 0	0
Deck over........	14' × 10'	140	0.15	(70 − 0) = 70	1470
Deck under.......	14' × 10'	140	0.20	(70 − 20) = 50	1400
Total heat loss, Btu per hr....					4141

The summer heat gain, based on 10 deg temperature rise (*i.e.*, 95 F in all spaces) and bare steel deck overhead would be calculated as follows:

Boundary	Dimensions	Area, A	Heat-transfer coef U	Temp difference t_d	Heat transmitted $H = A \times U \times t_d$
Outboard side....	10' × 8'	80	0.15	(120 − 95) = 25	300
Other 3 sides....	....	....	....	(95 − 95) = 0	0
Deck over........	14' × 10'	140	0.15	(140 − 95) = 45	915
Deck under.......	14' × 10'	140	0.20	(85 − 95) = −10	(−) 250
Total heat gain, Btu per hr....					965

Cubic volume (always based on empty volume) = 1,120 cu ft.

the top of the outer stack and then discharged through openings around its after perimeter, additional natural-draft "flue effect" will assist the ventilation.

Cargo-oil pump rooms on bulk-oil tankers require special consideration. The explosive and toxic vapors settling in the bilges and lower zones should be effectively removed by an exhaust duct system, extending from the space below the floor plates to outlets located well above the deck. A small steam jet eductor or an exhaust fan, in the duct, induces the upward flow of vapor. Care should be exercised to prevent any spark or flame in this duct. If a motor-driven fan is used, the motor should be located outside of the duct.

Heating of main machinery spaces is not generally required, as the heat radiated is generally sufficient to ensure a livable temperature. However, spaces housing the steering gear, emergency generator, storage batteries, etc., should be provided with some means for heating to protect the equipment from moisture and excessive cold. Where steam lines of suitable pressure are located near by, direct heating, either by steam radiators or by "unit heaters" (fin-tube radiator with a small electric fan blowing across the heating element) is economical. If there is no steam supply near the space, electric resistance heaters are suitable; however, it should be borne in mind that electrical energy is an expensive source of heat, and resistance heaters cannot be economically used to supply a large amount of heat. Storage-battery rooms should have only sufficient heating to prevent freezing; high temperatures will cause excessive evaporation of water from the electrolyte. Electrical resistor rooms or spaces should be heated sufficiently to prevent condensation of moisture from the sea air on the resistors and contactors. Frequently, contactor panels for deck machinery are fitted with a strip or coil of resistance wire at the base, which gives off sufficient warm air to rise across the contactors and keep them dry.

Calculations. For main machinery spaces, the problem is to determine the amount of heat given off by the machinery and equipment, so that adequate air may be supplied to remove this heat with a moderate temperature rise. The heat given off by a well-designed steam plant amounts to approximately $1\frac{1}{2}$ to $2\frac{1}{2}$ percent of the heat content of the fuel burned. To meet this figure, the boilers, steam piping, and auxiliary units must be well insulated, so that the surface temperature of the insulation does not exceed approximately 135 F. This surface temperature likewise will protect the operating personnel from burns.

The heat given up to the air can be determined with good accuracy by calculation. For insulated surfaces such as steam pipes, boiler casings, etc., the heat radiated from the surface may be determined as follows:

$$H_0 = K \times S \times (t_s - t_a)$$

where H_0 = heat radiated, Btu per hr
K = coefficient depending on the type of insulation covering, = 0.35 – 0.50
S = total surface over the insulation, sq ft
t_s = surface temperature of the insulation; should not exceed 135 F
t_a = ambient temperature of surrounding air; about 100 to 110 F

If the air velocity sweeping the surface is high, the heat radiated will be increased considerably. If the velocity of the air is 10 fps, the heat radiated will be approximately three times the value given by the formula above.

Heating requirements by radiation are figured as 240 Btu per sq ft of rating based on steam at 215 F. For other steam temperatures, the value of 240 Btu must be divided by the following factors:

For radiators:

$$C = \left(\frac{215 - 70}{T_s - T_r}\right)^{1.3}$$

For convectors:

$$C = \left(\frac{215 - 65}{T_s - T_i}\right)^{1.5}$$

where T_s = steam temperature, deg F
T_r = room temperature, deg F
T_i = inlet air temperature, deg F (5 deg lower than room temperature)

Example. *By Means of Steam.* Assuming a steam pressure = 50 psi (temperature 298 F, from steam tables), the convector requirements for room *A* (Fig. 2) are as follows:

$$240 \div \left(\frac{215 - 65}{298 - 65}\right)^{1.5} = \frac{240}{0.517} = 464 \text{ Btu per sq ft}$$

$$\frac{4141 \text{ Btu}}{464 \text{ Btu per sq ft}} = 9 \text{ sq ft convector surface to overcome heat loss and to maintain 70 F with 0 deg outside}$$

Steam required would be

$$\frac{\text{Total Btu per hr}}{\text{Latent heat per lb of 50-lb steam}} = \frac{4141}{910} = 4.55 \text{ lb condensate per hr}$$

By Means of Hot Water. Convector requirements for heating with **hot water** are based on a rating of 150 Btu/(sq ft)/(hr).

For room *A* this would be equal to $4141/150 = 27.6$ sq ft

By Means of Electric Heaters. Electric heaters are based on an output of 3413 Btu per kwhr.

For room *A* this would be $4141/3413 = 1.21$ kw heater

In room *A*, 4141 Btu per hr is the heat required to make up the loss through the room boundaries. If unheated outside air is introduced into the room, additional heat is required to raise this air from 0 to 70 deg, the temperature maintained in the room. This is calculated by the formula

$$\text{Btu} = 1.08 \times \text{cfm} \times t_d$$

In this case, t_d, the temperature difference, is equal to (70 − 0) or 70 F.

By Means of Hot-air Blast. Hot-blast heating is based on the following formula:

$$\text{Cfm} = \frac{55.6 \times H}{60 \times t_d} = \frac{H}{1.08 \times (t_i - t_r)}$$

where H = heat loss, Btu per hr

$t_d = t_i - t_r$ = difference between inlet air temperature and desired room temperature

55.6 = cfm of air heated 1 deg by 1 Btu at 70 deg which is the temperature at which the system capacity is rated

Assume room *A* to be separately heated and that a 5-min air change is desired with the heating.

$$\text{Cfm to be supplied} = \frac{1{,}120 \text{ cu ft}}{5 \text{ min}} = 224 \text{ cfm}$$

The heat given off by electrical machinery, such as motors or generators, is calculated by determining the Btu equivalent of the electrical loss in the motor or generator.

Example. A 50-hp motor, with an electrical efficiency of 88 percent and with 12 percent of the input energy lost as heat. The input energy, in horsepower, is $50 \div 0.88$ or 56.8 hp. As the output is 50 hp, the heat equivalent of $56.8 - 50$ or 6.8 hp is given off as heat. As 1 hp = 2546 Btu per hr, the total heat given off $= 6.8 \times 2546 = 17{,}300$ Btu per hr.

If a diesel engine is cooled either directly or indirectly by sea water, the heat given off by the cylinders and pistons is carried overboard by the sea water. However, if the engine is installed with an air-cooled radiator, as is generally the case with emergency generator units located above the water line, the heat given off by the radiator is considerable—about one-third of the total heat value in the fuel. In addition, the heat radiated from the exhaust manifold and the heat loss from the electrical generator should be considered. Tests made on air-cooled emergency generator units indicate that approximately one-half of the heat value in the fuel burned is given up to the surrounding air. This indicates the necessity for an adequate supply of ventilating air to spaces housing such a unit.

In determining the ventilation requirements of a main machinery space, the entire space should be arbitrarily divided up into a number of zones or compartments, and the heat liberated in each zone should be calculated in the manner indicated above. All auxiliaries, steam pipes, heaters, boiler surfaces, etc., should be considered. Having determined the total heat liberated in such a zone, the allowable temperature rise in that zone should be decided on, and the air flow to that zone determined, as follows:

$$C = \frac{H}{w \times 60 \times 0.24 \times t_r} = \frac{H}{14.4 \times w \times t_r}$$

where C = air required, cfm

H = total heat liberated in the zone, Btu per hr

w = specific weight of air at the supply temperature, lb per cu ft

t_r = desired temperature rise of the air, deg F

As the outside air temperature will not generally exceed 90 F, the value of w at this temperature is 0.0723 lb per cu ft, and the above may be written

$$C = \frac{0.96 \times H}{t_r}$$

Special Features. Although the heat in a main machinery space is liberated over a considerable vertical depth, the ventilating air should be generally supplied at the lower levels, so that the lower and operating spaces will be maintained at a comfortable temperature. The average temperature rise at working levels should not exceed about 20 F. The natural tendency of heated air to rise will sweep the heated surfaces above the lower levels.

Consideration should be given to the areas generally occupied by the operators. The operating station, for example, should be kept cooler than less frequented areas, such as behind the boilers.

If an exhaust system is installed, the inlets for heated air should be located close to any "hot spots" to lead the hot air away from the operating personnel.

HYPERBOLIC FUNCTIONS

The **hyperbolic sine, hyperbolic cosine,** etc., of any number x, are functions of x which are closely related to the exponential e^x, and which have formal properties very similar to those of the trigonometric functions, sine, cosine, etc. Their definitions and fundamental properties are as follows (see also p. 127; graphs, p. 175; table, p. 60; series, p. 161):

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x});\ \cosh x = \tfrac{1}{2}(e^x + e^{-x});\ \tanh x = \sinh x/\cosh x;$$
$$\operatorname{csch} x = 1/\sinh x;\ \operatorname{sech} x = 1/\cosh x;\ \coth x = 1/\tanh x;$$
$$\cosh^2 x - \sinh^2 x = 1;\ 1 - \tanh^2 x = \operatorname{sech}^2 x;\ 1 - \coth^2 x = -\operatorname{csch}^2 x;$$
$$\sinh(-x) = -\sinh x;\ \cosh(-x) = \cosh x;\ \tanh(-x) = -\tanh x;$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y;$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y;$$
$$\tanh(x \pm y) = (\tanh x \pm \tanh y)/(1 \pm \tanh x \tanh y);$$
$$\sinh 2x = 2 \sinh x \cosh x;\ \cosh 2x = \cosh^2 x + \sinh^2 x;$$
$$\tanh 2x = (2 \tanh x)/(1 + \tanh^2 x);$$
$$\sinh \tfrac{1}{2}x = \sqrt{\tfrac{1}{2}(\cosh x - 1)};\ \cosh \tfrac{1}{2}x = \sqrt{\tfrac{1}{2}(\cosh x + 1)};$$
$$\tanh \tfrac{1}{2}x = (\cosh x - 1)/(\sinh x) = (\sinh x)/(\cosh x + 1).$$

The hyperbolic functions are related to the rectangular hyperbola, $x^2 - y^2 = a^2$ (Fig. 12), in much the same way that the trigonometric functions are related to

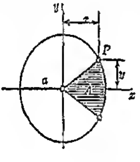


Fig. 11.

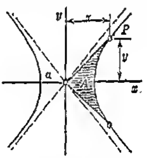


Fig. 12.

the circle $x^2 + y^2 = a^2$ (Fig. 11); the analogy, however, concerns not angles but areas. Thus, in either figure, let A represent the shaded area, and let $u = A/a^2$ (a pure number). Then for the coordinates of the point P we have, in Fig. 11, $x = a \cos u$, $y = a \sin u$; and in Fig. 12, $x = a \cosh u$, $y = a \sinh u$.

The **inverse hyperbolic sine** of y, denoted by $\sinh^{-1} y$, is the number whose hyperbolic sine is y; that is, the notation $x = \sinh^{-1} y$ means $\sinh x = y$. Similarly for $\cosh^{-1} y$, $\tanh^{-1} y$, etc. These functions are closely related to the logarithmic function, and are especially valuable in the integral calculus. For graphs, see p. 175.

$$\sinh^{-1}(y/a) = \log_e(y + \sqrt{y^2 + a^2}) - \log_e a;$$
$$\cosh^{-1}(y/a) = \log_e(y + \sqrt{y^2 - a^2}) - \log_e a;$$
$$\tanh^{-1}\frac{y}{a} = \tfrac{1}{2}\log_e \frac{a + y}{a - y};\quad \coth^{-1}\frac{y}{a} = \tfrac{1}{2}\log_e \frac{y + a}{y - a}.$$

The **Gudermannian** of x (written gd x) is an angle u such that $x = \log_e \tan(\tfrac{1}{4}\pi + \tfrac{1}{2}u)$. See Smithsonian Tables of the Hyperbolic Functions.

The **anti-gudermannian** of an angle u, denoted by $\operatorname{gd}^{-1} u$, is a number defined by $\operatorname{gd}^{-1} u = \log_e \tan(\tfrac{1}{4}\pi + \tfrac{1}{2}u) = \int \sec u\, du = \sinh^{-1}(\tan u) = \cosh^{-1}(\sec u) = \tanh^{-1}(\sin u) = 2 \tanh^{-1}(\tan \tfrac{1}{2}u)$. When u is small, $\operatorname{gd}^{-1} u = u + \tfrac{1}{6}u^3 + \tfrac{1}{24}u^5 + \tfrac{61}{5040}u^7 + \ldots$

ANALYTICAL GEOMETRY

THE POINT AND THE STRAIGHT LINE

Rectangular Co-ordinates (Fig. 1). Let $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$. Then, distance $P_1P_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$; slope of $P_1P_2 = m = \tan u = (y_2 - y_1)/(x_2 - x_1)$; co-ordinates of mid-point are $x = \frac{1}{2}(x_1 + x_2)$, $y = \frac{1}{2}(y_1 + y_2)$; co-ordinates of point $(1/n)$th of the way from P_1 to P_2 are $x = x_1 + (1/n)(x_2 - x_1)$, $y = y_1 + (1/n)(y_2 - y_1)$.

Let m_1, m_2 be the slopes of two lines; then, if the lines are parallel, $m_1 = m_2$; if the lines are perpendicular to each other, $m_1 = -1/m_2$.

Equations of a Straight Line.

1. Intercept Form (Fig. 2): $\dfrac{x}{a} + \dfrac{y}{b} = 1$. ($a$, b = intercepts of the line on the axes.)

2. Slope Form (Fig. 3): $y = mx + b$. ($m = \tan u$ = slope; b = intercept on the y-axis; see also Fig. 2, p. 174.)

3. Normal Form (Fig. 4): $x \cos v + y \sin v = p$. (p = perpendicular from origin to line; v = angle p makes with the x-axis.)

4. Parallel-intercept Form (Fig. 5): $\dfrac{y - b}{c - b} = \dfrac{x}{k}$. ($b$, c = intercepts on two parallels at distance k apart.)

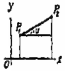


Fig. 1.

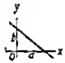


Fig. 2.


Fig. 3.

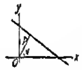


Fig. 4.


Fig. 5.

5. General Form: $Ax + By + C = 0$. [Here $a = -C/A$, $b = -C/B$, $m = -A/B$, $\cos v = A/R$, $\sin v = B/R$, $p = -C/R$, where $R = \pm\sqrt{A^2 + B^2}$ (sign to be so chosen that p is positive).]

6. Line Through (x_1, y_1) with Slope m: $y - y_1 = m(x - x_1)$.

7. Line Through (x_1, y_1) and (x_2, y_2): $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

8. Line Parallel to x-axis: $y = a$; to y-axis: $x = b$.

Angles and Distances.

If u = angle between two lines whose slopes are m_1, m_2, then

$$\tan u = \frac{m_2 - m_1}{1 + m_2m_1}.$$

If parallel, $m_1 = m_2$.
If perpendicular, $m_1m_2 = -1$.

If u = angle between the lines $Ax + By + C = 0$ and $A'x + B'y + C' = 0$, then

$$\cos u = \frac{AA' + BB'}{\pm\sqrt{(A^2 + B^2)(A'^2 + B'^2)}}.$$

If parallel, $A/A' = B/B'$.
If perpendicular, $AA' + BB' = 0$.

The equations of the bisectors of the angles between the two lines just mentioned are

$$\frac{Ax + By + C}{\sqrt{A^2 + B^2}} \mp \frac{A'x + B'y + C'}{\sqrt{A'^2 + B'^2}} = 0$$

Where heat-liberating units, such as auxiliary generators, are located toward the sides of the ship, the supply air ducts should discharge outboard of these units, as the natural tendency of the air to rise in the center up the machinery casing will induce air flow from the sides toward the center of the space. Supply outlets should not discharge directly toward switchboards or electrical controls, as the moisture in the air will affect these elements. Outlet terminals, particularly of larger sizes, should be fitted with adjustable dampers, which may be set to control the flow as required. This is particularly important for vessels operating in widely varying temperatures, as the machinery spaces may otherwise be unduly cold when in port in cold climates.

CARGO SPACES

General. Cargo spaces requiring ventilation include lower holds and 'tween-deck spaces for stowage of dry cargo or cased liquids. The cargo tanks of bulk-oil tankers are not ventilated when carrying cargo oil, but temporary means for ventilation of cargo tanks when empty are necessary in order to "gas-free" the tank and purify the air prior to entry by personnel. As the prime purpose of a cargo carrier is to deliver cargo in a satisfactory condition at its destination, proper care during transit is essential.

The condition of air in, and air supply to, dry cargo spaces has only recently received proper attention. Most ships afloat today are equipped only with natural ventilation of cargo spaces, depending on cowled intake and exhaust ventilators for air circulation. Some forms of cargo are "inert" and require no ventilation as long as the temperature of cargo and the air in the hold are constant; but very few trade routes lie completely in a zone of constant air and sea temperature. All sea air is humid and contains a considerable amount of moisture. If a ship is loaded in warm weather and the hatches are sealed prior to sailing, considerable moisture is held in the warm air. When the ship enters cooler weather, the moisture in the air condenses on the cooled ship structure or cargo, unless replaced with drier air. Similarly, when the ship is loaded in cold weather and proceeds to a warmer climate, if warm (and moisture-laden) air is pumped through the hold, its moisture will condense on the cold surfaces, and sweat appears.

Air of 100 percent relative humidity, at a temperature of 80 F, contains 156 grains water vapor per lb air. At 40 F it contains only 36 grains water vapor per lb air; hence a large amount of moisture will condense on the cooled surfaces in proceeding from warm to cold climates. Positive means of replacing the moisture-laden air should be provided.

Ventilation of cargo spaces is also necessitated by the nature of the cargo itself. Certain commodities like chocolate, lard, cheese, and alcoholic beverages are damaged by even moderate heat. Heat will also affect wet or damp hygroscopic cargoes like green lumber, grain, hides, and other vegetable or animal products, and cause them to release part of their inherent moisture as water vapor, which will settle on cooler surfaces or be absorbed by other hygroscopic cargo, thus damaging both commodities. Other commodities, chiefly of vegetable or animal origin, are subject to spontaneous heating and, unless kept at a proper temperature by removal of this heat, they will spoil. Still other cargoes will give off objectionable odors, which may be absorbed by other parts of the cargo carried. Adequate provision should be made for air circulation around and through such cargoes, so that heat or vapor may be released through the cargo itself. The designer must provide

found among various authorities. Despite such variations, friction-loss calculations are absolutely necessary in designing a system, and the data available adequately fulfill their purpose.

Friction loss is expressed in inches water gage. It may also be expressed in terms of velocity head, which is related to the velocity of standard air by the following formula:

$$H_v = \left(\frac{V}{4{,}005}\right)^2$$

where H_v = velocity head, in. water gage
V = velocity of air, fpm

Velocity head is frequently expressed in terms of so many diameters of straight circular pipe. Consequently any fitting may be designated as the frictional equivalent of a certain length of straight pipe expressed in diameters. The length of pipe equivalent to one velocity head depends on the duct construction, the usual range being between 39 and 50 diameters. For this discussion, however, loss will be expressed in terms of velocity head, H_v.

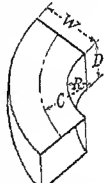


FIG. 13.—Elbow dimensions.

Straight-duct friction losses in inches water gage per 100 ft duct length are shown in Fig. 12. This chart plots pressure loss against cfm for various sizes of circular pipes. If a rectangular duct is being considered, it must first be converted by means of Table 4 to the circular duct having the same friction when carrying a given amount of air. The friction reading from Fig. 12, divided by 100 and multiplied by the length of duct in feet, will give the pressure loss of any given run of duct in inches water gage.

Before treating the subject of **elbows**, the following should be defined:

Center-line Radius Ratio. Center-line radius of an elbow divided by the width of the duct in the direction of the bend $= \frac{C}{W}$.

Inside-radius Ratio. Inside radius of an elbow divided by the width of the duct in the direction of the bend $= \frac{R}{W}$.

Aspect Ratio of Elbow. Depth of duct divided by the width of duct in the direction of the bend $= \frac{D}{W}$.

Aspect Ratio of a Duct. Long side divided by short side.

The pressure loss in an elbow varies with both the radius ratio and the aspect ratio. Figure 14 gives the loss in terms of velocity head, plotted against the inside-radius ratio. If the aspect ratio is greater than unity, it may be neglected; if less than unity, the value read from Fig. 14 should be multiplied by factor (*a*) taken from Fig. 15. If an elbow is less than 90 deg, the loss is less in direct proportion; if an elbow is more than 90 deg, the loss increases by approximately 75 percent of the greater angle. For example, the loss in a 45-deg bend will be 50 percent of that in a 90-deg bend of the same characteristics; for one of 135 deg the loss will be 100 + (50 × 0.75)

for the ultimate removal of such heat and vapor and the supply of replenishing air.

Other cargoes, inert of themselves, are seriously damaged by heat or moisture. Metals, machinery, and metal containers are rusted; foodstuffs are made unpalatable; textiles are discolored and mildewed; fibers and grains are fermented; and mineral or chemical products are dissolved by excessive moisture. For ships frequently carrying such cargoes, a system for artificially controlling the moisture content of the air may be desirable. Such a system is generally designed as an integral part of the cargo ventilating system.

When a cargo ventilating system is designed, the inlet to, and the outlet from, a given space should be as far removed from each other as feasible. The inlet or supply may be at one end of the hold and the outlet or exhaust at the other end to ensure complete circulation through the space. If this is not possible, the terminals may be carried out approximately to the sides of the ship by means of ducts, lying between deck beams if the ship is transversely framed. Ducts and terminals in cargo spaces should be well protected, to prevent damage while cargo is being loaded. Also, the terminals should be so arranged that stowage of cargo will not obstruct the egress or ingress of air. If the hold is of considerable size, several points of inlet and outlet should be provided.

Conditions. The nature of the commodities carried is so varied as to make precise statements of desirable conditions difficult, if not impossible. However, as a general guide, ventilating systems should provide an air change in 15 to 30 min, based on the grain capacity of the empty hold. This will permit the air to be changed, and any heat or vapor removed, at such a rate that cargo will not be damaged by usual variations in outside air temperature. If the spaces are to carry inflammable cargoes, or cargoes giving off inflammable vapors, special consideration is necessary.

Cargoes of "inherent vice," *i.e.*, those which are not inert and give off spontaneous heat or moisture, may require the inclusion of a system giving humidity control. Here again the establishment of accurate standards is difficult, owing to the varying nature of the cargo. Present practice for ships carrying 8,000 to 10,000 tons of cargo is to install a system that will permit the circulation of approximately 3,000 cfm air through the dehumidifying equipment. The latter should be designed to remove approximately 160 lb moisture per hr from 3,000 cfm outdoor air, the outdoor air being supplied at 88 F dry bulb and 80 F wet bulb, or a relative humidity of 75 percent. The relative humidity of the dehumidified air, assuming a temperature of 150 F, will be about $4\frac{1}{2}$ percent. A psychrometric chart, showing the relations among dry-bulb temperature, wet-bulb temperature, relative humidity, and moisture content of air, is shown in Fig. 1.

A dehumidifying system of the capacity stated will suffice to maintain cargo holds and their contents in good condition. It is rarely necessary to dehumidify all cargo holds simultaneously. Such a system is frequently operated on one or two holds at a time, until proper conditions in them are obtained; then the treatment is switched to another hold or holds. It may be unnecessary to treat certain holds at all during a voyage, depending on the nature of the cargo in them.

Methods. Natural ventilation of cargo spaces is satisfactory where the ship carries cargo unaffected by temperature or humidity changes, such as a bulk ore carrier. It is also fairly satisfactory where inert cargoes are

Table 4. Circular Equivalents of Rectangular Ducts for Equal Friction

$$d = 1.265 \sqrt[5]{\frac{(ab)^3}{a+b}}$$

where d = diameter of circular equivalent

a and b = sides of retangular duct

Side rectangular duct	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	24
2	3.0	3.4	3.7	3.9	4.1															
2.5	3.4	3.8	4.2	4.4	4.7	5.0	5.2													
3	3.8	4.2	4.6	4.9	5.2	5.5	5.7	6.0	6.2											
3.5	4.1	4.6	5.0	5.3	5.7	6.0	6.2	6.5	6.8											
4	4.4	4.9	5.3	5.7																
4.5	4.6	5.2	5.7	6.1	6.5	6.8	7.2	7.5	7.8											
5	4.9	5.5	6.0	6.4																
5.5	5.1	5.7	6.3	6.8	7.2	7.6	8.0	8.3	8.7											
6	5.3	6.0	6.6	7.1																
7	5.8	6.5	7.1	7.6																
8	6.1	6.9	7.6	8.2	8.8															
9	6.5	7.3	8.0	8.7	9.3	9.9														
10	6.8	7.7	8.4	9.2	9.8	10.4	11.0													
11	7.1	8.0	8.8	9.6	10.2	10.9	11.5	12.1												
12	7.4	8.3	9.2	10.0	10.7	11.4	12.0	12.6	13.2											
13	7.6	8.7	9.6	10.4	11.1	11.8	12.5	13.1	13.7	14.3										
14	7.9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]									
15	8.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]								
16	8.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]							
17	8.6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]						
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]					
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]				
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
38	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
42	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46	13.3	15.1	16.7	18.4	19.8	21.1	22.4	23.6	24.8	25.9	27.0	28.1	29.1	30.1	31.0	31.9	32.8	33.8	34.6	36.2
48	13.5	15.4	17.0	18.7	20.1	21.5	22.8	24.1	25.2	26.4	27.5	28.6	29.6	30.5	31.6	32.5	33.4	34.3	35.2	37.0
50	13.7	15.7	17.3	19.0	20.4	21.9	23.2	24.5	25.7	26.9	28.0	29.2	30.3	31.3	32.2	33.1	34.1	35.0	35.9	37.6

carried on routes of uniform temperature. However, the majority of general cargo ships operate in widely varying climates, most cargoes are susceptible to temperature and humidity changes, and natural ventilation is relatively inflexible. At best, it offers a means for circulating a limited amount of air, around and through the hold or space in fair weather. In rough or rainy weather, its usefulness is greatly decreased by the need for maintaining weathertightness and preventing rain, spray, or solid water from entering the inlet cowls. Frequently the need for ventilation is greatest at such times and, as a result, it is impossible to maintain the hold conditions at a satisfactory standard.

Where natural ventilation is fitted, several ventilating trunks must be fitted for each hold; one or more for supply air and an equal number for exhaust. Inlet and outlet heads may be of conventional cowl type, capable of being turned in any direction, so that the inlets may be trimmed into the wind and the outlets trimmed away from the wind to permit circulation. Such weather openings must be high enough off the deck to prevent rough seas from entering the cowls; and in rainy weather they must be trimmed away from the weather. Kingposts, where fitted, may be used as trunks, with proper heads at their tops.

Mechanical ventilation is far more satisfactory than natural ventilation, owing to its flexibility of use and operation. Smaller weather openings are possible. Ducts for supply and exhaust may be smaller, as the friction losses may be much greater; greater volumes of air are obtainable; and it is possible to operate the system in bad weather. Weather heads do not require constant trimming into or away from the wind.

Mechanical systems embrace the following types:

1. Mechanical supply, with natural exhaust.
2. Natural supply, with mechanical exhaust.
3. Mechanical supply, with mechanical exhaust.
4. Mechanical supply and exhaust, with humidity control.

Mechanical supply, with natural exhaust, is most frequently employed, as it overcomes the main objections to natural ventilation at minimum installed cost. Where kingposts are fitted, they may be utilized as exhaust outlets. However, the use of kingposts as exhaust trunks is likely to result in condensation of moisture within the kingpost, with the consequent dripping of condensed moisture to the bottom interior of the post.

Natural supply, with mechanical exhaust, is objectionable in that the natural inlets are subject to many of the undesirable features of natural ventilation, unless care is taken to prevent spray and rain from entering the inlets.

Mechanical supply, with mechanical exhaust, overcomes many of the disadvantages of natural ventilation, but its higher first cost is an objection, owing to the duplication of mechanical equipment on inlet and outlet. If the attendant ducts are properly arranged, a uniform circulation of air throughout the cargo space is obtainable, as the supply air may enter on one side of the cargo space and be withdrawn from the opposite side of the space. If the fans are reversible, even greater flexibility is obtained, as the flow direction may likewise be reversed. Where a cargo giving off odor, moisture, or heat is stowed on one side of the ship, it may be advisable to supply the air from the opposite side to avoid tainting or damaging other commodities in the same hold.

Mechanical supply and exhaust, with humidity control, has been recently developed and has much to recommend it, where cargoes of "inherent

or 137.5 percent of that for the 90-deg bend. For figuring pressure loss, a vaned elbow should be considered as a series of separate standard elbows, the vanes being so located that the radius ratio is the same in all of them. The resulting inside-radius ratios for elbows with 1, 2, or 3 vanes are given in Fig. 18. From these values, the friction loss can be read directly from Fig. 14.

Terminals and fittings must be considered as special problems in each case, since there are too many types and variations for much generalization. Table 5 lists friction losses for typical marine equipment. The values given can serve as guides in estimating losses for similar cases. The values given for vendor's equipment should be used only for preliminary determinations.

FIG. 14.—Friction loss in 90-deg elbows.

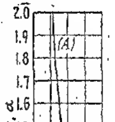


FIG. 15.—Aspect ratio factors.

When the equipment to be used is definitely known, exact loss figures should be obtained from the manufacturer.

Example. The following system loss is calculated by the friction-loss method. Figure 16 shows a typical duct layout for quarters. It will be assumed that these spaces are served by fan 2 shown in Fig. 6 and that the run to outlet G has the highest resistance of the runs on the system served by this fan. This run will, therefore, determine the fan static pressure to be specified. The pressure-loss calculations for this system are shown in Table 6.

Since the fan outlet size gives a reasonable velocity for the initial runs of duct, this size will be used for establishing the friction-loss factor for all straight ducts in the run. This appears in Table 6 as items 7 and 12 and is equal to 0.27 in. water gage per 100 ft of duct. The vertical line in Fig. 12 corresponding to this value will, therefore, hold throughout the system, and its intersection with the horizontal cfm lines will determine the proper size of round duct for any air quantity. The round duct size is then converted to a rectangular duct suitable for the installation by means of Table 4. For example, the proper size for 300 cfm is an 8 in. diam duct, which in turn is the frictional equivalent of a rectangular duct 5 by 11 in., 6 by 9 in., 4 by 14 in., etc. A 9 by 6 in. duct is chosen as the most suitable for the run between rooms *D* and *E*. The sources of other values used are explained in the notes in Table 6.

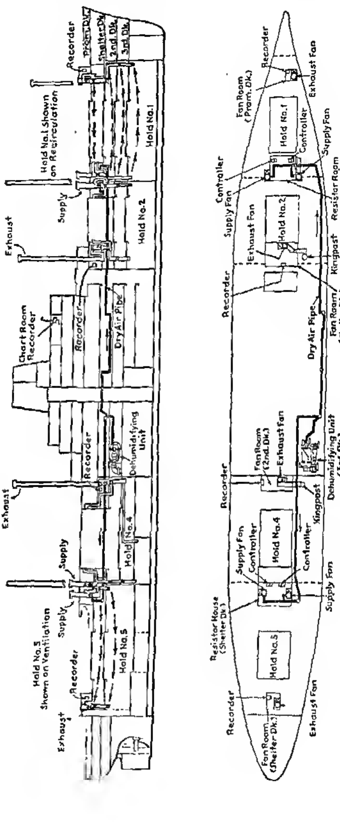


FIG. 7.—Dehumidification system on S.S. "African Comet."

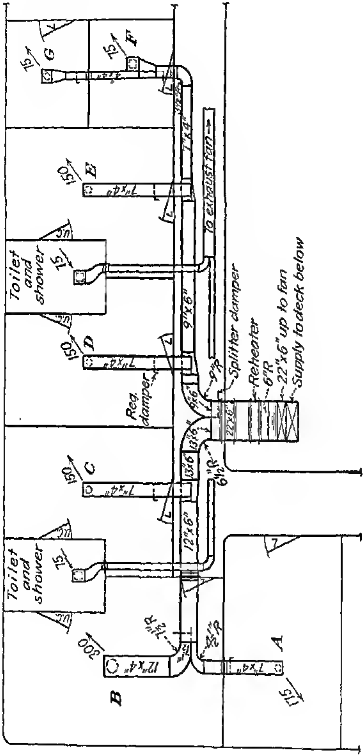


FIG. 16.—Typical duct layout.

vice" are carried or where extreme variations of outside temperature are met. This method employs a normal mechanical supply and exhaust system, with the addition of a central dehumidifying system selectively serving all or some of the holds.

A diagrammatic arrangement for a typical installation is shown in Fig. 7. The ship illustrated is a U.S. Maritime Commission cargo ship of the "African Comet" type. The installation there shown comprises a dehumidifying unit centrally located, which discharges dry air through a dry-air pipe to any of the holds arranged for control. Each hold is fitted with a supply and an exhaust fan, with ducts and dampers so arranged that the following are possible:

1. *Straight Ventilation.* Outside air drawn from atmosphere by supply fan and discharged to hold and 'tween deck. Exhaust air withdrawn from hold and 'tween deck by exhaust fan and discharged to atmosphere.

2. *Recirculation.* Dry air supplied from dehumidifying unit via dry-air pipe to suction of supply fan. Supply fan also takes suction from 'tween-deck space. The supply fan discharges a mixture of partly dehumidified air to the lower hold, absorbing moisture from the air therein, and the humidified air is withdrawn from the lower hold by the exhaust fan, which discharges part to the atmosphere, the balance returning to the supply fan inlet by way of the upper 'tween deck. The moisture removed from the hold air is that contained in the air discharged to the atmosphere. The volume of the air discharged to the atmosphere is, of course, equal to the volume of the dry air entering the hold. By proper control of the relative volumes of dry and recirculated air, the desired air condition is maintained.

Each hold is fitted, near its exhaust fan, with a recorder, which withdraws a continuous sample of the air being circulated and traces on a paper tape the actual temperature of the air leaving the hold and the relative humidity. As long as the dew point is below the temperature of the cargo, sweat will not be deposited on the cargo.

It is not necessary to supply dehumidified air from the central unit at all times, in order to maintain proper conditions of the air in the hold. Straight ventilation should be used whenever the temperature and humidity of outside air are suitable.

Refrigeration is another method of conditioning cargo spaces. This subject is covered elsewhere under the subject of Cargo Refrigeration (see p. 1882).

Calculations. A typical case showing the importance of ventilation to cargo holds follows.

Example. Consider a single hold, loaded with 1,000 tons of steel, at a temperature of 40 F. If the hold volume is 60,000 cu ft, the steel will occupy 4,600 cu ft, and the air the balance, or 55,400 cu ft. If the relative humidity during the loading period was 70 percent, the air will contain 25 grains moisture per lb air. As air at this condition weighs 0.079 lb per cu ft, the weight of the air in the hold at time of loading is 4,370 lb, of which about 16 lb is water vapor.

Assume the ship sails into a warm climate, where the air temperature is 80 F with a relative humidity of 90 percent. Air at this condition has a dew point of 76.5 F and contains 138 grains water vapor per lb. If air is then circulated through the hold, the weight of air therein will decrease from 4,370 to 3,940 lb, but the moisture content will have increased from 16 to 78 lb. If the temperature of the steel had not been raised above the dew point by ventilation, the moisture would condense on the cold steel and cause it to rust. In order to prevent sweat and condensation, the temperature of the cargo must be maintained above the dew point of the air in the hold.

Table 5. Friction-pressure Losses, Fittings and Terminals

Item	Figure No.	Loss, expressed in[b] H_v = velocity head	H = static pressure	Remarks
Cowl intake	26	0.8		
Mushroom intake	27	2.5		
Mushroom outlet	27	1.6		
Gooseneck, single or double	28, 29	....	....	Equivalent to 180-deg elbow
Double elbow or ogee	..	....	....	1.25 × sum of elbow losses
Square elbow with vanes	17	....	....	Equivalent to elbow with $\frac{R}{W} = 1$
Transforming elbow	..	....	....	Use average size
Louvered intake	30	1.0	....	Base velocity on free area
Adjustable terminal	35	1.2	....	When straight
Directional terminal	36	1.8		
Register	..	....	0.05–0.10	[a]
Diffuser	..	....	0.10–0.20	[a]
Duct opening, exhaust:				
With baffle	..	1.4	....	Free area = duct area
Without baffle	..	1.9	....	Free area = duct area
Duct ending in bulkhead, exhaust:				
With cone	..	0.25	....	Base on duct velocity
Without cone	..	0.50	....	Base on duct velocity
Duct ending in bulkhead, supply:				
With cone	..	0.8	....	Base on duct velocity
Without cone	..	1.2	....	Base on duct velocity
Branch, angle < 60°	21	0.3	....	Base on branch velocity
Branch, top takeoff	21	2.0	....	Base on branch velocity
Filter	..	....	0.25	For dirty filter
Duct heater	..	....	0.1–0.3	[a]
Wire screen, ½-in. mesh	..	0.25	....	Base on gross area
Insect screen	..	0.66	....	Base on gross area

[a] For usual velocities. Consult vendor if possible.
[b] Inches of water.

It has been stated that the run to outlet G has the greatest resistance on the system. Any other run, such as that to outlet A, will therefore, if based on a factor of 0.27 in., have a lower resistance and tend to draw more than its share of the air. This is rectified by the splitter damper provided at the split between the two main branches. If, however, a short branch is taken off near the fan, its resistance would be far less than that in the longest run. Such a duct would have to be restricted or "dampered down" considerably to reduce its air volume to the proper quantity. The same result can be accomplished more practically by reducing the size of the branch and thus increasing the air velocity and resistance. This should be done only to a point where the velocity will not exceed the reasonable limits given above.

The method of balancing the friction in the branches of a system can be illustrated by Fig. 16. If the fan were selected on the basis of the run to outlet G, the pressure at a point in the 22 by 6 in. duct between the reheater and the split in the ducts would be 1.02 in. minus 0.86 in. (cumulative total of items 1 to 9) or 0.16 in. For outlet A to be in balance with outlet G, the resistance between the 22 by 6 in. duct and outlet A must be made equal to that between the 22 by 6 in. duct and outlet G (total of items 10 to 13) or 0.16 in. Since the loss in the diffuser is known and that in the elbows can be estimated, these can be subtracted from 0.16 in. to give the loss for the 15 ft straight duct. Con-

The purpose of **ventilation** of a cargo hold, for inert cargoes such as the above, is to control their temperature and maintain it above the dew point of the air in the hold. The purpose of **humidity control** is to reduce the dew point of the air and thus prevent sweat and condensation either on the steel of the ship or on the cargo itself.

Example. A simplified typical calculation for the mechanical hold ventilating fans of a ship is as follows:

Volume of hold, cu ft (grain capacity) = 75,000
Air changes per hour, assumed = 3

$$\text{Air to be circulated, cfm} = \frac{75{,}000 \times 3}{60} = 3{,}750$$

Allow for fan design, cfm = 4,000
Static pressure loss, P_s, in. water:

Inlet head	0.2
Inlet duct friction	0.3
Loss through hold	0.5
Outlet duct friction	0.1
Outlet head	0.2
Total static pressure loss, in. water	1.3

Allow for fan design: $P_s = 1.5$ in.

$$\text{Air hp} = \frac{\text{cfm} \times P_s}{6{,}356} = \frac{4{,}000 \times 1.5}{6{,}356} = 0.95$$

The static efficiency of the fans at their rated performance varies from 50 to 70 percent, depending on the actual design. Assume here a static efficiency of 60 percent. Then,

$$\text{Bhp required of motor} = \frac{\text{air hp}}{\text{static efficiency}} = \frac{0.95}{0.60} = 1.58$$

Hence a standard rated 2-hp motor would be fitted.

Special Features. Ventilating ducts should be laid out to offer as little obstruction to cargo stowage as possible. They should lie between deck beams and frames. Ducts or trunks which do not lie close to the ship structure or which cause broken stowage are to be avoided, for they are likely to be damaged, and they reduce the cubic space available for cargo and the ship's potential earning capacity. Protection from damage when cargo is being worked should be provided by strong guards. The terminals should have ample guards to prevent blocking the air flow and should not discharge the air stream directly against the cargo. The possibility of moist inlet air or moisture condensed within the duct should be considered.

EQUIPMENT

Fans. In order to create a movement of air by mechanical means in a fan, the energy of the driving motor must be converted into kinetic or velocity energy by rotation of a rotor. The rotor may create air velocity by several means, the difference being mainly whether the velocity is created in a **centrifugal** direction (normal to the axis of the rotor) or in an **axial** direction (parallel to the axis of the rotor). Both centrifugal- and axial-flow fans are used in ventilation, the selection of type being governed by the requirements.

Centrifugal fans are broadly divided into two types, depending on the inclination of the blades comprising the rotor. The blades of one type are

Table 6. Friction-loss Calculations

Item	Size, in.	Area, sq ft	Flow, cfm	Velocity, fpm	Elbows: Aspect ratio[b]	Elbows: Inside-radius ratio[c]	Ducts: H_f (d)	Ducts: L (e)	Fittings: H_v (f)	Fittings: F (g)	Friction loss, H in. water
1. Louver, ½-in. mesh	64 × 56	20.6	11,340	550	....	....	....	..	0.02	1.25	0.03
2. Filter	60 × 100	41.7	11,340	272	....	....	....	..	....	....	0.25[h]
3. Preheater	....	....	....	....	....	....	....	..	....	....	0.15[h]
4. Fan intake	$d = 24$	3.14	5,670	1,800	....	....	....	..	0.20	0.5	0.10
5. Elbow, 90°	18 × 24	3.0	5,670	1,890	0.75	0.38	....	..	0.22	0.26	0.06
6. Elbow, 90°	18 × 24	3.0	5,670	1,890	0.75	0.50	....	..	0.22	0.19	0.04
7. Duct to heater	18 × 24	3.0[a]	5,670	1,890		....	0.27	35	....	....	0.10
8. Elbow, 90°	6 × 22	0.92	1,075	1,170	0.27	1.0	....	..	0.09	0.09	0.01
9. Reheater	....	....	....	....	....	....	....	..	....	....	0.12[h]
10. Elbow, 90°	6 × 10½	0.45	450	1,000	0.57	0.86		..	0.06	0.12	0.01
11. Elbow, 90°	4 × 7	0.194	150	775	0.57	0.50		..	0.04	0.22	0.01
12. Duct from heater	....	....	....	....	....	....	0.27	30	....	....	0.08
13. Terminal	$d = 4½$	0.11	75	682	....	....	....	....	0.03	2.05	0.06
Total, calculated											1.02
Use for design (allowing for dampers, etc.)											1.13

[a] Equivalent diam = 22.8 in. From Table 4.
[b] See Fig. 15.
[c] See Fig. 14.
[d] From Fig. 12.
[e] Length, ft.
[f] Velocity head $= \left(\frac{V}{4{,}005}\right)^2$.
[g] Loss in terms of H_v. From Table 5.
[h] From manufacturer.

verting this to the basis of 100 ft of duct, the diameter of duct to be used can be read from Fig. 12 and the rectangular equivalent from Table 4. A system with the principal branches balanced in this manner requires but slight equalizing with regulating dampers, and duct sizes are reduced to a minimum.

Duct Construction. Most ducts are constructed of galvanized sheet steel, the gage of which should be selected approximately on the basis of the dimension of the greater side.

Exposed vertical ducts:
No. 18 or No. 16 U.S.S. gage depending on size and chance of damage
Horizontal and concealed vertical ducts:

Over 24 in.	No. 16 U.S.S. gage
12–24 in.	No. 18 U.S.S. gage
6–12 in.	No. 20 U.S.S. gage
Less than 6 in.	No. 22 U.S.S. gage

The method of construction will depend on the available shop facilities, but usually ducts of No. 18 U.S.S. gage or heavier have riveted or spot-welded seams, while those of lighter metal are grooved, double-seamed, or Pittsburgh-locked. All seams must be made airtight with a sealing compound. Ducts should be smooth inside with joints lapped in the direction of the air flow. Sufficient flanged joints, fitted with suitable gasket, should be provided to make all ducts portable. Hangers should rigidly support all

curved **forward**; for the other type they are curved **backward**. This difference results in entirely different performance characteristics. Figures 8 and 9 show the type of blades and the performance characteristics for forward-curved and backward-curved blades, respectively. For forward-curved blades, the static pressure at no delivery is a maximum; as the air flow increases, the static pressure falls, then rises again to a peak, and again falls

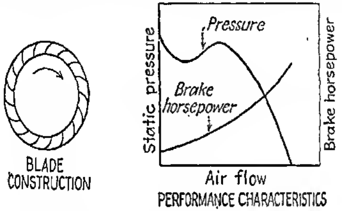


FIG. 8.—Centrifugal fan. Forward-curved blades.

as the flow increases still further. For the backward-curved blades, the static pressure at no delivery is a certain value; as the air flow increases, the static pressure rises to a peak and then falls off. The power requirements are also shown. The power for forward-curved blades continually rises as the flow increases; for backward-curved blades the power rises to a maximum and then decreases. Where a ventilating system, designed for a certain flow-pressure relation, may on occasion pass a larger flow at lower pressure, the use

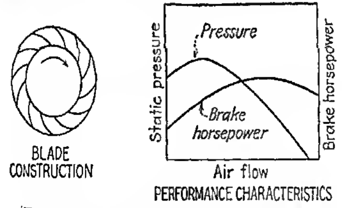


FIG. 9.—Centrifugal fan. Backward-curved blades.

of a forward-curved blade may overload the driving unit. For such systems the backward-curved blade should be used. However, most ventilating-duct systems are so designed that increased flow is obtained with only an attendant increase in static pressure; therefore the forward-curved blades are suitable. For a given volume and pressure condition, forward-curved-blade fans will operate at a lower tip speed and generally will be quieter than backward-curved fans.

ductwork, spacing to be not over 8 ft. All ducts with a side of 24 in. or more, and other ducts as necessary, should be stiffened with angle bars welded or riveted to the outside to prevent panting. In any case, where it is impossible to avoid a pipe, structural angle, or other obstruction passing through or cutting into a duct, a "teardrop"-shaped easement of sheet metal should be placed around it to lessen the disturbance of the air stream. A duct with an aspect ratio greater than 4 should have a dividing diaphragm to give stiffness and aid the air flow.

Plate trunks are used in lieu of sheet-metal ducts,

1. When a watertight compartment is penetrated, in which case the ventilation trunk must be of the same strength as the boundaries penetrated, to ensure the watertightness of the compartment violated.

2. In cargo holds and other places where lighter material would be subject to damage.

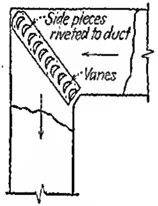


FIG. 17.—Square elbow with double radius turning vanes.

3. If the structure or space arrangements are such that it is advantageous to use a bulkhead as one side of the ventilation trunk. In this case the adjacent sides of the duct are welded to the bulkhead.

Plate trunks should be all welded for airtightness and should be either flanged for portability (small sizes only) or have handholes provided for painting and inspection.

Elbows should have an inside-radius ratio of 1 or greater whenever possible. Whenever the inside-radius ratio is less than 0.5 and in all cases in which the elbow discharges to or draws directly from the weather, vanes should be inserted. The vanes should run the full angle of the elbow and the full depth of the duct. They should be concentric to the inside and outside radii, except in a transforming elbow in which case they should divide the two sizes of ducts in the same proportion. Figure 18 shows the proper location of the vanes in any elbow.

For the case of 90-deg elbows, in which it is impossible to get any reasonable radius, there are available double-radius types of turning vanes which are installed diagonally in the square elbow. These vanes are designed so that the free area between them is equal to the cross-sectional area of the straight duct, thus giving a uniform velocity through the turn. These are sold in various sizes to suit different sizes of ducts.

Axial-flow fans, as used for ventilation, are broadly divided into two classes. **Propeller fans** are similar to the conventional domestic fan and are often built without a surrounding duct. For supply to or exhaust from a space such as a room or passage, they are mounted in an aperture cut into a bulkhead or deck. Having no connecting ducts, fans for such application are not used for high static pressures; their chief use is the moving of air

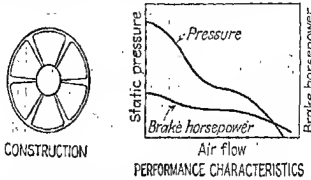


FIG. 10.—Propeller fan.

against relatively low resistance. Bracket fans, with the fan propeller directly mounted on the motor shaft and with merely a wire guard surrounding the blades, are used where local circulation of air within a space is desired. Fans of the propeller type, built inside a surrounding duct and connected to a supply or exhaust duct system, are capable of developing higher pressures and are conventionally termed **axial-flow fans.** They are built with fixed guide vanes at the inlet or outlet, to rectify the rotation of the air stream.

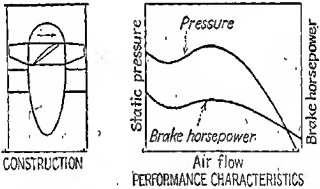
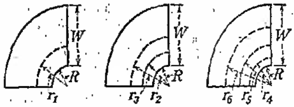


FIG. 11.—Axial-flow fan.

So defined, axial-flow fans are suitable for moving large volumes of air against moderate, and even high, static pressures; when working against considerable pressure, they are somewhat noisier than a centrifugal fan for the same duty. The blade type and typical characteristic curves for propeller and axial-flow fans are shown in Figs. 10 and 11, respectively.

Ducts. In laying out any duct system the following general rules should be observed:

1. Make runs as short as possible.



R = Inside radius of elbow
F = Vane factor
r = Radius of vane = R (1+F)

FIG. 18.—Equivalent of vaned elbows.

No. of vanes	1		2			3			
Inside-radius ratio $= \frac{R}{W}$	F_1	Equivalent ratio[a] $\frac{1}{F_1}$	F_2	F_3	Equivalent ratio[a] $\frac{1}{F_2}$	F_4	F_5	F_6	Equivalent ratio[a] $\frac{1}{F_4}$
0.10	....	....	1.20	3.90	0.83	0.75	2.08	4.42	1.33
0.15	....	....	1.00	2.87	1.00	0.56	1.56	3.11	1.78
0.20	1.21	0.83	0.80	2.30	1.25	0.46	1.21	2.30	2.17
0.25	1.00	1.00	0.72	1.92	1.39				
0.50	0.74	1.35							
0.75	0.53	1.88							
1.0	0.42	2.38							

[a] Use as inside-radius ratio for determining loss from Fig. 14.

Hangers may be of either angle or flat bar depending on the size of the duct and the distance from the deckhead. The bar should be as light as possible consistent with rigidity. A simple effective hanger consists of an angle welded vertically to the deckhead and bolted to the side angles of a

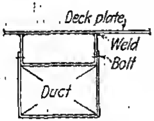


FIG. 19.—Flat-bar hanger.

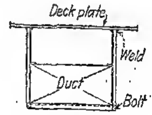


FIG. 20.—Angle-bar hanger.

connection flange by three or more of the flange bolts. Two other typical types are shown in Figs. 19 and 20.

Branch Connections. The preferred take-off for a branch is a split from the main duct. The division should be in proportion to the cfm in the two small ducts which can then be transformed to their proper size, based on equal friction, velocity, or other method.

2. Keep the number of turns and transformations to a minimum and make those that are necessary as easy and conventional as possible.

3. Round, square, and rectangular ducts are preferred in that order, with the last-named as nearly square as possible.

4. Check the structure, piping, wireways, and fixtures carefully.

5. Headroom and appearance should always be considered.

With these rules in mind, a single line diagram of the system is laid out, showing the general location of all outlets, elbows, branches, etc., and marked with the flow for every section of duct. The duct sizes should then be determined by one of two basic methods: the velocity method or the friction method.

The **velocity method** is based on designated velocities for the various duct sections, with gradual reduction as the air volume gets smaller. The friction losses for the various sections of the run are then calculated separately and added to determine the total. The actual velocities chosen should be governed by the particular case, but Table 3 may be used as a guide.

Table 3. Velocities in Ducts

Air Volume, Cfm	Velocity, Fpm
Less than 1,000	1,000–1,500
1,001–2,000	1,700
2,001–3,000	1,800
3,001–5,000	2,000
5,001–8,000	2,250
8,001 and over	2,500–3,000

This method is easy to apply and gives good results for small systems. For a large system, however, the duct sizes are apt to run somewhat larger than when figured by the friction-loss method.

The **friction-loss method** is based on the choice of duct sizes that will give equal friction per linear foot of duct throughout the length of the run. This method is generally preferred for systems of any complexity and is illustrated below.

The pressure loss for any run is equal to the sum of the pressure losses through all the elements in that run, from intake to discharge. The static pressure of any system, *i.e.*, the pressure that should be maintained by the fan in order to distribute the specified air, is calculated from the run having the highest resistance. This is generally, but not necessarily, the longest run on the system. If a mechanical supply system serves a space with natural exhaust, the fan must overcome the resistance in this natural system, in addition to the resistance in the supply system. Therefore, this must be added to, and considered part of, the run supplying the space. The same situation is, of course, true of a mechanical exhaust system with natural supply.

The flow of gases is a complex subject and, as such, will not be discussed here. The practical application, however, is used in the design of any ventilation system and will be touched on in relation to shipwork. It should be observed that values for friction losses cannot be regarded as exact quantities. Some are based on formulas that cannot be exact for all conditions met in practice; some are obtained from laboratory tests and will vary with the experimenter's interpretations of the results; others are assumptions based on known values for similar shapes, etc. Seemingly insignificant factors such as the type of paint used on the inside of a duct will have an effect on the friction loss. These conditions explain the differences to be

A round duct is generally connected at an angle to the main duct. Care should be taken in locating such a branch since, with supply especially, high-velocity or uneven flow in the main duct may interfere with flow in the branch.

The top take-off is useful in quarters systems where large mains with many branches are run hard under the deck beams. This fitting permits taking off

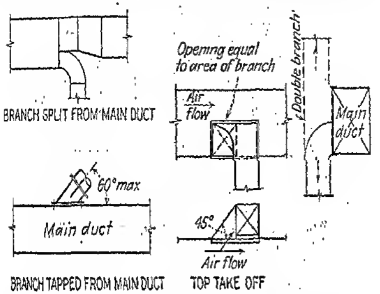


Fig. 21.—Typical branch connections.

and running the branch between the beams. It may be either single or double as shown.

Slip joint is used for connecting sections of sheet-metal duct in locations where there is not sufficient space to make up a flange connection. It is

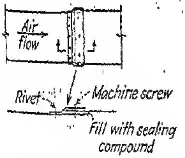


Fig. 22.—Slip joint.

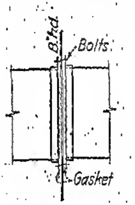


Fig. 23.—Flange connection.

frequently used in branches of a quarters system where the main duct in the passage is too close to the room bulkhead to permit a flange joint.

Flange connection is the most used joint between portable sections of duct and for bulkhead connections where no compensation for the cut is required. The joint is made airtight with gasket and sealing compound. Bolt spacing should be about every 3 in. The weight of the flange angle

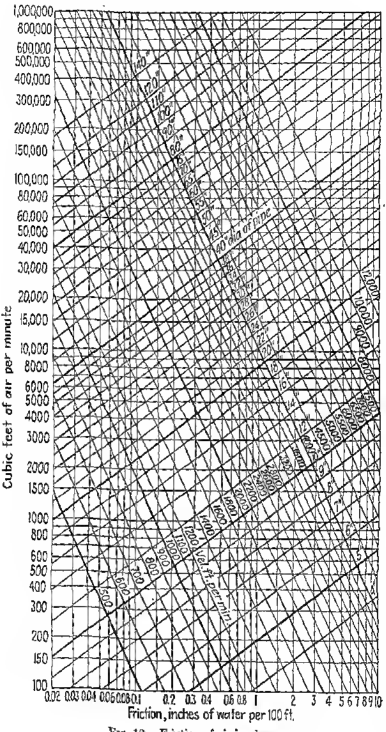


FIG. 12.—Friction of air in pipes.

varies from about 1 by 1 by ⅛ in. to 1½ by 1½ by ¼ in., depending on the size of duct.

Plate coamings must be used wherever a duct passes through or terminates at a deck. Their function is to give a watertight connection around the cut in the deck, to afford compensation for the steel cut away, and to

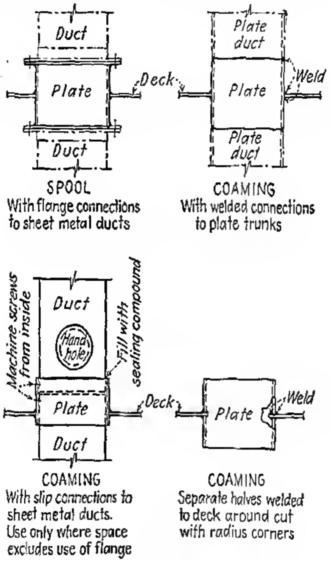


Fig. 24.—Typical coamings.

facilitate installation. They are also the preferred bulkhead connection. With sheet-metal ducts, the flanged coaming or spool is generally used. In cases where a rectangular cut is required in a strength member, a radius corner is generally called for. To meet this condition the corners of the coaming may be rounded, or the rectangular coaming may terminate at the deck outside of the cut. Coamings generally extend 3 in. above and below

The equation of a line through (x_1, y_1) and meeting a given line $y = mx + b$ at an angle u, is

$$y - y_1 = \frac{m + \tan u}{1 - m \tan u}(x - x_1).$$

The distance from (x_0, y_0) to the line $Ax + By + C = 0$ is

$$D = \left|\frac{Ax_0 + By_0 + C}{\sqrt{A^2 + B^2}}\right|$$

where the vertical bars mean "the absolute value of."

The distance from (x_2, y_2) to a line which passes through (x_1, y_1) and makes an angle u with the x-axis, is

$$D = (x_2 - x_1)\sin u - (y_2 - y_1)\cos u.$$

Polar Co-ordinates (Fig. 6). Let (x, y) be the rectangular and (r, θ) the polar co-ordinates of a given point P. Then $x = r\cos\theta$; $y = r\sin\theta$; $x^2 + y^2 = r^2$.

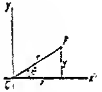


FIG. 6.

Transformation of Co-ordinates. If origin is moved to point (x_0, y_0), the new axes being parallel to the old, $x = x_0 + x'$, $y = y_0 + y'$.

If axes are turned through the angle u, without change of origin,

$$x = x'\cos u - y'\sin u, \quad y = x'\sin u + y'\cos u.$$

THE CIRCLE

(See also pp. 99, 103-105, 109)

Equation of Circle with center (a, b) and radius r:

$$(x - a)^2 + (y - b)^2 = r^2.$$

If center is at the origin, the equation becomes $x^2 + y^2 = r^2$. If circle goes through the origin and center is on the x-axis at point $(r, 0)$, equation becomes $x^2 + y^2 = 2rx$. **The general equation of a circle** is

$x^2 + y^2 + Dx + Ey + F = 0$; it has center at $(-D/2, -E/2)$, and radius $= \sqrt{(D/2)^2 + (E/2)^2 - F}$ (which may be real, null, or imaginary).

The equation of the radical axis of two circles, $x^2 + y^2 + Dx + Ey + F = 0$ and $x^2 + y^2 + D'x + E'y + F' = 0$, is $(D - D')x + (E - E')y + (F - F') = 0$. The tangents drawn to two circles from any point of their radical axis are of equal length. If the circles intersect, the radical axis passes through their points of intersection (see p. 100).

The equation of the tangent to $x^2 + y^2 = r^2$ at (x_1, y_1) is $x_1x + y_1y = r^2$. The tangent to $x^2 + y^2 + Dx + Ey + F = 0$ at (x_1, y_1) is $x_1x + y_1y + \tfrac{1}{2}D(x + x_1) + \tfrac{1}{2}E(y + y_1) + F = 0$. The line $y = mx + b$ will be tangent to the circle $x^2 + y^2 = r^2$ if $b = r\sqrt{1 + m^2}$.

Equations of Circle in Parametric Form. It is sometimes convenient to express the co-ordinates x and y of the moving point P (Fig. 7) in terms of an auxiliary variable, called a **parameter.** Thus, if the parameter be taken as the angle u which the radius OP makes with the x-axis, then the equations of the circle in parametric form will be $x = a\cos u$; $y = a\sin u$. For every value of the parameter u, there corresponds a point (x, y) on the circle. The ordinary equation $x^2 + y^2 = a^2$ can be obtained from the parametric equations by eliminating u.

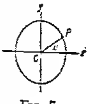


FIG. 7.

THE PARABOLA

The **parabola** (see also p. 107) is the locus of a point which moves so that its distance from a fixed line (called the **directrix**) is always equal to its distance from a fixed point F (called the **focus**). See Fig. 8. The point half-way from focus to directrix is the **vertex**, O. The line through the focus, perpendicular to the directrix, is the **principal axis**. The breadth of the curve at the focus is called the **latus rectum**, or **parameter**, $= 2p$, where p is the distance from focus to directrix. (Compare also Fig. 3, p. 174.)

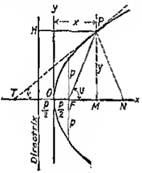


Fig. 8.

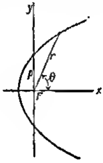


Fig. 9.

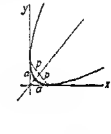


Fig. 10.

Any section of a right circular cone made by a plane parallel to a tangent plane of the cone will be a parabola.

Equation of Parabola, origin at vertex (Fig. 8): $y^2 = 2px$.

Polar Equation of Parabola, referred to F as origin and Fx as axis (Fig. 9): $r = p/(1 - \cos\theta)$.

Equation Referred to the Tangents at the ends of the latus rectum as axes (Fig. 10): $x^{1/2} + y^{1/2} = a^{1/2}$, where $a = p\sqrt{2}$.

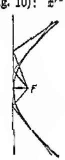


Fig. 11.

Fig. 12.

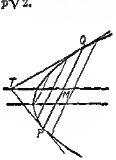


Fig. 13.

Equation of Tangent to $y^2 = 2px$ at (x_1,y_1): $y_1y = p(x + x_1)$. The line $y = mx + b$ will be tangent to $y^2 = 2px$ if $b = p/(2m)$.

The **tangent** PT at any point P bisects the angle between PF and PH (Fig. 8). A ray of light from F, reflected at P, will move off parallel to the principal axis. The **subtangent,** TM, is bisected at O. The **subnormal,** MN, is constant, and equal to p. The locus of the foot of the perpendicular from the focus on a moving tangent is the tangent at the vertex (Fig. 11). The locus of the point of intersection of perpendicular tangents is the directrix (Fig. 12). The locus of the mid-points of a set of parallel chords whose slope is m is a straight line parallel to the principal axis at a distance p/m,

the deck. The extent must be varied to suit conditions. Coaming should be at least equal to the thickness of the plate cut.

Wire-mesh screens must cover all openings into the ventilation system or cuts in bulkheads to prevent the entry and nesting of rats. Such screen should be No. 2 mesh or smaller, with wire not less than 0.064 in. diam. Screens shall be galvanized steel with a sheet-metal binding strip and be secured by machine screws or bolts spaced from 3 to 6 in. apart.

Insect screen, where required, shall be in addition to the ratproof screen and held in the same frame with it. Screen shall be No. 16 mesh with 0.014 in. diam wire.

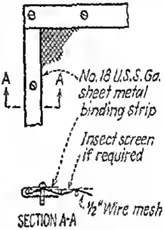


FIG. 25.—Method of holding wire mesh.

Standard cowls are the most used natural supply ventilators. They have the disadvantage that they must be trimmed to the wind; in bad weather they must be unshipped or covered. Although the air supplied depends on the wind, for design purposes it can be estimated on the basis of an 800 fpm velocity in the skirt, diameter D. Cowls up to about 15 in. can be fitted with grabs, trimmed by hand, and unshipped when necessary. Larger sizes of cowls or those not easily accessible from the deck are provided with a turning gear worked by a handle and must be trimmed away from the weather or covered as necessary.

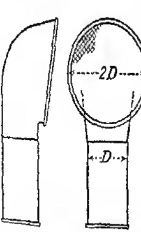


FIG. 26.—Standard cowl.

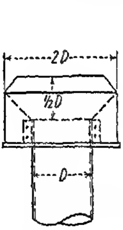


FIG. 27.—Standard mushroom.

Mushroom is a widely used ventilator head, owing to the fact that it can serve for supply or exhaust and because of its adaptability. Although both coaming and head are usually round, it can be made square, rectangular, or oval to suit conditions. The cone under the top of the mushroom is used with mechanical ventilation only. Although essentially weatherproof, a mushroom in a position exposed to seas can be fitted with a watertight damper to close down over the coaming.

quantity. These values are then tabulated on a data sheet. Set all regulating and splitter dampers in neutral position, check fan speed, and see that all conditions affecting ventilation are uniform in all compartments concerned. Doors and airports should be either all closed or all open—preferably all closed. Shutoff dampers must, of course, be wide open. After taking a full set of rough readings, balance among themselves the outlets on each individual branch. Then balance the branches one against the other, starting with those farthest from the fan and working back until the entire system is balanced proportionately. In the case of an air-conditioning system requiring adjustment down to design volume, this is accomplished after the balancing by one of three means: (1) by adjusting the fan speed, provided the required winter reduction can still be obtained below this setting; (2) by regulating the damper in the main duct; or (3) by choking down the outlets or branches individually, the last-mentioned being the least desirable. When the system is balanced, approval tests can be run and final rpm and electrical readings taken. The final test readings are made up as a report and should give location and dimensions of terminals, actual fpm, instrument correction, corrected fpm, specified cfm, and cfm delivered; also weather-opening readings—a traverse if one is taken; wet-bulb and dry-bulb temperatures; barometer; bow direction; wind direction and velocity (taken with anemometer); type of fan; motor serial number; horsepower; rated and actual rpm, volts, and amperes.

It should be noted that the weather-opening reading in cfm will frequently vary considerably from the sum of the discharge volumes. This is due to the difference in error for the different types of opening and the usual wind disturbance around the intake.

When the final tests have been accepted, the regulating and splitter dampers should be permanently locked in position by setscrew or nut and removal of handle, or by spot welding, to prevent tampering. Splitter damper rods should be either spot-welded or bent over before they are cut, to guard against their dropping into the duct if the screw should work loose.

Correction for standard air under extreme conditions may amount to about 15 percent. Generally, however, it will be less than 5 percent. It is meaningless, therefore, unless other errors are kept down to these limits. It is unnecessary in balancing unless air of widely varying temperatures is being delivered by different outlets of the same system. If, however, the tests are run for absolute values, the accuracy should be such as to warrant the air correction.

Standard air is generally taken as the weight at 70 F, 29.92 in. barometer, and 50 percent relative humidity. Since fans are rated on air of this weight, the cfm delivery will be greater with less dense air (warmer, less humid, or at lower pressure) and less with air of greater density. To obtain the volume of standard air, multiply the volume of measured air by the weight of measured air, in pounds per cubic foot, and divide by 0.07492 (the weight of 1 cu ft standard air). Properties of Air tables give the weight of dry air at any given temperature and barometric pressure, together with the increase in weight per degree wet-bulb depression at various temperatures.

Wind velocity and direction do not affect the weight of the air but should be recorded, since, if blowing directly at or away from a weather opening, it will affect the delivery of the fan.

Heat tests for temperatures maintained in various spaces under winter operation cannot be satisfactorily run unless the outside temperature is below 60 F. Likewise, cooling tests for temperature rise cannot be satis-

Gooseneck vents are made of standard pipe parts up to about 10 in. diam. In larger sizes, goosenecks and adaptations of the gooseneck are built of plate, generally with rectangular cross section. Small vents are generally fitted with covers, watertight or nontight as required. Large sizes generally have jack rods for canvas covers. Goosenecks may be used for either supply or exhaust.

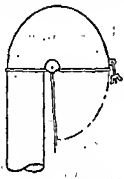


Fig. 28.—Pipe gooseneck with cover.

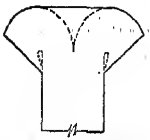


Fig. 29.—Double gooseneck.

Fig. 30.—Typical louver.

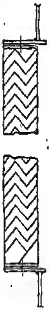


Fig. 31.—Light excluding louver

The **double gooseneck** shown in Fig. 29 is an adaptation frequently us for fan intake or discharge.

Louvers are placed in weather openings for protection against rain a spray. They are no protection against seas and should be provided wit plate or canvas cover depending on location. There are many commerc louvers suitable for shipwork. Those with movable blades are not genera heavy or tight enough to give adequate protection from seas.

The light-excluding louver finds some application on shipboard.

factorily run unless the outside air is over 70 F. For such tests thermometers should be placed so as to record temperatures of outside air, air entering the space, and representative temperatures of the air in the space. The readings on the regular duct thermometers should also be recorded. If outside temperatures are too high to permit a regular operating test, thermostatic controls may still be tested by cooling the elements with air or water of decreasing temperature and by noting the temperature at which the valve operates. Compensated controls can be checked by immersing the compensating bulb in a cooling bath of known temperature and by noting the reading on the duct thermometer following the heater being checked. Since the warm-air temperatures follow a straight-line variation as the outside air varies from 0 to 70 F, any two points, not too close together, will serve as a check. Table 7 shows a tabulation of points on the curves for a group of reheaters with compensated thermostatic controls. Intermediate points can be determined by interpolation.

Table 7. Reheater Temperatures

Outside air temperature, deg F	Temperature of air leaving reheater, deg F		
	Reheater 1	Reheater 2	Reheater 3
70	70	70	70
60	74	72	73
40	83	76	78
20	92	80	83
0	102	83	88

Natural ventilation is not generally tested quantitatively, owing to the great difficulty of getting consistent readings on outlets dependent on the wind. In cases where there is more than one outlet on a natural system, they can be roughly balanced if a steady breeze is blowing. A series of readings should be taken and averaged, testing the several outlets alternately so as to even out wind variations as much as possible.

Operating instructions covering all ventilation systems are very important in shipwork. Unlike shore installations, where the designer or contractor can be called in if something goes wrong, a ship is away from home port most of the time. Furthermore, if a vessel is sold, a complete change of operating personnel may take place thousands of miles from the building yard and the designer's office. Operation of the ventilation must, therefore, be made as simple as possible, and instructions must be as clear as possible.

Label plates should be provided to identify every fire damper, fan, fan controller, thermostatic bulb, heater, weather opening, and any other major part of the system. Also, identifying marks are frequently painted on vent ducts in engine rooms and similar locations.

A diagrammatic layout of the complete ventilation system for the vessel should be posted under glass for the use of the ship's personnel. It is generally located in or near the chief engineer's office. This should show, diagrammatically, all ductwork and equipment, the air quantity delivered by each outlet, the location of all fire dampers, and emergency control buttons. Fan identification should include system number, cfm winter and summer, and

Weather Closures. Every ventilation opening to the weather should be provided with a means of closing it against seas and weather. Openings located on the freeboard deck or other exposed positions require a cover with gasket or machine fit, which will pass a hose test for watertightness and which is of the same strength as the surrounding structure. Other openings, requiring protection from driven rain and spray but not from green seas, should have a nontight cover, *i.e.*, a plate cover without gasket, or be provided with jack rods to which a canvas cover can be lashed.

Diffusers are a commercial type of ceiling outlet, designed to deliver air at a high velocity but without objectionable drafts by means of rapid dis-

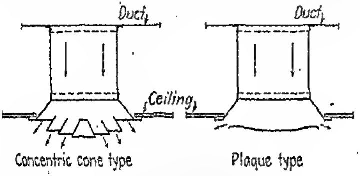


Fig. 32.—Typical diffusers.

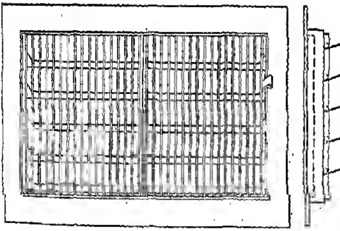


Fig. 33.—Register.

persion and mixing with the surrounding air. Diffusers are well suited for use in public rooms and staterooms. There are also types with directed air flow, suitable for group berthing spaces. The manufacturer should be consulted in the choice of sizes, but generally neck velocities of 1,000 to 1,200 fpm are acceptable. There are two basic types of diffuser: the concentric cone and the plaque types, as shown in Fig. 32. Owing to the low ceilings encountered in shipwork, diffusers are within reach of personnel and must, therefore, be of more rugged construction than is required for land installations. There are units that incorporate a diffuser and lighting fixture but, by location or construction, either lighting or ventilation efficiency is often sacrificed for the sake of appearance. The control damper can either be part of the diffuser or be located in the supply duct.

speed reduction. Heater identifying notes should cover winter cfm, temperature range, and type of control. Preferably on the same sheet with the diagram, written instructions should cover the following general items plus any additional features for a particular case, *i.e.*, air conditioning, etc.

1. *Warning.* All covers and dampers must be open before a fan is started.

2. *Emergency—Fire.* Shut off fans and close all fire closures for spaces involved. Give location of emergency stop buttons.

3. *Emergency—Weather.* Unship cowls and close weather openings as necessary to meet conditions.

4. *Summer Operation.* Fans to run at full capacity. All summer outlets to be open.

5. *Winter Operation.* Fans to run at winter operating speed, summer openings to be closed. Start heaters by first turning on main heating steam valves, then hand valves at preheaters, finally hand valves at reheaters. Include chart similar to Table 7, and explain the operation of the heaters. At the end of winter operation all hand valves shall be closed and all heaters, pipes, and traps shall be drained completely.

6. *Service and Maintenance. Filters* must be washed and recharged at regular intervals, the period depending on the vessel's service, but generally every 6 to 10 weeks. Cells should be replaced in rotation, so many every week or every two weeks depending on the total number and on the number of spare cells carried. Method of washing and charging should be described.

Motors should have grease and brushes checked, commutators and windings cleaned, once a month.

Heaters. Steam strainers at thermostatic control valves, thermometer bulbs, and thermostat elements should be cleaned at least twice a year.

7. *Fans.* A list of the systems with space served, speed reduction, and location of controls.

8. *Miscellaneous directions* for systems requiring special operation such as resistor house exhausts, emergency machine spaces, and air-conditioned spaces.

In addition to the diagram and operating instructions, it is good practice to provide additional instruction sheets to be framed in the fan rooms. These are generally small sheets covering only such of the items listed above as may apply to the equipment in the fan rooms.

Registers are a commercial type of outlet, generally bulkhead-mounted, designed to deliver air without objectionable drafts and under controlled conditions. They should have vertical vanes in the register face, which can be individually or group set to control the horizontal deflection of the air; behind these, horizontal vanes should be located, for vertical deflection and closure. The latter are controlled by a single handle (for private spaces) or by a removable key (for public spaces). The manufacturer should be consulted in choosing appropriate sizes, which are based on the following: (1) throw, or length of air travel; (2) dispersion, or vertical area covered; (3) deflection, or horizontal angle covered. A typical register is shown in Fig. 33.

Terminals. An **adjustable terminal** is a sheet-metal coned outlet which can be turned to deliver air in almost any direction. It is used principally in machinery and working spaces for supplying a controlled blast of air at high velocity (see Fig. 34).

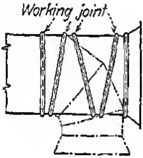


FIG. 34.—Adjustable terminal.

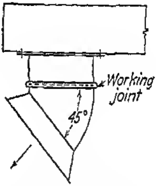


FIG. 35.—Directional terminal.

Directional terminal is a sheet-metal coned outlet which can be rotated through 360 deg to deliver air at a fixed angle. It is used principally in working spaces to supply a controlled blast of air for spot cooling (see Fig. 35).

Miscellaneous. **Dampers** and other volume-control devices are fitted for three purposes:

1. *Fire.* Dampers or closures which are easily accessible for closing off a ventilation opening, system, or part of a system, in case of emergency.

2. *Regulating.* Dampers or other devices which are used to apportion the proper volume of air to various terminals or branches when the system is being balanced and which are then permanently set in the proper position.

3. *Control or Shutoff.* Dampers that are adjustable by a crew member or passenger and permit control of the air volume supplied to a given space.

Fire dampers should be close fitting in the duct, controlled by a sturdy quadrant with bronze bearings, and of distinctive appearance. The most desirable type of fusible-link fire damper is one that may be closed manually without breaking the link. There are many types of **regulating devices** to meet various conditions. Most common are the splitter damper (see Fig. 36) which is suitable for any but very small ducts; commercial damper sets which may be locked by means of lock nut or screw; commercial vane type of volume controllers used principally with registers or grilles; and orifice plates with an opening of calculated size inserted between duct flanges.

ILLUMINATION

Based on Material Furnished by the Nela Park Department, General Electric Co.
From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

REFERENCES: Moon, "The Scientific Basis of Illuminating Engineering," McGraw-Hill. Barrows, "Light, Photometry, and Illuminating Engineering," McGraw-Hill. Luckiesh and Moss, "The Science of Seeing," Van Nostrand. Forsythe, "Measurement of Radiant Energy," McGraw-Hill. Cady and Dates, "Illuminating Engineering," Wiley.

Light rays have an effective range from 3800Å to 7200Å wave length (1 Ångstrom = 10^{-7} mm) and differ from radio, infrared, ultraviolet, X-ray, gamma, and cosmic radiations only in their wave lengths.

Photometric Units

Luminous Intensity (Candles). The luminous (*i.e.*, evaluated as to its effectiveness in producing visual sensation) intensity of any radiation in a given direction is its illuminating power in that direction. It is expressed in candles, the basic unit for which is the **International candle,** and is derived from the luminous intensities of certain carbon-filament lamps maintained in the Bureau of Standards and equivalent European laboratories. In 1940, a new unit and a new standard of luminous intensity were adopted by the International Commission on Illumination (I.C.I.) based on a black body at the temperature of freezing platinum. This has a brightness of 58.6 candles per sq cm, and the new unit is defined as one-sixtieth of this brightness. The change alters but slightly the units based upon the former International candle.

Lumens. One lumen is that quantity of luminous energy included in one steradian (unit solid angle) from a uniform point source of one candle power; or, more commonly, it is the luminous flux intercepted by a surface of 1 sq ft, all points of which are 1 ft distant from a uniform point source of one candle power.

Illumination (Foot-candles). Illumination is density of luminous flux, or the amount of luminous energy per unit of area. The unit is the foot-candle and is equivalent to the density of one lumen uniformly distributed over an area of 1 sq ft; or it is the illumination at all points which are 1 ft distant from a uniform point source of one candle power. The lux is the unit used in Europe.

Brightness (Candles per Square Inch, Foot-lamberts). The brightness of an object in the direction of viewing is the luminous intensity from the object per unit of area projected along the direction of viewing. One candle per square inch is the brightness of a projected square inch of surface from which the luminous intensity is uniform and averages one candle in the direction of projection. A foot-lambert is the brightness of a perfectly diffusing surface uniformly emitting or reflecting one lumen per square foot of surface. Since no surface is a perfect diffuser, the brightness of actual surfaces will vary with the angle at which they are viewed. The brightness for each angle of viewing is expressed as the foot-lambert brightness of a similarly appearing, perfectly diffusing surface. The average brightness in foot-lamberts of any surface is therefore the product of the illumination in foot-candles and the reflection or transmission factor of the surface.

The most common type of **shutoff damper,** other than those built into a diffuser or register, is the common pull type with open, closed, and one or two intermediate positions. This type is not suitable for large ducts or with high air velocities. For the latter, a quadrant or a barrel-type with plunger is satisfactory. If a damper must be located in a concealed duct, a remote-control or extension handle should be provided.

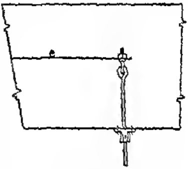


FIG. 36.—Splitter damper.

Air filters of commercial type are suitable for shipboard use, provided they are fireproof, verminproof, not subject to damage from salt water or vibration, and do not require special replacement or maintenance materials. These requirements are met by the viscous washable filter, which consists of a metal screen or frame and is periodically washed and recoated with oil. Washing and recharging equipment should be installed for the purpose. Filters are generally installed on the basis of about 800 cfm for each 20 by 20 in. filter cell.

Grease filters for use in canopies over ranges should meet about the same requirements as air filters, except that they are not oil-coated, and no washing and recharging equipment is required. They can be washed with either a steam or hot-water hose.

Blast heaters for duct installation are generally of the header type with nonferrous tubes and fins. Heaters handling straight outside air should be of the nonfreezing type, with internal steam-distributing tube construction, and be mounted with tubes vertical. Reheaters should be of single-tube construction with horizontally mounted tubes, properly pitched to provide drainage. Coils may be single or multiple row. The former are preferred owing to the lower resistance loss for a given air velocity.

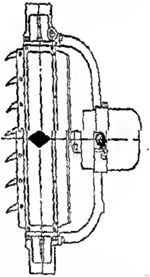


FIG. 37.—Typical unit heater.

Convectors should have a sheet-metal casing of the cabinet type, fully open at the bottom and with an outlet grille in the top only. The heating unit should be of the header type with tubes and fins to meet the heating requirements, arranged with piping connections at either end.

Unit heaters consist of a standard propeller-type fan, mounted integrally with a steam or hot-water coil, both fan and heating medium being controlled by a single switch. Adjustable horizontal vanes provide a vertical deflection of air. A typical unit heater is shown in Fig. 37.

Heating control valves should be of the modulating rather than the two-position type. Preheaters are controlled by a thermostatic bulb in the outlet duct, set for a definite temperature. Reheaters may have a room thermostat set for a definite temperature to be maintained in the space served, or a

Luminous Ratings. Light sources are rated in lumens by integrating the candle power in all directions. The **horizontal candle power** is the mean value in candles of the luminous intensity in all horizontal directions about the source. This value is but one of the mean angular candle-power values that may be determined for a source because the horizontal is but the 90 deg angular position in regard to the source axis. A candle-power distribution curve is shown in Fig. 1. The mean angular candle powers about this source are represented, *for each angular position*, by the distance of the curve from the origin in the direction being considered. The average of all candle powers is the **mean spherical candle power**, which, when multiplied by 4π (the number of steradians about a point), is numerically equal to the lumen output of the source. It is also frequently desired to know the lumens emitted in the angular zones. These are equal to the product of the zone constant (the number of steradians within the zone) and the average candle power of the zone. In practice, the candle power at the mid-elevation of each zone is used as representing the average for the zone.

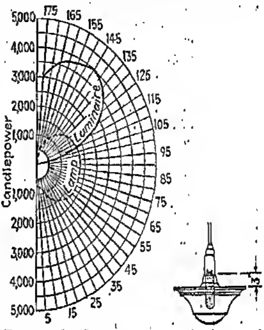


FIG. 1.—Candlepower Distribution of Lamp and Luminaire.

Equivalents and Conversion Factors.

4π lumens = the luminous flux emitted by a source of one mean spherical candle power

1 foot-candle = 10.76 lux

1 candle per sq in. = 452 foot-lamberts = 0.487 lamberts = 0.155 candles per sq cm.

The seeing task is the job to be done by the eyes. The visibility of an object depends upon its size, its brightness, its contrast with surroundings (both as to brightness and color), and the time available for seeing. The importance of time as a factor in seeing is seldom obvious and is usually given little consideration; yet, safety in the factory as well as on the highway is often closely related to it. The size, the color, and the contrast of an object can seldom be modified to make seeing easier. The remaining factor, brightness, is proportional to the illumination which is variable and which becomes the controllable factor that will determine the ultimate difficulty or ease of each visual task.

Color Vision. The visual response to a unit of radiant energy is not the same in quantity or in character for each wave length of radiation. The luminosity curve in Fig. 2 indicates the selectivity of the eye in terms of visual response per unit of energy radiated at each wave length. The curve represents the visual responses of the eye to wave lengths of the visible spectrum for brightnesses of surroundings such as are usual. The sensitivity

compensated duct thermostat set for an air temperature related to that of the outside air, as illustrated in Table 7.

The simplest control is the self-contained unit, consisting of a bulb and capillary tubing containing a fluid, the expansion of which operates a bellows in the regulating valve.

Pneumatic controls use compressed air as the power to operate the valve. These generally operate on air at 15 psi, obtained from either the ship's air supply or an independent compressor. Pneumatic control is somewhat more accurate than the self-contained unit and is capable of handling larger units, although this is seldom required in ship ventilation.

Electric controls use electric relays, solenoids, or motors to operate the valve. This is the most accurate type of control, but owing to its complication it is not so widely used as the other types described.

Regardless of the type of control used, careful adjustment is most important for proper results.

Hardware and fittings of all kinds should either be of nonferrous material or be protected against corrosion. Bolts and nuts, screws, and damper fittings should be brass, or cadmium-plated, or galvanized steel. Diffusers, registers, etc., should be cadmium- or zinc-plated, or bondorized for a priming coat finish. Working parts of registers, diffuser dampers, etc., should be of bronze. Fan and heater casings, filter frames, and other sheet-metal assemblies should be hot-dip galvanized after fabrication.

Spare parts should follow the marine provisions of the applicable rules, since ship requirements are very different than those for a land installation.

BALANCING, TESTING, AND OPERATION

No matter how well designed a ventilation system may be, it will not perform properly unless it is correctly balanced and operated. It is generally the responsibility of the ship designer or the builder: (1) to balance all ventilation systems and permanently set regulating devices; (2) to run tests that will demonstrate to the owner's satisfaction that the system is properly balanced and capable of design performance or better; (3) to furnish operating instructions for posting aboard the ship. The items of equipment most commonly used in balancing and testing ventilation systems are listed below, together with notes on their use.

Anemometer. A 4-in. rotating-vane type with range of approximately 200 to 3,000 fpm is most generally used. It is applicable to all work except where great accuracy is required near the limits of the range, or for readings outside the above range, for which cases other sizes should be used. A calibration curve comes with every instrument. The instrument should be checked periodically and recalibrated by the manufacturer as necessary.

When held in the air stream and timed for 1 min, an anemometer registers air velocity in feet per minute. It is used to measure inlet and outlet velocity readings at ventilation openings of definite area, whose smaller dimension is not less than the diameter of the rotating vanes. Readings must cover the entire area of the opening, with the face of the anemometer held parallel to the face of the opening and perpendicular to the air stream. The anemometer should be moved evenly over the opening, so that the cumulative total is a representative average for the entire area. For a very large opening or where utmost accuracy is required, a traverse should be made; *i.e.*, the total area is divided up into smaller areas, and a separate reading taken for each smaller area. The readings are then averaged after the anemometer correc-

is a maximum at approximately 5560Å wave length. If all the energy from a light source were expended at this single wave length, the resultant efficiency would be equivalent to 620 lumens per watt (lpw). The colors corresponding to various wave lengths are also indicated in Fig. 2. The eye is sensitive to any combination of wave lengths but it cannot distinguish the components. For example, a mixture of red and green lights, in neither of which there is any yellow, may produce the sensation of yellow.

Factor of Safety. The average characteristics of many eyes are used to define and determine a so-called normal one. Refractive and retinal deficiencies can usually be corrected, but nothing else can be done at the eye to increase the ease of seeing. Since actual eyes differ greatly from the average, factors of safety are necessary in illuminating engineering.

Glare. The effectiveness of the illumination on the task is reduced if there is extreme brightness within the field of view. Such brightness affects the sensitivity of the retina, reduces the size of the pupil and the brightness of the retinal image, and, hence, decreases the over-all visibility and produces discomfort and fatigue. Lighting equipment should eliminate glare as far as possible. It is recommended that no part of the light fixture or **luminaire**, within the normal field of view, be brighter than 500 foot-lamberts for exacting and prolonged tasks and 1,500 foot-lamberts for casual seeing tasks. (This range is approximately 1 to 3 candles per sq in.) The brightness of a luminaire becomes even more important if there are shiny surfaces, such as polished metal or glass desk tops, within the field of view because the luminaire will be reflected in these surfaces and will contribute brilliant reflected images.

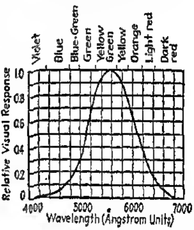


Fig. 2.—Luminosity Curve.

Light Distribution. An improper directional distribution of light is a frequent cause of glare. The illumination should be nearly the same on oblique, vertical, and horizontal surfaces; hence, shadows should not be either dense or sharply defined. Totally indirect lighting or that from large area low-brightness diffusing luminaires is preferable. With light coming from many directions, in none of which there is any extreme brightness, reflected glare is eliminated.

Some special applications require directional light distribution. In the inspection for very shallow scratches on metallic surfaces, for instance, the defects can best be seen by their specular reflection of a directional or semidirectional light source. Only the side of the scratch reflects light into the eye, the polished surface directs its reflection away from the eye making the background appear dark. When sharply directional lighting equipments as downlights, spotlights, projectors, and inspection units are used, they must be shielded and aligned so as to prevent direct and reflected glare.

Color Quality. Color quality is of varying importance. For accurate color matching, white light that has equal energy content at every visible wave length is ideal. For color discrimination, it is necessary that all wave lengths be included but that no wave length or group of them be given undue emphasis.

tion factor has been applied to each. When readings are taken, the anemom eter should be held in one hand, the hand being kept as much out of the a stream as possible, or on a stick secured to the base. In the latter case, second person should hold the stop watch, which may introduce some err unless the actions of both are coordinated. When readings are taken c weather openings exposed to the wind, a shield should be provided to reduce the effect of wind on the readings.

Velocities for various types of outlets should be determined as follows:

Wire-mesh Supply or Exhaust Openings. Figure the velocity on gross area of opening, making no allowance for screen.

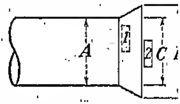


FIG. 38.—Bell or coned outle

Bell or Coned Outlet. (See Fig. 38.)

Supply. Velocity is figured on throat area A. If there is no screen, ho anemometer at position 1. If there is a screen, take readings at position over area C, which is the projection of A on the screen.

Exhaust. Velocity is figured on area B, and readings are taken in positic 2 over area B.

Registers. The manufacturer should be consulted as to the proper fact for a particular register and velocity range. In general, however, veloci should be figured on about 90 percent of the gross face area for a supply ar 100 percent for exhaust.

Diffusers. The anemometer can be used only with a test cone, or a chamb placed over diffuser. The manufacturer should be consulted as to the desig of the test device and the factors to be applied.

The anemometer is the most commonly used instrument in balancir and testing ship ventilation systems. Although, after calibration correction it may have an error up to about ± 10 percent depending on the size and ty of outlet tested, it gives reliable and consistent results and is easily handled c shipboard.

Stop Watch. A stop watch, for readings up to 1 min, is required for u with an anemometer.

Velometer. This instrument gives instantaneous readings for tl velocity of an air stream and reads directly in fpm. Calibration correctic factors are generally so small that they can be neglected in ship balancir and testing. The velometer comes with a set of jets, which are calibrat for the particular instrument and which are used only with that instrumen Regularly furnished impact jets are used for both supply and exhaust te minals. A duct jet is also desirable for taking velocity readings in duct heaters, by-passes, etc., in lieu of the more time-consuming but more accura pitot tubes. For any type of outlet, the velometer readings taken at vario points must be averaged, since the individual reading covers only a very sma area. To equal the accuracy of an anemometer and to obtain any degree consistency, considerable care should be taken with these readings. Th fact, in addition to the greater ease with which an anemometer can be handl in shipwork, limits the use of the velometer to testing outlets that cannot l measured with an anemometer and for rough checking. Principal amor these is the testing of diffusers. The diffuser manufacturer should be co sulted as to the method of test, the proper jet to use, and the factor to l applied. Six or eight readings at uniform intervals around the diffus should be taken and averaged. Care should be used to see that none of the

Specifying Foot-candles. Quality of illumination can be specified with considerable certainty but the specification of the amount of light cannot be so exact with present knowledge. The eye will function both in moonlight and in sunlight, which represents a variation in illumination from perhaps $\frac{1}{100}$ to 10,000 foot-candles—a range of a million to one. Eyes can be forced to work under inadequate and glaring lighting conditions, but the effects of the resulting eyestrain may be translated into nervousness, fatigue, indigestion, and other disorders. Since the optimum amounts of illumination are greater than those which are economically feasible, present practice is to establish minimum values for representative tasks. The eye and seeing processes are so constituted that the illumination must be doubled to achieve a significant and measurable improvement in seeing. Minimum recommended values of illumination are as follows for representative visual tasks. The values are in foot-candles of total illumination.

Industrial Interiors. *Assembly and inspection:* rough, 10; medium, 20; fine, 20 to 100; extra fine, above 100. *Machine shops:* rough bench and machine work, 10; medium bench, machine, buffing, and polishing work, rough grinding, ordinary automatic machines, 20; fine bench, machine, buffing, and polishing work, medium grinding, fine automatic machines, 50 to 100; extra fine bench and machine work, fine grinding, above 100. *Sheet metal works:* miscellaneous machines, ordinary bench work, 15; punches, presses, shears, stamps, welders, spinning, medium bench work, 20; tin-plate inspection, 50 to 100. *Woodworking:* rough sawing and bench work, 10; rough sanding, medium machine and bench work, sizing, planing, gluing, veneering, cooperage, 20; fine bench and machine work, fine sanding and finishing, 30 to 50.

Office Interiors. Corridors and stairways, 5; reception rooms, conference rooms, vaults, 10; mailrooms, file and index rooms, 20. *Desk work:* intermittent reading and writing, 20; reading blueprints and plans, 30; stenographic, reading shorthand notes, 30 to 50. *Art and drawing work:* rough sketching and drawing, 30; prolonged close work, computing, studying, drawing, drafting designing, 30 to 50. *Business machines:* (transcribing and tabulating) calculators, keypunch, bookkeeping, 50 to 100.

Supplementary Lighting. Severe visual tasks require higher levels of illumination than are practically obtainable with general lighting systems alone. It is therefore necessary to resort to supplementary lighting units. For every 10 foot-candles of supplementary lighting, at least 1 foot-candle of general illumination is needed if the ratio between task brightness and background brightness is not to become excessive and defeat the purpose of the greater illumination.

Electric Illuminants

Electric lamps convert electrical energy into light or radiant energy. The efficiency of the conversion is usually expressed in lumens per watt (lpw). As previously indicated, it is 620 lpw for a source that radiates all its energy at 5560Å wave length; it is approximately 200 lpw for a "white light" source that radiates all its energy equally at every visible wave length. These are optimum values; actual efficiencies are considerably less than these amounts.

Filament Lamps

Table 1 gives the lamp efficiencies, in lumens per watt, of past and present electric illuminants. The most important are those which contain tungsten

points fall opposite the supporting spider or other obstruction that might affect the air stream.

Sling or Hand-aspirated Psychrometer. This instrument is used to determine the wet- and dry-bulb temperature of air. These values for outside air should be recorded with every ventilation system volumetric test, in case it is desired to correct to standard air. It is also used in air-conditioning testing, in which case a psychrometric chart should be used to convert the readings to humidity values (see Fig. 1).

Barometer. The atmospheric pressure as indicated by the barometer should be recorded for volumetric tests, to permit correction to standard air if desired.

Thermometers. If a vessel's heating system is tested, straight mercury thermometers, with range of 0 to 120 deg, 32 to 180 deg, or 32 to 212 deg F are generally used in the spaces and locations required.

Pitot Tube. This is the standard instrument for measuring pressures in ventilation ducts. It consists of two concentric tubes, the outer ends of which have connections for a U-tube manometer, or differential pressure gage. Within the duct, the open end of the inner or impact tube is turned into the air stream. The outer or static tube has small holes drilled in the sides only.

If the impact tube is connected to one leg of a U-tube manometer, the other leg of the manometer being open to the atmosphere, the manometer will indicate the **impact pressure,** P_t. If the static tube is connected to one leg of a manometer, the other leg of the manometer being open to the atmosphere, the manometer will indicate the **static pressure,** P_s. If the impact tube is connected to one leg of the manometer and the static tube is simultaneously connected to the other leg of the manometer, the manometer will indicate the **velocity pressure,** P_v. The velocity pressure, measured in inches of water, is converted into velocity V, in fpm, by the following relation:

$$V = 1{,}096.2\sqrt{\frac{P_v}{d}} \text{ where } d = \text{density of air, lb per cu ft.}$$

Knowing the velocity and the sectional area A of the duct, the air volume Q, in cfm, is calculated by the relation $Q = A \times V$.

The pitot tube is used to determine the air flow Q in the main duct, the air delivery of the fan, and also the static pressure of a system or part of a system. A traverse should be made when measuring any but very small ducts. A small hole, about ½ in. diam, is provided in the duct for inserting the pitot tube at a section where fairly uniform flow is expected.

To determine the **total pressure,** P_t, of a fan, the pitot tube is inserted in the suction duct as shown in Fig. 39, and the suction impact pressure P_{it} is read, using the **impact tube.** The pitot tube is then inserted in the fan discharge duct as shown in Fig. 39, and the discharge **impact** pressure, P_{ot}, is read, using the impact tube. The total pressure, P_t, of the fan is the **sum** of the suction and discharge impact pressures, or $P_{it} + P_{ot}$.

To determine the static pressure, P_s, of a fan, the pitot tube is inserted in the fan suction duct as shown in Fig. 40, and the suction impact pressure, P_{it}, is read, using the impact tube. The pitot tube is then inserted in the fan discharge duct as shown in Fig. 40, and the discharge static pressure, P_{os}, is read, using the static tube. The static pressure, P_s, of the fan is the **sum**

Table 1. Relative Luminous Efficiencies of Electric Illuminants

	Lpw	Lumens	Life-hours
Edison's lamp	1.5		
Carbon lamps	3.4		
Gem lamps (metallized carbon filament)	4.25		
Tantalum lamps	4.8		
Osmium lamps	5.9		
Tungsten lamps (first)	7.85		
Mazda B (vacuum) lamps:			
6-watt	6.6	40	1,500
25-watt	11.0	275	750
Mazda C (gas-filled) lamps:			
40-watt	11.6	460	1,000
60-watt	13.9	830	1,000
100-watt	16.3	1,650	750
200-watt	18.5	3,700	750
300-watt	19.8	5,950	750
300-watt	19.1	5,730	1,000
500-watt	20.0	10,000	1,000
1,000-watt	21.0	21,000	1,000
1,500-watt	22.2	33,500	1,000
5,000-watt	32.7	165,000	75
Fluorescent Mazda lamps (daylight):			
15-watt	33.0*	495	2,500
20-watt	36.5*	730	2,500
30-watt	40.0*	1,200	2,500
40-watt	42.0*	1,700	2,500
Fluorescent Mazda lamps (white):			
15-watt	41.0*	615	2,500
20-watt	45.0*	900	2,500
30-watt	40.9*	1,450	2,500
40-watt	52.0*	2,100	2,500
100-watt	42.0*	4,200	2,000
30-watt green fluorescent Mazda lamp	75.0*	2,250	2,500
85-watt RF (rectified fluorescent)	47.0*	4,000	3,000
350-watt a-c Cooper-Hewitt	19.0	5,600	4,000
385-watt d-c Cooper-Hewitt	16.2	5,200	4,000
Mazda H high-intensity mercury lamps:			
100-watt (AH-4)	35.0*	3,500	1,000
250-watt (AH-2)	30.0*	7,500	2,000
400-watt (AH-1)	40.0*	16,000	2,000
1,000-watt (AH-6)	65.0*	65,000	75
180-watt sodium (NA-9)	55.5*	10,000	3,000

* Watts necessarily consumed in auxiliaries reduce these lpw values about 15 to 30 percent depending upon lamp type and size and the type of auxiliary equipment used.

filaments. These filaments radiate light by reason of their high temperature and are similar to black bodies that radiate energy in proportion to the fourth power of their absolute temperature. Tungsten is favorably selective, however, and differs from black-body radiators in that it radiates a greater proportion of the total energy in the visible spectrum.

Filaments in lamps used for illumination are made of very pure tungsten. Both tungsten and carbon are used in lamps for infrared applications. Although carbon melts at 6510 F and tungsten at only 6100 F, the latter has a much lower rate of evaporation and may be used effectively at higher operating temperatures. The efficiency of tungsten-filament lamps increases as the temperature of the filament is raised and as the wattage is increased, especially with gas filling and the larger sizes of filaments. Most filaments are wound or coiled into a helix so that their characteristics are similar to a large-diameter filament and advantage may be taken of gas-filled operation.

of the suction impact pressure, P_{it}, and the discharge static pressure, P_{os}, or $P_{it} + P_{os}$.

If no suction duct is fitted on the fan, suction readings are zero. The total pressure, P_t, of the fan $= P_{ot}$; and the static pressure, P_s, of the fan $= P_{os}$.

Differential Pressure Gage. A U-tube manometer, or other differential pressure gage, having a range of 0 to 5 in. water, is suitable for use with a

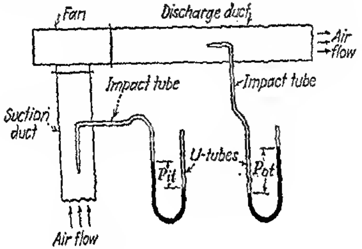


Fig. 39.—Fan total pressure.

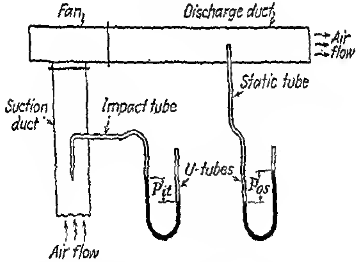


Fig. 40.—Fan static pressure.

pitot tube. If low velocities are to be measured, an inclined-tube manometer, or differential gage, with a range of 0 to 1 in. water, will permit greater accuracy in this range.

Tachometer. The fan speed should always be checked before taking air readings. A tachometer reading directly in rpm is best, but a speed counter, if timed with a stop watch, is satisfactory. An extension shaft is useful for fan openings protected by wire mesh. Speeds should never be

In some lamps, especially of lower wattage, this helix is formed into another, resulting in the so-called coiled-coil construction. This further increases the apparent diameter, shortens the apparent length of the filament, reduces the number of filament supports, and increases the efficiency. Filaments and filament mountings are adapted to specialized services such as those in which vibration or severe mechanical shock is present. Collector screens are employed in high-wattage small-bulb lamps to collect the vaporized filament particles which otherwise form a light-absorbing blackening on the bulb.

The primary purpose of the glass **bulb** as part of a lamp is to preserve the atmosphere around the filament, whether it be a nearly perfect vacuum or one of inert gases. If air or moisture is present in a lamp, the filament will be quickly oxidized and destroyed. Inert gases are introduced to reduce the evaporation of the filament. Although the gas conducts some heat away this loss is more than offset by the higher temperatures at which the filament may be operated. Bulb shapes and the type of glass used are modified for the intended service of the lamp. Softglass vacuum lamps and hardglass gas-filled lamps may be used outdoors without protection, but softglass gas-filled lamps are subject to bulb fracture if struck by rain, insects, or other moisture while hot. Hard glass is used in high wattage small-bulb lamps. Bulbs are available in clear and inside-frosted glass which has a specially acid-etched inner surface to break up the extremely directional flux from the filament without appreciable absorption. Less efficient are the coated bulbs in white and the coated or natural (transparent) colored bulbs which transmit the radiations of desired wave length and absorb all others. The directional distribution of light from a lamp may be controlled by incorporating into the bulb the desired lens or prism elements or by processing parts of the bulb with a metallic coating. Mazda projector and Sealed Beam lamps employ both of these control principles.

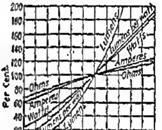


FIG. 3.—Operating Characteristic Curves for Large Gas-filled Filament Lamps.

The more common lamps have **threaded or screw bases** of the miniature, candelabra, intermediate, medium, and mogul sizes. The **bayonet base** is best represented by most automotive lamps. Those used in the headlights prior to the development of the Sealed Beam system were usually equipped with a **prefocusing** collar similar to that used on all lamps for accurate projection service. Most hard glass lamps have **bipost bases**; tubular lamps have screw, disk, or pin, depending upon the lamp design and application.

Filament-lamp Characteristics. Most lamps are designed for constant-voltage or multiple-circuit operation. Figure 3 shows the effect of other than designed voltage operation upon the characteristics of filament lamps. Although the positive resistance characteristic of the tungsten filament reduces the variation in wattage in comparison to the operating voltage, the change in lumen output of these sources is considerably greater than the wattage or voltage variation and a decrease in operating voltage results in a decreased operating efficiency.

Cost of Light. The operating voltages of filament lamps are designed as economical compromises between high operating efficiency and reasonable

taken with a tachometer on the blade side of a centrifugal or propeller fan, unless there is wire-mesh protection. There are some axial fans on which a tachometer cannot be used; it is poor practice on practically all axial fans, owing to the danger of injury and to the fact that opening the access door affects the speed.

Ammeter and Required Shunts. Electrical current readings should be taken with fan running at rated speed and full air delivery.

Voltmeter. Voltage at the fan should be read at the same time that the current is measured.

Strobotac. Speed readings on axial fans should be taken with a strobotac through an opening in the duct provided for the purpose and covered with transparent material. This is an instrument which utilized the stroboscopic principle to determine the rpm. Owing to the difficulties involved in this method, it is generally used only when a study of a fan is made. For routine regulation, the rheostat on an axial fan is usually set at a point suggested by the fan manufacturer or at rated current, on the assumption that this will give approximately rated speed.

Procedure. Before running any tests, the purpose to be achieved should be definitely ascertained and the desired accuracy decided upon. It is both wasted effort and poor engineering to apply corrections of 1 or 2 percent to one factor when other factors involved have errors of 10 or 20 percent. The accuracy required in any set of tests depends on the purpose of those tests. A study of a fan or system to determine its characteristics requires the greatest accuracy attainable; adjustment of air-conditioning systems calls for accurate balancing and air quantities within about 10 percent of specified. This accuracy is required of tests. Most straight ventilation systems require close balancing with only minor regard to the absolute air quantities.

In most shipwork, the purpose of tests is to show that the ventilating system is correctly balanced and to demonstrate its ability to function properly in service. Except for cases in which set conditions or a balance is required within a given space, this does not require that absolute air quantities be held down to the design figures. It is not sound practice, if a system is capable of delivering more than specified air volume without overloading the fan motor, to restrict it by means of regulating dampers which are permanently set, merely to show a test reading close to specifications. There are few cases in which some additional air is not advantageous under certain cooling load conditions. It is preferable, therefore, to allow a system, when permanently adjusted, to deliver its full capacity and leave it to the ship's personnel to reduce the air delivery by slowing the fan or closing the dampers provided for this purpose. In any case, if absolute air volumes are to be obtained, ship operating conditions must be maintained throughout all tests. This means giving over large sections of the ship to ventilation tests exclusively, a condition which is usually impossible during the outfitting period. It involves closing all hatches, cargo ports, air ports, doors inside and out; operating with ship's current all fans affecting a given section of the ship, and all fans in every fan room involved. Thus, since generally it is neither warranted nor feasible to adjust a straight ventilation system to specified air deliveries, the following procedure should be followed in balancing and testing such a system. Anemometer, stop watch, and velometer for diffusers will be used.

From cfm figures and areas in square feet as given on design drawings, determine the velocity in fpm for each terminal when delivering design air

lamp life which, for large gas-filled filament lamps, is inversely proportional to approximately the 13.1 power of the operating voltage (*B. of S. Paper* 502, "Characteristic Equations of Vacuum and Gas-filled Tungsten-filament Lamps)." The total cost of light in dollars per million lumen-hours depends upon the cost of the lamp delivered to the socket P (cents), the rate at which electrical energy is available R (cents per kwhr), and the light output of the source in lumens per watt E. The cost of light, in dollars per million lumen hours $= \frac{10}{E}\left(\frac{P}{WL} + R\right)$ where W is the average watts consumed by the lamp, and L is the rated life of the lamp in thousands of hours.

Vapor Lamps

During operation, the tube of a vapor lamp is filled with vapor molecules, ions, and electrons. Each gas or vapor has its own characteristics as regards color of light emission, such as blue-green for mercury, yellow for sodium, and red-orange for neon. The discharge takes place through the gas or vapor and between electrodes which are charged to differences in potential. Lamps with electrodes that are not preheated before starting are known as **cold-cathode** lamps. The electrode that has a negative potential is called the cathode; since most electrical illuminants are used in a-c circuits, each electrode is alternately a cathode and an anode. If an electrode is heated, it emits electrons more freely and these may be sufficiently accelerated by a relatively small potential to ionize the gas or vapor and establish the discharge. The electrodes of these **hot-cathode** lamps are frequently coated with oxides to provide greater electron emission at low temperatures and potentials. Coated electrodes are also used in cold-cathode lamps to provide electron emission and improve the potential gradient along the discharge.

Because of the potential gradient along the axis of a discharge, the electrodes are subject to bombardment and heating. The heated electrodes and the collision of molecules, ions, and electrons in the arc liberate additional ions and electrons in excess of the number necessary to maintain the desired current. The effect is that of a short circuit; the current tends to increase, and the result would be the destruction of the electrodes and the discharge tube were a current limiting device not used in series with these sources. Each lamp requires an auxiliary designed for its characteristics, and, in most cases, only one lamp may be operated from a single auxiliary. Because inductive devices are commonly used to limit the current in a vapor-lamp circuit, power factor becomes a consideration. The lamps, themselves, usually operate at a high power factor but a considerable reduction in this factor occurs in the auxiliary. Capacitors are generally employed with the auxiliary to improve the power factor.

Vapor lamps have long life, the length being generally determined by the life of the cathodes. These are reduced in emissivity during lamp operation, and finally the available potential is insufficient to establish the arc discharge under these conditions of reduced emission.

The **Cooper-Hewitt mercury-vapor lamp** was the first practical high-efficiency electric vapor lamp. The tube is of the cold-cathode type and depends upon an initial potential surge of several thousand volts to start. During operation the lamp remains cool and there is but slight increase in pressure. The light from these lamps is blue-green in appearance.

By increasing the operating temperature and pressure, the relative distribution of energy in the mercury spectrum is modified, and in some of the

and is called a **diameter** (Fig. 13). If M is the mid-point of a chord PQ, and if T is the point of intersection of the tangents at P and Q, then TM is parallel to the principal axis, and is bisected by the curve (Fig. 13).

To Construct a Tangent to a Given Parabola. (1) At a given point of contact, P (Fig. 14): Find T so that $OT = OM$, or $FT = FP$. Then TP is the tangent at P. Or, make $MN = p = 2(OF)$; then PN is the normal at P.

(2) From a given external point, Q (Fig. 15): With Q as center and radius QF draw circle cutting the directrix in H; draw HP parallel to principal axis; then P is required point of contact. As check, note that QP is the perpendicular bisector of FH.

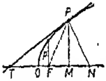


Fig. 14.

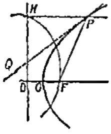


Fig. 15.

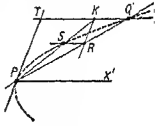


Fig. 16.

To Construct a Parabola. 1. Given Any Two Points, P and Q, the Tangent PT at One of Them, and the Direction of the Principal Axis OX. In Fig. 16, let K be a variable point on a line through Q parallel to OX. Draw KR parallel to PT (meeting PQ in R), and draw RS parallel to OX (meeting PK in S); then S is a point of the curve. Note. A line through P parallel to the principal axis bisects all chords parallel to the tangent PT.

2. Given the Vertex O and Focus F. (a) In Fig. 17 draw Oy perpendicular to OF, and slide the vertex of a right angle along Oy so that one side always passes through F; then the other side will always be a tangent to the parabola.

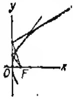


Fig. 17.

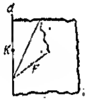


Fig. 18.

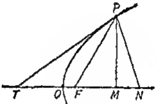


Fig. 19.

(b) Take a piece of paper (Fig. 18) with a straight edge, d, and mark a point F near the edge. Let K be a variable point of the edge, and fold the paper so that K coincides with F. The crease will be a tangent to the parabola which has focus F and directrix d.

(c) In Fig. 19, let M be a variable point of the principal axis, and lay off $MN = 2(OF) = p$. With F as center and radius FN draw a circle, cutting the perpendicular at M in P. Then P is a point of the curve, and PT and PN are the tangent and normal at P.

3. Given Two Tangents and Their Points of Contact, P and Q (Fig. 20). Divide TP and QT into any number of equal parts (here 4). Then the lines 11, 22, 33, . . . will be tangents to the parabola. This method is especially advantageous in drawing rather flat arcs.

The Radius of Curvature of $y^2 = 2px$ at a point $P = (x,y)$ is $R = (p + 2x)^{3/2}/\sqrt{p}$, or $R = p/\sin^3 v$, where v = the angle which the tangent at P makes with PF (Fig. 21). At the vertex, $R = p$. To construct the radius of curvature at any point P, lay off $PR = 2(PF)$ parallel to the principal axis, and draw RC perpendicular to the axis, meeting the normal, PN, in C. Then C is the center of curvature for the point P, and a circle about C with radius CP will coincide closely with the parabola in the neighborhood of P.

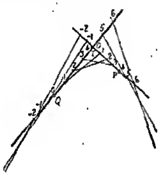


Fig. 20.

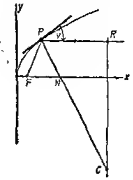


Fig. 21.

THE ELLIPSE

The ellipse (see also p. 107) has two **foci**, F and F' (Fig. 22), and two **directrices**, DH and $D'H'$. If P is any point of the curve, $PF + PF'$ is constant, $=2a$; and PF/PH (or PF'/PH') is also constant, $=e$, where e is the **eccentricity** ($e<1$). Either of these properties may be taken as the definition of the curve. The relations between e and the semi-axes a and b are as shown in Fig. 23. Thus, $b^2 = a^2(1 - e^2)$, $ae = \sqrt{a^2 - b^2}$, $e^2 = 1 - (b/a)^2$. The **semi-latus rectum** $= p = a(1 - e^2) = b^2/a$. Note that b is always less than a, except in the special case of the circle, in which $b = a$ and $e = 0$.

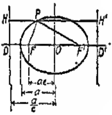


Fig. 22.

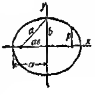


Fig. 23.

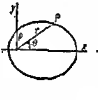


Fig. 24.

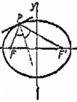


Fig. 25.

Any section of a right circular cone made by a plane which cuts all the elements of one nappe of the cone will be an ellipse; if the plane is perpendicular to the axis of the cone, the ellipse becomes a circle.

Equation of Ellipse, center as origin:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ or } y = \pm \frac{b}{a}\sqrt{a^2 - x^2}.$$

If $P = (x, y)$ is any point of the curve, $PF = a + ex$, $PF' = a - ex$.

Equations of the Ellipse in Parametric Form: $x = a \cos u$, $y = b \sin u$, where u is the eccentric angle of the point $P = (x,y)$. See Fig. 28.

present type H lamps, a weak continuous spectrum is obtained in addition to the distinct line spectrum. Because of the short arc length, these lamps operate at relatively low voltage and by adding a relatively small amount of argon they are also started at relatively low voltage. In some lamps the arc strikes between one of the main electrodes and a special starting electrode. The preliminary discharge then generates sufficient heat to vaporize the mercury and permit the mercury discharge to strike between the main electrodes. After several minutes' operation, the vapor pressure and the light output reach their maximums. A momentary current interruption or a sudden drop in voltage (without current interruption) will extinguish the discharge which will not strike again until the lamp has cooled and the vapor pressure has been reduced. This requires from a few seconds to 10 to 15 min depending upon the type of lamp. These lamps are comparable in size with filament sources and are adaptable to lighting equipment of similar design.

Sodium-vapor lamps are hot-cathode sources and contain neon to facilitate starting. The heat from the neon discharge vaporizes the sodium which then carries the current of the main discharge. The light is monochromatic and 5893Å in wave length. Lamps are available in 4,000, 6,000, and 10,000 lumen sizes and, although not restricted to highway and street lighting, have been used mostly in these applications. The over-all efficiency of sodium lamps and auxiliaries is approximately 45 lpw.

Fluorescent lamps are primarily low-pressure vapor lamps which make use of the ultraviolet energy from the mercury-arc discharge to activate phosphors coated on the inside bulb surface. Inside coating is essential because the glass bulb does not transmit ultraviolet radiations. Phosphors are selected which respond to radiations of 2537Å wave length as this radiation is efficiently produced by the low-pressure mercury-arc discharge. The color produced is determined by the atomic and crystalline structure of the phosphor. These lamps are hot-cathode types, a current being passed through both electrodes before they are subjected to the arc potential. They are especially efficient in the production of colored light and include one that closely approaches "daylight" at a color temperature of 6500 K.

RF lamps also contain fluorescent powders which are highly efficient in light production although not as accurately balanced for color quality as those used in Mazda F lamps. These RF lamps are a development of the former Cooper-Hewitt lamps and operate on a rectified current circuit similar to that of these earlier sources.

High-voltage Tubing. In 1915, Georges Claude patented his discovery that, for satisfactory operation, at least 23 sq in. of untreated electrode surface area are required for every ampere of current carried by high-voltage lamps of the neon type. This patent has expired and, in commercial practice, electrodes with areas of 150 to 180 sq in. per amp are used. Most high-voltage discharge tubing is of the cold-cathode type, although recent advances in the technique of using hot cathodes will probably make them of considerable importance in the future. Table 2 gives the important operating characteristics of the available sizes of these discharge tubes.

Arc Lamps

The use of **carbon electric-arc lamps** is today restricted to applications such as projection, searchlighting, and photochemical work. Arcs are efficient, rugged, and concentrated sources, the light from which can be controlled to a considerable degree in intensity, distribution, and color.

This type face is 8-point Bodoni.

The task is viewed through filters which are adjusted until the task is just discernible. At this condition, the visibility of the task may be compared with the visibility of the standard task and a specification made for the illumination necessary to make the difficulty of test and standard tasks equal. The visibility factors of size, brightness, and contrast are integrated by this meter. Since brightness is a factor of the integration, the meter evaluates the effectiveness of the illumination as well as indicates the necessary amount. It may, for instance, show that 10 foot-candles of suitable quality lighting are as effective as 100 foot-candles where the quality has been neglected.

The most frequently used foot-candle meter is the **Light Meter** which consists of a photovoltaic cell and a microammeter calibrated in foot-candles. It is a direct-reading instrument designed primarily for measuring illumination but may also be used to determine brightness, reflection factors, and transmission factors. Other portable visual instruments such as the **Macbeth illuminometer** are used in measuring candle power, illumination, and brightness; these depend upon visual observations, and accuracy depends on the observer's experience in "balancing" the brightness of screens as used in visual photometric instruments.

Several **brightness meters** are available. One of these devices contains a telescopic optical system so that readings may be taken from convenient locations. Brightness meters are visual comparison instruments and are not as easy for the layman to use as are the foot candle meters which are usually of the photoelectric type.

For d-c crater arcs, about 90 percent of the light is emitted from the crater.

Table 2. Operating Data for High-voltage Discharge Tubing*

Included is information as to the number of feet of tubing that may be operated from a *single transformer of specified characteristics.*

Transformer specifications					Color of tubing															
					Red						Blue						White or gold			
Secondary volts	Secondary milliamperes short-circuited	Volt-amp	Operating primary watts	Primary amp, open	Diameter of tubing, mm															
					7	9	10	11	12	15	7	9	10	11	12	15	9	10	11	12
15,000	60	875	400	8.0	...	28	32	36	43	60	....	34	38	44	54	70	..	13	18	23
15,000	30	450	210	4.0	...	27	31	34	42	58	....	32	36	42	50	68	9	12	16	22
12,000	30	350	175	3.2	...	21	24	28	33	46	....	26	29	34	39	54	7	9	12	17
12,000	25	280	145	2.5	...	18	23	25	30	40	....	22	27	31	36	49	..	7	10	15
9,000	30	280	130	2.5	...	14	17	19	22	32	....	18	20	23	28	40	..	6	9	12
9,000	18	200	80	1.8	9	12	15	17	20	28	11	15	18	20	24	34				
7,500	30	245	100	2.2	...	9	12	15	17	21	....	12	15	18	21	26				
7,500	20	150	72	1.3	7	9	12	15	17	..	8	12	15	18	21					
7,500	18	140	70	1.2	7	9	12	15	17	.	8	12	15	16	21					
6,000	30	170	86	1.5	...	....	9	11	13	17	....	....	11	13	15	22				
6,000	20	130	60	1.1	6	7	9	11	13	17	7	9	11	13	15	22				
5,000	30	160	70	1.4	...	....	8	9	11	15	....	....	9	11	13	18				
5,000	20	100	50	0.9	...	6	8	9	11	15	....	....	9	11	13	18				
5,000	18	95	48	0.8	5	6	8	9	11	15	6	8	9	11	13	17				
4,000	30	130	60	1.1	...	5	6	7	8	12	....	....	8	9	10	14				
4,000	20	75	38	0.6	...	5	6	7	8	...	....	6	8	9	10					
4,000	18	70	36	0.6	...	5	6	7	8	...	....	6	8	9	10					
3,500	18	70	35	0.6	2	3	4	5	6	..	3	4	5	6	7					
3,000	20	55	30	0.5	1.5	2	3	4	5	..	2	3	4	5	6					
2,000	20	45	22	0.4	1	1.5	2	3	4	..	1.5	2.5	3	4	5					

* From "Neon Signs" by Miller and Fink.

In **incandescent arcs**, the tip of the positive carbon takes on the form of a crater and is heated to incandescence. When carbons of this type are used in a-c circuits, each electrode forms a small crater but the light emitted from both does not equal the emission from the single large crater in d-c operation. Forsythe ("Measurement of Radiant Energy," McGraw-Hill) rates low-intensity d-c incandescent arcs at 15 lumens per arc watt, high intensity at 29, and superhigh intensity at 32; this corresponds to 12, 23, and 25 (respectively) lumens per watt over all.

In **luminescent or flame arcs**, most of the light is produced in the arc stream just as in the mercury-vapor lamp. With carbon impregnated with certain salts, the arc stream becomes the light source and the incandescence of the electrodes is comparatively unimportant. With d-c white-flame arcs with ½ in. (⅝ in.) diameter carbons, the performance is 57 (80) lumens per watt input. Since the light is produced in the arc stream, these carbons may be used in a-c circuits with transformer ballasts instead of resistances. The over-all efficiency for these arcs is 53 lpw for ½ in. and 51.1 for ⅝ in. carbons. Carbon arcs give a concentrated high-intensity light which cannot as yet be equaled by any other source. Table 3 gives the characteristics of some carbon arcs.

SOUND AND NOISE

BY

F. V. HUNT

From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

Sound is a longitudinal wave phenomenon representing the propagation of compressional waves in an elastic medium. The **velocity of propagation** V of sound waves depends on the ratio of the elastic modulus E to the density ρ of the medium, $V = \sqrt{E/\rho}$; thus $V = \sqrt{\frac{kp}{\rho}}$ for gases where k is the ratio of specific heats and p is the pressure. In air or other gases, the wave velocity is independent of the pressure, changes in density being just compensated by changes in elasticity. The velocity increases slightly with increasing relative humidity and varies directly as the square root of the absolute temperature. Thus the **normal velocity of sound in air, 1,126 fps,** increases by 0.1 percent per deg F at 68 F. The velocity of sound in liquids and solids is usually much higher than in gases (see Table 1).

Table 1. Velocity of Sound

Material	Sound vel, fps	Density, lb per cu ft	Density × velocity, lb per sq ft per sec
Aluminum	16,740	168	2.82×10^6
Brass	11,480	530	6.08×10^6
Copper	11,670	555	6.47×10^6
Iron and soft steel	16,410	486	7.98×10^6
Lead	4,026	1125	4.54×10^6
Brick	11,980	125	1.5×10^6
Cork	1,640	15	0.025×10^6
Wood	10,000–15,000	30–50	0.3×10^6–0.75×10^6
Water	4,794	62.4	0.299×10^6
Air, dry, CO_2 free, 32 F	1,088.5	0.0808	88.0
Hydrogen	4,165	0.00560	23.3
Water vapor, 212 F	1,328	0.0372	49.4

Approximate values from Smithsonian Tables.

The relation between **frequency** and **wave length** is, velocity = frequency × wave length. In air, at a frequency of 1,126 cycles per sec, the wave length is 1 ft.

The attentuation of sound **in transmission** through air is small. At high frequencies (above 3,000 cycles) and for low values of relative humidity, absorption in air may become large (Knudsen, *J. Acous. Soc. Am.*, 6, 1935, p. 199), but under ordinary conditions the absorption in air may be neglected. Except for absorption on reflection from material surfaces, sound waves are diminished in intensity principally in consequence of the distribution of the energy over wider areas as the sound waves are propagated from the source.

Perception of Sound

Audible Range. Sound waves having frequencies lying between 16 and 20,000 cycles per sec may be perceived as "sound" by the average young observer. Sound waves having frequencies lying below or above this range,

Table 3. Characteristics of Some Carbon Arcs[a]

Low-intensity and high-intensity projection and searchlight carbon arcs—d-c

	Carbon size, mm	Amperes	Volts	Horizontal candle power[b]	Crater light, lumens	Total light, lumens
Low intensity	10	20	55	5 400	16,500	16 500
	12	30	55	8,500	25 900	25 900
	13	40	55	11,900	35,800	35,800
High intensity (Suprex)	6	40	32	8,150	29,500	43 400
(Suprex)[c]	7	50	34	11,600 (30°)	41,200	60,500
(Suprex)	8	65	35	17 100 (30°)	56,200	82,600
Rotating positive	9	75	56	15,900	53,300	81,500
Rotating positive	11	85	58	19,000	65,000	100,000
Rotating positive	13.6	130	76	46,000	169,000	260,000
Rotating positive	16	150	76	62,000	215,000	330,000
Superhigh intensity:						
Rotating positive	13.6	180	75	68,000	249,000	383,000
Rotating positive	16	195	90	88,400 (20°)	294,000	555,000

[a] This table furnished by Research Laboratory of National Carbon Co., Inc., through the courtesy of A. C. Downes.

[b] Horizontal candle power determined directly in front of the crater on the axis of the positive carbon, except where an angle is indicated, in which cases the candle power was measured at this angle with the positive carbon axis. Candle power of crater light only, except for Suprex arcs where values are total candle power.

[c] The positive carbons in these high-intensity arcs are stationary and do not rotate.

Reflectors

Reflecting Materials. To use a reflecting material for light control, it is necessary to know the distribution of light reflected from the surface as well as the value of the reflectance. If the surface is microscopically smooth, the surface is shiny or **specular.** Polished metal and glass surfaces are specular, and the directional quality of the incident light is preserved in the reflected. If a surface is slightly irregular, the reflection from it is spread. The irregularities in depolished, etched, brushed, and stain-finished metallic, glass, and plastic surfaces render the over-all light control only moderately directional. If each element of a surface is positioned at random in regard to the general contour of the surface, the surface is **matte** and the reflection from it is **diffuse.** For any direction, the candle-power distribution and the projection of a unit area of the surface are both proportional to the cosine of the angle between the normal to the surface and the direction being considered; consequently, the ratio of the two is constant and the brightness is the same for all viewing angles. The reflection from magnesium carbonate is nearly perfectly diffused. Other surfaces of high diffusion are those of blotting paper, plaster, concrete, stone, terra cotta, matte porcelain, and matte paint.

No materials are perfectly specular or absolutely diffuse, all have distributions within these extremes. The term **spread reflection** indicates a distribution which is neither predominantly specular nor diffuse. Whereas most materials are characterized by one type of distribution or another, a few, such as shiny porcelain enamel, glazed terra cotta, structural glass, polished marble, and glass paint, have more than one. The major component is diffuse but is accompanied by a highly specular surface reflection that is [illegible] percent of the total reflectance. This latter component causes

called subaudible and supersonic waves, respectively, obey the same laws as audible sound waves. Within the audible frequency range, the ear is responsive to a wide range of intensities. In the most sensitive range, lying between 500 and 5,000 cycles, the ratio of sound intensity which the ear can tolerate to that which it can just detect is approximately 10^{12}. The minimum sound intensity that the ear can detect varies widely with the frequency. At 50 cycles, a sound, to be perceived, must be 10^6 times as intense as at a frequency of 3,000 cycles. A logarithmic scale unit, the **decibel**, is in common use for measurement and comparison of relative sound intensities. The decibel is defined by the statement: The sound intensity I_1 is N decibels higher in "intensity level" than the reference sound intensity I_0 if $N = 10 \log (I_1/I_0)$. One decibel is the minimum change in sound intensity detectable by the typical observer throughout the usual range of frequencies and intensity. The **reference intensity level** (A.S.A. Tentative Standard, 1940) is 10^{-16} watts per sq cm, and for a plane progressive sound wave in air corresponds to a variational pressure of 0.0002 dynes per sq cm. The reference intensity constitutes a sound stimulus which is approximately the minimum audible stimulus for the typical observer. The frequency and intensity range audible to the typical individual is illustrated in Fig. 1 between the extreme lower and upper curves representing, respectively, the threshold of audibility and the threshold at which sound perception merges into a feeling sensation within the ear.

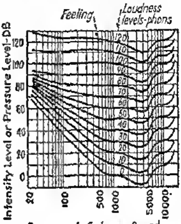


FIG. 1.—Loudness Contours.

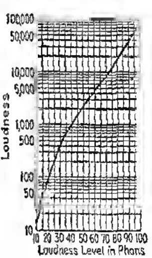


FIG. 2.—Relation between Loudness and Loudness Level.

Quality is a subjective attribute of sound by which equally loud sounds may be distinguished as different in kind. For musical tones, differences in quality arise from differences in the distribution of energy among the harmonics (overtones) of the fundamental frequency. For the unpitched sound constituting random noise, differences in quality represent differences in the distribution of energy in various parts of the acoustical spectrum. Differences in quality affect both the sensation of loudness of noise and its psychological annoyance. Shrill, high-pitched, and irregular sounds are usually judged less pleasant than low-pitched and regular sounds.

The **loudness** of a noise is not correlated simply with physically measureable attributes of the sound waves. To provide a quantitative basis for considering the subjective attribute, loudness, the term **loudness level**, in **phons**, is defined as the intensity level (decibels above standard reference level) of a pure tone having a frequency of 1,000 cycles which is judged by the

reflected images to be seen in these surfaces. Except for special effects, totally matte-surfaced materials are more satisfactory.

Table 4. Reflecting Materials

Type	Material	Per-cent reflec-tion	Type	Material	Per-cent reflec-tion
Specular	Aluminum-alloy film on glass	90-94	Spread	Aluminum Alzak, diffuse	71-80
	Silver plate	87-92		Oxidized and etched aluminum	70-82
	Glass mirror	80-90		Chromium, satin	50-56
	Aluminum foil	84-87		Stainless steel, satin	51-56
	Aluminum Alzak, specular	75-84		Aluminum, brushed	54-57
	Rhodium	70-90	Diffuse	Magnesium carbonate	93-98
	Aluminum, polished	60-72		White plaster	90-92
	Tin	68-71		White paint mat	75-90
	Chromium, specular	62-67		White semimat	72-85
	Stainless steel	55-65		White porcelain enamel mat	60-83
	Nickel	60-63		White and cream terra cotta	60-81
	Monel	57-62		White structural glass	74-49
	Aluminum, mill finish	52-55		Limestone	36-58
	Black structural glass	4-5		Sandstone	20-42
			Specular and diffuse	Porcelain enamel, glossy	60-80

Transmission. Materials that transmit light without modifying the directional distribution of the light are termed **transparent.** Clear fused quartz, clear glass, and clear plastics are of this classification. All other transmitting materials are **translucent** and, according to the distribution of light from the surfaces, are either spread or diffuse. The former are low-

Table 5. Translucent Materials

Type	Material	Per-cent trans-mis-sion	Type	*Material*	Per-cent trans-mis-sion
Concentrated	Fused quartz	91-94	Diffuse	Opalescent glass	55-85
	Glass, clear	80-90		Cased opal glass	30-53
				Alabaster	27-50
Spread	Glass, clear mat	62-89		Solid opal glass	12-40
	Glass, clear configurated	65-87		Enameled glass	27-38
	Plastic, clear mat	60-80		White plastic, diffuse	3-35
	Alabaster glass, configurated	60-70		Marble, impregnated	0-22
			Colored glass	Blue	1-5
				Red	7-15
				Green	10-15
				Amber	40-60

diffusion materials such as frosted, configurated, and alabaster glasses, similar appearing gelatins and plastics, light-density papers, and scrim-type fabrics. The latter group includes homogeneous and cased opal glass,

typical observer to sound as loud as the sound in question. The validity of this definition rests upon the experimental observation that various observers can, with tolerable precision, agree upon the intensity of a pure tone that sounds as loud as a complex noise in question. For pure tones having frequencies other than 1,000 cycles, the "equal loudness" contours of Fig. 1 show the requisite level that will sound as loud as the reference tone of 1,000 cycles. For such pure tones, the loudness scale of Fig. 2 shows the relation between **loudness numbers** and the corresponding loudness levels, in phons. Loudness numbers obtained from this chart behave arithmetically, *i.e.*, two sounds having individual loudness numbers of 1,000 and 2,500 will, when sounded together, have a loudness corresponding to 3,500 loudness units. This chart is not reliable for complex tones having a wide spectral distribution, on account of the phenomenon of masking. The phenomenon of **masking** represents the ability of one tone present at the ear to render the ear incapable of perceiving another tone even though the second tone differs markedly in frequency from the first. The partial deafening of the ear through masking by noise affords a direct quantitative measure of the interfering effect of the noise. If the masking is measured at several frequencies throughout the audible range, the loudness of the noise may be computed; conversely, if the intensity spectrum of the noise is known, the masking at any frequency may be computed (see Fletcher and Munson, *J Acous. Soc. Am*, 9 1937 p. 1).

Noise

Noise embraces those sounds which are audibly unpleasant either by virtue of their loudness, their pitch, or their quality. Most noises are essentially unpitched and include components distributed throughout the audible frequency range. Noise usually arises from the vibration of surfaces in contact with air. When these surfaces are excited periodically, large amounts of energy may occur in particular regions of the frequency spectrum. Typical examples of noises involving regular excitation are the clash of gear teeth, the vibrations induced by unbalance in rotating machinery, the 60 cycle hum in electrical apparatus, and noise induced by the explosions in internal-combustion engines. In addition to the frequency of fundamental excitation, noises generated by such equipment contain also the resonant frequencies of the particular vibrating surfaces involved, such as the gear teeth and the motor frame. In general, small surfaces are sources of high-frequency noises, and large surfaces radiate low frequencies. Irregular excitation of vibrating surfaces, as by hammer blows or by scraping, gives rise to noises whose frequency spectrum is chiefly determined by the vibrational properties of the surfaces themselves. Hissing sounds, as of gases escaping from an orifice or originating in turbulence produced by irregularities in ducts, predominate in high frequencies and are, in general, very complex.

Noise measurements are usually carried out by means of a **sound-level meter.** This is a device involving a microphone, a vacuum-tube amplifier, and an indicating or recording instrument. The microphone is usually responsive to the sound pressure existing at the microphone. No instruments are available for objective measurements of the subjective loudness or of the sound energy flow in any sound fields except ideally simple ones. The human ear being also responsive to pressure, it is appropriate to employ a pressure-responsive instrument in estimating the objectionable amount of noise present. Bulletin Z-24.3 (1936) of the A.S.A. describes "American Tentative Standards for Sound Level Meters," and instruments meeting these specifica-

enameled glass, dense plastics, treated marble, heavier weights of paper, and treated closely woven fabrics.

Reflector Contours. Various geometric forms (Fig. 4) are employed for controlling the flux distribution from a source. In general, surfaces of revolution are used for illuminants that approach a "point source" condition and extended troughs for the linear sources. The cross section form of the surface defines the surface. All rays intercepted by a *parabolic* surface, from a point at the focus of the surface, are redirected parallel to the axis of the surface. The rays from a focus of an *elliptical* surface are redirected by the surface through the other focus of the ellipse. The rays of light from a source at the focus or center of a *circular* contour are redirected through the source. If the source is moved toward the circular contour, the resulting distribution is a combination of the characteristic distributions from parabolic and elliptical surfaces. Combinations of these basic forms may be made to obtain nearly any type of flux distribution. The distribution of light from these surfaces is not only a function of the surface contour, but also of the character of the

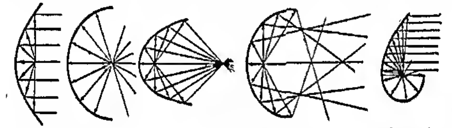


FIG. 4.—Geometric Forms Used as Reflector Contours.

surface. Accurate control is obtained only with specular surfaces; if matte surfaces are used, the contour is ineffective in controlling the light distribution although it may have a decided influence upon the total light output.

Selection of Luminaires. The luminaire, or lighting fixture, must have a distribution to provide the required amounts of horizontal, vertical, and oblique surface illumination; to reduce the harshness of shadows as much as practical; and to conform with the proportions of the room in which the installation is to be used. The brightness of the lower hemisphere should not produce direct and reflected glare. When totally or semi-indirect luminaires are used, the ceiling is the apparent source of light and may become objectionably bright when high values of illumination are provided. Combinations of general and supplementary systems are used to provide ranges of foot-candles above those available with general lighting only. In commercial interiors, downlighting is frequently used, and since it is usually designed for specific limited areas, it is considered and treated as a type of supplementary lighting.

Calculation of Illumination

Point-by-point Method of Illumination Calculation. The illumination provided by a luminaire may be calculated from a candle-power distribution curve as in Fig. 5. The calculations are based upon the "inverse square law" and are valid only for the point in consideration; the method is

tions have been produced by Electrical Research Products, Inc., General Radio Co., General Electric Co., and others. These instruments measure the total sound pressure level at the microphone, including all-frequency components, and indicate or record the result in decibels referred to the standard reference level. The **sound analyzer** is an instrument for obtaining or recording the pressure of the sound waves at each frequency. By its use, the outstanding components of a heterogeneous noise can be readily identified and can frequently be correlated with periodic excitations occurring in the apparatus producing the noise. Noise measurements serve important functions in controlling the design of new apparatus or machinery and in providing routine inspection tests for finished products. In specific applications, it is frequently observed that one portion of the frequency spectrum may be more indicative than another as a criterion of specific troubles. For these applications, the functions of the sound analyzer may be supplemented or replaced by a set of **electrical filters** which can be introduced in the amplifier of the sound-level meter and which permit the noise lying within restricted frequency ranges to be observed independently.

In making determinations of the frequency of outstanding components of a complex noise, it is convenient to employ an auxiliary sound source of variable frequency such as a **beat-frequency oscillator** connected to head telephones or a small loudspeaker. The frequency of the oscillator is varied until slow beats are observed between the variable source and the component of the noise in question.

In identifying the vibrating surface of a piece of machinery responsible for a particular component of the disturbing noise, a **vibration pickup** may be used to explore the surface of the machine. The pickup may be of the piezoelectric crystal type or the electromagnetic type and may usually be substituted for the microphone at the input of the sound-level meter. The

Table 2. Typical Sound Levels

Decibels	
—120	Threshold of feeling
	Thunder, artillery
Deafening—110	Nearby riveter
	Elevated train
—100	Boiler factory
	Loud street noise
Very loud — 90	Noisy factory
	Truck unmuffled
— 80	Police whistle
	Noisy office
Loud — 70	Average street noise
	Average radio
— 60	Average factory
	Noisy home
Moderate — 50	Average office
	Average conversation
— 40	Quiet radio
	Quiet home or private office
Faint — 30	Average auditorium
	Quiet conversation
— 20	Rustle of leaves
	Whisper
Very faint— 10	Sound-proof room
	Threshold of audibility
— 0	

any point, the candle power at the point P is determined from the distribution curve. The illumination I in foot-candles at P and normal to SP at the distance D ft is $E = I/D^2$. By **Lambert's law**, the illumination on a surface is proportional to the cosine of the angle of the ray with the normal to the surface; the illumination at P, but normal to OP, $= I \cos PSO/D^2 = I \cos^3 PSO/h^2$.

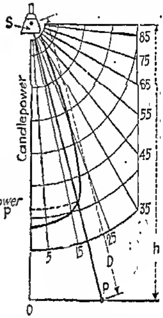


FIG. 5.—Illumination at Any Point by a Single Luminaire.

Equipment Spacing and Layout. By calculating the illumination at representative points across OP, it is possible to determine the "trend" of illumination across OP. This will establish the illumination distribution provided by a single unit. If other units are added, the illumination at each point is the sum of the components from each unit. This is represented in Fig. 6 in which the spacing of units has been made twice the mounting height and a 7 to 1 variation in illumination results. A mounting height-spacing ratio of 1 to 1, as shown in Fig. 7, gives more uniform illumination and reduces shadows. The only practical method of gaining a wider spacing distance without detrimental effect upon the illumination distribution is to increase the mounting height. For a range of ceiling heights, Table 6 gives typical spacing distances and mounting heights for direct-lighting units and suspension lengths of indirect units. Closer spacings than those in this table are often desirable either to obtain a symmetrical layout in regard to arrangement of bays, etc., or for equipments of more concentrating distribution.

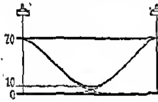


FIG. 6.—Non-uniform Illumination Resulting from Wide Spacing of Units.

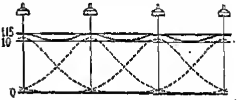


FIG. 7.—Uniform Illumination Distribution.

Lumen Method of Illumination Calculation. The point-by-point method of illumination calculations has no provisions for determining those components of the work-plane illumination not directly incident from the luminaire; the method is most valuable in determining the illumination from concentrating direct-lighting equipment such as downlights. The coefficient of utilization is the ratio of all lumens (direct or indirect) available on the work plane to those emitted by the lamps.

Large rooms use light more efficiently than small ones; in the large ones, the ratio of wall area to floor area is less and proportionately less light is

vibration amplitude is indicated directly on the sound-level-meter scale. Although not adapted to quantitative measurements, a physician's stethoscope (sometimes modified by attaching a probe rod to the center of the diaphragm) provides a sensitive detector for low-frequency vibrations.

Great care should be exercised in the maintenance of noise-measuring equipment, and attention should be given to selecting equipment that will remain unaffected by variations in temperature and humidity. When several sources of noise contribute to a complex sound, great precision is required to detect the change in the total noise produced by alteration or removal of a single component; *e.g.*, two noise sources, each capable of producing a sound level of 90 db, together produce a sound level of only 93 db. Conversely, in a composite noise produced by two noise sources, the complete quieting of one of these will produce only 3 db reduction in the measured sound level (corresponding to approximately 20 percent reduction in the subjective loudness).

Typical sound levels are shown in Table 2.

Noise Control

Control of noise is secured by following one or more of three different procedures. The primary source of the noise may be removed or altered by design changes; the transmission paths by which the sound is propagated from the source to the listener may be reduced in effectiveness; or the sound level produced at the listening position may be reduced through the process of quieting.

Source Control. Design Changes. The most effective method of controlling noise at its source is to alter the design of the offending machinery so as to reduce the vibration of external surfaces. This may involve redesigning cams to avoid sharp impacts, substitution of helical for spur gearing, provision of elastic cushioning for reciprocating parts, or the redesign of shafts or plates to shift the location of resonant frequencies. The general procedure is to seek out the vibrating surface responsible for the noise production and to attempt an alteration of design that will reduce the excitation of vibration or that will prevent the effective radiation of sound from the surface to the outside air.

Damping. When it is not possible to redesign the machinery to avoid impact or other excitation of vibrating surfaces, their vibrations may be reduced by application of lagging or other forms of viscous damping. The lagging may consist of non-hardening plastic mixtures such as putty, asphalt, or tar, which may be used as loading for cloth or felt for greater ease in handling and application and to avoid subsequent flow after application to the surface being damped. The weight of the damping layer depends on the weight and size of the vibrating panel; thus relatively rigid lagging is suitable for stiff panels having high resonant frequencies, and thicker and softer coatings are required for the suppression of low-frequency vibrations. The use of common acoustical surface absorbents for reduction of sound radiation from vibrating surfaces is not effective except for absorption of the sound after it has been radiated.

Isolation. The function of vibration isolation in quieting *machinery noises at their origin* lies in preventing the vibration of one portion of the machine from setting into vibration other portions of the machine which may be more effective radiators of sound. If the normal operation of the machine involves impacts or sudden accelerations, these force reactions should be taken up within the structure and not be allowed to reach the external

Table 6. Spacing and Mounting Height Data
(Dimensions in feet)

Allowable spacing between luminaires					Mounting height of luminaires		
Clear ceiling height	Spacing dimensions			Approximate area per outlet[b]	Spacing between units[c]	Mounting height for direct units[d]	Suspension length for indirect units[d]
	Between units		Units to wall maximum[a]				
	usual	maximum					
8	7	7½	3	50–60	7	8	1–3
9	8	8	3	60–70	8	8½	1–3
10	9	9	3½	70–85	9	9	1–3
11	10	10½	3½	85–100	10	10	1½–3
12	10–12	12	3½–4	100–150	11	10½	2–3
13	10–12	13	3½–4	100–150	12	11	2–3
14	10–13	15	4–5	100–170	14	12½	2½–4
15	10–13	17	4–5	100–170	16	14	3–4
16	10–13	19	4–6	100–170	18	15	3–4
18	10–20	21	4–6	100–400	20	16	4–5
20	18–24	24	5–7	300–500	22	18	4–5
					24	20	4–6

[a] Apply for rooms in which desks or benches are against the wall. Spacing for other rooms may be half the spacing between units.
[b] For illumination calculations, determine actual area which is the product of the spacing dimensions, such as $5 \times 7 = 35$ sq ft.
[c] Actual spacing dimensions of the installation.
[d] Also for semiindirect units.

redirected and absorbed by the walls. In Table 7 are **room index values** for a variety of sizes and shapes of rooms. These indices are then applied in Table 8 to determine the **coefficient of utilization** of a complete installation.

Since walls and ceilings redirect light, their reflectances influence the utilization. The distribution of flux from a luminaire and the reflectances of these room surfaces must be simultaneously considered; for it is obvious that as a luminaire directs more and more flux toward these surfaces they become increasingly important in determining the ultimate coefficient of utilization. Lighting equipment (sources, luminaires, painted surfaces) depreciates and must be maintained continually. An average maintenance factor is given for each of the representative luminaires; this factor should be modified for conditions other than average.

Spacing information as contained in Table 6 is applicable to any installation, regardless of the method of illumination calculation that is used. By using these data, those in Tables 7 and 8, and the following equation, it is possible to determine the foot-candles of illumination that will be obtained on the horizontal work plane (2½ to 3 ft above the floor) with a given size of source:

$$\text{Foot-candles} = \frac{\text{lamp lumens} \times \text{coefficient of utilization} \times \text{maintenance factor}}{\text{area of working plane in sq ft per lamp}}$$

The desired foot-candles of illumination are usually known, having been

surfaces in communication with the air, nor should they be allowed to reach a floor or mounting which would provide good radiating surfaces, or which would permit transmission of mechanical vibrations to other portions of the building with consequent radiation as noise.

Transmission Control. Isolation. If the vibrations of noisy machinery cannot be suppressed at the source, their transmission to the listener should be impeded. For the higher frequencies constituting noise, the most effective isolation method is the introduction of elastic discontinuities in the structure transmitting the noise (measured by the difference between density-velocity products as given in Table 1). The discontinuities may be obtained by the use of felt, cork, rubber, or springs in machinery mountings, or by the introduction of alternate lead and cork sheeting at masonry junctions. The isolation treatment should be applied as close to the source as possible in order to eliminate sound radiation from the structures transmitting the vibrations. Where this is not possible, the listening space itself is to be isolated. Thus **quiet rooms**, constructed especially for noise measurements, are usually built as separate structures isolated from the main building by felt, cork, or other springing.

Filtration. Some problems of noise transmission through air lend themselves to solution by methods of filtration. Typical examples are the transmission of sound in ventilating ducts and the noise production at engine exhaust pipes. In each of these cases, the steady flow of gas must not be impeded, but the alternating flow, representing sound transmission, must be effectively suppressed.

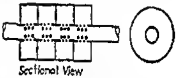


FIG. 3.—Exhaust Muffler.

For ventilating ducts, an acceptable degree of noise suppression may be obtained by lining the ducts (on at least two non-opposite walls) with an efficient sound absorbent for a distance of 10 to 15 ft from both the inlet and the outlet. Where the length of duct available is insufficient, or where additional noise suppression is required, baffles, covered with absorbing material, may be introduced in the duct. A plenum chamber, used to serve several ducts, should be lined with sound absorbents. If the air velocities are high, it may be necessary to introduce additional baffles at bends in the ducts to avoid noise production through turbulence.

Exhaust mufflers are usually modifications of the elementary low-pass acoustical filter, comprising a through tube to which closed cavities are coupled through small holes at intervals along the tube. Typical structures of this type (Fig. 3) produce little increase in back pressure and considerable attenuation of sound waves having frequencies above a cutoff frequency determined by the size of the holes and cavities. Porous packing, such as steel wool, in the side cavities or a studied irregularity in the size and spacing of the cavities will increase the uniformity of noise suppression, whereas increasing the number of side cavities and the length of the muffler will increase the amount of suppression. Baffles in the tail pipe or irregular obstructions producing devious flow paths (*e.g.*, the stone-filled pit for stationary engine exhausts) produce muffling action at the expense of appreciable increase in exhaust back pressure.

Shielding of air-borne noise must be done by sound-opaque screens large in comparison with the wave length of the sounds whose transmission they are to impede. This is seldom possible in building interiors except by utilization of building partitions as screens. Sound is transmitted through such parti-

Table 7. Room Indices
(All dimensions in feet)

Ceiling height		Up to 10	10 to 12	12 to 14	14 to 17	17 to 21	21 to 25	25 to 31	31 to 37	37 to 50		
Mounting height		**Up to 8**	**8 to 9**	**9 to 10**	**10 to 12**	**12 to 14**	**14 to 17**	**17 to 21**	**21 to 25**	**25 to 31**	**31 to 37**	**37 to 50**
Width	**Length**	**Room indices**										
8½–9½	8–14	*H*	*I*	*J*	*J*							
	20–30	*G*	*G*	*H*	*I*	*J*	*J*					
	42–up	*E*	*F*	*G*	*H*	*I*	*J*	*J*				
9½–11	10–20	*G*	*H*	*I*	*J*	*J*	*J*					
	30–60	*F*	*G*	*G*	*H*	*I*	*J*	*J*				
	60–up	*E*	*F*	*F*	*H*	*H*	*I*	*J*				
11–13	10–14	*G*	*H*	*I*	*I*	*J*	*J*					
	20–42	*F*	*G*	*G*	*H*	*I*	*J*	*J*				
	42–up	*E*	*F*	*F*	*G*	*H*	*I*	*J*				
13–16	14–20	*F*	*G*	*H*	*H*	*I*	*J*	*J*				
	30–60	*E*	*F*	*F*	*G*	*H*	*I*	*J*	*J*			
	90–up	*D*	*E*	*E*	*F*	*F*	*G*	*I*	*J*	*J*		
16–19	14–20	*E*	*F*	*G*	*H*	*I*	*J*	*J*				
	30–42	*D*	*E*	*F*	*G*	*H*	*H*	*J*	*J*	*J*		
	110–up	*C*	*D*	*E*	*E*	*F*	*G*	*H*	*I*	*J*	*J*	
19–22	20–30	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*J*			
	42–60	*D*	*D*	*E*	*E*	*F*	*G*	*I*	*J*	*J*	*J*	
	90–up	*C*	*D*	*D*	*E*	*F*	*F*	*H*	*I*	*I*	*J*	*J*
20–27	20–30	*D*	*E*	*E*	*F*	*G*	*H*	*I*	*J*	*J*		
	42–60	*C*	*D*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*J*	
	90–up	*C*	*C*	*D*	*E*	*E*	*F*	*G*	*H*	*I*	*J*	*J*
27–34	30–42	*C*	*D*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*J*	
	60–90	*B*	*C*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*J*
	90–up	*B*	*C*	*C*	*D*	*B*	*E*	*F*	*G*	*H*	*I*	*J*
34–40	30–42	*B*	*C*	*D*	*E*	*F*	*F*	*H*	*I*	*I*	*J*	
	60–90	*A*	*C*	*C*	*C*	*E*	*E*	*F*	*H*	*H*	*J*	*J*
	140–up	*A*	*B*	*C*	*C*	*D*	*E*	*F*	*F*	*G*	*H*	*I*
40–46	42–60	*A*	*E*	*C*	*C*	*E*	*F*	*G*	*H*	*I*	*I*	*J*
	90–140	*A*	*E*	*B*	*C*	*D*	*D*	*E*	*F*	*G*	*H*	*J*
	140–up	*A*	*A*	*B*	*C*	*D*	*D*	*E*	*F*	*G*	*H*	*I*
46–56	42–60	*A*	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
	90–140	*A*	*A*	*A*	*C*	*C*	*D*	*E*	*F*	*F*	*G*	*I*
	140–up	*A*	*A*	*A*	*C*	*C*	*D*	*E*	*E*	*F*	*G*	*I*
56–68	60–90	*A*	*A*	*A*	*B*	*C*	*D*	*E*	*E*	*G*	*H*	*I*
	90–140	*A*	*A*	*A*	*E*	*C*	*C*	*D*	*E*	*F*	*G*	*H*
	140–up	*A*	*A*	*A*	*E*	*C*	*C*	*D*	*B*	*E*	*F*	*H*
68–90	60–90	*A*	*A*	*A*	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*I*
	90–140	*A*	*A*	*A*	*A*	*E*	*C*	*D*	*E*	*F*	*F*	*H*
	140–up	*A*	*A*	*A*	*A*	*E*	*E*	*C*	*D*	*E*	*F*	*G*
90 or more	60–90	*A*	*A*	*A*	*A*	*E*	*C*	*D*	*B*	*F*	*C*	*H*
	90–200	*A*	*A*	*A*	*A*	*A*	*E*	*C*	*D*	*E*	*F*	*G*
	200–up	*A*	*A*	*A*	*A*	*A*	*B*	*C*	*C*	*D*	*E*	*F*

tions principally by minute flexure of the wall as a whole in response to the incident sound pressure on the noisy side, with consequent reradiation on the quiet side. Reduction of sound transmission is obtained by increasing the mass per unit area of the partition, by constructing the partition of material having large viscosity for bending, such as Thermax, or by the use of double partitions, vibrationally isolated.

Sound-transmission loss is usually greater for high frequencies than for low and is measured by comparing the average sound level on each side of the partition under standardized conditions, as described in reports of the A.S.A. Subcommittee Z-24. Average values of transmission loss, for frequencies from 125 to 4,000 cycles, for typical partitions are shown in Table 3. In any specific case, a more exact measure of the effectiveness of an insulating partition can be obtained by direct comparison of the transmission-loss vs. frequency curve for the partition and the intensity vs. frequency curve for the noise.

Table 3. Sound-transmission Loss in Building Partitions

Wall	Thickness, in.	Weight, lb per sq ft	Transmission loss, db
Wood	0.2	0.45	18.5
Plate glass	0.25	3.2	27.0
Hollow gypsum tile, unplastered	3	11.1	27.2
Brick wall, unplastered	.....	22.0	33
Brick wall, plastered	6	46	43
Brick wall, plastered	10.5	93	49
Double wall; metal lath, ½ in. gypsum plaster, on staggered 2 × 4 in. wood studs	7.5	19.8	44
Double 3 in. hollow gypsum tile, unplastered, 3 in. air space	9	22.0	42.6
1 in. Thermax nailed over building paper to 3 in. Thermax laid up in mortar, ½ in. plaster on both sides	5	15	47
Double 2 in. solid-gypsum tile, unplastered, completely isolated structurally by separate foundations, 4 in. air space	8	20.4	59

Based on Sabine, "Acoustics and Architecture," McGraw-Hill.

In general, double partitions (including floated floor constructions) provide greater transmission loss than equally heavy concrete, masonry, or brick walls but, except for special designs, less transmission loss than equally thick masonry walls. Double walls must be constructed carefully to avoid loss of vibration isolation through mechanical bridging between the opposite surfaces. Sound-absorbing fillers (mineral wool, etc.) are usually detrimental to sound insulation if in contact with both interior surfaces, and a single bridging nail may alter significantly the insulating efficiency. For maximum effectiveness, one of the wall surfaces should be hung structurally free at all four edges with the boundary cracks sealed, with felt or asphalt compounds, against sound leakage. Through piping should be made vibrationally discontinuous by introducing canvas or metallic sylphon sections, and clearance holes at the walls should be sealed. Sound leakage through small clearance cracks contributes to the low transmission loss of ordinary doors. Special self-sealing "sound-proof" doors are required to maintain the effectiveness of an efficient sound-insulating partition.

determined by a survey of the seeing tasks to be performed. The size of source is therefore the unknown factor to be determined with the preceding equation. A lamp of approximately the output thus determined is selected from a lamp manufacturer's catalogue.

When fluorescent luminaires are used, the lumen output per luminaire is generally known so that the number of luminaires to be installed becomes

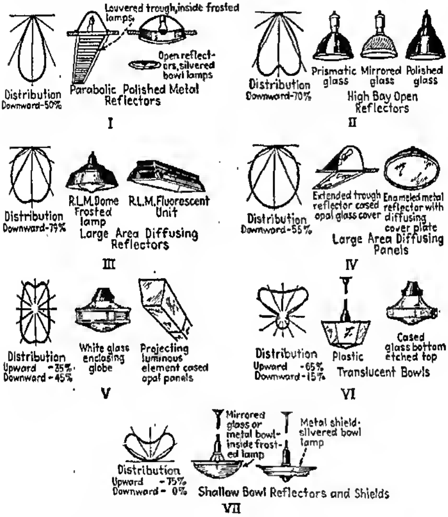


FIG. 8.—Typical Luminaires and Light Distributions.

the factor to be determined. The equation above may be written for these calculations as

$$\text{Number of luminaires to be installed} = \frac{\text{total lamp lumens required}}{\text{lamp lumens per luminaire}}$$

The layout is then planned to accommodate this number of units and in general the resulting spacings will not exceed the limits established in Table 6.

Illumination Instruments

The **Luckiesh-Moss visibility meter** appraises the difficulty of any seeing task in terms of the difficulty of reading 8-point Bodoni book monotype illuminated to 10 foot-candles.

Quieting. The sound level established in a room by a noise source is *higher than the same source would produce in free space on account of* successive reflections of sound at the walls. It is the function of **quieting** to avoid such enhancement of noise by providing a high degree of sound absorption at all interior reflecting surfaces exposed to the noise. Commercially available sound-absorbing materials may be cemented to flat surfaces or secured to wood or metal furring strips. They derive their absorbing property either from capillary porosity of the surface or from the dissipative vibration of surface layers. The effectiveness of sound absorbers varies with frequency, usually being greater for high and intermediate than for low frequencies. It may be measured by determining the **absorption coefficient,** defined as the fraction of sound energy diffusely incident on the material that is not reflected, or by determining the **specific acoustic impedance** of the material. The measured absorption coefficient is not

Table 4. Sound-absorption Coefficients

Maker, material, thickness	Absorption coefficient at indicated frequencies						Noise-reduction coefficient	Weight, lb per sq ft	Surface	A.M.A. Test No.
	128	256	512	1024	2048	4096				
Armstrong Cork Co.										
Corkoustic B4, 1¼ in.	0.08	0.13	0.51	0.75	0.47	0.46	0.45	0.68	Oil-base paint	39-30
Tekoustic B5, 1½ in.	0.23	0.49	0.99	0.73	0.46	0.44	0.65	1.52	Oil-base paint	285
Johns-Manville Sales Corp.										
Transite Acoustical unit with Transite facing, 1⅜ in.	0.28	0.55	0.83	0.91	0.76	0.67	0.75	2.20	576-5/32 in. holes per sq ft; any paint	39-59
Permacoustic, 1 in.	0.21	0.47	0.74	0.72	0.75	0.73	0.65	2.36	Unpainted	300
Airacoustic duct lining, 1½ in.	0.44	0.44	0.74	0.80	0.93	0.74	0.75	1.6	Unpainted	39-94
National Gypsum Co.										
Acoustex 70R, 1⅛ in.	0.24	0.33	0.74	0.96	0.80	....	0.70	2.67	Oil-base paint	180
U. S. Gypsum Co.										
Acoustone D, 13/16 in.	0.20	0.40	0.84	0.86	0.85	0.88	0.75	1.53	Unpainted	402
Celotex Corp.										
Acousti-Celotex C4, 1¼ in.	0.37	0.43	0.98	0.79	0.57	....	0.70	1.50	441-3/16 in. holes per sq ft, any paint	138
Absorbex-Type A, 1 in.	0.18	0.26	0.63	0.96	0.77	....	0.65	2.63	Oil-base paint	207
Q-T Ductliner, 1 in.	0.43	0.37	0.69	0.78	0.78	0.70	0.65	1.52	Unpainted	290
Brick wall, painted	0.012	....	0.017	....	0.023	....	0.02			
Concrete wall or floor	0.01	....	0.015	....	0.02	....	0.02			
Wood floor	0.05	....	0.03	....	0.03	....	0.03			
Cork or rubber tile on concrete	....	....	0.03-0.08	....	....	....	0.05			
Glass	0.035	....	0.027	....	0.02	....	0.02			

This tabulation is based on *Bull.* 7 (April, 1940), Acoustical Materials Assoc. All samples were cemented to plasterboard for test, except that the Transite unit is nailed to 1 × 2 in. wood furring, 12 in. O.C., and the duct linings are laid on 24 gage sheet iron, nailed to 1 × 2 in. wood furring, 24 in. O.C.

Table 8. Coefficients of Utilization

Type of equipment (see Fig. 8)	Maintenance factor (average conditions)	Ceiling	Very light		Fairly light		Fairly dark	
		Walls	Fairly light	Very dark	Fairly light	Very dark	Fairly dark	Very dark
		Room index	Coefficients of utilization					
I	0.70	J	0.29	0.26	0.28	0.26	0.28	0.26
		I	0.34	0.32	0.34	0.32	0.32	0.31
		G	0.39	0.38	0.38	0.37	0.38	0.36
		E	0.43	0.42	0.42	0.40	0.41	0.40
		C	0.46	0.44	0.45	0.43	0.43	0.42
		A	0.47	0.46	0.46	0.44	0.44	0.44
II	0.70	J	0.40	0.36	0.39	0.36	0.39	0.36
		I	0.48	0.46	0.47	0.45	0.46	0.43
		G	0.55	0.54	0.54	0.52	0.52	0.51
		E	0.60	0.58	0.59	0.57	0.57	0.56
		C	0.65	0.61	0.63	0.60	0.60	0.60
		A	0.66	0.64	0.64	0.62	0.62	0.62
III	0.75	J	0.37	0.28	0.37	0.28	0.31	0.28
		I	0.46	0.38	0.45	0.37	0.41	0.37
		G	0.54	0.47	0.53	0.47	0.48	0.47
		E	0.62	0.56	0.61	0.56	0.57	0.56
		C	0.69	0.63	0.67	0.63	0.64	0.62
		A	0.74	0.69	0.72	0.68	0.68	0.67
IV	0.70	J	0.26	0.19	0.25	0.19	0.21	0.19
		I	0.32	0.26	0.31	0.26	0.28	0.26
		G	0.38	0.33	0.37	0.32	0.34	0.32
		E	0.43	0.39	0.42	0.38	0.40	0.38
		C	0.48	0.43	0.46	0.43	0.44	0.43
		A	0.51	0.48	0.50	0.47	0.47	0.46
V	0.75	J	0.24	0.17	0.22	0.16	0.16	0.14
		I	0.30	0.23	0.27	0.20	0.21	0.19
		G	0.37	0.30	0.33	0.27	0.27	0.25
		E	0.45	0.37	0.40	0.33	0.32	0.30
		C	0.52	0.43	0.45	0.38	0.37	0.35
		A	0.57	0.50	0.50	0.44	0.41	0.40
VI	0.65	J	0.17	0.12	0.13	0.09	0.07	0.06
		I	0.22	0.16	0.16	0.12	0.09	0.08
		G	0.29	0.21	0.21	0.16	0.12	0.11
		E	0.35	0.28	0.26	0.20	0.15	0.14
		C	0.41	0.34	0.30	0.25	0.18	0.17
		A	0.47	0.41	0.34	0.30	0.22	0.20
VII	0.65	J	0.15	0.10	0.10	0.07	0.04	0.04
		I	0.19	0.14	0.13	0.09	0.06	0.05
		G	0.25	0.10	0.17	0.13	0.08	0.08
		E	0.31	0.25	0.21	0.16	0.10	0.10
		C	0.36	0.30	0.24	0.20	0.13	0.12
		A	0.42	0.37	0.28	0.25	0.16	0.14

a property of the material alone, but depends partly on the size and mounting of the test sample and the size and shape of the test chamber; thus comparison of the coefficients for different materials should be based only on measurements made under identical conditions. Such measurements on a wide variety of materials have been made available by the Acoustical Materials Assoc. (Chicago, Ill.), although it is to be expected that the absorption coefficients effective in various practical applications may differ somewhat from the published values.

For ordinary noise quieting, the average of absorption coefficients measured at frequencies of 256, 512, 1,024, and 2,096 cycles, called the **noise-reduction coefficient**, may be used. Typical values of this coefficient for representative materials are given in Table 4. In making quantitative estimates of noise reduction, the **total sound absorption** of the room boundaries may be computed by multiplying the noise-reduction coefficient of each different *material present by the total exposed area of that material* and summing up the resulting products. The noise reduction is then given by

$$\text{Noise reduction in decibels} = 10 \log \frac{\text{total absorption after treatment}}{\text{total absorption before treatment}}$$

When the frequency spectrum of the offending noise is known, greater precision in calculation of total absorption is obtained by replacing the noise-reduction coefficient by the absorption coefficient measured at the frequency of maximum loudness level from the noise source. Subjective judgments of the loudness reduction obtained by quieting can be estimated by using the noise reduction in decibels in connection with the loudness chart of Fig. 2. Typical values of noise reduction are given in Table 5.

Table 5. Noise Reduction

Factor by which total sound absorption is increased by quieting treatment	Noise reduction, db	Percentage decrease in loudness, when original loudness level is		
		60 phons	80 phons	100 phons
2	3	18	21	23
4	6	33	38	40
6	7.8	42	46	48
10	10	50	53	57
20	13	59	62	67

Based on A.S.A. Tentative Standard (1940) for loudness reduction.

In general, the larger the area of absorbing material introduced, and the higher its noise-reduction coefficient, the more effectively the noise is reduced. No amount of quieting treatment can reduce the level of the noise received directly from the source. If full coverage of walls and ceiling is not possible, distribution of the material in several small patches is more effective than the same total area of material concentrated in one location. Similarly, the same area of material is more effective when applied to non-opposite walls and ceiling then when concentrated on either of these areas, and more effective when located near the edges and corners of a given area than when located in the center.

Polar Equation, focus as origin, axes as in Fig. 24: $r = p/(1 - e \cos\theta)$.

Equation of the Tangent at (x_1,y_1): $b^2x_1x + a^2y_1y = a^2b^2$.

The line $y = mx + k$ will be a tangent if $k = \pm\sqrt{a^2m^2 + b^2}$. The normal at any point P bisects the angle between PF and PF' (Fig. 25). The locus of the foot of the perpendicular from the focus on a moving tangent is the circle on the major axis as diameter (Fig. 26). The locus of the point of intersection of perpendicular tangents is a circle with radius $\sqrt{a^2 + b^2}$ (Fig. 27).

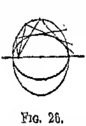


Fig. 26.

Fig. 27.

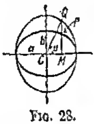


Fig. 28.

Fig. 29.

Ellipse as a Flattened Circle. Eccentric Angle. If the ordinates in a circle are diminished in a constant ratio, the resulting points will lie on an ellipse (Fig. 28). If Q traces the circle with uniform velocity, the corresponding point P will trace the ellipse, with varying velocity. The angle u in the figure is called the eccentric angle of the point P.

Conjugate Diameters are lines through the center, each of which bisects all the chords parallel to the other (Fig. 29). If m_1 and m_2 are the slopes, then $m_1m_2 = -b^2/a^2$. One pair of conjugate diameters are the diagonals of the rectangle circumscribing the ellipse. The eccentric angles of the ends of two conjugate diameters differ by 90 deg. Thus (Fig. 30), if CQ and CQ' are perpendicular radii in the circle, CP and CP' will be conjugate semi-diameters in the ellipse. A parallelogram formed by tangents drawn parallel to a pair of conjugate diameters has a constant area, $= 4ab$ (Fig. 31). Also, if a',b' are conjugate semi-diameters, and w the angle between them, then $a'^2 + b'^2 = a^2 + b^2$ and $a'b' = ab/\sin w$.

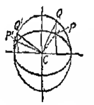


Fig. 30.

Fig. 31.

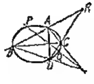


Fig. 32.

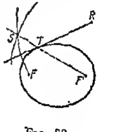


Fig. 33.

To Construct a Tangent to a Given Ellipse. (1) At a Given Point of Contact, P. Bisect the angle between the focal radii PF and PF' (Fig. 25).

(2) From a Given External Point, R. (*a*) Through R draw any two lines cutting the ellipse, one in A and B, the other in C and D (Fig. 32). Through the point of intersection of AD and BC and the point of intersection of AC and BD, draw a line cutting the ellipse in P and Q. Then P and Q are the required points of contact. (*b*) With R as a center and radius RF, draw an arc; with F' as center and radius $2a$ draw an arc, intersecting the first in S; and let SF' meet the curve in T. Then T is the point of contact (Fig. 33).

To Construct an Ellipse, Given a and b. (1) In Fig. 34, with O as center, draw circles with radii a and b (and also a third circle with radius $a + b$). Let a variable ray through O cut these circles in J, K (and S); through J and K draw parallels to the axes, meeting in P. Then P is a point of the ellipse (and SP is the normal at P).

(2) In Fig. 35, let P divide a line AB so that $PA = a$ and $PB = b$. Then if A and B slide on the axes, P will describe an ellipse.

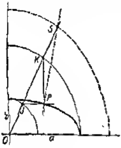


FIG. 34.

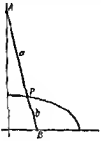


FIG. 35.

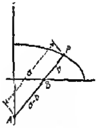


FIG. 36.

(3) In Fig. 36, let PBA be a straight line such that $PA = a$ and $PB = b$. Then if A and B slide on the axes, P will trace an ellipse. (Use a strip of paper, with the points P, B, and A marked on it.)

(4) Find the foci, F and F', by striking an arc of radius a with center at B (Fig. 37). Drive pins at F, F', and B, and adjust a loop of thread around them. Then remove the pin at B, and replace it by a pencil point; by moving the pencil so as to keep the string taut, the complete ellipse can be drawn at one sweep. Or, use a mechanical ellipsograph.

(5) and (6). Apply methods (1) and (2) of the following paragraph to the special case in which OP and OQ are perpendicular semi-axes.

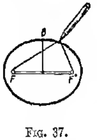


FIG. 37.

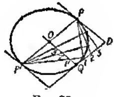


FIG. 38.

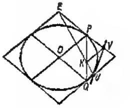


FIG. 39.

To Construct an Ellipse, Given a Pair of Conjugate Semi-diameters, OP and OQ. (1) Complete the parallelogram, as in Fig. 38. Divide QD and QO into n equal parts, $1, 2, 3, \ldots$ and $1', 2', 3', \ldots$ Connect P with $1, 2, 3, \ldots$, and P' with $1', 2', 3' \ldots$. The points of intersection of corresponding lines will be points of the ellipse.

(2) Take any point K on PQ (Fig. 39). Draw EKU, and draw KV parallel to OP. Then UV will be a tangent. By varying K along PQ as many tangents may be drawn as desired, thus "enveloping" the ellipse.

(3) Through P (Fig. 40), draw a perpendicular PT to OQ, and lay off $PR = PS = OQ$. Then if the line RPT is made to slide with one end on OR and the other on OQ, P will trace the ellipse. Further, the bisectors of the angle ROS show the directions of the principal axes, and $OR + OS = 2a$ and

SECTION 11

PUMPS AND COMPRESSORS

BY

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CONTENTS

speeds are slightly higher (1,500 to 4,500). The inlet-vane angle must decrease with the radius to ensure smooth entry of the water, so that it resembles a Francis turbine wheel in shape. It may be made double-suction.

3. *Mixed-flow-type Impeller.* The head developed in this type is due partly to the centrifugal and partly to the push of the vanes. The discharge is partly radial and partly axial, which is the reason for the name. The impeller is made screw-shaped (doubly curved) for the same reason as the Francis impeller. The specific speed range is usually between 4,500 and 8,000.

4. *Propeller-type Impeller.* Practically all the head developed by this type is due to the push of the vanes. The flow is entirely axial. It has the

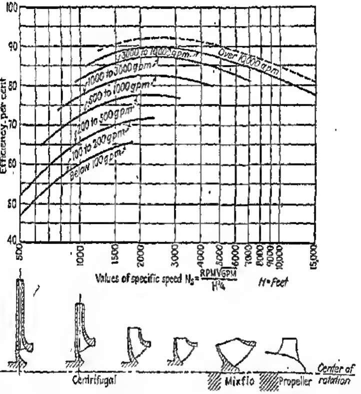


FIG. 11.—Approximate relative impeller shapes and efficiencies as related to specific speed.

highest specific speed (above 8,000) and is used for low heads (3 to 40 ft), low rpm (200 to 1,800), and large capacities.

The various types of impellers are illustrated on the lower part of Fig. 11. The upper portion of the chart gives the approximate efficiencies that can be obtained for a given specific speed and capacity. The efficiency thus obtained may be used for estimating purposes. To illustrate, it is desired to predict the brake horsepower required to drive a two-stage main condensate pump delivering 100 gpm water against a head of 200 ft while running at 3,000 rpm. The head per stage is $\frac{1}{2} \times 200$ or 100 ft, and the specific speed of each stage is $n_s = 3{,}000\sqrt{100}/100^{3/4} = 950$ (this may be found from Fig. 10). The efficiency as read from Fig. 11 is about 60 percent. The water horsepower is

MARINE PUMPS

BY

AUSTIN H. CHURCH

REFERENCES: "Standards of the Hydraulic Institute." Church, "Centrifugal Pumps and Blowers," Wiley. Pfleiderer, "Die Kreiselpumpen," Springer. Labberton, "Marine Engineering," Chap. 8, McGraw-Hill. Kristal and Annette, "Pumps," McGraw-Hill.

The three main types of marine pumps are the centrifugal, reciprocating, and rotary. Centrifugal pumps are relatively small for a given capacity and head; hence they are also cheaper, lighter, and require less floor space.

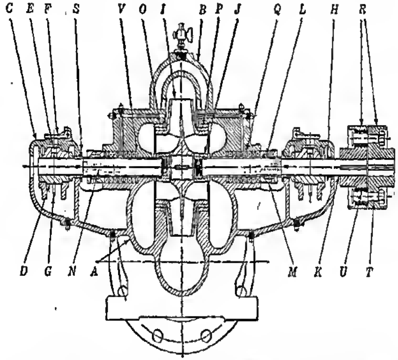


FIG. 1.—Dayton-Dowd single-stage double-suction pump. *A*, lower casing; *B*, upper casing; *C*, bearing bracket cap; *D*, bearing, lower half; *E*, bearing, upper half; *F*, oil-hole cover; *G*, oil ring; *H*, oil guard; *I*, impeller; *J*, impeller nut; *K*, shaft; *L*, shaft sleeve; *M*, gland halves; *N*, gland bolts; *O*, casing wearing ring; *P*, impeller wearing ring; *Q*, lantern ring; *R*, coupling halves; *S*, thrust collar and water slinger; *T*, coupling pins and nuts; *U*, coupling bushings; *V*, packing-box bottom.

The wear is not so great as for reciprocating pumps since the clearances are larger. They have no internal valves, are easily balanced, and deliver fluid without pulsations. Reciprocating units generally have higher efficiencies,

$\frac{wH}{550} = \frac{100 \times 8.34 \times 200}{60 \times 550} = 5.06$ (1 gal water weighs 8.34 lb), and the brake horsepower required will be

$$\frac{\text{water hp}}{\eta} = \frac{5.06}{0.6} = 8.4$$

The shape of the performance curves varies with the impeller type. This is illustrated by Fig. 12, where the head, horsepower, and efficiency are plotted on a percentage basis against the percentage flow for typical impeller types.

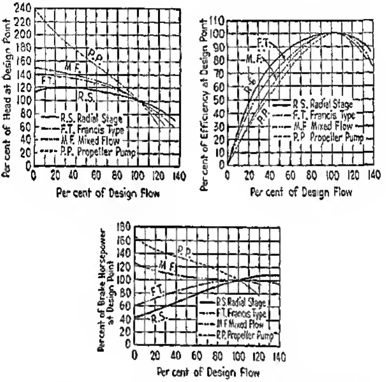


FIG. 12.—Approximate performance curves of various impeller types at partial loads.

Cavitation and Suction Head. If the suction head on a pump becomes too low, the water may vaporize and the flow will be reduced or stopped. This condition known as cavitation is usually accompanied by corrosion and erosion of the parts and noisy operation. It occurs when the absolute pressure of the water reaches the vapor pressure.

The suction head, H_{sv}, of a pump is the equivalent total head at the pump center line corrected for vapor pressure. The following four factors enter into its determination: (1) H_p is the head corresponding to the absolute pressure on the surface of the liquid from which the pump draws. This will be the atmospheric pressure if the tank is open, or the absolute pressure in the heater or closed tank from which the pump takes liquid. (2) H_z is the

particularly when handling small flows against high heads, and do not have to be primed. Rotary pumps do not have valves and are simple, compact, and cheap. They are generally used to develop high pressures (25 to 400 psi) with small flows (1 to 2,000 gpm) and are particularly effective when handling viscous fluids.

CENTRIFUGAL PUMPS

A centrifugal pump consists essentially of one or more impellers, equipped with vanes, mounted on a rotating shaft, and enclosed by a casing. Fluid enters the impeller axially near the shaft and has energy, both potential and kinetic, imparted to it by the vanes. As the fluid leaves the impeller at a relatively high velocity, it is collected in a volute or series of diffusing passages which transforms the kinetic energy to pressure. This is, of course, accompanied by a decrease in the velocity. After the conversion is accomplished, the fluid is discharged from the pump.

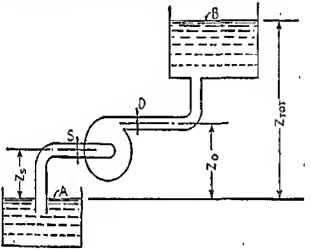


FIG. 2.—Head on a pump.

The water may enter the impeller from one side only (single-suction) or from both sides (double-suction) as illustrated in Fig. 1, which represents a typical pump of this type.

External Head Required. Before a pump can be ordered or designed, the external head against which it must operate must be found. This is made up of the sum of the friction and eddy losses occurring in the suction and discharge lines and the total increase in elevation or pressure head acquired by the liquid. Referring to Fig. 2, $H_{tot} = \Delta H_{AS} + \Delta H_{DB} + z_{tot}$. The pipe losses ΔH_{AS} and ΔH_{DB} include the friction loss plus the turbulence losses at the pipe entrance and exit, at fittings, bends, etc., while z_{tot} represents the sum of the actual lift and/or its pressure equivalent. Additional losses will occur in the pump itself (between S and D), but they will be cared for in the design and do not affect the external head required. Hence the head developed by the pump must be the difference between the total head at the center of the discharge flange and that at the center of the suction flange. Referring again to Fig. 2,

$$H_{tot} = z_D - z_S + \frac{P_D - P_S}{\gamma} + \frac{V_D^2 - V_S^2}{2g}$$

height in feet of the fluid surface above or below the impeller center line. (3) H_{vp} is the head corresponding to the vapor pressure of the liquid at the existing temperature. (4) H_f is the head lost due to friction and turbulence between the surface of the liquid and the pump suction flange.

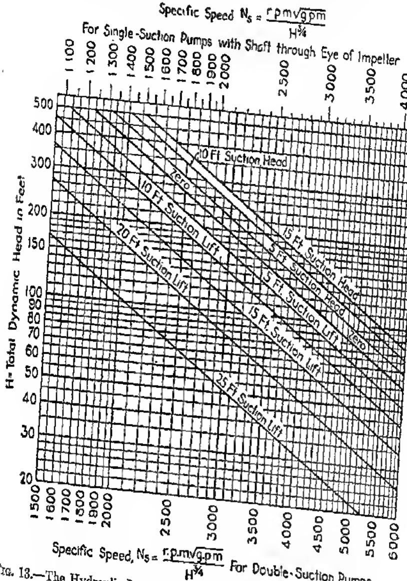


FIG. 13.—The Hydraulic Institute upper limits of specific speeds for single-stage pumps handling clear water at sea level at 85 F.

The suction head is the algebraic sum of these factors. Any term that would reduce the total suction head is negative. The vapor pressure and the loss due to friction and turbulence are always negative since they decrease the total suction head. Thus, the equation for the suction head is

$$H_{sv} = H_p \pm H_s - H_{vp} - H_f$$

where z = elevation, ft
P = pressure, lb per sq ft
γ = specific weight of liquid, lb per cu ft
V = velocity, fps
g = acceleration due to gravity = 32.2 ft per sec per sec
S and D = center lines of the suction and discharge flanges, respecti

Example. A boiler-feed pump draws water from an open heater and disc into a boiler at a pressure of 600 psi gage. The friction and eddy losses in the su amount to 2 ft head, while those in the discharge line are 4 ft. The water has weight of 59.2 lb per cu ft, and the total lift of the water is 8 ft,

$$\text{Equivalent lift of the pump, } z_{st} = \frac{600 \times 144}{59.2} + 8 = 1{,}468 \text{ ft}$$

$$\text{Total external head on the pump, } H_{ist} = 2 + 4 + 1{,}468 = 1{,}474 \text{ ft}$$

In considering the action of the water in the pump, the various ve must be taken into account. Figure 3 illustrates these velocities at t

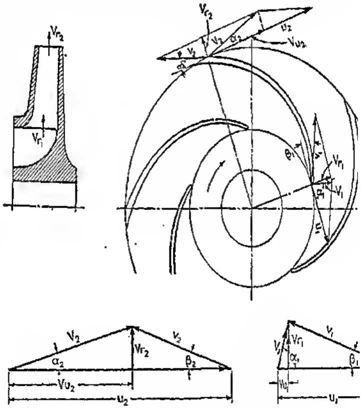


FIG. 3.—Virtual inlet and outlet velocity diagrams of pump impe

and outlet of the vane passage. The velocity of a point on the designated as u, the absolute velocity of the water (relative to the as V, and the velocity of the water relative to the impeller as v. Th between v and u corresponds to the angle of the vane and is denote the angle between u and V representing the absolute angle at wh

It may be noted that the suction velocity head, $V_{su}^2/2g$, is not included in the equation. The net suction head, H_{sv}, will appear in two forms at the suction flange, *i.e.*, as velocity and pressure heads. As the equation gives the total head and not the static pressure head, the velocity head is not included. The term H_{sv} is known as the net positive suction head over the vapor pressure and is frequently designated as NPSH.

In designing a pump installation there are two types of suction head, or NPSH, to be considered. One is the available suction head of the system, and the other is the required suction head of the pump to be placed in the system. The former is determined by the marine engineer and is based

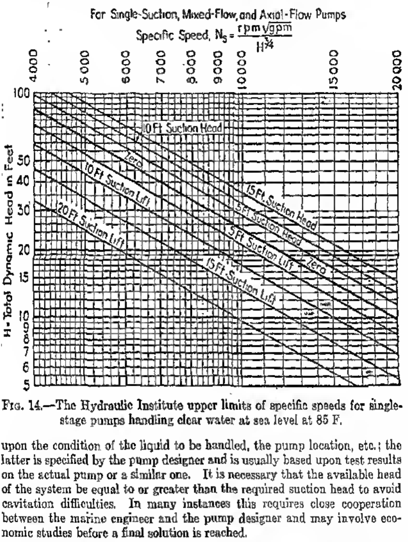


FIG. 14.—The Hydraulic Institute upper limits of specific speeds for single-stage pumps handling clear water at sea level at 85 F.

upon the condition of the liquid to be handled, the pump location, etc.; the latter is specified by the pump designer and is usually based upon test results on the actual pump or a similar one. It is necessary that the available head of the system be equal to or greater than the required suction head to avoid cavitation difficulties. In many instances this requires close cooperation between the marine engineer and the pump designer and may involve economic studies before a final solution is reached.

Example. A condensate pump draws water from a condenser in which a 27.5 in. vacuum is maintained. The friction and turbulence loss in the piping between the two is estimated to be 4 ft. If the required suction head of the pump is 12 ft and this is made equal to the available suction head of the system, what minimum height of water level in the condenser above the pump center line must be maintained? The condenser

water enters or leaves the impeller is α. The tangential component of V is V_u, while the radial component is V_r. The subscripts 1 and 2 designate the inlet and outlet edges of the vane passage, respectively.

Developed Head. If it is assumed that the water follows the vanes, the head developed by the impeller is

$$H_{vir} = \frac{u_2^2 - u_1^2}{2g} + \frac{v_1^2 - v_2^2}{2g} + \frac{V_2^2 - V_1^2}{2g}$$

where $u_2^2 - u_1^2/2g$ = head due to centrifugal action
$v_1^2 - v_2^2/2g$ = head due to the change in the relative velocity
$V_2^2 - V_1^2/2g$ = head due to the change in the absolute velocity

The first two terms represent the pressure head that is developed in the impeller, while the last term is the velocity head developed in the impeller and converted to pressure in the volute or diffuser.

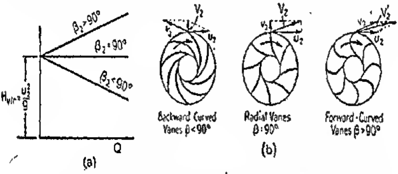


FIG. 4.—Virtual head-capacity curves and outlet velocity diagrams for various vane angles.

The foregoing equation may also be written $H_{vir} = (1/g)(u_2V_{u2} - u_1V_{u1})$. It is generally assumed that the water enters the impeller radially, hence $V_{u1} = 0$, and the latter equation reduces to $H_{vir} = u_2V_{u2}/g$.

It may be seen from these equations that the virtual head is closely linked with the vane angle at the outlet. It can be shown that this virtual head will have a straight-line relation with the flow through the pump. If the outlet angle β_2 is greater than 90 deg, the head will increase with the flow; if 90 deg, it will be constant; if less than 90 deg, it will decrease. This is illustrated by Fig. 4, which also shows the outlet velocity diagrams for various vane angles with the same radial outlet velocity component. It should be noted that the water leaves the impeller with a greater absolute outlet velocity V_2 for forward-curved vanes than for backward-curved vanes. Since the efficiency of converting velocity to pressure is related low, backward-curved vanes are always used to reduce the losses. Generally the outlet vane angle is made between 15 and 40 deg. This also aids in securing a stable characteristic curve and reduces the possibility or severity of pulsation or surging.

As the water contained in the vane passage is being accelerated, owing to the impeller rotation, it will develop a circulatory flow in the opposite direction. This flow will reduce the tangential component of the absolute outlet velocity from V_{u2} to V_{u2}' in the ratio $V_{u2}'/V_{u2} = 0.65$ to 0.75 depending

absolute pressure is 30.0 − 27.5 = 2.5 in. Hg, or 1.227 psia. This will also be the vapor pressure of the water. The corresponding specific gravity of the water is 0.9933. The

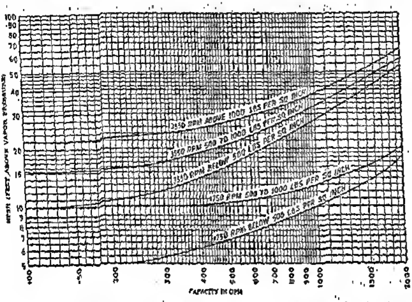


FIG. 15.—The Hydraulic Institute required net positive suction head for single-suction centrifugal hot-water pumps.

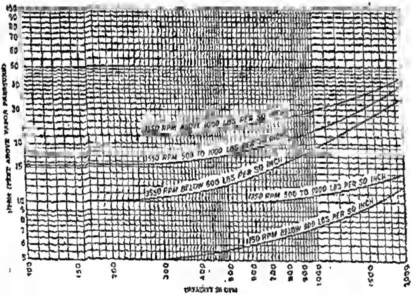


FIG. 16.—The Hydraulic Institute required net positive suction head for double-suction centrifugal hot-water pumps.

head corresponding to a given pressure in pounds per square inch is 2.31 × p/sg, hence $H_p = H_{vp} = 2.31 \times 1.227/0.9933 = 2.86$ ft. $H_f = 4$ ft. $H_{rs} = 12$ ft.

$$H_{rs} = H_p + H_z - H_{vp} - H_f$$
$$12.0 = 2.86 + H_z - 2.86 - 4.0$$

or $H_z = 16$ ft, which is the minimum height of the water level in the condenser above the pump center line.

upon the impeller type, vane shape, etc. This reduces the effective head developed by the impeller. It is further reduced by the fluid friction and turbulence losses. The former is zero at no flow and increases approximately with the square of the flow, while the latter is a minimum at the design flow and increases as the flow varies from this condition. This is illustrated in Fig. 5, the top curve being the virtual head for backward-curved vanes and the bottom the actual characteristic *curve of the pump.*

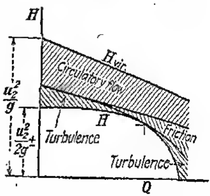


FIG. 5.—Development of the actual head-capacity curve from the virtual.

From tests it is found that, for radial-type impellers, the head at the point of maximum efficiency is between 0.8 and 1.4 of $u_2^2/2g$. This fact can be used to determine the approximate outside diameter of the impeller D_2 to develop a specified head as $u_2 = \pi D_2 n/720$. This may be written $D_2 = 1{,}840\phi\sqrt{H}/n$, where ϕ is a coefficient varying between 0.9 and 1.2, n is the rpm, and H the delivered head at the point of maximum efficiency.

When an impeller is rotated in water, power is absorbed. This is known as **disk friction.** The amount of power lost, as calculated by A. H. Gibson, is given by the equation

$$\text{hp}_{DF} = \frac{(D_2/12)^{4.83} n^{2.83}}{8.75(10)^8}$$

where n = speed, rpm
D_2 = rim diameter, in.

From this equation it may be noted that less power is absorbed if a small-diameter impeller is rotated at high speed than if a slow-speed large-diameter

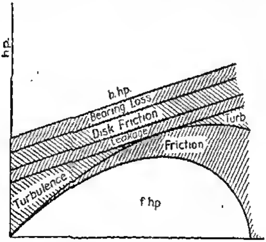


FIG. 6.—Development of actual brake-horsepower curve from the virtual.

The Hydraulic Institute has issued charts to estimate the required suction lift for pumps based upon the specific speed, total head, operating speed, etc., which are reproduced as Figs. 13 to 18. These curves are not to be taken as the very minimum required suction lift that can be obtained with special design, but they are a good guide in that they represent average results based upon the experience of a large number of pump manufacturers.

The use of the charts is simple and may be illustrated by an example.

Example. A double-suction pump operating at 3,560 rpm delivers 1,200 gpm water against a total head of 200 ft. What suction head on the pump is required for satisfactory operation?

The specific speed based upon the total flow is $n_s = \frac{n\sqrt{Q}}{H^{3/4}} = \frac{3{,}560\sqrt{1{,}200}}{200^{3/4}} = 2{,}320$ rpm, or it could be read from Fig. 10. Referring to Fig. 13, the point corresponding to a specific speed of 2,320 for double-suction pumps and a total dynamic head of 200 ft indicates about 12 ft suction lift as the safe maximum. If the same conditions were applied to a single-suction pump, the safe maximum suction condition would require at least a 7 ft positive head, *i.e.*, H_s would have to be at least $+7$ ft rather than -12 ft.

Figures 15 and 16 give the required NPSH for single- and double-suction hot-water pumps handling water with temperatures up to 212 F. For temperatures above 212 F, the NPSH as found from the figures is increased by the amount shown on Fig. 17. If the operating speed of the pump is within 25 percent of those shown on Figs. 15 and 16, the flow through the pump is corrected according to the relationship, rpm $\sqrt{\text{gpm}}$ = a constant.

Figure 18 gives the required NPSH for condensate pumps with a shaft passing through the eye of the impeller. It applies to pumps having a maximum of three stages, the lower scale representing single-suction pumps and the upper scale double-suction or a double-suction first-stage impeller. The speed-capacity relationship given above may be applied to this curve also with the same limitation.

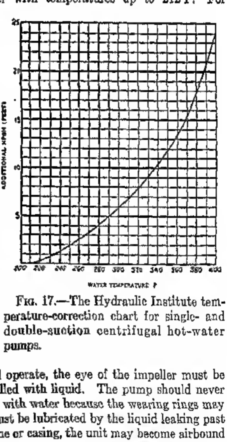


FIG. 17.—The Hydraulic Institute temperature-correction chart for single- and double-suction centrifugal hot-water pumps.

Priming. Before a pump will operate, the eye of the impeller must be submerged and the suction line filled with liquid. The pump should never be run unless the impeller is filled with water because the wearing rings may rub and seize; also the packing must be lubricated by the liquid leaking past it. If air leaks into the suction line or casing, the unit may become airbound and lose its prime, *i.e.*, cease delivery.

If water is available from an outside source, a check or foot valve may be placed in the suction line and the pump filled from this source. When the

wheel is used. Hence high-speed machines have less loss than the slow from this standpoint.

Another power loss is due to the leakage. Some of the water (2 to 10 per cent) leaving the impeller will leak past the wearing rings to the suction and must be pumped again. On multistage machines there will also be some leakage past the diaphragms between the stages.

The useful power delivered by the pump (known as **water horsepower**) is water hp $= wH/550$, where w is the weight flow in lb per sec and H is the delivered head in feet. The power required by the pump is the sum of this water horsepower, the power consumed in friction and turbulence, leakage

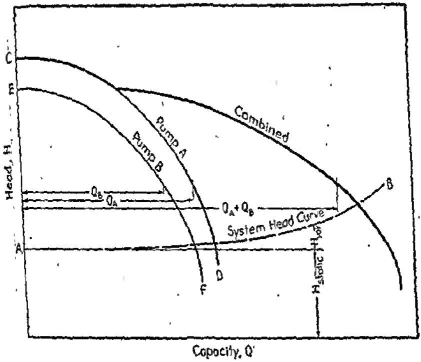


Fig. 7.—Head-capacity curves of pumps operating in parallel.

loss, disk friction, and bearing losses. Thus the brake horsepower required of the driver is made up of these components as shown in Fig. 6. The over-all efficiency of the pump for any flow is

$$\eta = \frac{\text{water hp}}{\text{bhp}}.$$

It is frequently desirable to predict the performance of a pump when operating at speeds other than that at which it is tested. The basic relationships applicable to this are $Q = Q'n'/n$; $H = H'(n'/n)^2$; bhp $=$ bhp$'(n'/n)^3$, where the primed values represent test conditions and the unprimed values the condition desired. The pump will follow these relationships over a fairly large range, with the efficiency remaining practically constant.

If the viscosity of the liquid is increased, the head will be reduced and the brake horsepower increased, thus lowering the efficiency. The exact amount of this effect is difficult to predict.

discharge line remains filled with water after the pump is stopped, this water may be used for priming by opening a by-pass valve around the pump (shown dotted in Fig. 19).

For marine installations, the priming is frequently made automatic by means of a central priming system connected to the condensate, bilge, ballast, etc., pumps that require it. The air may be removed by means of an air ejector operated by high-pressure air, steam, or water (Fig. 19), or a wet

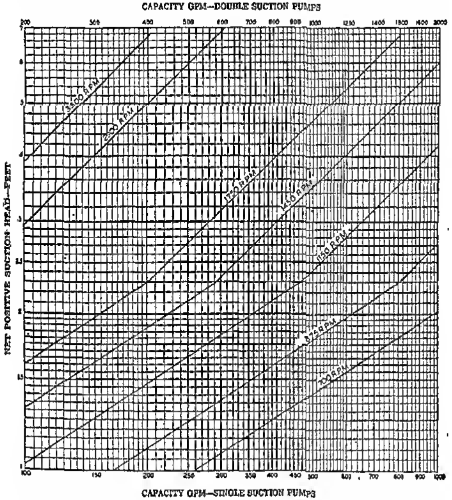


FIG. 18.—The Hydraulic Institute capacity and speed limitations for condensate pumps with a shaft through the eye of the impeller.

vacuum pump may be used. These are automatically controlled by means of a solenoid or float valve which actuates the air pump or ejector if the water level falls below a predetermined safe value.

A common type of wet vacuum pump is the Nash Hytor (Fig. 20). A rotor (5) in hydraulic balance revolves freely without contact in an elliptical casing (6) containing a liquid, usually water (4). This rotor (5) is a circular casting consisting of a series of blades that project from a cylindrical hub to form pockets or chambers. Ports are arranged at the bottom of each chamber. A cone-shaped casting containing two inlet and two outlet ports, as at (1), fits without contact into the rotor hub.

Pump Systems. It may be desirable to install several small pumps in either parallel or series to secure more flexible operation. In planning such installations, it is necessary to construct first a head-capacity curve for the system. The **system head curve** is the sum of the static head (difference in elevation and/or its pressure equivalent) plus the variable head (friction and turbulence in the pipe, bends, fittings, heaters, etc.). The former is usually constant for a given installation, whereas the latter increases approximately with the square of the flow. The resulting curve is AB in Figs. 7 and 8.

Pumps in Parallel. For pumps to operate satisfactorily in parallel they must be working on the stable part of their characteristic curves, *i.e.*, to the right of the pulsation point. The system head-capacity curve AB shown in Fig. 7 starts at H_{static} when the flow is zero and rises approximately parabolically with increased flow. Curve CD represents the head-capacity curve of pump A operating alone; the similar curve for pump B is represented by EF. Pump B will not start delivery until the delivery pressure of pump A falls below that of the shutoff head of B (point E). The combined delivery for a given head is equal to the sum of the individual capacities of the two pumps at that head. For a given combined delivery head, the capacity is divided between the pumps as noted on the figure by Q_A and Q_B. The combined characteristic curve shown on the figure is found by plotting these summations. The combined brake horsepower curve can be found by adding the brake horsepower of pump A corresponding to Q_A to that of pump B corresponding to Q_B, and plotting this at the combined flow. The efficiency curve of the combination may be determined by dividing the combined fluid horsepower, $\gamma(Q_A + Q_B)H/550$, by the corresponding combined brake horse-power (Q taken as cubic feet per second, and γ as pound per cubic foot).

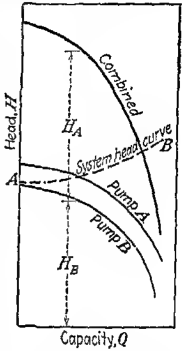


FIG. 8.—Head-capacity curves of pumps operating in series.

Pumps in Series. If two pumps are operated in series, the combined head for any flow is equal to the sum of the individual heads, as illustrated in Fig. 8. The combined brake horsepower curve may be found by adding the horsepowers given by the curves for the individual pumps. Points on the combined efficiency curve are found by dividing the combined water horsepower, $\gamma(H_A + H_B)Q/550$, by the combined brake horsepower, where the symbols have the meanings given above.

Pulsation. Under certain operating conditions, a pump will experience unstable operation and deliver, alternately, large and small quantities of fluid. This phenomenon is known as pulsation or surging and should be avoided in both design and operation, as it puts a heavy strain on the unit and system.

Starting at point A, the chambers are full of water. The water, turning with the rotor and constrained to follow the casing (6) by centrifugal force, alternately recedes from (4) and is forced back into the rotor (3) twice in a revolution. As the water recedes from the rotor (7), it draws air from the pump inlet into the cone, through the cone inlet port, and into the rotor by means of the ports in the bottom of the rotor chambers. When the water is

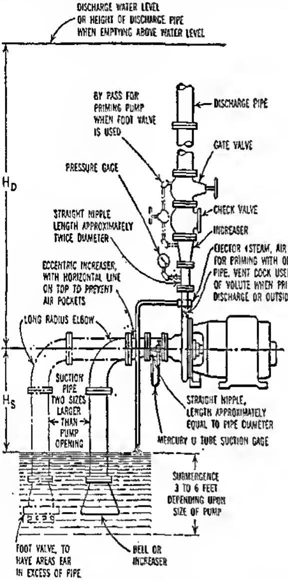


FIG. 19.—Typical pump installation and methods of priming.

forced back into the rotor by the converging casing, the air is discharged through the ports at the bottom of the rotor chambers, through the cone outlet ports, and out the pump discharge.

A small amount of seal water must constantly be supplied. Most of the water stays in the pump. Excess water is carried over with the air and is usually run to waste.

Figure 9 shows a typical characteristic curve for a pump operating at constant speed. Assume that the machine is delivering liquid to a tank or pipe system from which it is removed for use. If the demand on the system is increased, the delivered flow from the unit is also increased and the pressure in the system is reduced, *i.e.*, the operating point moves to the right toward M and the delivery pressure is decreased.

If the demand gradually decreases, the system and delivery pressure increase up to point P on the curve. For further decrease of demand, the system pressure will be higher than the delivery pressure (since the latter starts to drop off) and the fluid tends to flow back into the machine. Hence delivery from the unit stops, and the operating point jumps from P to S. If fluid is still being removed from the system, the pressure will gradually drop below that at S. When this occurs, the unit starts to deliver fluid but will not build up pressure rapidly enough to follow along the curve from S to P but instantaneously begins to deliver a flow corresponding to the pressure against it, which is that at point B. The increased flow from the pump causes the pressure to rise rapidly along the curve from B to point P, and delivery again ceases until the system pressure drops below S. This cycle repeats itself until the condition causing it is removed, *i.e.*, until the flow from the system is greater than the flow from the pump corresponding to point P.

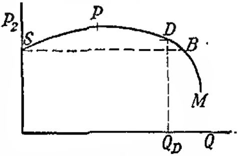


FIG. 9.—Pulsation or surging.

Point P is known as the pulsation point and is the highest point on the characteristic curve. Stable conditions prevail on the curve to the right of this point, hence it should be as far to the left as possible. As noted above, backward-curve vanes are the best from this standpoint.

Specific Speed. Specific speed is a term used to classify impellers on the basis of their performance and proportions regardless of their actual size or the speed at which they operate. Since it is a function of the impeller proportions, it is constant for a series of homologous (having the same angles and proportions) impellers or for one particular impeller operating at any speed.

It is defined as the speed in rpm at which an impeller would operate if reduced proportionately in size so as to deliver a rated capacity of one gpm against a total head of 1 ft. It is designated by the symbol n_S and found from the equation $n_S = n\sqrt{Q}/H^{3/4}$, where Q is the flow in gpm and H is the head in feet at the point of maximum efficiency.

It is not necessary to grasp the physical significance of the definition of specific speed to use it intelligently. It should be considered as a type characteristic of the impeller which specifies its general proportions and performance characteristics rather than as an rpm for special conditions of operation.

The chart of Fig. 10 may be used to find the specific speed with sufficient accuracy for practical purposes and avoids the necessity of calculating the head to the three-fourth power. The point located by plotting the total head and capacity at the point of maximum efficiency is moved parallel to the

Installation. Certain precautions should be observed in planning a pump installation and during the erection period. The suction line should be as short and straight as possible. Any elbows should have large radii. For pumps operating with suction lifts, no valves other than a foot valve should be used. The diameter of the suction line is made one or two sizes larger than the pump flange size. All these precautions ensure the maximum available suction head on the pump. When an oversized line is used, an eccentric reducer is placed between it and the pump flange to avoid high spots at which the air might collect (Fig. 19). The inlet end of the suction line should be 3 to 6 ft below the minimum water level in the sump to prevent air from being drawn into the pipe with the water.

It is desirable to have as long a length of straight pipe between the elbow and suction flange as possible to even out the flow of water as it enters the pump. The pump should be placed to secure the greatest possible suction head and yet be available for inspection and repair.

Both the suction and discharge lines should be independently supported so that no strains will be thrown on the casing which may cause distortion and rubbing. The foundation should be heavy to reduce vibrations and should be rigid enough to avoid any twisting or misalignment. When the pump is in place, it is aligned both radially and axially with the driver by means of wedges and shims so that it turns freely. If the shaft is not properly aligned, there will be vibrating and excessive wear on the bearings, packing, and wearing rings.

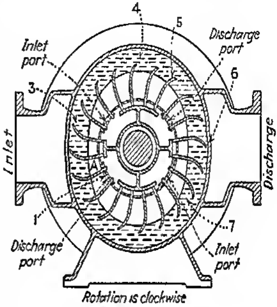


FIG. 20.—Nash-Hytor vacuum pump.

Mechanical Details. The pump shaft is usually protected by sleeves, particularly where it passes through the packing boxes, to prevent scoring and corrosion. For some applications with sea water monel metal shafts may be used, but they are more expensive than ordinary steel. The sleeves fit close to the shaft and are threaded to it as shown in the sectional drawings. The direction of the thread is counter to the direction of rotation and the sleeve is locked in place.

Both journal and antifriction bearings are used for pumps; the former are found more frequently on multistage machines, while the latter are extensively used on single-stage units.

Where the shaft enters the pump casing, packing or stuffing boxes are provided to prevent leakage. These boxes are filled with a soft packing which is compressed against the shaft by a gland which is split to facilitate replacement of the packing without removing the pump cover. For cold-water service, graphited asbestos packing is commonly employed; for hot-

dashed lines to the corresponding speed in rpm. The specific speed is read at the top of the chart directly above this final point. The procedure is illustrated by the heavy dashed lines for a pump delivering 1,200 gpm against a head of 120 ft when operating at 3,500 rpm. The specific speed is then 3,350.

For double-suction impellers the flow should be taken as one-half that handled by the pump. For multistage units the specific speed is taken as that of one stage, *i.e.*, the total head of the pump is divided by the number of stages to find the head to be used in the equation or on the chart.

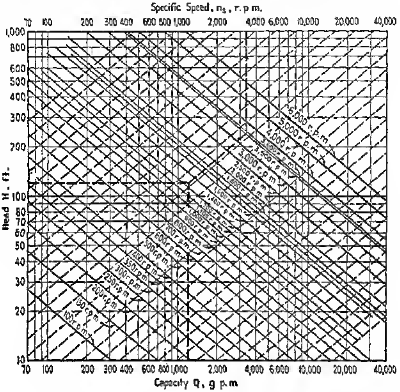


FIG. 10.—Chart for determination of specific speed.

Impeller Classification. One use of specific speed is to classify the various types of pump impellers. Each type has a range of specific speed for which it is best suited. This range is only approximate; there is no sharp line of demarcation between them.

1. *Radial-type Impeller.* The head is developed largely by the action of centrifugal force. They are used for medium and high heads (above about 150 ft). It is the conventional type of impeller and is used in practically all multistage machines. The range of specific speed is generally between 500 and 3,000. The ratio of discharge diameter to inlet eye diameter is usually in the neighborhood of 2. When larger volumes must be handled, a double-suction impeller is used.

2. *Francis-type Impeller.* For lower heads an axial-inlet, radial-discharge impeller is frequently used. For a given capacity and head this type operates at a higher speed than the conventional radial-type impeller. The specific

water service, metallic packing is frequently used. Pumps operating under suction lift have the packing divided into two parts by means of a lantern ring (item Q, Fig. 1). Water is introduced into the ring from the pump discharge line or some other pressure source to prevent air from leaking into the pump.

The impeller is usually cast in one piece and made of cast iron or bronze. It is fixed to the shaft with a light press fit, keyed, and balanced. It may be made "open" with no side plates or shrouds, "semiopen" with a shroud on the side away from the inlet only, or "enclosed" with both a front and back shroud. Open and semiopen impellers have no wearing rings, and the leakage is relatively greater unless a very closed axial clearance between the impeller and the casing is maintained. The enclosed impeller is by far the most common type and will maintain higher efficiencies for longer periods of time generally.

The wearing rings operate with close clearances to reduce the leakage and are generally made of different materials to prevent seizure in case rubbing occurs between them.

For cold-water service, the casing is usually made of cast iron for low pressures and of semisteel for discharge pressures exceeding about 100 psi. Pumps handling sea water are generally made of all bronze, *i.e.*, both the casing and impeller. Care should be taken in this case to use only one material. Although sea water has a relatively slight effect on cast iron alone or bronze alone, it will corrode the cast iron owing to electrolytic action if they are used together.

Volutes are generally used on pumps developing low heads, while diffusers are more commonly used on high-head pumps and multistage machines. If the outside diameter of the impeller is relatively large, it is not generally economical to make the casing large enough to secure sufficient conversion of the velocity head by means of a volute. Pumps having volutes generally have higher efficiencies at partial loads than diffuser pumps, since there are not so many points at which eddies may be set up.

Applications. Boiler-feed pumps must supply a dependable uninterrupted stream of water to the boilers. There must not be any cavitation; and there must be sufficient flow to prevent the water from flashing into steam while passing through the pump. To increase the suction head of high-speed units, a feed booster pump may be placed before the feed pump if a closed feed-water system is used. When operating at reduced capacity or with no flow to the boiler, a by-pass line is used to recirculate from 5 to 10 percent of the design flow back to the heater and thus prevent it from flashing into steam. For pressures below about 400 psi, the pump may be made single-stage; above this, it is usually multistage. The specific speed of the stages is relatively low, hence standard radial-type impellers are used (Fig. 21). These pumps are usually motor or steam-turbine driven at speeds of 3,500 rpm or above. Frequently they have a dual drive with a steam turbine to be used for emergency operation if the current fails. The flow through the pump is generally regulated by a water-level control which throttles the flow to the boiler.

Condensate pumps (Fig. 22) operate under a nearly constant suction and total head. The capacity is based upon the steam flow to the condenser. The impeller is either of the Francis or radial type and is usually motor-driven at a constant speed between 1,000 and 1,750 rpm, depending upon the capacity and submergence. Since the temperature of the con-

$OR - OS = 2b$. Also, if a line through P perpendicular to RS (and therefore tangent to the ellipse at P) meets the minor axis in M, a circle with M as center and MR or MS as radius will cut the major axis in the two foci.

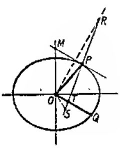


Fig. 40.

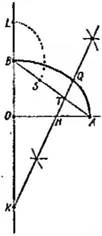


Fig. 41.

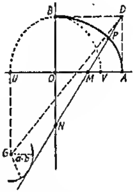


Fig. 42.

To Construct an Ellipse Approximately by Circular Arcs. [Methods (1) and (2) employ two radii, (3) and (4) employ three radii.] (1) In Fig. 41, lay off $OL = OA$ and $BS = BL = a - b$. Bisect SA in T, and draw THK perpendicular to BA. Then H is one center, with radius HA, and K is the other center, with radius KB. The junction point Q of the two arcs will fall a little outside the true ellipse.

(2) In Fig. 42, lay off $OU = OV = OB = b$. Draw UG perpendicular to the axis and DG at 45°. With G as center draw an auxiliary arc with radius

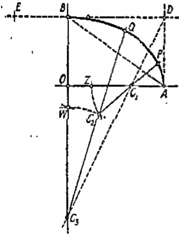


Fig. 43.

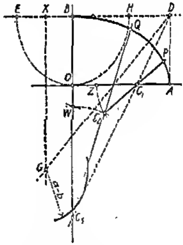


Fig. 44.

$= AV = a - b$, and through D draw DMN just touching this arc. Then M is one center (with radius MA) and N is the other center (with radius NB). The junction point P of the two arcs will be a true point of the ellipse. [E. V. Huntington.]

(3) Through D (Fig. 43) draw DC_1C_3 perpendicular to AB. Call $C_1A = r_1$ and $C_3B = r_3$. Lay off $BE = BO$ ($=b$), and on ED as diameter draw a semicircle cutting the minor axis in W; then $BW = \sqrt{ab} = r_2$. Lay off $AZ =$

BW. From C_1 with radius $C_1Z(=r_2-r_1)$, and from C_3 with radius C_3W $(=r_3-r_2)$, draw arcs intersecting in C_2. Draw C_1C_2 extended and C_2C_3 extended. Then draw in the three arcs, with centers at C_1, C_2, C_3 and radii r_1, r_2, r_3. NOTE. Since r_1 and r_3 are the radii of curvature of the ellipse at A and B, this construction gives a curve which is a little too sharp at A and a little too flat at B. A more accurate construction is the following:

(4) In Fig. 44, lay off $BE = BH = BO = b$. Through the mid-point X of BE draw XG perpendicular to the axis, and through D draw DG at an angle of 45 deg. From G as center draw an auxiliary arc with radius $= DH$ $(= a - b)$, and through D draw DC_1C_3 just touching this arc. Take C_1A as r_1 and C_3B as r_3. On DE as diameter draw a semi-circle cutting the minor axis in W, and take $BW(=\sqrt{ab})$ as r_2. Lay off $AZ = BW$. From C_1 with radius $C_1Z(=r_2-r_1)$, and from C_3 with radius $C_3W(=r_3-r_2)$, draw arcs intersecting in C_2. Then C_1, C_2, C_3 are the required centers. [E. V. Huntington.]

Radius of Curvature of Ellipse at Any Point $P = (x, y)$ **is** $R = a^4b^4(x^2/a^4 + y^2/b^4)^{3/2} = p/\sin^3 v$, where v is the angle which the tangent at P makes with PF or PF'. At end of major axis, $R = b^2/a = MA$; at end of minor axis, $R = a^2/b = NB$ (see Fig. 45). To construct the radius of curvature at any other point P (Fig. 46), draw the normal at P (by bisecting the angle between PF and PF') and let it meet the major axis in N. At N draw a perpendicular to PN meeting PF in H. At H draw a perpendicular to PH meeting PN in C. Then C is the center of curvature for the point P, and a circle about C with radius CP will coincide closely with the ellipse in the neighborhood of P. [Note. If the circle of curvature meets the ellipse in Q, then the tangent at P and the line PQ are equally inclined to the axis.]

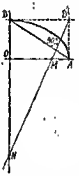


FIG. 45.

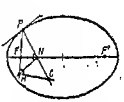


FIG. 46.

THE HYPERBOLA

The **hyperbola** (see also p. 107) has two **foci**, F and F', at distances $\pm ae$ from the center, and two **directrices**, DH and $D'H'$, at distances $\pm a/e$ from the center (Fig. 47). If P is any point of the curve, $|PF - PF'|$ is constant, $= 2a$; and PF/PH (or PF'/PH') is also constant, $= e$ (called the **eccentricity**), where $e > 1$. Either of these properties may be taken as the definition of the

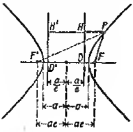


FIG. 47.

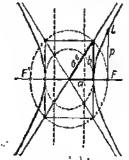


FIG. 48.

densate is close to the boiling point (at the condenser pressure), the available suction head should be carefully investigated for each installation. The pump should be kept as low as possible to reduce the suction lift, and the suction line should be made large to reduce the losses before the impeller.

Circulating pumps operate under constant head and with nearly a constant flow. The flow requirement is determined by the inlet water temperature and the amount of heat to be removed from the steam. The impeller may be either a double-suction (Fig. 1) radial, mixed-flow, or propeller type. The latter form is frequently used in marine work where space and weight are important. The head developed is usually quite low since the water in the discharge line from the condenser usually balances that

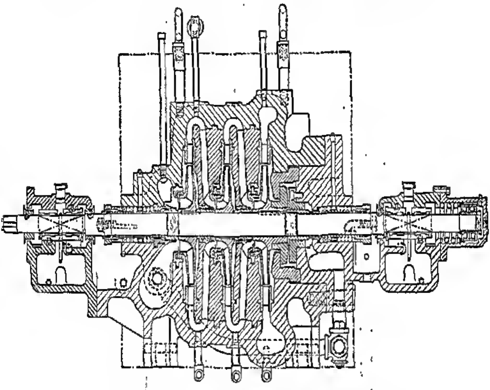


FIG. 21.—De Laval three-stage boiler-feed pump.

in the suction line. Hence the head required is only that needed to overcome the friction and eddy losses in the condenser and pipe lines. These pumps are usually motor-driven. Since the flow is large and the head low, the specific speed is high.

The chief feature of a **bilge pump** is that it must be able to pass large solids. The impeller is extra wide and has few vanes. This results in relatively low efficiencies since the liquid cannot be given much guidance. The impeller is made of cast iron and may be either semienclosed or fully enclosed (Fig. 23). The speeds range up to 1,750 rpm and the heads are low (20 to 40 ft). To prevent the packing boxes and impeller rings from being packed with dirt, provision may be made to flush them continuously with clear water. The suction lift should be low because the liberated gases tend to cause the pump to become gas bound. For this reason the inlet portion is well rounded to decrease the flow resistance, and the shaft may be vertical

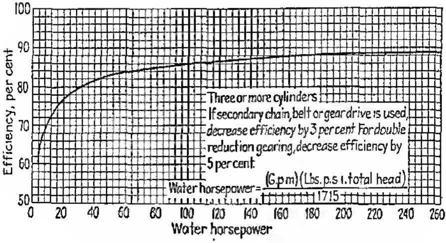


FIG. 32.—Motor-driven reciprocating-pump mechanical efficiency.

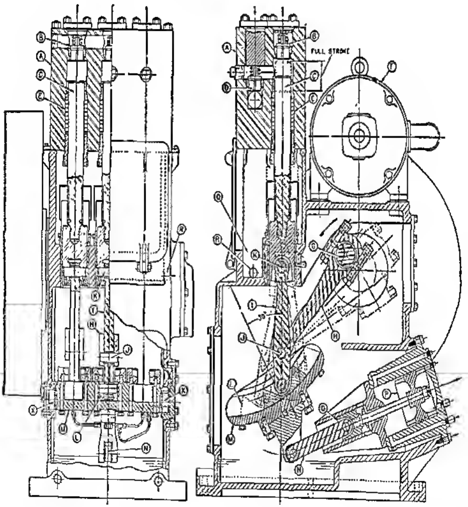


FIG. 33.—Aldrich-Groff controllable capacity pump.

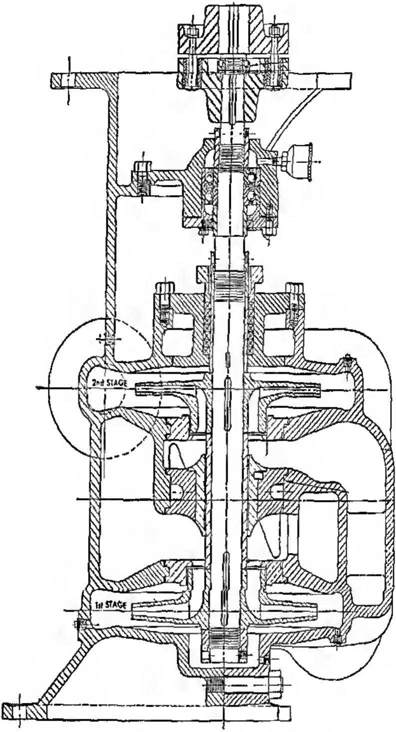


FIG. 22.—Worthington two-stage main condensate pump.

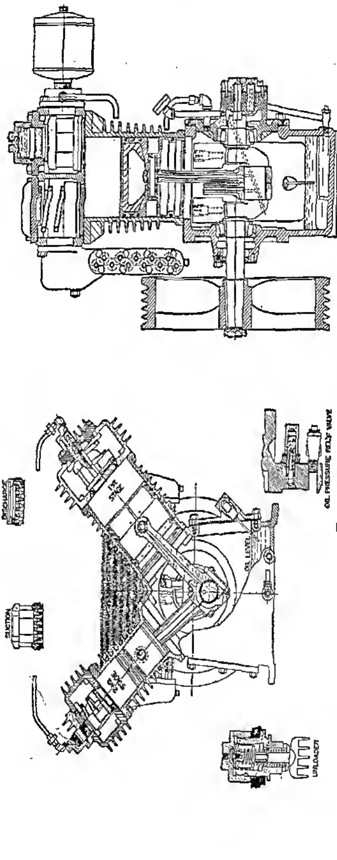


Fig. 34.—Worthington air compressor.

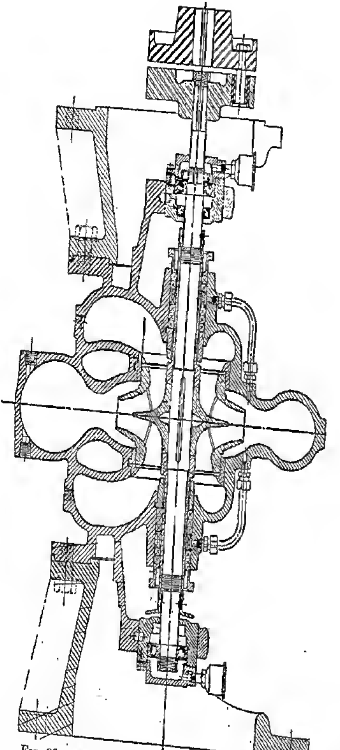


Fig. 23.—Worthington ballast and bilge pump.

may be used. The air may be cooled between the stages, either by sea water for larger sizes or by circulating air for the smaller ones. Figure 34 illustrates a motor-driven compressor of this latter type. The operation is regulated by a pressure-regulating governor. (See also page 1634.)

ROTARY PUMPS

The chief use of rotary pumps is to handle fuel and lubricating oils. They are simple, small, lightweight, and rugged. They have no valves but do require close clearances to maintain efficiency; hence they will not give satisfactory operation when used with abrasive fluids. As the leakage is reduced when the fluid handled has a high viscosity, they are particularly

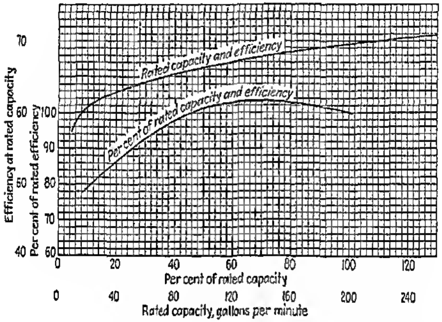


Fig. 35.—Fuel-oil booster-pump efficiencies, rotary or constant-displacement type.

adapted to that field. They are self-priming and will handle fluids containing entrained air or gas, or fluids having relatively high temperatures and low suction pressures which may cavitate. Discharge pressures of 2,000 psi and above can be obtained with capacities up to 3,500 gpm.

Only the three types most commonly used in the marine field will be considered: the screw, eccentric piston, and gear types. Figure 35 gives the approximate efficiency at rated capacities and percentage of rated efficiency for fuel-oil booster pumps of these types; Fig. 36 gives similar values for fuel-oil service pumps. These pumps frequently have a jacket for heating the viscous oil or for water cooling if desired.

Screw Pump (Fig. 37). The driven element is the center rotor which has a helical thread. Two other idler rotors mesh with it. Fluid is trapped in the spaces between the rotors and is discharged at higher pressure. The pump shown has right and left helices to balance the end thrust. The fluid enters at the ends of the rotors and is discharged at the center.

to place the impeller closer to the water level. The same type of pump is used to handle ballast.

Fire pumps may be single-stage, single- or double-suction, or as two-stage units with opposed impellers (Fig. 24). They develop a pressure of about 125 psi and are generally made in standard capacities of 500, 750, 1,000, and 1,500 gpm. **Flushing pumps** are of the same type, but generally develop

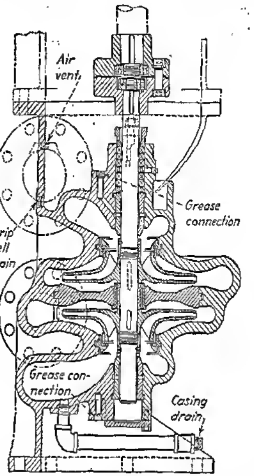


Fig. 24.—De Laval fire pump.

less pressure. Since they handle sea water, they are bronze fitted. As they are used only occasionally and the flows are not large, the efficiency is not too important. Reliability and low first cost are more essential.

RECIPROCATING PUMPS

Classification. Reciprocating pumps are divided into two general classes: direct-acting or steam pumps, and power or motor-driven pumps. The former have two cylinders in line, one steam and the other fluid. They are connected by a rod as shown in Fig. 25. The latter are frequently driven through gears or a belt, and the power is taken from a crank (Fig. 26). They are designated as simplex, duplex, triplex, etc., depending upon whether

Eccentric Piston Pump (Fig. 38). An eccentric is keyed to the driven shaft. The strap that forms the plunger has a close clearance with the pump casing. As the eccentric rotates, fluid is drawn in at the suction side and discharged above.

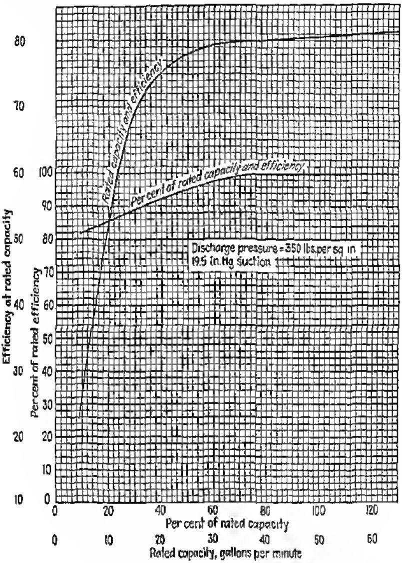


FIG. 36.—Fuel-oil service-pump efficiencies, rotary or constant-displacement type.

Gear Pump (Fig. 39). Pumps of this type consist of a pair of gears (spur or helical) enclosed in a close-fitting casing. One gear drives, while the other idles. Fluid is trapped in the space between the teeth and discharged at a higher pressure.

In designing the installation of fuel-oil pumps, it is necessary to ensure that the suction pressure is kept above the vapor pressure of the oil handled

one, two, three, etc., fluid cylinders are used. Multicylinder pumps will have a more even torque fluctuation than the single-cylinder type. The pump piston may be either single- or double-acting. For marine work they are generally mounted vertically to save floor space.

Mechanical Details. The steam end of direct-acting pumps generally has a cast-iron liner inserted in a steel cylinder, while the water end usually has a bronze liner fitted to a cast-iron cylinder. The rods are made of stainless steel when fresh water is handled, or of monel metal for salt water.

The valve on the steam end of direct-acting pumps is generally of the conventional *D* slide-valve or piston type, and usually operates without cutoff, *i.e.*, there is no expansion of the steam in the cylinder. As the exhaust steam is frequently used for feed-water heating, this practice is not too wasteful

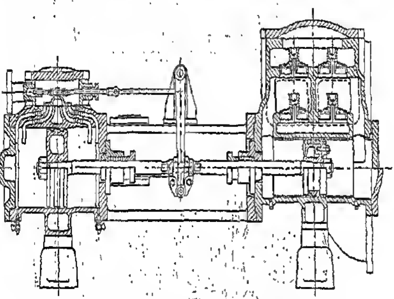


Fig. 25.—Worthington steam-driven reciprocating pump.

and it ensures dependable action. The valve is actuated by the piston rod, shown in Fig. 25.

For the moderate pressures usual in marine work, the water valves are usually of the spring-loaded disk or poppet type. The valve seat is fixed to the frame either by threads or by a slight taper. The disks are made of rubber for low-pressure cold water; while for low-pressure hot water and for lubricating- or fuel-oil services, metal disks are employed. For all high-pressure service the disks are generally made of fabric impregnated with phenolic-base gum.

Design Factors. The size of direct-acting pumps is specified by three numbers in the following order: steam cylinder diameter, water cylinder diameter, and stroke, all in inches.

The leakage past the piston or plunger, the valves, and the packing is known as slip and generally accounts to 1 to 5 percent, depending upon the pump type, clearances, etc.

The **efficiency** of reciprocating pumps is the ratio of the fluid horsepower to the indicated steam horsepower. The water hp = gpm × head, in psi/1,715;

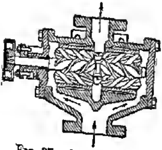


FIG. 37.—Screw pump.

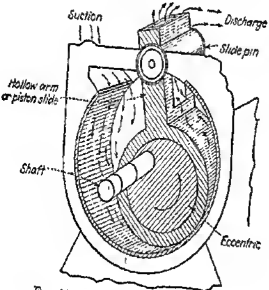


FIG. 38.—Rotating-plunger pump.

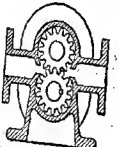


FIG. 39.—Gear pump.

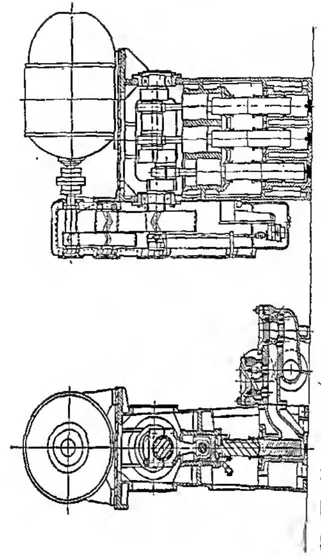
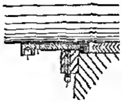


FIG. 26.—Worthington motor-driven reciprocating power pump.



to prevent cavitation and vapor binding, which would reduce the pump capacity and result in generally unsatisfactory operation. Figure 40 gives the vapor pressure of various oils as a function of the temperature. The

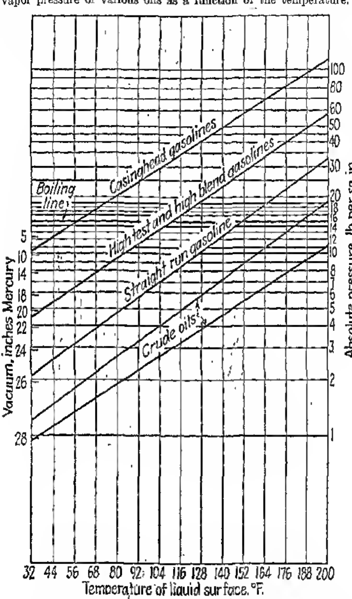


FIG. 40.—Vapor pressure chart for various fuels.

absolute suction pressure should be a safe margin above the values given on the chart for satisfactory operation.

GENERAL

The approximate discharge pressures in pounds per square inch of marine pumps are given in the accompanying table. These values may be used for preliminary estimates of heat balance and costs, but they will of course vary with different ships.

d the indicated steam horsepower = $pLAn/198{,}000$, where p is the steam essure in pounds per square inch, A the cylinder area in square inches, L the roke in inches, and n the *rpm* or number of double strokes per minute. pproximate efficiencies which may be used for estimating purposes may be ken from Fig. 27.

The rubbing speed of a pump in feet per minute is $V = Ln/6$, where the mbols have the meanings given above. Conservative operating speeds, nown as **basic speeds,** for direct-acting pumps handling cold water or quids of a viscosity not over 250 SSU are given in Fig. 28 issued by the Iydraulic Institute, and apply to both simplex and duplex pumps. For igher temperatures or viscosities, the speed is reduced in accordance with 'ig. 29. These charts represent average commercial practice rather than he values to be used for specially designed pumps.

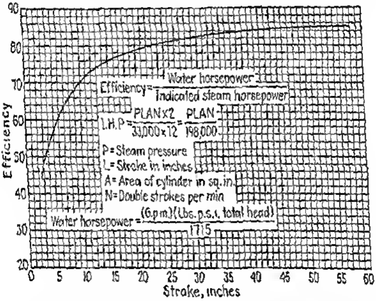


FIG. 27.—Steam-driven reciprocating pump mechanical efficiency.

The force developed on the steam end should be at least 50 percent greater than that on the water end to secure reliable operation, *i.e.*, $\frac{p_s}{p_w}\left(\frac{D_s}{D_w}\right)^2 = 1.5$, where p is the pressure, and D the cylinder diameter, and the subscripts s and w refer to the steam and water ends, respectively. If a factor of 94.5 percent is assumed for the slip and volume loss due to the rod, the flow from a double-acting pump is $Q = 0.00645D_w^2Ln = 0.0385D_w^2V$, the symbols being those given above with the addition that D_w is the diameter of the water cylinder in inches.

To prevent cavitation and consequent flow stoppage in the suction line, the **suction lift** (the vertical distance from the water surface to the top of the pump discharge-valve deck) must be kept within safe limits. Figure 30 issued by the Hydraulic Institute gives these values as a function of the water temperature. Curves T, N, and M are, respectively, the theoretical, normal, and maximum possible suction lifts at sea level. The horizontal distance between T and N represents the additional head required to prevent cavita-

Pump	Discharge Pressure
Ballast	35
Bilge	35
Circulating	10
Condensate booster, or feed booster	50–80
Feed (main)	$1.05P_b + C$ [a]
Fire	125
Flushing	50
Fresh water	75
Fuel-oil service	400
Fuel-oil transfer	*35*
Ice water circulating	20
Lubricating-oil service	50[b]
Refrigeration-condenser circulating	25
Sanitary or flushing	85

[a] P_b is the boiler pressure, and C a constant that equals 65 if an economizer is used; 50, if it is not.

[b] The capacity of these pumps in gpm approximately equals

$$30 + \sqrt{(7.15)(\text{shp}) + 1{,}300},$$

where shp is the ship shaft horsepower.

tion. It includes an allowance for air in the water and the hydraulic losses usually found in a short suction line with one bend and a fluid velocity not greater than 3 fps. For water temperatures greater than 212 F the same differential of 12 ft between curves *T* and *N* must be maintained.

Basic speed curves for simplex, duplex, and triplex power pumps are given in Fig. 31, as issued by the Hydraulic Institute. The temperature and

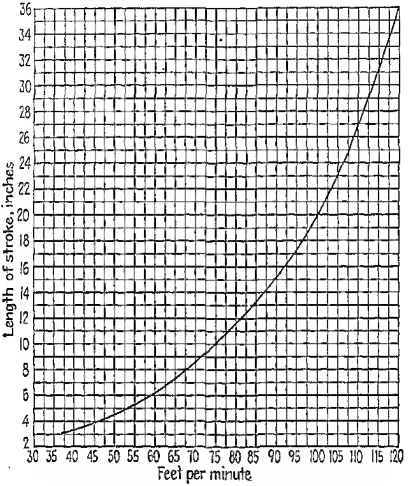


FIG. 28.—The Hydraulic Institute curve of basic speeds for simplex and duplex direct-acting steam pumps.

viscosity correction curve of Fig. 29 also applies to this figure. Figure 32 gives the approximate efficiency for pumps of this type, which is defined as the ratio of the water horsepower to the required horsepower input.

A relatively new type of power pump having a variable stroke is the Aldrich-Groff Powr-Savr, illustrated in Fig. 33, which is particularly suitable *for pressures over 250 psi at the discharge, as for boiler-feed pumps.* The pump is driven by a constant-speed motor, *F*, through herringbone reduction gearing from motor to crankshaft, *G*, which operates the connecting rods, *H*. The other end of each rod, *H*, is pivotally connected to links, *I*, by the con-

AIR AND GAS COMPRESSORS

BY

PAUL DISERENS

From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

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REFERENCES: Simons, "Compressed Air," McGraw-Hill. Peele, "Compressed Air Plants," Wiley. Harris, "Compressed Air," McGraw-Hill. Innes, "Air Compressors and Blowing Engines," Van Nostrand. Von Ihering, "Die Gebläse," Springer, Berlin. Hirsch, "Die Luftpumpen," Jänecke, Hanover. Ostertag, "Theorie und Konstruktion der Kolben- und Turbo-Kompressoren," Springer. Bouché, "Kolbenverdichter," Springer.

Types of Compressors. For pressures below that of the atmosphere, many types of compressors are available. **Fans** (see p. 1668) are usually used for exhaust systems for handling foul air and shavings or other light material. **Jet blowers** (p. 1639) are also used, and for the low pressures occurring in condensers, piston and special centrifugal or impeller types are employed.

For compressing large quantities of air to pressures of 5 or 10 lb, and in some cases pressures as high as 100 lb and over, **centrifugal compressors** (turboblowers, p. 1656) are being increasingly used.

Piston compressors are built for pressures as low as 1 lb above atmospheric, and single cylinders are used with "single-stage" compression to pressures of 80 or 100 lb per sq in. gage. When **higher pressures** are desired, the compression is divided into stages, with intercoolers between the cylinders. Two-stage compressors are used for pressures from 80 or 100 to 500 lb, three-stage for pressures up to 1,200 lb (in small units up to 2,000 or 2,500 lb), and four-stage for pressures of 2,500 to 5,000 lb per sq in. or even higher. All these types of piston compressors may be driven by steam or internal-combustion engines, electric motors, or water wheels.

Table 1. Density of Air at Various Temperatures, Pressures, and Degrees of Humidity

Temperature, F	Weight of 1 cu ft of dry air (at 14 lb per sq in. or 28.5 in. of Hg), lb	Increase or decrease of weight for each 0.1 lb change in pressure, lb	Increase or decrease of weight for each 1 in. of Hg change of pressure, lb	Decrease of weight for each 10 percent increase in relative humidity, lb
32	0.07688	0.000549	0.002698	0.000019
35	0.07642	0.000546	0.002681	0.000021
40	0.07565	0.000540	0.002654	0.000025
45	0.07490	0.000535	0.002628	0.000030
50	0.07417	0.000530	0.002602	0.000035
55	0.07340	0.000525	0.002580	0.000040
60	0.07272	0.000520	0.002554	0.000051
65	0.07203	0.000515	0.002530	0.000059
70	0.07134	0.000510	0.002506	0.000070
75	0.07068	0.000505	0.002482	0.000081
80	0.07003	0.000500	0.002457	0.000095
85	0.06938	0.000495	0.002432	0.000111
90	0.06875	0.000490	0.002408	0.000127
95	0.06811	0.000485	0.002384	0.000147
100	0.06752	0.000480	0.002359	0.000172
105	0.06694	0.000475	0.002334	0.000199

DATA ON AIR

Specific Weight. Table 1 will be found useful in determining the density of air at various temperatures, pressures, and degrees of humidity.

necting rod pins, J, and serves to oscillate them about the axis of th head pins, K, as centers. To the bottom end of each of the link pivotally connected a bronze guide block, L, which slides back an within a smooth curved track on the "stroke transformer," M, whi casting of semisteel. By tilting the stroke transformer, M, the mag of the stroke may be varied. This may be done automatically by me

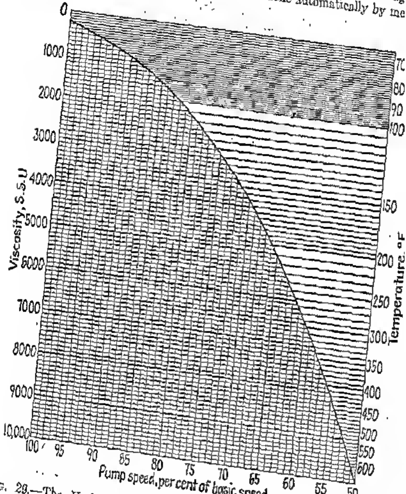


FIG. 29.—The Hydraulic Institute correction chart for temperature or viscosity.

the hydraulic servo-piston, P, which is actuated by a pressure governor or a remotely located water-level regulator of conventional design. The figure illustrates the stroke transformer in the position for full stroke.

High-pressure Air. This is used on shipboard to operate tools, clean machinery, etc. The compressor is of the reciprocating type, either single- or double-stage, and develops discharge pressures of 80 to 125 psi. It is usually driven by a motor, but occasionally a geared steam turbine or engine

Table 2 gives the density of saturated air at various temperatures and pressures.

Table 2. Density of Saturated Air in Pounds per Cubic Foot at Different Barometric Pressures

Temp, F	Barometer readings, inches of mercury								
	28.5	29.0	29.5	29.7	29.9	30.1	30.3	30.5	31.0
30	0.07703	0.07839	0.07974	0.08028	0.08083	0.08137	0.08191	0.08245	0.08381
35	0.07621	0.07756	0.07890	0.07943	0.07997	0.08051	0.08104	0.08158	0.08292
40	0.07541	0.07674	0.07806	0.07859	0.07913	0.07966	0.08019	0.08072	0.08205
45	0.07461	0.07592	0.07724	0.07776	0.07829	0.07881	0.07934	0.07986	0.08118
50	0.07381	0.07512	0.07642	0.07694	0.07746	0.07798	0.07850	0.07902	0.08032
55	0.07302	0.07431	0.07560	0.07612	0.07663	0.07715	0.07766	0.07818	0.07947
60	0.07224	0.07352	0.07479	0.07530	0.07581	0.07632	0.07683	0.07734	0.07862
65	0.07145	0.07272	0.07398	0.07449	0.07499	0.07550	0.07600	0.07651	0.07777
70	0.07067	0.07192	0.07317	0.07367	0.07417	0.07467	0.07518	0.07568	0.07693
75	0.06988	0.07112	0.07236	0.07286	0.07335	0.07385	0.07434	0.07484	0.07608
80	0.06909	0.07032	0.07155	0.07204	0.07253	0.07302	0.07351	0.07400	0.07523
85	0.06829	0.06950	0.07072	0.07121	0.07170	0.07218	0.07267	0.07316	0.07437
90	0.06748	0.06868	0.06989	0.07037	0.07085	0.07133	0.07182	0.07230	0.07351
95	0.06665	0.06785	0.06904	0.06952	0.07000	0.07048	0.07095	0.07143	0.07263
100	0.06581	0.06700	0.06818	0.06866	0.06913	0.06960	0.07008	0.07055	0.07174

Specific Heat. The instantaneous specific heat at constant pressure of **dry air** is given by F. G. Swann as $0.24112 + 0.000009t$, and the specific heat of **water vapor** as $0.4423 + 0.00018t$, where t is the temperature, F. The specific heat of **moist air** with any degree of saturation may then be found by multiplying the weight of air by its specific heat and adding to this the product of the weight of water vapor and its specific heat and dividing the sum by the weight of the mixture. Table 3 is calculated on that basis.

For dry air at high temperatures, see p. 367.

Table 3. Specific Heats of Dry and Saturated Air

Temp, F	Specific heat of		Temp, F	Specific heat of	
	Dry air	Saturated air		Dry air	Saturated air
60	0.2417	0.244	85	0.2419	0.2474
65	0.2417	0.2447	90	0.2419	0.2486
70	0.2417	0.2452	95	0.2420	0.2498
75	0.2418	0.2458	100	0.2420	0.2512
80	0.2418	0.2466	105	0.2420	0.2526

The specific heat of air having any relative humidity may be found from Table 3 by interpolation. For example, for air of 40 percent relative humidity and at 80 F, the specific heat will be $0.2418 + 0.40 \times (0.2466 - 0.2418)$, or 0.2437.

The variation of the specific heat of air with pressure has been investigated by Holborn and Jakob (*Z. Ver. deut. Ing.*, 58, p. 1436). The mean specific heat at constant pressure for the temperature range 20 to 100 C (68 to 212 F) is given by the equation $10^4C_p = 2413 + 2.86p + 0.0005p^2 - 0.00001p^3$, where p is the pressure in kg per sq cm. Values of the specific heat of air at constant pressure C_p as determined by various observers are tabulated at the top of page 1637.

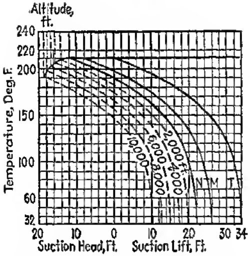


Fig. 30.—Suction lifts for reciprocating pumps.

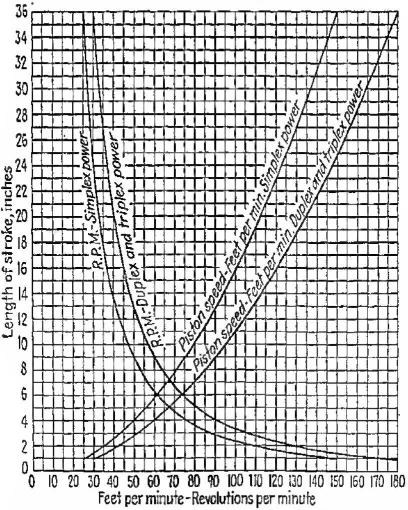
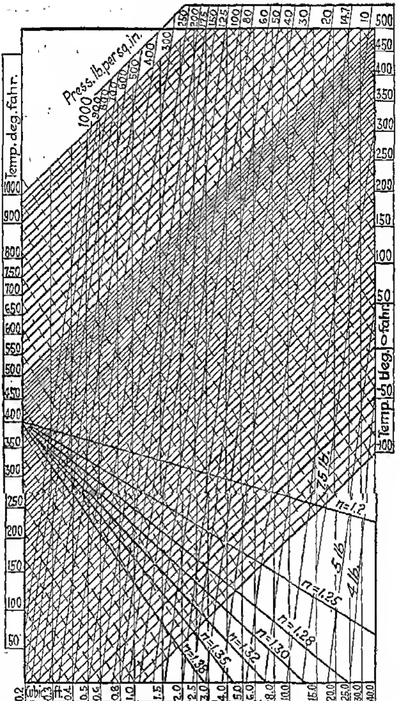


Fig. 31.—The Hydraulic Institute curve of basic speeds for simplex, duplex, and triplex direct-acting power pumps.



Adapted from Prof. C.R. Richards, Entropy-Log. Temp. Diagram, Univ. of Ill. Eng. Experiment Station Bulletin No. 63.

FIG. 1.—Chart for Calculations on Compressed Air.

curve. The curve has two branches which approach more and more nearly two straight lines called the **asymptotes**. Each asymptote makes with the principal axis an angle whose tangent is b/a. The relations between e, a, and b are shown in Fig. 48: $b^2 = a^2(e^2 - 1)$, $ae = \sqrt{a^2 + b^2}$, $e^2 = 1 + (b/a)^2$. The semi-latus rectum, or ordinate at the focus, is $p = a(e^2 - 1) = b^2/a$.

Any section of a right circular cone made by a plane which cuts both nappes of the cone will be a hyperbola. (Compare also Fig. 3, p. 174.)

Equation of the Hyperbola, center as origin:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ or } y = \pm \frac{b}{a}\sqrt{x^2 - a^2}.$$

If $P = (x,y)$ is on the right-hand branch, $PF = ex - a$, $PF' = ex + a$. If P is on the left-hand branch, $PF = -ex + a$, $PF' = -ex - a$.

Equations of Hyperbola in Parametric Form. (1) $x = a \cosh u$, $y = b \sinh u$. (For tables of hyperbolic functions, see pp. 60 and 61.) Here u may be interpreted as A/ab, where A is the area shaded in Fig. 49.

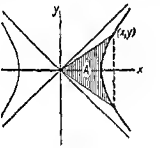


Fig. 49.

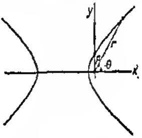


Fig. 50.

(2) $x = a \sec v$, $y = b \tan v$, where v is an auxiliary angle of no special geometric interest.

Polar Equation, referred to focus as origin, axes as in Fig. 50:

$$r = p/(1 - e \cos \theta).$$

Equation of the Tangent at (x_1,y_1): $b^2x_1x - a^2y_1y = a^2b^2$.

The line $y = mx + k$ will be a tangent if $k = \pm\sqrt{a^2m^2 - b^2}$. The tan-

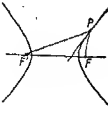


Fig. 51.

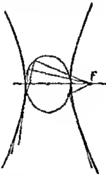


Fig. 52.

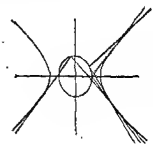


Fig. 53.

gent at any point P (Fig. 51) bisects the angle between PF and PF'. The locus of the foot of the perpendicular from the focus on a moving tangent is the circle on the principal axis as diameter (Fig. 52). The locus of the point of intersection of perpendicular tangents is a circle with radius $\sqrt{a^2 - b^2}$, which will be imaginary if $b > a$ (Fig. 53).

Properties of the Asymptotes. (Fig. 54.) If P is any point of the curve, the product of the perpendicular distances from P to the two asymptotes is constant, $= a^2b^2/(a^2+b^2)$. Also, the product of the oblique distances (the distance to each asymptote being measured parallel to the other) is constant, and equal to $\frac{1}{4}(a^2+b^2)$. If a line cuts the hyperbola and its asymptotes, the parts of the line intercepted between the curve and the asymptotes are equal. The part of a tangent intercepted between the asymptotes is bisected by the point of contact. The triangle bounded by the asymptotes and a variable tangent is of constant area, $= ab$. If a line through Q perpendicular to the principal axis meets the asymptotes in R and S (see Fig. 54), then $\overline{QR} \times \overline{QS} = b^2$. If a line through Q parallel to the principal axis meets the asymptotes in U and V, then $\overline{QU} \times \overline{QV} = a^2$.

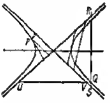


FIG. 54.

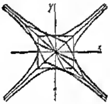


FIG. 55.

Conjugate Hyperbolas are two hyperbolas having the same asymptotes with semi-axes interchanged (Fig. 55). The equation of the hyperbola conjugate to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

Conjugate Diameters are lines through the center, each of which bisects all the chords parallel to the other—a chord which does not meet the given hyperbola being understood to be terminated by the conjugate hyperbola (Fig. 55). If m_1 and m_2 are the slopes, then $m_1m_2 = b^2/a^2$. Each asymptote, regarded as a diameter, is its own conjugate. If a parallelogram is formed *by tangents drawn parallel to a pair of conjugate diameters, its vertices will* lie on the asymptotes, and its area will be constant $= 4ab$. If a', b' are conjugate semi-diameters, and w the angle between them, then $a'^2 - b'^2 = a^2 - b^2$, and $a'b' = ab/\sin w$.

Equilateral Hyperbola ($a = b$). Equation referred to principal axes (Fig. 56): $x^2 - y^2 = a^2$. NOTE. $p = a$. Equation referred to asymptotes as axes (Fig. 57): $xy = a^2/2$. (See also Fig. 3, p. 174.)

Asymptotes are perpendicular. Eccentricity $= \sqrt{2}$. Any diameter is equal in length to its conjugate diameter.

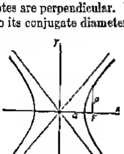


FIG. 56.

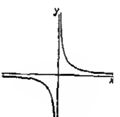


FIG. 57.

Temp. F	−184				−58			
Press. Atm	10	20	40	70	10	20	40	70
C_p	0.2719	0.3221	0.4791	0.7771	0.2440	0.2521	0.2741	0.3121

Temp. F	122			212				752
Press, atm	20	100	220	1	20	100	220	1
C_p	0.2480	0.2719	0.2961	0.2404	0.2471	0.2600	0.2841	0.2430

For the specific heats of industrially important hydrocarbon and other gases, *see pp. 301, 367.*

Calculations of the changes in the pressure, volume, and temperature of air during compression or expansion are facilitated by the diagram Fig. 1, adapted from the "Entropy-Log Temperature Diagram for Air" by Prof. C. R. Richards (*Univ. Illinois Eng. Expt. Sta., Bull.* 63).

In Fig. 1 the vertical lines represent the volume in cubic feet occupied by 1 lb of air. The lines slightly inclined to the vertical represent the absolute pressure in pounds per square inch. The lines at an angle of 45 deg represent temperature in degrees Fahrenheit, and the broken inclined lines represent entropy. These last lines are paralleled in finding the results of an adiabatic change, and if the change follows a $p_1V_1^n = p_2V_2^n$ path, the effects are studied by paralleling lines for various values of n at the lower right-hand corner of the figure, all of which pass through the 400 point on the left of the figure. For example, the adiabatic expansion of 1 lb of air from a pressure of 100 lb abs per sq in. and a temperature of 160 F to a pressure of 14.7 lb abs will result in a final temperature of −100 F. This expansion follows the dotted line of the diagram, but if the expansion followed the equation $p_1V_1^{1.2} = p_2V_2^{1.2}$, the final temperature for any pressure range would be found by following a line parallel to the line marked $n = 1.20$ from the upper to the lower pressure. If such an expansion occurred between 100 lb abs per sq in. and 160 F and 14.7 lb abs, the resulting temperature would be −5 F.

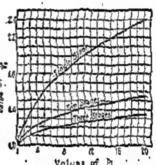


FIG. 2.—Compression Temperatures.

With adiabatic compression, the final temperature T_2 (F abs) is given by the formula $T_2 = T_1\,(p_2/p_1)^{0.283}$ for single-stage compression; by $T_2 = T_1\sqrt{(p_2/p_1)^{0.283}}$ for two-stage compression and perfect intercooling; and by $T_2 = T_1\sqrt[3]{(p_2/p_1)^{0.283}}$ for three-stage. Values of T_2 may be obtained readily from the chart devised by F. W. O'Neill and shown in Fig. 2, in which the ordinates give the factors by which the initial absolute temperature T_1 (F) is to be multiplied to obtain the final absolute temperature T_2 for various ratios of p_2 to p_1 (see also Table 17, p. 315).

BLOWERS AND COMPRESSORS

Rotary blowers are built for air pressures varying from 6 oz to 10 lb or even 12 lb per sq in. The best efficiencies of this type of blower, however, are

The loss in capacity for stage compression will be represented by the following formulas, in which p_1 is the initial and p_2 the final pressure:

$$\text{Two-stage: } C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{2\times 1.4}} - 1\right]; \qquad \text{Three-stage: } C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{3\times 1.4}} - 1\right]$$

Figure 21 represents graphically the part within the brackets of the preceding equations. Knowing the actual clearance and the pressure range, the effect of this clearance on capacity can be found. For example, with a pressure ratio of $p_2/p_1 = 8$, the chart shows this value for single-stage compression to be 3.3. The capacity for these conditions and a 4 percent clearance will be $1 - (0.04 \times 3.3) = 1 - 0.132$, or 86.8 percent of the piston displacement. Clearances in the larger sizes of compressors vary from 2 to 4 percent; in smaller machines they are greater, being in some cases as high as 5 or 6 percent.

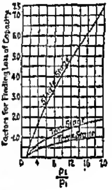


Fig. 21.

The indicator card would be a true method of measuring the volumetric efficiency if the temperature of the air after being drawn into the cylinder were the same as that of the atmosphere and if the pressure at the end of the suction stroke were the same as that of the atmosphere. This is never the case.

The **compression efficiency** is the ratio of the work required to compress adiabatically all the air or gas delivered by the compressor to the work done within the compressor cylinder, as shown by the indicator cards.

Compression efficiency is sometimes expressed in terms of isothermal compression, though this practice is discouraged by the A.S.M.E. in its Power Test Code. Table 4 gives conversion factors or multipliers for converting compression efficiency adiabatic base to corresponding efficiencies based on *isothermal compression with perfect intercooling.*

Table 4. Factor for Converting Compression Efficiency Adiabatic Base to Compression Efficiency Isothermal Base

Ratio of compression	Single stage	*Two stage*	Ratio of compression	*Two stage*	*Three stage*
1.5	0.946		9.0	0.864	
2.0	0.920		10.0	0.859	
2.5	0.890		11.0	0.854	
3.0	0.864		12.0	0.838	
3.5	0.838		13.0	0.833	
4.0	0.813		14.0	0.828	0.894
4.5	0.795		16.0	0.813	0.889
5.0	0.780		18.0	0.798	0.882
5.5	0.770		20.0	0.793	0.878
6.0	0.759		22.0		0.873
6.5	0.752	0.888	25.0		0.862
7.0	0.746	0.883	30.0		0.859
7.5	0.738	0.879	35.0		0.850
8.0	0.732	0.874	40.0		0.842
8.5	0.725	0.869	45.0		0.835

usually secured below 5 lb pressure, but the simplicity of the machine gives it an advantage over compressors of the piston type and frequently warrants its installation for the higher pressures indicated when designed for this purpose. As the machine operates by displacement, it is usually preferred to a centrifugal blower for cupola practice because its positive action will maintain the air supply if the cupola tends to clog. For other uses of air at pressures below 8 oz, the fan is ordinarily more economical.

Blowers of this type may be arranged to give either **constant volume or constant pressure** and to handle either liquids or gases. They consist of a casing containing one or more revolving impellers of various forms of design.

Figure 3 illustrates a cross section of a **Roots blower.** The two impellers are symmetrical and are driven in opposite directions by gears outside the casing. The impellers do not touch each other or the casing, but the clearance is reduced to a minimum in order to reduce slip or leakage. The amount of this **slip or leakage** may be determined by operating the machine with a closed discharge, at a speed sufficient to maintain the required discharge pressure. The amount is usually largest in machines of smallest capacity, *i.e.*, a machine displacing 0.75 cu ft per rev at a pressure of 1 lb will have a slip of from 60 to 70 rev, while a machine having a capacity of 300 cu ft per rev will have a slip of from 3 to 5 rev. For intermediate capacities, the slip will vary proportionally and increase with higher pressures as the square root of the discharge pressure, *i.e.*, at 4 lb pressure the slip will be approximately twice that at 1 lb.

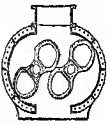


FIG. 3.—Roots Blower.

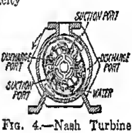


FIG. 4.—Nash Turbine Vacuum Pump.

Blowers of this type built by the Roots-Connersville Co. are suitable for capacities ranging from 5 to 10,000 cfm at pressures up to 5 to 10 lb per sq in. at speeds ranging from 250 for the larger sizes to 1,200 rpm for the smaller sizes. They are commonly used for cupola service, for sewage disposal plants, for low-pressure gas boosters in manufactured gas plants, and for blower service in general.

In most blower work, the so-called hydraulic formula for horsepower will be found satisfactory: hp $= Q(p_2 - p_1)/33{,}000$, where Q is the cu ft of air compressed per min, p_1 the initial pressure, and p_2 the final pressure, lb per sq ft. To get the actual horsepower at the shaft, the horsepower should be divided by the efficiency, which will vary from 0.80 to 0.90.

In the **liquid packing ring type** of blowers used for vacuum pumps or air compressors, water forms a seal between the suction and the discharge chambers. The turbine vacuum pump, shown in Fig. 4 (Nash Engineering Co., South Norwalk, Conn.), uses a bulged out casing and obtains two impulses in every revolution. A rotor revolves in an elliptical casing partly filled with water. The water approaches and recedes from the hub and in this manner acts as a liquid piston displacing the air in the spaces between the vanes of the impeller.

Starting at the point near the suction port, the chamber or space between the vanes is full of water. As the vanes rotate, the elliptical casing permits the water to be thrown out by centrifugal force and air is drawn in. After the inlet ports are passed, the water is forced back into the rotor by the elliptic casing, gradually compressing the air. When the rotor spaces

The mechanical efficiency of an air compressor is the work done in the air cylinders divided by the work done in the engine cylinders if driven direct by steam or gas engine, or by the work delivered at the belt if the compressor is belt driven.

Actual Values of Efficiencies. Tests of piston compressors show extreme variations of mechanical efficiency from 76 to 97 percent, with approximate averages for the more common sizes of 90 percent.

The volumetric efficiency of piston compressors varies from 50 to 75 percent for single-stage 100 lb machines to 80 to 90 percent for multistage machines in which the first-stage pressure does not exceed 30 lb per sq in. Where actual measurement of the air delivered by a compressor cannot be made, the volumetric efficiency may be very closely determined from the vacuum that can be developed by the compressor with the intake completely closed. The test to establish the vacuum developed with closed suction may be made by closing the intake of the compressor and allowing it to discharge into the atmosphere, or, if it is a multistage compressor, by allowing the first-stage cylinder to discharge into the atmosphere.

$$e = 1 - P_2(b - v)/P_a b$$

where e = volumetric efficiency, per cent; P_2 = absolute pressure under which compressor (or first stage) normally operates, lb per sq in. abs; P_a = normal suction pressure, lb per sq in. abs; v = vacuum developed on closed suction, in. of mercury; b = barometer, in. of mercury.

Average values of volumetric efficiency for single-stage compressors with various percentage clearances, taking into account leakage, heating, and other factors, are given in Fig. 22.

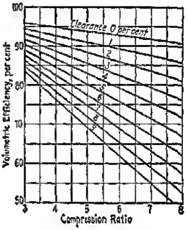


FIG. 22.—Volumetric Efficiency at Various Compression Ratios and for Various Percentages of Cylinder Clearance.

Compression efficiency varies with the number of compressions as well as all factors affecting volumetric efficiency, except clearance. Average or good values, calculated on an adiabatic base, are given below for atmospheric intake pressure.

Discharge air pressure, lb per sq in., gage	10	20	30	40	60	80	100
Compression efficiency (adiabatic base)	0.74	0.82	0.88	0.89	0.89	0.90	0.90

Horsepower Required to Compress a Gas (see p. 319). The horsepower required to compress a gas in a single-stage compressor is

$$\text{Hp} = \frac{144}{33{,}000}\frac{n}{n-1}p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

where V_1 represents the volume (cu ft) of gas compressed per minute and p_1 and p_2 the pressures lb per sq in. abs.

Values of $\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ may be obtained from Table 16, p. 314. Since this tabulation is for expansion from p_1 to p_2, the values of p_1/p_2 of the tabulation are the same as p_2/p_1 in the preceding expression.

have reached the outlet ports the air has been compressed to the terminal pressure and is then discharged completely by the reentering water.

Liquid-packed ring-type rotary compressors are suitable for pressure ranges up to approximately 75 lb per sq in. and vacuum up to approximately

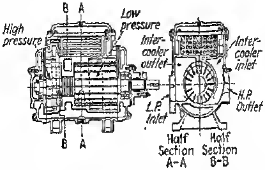


FIG. 5.—Two-stage Sliding-vane Compressor. (Allis-Chalmers "Ro-Twin.")

27 in. Hg in a single stage. Two-stage equipment is available for higher pressures. Standard sizes are available over a wide range of capacities. They are used to advantage where the air or gas to be compressed must be oil free, for example, in certain chemical processes or sewage disposal.

Sliding-vane rotary compressors, originally developed more than 25 years ago, have been used extensively in Europe and have been introduced in the United States by Yeomans Brothers Co., the Fuller Co., and the Allis-Chalmers Co. The principle of operation is shown diagrammatically in Fig. 5. Compressors of this type are built for discharge pressures up to 125 lb per sq in. in capacities up to about 2,000 cfm. Single-stage machines are generally used for pressures up to 50 lb per sq in. and two-stage machines for higher pressures. They are designed to operate at standard electric motor speeds ranging from 3,600 in the smaller sizes to 450 rpm in the larger sizes.

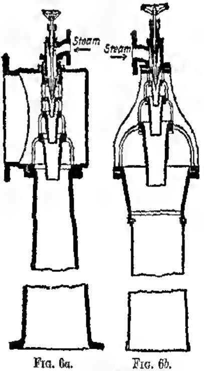


FIG. 6a. FIG. 6b.

FIG. 6a.—Körting Exhauster.

FIG. 6b.—Körting Steam-jet Air Compressor.

Steam-jet Blowers. Steam jets have long been used for "blowing" or exhausting in order to maintain combustion in locomotive boilers, usually employing the exhaust from the engines through properly shaped "nozzles." This type of air compressor or exhauster also finds extended application for emergency use and at times in permanent installations for removing foul air from mines, factories, ship holds, and for gas exhausters, for securing forced draft, and for handling gases under low pressures in certain

Representing the intercooler pressure by p_i, the work done in both cylinders of a two-stage compressor will be

$$\text{Hp} = \frac{144}{33{,}000}\frac{n}{n-1}p_1V_1\left[\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}} - 2\right]$$

With perfect intercooling, $p_1V_1 = p_iV_i$. The preceding expression for the total work will be a minimum when $p_i = \sqrt{p_1p_2}$.

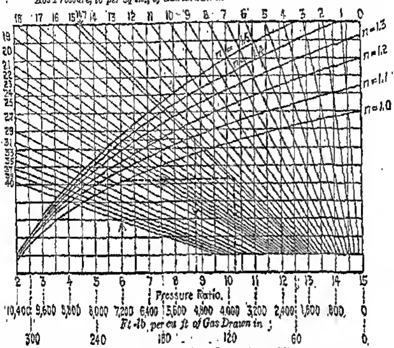


FIG. 23.—Chart for Determining the Work Done in Single-stage Gas Compression (Lucke).

The proper intercooler pressures for three-stage compression are

First intercooler, $p = \sqrt[3]{p_1^2p_2}$; second intercooler, $p = \sqrt[3]{p_1p_2^2}$

The minimum work done in compressing a gas is given by

$$\text{Hp} = 2 \times \frac{144}{33{,}000}\frac{n}{n-1}\cdot p_1V_1\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{2n}} - 1\right] \text{ for two-stage compressors}$$

$$\text{and hp} = 3\times\frac{144}{33{,}000}\frac{n}{n-1}\cdot p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{3n}} - 1\right] \text{ for three-stage compressors.}$$

With perfect intercooling, the volumes of the cylinders should be inversely as the pressures of the gas admitted to them.

Professor Lucke ("Engineering Thermodynamics") gives a number of charts for solving graphically many of the problems of gas compression, of which Fig. 23 is a very convenient one for determining the work per cubic foot and the horsepower per 1,000 cu ft of gas compressed in a single stage to any pressure up to 15 times its initial pressure and for various kinds of compression curves from the isothermal to the adiabatic.

chemical industries. Among its advantages are simplicity, ease of operation, small space, minimum of repairs, and ease of regulation.

Figure 6*a* shows a **Körting exhauster** for removing air, and Fig. 6*b* a Körting compressor of the steam-jet type. These machines are built in capacities of 10 to over 20,000 cfm and usually operate with steam at from 60 to 100 lb gage pressure. In some cases, they are operated by water under pressures from 1 to 45 lb gage. A. von Ihering ("Die Gebläse") reports some tests as to **steam consumption** which indicate 0.19 to 0.37 lb of steam required per 1,000 cu ft of air handled, the larger sizes being the more economical.

Steam-jet exhausters are used principally for lifting liquids as, for example, in creosoting plants, for forcing or inducing air or gases through liquids, for priming centrifugal pumps, etc. A typical performance is shown in the following table:

1 In. Steam Jet Primer
(Steam-operated air exhauster, cubic feet of free air per minute)

Steam pressure, lb per sq in. gage	Vacuum, in. of mercury (barom 30 in.)						
	1¼	5	8	10	15	23	26
25	40	9	0				
50	40	17	12	9	4	0	
75	40	19	14	11	7	2	0
150	40	20	15	13	8	2	0

Water-jet exhausters are also used for priming and other purposes. Typical performance data for water under 70 ft head are given below. The second line gives the free air exhausted per 100 gal of water.

Vacuum in in. of mercury (barom 30 in.).....	4	5	7	10	15	20	25	30
Air in cu ft per 100 gal of water............	23	18	13	8.5	4.5	2.2	0.8	0

Hydraulic Compressors. Several devices have been made for utilizing falling water for the purpose of compressing air without the use of any mechanical moving parts. The most successful of these is the **Taylor compressor,** shown diagrammatically in Fig. 7. In the figure, air tubes are represented at C, all terminating at the conical entrance B to the downflow pipe E. The water supply is furnished through the flume D. As the water falls, it draws air through the small tubes, carrying it down to the separating tank G, where it is liberated at a pressure depending on the weight of water in the vertical pipe H. The compressed air is then conducted through the pipe K to the place where it is to be used. The vertical distance from M to the tail race L represents the fall of water that is available.

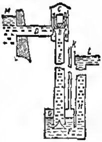


Fig. 7.—Taylor Hydraulic Air Compressor.

In this system, the compression is isothermal and the compressed air is saturated with moisture. The oxygen content of the air is reduced, which renders the air less beneficial for purposes of mine ventilation if the exhaust from the air tools is planned to assist ventilation. The system offers a very simple solution for utilizing water powers when the market for compressed air justifies its installation. It has the advantage of simplicity with a minimum of operating expense, and very high efficiencies are secured. The first cost of the installation is likely to be high.

In Fig. 23 the diagonal lines represent various absolute pressures for the gas drawn in, and the curved lines apply to various kinds of compression curves. The lower horizontal scale gives pressure ratios, work, and horsepower. In using the curves, follow vertically from the pressure ratio to the n curve, horizontally to the inlet pressure line, and vertically downward to the horizontal axis where the work and horsepower may be read. For example, if the compression ratio is 6, the compression curve follows the equation $p_1V_1^{1.4} = p_2V_2^{1.4}$, and the gas is at 14.7 lb per sq in. abs; there will be required 4,960 ft-lb of work per cu ft of gas compressed, or 152 hp per 1,000 cu ft of gas per min.

If the compression for the same pressure range (or to 88.2 lb per sq in. abs) had followed the isothermal compression curve, it would have required 3,880 ft-lb of work per cu ft, or 120 hp per 1,000 cfm.

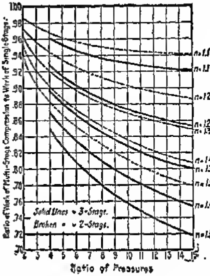


Fig. 24.—Chart for Determining the Work Done in Two- and Three-stage Gas Compression (Lucke).

Figure 24 enables calculations to be made for work and horsepower for two- and three-stage compression, when used in connection with Fig. 23. The dotted lines represent two-stage and the full lines three-stage compression and are marked according to the character of the compression curve. The horizontal scale shows pressure ratios and the vertical scale ratio of work or horsepower for two- or three-stage compression to the work that would be required for single-stage compression as determined from Fig. 23. For example, for a pressure ratio of 8, or a discharge of 117.6 lb per sq in. abs on a suction pressure of 14.7 lb, the work for a compression following the equation $p_1V_1^{1.4} = p_2V_2^{1.4}$ would be 85.2 percent of the work for the same conditions single-stage if two-stage were used and 81 percent of it if three-stage compression were used.

Effect of Altitude. As the density of the atmosphere decreases with the altitude, a compressor located at a high altitude will take in a smaller weight of air at each stroke. The reduction of pressure at the inlet affects the power expended in compressing the air, but the decrease in power required does not vary in the same ratio as the decrease in capacity. For this reason, compressors to be used at high altitudes should have the steam and air cylinders properly proportioned to meet the varying conditions at different levels. Table 5 shows the variation in capacity and horsepower for various altitudes. The altitudes given are heights above mean sea level and are subject to correction for temperature and latitude. From the table, it can be seen that for a two-stage compressor discharging at 100 lb pressure when operating at an altitude of 8,000 ft the weight of air compressed will be only 76 percent of that at sea level and the horsepower required will be 85 percent of that at sea level.

Figures 25 and 26 show the performance of typical single- and two-stage compressors, respectively, when operating at sea level and different altitudes

In 1906, a large plant of this type was installed at the Victoria Copper Mine, near Rockland, Mich., consisting of three units with a total capacity of 34,000 to 36,000 cu ft of free air per minute. A series of tests made on a single intake head by Prof. F. W. Sperr gave the results shown below.

Air measurements				Water measurements			
Free air, cu ft per min	Abs pressure, lb per sq in.		Horse power	Cu ft per min	Head, ft	Horse power	Efficiency, percent
	Free air	Compressed air					
10,580	14	128	1,430	13,057	70.5	1,741	82.17
11,930	14	128	1,623	14,820	70.0	1,961	82.27
9,238	14	128	1,248	12,710	70.6	1,700	73.50

Figure 8 illustrates some of the dimensions of a Taylor hydraulic compressor installed near Cobalt, Ontario, Canada. This was designed for a capacity of 40,000 cu ft of free air per minute to be compressed to a gage pressure of 120 lb. The compressed air is conducted to mines through 9 miles of 20 in. pipe leading to two 12 in. lines with a total distributing line of 21 miles in length. The

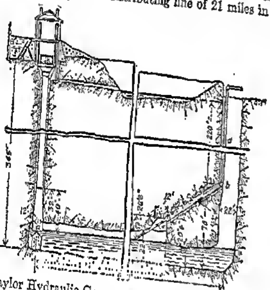


FIG. 8.—Taylor Hydraulic Compressor Installation near Cobalt, Ont.

water is admitted through suitable gates to two "heads" each 16 ft diam and containing 66 pipes each 14 in. diam. The size of the heads is reduced in diameter to about 8 ft, and the whole apparatus can be raised or lowered as required by operating conditions. A cone *a* assists in separating the air and water, and the long horizontal tunnel permits quite complete separation. The compressed air is removed through the pipe *c* and the water freed from the entrained air escapes through the vertical shaft *b*. Pipe *e* acts as a relief for a surplus of compressed air. Its end is normally below the surface of the water in the tunnel, but if the amount of air should accumulate it would be exposed and permit the escape of the surplus air without seriously affecting the normal air pressure of the distribution system.

Piston Compressors and Blowers

For pressures ranging from 5 lb per sq in. to the maximum workable pressure, compressors of the reciprocating piston type are most commonly

for various discharge pressures. Volumetric efficiency shown on the charts is defined on p. 1646 and should not be confused with the quantity coefficient tabulated in Table 5.

Table 5. Variation with Altitude of Quantity and Horsepower Coefficients for Two-stage Air Compression
(Quantity coefficient gives variation in weight of air compressed)

Altitude, ft	Barom press, lb per sq in.	Terminal gage pressure, per sq in.						Altitude, ft	Barom press lb per sq in.	Terminal gage pressure, per sq in.					
		70		100		150				70		100		150	
		Hp coeff	Quantity coeff	Hp coeff	Quantity coeff	Hp coeff	Quantity coeff			Hp coeff	Quantity coeff	Hp coeff	Quantity coeff	Hp coeff	Quantity coeff
Sea level	14.72	1.00	1.00	1.00	1.00	1.00	1.00	8,000	10.85	0.86	0.77	0.85	0.76	0.84	0.76
1,000	14.17	0.98	0.97	0.98	0.97	0.98	0.97	9,000	10.45	0.85	0.75	0.83	0.74	0.82	0.73
2,000	13.64	0.97	0.94	0.96	0.94	0.96	0.93	10,000	10.06	0.83	0.72	0.82	0.71	0.80	0.70
3,000	13.13	0.95	0.91	0.94	0.91	0.94	0.90	11,000	9.69	0.82	0.70	0.80	0.69	0.79	0.68
4,000	12.64	0.93	0.88	0.92	0.88	0.92	0.87	12,000	9.33	0.80	0.68	0.78	0.67	0.77	0.68
5,000	12.17	0.91	0.85	0.91	0.85	0.90	0.84	13,000	8.98	0.78	0.65	0.77	0.64	0.75	0.63
6,000	11.71	0.90	0.82	0.89	0.82	0.88	0.81	14,000	8.64	0.77	0.63	0.75	0.62	0.74	0.61
7,000	11.27	0.88	0.80	0.87	0.79	0.86	0.78	15,000	8.32	0.75	0.61	0.74	0.60	0.72	0.59

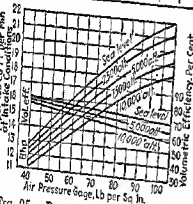


Fig. 25.—Representative Single-stage Compressor Performance.

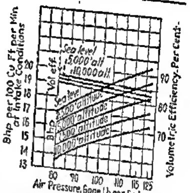


Fig. 26.—Representative Two-stage Compressor Performance.

REGULATION, REHEATING, LUBRICATION

Unloading Devices. Many compressors operate at constant speed, independent of the demands for compressed air; and, in order to secure economy of operation for this condition, various types of "unloaders" have been designed. For small single-stage compressors, some builders provide an unloading valve connected by piping to the inlet valves. When the predetermined pressure is exceeded, the unloader raises the inlet valves from their seats and prevents further compression of air until the pressure falls a few pounds, when the unloader allows the valves to resume their seats and the work of compression is again taken up. An inlet valve so equipped is shown in Fig. 27. Other manufacturers use a similar device to keep the

used. For the lower pressures, compression is generally accomplished in a single stage. For large size cylinders (300 to 5,000 cfm), the limit of pressure per stage is generally set at not more than 3 to 4 compressions; for relatively small sizes (50 to 300 cfm) the limit may be as high as 7 or 8 compressions, and for very small capacities (1 to 50 cfm) 15 compressions is not uncommon.

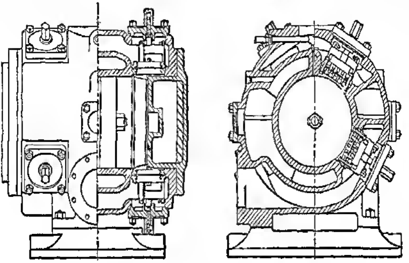


FIG. 9.—Double-acting Air Compressor with Feather Valves.

When the total pressure does not exceed 500 to 600 lb per sq in., cylinders of the double-acting type are generally used. A typical design is that shown in Fig. 9, which illustrates a Worthington feather-valve cylinder of moderate size. For very high pressure, particularly for the third and fourth stage of multistage compressors, cylinders of the single-acting plunger type give the

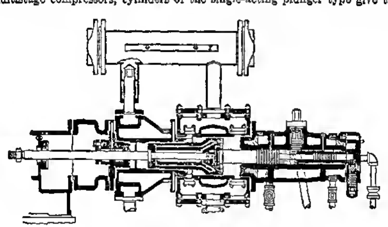


FIG. 10.—Four-stage 4000 lb Compressor.

best service. The plungers may be either ring-packed or packed with soft or semipliable metallic packing using gland and stuffing-box construction. Figure 10 shows a four-stage 4,000 lb compressor. Compressor cylinders and cylinder heads should be as completely jacketed as possible. The air ports and passages are generally proportioned to limit the average air velocity through them to not more than 2,000 fpm. For compressors having a piston

inlet closed when the predetermined pressure is reached. For larger compressors, a *double-beat valve* is used, which is placed on the air-inlet duct and controlled by air pressure from the air receiver. This valve is set to shut off completely all the incoming air when the receiver pressure rises above a predetermined point. In order to relieve the high-pressure cylinder during intervals of unloading, high-pressure relief valves are used.

A total-closure suction unloader is shown in Fig. 28. The discharge pressure acting on the piston *A* compresses the spring *B*; when the port *C* is uncovered, the air has access to the under side of the valve *D* and closes it.

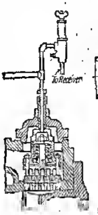


FIG. 27.—Unloading Inlet Valve.

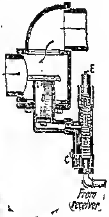


FIG. 28.—Suction Unloader.

Another widely used type of compressor regulation is the variable clearance control. The compressor cylinder (Fig. 29) is equipped with clearance valves *A*, *B*, *C*, and *D*. These valves open communication between the cylinder and the clearance pockets *E*, *E*. The clearance valves are piped to a regulator controlled by the air pressure.

A third type of compressor for variable volume at constant speed is shown in Fig. 30. In this a combination of clearance and suction-valve by-pass control

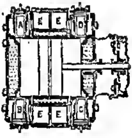


FIG. 29.—Variable Clearance Compressor.

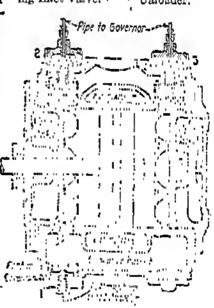


FIG. 30.

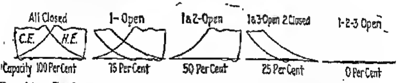


FIG. 31.—Cards from Compressor with Combination Clearance and Suction-Valve By-pass Control.

is employed. The first step of unloading is effected by means of a clearance pocket on one end of the cylinder, the second by holding open the suction valves on the same end, the third by holding open the suction valves on the other end and permitting the first

speed of not more than 700 fpm, the intake pipe connection varies in diameter from $\frac{3}{8}$ to $\frac{1}{2}$ of the cylinder diameter and the discharge connection from $\frac{1}{3}$ to $\frac{1}{2}$ of the cylinder diameter.

Piston speeds, commonly used for air and gas compressors, have been greatly increased since the practical elimination of the mechanically operated valve and the introduction of modern lightweight plate and strip valves. Although some compressors are now built for piston speeds as high as 830 fpm, it is considered better practice to limit speeds to 650 to 750 fpm, with the lower limit for smaller units and the higher limit for larger units. Rotative speeds are fixed to correspond to these piston speeds and vary from 800 for 6 in. stroke compressors to 150 rpm for 30 in. stroke compressors.

The large quantities of air required for blast furnaces and Bessemer converters were, until recently, usually supplied by piston compressors of large capacity, driven either by steam or gas engines. The turboblower directly driven by steam turbines or electric motors has supplemented almost completely the reciprocating piston blowing tubs for this service.

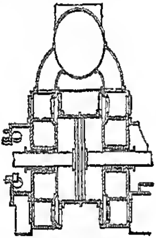


FIG. 11.—Blowing Tub.

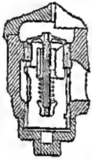


FIG. 12. Inlet Valve.

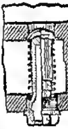


FIG. 13. Discharge Valve.

Ingersoll-Rand.

Piston compressors used as blowers for blast furnaces and Bessemer converters usually operate at discharge pressures of 15 or 20 lb gage with piston speeds as high as 800 fpm. In order to provide sufficient valve area, it is necessary to increase cylinder clearance to $9\frac{1}{2}$ to $11\frac{1}{2}$ percent of the piston displacement. Figure 11 shows the head and valve arrangement of a typical small-sized blowing tub.

Compressor valves are generally of the automatic type as distinguished from the mechanically actuated type. For small compressors, or the high-pressure stages in high-pressure compressors, poppet valves of the single-beat cup type are most extensively used (Figs. 12 and 13). For all other services, some one of the various lightweight plate or strip valves are used. Plate valves may be classified into three groups: (1) rigidly attached disk valves with integrally connected rings and spring element, (2) semi-attached strip or ribbon valves, (3) unattached and independent plate valves. The Ingersoll, the Worthington Feather Valve, and the Chicago Pneumatic Simplate are typical examples of valves falling in the three preceding classifications.

In the **Ingersoll valve** (Fig. 14), the valve disk is made from a thin sheet of steel. The outer portions of this disk are used as valve rings, and the inner portion is used to give the disk elasticity and to guide it. The valve

suction valves to function, and the last step or complete unloading is effected by holding open all suction valves, completely by-passing all air entering the cylinder. Typical indicator cards are shown in Fig. 31.

The performance of a 15½ and 9¾ × 10 direct-connected compressor equipped with combination clearance and by-pass volume control and operated at 360 rpm against 100 lb gage discharge pressure with atmospheric intake at sea level is as follows:

Item	Full load	¾ load	½ load
Displacement, cfm	782.		
Volumetric efficiency, percent	84.1		
Actual capacity, cfm	664.	498.	332.
Total bhp	126.7	98.5	71.5
Bhp per 100 cfm	19.10	19.78	21.5
Total electrical hp	135.	105.	77.
Electrical hp per 100 cfm	20.3	21.1	23.2

Regulators. **Variable capacity control** is obtained in various ways. One method uses solenoid-operated three-way valves to control the admission of air pressure to the compressor unloading elements. The solenoids, mounted on the compressor, are arranged to unload when deenergized and, therefore, may be electrically interlocked with a motor starter for **automatic initial unloading.** The electric circuit to each solenoid is completed through a governor instrument which may be mounted remotely from the compressor.

A **combined speed and pressure** governor (Ingersoll-Rand) for steam-driven compressors is illustrated in Fig. 32. It is of the hydraulic type, operated by means of a small oil pump *A* driven from the compressor shaft. Raising or lowering of the rack and weight *B*, which is a part of the governor, controls the cutoff position of the steam valves and varies the speed of the compressor. Movement of the weight is accomplished by a change in the pressure of the oil under the plunger. The pressure variation is a resultant of the simultaneous action of two separate methods of regulation. Pump *A*, which is chain driven from the main shaft, produces a flow of oil under pressure acting against the governor plunger. Variation in steam pressure produces a change in the oil pressure and causes a movement of the weight and rack and changes the cutoff in such a way as to restore the required speed. Variation in the demand for air or gas also affects this oil pressure by means of a diaphragm-operated valve *D* which by-passes a portion of the flow of oil. The diaphragm is connected to receiver pressure.

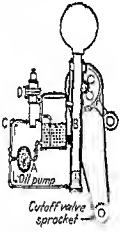


Fig. 32.—Combined Pressure and Speed Governor. (Ingersoll-Rand.)

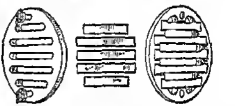
Water Jackets, Intercoolers, Receivers, and Aftercoolers. In good modern practice, the number of square feet of cooling surface per 100 cu ft of free air per minute with brass tubes and for compression to 80 to 125 lb gage is about as follows:

Cooling water temp., F	60	70	80	90
Cooling surface, single-stage, sq ft	16	17	20	23
Cooling surface, two-stage, sq ft	13	13	14	15

is held rigidly through the elastic central portion. The spring loading of the valve is effected by means of spring elements forming part of the cushion plate. The lift of the valve does not exceed ⅛ in. In the **Ingersoll channel valve**, two or more parallel valve members, shaped like a channel, lift uniformly from their seat for their entire length. The back of the channel forms the seat, against which it is held by a flat ribbon spring, bowed against the valve guard at the middle and against the channel at the two ends. The flat spring is the width of the channel opening, which forms the spring retainer. The valve channel is retained in position by recesses in the guard at the ends. The lift of the valve is approximately ⅛ in. The **Worthington feather valve** is shown in Fig. 15. The valve proper is made up of a series of thin flexible strips of ribbon steel, held in position over rectangular openings in the seat by curved milled guards in the valve cover. The valve strips have a lift of about ⅛ in. at the middle and about ⅛ in. at the ends, giving large lift areas. Ribbon steel springs are placed over each valve strip, bowed against the guard at the middle and against the valve strip at the ends. In opening, the spring allows the strip to lift uniformly for half its lift and then flex against the curved guard for the rest of its travel. The **Chicago pneumatic simplate valve** is shown in Fig. 16. The valve seat has circular ports over which are placed, concentrically, thin ring valve elements. These are held down by means of a number of small helical springs. The lift of the outer or larger rings is generally not more than ⅛ in., and that of the inner or smaller rings is sometimes made somewhat less.

Valve Cushion plate Seat

FIG. 14.—Ingersoll High-speed Valve.



Seat Valves Cover

FIG. 15.—Worthington Feather Valve.

Seat Valves Cover

FIG. 16.—Simplate Compressor Valve.

The area through the valves for any compressor depends upon the piston speed and also on the rotative speeds. In some designs, the area through the intake valves is made larger than that through the discharge valves. Generally, however, it is made the same. In low-pressure compressors, or blowing tubs, the valve area is made large enough to give an average velocity of 3,500 to 4,000 fpm. For higher pressures (30 lb per sq in. and over), 5,000 fpm is considered satisfactory. For high rotative speeds and relative low piston speeds, the limits given above may be somewhat exceeded, and for low rotative speeds and high piston speeds they represent the limit of good practice.

Vacuum Pumps. A vacuum pump is essentially a compressor. It takes its suction at low absolute pressure, compresses through a large number of compressions, and generally discharges to the atmosphere. For condenser service, a vacuum pump must develop a vacuum varying from 27 to 29½ in. of mercury. For other services, for example pumping gas out of oil wells, vacuums from 15 to 28 in. of mercury are common. Vacuum pumps for

The number of gallons of cooling water required for aftercoolers (A), intercoolers (I), and cylinder jackets (J), per 100 cu ft of free air compressed, is given by The Compressed Air Society as follows:

Apparatus	No. of stages	Compression pressure, lb gage	Water temperature, F			
			60	70	80	90
A or I	2	80–100	2.5	3.0	3.5	4.0
I and J	2	80–100	2.9	3.4	4.0	4.5
A	1	80–100	4.0	4.5	5.2	6.0
J	2	80–100	0.85	1.0	1.2	1.4
J	1	40	0.5	0.6	0.7	0.9
J	1	60	0.6	0.7	0.8	1.0
J	1	80	0.7	0.8	0.9	1.1
J	1	100	0.8	0.9	1.0	1.2

Receivers are used to supply a reservoir of air, to equalize the pulsations in the air coming from the compressor, to collect the water and grease held in suspension by the compressed air as it leaves the compressor, to reduce the friction of air in the pipe system, and to cool the air as thoroughly as possible before entering the transmission system. To facilitate the removal of water from the compressed air, an **aftercooler** is frequently used. It precipitates the water and thus reduces difficulties in transmission lines and tools. When the transmission pipe line is long, receivers should be placed at both ends of the pipe.

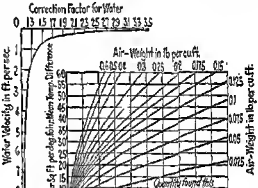


Fig. 33.—Heat Transfer Rate for Aftercooler. (Schutte and Koerting Co.)

Figure 33 gives the heat transfer in Btu per hr per sq ft of surface per deg F mean temperature difference between the air and water with different air velocities in fps for aftercoolers. Thus for an air velocity of 50 fps and for an air density of 0.2 lb per cu ft, the heat transfer rate is about 48. This value, however, must be corrected for the velocity of the water. On the assumption that the latter is 1 fps, the correction factor from the curve is 1.4. The corrected heat transfer rate is $48/1.4 = 34$ Btu.

Reheaters. Heating the air just before expansion may increase the efficiency of the system and, in addition, will increase the temperature at the end of expansion and prevent the freezing up of the motor.

In quarry work, **stoves** are sometimes used for preheating the air. In locomotive work for mines and surface use, hot water is frequently employed for this purpose.

Reheaters are usually capable of raising the temperature of the air to from 300 to 500 F, although common practice shows temperatures of 250 to 350. In figuring on reheaters, it is usual to assume that 1 lb. of coal will

high vacuum are either of the close-clearance type or of the flash-port type. An example of the latter developed by Wheeler is shown in Fig. 17. One suction valve is used which turns alternately to admit air to one end of the cylinder and then to the other. A flash port, cored in the suction valve, opens communication between the clearance space of one end of the cylinder and the compression of the other end, just as the suction and discharge passages are closed. A row of poppet valves controls the discharge. The close-clearance type of pump is identical in construction with that of any air compressor. The valves, usually of the plate or strip type, are placed in the cylinder heads in order to reduce clearance to a minimum. For high vacuum, two-stage pumps are most commonly used. Figure 18 shows a single-cylinder two-stage vacuum pump developed by the Worthington Corp., in which the air is first taken into one end of the cylinder and then discharged directly into the other end of the cylinder, which has a large clearance volume, where it is compressed and discharged into the atmosphere. Pumps of this type develop a vacuum, on closed suction, equivalent to a pressure of less than 0.1 in. of mercury absolute.

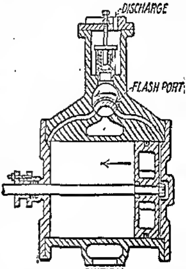


FIG. 17.—Wheeler Flash-port Vacuum Pump.

The capacity of single-stage vacuum pumps, or the volumetric efficiency, may be very approximately measured by the vacuum developed with the intake completely closed. $Q = D(V - V_w)/B$, where Q is capacity, cfm; D the displacement; V the vacuum developed on closed suction; V_w the working vacuum; and B the barometric pressure, all in in. of mercury. The air-handling capacity of vacuum pumps on condenser service must be calculated not from the observed vacuum but from the partial pressure of the air taking into account the vapor pressure of the steam entering the cylinder with the air.

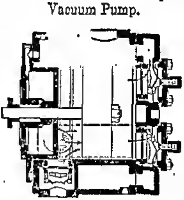


FIG. 18.—Single-cylinder Two-stage Vacuum Pump.

Injection compressors for Diesel engines may be either two or three stage; the latter, however, is preferable. In order to regulate the capacity, one of three methods is generally used: (1) throttle valve in the compressor intake, (2) bleeder valve from the first stage discharge, (3) clearance pockets in the low-pressure cylinders with valves for opening or closing them as occasion demands.

AIR COMPRESSION

For the theory of air compression, see p. 319.

Mean Effective Pressure in Multistage Compression. The mean effective pressure, with complete intercooling, referred to the low pressure piston, is expressed by the formula

$$p_m = p_1\left\{\frac{Nn}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{Nn}} - 1\right]\right\}$$

give from 8,000 to 10,000 Btu to the air. As the specific heat of air is approximately 0.24, 1 lb of coal will raise the temperature of approximately 100 lb (or 1,200 cu ft) of free air 300 deg.

The increase in efficiency resulting from reheating is greater with tools that use air expansively than with machines taking in air for full stroke. Sometimes it is not desirable to have the air entering a tool at a temperature above 300 F because of the effect of this high temperature on the lubrication. For these conditions, small portable hot-water stove-type reheaters are available in capacities of 62 to 800 cu ft of air per min.

Lubrication (See p. 1902). If oil is fed too rapidly in the air cylinders or if it is of unsuitable quality, there is a gradual accumulation of carbon, which interferes with the free movement of the valves and may actually choke the passages and produce high temperatures sufficient to produce ignition or **explosion.**

Explosions have taken place from the introduction of kerosine or naphtha into the air cylinder for the purpose of cleaning the valves and cutting away the carbon deposits. This is a very effective way of **cleaning valves and pipes,** but is a source of danger and should be absolutely prohibited.

In order to reduce the danger of excessive temperatures, fusible **safety alarm plugs** may be inserted in the discharge line. These are usually set for a temperature of 350 F for a single-stage compressor working at 40 lb gage pressure, for a two-stage compressor at 100 lb gage, and for a three- or four-stage compressor delivering at 1,000 lb gage. A 500 F plug is furnished for use with a single-stage compressor discharging at 100 lb gage pressure.

where N represents the number of stages and n is the exponent of the compression curve. Figure 19, from a chart plotted by F. W. O'Neill, shows the relation between the mean effective pressure and the initial pressure for various pressure ratios p_2/p_1 with adiabatic compression and complete intercooling. For isothermal compression, $p_m = p_1 \log_e (p_2/p_1)$.

Wet vs. Dry Compression. The ideal method of compressing air when it is to be stored or allowed to cool before being used is the isothermal, and in the earlier types of compressors this was attempted by so-called wet compressors, by means of which it was possible to secure compressions approximating $p_1V_1^{1.2} = p_2V_2^{1.2}$. The mechanical difficulties involved and the necessary low speeds with consequent small capacity have led to the use of modern "dry compressors," which in small sizes have cylinders with cast-iron ribs for radiating heat and in large sizes have water jackets surrounding the cylinder. The cooling thus secured is sufficient to keep temperatures from being excessive, but as a rule the compression curves are above $pV^{1.35}$ = constant. Dry compression has the advantage of higher speeds and larger capacities.

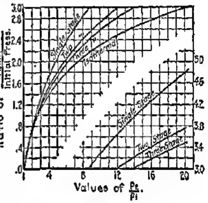


Fig. 19.—Mean Effective Pressures in Air Compression.

Efficiencies. The **volumetric efficiency** of an air or gas compressor is the ratio of free air at intake pressure and temperature actually drawn in, to the piston displacement. The principal sources of volumetric loss are wiredrawing, heating of the air during admission to the cylinder, leakage past valves and pistons, and reexpansion of the clearance air. Except for the last item, none of these losses can be directly measured, so that a determination of volumetric efficiency involves the actual measurement of the air delivered. The low-pressure orifice is generally used for this purpose (see A.S.M.E. Power Test Code for Compressors). Figure 20 shows a typical compressor card. The clearance loss is proportional to MK. If the clearance expansion line follows the equation $p_JV_J^n = p_KV_K^n$, where the clearance $C = V_J/L$, then $V_K/L = (p_J/p_K)^{\frac{1}{n}}C$. The volumetric loss due to clearance may also be written

Fig. 20.—Air Compressor Card.

$$\frac{V_K - V_J}{L} = C\left[\left(\frac{p_J}{p_K}\right)^{\frac{1}{n}} - 1\right]$$

Example. If p_J is 94.7 lb per sq in. abs and C is 2 percent, the volumetric loss due to clearance will be $0.02\left[\left(\frac{94.7}{14.7}\right)^{\frac{1}{1.4}} - 1\right] = 0.0556$.

CENTRIFUGAL COMPRESSORS

BY

JOHN AVERY

From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

REFERENCES: Eck-Kearton, "Turbo-Gebläse und -Kompressoren," Springer. Kearton, "Turbo-blowers and Compressors," Pitman. Ostertag, "Kolben und Turbokompressoren," Springer. Zerkowitz, "Thermodynamik der Turbomaschinen," Oldenbourg. McBride, "Centrifugal Compressors," *Gen. Elec. Rev.*, May, 1932. A.S.M.E. Test Code for Centrifugal Compressors, Exhausters and Fans, Part I.

Centrifugal compressors are similar in design and construction to centrifugal pumps. The pressure that can be produced depends on the peripheral speed of the impeller wheel, the density of the fluid handled, the type of impeller blading used, and the number of wheels. Since air and other gases are relatively light, the centrifugal compressor must run at high speed. The compressibility of gases introduces a phenomenon, not present in centrifugal pumps, known as "pumping," or "surging," which occurs at low load.

Advantages. The high speed of centrifugal compressors makes them especially suitable for drive by steam turbines or electric motors. They can generally be direct-connected to their drives, but in some cases speed-increasing gears must be employed, particularly when the volumes to be handled are relatively small.

Centrifugal compressors for pressures below 1 lb per sq in. are generally known as *blowers* or *fans*. In these machines, the kinetic energy of the gas at the impeller exit is usually allowed to dissipate itself in eddies. For pressures greater than 1 lb per sq in., the kinetic energy is normally recovered as pressure by means of a diffuser. For air pressures of 6 lb per sq in. and under, a single impeller is generally sufficient. Higher pressures generally require two or more impellers in series. More than one impeller may also be required for pressures under 6 lb if the gas handled is lighter than air. Multi-stage compressors for pressures up to 30 to 35 lb gage are generally uncooled. For higher pressures, they are provided with special means for cooling the gas during its passage from stage to stage.

The centrifugal compressor is characterized by small dimensions, relatively light weight, and freedom from vibrations. The only rubbing parts are the bearings. The internal clearances between the rotor and the stationary parts are generally quite large. The air or gas is delivered in a steady stream, free of pulsations. No oil enters the air or gas stream, a desirable feature in many cases, particularly for chemical plants.

The centrifugal compressor is very flexible and easy to regulate, responding rapidly to changes in demand and lending itself readily to such forms of regulation as constant volume, constant suction pressure, and constant discharge pressure.

Limitations. Although a centrifugal compressor will maintain a fairly constant pressure over a wide range of quantities, there is, for every speed, a certain quantity below which the operation will be unstable. This *breakdown point* or *pumping limit* can be pushed back toward lighter loads by making the discharge vanes very few and their inlet angle small or by omitting them entirely. Also, when working on that part of the pressure curve where the pressure remains constant or increases with the quantity, there are usually pressure surges or pulsations which, although slight in themselves, may be greatly intensified by a sort of resonance effect if the volume of the inlet and discharge piping happens to have a certain critical value. A slight throttling

To Construct a Tangent at any given point P of a hyperbola. In Fig. 58, draw PA and PB parallel to the asymptotes, and take $OS = 2(OA)$ and $OT = 2(OB)$. Then ST is the tangent at P.

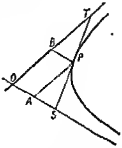


Fig. 58.

To Construct a Hyperbola, given the asymptotes and any point P

(1) In Fig. 59 let TPT' be a variable line through P, and lay off $T'P' = TP$; then P' is a point of the curve.

(2) In Fig. 60, draw PA and PB parallel to the asymptotes. Lay off $OA' = n(OA)$ and $OB' = (1/n)(OB)$, where n is any number; and through A' and B' draw parallels to the axes; these will meet in a point P' of the curve.

Fig. 59.

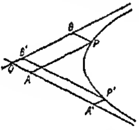


Fig. 60.

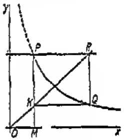


Fig. 61.

(3) (Fig. 61.) Take any point K in the ordinate PM, and draw OK meeting the line through P parallel to the x-axis in R. Draw a parallel to the x-axis through K and a parallel to the y-axis through R, meeting in Q. Then Q is a point of the curve.

THE CATENARY

The catenary is the curve in which a flexible chain or cord of uniform density will hang when supported by the two ends. Let w = weight of the chain per unit length; T = the tension at any point P; and T_h, T_v = the horizontal and vertical components of T. The horizontal component T_h is the same at all points of the curve.

The length $a = T_h/w$ is called **the parameter** of the catenary, or the distance from the lowest point O to the directrix DQ (Fig. 62). When a is very large, the curve is very flat. For methods of finding a in any given case, see problems 1–6 below.

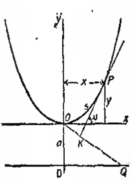


Fig. 62.

The rectangular equation, referred to the lowest point as origin, is $y = a\ [\cosh\ (x/a) - 1]$. (For table of hyperbolic functions, see p. 60.) In case of

very flat arcs (a large), $y = \frac{x^2}{2a} + \ldots$; $s = x + \frac{1}{6}\frac{x^3}{a^2} + \ldots$, approximately, so that in such a case the catenary closely resembles a parabola.

If the perpendicular from O to the tangent at P meets the directrix in Q, then $DQ = \text{arc } OP = s$ and $OQ = y + a$. The radius of curvature at P is $R = (y + a)^2/a$, which is equal in length to the portion of the normal intercepted between P and the directrix.

Problems on the Catenary (Fig. 62). When any two of the four quantities x, y, s, T/w are known, the remaining two, and also the parameter a, can be found, as follows:

(1) GIVEN x AND y. Compute y/x, and find from Table 1 the value of the auxiliary variable z. Then compute $a = x/z$, $s = a \sinh z$, and $T = wa \cosh z$. Or, having z, find s/x and wx/T by using Tables 3 and 2 inversely, and hence (since x is known) compute s and T/w without the use of a.

TABLE 1. GIVING z WHEN y/x IS KNOWN. THEN $a = x/z$

y/x	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0200	0.0400	0.0600	0.0800	0.0999	0.1199	0.1398	0.1597	0.1795
0.1	0.1993	0.2191	0.2389	0.2586	0.2782	0.2978	0.3173	0.3368	0.3562	0.3756
0.2	0.3948	0.4140	0.4332	0.4522	0.4712	0.4901	0.5089	0.5276	0.5463	0.5648
0.3	0.5833	0.6016	0.6199	0.6381	0.6561	0.6741	0.6919	0.7097	0.7274	0.7449
0.4	0.7623	0.7797	0.7969	0.8140	0.8311	0.8480	0.8647	0.8814	0.8980	0.9145
0.5	0.9308	0.9471	0.9632	0.9792	0.9951	1.0109	1.0266	1.0422	1.0576	1.0730
0.6	1.0883	1.1034	1.1184	1.1334	1.1482	1.1629	1.1775	1.1920	1.2064	1.2207

NOTE. $y/x = (\cosh z - 1)/z$.

(2) GIVEN x AND T/w. Compute wx/T, and find from Table 2 the value of the auxiliary variable z. Then compute $a = x/z$, $y = a(\cosh z - 1)$ and $s = a \sinh z$. Or, having z, find y/x and s/x by using Tables 1 and 3 inversely, and hence (since x is known) compute y and s without the use of a.

TABLE 2. GIVING z WHEN wx/T IS KNOWN. THEN $a = x/z$

wx/T	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0100	0.0200	0.0300	0.0400	0.0501	0.0601	0.0702	0.0803	0.0904
0.1	0.1005	0.1107	0.1209	0.1311	0.1414	0.1517	0.1621	0.1725	0.1830	0.1936
0.2	0.2042	0.2149	0.2256	0.2365	0.2474	0.2584	0.2695	0.2807	0.2920	0.3035
0.3	0.3150	0.3267	0.3385	0.3505	0.3626	0.3749	0.3874	0.4000	0.4129	0.4259
0.4	0.4392	0.4528	0.4666	0.4806	0.4950	0.5097	0.5248	0.5403	0.5562	0.5726
0.5	0.5894	0.6068	0.6249	0.6436	0.6632	0.6836	0.7051	0.7277	0.7517	0.7775
0.6	0.8053	0.8357	0.8695	0.9082	0.9541	1.0132	1.1110	......	......	......

NOTE. $wx/T = z/\cosh z$. If wx/T is less than 0.6627, there are two values of z, one less than 1.200 and one greater than 1.200; *only the smaller of these values is tabulated.* If wx/T is greater than 0.6627, the problem is impossible.

(3) GIVEN x AND s. Compute s/x, and find from Table 3 the value of the auxiliary variable z. Then compute $a = x/z$, $y = a(\cosh z - 1)$, and $T = wa \cosh z$. Or, having z, find y/x and wx/T by using Tables 1 and 2 inversely, and hence (since x is known) compute y and T/w without the use of a.

of the inlet will always stop these pulsations by making the pressure curve slightly drooping.

Theory. The theory of the centrifugal compressor follows closely the theory of the centrifugal pump, the main differences being due to the compressibility of gas.

A rapidly rotating impeller wheel (Fig. 1) draws in the gas at its center, or eye, and discharges it at high velocity into an annular diffuser. The diffuser reduces the velocity of the gas and converts its kinetic energy into pressure.

The impeller wheel may take one of several forms. The most commonly employed forms are the **backward bladed**, in which the blades curve or slope away from the direction of rotation, and the **radial bladed**, in which the blades are straight and extend radially. In some few cases, a combination of the two types is used. Each type has features of advantage for specific conditions. In general, the radial-bladed wheel produces a higher pressure for a given peripheral speed and may be operated at higher speeds than the backward-bladed wheel. On the other hand, the backward-bladed wheel gives a somewhat steeper pressure-volume characteristic, has a lower *pumping* limit, and possesses a non-overloading power-input characteristic which is of particular value with motor-driven units.

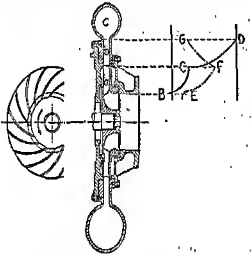


FIG. 1.—Single-stage Centrifugal Compressor with Backward-bladed Impeller *a*, Unbladed Diffuser *b*, and Spiral Volute *c* (*E-F-G* = Velocity Variation; *B-C-D* = Pressure Variation).

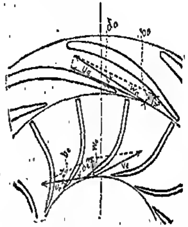


FIG. 2.—Velocity Diagram of Radial-bladed Impeller with Bladed Diffuser.

Figure 1 shows a typical single-stage centrifugal compressor with backward-bladed wheel, illustrating the distribution of velocity and pressure in the wheel and diffuser. Figure 2 shows a radial-bladed wheel in combination with a bladed diffuser.

Notation. Referring to Fig. 2, let

$D_e(D_a)$ = impeller inlet (exit) diameter, ft

$u_e(u_a)$ = impeller inlet (exit) peripheral velocity, ft per sec

$w_e(w_a)$ = absolute inlet (exit) velocity of gas, ft per sec

$v_e(v_a)$ = relative inlet (exit) velocity of gas, ft per sec

$b_e(b_a)$ = impeller inlet (exit) angle, deg

d_e = angle between w_e and u_e, deg

during compression, either by means of water jackets or intercoolers. They are especially suitable for plants where steam is available, enabling them to be direct-connected to steam turbines. Six to eleven stages are commonly employed (Fig. 9).

Centrifugal Compressor Applications

Among applications of interest to Marine Engineers are the following:

Refrigeration Compressors. Centrifugal compressors are used for mechanical refrigeration units in large capacities handling ammonia, methylene chloride, ethyl chloride, Freon 11, and water vapor.

They have found greatest application in connection with water-cooling systems for air-conditioning work. For this service, either Freon 11 or water vapor is generally used in this country. As a consequence of the high density of Freon 11 (the weight is about 4.5 times the weight of air) and the low ratio of specific heats, large pressure ratios can be produced in a few stages. For most water-cooling applications, two or three stages are sufficient. The design of such compressors involves special methods of preventing leakage of refrigerant and air infiltration.

Internal-Combustion Engines. Centrifugal compressors are used to supply scavenging air to large two-stroke Diesel engines. For this service, pressures from 1.75 to 3 lb gage are required for conventional stationary engines and higher pressures—up to 5 or 6 lb—for high-speed marine or traction engines. Single-stage compressors driven by electric motors are usual, although in some cases centrifugal compressors have been geared to the engine shaft. For supercharging gasoline and Diesel engines, centrifugal compressors driven by exhaust gas turbines are quite commonly used.

d_a = angle between w_a and u_a, deg, or inlet angle of discharge vanes, if any
p_1 = initial pressure of the gas, including the velocity energy (if any), lb per sq in.
T_1 = temperature of gas corresponding to p_1, deg F abs
d_1 = density of gas corresponding to p_1, lb per cu ft
p_2, T_2, d_2 = corresponding values for the final conditions of the gas as it leaves the compressor
k = ratio of specific heats of gas, taken as 1.395 for air
$Y = (T_2/T_1) - 1 = (p_2/p_1)^{(k-1)/k} - 1$; for air $= (p_2/p_1)^{0.283} - 1$
s = specific gravity of gas at inlet conditions, referred to that of "free air" as unity
Q = quantity of inlet gas, cu ft per sec
e_h = hydraulic efficiency, referred to adiabatic compression
H = the theoretical (or total) head, or height against which the gas is raised, ft, including all hydraulic losses
N = rpm.

For values of Y see Table 17, p. 315.

Total Pressure Rise. The fundamental equation giving the value of H is as follows:

$$H = (1/g)(u_a w_a \cos d_a - u_e w_e \cos d_e)$$
$$= (1/2g)(u_a^2 - u_e^2 + w_a^2 - w_e^2 + v_e^2 - v^2)$$

For single-stage compression,

$$Y = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1 = \frac{e_h d_1 (k-1) H}{144 p_1 k}. \quad \text{This is equal to } \frac{e_h d_1 (k-1) u_a^2}{4631 p_1 k}$$

when $d_e = 90$ deg (no inlet guide vanes) and $b_a = 90$ deg (radial discharge). Assuming $p_1 = 14.7$, $T_1 = 520$, $k = 1.395$, $e_h = 0.72$, and substituting, $p_2 = 14.7[1 + (u_a^2 s/4{,}300{,}000)]^{3.54}$. For small pressure rise, with $d_e = 90$ deg, $p_2 - p_1 = 0.0000165 e_h u_a^2 s[1 + (v_a/u_a) \cos b_a]$.

Fluid input horsepower is the horsepower applied to the gas and is independent of the actual pressure rise obtained. Fluid input hp $= Qd_1H/550$. For $d_e = 90$ deg, this becomes $0.00000432 Q s u_a^2 [1 + (v_a/u_a) \times \cos b_a]$.

Theoretical Horsepower. This is the horsepower necessary to compress (and deliver) Q cu ft of gas per sec from p_1 to p_2. When the compressor is not artifically cooled, it is common to refer its performance to theoretical **adiabatic horsepower.** The formula for theoretical adiabatic horsepower is

$$P_g = \frac{144k}{550(k-1)} p_1 Q \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right]$$

For normal air ($k = 1.395$), this formula becomes

$$P_g = 0.9252 p_1 Q[(p_2/p_1)^{0.283} - 1] = 0.9252 p_1 Q Y$$

Values of Y for air are given on p. 315. A more extensive tabulation is given in the A.S.M.E. Test Code for Centrifugal Compressors. Table 1 may be used for rough horsepower calculations, when the inlet pressure (p_1) is 14.7 lb abs. The figures given refer to an inlet volume of 100 cfm.

Table 1. Theoretical Adiabatic Horsepower

Final press, lb. per sq in. gage......	1	2	3	5	10	15	20	30	40	50
Theoretical hp per 100 cfm........	0.425	0.83	1.22	1.95	3.58	4 99	6.25	8.4	10.3	11.9

CENTRIFUGAL AND PROPELLER FANS

BY

H. F. HAGEN

From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

REFERENCES: Innes, "The Fan," Technical Publishing Co. Lorenz, "Neue Theorie und Berechnung der Kreiselrader," Oldenbourg. Keller and Marks, "The Theory and Performance of Axial-flow Fans," McGraw-Hill. Spannhake, "Centrifugal Pumps, Turbines, and Propellers," The Technology Press, Cambridge, Mass.

Symbols: p = total pressure, in. of water, above barometric pressure
p_s = static pressure, in. of water, above barometric pressure
p_v = velocity pressure, in. of water
V = absolute velocity, fps
v = velocity of air relative to fan wheel, fps
V_u = rotative component of velocity, fps
d = density of air, lb per cu ft
S = area, sq ft
Q = volume, cfm
T = absolute temperature, deg F
P_b = absolute or barometric pressure, in. of mercury
Air hp = air horsepower
bhp = brake horsepower
e = mechanical efficiency
N = rpm
D = diam of fan wheel, ft
A = fan outlet area, sq ft

FUNDAMENTAL FORMULAS

Pressure, as usually understood, is called **static pressure** in fan engineering and is measured above barometric pressure. The pressure set up by velocity impingement is called **velocity pressure**; the sum of the static pressure and the velocity pressure is the **total pressure.**

Pressure in pounds per square foot is equal to the energy per unit volume in foot-pounds. Static pressure is the potential energy, velocity pressure is the kinetic energy, and total pressure is the total energy per unit volume.

Fan pressures are determined from readings of duct pressures. The **total pressure rise** p of a fan is the increase in total pressure through the fan as indicated by a differential reading between two impact tubes facing the air current, one in the fan inlet and one in the fan outlet.

The **static pressure** of a fan p_s is the total pressure rise p diminished by the velocity pressure in the fan outlet.

The **velocity pressure** of a fan p_v is the velocity pressure in the fan outlet.

Velocity. The velocity pressure, p_v, is conventionally expressed in in. of water. The relation between p_v and the velocity of a gas of density d lb per cu ft is given by

$$V = 18.3\sqrt{p_v/d} \text{ fps} = 1096\sqrt{p_v/d} \text{ fpm}$$

Air horsepower, or fan power output, is the horsepower determined from the product of the volume of air Q and the pressure rise p

$$\text{Air hp} = \frac{62.3pQ}{12 \times 33{,}000} = 0.0001575pQ$$

where p is in in. of water.

For a small pressure rise, the equation hp $= 0.2616Q(p_2 - p_1)$ may be used. For higher pressures, this equation holds when the mean effective pressure p_e is substituted for $(p_2 - p_1)$. For more accurate values, see under Air Compression, p. 319.

Table 2 gives values of p_e for various values of $p_2 - p_1$ when $p_1 = 14.7$.

Table 2. Mean Effective Pressures

$(p_2 - 14.7)$ lb per sq in.	1	2	3	5	10	15	20	30	40
p_e, lb per sq in.	0.97	1.9	2.8	4.5	8.2	11.4	14.2	19.1	23.2

When the centrifugal compressor is water-cooled—which is generally the case for pressures above 35 lb per sq in gage—it is common to refer the performance to theoretical **isothermal compression** rather than to adiabatic compression. In this case, the theoretical horsepower is

$$P_g = 0.0035Qp_1 \log_{10}(p_2/p_1)$$

Table 3 shows the theoretical horsepower for isothermal compression when the inlet pressure p_1 is 14.7 lb abs. It also shows the ratio of the theoretical isothermal horsepower to theoretical adiabatic horsepower.

Table 3. Theoretical Isothermal Horsepower

Final press, lb per sq in, gage	30	35	40	50	60	70	80	100	120
Isothermal hp per 100 cfm	7.1	7.8	8.4	9.5	10.4	11.2	12.0	13.2	14.2
$\frac{\text{Isothermal hp}}{\text{Adiabatic hp}}$	0.845	0.83	0.815	0.799	0.782	0.766	0.755	0.733	0.714

Losses

Hydraulic Losses. The actual total head produced is always less than the theoretical head H because of the hydraulic losses occasioned by friction of the gas in passing through the compressor, because of shocks and eddies caused by abrupt changes in velocity or direction of flow, and because of leakage losses. The ratio of the actual total head produced to the head theoretically obtainable is termed the **pressure coefficient.**

The **hydraulic efficiency** is the ratio of the theoretical horsepower to the fluid input horsepower, or $e_h = 509p_1Y/d_1H$.

For $d_e = 90$ deg,

$$e_h = \frac{214{,}166p_1Y}{su_a^2[1 + (v_s/u_a)\cos b_a]}$$

With a small pressure rise, and $d_e = 90$ deg,

$$e_h = \frac{60{,}600(p_2 - p_1)}{su_a^2[1 + (v_a/u_a)\cos b_a]}$$

The hydraulic losses affect the pressure rise as well as the power required; **mechanical losses** affect the horsepower also. The mechanical losses are *bearing losses* due to friction and *rotation loss*, or disk friction of the impellers.

The order of magnitude of these losses is indicated by the following values, from Kearton, for an uncooled centrifugal compressor, expressed as percent of the power input:

Hydraulic losses: surface friction of gas, 5.2; shock and eddy losses, 5.5; loss due to increase in indicated work due to non-cooling, 1.5; leakage, 2.0. *Mechanical losses:* disk friction, 7.23; bearing friction, 1.57. Total losses, 23.00.

The ratio [illegible] ɔwer required at the compressor coupling, which includes all [illegible] hanical as well as hydraulic, to the theoretical horsepower [illegible] haft efficiency e_s.

In many installations, the fan velocity pressure is wasted and only the static pressure is useful. For static-pressure power p_s is substituted for p.

The efficiency of a fan is the ratio between the horsepower output (air hp) and the horsepower input (bhp). e = air hp/bhp.

The static efficiency of a fan is the ratio between the static-pressure power and the horsepower input.

Standard air density is 0.075 lb per cu ft. This figure is the weight of 1 cu ft of air under average conditions of temperature, humidity, and barometric pressure. Manufacturers' publications of tables and curves are usually based on this standard air density.

The approximate air density is $d = 1.325P_b/T$, where P_b is the barometric pressure in in. of mercury and T is in deg F abs. For most fan testing and calculation, the density determined by this formula will introduce no appreciable error.

Fan pressures and horsepowers vary directly as the air density

Fan Characteristics

The performance of a fan can best be presented graphically. The accepted chart uses volumes as abscissas and pressures, horsepower inputs, and

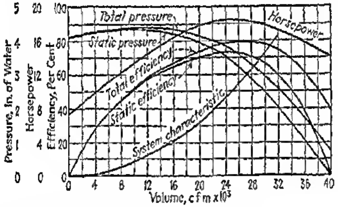


FIG. 1.—Fan Characteristics.

efficiencies as ordinates. The forms of the pressure and horsepower curves depend on the type of blading. They are characteristic of the particular type. Figure 1 shows a typical standard form of plotting of fan performance, with total pressure, static pressure, horsepower, and total and static efficiencies. Curves of the Fig. 1 plotting are necessarily drawn for a given size fan at a given speed.

Other plottings of more general application are also used. Fans function in close agreement with dimensional theory, and dimensionless plotting of fan curves is common practice.

A simple form of dimensionless plot is shown in Fig. 2. The abscissas are in percent of volume at zero static pressure, the wide open volume. The ordinates of the pressure curves are in percent of maximum pressure. Similarly horsepowers are in percent of maximum. Efficiencies are without dimension and are not changed. With the percent scales for abscissas and ordinates of a similar order of magnitude, this plotting preserves the characteristic shapes of the curves. For other forms of plotting, see p. 1682.

System Characteristics. The pressure required to deliver air through a resistance depends on the volume of flow. The relation between volume and

Speeds of Compressors. The larger the volume to be handled, the larger must be the diameter of the impeller, if proper proportions are to be maintained. On account of centrifugal stresses, tip speeds are limited by the available materials for the impellers, and the larger the impeller diameter, the lower the speed. The following table gives representative maximum rpm for various sizes of compressors:

Table 4. Representative Compressor Speeds

Inlet, vol, cfm.	1,300	2,300	4,000	7,000	12,000	20,000	28,000	40,000	60,000	90,000
Speed, rpm.	25,000	19,000	14,000	10,000	8,400	6,600	5,500	4,500	3,800	2,700

In practice, higher or lower speeds may be employed, depending on the type of impeller and the available driving units. Extremely high speeds are obtained only with exhaust gas-turbine drive.

Efficiency of Compressors. For estimating purposes, a round figure of 70 percent may be taken as the adiabatic efficiency of good uncooled centrifugal compressors at designed load conditions. The following table gives representative commercial efficiencies of centrifugal compressors of good proportions for various rated capacities.

Table 5. Representative Compressor Efficiencies (No Water Cooling)

Inlet vol, cfm.	1,300	2,300	4,000	7,000	12,000	20,000	28,000	40,000	60,000	90,000
Adiabatic eff.	0.60	0.635	0.655	0.675	0.69	0.71	0.715	0.72	0.735	0.74

Higher efficiencies have been obtained in practice; blast-furnace blowers and single-stage blowers of large capacity have shown well over 80 percent adiabatic efficiency on test. As a rule, however, extremely high efficiency calls for expensive refinements which are generally not commercially justifiable. Abnormally long and narrow impellers may show poorer efficiencies than indicated above, such wheels being sometimes used to avoid high rpm.

The performance of water-cooled compressors is generally referred to theoretical isothermal compression. Three methods of cooling may be used.

1. Cooling of the Diffusers Only. This method is generally used with moderate pressure (30 to 40 lb gage) and serves mainly to reduce the final temperature of the compressed gas. The saving in horsepower is not appreciable.

2. Cooling by Means of Water Jackets. This method is used for pressures from 35 to 100 lb gage and over. The available cooling surface is limited and jacket cooling is generally used up to about 70 lb only. Figure 9 shows a compressor for pressures up to 70 lb gage with jacket cooling.

3. Cooling by Means of Intercoolers. With this method, the compressed gas is led out of the compressor at intervals during compression and returned to the succeeding stages after cooling.

The following table shows representative efficiencies (referred to isothermal compression) of water-cooled centrifugal compressors, for various rated capacities:

Table 6. Representative Compressor Efficiencies
(With Water Cooling)

Inlet vol, cfm.	5,000	7,500	10,000	20,000	30,000
Isothermal eff, waterjacketed comp., 70 lb gage.	54	57	59	60	60
Isothermal eff, intercooled comp., 100 lb gage.	60	63	64.5	66	66

Centrifugal Compressor Constants and Characteristic Curves

Quantity Constant. The quantity of gas delivered by a centrifugal compressor is proportional to $u_a D_a b_a$, or quantity constant $= Q/u_a D_a b_a$.

pressure can be plotted to give a curve, the system characteristic. The system characteristic is best plotted with volumes as abscissas and pressures as ordinates. It is shown on the fan characteristic chart Fig. 1. The crossing point of the system characteristic and the fan pressure characteristic is the point of operation.

The fundamental system characteristic is that of the fixed resistance system. In such a system, the pressure varies as the square of the volume, giving a simple parabolic curve. Most discussions of the relation of system and fan characteristics are limited to this parabolic form. Other characteristics are empirical, and are of limited use.

Compound system characteristics are frequently encountered. Further consideration may permit their analysis into a combination of fixed resistance systems. An important one is the multiple-nozzle arrangement for steam-boiler combustion. The resistance is due to the duct losses, which are substantially parabolic, *to which is added the constant pressure required for* nozzle combustion. The addition can be made for each nozzle giving isolated points each one of which determines its own fundamental parabolic *characteristic.*

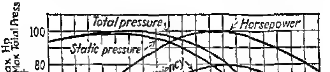


FIG. 2.—Dimensionless Plot of Fan Characteristics.

Equivalent Orifice. The equivalent orifice is a useful concept. If the total pressure delivered to a system is converted to velocity, the equivalent orifice of the system is the area which, multiplied by that velocity, gives the volume flowing through the system. Physically, the equivalent orifice may be considered an ideal nozzle of 100 percent coefficient of discharge. The term is an old one in fan engineering and originally was the equivalent thin plate orifice of the same effective area as above. Allowance was made for the vena contracta with a coefficient of 0.625. This coefficient is subject to modification by stream forms, and the ideal nozzle concept avoids such confusion.

Orifice ratio O is a dimensionless number that serves to relate the performance of geometrically similar fans of different sizes.

$$O = Q/D^2\sqrt{2gP/d}$$

where P is in pound foot units.

This true orifice ratio is equal to the equivalent orifice $Q/\sqrt{2gP/d}$ divided by D^2, or by the fan-outlet area which is proportional to D^2, since the square

Compressor Constant. A compressor model can be used with practically the same efficiency for various combinations of quantity and pressure such that $K = Q/\sqrt{p_e}$, where K is the compressor constant, Q and p_e are the desired quantity and mean effective pressure, respectively. The compressor constant can be more conveniently written as $K = QN^2/p_e^{3/2}$.

Similar Compressors. Two compressors are similar when they have the same compressor constant. In similar compressors, all impeller and discharge-vane linear dimensions are in the same ratio as their impeller diameters, and their impeller and discharge vane angles are, respectively, equal. For the same rpm, the quantities delivered by similar compressors will vary as the cubes of their diameters, the pressures will vary as the squares of their diameters, and the shaft powers will vary as the fifth powers of their diameters. For the same wheel speed, the quantity and the power will vary as the square of the diameter, but the pressure will remain constant.

Compressor Coefficients. In order to make the tests on different compressors, or on the same compressor under different circumstances, comparable on a common basis, various coefficients are computed corresponding to the given observations and these coefficients are plotted as **characteristic curves.** From these curves the pressure, power, and hydraulic and shaft efficiencies can readily be computed for any quantity of gas and any rpm. The departure of the ratio v_a/u_a from that value for which the compressor was designed determines largely the efficiency of operation. Therefore, v_a/u_a or its equivalent Q/u_a is generally used as the abscissa for the characteristic curves of a compressor, and it is designated as the **load coefficient** C_e. It is also frequently represented by Q/N. The **fluid input coefficient** (C_i) represents the horsepower corresponding to any given observation divided by the cube of the wheel speed. For the case of axial impeller inlet and radial impeller exit, $C_i = 0.00000432Qs/u_a \doteq 0.00000432C_e s$. Evidently the characteristic curve of C_i against C_e is a straight line making an angle with C_e whose tangent is $0.00000432s$; it may be drawn independently of the actual observations. In general, $C_i = 0.00000216sQV^2/u_a^3$, where $V^2 = 2gH = u_a^2 + w_a^2 - v_a^2 - u_e^2 - w_e^2 + v_e^2$. The **pressure coefficient** for normal air corresponding to the observed pressure rise ($p_2 - p_1$), is $C_p = YT_1/u_a^2$. The **theoretical power coefficient** is $C_t = 0.02571C_eC_p$. The characteristic curve for C_t should be computed and drawn from readings from the *smooth* curve of C_p against C_e. The **shaft power coefficient** is C_s = shaft hp$/u_a^3$. The **rotation loss coefficient** is $C_r = 0.0737 \times 10^{-6} D_a^2 d_m$, where d_m is the average density of the gas between p_1 and p_2, lb per cu ft. By adding the values of C_r to the values of C_i, a characteristic curve (practically a straight line) of fluid input plus rotation loss is obtained, and this curve will, in a correctly designed compressor, nearly touch the shaft power characteristic curve at the value of Q/u_a corresponding to the rated load of the compressor.

Hydraulic and Shaft Efficiencies. The ratio of C_t to C_i for any value of C_e gives the hydraulic efficiency e_h for that particular load. (For $C_e = 0$, the general formula $e_h = 6{,}080\ C_p$ must be used.) Similarly, ratios of C_t to C_s give values of e_s, the shaft efficiency. The efficiency curves thus obtained will of course be smoother and more reliable than if the efficiencies were computed directly from the individual observations of pressure and power.

Uses of Characteristic Curves. Besides affording smooth curves of hydraulic and shaft efficiencies, a set of characteristic curves as shown in

of the wheel diameter is directly proportional to the fan areas in any line of geometrically similar fans.

The conventional orifice ratio is simpler to calculate and has a different numerical value. It departs from the fundamental units, and the ratio between equivalent orifice and fan area is less apparent. For convenience g is considered constant, d taken as standard air, and p in inches of water. The ratio remains dimensionless only if the dimensions of the changes are understood.

$$O = Q/D^2\sqrt{p}$$

At constant orifice ratio and constant density, for fans of geometrically similar design, the following relations are valid:

$$Q \propto D^3N$$
$$p \propto D^2N^2$$
$$\text{Hp} \propto D^5N^3$$

For any given fan, D is constant, and at constant orifice ratio the fan laws are (1) volume varies directly as the speed, (2) pressure varies as the square of the speed, and (3) horsepower varies as the cube of the speed.

For fans of different sizes but at the same tip speed πND, the volume and horsepower vary as D^2, pressure remaining constant.

For a given orifice ratio, the velocity relations of air flow to blade speed are constant, and the efficiency of geometrically similar fans is independent of size or speed throughout a wide range. This principle is the basis of the fan industry and is used by fan manufacturers in preparing the tables and charts that they publish. An example of such a chart is the Hagen chart, Fig. 17.

DESIGN OF CENTRIFUGAL FANS

Casing. The fan casing collects the air delivered from the impeller and directs the flow into the connected duct. It is commonly a *spiral* or *scroll*. The sides of the casing are plane and parallel. In such a space, the law of the constancy of the moment of momentum, or circulation, requires the spiral to be logarithmic.

$$\theta - \theta_0 = K \log (r/r_0)$$

Figure 3 shows a typical layout of fan spiral.

More complex forms of housing are sometimes used for various reasons. But performance at high efficiency is more readily secured with the simpler logarithmic casing.

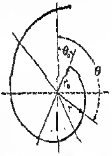


Fig. 3.—Logarithmic Spiral.

The value of the constant K can be determined aerodynamically as the ratio of the tangential component to the radial component of the air velocity leaving the fan wheel. This rational procedure usually results in a casing which is much too large for commercial consideration.

Experiment shows that the inner edge of the fan outlet may be located on the center line of the fan and the spiral swept from this point around to form an outlet substantially square. The outlet area should give a velocity pressure 10 percent of the static at the specified volume. K is then determined by trial and error to give the required sweep. Figure 4a shows a conventional spiral so determined.

Fig. 3 enables one readily to draw reliable pressure and power curves against quantity for any given wheel speed or rpm. *In that case, the load-coefficient* scale may be replaced by a quantity scale, and the readings of the C_p and the C_s curves will give the data for the corresponding pressures and powers. (In Fig. 3 the wheel speed has, in all the coefficients, been replaced by the rpm.) The system of characteristic curves is found to hold true for various sizes of centrifugal compressors, centrifugal blowers, and centrifugal pumps, giving in each case consistent curves regardless of the actual speeds, pressures, and powers. Only in ventilating-fan blowers do there seem to be occasionally some serious discrepancies near the point of maximum efficiency.

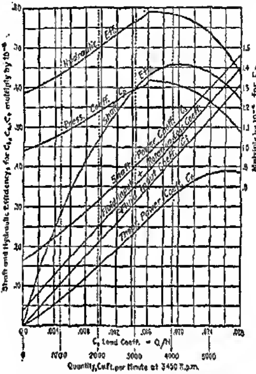


FIG. 3.—Characteristic Curves of a Centrifugal Compressor.

Numerical Examples

Power Required. Find the power required to compress adiabatically 20,000 cu ft of air per min from atmosphere to 30 lb per sq in. gage, the shaft efficiency of the compressor being 75 percent.

SOLUTION: From Table 1, the theoretical horsepower is 8.4 × 20,000/100 = 1,680 and the shaft hp is 1,680/0.75 = 2,240.

Equivalent Suction Pressure. What suction can be obtained with a compressor rated to deliver 2,500 cu ft of air per min against 2 lb per sq in. gage?

SOLUTION: The compressor is rated for an *initial* pressure of 14.7 lb per sq in. Since the *pressure ratio* depends only on the wheel speed, the hydraulic efficiency, and the initial temperature, all of which are supposed to remain the same, the initial suction pressure is (14.7/16.7) × 14.7 = 12.94 lb per sq in. abs and the suction obtained is 14.70 − 12.94 = 1.76 lb per sq in.

Equivalent Pressure when Compressing Gas. What pressure can be obtained when compressing water gas with a standard unit rated 25,000 cu ft of air per min

Modern design allows wide variation in fan outlet area for a given wheel. But a good proportion, which conforms to the 10 percent rule above, is

$$A = 1.5D^2$$

Calculations for casing of Fig. 4a. $D = 2$ ft; $A = 6$ sq ft; $\sqrt{A} = 2.45$ ft $= r_1$; 3 in. = distance to cutoff; $r_0 = \frac{D}{2} + \frac{3}{12} = 1.25$ ft; $\theta_0 = 90$ deg; $\theta_1 = 360$ deg; $\theta_1 - \theta_0 = K \log (2.45/1.25)$ deg; $K = 922$. Equation of spiral $\theta - 90 = 922 \log (r_1/1.25)$.

Cutoff. There must be a transition from the spiral flow in the housing to the straight line flow in the connected duct. Figure 4b shows the application of the principle of directing vanes to accomplish this result. Tests show that the value of this multiple vaning is negligible. The inner vane, the cutoff sheet, is worth while; the commercial casing most used is as indicated in Fig. 4c. The cutoff sheet extends into the outlet 10 to 40 percent, the lesser amount for the backwardly curved blade fan wheels and the greater amount for forwardly curved blade fans.

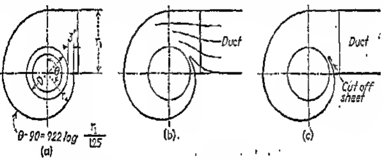


FIG. 4.—Fan Casing Scrolls.

The cutoff sheet continues the curve of the spiral. The nearest point of the cutoff sheet to the wheel should be from 3 to 5 in. in a fan of 3 ft diam. Larger diameters require a greater distance, and smaller wheels permit less. Exact proportion is not necessary as noise rather than efficiency is the determining factor in fixing this dimension.

Width of Casing. The rule given for the size of the fan outlet, *i.e.*, a velocity pressure 10 percent of the static, and the desideratum of a substantially square outlet determine the width of the casing as well as the angle, or throw, of the spiral.

Commercial fans are probably the best guide to theoretically indeterminate dimensions. Fan housings are frequently standardized for a given wheel diameter and the same housing used for fan wheels of different types. Tests indicate consistently good performance. The discovery—a purely empirical one—that forwardly curved blade and backwardly curved blade wheels would operate successfully in the identical housing has indicated that the amount of the cross-sectional area in the housing is more important than the disposition of that area.

The flow in the casing is a complex action of a spiral flow around the fan shaft center line and a vortex around the spiral center line of the casing. This latter vortex continues spinning down the outlet duct. It can be changed, and sometimes eliminated, by changing the position of the wheel axially in the housing. Such change does not affect the true fan performance. The

and 15 lb per sq in. gage, and what power will be required if it requires 2,000 hp to compress the air to 15 lb pressure?

SOLUTION: The density of water gas is 0.05167, and its specific gravity (compared with air) is 0.677. The mean effective pressure (mep) corresponding to 15 lb per sq in. is 11.4 for air (see Table 2). For water gas, everything remaining the same, mep = $11.4 \times 0.677 = 7.72$ lb per sq in., and the corresponding final pressure is 9.3 lb per sq in. gage. The theoretical power for the air rating (from Table 1) is $4.99 \times 25{,}000/100 = 1{,}248$ hp. For water gas, the theoretical horsepower is $1{,}248 \times 0.677 = 845$ hp, and the actual power $= 2{,}000 \times 0.677 = 1{,}354$ hp.

Equivalent Rating at Other Speeds. A standard centrifugal compressor rated 4,500 cu ft of air and 15 lb per sq in. pressure is to be speeded up from 3,450 to 4,000 rpm. What increase of pressure and of quantity will result?

SOLUTION: The mep for 15 lb per sq in. is 11.4. At 4,000 rpm, it will increase to $(4{,}000/3{,}450)^2 \times 11.4 = 15.33$, and the corresponding final pressure from Table 2 is 22.05 lb per sq in. abs. Also the new quantity will be $(4{,}000/3{,}450) \times 4{,}500 = 5{,}220$ cu ft per min if the same ratio of u_e/v_e, and therefore the same hydraulic efficiency is to be maintained. See Quantity Constant and Load Coefficient, pp. 1660, 1661.

General Problem. What standard compressor can be used to exhaust 18,500 cu ft of anthracite producer gas per minute against a suction of 7 lb per sq in.? The compressor is to be installed 2,000 ft above sea level. What horsepower is required?

SOLUTION: The barometer at 2,000 ft altitude is 13.56. The compressor is therefore required to compress the gas from $13.56 - 7.00 = 6.56$ lb per sq in. abs to 13.56 lb. This is equivalent to compressing the gas at sea level to a final pressure of $(13.56/6.56) \times 14.7 = 30.4$ lb per sq in. abs, or 15.7 lb gage. The density of anthracite producer gas is 0.065 lb per cu ft. The mep corresponding to 15.7 final pressure is (Table 2) 11.9 lb per sq in. The corresponding mep for air is $(0.0764/0.065) \times 11.9 = 14.0$, and the final pressure from Table 2 is 19.4 lb per sq in. gage. Suppose the nearest standard compressor is rated 16,000 cu ft per min, and 15 lb per sq in. at 3200 rpm. The mep for 15 lb is 11.4 and for 19.4 is 14.0. The standard compressor must therefore be speeded up to $3200 \times \sqrt{14/11.4}$, or 3,540 rpm. For a constant "load coefficient" Q/N, the new quantity will be $16{,}000 \times 3{,}540/3{,}200 = 17{,}700$. So if it is desired to use the standard compressor and save the extra cost of a special size, then only 17,700 cu ft per min of the gas can be exhausted with a suction of 7 lb per sq in. gage; or the compressor can be speeded up to handle 18,500 cu ft per min, but the suction will be somewhat above 7 lb. If the horsepower required by the standard compressor is 1,300 (corresponding to a shaft efficiency of 0.615) the horsepower for the desired conditions will be $1{,}300 \times (17{,}700/16{,}000) \times (11.9/11.4) = 1{,}495$.

Relative Proportions of Parts. The **impeller hub diameter** is generally 1 to 2 in. larger than the shaft diameter and usually between $\frac{1}{4}D_a$ and $\frac{1}{3}D_a$. The **casing inlet diameter** D_c is determined by the velocity desirable to allow in the annulus between the casing inlet and the impeller hub and varies generally between $\frac{1}{3}D_a$ and $\frac{2}{3}D_a$. For an axial-inlet type D_c (in.) $= 12(Q/KN)^{\frac{1}{3}}$, where Q = quantity, cu ft per min, N = rpm, and $K = 1$ for single-inlet impellers and 2 for double-inlet impellers. In radial-inlet-type impellers the **impeller inlet diameter** is $\frac{3}{4}$ to $1\frac{1}{2}$ in. larger than D_c, to allow the air to turn into a radial direction before striking the impeller blades. In axial-inlet-type impellers, the outer diameter of the impeller inlet blades is just sufficiently smaller than the casing inlet diameter to allow for mechanical clearance. The total **axial width of blade** at the inlet (on both sides of the impeller web in a double-inlet impeller) is $\frac{1}{6}D_a$ to $\frac{1}{3}D_a$; at exit, $\frac{1}{6}D_a$ to $\frac{2}{3}D_a$. Frequently the impeller exit width is designed to give a sufficiently small discharge-vane angle, and then the blade is made of constant over-all width (parallel edges). The **number of impeller blades** z_i should be as small as possible consistent with the proper guiding of the gas. The usual number is between 16 and 24, with an equal number of half blades at the outer periphery, if necessary.

determination of the axial location of the wheel in the casing is a matter of structural advantage.

Fan Inlet. The fan inlet is a converging passage, preferable curved, leading from the ambient air or from an inlet duct to a smaller diameter at the entrance to the fan wheel. Some designs are improved by a small expansion of the inlet piece immediately before the wheel—the so-called *Venturi passage.* The upstream large end of the inlet should have an area substantially equal to the fan outlet. The smaller end at the wheel is determined by the wheel proportions and may reduce in area of passage as much as 40 percent.

The clearance between inlet and impeller is important; the clearance requirements are different with different fan types. In the backwardly curved blade fan, the inlet clearance should be small, of the order of ⅛ in. in a 3 ft diam fan. The volume recirculated through a large clearance reduces the fan output materially if clearance is as much as ½ in. With radial-blade fans, moderate clearance variations have little effect. And with the forwardly curved blade type, a large clearance not only does not harm the performance but actually is necessary for stability to minimize pulsation.

Inlet Ducts. Recent investigations have emphasized the important effect on fan performance of the actual inlet connections. The ducts leading to and from a fan are designed usually to escape beams and other limitations of space, and such duct work may seriously affect fan performance. The variation in the form of ducts is so great that no rules can be given for particular application. Figures 5, 6, and 7 show three general forms.

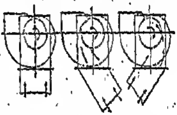


FIG. 5. FIG. 6. FIG. 7.
Inlets of Fan Casings.

In Fig. 5, the air is brought straight up to an inlet box, making a right-angle bend to get into the fan inlet. The loss in efficiency, which in this case shows itself in a drop in pressure, will amount to approximately 15 percent. This large loss is not explainable by any friction or eddy-current loss in the connection itself but is due to the poor distribution of the air flow into the wheel.

In Fig. 6, the air is led into the inlet box at an angle and will tend to produce a spin in the direction of wheel rotation. This action will reduce the horsepower taken by the fan and will decrease the efficiency. This decrease in efficiency is due to decrease in pressure of a much larger percentage than the decrease in horsepower. In extreme cases, this form of connection has reduced the pressure to one-half that which the fan is capable of supplying with open inlets.

In Fig. 7, the connection is brought in from the other direction, producing a spin against the wheel rotation. In this case, there is little or no decrease in the fan pressure. The decrease in efficiency is large, however, and shows itself as an increased horsepower. An increase in horsepower of 25 percent is quite usual and may go as high as 50 percent.

Fans should be selected from tests made with the inlet connection in place. Laboratory research can determine suitable inlet ducts, called conventionally inlet boxes, of such proportions that the fan performance is affected only slightly. With such a box, even angular connections, with suitable vaning to give straight flow into the tested inlet box, will give satisfactory results.

The absolute velocity of the gas w_e is computed from Q, D_e, and the axial width of the blade (radial-inlet type). u_e is computed from D_e and the rpm. The ratio w_e/u_e gives the tangent of the impeller inlet angle b_e (absence *of impeller inlet guides, which is the usual case, is assumed*). For the axial-inlet type, w_e is the *average* velocity in the annulus between the hub and the casing inlet and u_e and b_e vary with the diameter, giving a helical edge. The **impeller inlet passage height** is the perpendicular from the tip of a blade to the surface of the next blade. Its theoretical value is $(\pi D_e \times \sin b_e)/z_i$, and this should be increased by 10 to 30 percent. The rpm is generally determined by that of the driver or by other considerations. From the formulas given before for pressure rise, the wheel speed u_a and the outer **impeller diameter** D_a can then be determined by assuming a reasonable value for the hydraulic efficiency e_h. The rotation loss should now be computed to make sure that it is not excessive in comparison with the theoretical horse power. The impeller stresses should also be computed to see that they are not excessive for the available material. Increasing the number of stages will reduce both the rotation loss and the stresses. The inner diameter of the **discharge vanes** should be $1.1D_a$ *to* $1.2D_a$. Three to six vanes are ample for most purposes. The overlap of one vane on the other, or the definite passage between vanes, need not be very great. The **discharge vane angle** d_a is found in the same way as explained for b_e, the inlet gas quantity being multiplied by a compression factor according to the wheel speed u_a, as per Table 7. Generally, d_a is made between 3 and 5 deg. The theoretical **passage height** of the discharge vanes (perpendicular to the axial width) is $(\pi D_a \sin d_a)/z_d$, where z_d is the number of discharge vanes (same axial width of impeller and discharge vanes is assumed). The actual passage height should be 50 to 100 percent larger.

Table 7. Compression Factors

Impeller wheel speed, ft per sec.	200	250	300	350	400	450	500	550	600	650	700
Compression factor	0.989	0.982	0.974	0.965	0.955	0.944	0.931	0.917	0.903	0.888	0.871

Centrifugal-compressor Tests. Instructions for testing centrifugal compressors and for correcting test results to guaranteed conditions are given in the A.S.M.E. Test Code for Centrifugal Compressors and Exhausters, Part 1.

For **measuring volumes**, a nozzle with well-rounded inlet and smooth polished internal surfaces, proportioned as defined in the A.S.M.E. Test Code, should be used (see p. 1805). The diameter should be selected so that the required range of volume can be obtained with a total pressure drop (including velocity head) of not less than 10 or more than 40 in. of water. If greater pressure drops are required, the usual simplified formulas require correction. For air under usual atmospheric conditions, the following table will serve to give a preliminary estimate of the necessary nozzle diameter.

Diam of nozzle, in.	2½	3¼	4¼	5½	6¼	8½	11½	16	20
Cfm at 10 in. press drop	425	720	1,225	2,050	2,650	4,900	0,000	17,500	27,500
Cfm at 40 in. press drop	850	1,440	2,450	4,100	5,300	9,800	18,000	35,000	55,000

Classification

Centrifugal compressors are generally employed for pressures in excess of 1 lb gage. For lower pressures, conventional types of fans and blowers are used. Three basic classifications are in general use: Single-stage, usually

Untested inlet connections built by the tinsmith should be avoided or proper allowance made.

Outlet Connections. The transition from the spiral flow in the housing to the flow in the duct is a turbulent change with an action similar to that of abrupt expansion. Outlet ducts continuing the fan outlet area serve to convert a portion of the high velocities into static pressure. Expanding outlet connections naturally convert a large portion of the initial velocity pressure, but, due to the varying directions of the fan outlet flow, they are not as effective as would appear. They return only 50 to 70 percent of the reduction in the velocity pressures as gain in static.

Diffuser Casing. The diffuser casing permits a circumferential discharge of air, a discharge suitable when the air is delivered into a large open space. The diffuser consists of two stationary plates continuing the sweep of the side plates of the wheel and leading the air outward to a large diameter.

The diffuser functions to convert velocity into static pressure by increasing the radius of the air spin and by increasing area. In the diffuser, no energy is imparted to the air, and constant energy requires constant circulation. Circulation is the product of tangential velocity *times the circumference*, and *constancy of circulation* requires a tangential component varying inversely with the radius.

A diffusion of the radial component of the velocity also takes place owing to an increase of area with increasing radius, but no quantitative predetermination of conversion is possible as the discharge from a fan impeller is not continuous. The discharge consists of jets from the spaces between blades, and the radial expansion of these jets does not follow any simple formula. Figure 8 shows a typical arrangement of fan and diffuser casing.

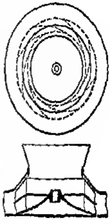


FIG. 8.—Fan and Diffuser Casing.

In general, diffusers and fan housings are designed empirically with some help from the theorem of constant circulation.

Fan Wheels. Fan-wheel proportions are subject to wide variation. Stresses in material or the combination of specified duty and speed are often determining factors. The significant proportions are the number of blades, the ratio of inlet diameter to wheel diameter, and the ratio of axial width to diameter. This latter ratio is important only as a maximum; reduced widths are always allowable.

The following table gives the proportions of fan wheels according to commercial practice:

Type wheel	Number of blades	Ratio of inlet diam to wheel diam	Ratio of width to diam
Forwardly curved	60	0.88	0.55
Radial tip	16-24	0.78	0.35
Backwardly curved	8-16	0.75	0.26
Paddle wheel	6-12	0.50-0.70	0.38-0.45

employed for air pressures from 1 to 6 lb gage, although pressures as high as 15 lb gage have been obtained with a single wheel; multistage, air-cooled, ordinarily employed for air pressures from 6 to 30 lb gage; and multistage, water-cooled, ordinarily used for air pressures from 30 to 125 lb gage.

Single-stage centrifugal compressors (turbo-blowers). Three general constructions are used.

OVERHUNG TYPE. The impeller wheel is overhung on the extended shaft of the driving machine (see Fig. 4). A very compact unit results.

PEDESTAL TYPE. The impeller wheel is supported overhung on its own shaft and has its own bearings, contained in a bearing pedestal which also may support the blower casing. The shaft is flexibly coupled to the driving machine. Figure 5 is an example of this construction.

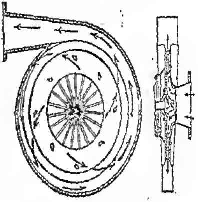


FIG. 4.—Ingersoll-Rand Single-stage Compressor.

DOUBLE-INLET TYPE. A double-sided impeller is mounted on a shaft supported by a bearing at each end, the shaft being flexibly coupled to the driving machine (see Fig. 6).

Single-stage centrifugal compressors are commonly used for supplying air to foundry cupolas and to oil and gas-fired furnaces; scavenging large two-stroke Diesel engines; supercharging four-stroke Diesel engines; pneumatic conveying of light materials; ventilating mines and tunnels; gas boosting and exhausting; emptying submarine ballast tanks; blowing coating on paper.

Multistage Centrifugal Compressors (Air-cooled). These machines are usually employed for air pressures from 6 to 30 lb gage, but, when handling gases lighter than air, may be used for lower pressures. They are built in two general types.

SINGLE INLET. The air or gas enters at one end and passes successively through the various wheels, to be discharged at the other end (see Fig. 7).

DOUBLE INLET. The number of impellers is doubled and arranged in two opposed groups. The air or gas is then drawn in at both ends and discharged at the center (see Fig. 8). This type is relatively rare in the United States, being used for exceptionally large volumes at low pressures and for larger sizes of refrigeration compressors. It is quite common in Europe.

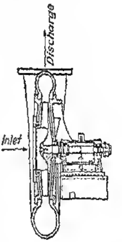


FIG. 5.—Pedestal-type Single-stage Compressor. (Roots-Connersville Blower Corp.)

For special cases, double-inlet and single-inlet impellers may be grouped in the same casing, but this construction is relatively rare.

Multistage centrifugal compressors of the uncooled type are commonly used for the following applications: blast-furnace blowing; copper-converter blowing; Bessemer-converter blowing; gas exhausting and boosting; aeration of

Fan-wheel Theory. The theory of the design of fan wheels is highly developed. The Euler equation for pressure is

$$p = (d\omega/g)[r_1V_{u_1} - r_0V_{u_0}]$$

where r_1 and V_{u_1} are the radius and rotative air velocity at exit; r_0 and V_{u_0} the quantities at entrance; and ω the rotative speed of the impeller, radians per sec. The Euler equation applies to any wheel, centrifugal or propeller.

A newer analysis, applied chiefly to propeller fans, is derived from modern aeronautics and is based on the theory of circulation around an airfoil. In airfoil theory, Γ, the circulation, is

$$\Gamma = \tfrac{1}{2}C_Lbv$$

where C_L is the lift coefficient, b the length of the chord, and v the effective velocity relative to the airfoil (see p. 1532).

Applied to a fan with a number of blades Z,

$$p = d\omega Z\Gamma/2\pi g$$

This expression for p may be equated to that of the Euler equation which, for a propeller fan with r_1 equal to r_0, becomes

$$p = d\omega r(V_{u_1} - V_{u_0})/g$$

With circulation Γ per blade,

$$r(V_{u_1} - V_{u_0}) = Z\Gamma/2\pi$$

The required circulation is then

$$\Gamma = 2\pi r(V_{u_1} - V_{u_0})/Z = C_Lbv/2$$

Values of C_L are available for published tests of airfoils (see p. 1535).

Airfoil circulation is changed by the proximity of other blades, the so-called **cascade effect**, and few quantitative data on cascade effect are available in form suitable to engineers. The practical application of airfoil wind-tunnel tests to fan-blade action is at present limited to those cases with negligible cascade effect, *i.e.*, with a spacing to chord ratio of not less than 1.5 to 2 and a low angle of attack of the order of 4 deg or less.

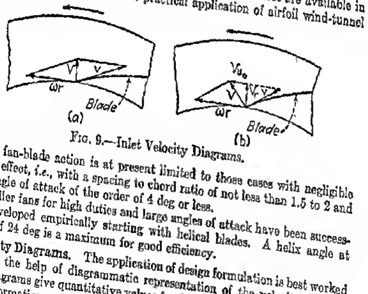


FIG. 9.—Inlet Velocity Diagrams.

Propeller fans for high duties and large angles of attack have been successfully developed empirically starting with helical blades. A helix angle at the tip of 24 deg is a maximum for good efficiency.

Velocity Diagrams. The application of design formulation is best worked out with the help of diagrammatic representation of the velocity vectors. These diagrams give quantitative values for design and, properly interpreted, much information as to the fan characteristics of different types of blading.

For centrifugal fans, the Euler analysis gives satisfactory results. The Euler theory requires separate diagrams for inlet and outlet. The inlet diagrams are always the same, as they are independent of the blading.

yeast and sewage; circulating gases in chemical plants; compressing refrigerants; pneumatic conveying.

Multistage Centrifugal Compressors (Water-cooled). Machines of this type are in common use in Europe and South Africa and are employed

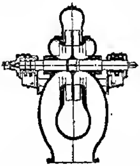


Fig. 6.—De Laval Double-inlet Single-stage Centrifugal Compressor.

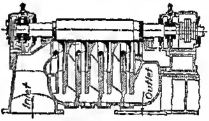


Fig. 7.—Half Section of Allis-Chalmers Single-inlet Multistage Centrifugal Compressor.

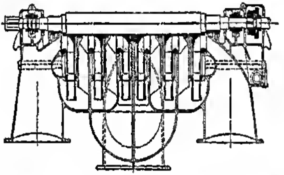


Fig. 8.—Half Section of Allis-Chalmers Double-inlet Multistage Centrifugal Compressor.

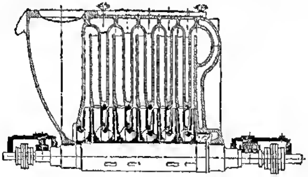


Fig. 9.—Half Section of Brown Boveri Single-inlet Multistage Water-jacketed Centrifugal Compressor.

to some extent in America for supplying large volumes of compressed air at pressures from 40 to 125 lb gage for mining operations, large manufacturing plants, chemical plants, shipyards, etc. They are generally similar to multistage blowers of the uncooled type, but have provision for cooling the air

Figure 9*a* shows the usual diagram for air approaching the wheel with velocity V, which combines with ωr, reversed, to give the velocity v relative to the wheel or blade. The direction of v determines the angular setting of the blade. The entrance portion of the blade may be tangent to v or inclined a few degrees from tangency dependent upon the particular blade shape contemplated.

Figure 9*b* shows the inlet velocity diagram for a positive value of an inlet spin V_{u_0} positive in the same direction as the wheel rotation. With reverse inlet spins the action produces unsatisfactory and often unstable characteristics.

Figures 10*a*, *b*, and *c* are the outlet velocity diagrams for backwardly curved, radial, and forwardly curved blades, respectively.

The height of the diagram, indicated by the velocity vector V_r, is determined in each of these cases—and also for any special inlet diagram such as 9*b*—from the design volume and the effective area through which that volume flows.

In Fig. 10*b*, the relative velocity v is tangent to the blade, ωr is perpendicular to the wheel radius. The actual air velocity V is the resultant of v and ωr. V_{u_1}, the rotative component of V, is projected from V to the direction of ωr.

FIG. 10.—Outlet Velocity Diagrams.

Velocity diagrams present a rational basis for comparing the characteristics of the three types of blading. Characteristics are determined at constant speed so the ωr vector will not change. If the volume decreases, the vector V_r decreases and, as the direction of v does not change for a given blade, V_u increases with backwardly curved blades, is constant with radial blades, and decreases with forwardly curved blades.

As the pressure of the fan depends on the product of $V_u \times \omega r$, the pressure change is the same as the change in V_u, and the pressure characteristic of the backwardly curved blade fan rises with decreasing volume, is constant for the radial blade, and decreases or falls for the forwardly curved blade. Frequently the slope of a fan characteristic is discussed in the opposite sense of increasing volumes with the designation *drooping* for the backwardly curved and *rising* for the forwardly curved blade. The decreasing volume basis is preferable as directly applicable to considerations of type desirability.

The desirable qualities of a fan are stability, successful operation in parallel, freedom from pulsation and, from the driver point of view, a non-overloading horsepower demand.

Stability. The factors of fan output are pressure and volume. Stability of operation requires that the factors change oppositely; *i.e.*, a reduction in volume should be opposed by an increase in pressure. The backwardly curved blade fan is inherently stable. The radial-blade fan is stable as to static pressure but not so where total pressure may control the action. The forwardly curved blade fan is inherently unstable.

Duct work has a characteristic also. With most ducts, the pressure decreases so much with decreased volume that even the characteristic of the

TABLE 3. GIVING z WHEN s/x IS KNOWN. THEN $a = x/z$

s/x	0	1	2	3	4	5	6	7	8	9
1.000	......	0.0245	0.0346	0.0424	0.0490	0.0548	0.0600	0.0648	0.0693	0.0735
1	0.0774	0.0812	0.0848	0.0883	0.0916	0.0948	0.0980	0.1010	0.1039	0.1067
2	0.1095	0.1122	0.1149	0.1174	0.1200	0.1224	0.1249	0.1272	0.1296	0.1319
3	0.1341	0.1363	0.1385	0.1407	0.1428	0.1448	0.1469	0.1489	0.1509	0.1529
4	0.1548	0.1567	0.1586	0.1605	0.1623	0.1642	0.1660	0.1678	0.1696	0.1713
1.005	0.1731	0.1748	0.1765	0.1782	0.1799	0.1815	0.1831	0.1848	0.1864	0.1880
6	0.1896	0.1911	0.1927	0.1942	0.1958	0.1973	0.1988	0.2003	0.2018	0.2033
7	0.2047	0.2062	0.2076	0.2091	0.2105	0.2119	0.2133	0.2147	0.2161	0.2175
8	0.2188	0.2202	0.2215	0.2229	0.2242	0.2255	0.2269	0.2282	0.2295	0.2308
9	0.2321	0.2334	0.2346	0.2359	0.2372	0.2384	0.2397	0.2409	0.2421	0.2434
1.01	0.2446	0.2565	0.2678	0.2787	0.2892	0.2993	0.3091	0.3186	0.3278	0.3367
2	0.3454	0.3539	0.3621	0.3702	0.3781	0.3859	0.3934	0.4009	0.4082	0.4153
3	0.4224	0.4293	0.4361	0.4428	0.4494	0.4559	0.4623	0.4686	0.4748	0.4809
4	0.4870	0.4930	0.4989	0.5047	0.5105	0.5162	0.5218	0.5274	0.5329	0.5383
1.05	0.5437	0.5490	0.5543	0.5595	0.5647	0.5698	0.5749	0.5799	0.5849	0.5898
6	0.5947	0.5996	0.6044	0.6091	0.6139	0.6186	0.6232	0.6278	0.6324	0.6369
7	0.6414	0.6459	0.6504	0.6548	0.6591	0.6635	0.6678	0.6721	0.6763	0.6806
8	0.6848	0.6889	0.6931	0.6972	0.7013	0.7053	0.7094	0.7134	0.7174	0.7213
9	0.7253	0.7292	0.7331	0.7369	0.7408	0.7446	0.7484	0.7522	0.7559	0.7597
1.10	0.7634	......	......	......	......	......	NOTE: $s/x = \sinh z/z$			

(4) GIVEN y AND s. Then $\dfrac{T}{w} = \dfrac{s^2}{2y} + \dfrac{y}{2}$, $x = \left(\dfrac{s^2}{y} - y\right) \tanh^{-1}\left(\dfrac{y}{s}\right)$; $a = \dfrac{s^2}{2y} - \dfrac{y}{2}$. Or, if y/s is small, $x = s\left[1 - \dfrac{2}{3}\left(\dfrac{y}{s}\right)^2 - \dfrac{2}{15}\left(\dfrac{y}{s}\right)^4 - \dots\right]$.

(5) GIVEN y AND T/w. Then $a = \dfrac{T}{w} - y$, $x = \left(\dfrac{T}{w} - y\right) \cosh^{-1} \dfrac{T/w}{(T/w) - y}$, $s = \sqrt{2y(T/w) - y^2}$. Or, if $y/(T/w)$ is small,

$$x = \sqrt{\frac{2yT}{w}}\left[1 - \frac{7}{12}\frac{wy}{T} - \dots\right], \quad \frac{s-x}{s} = \frac{1}{3}\frac{wy}{T}, \text{ approximately},$$

$$s = \sqrt{\frac{2yT}{w}}\left[1 - \frac{1}{4}\frac{wy}{T} - \frac{1}{32}\left(\frac{wy}{T}\right)^2 - \frac{1}{128}\left(\frac{wy}{T}\right)^3 - \dots\right].$$

(6) GIVEN s AND T/w. Then $x = \dfrac{T}{w}\sqrt{1 - \left(\dfrac{ws}{T}\right)^2} \tanh^{-1}\left(\dfrac{ws}{T}\right)$; $y = \dfrac{T}{w} - \dfrac{T}{w}\sqrt{1 - \left(\dfrac{ws}{T}\right)^2}$, $a = \dfrac{T}{w}\sqrt{1 - \left(\dfrac{ws}{T}\right)^2}$. Or, if ws/T is small,

$$x = s\left[1 - \frac{1}{6}\left(\frac{ws}{T}\right)^2 - \frac{11}{120}\left(\frac{ws}{T}\right)^4 - \dots\right], \quad y = s\left[\frac{1}{2}\left(\frac{ws}{T}\right) + \frac{1}{8}\left(\frac{ws}{T}\right)^3 + \dots\right],$$

$$a = \frac{T}{w}\left[1 - \frac{1}{2}\left(\frac{ws}{T}\right)^2 - \frac{1}{8}\left(\frac{ws}{T}\right)^4 - \dots\right].$$

Given the Length $2L$ of a Chain Supported at Two Points A and B not in the Same Level, to find a. (See Fig. 63; b and c are supposed known.) Let $(\sqrt{L^2 - b^2})/c = s/x$; enter Table 3 with this value of s/x, and find the corresponding value of the auxiliary variable z. Then $a = c/z$.

NOTE. The co-ordinates of the mid-point M of AB (see Fig. 63) are $x_0 = a \tanh^{-1}(b/L)$, $y_0 = (L/\tanh z) - a$, so that the position of the lowest point is determined.

Correction for Sag in Chaining Uphill (Fig. 64). Let l = length of tape (corrected for stretch and temperature), w = weight per unit length of tape, A = angle between the chord AB and the horizontal.

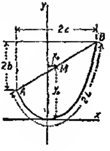


FIG. 63.

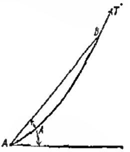


FIG. 64.

If the tension P at the upper end is known, compute wl/P and find k from Table 4. If the tension Q at the lower end is known, compute wl/Q and find k from Table 5. In either case, chord $AB = l - kl$.

TABLE 4. GIVING k

$\frac{wl}{P}$	$A=0°$	10°	20°	30°	40°	50°	60°	70°	80°
.01	.00000	000	000	000	000	000	000	000	000
.02	002	002	001	001	001	001	000	000	000
.03	004	004	003	003	002	002	001	000	000
.04	007	006	006	005	004	003	002	001	000
.05	011	010	009	008	006	004	003	001	000
.06	.00015	015	013	012	009	006	004	002	000
.07	020	020	018	016	012	009	005	003	001
.08	027	026	024	021	016	012	007	003	001
.09	034	033	031	026	021	015	009	004	001
.10	042	041	038	033	026	019	011	005	001
.11	.00051	050	046	040	032	023	014	007	002
.12	060	060	055	048	038	027	017	008	002
.13	070	070	055	057	045	032	020	009	002
.14	082	081	076	066	053	038	023	011	003
.15	094	094	087	076	061	044	027	013	003
.16	.00107	107	100	087	070	050	031	015	004
.17	121	121	113	099	079	057	035	017	004
.18	136	136	128	112	090	065	040	019	005
.19	151	152	143	125	101	073	045	021	006
.20	168	168	159	140	113	082	050	024	006

TABLE 5. GIVING k

$\frac{wl}{Q}$	$A=0°$	10°	20°	30°	40°	50°	60°	70°	80°
.01	.00000	000	000	000	000	000	000	000	000
.02	002	002	001	001	001	001	000	000	000
.03	004	004	003	003	002	001	001	000	000
.04	007	006	006	005	004	003	002	001	000
.05	011	010	009	008	006	004	002	001	000
.06	.00015	014	013	011	008	006	004	002	000
.07	020	020	018	015	011	008	005	002	001
.08	027	026	023	019	015	011	006	003	001
.09	034	032	029	024	019	013	008	004	001
.10	042	040	036	030	023	016	010	004	001
.11	.00051	048	043	036	028	019	011	005	001
.12	060	057	051	043	033	023	014	005	002
.13	070	067	060	050	038	026	016	007	002
.14	032	078	069	057	044	030	018	008	002
.15	094	089	079	066	050	035	021	010	002
.16	.00107	101	090	074	057	039	022	011	003
.17	121	114	101	084	064	044	026	012	003
.18	136	128	113	092	071	049	029	013	003
.19	151	142	125	103	079	054	032	015	004
.20	168	157	138	114	087	060	035	016	004

NOTE. $k = 1 - \{[1 - \sqrt{1 - 2m \sin u + m^2}]/[m \sin A]\}$, where $m = wl/P$ and u is given by

$[1 - \sqrt{1 - 2m \sin u + m^2}] \sec u = [\sinh^{-1}(\tan u) - \sinh^{-1}(\tan u - m \sec u)] \tan A$.

Also, $Q = P - wl(1 - k) \sin A$, where k is the value in Table 4 corresponding to the given values of P and A.

Correction for Stretch in Chaining Uphill. Let L = unstretched length of tape at working temperature, w = weight per unit length of tape, A = angle

forwardly curved blade fan has excess pressure requirement at reduced volume and because of duct reaction becomes stable in spite of its own unfavorable characteristic.

Parallel Operation. Two or more fans operating on a single duct system must have suitable characteristics for such operation and be properly selected as to point of operation. Parallel operation requires that the sum of the outputs of the fans equal the duct capacity and that this equality exist for only one set of conditions. *In mathematical phraseology, the characteristics* of the fans and duct must be satisfied and uniquely.

Figure 11 shows the graphical analysis for parallel operation of two fans. The duct characteristic is the usual parabola, and the fan characteristic is typical of the backwardly curved blade type. One fan operating on the duct will deliver the volume and pressure of their intersection A. If an identical fan is added to the system, the volume and pressure will be greater. The exact point of operation may be determined by constructing the "limit curve." At a series of pressures such as P, plot the volumes L which when

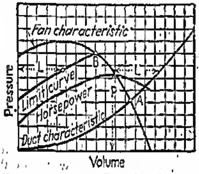


FIG. 11.—Stable Parallel Operation of Two Fans.

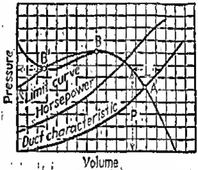


FIG. 12.—Unstable Parallel Operation of Two Fans.

added to the volume of the fan will equal the volume of the duct characteristic. Draw the limit curve through the points. The intersection B of limit curve and fan characteristic gives the point of operation of each of two identical fans.

For successful operation, there should be only one such point. Further, the intersection angle should not be small or there may be hunting. The requirements are best presented by a comparison of Fig. 11, which indicates successful paralleling, with Fig. 12, which shows an incorrect combination. Figure 12 shows the analysis for forwardly curved blade fans. The limit curve is constructed in the same manner as in Fig. 11, but in Fig. 12 there are two intersections and between them a superposition of the two curves. Any attempt to parallel fans with such characteristics will be successful only by accident. The point of equal load division in both Figs. 11 and 12 was selected for efficient operation and correctly represents the comparison of the parallel operation of the two types. It is apparent that, with a different relation between duct and fan characteristics which would permit the equal division of load to occur at a larger orifice on the fan curve, successful parallel operation is possible even with the forwardly curved blade fan, but only at a sacrifice, as this region of the fan output is in the range of low efficiency.

Figures 11 and 12 show also an important advantage of the self-limiting horsepower of the backwardly curved blade fan. When two fans are installed on one duct system, the reliability of partial load operation with one fan in

Permeability (μ) is the ratio of the magnetic-flux density to the magnetizing force ($=B/H$). Actually it is the ratio of the magnetic flux in any element of a medium to the flux which would exist if that element were replaced with air, the mmf acting on the element remaining unchanged.

Reluctivity (ν) is the reluctance between any two parallel faces of 1 cm-cube of the medium. It is the reciprocal of permeability.

Reluctance ($\mathcal{R}$) is resistance to magnetic flow. In a homogeneous medium of uniform cross section cgs reluctance is equal to the length divided by the product of the area and permeability, the length and area being expressed in centimeter units ($\mathcal{R} = L/A\mu$).

Permeance ($\mathcal{P}$) is the reciprocal of reluctance.

Current (I, i). The practical unit of current is the **ampere**, which is equal to one-tenth the absolute unit of current and is the current that flows through a conductor having a resistance of 1 ohm and a difference of potential of 1 volt between its ends. The absolute unit of current is that current which flowing in a conductor perpendicular to the lines of force in a field of unit intensity causes to be exerted on each centimeter of the conductor a (unit) force of 1 dyne. One ampere (direct current) will deposit 0.001118 g of silver per sec from a standard silver solution.

Quantity (Q). The practical unit of quantity is the **coulomb** and is that quantity which passes a cross section of the conductor in 1 sec when the rate of flow is 1 amp.

Potential Difference or Electromotive Force (E, V, emf). Electromotive force tends to cause flow of electricity. The practical unit of electromotive force is the **volt**. A conductor cutting flux at the rate of 1 maxwell per sec has induced in it one absolute unit of emf. The volt is 10^8 times the absolute unit.

Resistance (R, r). The practical unit of resistance is the **ohm** (Ω) and is that resistance through which the fall of potential is 1 volt when the current is 1 amp. The ohm equals 10^9 absolute units of resistance.

Conductance (G, g). Conductance is the reciprocal of resistance and is expressed in reciprocal ohms or **mhos** ($\mho$) ("mho" is ohm spelled backward).

Capacitance (C). The practical unit of capacitance is the **farad** (f) and is that capacitance the potential of which will be raised 1 volt by the addition of a charge of 1 coulomb. As the farad is too large a unit for practical purposes, the **microfarad** (μf), which is one-millionth of a farad, is generally used. For capacitors such as are used for radio purposes the **micro-microfarad** ($\mu\mu f$), or 10^{-12} farad, is a more suitable unit. The magnitude of the microfarad is 9×10^5 that of the cgs electrostatic unit (statfarad).

Dielectric Constant or Permittivity (κ). The dielectric constant of vacuum is unity and that for air is practically the same. The dielectric constant is the ratio of the conductivity of a dielectric for electrostatic lines to that for vacuum. Therefore, it is the ratio of the electrostatic capacitance of a given capacitor with a certain dielectric to the capacitance of the same capacitor with vacuum as the dielectric.

Self-inductance (L). The practical unit of self-inductance is the **henry**. The henry is also called the **coefficient of self-induction**. An electric circuit has an inductance of 1 henry when a rate of change of 1 amp per sec will induce an emf of 1 volt. It also follows that in such a circuit 1 amp will produce 10^8 cgs linkages of magnetic lines (product of turns and flux) in the circuit, since a change of 10^8 linkages per second is required to induce 1

the event of the failure of the other unit is usually of importance. With a self-limiting-horsepower fan, there is no possibility of overloading the driving motor. With the other types of fans, the excess load with single-fan operation on a parallel system must always be considered in selecting motors.

Pulsation. Fans used for high pressures can, under some conditions, develop a severe pulsation. No exact pressure can be stated below which this action cannot occur, but it seldom happens below 15 in. of water. In most fans at one-sixth to one-third of the design volume, there is a *blowback* out of a portion to the inlet and a definite puffiness in the discharge. Fans should not be used for the higher pressures at the blowback point.

Pulsation is most severe when the fan discharges into a large duct or chamber with a volume content 5 percent or more of the volume per minute of normal fan output. A chamber of this size can be set into violent pulsation by a fan, not only at the blowback point, but also at any portion of its characteristic curve where the slope of the curve shows decreasing pressure with decreasing volume. Pulsation can also be set up by an apparently stable fan in double-inlet arrangement when there is a large difference between the flows to the two inlets.

DESIGN OF PROPELLER FANS

Velocity diagrams for propeller fans are usually drawn for a uniform axial velocity. Figure 13 shows a typical propeller fan diagram where ωr is the reversed speed of the blade section under consideration, V_0 the axial velocity of the air, V_1 the air velocity leaving the impeller, and v_∞ the mean relative velocity drawn to bisect V_{u_1}. v_∞ is the theoretically correct velocity to use in the equation relating the circulation and the lift coefficient.

FIG. 13.—Velocity Diagram for Propeller Fan.

$$\Gamma = \tfrac{1}{2}C_L v_\infty b$$

Propeller fans are inherently of the backwardly curved blade type and have characteristic curves somewhat similar to those of that type of centrifugal fan. The propeller curves are changed in the low-volume range by an outward radial flow progressively increasing to a maximum at zero delivery. When no air is delivered from the impeller, the air flow consists of two eddies entering the impeller from both sides in the region near the hub and leaving near the tip. This action increases both pressure and horsepower. Figure 14 shows the characteristic curves of an efficient propeller fan. This fan has a small pitch angle, 15 deg at the tip. The fan performance is perfectly stable throughout the full range of the curve. This quality of stability will hold for most propeller fans up to 17 deg and with careful design, with constant circulation at the design point, even up to 20 deg. For steeper pitches, instability occurs with a break in the curve as indicated in Fig. 15.

The small-pitch angle fan has the advantage of high efficiency and stable operation. Its disadvantages are low capacity and high speed. Because of this high speed, these fans are noisy for a given duty and for the higher pressures present a difficult structural design problem.

Steep-pitch fans deliver large volumes, are less noisy, and can be built for high pressures—but all at a sacrifice of stability and efficiency.

The commercially desirable propeller fan is the medium-pitch type from 17 to 20 deg. It usually has wide blades, taking up, in projected area, more than 50 percent of the circle. The cascade ratio, spacing to chord, is less than unity.

volt. One henry is equal to 10^9 absolute units of self-inductance. If the permeability is constant, $L = n\phi 10^{-8}/I$ henry, where $n\phi$ is the cgs linkages and I is the amp.

Mutual Inductance (M). When a change of current in one circuit induces an emf in a second circuit, the two circuits are said to have **mutual inductance**. The unit of mutual inductance is the henry. When a change of current of 1 amp per sec in either of two separate circuits induces an emf of 1 volt in the other circuit, their mutual inductance is 1 henry. Also, 1 amp in one circuit produces 10^8 cgs magnetic linkages in the other circuit when their mutual inductance is 1 henry. If M is the mutual inductance of two circuits and k is the **coefficient of coupling** (*i.e.*, the proportion of flux produced by one circuit which links the other) then $M = k\sqrt{L_1L_2}$, where L_1 and L_2 are the respective self-inductances of the two circuits.

Table 1. Electrical Units

Symbol	Quantity	Equation	Practical unit	Value of practical units, electromagnetic units
I, i	Current	$I = E/Z,\ I = Q/t$	Ampere	10^{-1}
Q, q	Quantity	$Q = It$	Coulomb	10^{-1}
			Ampere-hour	360
E, e	Electromotive force	$E = W/Q$	Volt	10^8
R, r	Resistance	$R = P/I^2 = E/I$	Ohm	10^9
ρ	Resistivity	$\rho = RA/L$	Ohms per circular mil-foot	
			Ohms per centimeter cube	10^9
λ	Conductivity	$\lambda = 1/\rho$	Mhos per unit volume	
C	Capacitance	$C = Q/E$	Farad	10^{-9}
L	Inductance	$L = n\phi 10^{-8}/I$	Henry	10^9
	Time constant	L/R	Second	1
			Henry per ohm	1
T	Period of cycle	$T = 1/f$	Second	1
f	Frequency	$f = 1/T$	Cycles per second	1
ω	Angular velocity	$\omega = 2\pi f$	....	......
X_L	Inductive reactance	$X_L = 2\pi fL$	Ohm	10^9
X_C	Capacitive reactance	$X_C = 1/2\pi fC$	Ohm	10^9
X, x	Reactance	$X = X_L - X_C$	Ohm	10^9
Z, z	Impedance	$Z = \sqrt{R^2 + X^2}$	Ohm	10^9
G, g	Conductance	$G = R/Z^2$	Mho	10^{-9}
B, b	Susceptance	$B = X/Z^2$	Mho	10^{-9}
Y, y	Admittance	$Y = \sqrt{G^2 + B^2} = 1/Z$	Mho	10^{-9}
P	Electric power	$P = EI = I^2R$	Watt	10^7
		$= EI \cos\theta$ (a-c)	Watt	10^7
W	Electric energy	$W = Pt$	Joule	10^7
			Watthour	36×10^9
			Kilowatt-hour	36×10^{12}
	Power factor (p.f.)	$\dfrac{EI\cos\theta}{EI} = \dfrac{\text{real } P}{\text{apparent } P}$		
	Reactive factor	$\dfrac{EI\sin\theta}{EI} = \dfrac{\text{reactive } P}{\text{apparent } P}$		

n = number of turns; t = time in seconds; f = frequency; ϕ = flux.

Special propeller fans have been built for high efficiency and in large sizes for wind tunnels. These fans are the opposite of the commercial type. The blades are narrow compared with their radial length. The cascade ratio is 2 or more, and the lift coefficient 0.6 or less corresponding to a low angle of attack. For a given duty, such fans may be twice the diameter of the equivalent commercial fan, but with a gain of approximately 10 percent in efficiency.

Guide Vanes for Propeller Fans. The discharge from the propeller wheel necessarily has a rotation unless guide vanes are used in the inlet. The tangential component of the discharge velocity serves no useful purpose. Stationary deflecting or diffusion vanes past the wheel convert the tangential velocity into static pressure. The vanes are preferably airfoil shapes with a total circulation equal and opposite to the total circulation produced by the propeller. The number of diffusion vanes should be prime to the number of propeller blades.

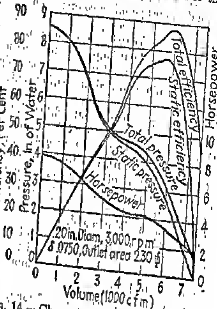


FIG. 14.—Characteristic Curves of Stable Propeller Fan.

Cascade effect must be allowed for in designing diffusion vanes also. The allowance calculations are complex. They are based on the increase of angle of attack for zero lift resulting from cascade with additional corrections for thickness and camber. A much simpler correction has given satisfactory results—within 8 percent of expected performance. This approximate method makes the correction entirely by increased camber. Figure 16 indicates the procedure. V_d is the average discharge velocity leaving the propeller, V_a the axial component of this velocity, and v_∞ the mean velocity, a factor of the vane circulation.

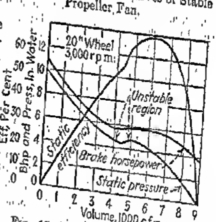


FIG. 15.—Characteristic Curves of a Propeller Fan Having an Unstable Region.

v_∞ is extended, intersecting DE, parallel to V_a, at B. A smooth arc tangent to V_d at A and to DE at B gives, to an unknown scale, the b of the equation for circulation. A suitable C_L may be selected from airfoil data and the curve AB rotated about A through the required angle of attack α to the position AC. AC is the corrected mean line of the airfoil. The scale is determined by the length b calculated from the known Γ.

$$\Gamma = \tfrac{1}{2}C_L v_\infty b$$

The chord of the selected airfoil now becomes warped to the curved line AC.

Power (P). The practical unit of power is the watt. One watt is produced when 1 amp flows under an emf of 1 volt. One watt equals 10^7 ergs per sec. One kilowatt equals 1,000 watts.

1 watt = 0.00134 hp = 44.25 ft-lb per min = 0.2389 g-cal per sec = 0.737 ft-lb per sec, = 0.0569 Btu per min.

Watts = volts × *amp* × *cosine of angle of phase* difference (or power factor) with alternating currents.

Volt-amperes or **apparent power** = volts × amp (a-c).

Energy (W). **Watthours** and **kilowatt-hours** are the units of energy used in commercial electrical work.

1 whr = 3,600 joules = 2,655.4 ft-lb = 860 IT g-cal = 3.413 Btu = 0.001341 hp-hr.

The practical unit of electrical energy is the **joule**. One watt-second equals one joule. One joule is produced when a steady current of 1 coulomb per sec., *i.e.*, 1 amp flows through a resistance of 1 ohm for 1 sec. One joule is equivalent to 0.2389 g-cal and also equals 10^7 ergs. (An erg is a dyne-centimeter.)

CONDUCTORS AND RESISTANCE

Materials. The materials generally used for the transmission and distribution of electrical energy are **copper, aluminum, iron,** and **steel.** For resistors and heaters, iron, steel, commercial alloys, and carbon are most used.

Copper is the most commonly used electrical conductor; it is always used for insulated wires and cables.

Aluminum is used to considerable extent for high-voltage transmission lines, because its weight is one-half that of copper for the same conductance. Moreover, the greater diameter reduces corona loss. It has 1.4 times the linear coefficient of expansion so that changes in sag with temperature are greater. Because of its lower melting point spans fail more readily with arc-overs. In aluminum cable steel-reinforced (A.C.S.R.), the center strand is a steel cable, which gives added tensile strength. Aluminum is used occasionally for bus bars because of its large radiating surface for a given conductance.

Steel, either galvanized or copper clad, is used occasionally for high-voltage transmission spans where tensile strength is more important than high conductance. Steel is also used for third rails.

Resistivity or **specific resistance** is the resistance of a sample of the material having both a length and cross section of unity. The resistance of a **centimeter-cube** and **circular-mil-foot** (see below) are the two most common units of resistivity. If l is the length of a conductor of uniform cross section a, then its resistance $R = \rho l/a$ (1) where ρ is the resistivity. With a cir-mil-ft ρ is the resistance of a cir-mil-ft and a is the cross section in cir mils. Since $v = la$ is the volume of a conductor $R = \rho l^2/v = \rho v/a^2$ (2).

A **circular-mil** is a unit of area equal to that of a circle whose diameter is 1 mil (0.001 in.). It is the unit of area which is used almost entirely in this country for wires and cables. To obtain the circular mils of a solid *cylindrical* conductor, square its diameter expressed in mils. For example, the diameter of 000 A.W.G. solid copper wire is 410 mils and its cross-section is $(410)^2$, or 168,100, cir mils. The diameter in mils of a solid cylindrical conductor is the square root of its cross-section expressed in cir mils.

A **circular-mil-foot** is a conductor having a length of one foot and a uniform cross section of one circular mil. In terms of the copper standard (see Table 4, p. 1692) the resistance of a circular-mil-foot of copper at 20 C is

For diffusion vane determination, this approximate method of cascade correction is satisfactory.

The same method is applicable to propeller blades but tends to give slightly higher values of C_L than are shown by the wind-tunnel tests.

Axial-flow Multistage Compressors. Axial-flow impellers and diffusion or guide vanes permit a compact multistage arrangement for high pressures. Rotative speeds need not be excessive. Efficiencies are higher than those of multistage centrifugal compressors. With a compression ratio of 4, efficiencies of about 85 percent have been obtained.

The disadvantage of the axial-flow design is its tendency to pulsate within a normal range of output. To limit the number of stages and the rotative speed, a steeply pitched blade with a large angle of attack is desirable. The pressure characteristic consequently exhibits the unstable break or dip characteristic of steep-pitch propellers, as previously referred to in connection with Fig. 15, and the break is located at from 50 to 80 percent of the design capacity. Centrifugal compressors also have a pulsation range, but at 20 to 30 percent of normal output, a percent of rating which for most applications is below the useful demand.

The field of the axial-flow compressor is therefore limited to fans with a substantially fixed resistance system (see p. 1947). An instance of such duty is the compressor end of a constant-output gas turbine.

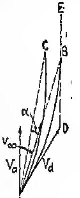


FIG. 16.—Determination of Airfoil Camber for Propeller Fan Vanes and Blades.

Multistage Compressor Theory. The same general laws apply to the action of compressors as apply to low-pressure fans. For fans, however, an implied constant density simplifies the statement of the laws. Compression requires more exact and specific discussion.

The simple fan laws (p. 1671) become for the compressor

(1) the *inlet* volume varies directly with the speed;

(2) the *work per pound* varies as the square of the speed; and

(3) the *horsepower* varies as the cube of the speed.

As the changes in velocity pressure through an axial flow compressor are a negligible proportion of the developed pressure, the increase in static pressure may be considered identical with total pressure rise.

Compressors are rated on inlet volume, pressure rise, and an efficiency based on the corresponding adiabatic work. This adiabatic efficiency decreases with increased speed and the consequent increased pressure ratio. The true efficiency of a bladed impeller is substantially unaffected by speed.

Adiabatic work and efficiency are suitable criteria from the standpoint of the purchaser, but the designer can work more readily and intelligently from a formula that allows for added heat due to an efficiency less than unity.

Multistage-compressor Design. Practicability requires minimum axial lengths per stage. Compressor impellers, therefore, have a large number of narrow blades, resembling a Parsons-turbine-rotor element. The number of blades is usually 60 to 100. Diffusion vanes are likewise many and narrow and prime to the number of impeller blades. Blades and vanes should be of streamline cross section. As streamlining in metal requires expensive die work, a uniform impeller blade and diffusion vane is preferably used for all stages. The blade height may be reduced in the later stages to allow for compression and maintain uniform entrance velocity.

10.371 ohms. As a first approximation 10 ohms may frequently be used (also see p. 1760).

At 60 C a circular-mil-inch of copper has a resistance of 1.0 ohm. This is a very convenient unit of resistivity for magnet coils since the resistance is *merely the length of copper in inches divided by its cross section in cir mils* (see p. 1772).

Table 2. Properties of the Metals and Alloys

Metals	Resistivity, 20 C		Temperature coefficient of resistance at 20 C	Metals	Resistivity, 20 C		Temperature coefficient of resistance at 20 C
	Cm-cube (microhms)	Cir-mil-ft (ohms)			Cm-cube (microhms)	Cir-mil-ft (ohms)	
Aluminum	2.828	17.01	0.0039	Monel metal	43.1	259	0.0019
Antimony	41.7	251.0	0.0036	Mercury	95.783	576	0.00089
Bismuth	110.0	662.0	0.004	Nickel	8.45	50.8	0.0041
Brass	61.5	370	0.0015	Platinum	10.001	60.2	0.003
Carbon:				Platinum-silver,			
Amorphous	3,800 to 4,100	....	(−)	2Ag + 1Pt	24.33[a]	146.4[a]	0.00031
Retort				Silver	1.629	9.8	0.0038
(graphite)	720 to 812[b]	....	(−)	Steel: soft	15.9	95.6	0.0016
Copper				Glass hard	45.7	275	
(drawn)	1.724	10.37	0.00393	Silicon 4 per			
Gold	2.44	14.7	0.0034	cent	51.15	308	
Iron:				Transformer	11.09	66.7	
Electrolytic	9.96	59.9	....	Tin	11.50	69.2	0.0042
Cast	74.4 to 97.8	448 to 588	....	Tungsten	5.51	33.7	0.005
Lead	22.0	132	0.00387	Zinc	5.75[a]	34.7[a]	0.0037

Max. working temperatures, C, Cu, 260; Ni, 600; Pt, 1500.
[a] 0 C. [b] Furnace electrodes, 3000 C.
(See Table 24, p. 1766, for properties of resistor alloys.)

Temperature Coefficient of Resistance. The resistance of the pure metals increases with temperature. The resistance at any temperature t C is $R = R_0[1 + \alpha t]$ (3) where R_0 is the resistance at 0 C and α is the temperature coefficient of resistance. For copper, $\alpha = 0.00427$.

With any initial temperature t_1, the resistance at temperature t C is $R = R_1[1 + \alpha_1(t - t_1)]$ (4) where R_1 is the resistance at temperature t_1 C and α_1 is the temperature coefficient of resistance at temperature t_1 (see Eq. 6).

Inferred Absolute Zero. Between 100 C and 0 C the resistance of copper decreases at a rate which is practically uniform and which if continued would give a resistance of zero at −234.5 C (an easy number to remember). If the resistance at t_1 C is R_1 and the resistance at t_2 C is R_2, then

$$R_2/R_1 = (234.5 + t_2)/(234.5 + t_1) \tag{5}$$

For any initial temperature t_1 the value of α_1 is

$$\alpha_1 = 1/(234.5 + t_1) \tag{6}$$

Example. The resistance of a copper coil at 25 C is 4.26 ohms. Determine its resistance at 45 C. Using Eq. (4) and $\alpha_1 = 1/(234.5 + 25) = 0.00385$, $R = 4.26\,[1 + 0.00385(45 - 25)] = 4.59$ ohms. Using Eq. (5) $R = 4.26\,(234.5 + 45)/(234.5 + 25) = 4.26 \times 1.077 = 4.59$ ohms.

American Wire Gage (A.W.G.). The A.W.G. (formerly Brown & Sharpe gage) is based on a constant ratio between diameters of successive gage numbers. The ratio of any diameter to the next smaller is 1.123, and the corresponding ratio of cross sec-

The ratio e of the temperature rise during adiabatic compression to the actual temperature rise has been observed to be practically constant as the speed of a fan changes. On the assumption that this is the case, the work H done in compressing 1 lb of air is

$$H = \frac{p_0V_0}{e}\left\{\frac{k}{k-1}\left[\left(\frac{p_1}{p_0}\right)^{(k-1)/k} - 1\right] - (1 - e)\log_e\frac{p_1}{p_0}\right\}$$

where p_0 and p_1 are the inlet and outlet absolute pressures (lb per sq ft) respectively, and k is the ratio of specific heats ($= 1.4$ for air). A usual value for e is 0.8 with good design. The work per stage h is given by H/z where z is the number of stages.

From the Euler equation for axial flow,

$$h = \omega r(V_{u_1} - V_{u_0})/g$$

V_{u_0}, the tangential spin at the inlet of a stage, may be zero; may be positive spin with the wheel, diminishing the possible work; or may be negative spin against the rotation of the wheel, increasing the work. This last direction of V_{u_0} will increase the tendency to and the violence of pulsation. With the V_{u_0} determined, preferably zero, V_{u_1} is evaluated.

The total circulation Γ can then be calculated. $\Gamma = 2\pi r(V_{u_1} - V_{u_0})$, and $\Gamma_b = \Gamma/Z$, where Γ_b is the circulation per blade. The blades and vanes may be designed from the lift coefficients of a high lift air foil in accordance with the procedure described for propeller fans.

SELECTION OF FAN SIZE

Fan engineering is based on tests of models. The size of the model should not be too small; 30 to 36 in. in diameter is convenient. Curves of test results may be confidently used to predict the performance of geometrically similar fans of any size larger than the model. Sizes smaller than 30 in. show poorer results and should be tested in each size.

The curves of Fig. 1 are for a 34 in. fan. The use of such curves is best explained by an example:

Required a geometrically similar fan to deliver 32,000 cfm at 0.8 in. of water, static pressure, with air at 200 F.

Select first from Fig. 1 a point of operation. The peak of the total efficiency is a desirable point.

Volume 27,000 cfm; static pressure, 3.05 in.; speed, 1,100 rpm. The curve is drawn for a density of 0.075, whereas the example has the lower density $d = 1.325 \times 29.92/660 = 0.06$. As the pressure of a fan varies as the air density, the equivalent pressure at the 0.075 of the test curve is $0.8 \times 0.075/0.06 = 1$ in. The equivalent performance of the model at this pressure is

$$\text{Speed} = 1{,}100\sqrt{1/3.05} = 630 \text{ rpm}$$
$$\text{Volume} = 27{,}000 \times 630/1{,}100 = 15{,}450 \text{ cfm}$$

At the same tip speed, the larger fan must have the same air velocities so the ratio of the two fan areas, or diameters squared, must be in proportion to the volumes. $D_L^2/D_m^2 = K^2 = 32{,}000/15{,}450 = 2.07$, where D_L is the required diameter of the larger fan, D_m the diameter of the model, and K the ratio. $D_L = 34\sqrt{2.07} = 49$ in. The speed of the large fan is $630/\sqrt{2.07} = 437$ rpm. The horsepower with 71 percent static efficiency $= 0.0001575 \times 32{,}000 \times 0.8/.71 = 5.68$.

The problem is usually complicated by the necessity of fitting the required duty to some standard diameter. The sizes given in Fig. 17 are a typical commercial standardization. A 48.5 in. wheel is the closest to the 49 in. of the tentative calculation.

Table 3. Working Table, Standard Annealed Copper Wire, Solid

American Wire Gage (B. & S.)

Gage No.	Diam, mils	Cross section		Ohms per 1,000 ft		Ohms per mile	Pounds per 1,000 ft
		Circular mils	Square inches	25 C (= 77 F)	65 C (= 149 F)	25 C (= 77 F)	
0000	460.0	212,000	0.166	0.0500	0.0577	0.264	641.0
000	410.0	168,000	0.132	0.0630	0.0727	0.333	508.0
00	365.0	133,000	0.105	0.0795	0.0917	0.420	403.0
0	325.0	106,000	0.0829	0.100	0.116	0.528	319.0
1	289.0	83,700	0.0657	0.126	0.146	0.665	253.0
2	258.0	66,400	0.0521	0.159	0.184	0.839	201.0
3	229.0	52,600	0.0413	0.201	0.232	1.061	159.0
4	204.0	41,700	0.0328	0.253	0.292	1.335	126.0
5	182.0	33,100	0.0260	0.319	0.369	1.685	100.0
6	162.0	26,300	0.0206	0.403	0.465	2.13	79.5
7	144.0	20,800	0.0164	0.508	0.586	2.68	63.0
8	128.0	16,500	0.0130	0.641	0.739	3.38	50.0
9	114.0	13,100	0.0103	0.808	0.932	4.27	39.6
10	102.0	10,400	0.00815	1.02	1.18	5.38	31.4
11	91.0	8,230	0.00647	1.28	1.48	6.75	24.9
12	81.0	6,530	0.00513	1.62	1.87	8.55	19.8
13	72.0	5,180	0.00407	2.04	2.36	10.77	15.7
14	64.0	4,110	0.00323	2.58	2.97	13.62	12.4
15	57.0	3,260	0.00256	3.25	3.75	17.16	9.86
16	51.0	2,580	0.00203	4.09	4.73	21.6	7.82
17	45.0	2,050	0.00161	5.16	5.96	27.2	6.20
18	40.0	1,620	0.00128	6.51	7.51	34.4	4.92
19	36.0	1,290	0.00101	8.21	9.48	43.3	3.90
20	32.0	1,020	0.000802	10.4	11.9	54.9	3.09
21	28.5	810	0.000636	13.1	15.1	69.1	2.45
22	25.3	642	0.000505	16.5	19.0	87.1	1.94
23	22.6	509	0.000400	20.8	24.0	109.8	1.54
24	20.1	404	0.000317	26.2	30.2	138.3	1.22
25	17.9	320	0.000252	33.0	38.1	174.1	0.970
26	15.9	254	0.000200	41.6	48.0	220	0.769
27	14.2	202	0.000158	52.5	60.6	277	0.610
28	12.6	160	0.000126	66.2	76.4	350	0.484
29	11.3	127	0.0000995	83.4	96.3	440	0.384
30	10.0	101	0.0000789	105	121	554	0.304
31	8.9	79.7	0.0000626	133	153	702	0.241
32	8.0	63.2	0.0000496	167	193	882	0.191
33	7.1	50.1	0.0000394	211	243	1,114	0.152
34	6.3	39.8	0.0000312	266	307	1,404	0.120
35	5.6	31.5	0.0000248	335	387	1,769	0.0954
36	5.0	25.0	0.0000196	423	488	2,230	0.0757
37	4.5	19.8	0.0000156	533	616	2,810	0.0600
38	4.0	15.7	0.0000123	673	776	3,550	0.0476
39	3.5	12.5	0.0000098	848	979	4,480	0.0377
40	3.1	9.9	0.0000078	1,070	1,230	5,650	0.0200

tions is $(1.123)^2 = 1.261$ or 1¼ approximately. $(1.123)^6$ is 2.0050 so that diameters differing by 6 gage numbers have a ratio of approximately 2; cross sections differing by 3 gage numbers also have a ratio of approximately 2. The ratio of cross sections differing by two numbers is $(1.261)^2 = 1.590$, or 1.6 approximately. The ratio of

With a known wheel diameter, the procedure becomes inverse. $K = D_L/D_m = 48.5/34 = 1.425$. The equivalent volume for the model is $Q/K^2 = 32{,}000/2.03 = 15{,}740$. But this volume is at a speed different from that of the model test curve, as the pressure corresponding to this equivalent volume is still 0.8 at 200 F corrected to 0.075 density, or 1 in.

A parabola plotted on Fig. 1 with vertex at the origin of coordinates, passing through 15,740 cfm and 1 in. static pressure, is the locus of the volume and pressure values for changing speed with constant orifice. The orifice is set by the calculated equivalent

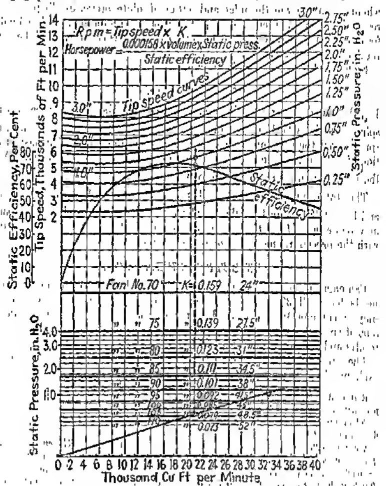


FIG. 17.—Hagen Fan Chart.

volume and the given pressure corrected. The intersection of this constant orifice curve and the static pressure characteristic of the fan gives the volume and pressure that the fan would deliver through that orifice at the speed of the test. The values at the intersection are: volume, 27,400 cfm; static pressure, 3 in; static efficiency, 70.5 percent; rpm, 1,100.

The speed of the test fan for the equivalent conditions is in proportion to the volumes, or the speed is $1{,}100 \times 15{,}740/27{,}400 = 633$ rpm. For the 48.5 in. standard fan, the speed is $633/K = 633/1.425 = 444$ at which speed and handling 200 F air the fan will deliver the required 32,000 cfm against 0.8 in. static pressure at a static efficiency of 70.5, taking 5.71 hp.

Fan Charts. Dimensionless plottings of fan data can be based on many different ratios to serve specific purposes. These plottings are not character-

cross sections differing by 10 numbers is approximately 10. The gage ordinarily extends from No. 40 to 0000 (4/0). Wires larger than 0000 must be stranded and their cross section is given in cir mils.

Table 4. Bare Concentric Lay Cables of Standard Annealed Copper

A. W. G. No.	Circular mils	Ohms per 1,000 ft		Pounds per 1,000 ft	Standard concentric stranding		
		25 C (= 77 F)	65 C (=149 F)		Number of wires	Diameter of wires, mils	Outside diameter, mils
	2,000,000	0.00539	0.00622	6,180	127	125.5	1,631
	1,700,000	0.00634	0.00732	5,250	127	115.7	1,504
	1,500,000	0.00719	0.00830	4,630	91	128.4	1,412
	1,200,000	0.00899	0.0104	3,710	91	114.8	1,263
	1,000,000	0.0108	0.0124	3,090	61	128.0	1,152
	900,000	0.0120	0.0138	2,780	61	121.5	1,093
	850,000	0.0127	0.0146	2,620	61	118.0	1,062
	750,000	0.0144	0.0166	2,320	61	110.9	998
	650,000	0.0166	0.0192	2,010	61	103.2	929
	600,000	0.0180	0.0207	1,850	61	99.2	893
	550,000	0.0196	0.0226	1,700	61	95.0	855
	500,000	0.0216	0.0249	1,540	37	116.2	814
	450,000	0.0240	0.0277	1,390	37	110.3	772
	400,000	0.0270	0.0311	1,240	37	104.0	728
	350,000	0.0308	0.0356	1,080	37	97.3	681
	300,000	0.0360	0.0415	926	37	90.0	630
	250,000	0.0431	0.0498	772	37	82.2	575
0000	212,000	0.0509	0.0587	653	19	105.5	528
000	168,000	0.0642	0.0741	518	19	94.0	470
00	133,000	0.0811	0.0936	411	19	83.7	418
0	106,000	0.102	0.117	326	19	74.5	373
1	83,700	0.129	0.149	258	19	66.4	332
2	66,400	0.162	0.187	205	7	97.4	292
3	52,600	0.205	0.237	163	7	86.7	260
4	41,700	0.259	0.299	129	7	77.2	232

From *N.B.S., Circular* 31. See Table 20, p. 1763, for the carrying capacity of wires.

The diameter of No. 10 wire is 102.0 mils. As an approximation this may be considered as being 100 mils; the cross section is 10,000 cir mils; the resistance is 1 ohm per 1,000 ft; and the weight of 1,000 ft is 31.4 (10π) lb. Also the weight of 1,000 ft of No. 2 is 200 lb. These facts give many short cuts in estimating resistances and weights of various gage numbers.

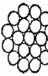


Fig. 1.—*Makeup of a 19-strand Cable.*

Lay Cables. In order to obtain sufficient flexibility, wires larger than 0000 are stranded, and they are designated by their circular mils. Smaller wires may be stranded also since sizes as small as No. 4 when insulated are usually too stiff for easy handling. *Lay cables are made up geometrically as shown in* Fig. 1. Six strands will just fit around the single central conductor; the number of strands in each succeeding layer increases by 6. The number of strands that can thus be layed up are 1–7–19–37–61–91–127, etc. In order to obtain sufficient flexibility with large cables, the strands themselves frequently consist of stranded cable.

The **resistance of cables** is readily computed from Eq. (1), using the cir-mil-ft as the unit of resistivity.

istic curves in that they tend to the same form or shape for all types of fans. They are useful as criteria of the value of a design or for convenience in calculation. Most generally used is a plot of efficiencies and tip speeds as ordinates with orifice ratios as abscissas.

The upper portion of the Hagen Chart, Fig. 17, is a plot of this type. No scale for orifice ratio is given on the chart as the correct abscissa is provided for by the lower portion of the chart. This lower portion has volumes as abscissas and pressures as ordinates spaced as the square roots of the pressures. A diagonal line to the origin through a point of desired volume and pressure is the hypotenuse of a triangle of which the volume abscissa and the ordinate length of square root of pressure are the two sides.

Fan lines are located on the chart at distances from the base line proportional to $1/D^2$. The same diagonal, extended if necessary, intersects any fan line forming a triangle similar to the pressure volume triangle. From these two similar triangles the abscissa intercept along the fan line must equal $V/D^2\sqrt{p_s}$, that is, the orifice ratio. The function of the lower portion of the chart is to determine conveniently the orifice ratio for a given volume and pressure of any of the standard size fans of the manufacturers line.

Quantitative results depend on the scales. In constructing the chart the scales for volume and pressure may be selected arbitrarily for ease of reading. The scale for the location of the fan lines proportional to $1/D^2$ may also be selected arbitrarily so as to use the space on the chart to good advantage. These three scales fix the scale of orifice ratio. This scale can be calculated arithmetically if desired but it is simpler to construct the chart from a fan-test curve such as Fig. 1.

The fan line of the test-fan size is drawn in for construction purposes. A volume and corresponding pressure may then be selected from the test curve and located on the chart determining the diagonal. The intersection of this diagonal with the fan line of the test fan size gives the abscissa of orifice ratio to correct scale value. At that same abscissa on the upper portion of the chart the test efficiency may be plotted, also the tip speed of the test. Additional tip speed points are plotted at this same abscissa spaced as the square roots of the pressure. This procedure is repeated for enough points taken from the test curve to permit accurate determination of the curves of the chart.

Solving the preceding problem by the use of the chart (Fig. 17) the procedure is as follows: Through the 32,000 volume and the corrected 1 in pressure point, a straight line is drawn to the origin intersecting fan lines of the standardized sizes. Above any of these intersections can be taken off efficiency and tip speed for that particular fan. For example, above the 48.5 in. intersection the efficiency is 70.5 and the tip speed for 1 in. static is 5,600 fpm. The rpm is $5{,}600/(48.5\pi/12) = 442$.

Example. Determine the resistance of 3,500 ft of 800,000-cir mil cable at 20 C. *Ans.* ρ (of a cir mil-ft) = 10.37. R = 10.37 × 3,500/800,000 = 0.0454 ohm.

ρ = 10 ohms per cir-mil-ft is often sufficiently accurate for practical purposes.

ELECTRICAL CIRCUITS

Ohm's law states that, with a steady current, the current in a circuit is *directly* proportional to the *total* emf acting in the circuit and is *inversely* proportional to the total resistance of the circuit. The law may be expressed by the following three equations:

$$I = E/R \quad (7); \qquad E = IR \quad (8); \qquad R = E/I \tag{9}$$

where E is the emf, volts; R the resistance, ohms; and I the current, amp.

Series Circuits. The combined resistance of a number of series-connected resistances is the sum of the separate resistances. When batteries or other sources of emf are connected in series the total emf of the combination is the sum of the separate emfs. The open-circuit emf of a battery is the total generated emf, and may be measured at the battery-terminals only when no current is being delivered by the battery. The internal resistance is the resistance of the battery alone. The current which will flow in a circuit connected in series with a source of emf is $I = E/(R + r)$, where E is the open-circuit emf, R the external resistance, and r the internal resistance of the source of emf.

Parallel Circuits. The combined conductance of a number of parallel-connected resistances is the sum of the separate conductances.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \tag{10}$$

The equivalent resistance for two parallel resistances R_1, R_2, is

$$R = R_1R_2/(R_1 + R_2) \tag{11}$$

The equivalent resistance for three parallel resistances, R_1, R_2, R_3, is

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \tag{12}$$

and for four parallel resistances, R_1, R_2, R_3, R_4,

$$R = \frac{R_1R_2R_3R_4}{R_1R_2R_3 + R_2R_3R_4 + R_3R_4R_1 + R_4R_1R_2} \tag{13}$$

To obtain the resistance of combined series and parallel resistances, the equivalent resistance of each parallel portion is obtained separately and then these equivalent resistances are added to the series resistances according to the principles stated above.

Kirchhoff's laws (derived from Ohm's law) make it possible to solve many circuit networks that would otherwise be difficult of solution. The first law states that: *In any branching network of wires the algebraic sum of the currents in all the wires that meet at a point is zero.* The second law states that: *The sum of all the electromotive forces acting around a complete circuit is equal to the sum of the resistances of its separate parts multiplied each into the strength of the current that flows through it, or the total change of potential around any closed circuit is zero.*

In applying Kirchhoff's laws the following rules should be observed: Currents flowing toward a junction should be preceded by a plus sign. Cur-

rents flowing away from a junction should be preceded by a minus sign. A rise in potential should be preceded by a plus sign. (This occurs in going

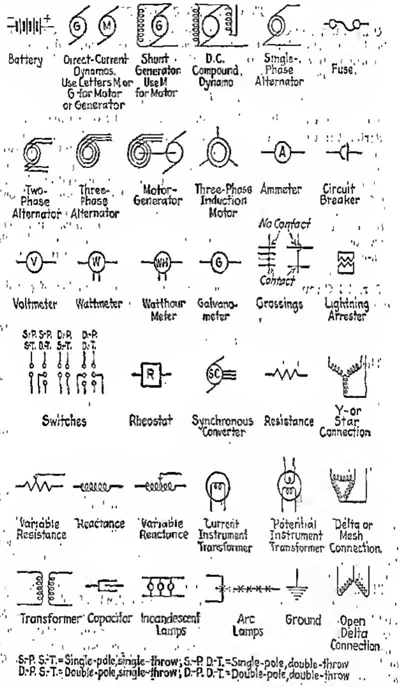


FIG. 2.—Diagrammatic Symbols for Electrical Machinery and Apparatus.

through a source of emf from the negative to the positive terminal, and in going through a resistance in opposition to the current flow.) A drop in

SECTION 12

ELECTRICAL ENGINEERING

BY

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

CONTENTS

potential should be preceded by a minus sign. (This occurs in going through a source of emf from the positive to the negative terminal and in going through a resistance in conjunction with the current flow.)

The application of Kirchhoff's laws is illustrated by the following example.

Example. Determine the three currents I_1, I_2, and I_3 in the circuit network (Fig. 3). The arrows show the assumed directions of the three currents.

Applying Kirchhoff's second law to circuit *a b c d e a*,

$$+4 + 0.2I_1 + 0.5I_1 - 3I_2 + 2 - 0.1I_2 + I_1 = 0$$

or,

$$+6 + 1.7I_1 - 3.1I_2 = 0 \qquad \text{(I)}$$

and for *e d c f g e*,

$$-2 + 0.1I_2 + 3I_2 + I_3 + 3 + 0.3I_3 = 0$$

or,

$$+1 + 3.1I_2 + 1.3I_3 = 0 \qquad \text{(II)}$$

Applying Kirchhoff's first law to junction *c*,

$$-I_1 - I_2 + I_3 = 0 \qquad \text{(III)}$$

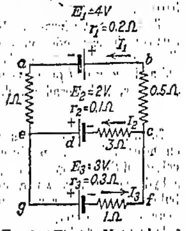

FIG. 3.—Electric Network and Kirchhoff's Laws.

Solving (I), (II), and (III) simultaneously gives $I_1 = -2.56$, $I_2 = +0.53$, and $I_3 = -2.03$. The minus signs before I_1 and I_3 show that the actual directions of flow of these two currents are opposite the assumed directions.

Electrical Power. With direct currents the electrical power is given by the product of the volts and amperes. That is, $P = EI$ **watts**. (14) Also, by substituting for E and I Eqs. (8) and (7), $P = I^2R$ watts (15); $P = E^2/R$ watts (16).

The practical unit of power is the **watt** $= 10^7$ ergs per sec. The watt is too small a unit for many purposes. Hence, the **kilowatt** (kw) (1,000 watts) is used. 746 watts $= 1$ hp. $= 0.746$ kw. 1 kw $= 1.340$ hp.

Joule's Law. When an electric current flows through a resistance, the number of heat units developed is proportional to the square of the current, directly proportional to the resistance, and directly proportional to the time that the current flows. $h = 0.2389i^2rt$, where h represents the number of IT gram-calories; i the current, amp; r the resistance, ohms; and t the time, sec. h (in Btu) $= 0.0009480i^2rt$. See p. 1689.

Electrical Energy. The fundamental unit of electrical energy is the erg. The practical unit is the **joule** or **watt-second** $= 10^7$ ergs. This unit is too small for most practical purposes, hence the **kilowatt-hour** (kwhr) is used.

BATTERIES

In an **electric cell** or **battery**, chemical energy is converted into electrical energy. Strictly speaking, the word battery applies to an assembly of cells, but the word has come to mean single units or cells. A battery is based on the fact that a potential difference exists between different elements. When two different elements are immersed in electrolyte an emf exists tending to send current within the cell from the negative pole, which is the more highly electropositive, to the positive pole. The poles or electrodes of a battery form the junction with the external circuit.

If the external circuit is closed, current flows from the battery at the *positive electrode* or *cathode*, and enters the battery at the *negative electrode* or *anode*.

ELECTRICAL ENGINEERING

BY

CHESTER L. DAWES

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MAGNETIC AND ELECTRICAL UNITS

Systems of Units. There are two fundamental systems of electrical units based dimensionally on the centimeter, gram, and second (cgs), the electrostatic and the electromagnetic. The cgs electrostatic system is derived from the force exerted between two unit charges of electricity at points 1 cm apart in a medium of unit dielectric constant. The cgs electromagnetic system is derived from the force exerted between two unit magnetic poles placed at points 1 cm apart in a medium of unit magnetic permeability. The practical units such as the volt and ampere are derived from the units of the electromagnetic system, differing from them only in their magnitudes.

The mks system has recently been adopted as a standard. It is based dimensionally on the meter, kilogram, and second. The electrical units are those of the practical system.

Magnetic calculations are usually made in the cgs electromagnetic system.

Unit magnetic pole is one that will repel an equal and like pole with a force of 1 dyne (see p. 1701) at a distance of 1 cm in a medium of unit permeability.

Magnetic potential is measured by the work involved in moving a unit magnet pole from the boundary of the field to the point at which the potential is desired.

Magnetic field intensity (H) is measured by the force in dynes exerted on a unit pole. In media whose permeability is unity the field intensity is given by the number of lines of force per square centimeter taken normal to their direction. The cgs unit of field intensity is the **oersted**.

Magnetic flux (Φ, ϕ) is the magnetic flow that exists in any magnetic circuit. The cgs unit is the **maxwell**.

Magnetic flux density (B) is the ratio of the flux in any cross section to the area of that cross section, the cross section being taken normal to the direction of flux. One maxwell per square centimeter is equal to 1 **gauss**.

Magnetomotive force ($\mathcal{F}$ and mmf) is that which tends to produce magnetic flux and corresponds to electromotive force in the electric circuit. The cgs unit is the **gilbert** which is equal to $0.4\pi nI$ where nI are the ampere-turns. The **ampere-turn** is frequently a convenient unit.

In a **primary battery** the chemically reacting parts require renewal; in a **secondary battery**, the electrochemical processes are reversible to a high degree and the chemically reacting parts are restored after partial or complete discharge by reversing the direction of current flow through the battery.

Electromotive force of a battery is the total potential difference existing between the electrodes on open circuit. When current flows, the potential difference across the terminals drops because of the resistance drop within the cell and because of **polarization.** If E is the emf of the cell, r the internal resistance, V the terminal voltage, when current I flows, then

$$V = E - Ir \tag{17}$$

Polarization. When current flows in a battery, hydrogen is deposited on the cathode. This produces two effects, both of which reduce the terminal voltage of the battery. The hydrogen in contact with the cathode constitutes a hydrogen battery which opposes the emf of the battery; the hydrogen bubbles reduce the contact area of the electrolyte with the cathode, thus increasing the battery resistance. The most satisfactory method of reducing polarization is to have present at the cathode some compound that supplies negative ions to combine with the positive hydrogen ions at the plate. In the Leclanché cell, manganese peroxide in contact with the carbon cathode serves as a depolarizer, its oxygen ion combining with the hydrogen ion to form water.

Primary Batteries

The **Leclanché** cell has carbon for the positive and zinc for the negative electrode with sal ammoniac as solution. It is used where only low values of current and intermittent service are desired. In the improved type the cathode is a porous carbon cup in which are packed lumps of manganese dioxide to serve as an insoluble oxidizing agent. The rather high internal resistance has been reduced by using a zinc cylinder for the anode and placing it about the carbon and as near as possible. The emf of this cell is about 1.4 volts, but the terminal voltage drops to approximately 1 volt when in service. This type of cell now is only of importance in that it forms the basis of the dry cell.

The **gravity cell** consists of a heavy zinc crowfoot as anode suspended on the top of a battery jar and a cathode of sheet copper in the bottom. The electrolytes are zinc sulphate about the anode and copper sulphate about the cathode. The two electrolytes are kept separate by gravity. This is a closed-circuit battery. The emf is about 1 volt.

The **Edison primary cell** has a cathode of copper oxide, an anode consisting of zinc plates, and an electrolyte of caustic soda ($NaOH$). The emf is about 0.9 volt and the terminal voltage is about 0.75 volt when in service. This battery can be used for either open- or closed-circuit work.

The foregoing primary batteries are practically obsolete so far as general use is concerned. They have been replaced by dry batteries, storage batteries, bell-ringing transformers, etc.

Dry Batteries. The dry battery is a development of the Leclanché battery and consists of a zinc container which serves as the negative electrode and is lined with specially prepared paper, or some similar absorbent material, to prevent the mixture of carbon and manganese dioxide, which is tamped tightly around the positive carbon electrode, from coming in contact with the zinc. The absorbent lining and the mixture are moistened with a solution of zinc chloride and sal ammoniac. In smaller cells the manganese-carbon mixture is often molded into a cylinder around the carbon electrode, the whole is then set into the zinc cup, and the space between the mold of mix and the zinc is filled with electrolyte made into a paste in such a manner that it can

between chord AB and the horizontal, F = area of cross-section, E = Young's modulus of elasticity (for steel, E = 29,000,000 lb per sq in.), l = stretched length (along curve).

If the tension P at the upper end is known, compute wL/P and find m from Table 6. Then $l = L + (LP/FE)(1 - m)$.

If the tension Q at the lower end is known, compute wL/Q and find n from Table 7. Then $l = L + (LQ/FE)(1 + n)$.

TABLE 6. GIVING m

$\frac{wL}{P}$	$A = 0°$	10°	20°	30°	40°	50°	60°	70°	80°	90°
.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	.001	.010	.018	.026	.033	.039	.044	.047	.049	.050
.20	.003	.021	.038	.053	.067	.078	.088	.094	.099	.100

TABLE 7. GIVING n

$\frac{wL}{Q}$	$A = 10°$	20°	30°	40°	50°	60°	70°	80°	90°
.00	.000	.000	.000	.000	.000	.000	.000	.000	000
.10	.008	.016	.024	.032	.038	.043	.047	.049	.050
.20	.014	.031	.047	.062	.075	.086	.094	.099	.100

OTHER USEFUL CURVES

The Cycloid is traced by a point on the circumference of a circle which rolls without slipping along a straight line. **Equations of cycloid**, in parametric form (axes as in Fig. 65): $x = a$ (rad $u - \sin u$), $y = a(1 - \cos u)$, where a is

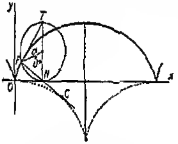


FIG. 65.

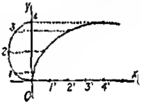


FIG. 66.

the radius of the rolling circle, and rad u is the radian measure of the angle u through which it has rolled. The tangent and normal at any point pass through the highest and lowest points of the corresponding position of the generating circle. The **radius of curvature** at any point P is $PC = 4a \sin(u/2) = 2\sqrt{2ay}$ = twice the length of the normal, PN. The **evolute**, or locus of centers of curvature, is an equal cycloid. ***To construct** a cycloid (Fig. 66)*, divide the semi-circumference of the generating circle into n equal parts (here 4) and lay off these arcs along the base (from O to $4'$). Describe arcs with centers at $1'$, $2'$, . . . and radii equal to the chords $O1$, $O2$, . . ., and sketch the cycloid as a curve tangent to all of these arcs. Or, on horizontal lines through 1, 2, . . . lay off distances equal to $O1'$, $O2'$, etc.; the points thus reached will lie on the cycloid.

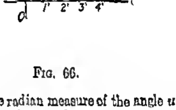
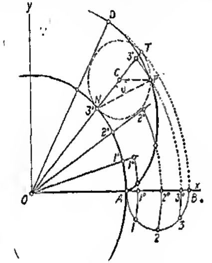


FIG. 67.

The area of one arch $= 3\pi a^2$, length of arc of one arch $= 8a$. Area bounded by the ordinate of the point P corresponding to any value of u is a^2 ($\frac{3}{2}$ rad $u - 2 \sin u + \frac{1}{4} \sin 2u) = \frac{3}{2} ax - \frac{1}{2} y\sqrt{(2a - y)y}$. Length of arc $OP = 4a (1 - \cos \frac{1}{2} u) = 4a - 2\sqrt{2a(2a - y)}$.

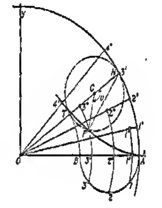
The Trochoid is a more general curve, traced by any point on a radiu of the rolling circle, at distance b from the center (Fig. 67). It is a prolat trochoid if $b < a$, and a curtate or looped trochoid if $b > a$. The equations ir either case are $x = a$ rad $u - b \sin u$, $y = a - b \cos u$.

The Epicycloid (or Hypocycloid) is a curve generated by a point on th circumference of a circle of radius a which rolls without slipping on th outside (or inside) of a fixed circle of radius c. For the **equations**, pu $b = a$ in the equations of the epi- (or hypo-) trochoid, below. The norma at any point P passes through the point of contact N of the correspondin position of the rolling circle. To construct the curve (Figs. 68 and 69)



Epicycloid.

FIG. 68.



Hypocycloid.

FIG. 69.

divide the semi-circumference of the rolling circle into n equal parts, by point 1, 2, 3 . . ., and lay off these arcs ($A1$, $A2$, $A3$) along the circumference of th base circle, as $A1'$, $A2'$, $A3'$, ... Describe circles with centers at 1′, 2′, 3′ . . . and radii equal to the chords $A1$, $A2$, $A3$, . . .; then the required curve will be tangent to all these circles. Or, with O as center, draw arc through 1, 2, 3, . . ., meeting the radius OA in 1^0, 2^0, 3^0, . . ., and th radii $O1'$, $O2'$, $O3'$, . . . in 1″, 2″, 3″, . . .; then from 1″, 2″, 3″, . . . lay off arcs equal to 1^01, 2^02, 3^03, . . . respectively; the points thu reached will be points of the curve.

The area $OAP = \dfrac{a(c \pm a)(c \pm 2a)}{2c}$ (rad $u - \sin u$), where the upper sig applies to the epicycloid, the lower to the hypocycloid, and rad u = th radian measure of the angle u shown in Figs. 68 and 69. Arc AP = $(4\,a/c)(c \pm a)(1 - \cos \tfrac{1}{2} u)$; arc $AD = (4a/c)(c \pm a)$. [In Fig. 69, $D = 4''$.

Radius of curvature at any point P is $R = \dfrac{4a(c \pm a)}{c \pm 2a} \sin \tfrac{1}{2}u$; at A, $R = 0$; at D, $R = \dfrac{4a(c \pm a)}{c \pm 2a}$.

Special Cases. If $a = \tfrac{1}{2}c$, the hypocycloid becomes a straight line, diameter of the fixed circle (Fig. 70). In this case the hypotrochoid traced by any

be solidified by either standing or heating. The emf of a dry cell when new is 1.4 to 1.6 volts.

In **block assembly** the dry cells, especially in the smaller sizes, are assembled in series and sealed in blocks of insulating compound with only two terminals and sometimes intermediate taps brought out. This type of battery is used for radio B and C batteries. Another construction is to build the battery up of layers in somewhat the manner of the old voltaic pile. Each cell consists of a layer of zinc, a layer of treated paper, and a flat cake of the manganese-carbon mixture. The cells are separated by layers of a special material which conducts electricity, but which is impervious to electrolyte. A sufficient number of such cells are built up to give the required voltage and the whole battery is sealed into the carton.

Dry cells and batteries fall generally into three classes: (1) the **large size dry cells** which are usually approximately 2½ in. diam by 6 in. height; (2) **flashlight batteries** which are of small size, usually 1¼ in. diam by 2¼ in. height or smaller; and (3) **radio B batteries** which consist usually of 15 or 30 cells permanently connected into a battery which is used chiefly to supply the B battery current for radio-receiving sets.

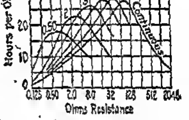


FIG. 4.—Service Efficiency Curves to Cut-off Voltage at Eight-tenths Volt for Various Periods of Discharge.

The **efficiency** of a standard-size dry battery depends on the rate at which it is discharged. Up to a certain rate the lower the discharge rate, the greater the efficiency. Above this rate the efficiency decreases. Figure 4 (*N.B.S. Circular 79*, p. 39) shows the service that may be obtained from an ordinary 6 in. dry cell. Thus, if a battery of dry cells is discharged through 16 ohms resistance for 15 min per hr to 0.8 volt, it will, from Fig. 4, give 28 hr of service per ohm. Since it is discharged through 16 ohms, the hours of service will be 448, and since it is on service 15 min per hr, the total life of the cell will be 1,792 hr, or about 75 days.

When used efficiently a 6 in. dry cell will give over 30 amp-hr of service. As ordinarily used, however, the dry cell gives no more than 8 to 10 amp-hr of service and at times even less. The 1¼ by 2¼ in. flashlight battery is usually employed with a lamp taking 0.25 to 0.35 amp. Under these conditions 3 amp-hr or thereabouts may be expected if the battery is used for not more than an hour or so a day. The so-called "heavy-duty" radio battery will give about 8 to 10 amp-hr when efficiently used.

For the best results 6 in. dry cells should not be used for current drains of over 0.5 amp except for very short periods of time. Flashlights should not be used for higher than the preceding current drain, and heavy-duty radio batteries will give best results if the current drain is kept below 25 milliamp.

Dry cells should be stored in a cool, dry place. Extreme heat during storage will shorten their life. The cell will not be injured by being frozen but will be as good as new after being brought back to normal temperature. In extreme cold weather dry cells may not give more than half of their normal service. At a temperature of about −30 they freeze solid and give neither voltage nor current.

The amperage of a dry cell by definition is the current that it will give when it is short-circuited through an ammeter which with its leads has a resistance of 0.01 ohm. For correct results, the amperage of a dry cell must be taken at about 70 F.

The **Weston cell** is a primary cell used as a standard of emf. It consists of a glass H-tube in the bottom of one leg of which is mercury which *forms* the positive or cathode; in the bottom of the other leg is cadmium amalgam forming the anode. The electrolytes consist of mercurous sulphate and cadmium sulphate. The emf of the unsaturated, or working, cell is approximately 1.0186 volts. The saturated cell can be reproduced to 2 or 3 parts in 100,000 and hence is a very accurate standard. No appreciable current

Equations (32) and (33) are used for calculating the capacitance of overhead transmission lines. When computing charging current, use voltage between lines in (32) and to neutral in (33).

Capacitances in Parallel. The equivalent capacitance of capacitances in parallel (Fig. 17)

$$C = C_1 + C_2 + C_3 \tag{34}$$

Capacitances in parallel are all across the same voltage. If the voltage is E, then the total quantity $Q = CE$ and $Q_1 = C_1E$, etc.

Capacitances in Series. The equivalent capacitance C of capacitances in series (Fig. 18) is found as follows:

$$1/C = 1/C_1 + 1/C_2 + 1/C_3 \tag{35}$$

If the capacitances are not leaky, the charge Q is the same on each. $Q = CE$, $E_1 = Q/C_1$, $E_2 = Q/C_2$, etc.

Insulators and Dielectrics. Insulating materials are applied to electric circuits to prevent the leakage of current. Insulating materials when used with high voltage must not only have a high resistance to leakage current, but must also be able to resist dielectric puncture. That is, in addition to being a good insulator, the material must be a good dielectric. Insulation resistance is usually expressed in megohms (10^6 ohms) and the resistivity given in megohms per centimeter cube. The dielectric strength is usually given in terms of voltage gradient, common units being volts per mil, volts per millimeter, and kilovolts per centimeter. Insulation resistance decreases very rapidly with increase in temperature. Absorbed moisture reduces the insulation resistance, and moisture and humidity have a large effect on surface leakage.

Transients

Induced EMF. If a flux ϕ linking n turns of conductor changes, an emf

$$e = -n(d\phi/dt)10^{-8} \text{ volts} \tag{36}$$

is induced.

Self-inductance. Let a flux ϕ link n turns. The linkages of the circuit are $n\phi$ maxwell-turns. If the permeability of the circuit is assumed constant, then the number of these linkages per ampere ($\times 10^{-8}$) is the **self-inductance** or **inductance** of the circuit. The unit of inductance is the **henry**. The inductance is

$$L = n\phi/(i \times 10^8) \text{ henrys} \tag{37}$$

If the permeability changes with the current

$$L = n(d\phi/di)10^{-8} \text{ henrys} \tag{38}$$

The energy stored in the magnetic field

$$W = \tfrac{1}{2}Li^2 \text{ joules} \tag{39}$$

EMF of Self-induction. If Eq. (37) be written $Li = n\phi 10^{-8}$ and differentiated with respect to the time t, $L(di/dt) = n(d\phi/dt)10^{-8}$ and from Eq. (36)

$$e = -L(di/dt) \text{ volts} \tag{40}$$

This is the emf of self-induction. In a circuit, if a rate of change of current of 1 amp per sec induces an emf of 1 volt, the inductance is 1 henry.

Current in Inductive Circuit. If a circuit containing resistance R and inductance L in series is connected across a steady voltage E, the voltage E must supply the iR drop in the circuit and at the same time overcome the emf

can be taken from the cell, hence some null method must be used to utilize its emf (see p. 1997).

Storage Batteries

In a **storage battery** the electrolytic action must be **reversible**. There are two types of storage batteries, the lead-lead-acid type and the nickel-iron-alkaline type (Edison battery).

In the manufacture of the **lead-lead-acid cells** there are two general types of plates or electrodes. In the **Planté type** the active material is electrically formed of pure lead by repeated reversals of the charging current. In the **Faure or pasted plate** the active material is obtained by applying paste to supporting grids, lead peroxide to the positive and lead oxide to the negative plate.

In order to obtain high capacity per unit weight it is necessary to expose a large plate area to the action of the acid. This is done in the Planté plate by ploughing with sharp steel disks and by using corrugated helical inserts as active positive material (Manchester plate). In the pasted plate a large area of the material is necessarily exposed to the action of the acid.

The chemical reactions in a lead cell may be expressed by the following equation, based on the double sulphation theory:

$$\underset{\substack{\text{Positive}\\\text{plate}}}{PbO_2} + \underset{\substack{\text{Negative}\\\text{plate}}}{Pb} + \underset{\substack{\text{Sulphuric}\\\text{acid}}}{2H_2SO_4} \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} \underset{\substack{\text{Positive and}\\\text{negative plates}}}{2PbSO_4} + \underset{\text{water}}{2H_2O}$$

Between the extremes of complete charge and discharge, complex combinations of lead and sulphate are formed. After complete discharge a hard insoluble sulphate forms slowly on the plates, and this is reducible only by slow charging. This sulphation is objectionable and should be avoided.

Specific Gravity. Water is formed with discharge, and sulphuric acid is formed on charge, consequently the specific gravity must decrease on discharge and increase on charge. The variation of the specific gravity for a stationary battery is shown in Fig. 5. With starting and vehicle batteries it is necessary to operate the electrolyte from between 1.280 to 1.300 when fully charged to as low as 1.100 when completely discharged. The condition of charge of a battery may be determined by its specific gravity.

FIG. 5.—Variations of Specific Gravity in a Stationary Battery.

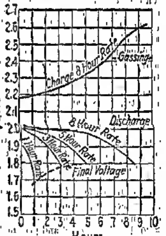


FIG. 6.—Voltage Curves on Charge and Discharge for Lead Cell.

of self-induction. That is $E = Ri + Ldi/dt$. A solution of the differential equation gives

$$i = (E/R)(1 - \epsilon^{-Rt/L}) \text{ amp} \tag{41}$$

where ϵ is the base of the natural system of logarithms.

Figure 19 shows this equation plotted when $I_0 = 0.5$ amp, $R = 20\Omega$, $L = 0.6$ henry. It is to be noted that inductance causes the current to rise slowly to its Ohm's law value. When $t = L/R$, the current has reached 63.2 percent of its Ohm's law value. L/R is the **time constant** of the circuit.

If a circuit containing inductance and resistance in series be short-circuited when the current is I_0, the equation of current becomes

$$i = I_0\epsilon^{-Rt/L} \text{ amp} \tag{42}$$

Figure 20 shows this equation plotted when $I_0 = 0.5$ amp, $R = 20\Omega$, $L = 0.6$ henry. It is seen that inductance opposes the decay of current. Inductance always opposes change of current.

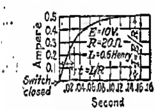


FIG. 19.—Rise of Current in Inductive Circuit

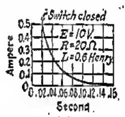


FIG. 20.—Decay of Current in Inductive Circuit.

Mutual Inductance. If two circuits are so related to each other geometrically that any portion of the flux produced by the current in one circuit links the other circuit, the two circuits possess *mutual inductance*. It follows that a change of current in one circuit causes an emf to be induced in the other. Let e_2 be induced in circuit 2 by a change di_1/dt in circuit 1. Then

$$e_2 = Mdi_1/dt \text{ volts} \tag{43}$$

M is the mutual inductance of the two circuits. Also a change of current di_2/dt in circuit 2 induces an emf e_1 in circuit 1, $e_1 = Mdi_2/dt$.

Current in Capacitive Circuit. If a capacitance C farads and a resistance R ohms be connected in series across the steady voltage E, the current

$$i = (E/R)\epsilon^{-t/CR} \text{ amp} \tag{44}$$

If a capacitor charged to voltage E be discharged through resistance R, the current

$$i = -(E/R)\epsilon^{-t/CR} \text{ amp} \tag{45}$$

Except for sign, these two equations are identical and are of the same form as (42).

Resistance, Inductance, and Capacitance in Series. If a circuit having resistance, inductance, and capacitance in series is connected across a source of steady voltage a transient condition results. If $R > \sqrt{4L/C}$ or $R = \sqrt{4L/C}$ the transient dies out rapidly without oscillation. The value of $R = \sqrt{4L/C}$ is the **critical damping resistance.** If $R < \sqrt{4L/C}$ the transient oscillates at a frequency equal very nearly to $1/(2\pi\sqrt{LC})$. If a capacitor is discharged into resistance and inductance in series, these same conditions exist.

Battery electrolyte may he made from concentrated sulphuric acid (oil of vitriol, sp gr 1.84) by *pouring the acid into the water* in the following proportions.

Table 5. Parts Water to One Part Acid

Specific gravity	1.200	1.210	1.240	1.280
Volume	4.3	4.0	3.4	2.75
Weight	2.4	2.2	1.9	1.5

Table 6. Freezing Temperatures of Sulphuric Acid

Specific gravity	1.180	1.200	1.240	1.280
Freezing temp, F	−6	−16	−51	−90

Voltage. The emf of a lead cell when fully charged and idle is 2.05 to 2.10 volts. Discharge lowers the voltage in proportion to the current flowing. Complete *discharge* is reached at 1.7 volts. When *charging* at constant current and normal rate, the terminal voltage gradually increases from 2.2 to 2.35 volts, then increases rapidly to between 2.5 and 2.7 volts (Fig. 6). This latter interval is known as the *gassing period*. When this period is reached, the charging rate should be reduced in order to avoid waste of power and unnecessary deterioration of the plates.

Practically all batteries have a **normal rating** based on the 8-hr rate of discharge. Thus a 320 amp-hr battery would have a normal rate of 40 amp. The amp-hr capacity of batteries falls off rapidly with increase in discharge rate.

Table 7. Effect of Discharge Rate on Battery Capacity

Discharge rate, hr	8	5	3	1	$\frac{1}{3}$	$\frac{1}{10}$
Percentage of rated capacity. Planté type	100	88	75	55.8	37	19.5
Percentage of rated capacity. Pasted type	100	93	83	63	41	25.5

The following rule may be observed in **charging a lead battery:** The charging rate in amp may always be made equal to the number of amp-hr out of the battery. For example, if 200 amp-hr are out of a battery, a charging rate of 200 amp may be used until the amp-hr out of the battery are reduced appreciably.

There are two common methods of charging, the **constant-current** method and the **constant-potential method.** Figure 7 shows a common method of charging with constant current, the battery being in series with resistance across d-c mains. Several batteries may be connected in series. If the polarity of the two wires is not known, it may be determined by placing them in a container of salty water; the negative wire will generate gas more freely than the other. By moistening a strip of litmus paper with salty water and holding the wires from the charging system close together on this strip of paper, an acid reaction develops a red spot under the positive terminal and an alkaline reaction produces a blue spot under the negative.

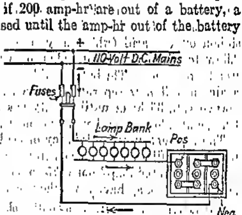


FIG. 7.—Charging a Starting Battery from 110-volt Mains.

The constant-potential method is to be preferred, since the rate automatically tapers off as the cell approaches the charged condition. Without resistance the terminal voltage should be 2.3 volts per cell, but it is preferable to use 2.4 to 2.5 volts per cell with a low resistance in series.

ALTERNATING CURRENTS

Sine Waves. In the following discussion of alternating currents sine waves of voltage and current will be assumed. That is, $e = E_m \sin \omega t$ and $i = I_m \sin (\omega t - \theta)$ where E_m and I_m are maximum values of voltage and current; ω, the angular velocity in radians per second, is equal to $2\pi f$ where f is the frequency; and θ is the angle of phase difference.

Cycle; Frequency. When any given armature coil has passed a pair of poles, the emf or current has gone through 360 electrical degrees or 1 cycle. An alternation is one-half a cycle. The frequency of an alternator in cycles per second

$$f = NP/120 \tag{46}$$

where N is the speed in rpm and P the number of poles. Standard frequencies are 60 cycles for lighting and power and 25 cycles for power. Frequencies of 40 and 50 cycles are used in some localities, and 12½ and 15 cycles are used abroad in railway work. One hundred and thirty-three cycles was extensively used some years ago but is rarely used at the present time.

The effective value of a wave (root-mean-square value) produces the same heating in a given resistance as a direct current of the same value. Since the heating effect of a current is proportional to i^2r, the effective value is obtained by squaring the ordinates, finding their average value, and extracting the square root, i.e., the effective value

$$I = \sqrt{1/T \int_0^T i^2 dt} \text{ amp}$$

where T is the time of a cycle. The effective value I of a sine wave equals $(1/\sqrt{2})I_m = 0.707I_m$.

The Average Value of a Wave. The average value of a sine wave over a complete cycle is zero. For a half-cycle the average is $(2/\pi)I_m$, or $0.637\ I_m$, where I_m is the maximum value of the sine wave. The average value is of little practical importance. A d-c measuring instrument gives the average values of a pulsating wave. The average value is of use only when the effects of the current are proportional to the number of coulombs passing, as in electrolytic work.

Form Factor. The form factor of a wave is the ratio of effective value to average value. For a sine wave this is $\pi/(2\sqrt{2}) = 1.11$. This factor is important in that it enters equations for induced emf.

Inductive reactance, $2\pi fL$ or ωL, opposes an alternating current in passing through an inductance L. It is expressed in ohms. Reactance is usually denoted by the symbol X. Inductive reactance is denoted by X_L.

The current through an inductive reactance X_L when connected across the voltage E is

$$I = E/X_L = E/(2\pi fL) \text{ amp} \tag{47}$$

This current lags the voltage by 90 deg. Inductance absorbs no energy. The energy stored in the magnetic field during each half-cycle is returned to the source during the same half-cycle.

Capacitive reactance is $1/(2\pi fC) = 1/\omega C$ and is denoted by X_C, where C is in *farads*. If C is given in microfarads, $X_C = 10^6/2\pi fC$. The current through a capacitive reactance X_C when connected across voltage E is

$$I = E/X_C = 2\pi fCE \text{ amp} \tag{48}$$

This current leads the voltage by 90 deg. Capacitance absorbs no energy.

When a battery is being charged, its terminal voltage $V = E + Ir$. (18) Compare with Eq. (17).

When a battery is fully charged, any rate will produce gassing, but the rate may be reduced to such a low value that gassing is practically harmless. This is called the **finishing rate.**

Stationary Batteries. The containing tanks are glass jars in the smaller sizes and lead-lined wooden tanks in the large sizes. The lead must be burned, not soldered, and the wood painted with asphaltum or other acid-resisting paint. When batteries are to be used for **regulating duty**, Planté positive plates should be used because of their long life. When used for **stand-by service**, pasted positive plates should be used because of their high rate of discharge. This is the more common use of batteries in central-station service. Pasted negatives are used in practically all stationary batteries. The separators are of thin wood veneer.

Because of the necessity of having very high discharge rates for their size and weight, **starting and lighting batteries** employ pasted plates. The separators may be wood veneer alone, or in the better types the wood may be reinforced with perforated hard rubber. The Electric Storage Battery Co. has developed a sheet-rubber separator having microscopic pores, hence the name "mipor." Also "fiberglas" fabric, woven from spun-glass fibers, is being used for separators. The jars are made of hard rubber sealed with asphaltum.

The **Exide Ironclad** battery is designed for propelling electric vehicles. The positive plate consists of a lead-antimony frame supporting perforated hard-rubber tubes. An irregular lead-antimony core runs down the center of each tube, and the lead peroxide paste is packed into these tubes so that shedding of active material from the positive plate cannot occur. Pasted negative plates are used.

A storage battery **removed from service** for less than 9 months should be charged once a month if possible, but if not, it should be given a heavy overcharge before discontinuing service. If removed for a longer period, siphon off the acid (which may be used again) and fill with fresh water. Allow to stand 15 hr and siphon off water. Remove and throw away the wood separators. The battery will now stand indefinitely. To put in service again, install new separators, fill with acid (sp gr 1.210), and charge at normal rate 35 hr or until gravity has ceased to rise over a period of 5 hr. Charge at a low rate a few hours longer.

The **ampere-hour efficiency** of lead batteries is 90 to 95 percent. The **watthour efficiency** obtained from full charge to discharge at the normal rate and at rated amp-hour is 75 to 85 percent. Batteries which do regulating duty only may have a watthour efficiency as high as 95 percent.

The **Edison storage cell** has a positive plate of nickel pencils filled with green nickel oxide and a negative plate of flat nickel-plated steel stampings containing iron in finely divided form. The active material for the positive plate is nickel hydrate and for the negative plate iron oxide. The electrolyte is a 21 percent solution of potassium hydrate with a little lithium hydrate. The initial emf is about 1.5 volts and the average emf about 1.1 volts throughout discharge. On account of the high internal resistance of the cell the battery is not efficient from the energy standpoint, 60 percent being the efficiency usually attained in practice. The jar is welded nickel-plated steel. The battery is compact and extremely light and strong and, for these reasons, is particularly adapted for propelling electric vehicles and for train-lighting systems.

Precautions in the **care of storage batteries**: An ammeter should not be connected directly across the terminals to test the condition of a cell; a battery should not be left to stand in a discharged condition; a flame should not be brought in the vicinity of a

The energy stored in the dielectric field during each half-cycle is returned to the source during the same half-cycle.

Impedance opposes the flow of alternating current and is expressed in ohms. It is denoted by Z. With resistance and inductance in series

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2} \text{ ohms} \tag{49}$$

With resistance and capacitance in series

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + [1/(2\pi fC)]^2} \text{ ohms} \tag{50}$$

With resistance, inductance, and capacitance in series

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + [2\pi fL - 1/(2\pi fC)]^2} \text{ ohms} \tag{51}$$

The current

$$I = E/\sqrt{R^2 + [2\pi fL - 1/(2\pi fC)]^2} \text{ amp} \tag{52}$$

Vector Representation. Sine waves of voltage and current can be represented by vectors, these vectors being proportional in magnitude to the waves that they represent. The angle between two vectors is also equal to the angle existing between the two waves that they represent.

Vectors may be combined as forces are combined in mechanics. Both graphical methods and the methods of complex algebra are used. Impedances and also admittances may be similarly combined, either graphically or symbolically. The usual method is to resolve the impedances into their component resistances and reactances, then combine all resistances and all reactances, from which the resultant impedance is obtained. Thus $Z_1 + Z_2 = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}$, where r_1 and x_1 are the components of Z_1, etc.

Phase Difference. With resistance only in circuit the current and the voltage are in phase with each other; with inductance only in circuit the current lags the voltage by 90 deg; with capacitance only in circuit the current leads the voltage by 90 deg.

With resistance and inductance in series the voltage leads the current by angle θ where $\tan\theta = X_L/R$. With resistance and capacitance in series the voltage lags the current by angle θ where $\tan\theta = -X_C/R$.

With resistance, inductance, and capacitance in series the voltage may lag, lead, or be in phase with the current.

$$\operatorname{Tan}\theta = (X_L - X_c)/R = (2\pi fL - 1/2\pi fC)/R \tag{53}$$

If $X_L > X_c$, the voltage leads; if $X_L < X_c$, the voltage lags; if $X_L = X_c$, the current and voltage are in phase.

Power Factor. In a-c circuits the power $P = I^2R$ where I is the current and R the effective resistance (p. 1711). Also the power

$$P = EI\cos\theta \text{ watts} \tag{54}$$

where θ is the phase angle between E and I. Cos θ is the **power factor** (p.f.) of the circuit. It can never exceed unity and is usually less than unity.

$$\cos\theta = P/EI \tag{55}$$

P is often called the true power. The product EI is the volt-amp and is often called the apparent power.

Active or **energy current** is the projection of the total current on the voltage vector. $I_e = I\cos\theta$. Power $= EI_e$.

battery that is being charged; the battery should not be allowed to become heated when charging; water should never be added to the concentrated acid—always acid to the water; acid should never be equalized except when the battery is in a charged condition; a battery should never be exposed to the influence of external heat; voltmeter tests should be made when the current is flowing; batteries should always be kept clean. To replace acid lost through slopping, use a solution of 2 parts concentrated sulphuric acid in 5 parts water by weight, unless a hydrometer is at hand to enable the solution to be made up according to the specifications of the makers of the cell.

MAGNETISM

The Magnetic Circuit. The magnetic circuit is analogous to the electric circuit in that the flux Φ is proportional to the magnetomotive force ($\mathfrak{F}$) and inversely proportional to the reluctance ($\mathfrak{R}$) or magnetic resistance. Thus $\Phi = \mathfrak{F}/\mathfrak{R}$ (19). [Compare with Eq. (9), p. 1693.] Φ is in maxwells, $\mathfrak{F}$ in gilberts, and $\mathfrak{R}$ in cgs reluctance units.

Table 8. Magnetic Units

Symbol	*Quantity*	*Equation*[a]	*Practical unit*
m	Pole strength	$\mathfrak{F}/\mathfrak{R}$	
H	Field intensity	$H = F/m$	Oersted or dynes per cm
Φ, ϕ	Flux	$\Phi = \mu HA$	Maxwell
B	Flux density	$B = \Phi/AI$	Gauss
H	Magnetizing force	$H = 4\pi nI/10L$	Gilberts per cm
$\mathfrak{F}$	Magnetomotive force	$\mathfrak{F} = 4\pi nI/10$	Gilberts
ν	Reluctivity	$\nu = 1/\mu$	
$\mathfrak{R}$	Reluctance	$\mathfrak{R} = \nu L/A = \mathfrak{F}/\Phi$	
μ	Permeability	$\mu = \mathfrak{B}/H$	
$\mathcal{P}$	Permeance	$\mathcal{P} = 1/\mathfrak{R}$	

[a] L = length; A = sectional area; F = force, dynes; n = number of turns.

The magnetic path is large in cross section as compared with its length; magnetic paths are usually irregular and geometrically indeterminate as in air gaps having slots and teeth on one or both sides of the gap. Hence the reluctance can only be approximated. Magnetic flux cannot be confined to definite magnetic paths, but a considerable proportion usually takes paths external to the circuit giving magnetic leakage (see Fig. 10). The permeability of iron varies over wide ranges with the flux density and with the previous magnetic condition. These variations of permeability cannot be expressed by any simple equation. The foregoing factors prevent the obtaining of the high accuracy in magnetic calculations that is obtainable in electrical calculations, but with experience it is possible to design magnetic circuits with a very fair degree of accuracy.

The **magnetomotive force** $\mathfrak{F}$ in Eq. (19) is expressed in gilberts $= 0.4\pi nI$ where n is the number of turns linked with the circuit and I is the current in amp. The unit of reluctance is the reluctance of 1 cm cube of air. The total reluctance is proportional to the length and inversely proportional to the cross-sectional area of the magnetic circuit, which is analogous to electrical resistance. Hence the reluctance of any given path of uniform cross section A is $l/A\mu$ where l is the length of the path in cm, A its cross section in sq cm, and μ is the permeability. Reluctances in series are added to obtain their combined reluctance. Ohm's law of the magnetic circuit becomes

$$\Phi = \frac{0.4\pi nI}{l_1/A_1\mu_1 + l_2/A_2\mu_2 + l_3/A_3\mu_3 \ldots} \qquad (20)$$

Reactive, quadrature, or wattless current $I_w = I \sin \theta$ and is the component of the current that contributes no power but increases the I^2R losses of the system. In power systems it should be made as low as possible or eliminated entirely.

The vars (volt-amp reactive) are equal to the product of the voltage and reactive current. Vars $= EI_w$. Kilovars $= EI_w/1{,}000$.

Effective Resistance. When alternating current flows in a circuit, the losses are ordinarily greater than are given by the losses in the ohmic resistance alone. For example, alternating current tends to flow near the surface of conductors (skin effect). If iron is associated with the circuit, eddy current and hysteresis losses result. These power losses may be accounted for by increasing the ohmic resistance to a value R, where R is the effective resistance. $R = P/I^2$. Since the iron losses vary as $I^{1.6}$ to I^2, little error results from this assumption.

Solution of Series-circuit Problem. Let a resistance R of 10 ohms, an inductance L of 0.06 henry, and a capacitance C of 60×10^{-6} farad be connected in series across 120-volt 60-cycle mains (Fig. 21). Determine: (1) the impedance; (2) the current; (3) the voltage across the resistance, the inductance, the capacitance; (4) the power factor; (5) the power; (6) the angle of phase difference.

(1) $\omega = 2\pi 60 = 377$. $X_L = 0.06 \times 377 = 22.6\Omega$; $X_c = 1/(377 \times 0.000060) = 44.2\Omega$; $Z = \sqrt{(10)^2 + (22.6 - 44.2)^2} = 23.8\Omega$; (2) $I = 120/23.8 = 5.04$ amp.; (3) $E_R = IR = 5.04 \times 10 = 50.4$ volts; $E_L = IX_L = 5.04 \times 22.6 = 114.0$ volts; $E_c = IX_c = 5.04 \times 44.2 = 223$ volts; (4) $\tan \theta = (X_L - X_c)/R = -21.6/10 = -2.16$, $\theta = -65.2°$, $\cos = \text{p.f.} = 0.420$; (5) $P = 120 \times 5.04 \times 0.420 = 254$ watts; $P = I^2R = (5.04)^2 10 = 254$ watts (check); (6) From (4) $\theta = -65.2°$. Voltage lags. The vector diagram to scale of this circuit is shown in Fig. 22. Since the current is common for all elements of the circuit, it is laid horizontally along the axis of reference.

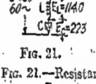


FIG. 21.

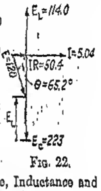


FIG. 22.

FIG. 21.—Resistance, Inductance and Capacitance in Series.

FIG. 22.—Vector Diagram for Series Circuit.

Resonance. If the voltage E and the resistance R [Eq. (52)] are fixed, the maximum value of current occurs when $2\pi fL - 1/2\pi fC = 0$. The circuit so far as its terminals are concerned behaves like a non-inductive resistance. The current $I = E/R$, the power $P = EI$, and the power factor is unity.

The voltage across the inductance and the voltage across the capacitance are opposite and equal and may be many times greater than the circuit voltage. The frequency

$$f = 1/(2\pi\sqrt{LC}) \text{ cycles} \tag{56}$$

is the natural frequency of the circuit and is the frequency at which it will oscillate if the circuit is not acted upon by some external frequency (p. 1984). This is the principle of radio sending and receiving circuits. Resonant conditions of this type should be avoided in power circuits, as the piling up of voltage may endanger apparatus and insulation.

Example. For what value of the inductance in the circuit (Fig. 21) will the circuit be in resonance, and what is the voltage across the inductance and capacitance under these conditions?

From Eq. (56) $L = 1/(2\pi f)^2C = 0.1173$ henry. $I = E/R = 120/10 = 12$ amp. $L\omega I = I/C\omega = 0.1173 \times 377 \times 12 = 530$ volts. This voltage is over four times the line voltage.

where l_1, A_1, μ_1, etc., are the lengths, cross sections, and permeabilities of each series part of the circuit.

Magnetization and Permeability Curves. The magnetic permeability of air is a constant and is taken as unity. The permeability of iron and other magnetic substances varies with the flux density. A curve of permeability as a function of flux density for cast steel is shown in Fig. 8. No satisfactory equation has been found to express the relation between magnetizing force and flux density and between permeability and flux density. Hence, if an attempt is made to solve Eq. (20) for flux, the factors μ_1, μ_2, etc., are unknown since they are functions of the flux density, which is being determined. The simplest method is one of trial and error, i.e., a value of flux, and the corresponding permeability, is first assumed; the equation solved for the flux, and if the computed flux differs widely from the assumed flux, a second approximation is made, etc. Fortunately, in nearly all magnetic designs either the flux or flux density is the independent variable, and it is required to find the necessary amp-turns to produce them. Let the flux $\Phi = BA$ where B is the flux density. Then

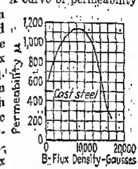


FIG. 8.—Permeability Characteristic of Cast Steel.

$$\Phi = BA = 0.4\pi nI/(l/A\mu)$$

and

$$nI = Bl/0.4\pi\mu = 0.796Bl/\mu \tag{21}$$

Eq. (21) shows that the necessary amp-turns are proportional to the *flux density* and the length of path and are inversely proportional to the permeability.

With air and non-magnetic substances μ [Eq. (21)] becomes unity and

$$nI = 0.796Bl \tag{22}$$

in centimeter units. With inch units

$$nI = 0.313B'l' \tag{23}$$

where B' is the flux density in lines per sq in. and l' the length of the magnetic path in inches.

Example. The average flux density in the air gap of a generator is 40,000 lines per sq in., and the effective length of the gap is 0.2 in. How many ampere-turns per pole are necessary for the gap?

$$nI = 0.313 \times 40{,}000 \times 0.2 = 2{,}500$$

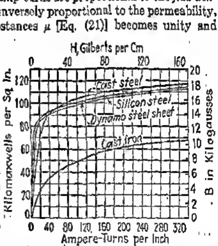


FIG. 9.—Typical Magnetization Curves.

Since the relation of μ to flux density in Eq. (21) is not simple, the relation of ampere-turns per unit length of magnetic circuit to flux density is ordinarily shown graphically. Typical curves of this character are shown in Fig. 9, inch units being used although scales of kilogausses and gilberts per cm are also given. To find the kilolines per sq cm (kilogausses), divide the ordinate scale by 6.45. To determine the number of ampere-turns necessary to produce a given total flux in a magnetic circuit composed of several parts in series having

Parallel Circuits

Parallel circuits are used for nearly all power distribution. With several series circuits in parallel it is merely necessary to find the current in each and add all the currents vectorially to find the total current. Parallel circuits may be solved analytically.

A series circuit has a resistance r_1 and an **inductive reactance** x_1. The **conductance**

$$g_1 = r_1/(r_1^2 + x_1^2) = r_1/Z_1^2 \text{ mhos} \tag{57}$$

the **susceptance**

$$b_1 = x_1/(r_1^2 + x_1^2) = x_1/Z_1^2 \text{ mhos} \tag{58}$$

Conductance is not the reciprocal of resistance; susceptance is not the reciprocal of reactance. With inductive reactance the susceptance is negative; with capacitive reactance the susceptance is positive.

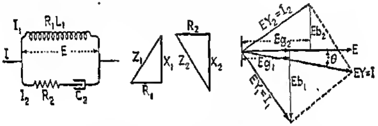


FIG. 23.—Parallel Circuit and Vector Diagram.

If a second circuit has a resistance r_2 and a **capacitive reactance** x_2 in series, $g_2 = r_2/(r_2^2 + x_2^2) = r_2/Z_2^2$; $b_2 = x_2/(r_2^2 + x_2^2) = x_2/Z_2^2$. The total conductance $G = g_1 + g_2$; the total susceptance $B = -b_1 + b_2$. The **admittance**

$$Y = \sqrt{G^2 + B^2} = 1/Z \text{ mhos} \tag{59}$$

The energy current is EG; the reactive current is EB; the power

$$P = E^2G \text{ watts} \tag{60}$$

The power factor

$$\text{p.f.} = G/Y \tag{61}$$

Also the following relations hold:

$$r = g/(g^2 + b^2) = g/Y^2 \text{ ohms} \tag{62}$$

$$x = b/(g^2 + b^2) = b/Y^2 \text{ ohms} \tag{63}$$

Solution of a Parallel-circuit Problem. In the parallel circuit of Fig. 23 it is desired to find the joint impedance, the total current, the power in each branch, the total power and the power factor, when $E = 100$, $f = 60$, $R_1 = 2$ ohms, $R_2 = 4$ ohms, $L_1 = 0.00795$ henry, $X_1 = 2\pi fL_1 = 3$ ohms, $C_2 = 1{,}326 \times 10^{-6}$ farad, $X_2 = 1/2\pi fC_2 = 2$ ohms, $Z_1 = \sqrt{2^2 + 3^2} = 3.6$ ohms, and $Y_1 = 1/3.6 = 0.278$ mho. Solution: $g_1 = R_1/(R_1^2 + X_1^2) = 2/13 = 0.154$; $b_1 = -3/13 = -0.231$; $Z_2 = \sqrt{16 + 4} = 4.47$; $Y_2 = 1/4.47 = 0.224$; $g_2 = R_2/(R_2^2 + X_2^2) = 4/(16 + 4) = 0.2$; $b_2 = 2/20 = 0.1$ mho; $G = g_1 + g_2 = 0.154 + 0.2 = 0.354$ mho; $B = b_1 + b_2 = -0.231 + 0.1 = -0.131$ mho; $Y = \sqrt{G^2 + B^2} = \sqrt{0.354^2 + (-0.131)^2} = 0.377$ mho, and joint impedance $Z = 1/0.377 = 2.65$ ohms. Phase angle $\theta = \tan^{-1}\dfrac{-0.131}{0.354} = -20.3°$. $I = EY = 100 \times 0.377 = 37.7$ amp; $P_1 = E^2g_1 = 100^2 \times 0.154 = 1{,}540$ watts; $P_2 = E^2g_2 = 100^2 \times 0.2 = 2{,}000$ watts; total power $= E^2G = 100^2 \times 0.354 = 3{,}540$ watts. Power factor $= \cos\theta = \dfrac{3{,}540}{100 \times 37.7} = 93.8$ percent.

various lengths, cross sections, and permeabilities, determine the flux density if the cross section is fixed, or otherwise choose a cross section to give a suitable flux density. From the magnetization curve obtain the ampere-turns necessary to drive this *flux density* through a unit length of the portion of the circuit considered and multiply by the length. Add together the ampere-turns required for each series part of the magnetic circuit to obtain the total ampere-turns necessary to give the assumed flux.

It is desirable to operate magnetic circuits at as high flux densities as is practicable in order to reduce the amount of iron and copper. The air gaps of dynamos are operated at average densities of 40,000 to 50,000 lines per sq in. Higher densities increase the exciting ampere-turns and tooth losses. At 45,000 lines per sq in. the flux density in the teeth may be as high as 120,000 to 130,000 lines per sq in. The flux densities in transformer cores are limited as a rule by the permissible losses. At 60 cycles and with silicon steel the maximum density is 60,000 to 70,000 lines per sq in.; at 25 cycles the density may run as high as 70,000 to 80,000 lines per sq in. With laminated cores, the *net* iron is approximately 0.9 the gross cross section.

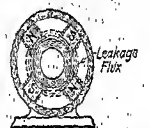


FIG. 10.—Magnetic Circuit of 4-pole Dynamo Showing Leakage Flux.

Magnetic Leakage. It is impossible to confine all magnetic lines to any desired path since there is no known insulator of magnetic lines. Figure 10 shows the magnetic circuit of a modern four-pole dynamo. A considerable proportion of the useful magnetic flux leaks between the pole shoes and cores, rather than across the air gap. The ratio of the maximum flux, which exists in the field cores, to the useful flux (*i.e.*, the flux that crosses the air gap) is the **coefficient of leakage.** This coefficient must always be greater than unity and in carefully designed dynamos may be as low as 1.15. It is frequently as high as 1.30. Although the geometry of the leakage-flux paths is not simple, the leakage flux may be determined by approximations with a fair degree of accuracy.

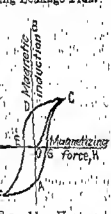


FIG. 11.—Hysteresis Loop.

Magnetic Hysteresis. The magnetization curves shown in Fig. 9 are called **normal curves.** They are taken with the magnetizing force continuously increased from zero. If at any point the magnetizing force be decreased, a greater value of flux density for any given magnetizing force will result. The effect of carrying iron through a complete cycle of magnetization, both positive and negative, is shown in Fig. 11. The resulting curve is called a **hysteresis loop.** It is seen that the flux density lags the magnetizing force. When the positive magnetizing force H returns to zero at O, the induction OD remains. This is called the **remanence.** In order to reduce the induction to zero a negative magnetizing force OE is necessary. OE is the **coercive force.** Permanent magnets operate on the portion DE of the loop, and the area ODE is a criterion of permanent-magnet material. The energy dissipated per cycle is proportional to the area of the loop and is equal to $\frac{1}{4\pi}\int HdB$ ergs per cycle. For moderately high densities the energy loss per cycle varies

With parallel circuits, unity power factor is obtained when the algebraic sum of the quadrature currents is zero. That is, $b_1 + b_2 + b_3 \cdots = 0$.

Three-phase Circuits. Alternating-current generators are usually wound with three armature circuits which are spaced 120 electrical degrees apart on the armature. Hence these coils generate emfs 120 electrical degrees apart. The coils are connected either in Y (star) or in Δ (mesh) as shown in Fig. 24. Whether Y- or Δ-connected, with a balanced load, the three coil emfs E_c and the three coil currents I_c are equal. In the Y connection the line and coil currents are equal, but the line emfs AB, BC, CA are $\sqrt{3}$ times the coil emfs OA, OB, OC, since each is the vector sum of two coil emfs. In the delta connection the line and coil emfs are equal, but I, the line current, is $\sqrt{3}I_c$, the coil current, *i.e.*, it is the vector sum of the currents in the two coils connected to the line. The power of a coil is $E_cI_c \cos \theta$, so that the total power is $3E_cI_c \cos \theta$. If θ is the angle between *coil* current and *coil* voltage, the angle between line current and line voltage will be $(30° \pm \theta)$. In terms of line current and emf, the power is $\sqrt{3}EI \cos \theta$. A fourth or neutral conductor connected to O is sometimes used with the Y connection. The neutral point O is frequently grounded in transmission and distribution circuits. The coil emfs are assumed to be sine waves. Under these conditions they balance, so that in the delta connection the sum of the two coil emfs at each instant is balanced by the third coil emf. Even though the third, ninth, fifteenth . . . . harmonics, $3(2n + 1)f$, where $n = 0$ or an integer, exist in the coil emfs, they cannot appear on the external line of the three-phase Y-connected circuit, except when the neutral conductor is used. In the delta circuit, the same harmonics $3(2n + 1)f$ cause a local current to circulate around the mesh. This may cause a very appreciable heating. In a three-phase system the power

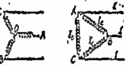


FIG. 24.—Three-phase Connections.

$$P = \sqrt{3}EI \cos \theta \tag{64}$$

the power factor is

$$P/\sqrt{3}EI \tag{65}$$

and the kilovolt-amperes

$$\sqrt{3}EI/1000 \tag{66}$$

Two-phase Circuits. Two-phase generators have two windings spaced 90 electrical degrees apart on the armature. These windings generate emfs differing in time phase by 90 deg. The two windings may be independent and power transmitted to the receiver though the two single-phase circuits are entirely insulated from each other. The two circuits may be combined into a two-phase three-wire circuit such as is shown in Fig. 25, where OA and OB are the generator circuits and $A'O'$ and $B'O'$ are the receiver circuits. The wire OO' is the common wire and under balanced conditions carries a current $\sqrt{2}$ times the current in wires AA' and BB'. For example, if I_c is the coil current, $\sqrt{2}I_c$ will be the value of the current in the common conductor OO'. If E_c be the voltage across OA or OB, $\sqrt{2}E_c$ will be the voltage across AB. The power of a two-phase circuit is twice the power in either coil if the load is balanced. Normally, the voltages OA and OB are equal, and the current is the same in both coils. Owing to non-symmetry and the high degree of

according to the **Steinmetz law**

$$W = \eta B^{1.6} \text{ ergs} \tag{24}$$

Table 9 gives values of η for common magnetic steels.

Table 9. Steinmetz Coefficient

Hard tungsten steel	0.058	Annealed cast steel	0.008
Hard cast steel	0.025	Ordinary sheet iron	0.004
Forged steel	0.020	Pure iron	0.003
Cast iron	0.013	Annealed iron sheet	0.002
Electrolytic iron	0.009	Best annealed sheet	0.001
Soft machine steel	0.009	Silicon steel sheet	0.001

A permanent increase in the hysteresis constant occurs if the temperature of operation remains for some time above 80 C. This phenomenon is known as **aging** and may be much reduced by proper annealing of the iron. Silicon steels containing about 3 percent silicon have a lower hysteresis loss, somewhat larger eddy-current loss, and are practically non-aging.

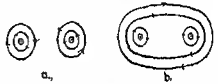


FIG. 12.—*a*. Currents in Opposite Directions. *b*. Currents in Same Direction.

Eddy-current losses, also known as Foucault-current losses, occur in iron subjected to cyclic magnetization. Eddy-current losses are reduced by laminating the iron, which subdivides the emf and increases greatly the length of path of the parasitic currents. Eddy currents have also a screening effect, which tends to prevent the flux penetrating the iron. Hence laminating also allows the full cross section of the iron to be utilized unless the frequency is too high.

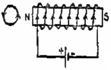


FIG. 13.—Directions of Current and Poles in Solenoid.

Relation of Direction of Magnetic Flux to Current Flow. The direction of the magnetizing force of a current is at right angles to its direction of flow. Magnetic lines about a cylindrical conductor carrying current exist in circular planes concentric with and normal to the conductor. This is illustrated in Fig 12*a*. The ⊕ sign, corresponding to the feathered end of an arrow, indicates current flowing away from the observer; a ⊙ sign, corresponding to the tip of an arrow, indicates current flowing toward the observer.

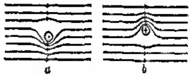


FIG. 14.—Effect of Current on Uniform Magnetic Field.

Corkscrew Rule. The direction of the current and that of the resulting magnetic field are related to each other as the forward travel of a corkscrew and the direction in which it is rotated. **Hand Rule.** Grasp the conductor in the right hand with the thumb pointing in the direction of the current. The fingers will then point in the direction of the lines of flux.

The applications of these rules are illustrated in Figs. 12*a* and 12*b*. If the currents in parallel conductors flow in opposite directions (Fig. 12*a*), the conductors tend to move apart; if the currents in parallel conductors flow in the same directions (Fig. 12*b*), the conductors tend to come together. The magnetic lines act like stretched rubber bands and, in attempting to contract, tend to pull the two conductors together.

unbalancing of this system even under balanced loads, it is not used at the present time for transmission and is little used for distribution.

Quarter-phase Circuit. A quarter-phase circuit is shown in Fig. 26. The windings AC and BD may be independent or connected at O. The voltages AC and BD are 90 deg apart as in two-phase circuits. If a neutral wire OO' be added, three different voltages may be obtained: Let E_1 = voltage $OA = OB = OC = OD$. Voltages AB, BC, CD, $DA = \sqrt{2}E_1$. Voltage $AC = BD = 2E_1$. Because of this multiplicity of voltages and the fact that polyphase power apparatus and lamps may be connected at the same time, this system is still used to some extent in distribution.

FIG. 25.—Two-phase Three-wire Circuit.

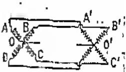


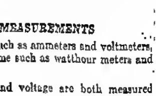
FIG. 26.—Quarter-phase Circuit.

Advantages of Polyphase Power. The advantages of polyphase power over single-phase power are as follows: The output of alternators and most other rotating machinery is from 60 to 90 percent greater when operated polyphase than when operated single phase; pulsating fluxes and corresponding iron losses which occur in many common types of machinery when operated single phase are negligible when operated polyphase; with balanced polyphase loads polyphase power is constant whereas with single phase the power fluctuates over wide limits during the cycle. Because of its minimum number of wires and the fact that it is not easily unbalanced, the three-phase system has for the most part superseded the other polyphase system. (See p. 1739. for polyphase connections of transformers.)

ELECTRICAL INSTRUMENTS AND MEASUREMENTS

Electrical measuring devices that merely indicate, such as ammeters and voltmeters, are called **instruments**; devices that totalize with time such as watthour meters and ampere-hour meters are called **meters**.

Direct-current Instruments. Direct current and voltage are both measured with an indicating instrument based on the principle of the D'Arsonval galvanometer. A coil with steel pivots and turning in jewel bearings is mounted in a magnetic field produced by permanent magnets. The motion is restrained by two small flat coiled springs which also serve to conduct the current to the coil. *The deflections of the coil are read with a* light aluminum pointer attached to the coil and moving over a graduated scale. The same instrument may be used either for current or voltage, but the method of connecting in circuit is different in the two cases. Usually, however, the coil of an instrument to be used as an ammeter is wound with fewer turns of coarser wire than an instrument to be used as a voltmeter and so has lower resistance. The instrument itself is frequently called a **millivoltmeter**. It cannot be used alone to measure voltage of any magnitude since its resistance is so low that it would be burned out if connected across the line. Hence a resistance r' in series with the coil is necessary as indicated in Fig. 27(a) in which r_c is the resistance of the coil. From 0.2 to 750 volts this resistance is usually within the instrument. For higher voltages an external resistance R called an extension coil or multiplier [Fig. 27(b)] is necessary. Let e be the reading of the instrument in volts [Fig. 27(b)], r the resistance of the instrument, R the resistance of the multiplier. Then the total voltage is



(a) Internal resistance (b) With multiplier

FIG. 27.—Voltmeter.

FIG. 28.—Millivoltmeter with Shunt.

The relation of the direction of current in a solenoid helix to the direction of flux is shown in Fig. 13. Figure 14 shows the effect on a uniform field of placing a conductor carrying current in that field and normal to it. In *a* the current flows toward the observer. By applying the corkscrew rule it is seen that the current weakens the field immediately above it and strengthens the field immediately below it. The reverse is true in *b* where the current flows away from the observer.

Figure 14 is illustrative of the force developed on a conductor carrying current in a magnetic field. In *a* the conductor will tend to move upward owing to the stretching of the magnetic lines beneath it. Similarly, the conductor in *b* will tend to move downward. This principle is the basis of motor action (p. 1724).

(For magnets and solenoids see p. 1767.)

DIELECTRIC CIRCUIT

Dynamic and Static Electricity. Electricity in motion such as an electric current is dynamic electricity; electricity at rest is static electricity. The two are identical physically. Since static electricity is frequently produced at high voltage and small quantity, the two are frequently considered as being two different types of electricity.

Capacitors or Condensers

Capacitors (formerly condensers). Two conducting bodies separated by insulation constitute a capacitor. If a positive charge is placed on one plate of a capacitor, an equal negative charge is induced on the other. The medium between the capacitor plates is called a **dielectric**. The dielectric properties of a medium relate to its ability to conduct *dielectric lines*. This is in distinction to its **insulating** properties which relate to its property to conduct *electric current*. For example, air is an excellent insulator but ruptures dielectrically at low voltage. It is not a good dielectric so far as breakdown strength is concerned.

With capacitors

$$Q = CE \quad (25); \qquad C = Q/E \quad (26); \qquad E = Q/C \quad (27)$$

where Q is the quantity in coulombs, C the capacitance in farads, and E the voltage. The unit of capacitance in the practical system is the *farad*. The farad is too large a unit for practical purposes, so that the *microfarad* is used. However, in voltage, current, and energy relations the capacitance must be expressed in farads.

The energy stored in a capacitor

$$W = \tfrac{1}{2}QE = \tfrac{1}{2}CE^2 = \tfrac{1}{2}Q^2/C \text{ joules} \qquad (28)$$

Capacitance of Capacitors. The capacitance of a parallel-plate capacitor (Fig. 15) is

$$C = \kappa A/(4\pi d \times 9 \times 10^5) \text{ microfarads} \qquad (29)$$

where κ is the dielectric constant, A is the area of one plate, sq cm; and d the distance between plates, cm.

The capacitance of **concentric cylindrical** capacitors (Fig. 16) is

$$C = 0.2171\kappa l/[9 \times 10^5 \log_{10} (R_2/R_1)] \text{ microfarads} \qquad (30)$$

where κ is the dielectric constant and l the length, cm. Also

$$C = 0.03882\kappa/\log_{10} (R_2/R_1) \text{ microfarads per mile} \qquad (31)$$

Equation (31) is useful in that it is applicable to cables.

$$E = e(R + r)/r \qquad (67)$$

It is obvious that by using suitable values of R a voltmeter may be made to have several scales.

Instruments themselves can only carry currents of the magnitudes of 0.01 to 0.06 amp. To measure larger values of current the instrument is provided with a shunt R (Fig. 28). The current divides inversely as the resistances r and R of the instrument and the shunt. A low resistance r' within the instrument is connected in series with the coil. This permits some adjustment of the deflection so that the instrument can be adapted to its shunt. In most cases most of the current flows through the shunt, and the current in the instrument is negligible in comparison. Up to 50 and 75 amp the shunt may be incorporated within the instrument. For larger currents it is usually necessary to have the shunt externa. to the instrument and connect the instrument to the potential terminals of the shunt by means of leads. Any given instrument may have any number of ranges by providing it with a sufficient number of shunts. The range of the usual instrument of this type is approximately 50 millivolts. Although the same instrument may be used for voltmeter or ammeter, the moving coils of voltmeters are usually wound with more turns of finer wire. They take approximately 0.01 amp so that their resistance is approximately 100 ohms per volt. Instruments used as ammeters alone operate with 0.04 to 0.06 amp.

Permanent-magnet moving-coil instruments may be used to measure unidirectional pulsating currents or voltages and in such cases will indicate the average value of the periodically varying current or voltage.

Alternating-current Instruments. Instruments for alternating currents may be divided into four types: electrodynamometer, electromagnetic, hot-wire, and electrostatic. Instruments of the **electrodynamometer** type (the most precise) operate on the principle of one coil carrying current, turning in the magnetic field produced by a second coil carrying current taken from the same circuit. If these circuits or coils are connected in series, the torque exerted on the moving system for a given relative position of the coil system is proportional to the square of the current strength and is not dependent on the direction of the current. Consequently, the instrument will have a compressed scale at the lower end and will usually have only the upper two-thirds of the scale range useful for accurate measurement. Instruments of this type ordinarily require 0.04 to 0.08 amp or more in the moving-coil circuit for full-scale deflection. They read the effective or root-mean-square value of the alternating or pulsating current. The wattmeter operates on the electrodynamometer principle. The fixed coil, however, is energized by the current of the circuit, and the moving coil is connected across the potential in series with high resistance. Unless shielded magnetically the foregoing instruments will not, in general, indicate as accurately on direct current as on alternating current because of the effects of external stray magnetic fields. Also reversed readings should be taken. **Electromagnetic** or **soft-iron** instruments consist of a fixed coil which actuates magnetically a light movable iron vane or core; they are rugged, inexpensive, and may be had in ranges of 30 to 750 volts and 0.05 to 100 amp. They measure effective values and have compressed scales as in the case of electrodynamometer instruments. Instruments of the **hot-wire** type depend on the expansion of a wire which is heated by the current or part of the current to be measured. They are used principally in radio work where it is necessary to measure currents of high frequencies. Electrostatic instruments which depend for indication on the attraction of oppositely charged bodies are used occasionally and for high-voltage measurements but have only a very limited field of application, being relatively more expensive and less accurate than the preceding types. The foregoing instruments may all be used with direct current but are less accurate than the permanent-magnet type of instrument.

Alternating-current instruments of the **induction** type (Westinghouse Co.) must be used on a-c circuits of the frequency for which they have been designed. They are rugged and relatively inexpensive and are used principally for switchboards where a long-scale range and a strong deflecting torque are of particular advantage.

Power Measurement in Single-phase Circuits. Wattmeters are not rated primarily in watts, but in amperes and volts. For example, with low power factor the current and voltage coils may be overloaded and yet the needle be well on the scale. The current coil may be carrying several times its rated current, and yet the instrument read zero because the potential circuit is not closed, etc. Hence it is desirable to

The capacitance of two **parallel cylindrical conductors** D cm between centers and having radii of r cm is

$$C = 0.01941/\log_{10}(D/r) \text{ microfarads per mile} \quad (32)$$

In practice, the capacitance to neutral or to an infinite conducting plane midway between the conductors and perpendicular to their plane is usually

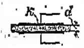


FIG. 15.—Parallel-plate Capacitor.

FIG. 16.—Concentric-cylinder Capacitor.

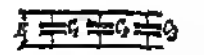


FIG. 17.—Capacitances in Parallel.

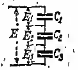


FIG. 18.—Capacitances in Series.

used. The capacitance to neutral

$$C = 0.03882/\log_{10}(D/r) \text{ microfarads per mile} \quad (33)$$

Table 10. Insulating and Dielectric Properties of Insulating Materials

	Insulation resistance, megohm cm cube	Dielectric constant	Rupturing strength	
			Volts per mil	Kilovolts per cm
Asbestos board (ebonized)	1.0×10^7	....	55	22
Bakelite	$5\text{-}30 \times 10^{11}$	4.5-5.5	450-1,400	180-550
Ebonite	$10^9\text{-}10^{12}$	1.9-3.5	1,000-2,000	390-780
Empire cloth	....	3.5-5.5	400-1,100	160-430
Empire paper	....	....	1,140	450
Fiber	5×10^3	....	50-200	20-80
Fuller board	11×10^9	....	120-440	47-170
Glass	17×10^9	....	760-3,800	300-1,500
Flint	....	6.61-9.90		
Jena-boron	....	5.5-8.1		
Gutta percha	25×10^8	2.9-4.9	200-510	80-200
Linen, varnished (see Empire cloth)	....	....		
Marble	$10^2\text{-}10^4$	8.3	50-100	20-39
Mica				
India	10^4	7.07-7.90	4,000	1,580
Canada	$0.44\text{-}22 \times 10^4$	2.9-3.0	1,270-3,800	500-1,500
So. America	39×10^6	5.9	3,800	1,500
Mica segment plate	....	....	900-1,200	350-470
Mica high-heat plate	....	....	1,000	390
Oils				
Castor	6.6×10^4	4.7	330-480	130-190
Cottonseed	10^4	3.2	300-400	120-160
Lard	....	....	102-355	40-140
Linseed	6×10^5	3.3	300-400	120-160
Mineral	21×10^{10}	2.0-2.2	300-400	120-160
Paraffin	$1{,}000 \times 10^{12}$	2.41	410-550	160-215
Paper	....	1.7-2.6	110-230	43-90
Paper, treated	....	2.5-4.0	500-750	20-300
Paraffin, solid	1×10^{10}	1.9-2.3	300	120
Porcelain	3×10^8	5.7-6.8	240-300	95-120
Rubber (vulcanized)	$10^{14}\text{-}10^{16}$	2.0-3		
Rubber (compounds)	$10^{14}\text{-}10^{16}$	2.5-6	300-500	120-200
Slate	$10^2\text{-}10^4$	6-7	5-10	2-3.9
Transil oil	....	2.4-2.6	300	120
Vaseline	....	2.16-2.2	230-330	90-130

use both an ammeter and a voltmeter in conjunction with a wattmeter when measuring power [Fig. 29(a)]. The instruments themselves consume appreciable power, and correction is oftentimes necessary unless these losses are negligible compared with the power being measured. For example, in Fig. 29(a), the wattmeter reads the I^2R loss in its own current coil (1 to 2 watts) and in the ammeter, as well as the loss in the voltmeter ($=E^2/R$ where R is the resistance of the voltmeter). The losses in the ammeter and voltmeter may be eliminated by short-circuiting the ammeter and disconnecting the voltmeter when reading the wattmeter. If the wattmeter is connected as shown in Fig. 29(b), it measures the power taken by its own potential coil (E^2/R_p) which at 110 volts is 5 to 7 watts. (R_p is the resistance of the potential circuit.) Frequently correction must be made for this power.

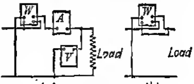


FIG. 29.—Connection of Instruments to Single-phase Load.

Power Measurement in Polyphase Circuits. Three-wattmeter Method. Let ao, bo, and co be any Y-connected three-phase load (Fig. 30). Three wattmeters with their current coils in each line and their potential circuits connected to neutral measure the total power, since the power in each load is measured by one of the wattmeters. The connection oo' may, however, be broken, and the total power is still the sum of the three readings. That is, the power $P = W_1 + W_2 + W_3$. This method is applicable to any system of n wires. The current coil of one wattmeter is connected in each of the n wires. The potential circuit of each wattmeter is connected between its own phase wire and a junction in common with all the other potential circuits. The wattmeters must be connected symmetrically, and the readings of any which read negative must be given the negative sign.

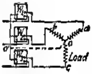


FIG. 30.—Three-wattmeter Method.

In the general case any system of n wires requires at least $n - 1$ wattmeters to measure the power correctly. The $n - 1$ wattmeters are connected in series with $n - 1$ wires. The potential circuit of each is connected between its own phase wire and the wire in which no wattmeter is connected. The application of this method to any four-wire system is shown in Fig. 31.

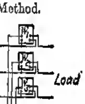


FIG. 31.—Power Measurement in n-wire System.

Three-phase Systems. The three-wattmeter method (Fig. 30) is applicable to any three-phase system. It is commonly used with the three-phase four-wire system. If the loads are balanced, $W_1 = W_2 = W_3$ and the power $P = 3W_1$. Hence W_2 and W_3 may be omitted if their potential circuits are replaced by two resistances enclosed in a Y-box. Each resistance in the Y box must be equal to the resistance of the wattmeter potential circuit.

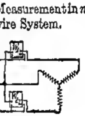


FIG. 32.—Two-wattmeter Method.

The **two-wattmeter** method is most commonly used with three-phase three-wire systems. The method of connecting the wattmeters is shown in Fig. 32. The current coils may be connected in *any* two wires, the potential circuits being connected to the third. It will be recognized that this is adapting the method of Fig. 31 to three wires. With balanced loads the readings of the wattmeters are $W_1 = EI \cos (30° + \theta)$, $W_2 = EI \cos (30° - \theta)$, and $P = W_2 \pm W_1$. θ is the angle of phase difference between coil voltage and current. Since

$$W_1/W_2 = \cos (30° + \theta)/\cos (30° - \theta) \tag{68}$$

the power factor is a function of W_1/W_2. Table 11 gives values of power factor for different ratios of W_1/W_2.

$P = W_2 + W_1$ when $\theta < 60°$.

When $\theta = 60°$, p.f. $= \cos 60° = 0.5$, $W_1 = \cos (30° + 60°) = 0$, $P = W_2$. When $\theta > 60°$, p.f. < 0.5, $P = W_2 - W_1$.

Also $$\tan \theta = \sqrt{3}(W_2 - W_1)/(W_2 + W_1) \tag{69}$$

point rigidly connected with the rolling circle (not necessarily on the circumference) will be an ellipse. If $a = \frac{1}{4}c$, the curve generated will be the four-cusped hypocycloid, or **astroid,** (Fig. 71), whose equation is $x^{2/3} + y^{2/3} = c^{2/3}$. If $a = c$, the epicycloid is the **cardioid,** whose equation in polar coordinates (axes as in Fig. 72) is $r = 2c(1 + \cos\theta)$. Length of cardioid $= 16c$.

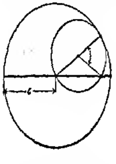
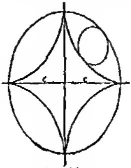
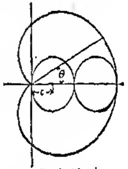


FIG. 70.



Astroid.
FIG. 71.



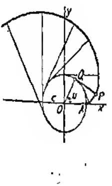
Cardioid.
FIG. 72.

The Epitrochoid (or Hypotrochoid) is a curve traced by any point rigidly attached to a circle of radius a, at distance b from the center, when this circle rolls without slipping on the outside (or inside) of a fixed circle of radius c.

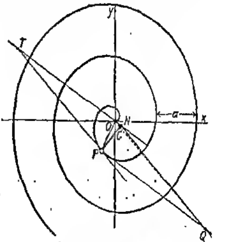
The equations are $x = (c \pm a)\cos\left(\frac{a}{c}u\right) \mp b\cos\left[\left(1 \pm \frac{a}{c}\right)u\right]$,

$y = (c \pm a)\sin\left(\frac{a}{c}u\right) - b\sin\left[\left(1 \pm \frac{a}{c}\right)u\right]$, where $u =$ the angle which the moving radius makes with the line of centers; take the upper sign for the epi- and the lower for the hypo-trochoid. The curve is called prolate or curtate according as $b < a$ or $b > a$. When $b = a$, the special case of the epi- or hypocycloid arises.

The Involute of a Circle is the curve traced by the end of a taut string which is unwound from the circumference of a fixed circle, of radius c. If QP



Involute of Circle.
FIG. 73.



Spiral of Archimedes.
FIG. 74.

is the free portion of the string at any instant (Fig. 73), QP will be tangent to the circle at Q, and the length of QP = length of arc QA; hence the construc-

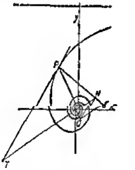
tion of the curve. The equations of the curve in parametric form (axes as in figure) are $x = c(\cos\ u + \text{rad}\ u \sin u)$, $y = c(\sin\ u - \text{rad}\ u \cos u)$, where rad u is the radian measure of the angle u which OQ makes with the x-axis. Length of arc $AP = \frac{1}{2}c(\text{rad}\ u)^2$; radius of curvature at P is QP. Polar equations, in terms of parameter v(= angle POQ), are $r = c \sec v$, rad $\theta = \tan v - \text{rad}\ v$. Here, $r = OP$, and rad θ = radian measure of angle AOP (Fig. 73).

The Spiral of Archimedes (Fig. 74) is traced by a point P which, starting from O, moves with uniform velocity along a ray OP, while the ray itself revolves with uniform angular velocity about O. Polar equation: $r = k\,\text{rad}\ \theta$, or $r = a\ (\theta/360°)$. Here $a = 2\pi k$ = the distance, measured along a radius, from each coil to the next.

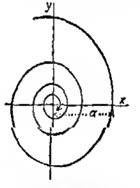
In order to construct the curve, draw radii $O1, O2, O3, \ldots$ making angles $\frac{1}{n}(360°)$, $\frac{2}{n}(360°)$, $\frac{3}{n}(360°)$, . . . with Ox, and along these radii lay off distances equal to $\frac{1}{n}a$, $\frac{2}{n}a$, $\frac{3}{n}a$, . . .; the points thus reached will lie on the spiral. The figure shows one-half of the curve, corresponding to positive values of θ.

Construction for tangent and normal: Let PT and PN be the tangent and normal at any point P, the line TON being perpendicular to OP. Then $OT = r^2/k$, and $ON = k$, where $k = a/(2\pi)$. Hence the construction.

The radius of curvature at P is $R = (k^2 + r^2)^{3/2}/(2k^2 + r^2)$. To construct the center of curvature, C, draw NQ perpendicular to PN and PQ perpendicular to OP; then OQ will meet PN in C. Length of arc $OP = \frac{1}{2}k\,[\text{rad}\ \theta\sqrt{1 + (\text{rad}\ \theta)^2} + \sinh^{-1}(\text{rad}\ \theta)]$. After many windings, arc $OP = \frac{1}{2}r^2/k$, *approximately*.



Hyperbolic Spiral.
FIG. 75.



Logarithmic Spiral.
FIG. 76.

The Hyperbolic Spiral is the curve whose polar equation is $r = a/\text{rad}\ \theta$. To construct the curve, take a series of points along Ox (Fig. 75); through each of these points, with center at O, draw an arc extending into the upper half of the plane; and along each of these arcs lay off a length $= a$. The points thus reached will lie on the curve. A line parallel to the x-axis, at distance a, is an asymptote of the curve. The curve winds around and around the point O without ever reaching it (asymptotic point). The figure shows one-half of the curve, corresponding to positive values of θ. If PT and PN are the tangent and normal at any point P, the line TON being perpendicular to OP,

Table 11. Ratio W_1/W_2 and Power Factor

W_1/W_2	Power factor	W_1/W_2	Power factor	W_1/W_2	Power factor	W_1/W_2	Power factor
+1.0	1.000	+0.4	0.804	−0.1	0.427	−0.6	0.142
+0.9	0.996	+0.3	0.732	−0.2	0.360	−0.7	0.102
+0.8	0.982	+0.2	0.656	−0.3	0.296	−0.8	0.064
+0.7	0.956	+0.1	0.576	−0.4	0.240	−0.9	0.020
+0.6	0.918	0.0	0.50	−0.5	0.188	−1.0	0.000
+0.5	0.866						

In a **polyphase wattmeter** the two single-phase wattmeter elements are combined to act on a single spindle. Hence the adding and subtracting of the individual readings are done *automatically*. *The total power* is indicated on one scale. This type of instrument is almost always used on switchboards. The connections of a portable type are shown in Fig. 33.

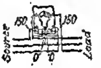


FIG. 33.—Connection for Polyphase Wattmeter in Three-phase Circuit.

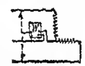


FIG. 34.—Measurement of Two-phase, Three-wire Power with Single Wattmeter.

In the foregoing instrument connections, Y-connected loads are shown. These methods are equally applicable to delta-connected loads. The two-wattmeter method (Fig. 32) is obviously adapted to the two-phase three-wire system (*Figs.* 25 *and* 34). The power in this system may also be measured with one wattmeter. Its current coil is connected in the common wire (Fig. 34), and the potential circuit is connected *first* to one outer wire and then to the other. The power is the sum of the two readings.

Measurement of Energy

Watthour meters record the energy taken by a circuit over some interval of time. Correct registration occurs if the angular velocity of the rotating element at every instant is proportional to the power. The method of accomplishing this with d-c meters is illustrated in Fig. 35. The meter is in reality a small motor. The field coils FF are in series with the line. The armature A is connected across the line, usually in series with a resistance R. The movable field coil F' is in series with the armature A and serves to compensate for friction. C is a small commutator, either of copper or of silver, and the two small brushes are usually of silver. An aluminum disk, rotating between the poles of permanent magnets M, acts as a magnetic brake the torque of which is proportional to the angular velocity of the disk. A small worm and the gears G actuate the recording dials.

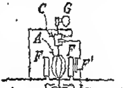


FIG. 35.—Direct-current Watthour Meter.

The following relation, or an equivalent, holds with most types of meter. With each revolution of the disk, K whr are recorded, where K is the **meter constant** found usually on the disk. It follows that the average watts W over any period of time t sec is

$$W = 3{,}600KN/t \qquad (70)$$

where N is the revolutions of the disk during that period. Hence, the meter may be calibrated by connecting standardized instruments to measure the average power taken by the load and by counting the revolutions N for t sec. Near full load, if the meter registers fast, the magnets M should be moved outward radially; if it registers slow, the magnets should be moved inwards. If the meter registers fast at light (5 to 10 percent) load, the starting coil F' should be moved further away from the armature; if it registers slow, F' should be moved nearer the armature. A meter should not register more than 1.5 percent fast or slow, and with calibrated standards it can be made to register to within 1 percent of correct.

in current will produce a much greater proportionate increase in torque (see Fig. 52). This makes the motor particularly well adapted to traction work, cranes, hoists, elevator service, and other types of work which require large starting torques. A study of Eq. (80) shows that with increase in current the numerator changes only slightly, whereas the change in the denominator is nearly proportional to the change in current. Hence the speed of the series motor is practically inversely proportional to the current. With overloads the speed drops to very low values (see Fig. 51). With decrease in load the speed approaches infinity, theoretically. Hence the series motor should always be connected to its load so that it cannot reach unsafe speeds (see Speed Control of Motors, p. 1728).

Differential Compound Motors. The cumulative compound winding of a generator becomes a differential compound winding when the machine is used as a motor. The differential motor, however, is not ordinarily used except occasionally to drive certain textile machines. Its speed may be made more nearly constant than that of a shunt motor, or, if desired, it may be adjusted to increase with increasing load.

Since the speed of the shunt motor is sufficiently constant for most purposes and the differential motor tends toward instability, particularly in starting and on overloads, the differential motor is little used.

Cumulative compound motors develop a more rapid increase in torque with load than shunt motors (Fig. 52); on the other hand, they have much poorer speed regulation (Fig. 51). Hence they are used where larger starting torque than that developed by the shunt motor is necessary, as for example with elevators. They are particularly useful where large and intermittent increases of torque occur as in drives for shears, punches, rolling mills, etc. In addition to the sudden increase in torque which the motor develops with sudden applications of load, the fact that it slows down rapidly and hence causes the rotating parts to give up some of their kinetic energy is another important advantage in that it reduces the peaks on the power plant.

Table 13. Test Performance of Compound Wound D-c Motors
(Westinghouse Electric & Mfg. Co.)

Hp	115 volts		230 volts		550 volts	
	Amp	Efficiency at full load	Amp	Efficiency at full load	Amp	Efficiency at full load
1	8.4	77.0	4.21	77.0	1.86	73.0
2	16.4	79.0	8.4	78.0	3.21	82.0
5	40.8	79.5	20.3	80.0	8.4	81.0
10	76.4	85.0	37.7	86.0	15.7	86.5
25	186.0	87.0	92.7	87.5	38.3	88.5
50	...	...	182.5	88.5	75.8	89.5
100	...	...	359.0	90.5	149.0	91.0
200	...	...	709.0	91.5	295.0	92.0

Commutation. The brushes on the commutator of either a motor or generator should be set in such a position that the induced emf in the armature coils undergoing commutation, and hence short-circuited by the brushes, is zero. In practice, this condition can at best be only approximately realized. Frequently conditions are such that it is far from being realized. At no load, the brushes should be set in a position corresponding to the

The induction watthour meter is used with alternating current. Although the d-c meter registers correctly with alternating current, it is more expensive than the induction type, the commutator and brushes may cause trouble, and at low power factors compensation is necessary. In the induction watthour meter the driving torque is developed in the aluminum disk by the joint action of the alternating magnetic flux produced by the potential circuit and by the load current. The driving torque and the retarding torque are both developed in the same aluminum disk, hence no commutator and brushes are necessary. The rotating element is very light, and hence the friction torque is small. Equation (70) applies to this type of meter. When calibrating, the average power W for t sec is determined with a calibrated wattmeter. The friction compensation is made at light loads by changing the position of a small hollow stamping with respect to the potential lug. The meter should also be adjusted at low power factor (0.5 is customary). If the meter is slow with lagging current, resistance should be cut out of the compensating circuit; if slow with leading current, resistance should be inserted.

Power-factor Measurement. The usual method of determining power factor is by the use of voltmeter, ammeter, and wattmeter. The wattmeter gives the watts of the circuit, and the product of the voltmeter reading and the ammeter reading gives the volt-amperes. The power factor is the ratio of the two [see Eqs. (55) and (65)]. Also single-phase and three-phase power-factor indicators, which can be connected directly in circuit, are on the market.

Instrument Transformers

With voltages higher than 600 volts, and even at 600 volts, it becomes dangerous and inaccurate to connect instruments and meters directly into power lines. It is also

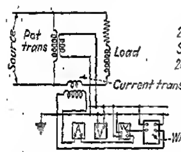


FIG. 36.—Single-phase Connections of Instruments with Transformers.

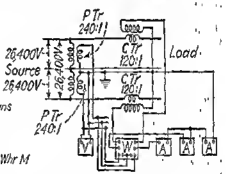


FIG. 37.—Three-phase Connections of Instruments and Instrument Transformers.

difficult to make potential instruments for voltages in excess of 600 volts and ammeters in excess of 100 amp rating. To insulate such instruments from high voltage and at the same time to permit the use of low-range instruments, instrument transformers are used. **Potential transformers** are identical to power transformers except that their volt-ampere rating is low, being 40 to 500 watts. Their primaries are wound for line voltage and their secondaries for 110 volts. **Current transformers** are designed to go in series with the line, and the rated secondary current is 5 amp. The secondary of a current transformer should always be closed when current is flowing; it should never be allowed to become open circuited under these conditions. When open circuited the voltage across the secondary becomes so high as to be dangerous and the flux becomes so large in magnitude that the transformer overheats. The secondaries of both potential and current transformers should be well grounded at one point (Figs. 36 and 37). Instrument transformers introduce slight errors because of small variations in their ratio with load. Also there is slight phase displacement in both current and potential transformers. The readings of the instruments must be multiplied by the instrument transformer ratios. The scales of switchboard instruments are usually calibrated to take these ratios into account.

geometrical neutral of the machine, for under these conditions the induced emf in the coils short-circuited by the brushes is zero. As load is applied, two factors cause sparking under the brushes. The mmf of the armature, or armature reaction, distorts the flux; when the current in the coils undergoing commutation reverses, an emf of self-induction Ldi/dt tends to prolong the current flow which produces sparking. In a generator, armature reaction distorts the flux in the direction of rotation and the brushes should be advanced. In order to neutralize the emf of self-induction the brushes should be set a little ahead of the neutral plane so that the emf induced in the short-circuited coils by the cutting of the flux at the fringe of the next pole is opposite to this emf of self-induction. In a motor the brushes are correspondingly moved backward in the direction opposite rotation.

Theoretically, the brushes should be shifted with every change in load. This is impracticable except occasionally with large machines under constant supervision. Hence the brushes are set in some position intermediate between the no-load and full-load positions. With reversible motors, such as railway motors, the brushes must remain in the no-load neutral. Although the emfs induced in the coils undergoing commutation are relatively small, the resistance of the coils themselves is low so that unless further resistance is introduced the short-circuit currents would be large. Hence, with the exception of certain low-voltage generators, carbon brushes that have relatively high resistance are used almost universally.

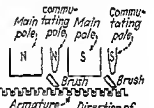


FIG. 53.—Commutating Poles in Motor.

If **commutating poles** are used, the brushes may be allowed to remain in the no-load neutral and at the same time commutation is improved greatly over the entire range of load. **Commutating poles** (or **interpoles**) are small poles between the main poles (Fig. 53) and are excited by a winding in series with the armature. Their function is to neutralize the flux distortion in the neutral plane caused by armature reaction and also to supply a flux that will cause an emf to be induced in the conductors undergoing commutation, opposite and equal to the emf of self-induction. Since armature reaction and the emf of self-induction are both proportional to the armature current, saturation being neglected, they are neutralized theoretically at every load. Commutating poles have made possible d-c generators and motors of very much higher voltage and larger kw ratings than would otherwise be possible.

Commutation difficulties are frequently encountered even in commutating-pole machines. The commutating poles may be connected incorrectly. In a motor, passing from a N main pole in the direction of rotation of the armature, a N commutating pole should be encountered as shown in Fig 53. In a generator under these conditions a S commutating pole should be encountered. The test may be easily made with a compass. If poor commutation is caused by too strong interpoles, the winding may be shunted. If the poles are too weak and the shunting cannot be reduced, they may be strengthened by inserting sheet-iron shims between the pole and the yoke.

Speed Control of Motors

Shunt Motors. In Eq. (79) the speed of a shunt motor $N = K_sE/\phi$ where K_s is a constant involving the design of the motor such as conductors on armature surface and number of poles. Obviously, in order to change

Figure 36 shows the use of instrument transformers to measure the voltage, current, power, and kwhr of a single-phase load. Figure 37 shows the connections that would be used to measure the voltage, current, and power of a 26,400-volt, 600-amp three-phase load.

High-voltage Testing. The most convenient method of measuring high voltage is to measure the voltage induced in a voltmeter coil of few turns interwoven in the high-voltage winding of the step-up transformer but insulated from it. For moderately high voltages (up to 200 kv) the ratio of the voltmeter-coil voltage to the total voltage is in proportion to the number of turns on the two windings. At higher voltages, however, capacitance effects between turns give large errors when this method is used. Sphere gaps may be used for these higher voltages. Calibration data for sphere gaps are given in the A.I.E.E. Standardization Rules. To prevent injuries due to overvoltage it is frequently deemed advisable to connect a sphere gap in parallel with the specimen being tested. The gap is set to a slightly higher voltage than that at which it is desired to test the specimen. Potential transformers are also used to measure high voltages, but they are very expensive for the higher voltages.

Measurement of Resistance

Voltmeter-ammeter Method. A common method of measuring resistance, known as the voltmeter-ammeter or fall-in-potential method, makes use of an ammeter and a voltmeter. In Fig. 38, the resistance to be measured is R. A current I flows through the resistance and ammeter in series, and the drop in potential across the resistance is measured by the voltmeter E. The current shunted by the voltmeter is so small that it may generally be neglected. A correction may be applied if necessary, for the resistance of the voltmeter is generally given with the instrument. The potential difference divided by the current gives the resistance included between the voltmeter leads. As a check, determinations are generally made with several values of current, which may be varied by means of the controlling resistance r. If the resistance to be measured is the armature of a d-c machine and the voltmeter leads are placed upon the brush holders, the resistance determined will include that of the brush contacts. To measure the resistance of the armature alone, the voltmeter leads should be placed directly on the commutator segments upon which the brushes rest but not under the brushes.

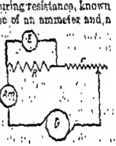


FIG. 38.—Voltmeter-ammeter Method of Resistance Measurement.

Insulation Resistance. Insulation resistance is so high that it is usually given in megohms (10^6 ohms) rather than in ohms. Insulation resistance tests are important, for, although they may not be conclusive they frequently reveal flaws in insulation, poor insulating material, presence of moisture, etc. Such tests are applied to the insulation of electrical machinery from the windings to the frame, to underground cables, to insulators, capacitors, etc.

For moderately low resistances, 1 to 10 megohms, the voltmeter method given in Fig. 39, which shows insulation measurement of the field winding of a generator, may be used. To measure the current flowing when a voltage E is impressed across the resistance R, a high-reading voltmeter V is connected in series with R. The current that flows under this condition with the switch connecting S and A, is $E/(R+r)$ where r is the resistance of the voltmeter. A high-reading voltmeter is necessary, for its resistance is higher than that of a low-reading instrument. Since the method is in reality a comparison of the unknown insulation resistance with the known resistance of the voltmeter, the latter must be comparable to the former, or the deflection of the instrument will be so small that the results will be inaccurate. To determine the impressed voltage E, the

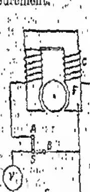


FIG. 39.—Voltmeter Method of Insulation-resistance Measurement.

the speed of a motor, without changing its construction, two factors may be varied, the counter emf E and the flux ϕ.

Armature-resistance Control. The counter emf $E = V - I_aR_a$ where V is the terminal voltage, assumed constant. R_a must be small in order that the armature heating may be maintained within permissible limits. Under these conditions the speed change with load is small. By inserting external resistance, however, into the armature circuit the counter emf E may be made to decrease rapidly with increase in load. That is, $E = V - I_a(R_a + R)$ where R is the external resistance. The resistance R must be inserted in the *armature* circuit only. The advantages of this method are its simplicity, the full torque of the motor is developed at any speed, and the method introduces no commutating difficulties. Its disadvantages are the very poor speed regulation with change of load (Fig 54), the very low efficiency, particularly at the lower speeds and the fact that provision must be made to dissipate the comparatively large power losses in the series resistor. Figure 54 shows typical speed-load curves without and with a series resistance in the armature circuit. The armature efficiency is nearly equal to the ratio of the operating speed to the no-load speed. Hence at 25 percent speed the armature efficiency is practically 25 percent. Frequently the controlling and starting resistances are one and the device is called a **controller**. Starting rheostats themselves are not designed to carry the armature current continuously and must not be used as controllers. The armature resistance method of speed control is frequently used to regulate the speed of ventilating fans where the power demand diminishes rapidly with decrease in speed.

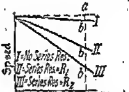


FIG. 54.—Speed-load Characteristics with Armature-resistance Control.

Control by Changing Impressed Voltage. From Eq. (80) it is evident that the speed of a motor may be changed if V is changed by connecting the armature across different voltages. Speed control by this method is accomplished by having mains (usually four), which are maintained at different voltages, available at the motor.

The shunt field of the motor is generally permanently connected to one pair of mains, and the armature circuit is provided with a controller by means of which the operator can readily connect the armature to any pair of mains. Such a system gives a series of distinct and widely separated speeds and generally necessitates the use of field-resistance control, in combination, to obtain intermediate speeds. This method, known as the **multivoltage method**, has the disadvantage that the system is expensive, for it requires several generating machines, a somewhat complicated switchboard, and a number of service wires. The system is used somewhat in machine shops and is extensively used for d-c elevator starting and speed control.

In the **Ward Leonard method**, speed control is obtained by applying variable voltage to the armature. Power for the working motor is obtained from a motor-generator set which runs at constant speed from the supply mains. The field circuit of the generator is excited from the supply mains. The resistance of its field rheostat is sufficient to vary the field current from full value to nearly zero. The armature of the working motor is connected to the armature of the variable-voltage generator. Its field excitation is constant, being connected across the supply mains. Hence any speed from

same voltmeter is used. The switch S connects S and B for this purpose. With these two readings, the unknown resistance is

$$R = r(E - e)/e \tag{71}$$

where e is the deflection of the voltmeter when in series with the resistance to be measured as when S is at A. If a special voltmeter, having a resistance of 100,000 ohms per 150 volts, is available, a resistance of the order of 2 to 3 megohms may be measured very accurately.

When the insulation resistance is too high to be measured with a voltmeter, a sensitive galvanometer may be used. The connections are shown in Fig. 40. The battery should have an emf of at least 100 volts. Radio B batteries are convenient for this purpose. The method involves comparing the unknown resistance with a standard 0.1 megohm. To calibrate the galvanometer the cable is short-circuited (dotted line) and the switch S is thrown to position a. Let the galvanometer deflection be D_1 and the reading of the Ayrton shunt S_1. The short circuit is then removed. The 0.1 megohm is left in circuit since it is usually negligible in comparison with the unknown resistance X. Let the reading of the galvanometer now be D_2 and the reading of the shunt S_2. Then

$$X = 0.1\, S_2D_1/S_1D_2 \tag{72}$$

When the switch S is thrown to position b, the cable is short-circuited and discharges electrostatically.

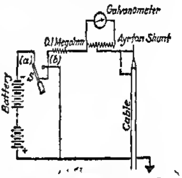


Fig. 40.—Measurement of Insulation Resistance with Galvanometer.

The **Megger** is an instrument that indicates insulation resistance directly on a scale. It consists of a small hand-driven generator which generates approximately 500 volts. A clutch slips when the voltage exceeds the rated value. The current through the unknown resistance flows through a moving element consisting of two coils fastened rigidly together, but which move in different portions of the magnetic field. A pointer attached to the spindle of the moving element indicates the insulation resistance directly. These instruments have a range up to 2,000 megohms, and are very convenient where portability and convenience are desirable.

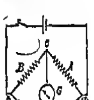


Fig. 41.—Wheatstone Bridge.

The insulation resistance of electrical machinery is of doubtful significance as far as dielectric strength is concerned. It varies widely with temperature, humidity, and cleanliness of the parts. When the insulation resistance falls below the prescribed value, it may, in most cases of good design, be brought to the required standard by cleaning and drying the machine. Hence it may be useful in determining whether or not the insulation is in proper condition for a dielectric test. The A.I.E.E. Standards (5–451) specify minimum value of insulation resistance in megohms = $\dfrac{\text{rated voltage}}{\text{rating in kw} + 1{,}000}$. If the operating voltage is higher than the rated voltage, the operating voltage should be used. The rule specifies that a d-c voltage of 500 be used in testing. If not, the voltage should be specified.

Wheatstone Bridge. Resistances from a fraction of an ohm to 100,000 ohms and more may be measured with a high degree of precision with the Wheatstone bridge (Fig. 41). The bridge consists of four resistances $ABCX$ connected as shown. X is the unknown resistance; A and B are ratio arms, the resistance units of which are in even decimal ohms as 1 – 10 – 100, etc. C is the rheostat arm. A battery or low-voltage source of direct current is connected across ab. A galvanometer G of moderate sensitivity is connected across cd. The values of A and B are so chosen that three or four significant figures in the value of C are obtained. As a first approximation it is well to

zero up to full speed may be obtained by adjusting the generator field current. Because of its cost, complications, and low over-all efficiency, the applications of this system are limited. It has been employed in the operation of large newspaper printing presses and the turning of gun turrets on battleships.

The Ilgner system is based on a principle similar to that of the Ward Leonard system except that the d-c generator is driven with a slip-ring induction motor having external resistance which is automatically controlled with a regulator. The armatures of the generator and work motor are connected electrically, and both are separately excited. Since the set is used principally to supply power to hoists and acts as a balancing set, the motor-generator set is provided with a heavy flywheel. When a heavy load is applied, the regulator inserts resistance in the rotor circuit which slows the motor down and causes the flywheel to give up some of its kinetic energy, thus tending to equalize the loads on the power system.

Control by Changing Field Magnetism. From Eq. (79) it is seen that the speed of a motor is inversely proportional to the magnetic flux ϕ. The flux can be changed either by varying the shunt-field current or by varying the reluctance of the magnetic circuit. The variation of the **shunt-field current** is the simplest and most efficient of all the methods of speed control.

With the ordinary motor, speed variation of 1.5 to 1.0 is obtainable with this method. If attempt is made to obtain greater ratios, severe sparking at the brushes results, owing to the field distortion caused by the armature mmf becoming large in comparison with the weakened field of the motor. Speed ratios as high as 5:1 are, however, obtainable with motors having commutating poles (see p. 1728). Commutating poles not only give the correct mmf for proper commutation but when they are used the brushes may remain in the geometrical neutral. Hence there are no demagnetizing armature ampere-conductors acting on the field to weaken it and thus increase the distortion and speed instability. Since the field current is a small proportion of the total current (1 to 5 percent), the rheostat losses in the field circuit are always small. Hence this method is efficient. Also for any given speed adjustment the speed regulation is excellent, which is another advantage. Because of its simplicity, efficiency, and excellent speed regulation, the control of speed by means of the field current is by far the most common method.

Removable Armature Method. In the Lincoln motor the speed is controlled by changing the position of the armature with respect to the field. For example, if it is desired to increase the speed, the armature is in part moved axially out of the field by means of a hand wheel. This obviously reduces the flux entering the armature. The advantages of this method are that very fine adjustment of speed is obtainable and, at the higher speeds, there is no tendency toward instability such as occurs with field rheostat control. This type of motor is more complicated than the simple motor. It has found extensive application in driving machine tools where fine speed control is essential.

Speed Control of Series Motors. The series motor is fundamentally a variable-speed motor, the speed varying widely from light load to full load and more (see Fig. 51 and p. 1726). From Eq. (80) the speed for any value of ϕ, or current, may be changed by varying the impressed voltage. Hence the speed may be controlled by inserting resistance in series with the motor. This method, which is practically the same as the armature-resistance control

make A and B equal. When the bridge is in balance,

$$X/C = A/B \tag{73}$$

The positions of the battery and galvanometer are interchangeable. There are many modifications of the bridge which adapt it to measurements of very low resistances and also to a-c measurements.

Potentiometer. The principle of the potentiometer is shown in Fig. 42. ab is a slide wire, and bc consists of a number of equal individual resistances between contacts. A battery Ba the emf of which is approximately 2 volts supplies current to this wire through the adjustable rheostat R. A slider m makes contact with ab, and a contactor m' connects with the contacts in bc. A galvanometer G is in series with the wire connecting to m. By means of the double-throw double-pole switch Sw either the standard cell (Weston cell p. 1697) or the unknown emf may be connected to mm' through the galvanometer G. The potentiometer is standardized by throwing Sw to the standard-cell side, setting mm' so that their positions on ab and bc correspond to the emf of the standard cell (see p. 1697). The rheostat R is then adjusted until G reads zero. The unknown emf is measured by throwing Sw to $E.M.F.$ and adjusting m and m' until G reads zero. The advantage of this method of measuring emf is that when the potentiometer is in balance no current is taken from either the standard cell or the source of E.M.F. Potentiometers seldom exceed 1.6 volts in range. To measure voltage in excess of this, a *volt box* which acts as a multiplier is used. To measure current, the voltage drop across a standard resistance of suitable value is measured with the potentiometer. For example, with 50 amp a 0.01-ohm standard resistance gives a voltage drop of 0.5 volt which is well within the range of the potentiometer.

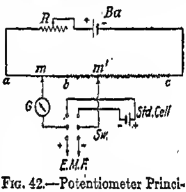


Fig. 42.—Potentiometer Principle.

Potentiometers of low range are used extensively with thermocouple pyrometers. Figure 42 merely illustrates the principle of the potentiometer. There are many modifications, conveniences, etc., not shown in Fig. 42.

Direct-current Generators

All electrical machines are comprised of a magnetic circuit of iron (or steel) and an electric circuit of copper. In a generator the armature conductors are rotated so that they cut the magnetic flux coming from and entering the field poles. In the d-c generator (except the unipolar type) the emf induced in the individual conductors is alternating, but this is rectified by the commutator and brushes, so that a unidirectional current flows to the external circuit.

The induced emf in a generator (or motor)

$$E = \phi ZNP/60P'10^8 \text{ volts} \tag{74}$$

where ϕ is the flux entering the armature from one north pole; Z the total number of conductors on the armature; N the speed, rpm; P the number of poles; and P' the number of parallel paths through the armature. Since with a given generator, Z, P, P' are fixed, the induced emf

$$E = K\phi N \text{ volts} \tag{75}$$

where K is a constant. When current flows from the armature, the terminal volts

$$V = E - I_aR_a \tag{76}$$

where I_a is the armature current and R_a the armature resistance including the brush and contact resistance which vary somewhat.

method for shunt motors, has the same objections of low efficiency and poor regulation with fluctuating loads. It is extensively used, however, in controlling the speed of hoist and crane motors.

The series-parallel system of series-motor speed control is almost universally used in electric traction. At least two motors are necessary. The two motors are first connected in series with each other and with the starting resistance. The starting resistance is gradually cut out, and the motors reach approximately half speed. Both motors take the same current, and each can develop full torque. This condition of operation is efficient since there is no external resistance in circuit. When the controller is moved to the next position, the motors are connected in parallel with each other and each in series with starting resistances. Full speed of the motors is obtained by gradually cutting out these resistances. Connecting the two motors in series on starting reduces the current to one-half the value that would be required for a given torque were both motors connected in parallel on starting. The power taken from the trolley is halved, and an intermediate running speed is efficiently obtained.

In the **multiple-unit** method of speed control which is used for electric railway trains, the starting contactors, reverser, etc., for each car are located under that car. The relays operating these control devices are actuated by energy taken from the train line consisting usually of seven wires. The train line runs the entire length of the train, the connections between the individual cars being made through the couplers. The train line is energized by the action of the motorman operating any one of the small master controllers which are located in each car. Hence corresponding relays, contactors, etc., in every car all operate simultaneously. High accelerations may be reached with this system because of the large tractive effort exerted by the wheels on every car.

Alternating-current Generators

Construction. In the usual alternator the armature or stator is the stationary member. This construction has many advantages. It is possible to make the slots any reasonable depth, since the tooth necks increase in cross section with increase in depth of slot; this is not true of the rotor. The large slot section which is thus obtainable gives ample space for copper and insulation. The conductors from the armature to the bus bars can be insulated throughout their entire lengths, since no rotating or sliding contacts are necessary. The insulation in a stationary member does not deteriorate as rapidly as that on a rotating member, for it is not subjected to centrifugal force or to any considerable vibration.

The **rotating member** is ordinarily the field. There are two general types of field construction, the **salient-pole type** and the **cylindrical or non-salient pole type**. The salient-pole type is used almost entirely for slow and moderate-speed generators since this construction is the least expensive and permits ample space for field copper.

It is not practicable to employ salient poles in high-speed turboalternators because of the excessive windage and the difficulty of obtaining sufficient mechanical strength. The **cylindrical type** consists of a cylindrical steel forging with either parallel or radial slots in which the field copper, usually in strip form, is placed. The fields are ordinarily excited at low voltage, 125 and 250 volts, the current being conducted to the rotating member by means of slip rings and brushes. The field power is ordinarily only 1 or 2 percent the rated power of the machine (see Table 14, p. 1730).

There are three standard types of d-c generators: the **shunt generator**, the **series generator**, and the **compound generator**. The series generator is nearly obsolete.

Shunt Generator. The field of the shunt generator in series with its rheostat is connected directly across the armature as shown in Fig. 43. This machine maintains approximately constant terminal voltage over its working range of load. An external characteristic of the generator is shown in Fig. 44. As load is applied the terminal voltage drops owing to the armature-resistance drop [Eq. (76)] and armature reaction which decreases the flux. The drop in terminal voltage reduces the field current which in turn reduces the flux, hence the induced emf, etc. At some point B, usually well above rated current, the foregoing reactions become cumulative and the machine commences to break down. The current reaches a maximum value and then decreases to nearly zero at short circuit. With large machines, point B is well above rated current, the operating range being between O and A. The voltage may be maintained constant by means of the field rheostat. Automatic regulators which operate through the field rheostat are frequently used to maintain constant voltage (see p. 1735). Shunt generators are commonly used in city substations which are all tied together through the network of feeders and mains. Their stability when in parallel is a distinct advantage for this service. If a generator fails to build up (1) the load may be connected; (2) the field resistance may be too high; (3) the field circuit may be open; (4) the residual magnetism may be insufficient; (5) the field connection may be reversed.

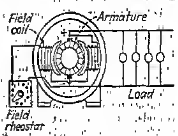


FIG. 43.—Shunt Generator.

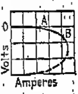


FIG. 44.—Shunt-generator Characteristic.

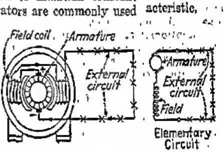


FIG. 45.—Series Generator.

Series Generator. In the series generator (Fig. 45) the entire load current flows through the field winding, which consists of relatively few turns of wire of sufficient size to carry the entire load current without undue heating. The field excitation, and hence the terminal voltage, depends on the magnitude of the load current. An external characteristic showing the relation between load current and terminal voltage is given in Fig. 46. When used as a booster the series generator operates on the left-hand side of the characteristic (Fig. 46); when used as a constant-current generator it operates on the right-hand side of the characteristic. The principal application of series generators was constant-current series arc lighting. They have been replaced almost entirely by constant-current transformers operating in combination with mercury-arc rectifier tubes.

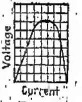


FIG. 46.—External Characteristic of Series Generator.

Compound-wound Generators (Fig. 47). By the addition of a series winding to a shunt generator the terminal voltage may be automatically

Classes of Alternators. Alternators may be divided into three general classes: (1) the slow-speed engine driven type; (2) the moderate speed water-wheel-driven type; and (3) the high-speed turbine-driven type. In (1) a hollow box frame is used as the stator support and the field consists of a spider to which a large number of salient poles are attached, usually bolted. The speed seldom exceeds 75 to 90 rpm, although it may run as high as 150 rpm. Water-wheel alternators also have salient poles which are usually dovetailed to a cylindrical spider consisting of steel plates riveted together. Their speeds range from 80 to 600 rpm and sometimes higher, although the 9,000 kva Keokuk alternators rotate at only 58 rpm, operating at a very low head. The speed rating of direct-connected water-wheel alternatore decreases with decrease in head. It is desirable to operate alternators at the highest permissible speed since the weight and costs diminish with increase in speed. Water-wheel-driven alternators must be able to run at double speed, as a precaution against accident should the governor fail to shut the gate sufficiently rapidly in case of opening of the circuit breakers or should the governing mechanism become inoperative.

Turbine-driven alternators operate at speeds of 720 to 3,600 rpm. Direct-connected exciters, belt-driven exciters from the alternator shaft, and separately driven exciters are used. In large stations separately driven (usually motor) exciters supply the excitation energy to excitation bus bars. Steam-driven exciters and storage batteries are frequently held in reserve. With slow-speed alternators the belt-driven exciter is frequently used because it can be driven at higher speed, thus reducing the cost.

Alternator Design. At the present time single-phase alternators are seldom built. For single-phase service two phases of a standard three-phase Y-connected alternator are used. A single-phase load or unbalanced three-phase load produces flux pulsations in the magnetic circuits of alternators, which in the past have caused serious difficulties. Two-phase windings consist of two similar single-phase windings displaced 90 electrical space degrees on the armature and ordinarily occupying all the slots on the armature. The most common type of winding is the three-phase lap- or wave-wound two-layer barrel-type of winding. In three-phase windings three windings are spaced 120 electrical space degrees apart, the individual phase belts being spaced 60 deg apart. Usually, all the slots on the armature are occupied. Standard voltages are 550, 1,100, 2,200, 6,600, 13,200, and 20,000 volts. It is much more difficult to insulate for 20,000 volts than it is for the lower voltages. However, if the power is to be transmitted at this voltage, its use would be justified by the saving of transformers. In machines of moderate and larger ratings it is common to generate at 6,600 and 13,200 volts if transformers must be used. The higher voltage is preferable, particularly for the higher ratings, because it reduces the cross section of the connecting leads and bus bars.

The **standard frequencies** in the country are 60 and 50 cycles for light and power; 50 cycles is less common than 60 cycles, its principal use being in southern California. Lower frequencies are not desirable for lighting loads because of the objectionable flicker of the lamps. For power purposes only, 25 cycles is frequently used. For example, power for conversion to direct current at railway substations and power for railway electrification is usually 25 cycles; induction motors, synchronous converters, and series commutating motors operate more satisfactorily at that frequency. The frequency of a synchronous machine

$$f = P \times \text{rpm}/120 \qquad (84)$$

maintained very nearly constant, or, by properly proportioning the series turns, the terminal voltage may be made to increase with load to compensate for loss of voltage in the line, so that approximately constant voltage is maintained at the load. If the shunt field is connected outside the series field (Fig. 47), the machine is long shunt; if the shunt field is connected inside the series field, *i.e.*, directly to the armature terminals, it is short shunt. So far as the operating characteristic is concerned, it makes little difference which way a machine is connected.

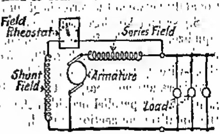


FIG. 47.—Compound-wound D-c Generator.

Compound-wound generators are chiefly used for small isolated plants and for generators supplying a purely motor load subject to rapid fluctuations such as in railway work. When first putting a compound generator in service the shunt field must be so connected that the machine builds up. The series field is then connected so that it aids the shunt field. Sometimes the residual magnetism is reversed by a short circuit feeding back through the series fields in the reversed direction. The generator will not then build up until the residual magnetism is reversed, temporary external excitation usually being necessary. Figure 48 gives the characteristics of a 200-kw 600-volt compound-wound generator.

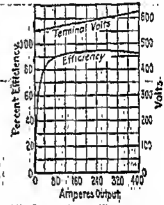


FIG. 48.—Characteristics of a 200-kw Compound-wound D-c Generator.

Parallel Operation of Shunt Generators. It is desirable to operate generators in parallel in order that the station capacity may be adapted to the load. Shunt generators, because of their drooping characteristics (Fig. 44), are inherently stable when in parallel. To connect shunt generators in parallel it is necessary that the switches be so connected that like poles are connected to the same bus bars when the switches are closed. Assume one generator to be in operation; to connect another generator in parallel with it, the incoming generator is first brought up to speed and its terminal voltage adjusted to a value slightly greater than the bus-bar voltage. This generator may then be connected in parallel with the other without difficulty. The proper division of load between them is adjusted by means of the field rheostats and is maintained automatically if the machines have similar voltage-regulation characteristics.

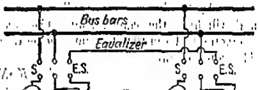


FIG. 49.—Connections for Compound-wound Generators Operating in Parallel.

Parallel Operation of Compound Generators. As a rule, compound generators have either flat or rising voltage characteristics. Therefore, when

where P is the number of poles. Alternators are rated in kilovolt-amperes (kva) rather than in kilowatts, for heating, which determines the rating, is dependent only on the current and is independent of power factor. If the kilowatt rating is specified, the power factor should also be specified.

Induced EMF. The induced emf per phase in an alternator

$$E = 2.22k_b k_p \Phi f Z 10^{-8} \text{ volts per phase} \tag{85}$$

k_b is the breadth factor or belt factor (usually 0.9 to 1.0) and depends on the *number of slots per pole per phase*; k_p *is the pitch factor* $= 1.0$ for full pitch; Φ is the total flux entering the armature from one north pole and is assumed to be sinusoidally distributed along the air gap; f is the frequency; and Z is the number of series conductors per phase.

Regulation. The terminal voltage of an alternator at constant frequency *and field excitation depends not only on the current load but on the power* factor as well. This is illustrated in Fig 55 which shows the voltage-current characteristics of an alternator with lagging current, leading current, and in-phase current (p.f. = 1.00). With leading current the voltage may actually rise with increase in load; the rate of voltage decrease with load becomes greater as the lag of the current increases. The regulation of an alternator is defined by the A.I.E.E. Standard Rules (No. 7) as follows: *In constant-potential alternators, the regulation is the rise in voltage (when the specified load at specified power factor is reduced to zero) expressed in percentage of rated voltage.* For example, in Fig. 55 the regulation under each condition is

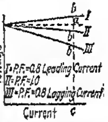


Fig. 55.—Alternator Characteristics.

$$100(ac - bc)/bc \tag{86}$$

With leading current the regulation may be negative.

Three factors affect the regulation of alternators, the **effective armature resistance**, the **armature leakage reactance**, and **armature reaction.** With alternating current the armature loss is greater than the value obtained by multiplying the square of the armature current by the ohmic resistance. This is due to hysteresis and eddy-current losses in the iron adjacent to the conductor and to the alternating flux producing losses in the conductors themselves. Also the current is not distributed uniformly over conductors in the slot, but the current density tends to be greatest in the top of the slot. These factors all have the effect of increasing the resistance. The ratio of effective to ohmic resistance varies from 1.2 to 1.5. The **armature leakage reactance** is due to the flux produced by the armature current linking the conductors in the slots and also the end connections.

The armature mmf reacts on the field to change the value of the flux. With a single-phase alternator and with an unbalanced load on a polyphase alternator this mmf is pulsating and causes iron losses in the field structure. With polyphase machines under a constant balanced load the armature mmf is practically constant in magnitude and fixed in its relation to the field poles. Its direction is determined by the power factor of the load.

A component of current in phase with the no-load induced emf merely distorts the field by strengthening the trailing pole-tip and weakening the leading pole-tip. A component of current lagging the induced emf by 90 deg weakens the field without distortion. A component of current leading the induced emf by 90 deg strengthens the field without distortion. Ordinarily, both cross-magnetization and one of the other components are acting simultaneously.

connected in parallel, they are inherently unstable. Stability may, however, be obtained by using an equalizer connection (Fig. 49) which connects the terminals of the generator at the junctions of the series fields. This connection is of low resistance so that any increase of current divides proportionately between the series fields of the two machines. The equalizer switch (E.S.) should be closed first and opened last, if possible. In practice, the equalizer switch is often one blade of a three-pole switch, the other two being the bus switch S, as in Fig. 49. When compound generators are used on a three-wire system, two series fields—one at each armature terminal—and two equalizers are necessary. It is possible to operate any number of compound generators in parallel provided their characteristics are not too different and the equalizer connection is used.

Table 12. Approximate Test Performance of Compound Wound D-c Generators with Commutating Poles
(Westinghouse Electric & Mfg. Co.)

Kw	Voltage	Amp	Efficiency, percent		
			½ load	¾ load	4/4 load
5	125	40	78.0	81.0	82.0
10	125	80	83.5	87.0	88.0
25	125	200	86.5	88.5	89.5
50	125	400	86.0	89.0	90.0
100	125	800	88.7	89.3	89.0
200	125	1,600	89.1	89.6	89.4
400	250	1,600	91.7	91.9	91.7
1,000	250	4,000	92.1	92.6	92.1

(See Table 22, p. 1764, for the approximate full-load currents of motors.)

Direct-current Motors

Motors operate on the principle that a conductor carrying current in a magnetic field tends to move at right angles to that field (see Fig. 14, p. 1980). The ordinary d-c generator will operate entirely satisfactorily as a motor and will have the same rating. The conductors of the motor rotate in a magnetic field and therefore must generate an emf just as does the generator. The induced emf

$$E = K\phi N \tag{77}$$

where K is a constant, ϕ the flux entering the armature from one north pole, and N the speed in rpm [see Eq. (75), p. 1997]. This emf is in opposition to the terminal voltage and tends to oppose current entering the armature. Its value is

$$E = V - I_a R_a \tag{78}$$

where V is the terminal voltage, I_a the armature current, and R_a the armature resistance [compare with Eq. (76)]. From Eq. (77) it is seen that the speed

$$N = K_s E/\phi \tag{79}$$

when $K_s = 1/K$. This is the fundamental speed equation for a motor. By substituting in Eq. (78)

$$N = K_s(V - I_a R_a)/\phi \tag{80}$$

which is the general equation for the speed of a motor.

The foregoing effects are called **armature reaction**. Frequently the effects of armature reactance and armature reaction can be combined into a single quantity.

It is difficult to determine the regulation of an alternator by actual loading, even when in service, owing to the difficulty of obtaining, controlling, and absorbing the large balanced loads. Hence methods of **predetermining regulation without actually loading** the machine are used.

Synchronous-impedance Method. Both armature reactance and armature reaction have the same effect on the terminal voltage. In the synchronous-impedance method the alternator is considered as having no armature reaction, but the armature reactance is increased a sufficient amount to account for the effect of armature reaction. The vector diagram for a current I lagging the terminal voltage V by an angle θ is shown in Fig. 56. The power factor of the load is $\cos \theta$; IR is the effective armature-resistance drop and is parallel to I; IX_s is the synchronous-reactance drop and is at right angles to I and leading it by 90 deg. IX_s includes both the reactance drop and the drop in voltage due to armature reaction.

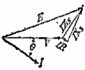


FIG. 56.—Vector Diagram for Synchronous-impedance Method.

That part of IX_s which replaces armature reaction is in reality a fictitious quantity. The synchronous-impedance drop is given by IZ_s. The no-load or open-circuit voltage

$$E = \sqrt{(V \cos \theta + IR)^2 + (V \sin \theta \pm IX_s)^2} \text{ volts} \tag{87}$$

All quantities are per phase. The negative sign is used with leading current.

$$\text{The regulation} = 100(E - V)/V \tag{88}$$

(see p. 1733). With leading current E may be less than V and a negative regulation results.

The synchronous impedance is determined from an open-circuit and a short-circuit test, made with a weak field. The voltage E' on open circuit is divided by the current I' on short circuit for the same value of field current.

$$Z_s = E'/I', \qquad X_s = \sqrt{Z_s^2 - R^2} \text{ ohms} \tag{89}$$

Since the synchronous reactance is determined at low saturation of the iron and used at high saturation, the method gives regulations that are too large; hence it is called the *pessimistic method*.

MMF Method. In the mmf method the alternator is considered as having no armature reactance but the armature reaction is increased by an amount sufficient to include the effect of reactance. That part of armature reaction which replaces the effect of armature reactance is in reality a fictitious quantity. To obtain the data necessary for computing the regulation, the alternator is short-circuited and the field adjusted to give rated current in the armature. The corresponding value of field current I_2 is read. The field is then adjusted to give voltage E' equal to rated terminal voltage $+ IR$ drop ($= V + IR$ vectorially, Fig. 57) on open circuit and the field current I' read.

I_2 is 180 deg from the current vector I, and I' leads E' by 90 deg (Fig. 57). The angle between I' and I_2 is $90 - \theta + \phi$, but since ϕ is small it can usually be neglected. The vector sum of I_2 and I' is I_0. The open-circuit voltage E corresponding to I_0 is the no-load voltage and may be found on the saturation curve. The regulation is then found from Eq. (88). This method gives a value of regulation less than the actual value and hence is called the *optimistic*

The torque developed by an armature is proportional to the flux and to the armature current. That is

$$T = K_1 \phi I_a \tag{81}$$

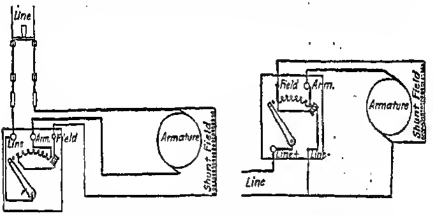
when K_1 is a constant. The torque at the pulley is slightly less than the developed torque by the torque necessary to overcome the rotational losses. Let VI be the motor input. The output is $VI\eta$ where η is the efficiency. The horsepower

$$P_H = VI\eta/746 \tag{82}$$

and the torque

$$T = 33{,}000 P_H/2\pi N. \tag{83}$$

Shunt Motor. In the shunt motor (Fig. 50) the flux is substantially constant and I_aR_a is 2 to 6 percent of V. Hence, from Eq. (80), the speed varies only slightly with load so that the motor is adapted to work requiring constant speed. The speed regulation of constant-speed motors is defined by the A.I.E.E. Standards as follows: In constant-speed d-c motors the regulation



(a) Three-point Box. (b) Four-point Box.

FIG. 50.—Connections for Shunt D-c Motors and Starters.

is the ratio of the difference between rated-load and no-load speeds to the rated-load speed at the final temperature attained at operation under rated load for the time specified in the rating. For example, in Fig. 54 the regulation under each condition is practically $(ac - bc)/bc$ (see Fig. 51). Also from Eq. (81) it is seen that the torque is practically proportional to the armature current (see Fig. 52). The motor is able to develop full-load torque and more on starting, but the ordinary starter is not designed to carry the current necessary for starting under load. The shunt motor is used to drive constant-speed line shafting, for machine tools, etc. Since its speed may be efficiently varied it is very useful when adjustable speeds are necessary, such as individual drive for machine tools (see p. 1751).

Shunt-motor Starters. At standstill the counter emf of the motor is zero and the armature resistance is very low. Hence, except in motors of very small size, series resistance in the armature circuit is necessary on starting. The field must, however, be connected across the line in order that it may obtain full excitation.

Figure 50 shows the two common types of starting boxes used for starting shunt motors. The armature resistance remains in circuit only during starting. In the **three-point box** (Fig. 50(a)) the starting lever is held, against the force of a spring, in the running position, by an electromagnet in

method. The actual regulation lies somewhere between the values obtained by the two methods but is more nearly equal to the value obtained by the mmf method.

Voltage regulators operate to maintain the bus-bar voltage constant and act usually through the field of the exciter. In the Tirrill regulator the field resistance of the exciter is short-circuited temporarily by contacts when the bus-bar voltage drops. Actually, the contacts are vibrating continuously, the time that they are closed depending on the value of the bus-bar voltage. The General Electric Co. manufactures a direct-acting regulator in which the regulating rheostat is part of the regulator itself. The rheostat consists of stacks of graphite plates, each plate being pivoted at the center. Tilting the plates changes the path of the current through the rheostat and thus changes the resistance. The plates are tilted by a sensitive torque armature which is actuated by variations of voltage from the normal value.

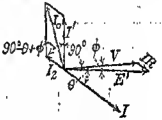


FIG. 57.—Vector Diagram for MMF Method.

Parallel Operation of Alternators. The kilowatt division of load between alternators in parallel is determined entirely by the speed-load characteristics of their prime movers and not by the characteristics of the alternator's themselves. No appreciable adjustment of kilowatt load between alternators in parallel can be made by means of their field rheostats, as with d-c generators. Consider Fig. 58, which gives the speed-load characteristics in terms of frequency of two alternators, No. 1 and No. 2, these characteristics being of course the speed-load characteristics of their prime movers. These speed-load characteristics are drooping, which is necessary for stable parallel operation. The total load on the two machines is $P_1 + P_2$ kw. Both machines must be operating at the same frequency f_1. Hence alternator 1 must be delivering P_1 kw, and alternator 2 must be delivering P_2 kw (the small alternator losses being neglected). If, under the foregoing conditions, the field of either machine is strengthened it cannot deliver a greater kilowatt load, for its prime mover can deliver more power only by dropping its speed. This is impossible, for both alternators must operate always at the same frequency f_1. For any fixed total power load, the division of kilowatt load between alternators can be changed only by modifying in some manner the speed-load characteristics of their prime movers, such, for example, as changing the tension in the governor spring. Alternators in parallel are of themselves in stable equilibrium. If the driving torque of one machine is increased, the resulting electrical reactions between the machines cause a circulating current to flow between machines. This current puts more electrical load on the machine whose driving torque is increased and tends to produce motor action in the other machines. In an extreme case, the driving torque of one prime mover may be removed entirely and its alternator will operate as a synchronous motor, driving the prime mover mechanically.

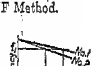


FIG. 58.—Speed-load Characteristics of Alternators in Parallel.

Variations in driving torques cause currents to circulate between alternators, transferring power which tends to keep the alternators in synchronism. If the power transfer takes the form of recurring pulsations, it is called **hunting**. Hunting may be reduced by building heavy copper grids called **amortisseur**

series with the field circuit, so that, if the field circuit is interrupted or the line voltage becomes too low, the lever is released and the armature circuit is opened automatically. In the **four-point starting box** the electromagnet is connected directly across the line, as shown in Fig. 50(*b*). In this type the arm is released instantly upon failure of the line voltage. In the other type some time elapses before the field current drops enough to effect the release. Some starting rheostats are provided with an overload device so that the circuit is automatically interrupted if too large a current is taken by the armature. The four-point box is used where a wide speed range is obtained by means of the field rheostat. The electromagnet is not then affected by changes in field current.

In large motors and in many small motors, automatic starters are frequently used. The advantages of the automatic starter are that the current is held between certain maximum and minimum values so that the circuit does not become opened by too rapid starting as may occur with manual operation; the acceleration is smooth and nearly uniform; relay contactors

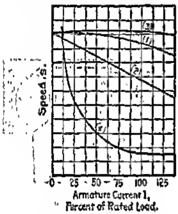


FIG. 51.

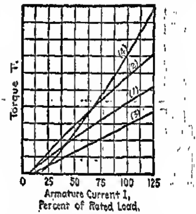


FIG. 52.

Speed and Torque Characteristics of D-c Motors.

(1) Shunt motor; (2) compound motor; (3) differential compound motor; (4) series motor.

of large current capacity are not difficult to design; the resistors and contactors may be placed in any convenient place; the controllers need only be designed to carry the small currents necessary to operate the relays. In modern automatic starters the resistances in series with the armature are successively short-circuited by contactors operated by relays. Any one relay is prevented from operating and cutting out further resistance until the current has dropped to some predetermined value. Since workmen can stop and start a motor merely by the pushing of a button, there results considerable saving by the shutting down of the motor when it is not needed. Automatic starters are very essential to elevator motors in order that smooth rapid acceleration with frequent starting and stopping may be obtained. Also automatic starting is very necessary with multiple-unit operation of electric-railway cars and with rolling-mill motors which are continually subjected to rapid acceleration, stopping, and reversing.

Series Motor. In the series motor the armature and field are in series. Hence, if saturation is neglected, the flux is proportional to the current and the torque [Eq. (81)] varies as the current squared. Therefore any increase

Table 14. Performance Data for Alternators
HORIZONTAL-COUPLED OR BELTED-TYPE ENGINE-DRIVEN GENERATORS
(Westinghouse Electric & Mfg. Co.)
80 percent power factor, three phase, 60 cycle, 240 to 2,400 volts

Kva	Poles	Speed, rpm	Kw excitation	Efficiency (percent)			Approx net wt, lb
				½ load	¾ load	4/4 load	
25	4	1,800	0.8	81.5	85.7	87.6	900
93.8	8	900	2.0	87.0	89.5	90.9	2,700
250	12	600	5.0	90.0	91.3	92.2	6,000
500	18	400	8.0	91.7	92.6	93.2	10,000
1000	24	300	14.5	92.6	93.4	93.9	18,600
3125	48	150	42.0	93.4	94.2	94.6	52,000

TURBODRIVEN DIRECT-CONNECTED TYPE

Kva	Poles	Speed, rpm	Kw excitation at 125v	Percent eff at 80 percent p.f.			Cu ft air per min
				½ load	¾ load	4/4 load	
625	2	3,600	12	92.4	94.0	94.6	2,200
1,250	2	3,600	15	92.9	94.5	95.2	3,500
2,500	2	3,600	19	94.4	95.7	96.3	5,000
4,375	2	3,600	26	95.1	96.2	96.8	10,500
9,375	2	3,600	45	95.1	96.2	96.8	17,000
12,500	2	3,600	60[a]	95.1	96.2	96.8	23,500
25,000	2	3,600	100[a]	95.2	96.4	97.0	25,000
37,500	2	3,600	120[a]	95.7	96.8	97.4	55,000

[a] Excitation for these ratings is usually at 250 volts.

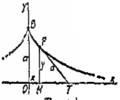
or damper windings into the pole faces. Turbine- and water-wheel-driven alternators are much better adapted to parallel operation than are alternators which are driven by reciprocating engines, because of their uniformity of torque.

Increasing the field current of an alternator in parallel with others causes it to deliver a greater lagging component of current. Since the character of the load determines the total current delivered by the system the lagging components of current delivered by the other alternators must decrease and may even become leading components. Likewise if the field of one alternator is weakened it delivers a greater leading component of current and the other machines deliver components of current which are more lagging. These leading and lagging currents do not affect appreciably the division of kilowatt load between the alternators. They do, however, cause unnecessary heating in the armatures of the alternators. The fields of all alternators should be so adjusted that the heating due to the quadrature components of currents is a *minimum. With two alternators having equal armature resistances this* occurs when both deliver equal quadrature currents.

Armature reactance between machines in parallel is desirable. If not too great it stabilizes their operation by producing the synchronizing action. Alternators with too little reactance are sensitive, and, if connected in parallel with slight phase displacement or inequality of voltage, considerable disturbance results. Armature reactance also reduces the current on short circuit. Frequently, external power-limiting reactances are connected in

then $OT = a$, and $ON = r^2/a$. **Hence a construction for the tangent and normal.** Radius of curvature at P is $R = r/\sin^3 v$, where v = angle between OP and the tangent at P. **Construction:** At N draw a perpendicular to PN, meeting PO in Q; at Q draw a perpendicular to PQ, meeting PN in C; then C is the center of curvature for the point P.

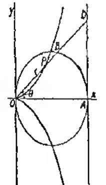
The Logarithmic Spiral (Fig. 76), is a curve which cuts the radii from O at a constant angle v, whose cotangent is m. Polar equation: $r = ae^{m \text{ rad } \theta}$. Here a is the value of r when $\theta = 0$. For large negative values of θ, the curve winds around O as an asymptotic point. If PT and PN are the tangent and normal at P, the line TON being perpendicular to OP (not shown in fig.), then $ON = rm$, and $PN = r\sqrt{1+m^2} = r/\sin v$. Radius of curvature at P is PN. The evolute of the spiral is an equal spiral whose axis makes an angle $\frac{1}{2}\pi - (\log_e m)/m$ with the axis of the given spiral. Area swept out by the radius r from $r = 0$ (where $\theta = -\infty$) to $r = r$, is $A = r^2/(4m)$ = half the triangle OPT. Length of arc from O to $P = s = r/\cos v = PT$.



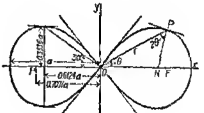
Tractrix.
Fig. 77.

The Tractrix, or Schiele's Anti-friction Curve (Fig. 77), is a curve such that the portion PT of the tangent between the point of contact and the x-axis is constant $= a$. Its equation is $x = \pm a\left[\cosh^{-1}\frac{a}{y} - \sqrt{1 - \left(\frac{y}{a}\right)^2}\right]$, or, in parametric form, $x = \pm a\,[t - \tanh t]$, $y = a/\cosh t$. (For tables of hyperbolic functions, see p. 60.) The x-axis is an asymptote of the curve. Length of arc $BP = a \log_e (a/y)$. The evolute (locus of centers of curvature) is the catenary whose lowest point is at B, and whose directrix is Ox.

The Cissoid (Fig. 78) is the locus of a point P such that OP, laid off on a variable ray from O, is equal to BD, the portion of the ray lying between a fixed circle through O and a fixed tangent at the point A opposite O. If a is the radius of the circle, the polar equation is $r = 2a \sin^2\theta / \cos\theta$. Rectangular equation, $y^2(2a - x) = x^3$.



Cissoid.
Fig. 78.

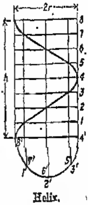


Lemniscate.
Fig. 79.

The Lemniscate (Fig. 79) is the locus of a point P the product of whose distances from two fixed points F, F' is constant, equal to $\frac{1}{2}a^2$. The distance $FF' = a\sqrt{2}$. Polar equation is $r = a\sqrt{\cos 2\theta}$. Angle between OP and the normal at P is 2θ. The two branches of the curve cross at right angles at O.

Maximum y occurs when $\theta = 30°$ and $r = a/\sqrt{2}$, and is equal to $\frac{1}{4} a\sqrt{2}$. Area of one loop $= a^2/2$.

The Helix (Fig. 80) is the curve of a screw thread on a cylinder of radius r. The curve crosses the elements of the cylinder at a constant angle, v. The pitch, h, is the distance between two coils of the helix, measured along an element of the cylinder; hence $h = 2\pi r \tan v$. Length of one coil $= \sqrt{(2\pi r)^2 + h^2} = 2\pi r/\cos v$. To construct the projection of a helix on a plane containing the axis of the cylinder, draw a rectangle, breadth $2r$ and height h, to represent the plane, with a semicircle below it, as in the figure, to represent the base of the cylinder. Divide h into equal parts (here 8), numbered from 1 to 8; think of the circumference as also divided into 8 equal parts, represented on the semicircle by numbers from $1'$ to $4'$ and back again from $4'$ to $8'$. Then the point of intersection of a horizontal line through 1, 2, . . . with a vertical line through $1'$, $2'$, . . . will be a point of the required projection. If the cylinder is rolled out on a plane, the development of the helix will be a straight line, with slope equal to $\tan v$.



Helix.
Fig. 80.

series to protect the alternators and equipment from injury that would result from the tremendous short-circuit currents. For these reasons, poor regulation in large alternators is frequently considered to be an advantage rather than a disadvantage.

TRANSFORMERS

Transformer Theory. The transformer is a device that transfers energy from one electric circuit to another without change of frequency and usually, but not always, with a change in voltage. The energy is transferred through the medium of a magnetic field. The energy is supplied to the transformer through the primary coil; the energy is delivered by the transformer by means of the secondary coil. Both coils link the same magnetic circuit. With no load on the secondary, a small current, called the exciting current, flows in the primary and produces the alternating flux. This flux links both primary and secondary and induces the same volts per turn in each. With a sine wave the emf

$$E = 4.44\,\Phi_m n f 10^{-8} \text{ volts} \tag{90}$$

where Φ_m is the maximum instantaneous flux, n the turns on either winding, and f the frequency. Equation (90) may also be written

$$E = 4.44\,B_m A n f 10^{-8} \text{ volts} \tag{91}$$

B_m is the maximum instantaneous flux density in the iron and A the net cross section of the iron. B_m is practically fixed. In large transformers with silicon steel it varies between 60,000 and 75,000 lines per sq in. at 60 cycles, and between 75,000 and 90,000 lines per sq in. at 25 cycles. It is desirable to operate the iron at as high density as possible in order to minimize the weight of iron and copper. On the other hand, with too high densities the eddy-current and hysteresis losses become too great, and with low frequency the exciting current may become excessive. It follows from Eq. (90) that

$$E_1/E_2 = n_1/n_2 \tag{92}$$

where E_1 and E_2 are the primary and secondary emfs and n_1 and n_2 are the primary and secondary turns. Since the impedance drops in the ordinary transformers are small, the terminal voltages of primary and secondary are also practically proportional to their number of turns. As the change in voltage in the ordinary constant-potential transformer over its range of operation is small (1.5 to 3 percent), the flux must remain substantially constant and the exciting current must remain substantially constant. Therefore, the added ampere-turns produced by any secondary load must be balanced by opposite and equal primary ampere-turns. Since the exciting current is small compared with the load current (1.5 to 8 percent) and the two are usually out of phase, the exciting current may ordinarily be neglected. Hence,

$$n_1 I_1 = n_2 I_2 \quad (93); \qquad \text{and } I_1/I_2 = n_2/n_1 \quad (94)$$

where I_1 and I_2 are the primary and secondary currents.

When load is applied to the secondary of a transformer, the secondary ampere-turns reduce the flux slightly. This reduces the counter emf of the primary, permitting more current to enter and thus supply the increased power demanded by the secondary.

Both primary and secondary coils must necessarily have resistance. All the flux produced by the primary does not link the secondary; the back ampere-turns of the secondary produce some flux which does not link the

may, however, deliver mechanical power and at the same time take either leading or lagging current. Its common applications are drives for motor-generator sets, ammonia compressors in refrigerating plants, rubber mills, and air compressors. The motor should not be used where fluctuations of torque are too violent. As a rule, it should not be used in small sizes (under 50 hp) since it requires d-c excitation, is more difficult to start than induction motors, and falls out of step quite readily when system disturbances occur.

If situated near an inductive load the motor may be overexcited, and its leading current will neutralize entirely or in part the lagging quadrature current of the load. This reduces the I^2R loss in the transmission lines and also increases the kilowatt ratings of the system apparatus. **The synchronous condenser and motor can also be used to control voltage** and to stabilize power lines. If the condenser or motor is overexcited, its leading current flowing through the line reactance causes a rise in voltage at the motor; if it is underexcited, the lagging current flowing through the line reactance causes a drop in voltage at the motor. Thus within limits it becomes possible to control the voltage at the end of a transmission line by regulating the fields of synchronous condensers or motors. Long 220-kv lines and the 287-kv Boulder Dam–Los Angeles line require several thousand kva in synchronous condensers floating at their load ends merely for voltage control. If the load becomes small, the voltage would rise to very high values if the synchronous condensers were not underexcited, thus maintaining nearly constant voltage.

The synchronous motor is started as an induction motor through the action of a starting or damper winding similar to the squirrel-cage rotor winding of an induction motor. Copper or alloy bars are inserted in the faces of the salient d-c field poles, and their ends are brazed to copper segmental end-rings bolted together to form a continuous ring. The process of starting a synchronous motor is simply one of accelerating the motor to as high a speed as it will reach as an induction motor with its damper winding and then applying field excitation in order to pull the rotor into synchronism. Because of its

Table 16. Performance Data for Coupled-type Synchronous Motors
(Westinghouse Electric & Mfg. Co.)
Unity power factor, three phase, 60 cycle, 2,300 volts

Hp	Poles	Speed, rpm	Full-load, amp	Kw excitation	Efficiencies, percent			Approx net wt, lb
					½ load	¾ load	4/4 load	
50	4	1,800	10.3	0.8	86.5	89.6	91.0	1,200
100	8	900	20.4	1.5	88.5	91.0	92.1	2,400
250	12	600	50.2	2.5	90.7	93.5	93.4	4,600
500	18	400	99.3	5.0	92.9	93.5	94.3	7,150
1,000	24	300	197	8.4	93.7	94.6	95.0	15,650
4,000	48	150	781	25.0	94.9	95.6	95.9	54,500
80 percent power factor, three phase, 60 cycle, 2,300 volts								
50	4	1,800	13.2	1.1	84.0	87.8	88.8	2,100
100	8	900	25.8	2.0	87.0	89.5	90.6	3,000
250	12	600	63.6	2.8	89.5	91.2	92.1	6,100
500	18	400	126	7.2	92.4	93.4	93.6	9,500
1,000	24	300	248	11.6	93.3	94.2	94.4	17,500
4,000	48	150	980	40.0	94.6	95.3	95.5	115,000

primary. These leakage fluxes produce reactance in each winding. The combined effect of the resistance and reactance produces an impedance drop in each winding when current flows. These impedance drops produce a slight drop in voltage with load.

Transformer Testing. Transformer regulation and losses are so small that it is far more accurate to compute the regulation and efficiency than to determine them by actual measurement. The necessary measurements and computations are comparatively simple, and little power is involved in making the tests. In the **open-circuit** test, the power input to either winding is measured at its rated voltage. Usually it is more convenient to make this test on the low-voltage winding, particularly if it is rated at 110, 220, or 550 volts. The open-circuit power practically all goes to supply the core losses, consisting of eddy-current and hysteresis losses. Let this value of power be P_0. The eddy-current loss varies as the square of the voltage and frequency; the hysteresis loss varies as the 1.6 power of the voltage, and directly as the frequency (see p. 1704). In the **short-circuit** test one winding is short-circuited, and the current in the other is adjusted to near its rated value. The voltage V_c, the current I_1, and the power input P_c are measured. When one winding of a transformer is short-circuited, the voltage across the other winding is 3 to 4 percent of rated value when rated current flows. Since a voltage range of from 110 to 250 volts is best adapted to measuring instruments, that winding whose rated voltage, multiplied by 0.03 or 0.04, is closest to this voltage range should be used for making the short-circuit test, the other winding being short-circuited. Practically all the power on short circuit goes to supply the copper loss of primary and secondary. If the measurements are made on the primary,

$$R_{01} = P_c/I_1^2 \quad (95); \qquad Z_{01} = V_c/I_1 \quad (96); \qquad X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} \quad (97)$$

where R_{01}, Z_{01}, and X_{01} are the equivalent resistance, impedance, and reactance referred to the primary. Also $R_{02} = R_{01}(n_2/n_1)^2$; $Z_{02} = Z_{01}(n_2/n_1)^2$; $X_{02} = X_{01}(n_2/n_1)^2$, these quantities being the equivalent resistance, impedance, and reactance referred to the secondary. If the d-c resistances, R_1 and R_2, of the primary and secondary are measured,

$$R_{01} = R_1 + (n_1/n_2)^2R_2 \quad (98); \qquad R_{02} = R_2 + (n_2/n_1)^2R_1 \quad (99)$$

The a-c or effective resistances are usually 10 to 15 percent greater than these values.

Regulation. The regulation may be computed from the foregoing data as follows:

$$V_1' = \sqrt{(V_1 \cos\theta + I_1R_{01})^2 + (V_1 \sin\theta \pm I_1X_{01})^2} \qquad (100)$$

$$\text{Regulation} = 100(V_1' - V_1)/V_1 \qquad (101)$$

V_1 = rated primary terminal voltage; $\cos\theta$ = load power factor; I_1 = rated primary current; R_{01} = equivalent resistance referred to primary; X_{01} = equivalent reactance referred to primary. The (+) sign is used with lagging current and the (−) sign with leading current. Equations (100) and (101) are equally applicable to the secondary if the subscripts are changed.

Efficiency. The only two losses in a constant-potential transformer are the core loss in watts P_0 which is practically independent of load and P_c the copper loss in watts which varies as the load current squared. The efficiency for any current I_1 is

$$\eta = V_1I_1 \cos\theta/(V_1I_1 \cos\theta + P_0 + I_1^2R_{01}) \qquad (102)$$

salient poles, the synchronous motor usually pulls into synchronism without d-c field excitation.

As with the larger sizes of induction motors, synchronous motors are usually started at reduced voltage, a compensator (Fig. 62) ordinarily being used. Sometimes the stator winding is connected in Y at starting and in Δ when running. In order to minimize line disturbances, the field is ordinarily connected while reduced voltage is being applied to the stator, and the connection to the running position is made quickly so that the motor does not have opportunity to drop out of step. All starting functions may be automatically performed by the operation of relays.

It is possible to design synchronous motors for any required values of starting and pull-in torque and, by special methods, meet the low-starting inrush current limitations imposed by power companies.

Synchronous Converter

The synchronous converter is essentially a d-c generator with slip rings connected by taps to equidistant points in the armature winding. Hence alternating current may also be taken from and delivered to the armature. The machine may be single-phase, in which case there are two slip rings and two slip-ring taps per pair of poles; it may be three-phase, in which case there are three slip rings and three slip-ring taps per pair of poles, etc. Converters are usually used to convert alternating to direct current, in which case they are said to be operating **direct**; they may equally well convert direct to alternating current, in which case they are said to be operating **inverted**. A converter will operate satisfactorily as a d-c motor, a synchronous motor, a d-c generator, an alternator, or it may deliver direct and alternating current simultaneously when it is called a **double-current generator**.

The rating of a converter increases very rapidly with increase in the number of phases owing, in part, to better utilization of the armature copper and also because of more uniform distribution of armature heating.

Table 17. Relative Outputs of Converter

Power factor, percent	Continuous-current generator	Single-phase converter	Three-phase converter	Four-phase converter	Six-phase converter
100	100	85	132	161	194
95.5	100	78	120	145	170
90	100	74	109	128	145

Because of the materially increased rating, converters are nearly all operated six-phase. The rating decreases rapidly with decrease in power factor, and hence the converter should operate near unity power factor (see Table 17). The diametrical a-c voltage is the a-c voltage between two slip-ring taps 180 electrical degrees apart. With a two-pole closed winding, that is, a winding that closes on itself when the winding is completed, the diametrical a-c voltage is the voltage between any two slip-ring taps diametrically opposite each other.

With a sine-voltage wave, the d-c voltage is the peak of the diametrical a-c voltage wave. The voltage relations for sine waves are as follows: d-c volts, 141; single-phase (diametrical), 100; three-phase, 87; four-phase, diametrical, 100; four-phase, adjacent taps, 71; six-phase, diametrical, 100; six-phase, adjacent taps, 50. These relations are obtained from the sides of polygons inscribed in a circle having a diameter of 100 volts, as shown in Fig. 64.

Equation (102) applies equally well to the secondary if the subscripts are changed. The maximum efficiency occurs when the core and copper losses are equal.

All-day Efficiency. Since transformers must usually be on the line 24 hr per day, part of which time the load may be very light, the all-day efficiency is important. This is equal to the total energy or watthour output divided by the total energy or watthour input for the 24 hr. That is,

$$\eta = \frac{(V_1 I_1 \cos \theta_1)t_1 + \cdots}{(V_1 I_1 \cos \theta_1)t_1 + \cdots + (I_1^2 R_{e1})t_1 + \cdots + 24P_0} \tag{103}$$

where t_1 is the time in hours that load $V_1 I_1 \cos \theta_1$ is being delivered, etc.

Polyphase Transformer Connections. Three-phase transformer banks may be connected Δ-Δ, Δ-Y, Y-Y, and Y-Δ. The Δ-Δ connection is very common particularly at the lower voltages and has the important advantage that the bank will operate V-connected if one transformer is disabled. The Δ-Y connection is advantageous for stepping up to high voltages since the secondary of the transformers need be wound only for 58 percent ($1/\sqrt{3}$) the line voltage; it is also necessary when a four-wire three-phase system is obtained from a three-wire three-phase system since "a floating neutral" on the secondary cannot occur. The Y-Y system may be used for stepping up voltage. It should not be used for obtaining a three-phase four-wire system from a three-phase three-wire system, because of the "floating neutral" on the secondary and the resulting high degree of unbalance of the secondary voltages. The Y-Δ system may be used to step down high voltages, the reverse of the Δ-Y connection. In the Δ-Y and Y-Δ systems the ratio of line voltages is obviously not that of the individual transformers. Because of different phase displacement between primaries and secondaries, a Δ-Δ bank cannot be connected in parallel (on both sides) with a Δ-Y bank, etc., even if they both have the correct voltage ratios between lines (see p. 1713).

Three-phase transformers combine the magnetic circuits of three single-phase transformers so that they have parts in common. A material saving in cost, in weight, and in space results, the greatest saving occurring in the case and oil. The advantages of three-phase transformers are often outweighed by their lack of flexibility. The failure of a single phase shuts down the entire transformer. With three single units, one unit may be readily replaced with a single spare. The primaries of single-phase transformers may be connected in Y or Δ at will and the secondaries properly phased. The primaries, as well as the secondaries of three-phase transformers, must be phased.

For the transformation of moderate amounts of power from three-phase to three-phase, two transformers employing either the V- or the **T-connection** (Fig. 59) may be used. The ratings of these systems are only 58 percent of the rating of the system using three similar transformers, one for each phase.

To transform from two- to three-phase or the reverse, the **T-connection** (Fig. 60) is used. To make the secondary voltages symmetrical a tap (called a **Scott tap**) is brought out at 87 percent ($\sqrt{3}/2$) of the primary winding of the auxiliary transformer as shown in Fig. 60. With balanced no-load voltages the voltages become slightly unbalanced even under a symmetrical load, owing to unequal phase differences in the individual coils. The three-phase neutral 0 is one-third the distance along the auxiliary transformer from the junction (see also pp. 1713, 1714).

At unity power factor and 100 percent efficiency, the ratio of alternating to direct current is as follows: 2 slip rings, 1.41; 3 slip rings, 0.94; 4 slip rings, 0.71; 6 slip rings, 0.47. At efficiency η and power factor p.f., divide by these quantities. Twenty-five cycle converters are slightly more efficient than 60-cycle converters.

The d-c voltage of converters may be controlled a limited amount by varying the field excitation. This, however, changes the power factor simultaneously. Converters are compounded. In split-pole converters, the space distribution of the field magnetism is altered by subdividing the field poles into two or three sections. By varying the ampere-turns of the sections the wave form and ratio of transformation may be varied to a limited extent. With large units, the most satisfactory method is to use an alternator of smaller rating and of the same number of poles mounted on the same shaft. This alternator, called a **booster**, may boost or buck the converter voltage. Converters operate satisfactorily in parallel. When used to convert alternating to direct current, the machine must be in synchronism with the alternating supply. The converter may be started from the a-c end in much the same manner that synchronous motors are started. Occasionally the machine is brought to speed as a d-c motor and synchronized. When operated inverted (direct current to alternating current) some centrifugal or electrical device must be employed to prevent the converter from running away since a highly inductive load weakens the field through armature reaction and causes the speed to increase.

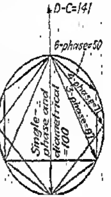


FIG. 64.—EMF Relations in Converter.

Converters are cheaper, more efficient, and occupy less floor space than motor-generator sets. They are much less flexible in the matter of voltage and power-factor control. Where they cannot operate near unity power factor and where otherwise transformers are not necessary, their advantage over a motor-generator set is doubtful.

Industrial synchronous converters have efficiencies at one-half (full) load from 90 to 92.5 (92.4 to 94.3) percent, the larger sizes having the higher efficiencies. Synchronous booster efficiencies at one-half (full) load vary similarly from 91.3 to 93 (93.8 to 94.4) percent (see also Fig. 67).

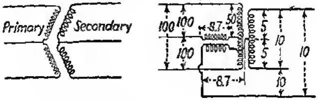
For the conversion of alternating to direct current at 600 volts (d-c) and higher, mercury-arc metal-tank rectifiers, rather than motor-generator sets and synchronous converters, are being used in new installations (see p. 1753).

Rating of Electrical Apparatus

The rating of electrical apparatus is almost always determined by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. It is permissible, as far as temperature is concerned, to overload the apparatus so long as the safe temperature is not exceeded. The A.I.E.E. Standards, No. 5, July, 1925, classify **insulating materials** in three general classes: Class O—cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in oil; Class A—cotton, silk, paper, and similar organic materials when so treated or impregnated as to increase the thermal limit, or when permanently immersed in oil, and also enameled wire; Class B—inorganic materials such as mica and asbestos in built-up form combined with binding substances.

The limiting temperature rises are as follows: Class O—no agreement reached but maximum temperatures should not exceed 100 C. Class A—(1) armature windings, wire field windings and all windings other than (2), 55 C; (2) single-layer field windings with exposed uninsulated surfaces and bare copper windings, 65 C; (3) cores and mechan-

An autotransformer, also called compensator or balancing coil, consists essentially of a single coil linking a magnetic circuit. Part of the energy is transformed, and the remainder flows through conductively. Suitable taps are provided so that, if the primary voltage is applied to two of the



V-connection. T-connection.

FIG. 59.—Transformer Connections for Transforming Moderate Amounts of Three-phase Power.

taps, a voltage may be taken from any other two taps. The ratio of voltages is equal practically to the ratio of the turns between their taps. An autotransformer should be installed only when the ratio of transformation is not large. The ratio of power transformed to total power is $1 - n$, where n is the

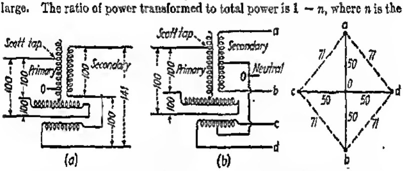


FIG. 60.—Connections for Transforming from Three-phase to Two- and Four-phase.

ratio of low-voltage to high-voltage emf. This gives the saving over the ordinary transformer and is greatest when the ratio is not far from unity. Figure 61(*a*) shows 100 kw being changed from 3,300 to 2,300 volts; 30.3 kw only are being actually transformed, and the remainder of the power flows through

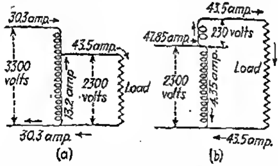


FIG. 61.—Autotransformer.

conductively. Figure 61(*b*) shows how an ordinary 10:1, 10-kw lighting transformer may be connected to boost 110 kw 10 percent in voltage. In Fig. 61(*b*), however, the 230-volt secondary must be insulated for 2,300 volts to the core and ground. The voltage may likewise be reduced by reversing

ical parts in contact with or adjacent to insulation, 55 C; (4) commutators and collector rings, 65 C. Class B—all temperatures 20 C higher than Class A. The temperature measurements are determined with a thermometer. Ordinarily an ambient temperature of 40 C is used.

Efficiency of Electrical Apparatus

The losses in electrical apparatus are iron losses, copper or I^2R losses, and, where there are revolving parts, friction losses. In constant-speed machinery or machinery operating under constant voltage as is generally the case, the iron losses are nearly constant regardless of load. The copper losses are proportional to the square of the load current. To determine the efficiency of electrical apparatus the output and input are sometimes measured. This is frequently impracticable, and the method is open to the objection that it is difficult to measure mechanical input or output accurately. A more accurate and satisfactory method is to measure the electrical input or output, depending on whether the machine is motor or generator, and calculate the losses.

In a motor or generator, the constant losses are hearing and brush friction, windage, armature hysteresis and eddy-current losses (called stray power), and the field copper losses. The stray power may be determined very closely by running the machine light, whether motor or generator, at the same induced emf and speed as under load conditions and measuring the armature input. From the measured resistance of the armature (including brushes and brush-contact resistance) the armature $I_a^2R_a$ losses may be calculated for any assumed output or input. If the machine is a motor, the output is

$$(\text{Input} - \text{losses}) = VI - I_a^2R_a - VI_f - S_p \qquad (109)$$

where V is the terminal voltage, I the line current, I_f the shunt-field current, and S_p the stray power. Since I includes the field current I_f,

$$\text{Output} = VI_a - I_a^2R_a - S_p \qquad (110)$$

With a generator,

$$\text{Input} = (\text{output} + \text{losses}) = VI + I_a^2R_a + VI_f + S_p \qquad (111)$$

The efficiency η = (input − losses)/input = output/(output + losses) may be calculated from Eqs. (109), (110), and (111). Since shunt motors are frequently used to drive pumps, compressors, and similar apparatus, Eq. (110) is very useful in determining their outputs (see pp. 1724 and 1727 for the efficiencies of generators and motors).

Industrial Applications of Motors

Alternating or Direct Current. The induction motor, particularly the squirrel-cage type, is preferable to the d-c motor for constant-speed work, for the initial cost is less and the absence of a commutator reduces maintenance. Also there is less fire hazard in many industries, such as saw mills, textile mills, and powder mills. The use of the induction motor in such places as cement mills is advantageous since with d-c motors the grit makes the maintenance of commutators difficult.

For variable-speed work like cranes, hoists, elevators, and for adjustable speeds, the d-c motor characteristics are superior to induction-motor characteristics. Even then, it may be desirable to use induction motors since their less desirable characteristics are more than balanced by their simplicity and the fact that a-c power is available. Direct-current power is supplied at 115 and 230 volts, 230 volts being preferable because of the saving in copper. In certain railway shops where 550 volts is available, 550 volt motors may be used, but their use, particularly in small sizes, is undesirable because of com-

the 230-volt coil. An autotransformer should never be used when it is desired to keep dangerous primary potentials from the secondary. It is used for starting induction motors (Fig. 62) and for a number of similar purposes.

Data on Transformers. Single-phase 55-deg self-cooled oil-insulated transformers for 2,300 volt primaries, 230—115-volt secondaries, and in sizes from 5 to 200 kva for 60(25) cycles have efficiencies from one-half to full load of about 98 (97–98.7) percent and regulation of 1.5 (1.1 to 2.1) percent with p.f. = 1, and 3.5 (2.7 to 4.1) percent with p.f. = 0.8. Power transformers with 13,200-volt primaries and 2,300-volt secondaries in sizes from 667 to 5,000 kva and for both 60 and 25 cycles have efficiencies from one-half to full load of about 99.0 percent and regulation of about 1.0 (4.2) percent with p.f. = 1(0.8).

Alternating-current Motors

Polyphase Induction Motor. The polyphase induction motor is the most common type of motor used. It ordinarily consists of a stator which is wound in the same manner as an alternator stator. If two-phase current is supplied to a two-phase winding or three-phase current to a three-phase winding, a rotating magnetic field is produced in the air gap. The number of poles which this field has is the same as the number of poles that an alternator employing the same stator winding would have. The speed of the rotating field, or the *synchronous speed*,

$$N = 120f/P \text{ rpm} \qquad (104)$$

where f is the frequency and P the number of poles.

There are two general types of rotors. The **squirrel-cage type** consists of heavy copper bars short-circuited by end rings, or the bars and end rings may be an integral aluminum casting. The **wound rotor** has a polyphase winding of the same number of poles as the stator, and the terminals are brought out to slip rings so that external resistance may be introduced. The rotor conductors must be cut by the rotating field, hence the rotor cannot run at synchronous speed but must slip. The *slip*, $s = (N - N_2)/N$ (105) where N_2 = the rotor rpm. The rotor frequency

$$f_2 = sf \qquad (106)$$

The torque is proportional to the air-gap flux and the components of rotor current in space-phase with it. The rotor currents tend to lag the emfs producing them, because of the rotor-leakage reactance. From Eq. (106) the rotor frequency is low when the motor is running near synchronous speed, and hence there is a large component of rotor current in space-phase with the flux. With large values of slip the increased rotor frequency increases the lag of the rotor currents behind their emfs, and hence considerable space-phase difference between these currents and the flux develops. Therefore, even with large values of current the torque may be small. The torque of the induction motor increases with slip until it reaches a maximum value called the **breakdown torque**, after which the torque decreases (see Fig. 63). The breakdown torque varies as the square of the voltage, inversely as the stator impedance and rotor reactance, and is independent of the rotor resistance.

The squirrel-cage motor develops but little torque on starting ($s = 1.0$) even though the current may be three to seven times rated current. For any value of slip the torque of the induction motor varies as the square of the voltage. The torque of the squirrel-cage motor on starting is inherently low

mutator difficulties. Alternating current is almost always 60 cycles, three-phase, and 220, 440, and 550 volts are all used for smaller motors. For larger motors, 1,150, 2,300, and even 6,600 volts may be used. Where both lights and motors are to be supplied from the same a-c system, the 208–120 volt four-wire three-phase system is now in common use. This gives 208 volts 3-phase for the motors, and 120 volts to neutral for the lights.

Electric Drives

Cranes and Hoists. The d-c series motor is best adapted to cranes and hoists. When the load is heavy the motor slows down automatically and develops increased torque thus reducing the peaks on the electrical system. With light loads, the speed increases rapidly, thus giving a lively crane. The series motor is also well adapted to moving the bridge itself and also the trolley along the bridge. Where alternating current only is available and it is not economical to convert it, the slip-ring type of induction motor, with external-resistance speed control, is the best type of a-c motor. Squirrel-cage motors with high-resistance end rings to improve the starting torque are used occasionally (also see Ilgner system, p. 1730).

Woodworking Machinery. Circular saws are usually driven by a belted squirrel-cage induction motor running at 1,140 and 1,720 rpm, the speeds of the saws being much greater than those of the motors. **Band saws** have considerable flywheel effect and require motors having high starting torque. Slip-ring motors may be used, but squirrel-cage motors having 7 to 8 percent slip are desirable. An added advantage is the fact that with heavy load, the motor slows down, thus utilizing the flywheel energy of the saw. **Planers** may be driven at high speed by belted squirrel-cage induction motors. In modern practice the motor is an integral part of the machine, using direct drive. Since speeds of 6,000 to 10,000 rpm are necessary and at 60 cycles the maximum speed obtainable is 3,600 rpm, the two-pole motors are supplied at higher frequencies by frequency changers. This is economical only with a number of planers.

Pumps. Single-acting **reciprocating pumps** should be driven with compound motors and duplex and triplex pumps with shunt motors, if direct current is used. Squirrel-cage and slip-ring motors are satisfactory with a-c supply. To reduce starting torque a by-pass in the pump is frequently opened until the motor comes up to speed. Constant head requires constant torque, and variable capacity under these conditions necessitates variable speed. For efficient operation, field control should be used with d-c motors and pole changing with induction motors.

Centrifugal pumps may be driven by shunt, compound, squirrel-cage, and slip-ring motors. Since such pumps require very small starting torque, general-purpose squirrel-cage motors make an ideal drive.

Compressors may be driven with shunt, squirrel-cage, slip-ring, and synchronous motors. With 100 hp and greater, direct connection is preferable. Many types of compressors require flywheel effect, particularly with synchronous-motor drive. Synchronous motors are being widely used for compressor drive because of their desirable power-factor characteristics. High-torque synchronous motors have been developed which can replace most induction motors of 50 hp and greater.

Rectifiers

Direct current is frequently necessary for such purposes as electric railways, electrolytic work, and charging batteries, when only alternating current is available. With large amounts of power either induction or synchronous motor-generator-sets or synchronous converters and also mercury-arc rectifiers may be used for converting the power. For small amounts of power, rectifiers are, as a rule, less expensive and more efficient.

With a single rectifier unit [Fig. 65(a)] the negative half-wave is eliminated entirely, and, with a noninductive load, current flows only on alternate half-cycles [Fig. 65(b)]. This is called **half-wave** rectification. With two rectifier units and a center-tap connection o in the secondary of the trans-

but is still further reduced in the larger motors because of the necessity for applying reduced voltage.

Polyphase squirrel-cage motors are used for constant-speed work. They are being used more commonly on account of their rugged construction and the absence of moving electrical contacts which makes them suitable for operation when exposed to inflammable dust or gas. General-purpose squirrel-cage motors have starting torques of about 1.5 times full load torque at rated voltage. The highest torques occur at the higher rated speeds. The locked rotor currents vary between four and seven times full-load current. Special motors with high-resistance rotors are built up to approximately 50 hp usually for intermittent duty on elevators, punch presses, and similar applications. These generally give the maximum torque at starting and operate at approximately 15 to 20 percent below synchronous speed at full load. They give rapid acceleration, and their starting current is relatively low. Up to 7.5 hp, the squirrel-cage motor may usually be connected directly across the line. If the motor is protected from overload by fuses, a double-throw switch should be used so that the large starting current does not flow in the fuses. In modern practice, motors are protected by temperature overload relays which operate thermally to trip the circuit breaker. Since a time element is involved in the operation of such relays, they do not respond to large starting currents, because of their short duration.

By some modifications in conventional design, motors as large as 200 hp can be connected directly across the line without the current exceeding 3.5 times rated current. A **double-squirrel-cage motor** is an example. In this type of motor there is a high-resistance winding in the top of the rotor slots and a low-resistance winding in the bottom of the slots. The low-resistance winding is made to have a high leakage reactance, either by separating the windings with a magnetic bridge or by making the slot very narrow in the area between the two windings. On starting, because of the high reactance of the low-resistance winding, most of the rotor current will flow in the high-resistance winding, giving the motor a large starting torque. As the rotor approaches the low value of slip at which it normally operates, the rotor frequency and hence the rotor reactance become low and most of the rotor current flows in the low-resistance winding. Hence the motor operates with a low value of slip. It thus has the excellent operating characteristics of the single-squirrel-cage motor and at the same time has a large starting torque.

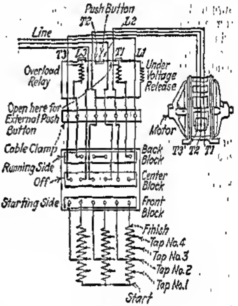


FIG. 62.—Connections of General Electric Compensator.

In order to reduce the line current in the larger ratings a polyphase **auto-transformer or compensator** is used (Fig. 62). On starting, the three

former [Fig. 66(a)], current can flow to the positive terminal of the load through unit 1 when terminal a is positive and, likewise, through unit 2 to the positive terminal of the load when terminal b is positive. This gives **full-wave** rectification [Fig. 66(b)]. If a smoothing inductance is connected in series with the load, the current is prevented from going to zero by the inertia effect of the inductance, and the pulsations in the rectified wave become small ripples [Fig. 66(c)].

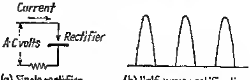


FIG. 65.—Single Rectifier Unit and Half-wave Rectification.

Full-wave rectification without a center tap may be obtained by means of the **bridge circuit** (Fig. 67). Four rectifying units are connected to form four bridge arms ab, ac, bd, and cd as shown. When line $a'a$ is positive, the current path is a, b; load, c, d. When line $d'd$ is positive, the current path is d, b; load, c, a. Current thus always enters the load at the positive terminal. The same full wave as shown in Fig. 66(b) will thus be obtained, and smoothing inductance may be used to reduce the current pulsations as in Fig. 66(c).

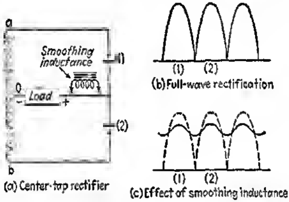


FIG. 66.—Two Rectifier Units and Center-tap Connection.

The **mercury-arc rectifier** depends on the rectifying action of mercury vapor for its operation. Figure 68 shows a typical single-phase rectifier. The glass tube is exhausted to a vacuum of 0.001 to 0.0005 mm. The iron cathode C is in contact with a pool of mercury. The anodes A_1 and A_2 are graphite. Current enters anodes A_1 and A_2 during alternate half-cycles, flows to the cathode and out to the load. Inductance in series with the load is necessary to prolong the current flow beyond the half-cycle of voltage and thus maintain the arc. The rectifying action is due to the fact that the arc maintains the cathode hot spot in the pool of mercury, making it a source of high electron emission. The electrons are attracted to each anode as it becomes positive and result in current flow from anode to cathode during this time. The direction of current flow is opposite to the direction of flow of the electrons. A starting anode A_3 in series with resistance R is necessary to start the arc. Figure 69 shows a three-phase rectifier being supplied by three secondaries of a three-

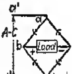


FIG. 67.—Bridge Circuit for Full-wave Rectification.

coils are connected in Y across the line, and the motor is connected across the taps which reduce voltage at the motor terminals. A double-throw switch is so arranged that the motor, after it has been started, can be connected across the full line potential and the autotransformer disconnected from circuit. The double-throw switch, which is equipped with sliding, self-wiping contacts, is immersed in oil. Means are provided for preventing the switch being thrown to the running position without first being thrown to the starting position. The taps are generally brought out from the windings so as to give voltages of approximately 40, 58, 70, and 80 percent of line voltage. To limit the current to as low a value as possible, the lowest taps that will give the motor sufficient voltage to supply the required starting torque should be used. *As the torque of an induction motor varies as the square of the voltage*, the compensator produces a very low starting torque.

Resistances in series with the stator may also be used to start squirrel-cage motors. They are inserted in each phase and are gradually cut out as the motor comes up to speed. The resistors are generally made of wire-type resistance units or of graphite disks inclosed within heat-resisting porcelain-lined iron tubes. The disadvantage of resistors is that if the motor is started slowly the resistor becomes very hot and may burn out. Resistor starters are less expensive than autotransformers. Their application is to motors that start with light loads at infrequent intervals.

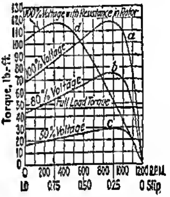


Fig. 63.—Speed-torque Curve of 10-hp, 60-cycle, 1140-rpm Induction Motor. Curves (*a*), (*b*), (*c*)—Squirrel-cage Characteristics; Curve (*d*) Resistance in Wound-rotor Motor.

By introducing resistance into the rotor circuit through slip rings, the rotor currents may be brought nearly into phase with the air-gap flux and, at the same time, any value of torque up to maximum torque obtained. As the rotor develops speed, resistance may be cut out until there is no external resistance in the rotor circuit. The speed may also be controlled by inserting resistance in the rotor circuit. However, like the armature-resistance method of speed control with shunt motors (see p. 1729), *it is inefficient and gives poor speed regulation.* Figure 63 shows graphically the effect on the torque of introducing resistance into the rotor circuit. The wound-rotor motor is used where large starting torque is necessary as in railway work, hoists, and cranes. It has better starting characteristics than the squirrel-cage motor, but, because of the necessarily higher resistance of the rotor, it has greater slip even with the rotor resistance all cut out. Obviously, the wound rotor, controller, and external resistance make it more expensive than the squirrel-cage type.

One **disadvantage of induction motors** is that they take lagging current and the power factor at half load and less is low. The speed-load and torque-load characteristics of induction motors are almost identical with those of the shunt motor. The speed decreases slightly to full load, the slip being from 10 percent in small motors to 2 percent in very large motors. The torque is almost proportional to the load nearly up to the breakdown torque. The power factor is 0.8 to 0.9 at full load. The direction of rotation of any three-phase motor may be reversed by interchanging any two stator wires.

Speed Control of Induction Motors. The induction motor inherently is a constant-speed motor. From Eqs. (104) and (105) the rotor speed

phase transformer system. Current enters the tube at any of the three graphite electrodes A, A', A'', and leaves at the cathode C. *Multiphase rectifiers require no inductance to sustain the* arc, although a smoothing inductance (Fig. 69) is usually connected in the load circuit to eliminate the current ripples. Single-phase glass-tube units are available up to 50 amp at 110 volts. The voltage drop across the arc is nearly constant at approximately 25 volts, irrespective of current, so that the efficiency of mercury arcs increases with voltage.

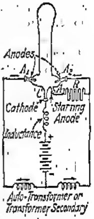


FIG. 68.—Single-phase Mercury-arc Rectifier.

Mercury-arc rectifiers with metal tanks for supplying large amounts of power have been developed. These usually operate six-phase. They are built in units of 300 kw at 250 volts up to 3,000 kw at 600 volts and to 4,000 kw at 3,000 volts and have been used successfully for street-railway and other d-c power supply. At 600 volts and over, they compare very favorably with the synchronous converter (see Fig. 70).

For 600 volts and higher the anodes, usually 6, 12, or 18 *in number, are mounted within a single tank and a single* mercury pool constitutes the common cathode. The voltage drop from anode to cathode is from 20 to 25 volts and does not vary with the current but remains nearly constant. Hence the efficiency increases with increase in voltage. At 250 volts the stated values of voltage drop would give an efficiency in the neighborhood of 90 percent. To decrease the voltage drop the **ignitron** is used for the lower voltages. Each anode and its cathode are mounted within a single steel tank; the cathode is not usually insulated from the tank. The single tank permits much closer spacing of anode and cathode and reduces the voltage drop. The arc is ignited each half-cycle by a high-resistance rod *which dips into the mercury pool*, the rod being energized at the proper instant. Its action is not unlike that of a spark plug.

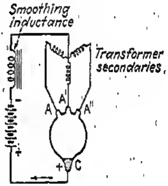


FIG. 69.—Three-phase Mercury-arc Rectifier.

Figure 70 shows the over-all efficiencies of metal-tank rectifiers, a synchronous converter, and a synchronous motor-generator set. All units have the same power rating of 3,000 kw; the losses in the transformers and auxiliary equipment are included.

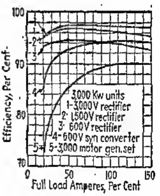


FIG. 70.—Comparative Efficiencies of Converting Machinery.

The **Tungar rectifier** consists of a glass bulb filled with inert argon gas at low pressure, a tungsten filament which can be heated and used as a cathode, and an additional graphite electrode which is the anode (Fig. 71). This rectifier operates on the principle of electron emission and ionization by collision. *When the anode is positive with respect to the filament, electrons* emitted by the hot filament are attracted to the anode and ionize, by collision, the molecules of the gas. The negative ions go to the anode and the positive

$$N_2 = 120f(1 - s)/P \tag{107}$$

The speed can be changed only by changing the frequency, poles, or slip. In some applications where the motors constitute the only load on the alternators, as with electric propulsion of ships, their speed may be changed by changing the frequency. Even then the range is limited, for both turbines and alternators must operate near their rated speeds for good efficiency. By employing two distinct windings or by reconnecting a single winding by switching it is possible to change the number of poles. Complications prevent more than two speeds being readily obtained in this manner. Elevator motors frequently have two distinct windings, and in the U. S. S. "Tennessee" the motors also have two windings, one wound for 24 and the other for 36 poles. In the U. S. S. "New Mexico" the windings are reconnected for 24 and 36 poles by switching. Another objection to changing the number of poles is the fact that the design is a compromise, and sacrifices of desirable characteristics usually are necessary at both speeds.

Table 15. Induction Motor Data
(Westinghouse Electric & Mfg. Co.)
Three phase, 220 volt, 60 cycle, 1,750 rpm

Horse power	Weight, lb	Amp	Power factor, percent			Efficiency, percent		
			½ load	¾ load	4/4 load	½ load	¾ load	4/4 load
				Squirrel-cage Type				
1	65	3.31	60	71	78	71	75	76
2	100	5.70	71	81	86	78	80	80
5	159	13.4	76	84	87	83	84	84
10	255	26.2	81	87	88	85	86	85
20	418	52.2	85	88	89	84	85	84
40	804	98	86	88.5	89.5	89.5	90	89.5
100	1,769	238	85	89.5	90.5	89	90.5	91
200	3,225	463	91	93	94	87	90	90
				Wound-rotor Type				
5	220	14.3	72.5	80	82.5	78	79	79.5
10	336	26.6	69	79	83	83	84.5	85
25	578	62.9	75	83.5	87	84	86	86.5
50	991	118.4	84	89	90	86	88	88
100	2,618	233	88	90.5	89.5	86	88	88
200	3,902	473	89	91	92	87	89	90
				Three phase, 2,300 volt, 60 cycle, 1,775 rpm Squirrel-cage Type				
300	3,200	67	87.5	89.3	90.6	90.0	91.8	92.7
700	5,200	151	90.2	92.0	92.9	91.6	93.0	93.6
1,000	7,700	212	91.2	92.8	93.7	92.2	93.4	94.0
				Wound-rotor Type				
300	3,900	67	84.7	89.0	90.0	90.0	91.8	92.7
700	5,750	151	88.5	91.8	92.6	91.6	93.0	93.6
1,000	8,450	212	90.0	92.8	93.5	92.2	93.4	94.0

(See Table 22, p. 1764 for the approximate full-load currents of motors.)

ions to the filament, current flow thus being from anode to filament. During the next half-cycle when the filament is positive with respect to the anode, the electrons are attracted into the filament, ionization ceases, and no current flows. Such rectifiers have a maximum rating of 75 volts and 6 amp, and the efficiency is 70 percent approximately. They are used extensively for charging storage batteries.

Electrolytic rectifiers consist of an aluminum plate and a lead plate immersed in ammonium phosphate or sodium borate solution. Current can pass from the lead plate, but not from the aluminum to the solution. By using an aluminum plate on each side of the lead and connecting the load between the lead plate and the center of the transformer winding, full-wave rectification is obtained.

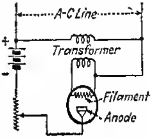


FIG. 71.—Connections for Tungar Rectifier.

The **copper oxide** or **Rectox** rectifier operates on the principle that a layer of cuprous oxide on the surface of copper permits the passage of electrons from the copper to the oxide but prevents their passage in the opposite direction. The conventional direction of current flow is opposite that of the electrons. In the commercial rectifier, the units consist of washers 1⅛ to 1½ in. diam, mounted on an insulating rod. A soft metal washer is placed between copper washers to produce a more uniform pressure on the oxide. The connections are shown in Fig. 72, the "bridge circuit" (Fig. 67) being used since it gives full-wave rectification. This type of rectifier is adapted only to small power and optimum conditions; the efficiency is 70 to 75 percent. Since it is inexpensive and requires negligible maintenance, it is the most widely used rectifier for small amounts of power.

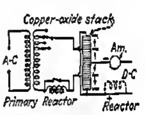


FIG. 72.—Copper-oxide Rectifier with Bridge Circuit.

Switchboards

Switchboards may, in general, be divided into two classes: **direct-control** and **remote-control boards**. With direct-control boards the switches, bus bars, meters, and other apparatus are mounted on or near the board; with remote-control boards, all bus bars, switches, and similar equipment are located some distance from the board, usually in separate compartments or rooms, and only the control-circuit apparatus is on the board. For low-voltage circuits up to and including 750 volts, direct-control panels are generally used. For circuits of 1,100 volts and over, no live switch parts or apparatus should appear on the face of the board and all switches should be of the remote-control type. Most **switchboard panels** are now standardized as to size and thickness. Standard switchboards for central stations are usually 90 in. high, and each panel consists of two or three slabs. The lengths of the General Electric Co. standard slabs for a two-piece panel are 62 and 28 in., respectively, for upper and lower slabs, and the corresponding dimensions for Westinghouse slabs are 65 and 25 in. Marble is still used for switchboard panels, but its use is becoming less common. Slate, which is less expensive than marble, is used extensively and may be made moisture- and oil-proof by black enameling or by finishing in black marine. Rolled sheet metal and ebony asbestos board are used considerably.

The change of slip by introducing resistance into the rotor circuit has been discussed under the wound-rotor motor (see p. 1743). It is possible to introduce counter emfs into the rotor circuit at slip frequency, by means of special commutator machines. The power, which in resistance control is dissipated as heat, is converted into mechanical power and a portion returned to the line. This method of control is only practicable in very large units and its applications are limited. In the **concatenation method** of speed control the rotors of two wound-rotor motors are mechanically coupled together. The stator of the second motor is supplied with power from the rotor slip rings of the first motor. If both motors have the same number of poles and equal number of turns in both stator and rotor, the set will operate efficiently at a little less than half synchronous speed. Because of its complications this method is little used in this country.

Induction Generator. If an induction motor, while connected to a source of power, is driven above synchronous speed, it becomes an **induction, or asynchronous, generator** and returns electrical energy to the line without any change in connections. When driven above synchronous speed, the rotor conductors cut the rotating field in a direction opposite to that when operating as a motor, and hence the mechanical power applied to the shaft is converted into electrical power. The load increases with the negative slip; this permits induction generators to be driven by prime movers without governor control. On short-circuit, the induction generator has the desirable characteristic of not delivering any power. It must always be used in parallel with some synchronous apparatus. Since it must take lagging current from the line for its own excitation, and in addition cannot deliver any lagging current to the system, the induction generator is little used for power supply. Induction motors are frequently used in railway work, especially in mountain systems where it is advantageous on the down-grades to permit the motors to operate as induction generators, thus acting as brakes and in addition returning energy to the line.

Single-phase Induction Motor. If one phase of a three-phase motor is opened while the motor is running, it will continue to operate, but with a rating of only about 60 percent of its three-phase rating. Likewise, if one phase of a two-phase motor is similarly opened, the motor will continue to operate at one-half its two-phase rating. In both cases the motor continues to operate single phase. It will not, however, start single phase. The single-phase motor runs in the direction in which it is started. There are several methods of starting single-phase induction motors. Short-circuited turns, or "shading coils," may be placed around the pole tips which retard the time phase of the flux in the pole tip, and thus a weak torque in the direction of rotation is produced. A high-resistance starting winding placed to produce poles between the main poles provides a rotating field which is weak but is sufficient to start the motor. This is called the "split-phase" method. In order to minimize overheating this winding is ordinarily cut out by a centrifugal device when the armature reaches speed. In the larger motors a repulsion-motor start is used. The rotor is wound like an ordinary d-c armature with a commutator, but with short-circuited brushes pressing on it axially rather than radially. The motor starts as a repulsion motor, developing high torque. When it nears its synchronous speed, a centrifugal device pushes the brushes away from the commutator, and at the same time causes the segments to be short-circuited, thus converting the motor into a single-phase induction motor.

Switchboards should be erected at least 3 or 4 ft from the wall. For panels supplying circuits of 750 volts or less the frames should be insulated from ground, and for higher voltages all frames should be grounded. For low-potential work, the conductors on the rear of the switchboard are usually made up of flat copper strip, known as **bus-bar** copper. The size required is based upon a current density of about 1,000 amp per sq in. Table 18 gives the sizes of horizontal bus bars for various currents. When the current is greater than the values given in the table, a laminated bar is built up of thin strips separated from each other to give a greater radiating surface.

The current-carrying capacity is calculated on the basis of 50 percent load factor for densities which under average conditions of radiation would give a temperature rise of about 10 C. With a load factor of 100 percent the current densities given should be halved. For vertical bus bars the values of current should be reduced 15 to 20 percent.

Table 18. Copper Bar Data
(The Cutter Co.)

Size, in.	Amperes	Amp per sq in.	Ohms per 1,000 ft	Wt, lb per ft	Size, in.	Amperes	Amp per sq in.	Ohms per 1,000 ft	Wt, lb per ft
1 × ¼	433	1,732	0.0336	0.97	2½ × ½	1,500	1,200	0.00672	4.86
1¼ × ¼	530	1,696	0.0269	1.21	2½ × ⅝	1,715	1,097	0.00537	6.07
1½ × ¼	626	1,669	0.0223	1.45	2 × ½	1,222	1,222	0.00840	3.89
1¾ × ¼	725	1,657	0.0192	1.70	No. 0000 A.W.G.	267	1,606	0.0505	0.64
1¼ × ⅜	676	1,442	0.0179	1.82	½ in. round	305	1,552	0.0428	0.76
1½ × ⅜	798	1,418	0.0149	2.18	⅝ in. round	426	1,388	0.0273	1.18
1¾ × ⅜	916	1,395	0.0128	2.54	¾ in. round	560	1,267	0.0190	1.71
2 × ⅜	1,035	1,380	0.0112	2.91	1 in. round	861	1,096	0.0107	3.05
2¼ × ⅜	1,154	1,367	0.00995	3.27					

Equipment of Standard Panels. Following are enumerated the various parts required in the equipment of standard panels for varying services:

Generator or Synchronous-converter Panel, Direct-current Two-wire System: 1 circuit breaker; 1 ammeter; 1 handwheel for rheostat; 1 voltmeter; 1 main switch (3-pole s.t. or d.t.) or 2 s.-p. switches.

Generator or Synchronous-converter Panel, Direct-current Three-wire System: 2 circuit breakers; 2 ammeters; 2 handwheels for field rheostats; 2 field switches; 2 potential receptacles for use with voltmeter; 3 switches; 1 four-point starting switch.

Generator or Synchronous-motor Panel, Three-phase Three-wire System: 3 ammeters; 1 three-phase wattmeter; 1 voltmeter; 1 field ammeter; 1 d.p. field switch; 1 handwheel for field rheostat; 1 synchronizing receptacle (4-pt.); 1 potential receptacle (8-pt.); 1 field rheostat; 1 triple-pole oil switch; 1 power-factor indicator; 1 synchronizer; 2 series transformers; 1 governor-control switch.

Synchronous-converter Panel, Three-phase: 1 ammeter; 1 power-factor indicator; 1 synchronizing receptacle; 1 triple-pole oil circuit breaker; 2 current transformers; 1 potential transformer; 1 watthour meter (polyphase); 1 governor control switch.

Induction Motor Panel, Three-phase: 1 ammeter; series transformers; 1 oil switch.

Feeder Panel, Direct-current, Two-wire and Three-wire: 1 s.-p. circuit breaker; 1 ammeter; 2 s.-p. main switches; potential receptacles (1 four-point for 2-wire panel; 1 four-point and 1 eight-point for 3-wire panel).

Feeder Panel, Three-wire, Three-phase and Single-phase: 3 ammeters; 1 automatic oil switch (3-pole for three-phase, 2-pole for single-phase); 2 series transformers; 1 shunt transformer; 1 wattmeter; 1 voltmeter; 1 watthour meter; 1 handwheel for control of potential regulator.

Exciter Panel (for 1 or 2 exciters): 1 ammeter (2 for 2 exciters); 1 field rheostat (2 for 2 exciters); 1 four point receptacle (2 for 2 exciters); 1 equalizing rheostat for regulator.

Phase Converter. If a polyphase induction motor be operating single-phase, polyphase emfs are generated in its stator. Such a machine can be utilized, therefore, for converting single-phase power into polyphase power and when so used is called a **phase converter.** Unless corrective means are utilized, the polyphase emfs at the machine terminals are somewhat unbalanced. The phase converter is used principally on railway locomotives, for it makes possible the use of three-phase driving motors with but a single trolley wire.

Alternating-current Commutator Motors. Inherently simple a-c motors are not adapted to high starting torques and variable speed and do not have good operating characteristics, particularly with regard to commutation. There are a large number of types of commutating motor that have been developed to meet the requirement of high starting torque and adjustable speed, particularly with single phase. These usually have been accompanied by compensating windings, centrifugal switches, etc., in order to overcome low power factors and commutation difficulties. With proper compensation, commutator motors may be designed to operate at a power factor of nearly unity or even to take leading current.

One of the simplest of the **single-phase commutator motors** is the a-c series railway motor such as is used on the New York, New Haven, and Hartford Railroad. It is based on the principle that the torque of the d-c series motor is in the same direction irrespective of the polarity of its line terminals. This type of motor must be used on low frequency, not over 25 cycles, and is much heavier and more costly than an equivalent d-c motor. The torque and speed curves are almost identical with those of the d-c series motor. Unlike most a-c apparatus the power factor is highest at light load and decreases with increasing load. Such motors operate with direct current even better than with alternating current. **Universal motors** are small simple series motors, usually of fractional horsepower, and will operate on either direct or alternating current, even at 60 cycles. They are used for vacuum cleaners, electric drills, and small utility purposes.

Synchronous Motor. Just as d-c shunt generators operate as motors, an alternator if connected across a-c bus bars to which power is being supplied will operate as a motor if its driving torque is removed, and mechanical power may be delivered by its shaft. Each conductor on the stator must be passed by an alternate pole every half-cycle so that at constant frequency the rpm of the motor is constant and is equal to

$$N = 120f/P \text{ rpm} \qquad (108)$$

and the speed is independent of the load.

The synchronous motor has the desirable characteristic that its power factor can be varied over wide range merely by changing the field excitation. With a weak field the motor takes a lagging current. If the load is kept constant and the excitation increased, the current decreases and the phase difference between voltage and current becomes less until the current is in phase with the voltage and the power factor is unity. The current is then at its minimum value, and the corresponding field current is called the *normal excitation.* Further increase in field current causes the armature current to lead and the power factor to decrease. Thus *underexcitation* causes the current to *lag;* overexcitation causes the current to *lead.* Because of its adjustable power factor, the motor is frequently run light merely to improve power factor or to control the voltage at some part of a power system. When so used the motor is called a **synchronous condenser.** The motor

Switches. The current-carrying parts of switches are usually designed for a current density of 1,000 amp per sq in. At contact surfaces, the current density should be kept down to about 50 amp per sq in. Knife switches are used on low-tension circuits and, in most cases, are mounted on the front of the board. They should be mounted to throw vertically, with the blade

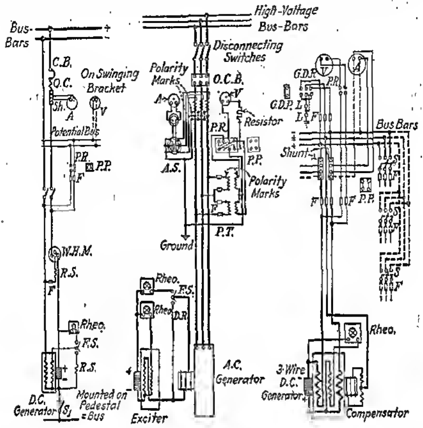


Fig. 73. Fig. 74. Fig. 75.

Switchboard Wiring Diagrams for Generators.

Fig. 73.—Diagram for 125-volt or 250-volt D-c Generator. Fig. 74.—Diagram for Three-phase Alternator and Exciter for Small or Isolated Plant. Fig. 75.—Diagram for Three-wire D-c Generator for Use in Small or Isolated Plant.

Symbols: *A*, ammeter; *A.S.*, three-way ammeter switch; *C.B.*, circuit breaker; *C.T.*, current transformer; *D.R.*, discharge resistance; *F.*, fuse; *F.S.*, field switch; *G.D.P.*, ground detector plug; *G.D.R.*, ground detector receptacle; *L.*, ground detector lamp; *O.C.*, overload coil; *O.C.B.*, oil circuit breaker; *P.P.*, potential plug; *P.R.*, potential receptacle; *P.T.*, potential transformer; *Rheo.*, rheostat; *R.S.*, resistance; *S*, switch; *Sh.*, shunt; *V.*, voltmeter; *W.H.M.*, watt-hour meter.

side of the switch dead or disconnected from the source of power when open, to lessen the danger of accidental contact. Plug switches are used on high-voltage circuits where the current is small, as with arc-lighting circuits. **Copper-brush switches** substitute a leaved copper brush with a wiping contact for the knife-blade contact and make use of an auxiliary break between carbon blocks to prevent burning of the copper leaves due to arcing. This type of switch is much used as a circuit breaker, being rendered automatic

DIFFERENTIAL AND INTEGRAL CALCULUS

DERIVATIVES AND DIFFERENTIALS

Derivatives and Differentials. A **function** of a single variable x may be denoted by $f(x)$, $F(x)$, etc. The value of the function when x has the value x_0 is then denoted by $f(x_0)$, $F(x_0)$, etc. The **derivative** of a function $y = f(x)$ may be denoted by $f'(x)$, or by dy/dx. *The value of the derivative at a given point* $x = x_0$ is the **rate of change** of the function at that point; or, if the function is represented by a curve in the usual way (Fig. 1), the value of the derivative at any point shows **the slope of the curve** (that is, the slope of the tangent to the curve) at that point (positive if the tangent points upward, and negative if it points downward, moving to the right).

The **increment** Δy (read: "delta y"), in y is the change produced in y by increasing x from x_0 to $x_0 + \Delta x$; that is, $\Delta y = f(x_0 + \Delta x) - f(x_0)$. The **differential**, dy, of y is the value which Δy would have if the curve coincided with its tangent. (The differential, dx, of x is the same as Δx when x is the independent variable.) Note that the derivative depends only on the value of x_0, while Δy and dy depend not only on x_0 but also on the value of Δx. The ratio $\Delta y/\Delta x$ represents the slope of the secant, and dy/dx the slope of the tangent (see Fig. 1). If Δx is made to approach zero, the secant approaches the tangent as a limiting position, so that the derivative $= f'(x) =$

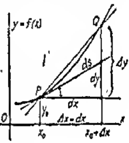


FIG. 1.

$$\frac{dy}{dx} = \lim_{\Delta x \doteq 0}\left[\frac{\Delta y}{\Delta x}\right] = \lim_{\Delta x \doteq 0}\left[\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}\right].\quad \text{Also, } dy = f'(x)\,dx.$$

The symbol "lim" in connection with $\Delta x \doteq 0$ means "the limit, as Δx approaches 0, of ..." [A constant c is said to be the limit of a variable u if, whenever any quantity m has been assigned, there is a stage in the variation-process beyond which $|c - u|$ is always less than m; or, briefly, c is the limit of u if the difference between c and u can be made to become and remain as small as we please.]

To find the derivative of a given function at a given point: (1) If the function is given only by a curve, measure graphically the slope of the tangent at the point in question; (2) if the function is given by a mathematical expression, use the following rules for differentiation. These rules give, directly, the differential, dy, in terms of dx; to find the derivative, dy/dx, divide through by dx.

Rules for Differentiation. (Here $u, v, w, \ldots$ represent any functions of a variable x, or may themselves be independent variables. a is a constant which does not change in value in the same discussion; $e = 2.71828$.)

1. $d(a + u) = du.$ 2. $d(au) = a\,du.$
3. $d(u + v + w + \ldots) = du + dv + dw + \ldots$
4. $d(uv) = u\,dv + v\,du.$
5. $d(uvw\ldots) = (uvw\ldots)\left(\frac{du}{u} + \frac{dv}{v} + \frac{dw}{w} + \ldots\right).$
6. $d\frac{u}{v} = \frac{v\,du - u\,dv}{v^2}.$
7. $d(u^m) = mu^{m-1}du.$

Thus, $d(u^2) = 2u\,du$; $d(u^3) = 3u^2 du$; etc.

8. $d\sqrt{u} = \dfrac{du}{2\sqrt{u}}$. 9. $d\left(\dfrac{1}{u}\right) = -\dfrac{du}{u^2}$.

10. $d(e^u) = e^u du$. 11. $d(a^u) = (\log_e a)a^u du$.

12. $d \log_e u = \dfrac{du}{u}$. 13. $d \log_{10} u = (\log_{10} e)\dfrac{du}{u} = (0.4343\ldots)\dfrac{du}{u}$.

14. $d \sin u = \cos u\,du$. 15. $d \csc u = -\cot u \csc u\,du$.

16. $d \cos u = -\sin u\,du$. 17. $d \sec u = \tan u \sec u\,du$.

18. $d \tan u = \sec^2 u\,du$. 19. $d \cot u = -\csc^2 u\,du$.

20. $d \sin^{-1} u = \dfrac{du}{\sqrt{1-u^2}}$. 21. $d \csc^{-1} u = -\dfrac{du}{u\sqrt{u^2-1}}$.

22. $d \cos^{-1} u = -\dfrac{du}{\sqrt{1-u^2}}$. 23. $d \sec^{-1} u = \dfrac{du}{u\sqrt{u^2-1}}$.

24. $d \tan^{-1} u = \dfrac{du}{1+u^2}$. 25. $d \cot^{-1} u = -\dfrac{du}{1+u^2}$.

26. $d \log_e \sin u = \cot u\,du$. 27. $d \log_e \tan u = \dfrac{2du}{\sin 2u}$.

28. $d \log_e \cos u = -\tan u\,du$. 29. $d \log_e \cot u = -\dfrac{2du}{\sin 2u}$.

30. $d \sinh u = \cosh u\,du$. 31. $d \operatorname{csch} u = -\operatorname{csch} u \coth u\,du$.

32. $d \cosh u = \sinh u\,du$. 33. $d \operatorname{sech} u = -\operatorname{sech} u \tanh u\,du$.

34. $d \tanh u = \operatorname{sech}^2 u\,du$. 35. $d \coth u = -\operatorname{csch}^2 u\,du$.

36. $d \sinh^{-1} u = \dfrac{du}{\sqrt{u^2+1}}$. 37. $d \operatorname{csch}^{-1} u = -\dfrac{du}{u\sqrt{u^2+1}}$.

38. $d \cosh^{-1} u = \dfrac{du}{\sqrt{u^2-1}}$. 39. $d \operatorname{sech}^{-1} u = -\dfrac{du}{u\sqrt{1-u^2}}$.

40. $d \tanh^{-1} u = \dfrac{du}{1-u^2}$. 41. $d \coth^{-1} u = \dfrac{du}{1-u^2}$.

42. $d(u^v) = (u^{v-1})(u \log_e u\,dv + v\,du)$.

Derivatives of Higher Orders. The derivative of the derivative is called the second derivative; the derivative of this, the third derivative; and so on. *Notation:* if $y = f(x)$,

$$f'(x) = D_x y = \frac{dy}{dx};\quad f''(x) = D_x^2 y = \frac{d^2y}{dx^2};\quad f'''(x) = D_x^3 y = \frac{d^3y}{dx^3};\quad \text{etc.}$$

NOTE. If the notation d^2y/dx^2 is used, this must not be treated as a fraction, like dy/dx but as an inseparable symbol, made up of a symbol of operation, d^2/dx^2, and an operand y.

The geometric meaning of the second derivative is this: if the original function $y = f(x)$ is represented by a curve in the usual way, then at any point where $f''(x)$ is *positive*, the curve is *concave upward*, and at any point where $f''(x)$ is *negative*, the curve is *concave downward* (Fig. 2). When $f''(x) = 0$, the curve usually has a point of inflection.

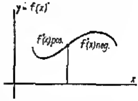


FIG. 2.

Differentials of Higher Orders. The differential of the differential is called the second differential; the differential of

in its action by the addition of tripping coils. Oil break switches are used in high-voltage work where the oil is necessary to extinguish the arc.

Circuit Breakers. Any of the foregoing switches equipped with a tripping device constitutes a circuit breaker. The tripping device is usually actuated by a solenoid which may be energized in various ways. In overload circuit breakers a solenoid coil connected in the main circuit trips the breaker when the current exceeds a certain value. If the coil is either connected in series with or across the circuit and is designed to trip the breaker when the current or voltage decreases beyond a certain value, the arrangements are, respectively, known as underload and undervoltage circuit breakers. By means of a combination shunt and series coils the circuit breaker may be made to trip when the energy reverses. Circuit breakers may trip when the difficulty is immediately cleared by a local breaker or fuse. In order that service shall not be interrupted unnecessarily, automatically reclosing breakers are used. After tripping, an automatic mechanism operates to reclose the breaker. If the short-circuit still exists, the breaker cannot reclose. The breaker thus attempts to reclose two or three times and then if the short-circuit still exists it remains permanently locked out.

In the rate-of-current-rise breakers the tripping circuit is nearly noninductive and is in parallel with a highly inductive branch. The more nearly a fault approaches a dead short-circuit, the greater the rate of current rise and the greater the proportion of current forced into the tripping circuit. Hence it follows that the greater the rate at which the current rises, the lower the value of total current at which the breaker trips. If motors or other loads seldom exceed their maximum ratings, fuses may be used for protection because of their low first cost. To avoid delays incurred by looking for and replacing fuses, the tendency at the present time is to use circuit breakers and thus reduce the time of shutdown to a minimum.

Power Distribution

Distribution Systems. The choice of the system of power distribution is determined by the type of power that is available and by the nature of the load. To transmit a given power over a given distance with a given power loss (I^2R), the weight of conductor varies inversely as the square of the voltage. Incandescent lamps will not operate economically at voltages much higher than 120 volts; the most suitable voltages for d-c motors are 230 and 550 volts, although 550 volts is practically obsolete, except for railway motors; for a-c motors, standard voltages are 220, 440, and 550 volts, three-phase.

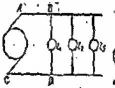


FIG. 76.—Parallel circuit.

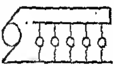


FIG. 77.—Loop circuit.

Parallel Circuits. Power is usually distributed at constant potential, and all the devices or receivers in the circuit are connected in parallel, giving a constant-potential system (Fig. 76). If conductors of constant cross section are used and all the lamps, L_1, L_2, etc., are operating, there will be a greater voltage IR drop in the portion of the circuit AB and CD than in the other portions; also the voltage will not be the same for the different lamps but will decrease along the mains with distance from the generating end.

Loop Circuits. A more nearly equal voltage for each load is obtained in the loop system (Fig. 77). The electrical distance from one generator

appliances. Pure nickel is used to satisfy the high requirements in the fabrication of radio tubes, such as the elimination of all gases and impurities in the metal parts. It has also other uses such as in incandescent lamps, for combustion boats, laboratory accessories, and resistance thermometers.

Carbon withstands high temperatures and has high resistance; its temperature coefficient is negative; it will safely carry about 125 amp per sq in. Amorphous carbon has a resistivity of 3,800 to 4,100 microhms per cc, retort carbon about 720 microhms, and graphite about 812 microhms per cc. The properties of any particular kind of carbon depend on the temperature at which it was fired. Carbon for rheostats may best be used in the form of compression rheostats.

MAGNETS

A **permanent magnet** is one that retains a considerable amount of magnetism indefinitely. Permanent magnets are used in electrical instruments, telephone receivers, radio loudspeakers, magnetos, and for any purpose where a constant magnetic field is desired. The steel used should have high retentivity, a high remanence, and a high coercive force (see Fig. 11, p. 1703). These properties are usually found with hardened steel and its alloys.

Five percent tungsten steel having small percentages of chromium and manganese and 0.63 to 1.0 percent carbon is frequently used. Special chromium-cobalt steels have been developed which give the following analysis in percent: "Alpha" quality—C, 0.56; Mn, trace; Si, 0.28; P, 0.016; S, 0.040; Cr, 3.31; Co, 30.18. Another successful alloy is C, 1.0; Cr, 3.50. An alloy, **Alnico**, consisting of aluminum, iron, nickel, and cobalt, developed by the General Electric Co., has permanent magnetic properties far superior to the foregoing alloys. The composition of Alnico II, which because of its high stability is commonly used with instruments and meters, is as follows: Al, 10; Ni, 17; Co, 12.5; Cu, 6.0.

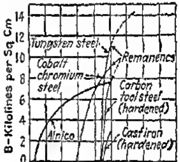


FIG. 86.—Characteristics of permanent-magnet materials.

Permanent magnets operate over the portion *DE* of the hysteresis loop (Fig. 11), and high remanence and high coercive force are both desirable. The area of *ODE* is proportional to the energy stored in the magnet and is a criterion of the material. In Fig. 86 are shown the characteristics of different magnet materials. It will be noted that although Alnico has less remanence than tungsten or cobalt-chromium steel the coercive force and area are very much larger.

The steels for permanent magnets are cut in strips, heated to a red-hot temperature and forged into shape, usually in a "bulldozer." If they are to be machined, they are cooled in mica dust to prevent air hardening. They are then ground, tumbled, and tempered. Permanent magnets are magnetized either by placing them over a bus bar carrying a very heavy direct current or by placing them across the poles of a powerful electromagnet.

Unless permanent magnets are subjected to artificial aging, they gradually deteriorate or weaken. With magnets for electrical instruments, where a constant field strength is imperative, artificial aging is accomplished by mechanical vibration or by immersion in oil at 250 F for a period of a few hours.

In an **electromagnet** the magnetic field is produced by an electric current. The core is usually made of soft iron or mild steel because, the permeability being higher, a stronger magnetic field may be obtained. Also since the retentivity is low, there is little trouble due to the sticking of armatures when

terminal to the other through any receiver is the same as that through any other receiver, and the voltage at the receivers may be maintained more nearly equal, but at the expense of additional conductor material.

Series-parallel Circuit. For incandescent lamps the power must be at low voltage (115 volts) and the voltage variations must be small. If the transmission distance is considerable or the loads are large, a large or perhaps prohibitive investment in conductor material would be necessary. In some special cases, lamps may be operated in groups of two in series as shown in Fig. 78. The transmitting voltage is thus doubled, and, for a given number of lamps, the current is halved, the permissible voltage drop (IR) in conductors doubled, the conductor resistance quadrupled, the weight of conductor material thus being reduced to 25 percent of that necessary for simple parallel operation.

Three-wire System. In the series-parallel system the loads must be used in pairs. To overcome this objection and at the same time to obtain the economy in conductor material of operating at higher voltage, the three-

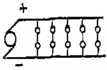


FIG. 78.—Series-parallel system.

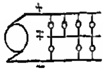


FIG. 79.—Three-wire system.

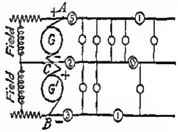


FIG. 80.—Three-wire system with two generators.

wire system is used. It consists merely of adding a third wire or neutral to the system of Fig. 78 as shown in Fig. 79.

If the neutral wire is of the same cross section as the two outer wires, this system requires only 37.5 percent of the copper required by an equivalent two-wire system. Since the neutral ordinarily carries less current than the outers, it is usually smaller and the ratio of copper to that of the two-wire system is even less than 37.5 percent (see Table 23).

When there is an equal number of lamps on each half of the system, there will be no current flowing in the middle or neutral wire, and the condition is the same as that shown in Fig. 78. When the number of lamps on the two sides is different, the neutral wire will carry a current equal to the difference of the currents in the outside wires. For example, if each of the lamps in the system shown in Fig. 80 takes 1 amp, the currents in each part of the system will be given by the numbers on the ammeters shown connected in the system.

The three-wire system shown in Fig. 79 is not practicable because no means are provided for holding the neutral at its correct potential. If, for example, four lamps are in operation on one side of the system and three on the other, as shown, the voltages on the two sides of the system become seriously unbalanced and the three lamps are subjected to overvoltage. One method of supplying the neutral is shown in Fig. 80 where each side of the system is supplied by a separate generator. This is open to the objection of the greater complications of two machines, greater cost, more floor space, and the lesser efficiency of two machines.

the circuit is opened. Electromagnets may have the form of simple solenoids, ironclad solenoids, plunger electromagnets, and electromagnets with external armatures.

A solenoid is a winding of insulated conductor and is wound helically; the direction of winding may be either right or left. A portative electro-

Table 25. Maximum Pull per Square Inch of Core for Solenoids with Open Magnetic Circuit

(From data by Underhill, *Elec. World*, **45**, 796, 881, 1906)

Length of coil, in., l	Length of plunger, in.	Core area, sq in., A	Total ampere-turns, $I \times n$	Max pull, lb per sq in., P	$1,000 \times C$	Length of coil, in., l	Length of plunger, in.	Core area, sq in., A	Total ampere-turns, $I \times n$	Max pull, lb per sq in., P	$1,000 \times C$
6	Long	1.0	15,000	22.4	9.0	12	Long	1.0	11,200	8.75	9.4
9	Long	1.0	11,350	11.5	9.1	12	Long	1.0	20,500	16.75	9.8
9	Long	1.0	14,200	14.6	9.2	18	36	1.0	18,200	9.8	9.7
10	10	2.76	40,000	40.2	10.0	18	36	1.0	41,000	22.5	9.8
10	10	2.76	60,000	61.6	10.3	18	18	1.0	18,200	9.8	9.7
10	10	2.76	80,000	80.8	10.1	18	18	1.0	41,000	22.5	9.8

magnet is one designed only for holding material brought in contact with it. **A tractive electromagnet** is one designed to exert a force on the load through some distance and thus do work. The **range** of an electromagnet is the distance through which the plunger will perform work when the winding is energized. After the core becomes saturated, the pull varies almost directly with the number of ampere-turns. For long range of operation, the plunger type of tractive magnet is best suited, for the length of core is governed practically by the range of action desired, and the area of the core is determined by the pull. **Solenoid and plunger** is a solenoid provided with a movable iron rod or bar called a plunger. When the coil is energized, the iron rod becomes magnetized and the mutual action of the field in the solenoid on the poles created on the plunger causes the plunger to move within the solenoid. This force becomes zero only when the magnetic centers *of the plunger and solenoid coincide.* If the load is attached to the plunger, work will be done until the force to be overcome is equal to the force that the solenoid exerts on the plunger. When the iron of the plunger is not saturated, the strength of magnetic field in the solenoid and the induced poles are both proportional to the exciting current, so that the pull varies as the *current squared.* When the plunger becomes highly saturated, the pull varies almost *directly* with the current.

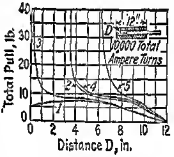


FIG. 87.—Pull of solenoid on plunger.

1. Coil and plunger; 2, coil and plunger with stop; 3, ironclad coil and plunger; 4 and 5, same as 3 with different lengths of stop.

The **maximum uniform pull** occurs when the end of the plunger is at the center of the solenoid and is equal to

$$F = CAnI/l \qquad (118)$$

Balancer Set. Another method of obtaining the neutral is to use a balancer set. This consists of two similar shunt or compound machines coupled together with the armatures connected in series across the outer lines as shown in Fig. 81. When the loads are balanced, there is no neutral current and the two machines merely run idle as motors, being connected in series across the line. If the load on one side of the system becomes greater than that on the other side, the machine on the more heavily loaded side operates as a generator and pumps some of the neutral current to its side of the line. The remainder of the neutral current goes through the other machine supplying it with the power that enables it to operate as a motor and and drive the generator. For example, in Fig. 81, the load on the positive side of the system is greater than that on the negative side. Hence, the machine G on the positive side is operating as a generator and the machine M is operating as a motor. If the machines are compounded so that when operating as a generator the winding is cumulative and hence is differential when a motor, the voltage unbalance with change in load can be made practically zero.

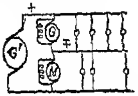


Fig. 81.—Motor-generator balancer.

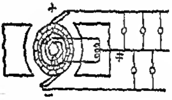


Fig. 82.—Three-wire generator.

Since balancer sets take power continuously, they are used for the most part on large systems of high diversity where the percentage unbalance is small.

Three-wire Generator. The three-wire generator is another common method of obtaining a neutral. It is a regular generator that would ordinarily be used to supply the outer wires with power. Two or more taps a and b are, however, brought out from the armature winding to two slip rings (Fig. 82). A compensator or reactance coil of low resistance is connected across the slip rings. The neutral of the three-wire system is connected to its center point. The voltage across the slip rings is alternating. Because of its choking action, but little alternating current flows in the compensator. Direct current flows back through the compensator to the armature. The inductance of the compensator has no effect on the steady direct current, and the resistance of the compensator is low so that there is little voltage drop due to the direct current. Two or more compensators with their neutrals connected may be used. With two such compensators, the second compensator is connected to slip rings that are tapped to the armature winding at points 90 electrical degrees from the first. In order to eliminate all but one slip ring, the reactance coil is sometimes mounted within the spider of the armature.

Wiring Calculations

Wiring Calculations for Direct-current Circuits. The determination of the proper size of conductor is influenced by a number of factors. Except for short distances, the minimum size of conductor recommended by the National Board of Fire Underwriters in Table 20, which is based on the maximum permissible current for each type of insulation, cannot be used;

where A is the cross-sectional area of the plunger, sq in.; n the number of turns; I the current, amp; l the length of the solenoid, in.; and C the pull, lb per sq in. per amp turn per in.. C depends on the proportions of the coil, the degree of saturation, the length, and the physical and chemical purity of the plunger. Table 25 gives values of C for several different solenoids.

Curve 1, Fig. 87, shows the characteristic pull of an open-magnetic circuit solenoid, 12 in. long, having 10,000 amp-turns or 833 amp-turns per in.

When a strong pull is desired at the end of the stroke, a stop may be used as shown in Fig. 88. Curve 2, Fig. 87, shows the pull obtained by adding a stop to the plunger. It will be noted that, except when the end of the plunger is near the stop, the stop adds little to the solenoid pull. The pull is made up of two components—one due to the attraction between plunger and winding and the other one due to the attraction between plunger and stop. The equation for the pull is

$$P = AIn[(In/l_a^2C_1^2) + (C/l)] \text{ lb} \tag{119}$$

where A is the area of the core, sq in.; n is the number of turns; l_a is the length of gap between core and stop; and C and C_1 are constants. At the beginning of the stroke the second member of the equation is predominant, and at the end of the stroke the first member represents practically the entire pull. Approximate values of C and C_1 are $C_1 = 2{,}660$ (for l greater than $10d$), $C = 0.0096$, where d is the diameter of the plunger, in.

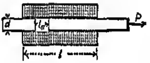


FIG. 88.—Solenoid with stop.

FIG. 89.—Conical plunger and stop.

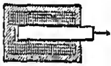


FIG. 90.—Ironclad solenoid.

The range of uniform pull may be extended by the use of conical ends of stop and plunger, as shown in Fig. 89. A stronger magnet mechanically may be obtained by using an ironclad solenoid (Fig. 90), in which an iron return path is provided for the flux. Except for low flux densities and short air gaps the dimensions of the iron return path are of no practical importance, and the fact that an iron return path is used does not affect the pull curve except at short air-gaps. This is illustrated in Fig. 87 where curves 3, 4, and 5 are typical pull curves for this same solenoid when it is made ironclad, each curve corresponding to a different position of the stop.

Mechanical jar at the end of the stroke may be prevented by leaving the end of the solenoid open. The plunger then comes to equilibrium when its middle is at the middle of the winding, thus providing a magnetic cushion effect. Electromagnets with external armatures are best adapted for short-range work, and the best type is the horseshoe magnet. The pull for short-range mangets is expressed by the equation

$$F = B^2A/72{,}134{,}000 \text{ lb} \tag{120}$$

where B is the flux density in lines per sq in. and A the area of the core in sq in. A greater holding power is obtained if the surfaces of the armature and core are not machined to an absolutely smooth contact surface. If the surface is slightly irregular, the area of contact A is reduced but the flux density B is increased approximately in proportion (if the iron is being oper-

the size of conductor must be larger in order that the voltage drop (IR) shall not be too great. With branch circuits supplying an incandescent-lamp load, this drop should not be more than a small percentage of the voltage between wires. Good practice in interior wiring allows a drop of not more than 3 volts in feeder circuits from the main switchboard to the farthest tablet board and a drop of not more than 1 volt from any tablet board to the farthest lamp.

The resistance of 1 cir mil-ft of commercial copper may be taken as 10.8 ohms. The resistance of a copper conductor may be expressed as $R = 10.8l/A$, where l is the length in feet and A the area in circular mils. If the length is expressed in terms of the transmission distance d (since the two wires are usually run parallel), the voltage drop IR to the end of the circuit is

$$e = 21.6Id/A \tag{112}$$

and the size of conductor in circular mils necessary to give the permissible voltage drop e is

$$A = 21.6Id/e \tag{113}$$

If e is expressed as a percentage x of the voltage E between conductors, then

$$A = 2{,}160Id/xE \tag{114}$$

Example. Find the size of conductor to supply power to a 10-hp 220-volt d-c motor 500 ft from the switchboard with 5 volts drop. Assume a motor efficiency of 86 percent. The motor will then require a current of $(10 \times 746)/(0.86 \times 220) = 39.4$ amp. From Eq. (113), $A = 21.6 \times 39.4 \times 500/5 = 85{,}100$ cir mils. The nearest standard wire is No. 1 (A. W. G.). See Table 19.

The calculation of the size conductor for d-c three-wire circuits is made in practically the same manner. With a balanced circuit no current flows in the neutral wire and the current in each outside wire will be equal to one-half the sum of the currents taken by all the receiving devices connected between neutral and outside wires plus the sum of the currents taken by the receivers connected between the outside wires. Using this total current and neglecting the neutral wire, make calculations for the size of the outside wires by means of Eq. (113). The neutral wire should have the same cross section as the outside wires in interior wiring.

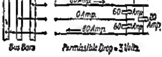


FIG. 83.—Three-wire 220–110-volt main.

Example. What size wire should be used for the three-wire main of Fig. 83? Allowable drop is 3 volts and the distance to the load center 40 ft; circuit loaded with two groups of receivers each taking 60 amp connected between the neutral and the outside wires, and one group of receivers taking 20 amp connected across the outside wires. Solution: Load $= (60 + 60)/2 + 20 = 80$ amp. Substituting in Eq. (113), cir mils $= 21.6Id/e = 21.6 \times 80 \times 40/3 = 23{,}030$ cir mils.

Referring to Table 20, No. 6 wire, which has an area of 26,250 cir mils, is the next size larger. This size of wire would satisfy the voltage-drop requirements, but rubber-insulated No. 6 has a safe carrying capacity of but 50 amp. The current in the circuit is 80 amp. Therefore, with rubber-insulated wire, No. 2, which has a carrying capacity of 90 amp, should be used. The neutral wire should be the same size as the outside wires. For exposed wiring with slow-burning or weatherproof insulation, No. 4 wire, which has a capacity of 90 amp, could be used.

Wiring calculations for a-c circuits are essentially the same as for d-c circuits, but other factors such as power factor, reactance, and skin effect may require consideration. Skin effect becomes pronounced only when very large

ated below saturation), and the pull is increased since it varies as the square of the density B. Non-magnetic stops should be used if it is desired that the armature may be released readily when the current is interrupted.

Lifting magnets are of the portative type in that their function is merely to hold the load. The actual lifting is performed by the hoisting apparatus. The magnet is almost toroidal in shape. The coil shield is of manganese steel which is very hard and thus resists wear and is practically nonmagnetic. The holding power

$$F = B^2A/72{,}134{,}000 \text{ lb} \tag{121}$$

where B is in lines per sq in. and A the area of the holding surface in sq in. It is difficult to calculate accurately the holding force of a lifting magnet for it depends on the magnetic characteristics of the load, the area of contact, and the manner in which the load is applied.

Rapid action in a magnet may be obtained by reducing the time constant (see p. 1708) of the winding and by subdividing the metal parts to reduce induced currents which have a demagnetizing effect when the circuit is closed. The movement of the plunger through the winding causes the winding and its bobbin to cut a magnetic field; if the bobbin is of metal and not slotted longitudinally, it is a short-circuited turn linked by a changing magnetic field and hence currents are induced in it. These currents oppose the flux and hence reduce the pull during the transient period. They also cause some heating. Where it is found impossible to reduce the time constant sufficiently, an electromagnet designed for a voltage much lower than normal is often used. In many electromagnets, the plunger, at the completion of its stroke, automatically connects in series with its winding a resistance of sufficient magnitude to reduce the current to a safe value. The solenoids on many automatic motor-starting panels are designed in this manner, as the extremely short time of overload produces very rapid action but does not injure the winding. When **slow action** is desired, it can be obtained by using solid cores and yoke and by using a heavy metallic spool or bobbin for the winding. A separate winding short-circuited on itself is also used to some extent.

Sparking at switch terminals may be reduced or eliminated by neutralizing the inductance of the winding. This neutralization is accomplished by winding a separate short-circuited coil with its wires parallel to those of the active winding. (This method can be used with d-c magnets only.) This is not economical, since one-half of the winding space is wasted. By connecting a capacitor across the switch terminals, the energy of the inductive discharge on opening the circuit may be absorbed. For the purpose of neutralizing the inductive discharge and causing a quick release, a small **reverse current** may be sent through the coil winding automatically upon opening the circuit. Tinfoil sleeves placed over the various layers of the winding absorb energy when the circuit is broken and reduce the energy dissipated at the switch terminals. This scheme can be used for d-c magnets only. **Sticking** of the parts of the magnetic circuit due to residual magnetism may be prevented by the use of non-magnetic stops. In the case of lifting magnets subjected to rough usage and hard blows (as in a steel works), these stops usually consist of plates of manganese steel, which are extremely hard and non-magnetic.

Alternating-current Tractive Magnets. Because of the iron losses due to eddy currents, the magnetic circuits of a-c electromagnets should be composed of laminated iron or steel. The magnetic circuit of large

conductors are used for alternating current. For interior wiring, conductors larger than 700,000 cir mils should not be used, and many prefer not to use conductors larger than 300,000 cir mils. Should the required copper cross section exceeds these values, a number may be operated in parallel.

For voltages under 5,000 the effect of line capacitance may be neglected. With ordinary single-phase interior wiring, where the effect of line reactance may be neglected and where the power factor of the load (incandescent lamps) is nearly 100 percent, the calculations are made the same as for d-c circuits. Three-wire a-c circuits of ordinary length with incandescent lamp loads are also determined in the same manner. When the load is other than incandescent lamps, it is necessary to know the power factor of the load in order

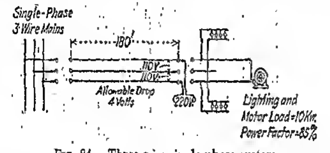


FIG. 84.—Three-wire single-phase system.

to make calculations. When the exact power factor cannot be accurately determined, the following approximate values may be used: incandescent lamps, 0.95 to 1.00; lamps and motors, 0.75 to 0.85; motors, 0.5 to 0.80. Equation (115) gives the value of current in a single-phase circuit.

$$I = \frac{P \times 1{,}000}{E \times p} \tag{115}$$

where I is the current, amp; P the kilowatts; E the load voltage; and p the power factor of the load. The size of conductor is then determined by substituting this value of I in Eq. (113) or (114).

Example. In Fig. 84, load = 10 kw; voltage of circuit = 220-110, three-wire; power factor = 0.85; distance = 180 ft; allowable drop = 4 volts. Substituting these values in (115), the current $I = (10 \times 1{,}000)/(220 \times 0.850) = 53.5$ amp. Substituting again in (113), cir mils = $21.6 \times 53.5 \times 180/4 = 52{,}000$. The next larger standard size of wire is No. 3 (52,630 cir mils), which will carry 80 amp (see Table 20).

For three-phase three-wire a-c circuits the current per wire

$$I = 1000P/\sqrt{3}\,Ep = 580P/Ep \tag{116}$$

Computations are usually made of voltage drop *per wire*. Hence, if reactance can be neglected, the conductor cross section in cir mils is one-half that given by Eq. (113). That is,

$$A = 10.8Id/e \text{ cir mils} \tag{117}$$

where e in (117) is the voltage drop per wire. The voltage drop between any two wires is $\sqrt{3}\,e$.

Example. In Fig. 85, load = 10 kw; voltage of circuit = 220; power factor = 0.85; distance = 360 ft; allowable drop per wire = 4 volts. Substituting in (116), $I = (580 \times 10)/(220 \times 0.85) = 31$ amp. Substituting in (117), $A = 10.8 \times 31 \times 360/4 = 30{,}100$ cir mils.

The next larger standard size wire No. 5 (33,100 cir mils) will carry with rubber insulation 55 amp and with other insulations 80 amp (Table 20) and is therefore ample in section for 31 amp. Three No. 5 wires would be used for this circuit.

Table 26. Diameters and Weights of Magnet Wire
(Rea Magnet Wire Co.)

Size, A. W. G.	Diameter, mils							Feet per pound				
	Bare	P.E.	S.C.E.	S.S.E.	S.C.C.	D.C.C.	S.S.C.	P.E.	S.C.E.	S.C.C.	D.C.C.	S.S.C.
10	101.9	103.9	109.9	.....	107.7	112.4	....	31.45	31.03	31.40	31.06	
11	90.7	92.6	97.6	.....	95.5	100.0	....	39.6	39.02	39.53	39.06	
12	80.8	82.7	87.7	.....	85.6	90.1	....	49.88	49.09	49.78	49.09	
13	72.0	73.7	78.7	.....	76.8	81.3	....	62.9	61.73	62.74	61.73	
14	64.1	65.8	70.8	.....	68.9	73.4	....	79.4	77.8	78.8	77.58	
15	57.1	58.7	63.7	.....	61.9	66.4	....	100.0	97.6	99.1	97.28	
16	50.8	52.4	57.4	53.7	55.6	60.1	52.6	126.1	122.9	124.7	122.1	126.4
17	45.3	46.8	51.8	48.2	50.1	54.6	47.1	159.4	154.6	156.5	152.7	159.2
18	40.3	41.7	46.7	43.2	45.1	49.6	42.1	200.8	194.6	196.5	190.8	200.8
19	35.9	37.3	42.3	38.8	40.7	45.2	37.7	252.8	243.9	246.9	238.7	253.2
20	32.0	33.3	38.3	35.2	36.8	41.3	33.8	319.0	305.8	310.6	297.6	318.5
21	28.5	29.7	34.7	31.7	33.3	37.8	30.3	401.6	381.7	387.6	370.4	401.6
22	25.3	26.5	31.5	28.5	29.6	33.6	27.1	507.6	483.1	489.7	469.5	505.1
23	22.6	23.7	28.7	25.6	26.9	30.9	24.4	639.0	602.0	615.8	581.0	636.9
24	20.1	21.2	26.2	23.1	24.4	28.4	21.9	803.0	758.0	766.9	719.0	800.0
25	17.9	18.9	23.4	20.9	22.2	26.2	19.7	1,012	943	960	901	1,009
26	15.9	16.9	21.4	18.65	20.2	24.3	17.7	1,275	1,176	1,201	1,116	1,269
27	14.2	15.2	19.7	16.95	18.5	22.5	16.0	1,608	1,477	1,515	1,377	1,595
28	12.6	13.5	18.0	15.35	16.9	20.9	14.4	2,024	1,832	1,880	1,718	2,008
29	11.3	12.1	16.6	14.05	15.6	19.6	13.1	2,554	2,299	2,347	2,110	2,519
30	10.0	10.8	15.3	12.75	14.3	18.3	11.8	3,221	2,857	2,899	2,591	3,154
31	8.9	9.6	14.1	11.5	13.2	17.2	10.7	4,057	3,559	3,615	3,175	3,968
32	8.0	8.6	13.1	10.6	12.3	16.3	9.8	5,012	4,367	4,456	3,861	4,973
33	7.1	7.7	12.2	9.55	11.4	15.4	8.9	6,452	5,388	5,525	4,566	6,238
34	6.3	6.8	11.3	8.75	10.6	14.6	8.1	8,130	6,532	6,658	5,405	7,788
36	5.0	5.5	9.5	7.35	8.8	12.8	6.8	12,887	9,158	10,881	6,711	12,019
38	4.0	4.4	8.4	6.25	7.8	11.8	5.8	20,492	13,038	13,550	8,425	18,215
40	*3.1*	*3.4*	*7.4*	*5.25*	*6.9*	*10.9*	*4.9*	*32,573*	*17,065*	*17,422*	*10,000*	*27,855*

P.E. = plain enamel; S.C.E. = single cotton-covered enamel; S.S.E. = single silk-covered enamel; S.C.C. = single cotton vered; D.C.C. = double cotton covered; S.S.C. = single silk covered.

Table 19. Resistance and 60 Cycle Reactance for Wires with Small Spacings

A. W. G. and size of wire, cir mils	Resistance in 1,000 ft of line (2,000 ft of wire), copper	Reactance in 1,000 ft of line (2,000 ft of wire) at 60 cycles per sec for the distance given in inches between centers of conductors										
		½	1	2	3	4	5	6	9	12	18	24
14- 4,107	5.06	0.138	0.176	0.218	0.220	0.233	0.244	0.252	0.271	0.284	0.302	
12- 6,530	3.18	0.127	0.159	0.190	0.210	0.223	0.233	0.241	0.260	0.273	0.292	
10- 10,380	2.00	0.116	0.148	0.180	0.199	0.212	0.223	0.221	0.249	0.262	0.281	
8- 16,510	1.26	0.106	0.138	0.169	0.188	0.201	0.212	0.270	0.238	0.252	0.270	0.284
6- 26,250	0.790	0.095	0.127	0.158	0.178	0.190	0.210	0.209	0.228	0.241	0.260	0.272
4- 41,740	0.498	0.085	0.117	0.149	0.167	0.180	0.190	0.199	0.217	0.230	0.249	0.262
2- 66,370	0.312	0.074	0.106	0.138	0.156	0.169	0.180	0.188	0.206	0.220	0.238	0.252
1- 83,690	0.248	0.068	0.101	0.132	0.151	0.164	0.174	0.183	0.201	0.214	0.233	0.246
0-105,500	0.196	0.063	0.095	0.127	0.145	0.159	0.169	0.177	0.196	0.209	0.228	0.241
00-133,100	0.156	0.057	0.090	0.121	0.140	0.153	0.164	0.172	0.190	0.204	0.222	0.236
000-167,800	0.122	0.052	0.085	0.116	0.135	0.148	0.158	0.167	0.185	0.199	0.217	0.230
0000-211,600	0.098	0.046	0.079	0.111	0.130	0.143	0.153	0.161	0.180	0.193	0.212	0.225
250,000	0.085	......	0.075	0.106	0.125	0.139	0.148	0.157	0.175	0.189	0.207	0.220
300,000	0.075	......	0.071	0.103	0.120	0.134	0.144	0.153	0.171	0.185	0.203	0.217
350,000	0.061	......	0.067	0.099	0.118	0.128	0.141	0.14[illegible]	0.168	0.182	0.200	0.213
400,000	0.052	......	0.064	0.096	0.114	0.127	0.138	0.146	0.165	0.178	0.197	0.209
500,000	0.042	......	......	0.090	0.109	0.122	0.133	0.141	0.160	0.172	0.192	0.202
600,000	0.035	......	......	0.087	0.106	0.118	0.128	0.137	0.155	0.169	0.187	0.200
700,000	0.030	......	......	0.083	0.102	0.114	0.125	0.133	0.152	0.165	0.184	0.197
800,000	0.026	......	......	0.080	0.099	0.112	0.122	0.130	0.148	0.162	0.181	0.194
900,000	0.024	......	......	0.077	0.096	0.109	0.119	0.127	0.146	0.159	0.170	0.191
1,000,000	0.022	......	......	0.075	0.094	0.106	0.117	0.125	0.144	0.158	0.176	0.188

For other frequencies the reactance will be in direct proportion to the frequency.

Where all the wires of a circuit, two wires for a single-phase, four wires for a two-phase, and three wires for a three-phase circuit, are carried in the same conduit or where the wires are separated less than 1 in. between centers, the effect of line (inductive) reactance may ordinarily be neglected. Where circuit conductors are large and widely separated from one another and the circuits are long, the inductive reactance may increase the voltage drop by a considerable amount over that due to resistance alone. Such problems are treated the same as high-voltage problems. Line reactance decreases somewhat as the size of wire increases and decreases as the distance between wires decreases.

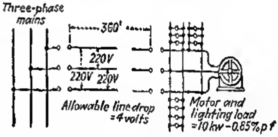


Fig. 85.—Three-phase lamp and induction-motor load.

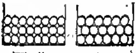
Wires

Insulated Wires. The following types of insulated wire are in common use for low-voltage wiring: rubber covered; synthetic (SN); weatherproof (WP); slow burning (SB);

magnets is usually built up of thin sheets of sheet metal held together by means of suitable clamps. Small cores of circular cross section usually consist of a bundle of soft iron wires. Since the iron losses increase with the flux density, it is not advisable to operate at as high a density as with direct current. The current instead of being limited by the resistance of the winding *is now determined almost entirely by the inductive reactance as the resistance* is small. With the removal of the load the current rises to high values. The pull of a-c magnets is nearly constant irrespective of the length of air gap.

In a **single-phase magnet** the pull varies from zero to a maximum and back to zero twice every cycle, which may cause considerable chattering of the armature against the stop. This may be prevented by the use of a spring or, in the case of a solenoid coil, by allowing the plunger to seek its position of equilibrium in the coil. **Chattering** may also be prevented by the use of a short-circuited winding or shading coil around one tip of the pole piece or by the use of polyphase. In a **two-phase magnet** the pull is constant and equal to the maximum instantaneous pull produced by one phase so long as the voltage is a sine function. In a **three-phase magnet** under the same conditions the pull is constant and equal to 1.5 times the maximum instantaneous pull of one phase. *Should the load become greater than the minimum instantaneous pull*, there will be chattering as in a single-phase magnet.

Heating of Magnets. The lifting capacity of an electromagnet is limited by the permissible current-carrying capacity of the winding, *which in turn is dependent on the amount* of heat energy that the winding can dissipate per unit time without exceeding a given temperature rise. Coils wound with wire having cotton insulation will, in general, be operating at a safe temperature if the average power expended does not exceed 0.5 watt per sq in. of radiating surface.



Winding space factor
FIG. 91. FIG. 92.

Design of Exciting Coil. Let n be the number of turns, l the mean length of turn in in. ($l = 2\pi r$, where r is the mean radius, in.), A the cross section of wire in cir mils. The resistance of a cir mil-ft of copper is practically 12 ohms at 60 C, or 1 ohm per cir mil-in. Hence the resistance, $R = nl/A$ ohms; the current, $I = EA/nl$; the amp turns, $nI = EA/l$; the power to be dissipated, $P = E^2A/nl$. From the foregoing equations the cross section of wire and the number of turns may be calculated.

Space Factor of Winding. The "space factor" is the ratio of the net volume of conductor in a given winding to the gross volume of the winding. Only in the theoretical case of uninsulated square or rectangular conductor may the space factor be 100 percent. For wire of circular section with insulation of negligible thickness, wound as shown in Fig. 91, the space factor will be 78.5 percent. When the turns of wire are "bedded," as shown in Fig. 92 (which is the case in most windings, particularly with smaller-sized wires), *there is a theoretical gain of about 7 percent* in space factor (Underhill, *Elec. World*, 53, 155, 1909). Experiments have shown that in most cases this gain is about neutralized in practice by the flattening out of the insulation of the wire due to the tension used in winding. When wound in a haphazard manner, the space factors of magnet wires vary according to size, substantially as follows:

	Double cotton covered					Single cotton covered		
Size, A. W. G.	0	5	10	15	20	25	30	35
Space factor, percent	60	53.8	45.5	35.1	32.2	32	25.7	16

and for wiring fixtures, rubber covered and heat resisting. Varnished cloth (V) and slow-burning weatherproof (SBW) are little used at the present time. **Rubber-covered wire** for voltages up to 600 volts has an unbroken rubber wall protected with cotton braid impregnated with moistureproof compound. Type letter (R) is a code grade rubber with a maximum operating temperature 122 F, (RP) a better grade or performance grade rubber 140 F, (RH) a heat-resistant rubber 167 F, (RW) a moisture-resistant rubber for wet locations 122 F, (RHT) a small diameter heat-resistant rubber 167 F for general building wiring, (RPT) a small diameter performance rubber 140 F

Table 20. Allowable Current-carrying Capacities of Conductors, Amp
Not More Than Three Conductors in Raceway or Cable[1]
(Based on Room Temperature of 86 F)

	A	B	C		A	B	C
Size AWG	Types R RW	Types SN RU RPT RP	Types RHT RH	Size, M cir mils	Types R RW	Types SN RU RPT RP	Types RHT RH
14	15	18	22	250	177	213	255
12	20	23	27	300	198	238	285
10	25	31	37	350	216	260	311
8	35	41	49	400	233	281	336
6	45	54	65	500	265	319	382
5	52	63	75	600	293	353	422
4	60	72	86	700	320	385	461
3	69	83	99	750	330	398	475
2	80	96	115	800	340	410	490
1	91	110	131	900	360	434	519
0	105	127	151	1,000	377	455	543
00	120	145	173	1,250	409	493	589
000	138	166	199	1,500	434	522	625
0000	160	193	230	1,750	451	544	650
				2,000	463	558	666

Correction Factor for Room Temperatures Over 86 F

Temp, deg F	104	113	122	131	140	158	167
Col. A	.71	.50	.00				
Col. B	.82	.71	.58	.41	.00		
Col. C	.88	.82	.75	.67	.58	.35	.00

For aluminum wire the allowable carrying capacity shall be taken as 84 percent of those given in the table for the respective sizes of copper wire with the same kind of covering.

[1] For 4 to 6 conductors the current is 80 per cent.
For 7 to 9 conductors the current is 70 per cent.

in sizes 14–10 for rewiring existing raceways for increased loads, (RU) a very thin high quality rubber 140 F in sizes 14–10 for rewiring existing raceways for increased loads. *Synthetic* (SN) is a *solid synthetic* compound of small diameter 140 F in sizes 14 to 4/0 for rewiring existing raceways for increased loads. **Slow-burning wire** has three braids of cotton, impregnated with a fire-resisting compound; its use is limited to temperatures over 185 F. **Weatherproof wire** is cheaper than other wires since the insulation consists merely of three braids of cotton impregnated with moistureproof compound. This wire is limited to outdoor use and must be supported on insulators as if the wire were bare, for the covering has no effective insulating properties. **Fixture wire** is made with either heat-resistant or rubber insulation. The rubber-covered wire is used for out-door fixtures and in fixtures where it is not subjected to extreme heat, as well as where the voltage between wires may be between 300 and 600 volts. The heat-resistant wire must be used in fixtures where the temperature exceeds 120 F. Fixtures should be wired with asbestos-covered heat-resistant fixture wire except that

Magnet wire is a soft insulated copper wire of high conductivity. It may be obtained in square, rectangular, and circular section, but the round or cylindrical wire is used almost entirely in the smaller sizes. Ribbons are frequently used in the larger sizes. Cotton covering is used on large and medium-sized wires but silk is used for the very small wires. Both cotton and silk insulation char at about 260 F and so are unsuitable for high-temperature operation. **Enameled magnet** wire is used where it is necessary to have the thinnest insulation possible and still retain high insulation resistance. Enamel covering is now highly resistant to abrasion and is used even on the larger sizes of wire. Enamel insulation will withstand a temperature of 400 F for long periods of time without deterioration. **Enamel and cotton** are now much used instead of the double textile coverings. The advantage lies in the greater space factor and increased dielectric strength. The cotton has no dielectric strength but acts as a spacer. With enamel it acts as a cushion to protect the enamel and as an absorbent when the winding is impregnated. The enamel has substantial dielectric strength.

Magnet wire provided with asbestos covering is used on magnets and spools operating at high temperature. Wire insulated in this manner may be operated at near red heat without injuring the insulation. **Fiberglas**, a thread spun from very fine glass fibers, is now being used extensively as high temperature insulation for wires. Its mechanical properties are far superior to those of asbestos, and it can withstand temperatures approaching red heat. The thread can also be woven into a fabric and used for insulation, particularly in tape form. **Formex**, an enameled wire, has great adherence, stretchability, and mechanical toughness.

Electron Tubes and Radio

Electron tubes depend for their operation on the fact that a hot body emits extremely small negatively charged particles of electricity called electrons. An **electron tube** with two electrodes is called a **diode** and consists of a hot filament (Fig. 93) and a plate P in a highly evacuated glass tube.

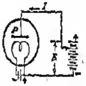


FIG. 93.—Two-electrode tube.

When an emf E is applied between the plate and the filament so that the plate is positive and the filament is negative, the electrons emitted by the filament are drawn to the plate. The current, however, flows from plate to filament since the conventional direction of current flow is opposite to the direction of flow of the electrons which are negative charges. The relation of the current to the voltage, for three different temperatures of the filament, is given in Fig. 94. At any given filament temperature T_1 the current increases rapidly at first with increase in voltage and then at some point such as *a* the current becomes essentially constant. This is **filament saturation** and occurs when the voltage is sufficiently high to draw all the electrons emitted by the filament at the given temperature. When the temperature of the filament is increased to temperatures T_2 and T_3, more electrons are emitted by the filament and saturation occurs at higher values of voltage. When the filament is positive and the plate negative, the electrons are all drawn to the filament and no current flows. Hence this tube has **unidirectional conduction** and may be used as a rectifier. It is the basis of

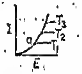


FIG. 94.—Current-voltage characteristics of two-electrode tube.

Table 21. Number of Conductors in Conduit or Tubing

Types R, RW, RP, RH, and RHT—600 V.

Size of wire, AWG-C.M.	Number of wires in one conduit									Size of wire cir mils	Number of wires in one conduit			
	1	2	3	4	5	6	7	8	9		1	2	3	4
	Minimum size of conduit, in.										Minimum size of conduit, in.			
No. 18	½	½	½	½	½	½	½	½	¾	550,000	1½	3	3½	4
16	½	½	½	½	½	½	¾	¾	¾	600,000	2	3	3½	4
14	½	½	½	½	¾	¾	¾	1	1	650,000	2	3½	3½	4
12	½	½	½	¾	¾	1	1	1	1¼	700,000	2	3½	3½	4½
10	½	¾	¾	¾	1	1	1¼	1¼	1¼	750,000	2	3½	3½	4½
8	½	¾	1	1	1	1¼	1¼	1¼	1¼	800,000	2	3½	4	4½
6	½	1	1¼	1¼	1½	1½	2	2	2	850,000	2	3½	4	4½
5	¾	1¼	1¼	1¼	1½	2	2	2	2	900,000	2	3½	4	4½
4	¾	1¼	1¼	1½	2	2	2	2	2½	950,000	2	4	4	5
3	¾	1¼	1¼	1½	2	2	2	2½	2½	1,000,000	2	4	4	5
2	¾	1¼	1½	1½	2	2	2½	2½	2½	1,250,000	2½	4½	4½	6
1	¾	1½	1½	2	2	2½	2½	3	3	1,500,000	2½	4½	5	6
0	1	1½	2	2	2½	2½	3	3	3	1,750,000	3	5	5	6
00	1	2	2	2½	2½	3	3	3	3½	2,000,000	3	5	6	
000	1	2	2	2½	3	3	3	3½	3½					
0000	1¼	2	2½	2½	3	3	3½	3½	4					
200,000	1¼	2	2½	2½	3	3	3½	3½	4					
250,000	1¼	2½	2½	3	3	3½	....	....	....					
300,000	1¼	2½	3	3	3½	3½	....	....	....					
350,000	1¼	2½	3	3½	3½	4	....	....	....					
400,000	1¼	3	3	3½	4	4	....	....	....					
450,000	1½	3	3	3½	4	4½	....	....	....					
500,000	1½	3	3	3½	4	4½	....	....	....					

[a] Under certain conditions more than nine wires are permitted in a single conduit (see Table 9, Chap. 10, National Electrical Code, 1940).

For rewiring existing raceways see Table 6, Chap. 10, National Electrical Code.

Table 22. Approximate Full-load Currents of Motors

Hp	Three-phase a-c motors			Single-phase a-c motors		Direct-current motors	
	Squirrel-cage and wound-rotor induction-type, amp			Amp		Amp	
	220V	440V	550V	110V	220V	115V	230V
½	2.5	1.3	1	7	3.5	4.5	2.3
¾	2.8	1.4	1.1	9.4	4.7	6.5	3.3
1	3.3	1.7	1.3	11	5.5	8.4	4.2
1½	4.7	2.4	2.0	15.2	7.6	12.5	6.3
2	6	3	2.4	20	10	16.1	8.3
3	9	4.5	4	28	14	23.0	12.3
5	15	7.5	6	46	23	40	19.8
7½	22	11	9	68	34	58	28.7
10	27	14	11	86	43	75	38
15	38	19	15	....	....	112	56
20	52	26	21	....	....	140	74
25	64	32	26	....	....	185	92
30	77	39	31	....	....	220	110
40	101	51	40	....	....	294	146
50	125	63	50	....	....	364	180
60	149	75	60	....	....	436	215
75	180	90	72	....	....	540	268
100	246	123	98	....	....	.....	357
125	310	155	124	....	....	.....	443
150	360	180	144				
200	480	240	195				

These values of full-load currents are average for all speeds and frequencies of continuous-duty motors.

the **kenotron** and **tungar rectifier** (p. 1753). It is also used as the detector in receiving sets (pp. 1778, 1780).

In the **three-electrode tube** or **triode** (Fig. 95) a grid is placed between filament and plate but very close the filament. A relatively small difference of potential applied between grid and filament has a much greater effect on the electron flow to the plate than when applied between filament and plate. A very small amount of energy applied to the grid can, therefore, control a very much greater amount of energy going to the plate. The energy applied to the grid may be zero as grid potential need not be accompanied by current. This characteristic of the tube makes it a very efficient amplifier and modulator of electric waves. It may also be used as a rectifier and detector. Figure 96 shows typical static relations between grid voltage and plate current in a triode with different plate voltages. The plate current I_p is plotted as a function of grid voltage for three different values of plate voltage E_1, E_2, E_3. In order to make the plate current zero, a negative

FIG. 95.—Three-electrode tube.

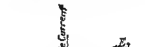


FIG. 96.—Static characteristics of three-electrode tube. (See Fig. 104.)

potential such as $-E_2'$ must be applied to the grid. If a small a-c voltage e_g *(Fig. 96) (see Fig. 99)* is applied to the grid, a corresponding variation of plate current i_p results. If the tube is made to operate on the straight portion of the characteristic by applying a negative polarizing voltage $-E_g$ to the grid by means of a C battery (see Fig. 95), the i_p wave is similar to the e_g wave and there is no distortion. When the grid is negative, the grid current I_g must be zero. Tubes are usually operated with a negative grid bias since it reduces distortion. They may, however, be made to operate on any desired portion of the characteristic by means of the C battery.

The plate current i_p (Fig. 96) represents a considerable increase of power over the grid input and may be utilized for operating a loudspeaker or the power may be further amplified.

In order to reduce the hum that occurs when a-c supply is used, the cathode frequently consists of a metallic cylinder with a heater inside. The heater usually is a hairpin loop of tungsten wire enclosed within the cylinder and insulated from it by refractory material (see Figs. 97 and 98 in which E_a may be replaced by a-c voltage). Obviously, the cyclic variation in the temperature of the cylinder and, therefore, in the emission is much less than for the filament alone.

Table 23. For Selecting Wire and Fuse Sizes for Motor Branch Circuits.

(See Table 22 for full-load motor current)

Full-load current rating of motor, amp	Minimum allowable size of copper wire, A. W. G. or cir mils			For running protection of motors, amp		Maximum allowable rating of branch circuit fuses, amp		
	Rubber, Type R	Type RP	Type RH	Max rating of N.E.C. fuses	Max setting of time-limit protective device	Squirrel-cage and synchronous motors, full-voltage starting[b], with code letters F to R[d]	Squirrel-cage and synchronous motors, reduced-voltage starting[c], with Code letters B to E[d]	Slip-ring a-c and d-c motors, with code letter A[d]
1	2	3	4	5	6	7	8	9
4	14	14	14	6[a]	5[a]	15	15	15
6	14	14	14	8[a]	7.5[a]	20	15	15
8	14	14	14	10[a]	10[a]	25	20	15
10	14	14	14	15[a]	12.5[a]	30	20	15
12	14	14	14	15[a]	15[a]	40	25	20
15	12	12	14	20[a]	18.75[a]	45	30	25
18	10	12	12	25[a]	22.5[a]	60	40	30
20	10	10	12	25[a]	25[a]	60	40	30
24	8	10	10	30	30	80	50	40
28	8	8	10	35	35	90	60	45
32	6	8	8	40	40	100	60	50
36	6	6	8	45	45	110	80	60
40	5	6	6	50	50	125	80	60
48	4	5	6	60	60	150	100	80
56	2	4	5	70	70	175	120	90
64	2	3	4	80	80	200	150	100
72	1	2	3	90	90	225	150	110
80	0	1	2	100	100	250	175	125
96	00	0	1	125	120	300	200	150
120	0000	000	0	150	150	400	250	200
140	250,000	0000	000	175	175	450	300	225
160	350,000	250,000	000	200	200	500	350	250
200	500,000	350,000	250,000	250	238	600	400	300

[a] For the running protection of 1 hp and less, the protection given in columns 7, 8, and 9 may be considered sufficient, but if the motor is located out of sight of the operator, the values in columns 5 and 6 should be used.

[b] Single-phase repulsion or split phase.

[c] High-reactance squirrel-cage motors, designed to limit the starting current by means of deep-slot secondaries or double-wound secondaries.

[d] Code letters marked on motor name-plates indicate a ratio of current taken with the rotor locked, to rated current. Letter A indicates 0 to 3.14; B to E, 3.15 to 4.99; F to R, 5.0 to 14.0 and higher. Adopted standard by N.E.M.A.

when the fixture wire will not be subjected to temperatures higher than 194 F, a cotton-covered heat-resistant fixture wire may be used. Table 20 gives the carrying capacity of insulated copper wire.

Resistor Materials

For use in rheostats, electric furnaces, ovens, heaters, and many electrical appliances, a resistor material with high melting point and high resistivity,

With radio-frequency amplification, the capacitance between the grid and plate is objectionable in that it causes feedback and limits the possible amplification of the tube. This capacitance is reduced to a very small value by the use of a screen-grid **tetrode** (Fig. 97). The screen grid completely surrounds the plate and is connected by a tap to the B battery so that its potential is negative with respect to the plate.

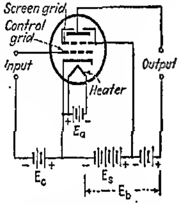


Fig. 97.—Screen-grid tetrode.

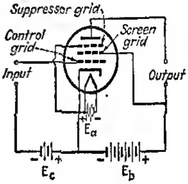


Fig. 98.—Pentode.

When the plate becomes negative with respect to the screen grid, secondary electrons emitted by the plate, owing to the impact of the primary electrons from the filament, are drawn to the screen grid. This reduces the plate current in the region of operation and curtails the power output of the tube. This effect may be eliminated by a **suppreseor** grid between the ecreen grid and the plate (Fig. 98). The suppressor grid is connected to the filament and hence is alwaye negative with respect to the plate. Therefore, any electrons emitted by the plate are driven back. Such a tube is a **pentode.**

Figure 99 ehows the connections of a tube for use as an **amplifier** with inductive coupling in which the emf e_g is amplied to e_g' (see Fig. 96). The plate current i_p flows through the primary N_1 of an amplifying transformer. Since the ratio of secondary to primary turns N_2/N_1 is something like 10:1, the secondary emf e_1' is much greater than that of the primary. For audio frequencies the amplifying transformer has a laminated iron core; with radio frequencies an air-core transformer is used. The amplified emf e_g' may be fed to a second amplifier. In this manner several stages of amplification are obtainable. Resistance and capacitive couplings are also used in amplifiers.

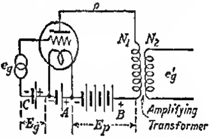


Fig. 99.—Vacuum tube as amplifier.

As the grid voltage e_g swings more negative (Fig. 96) distortion is produced in the amplified plate current i_p because of the curvature of the $I_p - E_g$ characteristic. In fact, with too great a negative swing of e_g a portion of the negative half of the i_p wave may become eliminated, and large distortion results. In order to reduce such distortion to a very small amount and increase the power output per tube, the **push-pull amplifier**, shown in Fig. 100, is used. There is a center tap o at the center of secondary ab of the input transformer to which the negative of the C battery is connected. The

which does not disintegrate or corrode at high temperatures, is necessary. These requirements are met by the nickel-chromium and nickel-chromium-iron alloys. For electrical instruments and measuring apparatus, the resistor material should have high resistivity, low temperature coefficient, and, for many uses, low thermoelectric power against copper. The properties of resistor materials are given in Table 24. Most of these materials are available in ribbon as well as in wire form. Cast-iron and steel wire are efficient and economical resistor materials for many uses, such as power-absorbing rheostats and motor starters and controllers.

Advance has a low temperature coefficient and is useful in many types of measuring instrument and precision equipment. Because of its high thermo-

Table 24. Properties of Metals, Alloys, and Resistor Materials[a]

Material	Composition	Specific gravity	Microhms cm-cube at 20 C	Ohms cir-mil-ft at 20 C	Temperature coefficient of resistance	Temperature range, deg C	Maximum safe working temperature, deg C	Approximate melting point, deg C
Advance	Cu; Ni 0.45	8.9	49	294	±0.00002	20–100	500	1210
Comet	Ni 0.30; Cr 0.06; Fe 0.65	8.15	95	570	0.00088	20–500	600	1480
Bronze, commercial	Cu; Sn	8.7	4.2	25	0.0020	0–100	....	1015
Hytemco	Ni 0.50; Fe 0.50	8.46	20	120	0.0045	20–100	600	1425
Magno	Ni 0.955; Mn 0.046	8.75	20	120	0.0036	20–100	400	1435
Manganin	Cu 0.84; Mn 0.12; Ni 0.04	8.19	46.2	290	±0.000015	15–35	75	1020
Monel metal	Ni 0.67; Cu 0.28	8.9	42.6	256	0.00198	20–100	425	1350
Nichrome	Ni 0.60; Fe 0.25; Cr 0.15	8.247	112	675	0.00017	20–100	930	1350
Nichrome V	Ni 0.80; Cr 0.20	8.412	108	650	0.00013	20–100	1100	1400
Nickel, pure	Ni 0.99	8.9	10	60	0.0050	0–100	400	1450
Platinum	Pt	21.45	10.616	63.80	0.00398	......	....	1755
Silver	Ag	10.5	1.622	9.755	0.00361	......	....	960
Tungsten	W	19.3	5.523	33.22	0.00524	......	....	3370

[a] Courtesy of Driver-Harris Co. (except monel metal).

electric power to copper, it is valuable for thermoelements and pyrometers. It is non-corrosive and is used to a large extent in industrial and radio rheostats. **Hytemco** is a nickel-iron alloy characterized by a high temperature coefficient and is used advantageously where self-regulation is required as in immersion heaters and heater pads. **Magno** is a manganese-nickel alloy which is used in the manufacture of incandescent lamps and radio tubes. **Manganin** is a copper-manganese-nickel alloy which, because of its very low temperature coefficient and its low thermal emf with respect to copper, is very valuable for high-precision electrical measuring apparatus. It is used for the resistance units in bridges, for shunts, multipliers, and similar measuring devices. **Nichrome V** is a nickel-chromium alloy free from iron, is noncorrosive, nonmagnetic, withstands high temperatures, and has high resistivity. It is recommended as material for heating elements in electric furnaces, hot-water heaters, ranges, radiant heaters, and high-grade electrical

positive of the B battery is connected to the center tap o' of the primary $a'b'$ of the output transformer. Hence so far as the direct current to the plate circuits is concerned the two halves of the primary $a'b'$ are in opposition, whereas the alternating voltages across the two halves are in phase with each other. Owing to the symmetry of the connection, even harmonics caused by the curvature of the characteristics are eliminated entirely and the odd harmonics are ordinarily so small as to be negligible. With this arrangement much greater power outputs with negligible distortion can be obtained.

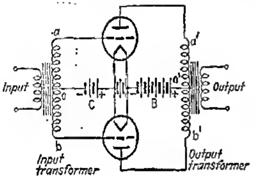


Fig. 100.—Push-pull amplifier

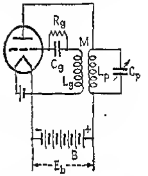


Fig. 101.—Oscillator.

A vacuum tube may become a power **oscillator** or a generator of alternating current. A tube may be made to oscillate by connecting an oscillatory circuit, consisting of inductance and capacitance in parallel, in the plate circuit. A portion of the energy of the plate circuit is fed back into the grid circuit, the direction of the connection being such that the emf due to the plate circuit is substantially in phase with the grid emf and so reinforces it. In Fig. 101 a simple oscillator is shown. The oscillatory circuit consisting of the inductance L_p and capacitance C_p in parallel is called the **tank circuit.** The frequency of oscillation $f = 1/2\pi \sqrt{L_pC_p}$. The grid circuit is coupled to the plate circuit by the mutual inductance M between L_p and L_g. A grid leak R_g prevents the accumulation of electrons on the grid which would block the tube. The load on the oscillator may be inductively coupled to L_p.

A radio wave such as is used for broadcasting is a high-frequency or r-f (radio-frequency) continuous wave having a frequency of 1,000,000 to 30,000,000 cycles per sec. This is called the **carrier wave.** In order to transmit speech and music this wave is modulated by a-f (audio-frequency) waves produced by the voice and by music. Figure 102 shows such a modulated wave. The envelope of the modulated carrier-frequency wave is the a-f wave, as shown in Fig. 102.

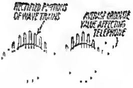


Fig. 102.—Rectified radio wave-train.

There are several methods of **modulating** the carrier wave. A common method is to introduce into the plate circuit of a tube, oscillating at the carrier frequency, an additional emf of audio frequency, the peak value of which is somewhat less than the steady plate voltage E_b. Figure 103 shows a simple plate-modulation circuit. The carrier-frequency tuned tank-cricuit L_pC_p and grid circuit are similar to those in Fig. 101. The secondary S of an audio-frequency transformer b is introduced into the plate circuit. The primary

this, the third differential; etc. These quantities are of little importance except in the case where dx = a constant. In this case

$$dy = f'(x)dx; \quad d^2y = f''(x)\cdot(dx)^2; \quad d^3y = f'''(x)\cdot(dx)^3; \ . \ . \ .$$

The first, second, third, etc., differentials are close approximations to the first, second, third, etc., differences (p. 115), and are therefore sometimes useful in constructing tables. Thus, denoting the first, second, third, etc., differences by D', D'', D''', etc., and, assuming always that dx = a constant,

$$\begin{aligned} D' &= dy + \tfrac{1}{2}d^2y + \tfrac{1}{6}d^3y + \tfrac{1}{24}d^4y + \ . \ . \ .; \quad d^3y = D''' - \tfrac{3}{2}D'''' + \ . \ . \ . \\ D'' &= d^2y + d^3y + \tfrac{7}{12}d^4y + \ . \ . \ .; \quad d^2y = D'' - D''' + \tfrac{11}{12}D'''' + \ . \ . \ . \\ D''' &= d^3y + \tfrac{3}{2}d^4y + \ . \ . \ .; \quad dy = D' - \tfrac{1}{2}D'' + \tfrac{1}{3}D''' - \tfrac{1}{4}D'''' + \ . \ . \ . \end{aligned}$$

Functions of Two or More Variables may be denoted by $f(x, y, \ . \ . \ .)$, $F(x, y, \ . \ . \ .)$, etc. The derivative of such a function $u = f(x, y, \ . \ . \ .)$ formed on the assumption that x is the only variable ($y, \ . \ . \ .$ being regarded for the moment as constants) is called the **partial derivative** of u **with respect to** x, and is denoted by $f_x(x,y)$, or $D_x u$, or $\dfrac{d_x u}{dx}$, or $\dfrac{\partial u}{\partial x}$. Similarly, the partial derivative of u with respect to y is $f_y(x,y)$, or $D_y u$, or $\dfrac{d_y u}{dy}$, or $\dfrac{\partial u}{\partial y}$.

NOTE. In the third notation, $d_x u$ denotes the differential of u formed on the assumption that x is the only variable. If the fourth notation, $\partial u/\partial x$, is used, this must not be treated as a fraction like du/dx; the $\partial/\partial x$ is a symbol of operation, operating on u, and the "∂x" must not be separated.

Partial derivatives of the second order are denoted by f_{xx}, f_{xy}, f_{yy}, or by $D_x^2 u$, $D_x(D_y u)$, $D_y^2 u$, or by $\dfrac{\partial^2 u}{\partial x^2}$, $\dfrac{\partial^2 u}{\partial x \partial y}$, $\dfrac{\partial^2 u}{\partial y^2}$, the last symbols being "inseparable." Similarly for higher derivatives. Note that $f_{xy} = f_{yx}$.

If increments Δx, Δy, (or dx, dy) are assigned to the independent variables x, y, the increment, Δu, produced in $u = f(x,y)$ is

$$\Delta u = f(x + \Delta x, y + \Delta y) - f(x,y);$$

while the **differential**, du, that is, the value which Δu would have if the partial derivatives of u with respect to x and y were constant, is given by

$$du = (f_x)\cdot dx + (f_y)\cdot dy.$$

Here the coefficients of dx and dy are the values of the partial derivatives of u at the point in question.

If x and y are functions of a third variable t, then the equation

$$\frac{du}{dt} = (f_x)\frac{dx}{dt} + (f_y)\frac{dy}{dt}$$

expresses the rate of change of u with respect to t, in terms of the separate rate of change of x and y with respect to t.

For the graphical representation of $u = f(x,y)$, see p. 178.

Implicit Functions. If $f(x,y) = 0$, either of the variables x and y is said to be an implicit function of the other. To find dy/dx, either (1) solve for y in terms of x, and then find dy/dx directly; or (2) differentiate the equation through as it stands, remembering that both x and y are variables, and then divide by dx; or (3) use the formula $dy/dx = -(f_x/f_y)$, where f_x and f_y are the partial derivatives of $f(x,y)$ at the point in question.

MAXIMA AND MINIMA

A Function of One Variable, as $y = f(x)$, is said to have a **maximum** at a point $x = x_0$, if at that point the slope of the curve is zero and the concavity

downward (see Fig. 3); a sufficient condition for a maximum is $f'(x_0) = 0$ and $f''(x_0)$ negative. Similarly, $f(x)$ has a **minimum** if the slope is zero and the concavity upward; a sufficient condition for a minimum is $f'(x_0) = 0$ and $f''(x_0)$ positive. If $f'(x_0) = 0$ and $f''(x_0) = 0$ and $f'''(x_0) \neq 0$, the point x_0 will be a **point of inflection**. If $f'(x_0) = 0$ and $f''(x_0) = 0$ *and* $f'''(x_0) = 0$, *the point* x_0 *will* be a maximum if $f''''(x_0) < 0$, and a minimum if $f''''(x_0) > 0$. It is usually sufficient, however, in any practical case, to find the values of x which make $f'(x) = 0$, and then decide, from a general knowledge of the curve, which of these values (if any) give maxima or minima, without investigating the higher derivatives.

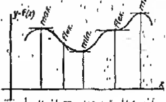


FIG. 3.

A Function of Two Variables, as $u = f(x,y)$, will have a **maximum** at a point (x_0,y_0) if at that point $f_x = 0$, $f_y = 0$, and $f_{xx} < 0$, $f_{yy} < 0$; and a **minimum** if at that point $f_x = 0$, $f_y = 0$, and $f_{xx} > 0$, $f_{yy} > 0$; provided, in each case, $(f_{xx})(f_{yy}) - (f_{xy})^2$ is positive. If $f_x = 0$ and $f_y = 0$, and f_{xx} and f_{yy} have opposite signs, the point (x_0,y_0) will be a "saddle point" of the surface representing the function (p. 178).

EXPANSION IN SERIES

The range of values of x for which each of the series is convergent is stated at the right of the series.

Arithmetical and Geometrical Series, and the Binomial Theorem. See p. 114.

Exponential and Logarithmic Series.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots; \qquad -\infty < x < +\infty.$$

$$a^x = e^{mx} = 1 + \frac{m}{1!}x + \frac{m^2}{2!}x^2 + \frac{m^3}{3!}x^3 + \cdots; \; a > 0, \; -\infty < x < +\infty,$$

where $m = \log_e a = (2.3026)(\log_{10} a)$.

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots; \qquad -1 < x < +1.$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots; \qquad -1 < x < +1.$$

$$\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right); \qquad -1 < x < +1.$$

$$\log_e\left(\frac{x+1}{x-1}\right) = 2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \cdots\right); \qquad x < -1 \text{ or } +1 < x.$$

$$\log_e x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right]; \qquad 0 < x < \infty.$$

$$\log_e(a+x) = \log_e a + 2\left[\frac{x}{2a+x} + \frac{1}{3}\left(\frac{x}{2a+x}\right)^3 + \frac{1}{5}\left(\frac{x}{2a+x}\right)^5 + \cdots\right]; \qquad \begin{cases} 0 < a < +\infty \\ -a < x < +\infty \end{cases}$$

current of this transformer comes from the microphone circuit consisting of a battery B and a microphone T. The capacitance C' bypasses the carrier-frequency circuit around the secondary S of the audio-frequency transformer b. Ordinarily there is not sufficient power in the microphone circuit, so that an amplifier (Figs. 99, 100) is inserted between the microphone and the transformer b.

The ordinary sound-producing devices cannot respond to the high-frequency modulated wave such as is shown in Fig. 102, and the frequency is far too high to be audible to the human ear. It is, therefore, necessary to **demodulate** such waves in order that the receiving devices may be actuated by a-f currents similar to those used for modulating. This is called **detection.**

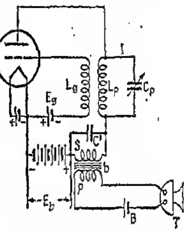


FIG. 103.—Plate modulation.

The detector must rectify or suppress one-half the wave shown dotted in Fig. 102. The high-frequency components of the rectified wave are by-passed by a capacitor such as C_p (Fig. 104). This leaves only the a-f wave shown by the solid line which is essentially the a-f wave which was transmitted. This wave then actuates the phones or speaker.

A common method of detecting is the **grid leak** shown in Fig. 104. A very high resistance R_G shunted by a small capacitance C_s is in series with the grid. The incoming carrier wave from the antenna flows through the inductance M which is in the grid circuit. Owing to the curvature of the grid-current characteristic I_g, Fig. 95, rectification is produced by the grid current flowing in C_s and R_G. Figure 104 shows a simple regenerative receiving set in which some of the plate energy is transferred to the grid circuit by mutual inductance. The plate circuit is coupled to the antenna inductance M by means of the coil M_p. A portion of the energy of the plate circuit is fed back into the grid circuit through the mutual induction of M_p on M reinforcing the received signal. This is called **regeneration.** A small capacitor C_p shunted around the phones by-passes the r-f portion of the rectified wave of Fig. 102. Two or more stages of amplification may be obtained by replacing the phones (Fig. 104) by e_g or the input of an amplifier such as is shown in Figs. 99 and 100. The phones or loudspeaker may then be connected in the plate circuit of the tube of the last stage of amplification.

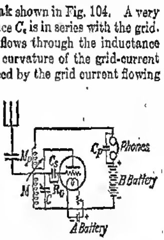


FIG. 104.—Simple regenerative receiving set.

The diode (see Fig. 93) is now in common use as a detector. It is very much less sensitive than the grid leak, but, with the high amplification now possible with heterodyne reception, this is no great disadvantage (see p. 1778).

Frequency modulation rather than the amplitude modulation just described is also being used. The amplitude of the carrier wave remains constant but its frequency is varied in accordance with the frequency of the

tion temperature produces more error in low-temperature than in high-temperature types.

Thermocouples of platinum and platinum-nickel alloy produce twice the voltage of a platinum-rhodium combination, but should not be subjected to temperatures above 2000 F. Thermoelectric couples are particularly adaptable for measuring temperatures in restricted spaces. The couples can be made thin enough to be inserted into spaces less than 1/10 in. wide. They are adaptable also for indicating temperatures of objects at a distance from the observer.

Any convenient size of wire may be used, but a good electrical contact at the junctions, and insulation of the rest of the wire, are important. Junctions may be merely twisted together; but for mechanical strength and to ensure good electrical contact in spite of oxidation, welded or soldered junctions are preferable. Insulations may be enamel, silk, cotton, or other fiber for low temperatures, and asbestos, porcelain, or other refractory for high temperatures. Pure metals should be used for thermocouple wires, and each batch of wire should be calibrated.

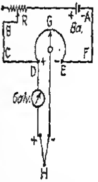


Fig. 4.—Potentiometer Circuit.

Thermocouple instruments may be either of the millivoltmeter type or of the potentiometer type. **Millivoltmeter instruments** have the advantage of being direct reading, but the readings are affected by the resistance of the leads. This difficulty is minimized by using a millivoltmeter of high internal resistance, but in any case the resistance of the thermocouple and leads should be as close as possible to that for which the instrument was calibrated. The accuracy of a high-grade millivoltmeter pyrometer with 6 in. scale and with calibrated couple and leads is usually within 1 or 2 percent of full-scale reading.

Potentiometer instruments measure the thermocouple voltage by balancing a known battery voltage against it. No current flows in the thermocouple or the lead wires, and hence their resistance does not affect the readings. A simple potentiometer circuit is shown in Fig. 4. The accuracy of thermocouple readings with a potentiometer instrument is limited by the length of the slide wire *DGE*, by the sensitiveness of the galvanometer, and by the accuracy of the wire calibration. Over-all accuracy varies from about 1 percent in commercial wide-range instruments to about 0.1 percent in the precision laboratory type used for comparative measurements with the same batch of wire.

Since the temperature measurement by a thermocouple is the difference in temperature between the two junctions, the **cold junction** or reference junction should be kept at a known temperature. In laboratory work, this is usually done by inserting the cold junction in a vacuum bottle with cracked ice and water. For industrial high-temperature work, the temperature of a cold junction buried 10 ft underground is approximately constant (within 5 F). Internal compensating devices may be built into the instrument to compensate for cold junction when the latter is at the instrument. These cold-junction **compensators** may be manually adjusted, or they may be automatic. (See p. 1721.)

Potentiometer-type industrial **temperature recorders** and controllers using thermocouples are increasing in number and variety because of their advantage of high accuracy, the usual guarantee on such instruments being less than 1 percent of full-scale deflection.

a-f wave. This method is advantageous in that quality is improved and static disturbances are in a large measure eliminated.

Radiation of Electrical Energy. When a capacitor is charged, the energy represented by the charge is stored in the dielectric medium of the capacitor; it is possible to utilize part of this energy for the purpose of radio communication. If a radio aerial or antenna (which, with the earth, forms a capacitor) is electrically charged, the stored energy will exist not only near the antenna but also out at a great distance. If an alternating voltage of high frequency is now applied between the antenna and the ground, this capacitor will be rapidly charged and discharged. If the frequency of reversal is sufficiently high, a second storage of energy in the field takes place before all the first energy stored at a great distance will have time to return to the antenna. Thus the antenna sends out successive energy impulses only a portion of which return to the system on each successive reversal; the unreturned energy impulses become ether waves the velocity of which is approximately that of light (3×10^8 m per sec in the ether or air) until intercepted or absorbed.

The **wave length** λ is the distance, usually expressed in meters, between successive wave crests. The **frequency** f of the radio wave propagation is the number of wave cycles per sec and may be obtained from the equation

$$\lambda f = 3 \times 10^8 \text{ m per sec} \tag{122}$$

For example, if the wave length is 300 m, the frequency $f = (3 \times 10^8)/(300 \times 10^3) = 1{,}000$ kilocycles (kc). **Continuous carrier waves** used in broadcasting are produced by tube oscillators which ordinarily have an oscillating circuit, composed of inductance and capacitance in parallel, connected in the plate circuit. Regenerative feedback (see Fig. 101) which sustains and reinforces the natural oscillations is obtained by coupling the plate circuit back to the grid. The antenna is coupled to the plate circuit. Modulation is accomplished by introducing the a-f emf produced by the microphone, but usually amplified, into either the grid or plate circuit (Fig. 103).

Superheterodyne Reception. In most modern receivers, superheterodyne reception is used. The method is based on the principle that a high-frequency current a may be converted to a lower frequency by superposing on it a second current of frequency a'. The frequency of the two, after passing through a detector tube, will result in an envelope frequency (see Fig. 102) equal to $a - a'$. This envelope frequency is called the beat frequency and, in receivers, the **intermediate frequency** (i-f). The intermediate frequency is obtained by a local oscillator superposing its frequency on the incoming modulated signal. If the incoming modulated frequency is 1,000 kc and the superposing frequency is 1,556 kc, the beat frequency is 1,556 − 1,000, or 556 kc. In tuning, the frequency of the local oscillator is adjusted to the incoming frequency so that the beat or intermediate frequency always remains the same. This simplifies the design of the amplifier and the circuits of the i-f stages since they can be adapted to fixed frequencies and thus operate under optimum conditions.

Superheterodyne Receiver. In Fig. 105 is shown a typical superheterodyne receiver as produced by the Browning Laboratories, Inc. For simplicity the tuning eye and details of the i-f amplifier have been omitted. The incoming signal from the antenna enters the tuned radio transformer T_1 which

Electrical-resistance thermometers employ a bridge circuit and may use a null or balance method of indication as already described for the thermocouple potentiometer. The sensitive element is a coil of fine wire, usually nickel or platinum, and the change in resistance of this wire with temperature is measured. The resistance thermometer is a highly accurate and sensitive temperature-measuring device and has the advantage of being adjustable in scale and range by the adjustment of the resistances in the bridge. A disadvantage of the resistance thermometer is its rather high cost, and its applications are usually limited to distant-reading installations in which precise measurements are needed or where the range covered is only a few degrees, as, for instance, in the accurate control of temperatures in rooms or storage spaces.

In the radiation pyrometer, a thermocouple or a thermopile is located at the focus of a lens within a lens tube (Fig. 5). When this device is sighted upon a large area such as an incandescent fuel bed or a furnace wall, the temperature attained by the thermocouple will be a function of the temperature of the object sighted upon and will be almost independent of the distance from the object, as long as the entire field of the lens is covered. The reading will depend upon the emissivity of the source of radiation. The instruments are calibrated for black-body conditions and will be approximately correct when sighted into a large furnace through a small opening. The instrument readings may be 10 to 50 percent low when sighted upon light-colored or metallic surfaces in the open. Radiation pyrometers are ordinarily used for temperatures above 1000 F, but by substituting a thermopile (series thermocouples) for a single couple the instrument may also be adapted for lower temperatures down to and including ordinarily atmospheric temperatures. The radiation pyrometer is made in both indicating and recording types and is also used as the sensitive element for control devices.

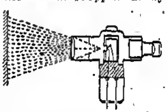


FIG. 5.—Radiation Pyrometer.

The optical pyrometer measures temperature by indicating the *brightness* of the radiation received from an object, in a narrow band of the visible spectrum. The optical pyrometer measures monochromatic radiation, and the radiation pyrometer measures total radiation. Most optical pyrometers are operated by visual comparison of the apparent brightness of the body under observation with the brightness of a standard light source. The brightness of the standard light is varied either by an absorption device such as a polarizing prism or by varying the filament current in an electric lamp. In the latter type, if the lamp filament and the object are both in focus in the same field, the filament will disappear when the current has been adjusted to give the filament the same brightness as the object. This type of instrument was used to determine the international temperature scale above the gold point. Optical pyrometers usually employ a milliammeter instrument for final reading, but potentiometer instruments are also available for more accurate work. The photoelectric pyrometer, which uses a photocell as the sensitive element, may be classed as an optical pyrometer.

The accuracy of optical pyrometers under favorable conditions may be within 1 percent. The optical pyrometer is used principally in the lower range of thermocouples, but it may be used for temperatures as low as 1200 F. When black-body conditions do not prevail, the optical pyrometer

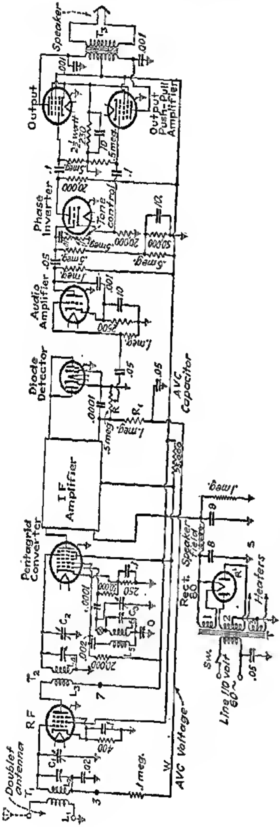


FIG. 105.—Heterodyne receiver. (Browning Laboratories, Inc.)

is subject to considerable error. For molten metals in the open, for instance, the instrument may read as much as 300 F low. Other sources of error are the presence of screens of smoke or gases and the necessity for visual setting which may introduce human error. In the determination of temperatures of luminous flames, the optical pyrometer has been applied with some success by using color screens of two different wave lengths, red and green.

Pyrometric cones are a simple and inexpensive form of fusion pyrometer. The cones are small pyramids, about 2 in. high. Each cone is prepared from mixtures of oxides and glass, to give a definite melting point. A series of cones with melting points 20 to 70 deg apart covers the range from about 1100 to 3600 F. The accuracy of the measurements is about the same as

Table 4. Melting Points of Sager Cones

No.	Deg C	Deg F	No.	Deg C	Deg F	No.	Deg C	Deg F
022	600	1110	7	1270	2320	30	1670	3040
016	750	1380	15	1435	2615	35	1770	3220
010	950	1740	20	1530	2790	39	1880	3420
02	1110	2030	26	1580	2880	42	2000 (about)	3600

the interval between successive cones, although it may not be as close because the behavior of the cones depends somewhat on the rate of heating and upon the furnace atmosphere. Pyrometric cones are used mainly in the ceramic industries.

Calibration of temperature instruments is usually accomplished by one of three methods: (1) The instrument to be calibrated may be compared with a standard instrument of suitable type. This should be done in a well-stirred fluid, with the sensitive elements of the two instruments fastened together. (2) The readings of the instrument may be determined at the melting or solidifying point of pure solid materials. The points usually used are ice, 32; tin, 449; lead, 621; zinc, 787; aluminum, 1220; sodium chloride, 1473; and copper 1982 F. Cooling curves (temperature vs. time) should be plotted and the point determined from the flat portion of the curve. (3) The readings of the instrument may be determined at the boiling points or saturation temperatures of pure liquids. Water is usually used and temperatures are taken from the steam tables. Pressures must be measured very accurately, as by dead-weight apparatus. The atmospheric boiling points of naphthalene, 425, and of sulphur, 832 F, are often included.

Accuracy of temperature measurement depends on instrument and on method of application. Almost any required degree of accuracy in scale and calibration may be obtained with glass-stem thermometers, electrical-resistance thermometers, or thermocouples, but large errors may be introduced by improper application. Most of these errors are due to: (1) stratification and poor mixing of the fluid; (2) radiation; (3) incomplete immersion or conduction. As the difference between measured temperature and ambient temperature increases, these errors become greater.

The Bureau of Standards, at Washington, D. C., will calibrate all types of thermometers and pyrometers. A fee schedule can be obtained on request. A certificate of calibration is provided by the bureau for each instrument.

Surface temperatures are best measured with thermocouples, but they may also be measured with thermometers or with radiation instruments. A thermometer pressed firmly against a hot surface and sealed with plastic

is adjusted to select the desired signal. The transformer increases the voltage impressed on the first r-f amplifier tube (RF). The signal is not only amplified in the plate current of the r-f tube but also by the effect of the tuned r-f transformer (T_2) which also couples the signal into a mixer tube (pentagrid converter). The local frequency is also fed into the converter by the oscillator O. This frequency is mixed with that of the modulated incoming signal, and several frequencies are produced in the plate circuit of the converter, but all the frequencies except the difference or beat frequencies between the oscillator and the incoming frequencies are eliminated by a tuned i-f transformer in the i-f amplifier. This difference or intermediate frequency is constant and varies in different circuits from 456 to 465 kc. The i-f amplifier amplifies the power of the i-f frequency which has all the modulation characteristics of the incoming signal. In addition, the amplifier adds greatly to the selectivity of the set because of its tuned coupled circuits. Tuning the incoming signal is accomplished by the simultaneous adjustment of the three variable capacitors C_1 and C_2, and C_3 which are all "ganged" on the same shaft.

After the i-f amplifier, the i-f signal is rectified by the diode detector (p. 1773), and the resulting audio frequency flows through a resistor R usually of the potentiometer type. This resistance acts as volume control. The audio frequency developed across the resistance is amplified by the audio amplifier and phase inverter. The phase inverter also provides the two suitable feed potentials for the push-pull amplifier. The output of the amplifier is supplied to the primary of the a-f transformer T_3 and the speaker is connected to its secondary.

Automatic volume control (a-v-c), is obtained by the voltage developed across the 1 meg resistance R_1 which charges the 0.05-microfarad a-v-c capacitor. This voltage is fed back over the wire W to terminal 3 and gives a more or less negative bias to the grids of the r-f and i-f tubes, according to the strength of the incoming signal.

The power supply system is shown at S. The current from the 110-volt 60-cycle supply goes into the primary of a 60-cycle transformer T. There are three secondaries. The top secondary 1, usually rated at 5 volts, supplies the filament of the rectifier tube R', this filament also acting as cathode. In the intermediate secondary 2, usually rated at 350 volts max to the grounded center tap, the outer terminals are connected to the two anodes of the rectifier tube R'. The lowest secondary 3 supplies the heaters of the tubes in the set and is rated at 6.3 volts. It also has a grounded center tap which fixes the potentials of the heaters. To the right of the rectifier is a filter in which the speaker field is used as a series reactance. The rectifier delivers direct current for the B or plate voltage of 250 volts. A potential of 90 volts is obtained for the screen grids by means of the drop in the 25,000-ohm resistance.

material will read low, the error varying from about 5 F when the true temperature is 50 F above ambient, to 20 F when the temperature is 200 F above ambient. More accurate surface-temperature measurements may be made with thermocouples, either attached to the surface with adhesive or imbedded in cement in a shallow groove. Fine wires (24 to 40 gage) should be used, and the *insulated lead wires should be in contact with the surface* for some distance from the couple. For high temperatures or for quick readings from inaccessible surfaces, a radiation thermopile or a radiation pyrometer may be used.

PRESSURE MEASUREMENTS

The preferred method of measuring **barometric pressure** is by means of a mercury column with a brass scale. Standard barometer reading at sea level *is 29.92 in. of mercury at 32 F or 30 in. at 58 F.* **Gage pressure** is the pressure indicated by a gage, above atmospheric. **Vacuum** is pressure below atmospheric and is commonly expressed in inches of mercury. **Absolute pressure** is the sum of gage and barometric pressures.

Table 5. Barometer Corrections for Temperature

To correct the observed reading of a mercury barometer or U tube to the 32 F standard, add or subtract the following values in inches of mercury.

Temperature of mercury column, deg F	Observed reading of mercury column, in. of mercury						
	20	22	24	26	28	30	32
−20	+0.09	+0.10	+0.11	+0.11	+0.12	+0.13	+0.14
0	+0.05	+0.06	+0.06	+0.07	+0.07	+0.08	+0.08
20	+0.02	+0.02	+0.02	+0.02	+0.02	+0.02	+0.02
40	−0.02	−0.02	−0.02	−0.03	−0.03	−0.03	−0.03
60	−0.06	−0.06	−0.07	−0.07	−0.08	−0.08	−0.09
80	−0.09	−0.10	−0.11	−0.12	−0.13	−0.14	−0.15
100	−0.13	−0.14	−0.15	−0.17	−0.18	−0.19	−0.20

Table 6. Barometer Corrections for Gravity

To correct the observed reading of a mercury barometer or U tube to the equivalent reading at standard gravity, subtract or add the following values in inches of mercury.

North latitude, deg	Elevation, ft							
	0	0	2,000	2,000	4,000	4,000	6,000	6,000
	Height of column, in. of mercury							
	30	28	28	26	26	24	24	22
25	−0.05	−0.05	−0.05	−0.05	−0.05	−0.05	−0.06	−0.05
30	−0.04	−0.04	−0.04	−0.04	−0.05	−0.04	−0.05	−0.04
35	−0.03	−0.03	−0.03	−0.03	−0.03	−0.03	−0.04	−0.03
40	−0.02	−0.01	−0.02	−0.02	−0.02	−0.02	−0.03	−0.02
45	−0.00	−0.00	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
50	+0.01	+0.01	+0.01	+0.01	−0.00	−0.00	−0.00	−0.00

SECTION 13

ENGINEERING MEASUREMENTS
MECHANICAL REFRIGERATION, ETC.

BY

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Section 13 except pp. 1882 to 1898 from "Mechanical Engineers' Handbook," edited by Lionel S. Marks

CONTENTS

The **corrected barometer reading** is the observed reading corrected for temperature, gravity, elevation, and instrument calibration. For absolute pressures considerably above atmospheric, the barometer reading as observed is usually sufficiently accurate. If low absolute pressures are to be determined, such as the exhaust pressure of a condensing steam turbine, all barometer corrections should be applied. Barometer corrections for temperature, gravity, and elevation are given in Tables 5, 6, and 7. The A.S.M.E. Power Test Codes require these three corrections for accurate work and require in addition a barometer calibration correction. The Code specifies that this *barometer calibration* is to be made by correcting the observed barometer reading (Tables 5 and 6) and reducing the result to sea level (Table 7). The difference between this result and the "weather-map reading" for the location of the observed barometer, obtained from the U.S. Weather Bureau, is the calibration correction. *Barometric pressures reported by the Weather Bureau* have already been corrected to 32 F and reduced to sea level. The reading of a local barometer for purposes of calibration should be made at the same time that the Weather Bureau takes its readings. These readings are taken at 8 A.M. and 8 P.M., 75th meridian time (E.S.T.).

Table 7. Barometer Corrections for Elevation

To correct the observed reading of a mercury barometer or U tube to the equivalent reading at a higher elevation, subtract the following values in inches of mercury for each 100 ft difference in elevation (add for lower elevation).

Mean elevation ft	Mean atmospheric temperature, deg F						
	−20	0	20	40	60	80	100
0	0.13	0.12	0.12	0.11	0.11	0.10	0.10
1,000	0.12	0.12	0.11	0.11	0.10	0.10	0.10
2,000	0.12	0.11	0.11	0.10	0.10	0.10	0.09
3,000	0.11	0.11	0.10	0.10	0.10	0.09	0.09
4,000	0.11	0.10	0.10	0.10	0.09	0.08	0.08
5,000	0.10	0.10	0.10	0.09	0.09	0.08	0.08
6,000	0.10	0.10	0.09	0.09	0.08	0.08	0.08
7,000	0.10	0.09	0.09	0.09	0.08	0.08	0.08

Following is a typical example of a barometer correction:

The barometer reading at the center line of the turbine casing in a power plant at Boston, Mass. is 29.80 in. of mercury. Barometer temperature 75 F. Barometer cistern located 16 ft below turbine center line. The temperature correction (Table 5) is −0.13 in. of mercury; the gravity correction (Table 6) is −0.01 in. of mercury; and the elevation correction (Table 7) is −0.02 in. of mercury. The calibration correction determined from Weather Bureau comparison is assumed to be +0.04 in. of mercury. The total net correction is the sum of these four or −0.12 in. of mercury. The barometric pressure at the turbine center line is then 29.68 in. of mercury.

Aneroid barometers are sometimes used on account of their compactness and portability. The aneroid is an exhausted chamber with corrugated diaphragm walls, the collapsing of which is resisted by a spring. The deflections of the diaphragms against the spring are indicated or recorded by a lever mechanism. A recording aneroid barometer is called a **barograph.** Because of hysteresis and aging, the aneroid barometer should be calibrated frequently against a mercury barometer. The aneroid barometer is also made in the more accurate null type in which the diaphragm is brought back to an initial position by changing the tension of the loading spring. This same motion rotates the scale to give the new reading.

Absolute-pressure gages may be made by using a mercury barometer tube as a vacuum gage. The vacuum connection is made at the mercury

cistern, or the barometer tube may be turned into a U and the vacuum connection made at the open end. Since the pressure in the closed end is practically zero, the mercury-level differential gives absolute pressure directly. Recording absolute-pressure gages of this type are available, the recording function being obtained by mounting a float in the high-pressure end of the U-tube barometer. Recording absolute-pressure gages are also made by differentially connecting an aneroid barometer and a spring-type vacuum gage.

Manometers are U-shaped tubes, either vertical or inclined, filled with a liquid of known density. The difference in pressure at the two ends of the U is determined from the displacement of the liquid. A manometer with one end open to atmosphere gives a direct reading of equivalent gage pressure or vacuum, *i.e.*, pressure above or below atmospheric. The U-tube manometer (Fig. 6) may be used in either upright or inverted position. The air or indicating liquid in the top of the inverted U tube must be trapped before inverting the instrument, or it may be pumped in against the existing pressure. A correction must be made for the difference in heights of the columns of indicating liquid above the manometric fluid in the two legs. Even with air above the manometric fluid, a correction should be made at high pressures—the density of air at 100 lb. pressure is about 0.01 of the density of water.

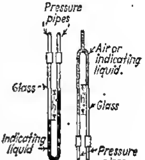


FIG. 6.—Upright and Inverted U-tube Manometers.

A manometer is easier to read if a large chamber or cistern is used as one leg of the U tube, as shown in Fig. 7. The readings may then be taken on the small-bore side only, with proper compensation for the slight change in level in the cistern. A cistern-type manometer with the small-bore tube inclined instead of vertical is often called a **draft gage**, because it is the instrument usually used in test work for measuring chimney draft. The increase in length of scale for a given vertical rise results in a more sensitive instrument. The multiplying factor for the scale of an inclined manometer or draft gage is equal to the cosecant of the angle the tube makes with the horizontal. The usual multiplying factors are from 2 to 1 to 10 to 1, but a multiplication of 20 to 1 may be used if the instrument is accurately leveled. The most common errors in the readings of inclined manometers are due to faulty setting of the zero of the scale or to inaccurate leveling of the instrument. They should always be calibrated by comparison with a micromanometer, preferably a hook gage device with water as fluid.

FIG. 7.—Cistern-type Manometer.

The **two-fluid manometer** is also a more sensitive gage than a simple U tube. Two fluids of different specific gravity are used, such as oil and water or mercury and oil. A two-fluid manometer for gas pressures is shown in Fig. 8. One of the cisterns may be open to the atmosphere. The heavier liquid may be either water or alcohol diluted with water to make its specific gravity only slightly greater than that of the lighter liquid (kerosine). The alcohol may be colored with an aniline dye that is insoluble in the kerosine. If the specific gravity of the dilute alcohol is 0.83 and that of the kerosine is

MECHANICAL MEASUREMENTS

BY

G. L. TUVE

(Originally prepared by J. A. Moyer)

REFERENCES: A.S.M.E. Codes on Instruments and Apparatus. Behar, "Fundamentals of Instrumentation," Instruments Publishing Co. Diederichs and Andrae, "Experimental Mechanical Engineering," Wiley. Glazebrook, "Dictionary of Applied Physics," Macmillan. Shoop and Tuve, "Mechanical Engineering Practice," McGraw-Hill. Smallwood and Keator, "Mechanical Laboratory Methods," Van Nostrand.

Errors in Measurement. An error is the difference between the observed value and the true value and may be expressed as a percentage of either. Errors may be due to the instrument, the method, or the observer. **Instrument errors** may be reduced by repeated calibration. Certain inaccuracies may be unavoidable, such as those due to the aging of glass or of magnets. Errors may be due to sluggishness or lack of sensitiveness, but sensitiveness beyond the requirements of the desired accuracy may result in slowness of operation.

The scales on most indicating instruments are 2½ to 6 in. long. Instrument and chart graduations closer than 20 to the inch are difficult to read. With these physical limitations, an accuracy of 1 or 2 percent of full scale is the best that can be expected with many common instruments. To determine the difference between two readings 1 in. apart on an instrument scale, within an accuracy of 1 percent, calls for readings accurate to a linear distance of 1/200 in. which is not possible without a magnifying lens. **Errors in method** of application or use of instruments are sometimes very large. **Personal errors** and blunders are minimized by the training and experience of observers. A lack of knowledge of instrument technique may introduce errors of considerable magnitude. **Accidental errors** still remain when all controllable conditions are kept as near constant as possible. Such errors may be treated by the laws of probability and the theory of least squares.

Relative accuracy of observations is usually much higher than absolute accuracy. Many instruments and methods are quite satisfactory for measuring *differences* in temperature, velocity, rate of flow, etc., but are not suitable for determining absolute values.

TEMPERATURE MEASUREMENTS

Instruments for measuring temperature are classified in Table 1, which also gives the temperature range and the degree of accuracy usually obtainable.

For temperatures below 1000 F, the choice is among a mercury thermometer, a thermocouple, a gas- or vapor-filled pressure-gage thermometer, and an electrical-resistance thermometer. For temperatures above 1000 F, the choice is among a thermocouple, a radiation pyrometer, and an optical pyrometer.

Liquid-in-glass thermometers are usually filled with mercury and have a vacuum in the capillary. Since the freezing point of mercury is −38 F, alcohol or pentane is used for very low temperatures. To measure temperatures above 600 F, the mercury is sealed under pressure, using nitrogen or carbon dioxide in the capillary. With special glass, this gives a range up to 1000 F, although difficulties from stem distortion may be encountered above 900 F. For greater ease in reading, the glass stem may be made with colored inserts, a colored background, or with a magnifying-lens front. Glass-stem thermometers are graduated for complete immersion of bulb and stem unless partial immersion is specified. When the stem of a total-immersion ther-

0.79, then the difference of gas pressures in inches of water is $H(0.83 - 0.79) = 0.04H$, *i.e.*, the gage shows a difference 25 times greater than a U tube with water. A small correction may be necessary for the difference in level of the surfaces in the two cisterns, but by making the cisterns large this correction may be avoided. The glass columns should not be over $\frac{3}{16}$ in. in internal diameter to avoid irregular meniscus.

The manometer liquids most commonly used are *mercury, water, oil, alcohol*. Table 8 gives the densities of these liquids at 32 and 70 F and also the conversion factors to change manometer readings into pressure in pounds per square inch.

Inverted-bell manometers are widely used for permanently mounted draft gages, and they operate like a gasometer storage tank. Functionally, they are modified U-tube manometers, the inside of the bell acting as one leg of the manometer (see Fig. 9). Vertical displacement of the bell is balanced by a weight, and the displacement may be greatly magnified by a lever mechanism and a long pointer. Thus a large easy-reading scale may be used.

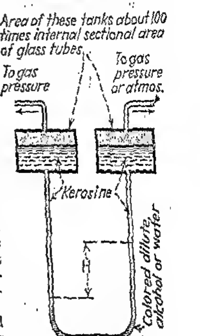


FIG. 8.—Differential Manometer.

Micromanometers are manometers fitted with some precision reading or magnifying device. They are usually graduated in increments of 0.001 in. of water, though they may in some cases be sensitive to 0.0001 in. Most micromanometers are of the inclined-tube type. The **Wahlen gage** is a two-fluid inclined-tube micromanometer, using fluids of very nearly the same density. (For details of construction see *Univ. Illinois, Eng. Expt. Sta. Bull.* 120.)

A **hook gage** is a means for accurately measuring a liquid level, and it is sometimes applied to manometers, in order to obtain more accurate readings

Table 8. Manometer Liquids and Conversion Factors

Manometer liquids	Specific gravity relative to water at 39 F[a]		To convert in. of liquid to lb per sq in. multiply by	
	Manometer temp 70 F	Manometer temp 32 F	Manometer temp 70 F	Manometer temp 32 F
Mercury	13.543	13.595	0.489	0.491
Water	0.9980	......	0.0361	
Kerosine (44–46 deg. A.P.I.)	0.799	0.812	0.0289	0.0293
Red draft gage oil (approx)	0.820	0.835	0.0296	0.0301
Methyl (wood) alcohol	0.790	0.810	0.0285	0.0293
Ethyl (grain) alcohol	0.789	0.805	0.0285	0.0291
Gasoline (58–60 deg. A.P.I.)	0.739	0.752	0.0267	0.0270

[a] Density of water at 39 F, 62.43 lb per cu ft.

mometer is only partially immersed, as in a thermometer well, a **correction for stem exposure** must be made. The A.S.M.E. Test Codes recommend the attachment by asbestos thread of a second thermometer with its bulb near the middle of the exposed portion of the stem and corrections made by the formula $K = 0.000088D(t_1 - t_2)$, where D is the number of degrees of exposed filament, t_1 the reading of the main thermometer, and t_2 the reading of the attached thermometer. When the stem is cooler than the bulb, the correction K, in degree Fahrenheit, is added.

Table 1. Range and Accuracy of Thermometers and Pyrometers

Type	Range, deg F	Accuracy, deg F
Glass thermometers:		
Ordinary glass, mercury-filled	−38 to 760	0.5 to 2
Jena glass, mercury and nitrogen-filled	−38 to 1000	2 to 10
Quartz glass and borosilicate glass, mercury and nitrogen-filled	−38 to 950	2 to 10
Ordinary glass, alcohol-filled	−95 to 150	1 to 2
Ordinary glass, pentane-filled	−300 to 70	1 to 2
Pressure-gage thermometers:		
Vapor-pressure type:		
Alcohol-filled	200 to 400	2 to 10
Ether-filled	*100 to 300*	*2 to 10*
Sulphur-dioxide-filled	20 to 250	2 to 10
Liquid-filled or gas-filled type:		
Alcohol-filled	−50 to 300	2 to 10
Mercury-filled	−38 to 1000	2 to 10
Nitrogen-filled	−200 to 1000	2 to 10
Bi-metallic thermometer	300 to 1000	Uncertain
Electrical resistance thermometer	*−400 to 1800*	*0.005 to 5[a]*
Thermocouple pyrometers:		
Base-metal	300 to 2000	2 to 20[a]
Rare-metal	300 to 2800	2 to 20[a]
Optical pyrometers	1400 up	20 for black-body conditions
Radiation pyrometers	*1000 up*	*20 to 30 for black-body conditions*
Fusion pyrometers	1100 to 3600	20 in best makes
Calorimetric devices	100 to 2500	Uncertain
Color-temperature charts	800 to 2900	Uncertain

[a] Depends upon indicating instrument.

External pressure corrections should be applied to glass-stem thermometers when the bulb is exposed to high pressures. The correction is about 0.01 F per lb per sq in. in the case of a high-grade thermometer with a bulb wall thickness of 0.5 to 0.7 mm.

Thermometer wells similar to the finned well of Fig. 2 and Table 2 are prescribed by the A.S.M.E. Codes for measuring the temperature of a gas or of a superheated vapor in a pipe. When the pipe contains a saturated vapor only, or a flowing liquid, the fins may be omitted. Thermometer wells should be filled with a non-viscous liquid of high conductivity. Water, alcohol, or kerosine may be used for low temperatures, mercury or oil for higher temperatures, and tin or solder for temperatures above 600 F. Exposed parts should be insulated.

For industrial applications, glass-stem thermometers with special metal protecting cases are available. In all applications, the direct immersion of the thermometer in the fluid is preferred, but where this is impracticable the bulb should at least be in good thermal contact with the wall of the thermometer well.

In **solid-expansion thermometers,** the sensitive element is a bimetallic strip or coil which is fixed at one end. The quality of these instruments

than are possible by visual observation of a meniscus. The essential feature of a hook gage is the piercing of the liquid surface by a sharp-pointed hook, raised from below. Just before the point pierces the skin of the liquid surface, a pimple is seen to rise above the point. The point is lowered until this pimple is barely discernible, and the position of the point, *i.e.*, the liquid level, is then read by means of a vernier or a micrometer screw. The standard Fan Test Code of the A.S.H.V.E. requires that inclined manometers used in fan tests be calibrated in place against a water-filled hook gage reading to 0.001 in. of water. See p. 281.

Commercial manometers are available with indicating, recording, and integrating devices. These devices are actuated by the level of the liquid in the manometer (usually mercury) using floats and lever arrangements or electrical or magnetic arrangements. It is thus possible to make the manometers distant reading.

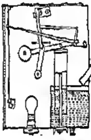


FIG. 9.—Inverted-bell Manometer.

Dead-weight Gages depend on balancing a known weight against the pressure to be measured. The weight is supported on a piston, and the pressure is usually transmitted to the bottom of the piston by oil, which also acts as seal and as lubricant for the piston. The piston is rotated with respect to the cylinder to prevent friction errors. Dead-weight gages are used as **gage testers** for calibrating other gages. They are also used in precision test work and for master gages in the boiler rooms of large steam stations. By spring loading a dead-weight gage, a very sensitive pressure gage for a small range may be obtained, as, for instance, a gage that reads 575 to 625 lb per sq in. for a 600 lb steam boiler; such gages are made distant reading by electric telemetering. The dead-weight gage is obtainable in any range from manometer pressures to 50,000 lb per sq in.

Bourdon-tube gages are the most common of the spring-type gages. A Bourdon tube is elliptical in cross section, bent into a circular arc. When internal pressure is applied to such a tube, it tends to straighten out. Single-spring and double-spring arrangements are used in pressure gages. A variety of Bourdon gages are available, from the common production-type gage selling at a dollar or two, to the special precision gage with machined alloy-steel tube, hardened sector, pinion, and bearings, and special adjustments including a micro zero setting. Such precision gages are accurate to about 0.25 percent of full-scale reading and are made in large sizes, usually 10 to 16 in. For accurate work, a Bourdon gage should be calibrated frequently on a dead-weight gage tester. In use, the gage should be protected from vibration, from excessive temperatures, and from corrosive liquids or gases. Gages of special materials are available, in case corrosion is a problem. For ammonia, an all-steel gage is required.

Diaphragm and **bellows gages** are used mainly for low pressures, though metallic-bellows gages are available for pressures as high as 200 lb per sq in. The diaphragm or bellows gage provides larger forces for actuating the indicating or recording mechanism than the bent-tube gage and, consequently, is especially suitable for measuring pressures in the manometer range or in the low Bourdon-type ranges. **Slack-diaphragm gages** use soft elastic diaphragms of leather, treated cloth, or rubber, externally spring loaded. Several commercial makes of draft gages operate on this principle and have the advantage of not involving any liquid or requiring accurate leveling.

varies from the domestic room thermometers costing but a few cents to laboratory thermometers of fair precision. Solid-expansion thermometers should be checked frequently, because the zero point may change with use.

Table 2. Dimensions of Thermometer Wells

Dimensions for Plain Thermometer Wells (Fig. 1), In.

Pipe size, in.	*A*	*B*	*C*	*D*	*E*	*G*	*H*	*J*	*P*
3–6	4 3/16	3 15/16	1 13/16	15/16	3/4	5/8	5/8	5/8	1/2
7–20	7 5/16	7	2 1/16	15/16	3/4	3/4	5/8	5/8	3/4

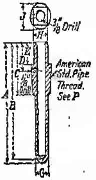


FIG. 1.—Plain Thermometer Well.

FIG. 2.—Finned Thermometer Well.

Dimensions for Finned Thermometer Wells (Fig. 2), In.

Pipe size, in.	*A*	*B*	*C*	*D*	*E*	*G*	*H*	*J*	*K*	*L*	*M*	*P*
3–6	4 9/32	4	2	1 13/16	15/16	0.525	5/8	7/8	1 1/16	3/32	1/16	1/2
7–12	7 1/8	7	2 1/4	2 1/16	15/16	5/8	3/4	1 1/16	7/8	5/32	3/32	3/4
13–20	10 5/16	10 3/16	2 1/4	2 1/16	15/16	5/8	1	1 5/16	1 1/8	5/32	3/32	1

Pressure-gage thermometers are the least expensive of the distant-reading and recording types and are widely used in industrial service. The operating fluid may be a gas, vapor, or liquid; the fluid bulb is connected by capillary tubing to a pressure-spring displacement unit, as shown in Fig. 3. Inexpensive units used for automobile instrument boards have an accuracy of only 5 to 10 percent of full-scale reading, while larger units for industrial and laboratory service may have a maximum error of 1 to 3 percent of full-scale reading. If the capillary and the spring tube of a gas- or liquid-filled unit are subjected to ambient temperatures widely different from that for which the instrument was calibrated a large error may result unless some compensating device is used; the vapor-pressure type (Fig. 3) is not affected by these variations. The sensitive bulb of a pressure-gage thermometer is comparatively large in size and is subject to radiation effects when used in air or gas; these radiation errors are minimized through the use of stainless steel bulbs or sockets, but it is best to locate the bulb where it will not "see" surfaces much hotter or colder than the temperature being measured. If temperatures beyond the range of the instrument are likely to be encountered, some type of overrange protection should be built into the instrument.

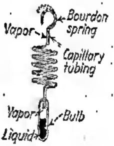


FIG. 3.—Vapor-pressure thermometer.

Thermocouples are pairs of wires, of dissimilar metals, connected at both ends. When the two junctions are subjected to different temperatures, an

MEASUREMENTS OF TIME AND SPEED

The ordinary spring-wound clock or stop watch is the most common instrument for measuring time, but with the accurate frequency control on electric utility systems, synchronous-motor timing devices are becoming more common. System frequency is usually held so close to 60 cycles per sec that synchronous timers are sufficiently accurate for all engineering uses.

High-speed studies may require more refined methods of timing than the ordinary stop watch or electric clock. By the use of the oscillograph and the motion-picture camera, moment-to-moment conditions in a rapidly changing system can be studied in detail. For measuring very short time intervals or longer intervals very accurately, both mechanical and electrical devices are available. The most common mechanical device is the tuning fork, usually arranged to draw a sine curve. By using forks of different frequencies, almost any desired subdivision of the second may be obtained. Chronographs may employ electromagnets to vibrate a pen or pencil, or the paper may be punctured at regular intervals by a high-tension electric spark produced by a mechanical interrupter. In a-c **cycle counters**, the ultimate division is a single a-c cycle.

Starting and stopping errors are the chief inaccuracies in timing speed measurements. The instruments should be mechanically or electrically connected so that a single impulse, such as the throwing of a switch or lever, will start both the timing and the displacement or counting instrument. Long runs are also desirable. An error of a single scale division on a ⅕ sec stop watch amounts to 1.33 percent error in a 15 sec run but only 0.11 percent error in a 3 min run.

Rotational speeds are measured by the counter, the tachometer, and the stroboscope. Photoelectric methods are used for certain types of counting. **Mechanical speed counters** may be either of the rotating or of the oscillating ratchet type. The rotating counter with a magnetic clutch connector and a synchronous electric timer operated by the same switch is probably the most accurate of the common speed-measuring devices and is used for calibrating other instruments. The rotating continuous counter may have direct-reading wheels of the cyclometer type or may operate dials or pointers through a gear train. The oscillating or stroke counter is adapted for low speeds only; rotary counters may be obtained for high-speed work, up to 5,000 rpm or more, but the speed limit for which a counter was designed should be ascertained before it is used for high speeds.

Hand counters are widely used in test work. These are small rotating counters arranged to be driven by a friction tip inserted in the shaft center or by some form of positive engagement. A hand counter or tachometer fitted with a surface wheel for obtaining linear surface speeds is sometimes called a **cutmeter**, because such instruments are used for determining cutting speeds in machine-tool work. Combined counter and stop-watch instruments are available in which the timing and the counting functions are started and stopped simultaneously; they are capable of giving accurate speed measurement with a hand instrument.

Tachometers give a direct and continuous indication of speed in rpm. They may be either permanently mounted or hand type and may be both indicating and recording. Their accuracy is affected by the mechanical condition of the instrument; frequent calibrations are required for accurate work. The force produced by the rotation is balanced against a calibrated spring or against the force of gravity. Electric and fluid-pump tachometers

electrical potential is set up between them. This voltage is almost directly proportional to the temperature difference, and hence a voltage-measuring instrument placed in the circuit will measure temperature. Table 3 gives

Table 3. Temperature-millivolt Relations for Thermocouples

Deg F above cold-junction temp	Millivolts				Deg F above cold-junction temp	Millivolts			
	Copper constantan	Iron constantan	Chromel alumel	Platinum 10% rhodium		Copper constantan	Iron constantan	Chromel alumel	Platinum 10% rhodium
50	1.11	1.44	1.08	0.148	700	....	21.15	15.86	3.067
100	2.23	2.88	2.20	0.313	800	....	24.23	18.20	3.597
150	3.40	4.36	3.34	0.493	900	....	27.33	20.56	4.136
200	4.65	5.84	4.50	0.687	1000	....	30.46	22.93	4.686
250	5.93	7.36	5.65	0.892	1200	....	36.96	27.66	5.817
300	7.23	8.87	6.77	1.108	1400	....	43.88	32.33	6.990
350	8.60	10.40	7.88	1.333	1600	....	50.92	36.88	8.204
400	9.99	11.94	8.99	1.565	1800	....	57.96	41.30	9.457
450	11.40	13.47	10.11	1.804	2000	....	65.00	45.57	10.749
500	12.87	15.01	11.24	2.048	2500	....	.....	55.53	14.068
600	16.03	18.08	13.53	2.549	3000	....	.....	.....	17.339

the emf generated at various temperatures with commonly-used thermocouples. The couples are not recommended for use near the maximum temperature unless protected by a metal or ceramic tube. In no case should they be used above the maximum temperature indicated. For use at temperatures below 500 F, copper-constantan (a copper-nickel alloy) thermocouples are best because of their resistance to corrosion and consequent long life with stable calibration. Between 500 F and 1200 F iron-constantan thermocouples are best and can be used in either an oxidizing or reducing atmosphere. The rusting of the iron at low temperatures in the presence of moisture make iron-constantan less desirable than copper-constantan for temperature ranges in which the latter can be used. For high-temperature work platinum-platinum 10 percent rhodium thermocouples should be used. In a reducing atmosphere the couples must be thoroughly protected. A slightly more sensitive high-temperature thermocouple consists of platinum-platinum 13 percent rhodium. Chromel-alumel thermocouples are more stable than iron-constantan but less stable than platinum-platinum rhodium thermocouples for high-temperature work.

The material used for the construction of protecting tubes is dictated by the maximum temperature encountered and the corrosive action to which it will be subjected. Iron or steel tubes can be used up to about 1300 F. The use of nickel-chromium alloy protecting tubes extends the temperature range up to about 2000 F. For higher temperatures various refractory materials (quartz, alundum, etc.) can be used. Silicon carbide tubes are especially resistant to the action of flame up to about 3000 F.

Pyrometers of the cheaper metals often have the leads of the same metals as the couples, so that the cold junction is at the terminals of the galvanometer, and the leads are usually long enough to permit the instrument being placed where the temperature can be maintained at about normal "room" temperatures. Variation in the cold-junction temperature from the calibra-

have the advantage of being adapted to distant reading. Hand tachometers with several sets of change gears are available, so that a wide range of speeds can be accurately measured with a single instrument.

The **stroboscope** utilizes the phenomenon of persistence of vision when an object is viewed intermittently. By viewing a cyclic motion at the same point in the cycle each time, the object appears to be motionless. By changing the frequency slightly, slow-motion in either direction can be obtained. The older stroboscopes interrupted the vision either by a tuning-fork arrangement or by a rotating perforated disk or cylinder. The neon-tube stroboscope is now displacing the other types, because of its convenience and because it is adapted to stroboscopic photography.

Neon-tube stroboscopes are commercially available for speed measurement, with indicating dials calibrated throughout the range from 700 to 14,000 rpm (the "Strobotac," General Radio Co.). These instruments are especially valuable where it is inconvenient to make a connection or contact with the rotating shaft or for light-powered machinery where the load to drive a speed-measuring instrument would affect the operation of the machine.

Speed measurements by the stroboscope necessitate its calibration in terms of frequency. The frequency of a neon-tube stroboscope is conveniently checked against the frequency of an a-c power system. The common neon-glow lamp obtainable at retail stores for a few cents may be operated from any ordinary utility power source for illuminating a stroboscopic disk. This apparatus can then be used for calibrating tachometers at the 60 cycle synchronous speeds, 3,600, 1,800, 1,200 rpm, etc. A satisfactory stroboscopic disk for such calibrations may be made from a 3 in. black disk bearing a single radial white line of some width. At 3,600 rpm this disk will show two lines, at 1,800 rpm it will show 4 lines, at 1,200 rpm it will show 6 lines. When the flash interval is neither a multiple nor an even fraction of the rpm, the geometric figure will appear to be moving.

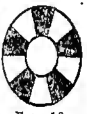


FIG. 10.—Stroboscopic Figure.

If it is desired to hold a rotating machine very exactly at a constant speed, this may be accomplished by constructing a disk with a number n of equally spaced black sectors as in Fig. 10. With 60 cycle current, the disk will appear to be stationary for any speed which is a multiple K of $120/n$. The actual speed may be determined by the use of an ordinary tachometer. The stroboscope would be used to permit close adjustment of the speed to the desired constant value. This arrangement can be used only for speeds of $120K/n$ where K and n are any integers.

WEIGHING DEVICES

The following devices are ordinarily used in engineering work to determine weight:

	Usual capacity, lb	Probable sensitiveness, lb.
Platform scales	100–2,000	1/16–1/2
Spring balances	5–200	1/16–1
Torsion balances	10	1/32–1/4
Automatic scales:		
1. Pendulum type	50	1/32
2. Spring type	200	1/16
Chemical balances	50 (g)	1 in 100,000

Series for the Trigonometric Functions. In the following formulæ, *all angles must be expressed in radians.* If D = the number of degrees in the angle, and x = its radian measure, then $x = 0.017453\ D$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots; \qquad -\infty < x < +\infty.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots; \qquad -\infty < x < +\infty.$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots; \qquad -\pi/2 < x < +\pi/2.$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \dots; \qquad -\pi < x < +\pi.$$

$$\sin^{-1} y = y + \frac{y^3}{6} + \frac{3y^5}{40} + \frac{5y^7}{112} + \dots; \qquad -1 \leqq y \leqq +1.$$

$$\tan^{-1} y = y - \frac{y^3}{3} + \frac{y^5}{5} - \frac{y^7}{7} + \dots; \qquad -1 \leqq y \leqq +1.$$

$\cos^{-1} y = \frac{1}{2}\pi - \sin^{-1} y; \quad \cot^{-1} y = \frac{1}{2}\pi - \tan^{-1} y.$

Series for the Hyperbolic Functions (x a pure number).

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots; \qquad -\infty < x < \infty.$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots; \qquad -\infty < x < \infty.$$

$$\sinh^{-1} y = y - \frac{y^3}{6} + \frac{3y^5}{40} - \frac{5y^7}{112} + \dots; \qquad -1 < y < +1.$$

$$\tanh^{-1} y = y + \frac{y^3}{3} + \frac{y^5}{5} + \frac{y^7}{7} + \dots; \qquad -1 < y < +1.$$

General Formulæ of Maclaurin and Taylor. If $f(x)$ and all its derivatives are continuous in the neighborhood of the point $x = 0$ (or $x = a$), then, for any value of x in this neighborhood, the function $f(x)$ may be expressed as a power series arranged according to ascending powers of x (or of $x - a$), as follows:

$$(1)\ f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + (P_n)x^n. \quad \text{(Maclaurin.)}$$

$$(2)\ f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n-1)}(a)}{(n-1)!}(x-a)^{n-1} + (Q_n)(x-a)^n. \quad \text{(Taylor.)}$$

Here $(P_n)x^n$, or $(Q_n)(x-a)^n$, is called **the remainder term**; the values of the coefficients P_n and Q_n may be expressed as follows:

$P_n = \{f^{(n)}(sx)\}/n! = \{(1-t)^{n-1}\ f^{(n)}(tx)\}/(n-1)!$

$Q_n = \{f^{(n)}[a + s(x-a)]\}/n! = \{(1-t)^{n-1} f^{(n)}[a + t(x-a)]\}/(n-1)!$

where s and t are certain unknown numbers between 0 and 1; the s-form is due to Lagrange, the t-form to Cauchy.

The error due to neglecting the remainder term is less than $(\bar{P}_n)x^n$, or

$(\overline{Q}_n)(x-a)^n$, where $\overline{P}_n$, or $\overline{Q}_n$, is the largest value taken on by P_n, or Q_n, when s or t ranges from 0 to 1. If this error, which depends on both n and x, approaches 0 as n increases (for any given value of x), then the general-expression-with-remainder becomes (for that value of x) a convergent infinite series.

The sum of the first few terms of Maclaurin's series gives a good approximation to $f(x)$ for values of x near $x=0$; Taylor's series gives a similar approximation for values near $x=a$.

Reversing a Series. If $y = x + bx^2 + cx^3 + dx^4 + ex^5 + \ldots$, then $x = y - by^2 + (2b^2-c)y^3 - (5b^3-5bc+d)y^4 + (14b^4-21b^2c+6bd+3c^2-e)y^5 + \ldots$, provided the latter series is converging.

Fourier's Series. Let $f(x)$ be a function which is finite in the interval from $x = -c$ to $x = +c$ and has only a finite number of discontinuities in that interval (see note below), and only a finite number of maxima and minima. Then, for any value of x between $-c$ and c,

$$f(x) = \tfrac{1}{2}a_0 + a_1\cos\frac{\pi x}{c} + a_2\cos\frac{2\pi x}{c} + a_3\cos\frac{3\pi x}{c} + \ldots$$
$$+ b_1\sin\frac{\pi x}{c} + b_2\sin\frac{2\pi x}{c} + b_3\sin\frac{3\pi x}{c} + \ldots$$

where the constant coefficients are determined as follows:

$$a_n = \frac{1}{c}\int_{-c}^{c} f(t)\cos\frac{n\pi t}{c}\,dt,\quad b_n = \frac{1}{c}\int_{-c}^{c} f(t)\sin\frac{n\pi t}{c}\,dt.$$

In case the curve $y = f(x)$ is symmetrical with respect to the origin, the a's are all zero, and the series is a sine series. In case the curve is symmetrical with respect to the y-axis, the b's are all zero, and a cosine series results. (In this case, the series will be valid not only for values of x between $-c$ and c, but also for $x = -c$ and $x = c$.) A Fourier's series can be integrated term by term; but the result of differentiating term by term will in general not be a convergent series.

NOTE. If $x = x_0$ is a point of discontinuity, $f(x_0)$ is to be defined as $\tfrac{1}{2}[f_1(x_0) + f_2(x_0)]$, where $f_1(x_0)$ is the limit of $f(x)$ when x approaches x_0 from below, and $f_2(x_0)$ is the limit of $f(x)$ when x approaches x_0 from above.

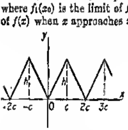


FIG. 4.

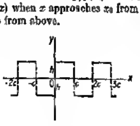


FIG. 5.

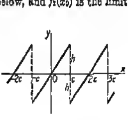


FIG. 6.

Examples of Fourier's Series. If $y = f(x)$ is the curve in Figs. 4, 5, and 6,

then, in Fig. 4, $\quad y = \dfrac{h}{2} - \dfrac{4h}{\pi^2}\left(\cos\dfrac{\pi x}{c} + \dfrac{1}{9}\cos\dfrac{3\pi x}{c} + \dfrac{1}{25}\cos\dfrac{5\pi x}{c} + \ldots\right)$,

in Fig. 5, $\quad y = \dfrac{4h}{\pi}\left(\sin\dfrac{\pi x}{c} + \dfrac{1}{3}\sin\dfrac{3\pi x}{c} + \dfrac{1}{5}\sin\dfrac{5\pi x}{c} + \ldots\right)$,

in Fig. 6, $\quad y = \dfrac{2h}{\pi}\left(\sin\dfrac{\pi x}{c} - \dfrac{1}{2}\sin\dfrac{2\pi x}{c} + \dfrac{1}{3}\sin\dfrac{3\pi x}{c} - \ldots\right)$.

Platform scales are best suited for most weighing operations. This type is quite sensitive when new, but its original sensitiveness is soon lost if the knife-edges are allowed to become rusty from exposure to dampness or become dulled from careless and excessive loading. All scales should be tested with standard weights and adjusted whenever used in important work. **Spring and torsion balances** are usually not very reliable and are used chiefly because they are easily portable and compact. Zero reading is not dependable. Careful calibrations are essential. **Automatic indicating scales**, particularly of the pendulum ("Toledo") type, are the best for many purposes such as weighing the fuel consumption of oil engines. A pointer on a dial indicates the weight continuously. Automatic scales operating with a spring mechanism are subject to the same faults as the usual types of spring balances. **Automatic scales** for coal bunkers are filled under the chute from the bunker till a predetermined weight has accumulated. The supply is then shut off, and the scoop trips and discharges. Such devices are usually integrating, and automatic scales for weighing fuel being conveyed to power houses and furnaces are recording.

MEASUREMENT OF AREAS

The areas of irregular figures such as indicator diagrams are generally determined either by measuring the lengths of ordinates drawn on the figure and inserting their values in certain formulas or "rules," or from readings of planimeters.

In the **ordinate method**, the figure (Fig. 11) is divided by parallel lines into an even number of strips of equal width w and the ordinates $y_0, y_1, y_2 \dots y_n$ measured. Letting n = number of strips (the greater the value of n, the greater the accuracy of the method), the area A may be approximately determined by using one of the following formulas:

FIG. 11.

1. **Trapezoid rule:**

$$A = w(\tfrac{1}{2}y_0 + y_1 + y_2 + \cdots y_{n-1} + \tfrac{1}{2}y_n)$$

2. **Durand's rule:**

$$A = w(0.4y_0 + 1.1y_1 + y_2 + y_3 + \cdots y_{n-2} + 1.1y_{n-1} + 0.4y_n)$$

3. **Simpson's rule** (see also p. 106):

$$A = \tfrac{1}{3}w(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$$

The various lengths required by the foregoing methods can be conveniently added by laying them off with dividers one after the other along a straight line and finally measuring the total length of the line.

Planimeters are instruments for measuring areas. The **Amsler polar planimeter** (Fig. 12) consists of two arms pivoted at O. At the end of one arm is a tracing point T, and at the end of the other a "fixed point" P. Attached to the tracing arm is a small graduated wheel W. In using the instrument, mark a starting point for the tracing point T on the contour of the figure to be measured and observe the reading of the graduated wheel W. If the figure is traced in a **clockwise** direction, back to the starting point, the area measured is found by subtracting the first reading from the last; but if the tracing point is moved around in a counterclockwise direction, the last reading must be subtracted from the first. The foregoing applies to small areas of only a few square inches, such as indicator cards, etc. For large figures, the area of the zero circle of the instrument enters into the

any form of tapered or rounded open-end tube will give accurate impact pressures, but it is always a question whether the static pressure measured is the true static pressure at the opening of the impact tube. Nevertheless when the Pitot tube is accurately positioned parallel to the axis of a continuous pipe, using straightening vanes and several diameters (5 to 20) of straight-pipe approach, the accuracy of the individual velocity readings is probably well within ±1 percent. Accurate calibrated manometers must be used. For obtaining the average velocity across the pipe, a traverse is necessary, and in a round pipe this is usually made on each of two diameters, with positions in the center of area of three or more concentric areas (preferably five or more, see Table 11), depending on the pipe size and the required accuracy (see Cole and Cole, "Pitot Tubes in Large Pipes," and Hubbard, "Investigation of Errors of Pitot Tubes," *Trans. A.S.M.E.*, Aug., 1939). For rectangular ducts, the readings are taken in the center of equal rectangular areas, using 16 to 64 areas, depending on the duct size. Pitot tubes have been designed which deviate greatly from the form shown in Fig. 22, using static openings facing downstream (see p. 279) or at an angle with the stream. Calibrations of such Pitot tubes are necessary. If a Pitot tube is preceded by a long run of straight pipe, approximate results may be obtained by fixing the tube in the center of the pipe and using a factor to obtain the average velocity. For turbulent flow, this factor (ratio of true average velocity to center velocity) varies within the range of about 0.77 to 0.92, being lower for rough pipes than for smooth ones. If this method is to be used, it is advisable to make a special determination of the factor. For the case of laminar flow, the factor is about 0.5. The impact tube is sometimes used separately, either with a static connection in the side wall of the pipe or in open streams where the static pressure is atmospheric. As long as the plane of the impact opening is normal to the flow, the size, shape, and wall thickness of the impact tube can be varied widely without affecting the accuracy, but the diameter of the tube should not be over 5 percent of the diameter of the air stream.

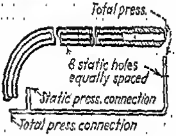


FIG. 22.—Standard Pitot Tube for Fan Testing (A.S.M.E. and A.S.H.V.E. Test Codes).

Table 11. Layout Measurements for Pitot-tube Traverses

Number of equal areas	Total number of readings	Distances from center of pipe to point of reading in percent of pipe diameter					
3	12	20.4	35.3	45.5			
4	16	17.7	30.5	39.4	46.6		
5	20	15.5	27.2	35.3	41.7	47.4	
6	24	14.5	25.0	32.3	38.2	43.3	47.9

MEASUREMENT OF POWER

Dynamometers, or instruments for measuring force or "power," are in general of two kinds: (1) those **absorbing** the power by friction and dissipating it as heat; (2) those **transmitting** or passing on the power they

calculation of areas. This correction has to be added whenever the fixed point is inside the area which is being measured. Its value can be determined by measuring a circle or other figure of known area. The difference between the known area and that recorded by the instrument is the area of the zero circle. The area of the zero circle is stamped conspicuously on many planimeters. For measuring figures of indefinite length and limited breadth, **roller planimeters** must be used. They are expensive and are not much used in America. Special planimeters are available (Coffin "averager") for determining directly the mean height of an indicator card.

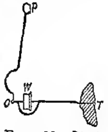


FIG. 12.—Planimeter.

Integrators for Circular Charts. Instruments have been developed for measuring the mean ordinate of circular charts having a constant radial scale. Such instruments are made by the Bailey Meter Co., Builders Iron Foundry, The Foxboro Co., Keuffel & Esser Co., and others.

For detailed information on the theory and operation of both polar and radial planimeters, see A.S.M.E. Power Test Codes, Instruments and Apparatus, Part 15, "Measurement of Surface Areas."

FLUID METERS

REFERENCES: Reports of the A.S.M.E. Research Committee on Fluid Meters: Part 1, "Theory and Application"; Part 2, "Description of Meters"; Part 3, "Selection and Installation." A.S.M.E. Power Test Codes, Instruments and Apparatus, Part 5, "Measurement of Quantity of Materials." Stewart and Doolittle, "Fluid Flow Measurement," *Instruments*, July, 1939, pp. 175–198. Report of the A.G.A.-A.S.M.E. Orifice Coefficient Committee (published by the A.S.M.E., 1935).

Selection of a meter for a given application depends on the rate of flow, the fluid to be metered, and the accuracy required (see Table 9). Quantity meters are usually more accurate than rate meters, especially for the lower rates of flow and for non-steady conditions. For large quantities and when the flow is free from sudden variations, the rate meters are accurate and are less expensive. They are also well adapted to combinations for indicating, recording, and totalizing the flow rate. Special selections are necessary for viscous liquids, for liquids containing sediment, or for gases or vapors with entrained particles of solids or liquids. Most commercial meters are designed for metering a specified fluid.

Gas and Air Meters

Bellows gas meters are generally used in connection with the sale of fuel gases, but they are also suited to other measurements of clean gases or air in the temperature range from 32 to 110 F. Such meters consist of a casing divided into two chambers by a vertical partition. Within each chamber is a measuring receiver in the form of a leather-walled diaphragm. Slide valves admit gas alternately into the flexible measuring receivers, and the reciprocating movements of filling and emptying actuate a set of counter dials. Connections are such that the movement of one bellows is at mid-position when the other is passing the end or dead point. Bellows meters are rated at a pressure drop of 0.5 in. of water and are usually adjusted to within ± 1 percent error, though the error may increase with use and be larger at the greater rates of flow. These meters are well suited for intermittent duty, and their accuracy is almost unaffected by variations in the

measure and wasting only a small part in friction. Devices for measuring power may be classified for convenience as follows:

Type	Approx limit of speed, rpm	Usual power limit, hp	Probable error, percent
Prony brakes:			
Block	2,000	10	1–5
Band	1,000	5	1
Wooden cleats on bands	1,000	200	½–1
Rope (¾-in.) with wooden cleats	1,000	50	1
Fluid friction dynamometers:			
Froude (ordinary "water" brake)	10,000	25,000	⅕–½
Westinghouse (turbine)	4,000	5,000	$\frac{1}{10}$–⅕
Alden	1,000	5,000	½–1
Fan brake	2,000	200	1–5
Electric dynamometers:			
Electric eddy-current brake	6,000	300	⅕–¼
Electric generator	750–4,000	30,000	$\frac{1}{10}$–⅕
Transmission dynamometers:			
Torsion	1,000–3,000	50,000	1–5
Kenerson	1,500	100	2

The speed limits given above are approximately the highest allowable. Limits of horsepower refer to the largest sizes made commercially. Any of these types (with the exception of the Westinghouse turbine) are made in sizes to absorb from 1 hp up. The probable error stated is very approximate and refers to the apparatus in fair adjustment.

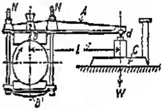


Fig. 23.

Absorption Dynamometers

A Prony brake consists of a lever A (Fig. 23) and blocks B, B' supported on a revolving drum or pulley. The blocks are held in place and tightened by the thumb nuts N, N. The tendency of the arm A to revolve is prevented by the resistance of a platform scales C, as shown, or by weights attached. If the pressure on the pedestal at d due to its own weight and that of the lever arm A is W_0 lb (determined by the weight on the scales when the brake block is supported at B on a three-cornered prism or on a small rod of circular section), the gross weight at the end of the brake arm with load is W lb, the length of the brake arm in feet is l, and the number of revolutions per minute is n, then brake horsepower $= 2\pi ln(W - W_0)/33{,}000$.

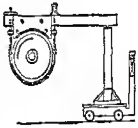


Fig. 24.

The disadvantages of any form of Prony brake are that they require continued adjustment for constant power and that water cooling is necessary for moderate and large powers, with consequent untidy conditions.

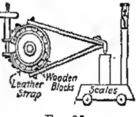


Fig. 25.

Figs. 23–25.—Types of Prony Brakes.

According to Bach, suitable dimensions for a brake of this type are given by the formula $bd =$ bhp $\times 12/k$, where d is the diameter of the brake pulley, in.; b the breadth of the brake blocks, in. (usually about 1.5 times the diam of the shaft); and k = ½

flow rate. They may be obtained with sheet-metal casings for low pressures and with cast iron casings for high-pressure lines.

Table 9. Classification of Fluid Meters

Common name or type	Fluids metered	Class of measurement	Maximum capacities	Accuracy ± %
Quantity meters:				
Weighers and tilting traps	Liquids	Weight	Unlimited	1
Disk water meters	Clean liquids	Volume	20–1,000 gpm	2–4
Piston meters	Clean liquids	Volume	1–1,000 gpm	0.5–2
Drum condensate meters.	Hot water	Volume	250–12,000 lb per hr	2–3
Bellows gas meters	Gases	Volume	150–17,000 cfh	1–3
Wet gas meters	Gases	Volume	100–100,000 cfh	0.5–2
Rotary gas meters	Gases	Volume	500–1,000,000 cfh	0.5–2
Rate meters (for steady flow only):				
Orifice plate	Any fluid	Head (constant area)	Unlimited	1–3
Flow nozzle	Any fluid	Head	Unlimited	1–3
Venturi meter	Any fluid	Head	Unlimited	1–3
Pitot tubes	Any clean fluid	Head	Unlimited	1–3
Weirs	Liquids (in open channels)	Head area	Unlimited	0.5–2
Area meters (variable orifice).	Clean fluids	Area (constant head)	Unlimited	1–3
Velocity meters	Air or water	Velocity	Unlimited	2–10
Thermal meters	Air and clean gases	Weight	25,000–2,000,000 cfh	1–2

Wet gas meters are usually of the revolving-drum type, sealed with water, but they may also be oil sealed and may employ a nutating bell instead of a revolving drum. With careful adjustment of the liquid level, the accuracy of laboratory-size wet meters with large dials may be well within ±1 percent. These meters should not be subjected to severe pulsations or rapid fluctuations of the flow rate, and the gas should be free from dust or corrosive constituents. Integrating registers of the multiple-clock-dial type are normally mounted in the meter case. Rotating-drum meters are suited for low pressures only.

Rotary dry-gas meters are adapted for very wide ranges of operating temperature and pressure and for high rates of flow. They are unaffected by pulsating or intermittent flow, will handle foul gases as well as clean gases, and operate on a pressure drop of 0.1 to 2.0 in. of water. These meters can register flow in either direction, but it is better to install them with the gas entering from the top to eliminate dirt settlement in the meter casing. Normal capacities vary from 500 to 1 million cu ft per hr, but larger meters can be furnished. These meters can be adjusted to have an error within 0.5 per cent, but an exact calibration is difficult.

Orifice, nozzle, Venturi, and Pitot meters for gas and air are widely used, especially for measuring high rates of flow (see Head Meters, pp. 1802 to 1807).

Thermal meters depend on observations of the change of temperature of gas or air when a known amount of heat is added. Electric thermal

for cooling air and varies from 2.5 to 5 for water cooling as the speed increases. F. A. Halsey (*Am. Machinist*, Apr. 4, 1912) states that the brake-drum surface for a water-cooled Prony brake should not be less than 0.09 sq ft per bhp, to avoid the danger of the blocks taking fire.

In Fig. 24, the lower block in the preceding figure is replaced by two steel bands and narrow cleats of maple or oak attached by wood screws inserted from the outside through the bands; the upper block is lined with similar cleats, the screws being countersunk. At least ½ in. spaces should be allowed between cleats for air circulation. In all such constructions for dynamometers, screws and nails should not touch the friction surface, as they are likely to cause the friction to be variable and the sound production is objectionable. Grooves may be cut in the inside surface of a few of the cleats and these grooves filled with grease to provide a little lubrication. A similar brake is shown in Fig. 25, where maple cleats are screwed to a leather belt. This type is more easily adjustable to different sizes of pulleys than the preceding designs, but it is not so durable. Washers should be placed on the heads of the screws fastening the cleats to the belt. If a brake wheel similar to Fig. 26 is used so that it can be satisfactorily cooled with water on its inside surface, the cleats should provide 5 to 10 sq in. of friction surface per bhp according as the speed ranges from low to high.

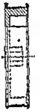
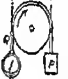


Fig. 26.

Band and rope brakes are also frequently used. The simplest form is shown in Fig. 27. If Q is the reading of the spring balance, lb; r the wheel radius, ft; P the applied weight, lb (both P and Q must be net); and n the rpm, then $\text{bhp} = 2\pi rn(Q - P)/33{,}000$. This type of brake is very accurate and sensitive, but it is suitable only for low powers. About the same friction surface must be allowed as given for wooden blocks by Bach's formula (see Prony Brakes, *ante*). A convenient type of **rope brake** with "stay" cleats is shown in Fig. 28, the rope being passed around the circumference of the pulley. In this arrangement, the cord or rope supporting the spring balance must have some point of attachment overhead. It is advisable to provide an anchoring rope or wire securely attached to the weights P, and its weight (or that part of it suspended) must then be added to P. Similarly, the weight of the rope between the spring balance and the point where it touches the pulley should be deducted from the readings of the spring balance. Usually four to six cleats are used on rope brakes. These cleats are often attached to the ropes by strong copper wire passed through a strand of the rope and over the backs of the cleats. The best arrangement is to place the



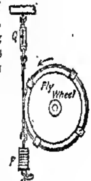


Fig. 27. Fig. 28.
Rope Brakes.

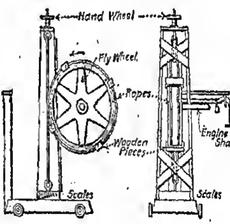


Fig. 29.—Rope Brake.

meters are made by Cutler-Hammer Inc., Milwaukee, and consist of an electric heater, two electric resistance thermometers, and a current regulator (Fig. 13). The current regulator automatically maintains 2 deg F difference between entrance and exit of the meter; hence wattmeters and watt-hour meters may be used to read flow directly. Capacities are 25,000 to 2 million cu ft per hour.

Area meters for gas and air are of two types, the variable-orifice and the multiple-orifice meters. Figure 14 is a diagram of a typical variable-orifice area meter. The Ingersoll-Rand and the Rotameter (Fischer and Porter Co.) are of the variable-orifice type, operating on a constant differential head as determined by the weight of the movable unit. These meters are self-adjusting for wide variations in the flow rate. The Toolometer (New Jersey Meter Co.) is a similar meter using multiple orifices instead of a single variable opening. Several manufacturers offer gate-type variable orifices or multiple-orifice units which are intended for manual adjustment. Since area meters are essentially orifice meters, they are subject to the advantages and disadvantages of orifice metering, but they have a much wider range of capacities. Their accuracy is usually within ± 2 percent.

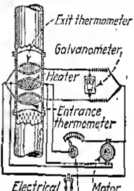


FIG. 13.—Electric-thermal Gas Meter.

Velocity meters of the open type are widely used for measuring air velocities and quantities, and these instruments are called anemometers. There are two types of anemometers, the mechanical and the thermal.

Rotating-vane anemometers usually have four to eight flat vanes mounted on a wheel, though a single propeller-shaped blade is sometimes used for high-speed work. The wheel is connected to a counting train, and the dials are calibrated to read directly in feet. A lever for setting all dials to zero may be included. The total dial reading for a timed run divided by the total elapsed time in minutes gives the air velocity in feet per minute. This type of instrument is especially suited for obtaining the average air velocity over a large area, as for instance in the stream from a fan or from an air-conditioning grille. The procedure is then to subdivide the area into small areas about the size of the instrument to be used and to hold the instrument successively in each small area for a short peroid (10 to 30 sec), stopping it only after all areas have been covered. If the volume of air is required, the average velocity is multiplied by the area. This result is an approximation unless the anemometer has been calibrated for the specific conditions of use. A rotating-vane anemometer should not be held in the hand during operation, but mounted at the end of a thin handle.

FIG. 14.—Orifice-and-plug Steam Meter.

The **rotating-cup anemometer** is used principally for meteorological observations. The standard Weather Bureau anemometer has three cups, with the arms set at 120 deg. In addition to the indicating dial on the instrument, reading in miles, the instrument may be equipped with electrical contacts to operate a counter every ½ mile, or the unit may contain an electric generator from which both indicating and recording instruments are energized.

rope on the wheel double. In Fig. 29, an arrangement is shown in which both ends of the rope are attached to a rigid frame supported on weighing scales. The reading of the scales (corrected for the weight of the frame) gives $Q - P$ directly. A double rope ¾ in. diam with six cleats each of about 12 sq in. on the "friction" side will absorb 50 bhp. For engines of smaller power, ½ in. rope with four cleats is used. By steeping the rope in a mixture of melted tallow and graphite, the frictional properties are improved. Manila or cotton ropes are used.

The water brake is a fluid-friction dynamometer somewhat similar in construction to a centrifugal pump. The dynamometer casing is supported on antifriction bearings and tends to revolve with the rotor. An attached brake arm, supported on scales, measures the turning moment, and the horsepower absorbed is calculated in the same way as for a Prony brake. One or more smooth rotors may be used in a water brake, or additional friction may be obtained by vanes and recesses on casing and rotor. The horsepower absorbed by a water brake varies approximately as the cube of the rpm and as the fifth power of the rotor diameter (see Culver, "Investigation of a Simple Form of Hydraulic Dynamometer," *Mech. Eng.*, Oct. 1937). Several commercial models of water brakes are available (Taylor Mfg. Corp., C. H. Wheeler Mfg. Co., Murray Iron Works, Bendix Products Corp., etc.). A water brake is sometimes mounted on the same shaft with an electric dynamometer to produce a dynamometer unit of sufficient capacity and flexibility to test large automotive and airplane engines.

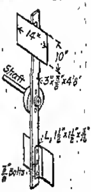


Fig. 30.—Fan Brake Dynamometer.

For the approximate determination of the horsepower of high-speed engines, a **fan brake dynamometer** consisting of two fan plates (Fig. 30) to be attached to the shaft is very convenient but is less reliable than any of the other methods described. The power is absorbed by the "fan" action of the plates on the surrounding air, which depends on the size of the plates, their distance from the center of rotation, and upon the cube of the rpm. Figure 31 shows curves for determining the bhp for varying speeds of a fan with rectangular blades 10 in. wide in a radial direction, 14 in. wide axially, and ½ in. thick. These curves are for atmospheric pressure and a temperature of 70 F. Errors are introduced by other pressures, temperatures, and the location of near-by objects. The numbers on the curves indicate inches from the center of one plate to the center of the other. This distance is varied by shifting the bolts. Ordinary weather changes may affect the power absorbed by a fan dynamometer by as much as 20 percent. It is convenient for comparative factory tests (see Merriam and Staples, "Aerodynamic Dynamometer," *Mech. Eng.*, July, 1938).

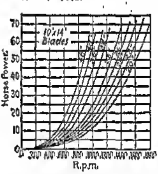


Fig. 31.—Power Absorbed by Fan Brake Dynamometer.

Electric dynamometers include calibrated generators, calibrated motors, cradle-mounted generators and motors, and magnetic-drag or eddy-current brakes. The calibration of an electric generator or an electric motor con-

Bridled-vane anemometers are direct-reading velocity meters consisting of one or more vanes, the movement of which is resisted by a calibrated spring. When the air velocity produces an impact force tending to move the vane, this force is resisted by the spring, and the equilibrium position of the vane indicates the air velocity. The range of the instrument may be changed by varying the size of the air openings and its versatility increased by using various types of impact fittings or jets at the end of a rubber connecting tube as in the case of the Velometer (Ill. Testing Laboratories).

The **heated-thermometer anemometer** is an ordinary thermometer with an electric resistance coil wound on the bulb (see "The Heated Thermometer Anemometer, C. P. Yaglou, *Jour. Ind. Hyg. Toxol.*, Oct., 1938, p. 497). By impressing a constant voltage across the coil and comparing the reading of this thermometer in the air stream with that of an unheated thermometer in the same stream, a difference is obtained which is a function of the air velocity. The range of the instrument is changed by changing the impressed voltage. After suitable calibration, an extreme range of air speeds from the minimum natural convection currents in a room to a mile-a-minute stream may be measured with the same instrument. The **heated-thermocouple anemometer** operates in a similar manner and has the additional advantage of being distant reading.

Calibrations of anemometers are made either by moving the instrument in still air or by mounting the instrument in a wind tunnel and comparing its readings with those of some other air meter. The still-air calibration is usually made with the instrument mounted at the end of a rotating arm, and for accuracy within 3 percent an arm at least 10 ft long should be used. Calibrations can also be made in the discharge of a large rounded-entrance nozzle, but they must not be attempted in the stream from an ordinary duct or from an orifice.

Steam meters are usually some form of head meter, such as the orifice, nozzle, or pitot tube (see Head Meters, pp. 1802 to 1807). The St. John steam flow meter (American District Steam Co.) is an area meter of the orifice-and-plug type, equipped with a direct pointer indicator and a strip-chart recorder (Fig. 14).

Liquid Meters

Rotary-disk meters are generally used for metering cold water on domestic and commercial service lines. They consist of a circular metering chamber, with a conical roof and floor (Fig. 15) divided into two equal compartments by a "nutating" disk. The disk does not rotate about its own axis, but the shaft on which it is mounted generates a cone with apex downward. Motion of the disk is guided by the two half-balls mounted upon it, and with each complete revolution a fixed volume of water passes through. The circular motion of the upper end of the disk shaft operates the counting gears. Disks are usually made of hard rubber molded over metal reinforcement. "Frost-proof" disk meters have a breakable bottom plate and a separable metering chamber. Standard sizes of disk meters are for ½ to 6 in. pipe line connections, with "normal" capacities from about 20 to 1,000 gpm. The error of a disk meter will usually be 2 or 3 percent, and they are intended for use only for clean water below 125 F, though they are also furnished with metal or with carbon disks for hot water and other clean liquids.

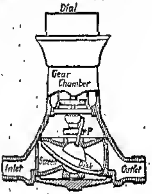


Fig. 15.—Rotary-disk Water Meter.

Piston meters are made in a variety of combinations of one, two, or more pistons, reciprocating or oscillating in fixed chambers. A typical piston

sists in determining the efficiency of the machine over a range of operating conditions. For convenience and for accuracy in interpolation, an input-output curve should be drawn. With the aid of this curve, the machine may then be used as a dynamometer over that range of operating conditions covered by the calibration, by measuring the electric power only.

The **electric-cradle dynamometer** (Fig. 32) is an electric motor or generator with rotor and stator mounted in concentric ball bearings, so that the stator is free to turn. The torque can then be measured by suitable scales. A separately excited d-c machine is usually used and can be used over a wide range of operating conditions both as a motor and as a generator.

Magnetic drag or eddy-current brakes are similar to cradle-mounted generators, but the electrical energy is dissipated within the machine itself. Load rheostats are therefore not necessary, and the entire installation is more compact and less expensive than that of a cradle-mounted d-c machine, but of course the motoring feature cannot be obtained in a magnetic brake. Commercial water-cooled magnetic brakes are available for automotive engine testing (Mid-West Dynamometer and Engineering Co., Electric Products Co., etc.).

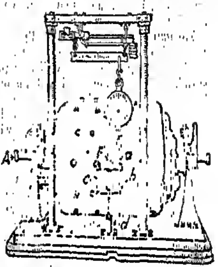


FIG. 32.—Electric-cradle Dynamometer.

A **chassis dynamometer** is an assembly that includes treadwheels and an absorption brake, so mounted that an automobile or a motor truck may be tested as a complete unit. The rear wheels of the automotive vehicle revolve upon the treadwheels of the dynamometer. Chassis dynamometers are made either with a ramp for floor mounting, or they may be located in a pit with the treadwheels at floor level and the controls at a convenient operating height. Several sizes are available, with capacities up to 400 hp and 100 mph (Bendix Products Corp.).

Airplane engines are often tested by being set up on a **torque stand** with a cradle attachment. The testing is done with the propeller attached. The engine is supported on ball or roller bearings in such manner that it is free to rotate, about the crankshaft as an axis, through a small angle. The torque required to prevent such rotation is measured.

Transmission Dynamometers

Torsion Dynamometers. When a shaft is subjected to a twisting moment, an angular twist is produced which is proportional to that moment. Thus, if the angle of twist is produced by a twisting moment T in in-lb and n is the rpm, then bhp $= 2\pi Tn/(12 \times 33{,}000)$.

Torsion meters, although applicable to large as well as small powers, have their most important applications for measuring shaft horsepower of marine turbines and engines. Several installations have been made of a torsion meter in which the angle of twist of the power-transmitting shaft is measured by the electric-gage principle. This gage uses reactance coils and a bridge circuit (see Hathaway and Lee, "The Electric Gage," *Mech. Eng.*, Sept., 1937).

meter for water is the Worthington meter (Fig. 16). Water is admitted alternately at the two ends by a slide valve moving on seats in a bottom plate containing the inlet and discharge ports. Piston meters are capable of a high degree of accuracy, but actual errors will depend on the type of service for which they are designed, on maintenance, and on the accuracy of adjustment.

Revolving drum condensate meters are common for metering purchased steam, and they usually operate at or near atmospheric pressure, with gravity discharge. The principle of operation is shown in Fig. 17. Standard registration for these meters is integration only but some of the larger sizes may be equipped with circular chart recorders. Sizes range from 250 to 12,000 lb per hour. An error of 2 or 3 percent may be expected. Approximately 18 in. static head is required to operate the meter, and the meter must be installed so as to be self-clearing or accuracy will be affected. Ordinary pulsations or intermittent flow will not affect the accuracy of the revolving drum meter.

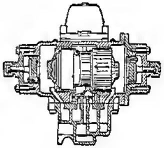


FIG. 16.—Worthington Piston Water Meter.

Rotary-displacement meters of the lobed impeller, gear, or screw type are sometimes used for water, though they are better adapted to oil and other lubricating liquids.

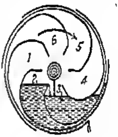


FIG. 17.—Condensate Meter.

Velocity meters for water are of two types: the **current meter** for measuring the velocity in open channels (see p. 278), and the enclosed **propeller or turbine type.** Enclosed velocity meters are made in a variety of designs and in sizes from ¾ to 12 in. pipe. The smaller meters are used chiefly for hot water or for dirty water. The large meters are suitable for high rates of flow only, and if accuracy at all rates is necessary, a **compound meter** consisting of both velocity meter and disk meter, with suitable automatic valves, is often installed.

Weir Meters. The weir is an open-channel or tank meter in which the registering device is operated by a change in height of the surface of the liquid (see pp. 259 to 263). In most mechanical engineering applications, the V notch or triangular weir is preferred because of its great capacity range (about 20 to 1). The 90 deg V notch is the most common, the 60 deg is sometimes used, and the "half-notch" (53 deg 8 min) and the "quarter-notch" (27 deg) have the advantage of even fractional capacities with the same meter register. Accuracy should be within 2 percent, and in some cases may be within 0.5 percent. Standard maximum heads for commercial meters are usually 4 to 15 in., though larger meters can be obtained (up to 4 million lb per hr). Secondary devices of most commercial meters are actuated by floats; indicators, recorders, and integrators are available. To obtain charts and scales with equal divisions or increments, special shapes of floats, cams, or tanks are used.

Head Meters

A head meter consists of two units, a primary device that produces a "differential head" or pressure difference, which varies as the square of the

For descriptions of various types of transmission dynamometers, see Diederichs and Andrae, "Experimental Mechanical Engineering."

Engine Indicators

Essential requirements for a good indicator are (1) a calibrated spring with a minimum of inertia and a high natural period of vibration; (2) a lightweight well-fitting lubricated piston; and (3) a light sturdy rod and linkage mechanism between piston and tracing point. For an exhaustive study of the development and use of indicators of all types, see De Juhasz, "The Engine Indicator," *Instruments*, June, 1932, to July, 1933 (14 chapters). Indicators are usually made in three sizes, giving cards approximately 5 × 2¾, 3½ × 2, and 2¼ × 1¼ in., and used, respectively, for low-, intermediate-, and high-speed work.

Indicator springs are usually double-coil spiral springs, calibrated in terms of the pressure required at the piston to produce a movement of 1 in. at the marking point. An indicator piston of ½ sq in. area is the normal or standard size. When used with pistons of other areas, the spring scale will be inversely as the piston area. *Indicator springs should be calibrated in place by the dead-weight test method.* This also furnishes a check on the pencil motion or magnifying gear, which should be free from lost motion as the pressure is increased and decreased. **Outside-spring indicators** are preferable for greater convenience in handling and because the spring is not subjected to considerable temperature changes. The indicator piston requires lubrication; for internal-combustion-engine service, a built-in lubricating arrangement, such as a grease feed, is desirable.

The paper drum is of light construction for minimum inertia, with an internal helical spring easily adjustable for tension. Indicators are available for recording numerous diagrams on a continuous strip of paper fed from a spool. In some cases, provisions may be made for driving the drum at uniform speed, for a pressure-time record. Special non-stretching **indicator cord** should be used. Stretch of the cord should be less than ½ percent with a 10 lb pull; a wire core will reduce the stretch by 50 percent. Indicator cord should be as short as possible, long connections being made by rigid links, wire or steel tape with slack take-up. **Integrating indicators** are available, using either mechanical or electrical integrators.

Precautions in the Use of Indicators. Unless an engine indicator is carefully handled, the diagrams taken with it may be in error from 5 to 10 percent. The following are the most important considerations:

1. Springs must be calibrated frequently.
2. The tension of the spring in the drum should be adjusted.
3. Before an indicator is used, all working parts should be cleaned and oiled. The piston and its rod should be examined before attaching the spring to determine by lifting the pencil lever and letting it fall whether these parts move easily. Attach the spring firmly, and observe carefully that there is no lost motion. This precaution is most important in indicators having a bead on the spring for making a ball-and-socket joint. To put this kind of spring in place properly, the piston rod should be screwed tightly into the piston when the lower adjusting nut is loose. Then screw up this nut just tight enough to permit a slight movement. If the nut in the piston has been properly adjusted, there should be no lost motion between the rod and the piston, and still there should be flexibility in this joint permitting the piston to adjust itself in the indicator cylinder.
4. Oil the piston with cylinder oil every time it is taken from the cylinder. Many careful engineers oil the piston regularly after taking about ten diagrams. A new piston usually requires more lubrication than one that is well worn.

rate of flow, and a secondary device, usually a manometer of some kind for measuring the differential head (see Pressure Measurements, p. 1792). The common head meters are (1) thin-plate sharp-edged orifice; (2) flow-nozzle or rounded-entrance nozzle; (3) Venturi tube; and (4) Pitot tube (or impact) tube). Selection of one of these four depends on requirements. **Pressure loss** caused by the *Pitot* tube is zero; for the other three devices, a loss of 10 to 90 percent of the differential pressure will occur. **Accuracy** of all four types of head meters should be within 2 percent if they are properly installed and operated. Head meters can be constructed for almost any desired capacity, and they may be used for any reasonably clean fluid with steady non-pulsating flow. Discharge coefficients are only slightly affected by the fluid used. Variations due to meter size, properties of the fluid metered, and rate of flow are all taken into account by stating orifice coefficients as a function of the dimensionless Reynolds number $DV\rho/\mu$ (see pp. 250 and 288). The Pitot tube is usually used with an indicating manometer only, but the other head meters are available with a wide variety of integrating and recording as well as indicating devices, both for near and distant reading.

Meter location is very important in the case of head meters, as upstream disturbances affect the flow. Various codes and instructions specify 5 to 40 diam of straight pipe upstream, depending on the diameter ratio and the fittings in the line. It is good general practice to allow 10 diam upstream and 5 diam downstream, if possible, and to use straighteners (egg crate or nest of tubes) several diameters upstream from the meter.

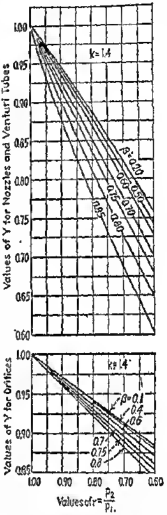


Fig. 18.—Values of the Expansion Factor Y (β = Ratio of Orifice or Throat Diameter to Pipe Diameter).

The flow equation for any head meter measuring a liquid, or measuring a compressible fluid with a differential head less than 40 percent of the upstream absolute pressure, is

$$Q = YMCA\sqrt{2gh} \qquad (1)$$

where Q = volume rate of flow, cu ft per sec (multiply by 60 for cfm); Y = compressibility factor or expansion factor (see Fig. 18); M = velocity of approach factor or "meter constant," the same for orifices, nozzles, and Venturi meters (see Table 10); C = coefficient of discharge, for values see discussion of each type of meter; A = measured area of minimum section of the throat, tube, or orifice, sq ft; g = acceleration of gravity = 32.174; and h = differential head in feet of the fluid flowing (conditions as of the upstream tap).

For large departures from atmospheric temperature, an additional correction for thermal expansion of the opening should be applied (about +0.5 percent for 400 F; see Fluid Meters, Part 1, Fig. 55). For derivations of

5. Adjust the handle on the pencil motion so that when the pencil is sharp it will draw a very fine line. If the pencil presses heavily on the paper, friction and shifting of the paper may distort the diagrams. Prepared metallic paper if used with blunt brass points increases pencil friction considerably above what it is when sharp lead pencils or ordinary "household" pins are used.

6. Adjust the length of the indicator cord, and turn over the engine by hand to make sure the cord is of proper length.

7. Immediately after a diagram has been taken, it should be examined and any irregularities or faults noted and corrected.

8. Mark diagrams plainly as regards head and crank ends of double-acting engines, the time, scale of spring, and name or initials of the person taking them.

9. Best results are always obtained with an indicator at each end of the cylinder of a double-acting engine. Tests show that sharp bends, long pipes, and restrictions of bore may cause errors in indicator diagrams as great as 20 percent (W. F. M. Goss, *Trans. A.S.M.E.*, 1896). This is a particularly important consideration in air and ammonia compressors having very small clearance and in long-stroke engines where the piping for a three-way cock for a single indicator would add considerably to the clearance.

10. The indicator cock should be kept closed, and the cord to the reducing motion should be unhooked except when a diagram is being taken.

High-speed operation introduces problems due to inertia and vibration. Mechanical indicators of the high-speed type embody stiff springs, light-weight parts, large pistons, and magnification of about 8 times between piston and pencil point. One successful model uses a cantilever spring (Fig. 33). Very small diagrams are also drawn by high-speed *indicators* (*¾ × 2 in.* or *smaller*). Stiff springs and light moving parts increase the natural frequency of vibration of the assembly. The spring mechanism should have a natural frequency of at least 10 to 12 oscillations per stroke of the engine.

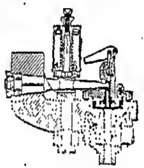


FIG. 33.—High-speed Indicator with Cantilever Spring.

For high-speed-engine research on internal-combustion engines, there are several modifications of the mechanical high-speed indicator. **Micro-indicators** trace very small diagrams which must be greatly magnified for analysis. **Optical indicators** obtain magnification by optical systems and record photographically. There are many well-developed optical indicators of which a number such as the Manograph, the Hopkinson, the Midgley, and the von Gehlen have been produced commercially. Optical indicators have a piston or a diaphragm in the pressure chamber, operating against a beam spring and transmitting a very small motion to a mirror which reflects light from a point source. A motion is imparted to the mirror in another plane (or to a second mirror in the system) by some type of connection to the engine piston or shaft. The card drawn by the light beam is magnified by the optical arrangement and can be either viewed on a ground glass screen or photographed.

In the **electrical indicator**, the characteristics of an electric circuit are made to change by the changes in pressure in the cylinder. The recording element is usually an oscillograph. The resistance, capacity, impedance, or potential in the electrical circuit may be varied by the pressure element. A carbon-pile rheostat suitable for the resistance method was developed at the Bureau of Standards (see *Tech. Paper* 240; also Martin and Caris, *Jour. S.A.E.*, July, 1928). The electrical-potential method differs from other indicator devices in that it utilizes a piezoelectric crystal instead of an indicator spring as a pressure-responsive element. With a vacuum-tube amplifier

Table 10. Velocity of Approach Factor M

For orifices, nozzles, and venturi tubes

Diameter ratio D_2/D_1	0.20	0.25	0.30	0.35	0.40	0.45
Approach factor M	1.001	1.002	1.004	1.008	1.013	1.021
Diameter ratio D_2/D_1	0.50	0.55	0.60	0.65	0.70	0.75
Approach factor M	1.033	1.049	1.072	1.103	1.147	1.209

flow equations and for equations applying to flow through orifices and nozzles with greater pressure drop, see pp. 253 to 259, and 353 to 358.

Thin-plate orifice meters are the most common head meters because they are relatively inexpensive, easy to construct and easy to install, and more data are available on the orifice than on any other metering device. A thin-plate concentric orifice is a flat diaphragm with a circular hole in the center. It may be clamped concentrically between the flanges in a pipe line, at the

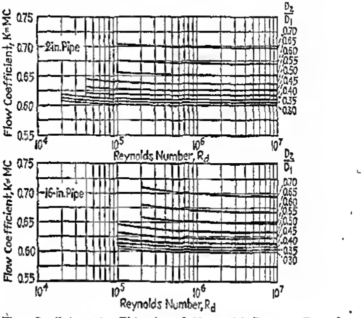


FIG. 19.—Flow Coefficients for Thin-plate Orifices with Pressure Taps One Diameter Upstream and One-half Diameter Downstream.

intake or the discharge end of a pipe or duct, or in the wall of a plenum chamber or a suction box. The hole must have a 90 deg corner at its upstream face, with no burrs or rounding, and the cylindrical edge should not exceed 5 percent of the orifice diameter. When thicker plates are used, the downstream corner may be beveled at 45 deg. Plates must be flat, strong enough to resist bulging, and non-corroding. **Pressure connections** recognized by the A.S.M.E. Code are as follows: (1) **flange taps**, with the centers of the holes 1 in. from the respective faces of the orifice plate; (2) **vena contracta taps** with the upstream hole located one pipe diameter from the upstream face and the downstream hole at the vena contracta; (3) **radius taps** with the upstream hole located one pipe diameter from the upstream face and the downstream hole one-half diameter from the downstream face. Drill size for the pressure hole at the inner surface of the pipe should not be over ¼ in. for 3 in. pipe or smaller, or over ½ in. in any case. The inner surface around the pressure hole should be smoothed with emery cloth. **Coefficient**

and a cathode-ray oscillograph, synchronized pressure-time curves may be obtained (R.C.A. Mfg. Co.). Electrical indicators have the advantage of distant reading.

In **multicycle indicators**, the indicator diagram is built up point-by-point from a large number of engine cycles. The Bureau of Standards Indicator (*N.B.S. Rept.* 107) operates on the balanced-pressure principle, with the cylinder pressure balanced against an external pressure source. The English Farnboro indicator, operating on this principle, has been refined and applied to airplane-engine research by the N.A.C.A. (N.A.C.A. *Tech. Note* 348). A balanced pressure indicator in which the balancing pressure is supplied by the engine itself and the cards are taken on an ordinary mechanical indicator has been developed by De Juhasz (*Instruments*, Mar., 1933).

Maximum pressure indicators are available for showing the maximum pressure in an internal-combustion engine cycle. A simple form uses a balancing spring to oppose the cylinder pressure. A contact and a neon-light flasher indicate the point of balance. The spring adjustment is a micrometer screw calibrated in pounds per square inch (Bacharach Industrial Instrument Co.).

Reducing Motions

To obtain the conventional indicator diagram, the drum of the indicator must be moved so as to give on a smaller scale an exact reproduction of the motion of the piston. A device accomplishing this is called a perfect reducing motion. Many devices used quite generally do not give a perfect motion.

The Pantograph (Fig. 34) is a theoretically perfect device and is actually accurate so long as there is no slackness at the numerous pinned joints. The point of attachment of the indicator cord B must be in the straight line joining the fixed point C with the connection on the cross head A. Figure 35 is a simple reduction motion that is nearly perfect when properly laid out. The pin D is attached to the cross head C, and the link BD connects the cross head to the oscillating arm AB. To convenient points, as 1, 2, or 3, the indicator cord is attached. If the arc XY in its extreme positions at the ends of the stroke reaches as much above the center line of motion of the pin D as it does below, the movement of the drum will be almost an exact duplicate of that of the piston. If the indicator motion is taken from the heavy dotted circular segment G instead of from a pin, the motion is more accurate. Figure 36 is also a simple device and is fairly accurate if AB is made equal to BC. The cord is attached to D.

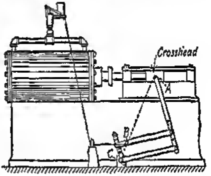


FIG. 34.—Pantograph Reducing Motion.

Figure 37 shows an accurate and simple reducing motion for attachment to the end of a crankshaft. The ratio of the lengths of the connecting rod and stroke of this device must be the same as the corresponding ratio in the engine. A similar device is shown in Fig. 38.

Reducing wheels may be attached either to the drum of the indicator or to some convenient part of the engine. Figure 39 shows a typical device. The large wheel receives the cord direct from the cross head,

of discharge C for a given orifice measurement depends on pipe size, diameter ratio, location of taps, and Reynolds number. Values are given in Fig. 19 for the product "$M \times C$" for "radius taps," applying to 2 and 16 in. pipe sizes, and interpolations may be readily made. (Fluid Meters, Part 1, gives extensive tables of coefficients to four places.) Diameter ratios greater than 70 percent (throat diameter to pipe diameter) are not recommended by the A.S.M.E. Code. Pressure loss due to the insertion of a thin-plate orifice is as follows:

Diameter ratio D_2/D_1	0.3	0.4	0.5	0.6	0.7
Pressure loss in percent of differential pressure	90	82	73	62	51

For metering viscous liquids (low Reynolds numbers), orifice coefficients change greatly, and available data are inadequate except for limited conditions (see Tuve and Sprenkle "Orifice Discharge Coefficients for Viscous Liquids," *Instruments*, Nov., 1933; Aug., Sept., 1935).

Intake and discharge orifices, located at the end of a pipe instead of in a continuous run, are very convenient, especially for air measurement.

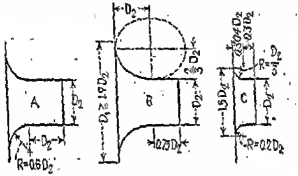


FIG. 20.—Types of Flow Nozzle.

A, A.S.M.E. Standard for displacement compressors.
B, A.S.M.E. long radius nozzle for low diameter ratios.
C, International Standards Association (I.S.A.) nozzle.

For a review of existing data, see Marks, "*Square-edged Inlet and Discharge* Orifices for Measuring Air Volumes in the Testing of Fans and Blowers," *Trans. A.S.M.E.*, Nov., 1936. In the absence of any generally accepted table of coefficients, it may be noted that a value of $C = 0.60$ is probably correct within 1.5 percent for either intake or discharge orifices, for Reynolds numbers above 100,000 for any diameter ratio up to $D_2/D_1 = 75$ percent, and for corner taps (at the orifice face), for flange taps (1 in. distant from face), or in the case of intake orifices for a tap 40 percent of the pipe diameter downstream. This tolerance is substantiated by Polson and Lowther (*Univ. Illinois, Eng. Expt. Stat. Bull.* No. 240) on orifices discharging from a plenum box.

The **flow nozzle** has the advantage over the thin-plate orifice of a more nearly constant coefficient of discharge, especially at high Reynolds numbers. But much less experimental work has been done on flow nozzles than on orifices, and moreover the form of the nozzle is not well standardized. Three proposed standards are shown in Fig. 20.

The A.S.M.E. Fluid Meters Report does not give any values of the coefficients for the short radius I.S.A. nozzle and gives only "tentative" coefficients for the long-radius nozzle. For high Reynolds numbers (above 400,000), a constant coefficient of discharge of 0.995 is given for both types A and B

and the cord on the small wheel is connected direct to the drum of the indicator. Ratio of diameters gives the reduction. Each reducing wheel is provided with a nest of rings to be put on the small wheel to change its diameter so that various reductions are possible to suit the stroke of the engine. The indicator drum is started and stopped by turning the knurled nut at the end of the shaft. The large wheel has a spring at its center to bring it back on the inward stroke. In engine testing for long periods, it is desirable to disconnect the cord connecting the large pulley with the cross head during the intervals between taking diagrams. Hooks like those shown in Figs. 40 and 41 are very convenient for this purpose, particularly if attachment of the cords can be made to a pin on the cross head. The Trill hook

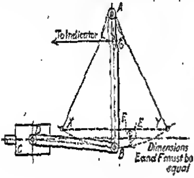


FIG. 35.

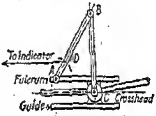


FIG. 36.

Simple Indicator Reducing Motions.

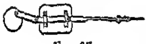


FIG. 37.

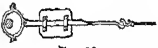


FIG. 38.

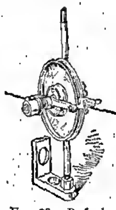


FIG. 39.—Reducing Wheel.

FIG. 40.

FIG. 41.

(Fig. 40) is intended to be held between the thumb and finger, about an inch from the end of the instroke, so that the pin or standard on the cross head strikes the straight part of the hook, and immediately the pin or standard will be caught. The hook shown in Fig. 41 is used in the same way. The hook is of spring brass and is held so that the pin or standard A on the cross head strikes the crotch when about 1 in. from the end of its stroke and opens the hook and assumes the dotted position when in operation. In disengagement, the hook is slipped off the end of the pin. Unless some special form of hook is used, as described, it is difficult to connect the cord when a diagram is to be taken, and the cord is likely to get tangled in moving parts of the engine or to be broken. Such difficulties can often be avoided, particularly in the case of such devices as Figs. 37 and 38, by continuing the cord from its

(Fig. 9). At lower values of the Reynolds number, the "tentative" coefficients of discharge C for type B are as follows:

Reynolds number ($DV\rho/\mu$) throat conditions	10,000	20,000	30,000	50,000	75,000	100,000	200,000	300,000
Nozzle coefficient, C	0.943	0.956	0.961	0.968	0.973	0.978	0.986	0.992

These coefficients are for pressure taps one diameter upstream and in the parallel throat portion downstream (not less than ¾ in. from the end). For nozzles with free open discharge, the tentative coefficients are about 0.5 percent lower. **Pressure loss** due to the insertion of a flow nozzle is about the same as that for an orifice, because there is no diverging tube to give an orderly transformation from velocity pressure to static pressure.

Venturi meters are usually made with diameter ratios of 25 to 50 percent, with an entrance cone of 21 deg, and a cylindrical throat section provided with piezometer openings (see p. 259). The standard Herschel design has an exit cone with a total included angle of 5 to 7 deg giving an over-all **pressure loss** of 10 to 20 percent of the differential pressure. This low

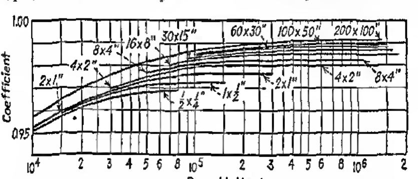


Fig. 21.—Discharge Coefficients for Venturi Tubes (as Manufactured by Builders Iron Foundry and Simplex Valve and Meter Co.).

pressure loss is the main reason for using a Venturi instead of a flow nozzle. Shorter diverging cones may be obtained; a short cone increases the over-all pressure loss but it does not affect the discharge coefficient. **Coefficients of discharge** for Venturi tubes depend on Reynolds number, but also vary *somewhat with diameter ratios and with pipe size.* With throat diameters of 1 in. or more, metering water, the coefficients are practically constant above 10 fps throat velocity, as follows:

Size of Venturi, in.	2 × 1	4 × 2	8 × 4	16 × 8	30 × 15	100 × 50
Discharge coefficient	0.976	0.980	0.983	0.986	0.988	0.990

Coefficients of discharge for general application in the range of Reynolds numbers from 10,000 to one million are given in Fig. 21. (For coefficients outside of this range see A.S.M.E. Fluid Meters Report, Part 1, p. 100.) The interior surfaces of Venturi tubes must be very carefully finished and must be kept clear of dirt or sediment. Very large Venturi tubes have been made with cones of smooth-surfaced concrete, and units as large as 42 × 24 ft are in service.

Pitot tubes are more often used for pipe traverses than for fixed-position meters (see pages 279 and 280). Many forms of Pitot tubes are used, and various standards in air and gases are somewhat different from those used in water. The standard Pitot tube for fan tests is shown in Fig. 22. Almost

point of attachment to the reducing motion closely past the indicator drum to a pipe or simple bracket that may serve as a stationary support. Between the indicator and this last support, a spiral or helical spring of rather light wire or a heavy rubber band is attached to the cord and will keep it taut and in motion. If now a ring is attached to the cord close to the indicator and between it and the reducing motion, this ring will be continually in motion but it will not be difficult to hitch into it the hook on the portion of cord connected to the indicator drum. This method is particularly recommended in every case where wire is used instead of cord.

Some indicators are provided with a detent or device for engaging a pawl in teeth on the circumference of the drum near the bottom and stopping the drum. Obviously, the pawl must engage when the cord is pulled out to the end of its stroke. On the return stroke, the cord will flap about and possibly catch on something and be broken on the next outward stroke. The best way to prevent this is to keep the string always taut by using a helical spring or rubber band attached to some fixed point, as explained above.

In all types of reducing motions except those in which the cord is taken off on a tangent to an arc of a circle, the direction of the cord as it moves back and forth at its point of attachment to the reducing motion must be parallel to the movement of the engine piston. Reducing motions are sometimes made inaccurate by not locating auxiliary pulleys properly to give the cord its proper direction.

DETERMINATION OF THE MOISTURE IN STEAM

Sampling steam is the most uncertain part of a determination of steam quality. The **standard sampling nozzle** should preferably be made of ½ in. pipe and should extend across the steam main, within ½ in. of the opposite wall. The end of the nozzle should be closed, the steam entering through a row of ⅛ in. holes, facing directly against the stream. For main steam pipes smaller than 5 in., the inlet holes are to be ½ in. apart; for larger sizes, six holes are to be used. The holes at each end should be about ½ in. from the pipe wall.

The most desirable **location for a sampling nozzle** is in a pipe in which the steam flows vertically downward, far removed from a valve, elbow, or other disturbing element. Second choice is in a pipe in which steam ascends vertically, far from a valve or fitting. A pipe bend or a horizontal pipe should be avoided, but if no other location is possible the sample should be taken at the entrance to a bend or immediately following a valve or other mixing device in a horizontal pipe. There can be no assurance that the sample will be representative in these latter cases.

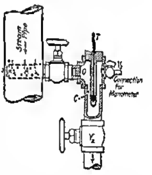


FIG. 42.—Throttling Calorimeter.

The **throttling calorimeter** is most commonly used for determining the moisture in steam. The calorimeter (Fig. 42) consists of a sampling nozzle A, a throttling orifice O, and a thermometer T located in a well which is entirely surrounded by the steam in the expansion chamber C. If the expansion chamber is not open to atmosphere, the pressure within the chamber is obtained from a manometer or gage connected at V_2. Two other essentials are the measurement of the pressure (or the temperature) in the saturated steam main and an ample insulation covering the calorimeter and connections.

INDETERMINATE FORMS

In the following paragraphs, $f(x)$, $g(x)$ denote functions which approach 0; $F(x)$, $G(x)$ functions which increase indefinitely; and $U(x)$ a function which approaches 1; when x approaches a definite quantity a. The problem in each case is to find the limit approached by certain combinations of these functions when x approaches a. The symbol $\doteq$ is to be read "approaches."

CASE 1. "$\frac{0}{0}$." To find the limit of $f(x)/g(x)$ when $f(x) \to 0$ and $g(x) \to 0$, use the theorem that $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$, where $f'(x)$ and $g'(x)$ are the derivatives of $f(x)$ and $g(x)$. This second limit may be easier to find than the first. If $f'(x) \doteq 0$ and $g'(x) \to 0$, apply the same theorem a second time: $\lim \frac{f'(x)}{g'(x)} = \lim \frac{f''(x)}{g''(x)}$; and so on.

CASE 2. "$\frac{\infty}{\infty}$." If $F(x) \to \infty$ and $G(x) \to \infty$, then $\lim \frac{F(x)}{G(x)} = \lim \frac{F'(x)}{G'(x)}$, precisely as in Case 1.

CASE 3. "$0 \cdot \infty$." To find the limit of $f(x) \cdot F(x)$ when $f(x) \to 0$ and $F(x) \to \infty$, write $\lim [f(x) \cdot F(x)] = \lim \frac{f(x)}{1/F(x)}$, or $= \lim \frac{F(x)}{1/f(x)}$; then proceed as in Case 1 or Case 2.

CASE 4. "0^0." If $f(x) \to 0$ and $g(x) \to 0$, find $\lim [f(x)]^{g(x)}$ as follows: let $y = [f(x)]^{g(x)}$, and take the logarithm of both sides thus:

$$\log_e y = g(x) \log_e f(x);$$

next, find $\lim [g(x) \log_e f(x)]$, $= m$, by Case 3; then $\lim y = e^m$.

CASE 5. "1^∞." If $U(x) \to 1$ and $F(x) \to \infty$, find $\lim [U(x)]^{F(x)}$ as follows: let $y = [U(x)]^{F(x)}$, and take the logarithm of both sides, as in Case 4.

CASE 6. "∞^0." If $F(x) \to \infty$ and $f(x) \to 0$, find $\lim [F(x)]^{f(x)}$ as follows: let $y = [F(x)]^{f(x)}$, and take the logarithm of both sides, as in Case 4.

CASE 7. "$\infty - \infty$." If $F(x) \to \infty$ and $G(x) \to \infty$, write $\lim [F(x) - G(x)]$ $= \lim \frac{\frac{1}{G(x)} - \frac{1}{F(x)}}{\frac{1}{F(x) \cdot G(x)}}$; then proceed as in Case 1. Sometimes it is shorter to expand the functions in series. It should be carefully noticed that expressions like $0/0$, ∞/∞, etc., do not represent mathematical quantities.

CURVATURE

The **radius of curvature** R of a plane curve at any point P (Fig. 7) is the distance, measured along the normal, on the concave side of the curve, to the **center of curvature**, C, this point being the limiting position of the point of intersection of the normals at P and a neighboring point Q, as Q is made to approach P along the curve. If the equation of the curve is $y = f(x)$,

$$R = \frac{ds}{du} = \frac{[1 + (y')^2]^{3/2}}{y''}$$

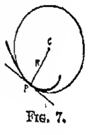


FIG. 7.

where $ds = \sqrt{dx^2 + dy^2}$ = the differential of arc, $u = \tan^{-1}[f'(x)]$ = the angle which the tangent at P makes with the x-axis, and $y' = f'(x)$ and $y'' = f''(x)$ are the first and second derivatives of $f(x)$ at the point P. Note that $dx = ds \cos u$ and $dy = ds \sin u$. The curvature, K, at the point P, is $K = 1/R = du/ds$; that is, the curvature is the rate at which the angle u is changing with respect to the length of arc s. If the slope of the curve is small, $K \approx f''(x)$.

If the equation of the curve in polar co-ordinates is $r = f(\theta)$, where r = radius vector and θ = polar angle, then

$$R = \frac{[r^2 + (r')^2]^{3/2}}{r^2 - rr'' + 2(r')^2},$$

where $r' = f'(\theta)$ and $r'' = f''(\theta)$.

The **evolute** of a curve is the locus of its centers of curvature. If one curve is the evolute of another, the second is called the **involute** of the first.

INDEFINITE INTEGRALS

An integral of $f(x)dx$ is any function whose differential is $f(x)dx$, and is denoted by $\int f(x)dx$. All the integrals of $f(x)dx$ are included in the expression $\int f(x)dx + C$, where $\int f(x)dx$ is any particular integral, and C is an arbitrary constant. The process of finding (when possible) an integral of a given function consists in recognizing by inspection a function which, when differentiated, will produce the given function; or in transforming the given function into a form in which such recognition is easy. The most common integrable forms are collected in the following brief table; for a more extended list, see B. O. Peirce's "Table of Integrals" (Ginn & Co.).

General Formulæ

1. $\int a\,du = a\int du = au + C$ 2. $\int (u + v)dx = \int u\,dx + \int v\,dx$

3. $\int u\,dv = uv - \int v\,du$ 4. $\int f(x)dx = \int f[F(y)]F'(y)dy,\ x = F(y)$

5. $\int dy \int f(x,y)dx = \int dx \int f(x,y)dy.$

Fundamental Integrals

6. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$; when $n \neq -1$

7. $\int \dfrac{dx}{x} = \log_e x + C = \log_e cx$ 8. $\int e^x dx = e^x + C$

9. $\int \sin x\,dx = -\cos x + C$ 10. $\int \cos x\,dx = \sin x + C$

11. $\int \dfrac{dx}{\sin^2 x} = -\cot x + C$ 12. $\int \dfrac{dx}{\cos^2 x} = \tan x + C$

13. $\int \dfrac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + C = -\cos^{-1} x + c$

14. $\int \dfrac{dx}{1 + x^2} = \tan^{-1} x + C = -\cot^{-1} x + c$

Rational Functions

15. $\int (a + bx)^n dx = \dfrac{(a + bx)^{n+1}}{(n + 1)b} + C$

The connection between sampling nozzle and orifice should be as short as possible.

The **steam quality** or relative dryness may be obtained from an enthalpy-entropy chart (see p. 326). Throttling is represented by a vertical line on the chart. The quality (or superheat) of the original steam sample is obtained by locating on the chart the downstream calorimeter temperature and absolute pressure, then tracing vertically to the line representing the upstream pressure. The temperature or moisture content at the latter intersection represents the condition of the original sample. For greater accuracy, a larger chart may be used, or the quality x may be calculated from the equation

$$x = (h_2 - h_f)/h_{fg}$$

where h_2 is the enthalpy of superheated steam at calorimeter pressure and temperature; h_f the enthalpy of the liquid of the high-pressure steam entering calorimeter; and h_{fg} the latent heat of vaporization of the high-pressure steam. The range of the throttling calorimeter is limited, but it increases with higher pressures. At 125 lb per sq in. abs, the range is 0 to 4.6 percent moisture, and at 400 lb its range is 0 to 7.3 percent. The accuracy of the throttling calorimeter itself is high. The A.S.M.E. Code states that its accuracy can be within 0.2 percent.

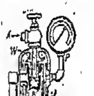


Fig. 43.—Separating Calorimeter.

Separating calorimeters may be used for determining the quality of steam which contains more moisture than can be determined by a throttling calorimeter. A simple form is shown in Fig. 43. The moisture is removed from the sample of steam by mechanical separation just as in the ordinary steam separator installed in the steam mains of a power plant. Steam enters at A, passes down through the vertical pipe into the perforated basin B, from which the dry steam escapes through a narrow slot near the top into the jacket J, while the moisture is deposited at the bottom of the vessel V. The volume or weight of the moisture can be determined from the height of the water in the gage glass G. Dry steam from the jacket J is discharged from the orifice O and must be condensed and weighed in a vessel containing cold water. The percentage of moisture is found by dividing the weight of water collected in the vessel V by the sum of the weight of steam condensed and the weight of water collected in V.

The separating calorimeter described above is effective in removing practically all the moisture in steam when the pressure is not lower than 25 lb per sq in. gage pressure. For lower pressures, such calorimeters may not take out more than 80 percent of the moisture. Consequently, for determinations of moisture in low-pressure steam, a throttling calorimeter should be attached to the discharge of the separating calorimeter. A throttling calorimeter discharging into the atmosphere has very little capacity when used with low steam pressures. By making it discharge into a receiver in which a high vacuum is maintained, the throttling portion of the calorimeter will evaporate 2 to 3 percent of moisture. A combination separating and throttling calorimeter for low-pressure steam is described in *Trans. A.S.M.E.*, 1910, p. 76.

The A.S.M.E. Power Test Codes, Instruments and Apparatus, Part 11, "Determination of Quality of Steam," describes various types of calorimeters including several forms of the throttling and separating types. Combina-

from these may, in some cases, not be disadvantageous, and where velocity-distance lag alone is present may even be helpful. In open-and-shut or proportional-with-reset controllers, good results can be obtained consistently only if the controllers are able to recognize minute changes in measured variable and institute effective corrective action on them. A correctly designed and built controller of simple construction will successfully handle applications that require a much more complicated mechanism, if the latter has been designed and built without sufficient attention having been paid to keeping the force-friction ratio amply large.

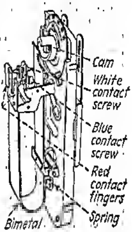


Fig. 1.—Thermostat.

Moving parts should be light, and mounting brackets should be rigid. Unbalanced forces, sometimes unavoidably present, should be prevented from shifting in direction. Mechanism should be arranged so that it takes up lost motion always in the same direction. Power must be provided to overcome friction or to reduce it to a negligible minimum. When mechanical force is necessary, relays should be used.

Force is necessary to produce motion of a final-control element or of a relay. The force developed by the permissible small amount of deviation of the measured variable in the ordinary industrial controller is almost negligible. In some types, such as pyrometer controllers, this force is actually so small as to be incapable of actuating even the most sensitive relay directly. In such cases, resort must be had to detecting the position of a pointer intermittently, which complicates the mechanism and may prevent successful results on processes that can change rapidly.

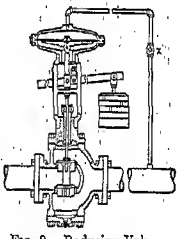


Fig. 2.—Reducing Valve.

Where the force developed by permissible small deviations of a controller element is large enough to be used directly, it is absolutely necessary that its full amount be used to move either the final control element directly, in the case of a self-actuated controller, or the first relay, in servo-actuated types, if the controller is to fulfill its function to its maximum possible extent.

Types of control mechanisms are illustrated in Figs. 1–4, showing different modes of control action and types of actuating mediums.

The common domestic **thermostat** shown in Fig. 1 is an example of an "on-and-off" controller of the "differential-gap" type. When temperature decreases, the bimetallic strip (red) moves toward the two contacts (blue and white). A relay in a standard electrical lockup circuit closes when both contacts are made and opens when both are broken. The red-to-white contact finger is flexible and after closing its contact permits further travel of the bimetallic strip until the red-to-blue contact closes. The difference in travel is ordinarily equivalent to approximately 1½ deg F, which is the "differential gap" and represents the temperature change necessary to start and stop the burner of the house heating system.

The weight-loaded reducing valve in Fig. 2 acts as a proportional regulator, as the change in effective area of the diaphragm through its stroke produces the proportional effect. By pinching off the valve x in the connecting line between the diaphragm and the pressure line, this type is made to act as a variable-rate controller, as described under B(3).

tions of the throttling and separating calorimeters are also covered in the Code. Specific instructions for steam sampling, for calibration of calorimeters, and for calculation of results are given.

ANALYSIS OF FLUE GASES AND EXHAUST GASES

For the analysis of flue gases in a power plant by chemical means, an **Orsat apparatus** is generally used. It consists of a graduated tube or burette (surrounded by a water jacket) to receive and measure the volume of the gas. This burette is connected by a manifold of glass or hard rubber to "pipettes" containing the liquids for absorbing CO_2, O_2, and CO. The pipettes are each a little more than half-filled with the absorbing liquid. Water well saturated with CO_2 is ordinarily used for displacement of gases. For accurate work, brine or mercury should be used. To absorb CO_2, the gases are first passed into a pipette containing a solution of one part by weight of KOH (caustic potash) in two parts by weight of water. Some form of surface-contact pipette or bubbling pipette should be used. Free oxygen is similarly absorbed by a mixture of pyrogallic acid and KOH, prepared by pouring 5 g of powdered pyrogallic acid into the pipette and then adding 100 cc of the KOH solution as prepared for the first pipette. Some engineers use solutions containing a much larger proportion of pyrogallic acid to make the absorption more rapid, but this is not recommended, as stronger solutions are likely to evolve CO in the presence of free oxygen. A more rapid oxygen reagent is chromous chloride. This absorbent may be purchased, all prepared, under various trade names.

The solution for absorbing CO is cuprous chloride (Cu_2Cl_2), which should be made in a glass bottle having a well-fitting glass stopper, greased to make it airtight. It can be made by pouring 25 g of copper oxide into the bottle and adding about 500 cc of commercial hydrochloric acid (sp. gr. about 1.1 being obtained by adding *to water* an equal volume of chemically pure acid). To this solution is added 150 to 200 g of copper wire cut in lengths to extend from the bottom to the top of the bottle. The solution is then allowed to stand in the bottle till clear, when it is ready for use in gas analysis. By filling the bottle from time to time with acid and adding copper wire as needed to replace that dissolved, a supply of the solution will always be on hand. The KOH solution will absorb 20 times its volume of CO_2 before requiring renewal, the pyrogallic solution will absorb efficiently only twice its volume of oxygen, and cuprous chloride only an equal volume of CO. The solution for absorbing oxygen, therefore, requires very frequent renewal.

The A.S.M.E. Power Test Codes, Instruments and Apparatus, Part 10, "Flue and Exhaust Gas Analyses," gives detailed information both on equipment and procedure.

The most common errors in the use of gas apparatus are due to leakage. The apparatus should be carefully tested before starting an analysis by filling each pipette with the solution up to a mark on its capillary tube, and then, after closing all stoppers and pinch cocks, carefully observing the volume of gas at atmospheric pressure on the scale of the burette. The water bottle is then placed on top of the case so that the gas in the burette and in the yoke will be under pressure for about 5 min, after which the volume of gas is again measured. If there is no leakage the volume will remain constant. If there is leakage a drop of water should be put at each place where there might be leakage. Bubbles of gas will indicate where the apparatus is not air-tight. Extreme care should be taken in arranging for and in collecting the samples of flue gas for analysis. A continuous sample should be collected at a uniform rate during the whole period of the test. Many engineers use for a collecting tube a ½ in. pipe open at the end,

The operation of Type B(6)—proportional control with automatic reset—may be followed from Fig. 3, which shows a pneumatically operated instrument.

Assume that the instrument has been in successful control and conditions are balanced so that 8 lb pressure exists in the controlled valve diaphragm motor line (15). The same pressure will exist in bellows (7) and (13).

If an upset occurs, making the element read lower, flapper (4) will uncover nozzle (6), reducing the pressure on diaphragm (11), since air can now flow out of nozzle (6) faster than it can flow in through restriction (12). This will cause bellows (11) to collapse and admit pressure through valve (17), until the increased pressure in bellows (7) causes it to pull the nozzle (6) against the flapper once more.

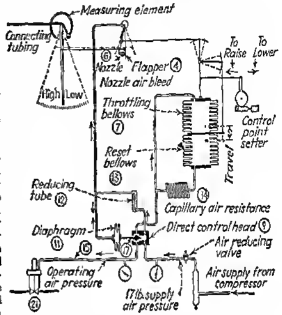


FIG. 3.—Proportional Control with Automatic Reset.

Bellows (7) is spring loaded so that the air pressure change is strictly proportional to the nozzle deflection, producing a uniform throttling range.

This increased pressure immediately starts to bleed through restriction (14) to bellows (13), which is equal in area to, and opposes the motion of, bellows (7). As the pressure in bellows (13) increases, nozzle (6) will be forced away from the flapper (4). However, when this happens, valve (17) in control head (9) will be opened further, raising the pressure in bellows (7) as much as is necessary to keep the nozzle just tangent to the flapper. This resetting effect will continue until the pressure on the diaphragm of the controlled valve (21) has increased (or decreased) enough to produce the new valve opening which will return the temperature (pressure, flow, liquid level) to the set point. When, and only when, this occurs, pressures in bellows (7) and (13) become equal, and equilibrium is again restored.

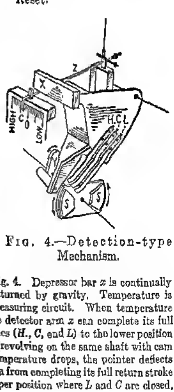


FIG. 4.—Detection-type Mechanism.

A detection-type mechanism is illustrated in Fig. 4. Depressor bar x is continually being raised by the motor-driven cam y and returned by gravity. Temperature is measured by the pointer c, of a potentiometer measuring circuit. When temperature rises, the pointer deflects toward "high," and the detector arm z can complete its full downward stroke. This throws the contact brushes (H., C, and L) to the lower position where H and C are closed by the contact disks S (revolving on the same shaft with cam y), thus reducing the heat supply. When the temperature drops, the pointer deflects toward "low," thereby preventing the detector arm from completing its full return stroke by gravity, holding the contact brushes in the upper position where L and C are closed, and thus increasing the heat supply. This produces off-and-on or two-position control action. Similar mechanisms are used to produce practically any of the modes of control discussed.

which is carefully located so that the open end will be in the unobstructed flow of the gases.

The A.S.M.E. Code states that probing the gas passage with a single tube gives questionable results and that a perforated pipe is worthless. The Code recommends taking individual simultaneous samples from different points in the cross-sectional area, analyzing these separately, and averaging the results. Other approved methods are: (1) composite sampling with equivalent flow from each individual tube; (2) a branched averaging sampler (or a spider) so constructed that each portion of the sample travels through the same length of pipe; (3) an open-end portable tube of ½ in. pipe, which is slowly and steadily moved across the area during sampling. With stationary sampling tubes, not less than one open-end tube is required for each 18 sq ft of gas passage, with *not less than 3 sampling points*. In large ducts, the sampling locations should be at least 3 ft from the walls.

For sampling from exhaust pipes of internal-combustion engines, a sampling tube of ⅜ in. pipe is to be extended to the middle of the exhaust pipe, with the end of the tube cut at an angle to its length and the opening turned against the gas flow. A small steam or water jet "ejector" or aspirator should be used for securing a continuous flow of gas into the collecting vessel. Usually it is most convenient to collect the gas over water, which must be saturated with the gas in order to avoid differential absorption of the constituents of the gas. Fresh samples should be taken directly from the collecting vessel into the analyzer if possible, without intervening storage time. To make sure that all the gas has been absorbed by each reagent, the process should be repeated until readings agree within $\frac{1}{10}$ percent. With fairly fresh solutions, 2 to 3 min are required to absorb all the CO_2 or CO, although 5 to 7 min may be required to determine the oxygen. It is desirable to replace the pyrogallic solution frequently.

Several semiautomatic **portable analyzers** are available, by which rapid determinations of CO_2 only may be made. These indicators actually measure the vacuum formed in a closed absorption chamber, and the vacuum gage is calibrated in percentage of CO_2.

One type of **automatic apparatus** in common use absorbs CO_2 continuously and registers the percentage content. In practically all such apparatus, KOH is the absorbent. Another type depends for its action on the difference between the specific weight of air and of flue gas, which is approximately proportional to the percentage by volume of CO_2 in the gas ("econometer" method). Of the two methods, that of absorption is the better. One automatic device depends upon the change of pressure in a stream of flue gas flowing through two apertures, when the CO_2 is absorbed.

Electric CO_2 Meter. **Thermal-conductivity methods** have come into use for measuring the amount of CO_2 in flue gases and also for evaluating the combustion conditions in internal-combustion engines.

The advantage of the method is that no materials are used which require replacement at frequent intervals. A suitable apparatus consists, in essential parts, of two gas cells marked *A* and *B* in Fig. 44, each of which contains a spiral of platinum wire, with provision for supplying electric current to each of the platinum spirals from a storage battery. The cell *A* is filled with air and sealed for a comparison standard. The other cell *B* is arranged so that the sample of flue gas to be tested passes through it. The current from the storage battery *S* is controlled to a constant value by means of the rheostat *R* and the ammeter *A*. The current from the battery *S* as shown in Fig. 44 passes through the platinum spirals *A* and *B* on one side and the "ratio coils" *C* and *D* on the other. If the gases surrounding the wires are alike in thermal conductivity, heat transmission to the metal walls of the two cells will be equal, and the platinum spirals will be heated to the same temperature.

The thermal conductivity of CO_2 differs from that of air by approximately 40 percent, and since the other constituents of flue gas, such as carbon monoxide, methane, nitrogen, and oxygen, have substantially the same conductivity as air, the difference in temperature under these conditions between the platinum spirals *A* and *B* will vary with the amount of CO_2 in the sample of flue gas being tested.

WELDING

BY

W. SPRARAGEN

REFERENCES: "The Welding Handbook," American Welding Society. Hale, "Welded Steel Construction," Pitman. Fish, "Arc-welded Frame Steel Structures," McGraw-Hill. "Procedure Handbook of Arc Welding Design and Practice," The Lincoln Electric Company. "The Welding Encyclopaedia," Welding Engineer Publishing Company. Owens, "Fundamentals of Welding," Penton Publishing Company. Miller, "Oxy-Acetylene Welding," Industrial Press. Wanamaker and Pennington, "Electric Arc Welding," Simmons Boardman.

Welding may be divided into forge welding, arc welding, gas welding, resistance welding, and thermit welding.

Arc Welding

Arc welding may be subdivided into metal-arc welding with bare electrodes or with coated (shielded-arc) electrodes, or in a reducing gas flame; atomic-hydrogen arc welding; carbon-arc welding, shielded or unshielded; and sub-

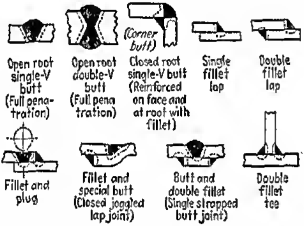


FIG. 1.—Forms of Welds and Joints.

merged arc. Further differentiation is based on whether the current is direct or alternating and whether the operation is manual, semiautomatic, or automatic.

In **metal-arc welding** an arc passes between a wire or rod of suitable composition, called the electrode, and the parts to be welded. The parts to be joined are made one side of an electric circuit, the other being the electrode. The metal is fused at both ends of the arc and the fused electrode deposited in the joint until the latter is properly filled. Various joints are illustrated in Fig. 1. Power is usually supplied by a transformer or by a generator delivering an open-circuit voltage ranging from 35 to 95 volts.

The arc should be short in order to avoid oxidation and the inclusion of harmful gases and to secure proper penetration. Some arc-welding generators are so designed that the arc will be extinguished after its length exceeds a safe limit. The normal arc voltage is so low (15 to 20 volts for bare wire and 22 to 40 volts for heavily coated wires) as compared with the striking or open voltage (35 to 95 volts) that machines of special design must be employed. A skillful operator can do good welding on a constant potential, 110 to 550 volt circuit, with a suitable resistance in series, but this process

The arrangement of battery, rheostat, and the four resistances, with the addition of the galvanometer G and the resistance F and H, as shown in Fig. 44, make up a circuit that is suitable for resistance measurements by the Wheatstone-bridge method. In the operation of this device, if the resistances are "balanced" when air is passed through the platinum spiral B, the bridge will be thrown out of balance when flue gas containing any appreciable amount of CO_2 passes through the platinum spiral B, and there results a deflection of the galvanometer G nearly proportional to the amount of CO_2 in the sample.

Heat is conducted away from the platinum spiral in the cell B by the flue gas less rapidly than from the filament in the air cell A. With the same flow of electric current in the two platinum spirals, the spiral exposed to the flue

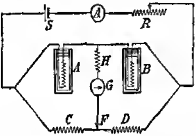


FIG. 44.—Diagram of Wheatstone Bridge Circuit.

gas will be hotter than the one in the air cell. Since the resistance of platinum changes with the temperature, the Wheatstone bridge will be unbalanced as soon as any CO_2 is present in the gas cell B. The scale on the galvanometer may be marked so that the pointer of the galvanometer will indicate on it directly the percentage of CO_2 in the flue gas. This type of apparatus may be made either as an indicating or a recording instrument. Convenient portable units of the thermal conductivity type are available, both for furnace gases and for engine-exhaust gases. The indicator scales on the latter may be calibrated in terms of air-fuel ratio instead of CO_2.

An electric **CO indicator** has been designed on the same general principles as the electric CO_2 meter. The operation of the CO meter is based on the catalytic action of a heated platinum wire in producing combustion of CO with oxygen. The essential parts of the instrument are two platinum wires which are heated by an electric current. These wires pass through the center of cylindrical chambers, each being in a separate metal block. One of the wires is the "measuring wire," which is in the chamber through which the sample of gas passes, and the other is the "comparison wire," which is maintained at a constant temperature by the electric current. A small amount of air is admitted to the cylindrical chamber surrounding the "measuring wire" to provide oxygen for combustion. The catalytic action causes the CO and the hydrogen in the flue gas to burn along the surface of the "measuring wire," thereby raising its temperature above that of the "comparison wire" and increasing the resistance of the "measuring wire" in almost exact proportion to the amount of CO and hydrogen gases in the sample of flue gas. The difference in resistance between the "measuring wire" and the "comparison wire" is measured by the Wheatstone-bridge method with a suitable galvanometer. The dial of the galvanometer is calibrated in percentage of CO.

is inefficient, as most of the power consumed is lost in the resistance. D-c generators may be constant-potential, constant-energy, or constant-current types.

The Welding Arc. The arc passes through incandescent vapor of the material forming the terminals of the arc gap. When direct current is used, one terminal (the anode) remains positive and the greater portion of the total heat is liberated at this terminal. When alternating current is used, approximately the same amount of heat is liberated at each terminal. *In the welding* of iron or steel the piece to be welded is usually made the positive terminal of the arc, but if very thin metal is being welded, it is advisable to reverse the polarity to prevent the arc burning through the metal. In welding with some coated electrodes and in the welding of manganese steel and bronze, it is often necessary to reverse the polarity and make the electrode positive. Excellent **arc welding** can be done **overhead** by a skilled operator.

The arc voltage determines the length of the arc with a given current. In manual welding with bare electrodes up to $\frac{3}{16}$ in. diam, it should seldom exceed 20 volts and with heavily coated wires it should not exceed 40 volts.

Shielded arc welding comprises those processes utilizing the electric arc for the fusion or deposition of metal in which the arc and metal are protected from the air. Shielding may be adapted to carbon-arc, metallic-arc, or hydrogen-arc processes in either the manual or automatic forms. Shielded-arc electrodes produce weld metal of high quality as a consequence of protection of the molten metal from the air. The effect of both oxides and nitrides in the weld metal is to cause brittleness or low ductility and, in some cases, low strength and poor corrosion resistance.

Shielding is accomplished by combinations of (1) the formation of a protruding flux sheath at the electrode tip; (2) the production of an inert or reducing atmosphere enveloping the arc and metal and (3) by providing a protective slag covering over the deposited weld metal. The molten slag in contact with the molten weld metal acts as a scavenger in removing oxides and other impurities from the weld. The fluidity of the metal is increased permitting higher rates of deposition.

Metallic deoxidizers are usually added to the coating to eliminate small amounts of oxygen and introduce into the weld metal various alloying elements for the production of high tensile strength, hardness, corrosion resistance, or other physical properties. The coatings cause the transfer of metal to take place as a stream of small globules rather than of large globules characteristic of bare metallic electrodes. **In automatic welding** it is necessary to effect some means of contact to the core metal for the introduction of *the welding current. Coatings are sometimes slit or pierced during the welding operation to provide for the contact.*

Carbohydrates are extensively used in coatings for steel electrodes for producing the **gaseous atmosphere.** Wood flour, pulp, paper, and cotton are commonly used and produce carbon monoxide and hydrogen in the arc.

Slag-forming materials for steel electrode coatings are mainly metallic oxides. Elements commonly employed are oxides or compounds of titanium, zirconium, silicon, magnesium, calcium, aluminum, iron, manganese, sodium, and potassium. Deoxidizers in common use are the ferroalloys of manganese, silicon, and titanium. These are largely oxidized during the welding process and form the corresponding oxides as constituents of the slag.

Stainless steel electrodes often contain small amounts of such materials as fluorides to aid in dissolving any chromium oxide which may be formed. Carbonaceous materials are kept at a minimum in these coatings to prevent an increase in the carbon content of the weld. Special elements for the stabilization of the stainless steel are sometimes included in the coating.

The most suitable mild steel **core wire** for shielded-arc electrodes is from high-grade rimmed steel. Specifications usually call for C, 0.13 to 0.18 percent; Mn, 0.40 to 0.60; Si, 0.06 max; S, 0.04 max; P, 0.04 max.

AUTOMATIC CONTROL

BY

J. B. McMAHON

REFERENCES: Wünsch, "Regler für Druck und Menge," Oldenbourg. Miteroff, "Principles underlying the Rational Solution of Automatic Control Problem," *Trans. A.S.M.E.*, May, 1935. Mason, "Quantitative Analysis of Process Lags," *Trans. A.S.M.E.*, May, 1938. Mason and Philbrick, "Automatic Control in the Presence of Process Lags," *Trans. A.S.M.E.*, 1940. Grebe, Boundy, and Cermak, "The Control of Chemical Processes," *Trans. Am. Inst. Chem. Eng.*, June, 1933. Bristol and Peters, "Some Fundamental Considerations in the Application of Automatic Control to Continuous Processes," *Trans. A.S.M.E.*, Nov., 1938. Ivanoff, "Theoretical Foundations of the Automatic Regulation of Temperature," *Jour. Inst. Fuel*, Feb., 1934. Maxwell, "On Governors," *Proc. Roy. Soc. (London)*, Mar. 5, 1868. Minorsky, "Directional Stability of Automatically-steered Bodies," *Jour. Am. Soc. Naval Eng.*, May, 1922. Callender, Hartree, and Porter, "Time-lag in a Controlled System," *Phil. Trans. Roy. Soc. (London)*, July, 1936.

Definitions. The A.S.M.E. Committee on Industrial Instruments and Regulators has tentatively defined an **automatic controller** (or regulator) as "an apparatus which measures the value of a quantity or condition which is subject to change with time and operates to maintain within limits this measured value." It consists essentially of two parts: (1) a **measuring device** comprising those elements which are involved in ascertaining and communicating to the controlling device an indication of the magnitude of the controlled variable, and (2) a **controlling device** comprising those elements which produce corrective action based upon indications supplied by the measuring means. A **variable** is a physical quantity subject to change with time.

The **set point** is that value of the variable which it is desired to maintain. A **final control element** is that portion of the controlling means which directly varies a control agent. A **control agent** is a medium in the process which is varied by the automatic controller and which affects the value of the controlled variable. The **adjusted flow** is the quantity rate of flow of the control agent.

Automatic Control and Automatic Operation. Confusion may arise from failure to distinguish between automatic operation and automatic control, particularly since the same equipment may be used in either service. A controller must (1) measure the variable (not necessarily producing a visual indication of it) and (2) produce corrective action when the variable departs from desirable limits, to bring it back within the desired limits.

Examples. A "safety controller" which shuts a process down when desirable limits are exceeded does not fulfill requirement 2. The common safety valve is a true automatic controller, as it fulfills both requirements. *Time-cycle controllers* which merely operate valves or other mechanisms in a certain fixed sequence are not true automatic controllers, since they do not measure a variable. A better name would be *time-sequence operator*. The simultaneous operation of two valves—as on fuel and combustion air—does not produce automatic ratio flow control of two fluids, unless each is operated by an independent flow controller which measures the effect of, and adjusts, its own valve independently of the other.

When a process or apparatus is such that constant attention is required, automatic control will produce results highly superior to hand control. Where occasional adjustments according to judgment, based on experience, will maintain the process or apparatus within the desired limits, manual

In arc welding in a reducing flame a cylindrical jet of gas surrounds the electrode and burns around the arc. Hydrogen, water gas, alcohol vapor, and a number of other gases have been tried with success. The welds are, in general, superior to the bare electrode welds, particularly as regards ductility. The operating voltage is higher than for simple metallic-arc welding so that the ordinary arc-welding generators will not serve.

In atomic-hydrogen arc welding a fine jet of hydrogen is forced through an arc formed between two tungsten electrodes. The arc breaks up the hydrogen molecules into hydrogen atoms, which recombine into molecules after passing through the arc, giving up the heat absorbed during dissociation in the arc. The result is a jet flame of hydrogen burning in a hydrogen atmosphere at a temperature higher than that of any other known flame but lower than that of the arc itself. The welding wire is fused in this flame and deposited in the joint exactly as in the case of gas welding. In cases where the plates to be welded are relatively thin, the edges are left square, butted together over a suitable backing, and fused together with the hydrogen jet without the use of any welding wire. The intensely reducing character of this hydrogen flame results in a nearly perfect weld, which, with suitable welding wire, is practically as good as the parent metal.

The magnitude of the arc current, and hence that of the hydrogen jet, is limited, because with heavy currents the tungsten electrodes burn away too rapidly. Thus the rate of welding-wire deposition is limited. Alloy steels, such as high-chromium and high-nickel steels, are readily welded by this process. As the heat applied to the joint is that of the atomic hydrogen jet flame and as this heat is controllable by varying the distance, much more delicate welding of thin parts can be done by this method than by direct arc welding with its higher temperature. In the latter case it is very difficult to avoid burning holes through thin stock.

The atomic-hydrogen arc process may be looked upon as the halfway point between arc and gas welding with some advantages over both, but as yet limited as to the rate of metal deposition. The arc voltage, both for striking and operating, is considerably higher than for either simple metal arc welding or for arc welding in a reducing flame, and alternating current is usually employed.

In carbon-arc welding, an arc is struck between a carbon (or graphite) electrode and the parts to be welded. The welding wire is fed into the arc and fused into the joint. For hand operation this requires two hands as in the case of gas welding or atomic-hydrogen arc welding. A fairly high rate of metal deposition is possible. In some cases the welding rod is laid in the joint groove and the carbon arc passed slowly along the joint until the fusion is complete. An automatic carbon-arc method, with a magnetically controlled arc, known as electronic-tornado welding, gives excellent results for thin plates in a flat position.

Automatic machines are available for nearly all the above described methods, although it is not easy to apply a welding-wire feeding mechanism to some of the coated electrodes, owing to the difficulty of getting electrical contact between the wire and the feeding rolls. Several methods have been devised for overcoming this difficulty. Oscillators are also available for varying the width of the weld.

Another method (Unionmelt) of producing shielded-arc welds automatically consists of applying powdered flux to the weld. This process employs welding currents of the order of 1,000 to 2,000 amp and makes possible the welding of plates 1½ in. thick and over in a single pass. A bare electrode

operation may be entirely adequate. In the latter case, satisfactory manual control of the end result may be secured much more readily by providing automatic controllers for all the individual variables affecting the end result, without direct application of automatic control to the end result itself.

The variables to be controlled should be selected with regard to (A) susceptibility to measurement; (B) significance of measurement; (C) susceptibility to automatic control, from the point of view of (1) economy, (2) consistent response to control action, and (3) consistent response to process; and (D) magnitude of process lags.

(A) Susceptibility to Measurement. Factors that can be measured in a laboratory may not be susceptible to continuous measurement by an industrial instrument; for example, definite chemical analysis may be the desired end result of a process, but, industrially, some physical effect of chemical variation, such as temperature, pressure, conductivity, or pH, may have to be used.

(B) Significance of Measurement. The variable, or variables, selected must be truly representative of changes in process conditions. Under constant operating conditions, some variables may become constant which do not vary quantitatively with changes in operating conditions. In the example under (A), all other variables affecting the end result must be held constant, so that the one selected may be truly significant. It might be said that the significant variable is the combined result of all the variables. As a further example, the temperature of vapors leaving the top, or of liquid leaving the bottom, of a fractionating column, which is separating a binary mixture into practically pure components, is not a satisfactory criterion for automatic adjustment of heat extracted from, or supplied to, the column; although it may be a satisfactory criterion of column operation.

(C) Susceptibility to Automatic Control. (1) Economy. In all cases, factors A and B must be satisfied for the successful application of automatic control. In many cases, however, A may be readily satisfied, but the obvious way of satisfying B may be very uneconomic. For example, the *characteristic curve* of a centrifugal pump and the kind of head against which the pump discharges (whether mostly static or mostly friction head) determine whether it is more economic to locate the controlled valve in the discharge of, or in a by-pass around, the pump.

(2) Consistent Response to Control Action. In complicated processes, response of adjusted flow rate to the corrective action of the controlling means must be reasonably consistent. For example, flow-line pressure drop and outside influences (such as the dumping action of steam traps and variations in pump speeds due to variable frictions) must not be excessive in their effects.

(3) Consistent Response of Process. Under some operating conditions, the measurement of the controlled variable may be significant but become insignificant under very slight changes in operating conditions. For example, both above and below ebullition, temperature is a significant measure of the quantity of heat in water or steam, but not at the point of change of state. Nor is temperature significant for the heat content of steel at the decalescence point.

(D) Process Lags. All automatic control is directed toward satisfying some demand by a corresponding supply. Process lag frequently prevents this from being done directly. As an example, the true measure of the load

is used, and the powdered flux is applied either between dams erected along the joint or from a hopper around the arc as the weld progresses. This process of welding produces extremely high welding speeds and welds with physical properties comparable to those made with the highest quality manual shielded-arc electrodes.

With automatic welding the arc length and other variables are accurately controllable, labor is saved, and the speed of welding can be greatly increased. It is readily applicable only to fairly simple shapes but has already found a large field of usefulness in production work.

In **semiautomatic arc-welding machines** the feed of the electrode and the arc length are under automatic control, but the "travel" or movement of the arc along the seam is under hand control. This largely eliminates the demand for skill on the part of the operator and is applicable to many situations where the full automatic machine could not be used.

Where **heavy sections** are to be **welded**, it is desirable to deposit the weld metal in a series of layers. When the first layer is deposited, all scale and slag should be removed before depositing the second layer. Peening of each layer with an air tool before depositing a new one not only removes the surface scale but stretches the deposited metal, removing any cooling shrinkage. Welds deposited in two or more layers are generally better than those deposited with a single layer, because the second and subsequent layers have a tendency to partly anneal the preceding layers, and a joint of improved structure will result.

Welding Technique and Procedures

A table of the average recommended welding currents for the gas-producing electrode is as follows:

		3/32	1/8	5/32	3/16	1/4	5/16	3/8
Electrode diam, in.		3/32	1/8	5/32	3/16	1/4	5/16	3/8
Amperes	Flat	60	120	150	175	250	325	425
	Vertical and overhead	60	110	140	160			

A table of the average recommended welding currents for the flat position, with heavy slag-producing electrodes is as follows:

Electrode diam, in.	1/8	5/32	3/16	1/4	5/16	3/8
Amperes	130–150	160–180	200–260	300–400	400–600	500–900

Typical automatic welding speeds obtained on butt- and fillet-welded joints with bare, wash-coated or dust-coated types of electrodes are given in Table 1.

Table 1. Automatic Welding Currents and Speeds

Plate thickness	Joint design	Electrode diam., in.	Current, amp	Passes	Speed, in. per min
18 gage	Square butt	1/8	110–180	1	25–33
14 gage	Square butt	1/8–5/32	150–210	1	18–26
10 gage	Square butt	5/32	180–230	1	12–20
3/16 in.	Square butt	3/16	225–300	1	8–13
1/4 in.	60° single V	3/16	225–300	2	5–8
1/2 in.	60° single V	3/16 & 1/4	240–350	3	2–3
3/4 in.	60° single V	3/16 & 1/4	240–350	5	1.5–2.0
1/4 in.	Fillet weld	3/16	200–280	1	16–19
3/8 in.	Fillet weld	1/4	250–300	1	8–10
1/2 in.	Fillet weld	1/4	250–300	2	5.8–6.5
3/4 in.	Fillet weld	1/4	260–325	3	2.5–3.0

(demand) on a turbogenerator may be the displacement of the governor measuring element, but it is usually impractical to attempt to control the fuel to the boiler directly from this effect, because of the intervening energy storage capacities and resistances to energy flow.

Self-regulation

Self-regulation may be positive, neutral, negative, or critical.

(*a*) **Positive processes** possess the property of being inherently self-balancing. Examples are (1) the temperature of water leaving a continuous hot-water heater varies with the demand, and (2) in an open tank through which water is flowing, every rate of inflow will produce the water height necessary to produce an outflow equal to the inflow.

(*b*) **Neutral processes** do not tend to balance themselves or to become progressively unbalanced. As an example, if a vessel, such as that in the last example, has superimposed on the water a high extraneous gas or vapor pressure, as in a high-pressure boiler drum, there will be very little relation between water height and rate of outflow. Any discrepancy between inflow and outflow will result in the vessel flooding or going dry. There will be little effective self-correction.

(*c*) In **negative processes**, any tendency to become unbalanced is progressive. For example, a liquid charged to a chiller by a centrifugal pump will tend to become more viscous if the rate of flow drops for any reason; this will tend to slow down the rate of flow, chilling it still more, etc. In most cases, this tendency will be balanced by the limiting viscosity change or by the increase of pump discharge pressure due to lowered rate of flow.

(*d*) In **critical processes**, the unbalancing tendency in (*c*) may be sufficient to overcome balancing factors. In the example under (*c*), an oil might possess great enough viscosity change with change of temperature to increase the resistance to flow faster than the balancing factors increase, resulting in complete stoppage of flow. Exothermic chemical reactions are frequently quite violent in their inherent unbalancing tendencies.

Process Lags

Process lags may be defined as the "retardation or delay in the value or condition of a process variable with respect to the immediate condition of another variable to which it is closely related." (Mason, "Quantitative Analysis of Process Lags," *Trans. A.S.M.E.* May, 1938.) These lags result from combinations of energy-storage capacities and energy-flow resistances in the process. They are of three forms: **Capacity lag** results from the ability of a process to store up energy; relatively large amounts in that part of the process being controlled are generally helpful to automatic control. **Transfer lag** results from the resistance offered to the flow of energy between two or more capacities; it is generally an unfavorable factor and is not dimensionally expressible, since it involves complicated functions of time. **Distance-velocity lag** results from any physical or mechanical characteristic of the process that requires time for the effect of an instantaneous change in a variable to be carried to that part of the process where it may affect the controlled variable; it is also generally an unfavorable factor.

Under balanced operating conditions, there is no analyzable necessity for automatic control. It is only under changing conditions that it becomes necessary. The function of automatic control is to restore balance; but the manner in which it successfully accomplishes this must take into account the reaction of the process to the restorative action. The reaction of the process will depend upon its lag characteristics, which are transient phenomena.

In a-c automatic welding for high-grade pressure vessels with shielded-arc electrodes the deposit per pass is about 1/8 in. The weld bottom of the groove is usually chipped out and rewelded from the back.

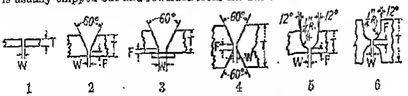


Fig. 2.—Proportions of Metal Arc Weld Grooves.

The dimensions in inches represented by the letters in Fig. 2 are as follows:

Groove	1	2	3	4	5	6
T	1/8–3/16	1/4 up	1/2 up	1/2 up	3/4 up	3/4 up
W	0–1/16	0–3/16	3/16–1/4	0–3/16	0–1/16	0–1/16
F	....	0–1/16	0–1/16	0–1/16	1/8 ± 1/32	1/8 ± 1/32

Table 2. Welding Speed. Atomic-hydrogen Hand Welding

Plate thickness, in.	Welding speed, in. per min			
	Lap and fillet	Butt	Corner	Edge
1/32		8–9	11–12	11–13
1/16	8–9	9–12	13–14	12–13
3/32	7½–8½	8–10	10–11	10–12
1/8	7–8	7–10	9½–10	9–10
5/32	5–6	5–7	6½–8	7½–9
3/16	3½–4½	3–3½	4–5	5½–6
1/4	2½–3½	2½–3	3–3½	4–4½
5/16	2–3	1½–2½	2½–3	3–4
3/8	2–2½	1–1½	2–3	2½–3½
1/2	1½–2	3/4–1	1½–2	2–2½

With 100 (125) [150] amp welding current the metal deposited per hour is 6–9 (7–8) [8–10] cu in.

Table 3. Physical Properties of Shielded and Unshielded Weld Metals

Physical properties	Base metal	Weld metal	
		Deposited with a metallic shielded arc electrode	Deposited with a metallic bare electrode
Tensile strength, lb per sq in.	55,000–70,000	60,000–75,000	50,000–60,000
Yield point, lb per sq in.	30,000–32,000	45,000–60,000	38,000–45,000
Elongation, % in 2 in.	30–40	20–40	5–10
Elongation, free bend, %	—	35–60	10–20
Reduction in area, %	60–70	35–65	8–20
Density, g per cc	7.85	7.80–7.85	7.50–7.70
Endurance limit, lb per sq in.	26,000–30,000	26,000–30,000	12,000–18,000
Impact, Izod, ft-lb	50–60	40–70	5–15

MODES OF OPERATION

Automatic control action can be divided into (A) on-and-off (open-and-shut) which produces maximum or zero value of the adjusted flow, with no intermediate quantities, and (B) throttling which may mean either proportional control (see definition below) or any type of control action intended to produce values of adjusted flow intermediate between zero and maximum. In this discussion, it will be used in the latter sense.

(A) **On-and-off control** produces a distinct and sudden change of adjusted flow and for many applications has a distinct advantage. The greater and more nearly instantaneous the correction following a disturbance, the sooner the value of the variable will recross the set point. Hence, it is more competent to deal with widely varying and upsetting conditions than are throttling types.

A(1) In its simplest form, an on-and-off controller moves its final control element completely through its stroke upon the minimum possible deviation of the controlled variable from the set point. This is referred to as **fast open-and-shut** action, or zero dead zone. Where process conditions require on-and-off action, it is essential that the dead zone be as nearly zero as can be obtained, in order to minimize the magnitude and period of the permanent cycle.

A(2) A variation of A(1) is frequently used to minimize mechanical wear of the final control element or to improve combustion efficiency, as in the case of domestic thermostats. In this case, the final control element is operated in one direction at one value of the controlled variable and is not operated in the opposite direction until the correction resulting from its action has brought the controlled variable back across a dead zone to another value. This dead zone or **differential gap** produces a definite cycle in the control, which frequently is larger than the amount of the differential, especially in the presence of velocity distance or transfer lag. Various **anticipators** are used to overcome this overrun, but, in general, they are successful only when carefully adjusted for a narrow range of load variation. The differential gap type of on-and-off control is one of the most widely used automatic controllers and is found in practically all domestic thermostats.

(B) The essential feature of **throttling control** is that the flow of the control agent is continuous. It simulates hand control and is frequently considered to be a superior type. The mechanism is generally more complicated and expensive than that of the on-and-off type. Each type fulfills a definite function, and the selection of either should be based on the requirements of the application.

Types of Throttling Control. B(1) **Limited on-and-off** (two-position) control mechanically is a variation of A(1) and A(2), obtained by limiting the stroke of the final control element or by restricting its effect upon the adjusted flow by by-passing part of the flow of control agent around the final control element. It has been most highly developed in electrically operated controllers, which are more difficult to make of throttling type than are those pneumatically operated. Where capacity lag is preponderant, and load range is small, it is very effective.

B(2) **Two-position with rate control** is a modification of on-and-off control of the differential gap type, wherein the action of the final control element is slowed down so that it is not able to complete its stroke during the over- or under-run period. There is no correspondence between the value of the controlled variable and the position of the final control element. As long as the value of the variable is outside the dead zone, the final control

Design Strengths

The bridge specifications of the A.W.S. (1940) permit design stresses in properly made butt joints, welded from both sides, of 13,500 lb per sq in. when the service stresses vary from zero to a maximum. When the service stresses are completely reversed, only 9,000 lb per sq in. is allowed. There is a penalty of 15 percent in allowed stress in case of single-V backed-up welds. In butt welds subjected to a pulsating shear varying from zero to a maximum a design value of 9,000 lb per sq in. is allowed. This is reduced to 6,000 lb if the shearing stress is completely reversed. The same 15 percent penalty applies to single-V backed-up welds.

Fillet welds subjected to tension, compression, or shear are allowed 7,200 lb per sq in. stress when the stress varies from zero to a maximum and 4,800 lb when the stress is completely reversed. Only a good grade of heavily covered electrods is permitted in bridgework.

The building code of the A.W.S. requires that welded joints shall be proportioned so that the stresses caused therein by loads specified in the Building Code shall not exceed the following values, expressed in thousands of pounds per square inch:

Kind of stress	For welds made with filler metal	
	Shielded arc	Bare or lightly coated
Shear on section through weld throat	13.6	11.3
Tension on section through weld throat	15.6	13.0
Compression (crushing) on section through throat of butt weld	18.0	18.0

Fiber stresses due to bending shall not exceed the values prescribed above for tension and compression, respectively. Stress in a fillet weld shall be considered as shear, for any direction of the applied stress. In designing welded joints, adequate provision shall be made for bending stresses due to eccentricity, if any, in the disposition or sections of base-metal parts.

Gas Welding

Gas welding is fusion welding in which heat is supplied by burning a mixture of oxygen and a suitable combustible gas. The gases are mixed in a blowpipo or torch which gives complete control of the welding flame.

Acetylene is almost universally used as the combustiblo gas because of its high flame temperature. This temperature, estimated to be about 6000 F, is so far above the melting point of all commercial metals that it provides a means for the rapid localized melting essential in welding. The oxyacetyleno flame is also used in cutting ferrous metals.

The **oxyhydrogen** flame is used in welding metals that have low melting points, such as lead, and in welding thin aluminum sheet.

It is essential that the weld should penetrate entirely through the metal. Beveled edges forming a V should be used where the thickness of the metal is such that it would be difficult to secure this penetration with butted edges. To build the weld up to the original surface, welding rods must be used to fill up the V.

Bronze Welding. (Braze Welding.) It is sometimes possible to produce sound strong joints in metals without actually melting the base metal. Thus in bronze welding the edges of the joint are heated to a dull red heat by oxy-acetylene. With the base metal at the proper temperature and with the aid of a suitable flux, molten bronze from a bronze welding rod will unite with the

element continues to move. This type of control is used to some extent where the process-lag characteristics are favorable but is relatively limited in its application.

B(3) In **variable rate of integrating control,** the rate mechanism of B(2) is supplemented by an effect that increases the speed of action of the final control element in proportion to the amount of deviation of the value of the variable from the set point. Independently, it has very little application, except in some simple pressure-regulation applications, but is combined with other types in widely used commercial controllers.

B(4) In the dynamic consideration of **proportional control,** the controlling means is caused to vary in proportion to variations of the measured variable, within a range known as the **proportioning band** (also called throttling range). Statically, the deviation of the controlling means from its midvalue is proportional to the deviation of the variable from midposition of the proportioning band. In all dynamic behavior, the proportionality is maintained regardless of the behavior of the measured variable, as long as it is within the proportioning band. The rate of change of the variable produces proportionate rates of change of the controlling means. This proportionality holds for all derivatives. It is a simulation of the self-regulation of the process itself, and where positive self-regulation is already present in the process, may be thought of as making it more effective; or where it is not present, may be thought of as furnishing it. This type is widely used; the common pressure-regulator or reducing valve falls within this classification.

B(5) **Multipositional Control.** Many ingenious mechanisms and electrical circuits have been developed to approximate proportional control by multiple-step action, with various combinations of B(2) and B(3). Where such action is useful, equally good or better results may be secured by the use of true proportional control.

B(6) **Proportional Control with Automatic Reset.** Since proportional control must permit deviations of the value of the variable in order to institute corrective action, combinations of B(2) or B(3) with B(4) are used to secure its desirable throttling action without permanent deviation.

Control systems may be (1) by means of a group of more or less related but unconnected instruments used to control all pertinent variables of a process or (2) by the use of several controllers interconnected to produce a single end result. This interconnection may be (a) **multiple,** in which case a **master controller** varies the set points of several independent controllers simultaneously according to some preestablished relation, as in several boiler combustion-control systems; or (b) **cascade,** in which type several controllers vary the set points of succeeding ones, in order to eliminate the effect of outside upsetting conditions and to make the control action of the first instrument in the cascade consistent in its effect. An example of this is a temperature controller varying the set point of a flow controller which varies the set point of the governor of a prime mover driving a pump, which is pumping the control agent.

(C) **Special Modes of Control.** C(1) **Relation Control.** It is frequently desired to keep the value of a variable in some direct relation to the value of another variable, although there is no reaction upon the second from the variation of the first.

Example. Control of relative humidity may be secured by suitably varying the set point of a wet-bulb controller according to variations in dry-bulb temperature. In "comfort cooling" of buildings, it is desirable to keep the indoor temperature only a

base metal to form a strong bond. A bronze weld is comparable in strength to a fusion weld.

With practically all the common metals except steel, it is necessary to use a suitable **flux**. The flux unites with the oxides to form an easily fusible slag. Special fluxes are necessary for fusion welding cast iron, for bronze welding, for fusion welding brass and bronze, for welding chromium alloys such as stainless steel, and for welding aluminum.

With a one to one mixture of oxygen and acetylene the resulting flame is **neutral**. The neutral flame has an inside portion consisting of a brilliant cone $\frac{1}{16}$ to $\frac{3}{4}$ in. long, surrounded by a faintly luminous envelope flame. When acetylene is in excess, the flame consists of three easily recognizable zones: a sharply defined inner cone, an intermediate cone of whitish color, and the bluish outer envelope. The length of the intermediate cone is a measure of the amount of excess acetylene. This flame is reducing, or carburizing.

When oxygen is in excess in the mixture, the flame resembles the neutral flame, but the inner cone is shorter, is "necked in" on the sides, is not as sharply defined, and acquires a purplish tinge. A slightly **oxidizing flame** is used in bronze welding and bronze surfacing, and a more strongly oxidizing flame is used in fusion welding brass and bronze.

By using a **carburizing** or excess acetylene **flame**, advantage can be taken of the facts (1) that hot steel readily absorbs carbon, (2) that high-carbon steel has a much lower melting point than low-carbon steel, (3) that carbon very effectively reduces iron oxide, and (4) that carbon disperses or migrates through hot steel at a rapid rate. Directing the flame backward over the completed weld but in such a way that the excess acetylene flame touches the scarf or V in advance of the welding puddle serves to reduce any iron oxide on the surface of the metal. The reduction of the oxide leaves a very porous spongy type of iron which very readily absorbs carbon from the flame, and the melting point falls as the carbon increases until, with fully carburized iron, it is nearly 700 F *below* the melting point of carbon-free iron. This procedure eliminates the gas-producing scale at a point in advance of the welding operation so that blowholes are prevented and adds carbon to the surface of the base metal. The carburized low melting-point surface of the scarf is easily brought to a sweating condition which is perfect for forming the union between the liquid metal from the rod and the base metal. There is no need for melting into the scarf to ensure the presence of clean metal for this union, and, as a result, the width of the groove need not be so great as in earlier methods; hence less rod must be melted to fill it, and this, together with the elimination of melting the scarf, leads to rapid easy welding at a reduced cost for welding materials. The carburized surface of the scarf is rapidly absorbed by the added metal, and the carbon is diffused uniformly through the weld while still at a high temperature, giving a uniform strong and ductile weld metal.

Machine Welding. Oxyacetylene welding by machines is applied in the manufacture of steel barrels, tubing, irrigation pipes, special forms, nickel and monel tubing. Multiple-flame water-cooled machine torches make welds at rates several times that possible by the most expert hand welders. Machine welding is done with and without welding wire, mostly *without*, on tubing and pipe, the welds being compressed in the process to make a flush or reinforced joint.

Gas Cutting

Gas cutting is employed for wrought iron, steel, and cast iron. The tips of the torch are usually made with a central orifice for the oxyacetylene cutting jet, around which are disposed several smaller orifices for the preheating flames. The latter are produced by burning mixed oxygen and acetylene, or fuel gas. Their function is heating the iron to the kindling temperature, 1500 F. The **oxygen cutting jet** burns the iron in its path,

certain amount below outdoor temperature to avoid undesirable shock to persons going outdoors. In chemical processes, it is frequently desirable to have a pressure or flow vary according to the variation in a temperature—or vice versa.

C(2) **Ratio Flow Control.** This is a special case of (1), in which the set point of a rate-of-flow controller is varied according to variations in the flow through another line; *e.g.*, varying the flow of an absorption oil according to the variations in flow of the gas from which it is absorbing a constituent, varying the rate of flow of air to sewage aerators in accordance with the rate of flow of raw sewage.

In some cases, the ratio between the two flows measured by a ratio-flow controller (the "primary" or uncontrolled flow and the "secondary" or controlled flow) is varied by another controller; *e.g.*, in some steam desuperheater applications, the velocity of steam and water is kept in ratio by a ratio-flow controller, with a temperature controller adjusting the ratio to secure the final exact temperature desired. This kind of combination is frequently desirable where load variations are abrupt and large and process lags are such as to prevent the possibility of quick action by the end-result controller. In such cases, the swings are taken care of approximately by the ratio-flow controller and exact adjustment under balanced operating conditions by the final controller. Where complicated process lags exist, even this setup may not produce good final results during transient conditions, since it does not take account of intermediate storages of energy and their transient influence.

C(3) **Averaging Control.** In many processes, it is desirable to keep upsets in one part from being passed on to another part. For this purpose, surge vessels are installed between them.

It is frequently desirable to control the storage in these chambers automatically, in such a way that the effect of a cycling or varying inlet flow to the vessel will be attenuated to the maximum possible extent, in order that the *average* change in flow rate will be passed along to the succeeding apparatus. This is a direct contradiction of the purpose of most automatic-control apparatus but is quite successfully handled by using a mechanism of the "proportional with reset" type, in which the proportion between variation of the value of the variable and corrective action is made relatively high (relatively wide throttling range) and the rate of reset is relatively slow.

The conditions should permit the actual outlet flow to be definitely and exactly responsive to the corrective action of the controlling means and not be affected by outside factors; when this is not possible, a "control system" should be used that will nullify outside effects also tending to influence the flow.

C(4) **Time or Program Control.** In many processes, operations require the gradual change of value of a variable according to a definite schedule. This may be secured by varying the set point of an automatic controller by means of a clock or motor-driven cam. In some cases, the cam setting is determined by the progress of the condition of the material being processed, rather than strictly according to time, in which case it might be said that each batch of material determines its own schedule.

Complicated process lags may prevent strict adherence to a time schedule, and where they exist, better over-all results may be secured by the use of time-flow (or other individual variable) controllers, rather than a time-schedule controller on the end-result variable, such as temperature.

C(5) **Combustion Control.** The requirements of boiler control include factors not ordinarily considered in the average industrial application. A discussion of its requirements will be found on p. 1160.

AUTOMATIC-CONTROL MECHANISMS

Friction, lost motion, and unbalance are the prime causes for trouble in automatic controllers. It is almost impossible to overemphasize this. In proportional controllers without automatic reset, the dead zone resulting

and as the torch is moved over the surface it makes a clean cut with narrow kerf comparable to that made with a metal saw.

The maximum thickness of metal that can be cut by high-temperature flames depends largely upon the gases used and the pressure of the oxygen. The thicker the material, the higher the pressure required. When using the oxyacetylene flame, it is possible to produce smooth cuts up to 10 in. or more in thickness. The oxyhydrogen flame will cut material up to 24 in. in thickness.

The torch-cutting process is widely used for fabricating steel plates and structural shapes, cutting field rivet holes, beveling for welding, cutting pipe to length and angle for welding, clearing away wrecks, scrapping old machinery and structures, cutting rivet heads and stay bolts, and removing plates from fireboxes.

Influence of Alloying Elements on Cutting. Alloying elements have two possible effects, they may increase the resistance of steel to cutting and they may give rise to harder cut surfaces. Where an alloy steel is difficult to cut by other means, an improvement is effected by tightly clamping a "waster" plate to the upper surface and cutting through both thicknesses. Another method is to weld a heavy bead on the upper surface along the proposed line of cut.

Steels up to 0.30 percent **carbon** can be cut without difficulty. Higher carbon steels should be preheated to about 600 F to prevent hardening. Graphite and cementite (Fe_3C) are detrimental, but cast irons containing 4 percent carbon may be cut by special technique.

Steels up to 14 percent **manganese** and 1.5 percent carbon are cut with difficulty and for best results should be preheated.

Silicon in amounts usually present has no effect. Silicon steel containing considerable amounts of carbon and manganese must be carefully preheated and postannealed for best physical properties.

Steels up to 5 percent **chromium** are cut without much difficulty when the surface is clean. Higher chromium steels, such as the 10 percent chromium steels, require special technique, and the cuts are rough. In general, carburizing preheating flames are desirable when cutting this type of steel.

Nickel up to 20 or 30 percent (if the carbon is not too high) may be cut. Up to about 7 percent nickel content, cuts are very satisfactory.

Aircraft quality chrome-**molybdenum** steel offers no difficulties. High molybdenum-tungsten steels, however, may be cut only by means of special technique.

Tungsten alloys up to 12 or 14 percent may be cut very readily, but with a higher percentage of tungsten cutting is difficult.

Vanadium in small amounts may improve cutting. The other common alloying elements have no appreciable influence.

Oxygen-cutting machines are capable of making oxygen cuts with jigsaw flexibility and of such high quality and accuracy as to require no further finishing. Cutting machines are frequently equipped with more than one cutting torch, centrally controlled and guided, and will oxygen cut a number of identical shapes simultaneously, thereby effecting marked economies where a high production rate prevails. Other machines will crawl around pipes, making one or two square or beveled cuts, as desired.

Oxyacetylene Cutting

Thickness, in....	¼	½	1	2	4	6	8	10
Speed, in. per min	18–28	14–24	10–19	7–14	6–9	5–6	3–5	2.5–4
Oxygen consumption, cfh......	50–90	90–125	125–190	175–260	300–390	390–500	500–640	600–750

Flame machining consists in removing material by surface oxidation. The operation is analogous to tool machining, but the tool in this instance

16. $\int \frac{dx}{a+bx} = \frac{1}{b}\log_e(a+bx) + C = \frac{1}{b}\log_e c(a+bx)$

17. $\int \frac{1}{x^2}dx = -\frac{1}{x} + C$ 18. $\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)} + C$

19. $\int \frac{dx}{1-x^2} = \tfrac{1}{2}\log_e \frac{1+x}{1-x} + C = \tanh^{-1}x + C,$ when $x < 1$

20. $\int \frac{dx}{x^2-1} = \tfrac{1}{2}\log_e \frac{x-1}{x+1} + C = -\coth^{-1}x + C,$ when $x > 1$

21. $\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(\sqrt{\frac{b}{a}}x\right) + C$

22. $\int \frac{dx}{a-bx^2} = \frac{1}{2\sqrt{ab}}\log_e \frac{\sqrt{ab}+bx}{\sqrt{ab}-bx} + C$

$= \frac{1}{\sqrt{ab}}\tanh^{-1}\left(\sqrt{\frac{b}{a}}x\right) + C$

(21, 22) when $a > 0$, $b > 0$

23. $\int \frac{dx}{a+2bx+cx^2} = \frac{1}{\sqrt{ac-b^2}}\tan^{-1}\frac{b+cx}{\sqrt{ac-b^2}} + C$ when $ac - b^2 > 0$;

$= \frac{1}{2\sqrt{b^2-ac}}\log_e \frac{\sqrt{b^2-ac}-b-cx}{\sqrt{b^2-ac}+b+cx} + C$

$= -\frac{1}{\sqrt{b^2-ac}}\tanh^{-1}\frac{b+cx}{\sqrt{b^2-ac}} + C,$

when $b^2 - ac > 0$;

24. $\int \frac{dx}{a+2bx+cx^2} = -\frac{1}{b+cx} + C,$ when $b^2 = ac$

25. $\int \frac{(m+nx)dx}{a+2bx+cx^2} = \frac{n}{2c}\log_e(a+2bx+cx^2) + \frac{mc-nb}{c}\int \frac{dx}{a+2bx+cx^2}$

26. In $\int \frac{f(x)dx}{a+2bx+cx^2}$, if $f(x)$ is a polynominal of higher than the first degree, divide by the denominator before integrating.

27. $\int \frac{dx}{(a+2bx+cx^2)^p} = \frac{1}{2(ac-b^2)(p-1)} \times \frac{b+cx}{(a+2bx+cx^2)^{p-1}}$

$+ \frac{(2p-3)c}{2(ac-b^2)(p-1)}\int \frac{dx}{(a+2bx+cx^2)^{p-1}}$

28. $\int \frac{(m+nx)dx}{(a+2bx+cx^2)^p} = -\frac{n}{2c(p-1)} \times \frac{1}{(a+2bx+cx^2)^{p-1}}$

$+ \frac{mc-nb}{c}\int \frac{dx}{(a+2bx+cx^2)^p}$

29. $\int x^{m-1}(a+bx)^n dx = \frac{x^{m-1}(a+bx)^{n+1}}{(m+n)b}$

$-\frac{(m-1)a}{(m+n)b}\int x^{m-2}(a+bx)^n dx$

$= \frac{x^m(a+bx)^n}{m+n} + \frac{na}{m+n}\int x^{m-1}(a+bx)^{n-1}dx$

IRRATIONAL FUNCTIONS

30. $\int \sqrt{a+bx}\,dx = \frac{2}{3b}\left(\sqrt{a+bx}\right)^3 + C$

31. $\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b}\sqrt{a+bx} + C$

32. $\int \frac{(m+nx)dx}{\sqrt{a+bx}} = \frac{2}{3b^2}(3mb - 2an + nbx)\sqrt{a+bx} + C$

33. $\int \frac{dx}{(m+nx)\sqrt{a+bx}}$; substitute $y = \sqrt{a+bx}$, and use 21 and 22

34. $\int \frac{f(x, \sqrt[n]{a+bx})}{F(x, \sqrt[n]{a+bx})}dx$; substitute $\sqrt[n]{a+bx} = y$

35. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C = -\cos^{-1}\frac{x}{a} + c$

36. $\int \frac{dx}{\sqrt{a^2+x^2}} = \log_e\left[x + \sqrt{a^2+x^2}\right] + C = \sinh^{-1}\frac{x}{a} + c$

37. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log_e\left[x + \sqrt{x^2-a^2}\right] + C = \cosh^{-1}\frac{x}{a} + c$

38. $\int \frac{dx}{\sqrt{a+2bx+cx^2}} = \frac{1}{\sqrt{c}}\log_e\left[b + cx + \sqrt{c}\sqrt{a+2bx+cx^2}\right] + C,$ when $c > 0$;

$= \frac{1}{\sqrt{c}}\sinh^{-1}\frac{b+cx}{\sqrt{ac-b^2}} + C,$ when $ac - b^2 > 0$;

$= \frac{1}{\sqrt{c}}\cosh^{-1}\frac{b+cx}{\sqrt{b^2-ac}} + C,$ when $b^2 - ac > 0$;

$= \frac{-1}{\sqrt{-c}}\sin^{-1}\frac{b+cx}{\sqrt{b^2-ac}} + C,$ when $c < 0$

39. $\int \frac{(m+nx)dx}{\sqrt{a+2bx+cx^2}} = \frac{n}{c}\sqrt{a+2bx+cx^2} + \frac{mc-nb}{c}\int \frac{dx}{\sqrt{a+2bx+cx^2}}$

40. $\int \frac{x^m dx}{\sqrt{a+2bx+cx^2}} = \frac{x^{m-1}X}{mc} - \frac{(m-1)a}{mc}\int \frac{x^{m-2}\,dx}{X} - \frac{(2m-1)b}{mc}\int \frac{x^{m-1}\,dx}{X},$ where $X = \sqrt{a+2bx+cx^2}$

41. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log_e\left(x + \sqrt{a^2+x^2}\right) + C$

$= \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + c$

42. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

is an oxygen jet. Metals may be removed at a rate of 10 to 15 lb per min. The oxygen consumption with present practice amounts to approximately 3 cu ft per lb of steel removed.

Carbon-arc cutting depends upon melting, except where a combination of the carbon arc with oxygen gas is used for certain work. Space must be left under the plate for the molten metal and slag to run out of the cut. The carbon arc is used for burning holes in metals and for cutting rivets. Current may vary from 450 amp for light work to 1,200 amp for general and heavy cutting. The cut made by the carbon arc is not smooth, but for scrap work it may sometimes be used economically.

Cutting metals under water can be accomplished by the electric arc or the oxyacetylene flame. The arc method combines the heat of the electric arc with the oxidizing flame of a stream of oxygen.

The torch method consists essentially of an oxyacetylene cutting torch surrounded by a protecting bell through which is forced compressed gas. Special methods are used to permit the lighting of the torch underwater.

Resistance Welding

In resistance welding (**Thomson process**) the parts to be joined, after proper shaping, are pressed together (Fig. 3). A large current is then passed through the joint until it has reached welding temperature. The current is

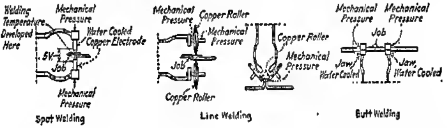


FIG. 3.—Diagrams of Circuits of Butt, Spot, and Seam Welding Machines.

stopped and further pressure, if needed, is applied, upsetting the joint and completing the weld.

As the electrical resistance of the contact surface is much greater than that of the solid metal, most of the heat is generated at the joint surface. *The larger the current, the more rapid the heating, the less the heat extends* back from the joint into the metal, and the less the upset.

The voltage required is so low and the current so high that the only convenient source is an a-c transformer built into the welder and as close as possible to the jaws which hold the parts and transmit the current to them. To conduct these heavy currents from any considerable distance involves either an excessive amount of copper or an excessive loss of power. Resistance welding is particularly adapted to production work, and special-purpose welders are used for large production.

Resistance welding is applied to most non-ferrous as well as to ferrous metal parts. For work of any considerable size, these machines are not readily portable, *i.e.*, the work must ordinarily be brought to the machine.

The type of resistance welding described above is the original form, known as **butt welding**, and has been applied to joint sections up to 36 sq in. in section.

With **non-ferrous** metals, with some exceptions such as platinum, the period of plasticity is short, and consequently a hand-operated pressure

If the temperature of the base metal in bronze welding is too low, the bronze does not spread out; if it is too high, the bronze collects in little balls which are driven away by the force of the flame. To avoid overheating, the torch should be held at a smaller angle to the deposited metal than in welding. Bronze welding of large joints should be done in several layers. The removal of graphite from the surfaces to be welded is important. Graphite may be removed by sealing the surface at 1650 F or by use of an oxidizing flame. A slightly oxidizing flame is usually recommended.

The flux should be oxidizing in character in order to remove graphite, and it should remove oxide films from the base metal, otherwise capillary flow of the bronze (tinning) will not occur without overheating. The customary filler rod for bronze welding is a brass containing 60 Cu, 40 Zn. If the zinc content is too low, the hot strength is correspondingly low; if zinc is too high, the deposited metal is too hard and brittle. Tin, iron, manganese, and silicon, about 1 percent, are often added to improve the flowing characteristics, decrease fumes, deoxidize, and increase the hardness. The V need be only wide enough so that the torch can be inserted. Double V joints are used for thick sections. In pipe welding the shear V joint is recommended.

With non-ferrous electrodes having a high melting point, covered electrodes (reversed polarity, short arc) are always deposited on cold metal. Beads not longer than 2 in. are deposited and peened lightly. After every second layer the deposit is caulked. For gas welding, bare rods are used with preheat and refined powdered borax. The usual high-melting-point non-ferrous electrode for cast iron is monel metal coated with deoxidizer and flux. Micrographic study reveals no hard zones in arc welds with monel electrodes. Good monel welds in cast iron have a tensile strength of 25,000 lb per sq in.

Malleable Cast Iron

Since the physical properties are dependent on certain heat-treatments which it undergoes, the heat of fusion welding will permanently change the structure and physical properties unless the temperature is kept below 1400 F. Bronze welding is the usual method of repair. The method is the same as for gray cast iron. The edges to be welded are beveled to 90 deg and cleaned. The scarfs should not be ground; grinding brings graphite to the surface and makes tinning difficult. The casting is then preheated to a black heat (locally for small castings) and bronze welded, using a tip one size larger than for similar work on steel and an oxidizing flame. Flux is used liberally. The rod may contain 59 Cu, 40 Zn, or 1 Fe, plus some Mn; lead is bad. Other rods such as manganese bronze and Cu-Zn-Si high-strength brass have also been found to be successful.

If the casting may be reannealed, *i.e.*, malleableized after welding, either arc or gas welding may be used as in cast-iron welding. The tensile strength and elongation of good arc, gas, and resistance welds in malleable cast iron which have been malleableized after welding are approximately 90 percent of those of the original malleable cast iron.

Wrought Iron

In the oxyacetylene welding of wrought iron a puddle of molten metal is maintained at the point of application of the filler rod. The end of the rod is immersed in the puddle until sufficient metal has been deposited. Then the edges of the molten puddle are fused into the surrounding colder metal. The weld progresses by repetition of this procedure. The puddle should be disturbed as little as possible to permit the slag to rise from the molten metal. The greasy appearance of the edges, that is noticeable in welding, corresponds to a temperature of 2100 to 2200 F, the correct temperature for welding being 2700 to 2750 F, a little higher than for mild steel. The filler rod can be low-carbon steel or a high-test rod. A neutral flame is best.

In metal-arc welding, slightly slower welding speeds should be used than for mild steel of the same thickness. In this way the metal is kept fluid longer, and gases and slag are eliminated. It may be necessary to use a slightly lower current than for the same thickness of mild steel, especially in thin sections where burning through is possible. Excessive penetration into the face of the metal should be avoided. High-grade unalloyed low-carbon electrodes should be used, bare or coated.

Carbon-arc welding and resistance welding of wrought iron are the same as those of mild steel.

butt-welder is not so satisfactory as a welder having spring or weight pressure, the full amount of which is exerted throughout the entire heating and welding operation. As soon as the abutting ends reach the welding or plastic stage, the spring or weight pressure forces them together, completing the weld and automatically breaking the primary circuit. This method is rarely used on parts having a crosssection over 0.25 sq in.

Welding Two Pieces of Material of Different Cross Section or of *Different Analyses*. *The tendency of a piece having a smaller cross-sectional area to heat the faster is offset by varying the projection of parts, i.e.,* by making the projection of the part having the larger sectional area somewhat longer than that of the smaller. Sometimes this can also be accomplished by preheating the larger section by means of a bridge or a piece of copper making contact with the die gripping the smaller part, so that when the circuit is closed the current flows and heats the larger part only.

In the **flash method** of butt welding the parts to be welded are clamped in the dies of the welding machine, giving equal or nearly equal projections to both parts, and usually the primary circuit is closed and the ends of the parts are brought together slowly. When these ends actually touch each other, they will flash, *i.e.*, minute particles of molten metal will fly off; this flashing is continued until the entire faces of the abutting ends have reached a welding heat when rapid and heavy pressure is applied, forcing the ends together and completing the weld, at the same time opening the primary circuit. With flash welding the power and time consumption is less, and the personal equation of the operator enters to a less degree than in butt welding. Some difficulty is encountered in flashing parts having cross-sectional areas over 8 to 10 sq in., where the ratio of their cross-sectional dimensions is over 50: 1, or less than 4: 1, although by preheating the parts in the latter case, this difficulty is greatly minimized. Flash welds are made both with cold and with preheated parts.

Cold flash is used for parts of an area well within the limits of the designed capacity of the welding machine. **Preheated flash** is used in welding parts having sections of an area closely approaching the design capacity of the welding machine and also where the physical or thermal characteristics of the parts are widely divergent.

In **welding non-ferrous metals** the cold flash is used entirely. The projection of the parts should generally be two or three times as great as in the welding of low-carbon steel. Platinum and similar metals are exceptions.

Strength of Resistance-welded Parts. In small sheet thickness up to 1/16 in. the strength of the spot weld is approximately 90 percent of that of the original material. In heavier thicknesses, such as 1/4 in., the strength ranges from 60 to 80 percent provided that the proper welding conditions are being used.

Flash-welded sections of ordinary structural steel will show 100 percent of the strength of the metal. In flash welding of steel, less current is required than in straight butt welding. *The voltage during the flashing period, however, is considerably higher. One* square inch cross section of iron or steel requires 30 kva for 10 sec. As the time required is inversely proportional to the power, if welded in 1 sec it will require 300 kva.

Great care must be exercised in butt welding of higher carbon steel. There is a danger of decarburization. Furthermore, heating should not be too sudden. Preheating to about 700 F outside of the machine is recommended. The pieces should be cooled slowly. Welds between low- and high-carbon steel can be readily made and possess high strength. *In welding of high-carbon steel*, higher pressures are necessary, for it does not flow so easily as low-carbon steel at welding heat.

Spot Welding. Where air-tightness is not required, a lap seam may be welded in spots by clamping the seam overlap between two (usually round)

Nickel Alloys

Gas Welding. The oxyacetylene flame used should be very slightly reducing with only a slight feather no longer than ⅛ in. showing beyond the tip of the luminous cone. The end of the welding rod should be kept well within the flame, so as to prevent its oxidation. Besides being reducing, the flame should be soft, rather than harsh, as is the case when too small a tip is used.

A flux is always used for monel (except for gas welding monel with the silicon monel gas-welding rod for pickling service), but none should be used with pure nickel. A flux recommended by manufacturers should be used. The silicon monel gas-welding rod requiring no gas-welding flux was developed particularly for the welding of equipment exposed to sulphuric acid service and specifically for monel pickling equipment for the pickling of steel.

After painting the flux on both sides of the joint, then adjusting the flame to slightly reducing conditions, and after beginning to weld, monel flows freely. Nickel, which is not fluxed, flows a little sluggishly. The appearance of properly made gas welds is quite similar to that of good steel welds. For the best results on high-nickel materials there should be little puddling; the molten pool should be kept quiet, with the tip of the luminous flame just touching its surface.

Welding rods are of the same composition as the alloy being welded if uniform corrosion resistance, with lack of galvanic effects, is desired. Some leeway is possible with the deoxidizing additions. The manufacturers of nickel furnish proper welding rods where these are specified. Wire should be bright annealed and free from oxide. Only wires tested for their weldability should be used for monel or nickel welding.

Electric Welding. Heavily extrusion-coated rods are used for monel and nickel metallic-arc welding. As the presence of aluminum is desirable in or near the molten pool of monel-weld metal, a small amount of aluminum is included in the monel-core wire. The use of a slightly alloyed monel rod results in developing strengths of 70,000 to 80,000 lb per sq in. in single-bead and multiple-bead butt joints, metallic-arc welded in flat, vertical, or overhead positions. This is the range of tensile strengths obtained in plate material. It is possible to get still higher values by using a core wire of type K monel. Monel and nickel electrodes carrying relatively thick flux coatings require reversed polarity, as do most of the high-strength steel electrodes.

In the metallic-arc welding of sheets of light to medium gage between 0.037 and 0.125 in. thick, it is desirable to clamp the sheets to restrain buckling. For lighter gages, it has been found that beads made without weaving are entirely satisfactory. Electrode sizes of $\frac{3}{32}$ (⅛)[$\frac{5}{32}$] in. with 50 to 70 (70 to 80) [80 to 140] amp are used for 12(9) [< 9] gage sheets.

Carbon-arc welding is frequently applied to monel and nickel sheet in the intermediate range of gages of 0.037, 0.050, and 0.062 in., and heavier.

The length of the arc is maintained between $\frac{1}{16}$ and ⅛ in., a relatively short arc when compared with the long one required for copper carbon-arc welding.

Aluminum and Aluminum Alloys

Gas Welding. The commonly used gas processes for welding aluminum and aluminum alloys are oxyhydrogen and oxyacetylene. A suitable flux should be used to remove the oxide coating that forms on the surface of aluminum alloys in the atmosphere. The flux is applied either dry or by mixing with tap water to the consistency of a thick paste. Practically all aluminum welding fluxes contain chlorides, fluorides, and sulphates. Residual deposits on the joints after welding will, in the presence of moisture, attack the base metal. Thorough cleaning is necessary. Commercially pure aluminum and metal of higher purity are generally welded with the same grade of metal as the parent material. The same practice is satisfactory for the aluminum-manganese alloy. In welding the aluminum-magnesium alloys or the aluminum-silicon-magnesium alloys, a filler rod containing approximately 95 percent Al and 5 percent Si is generally used. This rod has a substantially lower melting point than pure aluminum and permits the dissipation of some of the stress set up by solidification shrinkage and the thermal contraction that occurs in the weld zone as it cools. Cracks in the weld and the transition zone, on parts that are welded in jigs to hold proper alignment, can be minimized by using this rod.

electrodes and passing the necessary current between them and through the overlapping edges of the plates (Fig. 3). As the electrical resistance of the surface contact is least in the region under pressure, most of the current is confined to a spot of about the same area as that of the electrodes. For relatively thin metal this method is more rapid and economical than other methods of making a joint where mechanical strength alone is required. With a double row of staggered spots the joint strength may be made equal to that of the sheets themselves.

In thin metalwork of such shape and size that it is inconvenient to bring the work to a spot welder, the overlap is laid against a heavy flat electrode and the other electrode at the end of a flexible cable is pressed against the joint by hand. For the spot welding of heavy plates (½ in. and over) the currents and pressures are very large, and considerable heat is developed at the surfaces of contact between the electrodes and the plates. The electrodes must have high thermal as well as electrical conductivity. The combination of high pressure and temperature at the electrode surfaces makes it necessary to reshape them frequently. Three 1 in. plates have been successfully spot welded, although the operation cannot as yet be considered a commercially successful one.

The depth of the throat of a welding machine is determined by the width of the plates to be welded. With a single-spot welder, the alternating current must be carried around the throat, linking the long steel plate, with its high magnetic permeability. The resulting high reactance of this circuit requires a higher voltage than the weld itself, makes necessary a transformer of large capacity, and produces a load of low power factor. This obstacle can be largely surmounted by having two electrodes on each side of the overlap and welding two spots at once (Fig. 4). The transformer can be located close to the electrodes either below or above, or both, and the current passed up through one pair of electrodes and down through the other. The two spots should not be very close together, or too large a part of the current would be shunted through the plate on the transformer side. In any case a considerable amount is so shunted, resulting in waste of energy and undesirable heating of the plate. Where a short throat suffices, the single spot machine is preferable. Another method employs two transformers, one below and one above, where two spots are made at once.

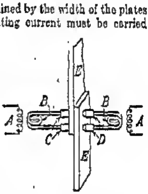


Fig. 4.—Wiring Diagram of Duplex Spot Welder.

Shot welding is a trade name applied to very rapid carefully-controlled spot welding of stainless steel.

In **projection welding**, projections are made on one or both of the work pieces and a flat die is used. This results in a smooth surface for the finished work.

Die points should have a diameter approximately equal to the total thickness of the two plates to be welded; the current used is about 50,000 amp per sq in. of die-point area; the pressure used is about 20,000 lb per sq in. of the die-point area. These figures will vary according to the stiffness of the plates, the distance between the welds, and the time required for making the welds.

In spot welding **high-carbon steel** it is desirable to employ a lower voltage than when welding the same thickness of stock of soft steel. The current is kept on for a longer time. Thin stock is easier to weld than heavier stock. Where slow welding is not permissible, it is desirable to heat-treat the welds.

As the aluminum alloys have a relatively high coefficient of thermal expansion compared with most weldable metals, it is desirable to minimize buckling and distortion from the welding heat by proper design and preparation of joints. Minimum distortion is obtained with edge or corner welds and in locations where the weld bead can be located on a crown or radius in the part at the weld. The location of welded seams or fittings in flat areas should be avoided.

Metal-arc Welding. Aluminum and the aluminum alloys can be fusion welded with the metallic arc. Because of the difficulty of controlling the arc it is not practical to butt weld material lighter than 0.081 in. thick in the shop or fillet weld joints on plate lighter than 1/8 in. thick. Electrode sizes are 1/8 (5/32) [3/16] in. with 45 to 85 (100 to 125) [175 to 225] amp and are used for <1/8 (1/8 to 1/4) [>1/4] in. sheets. Successful results are obtained by using a coated electrode on which a heavy coating is provided that functions both to remove the surface oxide on the metal and to stabilize the arc. As the 95 percent Al, 5 percent Si alloy provides superior fluidity at the welding temperatures, this alloy is used for the filler material in all the commercially available electrodes. No preliminary veeing is required on material up to 1/4 in. thick. Heavier materials should be veed to within 1/4 in. of the bottom of the section. Such preparation can sometimes be dispensed with on material 1/4 to 1/2 in. thick if a weld bead can be laid down from both sides of the section. In welding from one side only, it is desirable to confine the penetration bead by placing a metal backup strip below the joint. This strip should be provided with a cylindrical groove about 1/8 in. wide by 1/16 in. deep directly below the weld.

It is essential, if attack on the metal is to be prevented, to remove the residual welding flux on the joint after welding.

Electric-resistance Welding. The commercial aluminum alloys can be electric resistance welded. A machine should deliver at least 24,000 (35,000) [42,000] amp. to weld 1/16 (1/8) [3/16] in. thick material. A means of integrating the maximum current to permit the welding of intermediate thicknesses is also required. An autotransformer that will control the primary voltage from 20 percent full voltage to 100 percent voltage in 25 to 30 steps will provide adequate control of the current. For welding gages between 0.015 to 0.188 in. thick, periods varying from 2 to 25 cycles of the standard 60 cycle current wave are required. Good results can also be obtained on many types of work with contactor-type timers actuated either mechanically or by vacuum tubes.

For seam welding it is essential that full electronic equipment be used. The technique required involves delivery of the current to the work at a duty cycle of less than 30 percent. Timing cycles of 1 cycle on and 3 off, 1½ on and 4 off, or 2 on and 6 off are typical of the settings used for seam welding 0.032 to 0.064 in. material. Copper alloys with high electrical conductivity and mechanical strength have proved most satisfactory as tip materials. Water cooling to within 3/8 in. of the contact surface of the tips is necessary to ensure good tip life. It is necessary to clean only the area in contact with the welding tips, though, in a few cases, such as parts heat-treated in nitrate, it is desirable to clean one or both of the faying (contact) surfaces. It is also possible to clean the surface with an etching solution.

Aluminum alloys have low electrical resistance compared to ferrous alloys; currents ranging from 15,000 to 40,000 amp are necessary to cover the range of alloys and gages usually welded in an airplane factory. If these alloys are welded with high currents in short enough time it is possible to fuse the surfaces together without heating the outer surface or surrounding areas of the metal to an injurious temperature. Energy storage permits the development of the necessary high current without placing large maximum demand on the main lines. One type of equipment uses d-c power.

Resistance Butt Welding. Commercially pure aluminum, aluminum and 1.25 percent manganese alloy, and high-purity aluminum are readily electric butt welded and all the wrought alloys can be welded by this method.

The flash butt-weld process is adaptable to welding complicated extruded shapes of aluminum as well as the simpler cylindrical and rectangular shapes. Flashing is maintained for 1/2 to 1½ sec during which time 1/8 to 1 in. of aluminum is flashed from the parts being welded. The flashing period is terminated by the nearly simultaneous cutting off of power and a sudden increase in velocity of the moving die. The ends of

Special spot-welding electrodes are now used of a mixture of copper and special alloys such as tungsten, beryllium, and cadmium, the copper giving conductivity and the alloys rigidity and longer wear. A later practice uses chromium-plated copper electrodes.

In **seam welding** the overlapping edges of sheet metal are passed between two narrow roller electrodes (Fig. 3), the speed, current, and pressure being so adjusted as to produce a continuous seam weld by making a series of overlapping and closely-spaced spots. This method is limited to relatively thin sheets and is applicable to either straight or circular seams. It is employed in the manufacture of barrels, moderate-sized transformer tanks, and similar containers.

Tube Welding. Steel tubing may be made by the resistance welding of the edges of steel strip rolled up to form a tube. A single machine will take the flat strip, shape and weld it at a rate of 60 to 150 fpm. At these high speeds the fusion is confined to a thin surface layer, and the flash or upset is hardly perceptible. With alternating current of standard frequency (60 cycles per sec), high speeds result in an intermittent weld, for the heat is proportional to the square of the current and varies from zero to a maximum and back to zero, during each half cycle. This gives the appearance of a stitch on the outside of the tube, hence the common name of **stitch weld.** At a speed of 120 fpm (and 60 cycles) the stitches would be ½ in. apart. With a somewhat slower speed or higher frequency of current supply, the weld can be made continuous and airtight. No other process of tube manufacture can compete with this on the basis of cost. This process is at present confined to relatively thin-walled tubes. If the wall is thick as compared with the diameter, a relatively large amount of the current is shunted around the tube wall from the electrode rolls on either side of the seam.

Seam Welding

Total thickness, in.	0.016	0.032	0.059	0.098	0.157	0.197
Sec per ft of weld	9	15	22	33	45	55
Kva needed	6	8	12	16	25	30

The **current and energy required** in making welds can be varied over a wide range. In welding 1 sq in. of steel by the flash method the energy requirement may run from 30 to 350 kva, the voltage varying with these changes of current values. The average in such work may be 30 to 40 kw per sq in., or a total of 60 kw fed to the primary of the transformer. The power factor in such a case would probably not exceed 70 percent. The variables are many: the speed of work, the pressure applied, the current, the resistance of the piece, the heat conduction, the length of projection from the clamps, and the material of the clamps and their condition.

The optimum conditions for spot welding mild steel sheets 0.029 to 0.036 in. thick are a pressure of 15,000 lb per sq in. and exposure for 6 to 12 cycles with 12,000 to 13,500 amp for 60-cycle current and with electrodes ¼ in. diam. For stainless steel sheets 0.028 in. thick, the pressure is 50,000 lb per sq in. and the welding time 10 cycles or less with 9,000 amp.

Pulsation welding is a resistance welding method in which the welding current or heat is applied repeatedly for making a single weld, or simultaneous welds, in parts which are clamped between electrodes at rest. This is used when a single impulse of the same current is not sufficient to produce the weld.

The advantages of pulsation spot and projection welding are increased electrode life, the welding of thicker material with given equipment, and the production of better welds of improved appearance.

the parts being welded are forced together and rapid freezing of the weld takes place because of conduction of heat to the dies.

Characteristics of Welds in the Aluminum Alloys. When commercially pure and high-purity aluminum or the alloy with ¼ percent Mg are fusion welded, the welding heat removes the cold work in an area about two to four times the thickness of the section at the weld. *The tensile strength across a butt weld is not greater than the annealed* strength of the material regardless of the temper welded. Welds in this material are characterized by excellent ductility and a resistance to corrosion approximately the same as that of the parent material. An aluminum-magnesium-silicon-chromium alloy and an aluminum-silicon-magnesium alloy are used extensively in welded construction. In this case the welding heat partly obliterates the heat-treated structure, and the strength of the welds is intermediate between the annealed and heat-treated strength of the alloy.

Copper Alloys

For the welding of copper alloys, no general instructions are useful. Manufacturer's instructions should be sought or the A.W.S. Handbook consulted. See p. 629

Table 4. Data for Welding Fairly Clean Sheet Steel Stock
(Approximate)

SPOT WELDING

U. S. Standard gage No.	26	24	20	18	16	14	12	1/8"
Thickness, in.	0.0156	0.0250	0.0375	0.050	0.0625	0.0781	0.1094	0.125
Kw required	12	16	17	18	20	30	40	60
Time, sec	0.5	0.5	0.7	0.8	0.9	1.0	1.0	1.0

SLOW BUTT WELDING

Diameter, in.	1/4	3/8	1/2	5/8	3/4	1	1 1/4	1 1/2	1 3/4	2
Kw required	2.0	4	6	8	10	12	19	31	45	60
Time, sec	3	6	7	8	10	18	25	35	40	45

FLASH WELDING

Area of stock, sq in.	0.50	0.75	1.00	1.25	1.75	2.00	3.00	4.00
Kw required	25	30	60	80	200	250	350	500
Push-up pressure, lb.	800	1,750	3,150	5,000	9,600	12,500	35,000	60,000

These values will approximately hold for work where the greatest dimension of the welded section is not more than 4 in.

The following tabulation gives some examples of conditions for producing pulsation spot welds with 60-cycle current and clean surfaces.

Material	Tip diam, in.	Pressure, lb	Amp	Cycles		No. of pulsations
				On	Off	
Sheet steel, 1/8 to 1/8 in.	3/8	430	16,000	4	2	8
Steel bars, 1/4 × 2 in.	3/4	2,600	37,000	5	5	10
Steel bars, 1 × 3 in.	1	11,500	73,000	20	50	4 × 8
19 stacked steel sheets, 1/8 in. thick	3/4	3,300	58,000	13	31	5
Brass bars, 1/2 × 2 in.	1 1/16	4,000	86,000	22	50	4
Duralumin sheets, 1/8 to 1/8 in.	8° cone	1,200	35,000	4	2	6
Duralumin sheets, 1/4 to 1/4 in.	1 1/2 and 15° cone	2,200	56,000	5	2	12

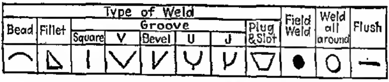


FIG. 5.—Fusion Welding Symbols.

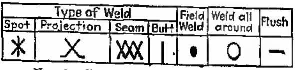


FIG. 6.—Resistance Welding Symbols.

MECHANICAL REFRIGERATION

BY

F. L. FAIRBANKS

(Revised by H. J. Macintire)

REFERENCES: Publications of the A.S.R.E.; "The Refrigerating Data Book," *Refrigerating Engineer*. Woolrith, "Handbook of Mechanical Refrigerating Engineering," Van Nostrand. Macintire, "Refrigeration Engineering," Wiley. Gottsche, "Taschenbuch für Kälte-Techniker," Hamburg. *Merkel and Bosnjakovic*, "Diagrammen und Tabellen zur Berechnung der Absorptions-Kältemaschinen," Berlin. Moyer and Fittz, "Refrigeration," McGraw-Hill.

Refrigeration Machines and Processes

(For general theory of refrigeration, see p. 349)

Refrigeration may be produced (1) through the absorption of heat by a gas that has been cooled by an expansion during which external work is performed, as in the air refrigerating machine; (2) through transferring heat by means of chemical machines or from a warmer body to a colder one (*e.g.*, refrigeration by cooled brine, water, etc.); (3) by melting or dissolving solid bodies, as in the melting of ice, the solution of salts in water and in chemical machines; (4) by evaporating liquids that have a relatively low boiling point temperature, as liquid ammonia and liquid carbon dioxide. The last type is the commercial method at the present time.

Air Machines (for theory, see p. 349). Air refrigerating machines are practically obsolete, because of their bulk, inefficiency, and operating difficulties. The **coefficient of performance** (the ratio of the useful refrigeration to the work of compression, see p. 350) of air machines is less than unity as compared with 4 or 5 in vapor machines. Table 1 gives results of tests of several of these machines. The expansion of air occurs in an engine that is preferably on the same shaft as the air compressor. The presence of water vapor in the air causes snow to form on the valves of the expansion cylinder.

Table 1. Test Results on Cold-air Machines
(Linde, *Trans. A.S.M.E.*, 14, p. 1416)

	System		
	Bell-Coleman	Lightfoot	Haslam
Air pressure in receiver, lb per sq in. abs.	61.0	65.0	64.0
Temperature of air entering compression cylinders, deg F	65.5	62.0	
Temperature of air after expansion, deg F	− 52.6	− 82.0	− 85.0
Ihp in compression cylinder	124.5	43.1	346.4
Ihp in expansion cylinder	58.5	28.0	176.2
Ihp in steam cylinder	84.4	24.6	332.7
Btu abstracted per hour per ihp of steam cylinder, at 20 F	668.0	1554.0	954.0

Chemical Machines. In most of these, the temperatures of water and brine are successively lowered by dissolving a salt (usually nitrate of ammonia) in water, the salt being recovered later by evaporating the water. They have no commercial importance.

Dry ice (solid CO_2) is useful as a refrigerating medium in special cases where the lack of drip is an advantage. The main objection to the general use of dry ice is its cost but

Aluminothermic (Thermit) Welding

Thermit is a trade name for a mixture of finely divided aluminum and iron oxide, which, upon reaction, produces one-half its weight of superheated molten steel and a molten alumina slag, both at a temperature of almost 5000 F. To this basic mixture are added oxides of other elements than iron, and in this way the analysis of the steel and its physical properties are controlled.

Thermit is an inert mixture until it is heated almost to the temperature of molten steel. A local heating initiates the reaction which then propagates rapidly through the mass of thermit and makes it possible to produce the desired amount of superheated molten steel in less than a minute.

The reaction is

$$3Fe_3O_4 + 8Al = 4Al_2O_3 + 9Fe$$

There are four varieties of thermit produced for the purpose of welding ferrous metals, *viz.*, plain thermit, cast-iron thermit, forging thermit, and wabbler thermit.

Plain thermit is used in making pipe welds where the material is used as a heating agent to bring the pipe ends to a welding temperature after which they are pressed together to form a pressure weld. **Forging thermit** *contains additions of nickel, manganese, and mild steel punchings to the* extent of about 17 percent and is used in the welding of all-steel sections. **Cast-iron thermit** has an addition of 3 percent ferrosilicon and 20 percent, mild-steel punchings and is used for the welding of cast-iron parts. **Wabbler thermit** is designed to produce a hard long-wearing, yet machinable, steel for building up worn wabbler ends of rolls and pinions.

In making a thermit weld the parts to be united are lined up and a space cut between these parts at the fracture. A wax pattern is then formed between and around the ends to be united, the wax forming a slight collar. The size of the gap and the dimensions of this collar are fixed by experience. The collar is not for the purpose of reinforcement, but rather to provide sufficient heat for the thorough fusion of the parts. A sand and clay mold is next rammed around the wax pattern and inside of a sheet-iron mold box, provision being made in the mold for heating gates and pouring gates and risers, the latter being placed at the highest points of the wax pattern and the others entering at the lowest points.

Figure 7 shows a longitudinal and a cross section of a typical mold. New molding material should be used next to the weld, gates, and risers, but previously used molding material may be used for backing.

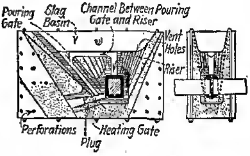


Fig. 7.—Mold for Making Thermit Welds.

Vent holes are provided in the mold to facilitate the escape of gases from the mold material. The parts are next heated to a bright red heat by means of a kerosine and air flame directed into the heating gate, which flame first melts out the wax and then while heating the parts to be welded also dries out the mold. In the meantime a suitable charge of thermit, which may be calculated from the weight of wax, is placed in a magnesite-lined crucible mounted over the pouring gate and when the preheating has been finished the heating gate is plugged and the thermit charge in the crucible ignited. In about 30 sec the reaction is

the ease in its handling, its low temperature of −110 F at atmospheric pressure, its non-corrosive and non-toxic properties, its high latent heat, and the absence of liquid drip make it desirable. The heat absorbed per pound of solid CO_2 during sublimation is 250.7 (246.3) [233.9] Btu at −140 (−110) [−70] F corresponding to 3.18 (14.7) [74.8] lb per sq in. abs. *The specific heat of the gas at constant pressure is about 0.2.*

Carbon dioxide is obtained commercially either by fermentation or by burning. The gas is compressed, usually in three stages, and cooled to atmospheric temperature, thereby forming a liquid at 900 to 1,100 lb per sq in. abs; the liquid is then throttled to about 5.3 atm pressure. During throttling, part of the CO_2 solidifies and is compressed to form a dense ice. The fraction that returns to the vapor phase is recompressed.

Water ice and common salt are still used commercially, as in the cooling of refrigerator cars and in passenger trains.

Vaporization Machines. Ammonia, water vapor, sulphurous acid, carbonic acid, ethyl chloride, propane, and other fluids are employed. These machines are of three types: vacuum machines, absorption machines, and compression machines.

In **water-vapor** refrigerating machines, the compression of the vapor is accomplished either by use of multistage centrifugal compressors or by the use of steam-ejector nozzles. Such machines are confined to comfort cooling, the cooling of drinking water, the cooling of rubber during the rolling process of its manufacture, and other applications which demand temperatures in the evaporator of 35 to 50 F, corresponding to pressures of the water vapor of 0.10 to 0.18 lb per sq in. abs; liquefaction is usually at about 100 F (1 lb per sq. in. abs), and cooling tower condenser water is frequently used.

In the water-vapor refrigerating machine, the return water from the sprays, or coils, and the make-up water enter an *evaporator* through a *throttle* (**expansion**) valve and the consequent vaporization of part of the water abstracts heat from the unvaporized part. The resulting cold water is pumped to the spray chamber or the cooling coils. The compression of the water vapor by multistage centrifugal compressors is similar to that used in the compression system. When steam jets are used, the machine consists of steam-ejector nozzles, an entrainment or mixing chamber, a booster ejector, and the water-cooled surface condenser. The steam pressure at the nozzles is preferably 150 to 100 lb per sq in. abs, in which case the weight of steam used is 20 to 35 lb per ton of refrigeration per hour, depending on the temperature of the chilled water desired in the evaporator and the steam pressure used in the nozzles. The amount of water supplied to the condenser is greater than in other refrigerating designs and is approximately 6 gal per ton of refrigeration per min. This type of refrigerating machine is built in sizes from 25 to 100 tons, but machines of 250 tons capacity have been installed.

Vapor machines using fluids other than water are of the compression type, the absorption type (p. 1866), or the adsorption type (p. 1874). The compression cycle is discussed on p. 350.

Liquids Used in Vapor Refrigerating Machines. In recent years, a number of refrigerants have been developed some of which are non-toxic and non-inflammable. Many of these are known by trade names, of which the following are perhaps the most prominent.

Freon 11 (Carrene 2), CCl_3F, Trichloromonofluoromethane
Freon 12, CCl_2F_2, Dichlorodifluoromethane
Freon 21, $CHCl_2F$, Dichloromonofluoromethane
Freon 113, $C_2Cl_3F_3$, Trichlorotrifluoroethane
Freon 114, $C_2Cl_2F_4$, Dichlorotetrafluoroethane

complete, and the contents of the crucible are converted into a superheated thermit steel at the bottom and alumina slag at the top. The tapping pin at the bottom of the crucible is struck a sharp upward blow releasing the thermit steel so that it runs from the crucible through the pouring gate of the mold and completely surrounds the ends to be welded giving up its superheat to melt these parts and solidifying with them. The thermit reaction is started by means of a teaspoonful of ignition powder which can be ignited with a match or a hot rod and which in turn gives off sufficient heat to start the reaction. The molds are usually constructed so as to provide a basin on top; the molten alumina slag overflows into this basin.

The tensile strength of unworked and non-heat-treated thermit steel is about 70,000 lb per sq in. with an elongation of 20 to 25 percent in 2 in. Other thermit steels have tensile strengths of 55,000 lb per sq in. with 40 percent elongation, and others over 100,000 lb per sq in. with practically no elongation. Mixtures can be placed in the crucible which will produce alloy steels of any desired composition.

The thermit process has been approved by the American Bureau of Shipping, by Lloyd's Register of Shipping, and by the other rating bureaus for use in the repair of stern frames and other important sections of steamers.

Thermit welding is used principally in the repair of cast-iron or steel sections over 3 in. thick; for lighter sections it is more expensive than oxyacetylene or electric welding. It is also used for the building up of worn parts.

In the steel mill, thermit welding is used in the repair of table rolls, roll mill housings, blooming-engine parts, crankshafts, connecting rods, charging peels, etc. By the use of wabbler thermit the worn surfaces on the driving ends of rolls, pinions, and other parts can be built up and the metal so accurately added as to require no machining; such a surface is just barely machinable and is very resistant to wear. The welding of heavy cast-iron parts introduces no difficulty when the thermit process is used, except that the high carbon content of the cast iron produces a high carbon content in the thermit steel weld, which may make machining difficult. The weld has at least as great strength as the cast-iron part.

One of the largest uses for thermit welding is the joining of rail ends in paved streets. In the railroad shop, thermit is used for the welding of locomotive frames, guide yokes, driving wheel centers, and other heavy parts.

Facing or Surfacing by Welding

Facing or surfacing by welding is a process of comparatively recent development which consists of welding onto wearing parts a facing, edge, or point of hard metal exceptionally resistant to abrasion or a metal suitable for the particular purpose as, for example, bearing or corrosion resistance. By this method, metal surfaces which normally wear away rapidly in service are protected by a layer of special alloy possessing unusual wear resistance. The process can be applied with equal advantage to new parts before their use or to old worn parts. The most important economy derived from hard facing is due to prolonged life of the parts. Hard-faced parts will outwear plain or unfaced ones 2 to 25 times, depending on the type of hard metal used and the service to which they are subjected. To meet the various requirements of hardness, toughness, shock-resistance, and other qualities, various hard-facing alloys of widely different compositions have been developed.

Most of the metals in common use except high-speed steels can be hard faced. Some metals and alloys require preheating or annealing, but generally the same precautions are followed as in the welding of these materials.

Silver Soldering

The term "silver solder" designates a type of hard solder or brazing alloy usually containing silver, copper, and zinc. Other base metals such as cadmium, tin, manganese, and nickel are also used. An important characteristic is the low melting point which is approximately 1160 to 1600 F, depending upon the composition. See p. 649.

Dielene, $C_2H_2Cl_2$, Dichlorethylene
Carrene 1, CH_2Cl_2, Methylene chloride
Arctic, CH_3Cl, Methyl chloride

A comparison of ideal performances of some of these refrigerants yields the results shown in Table 2. The results are for the standard temperature

Table 2. Ideal Performance of Refrigerants for Various Temperature Ranges

Refrigerant	Operating temp range, F	Suction press, lb per sq in. abs	Head press, lb per sq in. abs	Ratio of head to suction press	With dry and saturated suction vapor, per ton: Wt of vapor, lb per min	Piston displct, cfm	Theoretical hp	Temp at end of compression, F
Water	32-86	0.0886	0.613	6.92	0.196	646.0	0.60	282.0
	32-100	0.0886	0.946	10.68	0.199	655.0	0.78	353.7
	40-100	0.1217	0.946	7.77	0.19824	483.3	0.66	313.0
Carrene 1	5-86	1.28	10.07	8.56	1.485	74.0	0.96[a]	205.1[a]
(CH_2Cl_2)	20-100	1.92	13.25	6.90	1.520	47.72	0.91[a]	157.1[a]
Methylene chloride	40-100	3.38	13.25	3.92	1.493	27.76	0.63[a]	167.7[a]
Carrene 2	5-86	2.95	18.3	6.20	3.058	37.0	0.94	112.7
Freon 11, F-11	20-100	4.33	23.7	5.47	3.086	26.2	0.89	122.1
(CCl_3F)	40-100	7.034	23.7	3.37	2.945	16.08	0.63	114.2
Ethyl	5-86	4.65	27.10	5.83	1.405	24.0	0.95[a]	106.3[a]
Chloride	20-100	6.80	34.79	5.12	1.425	17.19	0.92[a]	116.1[a]
	40-100	10.79	34.79	3.22	1.375	10.73	0.63[a]	109.9[a]
Freon 21, F-21	5-86	5.45	30.5	5.60	2.364	20.87	0.94[a]	99.0[a]
($CHCl_2F$)	20-100	7.82	39.2	5.01	2.404	15.67	0.94[a]	110.4[a]
	40-100	12.339	39.2	3.18	2.315	10.19	0.68[a]	105.9[a]
Butane	5-86	8.2	41.6	5.07	1.619	16.16		
(C_4H_{10})	20-100	11.6	52.2	4.50	1.659	12.00		
	40-100	17.7	52.2	2.95	1.562	7.62		
Sulphur	5-86	11.81	66.45	5.63	1.415	9.08	0.97	191.4
dioxide	20-100	17.18	84.52	4.92	1.453	6.52	0.92	193.4
(SO_2)	40-100	27.10	84.52	3.12	1.444	4.17	0.63	162.6
Methyl	5-86	20.80	95.52	4.59	1.331	6.20	0.99	155.4
chloride	20-100	28.76	119.04	4.14	1.362	4.67	0.95	167.5
(CH_3Cl)	40-100	42.61	119.04	2.79	1.338	3.10	0.64	143.8
Freon 12, F-12	5-86	26.51	107.9	4.07	3.916	5.81	1.00	100.2
(CCl_2F_2)	20-100	35.75	131.6	3.68	4.054	4.54	0.97	112.5
	40-100	51.68	131.6	2.55	3.880	3.07	0.67	108.0
Ammonia	5-86	34.27	169.2	4.94	0.421	3.44	0.99	209.8
(NH_3)	20-100	48.21	211.9	4.40	0.421	2.49	0.94	212.8
	40-100	73.32	211.9	2.89	0.427	1.70	0.65	176.0
Propane	5-86	42.1	155.3	3.69	1.653	4.10	1.35[a]	92.9[a]
(C_3H_8)	20-100	55.5	187.0	3.37	1.73	3.29	1.32[a]	103.9[a]
	40-100	78.0	187.0	2.40	1.646	2.26	0.90[a]	101.5[a]
Carbon dioxide	5-86	332.0	1,043.0	3.14	3.528	0.94	2.12	160.3

[a] Values may be in error by a small amount.

Silver solders are free flowing, corrosion resistant, and produce strong joints that withstand severe shocks and vibration if used at ordinary temperatures. They can be used for joining practically all ferrous and non-ferrous metals and alloys except those having lower melting points, such as aluminum and zinc alloys.

WELDING PROCEDURES

Steel

Medium-carbon Steels. (Carbon from 0.30 to 0.45 by ladle test.) This class of steel may be welded by the electric-arc, electric-resistance, gas, and thermit processes. As the rapid cooling of the metal in the welded zone produces a harder martensitic or troostitic structure, it is desirable to hold the carbon as near 0.30 percent as possible. These hard areas are proportionately more brittle and difficult to machine. The cooling rate may be diminished and hardness decreased by preheating the metal to be welded to over 300 F, preferably to 500 F. The degree of preheating depends somewhat on the thickness of the section. Subsequent heating of the welded zone to 1100 to 1200 F will restore ductility and relieve strain.

High-carbon Steels. (Carbon from 0.45 to 0.80 by ladle test.) These steels are rarely welded except in special cases and in thick sections. The tendency for the metal heated above the critical range to become brittle is more pronounced than with lower or medium-carbon steel. Thorough preheating of metal in and near the welded zone to at least 500 F is essential. Subsequent annealing at 1350 to 1450 F is also desirable.

Low-alloy Steels. Those steels which are foolproof from the welding standpoint contain less than 0.15 per cent carbon. Steels containing less than 0.30 percent carbon may require a stress-relieving or tempering treatment and when fillet welded will generally develop a very narrow zone of reduced ductility. Steels containing 0.30 percent carbon or more require stress relieving, unless very special conditions are involved. Preheating is generally beneficial. The welding technique does not differ from that used in the welding of plain carbon steel.

Chrome-nickel austenitic steels are excellent material for welding and under satisfactory welding conditions produce strong, tough, and reasonably ductile welds. Alloys containing less than 0.07 percent carbon can be safely welded without any subsequent heat-treatment. In case of gas welding, it is safer to accelerate the cooling by means of an air blast, particularly if heavy sections ($\frac{3}{16}$ in. and above) are being welded. Alloys containing 0.07 to 0.10 percent carbon can be arc welded provided that the structures welded will not be used for service where high corrosion resistance is required.

No arc or gas welding should be attempted on alloys containing more than 0.10 percent carbon unless the complete structure can be subsequently heat-treated and rapidly cooled in an air blast.

Addition of titanium, columbium, or some of the other rare metals raises the safe carbon content of chrome-nickel alloys and permits arc welding of steel containing more than 0.10 percent carbon. In general, enough titanium or columbium is added to permit safe welding with the carbon content present. Such "stabilized" steel is not entirely proof from dangerous carbide precipitation.

Unstabilized alloys containing more than 0.07 percent carbon can be generally spot or seam welded without impairing their corrosion-resisting properties, provided that the welding time is made very short. The maximum safe length of the welding time will vary inversely with the carbon content of the alloy and somewhat with the welding technique used. With alloys with 0.10 percent carbon no dangerous carbid precipitation will occur if the time of the application of the welding current does not exceed 0.20 sec. With alloys having a carbon content of around 0.15 percent this figure should be reduced to 0.15–0.10 sec. maximum. All welding will produce some carbide precipitation, and the line of demarkation between the alloys that can and cannot be welded is a compromise between the extent of the carbide precipitation and the severity of the corrosion attack to which he welded product will be subjected. Chrome steels must never be arc welded with an unshielded arc. Special heavily coated electrodes must be used. Atomic-hydrogen arc by virtue of its protective blanket of hydrogen is satisfactory, but bare rod or carbon arc must never be used. No welding should be attempted without first thoroughly cleaning the surfaces to be welded or plating over an area extending not less than 1 in. on every side of the weld. Flash welding is, in general, quite satisfactory

range (5 to 86 F) and for other stated ranges also. Piston displacements are with no clearance. The theoretical coefficients of performance (see p. 350) for the various conditions covered in the table are obtainable by dividing 4.72 by the table theoretical hp. Data on some of the refrigerants not covered in Table 2 are given in Table 3. The expansion is through a throttle or expansion valve. The higher the compression ratio, the greater will be the superheat and the greater the resulting unproductive work of compression. Use of an expansion valve in place of an expansion cylinder results in a loss of refrigerating effect as well as in a loss of work which might

Table 3

	Saturation pressure lb per sq in. abs		Ratio of compression	Displacement per min per ton, cu ft
	At 5 F	At 86 F		
Freon 114 ($C_2Cl_2F_4$)	7.3	35.8	4.94	18.9
Methyl formate ($C_2H_4O_2$)	0.18	13.9	7.8	50
Freon 113 ($C_2Cl_3F_3$)	0.12	8	6.7	95
Dielene ($C_2H_2Cl_2$)	0.9	7.4	8.2	109
Trichlorethylene (C_2HCl_3)	0.16	1.72	9.2	512

Table 4. Properties of Propane and Butane

Temp, deg F	Propane (C_3H_8) (Heat measurements are from 0 F)						Butane (C_4H_{10}) (Heat measurements are from 0 F)					
	Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb		Entropy		Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb		Entropy	
			Liquid h_f	Vapor h_g	Liquid S_f	Vapor S_g			Liquid h_f	Vapor h_g	Liquid S_f	Vapor S_g
−70	7.37	12.9	−37.0	152.5	−0.086	0.400						
−60	9.72	9.93	−32.0	155.0	−0.074	0.393						
−50	12.6	7.74	−26.5	158.0	−0.061	0.389						
−40	16.2	6.13	−21.5	160.0	−0.049	0.384						
−30	20.3	4.93	−16.0	163.0	−0.036	0.380						
−20	25.4	4.00	−11.0	165.0	−0.024	0.377						
−10	31.4	3.26	−5.5	168.0	−0.012	0.374						
0	38.2	2.71	0	170.5	0.000	0.371	7.3	11.10	0	170.5	0.000	0.371
+10	46.0	2.27	5.5	173.5	0.012	0.370	9.2	8.95	5.5	174.0	0.01[illegible]	0.370
20	55.5	1.90	11.0	176.0	0.024	0.368	11.6	7.23	10.5	177.5	0.022	0.370
30	66.3	1.60	17.0	179.0	0.035	0.366	14.4	5.90	16.0	181.5	0.033	0.371
40	78.0	1.37	23.0	182.0	0.047	0.366	17.7	4.88	21.5	185.0	0.044	0.371
50	91.8	1.18	29.0	185.0	0.059	0.365	21.6	4.07	27.0	188.5	0.056	0.373
60	107.1	1.01	35.0	188.0	0.070	0.364	26.3	3.40	33.0	192.5	0.067	0.374
70	124.0	0.883	41.0	190.5	0.082	0.364	31.6	2.88	38.5	196.0	0.078	0.375
80	142.8	0.770	47.5	193.5	0.093	0.364	37.6	2.46	44.5	199.5	0.089	0.376
90	164.0	0.673	54.[illegible]	196.5	0.105	0.364	44.5	2.10	51.0	203.0	0.100	0.377
100	187.0	0.591	60.5	199.0	0.116	0.363	52.2	1.81	57.0	206.5	0.111	0.378
110	212.0	0.521	67.0	201.[illegible]	0.128	0.363	60.8	1.58	63.5	210.5	0.122	0.380
120	240.0	0.459	73.5	202.5	0.140	0.363	70.8	1.38	70.0	213.5	0.134	0.382
130	......	......	......	......	......	......	81.4	1.21	76.5	217.0	0.145	0.384
140	......	......	......	......	......	......	92.6	1.07	83.5	221.0	0.157	0.386

as most of the fused metal produced by the arc of the flash which could be expected to contain chromium oxides and nitrates is squeezed out by the upsetting operation and the weld obtained is therefore fairly ductile. Because of the amount of heating involved the straight butt-welding process should be avoided.

Chromium Iron and Steels. Welding of the chromium irons and steels can be divided into two classes: (1) welding in which the filler metal deposited has essentially the same chemical analysis as the base material; (2) welding in which the filler metal deposited is dissimilar in analysis and characteristics to the base metal, the filler metal commonly employed being an austenitic chromium-nickel steel of the 18 percent chromium, 8 percent nickel type. An austenitic chromium-nickel filler metal is used for all welds that cannot be annealed, including field and repair welds, and a filler metal of the same analysis as the parent metal for welds that can be annealed. For operation at high temperatures the expansion of the material must be considered. In a straight welded seam 30 ft long, there will be approximately 1¼ in. difference in expansion between the austenitic filler metal and chromium iron-base metal when heated to 1000 F. Operation at high temperature will result in warpage or high stresses and under repeated heating and cooling may result in failure through fatigue. The maximum corrosion-resisting properties of the austenitic steels are best realized by a rapid cool from temperatures higher than 1800 F, with either the unstabilized or stabilized steels. Since filler metal of the same analysis as the parent metal exhibits equivalent corrosion-resisting properties to the base material when properly controlled, it is recommended for all heat-treated welds under 18 percent chromium.

Austenitic Manganese Steel. In welding this material, it is important to remember that it is intrinsically a high-carbon metal, containing about the same amount of that element as the tool steel used for lathe tools. In arc welding with nickel-manganese steel rod, reversed polarity should be used, *i.e.*, the rod should be the positive pole. A nickel-manganese steel rod with a suitable coating welds smoothly, the arc sputters hardly at all, and it is possible to move the end of the rod steadily along without puddling the pool of metal. Some of the coated rods of this metal are self-feeding; the arc once started, the rod can be laid flat upon the work and the arc will travel steadily up the rod, like a slow-burning fuse, laying down a sound deposit. The slag coating over the deposited metal should break off readily at a light blow of the hammer. Peening can be started almost immediately as each length of rod is used up, as in the use of the bare rod.

It is best to lay the beads of each layer at right angles to those of the layer below, or at least at as high an angle as is feasible. In laying on metal to rebuild worn surfaces this is generally easy to accomplish. In repairing cracked or broken castings, especially heavy ones, the high angle may not always be easy to secure.

The current recommended for nickel-manganese steel rod is as follows:

Electrode size, in.	⅛	5⁄32	3⁄16	¼
Amperes	75–100	100–150	150–175	175–225

For gas welding, a tip should be selected large enough to melt down the metal with a soft flame, in order to minimize bubbling of the metal. A neutral flame should be used. A carbonizing flame makes the metal flow more freely but its use should be avoided.

Cast Steel

In good grades of cast steel with a carbon content below 0.25 percent, welding procedures are approximately the same as for rolled steel. With a carbon content above 0.25 percent or with special alloy compositions, precautions are necessary and sometimes special procedures. Segregations of phosphorus and sulphur should be removed from areas to be welded. The problem of overcoming shrinkage in repair of castings requires special care. Strains or stress relieving, especially repair work, is desirable. In higher carbon and alloy steels full annealing may be necessary.

For arc welding, a high-grade heavy-coated electrode is essential. A tough, general-purpose electrode (arc) or filler rod (gas) is used for plain carbon castings, but filler metal of the same composition as the casting is used for alloy cast steel.

Multilayer welding, high currents, and peening should be employed for low-carbon castings. Low currents and small diameter electrodes (and, in some cases, preheating

otherwise be available for operating the compressor. The magnitudes of these losses vary with the different refrigerants, and they are particularly high with a fluid that is compressed near to its critical pressure, as with carbon dioxide.

For the properties of water vapor see Table 20, p. 328, Table 21, p. 333, and Table 1, p. 374; for ammonia Table 25, p. 338 and Table 26, p. 341; for

Table 5. Properties of Freon 11 and Freon 12

Temp, deg F	Freon 11 (CCl_3F) (Heat measurements are from −40 F) Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb Liquid h_f	Vapor h_g	Entropy Liquid S_f	Vapor S_g	Freon 12 (CCl_2F_2) (Heat measurements are from −40 F) Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb Liquid h_f	Vapor h_g	Entropy Liquid S_f	Vapor S_g
−40	0.739	44.2	0.00	87.48	0.0000	0.2085	9.3	3.91	0.00	73.50	0.0000	0.1752
−30	1.03	32.3	1.97	88.67	0.0046	0.2064	12.0	3.09	2.03	74.70	0.00471	0.1739
−20	1.42	24.1	3.94	89.87	0.0091	0.2046	15.3	2.47	4.07	75.87	0.00940	0.1727
−10	1.92	18.2	5.91	91.07	0.0136	0.2030	19.2	2.00	6.14	77.03	0.01403	0.1717
0	2.55	13.9	7.89	92.27	0.0179	0.2015	23.9	1.64	8.25	78.21	0.01869	0.1709
10	3.35	10.8	9.88	93.46	0.0222	0.2003	29.3	1.35	10.39	79.36	0.02328	0.1701
15	3.82	9.59	10.88	94.09	0.0244	0.1997	32.4	1.23	11.48	79.94	0.02556	0.1698
20	4.34	8.52	11.87	94.69	0.0264	0.1991	35.7	1.12	12.55	80.49	0.02783	0.1695
25	4.92	7.58	12.88	95.30	0.0285	0.1986	39.3	1.02	13.66	81.06	0.03008	0.1692
30	5.56	6.75	13.88	95.91	0.0306	0.1981	43.2	0.939	14.76	81.61	0.03233	0.1689
35	6.26	6.07	14.88	96.51	0.0326	0.1976	47.3	0.862	15.87	82.16	0.03458	0.1686
40	7.03	5.45	15.89	97.11	0.0346	0.1972	51.7	0.792	17.00	82.71	0.03680	0.1683
45	7.88	4.90	16.91	97.72	0.0366	0.1968	56.4	0.730	18.14	83.26	0.03903	0.1681
50	8.80	4.42	17.92	98.32	0.0[illegible]	0.1964	61.4	0.673	19.27	83.78	0.04126	0.1678
55	9.81	4.00	18.95	98.93	0.0406	0.1960	66.7	0.622	20.41	84.31	0.04348	0.1676
60	10.9	3.63	19.96	99.53	0.0426	0.1956	72.4	0.575	21.57	84.82	0.04568	0.1674
70	13.4	2.99	22.0[illegible]	100.73	0.0465	0.1951	84.8	0.493	23.90	85.82	0.05009	0.1670
80	16.3	2.49	24.[illegible]	[illegible]	0.0504	0.1947	98.8	0.425	26.28	86.80	0.05446	0.1666
90	19.7	2.09	26.18	103.12	0.0542	0.1942	114.3	0.368	28.70	87.74	0.05882	0.1662
100	23.6	1.76	28.27	104.30	0.0580	0.1938	131.6	0.319	31.16	88.62	0.06316	0.1658
110	28.1	1.50	30.40	105.47	0.0617	0.1935	150.7	0.277	33.65	89.43	0.06749	0.1654
120	33.2	1.28	32.53	106.63	0.0654	0.1933	171.8	0.240	36.16	90.15	0.07180	0.1649
130	39.0	1.10	34.67	107.78	0.0691	0.1931	194.9	0.208	38.69	90.76	0.07607	0.1644

sulphur dioxide, Table 27, p. 342 and Table 28, p. 343; for carbon dioxide, Table 29, p. 344; for ethyl chloride and methyl chloride, Table 30, p. 344; for propane and butane, Table 4; for Freon 11 and Freon 12, Table 5; for dieline and Freon 21, Table 6; for Carrene 1, Table 7.

The volume of refrigerant to be handled is important with reciprocating compressors, as it determines the size of the compressor; but with centrifugal compression a large volume is not objectionable and may be a positive advantage for small units. A large compression ratio is undesirable in reciprocating compressors from the standpoint of clearance losses and may make the use of compound compression necessary.

and slow cooling) are necessary for high-carbon and special alloy castings. For gas welding, only large sections are preheated, locally or generally, to a bright red.

Cast Iron

Gas Welding. Preheating in gas-welding cast iron avoids shrinkage stresses, cracks at the junction zone, and hard zone next to the weld and saves gases. Preheating temperature 1300 F. A neutral flame is generally employed, although a slightly carburizing flame is sometimes advantageous, as, for example, when there is a deficiency in carbon.

In making a weld the crack is chipped out to form a 75 to 90 deg V, a little flux is spread over the starting point, and the flame is played on the V until the walls begin to melt. The metal of the outer walls of the V should not melt ahead of the metal at the root. The rod, dipped in flux, is then brought to a red heat and rubbed into the molten metal. The tip of the blue cone of the flame should be kept $\frac{3}{16}$ to $\frac{1}{8}$ in. away from the molten puddle to avoid hard spots and loss of silicon and carbon.

Cast-iron rods are used for gas welding. The A.W.S. recommends C, 3.0 to 3.5; Si, 3 to 3.5; Mn, 0.5 to 0.75; S, 0.1; P, 0.5 to 0.7.

Metal-arc Welding. Cast iron may be welded with ferrous electrodes in two ways: (1) hot, with preheating; (2) cold, without preheating. The purpose of preheating is

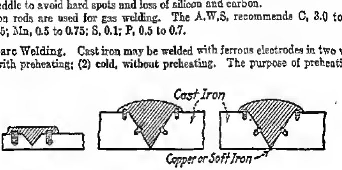


FIG. 8.—Cross Sections of Studded Joints, Showing Use on Thin and Thick Materials and the Use of a Copper or Soft Iron Strip at Bottom of Vee.

the same as in gas welding. The preheating temperature is usually 1000 to 1300 F. Cast-iron electrodes are used for hot welding. The composition is practically the same as for gas welding. Currents range from 300 to 1,500 amp, 400 to 600 amp being common. The voltage is 40 to 70.

The cold welding process is used for welds that need not be machined and that are not thick.

Mild steel-covered electrodes are generally used. The coating of the electrode may shield the arc and weld metal, or it may alloy with the steel core to produce a cast-iron deposit. The coating for the first purpose is the same as on ordinary mild steel-covered electrodes and is designed to avoid nitrides. Carbonaceous coatings are used for the second purpose. A coating of 40 to 60 percent graphite, 1 percent barium carbonate, and the rest carborundum, with a paste of water glass as a binder, has been found to be effective.

The amperage used for covered electrodes in cold welding is kept low in order to reduce heat, hardening, and cracking. The use of 80 to 90 amp with a $\frac{1}{8}$ in. electrode on $\frac{1}{2}$ in. material is common.

Much of the success of cold welding depends on proper preparation of the joint. Studding is a common method (Fig. 8). The size of studs and spacing depends on the thickness of the casting. Copper, lead, or soft-steel inserts are used to ensure machinability on one side.

Carbon-arc Welding. Preheating from 750 to 1300 F is used together with annealing or slow cooling. Graphite rods are generally used, and a flux containing equal parts sodium carbonate and sodium bicarbonate is useful. Filler rods are generally the same as for gas welding.

Thermit Welding. (See p. 1842.)

Bronze Welding. Preheating is usually applied to assure success and may be local or general depending on the casting. A black preheat is generally employed, although in some cases preheating to 1650 F has been found advantageous.

The large volumes required for Carrene 1, F-11, dieline, and water vapor can be handled satisfactorily by centrifugal compressors. When the evaporating pressure is below atmospheric, as in the case of Carrene, dieline, ethyl chloride, sulfur dioxide, water vapor, and butane, air leaks are likely to be excessive; where the refrigerant is nearly odorless (Freon 12 and methyl chloride) leaks into the atmosphere and loss of the refrigerant may be excessive.

Table 6. Properties of Dieline and Freon 21

Temp, deg F	Dieline ($C_2H_2Cl_2$) (Heat measurements are from 0 F)						Freon 21 ($CHCl_2F$) (Heat measurements are from −40 F)					
	Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb		Entropy		Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb		Entropy	
			Liquid h_f	Vapor h_g	Liquid S_f	Vapor S_g			Liquid h_f	Vapor h_g	Liquid S_f	Vapor S_g
−40	0.175	267	−10.8	127.0	−0.022	0.3062	1.36	32.1	0.00	114.6	0.0000	0.2730
−30	0.254	186	−8.1	129.3	−0.0162	0.3034	1.89	23.6	2.36	115.8	0.0055	0.2695
−20	0.356	134	−5.4	131.7	−0.0113	0.3003	2.58	17.7	4.71	117.0	0.0109	0.2663
−10	0.493	97.7	−2.7	133.8	−0.0057	0.2984	3.46	13.4	7.07	118.2	0.0162	0.2633
0	0.693	73.2	0	136.3	0.0000	0.2972	4.58	10.3	9.44	119.4	0.0214	0.2606
10	0.942	54.1	2.7	138.7	0.0057	0.2967	5.98	8.06	11.81	120.6	0.0265	0.2581
15	1.13	47.0	4.0	139.8	0.0090	0.2964	6.80	7.18	13.01	121.2	0.0291	0.2569
20	1.30	41.0	5.4	141.1	0.0113	0.2960	7.70	6.39	14.21	121.8	0.0316	0.2559
25	1.50	35.9	6.7	142.2	0.0145	0.2952	8.70	5.71	15.40	122.4	0.0341	0.2548
30	1.71	31.6	8.1	143.5	0.0162	0.2948	9.79	5.11	16.61	123.0	0.0365	0.2538
35	1.94	27.7	9.5	144.7	0.0200	0.2945	11.0	4.59	17.83	123.6	0.0390	0.2528
40	2.24	24.5	10.8	145.8	0.0220	0.2932	12.3	4.13	19.04	124.2	0.0414	0.2519
45	2.56	21.7	12.1	147.0	0.0240	0.2922	13.8	3.73	20.27	124.8	0.0439	0.2510
50	2.87	19.3	13.5	148.2	0.0265	0.2917	15.3	3.37	21.49	125.4	0.0463	0.2502
55	3.31	17.1	14.8	149.3	0.0290	0.2910	17.0	3.10	22.73	126.0	0.0487	0.2493
60	3.76	15.1	16.[illegible]	150.4	0.0313	0.2903	18.9	2.77	23.98	126.6	0.0511	0.2486
70	4.78	12.0	18.9	152.8	0.0357	0.2897	23.1	2.30	26.49	127.8	0.0559	0.2471
80	6.03	9.7	21.[illegible]	155.2	0.0405	0.2885	28.0	1.92	29.03	129.0	0.0606	0.2458
90	7.59	7.8	24.3	157.6	0.0445	0.2871	33.6	1.62	31.59	130.1	0.0652	0.2446
100	9.37	6.4	25.[illegible]	158.7	0.0483	0.2863	40.0	1.37	34.18	131.3	0.0699	0.2434
110	11.47	5.32	29.7	162.3	0.0521	0.2851	47.4	1.17	36.79	132.4	0.0745	0.2424
120	14.09	4.38	32.[illegible]	164.9	0.0559	0.2842	55.7	1.00	39.[illegible]	133.5	0.0791	0.2414
130	17.15	3.64	35.1	167.1	0.0595	0.2830	65.1	0.862	42.13	134.6	0.0837	0.2405

Water vapor can be used only for very mild refrigeration. With a centrifugal vapor compressor of 5 or 6 stages the variation in capacity with the temperature of the chilled water is about as follows:

Temp of chilled water, F	60	55	50	45	40	35
Percent rating, tons	142	119	100	84	69	58

Ammonia is used extensively in the United States for commercial work; in 1938 the Freon refrigerants had been adopted by only 15 percent of the new compressors of 26 to 100 tons capacity and only 2 percent of those over 100 tons. Ammonia is dangerous to life and corrodes copper and copper compositions. It has a high latent heat of vaporization, its pressures and

43. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log_e\left(x + \sqrt{x^2 - a^2}\right) + C$

$= \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + c$

44. $\int \sqrt{a + 2bx + cx^2}\,dx = \frac{b + cx}{2c}\sqrt{a + 2bx + cx^2}$

$+ \frac{ac - b^2}{2c}\int \frac{dx}{\sqrt{a + 2bx + cx^2}} + C$

TRANSCENDENTAL FUNCTIONS

45. $\int a^x dx = \frac{a^x}{\log_e a} + C$

46. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a}\left[1 - \frac{n}{ax} + \frac{n(n-1)}{a^2x^2} - \cdots \pm \frac{n!}{a^n x^n}\right] + C$

47. $\int \log_e x\,dx = x\log_e x - x + C$

48. $\int \frac{\log_e x}{x^2}\,dx = -\frac{\log_e x}{x} - \frac{1}{x} + C$

49. $\int \frac{(\log_e x)^n}{x}\,dx = \frac{1}{n+1}(\log_e x)^{n+1} + C$

50. $\int \sin^2 x\,dx = -\frac{1}{4}\sin 2x + \frac{1}{2}x + C = -\frac{1}{2}\sin x\cos x + \frac{1}{2}x + C$

51. $\int \cos^2 x\,dx = \frac{1}{4}\sin 2x + \frac{1}{2}x + C = \frac{1}{2}\sin x\cos x + \frac{1}{2}x + C$

52. $\int \sin mx\,dx = -\frac{\cos mx}{m} + C$ 53. $\int \cos mx\,dx = \frac{\sin mx}{m} + C$

54. $\int \sin mx\cos nx\,dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C$

55. $\int \sin mx\sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C$

56. $\int \cos mx\cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C$

57. $\int \tan x\,dx = -\log_e\cos x + C$ 58. $\int \cot x\,dx = \log_e\sin x + C$

59. $\int \frac{dx}{\sin x} = \log_e\tan\frac{x}{2} + C$ 60. $\int \frac{dx}{\cos x} = \log_e\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$

61. $\int \frac{dx}{1 + \cos x} = \tan\frac{x}{2} + C$ 62. $\int \frac{dx}{1 - \cos x} = -\cot\frac{x}{2} + C$

63. $\int \sin x\cos x\,dx = \frac{1}{2}\sin^2 x + C$ 64. $\int \frac{dx}{\sin x\cos x} = \log_e\tan x + C$

65.* $\int \sin^n x\,dx = -\frac{\cos x\sin^{n-1}x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

66.* $\int \cos^n x\,dx = \frac{\sin x\cos^{n-1}x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

67. $\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$

* If n is an odd number, substitute $\cos x = z$ or $\sin x = z$.

68. $\int \cot^n x\,dx = -\dfrac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx$

69. $\int \dfrac{dx}{\sin^n x} = -\dfrac{\cos x}{(n-1)\sin^{n-1} x} + \dfrac{n-2}{n-1}\int \dfrac{dx}{\sin^{n-2} x}$

70. $\int \dfrac{dx}{\cos^n x} = \dfrac{\sin x}{(n-1)\cos^{n-1} x} + \dfrac{n-2}{n-1}\int \dfrac{dx}{\cos^{n-2} x}$

71.* $\int \sin^p x \cos^q x\,dx = \dfrac{\sin^{p+1} x \cos^{q-1} x}{p+q} + \dfrac{q-1}{p+q}\int \sin^p x \cos^{q-2} x\,dx$

$= -\dfrac{\sin^{p-1} x \cos^{q+1} x}{p+q} + \dfrac{p-1}{p+q}\int \sin^{p-2} x \cos^q x\,dx$

72.* $\int \sin^{-p} x \cos^q x\,dx = -\dfrac{\sin^{-p+1} x \cos^{q+1} x}{p-1} + \dfrac{p-q-2}{p-1}\int \sin^{-p+2} x \cos^q x\,dx$

73.* $\int \sin^p x \cos^{-q} x\,dx = \dfrac{\sin^{p+1} x \cos^{-q+1} x}{q-1} + \dfrac{q-p-2}{q-1}\int \sin^p x \cos^{-q+2} x\,dx$

74. $\int \dfrac{dx}{a + b\cos x} = \dfrac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\dfrac{a-b}{a+b}} \tan \tfrac{1}{2}x\right) + C$, when $a^2 > b^2$

$= \dfrac{1}{\sqrt{b^2 - a^2}} \log_e \dfrac{b + a\cos x + \sin x\sqrt{b^2 - a^2}}{a + b\cos x} + C,$

$= \dfrac{2}{\sqrt{b^2 - a^2}} \tanh^{-1}\left(\sqrt{\dfrac{b-a}{b+a}} \tan \tfrac{1}{2}x\right) + C,$ when $a^2 < b^2$

75. $\int \dfrac{\cos x\,dx}{a + b\cos x} = \dfrac{x}{b} - \dfrac{a}{b}\int \dfrac{dx}{a + b\cos x} + C$

76. $\int \dfrac{\sin x\,dx}{a + b\cos x} = -\dfrac{1}{b} \log_e (a + b\cos x) + C$

77. $\int \dfrac{A + B\cos x + C\sin x}{a + b\cos x + c\sin x}\,dx = A\int \dfrac{dy}{a + p\cos y}$

$+ (B\cos u + C\sin u)\int \dfrac{\cos y\,dy}{a + p\cos y} - (B\sin u - C\cos u)\int \dfrac{\sin y\,dy}{a + p\cos y},$

where $b = p\cos u$, $c = p\sin u$ and $x - u = y$.

78. $\int e^{ax} \sin bx\,dx = \dfrac{a\sin bx - b\cos bx}{a^2 + b^2} e^{ax} + C$

79. $\int e^{ax} \cos bx\,dx = \dfrac{a\cos bx + b\sin bx}{a^2 + b^2} e^{ax} + C$

80. $\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1 - x^2} + C$

81. $\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1 - x^2} + C$

82. $\int \tan^{-1} x\,dx = x\tan^{-1} x - \tfrac{1}{2}\log_e (1 + x^2) + C$

83. $\int \cot^{-1} x\,dx = x\cot^{-1} x + \tfrac{1}{2}\log_e (1 + x^2) + C$

84. $\int \sinh x\,dx = \cosh x + C$

85. $\int \tanh x\,dx = \log_e \cosh x + C$

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* If p or q is an odd number, substitute $\cos x = z$ or $\sin x = z$.

specific volumes are convenient, and it is not miscible to any large extent in the usual mineral oils. Leaks are easily detected by the use of sulphur dioxide vapor. **Carbon dioxide** has been used as a safety refrigerant for a long time; its presence in large volume in a confined space is not a serious hazard unless the exposure is prolonged. Its condensing pressure is very high for condenser water initially at 70 F and higher. Its critical temperature is 87.8 F and consequently with cooling water of about 80 F it will not condense; the power consumption is high (see Table 2). **Sulphur dioxide** is used almost entirely in fractional tonnage machines in which it lends itself to the air-cooled condenser. It is not inflammable, but is an irritant and a toxic gas. It is practically harmless in the small household refrigerating machine but may be dangerous in multiple systems.

Table 7. Properties of Carrene 1
(Heat measurements are above 0 F)

Temp, deg F	Pressure, lb per sq in. abs	Specific volume of vapor, cu ft per lb	Enthalpy, Btu per lb		Entropy, Btu per lb	
			Liquid h_f	Vapor h_g	Liquid s_f	Vapor s_g
−10	.69	81.3	−3.4	161.8	−0.0072	0.3606
0	.98	58.6	0.0	163.2	0.0	0.3546
10	1.38	42.55	3.4	164.4	0.0072	0.3502
20	1.92	31.40	6.8	165.6	0.0151	0.3461
25	2.24	27.0	8.6	166.4	0.0188	0.3444
30	2.56	23.90	10.2	166.9	0.0222	0.3425
35	2.95	21.10	11.9	167.5	0.0256	0.3402
40	3.38	18.60	13.6	168.0	0.0285	0.3377
50	4.36	14.68	17.0	169.0	0.0350	0.3335
60	5.52	11.68	20.4	170.1	0.0410	0.3292
70	7.07	9.38	23.8	171.0	0.0406	0.3246
80	8.81	7.50	27.2	172.0	0.0520	0.3202
90	10.87	6.20	30.6	172.9	0.0570	0.3160
100	13.25	5.34	34.0	173.7	0.0620	0.3113
110	16.40	4.31	37.4	174.4	0.0652	0.3058
120	19.20	3.65	40.8	175.0	0.0714	0.3031
130	22.69	3.10	44.2	175.5	0.0756	0.2983
140	26.79	2.69	47.6	176.0	0.0795	0.2935

Unless sulphur dioxide is anhydrous, it will corrode the common metals used in the construction of pressure vessels. **Methyl chloride** (CH_3Cl), an anesthetic in amounts of 5 to 10 percent by volume, may be used in air-cooled condensers and is successful in large and small-sized units. It is miscible in mineral oils; water in the system will freeze in the expansion valve. Leaks may be detected by the use of acrolein. **Dichlorodifluoromethane** (F-12) is not toxic, inflammable, or irritant. An air-cooled condenser may be used, and leaks can be detected with a special torch. **Ethyl chloride** (C_2H_5Cl), **butane** (C_4H_{10}), and **propane** (C_3H_8) have as yet been used but little as refrigerants. The use of **dieline** ($C_2H_2Cl_2$) and **Carrene** (CH_2Cl_2) has been confined to the centrifugal compressors. Very low temperature refrigeration may use ethane, ammonia, or Freon 12. Other halides such as F-11 (CCl_3F), F-21 ($CHCl_2F$), and F-114 ($C_2Cl_2F_4$) are also available.

The refrigerant used for air-conditioning and commercial cooling is largely F-12 or methyl chloride; the similarity of physical properties permits the use

by a weak solution of ammonia in water. (6) The pump, which returns the rich liquor from the absorber to the generator. The Electrolux-Servel process (p. 1873) permits the omission of the pump or other moving part.

Between the generator and absorber, there are two channels of communication: one through the condenser, expansion valve, brine coil, absorber, and pump, as noted above; through the other, the weak solution in the generator passes directly to the absorber and there absorbs the gas coming from the brine coil. As in the compression system, there is a region of high pressure including the generator and condenser, and a region of low pressure including the brine coil and absorber. The pump is used to force the strong solution from the lower pressure of the absorber against the higher pressure in the generator.

For efficient operation of an absorption system, certain auxiliary organs are required. The **analyzer** forms the upper part of the generator and receives the rich solution from the pump. The solution descends over a series of disks or trays until it meets the boiling liquid in the still. The vapor rising from the still thus comes into intimate contact with the descending liquid and is enriched in ammonia and deprived of water.

The **rectifier** is placed between the generator and condenser. It consists of a coil surrounded by cooling water, and its function is to remove water vapor from the mixture of water and ammonia vapors driven off from the generator.

The **exchanger** is placed between the generator and absorber. In it, the hot weak solution passing from the generator to the absorber gives up heat to the cooler strong solution passing from the pump to the generator.

Ammonia Solutions. The analysis of an absorption system requires a knowledge of the properties of ammonia solutions. Certain of these properties have been investigated by Wilson (*Univ. Illinois, Eng. Expt. Stat. Bull.* 146).

The **concentration** of a solution of ammonia in water is defined as follows. Let y denote weight of water, Z the weight of ammonia in the solution: then the **weight concentration** is $x' = Z/(Z + y)$ and the **mol concentration** is $x = (Z/17)/[(y/18) + (Z/17)] = Z(0.944y + Z)$. (For definition of the "mol" see p. 301.)

The relation between pressure, temperature, and mol concentration, as determined by Wilson, is given by the equation

$$T/T' = 1 + 0.70358(1 - x^k)$$

with

$$k = \sqrt{x + 0.05}(1.347 - 2.9x + 1.77x^2)$$

In this equation, T denotes the temperature of the solution, T' the temperature of saturated ammonia corresponding to the pressure on the solution, and x the mol concentration.

Table 16 gives the pressure on the solution for various temperatures and various values of x, the mol concentration.

The vapor in contact with the solution will be a mixture of water vapor and ammonia vapor, each with its own partial pressure. Table 17 gives the partial pressure of the water vapor as determined by Wilson for various temperatures and mol concentrations of the liquid phase. The partial pressure of the ammonia vapor is obtained by subtraction. Thus for $t = 120$ deg and a mol concentration $x = 0.35$, Table 16 gives for the total pressure 45.62 lb per sq in., and for the pressure of the water vapor Table 17 gives 1.06 lb per

of either refrigerant in the same unit machine. Ammonia is still the favored refrigerant in the larger installations, or in the central-plant-type, with brine systems, and with direct-expansion systems, such as ice plants and where the ammonia is restricted to the operating plants. Sulphur dioxide is now used but little.

Basis of Rating of Refrigerating Machines. The commercial unit of capacity of a refrigerating machine is taken as the abstraction of an amount of heat equal to the heat of fusion of 1 ton (2,000 lb) of ice per day (24 hr). The determinations of the heat of fusion of ice by the Bureau of Standards show slight deviations between plate ice, can ice, and natural ice. The mean of 21 determinations is 79.63 cal per g, or 143.33 Btu per lb avoirdupois. This is equivalent to 286,600 Btu per ton, or the taking up of heat in a machine of unit capacity at the rate of 199.028 Btu per min. This is so close to the convenient round figures of 200 Btu per min, 12,000 Btu per hr, or 288,000 Btu per day, that these latter figures have been adopted as the **standard ton** by the A.S.R.E. and the A.S.M.E.

The rating of the tonnage of a refrigerating machine taken in connection with the plane of temperatures at which heat is to be taken up and that at which it is to be discharged (*i.e.*, temperature of the refrigerant in the evaporating or refrigerator coils and the highest temperature in the condenser) has not been fully agreed upon internationally. European engineers use +14 F at the refrigerating coil and +77 F at the condenser, but in America the **standard rating** is the number of standard tons of refrigeration under pressures which correspond to a saturation temperature of 5 F (−15 C) for the inlet pressure and 86 F (30 C) for the outlet pressure, these pressures being measured outside and within 10 ft of the refrigerating machine, measured along the inlet and outlet pipes, respectively. This is equivalent in ammonia machines to 34 lb abs and 169 lb abs, respectively. For the influence of change in the plane of temperatures on the ideal performance of refrigerating machines see Tables 2 and 3.

The refrigerating capacity of a machine is different from the actual **ice-making capacity** of a plant; the latter is considerably less, being 50 percent and upward of the refrigerating capacity, according to temperature of water, etc.

The Refrigeration Research Committee of the (British) Institution of Mechanical Engineers has adopted (1914) the calorie per second (= 342,860 Btu per 24 hr) as the unit of refrigeration, and has adopted as "standard conditions" a temperature range of the cooling water from 15 C (59 F) at inlet to 20 C (68 F) at outlet and a temperature range of the brine from 0 C (32 F) to −5 C (23 F). For direct-expansion systems, the standard vapor temperature in the refrigerator is to be taken as −10 C (14 F). The rated capacity of a machine is the number of units of refrigeration developed under the foregoing standard conditions.

Compression System

The influence of the head (or liquefaction) temperature on the theoretical coefficient of performance of a compression machine is as follows for **various refrigerants** with a common suction (or evaporation) temperature of 5 F.

The influence of increased head temperature is seen to be especially great for **carbon dioxide**; a consequence of its proximity to the critical temperature. Special procedures such as two-stage compression are necessary for carbon dioxide when cooling-water temperatures are high—as in the tropics. For example, with 300 lb suction pressure (temperature 0 F approx.) and with the compressed gases cooled to 95 F, the theoretical coefficients of

sq in.; hence the partial pressure of the ammonia vapor is 45.62 − 1.06 = 44.56 lb per sq in.

The tables of total and partial vapor pressures are required in analyzing conditions in the rectifier. The following example illustrates the process.

Let the pressure in the generator be 150 lb per sq in. and the average mol concentration $x = 0.30$. From Table 16, the solution will have this pressure when the solution attains the temperature 217 F approx. and from Table 17 the partial pressure of the water vapor above the solution is 11.1 lb per sq in. In the rectifier, let the mixture of vapors be cooled to 140 F. With a total pressure of 150 lb, the mol concentration of the condensed liquid must be 0.50, and from Table 17 for $t = 140$ deg and $x = 0.50$ the partial pressure

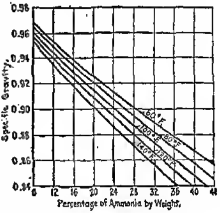

FIG. 1.—Relations between Temperature, Specific Gravity, and Weight Concentration of Aqua Ammonia.

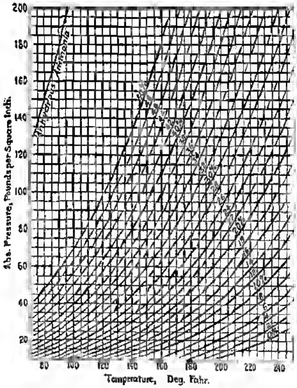

FIG. 2.—Relations between Temperature, Pressure, and Weight Concentration of Aqua Ammonia.

of the water vapor is 1.37 lb. The reduction in partial pressure from 11.1 to 1.37 lb means that only about ⅛ of the water vapor initially in the mixture

performance are as follows: discharge pressure 1,200 lb per sq in., 1.70; discharge pressure 1,400 lb, 2.095; in two stages (1) to 1,200 lb with cooling to 95 F and (2) to 1,400 lb, 2.25.

Table 8. Influence of Head Temperature on Coefficient of Performance
(Suction temperature 5 F)

Refrigerant		Ammonia	Carbon dioxide	Sulphur dioxide	Freon 12
Coeff. of performance with head temperature of	70 F	6.16	4.17	6.29	6.15
	86 F	4.77	2.14	4.69	4.71

Volumetric Efficiency. The volumetric efficiency of a compression machine is the ratio of the actual weight of vapor pumped by the machine to the weight calculated from the displacement of the compressor with gas entering saturated at its evaporating pressure. This efficiency, on account of heating of the gas coming to the cylinder and in the cylinder, of reexpansion from clearances, and of slip, varies from 60 to 85 percent. Compression machine builders under favorable conditions will guarantee a volumetric efficiency of 80 to 85 percent in machines of 50 tons and over; under good average working conditions, with a reasonable amount of care to maintain tight valves and tight piston rings, it should run from 75 to 80 percent.

Theoretical values of compressor displacement for ammonia are given in Table 9.

Table 9. Theoretical Volume of Dry Saturated Ammonia Gas (Cu Ft) Pumped per Minute to Produce 1 Ton of Refrigeration
(Add 33 percent for probable actual volume)

Suction pressure (gage) and corresponding temperature		Condenser pressures (lb per sq in. gage) and corresponding temperatures (deg F)								
Lb per sq in., P	Deg F, T	103 (65°)	115 (70°)	127 (75°)	139 (80°)	153 (85°)	166 (90°)	182 (95°)	198 (100°)	215 (105°)
1	−27	7.22	7.3	7.37	7.46	7.54	7.62	7.70	7.79	7.88
4	−20	5.84	5.9	5.96	6.03	6.09	6.16	6.23	6.30	6.43
6	−15	5.35	5.4	5.46	5.52	5.58	5.64	5.70	5.77	5.83
9	−10	4.66	4.73	4.76	4.81	4.86	4.91	4.97	5.40	5.08
13	−5	4.09	4.12	4.17	4.21	4.25	4.30	4.35	4.80	4.44
16	0	3.59	3.63	3.66	3.78	3.74	3.78	3.83	3.47	3.91
20	5	3.20	3.24	3.27	3.30	3.34	3.38	3.41	3.05	3.49
24	10	2.87	2.9	2.93	2.96	2.99	3.02	3.06	3.89	3.12
28	15	2.59	2.61	2.65	2.68	2.71	2.73	2.76	2.40	2.82
33	20	2.31	2.34	2.36	2.38	2.41	2.44	2.46	2.29	2.51
39	25	2.06	2.08	2.18	2.12	2.15	2.17	2.20	2.52	2.24
45	30	1.85	1.87	1.89	1.91	1.93	1.95	1.97	2.00	2.01
51	35	1.70	1.72	1.74	1.76	1.77	1.79	1.81	1.83	1.85

Example. Let head pressure = 198.9 lb per sq in. abs and back pressure = 30.42 lb per sq in. abs. Then h_{g1} = 611.8 Btu and h_f = 150.5 Btu. Also, V'' = 9.116 cu ft per lb. The weight of ammonia per min = 200/(611.8 − 150.5) = 0.434 lb, and cfm = 0.434 × 9.116 = 3.96. Assuming the volumetric efficiency of compressor = 80 percent, the actual piston displacement = 3.96/0.80 = 4.95 cu ft per ton of daily capacity.

The theoretical horsepower required to produce 1 ton of refrigeration under different operating conditions with ammonia is given in Table 10. The

is carried on to the condenser. The remainder is condensed and returned (as a strong aqueous solution) to the generator.

Table 16. Total Vapor Pressures of Aqua Ammonia, Lb per Sq In. Abs

Temp, deg F	Molal concentration of ammonia in the solutions in percentages										
	0	5	10	15	20	25	30	35	40	45	50
32	0.09	0.34	0.60	0.97	1.58	2.60	4.20	6.54	9.93	14.18	19.40
40	0.12	0.45	0.77	1.24	2.01	3.25	5.21	8.06	12.05	17.20	23.39
50	0.18	0.64	1.05	1.65	2.67	4.29	6.75	10.35	15.34	21.65	29.26
60	0.26	0.86	1.42	2.21	3.51	5.55	8.65	13.22	19.30	27.05	36.26
70	0.36	1.17	1.84	2.90	4.56	7.13	11.01	16.56	24.05	33.39	44.42
80	0.51	1.52	2.43	3.76	5.85	9.06	13.86	20.61	29.69	40.96	54.08
90	0.70	2.02	3.15	4.83	7.43	11.40	17.23	25.48	36.34	49.82	65.32
100	0.95	2.62	4.05	6.13	9.34	14.22	21.32	31.16	44.12	59.99	78.30
110	1.27	3.34	5.14	7.72	11.64	17.58	26.87	37.81	53.16	71.87	93.19
120	1.69	4.27	6.46	9.63	14.42	21.54	31.69	45.62	63.59	85.33	110.20
130	2.22	5.38	8.07	11.91	17.67	26.20	38.25	54.55	75.55	100.86	129.50
140	2.89	6.70	9.98	14.63	21.49	31.54	45.73	64.78	89.19	118.24	151.30
150	3.72	8.29	12.23	17.81	26.00	37.81	54.43	76.61	104.65	138.10	175.40
160	4.74	10.16	14.92	21.54	31.16	45.02	64.25	89.88	122.10	160.20	202.70
170	5.99	12.41	18.01	25.87	37.11	53.27	75.55	104.84	141.75	185.10	233.20
180	7.51	15.00	21.65	30.86	44.02	62.68	88.17	121.68	163.70	212.60	267.00
190	9.34	18.06	25.87	[illegible]	51.81	73.32	102.56	140.75	188.10	243.30	304.30
200	11.53	21.60	30.7[illegible]	43.14	60.62	85.33	118.68	161.81	215.20	277.00	345.50
210	14.12	25.61	36.26	50.58	70.72	98.80	136.42	185.10	245.10	314.50	390.70
220	17.19	30.27	42.47	59.00	81.91	113.81	156.41	211.24	278.20	355.10	439.60
230	20.78	35.59	49.60	68.4[illegible]	94.43	130.64	178.28	239.70	314.50	400.20	493.40
240	24.97	41.52	57.65	78.91	108.60	149.20	202.74	270.92	354.10	448.90	552.30
250	29.83	48.32	66.[illegible]	90.74	124.08	169.48	229.62	305.60	397.60	502.40	

Table 17. Partial Pressures of Water Vapor above Aqua Ammonia, Lb per Sq In. Abs

Temp, deg F	Molal concentration of ammonia in the solutions in percentages										
	0	5	10	15	20	25	30	35	40	45	50
32	0.09	0.084	0.079	0.074	0.070	0.065	0.060	0.056	0.051	0.047	0.042
40	0.12	0.115	0.108	0.101	0.095	0.089	0.083	0.076	0.070	0.064	0.058
50	0.18	0.17	0.16	0.15	0.14	0.13	0.12	0.11	0.10	0.094	0.085
60	0.26	0.24	0.23	0.21	0.20	0.19	0.17	0.16	0.15	0.13	0.12
70	0.36	0.34	0.32	0.30	0.28	0.26	0.25	0.23	0.21	0.19	0.17
80	0.51	0.48	0.45	0.42	0.40	0.37	0.34	0.32	0.29	0.27	0.24
90	0.70	0.66	0.63	0.58	0.55	0.51	0.47	0.44	0.40	0.37	0.33
100	0.95	0.90	0.85	0.79	0.74	0.69	0.64	0.59	0.55	0.50	0.45
110	1.27	1.20	1.14	1.07	1.00	0.93	0.86	0.80	0.73	0.67	0.60
120	1.69	1.60	1.51	1.42	1.33	1.24	1.15	1.06	0.97	0.89	0.80
130	2.22	2.10	1.98	1.86	1.74	1.62	1.51	1.39	1.28	1.17	1.05
140	2.89	2.73	2.57	2.42	2.26	2.11	1.96	1.81	1.66	1.52	1.37
150	3.72	3.51	3.31	3.11	2.91	2.72	2.52	2.33	2.14	1.95	1.76
160	4.74	4.48	4.22	3.97	3.71	3.46	3.22	2.97	2.73	2.49	2.25
170	5.99	5.66	5.34	5.02	4.70	4.38	4.07	3.75	3.45	3.15	2.84
180	7.51	7.10	6.69	6.30	5.89	5.49	5.10	4.71	4.33	3.94	3.57
190	9.34	8.83	8.32	7.82	7.32	6.83	6.34	5.86	5.38	4.91	4.44
200	11.53	10.90	10.27	9.65	9.04	8.43	7.83	7.23	6.64	6.06	5.48
210	14.12	13.35	12.58	11.82	11.07	10.32	9.59	8.86	8.13	7.42	6.71
220	17.19	16.25	15.32	14.39	13.48	12.57	11.67	10.78	9.90	9.03	8.17
230	20.78	19.64	18.51	17.40	16.29	15.19	14.11	13.03	11.97	10.91	9.87
240	24.97	23.60	22.25	20.91	19.58	18.26	16.95	15.66	14.38	13.12	11.86
250	29.83	28.20	26.58	25.00	23.39	[illegible]1.82	20.25	18.71	17.18	15.67	

Heat of Solution. A weak solution having the weight concentration x_1 absorbs ammonia liquid, and the concentration is raised to x_2. The heat

mean effective pressure in an ammonia compressor can be obtained approximately from the following table.

For $\dfrac{\text{Discharge pressure}}{\text{Suction pressure}}$ =	3.00	3.50	4.00	4.50	5.00	5.50	6.00	7.00	8.00
$\dfrac{\text{Mean effective pressure}}{\text{Suction pressure}}$ =	1.25	1.45	1.68	1.80	1.95	2.09	2.22	2.45	2.67

Table 10. Theoretical Horsepower to Produce 1 Ton of Refrigeration with Ammonia

Suction pressure (gage) and corresponding temperature		Condenser pressures (lb per sq in. gage), and corresponding temperatures (deg F)								
Lb per sq in, P	Deg F, T	103 (65°)	115 (70°)	127 (75°)	139 (80°)	153 (85°)	168 (90°)	184 (95°)	200 (100°)	218 (105°)
4	−20	1.059	1.130	1.205	1.283	1.361	1.443	1.525	1.609	1.691
6	−15	0.997	1.069	1.145	1.222	1.300	1.410	1.461	1.546	1.630
9	−10	0.903	0.978	1.045	1.118	1.193	1.260	1.347	1.435	1.509
13	−5	0.818	0.883	0.954	1.023	1.094	1.168	1.244	1.321	1.396
16	0	0.735	0.801	0.865	0.933	1.002	1.072	1.147	1.219	1.255
20	5	0.666	0.731	0.795	0.059	0.928	0.998	1.066	1.138	1.212
24	10	0.592	0.663	0.726	0.789	0.054	0.921	0.991	1.060	1.129
20	15	0.541	0.600	0.664	0.728	0.792	0.855	0.922	0.994	1.060
33	20	0.474	0.534	0.592	0.672	0.715	0.780	0.842	0.903	0.974
39	25	0.410	0.466	0.523	0.580	0.599	0.702	0.767	0.829	0.892
45	30	0.351	0.406	0.461	0.518	0.576	0.635	0.694	0.759	0.817
51	35	0.300	0.355	0.410	0.467	0.521	0.580	0.640	0.701	0.763

Condenser Pressure and Back Pressure. The lower the pressure and temperature in the condenser coil, and the higher the pressure and temperature in the evaporator (back pressure), the more economical will be the working of the plant. For these reasons, the cooling water in the condenser should be used as cold and in as large quantity as possible, and the back pressure should be as high as possible. In ammonia plants in which the temperature is to be kept at about 32 F, by direct expansion, a back pressure of about 33 lb gage (corresponding to about 20 F) is generally maintained. If brine circulation is used, the brine enters the room with a temperature of about 20 F and returns with a temperature of 24 to 26 F; the back pressure in the ammonia coils should be 25 to 28 lb gage, corresponding to a temperature of 10 to 15 F.

During the chilling stage in a packing house, the temperature in the room may rise in the beginning to 50 F and a higher back pressure—about 60 lb gage, corresponding to a temperature of about 40 F in the ammonia coil—is maintained. As the temperature falls in the room, the back pressure is decreased to the point corresponding to the desired temperature of the room, usually about 30 lb gage. If temperatures of 0 F and below are required, the back pressure should be 4 lb gage, corresponding to a temperature of −20 F. For ice making, the back pressure in ammonia coils is 20 to 28 lb gage, corresponding to a temperature of 5 to 15 F.

Wet Compression vs. Dry Compression. The considerable superheat of ammonia vapor at the end of compression may be reduced or prevented by carrying liquid ammonia into the compressor. If sufficient liquid is admitted to keep the vapor always in a saturated condition, the operation is known as **wet compression.** If the ammonia gas is dry or superheated when admitted to the compressor and there is no liquid injected during the compression stroke, the operation is known as **dry compression.** Wet compression is now practically obsolete.

The **multiple-effect system** may be used in plants where there are both high- and low-temperature systems. In this, the higher-temperature gases are admitted to the cylinder at or near the end of the suction stroke and raise the ultimate suction pressure to that of the high-temperature system,

developed depends on the mean concentration $x = \frac{1}{2}(x_1 + x_2)$. According to the experiments of Hilde Mollier, the heat of solution q_s per lb of ammonia is given by the equation $q_s = 345(1 - x) - 400x^2$. For $x = 0.59$, q_s reduces to zero and for all higher values of x, the heat of solution is zero. See Table 18.

Table 18. Heat of Solution of Liquid Ammonia
(Btu given up per lb of ammonia dissolved)

Concentration[a]	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution
0	347.4	11	302.8	21	253.8	31	197.6	41	135.0	51	63.0
1	343.8	12	298.2	22	248.4	32	191.9	42	127.8	52	55.8
2	340.2	13	293.6	23	243.0	33	186.1	43	120.6	53	48.6
3	336.6	14	289.0	24	237.6	34	180.4	44	113.4	54	41.4
4	333.0	15	284.4	25	232.2	35	174.6	45	106.2	55	34.2
5	329.4	16	279.4	26	226.4	36	168.1	46	99.0	56	27.4
6	325.0	17	274.3	27	220.7	37	161.6	47	91.8	57	20.5
7	320.6	18	269.2	28	214.9	38	155.2	48	84.6	58	13.7
8	316.2	19	264.2	29	209.2	39	148.7	49	77.4	59	6.8
9	311.8	20	259.2	30	203.4	40	142.2	50	70.2	60	0.0
10	307.4										

[a] Average concentration; percent of ammonia by weight.

The **heat that must be taken from the absorber** is made up of three parts: (1) the heat that must be abstracted from the entering vapor to convert it into liquid at the temperature of the absorber; (2) the heat of solution q_s; (3) the heat that must be absorbed in reducing the temperature of the weak solution to that of the strong solution leaving the absorber. The first of these is the difference of total heat of the ammonia in the initial and final states; the second is given by the preceding formula; the third is the product of the weight of weak solution per pound of ammonia, the specific heat, and the temperature difference. The specific heat averages about 1.05. The **heat of absorption** Q_a is the sum of (1) and (2). Values of this quantity are given in Table 19.

For example, let the concentrations be 0.25 and 0.35, whence per pound of ammonia circulated, 6.5 lb of weak solution enters the absorber at, say, 100 deg, and 7.5 lb of strong solution leaves the absorber at, say, 80 deg F. The pressure in the absorber is 20 lb abs., and ammonia gas enters at a temperature of 5 deg. Under these conditions the three quantities of heat are as follows: (1) To condense the gas, $h_1 - h_3 = 486.2$ Btu; (2) heat of solution $q_s = 345(1 - 0.30) - 400 \times 0.30^2 = 205.5$ Btu; (3) to reduce temperature of 6.5 lb of weak solution from 100 to 80 deg, taking the specific heat as 1.05, $6.5 \times 1.05 \times (100 - 80) = 136.5$ Btu. Total heat removed from the absorber $486.2 + 205.5 + 136.5 = 828.2$ Btu.

In the **generator** the **processes** are the reverse of those in the absorber. Heat is supplied from an external source and is used for the following purposes: (1) to separate the liquid ammonia from the solution; (2) to vaporize the liquid and superheat the vapor; (3) to raise the temperature of the solution. In practice, additional heat must be supplied to vaporize some of the water. The calculation follows the method indicated for the absorber.

Heat Balance. For 1 lb of ammonia passing through the expansion valve, the following quantities of heat are absorbed or rejected at various points of the system:

Q_1 = heat imparted to the fluid in the generator.

Q_2 = heat absorbed by fluid in the brine cooler.

thereby increasing the capacity of the machine to that corresponding to the high temperature gas. This system is of value only where the condensing water is of high temperature (say about 90 F) and, with ammonia, where there are in service refrigeration systems at two widely different temperature levels (say 20 to 30 deg apart), and further where these systems are so relatively proportioned that they can be handled in one machine. Any increase in load at one temperature reduces the capacity of the compressor for the other. The existing installations are most used for ice making with high-temperature condensing water. Carbon dioxide machines for use in tropical countries have also been built on this principle for the purpose of improving the capacity and efficiency of the machine. Part of the carbon dioxide coming from the condenser is throttled to about 450 lb per sq in. and is used for cooling the rest of the carbon dioxide which is then expanded down to some lower pressure, say 250 lb per sq in. The vapors at both pressures are compressed in a multiple-effect compressor. Under tropical conditions, an increase of 130 percent in refrigeration with an increase of 54 percent in required hp has been obtained (*Engineering*, Aug. 23, 1918, p. 201).

With a low-temperature system, and also for ice making, the compression may be carried out in two stages with intercooling either (1) by **compound compression** or (2) by the use of a **booster.** The compression ratio must be at least 5 or 6 to justify two-stage compression, except where there are two different refrigerating temperatures. In that case, the low-pressure cylinder compresses the low-temperature vapor only and the high-pressure cylinder admits both the high-temperature vapor and the discharge from the low-pressure cylinder. A compound compressor has both cylinders on the same shaft and operating at the same rpm. A *booster* is a comparatively cheap compressor, usually reciprocating, of relatively high rpm, and compresses the low-pressure vapor through a range of 25 to 50 lb per sq. in. The speed may be variable, and consequently the pressure of the booster discharge can be varied as the load and operating conditions vary. A booster increases the capacity of the high-pressure cylinder in direct ratio to the density of the high-pressure suction. A gain up to a maximum of 20 percent in efficiency over single-stage machines has been realised.

Tests by Horne (*A.S.R.E.*, 1922) on compound compression with a suction pressure of 13.8 to 14.8 lb per sq in. abs have yielded the following results:

Test Results on Compound Compression

Ratio of discharge to suction pressure (abs)....	8	10	12	14	16
Theoretical compression, ihp per ton with simple compression..........................	1.26	1.47	1.65	1.79	1.95
Theoretical compression, ihp per ton with compound compression......................	1.10	1.28	1.40	1.48	1.55
Actual compression, ihp per ton with compound compression........................	1.20	1.40	1.54	1.63	1.72

Intercooling was carried out between stages and approximately equal amounts of work were done in the two cylinders.

For temperatures below -25 F, a split system is sometimes used in which the condenser of a carbon dioxide system is cooled by brine or direct expansion from a standard ammonia system.

The compression system is most commonly used in the production of refrigeration. Electric current at low rates favors the motor-driven compressor so that a major proportion of installations are of this type. Diesel-driven compressors are available and are used in some localities, particularly

Q_3 = heat rejected by fluid in the condenser.
Q_4 = heat withdrawn from fluid in absorber.
Q_5 = heat equivalent of work of pump
Q_6 = heat loss by radiation, etc.

The following equation expresses the **heat balance** for the system:

$$Q_1 + Q_2 + Q_5 = Q_3 + Q_4 + Q_6$$

The heat Q_2 absorbed in the brine coils, as in the compression system, is $Q_2 = h_{g1} - h_{f2}$, where h_{g1} is the total heat of the saturated vapor corre-

Table 19. Ammonia Absorption Machine—Heat Removed in Absorber

The tabulated quantities are Btu per pound of ammonia absorbed. The pressure in the absorber is that corresponding to the temperature and pressure of the strong solution. The ammonia gas is assumed to be dry and saturated as it enters the absorber.

Concentration (per cent) of weak solution, x_2	Temperature of weak solution, deg F	Concentration (percent) x_1 and temperature (deg F) of strong solution											
		25			30			35			40		
		60°	80°	100°	60°	80°	100°	60°	80°	100°	60°	80°	100°
	80	853											
	100	954	838										
10	120	1056	939	821									
	140	....	1041	924									
	160	....	....	1026									
	80	892	....	....	828								
	100	1046	877	....	923	812							
15	120	1200	1032	862	1019	908	795						
	140	....	1186	1016	....	1004	891						
	160	....	....	1173	....	....	989						
	80	1035	....	....	863	....	....	799					
	100	1344	1021	....	1007	848	....	889	783				
20	120	1656	1332	1006	1153	993	831	979	873	766			
	140	....	1644	1318	....	1139	976	....	963	856			
	160	....	....	1636	....	....	1125	....	....	948			
	80	....	....	....	995	....	....	830	....	....	769		
	100	....	....	....	1286	980	....	966	814	....	852	752	
25	120	....	....	....	1579	1274	965	1102	950	798	936	836	735
	140	....	....	....	....	1568	1259	....	1087	934	....	920	819
	160	....	....	....	....	....	1559	....	....	1074	....	....	905
	80	....	....	....	....	....	....	952	....	....	796		
	100	....	....	....	....	....	....	1224	937	....	922	779	
30	120	....	....	....	....	....	....	1499	1211	921	1047	906	762
	140	....	....	....	....	....	....	....	1487	1197	....	1033	890
	160	....	....	....	....	....	....	....	....	1480	....	....	1020
	80	....	....	....	....	....	....	....	....	....	907		
	100	....	....	....	....	....	....	....	....	....	1161	891	
35	120	....	....	....	....	....	....	....	....	....	1416	1147	876
	140	....	....	....	....	....	....	....	....	....	....	1404	1132
	160	....	....	....	....	....	....	....	....	....	....	....	1397

sponding to the temperature of the fluid in the brine coil, and h_{f2} is the total heat of the liquid corresponding to the temperature of the liquid entering the expansion valve.

If it is assumed that the **fluid** enters the condenser in the dry saturated state, the heat Q_3 is simply the latent heat r corresponding to the temperature of the fluid in the condenser.

The weight G of strong solution (concentration x_1) circulated per lb of ammonia liberated is given by the equation

$$G = (1 - x_2)/(x_1 - x_2)$$

where x_2 is the weak liquor concentration.

in the Southwestern portions of the United States, but are not generally considered where central-station current is available at reasonable rates. The compression machine has displaced the absorption machine (except in isolated cases and more especially on low-temperature refrigeration) in consequence of its higher efficiency and better control of the operating cycle. The development of two-stage high-speed intercooled compressors has increased the advantage of the compression over the absorption machines.

Unit machines have been standardized and are available in capacities from 100 lb to 100 tons. The unit machine includes motor-driven compressor, condenser, controls, gages, and valves, arranged in a compact assembly at the factory, available from stock, ready for installation by connecting to the coils or devices to be cooled.

Electric Drives. Where **direct current** is available, capacity and temperature control is readily obtained by varying the speed of the motor, either manually or automatically, by means of a variable resistance in the shunt field. With **alternating current**, capacity and temperature control is obtained (1) by *intermittent operation*; (2) by *adjustable clearance pockets* in the compressor cylinder or heads; (3) by the use of an unloading device by means of which the compressor valves are held off their seats; and (4) by the use of motors wound for two or more speeds. The common practice is to use motors of repulsion induction-start type for the small machines, slip ring or wound secondary type for intermediate capacities, and self-starting synchronous motors for intermediate and large capacities. A **flywheel type motor** mounted directly on the compressor frame gives the necessary flywheel effect in the rotor.

Horsepower of Engine Driving Compressor. The indicated horsepower of the engine available should be about 20 percent greater than the indicated compressor horsepower for compressors up to 20 tons capacity, 15 percent greater from 20 to 100 tons, and 10 percent from 100 tons to 500 tons. For motor-driven compressors, the motor should have at least 25 percent greater power than the indicated compressor horsepower, and on small machines of 10 tons and under from 35 to 50 percent greater to supply sufficient starting torque, especially in automatic machines. This allowance should be made upon the horsepower of the gas compression based upon the highest probable condenser and suction pressure under which it may be necessary to operate.

The enclosed type of compressor, of the single-acting vertical design with 2, 3, or 4 cylinders, is commonly used, but the V type is also available with 4, 6, 8, and up to 16 cylinders. Piston diameters range from 1½ to 12 in. The stroke is short, frequently two-thirds the piston diameter. Valves are usually of the ring-plate type, made of chrome-vanadium steel, hardened and ground to a true surface; the lightweight spring-balanced cushioned poppet valve is still used. Lubrication is forced, and the oil pump is equipped with filters. Methyl chloride and Freon 12 as well as dieline are miscible in the usual lubricating oils, necessitating the use of a special oil trap. The pistons are supplied with oil rings as well as the usual compression rings.

The halide group of refrigerants (especially F-12) as well as methyl chloride operate well with air-cooled compressors with fins on the cylinders and permit a lighter construction. The water jackets on the compressor of a dry-compression ammonia machine change the compression curves only slightly. In single-acting machines, the compression line will usually fall slightly under the adiabatic, making a card about 4 percent smaller than the adiabatic. The control of the superheat is not attempted, the jacket water being used mainly to keep down the temperature of the lubricating oil and to increase the life of the gaskets. Jackets are sometimes

The pump forces G lb of rich liquor from the pressure p_1 in the absorber to the pressure p_2 in the generator. Hence if v is the volume of 1 lb of the rich liquor, $Q_5 = Gv(p_2 - p_1)/778$.

The heat Q_1 and the heat Q_4 may be calculated approximately by the method already indicated.

Tests by B. H. Jennings (*Refrig. Eng.*, Aug., 1935) of an absorption machine with 180 lb gage condenser and 25 lb gage absorber pressures show 35 lb of steam at 10 lb gage pressure used per ton of refrigeration per hour. The water requirements were 9.45 gpm per ton of refrigeration with 10 F rise of water temperature and 4.73 gpm with 20 F rise of temperature.

In the **intermittent absorption machine,** heat is applied to strong aqua in the generator and ammonia vapor is driven off and condensed. The liquid ammonia is stored in a receiver or evaporator until an automatic valve shuts off the heat. The generator is then cooled by water or air and then acts as an absorber for the vapor evaporated in the cooling coils. The temperature of evaporation will vary with the pressure existing in the absorber. There is no pump for the aqua, and one vessel is alternately absorber and generator.

Recent developments in Europe have produced two-stage absorption machines for very low temperatures with high efficiency. Another development is the resorption machine for moderate refrigeration with high efficiency.

In the **two-stage system,** the low stage absorbs suction gas from the evaporator at a temperature that may be as low as about −95 F (saturation pressure 1.52 lb per sq in. abs). The low-stage generator liberates the ammonia at about atmospheric pressure (−28 F). This vapor is now absorbed in the high-stage absorber from which point the procedure is that usual in the normal absorption process.

In the **resorption process,** the vapor from the generator is reabsorbed in weak liquor circulated from the evaporator and meeting this vapor in the resorber. From the resorber, the strong liquor goes to the evaporator which, as usual, sends its ammonia vapor to the absorber and returns its weak liquor to the resorber. This arrangement is suitable only for relatively high-temperature refrigeration but is claimed to be more efficient than the conventional system.

For further details and a list of references see A.S.R.E. Refrigerating *Data Book, 1939–1940, p. 47.*

There is no positive action of the absorption machine, and the capacity and efficiency depend to a large extent upon the skill and watchfulness of the operator. An absorption machine, under best conditions of design and operation, using low gas pressure, with a temperature in the refrigerator of not more than 0 F, and a capacity of 50 to 100 tons, gives refrigerating results practically equal to those of a steam-driven compression machine. For capacities exceeding 100 tons the compression machine is not only more positive but becomes more efficient. The absorption machine gives best comparative results as the suction pressures are reduced, the compression machine as they are raised. Absorption machines have been confined to comparatively small units (up to about 225 tons), while compression machines are now in successful operation with capacities as high as 1,000 tons.

Electrolux Servel Absorption Process. (Platen-Munters Patent.) The Electrolux Servel process is an ammonia-absorption process which eliminates the use of pumps or other moving parts. The gas pressure is uniform throughout the whole system and the difference in vapor pressure of ammonia in the condenser and in the evaporator is compensated by the

omitted on compound compressors with intercooling. The heat abstracted by the jacket water is seldom more than 2 or 3 percent of the indicated work.

Little attention is paid to keeping cylinder clearance low; it functions as a compressed spring and has little effect other than to reduce the pumping capacity.

Valves. The velocity through the suction valves should not exceed 4,000 fpm and through the discharge valves 10,000 fpm. With plate or feather valves, suction-valve velocities are kept under 2,500 and discharge valve under 6,000 fpm.

Connections. Suction and discharge connections to the cylinder are usually based on a velocity of not over 4,000 fpm, although the discharge pipe line to the condenser may be based on a velocity of 8,000 to 10,000 fpm, depending upon the length of line, fittings, etc. With receivers located close to the compressor inlet, the suction pipe velocities may be increased considerably.

Centrifugal Compressors. For refrigerants having liquefaction pressures of one atmosphere or less, the centrifugal compressor, with its capacity for handling large volumes, is preferable to the reciprocating compressor. In multistage compressors, there is also the advantage of interstage cooling. This type of compressor has been applied to water vapor, Carrene, dieline, Freon 12, and F-11.

Performance of Ammonia Compression Machines. Table 11 shows the effect on capacity and horsepower of varying the condenser and refrigerator temperatures.

Table 11. Variation in Capacity and Horsepower Required in Ammonia Compression Machines with Variation in Operating Conditions

(The values in this table are the ratios of the capacity and horsepower under the stated operating conditions as compared with capacity and horsepower when operating between 0 and 95.5 F.—York Mfg. Co.)

Condenser pressure, lb per sq in., gage (and corresponding temperature, deg F)	Suction gage pressure, lb per sq in. (and corresponding temp, deg F)											
	5 (−17.5°)		10 (−8.5°)		15.67 (0°)		20 (5.7°)		25 (11.5°)		30 (16.8°)	
	Tons	Hp	Tons	Hp	Tons	Hp	Tons	Hp	Tons	Hp	Tons	Hp
145 (82°)...	0.665	0.738	0.855	0.803	1.070	0.857	1.240	0.885	1.437	0.908	1.632	0.924
165 (89°)...	0.642	0.790	0.828	0.866	1.035	0.930	1.200	0.966	1.392	1.000	1.580	1.021
185 (95.5°)..	0.620	0.836	0.800	0.922	1.000	1.000	1.163	1.041	1.347	1.082	1.532	1.115
205 (101.4°).	0.600	0.882	0.775	0.977	0.972	1.063	1.125	1.110	1.307	1.160	1.485	1.256

Household refrigerating machines are designed for continuous automatic operation and for conservation of the charges of refrigerant and oil. These units are almost universally motor-driven compressors, the principal exception being the Electrolux-Servel absorption unit (see p. 1873). The compressor may be hermetically sealed, with the compressor and motor enclosed in the same casing, or may use a shaft seal, embodying some form of sylphon bellows, with an outside coil spring at the place where the crankshaft passes out of the casing. The compressor is generally reciprocating, but rotary types are being increasingly used either with a floating piston ring driven by an eccentric or with sliding blades. Lubrication is by internal forced-feed circulation in most cases. The condenser may be of the radiator, coil, or plate type, cooled by natural draft, by a fan on the main motor, or by a separate motor. The refrigerant is most commonly sulphur dioxide or Freon 12. Refrigerant feed control to the evaporator may be (1) float feed

presence of hydrogen in such quantity that the sum of the partial pressures of the hydrogen and of the ammonia vapor in the evaporator is equal to the ammonia pressure in the condenser. The general arrangement is shown in Fig. 3. Strong ammonia liquor (ammonia dissolved in water) in the lower portion, *L*, of the generator, *G*, is heated by a gas flame and the resulting slugs of ammonia vapor and ammonia liquor rise in the external tube *I* and discharge in the upper portion of the generator and are separated there. The ammonia vapor, carrying with it a small amount of water vapor, passes through the pipe *P*, to the rectifier *R*, where much of the water vapor is condensed and returns through *P* as a strong liquor to the generator. The ammonia vapor passes to the condenser *C* and returns as a liquid to the lower portion of the rectifier chamber. It then flows through the gas heat-exchanger *D*, where it is cooled, to the evaporator *E*. In the evaporator it meets

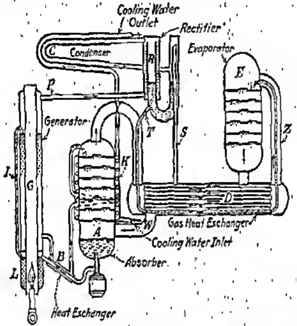

Fig. 3.—Electrolux Servel Absorption Process.

hydrogen and evaporates at a comparatively low partial pressure, and consequently at a low temperature. The evaporated ammonia, together with the hydrogen into which it has diffused, pass through the gas heat exchanger *D* and through the pipe *W* to the absorber *A*. In the absorber it meets weak liquor coming from the lower portion of the upper chamber of the generator, the weak liquor having been cooled on its way to the absorber by passing through the liquid heat-exchanger *B*. The resulting strong liquor formed in the absorber passes through the liquid heat-exchanger to the lower chamber of the generator. The hydrogen, which accompanied the low temperature ammonia vapor to the absorber, passes out by the pipe *Y* from the upper portion of the absorber and returns to the evaporator through the gas heat-exchanger *D*, being cooled (in passage) by the cold vapors passing from the evaporator to the absorber. A pipe *S* permits the return to the evaporator of any hydrogen which may have been carried in solution by the strong liquor in the absorber and liberated in the generator.

Tests made at the National Physical Laboratory (England) have shown, for a small unit, a ratio of refrigerating effect to heat supply of about 38 percent with 45 F cooling water and about 30 percent with 63 F cooling water.

(flooded system), (2) pressure-actuated diaphragm valve, or (3) by fixed orifice. The fixed orifice may be either a plate orifice or a capillary tube. Rotative speeds of the compressor vary from 300 to 600 rpm. The compressor is air-cooled except for the hermetically sealed unit (which is air-cooled indirectly) and has fins on the cylinder casing. The evaporator is usually made of stainless steel, tin-plated brass, or tin-plated copper. Controls are generally pressure-operated thermostatic switches. The power consumption of an average box (say 6 cu ft) is about 20 kw-hr per month. The heat gain of a refrigerator cabinet of the same size is about 3.3 Btu per hr per deg F.

Table 12. Test Results on Ammonia Machines at Quincy Market Cold Storage and Warehouse Co.[a]

Machine	Capacity of machine, tons	Capacity during test, tons	Suction temperature, deg F	Discharge pressure, lb gage	Steam conditions: Pressure, lb per sq in.	Steam conditions: Superheat, deg F	Steam conditions: Vacuum, in. of mercury	Compressor ihp per ton of refrigeration	Steam per ton, lb
1	1,000	750	+10	115	150	100	28	0 9	228
2	400	400	−10	115	150	100	28	1.25	390
2	400	400	+10	115	150	100	28	0.9	282
3	500	288	−27	139	140	125	28	1.34	426
4	500	370	−20	130	...	...	..	1.64	
5	150	150	−10	115	125	125	28	1.28	430
5	150	150	+10	115	125	125	28	0.92	376
6	100	100	−10	115	125	125	28	1.32	459
6	100	100	+10	115	125	125	28	0.94	389
7	225	225	−10	115	...	...	..	...	342.2
7	225	225	+10	115	...	...	..	...	316.8

1 and 2, cross-compound Corliss engines; 3, two-stage feather-valve compressor driven by uniflow engine; 4, two-stage electrically driven compressor; 5 and 6, tandem compound engines; 7, absorption machine.

[a] The discharge pressures in these tests are exceptionally favorable and result in low power consumptions.

The absorption machine in the preceding table was more economical than the 400 ton compression machine with the low-temperature suction gas and less economical than the same machine with high-temperature suction gas.

Station operation results show that absorption machines require 25 to 35 lb of steam per ton of refrigeration with 0 deg brine and 150 lb condenser pressure. Makers guarantee [illegible] to [illegible] lb of live steam per hour per ton of ice.

Condensers

(For heat transfer rates, see *Eng. Exp. Sta., Univ. Illinois*, Bull. 171, 186)

For unit types (F-12 and methyl chloride), pipe coil condensers are generally used. In central plant installations, particularly with ammonia, shell and tube condensers are most common.

Air-cooled condensers are common with low-pressure refrigerants, with and without fan circulation, for units up to 50 tons. Water-cooled condensers are used on larger plants and with ammonia or carbon dioxide.

The atmospheric condenser, in use on some ammonia machines, is usually a vertical return-bend coil of 2 in. pipe, 20 ft long over-all, usually 12 pipes high. Water flows downward over the pipes through a distributing device on top of each coil; gas usually enters at the bottom, flows upward through two or more lengths to remove superheat, then passes to the top and flows downward to the remainder of the coils. With 70 F condenser water maintaining 155 to 160 lb condenser pressure, a common rating is 1 ton of refrigeration for each 20 ft pipe, including return bend.

Advantages and Disadvantages of Absorption System. The absorption system would be favorably indicated where exhaust steam is available at about atmospheric pressure. The repair costs may be heavy when over driven and when using a corrosive water in the absorber and rectifier. Slight quantities of oil have a very bad effect upon its efficiency. Faulty rectification, due either to faulty design or operation, is one of the greatest sources of loss; this defect may reduce seriously the capacity of the machine. The best systems of rectification consist of an efficient analyzer, a mechanical separator, a cooling coil, and preferably a second separator with liquid drip after the cooling coil for the purpose of testing the results of rectification. At the full capacity of the machine, 1 to 5 percent of the water vapor is usually carried through the rectifier to the condenser and cooler. The analyzer, first separator, and cooling coils discharge their drips as returns to the generator; the drip from the final separator gives an indication of the condition of the rectified gas, which should be used as a basis for the regulation of the machine. Careful operation will make purging of the cooler necessary only once in three or four weeks.

Surfaces Required. The surface area of the generator coils varies with the steam pressure, approximate values for 60 deg F condensing water and 0 deg brine being 14 sq ft per ton for 5 lb (gage) steam pressure, 10 sq ft for 20 lb, and 6 sq ft for 50 lb. The rectifier surface varies from 1.5 to 4 sq ft per ton, the exchanger surface is about 6 sq ft per ton, the brine-cooler surface averages about 15 sq ft per ton.

Trouble is frequently found from the formation of permanent gases; this will occur when air is present in the system. The addition of 0.2 percent of sodium bichromate, based on the total weight of the aqua charge, eliminates this trouble (McKelvy and Isaacs, *Jour, A.S.R.E.*, Mar., 1918).

Adsorption. Certain inert solid materials, such as activated charcoal, the chlorides of calcium, barium, and strontium, ferric hydroxide, activated alumina, silica gel and other gels, have the ability to condense certain vapors by adsorption. An adsorbent for refrigeration purposes must be a good conductor of heat and be able to adsorb a large weight of the refrigerant per unit weight of the adsorbent material. Silica gel can be used with sulphur dioxide but not with ammonia; as it is a poor conductor of heat, the beds of the gel must not be thick. During adsorption, the liquefaction heat has to be removed; in commercial work, this is done by means of a water-cooled pipe surface.

In an adsorption process, the vapor adsorbed (SO_2, H_2O, etc.) passes from the vapor to the liquid phase directly without going into solution as in the absorption machine. This makes the adsorption machine simpler in design and operation.

Silica-gel Adsorption System of Refrigeration. Silica gel is a hard glassy material with the appearance of clear quartz sand. It is pure silicon dioxide and is chemically inert toward practically all substances and particularly all refrigerating mediums commonly used, with the exception of ammonia. It is made by treating sodium silicate with an acid to form a colloidal solution which sets as a gelatinous mass, and this, when washed, purified, and dried, becomes a silica sponge which is granulated to a size of 8 to 20 mesh. It has the property of adsorbing large quantities of vapors or liquids. While adsorbing, the heat of vaporization is liberated and must be carried away by air or other cooling agent if the temperature is to be kept down. The adsorbed substance is given up on activating the gel by heat.

The atmospheric drip condenser is of the same general construction as the atmospheric condenser, with the exception that each return bend has a trap drip connection through which the condensate is drained from each pipe. Commercial ratings in the drip type are usually approximately double that of the plain atmospheric type.

Combined with a cooling tower or water sprays, the atmospheric condenser becomes an evaporative condenser and is in general use for comfort cooling in cities where conservation of water supply is necessary.

Double-pipe condensers are usually constructed of 1¼ in. inner and 2 in. outer pipe, condensing water flowing through the inner pipe. In this type, ammonia gas enters at the top of the coil, liquid ammonia draining at the bottom; water enters at the bottom flowing countercurrent and out at the top.

Table 13. Heat Transmission through 2 × 3 in. Double-pipe Ammonia Condensers and Brine Coolers

Velocity of fluid in pipe, fpm		100	200	300	400	500	600	700
Btu per sq ft per deg F per hr	Condenser	150	235	290	340			
	Brine cooler	95	130	160	180	190	205	215

Table 14. Effect of Varying the Amount and Velocity of Condensing Water in a 1¼ × 2 in. Double-pipe Ammonia Condenser

Velocity of condensing water, fpm	Constant head pressure (185 lb per sq in., gage)		Constant capacity		
	Relative capacity, tons per 24 hr	Condensing water, gpm per ton of refrigeration	Head pressure, lb per sq in., gage	Horsepower per ton of refrigeration	Condensing water, gpm per ton refrigeration
100	0.67	1.160	225	2.04	0.78
150	1.00	1.165	185	1.71	1.17
200	1.34	1.165	165	1.54	1.55
250	1.64	1.180	155	1.46	1.94
300	1.88	1.240	148	1.40	2.33
400	2.40	1.300	140	1.33	3.11

Table 13 shows the rate of heat exchange with varying velocities of fluid for double-pipe (2 and 3 in.) ammonia condensers and brine coolers. Table 14 gives results of tests on a 1¼ × 2 in. double-pipe ammonia condenser, showing the effects of varying the amount and velocity of the condensing water at 70 F; gage suction pressure, 15.66 lb per sq in.

Shell and coil condensers have a cylindrical shell, usually of steel, inside of which are placed a number of helical coils with tails extending through stuffing boxes in either of the connecting headers. Ammonia gas enters the shell at the top; water enters at the bottom with countercurrent flow. The customary allowance is 12 to 16 sq ft of coil surface per ton, subject to the velocity and temperature of the water available.

Shell and tube condensers have a cylindrical steel shell filled with straight tubes expanded into tube sheets and are used both vertically and horizontally; size of tubes ranges from 1 to 2½ in., length of tubes up to 20 ft. Vertical condensers are generally open with the water flowing into an open water box on the top head, down through the tubes, and then through an open bottom head to a tank, or waste way. There is usually a device set into the tubes on the top head to distribute the water in a thin film on the surface of the tubes, sometimes with distributors to give it a spiral flow. Some vertical and all horizontal types have closed heads with water under pressure filling the tubes and with multiple passes; average velocities used

The internal volume of the gel is approximately 50 percent of its total volume, and in refrigerating practice, under normal conditions of operation, it will adsorb 25 to 35 percent of its own weight of SO_2.

Its use in commercial service (up to the present time) has been mainly freight-car refrigeration. The refrigerating system as applied to this service, consists of adsorbers, condenser, and evaporator with float valve feed to evaporator; the heat for activating the adsorbers is obtained from gas burners supplied from tanks of propane.

The silica gel is contained in ¾ in. steel tubes welded into headers. One adsorber carries 1,000 lb of silica gel, giving an ice-melting effect of 1 to 1½ lb per 24 hr per lb of silica gel, depending upon the evaporator and adsorber temperatures, with a possible 4 lb per pound of gel with forced circulation. The fuel consumption is approximately 135 lb of propane per ton of refrigeration.

The cycle of operation is as follows: the liquid refrigerant, coming from the condenser passes through an expansion valve, controlled by a float, into the

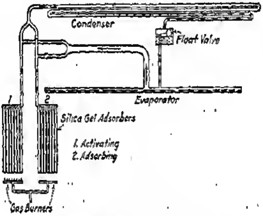

Fig. 4.—Silica-gel Refrigeration System.

evaporator (of the flooded type) and is there vaporized, absorbing heat from the chamber to be refrigerated. The vapor is then adsorbed in the pores of the silica gel. When the gel is approximately saturated, heat is applied to a battery of tubes carrying the gel and the refrigerant is driven out of the gel, as a vapor, to the condenser, where it is liquefied and returned to the evaporator. After the refrigerant content of the gel has been reduced to a certain limit, the heat is turned off and the gel is allowed to cool to a temperature at which it will again readsorb the vaporized refrigerant from the evaporator.

Multiple adsorbers will produce constant refrigeration, one or more acting as an adsorber while the other is being reactivated by evaporation to the condenser. The control of the fuel gas burners may be based upon a fixed timing or (by means of a thermostat) upon the temperature of the silica gel. The medium used is sulphur dioxide, but other fluids are possible. The control, as applied to refrigerator cars, is automatic, on a time basis, subject to the car thermostat, and consists mainly of control of the gas valve to the burners, with a thermostatic safety element.

The **Ice-O-Lator** of Prof. F. G. Keyes uses ammonia as the refrigerating fluid and employs, as a solid adsorbent, a calcium chloride sponge treated with other substances to increase its adsorbing power. Automatic control

are 3 to 6 fps. Commercial ratings vary from 8 to 15 sq ft effective surface per ton. This type of condenser has largely superseded other types in ammonia installations of sizes from 1 ton up.

For heat-transfer coefficients see Table 22.

Table 15. Capacities of 20 Ft Long Double-pipe Ammonia Condensers, 1¼ and 2 in. Pipe

(Tons refrigeration)

Pipes high	Gpm per coil	Water, ft head	Initial temperature of condensing water, deg F													
			60		65		70		75		80		85		90	
			Condensing pressure, lb per sq in. gage													
			145	165	165	195	175	205	185	215	185	225	185	215	205	235
4	5	1	2.0	3.0	2.0	2.5	1.5	2.5	1.5	2.0	1.3	2.0	1.0	1.3	1.0	1.5
	10	3	2.5	5.0	3.0	4.0	2.5	3.5	2.5	3.0	2.0	3.0	1.5	2.3	1.5	2.0
	20	9	4.5	7.0	5.0	7.0	4.5	6.0	4.0	6.0	3.0	5.5	2.0	4.0	2.5	4.0
6	15	9	5.4	8.4	6.0	8.4	5.4	7.8	4.8	7.2	3.6	6.6	2.4	4.8	3.0	4.2
	25	22	8.2	13	9.0	13	8.2	12	7.5	10.5	6.0	9.7	3.8	6.8	4.5	6.7
	35	43	10	16	11	15	10	14	9.8	14	7.0	12	5.0	9.1	5.6	9.1
8	15	11	7.8	12	8.5	12	8.0	11	6.6	10	5.2	9.1	3.8	6.5	3.9	6.5
	25	30	12	19	12	18	12	16	10	15	7.8	14	5.2	10	5.8	9.1
	35	58	14	22	14	20	14	19	12	18	9.8	16	6.5	15	7.0	12
10	15	14	9	13	9	13	9	12	8	11	6.0	10	4.0	7.3	4.5	8
	25	38	13	22	13	20	14	19	12	18	8.8	16	6.2	11	6.5	12
	35	73	15	24	16	23	15	22	14	20	11	18	7.3	13	8.0	16
12	15	17	9	15	9.7	15	9	13	9	12	6.5	12	5.0	8.5	.4	8
	25	45	14	23	16	22	14	20	13	19	9.3	17	6.6	12	.7	12
	35	88	17	29	19	27	16	25	16	23	12	21	9	14	.8	16

Ammonia Piping and Fittings

Three general types of fittings are now furnished by a number of manufacturers: (1) the **gland end**, in which the fitting is tapped with the regular pipe thread, outside of which is a recess for a rubber washer, which is pressed into the recess and against the pipe at the joint by a gland which slips over the pipe (now practically obsolete); (2) the **flanged end**, which usually makes a tongue-and-groove joint with its companion flange; (3) the **threaded end fitting**, which may or may not have a recess at the outer end for soldering. The defects of the soldering method are (*a*) when used on lines larger than 3 in., the vibration, expansion, and contraction ultimately break the soldered joint, allowing the fitting to leak; (*b*) when used on the discharge gas line of a compressor operating at a pressure of 175 to 200 lb, the temperature of the gas, which often reaches 300 F or more, brings the solder to a plastic state where it is of little or no value for maintaining a tight joint. Welding is coming into extensive use, wherever practicable, for all piping and coils, eliminating fittings, reducing costs, and maintaining tightness.

Absorption System of Refrigeration

The essential organs of a vapor absorption system are:

(1) The **generator** or still, in which ammonia is driven off from a solution of ammonia in water by a steam coil. The generator takes the place of the compressor in the compression system. (2) The **condenser**, in which the ammonia gas is condensed. (3) The **expansion valve**. (4) The **brine coil**, in which the ammonia, by vaporizing, absorbs heat from the brine. (5) The **absorber**, in which the ammonia returning from the brine coil is absorbed

cuts off the heating gas supply after a definite amount of ammonia has been driven off from the adsorbent and liquefied, and turns on a water valve for cooling the generator. Distillation takes 45 min, adsorption 2 to 6 hr. The general method of operation is similar to that of the silica-gel process described above.

Methods of Applying Refrigeration

Refrigeration is carried out either by direct expansion or by the use of brine. The direct-expansion system of cooling is regular practice with Freon, methyl chloride, and carbon dioxide installations and with ice-making plants using ammonia. With the **flash method of direct expansion**, the liquid, at approximately the condenser pressure, is fed through an expansion valve directly into the piping which is to be used for cooling. The liquid spray from the expansion valve passes through the coils at high velocity, maintaining a wetted inner surface, and is vaporized by the absorption of heat through the pipe walls. With the **flooded method** (used especially in ice making), the coils are filled with liquid and communicate with a drum placed above them in which a constant liquid level is maintained. The vapor formed in the coils goes to the drum.

In the **brine system**, the direct-expansion system is used for cooling brine and the cooled brine is pumped through pipe lines to the point where the cooling is to be done, the heat being absorbed by the brine and the brine returned to be again cooled. The brine-cooling vessel is usually either a double or triple pipe coil or of the shell-and-tube type.

Brine Coolers

There are three types of coolers in general use, the submerged coil, double pipe, and shell and tube.

The **submerged coil** in a brine tank is used little except in ice manufacture. The **double-pipe cooler** is usually of 2 in. inner or brine flow pipe

Table 20. Capacities of Double-pipe Brine Coolers, 20 Ft Long, 2 and 3 In. Pipes

(Tons refrigeration)

Pipes high	Gal per ton per min, 1.25 sp gr brine												
	12.5		7.5		3.75			2.5			2		
	Mean temperature difference, deg F												
	5	10	7.5	12.5	10	15	20	12.5	17.5	22.5	15	20	25
2	0.12	0.88	0.16	0.53	0.19	0.32	0.53	0.27	0.33	0.53	0.25	0.35	0.50
4	0.88	4.5	1.0	4.2	0.53	2.0	4.8	0.67	1.3	3.3	0.50	1.0	2.0
6	2.4	8.8	3.1	8.9	1.9	6.1	12	1.8	5.3	18	1.5	3.5	7.0
8	4.5	…	6.0	15	4.5	12	20	4.4	12	20	3.5	8.5	15
10	6.6	…	9.1	…	7.7	18	27	8.0	19	30	7.0	15	24
12	…	…	13	…	12	25	…	13	27	42	12	23	34
14	…	…	…	…	15	…	…	20	34	…	15	29	
16	…	…	…	…	18	…	…	25	…	…	18	34	
18	…	…	…	…	…	…	…	30	…	…	23		
20	…	…	…	…	…	…	…	…	…	…	28		

Figure brine leaving cooler at least 5 deg higher than temperature of ammonia. Add 20 percent to above capacities if coolers are flooded. Add 10 percent to above capacities if coolers are submerged in brine.

86. $\int \cosh x\,dx = \sinh x + C$ 87. $\int \coth x\,dx = \log_e \sinh x + C$

88. $\int \operatorname{sech} x\,dx = 2\tan^{-1}(e^x) + C$ 89. $\int \operatorname{csch} x\,dx = \log_e \tanh (x/2) + C$

90. $\int \sinh^2 x\,dx = \frac{1}{2}\sinh x \cosh x - \frac{1}{2}x + C$

91. $\int \cosh^2 x\,dx = \frac{1}{2}\sinh x \cosh x + \frac{1}{2}x + C$

92. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$ 93. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

DEFINITE INTEGRALS

The definite integral of $f(x)dx$ from $x = a$ to $x = b$, denoted by $\int_a^b f(x)dx$, is the limit (as n increases indefinitely) of a sum of n terms:

$$\int_a^b f(x)dx = \lim_{n=\infty} [f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \ldots + f(x_n)\Delta x],$$

built up as follows: Divide the interval from a to b into n equal parts, and call each part Δx, $= (b - a)/n$; in each of these intervals take a value of x (say $x_1, x_2, \ldots x_n$), find the value of the function $f(x)$ at each of these points, and multiply it by Δx, the width of the interval; then take the limit of the sum of the terms thus formed, when the number of terms increases indefinitely, while each individual term approaches zero.

Geometrically, $\int_a^b f(x)dx$ is the area bounded by the curve $y = f(x)$, the x-axis, and the ordinates $x = a$ and $x = b$ (Fig. 8); that is, briefly, the "area under the curve, from a to b." The **fundamental theorem** for the evaluation of a definite integral is the following:

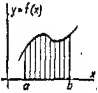

FIG. 8.

$$\int_a^b f(x)dx = \left[\int f(x)dx\right]_{x=b} - \left[\int f(x)dx\right]_{x=a};$$

that is, the definite integral is equal to the difference between two values of any one of the indefinite integrals of the function in question. In other words, the limit of a sum can be found whenever the function can be integrated.

Properties of Definite Integrals.

$$\int_a^b = -\int_b^a; \quad \int_a^c + \int_c^b = \int_a^b.$$

THE MEAN-VALUE THEOREM FOR INTEGRALS.

$$\int_a^b F(x)\, f(x)dx = F(X)\int_a^b f(x)dx,$$

provided $f(x)$ does not change sign from $x = a$ to $x = b$; here X is some (unknown) value of x intermediate between a and b.

THEOREM ON CHANGE OF VARIABLE. In evaluating $\int_{x=a}^{x=b} f(x)dx$, $f(x)dx$ may be replaced by its value in terms of a new variable t and dt, and $x = a$ and $x = b$ by the corresponding values of t, provided that throughout the interval the relation between x and t is a one-to-one correspondence (that is, to each value of x there corresponds one and only one value of t, and to each value of t there corresponds one and only one value of x).

DIFFERENTIATION WITH RESPECT TO THE UPPER LIMIT. If b is variable, then $\int^b f(x)dx$ is a function of b, whose derivative is

$$\frac{d}{db}\int_a^b f(x)dx = f(b).$$

DIFFERENTIATION WITH RESPECT TO A PARAMETER.

$$\frac{\partial}{\partial c}\int_a^b f(x,c)dx = \int_a^b \frac{\partial f(x,c)}{\partial c}dx.$$

Functions Defined by Definite Integrals. The following definite integrals have received special names, and their values have been tabulated; see, for example, B. O. Peirce's "Table of Integrals."

1. Elliptic integral of the first kind $= F(u, k) = \int_0^u \frac{dx}{\sqrt{1 - k^2 \sin^2 x}} (k^2 < 1)$

2. Elliptic integral of the second kind $= E(u, k) = \int_0^u \sqrt{1 - k^2 \sin^2 x}\, dx$ $(k^2 < 1)$

3, 4. Complete elliptic integrals of the first and second kinds; put $u = \pi/2$ in (1) and (2).

5. The Probability integral $= \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx$

6. The Gamma function $= \Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$

Approximate Methods of Integration. Mechanical Quadrature.

1. Use Simpson's rule. See p. 106, or, for greater accuracy, Weddle's Rule (see Scarborough, "Numerical Mathematical Analyses," p. 120, Johns Hopkins Press).

2. Expand the function in a converging power series, and integrate term by term.

3. Plot the area under the curve $y = f(x)$ from $x = a$ to $x = b$ on squared paper, and measure this area roughly by "counting squares," or more accurately, by the use of a planimeter, or by graphical means (see Fry, "Differential Equations," p. 69, Van Nostrand).

(4) Coradi's Mechanical Integraph ($240) provides a means of drawing on paper the curve $y = \int f(x)dx$, when the curve $y = f(x)$ is given, and can be used to facilitate the solution of certain differential equations. Full instructions for use with each instrument.

Double Integrals. The notation $\int\int f(x, y)dy\, dx$ means $\int \{\int f(x, y)dy\}dx$, the limits of integration in the inner, or first, integral being functions of x (or constants).

EXAMPLE. To find the weight of a plane area whose density, w, is variable, say $w = f(x, y)$. The weight of a typical element, $dx\, dy$, is $f(x, y)dx\, dy$. Keeping x and dx constant, and summing these elements from, say, $y = F_1(x)$ to $y = F_2(x)$, as determined by the shape of the boundary, the weight of a typical strip perpendicular to the x-axis is $dx\int_{y=F_1(x)}^{y=F_2(x)} f(x,y)dy$. Finally, summing these strips from, say, $x = a$ to $x = b$, the weight of the whole area is $\int_{x=a}^{x=b}\{dx\int_{y=F_1(x)}^{y=F_2(x)} f(x, y)dy\}$, or, briefly, $\int\int f(x, y)dy\, dx$.

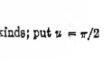

FIG. 9.

Table 21. Capacity of Multipass Shell-and-tube Brine Coolers, Flooded

(Tons refrigeration)

Diameter of shell, in.	Length of shell, ft	Velocity of brine through cooler, fpm: 75				200				400			
		Total brine, gpm	Mean temp diff, brine and ammonia, deg F			Total brine, gpm	Mean temp diff, brine and ammonia, deg F			Total brine, gpm	Mean temp diff, brine and ammonia, deg F		
			10	15	20		7½	12½	17½		5	10	15
26	6	55	5.7	8.6	11.5	140	7.74	12.9	18.1	290	7.15	14.3	21.2
26	9	55	8.5	12.8	17.2	140	11.5	19.3	26.0	290	10.7	21.4	32.0
26	12	55	11.4	17.2	22.9	140	15.5	25.8	36.2	290	14.3	28.6	42.9
34	9	90	13.6	20.5	27.4	230	18.5	30.7	43.1	440	17.1	34.1	51.1
34	12	90	18.1	27.3	36.4	230	24.6	41.0	57.4	440	22.6	45.3	68
34	18	90	27.2	41.0	54.8	230	37.0	62.0	86.2	440	34.1	68.2	101
42	12	190	33.1	50	66.5	510	45.0	74.8	105	970	41.5	83	123
42	18	190	50	75	100	510	67.4	112	157	970	62.2	124	187

Table 22. Over-all Coefficients of Heat Transfer

(Btu per sq ft per hr per deg F)

Can ice-making piping:	
Old-style feed, non-flooded	12–15
Flooded	20–30
Ammonia condensers:	
Submerged (obsolete except for CO_2)	30–40
Atmospheric, gas entering at top	60–65
Atmospheric, drip or bleeder	125–200
Flooded	125–150
Shell and tube	150–300
Double pipe	150–250
Baudelot coolers, counterflow, atmospheric type:	
Milk coolers	75
Cream coolers	60
Oil coolers	10
Water coolers, for direct expansion	60
Water coolers, for flooded	80
Brine coolers:	
Shell and tube	00–100
Double pipe	150–300
Cooling coils:	
Boiling refrigerant to air in unit coolers	4–8
Water to air in unit coolers	5–9
Brine to unagitated air	2–2½
Direct expansion	1½–2
Water cooler, shell and coil	15–25
Liquid ammonia cooler, shell and coil accumulator	45
Air dehydrator:	
Shell and coil (brine in coil), 1st coil	5.0
Shell and coil (brine in coil), 2d coil	3.0
Double pipe	6–7
Superheat remover, shell and tube	15–25

Forced circulation of the air increases the coefficient to 1½ to 2½ times the values for still air. One inch of frost decreases the value 25 percent.

and 3 in. outer pipe. The commercial rating is 15 to 20 ft length of coil per ton of refrigeration.

The **shell-and-tube cooler** is used with closed heads and is erected both vertically and horizontally; brine flows through the tubes and ammonia is in the shell. It is made in sizes from 1 to 350 tons with ratings of 8 to 15 sq ft effective surface per ton, varying with the temperature and brine velocities; tubes 1 to 2½ in. arranged multipass. This type of cooler has largely displaced all other types in recent installations.

Piping of Rooms. The size of pipe usually employed for piping rooms varies from 1 to 2 in.

For ships' provision rooms and vessels having only small cargo compartments, the direct-expansion system is usually adopted, as the mains are generally short and fairly readily accessible; also the financial liability to the vessel for cargo damage is much less than for a large cargo vessel because of the lesser value of the cargo carried.

On the contrary, the total tonnage of large cargo vessels employing the brine-distribution system is many times greater than that of the vessels employing the direct-expansion system of distribution, despite the fact that the earliest of such vessels were equipped with direct-expansion cooling. The requirements of operating reliability are met much more nearly with this system. It can be used for varying compartment temperatures quite readily and is more flexible in operation and requires less skilled supervision. It is a necessity in case of the centrifugal system of refrigeration where the refrigerant cannot be circulated to the various cooling units throughout the vessel. Competent authorities claim that it is the only system ever to be used for large refrigerated-cargo vessels, this being based on extensive experience with each, where cargoes are carried below freezing temperatures.

The air-cooling coil surface will accumulate ice which must be regularly removed to maintain efficient cooling. This can be readily accomplished on the brine system by circulating warm brine in a simple reliable manner through the cooling coils. It can be accomplished with the direct-expansion system of distribution by circulating hot gas through the cooling coils, but on a large cargo vessel this is not simple to operate. It requires much additional pipe and many connections, all of which are additional refrigerant-leakage hazards.

Further, when in use, unless the operator is quite skilled, it can completely upset the operational efficiency of the plant and thereby be a hazard against the safe carrying of the cargo. When the machinery used is of the reciprocating type and the selected refrigerant is Freon, it is imperative that the lubricating oil for any one compressor be returned to that compressor at approximately the same rate as it leaves with the discharge gas. Freon and oil are miscible; therefore, oil carried over from the compressor with the discharge gas will pass from the condenser in solution with the liquid Freon. It will not be concentrated until this liquid is boiled off in the evaporator coils, but it can return with the suction gas to the compressor as a foam.

It is not desirable to have liquid slop over from the evaporator shell in order to facilitate the return of this oil automatically with the suction gas, and it is difficult to get it back otherwise, unless means, external to the evaporator, for superheating are used, especially when the evaporator is operating under a reduced rate of loading. Further, the long run of suction piping and the position of the compressors in relation to the evaporator coils tend to prevent the ready return of this oil, and operational unreliability may result. This can be overcome by skilled operators if they are available.

If the percentage of oil in solution is allowed to accumulate in the evaporator shell, its presence there will raise the evaporator temperature for any given suction pressure and so reduce its efficiency, in addition to making it necessary to add more new oil to the compressor manually. This oil-return characteristic makes it essential to operate only one compressor on any evaporator unit or group of evaporator units; *i.e.*, no two compressors can operate satisfactorily in parallel on any evaporator unit or group of evaporator units.

When direct-expansion coils are used to transfer heat directly to the air stream, to the refrigerated space, or to brine, the refrigerant is fed into the coil usually in a direction counter to the direction of air or brine flow and

The average heat transmission in cold-storage rooms without forced circulation of the air is about 2 Btu per sq ft of outside metal surface per hr per deg F temperature difference with horizontal piping and 2.5 Btu with vertical piping. When forced air circulation is used, the transmission rate will increase to 20 Btu or more. In **brine circulation**, the brine, with the same back pressure, has a higher temperature than the ammonia, and consequently 1 to 1½ times as much pipe is used in brine circulation as in direct expansion for a given back pressure.

The extra cost of liberal piping allowance will often be offset by the consequent improvement in the efficiency of operation of the compressor. An expansion valve should be provided for every 400 ft length of 1 in. pipe, every 500 ft of 1¼ in. pipe, and every 1,000 ft of 2 in. pipe.

Values of **over-all coefficients of heat transfer** in Btu per hour per square foot per degree Fahrenheit for refrigerating practice are given in Table 22.

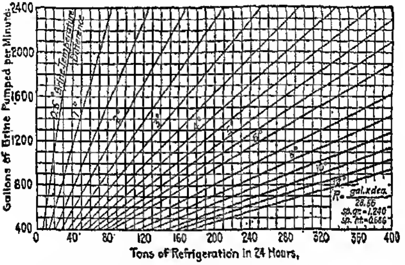

Fig. 5.—Refrigeration Produced by Brine.

Brine circulation is generally preferred to direct expansion, in consequence of the fear of danger from escaping ammonia or other refrigerant in case the pipes should leak. An advantage of the brine system is that there is always a considerable mass of refrigerated brine which can be drawn on in case the machinery should have to be stopped for any reason. In small plants, the general machinery may be stopped at night and only the brine pump be kept going to distribute the surplus refrigeration which has been accumulated in the brine during the day. Brine piping must consist of two lines, a flow and return, usually of the same size. Brine storage is seldom used because of its bulk, its first cost, and the practical inability to store much refrigeration. The modern automatic plant can handle the night load without difficulty.

The **cooling coils** in each refrigerator are in multiple with the supply and return pipes. It has been common practice to allow 120 to 150 running feet of 1¼ in. **pipe** per ton of refrigerating capacity in a **brine tank** for general refrigeration. The **tons refrigeration produced by brine** at various temperature differences and rates of pumping are given in Fig. 5.

distributed to the various coils by means of a distributor. As the liquid progresses through the coils, any oil accompanying the refrigerant is progressively concentrated as the refrigerant is evaporated. The final oil-refrigerant mixture leaving the coil has an amount of refrigerant absorbed in it which varies with the temperature-pressure condition. This mixture is quite homogenous in the form of foam. By properly dimensioning suction lines, this foam can be carried back to the compressor. This type of feeding is necessary in this type of application. This would not be strictly dry evaporation, and the heat-transfer coefficient on the refrigerant side is substantially the same as in the case of a flooded application, except that, as the refrigerant progresses through the coil, a considerable amount of surface is required for superheating the refrigerant. On this portion of the coil, the heat-transfer rate is quite low; therefore, external means of superheating are used as described below. This method is advantageous in that no oil is trapped anywhere beyond the condenser and the velocity of returning gaseous refrigerant sweeps the oil along with it and back to the compressor crankcase. However, when the loads are reduced, the distributor no longer functions so as to produce uniform feeding in all circuits nor does the necessary velocity exist to sweep the oil-Freon mixture, in the form of foam, back to the compressor.

Evaporators have been built having the refrigerant through the tubes and the brine through the shell. They work substantially as described above, but the indefinite distribution of refrigerant through respective feeds causes poor performance particularly at reduced capacity and, of course, poor performance means increased weight and space requirements. There is another point more definitely against this apparatus from a heat-transfer standpoint; namely, the change in evaporator temperature due to the pressure drop through individual feeds plus the liquid head penalty. This arrangement has never been satisfactory. Adequate brine velocity over tubes in the shell is more difficult of attainment than when the brine is fed through tubes.

Aside from the flooded brine cooler, where the refrigerant is in the shell and the brine is fed through the tubes, a type of evaporator often called **flash** or **spray** involves passing the brine through the tubes and spraying the refrigerant over the tubes by means of a refrigerant recirculating pump which takes its suction from the bottom of the brine-cooler shell and feeds it over the tubes through a distributing pipe. Oil return on this type of apparatus is accomplished by means of a still heated by discharge gas from the compressor, by electricity, or by other means. The liquid to be distilled is bled from the recirculating-pump discharge to the still and the resulting oil concentrate is returned to the compressor, the gas being returned to the suction line. This means of removing oil obviously entails a loss in refrigerating capacity, but the loss is so small that it can be neglected, being approximately 1 to 2 percent at most.

When flooded evaporators are used, it is necessary, as indicated above, to use means of superheating the refrigerant externally to the evaporator. The returning of oil from such an evaporator requires that a slight slop-over from the evaporator be maintained at all times, this liquid slop-over then being dried by superheating on the way to the compressor. Heat exchange between the high-pressure liquid line and the suction line is the usual method.

Float controls are not generally looked upon with favor when applied to marine brine coolers because of the pitch or roll of the ship. However, the same result may be accomplished by piloting the control valve with thermal expansion valves. Therefore, a direct-expansion evaporator does not necessarily always operate under conditions usually termed as dry evaporation.

Brine. Three kinds of brine have common use in refrigerating systems, sodium chloride (common salt), magnesium chloride, and calcium chloride. Calcium chloride is used most extensively because it is less corrosive and has a lower freezing point. It is supplied in 600 lb drums, either solid or granulated, and in tank cars containing 6,000, 8,000, or 10,000 gal of liquid at 1.350 sp gr. The properties of sodium chloride solutions are given in Table 23, of calcium chloride in Table 24, and of magnesium chloride in Table 25. Freezing points vary with the purity of the commercial salt and with contamination while in use. The Solvay Co. calcium chloride crystals are of approximately the following percentage composition: calcium chloride, 73.6; sodium chloride, 1.45; water, 24.9; constituents insoluble in water, 1.07. Table 26 gives the variation of specific gravity of brines with tempera-

Table 23. Properties of Sodium Chloride Solutions

For variation of sp gr with temperature see Table 26

Parts of NaCl by weight in 100 parts of the solution	Specific gravity at 60 F	Deg Beaumé	Weight per gal, lb	Weight per cu ft, lb	Freezing point, deg F	Specific heat at 14 F	32 F	50 F	68 F	80 F
6	1.044	6.06	8.71	65.1	25.5	....	0.924	0.927	0.929	0.932
8	1.058	8.00	8.82	66.0	22.9	....	0.902	0.906	0.909	0.912
10	1.073	9.91	8.95	66.9	20.2	....	0.882	0.887	0.890	0.893
12	1.088	11.78	9.08	67.8	17.3	....	0.865	0.869	0.873	0.876
14	1.104	13.63	9.22	68.8	14.1	....	0.848	0.853	0.857	0.859
16	1.119	15.45	9.33	69.8	10.6	0.827	0.834	0.839	0.842	0.844
18	1.135	17.25	9.47	70.8	6.7	0.815	0.821	0.825	0.828	0.830
20	1.151	19.02	9.60	71.8	2.4	0.804	0.809	0.813	0.815	0.817
22	1.167	20.78	9.74	72.8	−2.5	0.794	0.798	0.801	0.803	0.804
24	1.184	22.51	9.88	73.8	+1.4	0.784	0.788	0.791	0.792	0.793

Table 24. Properties of Calcium Chloride Solutions

For variation of sp gr with temperature see Table 26

Parts of $CaCl_2$ by weight in 100 parts of the solution	Specific gravity at 60 F	Deg Beaumé	Weight per gal, lb	Weight per cu ft, lb	Freezing point, deg F	Specific heat at −4 F	14 F	32 F	50 F	68 F	86 F
6	1.050	7.0	8.76	65.52	28.0	......	......	......	......	......	......
8	1.069	9.33	8.926	66.70	24.2	......	......	0.882	0.887	0.892	0.897
10	1.087	11.57	9.076	67.83	21.4	......	......	0.853	0.858	0.863	0.868
12	1.105	13.78	9.227	68.95	18.2	......	......	0.825	0.831	0.836	0.842
14	1.124	15.96	9.377	70.08	14.4	......	......	0.799	0.805	0.811	0.817
16	1.143	18.12	9.536	71.26	9.9	......	0.768	0.775	0.781	0.787	0.792
18	1.162	20.24	9.703	72.51	4.7	......	0.745	0.752	0.759	0.764	0.769
20	1.182	22.32	9.853	73.63	− 1.0	......	0.723	0.731	0.738	0.744	0.749
22	1.202	24.38	10.04	75.0	− 7.3	0.695	0.704	0.711	0.718	0.724	0.729
24	1.223	26.41	10.21	76.32	−14.1	0.678	0.686	0.693	0.900	0.706	0.712
26	1.244	28.41	10.38	77.56	−22.0	0.663	0.670	0.677	0.683	0.690	0.696
28	1.265	30.39	10.56	78.94	−32.0	0.649	0.656	0.662	0.669	0.675	0.682
30	1.287	32.34	10.75	80.35	−46.0	0.638	0.643	0.648	0.655	0.661	0.668

In short, flooded evaporation is preferable on account of the equal division of load at reduced capacity and on account of the more effective use of heat-transfer surface.

NOTE. This oil discussion refers to Freon or other hydrocarbon refrigerants that are miscible with oil and not to NH_3 or CO_2.

To summarize, the most reliable operational results would be obtained by using the brine-distribution system.

Types of Cargo. The cargo to be carried under refrigeration comes under two major types which differ essentially in their treatment:

1. Frozen cargo, such as meat in bulk (sides or quarters) or in individual frozen cuts packed in cartons or other containers, as they are received from the packing house. Butter, fish, and many other food products.

These are usually carried at 15 F or lower and will pack well in bulk in deep holds or 'tween decks, depending on the type of container used, except bulk meat which will always carry well packed in bulk if properly battened for air circulation through the cargo and to prevent the cargo from resting directly upon the surface of the insulation.

This kind of cargo will carry well with refrigeration applied by means of forced air circulation, because the forced air movement over and through the cargo, together with the temperature rise in the circulated air, will have little or no drying effect on the cargo, and neither shrinkage by drying nor mold growth from moisture will damage the cargo in transit, if it has been loaded thoroughly frozen and surface dry.

2. Chilled cargo, such as chilled meat, eggs, cheese, fruit, vegetables, or other edible products (not frozen), delivered to the vessel in a precooled condition (not at field temperature), is usually carried somewhat above the congealing point, approximately 40 F.

This kind of cargo (except chilled beef) will carry well with forced air circulation, but care must be used to have an air inlet temperature to the cargo high enough to avoid freezing the cargo with which it comes in contact. The amount of air circulated to take up insulation leakage and any heat generated within the cargo, together with fan heat, must be sufficient to maintain a small differential between air temperature delivered to the cargo and that returning from the cargo.

In general, these chilled cargoes are not suitable for stowage in deep holds as the weight of such stowage is too heavy for the lower tiers of goods to withstand. It is essential that adequate air passages be provided for the circulated air through the cargo and between the face of the insulation and the body of the cargo. This is accomplished by the use of portable gratings on the deck surface, permanent battens on the insulation surface, and the use of portable separating battens between the layers of cargo stowage, if the cargo package is of a nature which will, in itself, not provide for adequate air passages between and around the packages.

This battened stowage is of particular importance in the case of some of these products, because they themselves give off heat during carriage when the ripening process must be allowed to continue slowly.

In bulk meat, the most desirable edible beef is carried chilled—not frozen—and a very close temperature range or variation through the cargo has been found necessary, especially if the refrigerated voyage is over 10 days' duration. In past experience, the temperature variation permitted has been between 29½ to 30½ F, or a range throughout the entire cargo of only 1 F. Further,

ture. Table 27 gives the weight of commercial calcium chloride required to make brine of a stated specific gravity.

The density of brine is measured by a salinometer (or salometer) which is a simple hydrometer the indications on which are 4 times greater than on the corresponding Baumé scale (see p. 86).

Table 25. Properties of Magnesium Chloride Solutions

For variation of sp gr with temperature see Table 26

Parts of $MgCl_2$ by weight in 100 parts of the solution	Specific gravity at 60 F	Deg Baumé	Weight per gal, lb	Weight per cu ft, lb	Freezing point, deg F	Specific heat at					
						−4 F	14 F	32 F	50 F	68 F	86 F
6	1.051	7.06	8.776	65.61	25.0	......	......	0.912	0.914	0.917	0.919
8	1.069	9.32	8.926	66.74	21.8	......	......	0.882	0.885	0.889	0.892
10	1.086	11.53	9.068	67.80	17.9	......	......	0.854	0.858	0.862	0.866
12	1.105	13.73	9.227	68.99	13.1	......	0.822	0.827	0.832	0.837	0.841
14	1.123	15.88	9.377	70.11	7.3	......	0.795	0.801	0.806	0.812	0.817
16	1.142	17.99	9.536	71.30	0.4	......	0.770	0.776	0.782	0.788	0.794
18	1.161	20.08	9.694	72.48	−7.7	0.739	0.746	0.752	0.758	0.765	0.771
20	1.180	22.13	9.853	73.67	−17.3	0.717	0.724	0.730	0.737	0.743	0.750
22	1.200	24.17	10.02	74.92	−27.0	0.696	0.702	0.709	0.715	0.722	0.728
24	1.220	26.16	10.19	76.16	−14.0	0.676	0.682	0.689	0.695	0.702	0.708
26	1.241	28.14	10.36	77.48	−6.0	0.657	0.664	0.670	0.676	0.683	0.689
28	1.262	30.09	10.54	78.79	−1.0	......	0.645	0.652	0.658	0.664	0.671

It is undesirable to use a strength of solution of salt greater than is necessitated by its freezing temperature, as the specific heat (Tables 23, 24, and 25) decreases as the concentration of the brine increases, and consequently the stronger the brine, the less heat a given amount of it is able to convey between certain definite temperatures and the more power

Table 26. Specific Gravities of Brines

(To change to lb per cu ft multiply by 62.43; to change to lb per gal multiply by 8.35)

Parts by weight of salt in 100 parts of brine	Sodium chloride				Calcium chloride				Magnesium chloride			
	Temperature, deg F											
	14	32	50	68	14	32	50	68	14	32	50	68
6	......	1.046	1.044	1.041	......	1.053	1.051	1.049	......	1.053	1.051	1.049
8	......	1.061	1.059	1.056	......	1.071	1.069	1.066	......	1.070	1.069	1.067
10	......	1.077	1.074	1.071	......	1.070	1.087	1.084	......	1.089	1.087	1.084
12	......	1.093	1.090	1.086	......	1.108	1.106	1.103	1.108	1.107	1.105	1.102
14	......	1.108	1.105	1.101	......	1.127	1.124	1.121	1.127	1.126	1.123	1.121
16	1.128	1.124	1.121	1.116	1.150	1.147	1.144	1.140	1.147	1.145	1.142	1.139
18	1.144	1.140	1.136	1.132	1.170	1.167	1.163	1.159	1.166	1.164	1.161	1.158
20	1.161	1.157	1.152	1.148	1.190	1.187	1.183	1.179	1.186	1.183	1.181	1.178
22	1.178	1.173	1.169	1.164	1.211	1.208	1.203	1.199	1.206	1.203	1.201	1.197
24	1.195	1.190	1.185	1.180	1.233	1.229	1.224	1.219	1.226	1.224	1.221	1.218

is required to pump the brine. Moreover, brine which is too strong may cause clogging of pipes, etc., by depositing salt. On the other hand, if the solution is too weak it may not be able to withstand the temperature existing in the expansion coil, so that a layer of thin ice will form around the latter and interfere with the absorption of heat from the brine. The surface of the expansion coils in the brine tank should be inspected from time to time

it has been found necessary to have the least possible drying effect from the air; hence, no appreciable air velocity over the cargo can be used.

These factors indicate that, as long as these requirements for the carriage of this cargo are imposed upon the vessel by the shipper, it will not be possible to use forced air circulation for the cooling medium; and the cooling medium will have to remain convection air currents as in the past.

Bulk meat was a most important cargo in prewar years between the United States and European ports and between South American and European ports. The latter trade has been most exacting as to its refrigeration requirements because of the length of the voyage (18 to 21 days) under refrigeration. If the freight rates are equitable, the trade should still continue to be desirable. The cargo is carried suspended from meat rails running fore and aft and fastened to the overhead deck spaced at 12-in. centers between the overhead pipes of the brine-cooling system. This piping is spaced on alternate 8- and 4-in. centers, with the meat rail between the 8-in. center brine-pipe runs. The sides and bulkheads are piped all over at about 6-in. centers. The beef is in quarters and is muslin-covered with hind quarters suspended from the shank, and the fore quarters from the flank, by galvanized meat hooks and chains with hooks to slip over the meat rails. The meat is tightly packed to resist motion from the vessel's movement in a seaway. Brine at about 25 F is circulated through the pipe system to cool the space, and the cargo comes aboard quite chilled. The height from the deck to the underside of the 1¼-in. pipe rail is from 6 ft 9 in. to 7 ft 0 in., and no greater height is of any use. Therefore, the molded deck height for such cargo can be about 8 ft 0 in., depending upon the depth of the deck beams and the resulting depth of the insulation below the overhead deck. This is also a suitable deck height for refrigerated cargo in general.

The deck from which this chilled cargo is suspended may also have a bulk or package cargo supported on its upper surface at the same time. The deck, therefore, requires special strength treatment.

The construction of an efficient refrigerated-cargo vessel requires special design treatment to handle mixed refrigerated cargoes economically. Such a vessel could have a deep hold and two deep 'tween decks and be insulated only by an external envelope (as is the case of the ships refrigerated by turbo-compression units mentioned above), which is entirely adequate for the type of cargo it is designed to carry.

A preferable design for a refrigerated-cargo vessel would be made around the general condition that the cargo does not lend itself to deep stowage and has to be generally stowed by hand and carefully handled. The hatchways need not be large for refrigerated cargo, though at least one hatchway may be long for loading general cargo on a nonrefrigerated voyage.

A nonrefrigerated vessel of 40 ft 6 in. molded depth at the side may have a hold 19 ft 6 in. deep at the center and two 'tween decks of 9 ft 0 in. height, these dimensions depending on camber and shear. If this same hull is to be arranged for refrigerated cargo throughout, it could have a hold 13 ft 6 in. deep and three 'tween decks of 8 ft 0 in. height, depending, as before, on camber and shear. The refrigerated hull should be equipped with comparatively small hatchways and have cargo-loading ports in the upper deck space on each side. Then if two of the hatchways were trunked through the three upper 'tween decks to the hold ceiling, facilities would exist for loading and discharging refrigerated cargo at intermediate ports and, by the subdivision of insulated spaces resulting from such a design, varying temperatures in these cargo compartments and hatchway spaces could be maintained. All

to see if any ice has formed on them. In larger plants, it is customary to use a solution with a freezing point not less than 10 deg below the lowest temperature which will be obtained in the operation of the plant. In smaller isolated plants and where careful supervision is not ensured, it is customary to make the solution as strong as possible without being unstable, usually 1.240 to 1.250 sp gr. Magnesium chloride is unstable at high concentrations and low temperatures. Its decomposition may result in corrosion. For a given temperature, the concentration is less than for the other brines. It is not used in large systems.

Table 27. Weight of Commercial Calcium Chloride in Brine

Spec grav.	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32
Weight per gal, lb	1.41	1.70	2.00	2.30	2.59	2.90	3.20	3.50	3.83	4.13	4.46	4.78
Wt per cu ft, lb	10.55	12.72	14.96	17.20	19.37	21.69	23.94	26.18	28.62	30.89	33.36	35.75

Specific gravity is at 60 F for both brine and water. The weights are of 73 to 75 percent solid calcium chloride per gal of brine at 60 F. For flake 77 to 80 percent calcium chloride multiply the weights given by 0.94.

insulated spaces require adequate ratproofing the degree depending upon the type of insulation. All hatch coamings require heavy galvanized sheet-metal lining to protect them against cargo damage, particularly when loading a nonrefrigerated general cargo. All hatch plugs require similar metal treatment and should be constructed to be quite airtight and have overcovers to assist in this. They should also be protected at each deck level by the usual wooden hatch covers which would support the gratings for the cargo in the hatch wells. Where hatchways are trunked, the insulated doors to the compartments they serve should be well fitted with adjustable hinges and very substantial supports and fastenings. If double doors are used, they

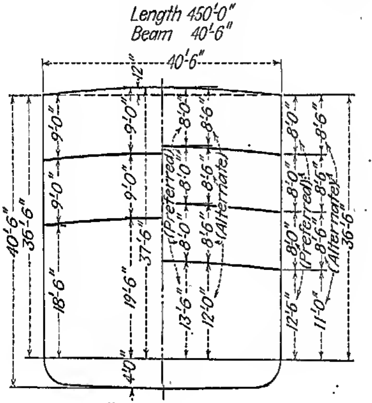

FIG. 3.

are more likely to remain tight if built with a portable center mullion, so that each door fits completely to its own aperture. All doors should open into the hatchway. All deck gratings should be in sections for easy removal for cleaning and should be constructed of finished lumber, shellac-coated, so that the space can be kept clean before loading. The bearers should be parallel to the direction of the designed air flow; they are usually athwartship with the slats fore and aft. Galvanized nails are used.

The fixed cargo battens on the face of the insulated surface should be arranged vertically and their corners mitered to guard against cargo damage. They keep the cargo from bearing on the insulation.

It is a designer's choice whether the designed air flow is from overhead down through the cargo, or from below the gratings upward through the cargo. Both systems have been used with success. A combination of these systems has also been used with success.

CARGO REFRIGERATION

BY

LLEWELLYN WILLIAMS

General. In general, refrigerated seagoing ships of the merchant marine are required to load and carry nonrefrigerated general cargo for a portion of their voyage from home port and on their return to home port, and a refrigerated cargo for some other part of this voyage. The vessel's cargo compartments and hatchways have, therefore, to be designed for the rapid and economical loading and discharging of these types of cargo. The whole vessel may be insulated and refrigerated throughout all its cargo spaces, or it may be subdivided so as to have some spaces available for general cargo only and the remainder equipped for refrigerated cargo. In either case it is economically desirable that the necessary deduction from the "bale" space available within the hull, for the insulation and refrigerating equipment and access thereto, be reduced to the minimum, since this deducted space shuts out both general and refrigerated cargo and thereby reduces the pay load that the vessel can handle.

Obviously, the weight of the refrigerating equipment and insulation should be kept as low as possible consistent with operational reliability, because this weight also reduces the pay load that the vessel can handle, particularly for that portion of the voyage when it is loaded with general cargo with a high weight per cubic measurement, such as steel rails.

As the United States is pretty much self-supporting for its food products, it is likely that our refrigerated vessels will be required to operate in a three-way manner; *i.e.*, they will load with American manufactured articles at their home port for the first leg of their foreign voyage; then, having discharged this cargo in foreign ports, they will load a refrigerated cargo for some densely populated country such as Europe or Asia; having discharged this cargo, they will load whatever cargo is available for import to the United States (generally much less than the vessel's carrying capacity) and will return home at least partly in ballast. Therefore, the greater the design stability of the vessel as a light ship, the less the ballast that will be required.

This requires that the refrigerating machinery be placed as far below the center of buoyancy as possible; and, to save loss of space and to simplify operational supervision, it is very desirable to have the refrigerating machines located within the main engine room and on the lower platform. This qualifies the necessity for a safe refrigerant applied to small space, lightweight refrigeration machines operating at high rotating speeds, and the use of a minimum number of such machine units consistent with reliable operation.

In the case of vessels using brine circulation as the means of distributing cold to the various cargo spaces, it will be desirable to locate the brine coolers (evaporators) and their pumps within an insulated space immediately adjacent to or as a part of the shaft tunnel abaft the engine room. The pump motors can be arranged in the main engine room with shafts extended through bulkhead stuffing boxes to the pumps located in the insulated brine-cooler space. The refrigerant condensers and their sea-water circulating pumps should be arranged within the main engine room. Thus the entire

The heat flow through the insulation is from the hot side to the cold side; therefore, heat entering the compartment first contacts the boundary air on the inner surface of the insulation and, if it is not immediately taken up by the stream of circulated air, will heat up the immediately adjacent cargo and perhaps damage it. Therefore, the air circulation should be directed to the removal of this heat.

Air-cooling Units. The air-cooling units, whether designed for brine-distribution or for direct-expansion distribution, can be completely factory-assembled units, each, when vertical, consisting of a galvanized metal lower coil section containing the cooling coils, and an upper fan section containing the fans, bearings, and an externally fitted motor for driving the fans. The fan drive can be either direct-connected or through V-belts with suitable arrangements for belt adjustment. This is the preferred system. The fans should, in general, be adequate for an air flow of one change per minute of total cargo-capacity air and have variable fan speeds or dampers, which will permit the air flow to be reduced to half the above. In some cases of chilled cargo, the maximum air flow above may need to be increased to conform to the permissible temperature rise in the air when balanced against the heat load. In handling frozen cargoes, a change of air every two minutes will be ample, and the heat imparted to the air by the fan will be reduced. In carrying chilled fruit, ample air is necessary for thorough ventilation of the cargo.

Some outside fresh air is desirable in handling fruit cargoes, but this air should be introduced over the coils, so that its surplus moisture will be deposited upon the cold surface of the coil and not upon the cold surface of the cargo. It is preferable to accomplish this by exhausting air from the end of the cargo space remote from the cooling unit and through separate ventilation openings. These openings should be fitted with insulated plugs so that they can be completely shut off when not in use. Some chilled cargoes, particularly apples, require a close regulation of this ventilating air for the whole voyage. The fruit gives off CO_2 gas very rapidly when being cooled down and the concentration should not be permitted to exceed 12 percent. The gas comes off at a greatly reduced rate after the fruit is cooled down, and it has been found desirable to have a permanent concentration of some 10 percent CO_2 gas to cut down fruit scald in transit.

In general, a provision of 5 percent of the total circulated air as fresh air for ventilating purposes will be ample; and the less that can be found practical for satisfactory use the better. Before loading a refrigerated cargo, all spaces should be thoroughly cleaned and the space cooled down to approximately the carrying temperature before loading. During loading, it is desirable to operate the fans continuously, but at their lowest air flow. When any compartment has been loaded, the air flow should be increased to conform to the temperatures required. The whole fan unit, including the coils, casing, and wheels, should be galvanized or tinned to guard against corrosion. All bearings should have suitable covered access openings. A suitable trapped scupper should be fitted to the drain pan and arranged for easy cleaning.

All refrigerant-flow-control devices and air-flow-control devices should be arranged outside the air stream. The cooling unit, as a whole, should always be outside the cargo space with suitable guarded air openings for the air flow from the cargo space to the unit and from the unit to the air-distributing duct system of the compartment. Large compartments should be protected by duplicate cooling units dividing the cooling load.

cold-producing portion of the refrigerating machinery plant will be immediately accessible to the main engine-operating staff, and their weight will be as far as possible below the center of buoyancy and will, in addition, cut out the least possible pay load.

In the choice of a refrigerant it is necessary or desirable to use one that can be readily obtained in any important foreign port as the value of refrigerated cargo (insured) is too high to rely for its safe carriage on having a "spare charge" aboard. This spare charge is a valuable asset but is not a guarantee that the cargo cannot be lost because the use of some special refrigerant, not in general use commercially, has been adopted for the plant. Further, in the interest of safe cargo carriage, it is desirable, on *refrigerated-cargo ships*, that the compression plant be subdivided into at least two parts (one spare), that these two plants be completely disconnected from each other in normal operation, and that only breakdown means for cross-connecting them be furnished with the vessel's spares.

By far the greater portion of refrigerated tonnage now in service uses CO_2 (carbonic anhydride) as its refrigerant, chiefly because it is generally recognized as safe and is universally procurable, since it is used for many commercial purposes other than refrigeration, besides being one of the oldest refrigerants in general use. Its chief disadvantages are its high operating pressures, high power requirements, and the large space and weight requirements of its condensers and evaporators.

The Freon refrigerants are comparatively new and, although they are universally used on U.S. refrigerated vessels, they are almost entirely "war-built" and form a relatively small percentage of the refrigerated-cargo space in use. These Freon refrigerants are easily the most desirable if they can be readily obtained in foreign ports.

Freon 12 is in general use in the United States for reciprocating-type machines, with the new Freon 22 threatening to displace its older brother, especially where low temperatures are desired. Freon 11 is the refrigerant generally adopted in U.S. refrigerated ships using the centrifugal type of refrigerating machine, and there are some large refrigerated-cargo U.S. ships which have been in satisfactory service through the tropics. More similar vessels are being built, each of which has approximately 350,000 cu ft insulated space available for refrigerated cargo of frozen meat carried at 15 F. Leakage of all the Freons is not easily detected.

When the refrigerated-cargo vessel has no passenger space and the refrigeration machinery can be located in a deckhouse, apart from the main engine room and navigation quarters, and can be adequately ventilated safely, anhydrous ammonia is a very desirable refrigerant. It is not a safe refrigerant, being highly noxious and, under certain conditions, both flammable and explosive. However, leaks in the system can be very easily located, and its operating pressures are reasonable, its power requirements low, its machinery weight also reasonable, but its weight would, of ventilation necessity, be located much above the vessel's center of buoyancy. It is universally readily procurable. New vessels being refrigerated in Great Britain are using it as the refrigerant.

As the choice of a refrigerant is so important for a cargo vessel, it might be desirable to summarize as follows:

First choice: For reciprocating-type machines, Freon 12 or Freon 22 whichever is the more readily procurable.

Second choice: For reciprocating-type machines, CO_2 (carbonic anhydride).

Third choice: For reciprocating-type machines, NH_3 (ammonia).

The cooling unit referred to is one in which the air is drawn over the cooling coils by the fans and discharged directly to the cargo space through the distributing duct system. In this case, the heat added by the fan losses raises the temperature of the air, reduces its relative humidity and, therefore, promotes the drying of the cargo by the circulated air. This is not of importance in the case of frozen cargo or completely enclosed package goods, but it is a detriment in the case of chilled-fruit cargoes, especially those carried in bulk or not paper-wrapped and in open slotted boxes. For such cargoes, it is very desirable to have the air drawn from the cargo space and then blown over the coils to the duct system so that the fan heat is taken up immediately by the cooling coil, and the drying effect on the cargo can be greatly reduced. This can easily be obtained if the unit is so constructed at the factory. Where the units required are too large for factory assembly, they may be fabricated in the ship from factory-assembled parts. This can also be done with smaller units if factory-assembled units are not available.

Where a vessel is trading through cold waters or discharging in cold climates, it may be quite desirable to fit a means of heating the air to each unit. This is also very useful in warming up and drying out the refrigerated compartments before general cargo is loaded and after the refrigerated cargo has been discharged. This can be done quite easily when a brine-distribution system has been adopted, and the brine heater normally installed for coil defrosting has been made large enough to furnish the heating requirements of the cargo spaces.

If the direct-expansion system of distribution is adopted, it will be necessary to fit each air-cooling unit with a heater. If electrical power is available, it would be the preferable method because it would then not be necessary to guard against freezing of the heating fluid when the heater was not in use, as would be the case with any other convenient methods.

Extreme care should be used to protect all electrical devices used against the action of sea air and the sweating that results in refrigerated compartments due to changes in temperature when a refrigerated cargo is being discharged or when a general cargo is being handled after the refrigerated cargo has been discharged. A desirable arrangement would be to center all electrical controls in the machinery space or in substations completely separated from the refrigerated cargo spaces and open to or ventilated from the outside air. Where large cooling units are fitted, this can be done readily by having the fan motors and their belt drive arranged in an access space insulated from the cargo space, and by locating all the controls in this space which would be ventilated from outside to remove the heat of the motor windings. Many vessels engaged in the fruit trade have used this method for years with complete satisfaction. Considering that the satsifactory carriage of a large amount of high-value cargo may depend upon the continuous operation of a 5-hp motor, it is obvious that the said motor should be installed so that it can be readily replaced at sea and in a position and manner which would subject it to the least risk against unsatisfactory operation.

Completely reliable temperature-indicating devices should be furnished and located where they are always readily accessible to the chief engineer and his staff. There should be at least one such connection for each compartment and, when air circulation is used, the temperature-sensitive member can be located in the return air stream, preferably in the fan chamber where it is easily accessible.

However, rugged and tested cased thermometers should always be provided, permanently mounted in the fan chamber and guarded against potential

The choice between centrifugal- and reciprocating-type machines is not simple as there are many conflicting characteristics of each to be considered. If the tonnage of each of the two machine units necessary as a minimum is not less than 120 at full power—which would be suitable for a vessel of from 250,000 to 300,000 cu ft of insulated space available for cargo—then the centrifugal equipment would be preferable if it is desirable to operate it by a direct-connected steam turbine without a speed-increasing gear box. When considered with its necessary steam condenser, vacuum-producing jet apparatus, and condensate pumps and because of the fact that it must be arranged in close-coupled form with its evaporator, refrigerant condenser, and steam condenser, both its weight and space requirements would probably be greater than for the reciprocating type and might well prevent its location within the main engine-room space and below the center of buoyancy of the hull. It cannot, at present, be economically produced for tonnages less than 120 full load in tropical waters. Such a compression unit in a refrigerated-cargo ship would be required to unload down to 25 percent capacity for continuous operation (say 30 tons output), after the cargo was at the desired temperature and the vessel still in tropical waters.

The centrifugal type is not positive in action, *i.e.*, it can operate and do no useful work, but it is easily and reliably operatable under extreme conditions by skilled, experienced personnel. However, it cannot be readily repaired on the voyage as the assembly work has to be very accurately executed. Vessels so equipped have given no serious trouble, but they must have experienced operators.

The reciprocating type of machine is positive in action, can be driven by a steam turbine through the medium of a speed-reducing gear box, can be produced in small or large size, can be readily unloaded, can be readily repaired during the voyage with spare parts, and is easy to operate efficiently by unskilled help. It requires slightly more power than the centrifugal and, of course, has many more parts subject to wear and breakage than the centrifugal, but it is more readily overhauled and repaired on the voyage or in foreign ports.

Both types can be operated by electric motor. The centrifugal type requires a speed-increasing gear box, and the reciprocating type can be direct-connected to its motor or driven through V-belts. If electrically driven, each type should have variable-speed characteristics built into the electric motor, which on shipboard is almost entirely d-c, 230-volt.

Distribution of Cooling Medium. The refrigerating plant produces liquid refrigerant or cold brine for distribution to the cargo-cooling mechanisms, which of necessity are arranged within the cargo spaces to be cooled. This distribution system is a very essential part of the over-all cooling system and should be designed in its simplest possible form to avoid troublesome operation and losses due to possible part failures.

The cooling units located within the cold spaces must have control devices readily accessible at all times.

The refrigerant itself can be distributed throughout the vessel from the central machine system to the cooling units and is then termed a **direct-expansion system of cooling.** The liquid refrigerant from the machine condenser system is piped throughout the vessel to each and every air-cooling unit in the cargo spaces. There it is expanded through suitable flow-regulating and protective devices to the air-cooling coils within each unit, where it is evaporated by extracting heat from the surrounding air

damage. There should be one for the return air and one for the delivery air, both readily accessible.

In cases where the shipper compels the vessel to hold very close temperature regulation as a part of the loading contract, it would be desirable that a temperature-recording device be also available for the observation of the chief engineer. This is especially true when a shipper places his own locked and sealed temperature recorder in the cargo space among the cargo, to be opened and examined when the cargo is being discharged. This has frequently been done with some cargoes, particularly with chilled beef.

If the heat removal method adopted is by direct expansion, the cooling coil should be entirely of nonferrous material and should be hot-tinned after coil assembly at the factory. This is to guard against internal corrosion of the coil and its control devices owing to water inadvertently getting into the refrigerant in the case of Freon.

Temperatures Required. Frozen cargoes are usually carried at 15 F or lower, when in bulk. In the future there will be large cargo offerings of quick-frozen cuts, *i.e.*, meat quick-frozen in the packing houses in cuts of varying weights for distribution to the consumer in the same condition as when shipped. These will be wrapped and packed in cartons. Similar packs of fish, fruit, and vegetable products will be shipped. The temperature required by the shipper will be low—probably around 0 F or even lower if such temperatures can be provided by the ship. Air circulation will be satisfactory for this type of cargo.

Chilled cargoes will require temperatures varying between 33 and 45 F, all requiring the control of relative humidities of the return air; therefore, the average relative humidity of the cargo space will be not less than 85 percent with 90 percent preferred. Lower relative humidities produce greater cargo-weight shrinkage and loss of product quality, whereas higher relative humidities can result in mold growth on parts of the cargo. The greatest hazard against the ship in the satisfactory carriage of such cargo is its being damp on the surface when being loaded. The vessel's personnel should make close inspection when such cargoes are being loaded, and they should also see to it that the cargo is properly packed and adequately dunnaged for air circulation. Insurance claims against the cargo by the shipper will result unless extreme care is exercised in loading. This is especially important because many honest stevedores are quite inexperienced in handling refrigerated cargo.

Refrigerating Tonnage. It is essential that the vessel be fitted with ample total refrigerating power, including spare power, for carrying cargoes through tropical climates and for loading cargoes in tropical ports, such as India, East Africa, North Australia, the East and West Indies.

It is simple to calculate the heat-leakage load by the usual heat-flow formula, using published values for the insulation materials to be used. However, when this has been done, the actual heat leakage will be found to be about twice as much as the calculation indicates, because the steel shell construction of the vessel is such that conventional calculation does not make due allowance for the varying heat-flow conditions as they actually exist.

Empirical heat-transfer rates as listed below can be used for any type of conventional insulation, such as is described below, used to enclose the space for refrigerated cargo. These rates cover the total heat leakage, and the summation will then approximate the actual heat leakage that will result

stream and then returned to the compression machines as suction gas for compression and liquefaction in the condenser system for recirculation through the closed system.

On a large cargo vessel this requires a complicated pipe system which should be accessible to the operating engineers for maintenance and repair. The pipe joints in such a system on shipboard are more vulnerable to leakage of refrigerant than on a corresponding land installation of similar size, because of the hull vibration, ever-present when operating at sea, and the necessity for tight-stowed cargo which renders accessibility extremely difficult where the pipes have to pass through cargo spaces and intervening watertight bulkheads. The refrigerant flow devices require numerous flanged joints which are always potential refrigerant leakers.

The direct-expansion system of distribution, however, is most attractive from a consideration of total weight and over-all operating power as no pumps are required to circulate the refrigerant. It is also very convenient

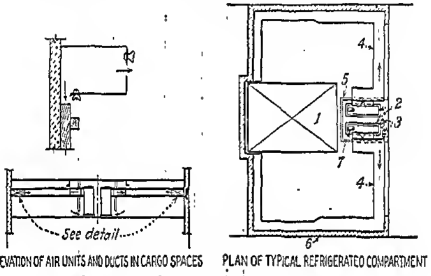

FIG. 1.—Typical refrigerated compartment showing air unit and duct arrangement.

for carrying widely varying temperatures in the several cargo compartments, as may be necessary with mixed cargoes. However, it does require more skilled operation to keep it in adjustment than the cold brine-distribution system.

The other method of cold distribution is the **brine-distribution system**, in which the refrigerating plant in addition to the machines, condensers, and liquid-storage receivers (common to each system) has brine coolers, in which the evaporating liquid cools a calcium chloride brine to the desired temperature, which is circulated by pumps and a pipe system to the various cooling units in the holds. Therefore, the brine coolers (there should be two for safety, one for stand-by in case of trouble), the brine pumps, and their motors add weight and require space in a vessel in excess of that required by the direct-expansion system.

The control at the air-cooling units is very simple and reliable, being merely flow-control valves which can be operated manually or automatically. The pipe lines can be fully welded and are not liable to leakage to the same extent as the lines carrying the refrigerant. In the case of a local fire there is

in practice. The heat flow per square foot of external surface per day per degree Fahrenheit difference in temperature is taken as

	Btu/(Day) (Deg F)
Area of upper deck surface exposed to sun (less hatches)	4
Area of hatches exposed to sun temperature	5
Area of ship's sides, tank tops, and bulkheads[a]	3
Area of all intermediate deck hatches	4

[a] If the ship is only partly insulated, this area should include intermediate decks adjacent to nonrefrigerated compartments.

The external temperatures are assumed to be

	Deg F
Top deck and hatches exposed to sun	120
Shell above water line	100
Shell below water line	90
Engine-room and boiler-room bulkheads	120

All deck and bulkhead ribbands are to be taken as additional hull areas.

Load calculations of the refrigerating requirements can vary quite widely, and the usual means of estimating heat loads as applied to land practice are insufficient for shipwork for several reasons:

1. There are extreme climatic changes and most frequently ships pass through the tropics—a fact that would require a design compatible with tropical conditions.

2. The insulation used in a steel ship hull is difficult to apply on account of structural conditions within the ship. For example, where the deck or bulkhead joins the ship's sides there is a heat path created which will increase heat losses. This is combated by the use of insulation, known as **ribbands**, extending inboard from the hull along the bulkhead or deck. Also, the hull insulation extends only a few inches inside the hull channels or frames (ribs).

3. Changing temperature causes the insulation to breathe, and it can very easily become saturated with moisture. Deterioration of insulation results.

4. The normal heat-transfer values for insulation do not generally apply because a certain portion of the ship is below water and subject to the heat-transfer coefficient of steel to water rather than steel to air as in the case of land practice.

5. The area of a ship's sides and top exposed to the sun is large in proportion to the volume contained and can easily become substantial. The upper deck exposed to sun and wind is frequently covered with a canopy when passing through the tropics to reduce heat leakage.

6. Many parts of refrigerated holds are subject to engine-room heat which at times may become quite high.

Heat-transfer factors for estimating the leakage load based upon actual practice have been given above. It is possible to arrive at a fairly accurate rule of thumb by dividing the stowage volume of a vessel by the tons of refrigeration installed so as to determine the cubic feet of stowage available per ton. This provides a rough check on heat leakage and at the same time presents a key to good commercial practice or a check on individual calculations. The factors are listed below and have been confirmed.

added hazard due to the brine; however, there can be a very serious added zard if the pipe lines carry the refrigerant. The brine flow can be comtely controlled from the central operating system and is usually so fitted

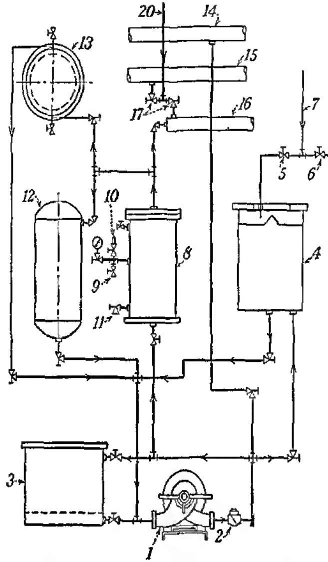

FIG. 2.—Schematic arrangement of warm brine piping. 1, Hot-brine pump; 2, check valve; 3, brine mixing tank; 4, hot brine return tank; 5, hot brine return valve; 6, cold brine return valve; 7, thirty-two hot or cold brine return lines from air-cooling coils; 8, brine heater; 9, steam inlet to heater; 10, relief valve; 11, condensate outlet; 12, brine storage tank; 13, brine cooler brine box; 14, brine main—from cold brine pump to cooler; 15, brine header—cold brine to coils; 16, hot brine header; 17, brine supply control valves; 20, hot or cold brine supply lines to air unit coils.

on either an open or a closed brine system. With this type of distribution system it is easier to prevent refrigerant leakage as all the refrigerant-carrying piping is within the operating space and is readily accessible at all times.

Insulation. Another reason why heat-transfer factors through insulation may vary widely is because the actual heat flow into the ship is a function of how the insulation is finally applied and what it is actually like having been in service. In view of this the American Bureau of Shipping makes an inspection of the insulation and machinery installation. The ship is cooled empty to a specified temperature and the temperature rise after six hours with machinery shut off is noted. The insulation value is then relative. Insulation is considered satisfactory if the temperature rise averages not more than ¾ deg per hr; however, more than this is allowed when the ship is classified. A typical method of insulation consists of four layers of rock-wool battens (Eagle Picher or Johns-Manville) between the ship's "ribs" with two layers of tongue-and-grooved boarding and paper, the latter covered with waterproofed plywood. The specification for such insulation reads in detail as follows. This is accepted as good practice.

All steel work behind insulation in refrigerated cargo spaces shall be thoroughly cleaned and given two coats of bituminous solution and one coat of enamel, applied hot, to a minimum thickness of 1/16 inch on vertical surfaces, and ⅛ inch on horizontal surfaces.

Insulation of the tank top shall consist of a 12 in. thickness of wooden grounds, laid in "grill" formation, spaced 18 in. centers, with all spaces completely filled with "insulation material." The bottom grounds shall be bolted to ¼ in. flat bars welded to the deck. Ends shall be adequately connected to each other. In addition, there shall be an adequate connection of grounds at crossings at sufficiently frequent intervals to prevent "tripping" and to reinforce each "layer" against lateral movement.

Insulation of boundary bulkheads and shell shall consist of "insulating material," applied to a minimum thickness of 12 in.; but completely filling the space between wood furring and steel plating. Adequate support shall be provided to prevent settling of insulation. The wood furring shall be of suitable thickness, bolted to ¼ in. flat bar clips welded to steel structure where bulkheads are flush or to the webs of frames and stiffeners and extending at least 2 in. inboard of inner edge of same. The lower ends shall be bolted to the grounds for the tank top or ribbands and the upper ends to the furring on the deckhead or the deck structure.

Insulation of deckhead under shelter deck shall consist of wood furring, fastened to the web of deck beams by screws or bolts, with spaces between completely filled with "insulating material," to the required thickness.

No insulation will be required on the underside or topside of 'tween decks, excepting for a three-foot ribband at shell and bulkheads, nor will insulation be required on intermediate bulkheads excepting for ribbands. Johns-Manville BX-18, or equal, shall be used for topside deck ribbands.

Adjacent decks for extent of 3 ft outside refrigerated spaces shall be fitted with 2 in. of Johns-Manville BX-18, or equal. Bulkheads and decks adjacent to machinery spaces shall be insulated on the machinery space side with at least 3 in. of approved insulation, and it shall extend for at least 3 ft beyond the refrigerated spaces. Ducts exterior to refrigerated cargo spaces and serving same shall be covered with insulation, 2 to 4 in. thick.

The tank top insulation shall be covered with 1½ in. of concrete, reinforced with 18-gage expanded metal. Over this shall be laid 1 in. of mastic composition coved up all around. It shall be compounded especially for use in refrigerated spaces for a range of 10 to 100 F. Gratings of wood, total height of 3¼ in., shall be fitted on the decks of all compartments and arranged in panels for easy removal. They shall be of spruce, with sleepers 1¾ by 3¾ in., top 1¾ in. thick by 4½ in. wide, and have ⅜ in. between boards. Top of grating boards shall be chamfered on each edge about ¼ in.

Sides and deckhead insulation shall be sheathed with two layers ⅞ in. T & G spruce, nailed to furring. There shall be two layers of 15-lb asphalt-saturated felt paper, one separating the two layers of T & G sheathing, one separating the T & G sheathing from the insulating material. Wood battens, 2 in., spaced 15 in. on centers, shall be nailed to the sheathing except in way of the coils.

DIFFERENTIAL EQUATIONS

An ordinary differential equation is one which contains a single independent variable, or argument, and a single dependent variable, or function, with its derivatives of various orders. **A partial differential equation** is one which contains a function of several independent variables, and its partial derivatives of various orders. The order of a differential equation is the order of the highest derivative which occurs in it. A solution of a differential equation is any relation between the variables, which, when substituted in the given equation, will satisfy it. The general solution of an ordinary differential equation of the nth order will contain n arbitrary constants. A differential equation is usually said to be solved when the problem is reduced to a simple quadrature, that is, an integration of the form $y = \int f(x)dx$.

Methods of Solving Ordinary Differential Equations

Differential Equations of the First Order

(1) If possible, separate the variables; that is, collect all the x's and dx on one side, and all the y's and dy on the other side; then integrate both sides, and add the constant of integration.

(2) If the equation is homogeneous in x and y, the value of dy/dx in terms of x and y will be of the form $\dfrac{dy}{dx} = f\left(\dfrac{y}{x}\right)$. Substituting $y = xt$ will enable the variables to be separated. Solution: $\log_e x = \displaystyle\int \frac{dt}{f(t) - t} + C.$

(3) The expression $f(x,y)dx + F(x,y)dy$ is an *exact differential* if $\dfrac{\partial f(x,y)}{\partial y} = \dfrac{\partial F(x,y)}{\partial x} (= P,$ say). In this case the solution of $f(x,y)dx + F(x,y)dy = 0$ is

$$\int f(x,y)dx + \int [F(x,y) - \int P dx]dy = C$$

or

$$\int F(x,y)dy + \int [f(x,y) - \int P dy]dx = C$$

(4) Linear differential equation of the first order: $\dfrac{dy}{dx} + f(x)\cdot y = F(x)$.

Solution: $y = e^{-P}\left\{\int e^{P}F(x)dx + C\right\}$, where $P = \int f(x)dx$.

(5) Bernoulli's equation: $\dfrac{dy}{dx} + f(x)\cdot y = F(x)\cdot y^n$. Substituting $y^{1-n} = v$ gives $\dfrac{dv}{dx} + (1 - n)f(x)\cdot v = (1 - n)F(x)$, which is linear in v and x.

(6) Clairaut's equation: $y = xp + f(p)$, where $p = dy/dx$. The solution consists of the family of lines given by $y = Cx + f(C)$, where C is any constant, together with the curve obtained by eliminating p between the equations $y = xp + f(p)$ and $x + f'(p) = 0$, where $f'(p)$ is the derivative of $f(p)$.

Differential Equations of the Second Order

(7) $\dfrac{d^2y}{dx^2} = -n^2y$. Solution: $y = C_1 \sin (nx + C_2)$

or $y = C_3 \sin nx + C_4 \cos nx$

(8) $\frac{d^2y}{dx^2} = +n^2y$. Solution: $y = C_1 \sinh (nx + C_2)$

or $y = C_3 e^{nx} + C_4 e^{-nx}$

(9) $\frac{d^2y}{dx^2} = f(y)$. Solution: $x = \int \frac{dy}{\sqrt{C_1 + 2P}} + C_2$, where $P = \int f(y)\, dy$.

(10) $\frac{d^2y}{dx^2} = f(x)$. Solution: $y = \int P dx + C_1 x + C_2$, where $P = \int f(x) dx$

or $y = xP - \int x f(x) dx + C_1 x + C_2$

(11) $\frac{d^2y}{dx^2} = f\left(\frac{dy}{dx}\right)$. Putting $\frac{dy}{dx} = z$, $\frac{d^2y}{dx^2} = \frac{dz}{dx}$, $x = \int \frac{dz}{f(z)} + C_1$ and $y = \int \frac{z dz}{f(z)} + C_2$; then eliminate z from these two equations.

(12) The equation for damped vibration: $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = 0$.

Case I. If $a^2 - b^2 > 0$, let $m = \sqrt{a^2 - b^2}$. Solution:

$y = C_1 e^{-bx} \sin (mx + C_2)$ or $y = e^{-bx}[C_3 \sin (mx) + C_4 \cos (mx)]$

Case II. If $a^2 - b^2 = 0$, solution is $y = e^{-bx}[C_1 + C_2 x]$.

Case III. If $a^2 - b^2 < 0$, let $n = \sqrt{b^2 - a^2}$. Solution:

$y = C_1 e^{-bx} \sinh (nx + C_2)$ or $y = C_3 e^{-(b+n)x} + C_4 e^{-(b-n)x}$

(13) $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = c$. Solution: $y = \frac{c}{a^2} + y_1$, where y_1 = the solution of the corresponding equation with second member zero [see (12) above].

(14) $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = c \sin(kx)$. Solution:

$y = R \sin(kx - S) + y_1$, where $R = c/\sqrt{(a^2 - k^2)^2 + 4b^2k^2}$,

$\tan S = \frac{2bk}{a^2 - k^2}$, and y_1 = the solution of the corresponding equation with second member zero [see (12) above].

(15) $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = f(x)$. Solution: $y = y_0 + y_1$, where y_0 = any particular solution of the given equation, and y_1 = the general solution of the corresponding equation with second member zero [see (12) above].

If $b^2 > a^2$, $y_0 = \frac{1}{2\sqrt{b^2 - a^2}} \left\{ e^{m_1 x} \int e^{-m_1 x} f(x) dx - e^{m_2 x} \int e^{-m_2 x} f(x) dx \right\}$,

where $m_1 = -b + \sqrt{b^2 - a^2}$ and $m_2 = -b - \sqrt{b^2 - a^2}$.

If $b^2 < a^2$, let $m = \sqrt{a^2 - b^2}$; then $y_0 =$

$\frac{1}{m} e^{-bx} \left\{ \sin (mx) \int e^{bx} \cos (mx) \cdot f(x) dx - \cos (mx) \int e^{bx} \sin (mx) \cdot f(x)\, dx \right\}$.

If $b^2 = a^2$, $y_0 = e^{-bx} \left\{ x \int e^{bx} f(x)\, dx - \int x \cdot e^{bx} f(x)\, dx \right\}$.

At least two "breather" plugs shall be fitted in each wall of each compartment as directed, consisting essentially of a ¾-in. shoulder bushing with a 100-mesh screen soldered in and arranged to screw into a threaded fitting in the sheating and readily replaceable with a plug.

In order to obtain the total heat load for the machinery, add to these calculated heat-leakage loads, the heat represented by the horsepower of all the fans, and the horsepower of the brine-circulating pumps (if brine distribution is used) together with an allowance for cooling down the cargo when received precooled but not received at the holding temperature. An allowance of 10 deg cooling in 48 hr for the whole cargo will be safe.

The summation of these items would represent the total refrigerated load and, if no complete spare unit is provided, the load which is estimated as a 24-hr load should have machinery available to absorb it in 18 hr of operation when sailing in tropical waters. Thus, if the summation amounted to 180 tons, then $180 \times {}^{24}\!/_{18} = 240$ tons refrigeration capacity would be provided by two 120-ton, three 80-ton, or four 60-ton machines. However, it would be necessary to have all the parts of these machines interchangeable and, in addition, carry a large complement of spare parts, to provide means of maintaining the operational efficiency of the equipment during the period of estimated shutdown. If two 240-ton machines were installed, either of which could absorb the heat load in 18 hr continuous operation, it is not necessary to carry extensive spares, but all parts of the machines should be interchangeable; and a fairly generous lot of spares should be carried.

In the event of the machinery being selected on the basis of 18-hr operation, the condensers, coolers, pipes, pumps, and all should be supplied to the full power of the machines, *i.e.*, 240 tons.

As the cargoes carried are very valuable and are always insured, the underwriters and classification societies will undoubtedly require the duplicate plant equipment, arranged as two completely separate units completely disconnected on the refrigerant side but common on the brine side.

As clearly stated, the above tonnages are not adequate for the carriage of cargoes not precooled, such as fruits loaded at field temperatures—bananas, for instance. These require quite special consideration, not only as to providing adequate machine capacity but also as to the stowage of the fruit, method of air circulation, etc.

The tonnage requirements as indicated from the calculations above referred to can be checked for any vessel by an examination of the ratio of insulated cubic capacity to refrigerating machinery tonnage as fitted to refrigerated-cargo vessels in the past. These figures can be obtained from Lloyd's register records of refrigerated ships. These records distinguish between vessels only partly refrigerated and having less than 80,000 cu ft insulated space and vessels wholly refrigerated or having insulated capacities of over 80,000 cu ft. From these records the following figures are found:

Twelve U.S. ships built 1918–1919, Lloyd's records 1924–1925, aggregate cubic capacity 3,420,000 ft, average per ship 285,000 cu ft. Total refrigerating-machine capacity 1,224 U.S. tons and average machine loading 2,790 cu ft per ton machine capacity. These machines all used CO_2 as the refrigerant and brine as the cold distribution medium, with all compartments fitted with brine piping distributed overhead and on all sides and bulkheads, and in hatchways. There was no forced air circulation used.

In the following list of ships from Lloyd's records, all machine tonnages have been reduced to U.S. tons, one ton being 12,000 Btu per hr between 0 F

Sales of Fractional Interests in Patents. Undivided fractions or territorial divisions of patent rights are advisable only in special cases, because with the best intentions the interests of joint or territorial owners are *liable to interfere.*

Licenses are permissions by the patentee to make, use, or sell the thing patented, and, broadly speaking, a license may carry any imaginable provisions the parties agree on, subject to certain restrictions upon agreements in restraint of trade, upon fixing resale prices, upon limiting the use of patented articles after their sale, and other restrictions of like character, as to which a lawyer should be consulted. License contracts should be reduced to writing. License by oral arrangement, or by implication from circumstances, is as binding on the parties as if reduced to writing, the difference lying in difficulty of proof.

Relations of employer and employee, when the latter makes an invention. Unless by express contract, or by implication from circumstances, the employee agrees that his patentable inventions should be the property of the employer, the employer has no title or claim to the invention or to the patent for it. Even if the employee has developed his invention in the time and with materials belonging to the employer, the claim of the employer is only for the value of time and materials and does not extend to the invention. Should the employer, with the employee's consent, express or implied, put the invention into use, license under the patent to continue use to the extent initiated is implied, but no such implication extends to any enlargement of the use. Many employers require or attempt to secure from their employees an express agreement that inventions shall belong to the employer, and a prospective employee should carefully read and understand any such agreement before signing.

Infringements of Patent. As the grant purports to give the exclusive right to make, sell, or use, so an unlicensed manufacture, sale, or use of the thing patented is an infringement, and the maker, seller, or user may be sued in a District Court of the United States. Suit must be brought in the district wherein the defendant resides, or where the defendant, though not residing, has a regularly established place of business, and has committed the act of infringement. In all but very exceptional cases, the courts will not grant preliminary injunctions in a suit on a patent which has not previously been held valid in another contested litigation.

Reissues of Patent. If, through inadvertence, accident, or mistake the original patent was defective, or claimed too much or too little, the error may be repaired by surrender of the patent and reissue, provided the reissue be for the same invention as was disclosed as such by the original and be applied for without unreasonable delay. The term of a reissued patent expires on the day when the original patent would have expired.

Marking Patented Articles. Unless the patentee has given particular notice of his patent, or general notice by marking the patented articles "*Patent*" followed by the number of his patent, he may not recover damages for infringement. For a patent issued prior to April 1, 1927, the notice "Patented" followed by the date of the patent is sufficient. If the patented articles themselves cannot be marked, the mark may be affixed to the packages in which they are contained. Marking requirements do not apply to process patents or to patents under which the patentee has not manufactured.

False Marking. Any person who, with intent to deceive the public, falsely marks an article or parcel with a patentee's mark, without leave or

evaporating temperature and 100 F condensing temperature. The figures are from Lloyd's Register, 1934.

Ship nationality	No. of ships	Total cubic of all ships	Average cubic per ship	Total U.S. tons of all machines	Average cubic per ton for all ships
U.S.A.	732	117,008,578	160,000	61,200	1,915
English	29	10,709,488	370,000	4,990	2,150
English	25	8,006,305	320,000	3,280	2,440
English	20	6,087,366	305,000	2,370	2,570
English	3	1,714,194	570,000	675	2,540

By far the greater number of these ships employed CO_2 as the refrigerant and brine as the distributing medium. The greater portion of the cooling work was taken up by exposed brine-pipe coils distributed over the insulated ceilings, sides, and bulkheads with hatch coils in the hatchways. Some compartments are fitted with forced air circulation for fruit carriage, but this would be a small percentage of the whole on general-purpose vessels. Purely fruit vessels, as those operating in the banana trades, would be cooled completely by air circulation over brine-pipe cooling coils arranged in batteries in the cooler casings.

In the case of ships loading nonprecooled cargoes such as fruit, these figures cannot be used. Such a vessel should have not over 1,000 cu ft cargo space per U.S. ton of machine capacity. In the case of a banana ship, about 20 percent higher machine capacity would be desirable, because the cooling-down time for the fruit from field temperature of 90 F to a delivery air temperature of 53 F should not exceed 24 hr. Allowance must be made for the large amount of heat given off by the fruit during the period of cooling down. Air temperatures lower than 53 F cannot be safely used, and *compartment temperatures should be reduced to 55 F as rapidly as possible* and in not more than 27 hr. The fans should operate at their greatest air capacity all during the cooling-down period, which includes the full loading period.

Cubic Capacity Required for Cargo Stowage. The cubic capacity of refrigerated space required for each long ton (2,240 lb) of cargo carried will differ for each type of cargo. It may be averaged with fair accuracy as follows:

	Cu Ft per Ton
Beef frozen, in quarters, not boned or pressed, and stacked in bulk with necessary dunnage	100
Beef chilled, in quarters, suspended in deck heights of not over 7 ft 0 in. clear	125
Beef boned and compressed with ½-in. strips between layers	54–70
Mutton frozen and stacked	90
Bananas stacked 2 high and 1 flat and allowing for the space of the necessary fruit bin boards	125–140
Oranges in boxes	110–120
Apples in boxes	110–120
Apples in barrels	120–125

authority, or so marks an unpatented article or parcel with patent marks, is liable to a fine of $100 for each offense, one-half to the use of the United States, and one-half to the person bringing the information to a District Court by proper action.

Proceedings in the U.S. Patent Office. Each application for patent found to be correct in form is examined in its turn. Rejections of claim or requirements of amendment must be answered by the applicant (or his attorney) within six months after the date of the official communication, or the application will be held to be abandoned. It can be renewed only by filing a fresh application, except in extraordinary circumstances.

After allowance of an application, the applicant has 6 months in which to pay the final fee. Lapse of 6 months without payment of the fee makes forfeiture of the application.

Interferences. When an application for patent is found to interfere with, *i.e.*, to present or claim substantially the same inventions as (1) another pending application, (2) a patent issued on a date less than one year prior to the date of the application in question, (3) a reissued patent, of which the original was granted on a date less than one year prior to the date of the application in question, the two interfering cases are impleaded in an action called an interference, the purpose of which is to determine, in the Patent Office, which of two or more rival claimants is the first inventor. The practice before the Patent Office in Interference is of such a character that an interference should be conducted only by legal counsel equipped with special experience in that practice.

Design Patents. The inventor of a new, original, and ornamental design for an article of manufacture may obtain a patent for such design. Design patents are granted for 3½ years (fee $10), 7 years (fee $15), or 14 years (fee $30). In other respects, the requirements for design patents are substantially similar to those for mechanical patents. Design patents protect only the ornamental design of an article, *i.e.*, its shape or ornamentation, as distinguished from its mechanical construction or its functional or utilitarian attributes.

Foreign Countries

International Convention for the Protection of Industrial Property. The important provision of the Convention is that any person who has duly applied for a patent in one of the contracting states shall have a right of priority for 12 months in making application in the other states. Such subsequent application is unaffected by any acts accomplished in the interval, as, for example, by publication of the invention or by the working of it.

The laws and regulations in foreign countries differ so much with respect to their requirements as to novelty and patentability, the effect of a public disclosure of the invention either in the country in question or elsewhere, who may obtain a patent, the term of the patent, the cost of a patent, working of the patent, taxes, importation, compulsory licenses, revocation, and other particulars that it is not practicable to make a useful summary in the space allotted to this note. An inventor interested in foreign protection should consult a patent attorney.

LUBRICATION

BY

RAYMOND HASKELL

The **maintenance of an oil film** between two sliding surfaces depends on the **viscosity** of the lubricant, the **pressure** per unit of area, the **speed** of rubbing, the mechanical **construction** of lubricated surface, and the conditions of **operation**. The last two considerations may be so varying as to make all formulas of doubtful value. The smoothness of surfaces, closeness of fit, ratio of clearance to diameter, oil grooving, and vibration may be as important in determining the proper oil viscosity to use as the load, speed, and general bearing dimensions. Truer and smoother surfaces allow closer clearances to be used and, hence, greater loads per unit area to be carried under the same conditions. It must be remembered that the temperature of oil film also will increase under these conditions, and the heavier loads and increased temperatures may cause distortion unless design takes this into account.

The increased **temperature** also may so reduce the operating viscosity of the oil as to bring the operation in the scope of boundary lubrication, *i.e.*, lubrication where the physical and chemical properties of the lubricant in relation to the bearing material become an important factor. If there is strong adhesion of the lubricant (**oiliness**) to the bearing surface (**wetting**), the resistance to removal may be great enough to prevent seizure or scoring, even though beyond the region of fluid film lubrication. If there is an actual mild chemical reaction between the lubricant and the surface, sufficient to form a thin but tough protecting film at high pressures, the lubricant is said to have extreme pressure characteristics. Both oiliness and extreme pressure properties (**film strength**) are functions not only of the lubricant, but also of the bearing material, temperature, and pressure. The coefficients of friction under oiliness or extreme pressure conditions are much higher than found where complete fluid film exists. The foregoing must not be confused with differences in load-carrying capacity depending on the character or structure of the bearing material—babbitt, bronze, brass, lead, silver, etc.—where oiliness or extreme pressure properties are not present in the oil. Some combinations of metals allow much higher load-carrying capacities than others with the same lubricant.

Lubrication of Special Machinery

Steam Cylinders. The characteristics necessary in an oil that is to be used in steam cylinders depend on the temperature and velocity of the steam and its percentage of moisture or degree of superheat. There must also be considered the type of engine, the system of oiling, the priming and foaming of the boiler, and whether the condensed steam is to be re-used or not.

High steam pressures mean high temperatures; high temperatures necessitate the use of a heavier bodied oil or a larger quantity of a lighter bodied oil, in order to maintain a film. An excess of oil is undesirable under high-temperature conditions as it may carbonize and cause gumming. It is advisable to use the heaviest bodied oil that will atomize completely under the operating conditions.

Wet Steam. Most steam is wet at some time during its passage through the engine, and even highly superheated dry steam, unless reheated, may become wet. Moisture in the steam is thrown to the cylinder walls and will displace the lubricating oil unless

MISCELLANEOUS

Lenses

Lenses are transparent bodies which, from the curvature of their surfaces, cause light waves traversing them to converge or diverge. Optical lenses have one or both surfaces of spherical curvature. Biconvex (), plano-convex (| and concavo-convex ((lenses are thicker at the center than at the edges and have a convergent effect. Biconcave)(, plano-concave)| and convexo-concave ((lenses are thinner at the center than at the edges and have a divergent effect. The distance from the center of a lens to the point at which incident plane light waves are brought to a focus is called the **focal length** (f). If u is the distance from the source of light to the lens, and v the distance from the lens at which the image is formed, then, for a biconvex lens, $1/f = 1/u + 1/v = (n-1)[(1/r_1) + (1/r_2)]$, where $r_1(r_2)$ is the radius of curvature of the lens surface nearer to (farther from) the source of light, and n is the index of refraction of the material of the lens ($=1.5$ to 2.0 for glass). For a biconcave lens, minus signs should precede the reciprocals of f, v, r_1, and r_2 in the formula. For a plano-convex or plano-concave lens, $r_2 = \infty$ and $1/r_2 = 0$. When $u = \infty$ (*i.e.*, the incident waves are plane), $f = v$; when $u = f$, $v = \infty$ (*i.e.*, the transmitted rays are parallel). The **magnifying power** of a lens is measured by $1/f$, the practical unit being that of a lens for which $f = 1$ meter. The magnifying power of a number of lenses in contact is the algebraic sum of their individual powers, the powers of diverging lenses being considered negative. Thus, for two *convex* lenses of focal lengths f_1 and f_2, $1/f = 1/f_1 + 1/f_2$; if the second lens be concave (*i.e.*, divergent), $1/f = 1/f_1 - 1/f_2$.

The **velocity of light** in a vacuum = 186,330 miles per sec. (Weinberg.)

Sizes of Type

The unit of height of a line of printer's type is the "point." 1 point = 1/72 in. Six-point type is consequently 6/72 = 1/12 in. high. The smaller sizes of type are 3½-point (Brilliant), 4-point (Excelsior), 4½-point (Diamond), 5-point (Pearl), and 5½-point (Agate). The sizes generally employed in books and periodicals are given below. The smaller sizes are not generally available.

6-point or Nonpareil	10-point or Long Primer
7-point or Minion	11-point or Small Pica
8-point or Brevier	12-point or Pica
9-point or Bourgeois	

Greek Alphabet

Alpha	Α α	Eta	Η η	Nu	Ν ν	Tau	Τ τ
Beta	Β β	Theta	Θ θ ϑ	Xi	Ξ ξ	Upsilon	Υ υ
Gamma	Γ γ	Iota	Ι ι	Omicron	Ο ο	Phi	Φ φ ϕ
Delta	Δ δ	Kappa	Κ κ	Pi	Π π	Chi	Χ χ
Epsilon	Ε ε	Lambda	Λ λ	Rho	Ρ ρ	Psi	Ψ ψ
Zeta	Ζ ζ	Mu	Μ μ	Sigma	Σ σ ς	Omega	Ω ω

the latter is compounded to resist such action. Fatty oils (lard, tallow, degras, etc.) have a stronger adhesion (oiliness) to metals than straight mineral oils and also will displace water from metal surfaces where mineral oils will not. They will also form an emulsion with the water, increasing the resistance to removal of the oil. Compounded oils (mineral oils plus fatty oils) act similarly to fatty oils and maintain oil films in presence of moisture better than straight mineral oils.

The amount of fatty oil necessary to preserve lubrication depends on the amount of water present and the nature of the mineral oil. Too much fatty oil may cause carbonization if steam is very hot, and so in superheated steam installations little or no compound is used unless the steam in the low-pressure cylinder is wet.

When condensed steam is to be used for ice making or other industrial purposes, either a pure mineral oil or one compounded with an easily removable fatty oil must be used. In some vertical marine engines, no oil is used, the walls being lubricated with condensed steam.

Recommended Properties of Lubricants for Steam Cylinders

Condition of steam	Saybolt viscosity, sec, at 210 F	Percentage of compound
Superheated steam	145–200	0–3
Saturated steam, below 150 lb per sq in	95–135	5–8
Saturated steam, above 150 lb per sq in	125–165	3–6
Very wet steam	95–135	8–10

Steam cylinder oils are generally applied by the **atomization method**, though direct application by oilers may be used on valves or other places where the steam cannot carry the oil satisfactorily. In the atomization method, oil is fed into the steam line by "spoons" a few feet back of the throttle valve. The "spoon" distributes the oil so that the steam breaks it up into a fine spray which is carried along with the steam and deposited on walls, piston rods, or other exposed surfaces. Sometimes more than one point of distribution is necessary.

The oil is fed to the spoon by a mechanical lubricator, which varies the feed according to the speed. Another type—hydrostatic—of lubricator operates by means of condensed steam displacing oil from a small supply tank into the oil feed line and to the spoon.

There is no fixed rule as to placing of spoons, except that they must be far enough away from the valve chamber to assure atomization, and not so far as to allow deposition in bends or on pipe walls.

Steam turbines demand oils of maximum stability and ability to separate quickly from water, even after long use. Turbines operate with a continuous oiling system in which the oil is regularly cleansed or filtered to remove water, impurities, or deteriorated oil. The deterioration is usually oxidation, owing to the hot oil being churned in the presence of air. Therefore, the best turbine oils are the least susceptible to oxygen.

As deterioration of oil depends on the temperature and presence of air and water, sufficient quantity of oil should be maintained in the oiling system to permit of a low average temperature, and also time for water and air to separate from the oil. If not possible to allow oil to "rest," coolers should be used, and the oil should be cleansed frequently.

When turbines alone are lubricated, viscosities between 140 and 300 SU at 100 F are used. If gears are lubricated from the same system, viscosities up to 500 SU at 100 F are employed.

Internal-combustion Engines. The larger internal-combustion engines are generally lubricated by two or three oils—one for cylinders, one for bearings, and, in case of the air injection or the two-cycle scavenging type, one oil

A.S.M.E. POWER TEST CODES

The A.S.M.E. has standardized the methods for testing the equipment ordinarily used in power plants. The Individual Test Codes include a code on General Instructions which applies to all individual tests and a code entitled Definitions and Values which states the units to be employed in reporting tests. There is also an extensive auxiliary section dealing with *instruments and apparatus* and *containing a description and analysis of* instruments commercially or otherwise available which may be used in tests of power plants and which covers instruments used for the measurement of pressure, temperature, head, quantities of material, electrical measurements, mechanical power, indicated horsepower, heat of combustion, chemical composition of fuels, oil, and products of combustion, quality of steam, time, speed, and other measurements.

The individual Test Codes include Displacement Compressors, Vacuum Pumps and Blowers, applying to rotary types of machines operated on a positive displacement principle; Centrifugal Compressors, Exhausters, and Fans; Atmospheric Water-cooling Equipment, applying only to equipment used for cooling the comparatively large amounts of water required for power or industrial purposes; Feed-water Heaters, applying to both open and closed boiler feed-water heaters; Steam-condensing Apparatus, including rules for determining the absolute pressure at the steam-inlet nozzle, the thermal transmittance of surface condensers, the amount of undercooling of the condensate, and the percentage of dissolved oxygen in the condensate; Reciprocating Steam Engines; Reciprocating Steam-driven Displacement Pumps, including both pump and engine; Stationary Steam-generating Units, including superheater, economizer, air heater, furnace, and fuel-burning equipment; Hydraulic Prime Movers; Internal-combustion Engines; Gas Producers, intended primarily for producers whose gas is to be used for power purposes; Steam Locomotives, covering laboratory and road tests; Refrigerating Systems, including reciprocating compression systems and absorption machines; Solid Fuels and Liquid Fuels, giving standard methods for the determination of those properties which indicate the value of the fuels when used in the generation of heat and power.

These Test Codes may be purchased from the A.S.M.E., 29 West 39th Street, New York City.

for the air compressors. In the small engines and especially in high-speed types, one oil suffices, though the oil may be different for different climatic conditions. When an oil is used for cylinders or air compressors, chemical stability is perhaps the most important characteristic; viscosity at the highest operating temperature is perhaps the next, though this value is becoming of less importance as greater precision of manufacture and smoother surfaces are obtained. The smoother the surface and the smaller the clearance, the lower the viscosity necessary, provided the temperature is not simultaneously greatly increased.

As speeds and powers are increased, more heat must pass into or through the lubricating oil from the piston. Heat conducted or radiated from the burning gases also tends to oxidize or polymerize the oil on the cylinder walls. The oil, therefore, must be refined or treated to withstand chemical changes and to prevent the formation of tars, varnishes, or lacquers which cause rings and valves to stick and also collect road dust and soot from the combustion chamber to form a sludge or hard carbon coating.

In a precisely made engine, oil consumption is low, but if rings get worn or stuck, not only do combustion gases "blow-by" and accelerate sludging, but also oil is pumped up into the combustion space faster than it can burn, forming carbon and fouling sparkplugs. This naturally increases oil consumption, and means a loss in power due to poor compression. New engines of the high speed automotive type, when operated at maximum efficiency of fuel, consume about one gal of lubricating oil to 150 to 200 gal of gasoline. Large buses are less efficient on account of stops and starts.

The crankshaft bearings of almost all internal-combustion engines are lubricated by a pressure circulating system. In some cases, the oil is carried even to the wrist pin by pressure, but most wrist pins are lubricated by spray from the crankshaft or excess oil from the cylinder walls. In some small engines, lubrication is almost entirely by splash from the crankshaft.

In some small two-cycle engines, as used in marine service, the lubricating oil is mixed with the fuels and feeds directly to the cylinder walls where the gasoline evaporates from it into the combustion space. Much larger quantities of oil are necessary under these circumstances. This same system, only using smaller quantities of oil, is sometimes employed for running in new engines, or even as a supposed safeguard in addition to the regular lubricating system.

Oil and bearing temperatures have risen rapidly as engine powers have been increased, to such extent that new bearing materials have become necessary. Some of these new bearing metals are very sensitive to the chemical action of oxidized lubricating oil at high temperatures and, hence, have introduced a new problem to oil producers, as well as bearing manufacturers. Such improvements have been made that bearings are now successfully carrying pressures several times as high as a few years ago, at much higher temperatures, and with thinner oil films.

Diesel-engine lubrication is little different from that of Otto cycle engines, except as to the cylinder walls. Lubrication here is complicated by the fact that although there is always an excess of air, nevertheless, in spite of this excess, complete combustion is rarely existent if operation is carried on in the most efficient manner. This results in considerable quantities of combustion-chamber soot being carried down into the lubricating oil. The successful oil must not allow this soot to coagulate or be held as a tarry mass which may cause sticking rings, valves, or even piston drag. This may

MUSCULAR ENERGY OF MEN AND ANIMALS

The accompanying table gives results obtained by Poncelet, Morin Rankine, and others. The work of Dr. F. W. Taylor shows that a maximum amount of shoveling may be accomplished by the use of a shovel taking up a load of 22 lb. Also that by the introduction of rest periods at stated intervals (determined from a study of the particular task) the amount of work done in a day by a laborer may be greatly increased above the figures given.

Nature of work	Weight moved or resistance overcome, lb w	Velocity of movement, ft per sec v	Work done per sec, ft-lb	Time of working, hr per day	Work done per day, ft-lb
RAISING WEIGHTS					
Man raising his own weight up a stair or ladder........	143	0.5	71.5	8	2,059,200
Man hoisting weight with rope and pulley, and lowering unloaded rope.........	40	0.66	26.4	6	570,240
Lifting weights by hand.....	44	0.56	24.6	6	531,360
Carrying weights on the back upstairs or a ladder, returning unloaded*............	143	0.13	18.6	6	401,760
Pushing loaded wheelbarrow up a 1:12 incline, returning unloaded................	132	0.065	8.6	10	309,600
Shoveling up earth: lift, 5 ft 8 in.....................	6	1.3	7.8	10	280,800
OPERATING MACHINES AND TOOLS					
Pushing or pulling horizontally and continuously (capstan or oar)...........	26.4	2.0	52.8	8	1,520,640
Pushing and pulling alternately in a vertical direction (pump)..................	13.2	2.5	33	10	1,188,000
Turning a crank.............	17.6	2.5	44	8	1,267,200
Horse† operating a horse gin, walking..................	99	3.0	297	8	8,553,600
Horse operating a horse gin, trotting..................	66	6.6	436	4½	7,063,200

* Dr. Taylor ("Principles of Scientific Management," p. 60) cites the instance of a laborer lifting and carrying 1,156 pigs of iron (each weighing 92 lb) up an incline into a car during a 10-hr day. Average distance of travel, 36 ft; total lift (probably not less than) 8 ft. Ft-lb of work (lifting) = 8 × 1156 × 92 = 850,816. Prior to a study of the task and the introduction of proper rest periods, the best day's accomplishment was the transporting of 305 pigs.

† Ox: w = 132, v = 2; mule: w = 66, v = 3; ass: w = 31, v = 2.6.

require a special type of oil or a special treatment, but, in any case, frequent drains are most desirable.

Airplane engines—especially those which are air-cooled—are subject to greater variations in temperature than automotive engines and, hence, have greater machine clearances. The maximum mean effective pressures and maximum temperatures are also quite high and consequently maximum stability is required in the lubricating oil. Usually a viscosity between 100 and 140 sec SU at 210 F is required. In Canada in winter, much lower values are found necessary on account of starting difficulties. Compounded oils are used, especially in Europe, but care must be taken that they do not break down to acid materials injurious to hard bearings. Some airplane engines are equipped with a combination oil heater and cooler so that oil inlet temperature can be kept constant. In order to expedite the warming up of oil in starting, a smaller quantity may be recirculated until the engine reaches desired temperature. Airplane engines work on the "dry-sump" principle. In this, there are two pumps: one pumps the oil out of the crankcase, the other pumps oil from the tank to the lubricating system.

Compressed-air Machinery. The cardinal point in the lubrication of compressed-air machinery is to supply only sufficient oil to keep the walls covered with a thin film. Trouble is usually caused by using too much oil rather than too little. The temperatures reached in air compressors are high, and an oil film subjected to these temperatures will gradually evaporate or decompose, in the latter case leaving a gummy mass which collects around valves and dead passages and may form a hard deposit with the dust from the entering air. It is advisable to use an oil with low distillation end temperature and not generally advisable to use blends of cylinder stocks. There are cases where there is excessive moisture, then it is necessary to use an oil that is compounded so as to produce an emulsion similar to that used in steam cylinders.

The Compressed Air Society suggests, as a minimum quantity of air-cylinder lubricant to maintain a film, about 1 drop of oil per minute for each 600 sq ft of cylinder wall swept by the piston per minute. This requires 1 drop per min in an 8 by 8, 6 drops in a 24 by 24, and 12 drops in a 42 by 42 in. compressor.

Refrigerating Machinery. An oil for use in refrigerating machinery should have a pour test sufficiently low to prevent its congealing on the coldest parts of the system, as some of it is unavoidably carried through to the evaporator coils. Pure mineral oils operate more satisfactorily than compounded oils, as the latter may separate or solidify under the low-temperature conditions. Compressors for ammonia and carbon dioxide are usually lubricated by applying the oil to the piston rod by means of an arrangement around the rod called an oil lantern. The function of this is not only to supply oil to the cylinder, but also to prevent gas from escaping and air from entering the cylinder. In the wet ammonia process, mineral oil clings well to the cylinder walls as the liquid dissolves in the oil, forming a sort of emulsion. In the dry-ammonia and carbon dioxide processes, the adhesion to the walls is not quite so good, and it may be necessary to use auxiliary oilers to get complete lubrication.

In case of machines using various refrigerants such as sulphur dioxide, methyl and ethyl chloride, and Freon, which in liquid state are soluble in oil; a light-bodied straight mineral oil is still satisfactory, as when acting as a lubricant it is in contact with the refrigerant when it is in a gaseous state. In

According to D. K. Clark, a laborer can exert 0.1 hp for 8 hr a day on a windlass or pump, and for periods of a few minutes as much as 0.5 hp. The maximum force which a man can exert in pushing or pulling is 110 to 130 lb; the greatest weight he can ordinarily carry is about 330 lb; the weight he is capable of sustaining ranges from 450 to 650 lb.

Performance in Transporting Loads Horizontally

(The quantity wv is not the work done)

Nature of transportation	Weight moved, lb w	Effective velocity,* ft per sec v	Transport per sec, ft-lb wv	Time of working, hr per day
Man walking unloaded	143	5.0	715	10
Man wheeling load w in a wheel barrow, returning unloaded	132	1.7	224	10
Man traveling with load w on back	88	2.5	220	7
Man carrying load w on back, returning unloaded	143	1.7	243	6
Horse drawing cart, loaded with w, walking	1,540	3.6	5,544	10
Horse drawing cart loaded with w, trotting	770	7.2	5,544	4½
Horse walking with loaded cart, returning empty	1,540	2.0	3,080	10
Horse carrying burden w, walking	264	3.6	950	10
Horse carrying burden w, trotting	176	7.2	1,267	7

* Distance through which w is transported ÷ total time including unloaded return, if any.

The draft of horses is reduced by working them in teams. The draft of a horse in a 2- (4-) [8-] horse team is but 98 (80) [49] percent of that exerted by the horse when worked alone. A 1:100 grade reduces the draft 10 percent; a 1:50 grade, 20 percent; a 1:30 grade, 35 per cent; a 1:20 grade, 60 percent, and a 1:10 grade, 75 percent.

this state, the solubility in oil is not sufficient to harm the lubricating ability of the oil. As the solubility depends on pressure, the gas alternately enters and leaves the oil with pressure changes. A satisfactory oil must allow this gas to come out without foaming. To prevent chemical change under operating conditions the system should be free from moisture and the oils dehydrated.

Refrigerating machinery should in all cases contain an oil separator, as it is impossible to prevent oil being carried out of compressors with the gas. The oil so collected can be re-used; it is sometimes purified before returning to the system.

Textile Machinery. In the textile industry, spindle oil and stainless oil are employed. Spindle oil is highly refined, free from gumming properties, and designed to prevent rusting. For light spindles an oil of about 50 to 60 sec, for heavy spindles 90 to 100 sec SU at 100 F should be used. Stainless oils are usually highly refined petroleum products combined with neatsfoot or other fixed oils. The object is to have an oil that will not show spots on the fabric and that can be easily removed by washing. This latter characteristic is especially essential where the fabric must be dyed or printed after it is formed.

Gears, especially those operating under heavy loads, should use a viscous lubricant that will not be easily displaced from the teeth by squeezing or rubbing, or thrown off by centrifugal force, and will not flow appreciably under ordinary temperatures. With steel mill roll pinions, this compound should have incorporated with it some product that will increase its adhesiveness and prevent its being washed away by water, even under pressure. In places such as cement mills where there is considerable dust flying, heavy oil and frequent feed may be required so as to wash off the accumulated dirt. If gears run in a bath, a heavy oil can be used.

Some modern gears are designed to operate continuously with heavier loads than can be safely carried by straight mineral oils. These gears require an extreme pressure type of lubricant, especially if there is any "shock" loading. These lubricants contain a chemical that reacts with the metal under the instantaneous pressure and temperature conditions so as to prevent welding of metal parts under the very extreme loads where mineral oils would be wiped off. The most active types of extreme pressure lubricants cannot be used under continuous heavy loading, as corrosion is apt to take place. A proper lubricant should only be active at the instant of extraordinary pressure or demand, which should be of short duration.

Chains, if heavy, should be lubricated similarly to ordinary gears. If light and enclosed in a bath, an oil of 300 to 500 sec Saybolt at 100 F should be used; if not enclosed, a viscosity of 150 to 200 sec at 210 F will usually avoid loss by throwing off due to centrifugal force.

Wire ropes should be treated hot with a heavy viscous straight mineral product that will penetrate the strands when hot, but not drop from the rope when at working temperatures. Lubricants must also be applied to the outside occasionally.

Lubricating Systems. Most circulating systems are pressure fed, i.e., the oil is forced from a pump directly to the bearings or gears. The pump may be directly connected or geared to the main shaft or may be driven by a separate motor. The advantage of the former is that the quantity of the oil fed depends on the speed of the engine and is positive. The advantage of the latter is that the oil can be circulated before the machine is started.

All systems have **cooling** and **filtering** or **cleansing** units. The cooling tank is similar to a steam condenser and is generally installed before the cleanser. In some plants, the oil is first passed through a heating tank where it is heated to expedite the separation of air and water. After settling, the oil may be cleansed by passing through filter cloths or by means of **centrifuges**. Some systems pass all the oil through the cleaning apparatus on each cycle, but it is common practice either to clean the oil intermittently or to by-pass a small percentage of the whole continuously through the filters or centrifuges.

In small units, the oil is generally cooled and allowed to settle and not filtered. There should be sufficient capacity in settling and supply tanks so that the oil comes to

SECTION 14

TESTING AND TRIALS OF MACHINERY

BY

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rest on each cycle and is used only a few times a day. Two entirely different lots of oil are often used—one remains at rest, while the other is being used.

All strainers should be installed in duplicate to provide for cleaning and should have a by-pass with relief valve so that if the strainers become clogged the supply of oil will continue. The water pressure in coolers should be less than the oil pressure, to prevent leakage of water into the oil. Some systems pump continuously from the sump or settling tanks to the supply tank in excess of the quantity of oil used, allowing the excess to overflow. Other systems pump only when the oil level in the supply tank gets low and stop automatically when the latter is full. The latter have the advantage of allowing the oil to remain quiet in the supply tank and give better opportunity for the settling out of water and impurities. Sometimes the lubricating pump runs slowly at all times and speeds up when the oil reaches the low level.

Specifications. The most generally used specifications at the present time are those formulated by the United States Navy. These, with their general usage, are given below. In addition to the specifications listed, the Navy Department demands that the oil pass a "work-factor" test. The oil is given a rating by this test according to the change in certain particular physical characteristics after a "run" in a particular type of bearing under

United States Navy Requirements for Lubricating Oils

Classification	Navy symbol	Flash point, deg F, min	Saybolt Universal viscosity, sec		Pour point, deg F, max	Fixed or fatty oils		Carbon	
			130 F	210 F		Per cent	Nature	Residue per cent, max	Neutralization no, max
Forced-feed oils	2075	315	70–90	....	−10	None	....	0.10	0.10
	2110	325	90–120	....	0	None	....	0.20	0.10
	2135	340	120–145	....	0	None	....	0.30	0.10
	2190	350	185–205	....	35	None	....	0.40	0.10
	2250	370	245–280	....	35	None	....	0.50	0.10
Forced-feed oils (automotive and general)	3050	390	....	45–55	0	None	....	0.30	0.10
	3065	410	....	60–70	5	None	....	0.50	0.10
	3080	440	....	75–90	15	None	....	0.80	0.10
	3100	460	....	90–105	25	None	....	1.00	0.10
	3120	480	....	115–125	30	None	....	1.25	0.10
Aviation oils	1042	390	....	40–44	0	None	....	0.30	0.10
	1047	400	....	44–50	0	None	....	0.30	0.10
	1065	420	....	52–68	0	None	....	0.60	0.10
	1080	450	....	76–84	0	None	....	0.90	0.10
	1100	470	....	93–103	10	None	....	1.20	0.10
	1120	490	....	115–125	20	None	....	1.50	0.10
	1150	510	....	140–160	30	None	....	1.80	0.10
Compounded marine-engine oil	4065	350	....	65–75	35	15–20	Blown rapeseed	....	3.00
Mineral marine-engine and cylinder oils	5065	350	....	65–75	35	None	....	....	1.00
	5150	490	....	135–165	60	None	....	3.00	0.15
	5190	525	....	180–220	60	None	....	4.00	0.15
Comp steam cylinder (tallow)	6135	475	....	120–150	60	5–7	....	2.50	1.00
(lard or tallow)	7105	450	....	95–110	40	9–10	....	2.00	1.00
Comp air-cylinder oils	8190	345	180–200	....	35	2–4	Lard oil	0.50	0.25

All lubricating oils shall equal or better the specifications shown in the foregoing table.

TESTS AND TRIALS OF MACHINERY

BY

D. C. MacMILLAN

TESTS PRIOR TO INSTALLATION

The American Bureau of Shipping and *the Marine Inspection Service of* the U.S. Coast Guard require that certain parts of machinery and piping be tested during manufacture or erection. The tests to be performed at the equipment manufacturer's plant or at the shipyard prior to installation (but not including material tests) are briefly described below:

Steam Engines. The following hydrostatic tests are to be witnessed by the American Bureau of Shipping surveyors:

1. High-pressure cylinder, liner, and valve chest to one and one-half times the boiler pressure.

2. Low-pressure and intermediate-pressure cylinders, receivers, and valve chests to one and one-half times the pressure of their relief valves, with a minimum of 30 psi.

Steam Turbines. The following tests are to be witnessed by the American Bureau of Shipping surveyors for all turbines over 135 shp:

1. Turbine casings are to be subjected to hydrostatic tests of one and one-half times the working pressure. For this purpose the casings may be suitably divided by temporary diaphragms for the proper distribution of the test pressures.

2. The turbine is to be tested to 15 percent above the maximum designed speed to operate the overspeed governor.

Diesels. The following hydrostatic tests are to be witnessed by the American Bureau of Shipping surveyor except as noted:

1. All relief valves are to be tested and set in the presence of the surveyor.

2. Cylinders and liners are to be tested to one-half the initial pressure. Where cylinders or liners are so designed that the parts subject to internal pressure may be accurately gaged for thickness of materials, the hydrostatic pressure may be reduced to 50 psi.

3. Water jackets are to be subjected to a pressure of 50 psi.

4. Air-compressor cylinders are to be subjected to a pressure equal to one and one-half times the maximum pressure.

5. Air-cooler coils are to be subjected to a pressure equal to one and one-half times the maximum pressure.

Tests of items under paragraphs (4) and (5) are to be made at the manufacturer's plant but need not be witnessed by the surveyor.

Condensers. The American Bureau of Shipping surveyor shall witness a hydrostatic test of the condenser body, with tubes and ferrules fitted, to 15 psi. (Also see Pressure Containers, p. 1915.)

Pumps. There are no requirements for tests prior to installation. However, it is general practice to test the steam ends of reciprocating pumps hydrostatically to one and one-half times the working pressure and the water ends to twice the working pressure. Centrifugal pump casings are generally tested to one and one-half times the shutoff pressure.

specified conditions. The less the oil changes under this test, the better the rating in its class.

Viscosities of lubricants are usually determined by the Saybolt Universal viscosimeter (see p. 244) although the Engler viscosimeter is also used. With this instrument, the outflow time in seconds for 200 cc of the oil is observed; when divided by the outflow time for 200 cc of water at 68 F, it gives results in Engler degrees. The outflow time of the water must be between 50 and 52 sec; the usual time for an instrument of normal dimensions is 51.3 sec. The **Redwood Standard** viscosimeter is the usual instrument in Great Britain; a modification of it, the **Redwood Admiralty** viscosimeter, is used in testing oil supplied to the British Navy.

There is a strong tendency at the present time to use **kinematic viscosity** expressed in **centistokes.** An approximate conversion table showing the relationship among the various viscosity units as developed by the Standard Oil Development Co. is given below. The values for the Saybolt and Redwood instruments vary slightly with the temperature of operation. The supplementary table indicates the magnitude of this variation.

Kinematic Viscosity Conversion Table
Centistokes to Engler, Saybolt, and Redwood Units

Centistokes	Engler degrees	Saybolt seconds at 130 F	Redwood seconds at 140 F	Centistokes	Engler degrees	Saybolt seconds at 130 F	Redwood seconds at 140 F	Centistokes	Engler degrees	Saybolt seconds at 130 F	Redwood seconds at 140 F
2.0	1.140	32.66	30.95	18.0	2.644	69.37	78.45	41.0	5.465	190.6	168.3
2.5	1.182	34.46	32.20	19.0	2.735	93.43	82.10	42.0	5.590	195.1	172.3
3.0	1.224	36.07	33.45	20.0	2.870	97.69	05.75	43.0	5.720	199.6	176.4
3.5	1.266	37.67	34.70	21.0	2.984	101.9	89.50	44.0	5.845	204.2	180.4
4.0	1.308	39.17	35.95	22.0	3.100	105.2	93.25	45.0	5.975	208.8	184.5
4.5	1.350	40.78	37.20	23.0	3.215	110.5	97.05	46.0	6.105	213.4	188.5
5.0	1.400	42.38	38.45	24.0	3.335	114.8	100.9	47.0	6.235	218.0	192.6
5.5	1.441	43.98	39.80	25.0	3.455	119.1	104.7	48.0	6.365	222.6	196.6
6.0	1.481	45.59	41.05	26.0	3.575	123.5	108.6	49.0	6.495	227.2	200.7
6.5	1.521	47.19	42.40	27.0	3.695	127.9	112.5	50.0	6.630	231.8	204.7
7.0	1.563	48.79	43.70	28.0	3.820	132.4	116.5	52.0	6.890	241.1	212.8
7.5	1.605	50.44	45.00	29.0	3.945	135.0	120.4	54.0	7.106	250.3	221.0
8.0	1.653	52.10	46.35	30.0	4.070	141.2	124.4	56.0	7.370	259.5	229.1
8.5	1.700	53.80	47.75	31.0	4.195	145.6	128.3	58.0	7.633	268.7	237.2
9.0	1.746	55.51	49.10	32.0	4.320	150.0	132.3	60.0	7.896	277.9	245.3
9.5	1.791	57.21	50.55	33.0	4.445	154.5	136.3	62.0	8.159	287.2	253.5
10.0	1.837	58.91	52.00	34.0	4.570	159.0	140.2	64.0	8.422	296.4	261.6
11.0	1.928	62.42	55.00	35.0	4.695	163.5	144.2	66.0	8.686	305.6	269.8
12.0	2.020	66.03	58.10	36.0	4.825	168.0	148.2	68.0	8.949	314.8	277.9
13.0	2.120	69.73	61.30	37.0	4.955	172.5	152.2	70.0	9.212	324.0	286.0
14.0	2.219	73.54	64.55	38.0	5.080	177.0	156.2	72.0	9.475	333.3	294.1
15.0	2.323	77.35	67.95	39.0	5.205	181.5	160.3	74.0	9.738	342.5	302.2
16.0	2.434	81.25	71.40	40.0	5.335	186.0	164.3	75.0	9.870	347.2	306.3
17.0	2.540	85.26	74.85								

Supplementary Kinematic Viscosity Conversion Table

Centistokes	2	6	10	20	30	40	50	60	70
Saybolt at 100 F	32.60	45.50	58.80	97.50	140.9	185.7	231.4	277.4	323.4
Saybolt at 210 F	32.83	45.82	59.21	98.18	141.9	187.0	233.0	279.3	325.7
Redwood at 70 F	30.20	40.50	51.70	85.40	123.7	163.2	203.3	243.5	283.9
Redwood at 200 F	31.20	41.50	52.55	86.90	126.0	166.7	208.3	250.0	291.7

Piping. Tests (material and hydrostatic) on all plain piping to be used for pressures over 150 psi are required by the American Bureau of Shipping at the manufacturer's plant. The surveyor shall witness the following tests on fabricated piping after bending and attachment of flanges:

Steam, boiler-feed, and blow-off pipes, valves, and fittings to twice the working pressure, but not more than the working pressure plus 1,000. For fusion-welded piping, see below.

The U.S. Coast Guard Marine Inspection Service requires that sections of main and auxiliary steam piping subject to boiler pressure, also feed piping, shall be tested hydrostatically with flanges attached to a test pressure equal to twice the maximum allowable working pressure of the boiler. This test is to be witnessed and certified by the inspector.

The Marine Engineering Regulations (August, 1943) of the U.S. Coast Guard also require that all welded Class I piping (piping for pressure over 125 psi gage, hot-water piping for temperatures over 200 F, and oil piping for temperatures exceeding 150 F regardless of pressure) be tested after fabrication to twice the maximum pressure to which the piping will be subjected in service.

Air Compressors. No tests required other than for compressors used with diesels, as noted above.

Boilers and Other Pressure Containers. Items included to be tested by the American Bureau of Shipping:

1. All boilers intended for working pressure above 30 psi.
2. All unfired pressure containers intended for working pressures above 100 psi.
3. Unfired pressure containers intended for working pressures of 100 psi and less whose shell diameters are more than 30 in. and which are necessary for the safe operation of the vessel, such as condensers, evaporators, feedwater heaters, coolers, and other similar pressure containers.

The American Bureau of Shipping surveyors are to witness hydrostatic tests on the above to a test pressure not less than one and one-half times the working pressure for containers of plate or pipe construction and twice the working pressure for cast shells. In no case shall the test pressure be less than 15 psi.

Requirements of the U.S. Coast Guard, Marine Engineering Regulations concerning boilers are quite detailed. Regarding evaporators, heaters, traps, separators, pressure vessels, and miscellaneous appliances, they require that shells made of plate construction be subjected to a hydrostatic test equal to one and one-half times the working pressure for which it is designed; that coils and all cast shells shall be tested to twice their respective working pressures; and that these tests be witnessed by an authorized inspector.

Fusion-welded Boilers and Other Pressure Containers. The American Bureau of Shipping requires that all fusion-welded boiler drums and other pressure containers, also fusion-welded piping, shall be subjected to a hydrostatic test pressure of one and one-half times the allowable working pressure and, while under this pressure, shall be given a thorough hammer or impact test. The hammer test shall consist of striking the plate at about 6-in. intervals on both sides and for the full length of the welded joints. The weight of the hammer shall be approximately 1 lb for each $\frac{1}{16}$ in. of the plate thickness but not to exceed 10 lb. The edges of the hammer should be rounded to prevent defacing the plates.

PATENTS FOR INVENTIONS

BY

ODIN ROBERTS

(Revised by Robert Cushman)

REFERENCES: Albert H. Walker, "Patents," Baker Voorhis & Co. William C. Robinson, "Patents," Little, Brown.

United States of America

What Subject-matter Is Patentable. Any original and useful art (*i.e.*, process or method), machine, article of manufacture, or composition of matter, or any improvement on either, or any asexually reproduced new variety of plant other than a tuber-propagated plant, which has not been

(1) Known to or used by others in the United States prior to the invention or origination;

(2) Described anywhere in any patent or printed publication prior to the invention or origination, or more than one year before application is made for patent in the United States;

(3) In public use or on sale in the United States more than one year before application is made for patent in the United States;

(4) Patented to the inventor in some other country, which by treaty or convention has established reciprocal relations with the United States in patent matters, upon an application filed more than 1 year prior to the patent application in the United States. (This applies to practically all civilized countries.)

Who May Apply for Patent. The original inventor, or inventors jointly, if more than one. The existence of joint invention can be determined only by the facts of each case. The only general rule is that if two or more have worked and consulted together in the development of an invention they are properly joined as applicants for patent. The *owner* of an invention, by assignment and sale from the inventor, may not *apply* for patent, but may receive the patent as assignee, provided the deed of assignment has been recorded in the Patent Office.

Term of Patent. Seventeen years from the date of issue. No extensions are granted under the general law.

The patent grant gives to the patentee for the term of the patent, the sole and exclusive right to manufacture, sell, and use the alleged invention patented.

The patent application should be prepared by a competent solicitor. For information concerning the forms and rules, obtain from the Commissioner of Patents, Washington, D.C., a copy of the Rules of Practice of the United States Patent Office.

Cost of Obtaining a Patent. The Patent Office fees are: Filing fee and final fee to obtain patent after allowance. Each fee is \$30 plus \$1 for each claim in excess of twenty. The cost of preparing drawings and specifications will, of course, vary with the subject-matter.

Sale, or assignment of patent, to be binding, must be by an instrument in writing. Acceptable forms are found in the Rules of Practice of the Patent Office. To provide for constructive notice to all, an assignment must be recorded in the Patent Office within 3 months of its date.

Following this test, the pressure shall be raised to twice the working pressure and held there for a sufficient length of time to permit a thorough inspection of all joints and connections. The U.S. Coast Guard requires a similar test.

Refrigerating Machinery. The American Bureau of Shipping requires that compressors, separators, condenser and evaporator coils, headers, connections, and all other parts subject to high pressure are to be tested by hydraulic pressure to three times the working pressure and afterward to an air pressure of one and one-half times the working pressure while submerged in water.

Gas condenser and evaporator casings are to be tested to a hydraulic pressure of not less than twice the working pressure. The tests required in this and the preceding paragraph are to be made at the plant of the manufacturer whose affidavit may be accepted by the surveyors.

Electrical Equipment. The American Bureau of Shipping requires that electrical propulsion generators and motors and auxiliary generators and motors of 100 kw and over shall be tested in the presence of and inspected by the surveyors at the plant of the manufacturer. For auxiliary machines of less than 100 kw the tests may be carried out by the manufacturer whose certificate of tests will be acceptable and should be submitted upon request from the Bureau. Sufficient tests shall be made to ensure that the generators and motors are in accordance with the requirements and, for original units of a type, shall include rated-load heat run, plot of saturation curve, regulation tests at operating temperature, cold resistance measurement, air-gap check, commutation check, end-play setting, insulation resistance, running balance, bearing temperatures, and high potential test.

For subsequent duplicate generators and motors the tests shall include plot of saturation curve, cold resistance measurement, air-gap check, commutation check, end-play setting, insulation resistance, running balance, bearing temperatures, and high potential test.

Spare rotors for propulsion apparatus and auxiliaries of 100 kw and over shall be dynamically balanced in the presence of a surveyor, and the tests shall include cold-resistance measurement, insulation-resistance and high potential tests. Spare coils shall be tested for short-circuited turns; in addition, the regular high potential test shall be applied if ground insulation is assembled on the coils.

Controls for propulsion equipment shall be inspected when finished and dielectric tests made on the various circuits. The satisfactory tripping and operation of all relays, contactors, and the various safety devices shall also be demonstrated.

Cables for the propulsion equipment shall be subjected to dielectric and insulation tests in the presence of a Bureau Surveyor. All other cables are to be tested to stated requirements by the manufacturer whose certificate of such tests may be accepted.

Detail insulation tests of generators, motors, cables, and controls are also specified.

INSTALLATION TESTS

General

Tests are required on all piping and equipment after installation in the vessel and are to be performed in the presence of the inspectors of approval agencies requiring the tests, such as the U.S. Navy, U.S. Maritime Com-

GRAPHICAL REPRESENTATION OF FUNCTIONS

For graphical methods in statistics, etc., see W. C. Brinton's "Graphical Methods for Presenting Facts."

EQUATIONS INVOLVING TWO VARIABLES

The Curve $y = f(x)$. To represent graphically any function, y, of a single variable, x, lay off the values of x as abscissae along a uniformly graduated horizontal axis, whose positive direction (as usually chosen) runs to the right, and at each point on this x-axis erect a perpendicular (called an ordinate) whose length represents the value of y at that point. The unit of measurement for the y-scale, whose positive direction (as usually chosen) runs upward, need not be the same as the unit for the x-scale. Draw a smooth curve through the extremities of the ordinates; this is the **graph** of the given function in rectangular co-ordinates, or the **curve** of the function.

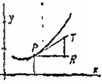


FIG. 1.

To measure graphically the rate of change of the function at any point P (Fig. 1), draw the tangent at P; then **rate of change** at $P = RT/PR$, where RT and PR are measured in units of the y-axis and x-axis, respectively. This ratio, which is positive if RT runs upward, negative if RT runs downward, is equal to the derivative of the function at the point P (see p. 157).

Graphs of Important Functions. Figs. 2–9 show the graphs (in rectangular co-ordinates) of the most important elementary functions, namely:

The **linear function,** $y = mx + b$ (Fig. 2).

The **power functions,** $y = x^n$ [n positive (parabolic type); n negative (hyperbolic type)] (Fig. 3).

The **exponential function,** $y = 10^x$ or $y = e^x$, and the **logarithmic function,** $y = \log_{10} x$ or $y = \log_e x$ (Fig. 4).

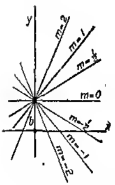
The **trigonometric functions** (Fig. 5), and the inverse trigonometric functions (Fig. 6).

The **hyperbolic functions** (Figs. 7 and 8) and the inverse hyperbolic functions (Fig. 9).

Various **special functions** (Figs. 10–12).

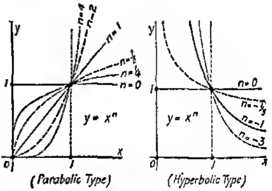
By a slight modification, each of these diagrams may be made to represent a somewhat more general function than that for which it is primarily intended. For, if x is replaced by $x - a$ in the equation, this merely requires re-numbering the x-axis so that each number is moved a units to the left; and similarly, if y is replaced by $y - b$ in the equation, this merely requires re-numbering the y-axis so that each number is moved b units downward. (Such a change is called a translation of the curve to the right, or upward.) Further, if x is replaced by x/c [or y by y/c] in the equation, it is merely necessary to multiply each of the numbers written along the x-axis [or y-axis] by c, in order to adapt the graph to the new equation. (Such a change is called a "stretching" of the curve along one of the axes.)

Empirical Curves. Any set of values of two variables x and y can be represented by plotting the points (x,y) on rectangular co-ordinate paper, and drawing a smooth curve through these points. The points which correspond to actual data should be clearly indicated by small circles or crosses, intermediate points being spoken of as interpolated points. While this process of graphically interpolating a continuous series of points *between* given values is usually fairly safe, the process of extrapolation—that is, extending the curve *beyond* the range of the given values—is dangerous.



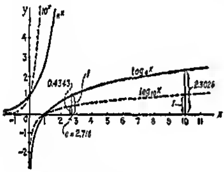
Linear function, $y = mx + b$.

Fig. 2.



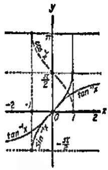
Power function, $y = x^n$.

Fig. 3.



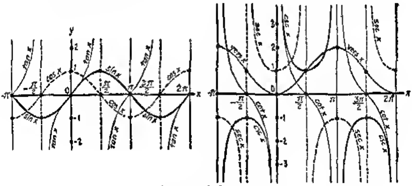
Exponential function (10^x or e^x).
Logarithmic function ($\log_{10} x$ or $\log_e x$).

Fig. 4.



Inverse trigonometric functions.

Fig. 6.



Trigonometric functions.

Fig. 5.

To Find a Mathematical Equation to Fit a Given Empirical Curve. This problem is one which in general requires much patience and ingenuity. Only the simplest cases can be mentioned here.

Case 1. If the given empirical curve is a straight line, then the law connecting the given values of x and y is $y = mx + b$, where m = the slope of the line, and b = the value of y at the point where the line crosses the y-axis. If

mission, U.S. Coast Guard Marine Inspection Service, and the American Bureau of Shipping. These tests will determine the success of the installation and are to be performed prior to the dock trials, except such tests as cannot be adequately made at the dock owing to local conditions may be performed at sea. Given below are installation test procedures for vessels built to these requirements.

Data sheets, giving the name of the vessel, hull numbers, date of test, observed data, name plate data, and space for the inspectors initials should be prepared for each item of equipment.

Preliminary Installation Tests

All piping systems should be carefully examined and tested before the official hydrostatic test is applied. The system should be examined to see that it conforms with the approved arrangement and diagrammatic plans, with attention paid to all small lines, pressure gages, thermometers, pressure switches, etc. Particular attention should be paid to see that check and stop-check valves are not installed backward in the lines. A preliminary hydrostatic test should be conducted on each system, and all leaks should be made tight.

Megger or equivalent tests should be made on each electric cable during the installation at the time connections are being made. These tests should be conducted in accordance with the procedure for insulation-resistance tests.

Before conducting any test, the shipyard operator should familiarize himself with the equipment and with all operating instructions furnished with it.

All engine-room and deck auxiliaries should be tested by the shipyard prior to the official tests. These tests may be conducted using a shore supply of current for the motors or steam for the steam-driven auxiliaries. The duration of the preliminary tests should be sufficient to indicate that the machinery is functioning properly. Before an electrical test is started, all connections should be carefully checked and insulation-resistance readings should be taken.

All motors throughout the ship for nonreversing service should be checked for proper direction of rotation. When the direction of rotation is established, a careful check of the brush rigging should be made and, if necessary, reaction-type brush holders should be reversed. Adjustments of this nature should be made in strict accordance with the instructions furnished by the motor manufacturer.

The lube-oil system should be properly cleaned and flushed in an approved manner.

Piping Systems

In general, these tests are to be conducted before the application of lagging obscures proper inspection for leaks, but in special cases the contractor may, upon written application, conduct them after portions of the lagging between pipe joints have been applied.

The sequence of piping system tests may be decided by the contractor to suit conditions, but sufficient notice shall be given to the interested parties so that they may prepare for and witness the tests. In this respect the contractor shall prepare a schedule of piping system tests giving the approximate date on which he plans to conduct each test.

overload condition shall be created without causing damage to the installation. Adjustments shall be made to effect operation of the protective devices at the specified overload rating applicable. A sample data sheet is given by Fig. 1.

Steam Reciprocating Pumps. Each reciprocating pump should be brought up to its rated double strokes per minute and total head. The designed total head should be obtained by properly throttling a valve in the discharge of the pump.

Readings should be taken of all data at least every 10 min during the continuous test that should be of at least ½-hr duration.

Any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of the pump should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of noise, gland leakage, correctness of alignment, length of stroke, and operation of the valve gear. The operation of the relief valve and its setting should be checked and, where a governor valve is fitted, its operation should also be checked.

Turbine-driven Pumps. The turbine-driven pumps should be operated at rated speed under service conditions. Readings should be taken of all data at least every 10 min during the continuous test that should be of at least ½-hr duration.

Any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of the pump or turbine should be carefully recorded at the time of occurrence in order that any serious defects may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, gland leakage, correctness of alignment, and adequacy of lubrication. When possible, an estimate of the amount of fluid pumped should be obtained.

The operation, settings, etc., of control and protective devices shall be demonstrated and checked.

Forced-draft Fans

Each forced-draft fan shall be operated for 1 hr at its maximum rated speed and pressure. The proper pressure shall be maintained by adjusting the control dampers or boiler registers. Readings shall be taken every 10 min of all data.

After completion of the above test, the fan shall be operated at the following conditions to demonstrate the characteristics over the operating range:

Registers open at normal speed.
Registers open at maximum speed.
Registers closed at normal speed.
Registers closed at maximum speed.

Any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of the equipment should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, and adequacy of lubrication.

If the fan is motor-driven, test requirements are the same as given above for Motor-driven Pumps, p. 1925.

Installation tests on piping systems should be conducted only after all hydrostatic shop tests have been completed as required by the U.S. Coast Guard, American Bureau of Shipping, and as noted on the piping detail drawings. If any welding is done on a pipe after it has been installed in the vessel, this pipe shall be retested as required by the U.S. Coast Guard, American Bureau of Shipping, and as noted on the piping detail drawings.

All equipment, such as heaters, coolers, or strainers in the system, which are subjected to the pressure of the system, shall be tested with the system. For equipment such as heaters or coolers pressure shall be applied to one side at a time so that adequate inspection for leaks may be made.

Label plates on valves, manifolds, etc., and drains and instruction plates on pipe hangers (where required) shall be installed in their entirety at the time of the test. If, for any reason, some of these items should be omitted, the same shall be specifically noted in the report.

The systems shall be cleaned thoroughly of all dirt, rust, scale, etc., before they are tested.

Pressure is to be applied to each system in its entirety and, in cases where this is not practical owing to tank suctions and the like, the joint nearest the tank, etc., shall be broken and blanked, so as to include as much of the system as it is possible in the test. Specific mention shall be made of those joints that have been excluded from the test so that their tightness may be observed under service conditions.

The pressure specified shall be maintained long enough to permit an adequate and detailed inspection of the system as well as to repair the leaks that occur. In the cases where it is necessary to remove the pressure to effect repairs, the test pressure shall be applied again after the leaks have been repaired.

Where piping is to be subjected to a test pressure in excess of the relief valve setting, the valves should be gagged or fitted with a blank on the inlet to the valve. An attempt should not be made to reset the relief valves to the higher test pressure.

All gages used for testing should be calibrated before conducting the test.

For a more detailed description of the Freon piping test, see procedure for testing Refrigeration Equipment, p. 1929.

Typical piping test requirements are given by Table 1.

Insulation-resistance Test

Megger or equivalent tests should be made of each cable during the installation at the time connections are being made to equipment. Circuit continuity should be checked at the same time to avoid errors. Record weather conditions, particularly humidity at the time of measurement.

A record of insulation resistance should be kept, and a tabulation should be made on the test form before the dock trials. The tabulation should then be submitted to the local maritime commission inspector for approval, on the basis of Par. 45.03 of *A.I.E.E.* 45.

The minimum insulation resistance of all generators and motors should be approximately 250,000 ohms. The minimum insulation resistance of fields of machines separately excited with voltage less than the rated voltage of the machine should be of the order of ½ to 1 megohm.

Megger tests shall include positive to negative, positive to ground, and negative to ground for power feeders. For lighting feeders, the same tests

Air Compressors

Each air compressor shall be operated at its designed speed and pressures. The compressor should be tested with all control apparatus in place and operating. It should be brought up to the rated speed and discharge pressure. Air should be bled from a convenient point in the system to permit the compressor to operate continuously for the duration of the test.

After completion of the heat-run test, the setting and operation of the unloading valves, relief valves, start-stop switch, as well as the ability of the compressor to charge the system from atmospheric pressure to the rated pressure, shall be demonstrated.

Motor test requirements are the same as given above for Motor-driven Pumps, p. 1925.

Oil Purifiers

The purifier shall be operated at its rated capacity with oil at proper temperature to demonstrate satisfactory operation and installation. The test should also demonstrate that the attached pumps are able to take suction through the normal or emergency suction and deliver the normal quantity of oil at the required pressure to the storage, settling, and sump tanks.

The motor should be brought up to its rated speed, and the discharge valve on the discharge pump should be throttled to obtain the designed discharge head. Readings should be taken every 15 min during continuous operation until the temperatures no longer show an increase. The time of continuous test shall be not less than 1 hr. Attention should also be paid to the water discharge to see if any oil is carried over with it to the sludge tank.

Any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of equipment should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, gland leakage, correctness of alignment, and adequacy of lubrication.

Motor test requirements are the same as given above for Motor-driven Pumps, p. 1925.

Emergency Diesel Generator

Generator tests similar to those described above for Turbogenerators (p. 1922) shall be made.

The automatic emergency diesel-engine starting mechanism and the bus transfer switch shall be tested under a simulated emergency condition. Such condition should be effected by opening the emergency switchboard breaker at the main switchboard while the main switchboard is energized.

In order to demonstrate the load characteristic of the machine, vary the generator load from no load to 125 percent load, and back to no load in 25 percent load steps. The generator field rheostat and the diesel governor should not be adjusted during this test. Particular note should be paid at this time to the engine exhaust. The engine at 125 percent load may show a light haze, but at all ratings at and below full load the exhaust should be practically clear.

The 4-hr heat-run test should not be started until the generator temperatures are fairly constant when the generator is operating under full load.

shall apply plus meggering the neutral leg to ground and between positive to neutral and negative to neutral.

Main Boilers

Boiler Hydrostatic Test. A hydrostatic test shall be conducted at one and one-half times the design boiler-drum pressure for such a period to permit a careful examination of every part of the boiler. All leaks developing during this test shall be reported together with the pressure at which they occurred and the method of correction. After completing the test, the boiler shall be opened up for inspection, and all pressure parts shall be examined.

125 Percent Steam Test. A test under steam shall be applied to the boiler and main and auxiliary steam piping at a pressure of one and one-quarter times the design pressure and a temperature approximately equal to the service temperature. For a typical case, the design boiler-drum pressure is 525 psi gage. The design (normal sustained operating) pressure of the main and auxiliary steam piping is 465 psi gage. The superheater safety-valve setting is 480 lb. The corresponding steam test pressure is 600 psi gage. Sufficient steam is to be bled from the superheater outlet by operating the turbogenerators or some other steam-driven unit, so that the normal temperature will be obtained. The duration of the test shall be of such a time as to permit an adequate inspection of the boiler and piping system, but in no case shall the test be less than 1 hr. A complete report shall be made on this test and submitted for approval. All leaks and other pertinent information shall be included in the report. For the steam test, the regular safety-valve springs should be replaced by springs designed to operate at the test pressure. After the completion of the test, the test springs should be replaced with springs designed to operate at the maximum allowable working pressure.

During the test, the tightness of the boiler furnace, uptakes, and smoke pipe shall be observed and a note made on the test report of the observed condition.

Cold-ship Emergency Starting. Cold-ship emergency starting test shall be demonstrated on at least one ship of each group of similar ships being built by each contractor. Steam shall be raised from cold boiler to normal operating conditions. A report shall be submitted stating the procedure used, the time of the various steps taken, and any difficulties encountered.

Safety-valves Setting. The valves shall be set under steam pressure in the presence of the various inspectors. The valves shall be popped by steam pressure so as to obtain the full designed lift and then permitted to reseat. Note the popping and reseating pressures and amount of simmering or chattering.

The blowdown of all valves shall not exceed 3 percent of the popping pressure.

Demonstrate the ability of the safety-valve easing gear to lift the safety valves at zero pressure.

Hydrostatic Testing of Condensers

The water sides of the main and auxiliary condensers shall be tested at the same time as the circulating-water piping to a pressure of 30 psi gage. All leaks that may occur shall be made tight under test conditions.

The data for the heat run should then be taken at ½-hr intervals. Immediately after the full-load test is completed, the equipment shall be operated at 125 percent load for 2 hr.

During the test any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of equipment should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, correctness of alignment, adequacy of lubrication, and cooling. Attention should also be paid to see if the generator room is properly ventilated so as not to cause any undue heating of the space.

At the conclusion of the test, measure the insulation resistance of the windings before the generator temperature can change appreciably.

The operation of the overspeed governor shall be demonstrated under the supervision of the manufacturer's representative. The main governor should be checked for its ability to return the engine to normal speed after it has been reduced or increased from that speed, and the time of such return noted.

Refrigeration Equipment

The following give the tests for a Freon-12 direct-expansion system:

Piping Test. Each system, on completion and before pipe covering is installed, should be tested as follows:

1. Charge with Freon to 10 lb. Test for leaks with halide detector.
2. Blank off bursting disk on relief valve.
3. Add bone-dry nitrogen or CO_2 to bring pressure to 315 lb. Retest for leaks. (Isolate compressor and all pressure controls and low-pressure gages.)
4. Seal system under pressure for 8 hr. Pressure drop should not exceed 5 lb if temperature remains approximately constant.
5. Release pressure to 150 lb, open compressor valves, test compressor and shaft seal for leaks.
6. Lower pressure to atmospheric pressure and replace bursting disk on relief valve.
7. Dehydrate with a portable dehydrator pump.
8. Pump lowest vacuum possible, using pumps first in parallel then in series.
9. Charge with Freon to 20 lb and retest for leaks.
10. Connect system cleaner charged with activated alumina or silica gel in the compressor suction line, using the connection provided. Circulate Freon, using compressor. Add Freon as necessary. Recharge cleaner when necessary.
11. Remove cleaner. Inspect all strainers, expansion valve, and suction regulators.

Operation Test. The refrigeration compressors shall be operated at their rated capacities. The units should be tested with all control apparatus in place and operating. Power for the motors shall be taken, using the ship's power and permanent cable installation. If winter conditions exist at the time of the test, the refrigerated compartments and surrounding spaces should be heated to not less than 70 F. All compressors should be started and brought up to speed with the entire refrigeration load connected. The com-

Table 1. Piping Installation Test Data

Name of Vessel:________ Observer:________ U.S.M.C. Hull No:________
Shipbuilder:________ Piping Tests Builder's Hull No:________

System	Design temp, deg F	Design press, psi gage	Working press, psi gage	Test press, psi gage	Test medium	Date	Approved by U.S.M.C. inspector	Remarks
Bilge suction	100	35	...	100	Air			
Bilge ovb discharge	100	125	...	188	Water			
Boiler blow	475	525	480	720	Water			
				600	Steam[a]			
Circulating main & aux[a]	85	20	20	30	Water[b]			
Clean ballast	100	125	...	188	Water			
Compressed air	125	140	125	210	Air[c]			
Diesel-oil piping	100	50	...	100	Water or oil			
Diesel-engine exhaust	600	20	10	50	Air			
Evaporating-plant piping	...	...	...	...	[d]			
Feed, hp, compound feed & feed testing	240	675	550	1,013	Water			
Feed l-p feed drains, & vents	240	100	70	150	Water			
Feed (condensate), air ejector, feed vents, & drains subject to vacuum	240	...	28.5 in. Hg	15	Air[e]			
Fire system	100	125	110	188	Water			
Fuel-oil heating coils & drains	407	45	35	500	[e]			
Fuel-oil service discharge	250	365	350	730	Water or oil			
Fuel-oil transfer[f]	100	15	15	100	Water			
Fuel-oil overflow & vent	100	...	...	...	[g]			
Fuel-oil filling[h]	100	90	75	135	Water			
Lubricating-oil piping	150	60	50	100	Water or oil			
Plumbing drains	100	20	10	[g]	Water			
Refrigeration piping	...	...	...	315	[f]			
Sprinkler system (wet)	100	125	110	188	Water[k]			
Sanitary system	100	100	85	150	Water			
Steam, main superheated	790	525	465	720	Water			
				600	Steam[a]			
Steam, hp aux	510	525	465	720	Water			
				600	Steam[a]			
Steam, 240 lb aux	450	265	240	398	Water			
				331	Steam			

partments should be brought down to and maintained at the designed temperatures.

Readings should be taken on all motor data every 15 min during continuous operation until the temperatures of the motors no longer show an increase. The time of continuous test on the motors shall be not less than 1 hr. Readings of the compartment temperatures, by dial thermometers installed, and the compressor suction and discharge pressures should be taken every hour. The remaining reading should be taken every 2 hr. The time when a compressor starts and stops owing to automatic pressure control should be noted. Temperatures in all surrounding spaces shall be taken.

Any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of equipment should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, correctness of alignment, any extreme variations in liquid level in the receiver or oil level in the compressor, and any undue or uneven frosting of the coils or air units.

The test should demonstrate the satisfactory operation and adjustment of all thermostats, pressure switches, solenoid valves, thermal expansion valves, suction regulators, and other controls.

Insulation Test. After the compartments have been maintained at the temperatures required for a period of at least 12 hr, the machines should be shut down and the compartments kept closed for 6 hr, and the temperature rise noted. Dial thermometers should be read every 2 hr, and temporary test thermometers should be read before and after the 6-hr period. Temperature drop should not exceed 1½ to 2 deg per hr when outside temperature is 70 F or above.

Shaft-turning Gear Motor

Oil should be circulated through the turbine and gear bearings by means of the main or stand-by lube-oil service pump. All bearings should be examined to see if they have sufficient lubrication before the turning gear is started. The motor should then be started and brought up to its rated speed. Readings should be taken at least every 15 min during continuous operation until the temperatures no longer show an increase. The time of continuous test should not be less than 1 hr.

Motor test requirements are the same as given above for Motor-driven Pumps, p. 1925.

Anchor Windlass

General. The cold insulation resistance of the motor armature, motor fields, and controller wiring shall be determined before starting tests.

Centigrade thermometers of proper range shall be wedged in the motor windings and taped or otherwise secured to the bearings. Readings shall be taken before each run and at 15-min intervals during tests. Measure the hot insulation resistance at the completion of each test before the motor cools appreciably.

The motor speed shall be measured at each point of the master switch for both hoist and lower.

Record the anchor weight, size of chain, chain and rope speeds, length of chain out of pipe, brake operation, input volts, amperes, and motor rpm, and any other pertinent data.

Table 1. Piping Installation Test Data—*(continued)*

System	Design temp. deg F	Design press, psi gage	Working press, psi gage	Test press, psi gage	Test medium	Date	Approved by U.S.M.C. inspector	Remarks
				263	Water			
Steam, 160 lb aux	446	175	160	219	Steam			
				210	Water			
Steam, 125 lb smothering	440	140	125	175	Steam			
				210	Water			
Steam, 125 lb whistle	440	140	125	175	Steam			
Steam, 35 lb steam out	407	45	35	65	Steam			
Steam, 35 lb ships heating[n]	407	45	35	65	Steam			
Steam, aux exhaust & turb bleeder	405	20	10	[i]	Steam			
Steam, gland seal	400	20	3	[i]	Steam			
Steam, escape piping	400	20	...	[i]	Steam[e]			
Washing & drinking	100	70	60	105	Water			
CO_2	100	...	...	[q]	CO_2 or air			

[a] Test with boiler steam and hydrostatic test.
[b] Test with main and aux condensers.
[c] Includes 2-hr static drop test at 125 psi.
[d] Test under service conditions when conducting evap test.
[e] All welds to be hammered while under hydrostatic test.
[f] *Inner bottom piping and all other piping protected* by fuel-oil and ballast overflow.
[g] Test to maximum head obtainable.
[h] Includes all fuel-oil filling and all piping not protected by fuel-oil or ballast overflow.
[i] Test under service conditions.
[j] Test with bone-dry nitrogen or CO_2.
[k] Test with fire system.
[l] Test with heating system.
[m] Includes fire and sanitary suction piping.
[n] All steam radiators and unit heaters to be cut in.
[o] Test with safety valve test on boilers.
[p] Test all vacuum piping with air at 15 psi and apply soap solution to all joints.
[q] Test header to 1,000 psi gage static pressure drop not to exceed 300 lb in 2 min. Also plug distribution lines and test at 600 psi gage.

Approved by: ____________

No-load Windlass Test. The anchor windlass shall be operated continuously at maximum motor speed for a period of ½ hr in each direction. During this period, the bearings and gear boxes shall be inspected for heating or other indications of unsatisfactory operation.

NOTE. This test to be run before anchors and chain are attached.

Load Test at Dock. The windlass shall be operated for 15 min by hoisting and lowering each anchor. Run out as much chain as the depth of dock permits, but not over 30 fathoms.

The test described above shall be conducted once, by lowering and hoisting both anchors simultaneously.

The ability of each mechanical brake to stop and hold each anchor and chain shall be demonstrated. Let go each anchor separately, under control of brake, and catch it before reaching bottom. The operation of locking head, for satisfactory performance, shall be noted.

The ability of the solenoid brake to stop and hold the anchor and chain shall be demonstrated, while hoisting and lowering at full speed.

The ability of each warping head to produce the specified line pull at the specified speed shall be demonstrated. Note the speed, and check the pull on a dynamometer or heavy-duty crane scale, recording maximum input volts and amperes.

The ability of each warping head to develop a high light-line speed as specified shall be demonstrated. (The light-line speed shall be recorded as the average time required to haul in 50 fathoms of ship's mooring line.) Record the rpm of warping head.

Lead a line from the warping head to a heavy-duty crane scale, secure the crane-scale hook, and then heave in with the warping head until the windlass stalls. Take readings of crane scale, line voltage, and line current. Operate the master switch so that the electric brake holds the load.

Check the operation of the interlock switches on the wildcats. With the wildcats disengaged and locked out for warping service, take readings of the motor speed at each point of the master switch for both hoist and lower.

The chain stoppers for each chain shall be tested with chain in a normal riding position. Check pawl for proper seating against shoulder of chain.

Demonstrate the ability of devil's claw to hold the anchor satisfactorily in stowed position.

Capstan

The general and no-load tests are similar to those described above for the Anchor Windlass (p. 1930), and the load test is similar to the load tests of the warping head of the anchor windlass.

Cargo Winches and Booms

General. All cargo-handling equipment and rigging shall be tested after installation and shall be in strict accordance with the requirements of the U.S.M.C. Register of Cargo Gear. The tests shall be made in the presence of Maritime Commission and American Bureau of Shipping representatives, and a certificate of tests obtained.

Measure the cold insulation resistance of the motor armature, motor fields, and controller wiring before starting tests.

Centigrade thermometers of proper range shall be wedged in the motor windings and taped or otherwise secured to the bearings. Readings shall

The steam side of the condenser shall be completely filled with fresh water up to the height of the turbine flange, and air at a pressure of 15 psi gage shall be placed upon the water. The manhole and handhole plates on the water box and return heads shall be removed, and all tubes and joints shall be made tight.

Turbogenerators

For an assumed installation of two 300-kw, three-wire, 240/120-volt, d-c, shunt-wound turbogenerators, the tests should be as follows:

Before starting, the cold insulation resistance of each generator should be determined by placing one lead of a Megger or other such device on an unpainted spot on the frame and the other on each of the wires to be tested. The line switch of the generator should be open and the machine at a standstill. This will indicate the insulation resistance of the generator and leads to the switchboard. In the event that the resistance is less than 250,000 ohms, the brushes should be raised from the armature and the shunt field disconnected to determine the faulty circuit. The unit should be examined to see if it is in proper working condition.

Thermometers of proper centigrade-scale range shall be attached by putty, tape, or other suitable means to the generator bearings and stator coils. Readings shall be taken before the generator is started and at each time readings are taken during the heat run.

The generator should be started and brought up to full speed. During this period, a careful check should be made of all items to see that the turbine is functioning properly. A preliminary check of the wiring should be made by applying a suitable load on each generator. The series fields should then be checked by shorting them with a carefully placed copper bar. When each series field is shorted, the voltage should drop slightly. Any rise in voltage should be corrected by stopping the machine and reversing the improperly connected series field or fields.

The operation of the speed-control governor shall be demonstrated as follows: After the governor has been set for the desired speed, the speed shall be increased by hand operation of the governor control shaft. The governor shall then bring the speed back within the normal tolerance. By similar action, the speed shall be reduced below the set speed, and the governor shall again bring the turbine back within the tolerance.

Each generator in turn should be brought up to rated speed and the main switchboard meters calibrated. The voltage should be varied from 75 to 125 percent rated volts by adjusting the field rheostat, and readings taken of the switchboard voltmeter, and a calibrated standard voltmeter connected on the main bus. Readings should be taken at 10-volt intervals and recorded. A load should then be applied and varied between 20 and 125 percent rated current. Readings should be taken of the switchboard ammeters and a calibrated standard ammeter, having its shunt connected in series with the switchboard shunt. One standard ammeter may be used during this test if the neutral is opened.

The circuit breakers should be tested by adjusting the tripping element to 128 percent rated load on the positive and negative, and 32 percent rated load in the neutral, with dashpot disconnected. The load should be increased gradually until the circuit breaker trips. The current in each leg should be recorded at the instant of tripping. When the circuit breaker has been set

be taken before each run, and at 15-min intervals during each test. Motor voltage and current should also be recorded.

Measure the peak inrush current while moving the master switch from "stop" to the last point both lowering and hoisting for each test.

With the winch running, trip the circuit breaker or open the line switch and observe that the winch does not start when power is returned to the line until the master switch is moved through the "off" position.

Before starting any tests, the preliminary inspections, adjustments, lubrication, etc., shall have been satisfactorily completed.

Demonstrate that the topping lifts are of sufficient length to provide at least three turns of wire rope on the gypsyhead when the boom is lowered into the boom crutch.

Demonstrate that the cargo falls are long enough to lower the hook to the tank top through the last quarter of the hatch nearest the boom. At least three turns of the rope should remain on the drum, with the end of the wire rope securely fastened to it.

The ability safely to withstand the stalled-motor torque (300 percent normal current), with overload adjusted to maximum setting, shall be demonstrated on one winch for each ship, either at factory or aboard ship. The torque should be applied to both the drum and the winch head. The manufacturer shall tag those winches on which the shop test has been satisfactorily completed.

In each test, the load, motor rpm, rope speed, hook speed, input volts and amperes, and other unusual or pertinent data shall be recorded. Where the ability of the winch to handle the various specified rope speeds has not been demonstrated at the plant of manufacture, it shall be demonstrated aboard ship.

All winches shall be tested aboard ship at 100 percent of load for five complete cycles through the maximum obtainable drift.

Record the loads, speed, cycle time, time for each operation in cargo cycle, and all other pertinent data as well as any unusual conditions or occurrences.

Whatever load is specified for the normal rope load at the drum shall be raised and lowered continuously for ½ hr with a maximum speed as near 0.6 maximum possible speed as practicable. The load shall be stopped in the hold 20 sec each time to simulate service conditions. Check rheostat heating. This test is to be carried out on one winch of each type.

During the cargo winch test the adequacy of the control shall be checked and any *objectionable features noted.*

No-load Test. Operate each winch for a period of ½ hr on hoist and ½ hr on lower. Where possible, this may be done by declutching the wire-rope drums, operating the gypsies only; otherwise, it will be necessary to remove the wire rope from the drums. Take readings every 15 min, four sets minimum, record electrical temperature, rpm, motor voltage, and current. Inspect the gear boxes and bearings for heating or other unsatisfactory operation.

Load Test. The following gives a typical test for a 5-ton boom and winch:

The 5-ton booms shall be tested with a load of 14,000 lb (6.25 tons—25 percent overload), using regular tackle. Test the boom at an angle of 15 deg to the horizontal. Hoist and lower the load through full drift for five cycles, and then swing inboard and outboard as far as obstructions will permit. On one boom at each hatch, the load will be lowered into the hatch to the tank top and then hoisted out.

trip at the desired load, the dashpot may be reconnected. The grounding eaker should be tested by connecting either positive or negative bus to ound through a suitable variable resistor having capacity equal to the rating the breaker. The test should be started with all resistance in the circuit id the ground current gradually increased until the breaker trips. All quired data should be recorded.

Parallel operation of the generators should be checked at this time in rder to make adjustments of the governors or any other adjustments ecessary to the successful conduct of the succeeding tests. With the enerators running at rated speeds and with terminal voltage and line switches pen, check the polarity of the machines before attempting to parallel them or the first time. If the polarities are opposite, it will be necessary to stop ither machine and either "flash" the field or change the connections. The atter is the more laborious but, if attempted, it should be accomplished by pplying the same principles as in changing the direction of rotation of a ompound motor.

To "flash" the field, connect the shunt field by temporary wires in such a vay that the polarity will be the same in relation to the other machine when paralleled. While the one machine is at a standstill, its shunt field should e connected as just outlined, then the machine should be brought up to speed and the polarities rechecked and the line switches closed. The temporary hunt field connections can be removed either before or after the machines are paralleled.

With the generators paralleled, adjust the load to about 75 percent rated oad on each generator. Divide the load evenly at this load with rated voltage, then, without making any further adjustments of the generator field rheostat or the governor of the prime mover, decrease the load until the least loaded machine carries approximately 15 percent of rated load and take readings; then increase the load to as nearly full load as possible and take readings as required.

If the generators fail to divide the load evenly at the above three points within 15 percent of the full-load rating of each generator, it will be necessary to make adjustments to correct this condition. The first adjustments should be made on the speed of the prime movers. The data will indicate which machine required adjustment. With few exceptions, the speed adjustments will correct any unequal division of load. In the event that this is ineffective, the manufacturer of the generators must correct the windings by adding external shunts or by any other approved method.

The proper functioning of overspeed protective devices must be demonstrated. This may be done while making preliminary adjustments or while the generators are operating in parallel.

Check the behavior of the governors and regulating devices during severe operating conditions by causing intermittent load swings from 25 to 75 percent load. This should be accomplished in one switching operation or step and should be repeated several times.

The foregoing tests are advisable in order that loss of time may be avoided in the following tests. Upon completion of the preliminary tests, the generators are ready for the individual heat runs or the parallel operation test, either of which may be run first. The order will be determined by existing conditions. If all machines are warmed up by reason of the above tests, the parallel test should be made. On the other hand, if a cold start is necessary, the individual heat runs should be made.

The ability of the mechanical and the electrical brakes, on each winch, to stop and to hold a 14,000-lb load shall be demonstrated. (Hold the electric brake open when testing the mechanical brake.)

With the boom topped so as to plumb the hatch, lower a load of about 1,000 lb at full speed, to demonstrate satisfactory operation of dynamic braking (to land loads without jarring or "inching").

The ability of rig to hoist and lower a hook load to 3,360 lb at a speed of 290 fpm, using a single whip, shall be demonstrated.

The ability of rig to hoist and lower a hook load of 6,720 lb at a speed of 220 fpm, using a single whip, shall be demonstrated.

The ability of rig to hoist and lower a hook load of 11,200 lb at a speed of 112 fpm, using a two-part purchase, shall be demonstrated.

Using two winches (burtoning), the complete cargo-handling cycle shall be demonstrated, using load of 6,720 lb; noting the time for each operation as well as the total time for the cycle.

Using a load of 6,720 lb on the hook and with a single whip, the load shall be raised and lowered continuously for 30 min with a maximum speed as near 0.6 maximum possible speed as practicable. The load shall be stopped in the hold 20 sec each time. Check the rheostat heating. This test is to be carried out on each winch of each ship.

Steering Gear

General. Record the cold insulation resistance of the motor armature, motor fields, and controller wiring before starting tests.

Centigrade thermometers of proper range shall be wedged in the stator windings and taped or otherwise secured to the bearings. Readings shall be taken before each run and at suitable intervals during each test. Measure the hot insulation resistance at the completion of each test before the motor cools appreciably.

During the following tests, the feeder transfer switch shall be thrown to each position to check both steering gear feeders.

In each test, note the motor rpm, input volts and amperes, oil pressures and temperatures, and the time required to move the rudder from bardover to hardover. The oil temperature rise during each test shall be noted.

In general, the steering gear shall be examined for satisfactory performance of limit stop settings, follow-up control functioning, shift-over arrangement, valves, fittings, and packing. Any unusual conditions relative thereto shall be noted.

Dock Test. The steering gear shall be operated at the dock continuously for 2 hrs, using each motor and pump unit for 1 hr. The rudder shall be moved 30 times from hardover to hardover (70 deg in 30 sec) during the test of each unit. The time required to change from one unit to the other shall be noted. During this run the follow-up control and all rudder-angle indicators shall be tested for accuracy at the following angles: 0, 5, 10, 15, 20, 25, 30, 35 deg (port and starboard).

Note the number of turns required at trick wheel to move the rudder from hardover to hardover (70 deg) in 30 sec. Repeat the above test for the wheel located on the aft steering stand, in the wheelhouse, and on top of the wheelhouse.

Operate the steering gear from wheelhouse and wheelhouse top, using telemotor.

Operate the steering gear from wheelhouse using the gyropilot (if fitted).

The parallel operation test shall be conducted to determine the ability of all machines to parallel by operating all generators simultaneously connected to the bus.

When the 2-hr load test is completed and with all machines in parallel, vary the load from 20 percent load to full load and back to 20 percent load in approximately 20 percent load steps. Do not adjust the field rheostats during this test. Record all data required.

Speed and load regulation data should be taken with all the machines running and immediately after the parallel tests. This is accomplished by disconnecting one generator from the bus and putting a full load on the remaining generator. After taking readings, open the circuit breaker of the loaded generator and take another set of readings. No adjustments of rheostats or governors shall be made between the two readings. Each generator should be tested in this manner, recording all data.

If for any reason the bus must be energized at all times, close the circuit breaker of the idle generator at the same time the load is removed from the generator under test.

The reverse-current relay of the generator circuit breakers should be checked at this time to determine the tripping current. This may be accomplished by reversing the ammeter leads while the generator to be tested is in parallel but carrying no load; reduce the speed of the turbine until the breaker trips. Record the reverse current.

The load test or heat run for each generator is accomplished by warming up the generator until temperatures are fairly constant with rated full load. Take data as required for 4 hr at full load, then increase the load to 125 percent rated load for 2 hr. Take readings at half-hour intervals during rated-load test and at 15-min intervals during the 125 percent load tests.

During the test any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of equipment should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, correctness of alignment, adequacy of lubrication, operation of the turbine glands, and operation of the lube-oil coolers.

At the completion of the load tests and before the generator temperature can change appreciably, determine the insulation resistance in exactly the same manner that the cold resistance was obtained.

The operation of the overspeed trip and low-oil-pressure trip shall be demonstrated under the supervision of the manufacturer's representative. The results of the tests shall be recorded on the data sheets. The governor should be checked for its ability to return turbine speed to normal after it has been reduced or increased from that speed, and the time of such return noted.

Pumps

General. Each power-driven pump shall be operated at its rated rpm or double strokes per minute, suction and discharge pressure. The working fluid for which the pump was designed shall be used when practicable but the fuel-oil service pumps must be tested using fuel oil only and the lube-oil service pump using lube oil. The main and auxiliary feed pumps, main and auxiliary condensate pumps, and evaporator pumps must be tested in conjunction with the system to which they are connected at design conditions

The ability of the steering gear to function with either set of two opposed cylinders in operation shall be demonstrated. Note the time required for change-over and for moving hardover to hardover (70 deg). Note the oil pressure.

Evaporating Plant

The salt-water-evaporating plant shall be operated at the designed capacity for 6 hr during the sea trials of the vessel. During the test, the evaporator shall be blown down continuously to the amount specified by the manufacturer.

The following data should be recorded as far as possible at every 30 min throughout the test: steam-pressure shell and tubes, density of the feed water, salinity of distillate, temperature of the circulating water inlet and outlet, shell temperature, quantity of fresh water (measured by taking soundings in tank to which distillate is drained).

The pumps may be tested in conjunction with this test or at some other time, but they must be tested with the evaporating plant operating at its normal rated conditions.

Whistle

The whistle shall be operated by both the mechanical and the electric control from all locations where a whistle control station is located. The whistle should be operated and then left inoperative for at least 4 hr. Particular note should be made of the sound, volume, and pitch produced by the first blast after 4 hr of inoperation. This blast should be sharp, distinct, and of volume equal to the subsequent blasts. The steam-supply pressure shall be recorded both before and during the test. Failure to meet this requirement shall be a cause for rejection.

The test shall also demonstrate the adequacy and effectiveness of moisture separation and drainage. Any carry-over of water through the whistle shall be considered an indication of inadequate drainage or ineffective moisture separation.

The automatic timing contactor shall be tested, using all the various timing cycles.

Lifting Gear

After complete installation of all machinery, equipment, and piping, the capacity, efficiency, and operation of each lifting gear shall be demonstrated if required by the U.S.M.C. representative. If required, each lifting gear shall be properly rigged for each item, the operation demonstrated, and a check made for interference. If rigging of the lifting gear is not required, each installation shall be examined for satisfactory installation.

TRIALS OF MACHINERY

Object. Machinery trials of vessels are held for the purpose of ascertaining the capabilities of the engines, boilers, and auxiliaries, either in conjunction with the contract requirements for new vessels, or to determine their efficiency under service conditions, the adequacy and workmanship of the installation, the extent of repairs necessary, the sufficiency of repairs that have been made, or the proper mode of operation and the most economical rates of performance under various conditions of service.

of vacuum, submergence, operating temperature, etc. The pumping equipment should be tested with all governors and control apparatus in place and operating. Power for the motors shall be taken using the ship's power and permanent cable installations.

The test should demonstrate that the pump is able to take its suction through the normal or emergency suction lines and deliver the required quantity of fluid at the required pressure by way of normal or emergency discharge lines, while performing its normal or emergency duties.

Motor-driven Pumps. Previous to starting the pump, the cold insulation resistance of the motor armature, motor fields, and controller wiring shall be taken. Thermometers of proper centigrade-scale range shall be attached by putty, tape, or other suitable means to the motor bearings and stator coils. Readings shall be taken before the motor is started and at each time readings are taken during the heat run.

The pump should be brought up to the rated speed and total head specified. The designed total head should be obtained by properly throttling a valve in the discharge line from the pump. This should be done for a centrifugal pump, since it is possible to place a load on the motor higher than the normal designed load if the pump is operated at its maximum rated rpm with a total head less than the designed value. This throttling should also be done for positive-displacement pumps, since this type of pump will not be loaded to its designed rating unless the total head equals the designed figure when the pump is operating at its rated speed.

Readings should be taken of all data at least every 15 min during continuous operation until the temperatures no longer show an increase. The time of continuous test shall be not less than 1 hr.

Any unusual operating conditions should be noted and carefully described on the data sheet at the time the test is run. Erratic behavior of equipment should be carefully recorded at the time of occurrence in order that any serious defect may be detected and serious failure prevented. Particular note should be made of the amount of vibration, noise, gland leakage, correctness of alignment, and adequacy of lubrication. When possible, an estimate of the amount of fluid pumped should be obtained. Where relief valves are fitted, their operation and setting should be checked.

Where no suction gages have been provided for centrifugal pumps, the static head on the pump suction should be obtained by reading the discharge pressure gage when the pump is not operating and the valve on the suction side used for the test is open. The pressure can be determined only after the pump has been properly primed, however.

At the completion of tests, field rheostats for adjustable-speed motors should be blocked so that motors cannot be operated when at full-load temperature beyond the speed or load corresponding to the rated motor output. This should be done only after the motor has reached a constant temperature.

The feeder switch on the switchboard or power-distribution panel should be opened while the motor is running at normal speed and held open until the motor stops. The switch should then be reclosed, and a note should be made on the test report as to whether or not the motor restarts.

Immediately after the motor is shut down and the feeder deenergized, the hot insulation resistance should be taken and recorded in the same manner as followed for cold insulation resistance.

The setting and operation of overload protective devices should be carefully checked during the test, preferably after the required heat run. A simulated

Trials Required by U.S. Navy. The U.S. Navy requires, in the case o new vessels, (1) a preliminary acceptance trial, which is held prior to the delivery of a vessel built under contract and (2) a final acceptance trial, which is held within a specified time after delivery of the vessel, to ascertain if there has appeared any defect, weakness, failure, or deterioration for which the contractor is responsible.

Trials Required by U.S. Maritime Commission. The U.S. Maritime Commission requires, in the case of new vessels, an acceptance trial, which is held prior to the delivery of a vessel built under contract. Final acceptance trials are generally not required; however, at the conclusion of the builder's guarantee period, usually 6 months, an inspection and examination of the machinery are made to ascertain if any defect or deficiency has appeared for which the builder is responsible.

BUILDER'S TRIALS

Before conducting acceptance trials, it is customary for the builders to give new vessels what is known as a builder's trial, in order to assure themselves that they are ready for the official trial. The builder's trial is generally an abbreviated version of the contract-acceptance trial.

DOCK TRIALS

It is generally required, as well as being the usual practice among builders, to conduct a 4- to 6-hr dock trial of machinery. Prior to starting the main engines, all auxiliary machinery necessary to the proper functioning of the main propelling plant will have been tested as required by the machinery installation tests. Previous to starting the trial, the main engines are gradually warmed up and run at slow or moderate speeds until satisfactory assurance is had that no serious difficulty will interfere with a trial of the engines at or near the highest revolutions it is possible to maintain with the vessel moored alongside the dock. On multiple-screw vessels, dock trial is generally conducted on each engine separately, in order to operate at the highest possible speed within the capacity of the mooring lines.

When undergoing dock trial, all data available are generally taken at 20- or 30-min intervals, depending on the length of the trial. Special care is taken to note the steam pressures, vacuum, lubricating-oil temperatures, condition of bearings, and general working condition of all machinery. On reciprocating engines, indicator cards are taken in both ahead and astern gear for computing horsepower and for checking valve settings. On turbine installations, torsion-meter readings are taken to determine power. Before the conclusion of the trial, the engines should be reversed and run astern for 10 to 15 min at the maximum speed obtainable, again reversed and run ahead for the remainder of the trial.

After dock trials, in the case of geared turbine-driven ships, the main gears should be carefully examined, removing all inspection plates in the gear casings that may be provided for this purpose, and a record made of the tooth contact. In the case of reciprocating-engine-driven ships, cylinder and valve-chest covers should be removed and examination made of the inside of the cylinders and the condition of the piston rings.

CONTRACT-ACCEPTANCE TRIALS

Purpose. The general purpose of a contract-acceptance trial is to determine whether the ship and her machinery will satisfactorily perform all the

Name of Vessel:______________ Observer:______________ U.S.M.C. Hull No:________

Date:______________ Builder's Hull No:________

Test of:______________

Motor-driven Pump

Time	Motor							Pump			Remarks
	Rpm	Line amp.	Line volts	Hp.	Temperature deg C			Press, psi		Est. gpm	
					Bearing	Bearing	Stator	Suction	Disch.		

Name Plate Data

Motor

Manufacturer______________
Winding______________
Amp______________
Volts______________
Type______________
Frame______________
Serial No.______________
Hp________ Rpm________
Time rating______________°C
Ambient temp.______________°C
Insulation resistance
Cold________megohms Hot________megohms

Pump

Manufacturer______________
Type______________
Serial No.______________
Capacity______________
T.D.H.______________

Special Valve Operation

Type______________
Design setting______________
Actual setting______________
Operation______________

Field amperes full load______________
Overload protective device setting________Trip________

Comments:

Approved by______________

FIG. 1.—Machinery installation test data form.

requirements of the contract. In order to determine this, the ship is subjected to trials to test her speed, power, fuel consumption, general strength, and condition of the vessel and machinery. Also, from these trials, useful data for use in future designs are obtained.

Type of Trials. The acceptance trials are generally composed of the following:

1. *Progressive speed, or standardization trial.*
2. Fuel-consumption trials.
3. Maneuvering trials.
4. Steering-gear tests.
5. Anchor-windlass test.
6. Miscellaneous trials and tests.

General Procedure. It is general practice to conduct the standardization trials first, followed by maneuvering trials, miscellaneous tests, and finally the fuel-consumption trials. During all trials, the vessel should be ballasted to the contract or specified trial displacement. The ship should be at this displacement during the middle run of the high-speed group of the standardization trial, and also at the middle of the fuel-consumption trials.

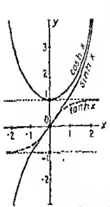
The complete trial schedule and program should be prepared well in advance, in order that all arrangements will be complete, personnel familiarized with their duties, and so that no time will be wasted while the ship is on the trial course. Trials each day should be run consecutively and without interruption, and the trial program, once approved or agreed to, should not be changed. Sudden changes in the trial schedule result in confusion among the ship and trial crews and result in poor or unsatisfactory performance.

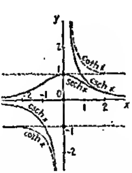
General Observations. In addition to the engineering data observed during the trials, general observations should be made of the operation of the main engines or turbines, boilers, bearings, piping systems, extent of any vibration, adequacy of light and ventilation, steadiness and seaworthiness of the vessel, and whether the vessel and machinery are well built and in accordance with contract plans and specifications and the state of completion of same.

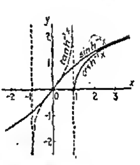
Description of Trials. 1. *Standardization Trials.* Standardization trials are conducted on new or untried vessels to determine corresponding values of speed, revolutions, and power over a reasonable range of speed (usually from one-half to maximum speed for merchant vessels, and from 9 knots to maximum for naval vessels). These trials are made at one displacement and under favorable conditions of wind and weather. From the results, it is determined whether or not the builder's guarantees have been met. Also, the designer utilizes the information in preparing subsequent designs and, from the results, the owner can generally make a close estimate of the performance to be expected in service.

Trials are conducted over a nautical mile course measured by ranges ashore. The measured mile at Rockland, Me., as shown by Fig. 2, is the best trial range on the Atlantic Coast, and the one generally used. The course off Point Vincente, Calif., is probably the best on the West Coast.

Before the start of the trials, the ship is ballasted to approximately contract displacement. Trials are started with the ship slightly heavy, so that, as fuel is expended, the ship will be at contract displacement during the high-speed runs. Generally, runs are made at slow speed first, gradually working up to high speed. A series of groups of runs are made as follows: Three

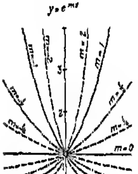


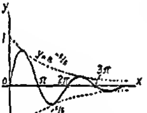




Hyperbolic functions and inverse hyperbolic functions.

FIG. 7. FIG. 8. FIG. 9.





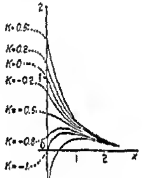


FIG. 10. FIG. 11. FIG. 12.

the points lie only approximately on a straight line, the best position for this line can usually be found by stretching a black thread among the points; or, assume a law of the form $y = mx + b$, and, by substituting in this formula n pairs of values of x and y, obtain n equations connecting the coefficients m and b; various pairs of these equations may then be solved for m and b, and the average of the results taken. Or, if great accuracy is required, all n of the equations may be solved for m and b by the method of least squares (p. 121).

If any law of the form $f(x,y) = m \cdot F(x,y) + b$ is suspected, where $f(x,y)$ and $F(x,y)$ are any expressions involving either x or y or both x and y, such a law may be tested by plotting $F(x,y)$ instead of x, and $f(x,y)$ instead of y, on rectangular cross-section paper, and seeing whether or not the points lie on a straight line. If they do, the form of the law is verified, and the values of m and b can be read from the figure as before. For example, if $y^2 = mxy + b$, a straight line will be obtained by plotting y^2 against xy. Again, if $xy = bx + my$, a straight line will be obtained by plotting y against y/x, since the equation may be written $y = b + m\,(y/x)$.

CASE 2. If a law of the form $y = cx^n$ is suspected, plot the points (x,y) on logarithmic paper (see below).

CASE 3. If a law of the form $y = c \cdot 10^{mx}$ [or $y = c \cdot e^{mx}$] is suspected, plot the points (x,y) on semi-logarithmic paper (see below).

CASE 4. If the given curve resembles the logarithmic curve, $y = \log x$, interchange x and y and proceed as in Case 3.

CASE 5. If the given curve is a wavy line, resembling a sine or cosine curve, try an equation of the form $y = a \sin bx$ or $y = a \cos bx$. If the heights of the waves diminish as x increases, try an equation of the form $y = ae^{-nx} \sin bx$. [NOTE. Any periodic function (satisfying certain simple conditions) can be expressed by a Fourier's series (p. 162)].

CASE 6. A great variety of functions can be represented approximately by a polynomial of the form $y = a + bx + cx^2 + dx^3 + ex^4 + \ldots$, the first three or four terms being usually sufficient. To determine the coefficients $a, b, c, \ldots$, most accurately, substitute in the formula all the given pairs of values of x and y, and solve the resulting equations for $a, b, c, \ldots$ by the method of least squares (p. 121).

CASE 7. Many simple curves can be represented approximately by an *equation of the hyperbolic form*, $xy = c + bx + ay$, where a, b, and c are determined by substituting the co-ordinates of three conspicuous points of the curve. The lines $x = a$ and $y = b$ are the asymptotes of the hyperbola. The equation may also be written $(x - a)(y - b) = k$, where $k = ab + c$.

Logarithmic Cross-section Paper. In this form of cross-section paper (Fig. 13), the distance from the origin to any point on the x- or y-axis is equal to the logarithm of the number written against that point. Thus, in Fig. 13 the distances (shown for clearness on two auxiliary scales X and Y) are the logarithms of the numbers written along x and y.

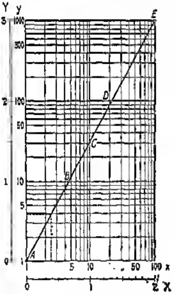


FIG. 13.

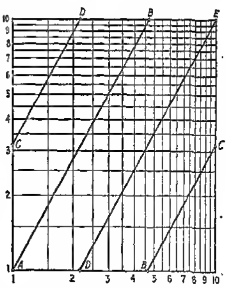


FIG. 14.

Accurately made logarithmic paper can be obtained from the principal dealers in draftsmen's supplies. Logarithmic paper can be easily constructed, in case of need, by copying the logarithmic scale from any ordinary slide rule. The actual figures along the x- and y-axes are usually left for the user to insert; in so doing, notice that the numbers . . ., 0.01, 0.1, 1, 10, 100, . . ., or such of them as may be needed to cover any given range of values, must be placed at the points of division which separate the main squares. It is often convenient, however, to omit the decimal point, num-

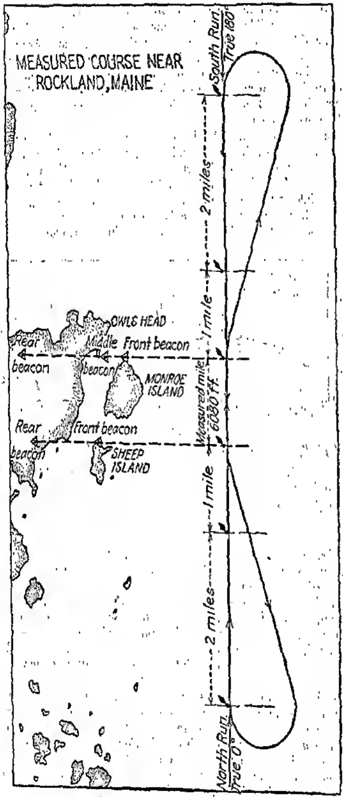


Fig. 2.—Rockland trial course.

Table 4. S.S. "Red Jacket." Endurance and Economy Trial Data
(Aug. 27, 1939)

General

Trial— % normal shp	110	100	83.3	66.7	50
Duration, hr	2	6	2	2	2

Propulsive Data

Displacement, tons	9,200	9,200	9,200	9,200	9,200
Rpm	98.90	96.68	92.11	85.56	78.64
Shp from torsion meter	6,793	6,130	5,111	4,051	3,035
Shp from model-basin curve	7,050	6,600	5,600	4,400	3,370
Speed, from standardization curve, knots[a]	17.7	17.4	16.8	15.9	14.7
Speed, from model-basin curve, knots[a]	17.7	17.4	16.8	15.9	14.8
Thrust, 1,000 lb, from meter	112.2	103.3	91.7	77.5	64.8

Fuel-oil Consumption

F.O. consumption, gal[b]	923.1	2491.0	718.2	590.7	485.8
F.O. consumption, gph	461.6	415.2	359.1	295.4	242.9
F.O. temperature, meters, deg F	109.2	106.7	104.2	100.8	98.4
F.O. lb per gal at meter temp[c]	8.1500	8.1575	8.1650	8.1758	8.1833
F.O. lb per hr as burned[d]	3,762.0	3,387.0	2,932.1	2,415.1	1,987.7
F.O. lb per hr (18,500 Btu/lb)	3,748.2	3,374.6	2,921.3	2,406.2	1,980.4
F.O. consumption, lb/shp/hr	0.5518	0.5505	0.5716	0.5940	0.6525
F.O. consumption guarantee, lb/shp/hr	None	0.600	None	None	None
F.O. consumption, lb/nautical mile[e]	211.2	193.7	173.4	151.8	134.7

Main Turbines

Throttle setting	Wide open			Partly closed	
Total no. nozzles	36	36	36	36	36
No. nozzles open	21[f]	15[g]	15[h]	15	15
Steam pressures, psi gage:					
Throttle	453	453	460	461	461
H-p inlet	444.8	450.3	454.7	395	311
Bleeder A	70.0	72.5	63.3	52.1	40.4
Bleeder B	11.6	15.0	13.3	10.5	8.1
Bleeder C (psia)	9.4	8.2	7.3	5.6	4.7
L-p inlet	29.8	25.5	18.9	11.3	4.0
Gland seal	4.7	3.6	3.9	2.9	3.0
Vacuum, in. Hg (gage)	28.46	28.42	28.37	28.60	28.50
Steam temperatures, deg F:					
H-p inlet	747	747	728	706	693
Bleeder A	532	515	473	435	424
Bleeder B	332	324	297	265	249
Bleeder C	192	189	180	173	162
H-p outlet	338	323	300	273	265
L-p inlet	350	333	300	269	268
L-p exhaust	93.4	93.7	92.3	86.2	88.4

[a] Based on rpm.
[b] Based on average of flow through two meters and the average meter calibration factor obtained before and after the trials.
[c] Based on oil analysis by U.S. Navy Yard, Washington, D.C.
[d] 18,432 Btu/lb from analysis of oil by U.S. Navy Yard, Washington, D.C.
[e] Based on speed from standardization curve.
[f] Handwheel for 3 additional nozzles, cracked open.
[g] Handwheel for 6 additional nozzles, opened 2 turns.
[h] Handwheel for 6 additional nozzles, cracked open.

consecutive runs, alternating in direction over the course and at as nearly constant rpm as possible, should be made for each speed point desired below full speed, and five consecutive runs, alternating in direction, should be made at the highest speed attainable. Each series of runs at the same speed *should be uninterrupted and performed in sequence. If it should be found* necessary to throw out any run, a sufficient number of additional runs should be made at that speed to produce at least three consecutive runs alternating in direction over the course.

The runs over the course should be made back and forth over the same water. Between runs, the ship should be taken well away from the measured *course, so as to ensure the attainment of steady speed on the next run.* The approach should be at least 3 miles, and the ship should be straightened on the course while at least 1 mile from the range as shown by Fig. 2.

During each group of runs, not only when running the measured course but throughout the intervals between runs, conditions should be maintained

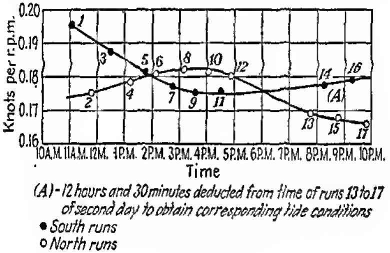


Fig. 3.—Standardization runs.

uniform; *i.e.*, the revolutions or power should be constant. On vessels equipped with governors, the desired rpm can be set and will be maintained irrespective of changes in steam pressure, etc. For turbine installations not equipped with governors, it is good practice to keep the number of nozzles open, the turbine inlet pressure and vacuum constant, and let the revolutions vary. Also, if the turbine is equipped with bleeders, these should not be changed. On reciprocating-engine installations, the cutoff and steam pressure and vacuum should be maintained constant during each group of runs.

The following data for each run should be recorded: (1) elapsed time on the course, (2) total revolutions of each engine while on the course, (3) indicator cards or torsion-meter readings at frequent intervals while on the course, (4) thrust measurements at frequent intervals, (5) wind direction and velocity. Generally, all engineering data are also observed once during each run.

At the end of *each run, computations should be made and the following* determined: (1) speed in knots, (2) rpm, (3) shp, (4) thrust, (5) shp/rpm^3, (6) speed/rpm, (7) thrust/rpm^2. Plots should be maintained of items (2), (3), and (4), on a base of item (1). Also, items (5) and (7) should be plotted

Table 4. S.S. "Red Jacket." Endurance and Economy Trial Data—(*continued*)

Lube-oil System					
Pressures, psi gage:					
Pump discharge	47.5	46.8	47.4	48.0	48.0
L.O. cooler inlet	39.8	39.3	39.5	40.2	40.2
L.O. cooler outlet	19.6	19.2	19.7	19.5	19.8
Strainer inlet	15	15	15	15	15
Supply to bearings	11	11	11	11	11
Temperatures, deg F:					
Oil to bearings	111	113	112	110	110
Oil to cooler	130.2	131.2	129.5	125.2	123
Oil from cooler	109.8	111.8	110.5	109.2	110
Water to cooler	63.8	68.8	68.0	60.2	66.0
Water from cooler	99.0	101.2	100	101	103
Main Turbine Lube Oil					
Gov. oil press, psi gage	66.4	69.5	65	58	54
Bearing temperatures, deg F:					
H.p. forward	120	122	121	122	115
H.p. aft	123	124	122	120	119
L-p. forward	120	120	118	115	114
L-p. aft	113	115	114	112	112
Main Reduction-gear Lube Oil					
Bearing temperatures, deg F:					
H-p—h-s pinion, forward	149	148	145	139	136
H-p—h-s pinion, aft	145	145	142	136	133
H-p intermediate gear, forward	123	124	123	120	118
H-p intermediate gear, aft	123	124	122	120	118
H-p—l-s pinion, forward	145	145	143	140	135
H-p—l-s pinion, aft	145	145	143	137	131
L-p—h-s pinion, forward	132	133	131	128	125
L-p—h-s pinion, aft	132	133	131	128	125
L-p intermediate gear, forward	124	124	122	118	116
L-p intermediate gear, aft	119	120	118	116	114
L-p—l-s pinion, forward	148	147	142	139	135
L-p—l-s pinion, aft	135	134	131	127	123
Main gear, forward	115	116	116	114	114
Main gear, aft	110	112	111	110	109
Main thrust bearing	118	119	118	116	114
Main Condenser					
Vacuum, in. Hg (gage)	28.46	28.42	28.37	28.60	28.50
Vacuum, in. Hg (Hg column)	28.50	28.45	28.40	28.52	28.50
Vacuum, temp, deg F (therm)	89.8	89.7	88.7	82.6	84.4
Condensate temp, deg F	84.6	86.3	84.5	78.8	78.6
Cooling-water inlet, deg F	63.8	68.9	67.2	60.9	63.6
Cooling-water outlet, deg F	73.2	77.8	75.0	66.7	69.4
Atmospheric pressure, in Hg	30.20	30.17	30.12	30.10	30.08
Auxiliary Condenser					
Vacuum temp, deg F	88.4	88.9	88.0	82.0	83.1
Cooling-water inlet, deg F	63.8	68.9	67.2	60.9	63.6
Cooling-water outlet, deg F	74.0	77.8	76.0	69.8	71.6

on item (2), and item (6) on time of day, as shown by Fig. 3, 4, and 5, for the standardization trial of the S.S. "Red Jacket." Runs in the same direction should fall on a fair curve; deviation from this curve indicates probable inaccuracies in measurements. Thus a continuous plot of such data will maintain a check on the trial instruments and observations. An explanation

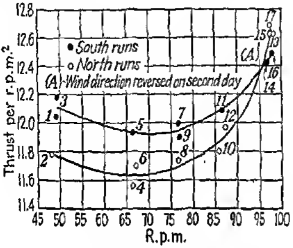


Fig. 4.—Standardization runs.

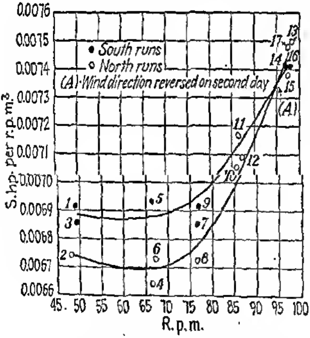


Fig. 5.—Standardization runs.

of the differences between these curves for the two directions and a method for analyzing standardization trial data will be found in "Speed and Power of Ships," by D. W. Taylor, 1943 ed., pp. 163–169.

2. *Fuel-consumption Trials.* These trials are conducted to determine the economy of operation of the machinery. For merchant vessels, fuel-con-

Table 4. S.S. "Red Jacket." Endurance and Economy Trial Data—(continued)

Main Air Ejector					
Vacuum (gage) in. Hg	28.5	28.5	28.5	28.8	28.5
Vacuum temp, deg F	78.8	84.3	85.0	81.8	79.8
Steam pressure, psi gage	214	210	209	205	203
Main Boilers (Average of two boilers)					
Pressures, psi gage:					
Drum	478	475	471	472	470
Superheater outlet	466	463	466	467	467
Desuperheater outlet	460	458	460	462	460
Temperatures, deg F:					
Superheater outlet (thermometer)	770	766	747	733	726
Superheater outlet (thermo couple)	770	765	749	736	725
Desuperheater outlet	468	466	469	469	463
Economizer inlet	306	308	304	296	284
Economizer outlet	372	375	366	359	350
Oil Burners					
Total No. burners in use	6	6	6	6	6
Size of tips	46	46	46	46	46
Combustion Air and Gases					
No. fans in use	2	2	2	2	2
Average fan rpm	1,802	1,592	1,434	1,176	1,083
Pressures, in. water:					
Fan outlet	8.70	7.07	5.85	4.05	3.75
Damper outlet	6.20	5.10	3.95	2.85	2.00
Burner inlet	4.60	3.80	2.80	2.00	1.45
Furnace	2.90	2.30	1.80	1.25	.85
Uptake	−0.30	−0.30	−0.30	−0.25	−0.30
Temperatures, deg F:					
Air to stack casing inlet	108	112	112	108	108
Air at fan	124	125	125	120	
Air to burner	289	295	291	283	276
Flue gas (thermometer)	274	273	265	253	240
Flue gas (pyrometer)	269	273	266	254	244
Gas analysis:					
CO_2, %	14.0	13.3	13.6	12.8	11.9
CO, %	0	0	0	0	0
O_2, %	2.9	4.2	3.7	5.1	6.0
Excess air, %	14.9	20.0	17.8	25.3	34.2
Fuel-oil System					
Pressures, psi gage:					
Stand-by pump discharge[1]	35	40	38	31	39
Service pump suction	6.2	6.3	5.0	6.5	10.2
Service pump discharge	318	305	280	265	241
Burner inlet	304	295	283	255	246
Burner return	73	56	48	35	30

[1] In use as booster pump.

sumption trials are generally run at normal and maximum designed shp and occasionally at reduced powers. Guarantees are made only at normal shp. For naval vessels, fuel-consumption trials are run at approximately 5-knot intervals from about 10 knots to maximum power, and guarantees are made at each point. Fuel-consumption trials involving guarantees generally last from 6 to 8 hr for merchant ships and 4 hr for naval vessels.

The trials are generally conducted at contract displacement in open sea. Readings of gages, thermometers, revolution counters, and all other trial-

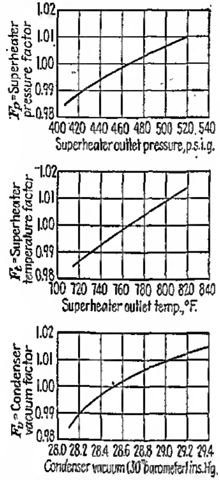


FIG. 6.—Fuel rate correction factors.

trip instruments are made at least every ½ hr during the trials and, in addition, the torsion meters are read about every 3 min so as to assure a continuous record of torque and revolutions. All meters, gages, tanks, and special equipment required in connection with the trials are tested or calibrated prior to the trials. Also, fuel-oil meters in addition to the regular ship's meter are generally used and are calibrated before and after the trials. Fuel-oil samples are taken during the trials and analyzed for heating value, etc. The trial data may be analyzed and reduced to standard conditions by a method similar to that described in a discussion appearing in the *Trans. Soc. Nav. Arch. and Mar. Engs.*, 1940, p. 354.

Table 4. S.S. "Red Jacket." Endurance and Economy Trial Data— *(continued)*

Fuel-oil System *(continued)*					
Temperatures, deg F:					
Settling tank[j]	122	118	114	111	108
Pump suction	110	107	104	102	99
At meters	109	107	104	101	98
To heater	123	127	132	142	150
From heater	213	215	215	216	214
To burners	208	209	209	211	209
Steam drains from heater	256	255	251	251	243
Feed System					
Feed pressures, psi gage:					
Condensate pump discharge	72.0	73.4	65.0	63.0	55.2
Main feed pump discharge	598	590	575	558	550
Feed regulator inlet	516	508	498	498	497
Feed regulator outlet	480	475	473	471	470
Steam pressures:					
First-stage heater, psia	9.4	8.6	7.3	5.6	4.7
D-c heater, psi gage	10 9	14.3	13.0	10.5	9.3
Third-stage heater, psi gage	74	72	63	56	45
Feed temperatures, deg F:					
Main condensate	84 6	86.3	84.5	78.8	78.6
Intercondenser outlet	87 6	90 2	89	81	85
Drain cooler outlet	91 8	94.0	92	86	87
First-stage heater outlet	173	172	162	152	148
After condenser outlet	189	187	181	174	175
D-c heater outlet	240	247	245	239	234
Third-stage heater outlet	309	310	302	295	285
Atmospheric drain tank	214	214	214	212	214
Contaminated Water Evaporator					
Steam pressure, psi gage	34 2	32.4	32	33	30.8
Vapor pressure, psi gage	23 2	24 3	23.5	23	23.4
Steam temperature, deg F	299	288	288	279	288
Vapor temperature, deg F	267	267	267	267	267
Make-up Water Evaporator					
Steam pressure, psi gage	[k]	53[l]	[k]	[k]	[k]
Vapor pressure, psi gage	[k]	17[l]	[k]	[k]	[k]
Steam temperature, deg F	[k]	318[l]	[k]	[k]	[k]
Vapor temperature, deg F	[k]	254[l]	[k]	[k]	[k]
Pump Data					
Main cond., disch. pres. psi gage	72.0	73.4	65.0	63.0	53.2
Main cond., submergence, ins	20.5	21.5	19.7	19.5	18.5
Main cond., rpm	1,759	1,811	1,768	1,773	1,625
Main feed, disch. pres, psi gage	598	590	575	558	550
Main feed, rpm	123	112	101	86	74
Main circulating, rpm	670	677	680	680	681
Auxiliary circulating, rpm	950	1,104	1,106	1,110	1,110

[j] No steam used during trials, since stand-by fuel-oil service pump, which was operating as a booster pump in order to meter oil accurately, used as equivalent amount of steam.

[k] Not in use.

[l] In use 1 hr during trial to make up lost feed water.

The results of fuel-consumption trials of naval vessels are accepted as run, the only corrections being those required for ship's heating, meter calibrations, and heat content of the fuel oil. However, the U.S. Maritime Commission has required, in all recent trials involving guarantees, that corrections be made for main steam pressure, temperature, and condenser vacuum differing from the design, in addition to the usual corrections for ship's heating, meters, and heating value of the oil. In order to accomplish this, correction factors were prepared by the contractor and approval secured before trials. The factors consisted of a set of curves that gave correction factors to be applied to the fuel rate for variations from designed boiler pressure and temperature and condenser vacuum, such as shown by Fig. 6, for designed conditions of 465 psi gage, 765 F, and 28.5 in. vacuum.

3. *Maneuvering Trials.* These trials are conducted to demonstrate the maneuvering ability of the vessel and generally consist of the following: (1) a test to determine the diameter of the turning circle of the vessel with the rudder in the hardover position, (2) a test to demonstrate the ability of the vessel to go from full speed ahead to full speed astern, (3) a backing test at full astern power to demonstrate that the engines will develop the specified astern power without undue heating, (4) a test to demonstrate the ability of the vessel to go from full speed astern to full speed ahead, (5) a test to determine the time required to go from full speed ahead to dead stop. These tests are usually conducted in the order named.

Other maneuvering trials are sometimes specified, such as a maneuver to determine the time required for the ship's heading to change a specified number of degrees from a base course with a specified amount of helm.

4. *Steering-gear Tests.* These tests are made to show that the main steering gear is capable of moving the rudder under all conditions. Generally the contract specifies that the steering gear is to move the rudder from hardover to hardover in a specified time with the vessel going ahead at full speed and also from hardover to hardover when developing full astern power without exceeding a specified oil pressure in the steering-gear cylinders.

Emergency steering-gear tests are also conducted to meet the requirements of the classification societies. The rudder must be moved from hardover to hardover, using emergency steering gear, with the vessel proceeding at 7 knots or at half speed, whichever is greater.

5. *Anchor-windlass Test.* This test is required to demonstrate the ability of the anchor windlass to handle the anchor under any condition. Generally, the contract specifies that the windlass will be tested in a specified depth of water, letting go both anchors at the same time, under control of the brake, and stopping at various depths; also hoisting one anchor with a specified length of chain at a specified speed, and hoisting both anchors with a specified length of chain at a specified speed.

6. *Miscellaneous Trials.* Miscellaneous trials, including torsiograph test, water-rate test, boiler-overload test, astern standardization trial, may be specified on certain types of vessels.

Generally, the first vessel of each class or type is given a torsiograph test to demonstrate that no serious torsional vibration exists in the main propelling shaft and mechinery. This test consists of operating the machinery at various rpm near the criticals and measuring the torsional vibration by means of a torsiograph.

When guarantees of water rates of the main propelling unit are made, the builder generally conducts a water-rate test on one ship to check the engine

Table 4. S.S. "Red Jacket." Endurance and Economy Trial Data—*(continued)*

Pump Data *(continued)*					
Lube-oil service, disch. pres. psi gage	47	47	47	48.5	48.6
Lube-oil service, rpm	1,680	1,688	1,685	1,695	1,688
F. O. service, disch. pres. psi gage	318	305	280	265	241
F. O. service, suct. pres. psi gage	6.2	8.3	10.0	9.5	3.0
F. O. service, rpm	1,842	1,867	1,875	1,889	1,890
F. O. stand-by, disch. pres. psi gage[i]	35	40	38	31	39
General service, disch. pres. psi gage	7.6	k	k	k	k
General service, strokes per min	45	k	k	k	k
Fire, disch. pres. psi gage	98	k	k	k	k
Fire, rpm	1,637	k	k	k	k
Ballast, rpm	1,877	k	k	k	k
Auxiliary Steam					
H-p desuperheated pres. psi gage	460	458	460	462	460
H-p desuperheated temp deg F	468	466	469	469	463
L-p desuperheated pres. psi gage	220	218	218	220	220
L-p desuperheated temp deg F	401	390	391	392	400
70 lb aux steam pres. psi gage	60	60	60	60	58
50 lb aux steam pres. psi gage	24	24	24	24	24
Auxiliary Generator					
No. in use	1	1	1	1	1
Rpm	1,200	1,200	1,200	1,200	1,200
Volts	240	240	240	240	240
Amperes	1,015	830	841	781	732
Kw (calculated)	243.6	199.2	201.8	187.4	175.7
Air Temperatures					
Engine room	101	103	104	97	98
Boiler room	103	104	104	100	98
Outside air[m]	71	73	75	76	73

[i] In use as booster pump.
[k] Not in use.
[m] Owing to air temperature, heating system was secured.

manufacturer's guarantees. During this test the condensate is weighed or metered, the power is measured by means of a torsion meter, and the steam conditions and vacuum are accurately determined. During this test the turbine is operated nonbleeding and noninduction.

It is the general practice on merchant vessels that have two boilers to specify that each boiler shall be good for 50 percent overload under emergency conditions in order that three-fourths power may be maintained with one boiler out of service. Therefore, a test is generally made to demonstrate that the boiler is capable of operating at this load. If the vessel is equipped with more than two boilers, then the overload is less. On naval vessels, the overload is usually 20 percent, and a test at this rating is generally made on one vessel of each class.

On special types of vessels, such as airplane carriers, it is essential to know the speed that can be maintained astern. Therefore, these vessels are standardized in the same manner as described above. Other types of naval vessels are generally run over the measured mile at maximum astern power to determine their approximate astern speed.

TRIALS OF S. S. "RED JACKET"

The requirements for and information obtained from acceptance trials may be illustrated from the following description* of the trials of the S. S. "Red Jacket." This ship is a U.S. Maritime Commission, C2-type vessel, having a single screw driven by double reduction-geared turbines supplied with steam from two water-tube D-type boilers and fitted with electric auxiliaries and automatic controls.

The contract required the following trials for this vessel:

Standardization Trials. The trials prescribed in the contract included

1. Standardization trial at deep draft, during which the vessel shall be loaded to a molded mean draft of 25 ft 9 in. with due regard to proper trim. Three consecutive runs will be made at each of the following speeds: at about 9, 12, 14, and 15½ knots, and five consecutive runs with the propelling machinery developing maximum shaft horsepower.

2. Standardization trial at light draft, during which the vessel shall be ballasted to a molded draft, approximately as follows:

Forward 16 ft 2 in., aft 22 ft 2 in., mean 19 ft 2 in. This condition of trim shall be obtained by the use of deep tanks, trim tanks, and such oil or salt water in the double-bottom tanks or by the use of such form of ballast as is found necessary. The speeds at which this trial shall be run will be at the number of revolutions established from model-tank experiments corresponding to 9, 12, 14, and 15½ knots and the number of rpm shown by model-tank experiments which can be obtained when developing the normal shaft horsepower of 6,000 under the above conditions of loading.

The results of these trials are given in Tables 2 and 3, and the average data are plotted on Fig. 7.

Endurance and Fuel-economy Trials. The contract required an 8-hr endurance trial in open sea in deep water, while having a molded draft of 16 ft 2 in. forward, 22 ft 2 in. aft, mean 19 ft 2 in. During this endurance trial the engines are to develop 6,000 shp, and the fuel consumption for all purposes will be carefully determined.

* Data from "Report of Progressive Speed at Deep and Light Draft, Maneuvering, Endurance and Fuel Economy Trials of the Single Screw Cargo Ship S. S. Red Jacket," U.S. Maritime Commission Trial Board, September, 1939.

On this trial the following auxiliaries shall be in operation:

1. All auxiliaries necessary to the main propelling plant during the trial
2. The refrigerating plant.
3. The electric generating plant to be run at not less than 170 kw.

Also, the specifications stated that "When the ship is at sea under normal operating and weather conditions, developing 6,000 shp and with one generator developing the required electrical load at this power (but not less than 170 kw), normal use of steam in heating of sanitary water and fuel oil, a sea-water temperature of 75 F and the boilers operated with 18,500 Btu

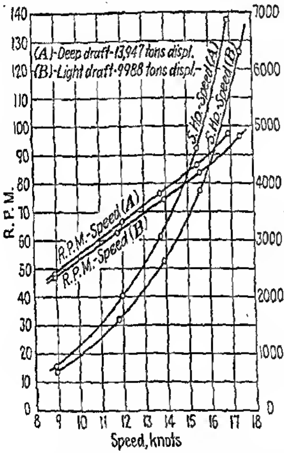


FIG. 7.—Standardization results.

fuel oil, an over-all consumption of 0.60 lb fuel oil per shp per hr is to be guaranteed."

As a development of the contract, owing to design changes, the 170 kw referred to above was changed to 185 kw. Also, it was agreed that the 8-hr endurance trial should consist of a 2-hr endurance trial at maximum designed shaft horsepower (6,600) and a 6-hr fuel-economy trial at normal designed shaft horsepower (6,000). Also, at the request of the U.S. Maritime Commission, it was agreed to run three trials of 2-hr duration each at reduced power.

The results and data obtained during these trials are given in Table 4.

Backing and Maneuvering Tests. In order to determine the backing and maneuvering ability of the vessel, the contract prescribed the following test:

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(For abbreviations used, see pp. xiii–xviii)

1. A ½-hr backing test at full astern power to demonstrate that the engines will develop the specified astern power without undue heating.

During this test the machinery operated without overheating at 62.4 rpm, developing 2,556 shp and a thrust of 60,400 lb, with steam conditions as follows:

Astern inlet pressure, psi gage	220
Astern inlet temperature, deg F	490
Exhaust temperature, deg F	103

2. A test to determine the ability of the vessel to go from full speed ahead to full speed astern and the time required to do so and the ahead reach.

With the vessel going ahead at normal speed (96 rpm) a signal was given for full astern power. The vessel was judged dead in the water 4 min 15 sec later. The plotted track showed an ahead reach over the ground of 1,680 yd, no correction being made for current.

3. A test to determine the ability of the vessel to go from full sternway to full speed ahead and the time required to do so and the stern reach.

With the vessel going astern at full astern power (67 rpm) a signal was given for full ahead power. The plotted track shows an astern reach of 620 yd in 3 min, no correction being made for current.

4. A test to determine the time required from full speed ahead to a dead stop.

With the vessel going ahead at normal speed (97 rpm), a signal was given to remove all power from the shaft. The vessel was judged dead in the water 39 min later. The plotted track shows a distance of 5,680 yd in 39 min.

5. A test to determine the diameter of the turning circle with the rudder in the hardover position, the speed of the vessel during this test to be that attainable with the propelling machinery developing 6,000 shp.

Turning circles were made with hard-left and hard-right rudder. The diameter was estimated as five ship lengths for each circle. The time to swing the ship's head through 360 deg was 6 min for the left circle and 6 min 37 sec for the right circle.

Steering-gear Tests. The tests prescribed in the contract included

1. Movement of the rudder from hardover to hardover (through 70 deg of arc) in 30 sec at full speed ahead.

2. Movement of the rudder from hardover to hardover at full speed astern in no specified time, without exceeding 1,500 psi oil pressure.

3. Movement of the rudder through full travel by means of the emergency gear (tackles led to the afterdeck winch).

Test 1 was performed with the vessel steaming ahead at full speed (96 rpm). The rudder reached the maximum travel specified (70 deg) in 30 sec, the required rate. The maximum pressure indicated was 1,000 psi.

Test 2 was performed with the vessel steaming astern at full speed (67 rpm). Rudder travel in this case was from 34½ deg left to 35½ deg right, a total of 70 deg in 25 sec. The maximum pressure was 850 psi.

Test 3 was performed with the vessel steaming ahead at 46 rpm. The emergency tackles were rigged in about 30 min by eight men. The rudder was moved from 34 deg right to 34 deg left in 30 sec.

Anchor-windlass Test. The tests specified for the anchor gear in the contract included

1. Letting go both anchors at the same time.

2. Hoisting one anchor with at least 30 fathoms of chain at a speed of 30 fpm.

3. Hoisting two anchors with at least 15 fathoms of chain attached to each anchor at a rate of 20 fpm.

Test 1 was performed by dropping both anchors in 32 fathoms of water.

Test 2 was performed at a chain speed of 32.1 fpm, 240 volts, 110 amp average current.

Test 3 was performed at a chain speed of 25.7 fpm, 240 volts, 160 amp average current.

Table 2. S.S. "Red Jacket." Deep-draft Standardization Trial Data
(Runs 1 to 12, Aug. 23, 1939, Runs 13 to 17, Aug. 24, 1939)

Run No.	Direction	Time at midpoint	Speed, knots	Rpm	Shp	Thrust, lb	Relative wind	
							Dir'n[a]	Vel, knots
1	S	11:04	9.58	49.11	819	29,050	F-45-P	7.02
2	N	11:47	8.45	48.18	754	27,350	F-15-S	7.69
3	S	12:29	9.27	49.42	827	29,750	F-10-P	8.69
	Mean of group		8.94	48.72	789	28,375		
4	N	1:14	11.87	66.49	1,952	51,100	F-0-0	5.10
5	S	1:14	12.09	66.48	2,036	52,750	F-10-S	17.52
6	N	2:10	12.12	67.05	2,029	52,550	F-30-P	5.27
	Mean of group		12.02	66.63	2,013	52,525		
7	S	2:45	13.58	76.70	3,092	70,550	F-10-S	21.69
8	N	3:10	13.95	76.58	3,021	68,850	F-56-P	3.49
9	S	3:37	13.49	76.94	3,150	70,450	F-10-S	21.96
	Mean of group		13.74	76.70	3,071	69,625		
10	N	4:04	15.59	85.95	4,480	87,150	F-15-P	8.73
11	S	4:31	15.18	86.47	4,636	90,400	F-10-S	24.90
12	N	4:54	15.75	87.37	4,729	91,350	F-30-P	7.75
	Mean of group		15.43	86.57	4,620	89,825		
13	N	8:14	16.61	98.24	7,109	121,850	F-0-0	19.78
14	S	9:46	17.22	96.89	6,747	116,700	F-5-P	12.26
15	N	9:16	16.36	97.52	6,845	120,100	F-10-S	19.47
16	S	9:47	17.54	97.97	6,969	119,850	F-5-P	14.28
17	N	10:17	16.19	97.53	6,943	120,650	F-0-0	16.82
	Mean of group		16.88	97.57	6,872	119,475		

Other data:

Draft, forward	25 ft 10¾ in.
Draft, aft	26 ft 1¾ in.
Displacement, sea water	13,990 tons
Corrections[b]	43 tons
Corrected displacement	13,947 tons
Sea water, spec. grav	1.0225
Sea-water temperature, deg F	59
Air temperature, deg F	75
Barometer	30.08
Weather	Bright
Wind	Calm
Sea	Smooth

[a] Forward or aft, degrees, port or starboard.

[b] Density and anchor and chain.

Table 3. S.S. "Red Jacket." Light-draft Standardization Trial Data
(Runs 1 to 12, Aug. 26, 1939. Runs 13 to 15, Aug. 27, 1939)

Run No.	Direction	Time at midpoint	Speed, knots	Rpm	Shp	Thrust, lb[b]	Relative wind	
							Dir'n[a]	Vel, knots
1	S	1:12	9.39	45.92	620	22,100	F-5-S	9.09
2	N	2:02	8.63	48.10	716	23,550	F-10-S	6.15
3	S	2:56	9.21	47.14	679	23,200	F-10-P	9.95
	Mean of group.....		8.97	47.32	683	23,100		
4	N	3:35	11.64	62.15	1,573	41,050	F-5-P	7.68
5	S	4:16	11.70	62.30	1,597	42,350	F-0-0	14.47
6	N	4:55	12.17	63.51	1,647	42,350	F-60-P	4.92
	Mean of group.....		11.80	62.57	1,604	42,025		
7	S	5:31	13.61	74.19	2,643	59,250	F-15-S	20.27
8	N	6:03	14.27	74.32	2,618	58,300	F-60-P	10.15
9	S	6:39	13.53	74.37	2,654	59,750	F-20-S	21.31
	Mean of group.....		13.92	74.30	2,633	58,900		
10	N	7:07	16.01	83.99	3,870	76,600	F-45-P	12.65
11	S	7:36	15.18	83.46	3,850	76,750	F-20-S	20.49
12	N	7:58	15.92	83.88	3,875	77,450	F-40-P	14.21
	Mean of group.....		15.57	83.70	3,861	76,888		
13	S	7:05	17.48	96.84	6,299	107,650	F-45-P	2.86
14	N	7:31	17.34	96.37	6,311	107,750	F-10-S	32.10
15	S	7:58	17.51	97.19	6,349	108,050	F-45-P	5.54
	Mean of group.....		17.42	96.69	6,318	107,800		

Other data:

Draft, forward	16 ft 4¾ in.
Draft, aft	22 ft 2¾ in.
Displacement, sea water	10,020 tons
Corrections[c]	32 tons
Corrected displacement	9,988 tons
Sea water, spec. grav	1.0222
Sea-water temperature, deg F	60
Air temperature, deg F	70
Barometer	30.10
Weather	Clear
Wind	Light southwesterly breeze
Sea	Calm

[a] Forward or aft, degrees, port or starboard.
[b] Corrected for trim.
[c] Density and anchor and chain.

bering each square independently from 1 to 10. The length of the side of one square is called the *unit* or *base* of the logarithmic paper; the larger the unit, the finer the possible subdivisions of the scale.

To plot a point (x,y) on logarithmic paper, for example, the point (3,5), means to find the point of intersection of the vertical line marked $x = 3$ and the horizontal line marked $y = 5$. In interpolating between two lines, account should be taken of the fact that the divisions are not of uniform length.

Any equation of the form $y = cx^n$ when plotted on logarithmic paper will be represented by a straight line whose slope is n. For, if $y_1 = cx_1^n$ and $y_2 = cx_2^n$, then $y_1/y_2 = (x_1/x_2)^n$, or $(\log y_1 - \log y_2)/(\log x_1 - \log x_2) = n$. The slope must be measured by aid of an auxiliary *uniform* scale.

EXAMPLE. Let $y = x^{3/2}$. When $x = 1$, $y = 1$; plot this point A on the logarithmic paper, and draw the straight line AB with a slope equal to $\frac{3}{2}$ (Fig. 13). By the aid of this line, the value of y for any value of x between 1 and 100 can be read off directly; for example, if $x = 2.50$, $y = 3.95$, as shown by dotted lines, so that $(2.50)^{3/2} = 3.95$. To find the value of y for any value of x outside this range, note that moving the decimal point 2 places in x is equivalent to moving it 3 places in y. The line shown in Fig. 13 is thus equivalent to a complete table of three-halves powers.

It will be noticed that this line crosses four squares of the logarithmic paper. By superposing these four squares the whole diagram may be condensed into a single square (Fig. 14), in which, however, the scales for x and y now give only the sequence of digits in the answer, the position of the decimal point having to be determined by inspection.

To determine whether a given set of values, x and y, satisfies a law of the form $y = cx^n$, plot the values on logarithmic paper, and see whether they lie on a straight line; if they do, then the given values satisfy a law of this form; moreover, the slope of the line gives the value of n, and the value of y when $x = 1$ gives the value of c.

If the plotted points fail to lie exactly in line, but form a curve slightly concave upward, try subtracting some constant b from all the y's, that is, move each point downward a distance equal to b units of the y-scale at that point. If it proves possible to choose b so that the resulting points lie in line, then the original values obey a law of the form $y - b = cx^n$, where n is again the slope of the line, and c is the value of $y - b$ when $x = 1$. (Conversely, if the curve is concave *downward*, try *adding* b to all the y's; that is, move each point *upward*; if the new points lie in line, the original values obey a law of the form $y + b = cx^n$.) Another method of "straightening" the curve consists of adding some constant, $\pm a$, to all the values of x, which has the effect of shifting all the points to the right or left (by varying amounts); if this method succeeds, the original values obey a law of the form $y = c(x + a)^n$.

Semi-logarithmic Cross-section Paper. This form of paper (Fig. 15) has a logarithmic scale along y and a uniform scale along x. The "**scale value**," k, of the paper is the number which stands, on the x-axis, at a distance from the origin equal to the width of one of the main horizontal strips. Thus, in Fig. 15, each number shown along the auxiliary scale Y is the logarithm of the corresponding number along y; and each number shown along the auxiliary scale X is $1/k$th of the corresponding number along x (here $k = 5$). The number k, which may be chosen at pleasure, should be taken equal to some simple integer, as 1, 2, or 5, or some integral power of 10.

In preparing the paper for use it is important to notice that the numbers . . ., 0.01, 0.1, 1, 10, 100, . . . (or such of them as may be needed in any given case) must be placed along the y-axis at the points which mark the main lines of division between the horizontal strips; while the numbers . . ., $-2k$, $-k$, 0, $+k$, $+2k$, . . . (or such of them as may be needed) must be placed along the x-axis at uniform intervals, each interval (from 0 to k, from k to $2k$, etc.) being equal to the width of one of the main horizontal strips. The width of one of these strips is called the *unit* or *base* of the semi-

logarithmic paper; the larger the unit, the finer the possible subdivisions of the scale.

To plot a point (x,y), as $x = 3$, $y = 5$, on semi-logarithmic paper means to find the point of intersection of the vertical line marked $x = 3$ with the horizontal line marked $y = 5$.

Any equation of the form $y = c \cdot 10^{mx}$ [or $y = c \cdot e^{mx}$] when plotted on semi-logarithmic paper with scale value k, will be represented by a straight line whose slope is km [or $0.4343\, km$.]. By a suitable choice of the scale value k, any given range of values of x can be brought within the size of the paper. Note that $e = 10^{0.4343}$.

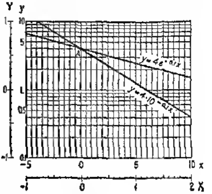


FIG. 15.

EXAMPLE. Given $y = 4 \cdot 10^{-0.5x}$ [or $y = 4 \cdot e^{-0.1x}$]. In Fig. 15, when $x = 0$, $y = 4$. By plotting this point (A) on the semi-logarithmic paper, with scale value 5, and drawing through it a straight line with slope equal to -0.5 [or -0.217] a graphical representation is obtained from which, for any value of x, the corresponding value of y can be read off. If it is desired to condense the figure, several horizontal strips may be superposed on a single strip; this of course renders the decimal point in the y-scale undetermined (unless a separate y-scale is provided for each section of the graph).

In order to determine whether a given set of values of x and y satisfy a law of the form $y = c \cdot 10^{mx}$ [or $y = c \cdot e^{mx}$], plot the values of x and y on semi-logarithmic paper, with a suitable scale value k, and see whether they lie on a straight line; if they do so, the law is satisfied, and the values of m and c may be found as follows: m = the slope of the line divided by k [or the slope of the line divided by $0.4343k$], and c = the value of y when $x = 0$.

If the plotted points fail to lie exactly in line, but form a curve slightly concave upward, try subtracting some constant b from all the y's, and plot the values thus modified; if b can be so chosen that the revised points lie in line, then the original values obey a law of the form $y - b = c \cdot 10^{mx}$ [or $y - b = c \cdot e^{mx}$], where m and c are to be found as before. If the curve is concave downward, add b, instead of subtracting; and replace $y - b$ by $y + b$ in the law.

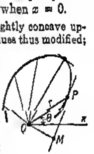


FIG. 16.

Curves in Polar Co-ordinates. Any function, r, of a single variable, θ, can be represented by a curve in polar co-ordinates (p. 137). Lay off the given values of θ as angles, the initial line Ox running toward the right, and the counterclockwise direction about the origin being taken as positive. Along the terminal side of each angle θ, lay off the corresponding value of r, forward if r is positive, backward if r is negative; and pass a smooth curve through the points thus determined.

The rate of change of r with respect to θ at a given point P is represented graphically as follows (Fig. 16): On the tangent at P drop a perpendicular OM from the origin: then $r(MP/OM)$ represents the rate of change, $dr/d\theta$, provided θ is measured in radians. Specially ruled polar co-ordinate paper is supplied by dealers in drafting supplies.

EQUATIONS INVOLVING THREE VARIABLES

The Surface $z = f(x, y)$. Any function, z, of two variables, x and y, may be represented by a surface, as follows: Plot the given pairs of values of x and y as points in a horizontal x, y plane, called the base plane; at each of these points erect an ordinate, parallel to a vertical axis z, and representing by its length the value of z at that point. Then conceive a smooth surface

passed through the extremities of these ordinates: this surface is said to represent the function. In practice, the ordinates may be made by implanting stiff vertical rods in a horizontal board of soft wood which serves as the base plane; the surface may then be constructed by filling in the spaces with plaster of Paris. Or, more simply, pieces of cardboard may be cut out to represent parallel plane sections of thn surface, and then stood on edge in slots cut in the board to receive them. The units employnd along x, y, and z need not he equal to each other.

Contour-line Charts. All the points of a surface $z = f(x, y)$ which are at any given height above the base plane form a curve on the surface, called a contour line of the surface. If each of these contour lines be projected on the base plane, and each labeled with the value of z to which it corresponds, a complete representation of the function $z = f(x, y)$ is obtained, all in one plane. A topographical map, with contour lines showing elevations above the sea, and a weather map, with contour lines showing barometric pressure, are familiar examples. If there are several values of z corresponding to any given point (x, y), there will be several contour lines whose projections pass through that point.

Contour-line Charts for Simultaneous Equations [of the form $z = f(x,y)$, $w = F(x,y)$]. In Fig. 17, plot the function $z = f(x,y)$ by contour lines on an x,y plane, and plot the function $w = F(x,y)$ by contour lines on the same x,y plane. Then every point on the diagram (either directly or by interpolation) is the intersection of four curves—an x-curve, a y-curve, a z-curve, and a w-curve. Here, by "curve" is meant any line, straight or curved. By the aid of such a diagram, when the values of any two of these four variables are given, the values of the other two can be found. The method of use consists simply in entering the diagram along the two given curves (or lines), tracing them to their point of intersection, and then coming out again along the two curves (or lines) whose values are required. The best manner of numbering the curves is indicated in the figure.

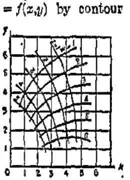


FIG. 17.

ALIGNMENT CHARTS

Alignment Charts for Three Variables, t, u, v. Any relation between three variables, t, u, v, which can be thrown into one of the forms listed in later paragraphs, can be represented graphically by a very convenient form of diagram called an alignment chart. In the simplest form of an alignment chart for three variables there are three scales (straight or curved), along which the values of the three variables, t, u, v, are marked in such a way that any three values of t, u, v which satisfy the given equation are represented by three points which lie in line. Hence, if the values of any two of the variables are given, the corresponding value of the third can be found by simply drawing a straight line through the two given points and reading the value of the point where it crosses the third scale.

The most important methods of constructing alignment charts for three variables are described below. Where several methods are applicable in a given case, the best one must be determined largely by trial. For further information, see M. d'Ocagne, "Traité de Nomographie," Gauthier-Villars, Paris; Carl Runge, "Graphical Methods," Columbia University Press; J. B. Peddle, "Construction of Graphical Charts," McGraw-Hill; J. Lipka, "Graphical and Mechanical Computation," Wiley; Hewes and Seward, "The Design of Diagrams for Engineering Formulas and the Theory of Nomography," McGraw-Hill.

Method 1. Given, an equation which can be thrown into the form

$$f_1(u) + f_2(v) = f_3(t),$$

where $f_1(u)$ is a function of u alone, $f_2(v)$ a function of v alone, etc. An alignment chart may be constructed as follows:

Choice of Moduli to Fit Size of Paper. Let $f_1(u')$ be the smallest and $f_1(u'')$ the largest value of $f_1(u)$ likely to be needed, and let h be the height of the available space on the paper. Then find a simple number, m_1, such that m_1 times $f_1(u'') - f_1(u')$ shall not exceed h. Similarly, find a simple number m_2 such that m_2 times $f_1(v'') - f_1(v')$ shall not exceed h.

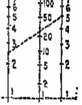
Also, compute a third modulus, m_3, by the formula

$$m_3 = (m_1 m_2)/(m_1 + m_2).$$

Construction of the First Two Scales. Draw two parallel vertical axes, at any distance, k, apart. On the first axis, marked u, starting with any convenient origin, lay off the distances $x = m_1 f_1(u)$ for successive values of u, *labeling each point thus plotted with the corresponding value of u.* Similarly, on the second axis, marked v, starting with any convenient origin, lay off $y = m_2 f_2(v)$ for successive values of v, labeling each point with the corresponding value of v. The u-scale and the v-scale are thus completed.

Construction of the Third Scale. Draw a third line, t, parallel to the first two lines, dividing the distance k in the ratio m_1/m_2; that is, the distance from u to t is $m_1 k/(m_1 + m_2)$. Compute the value t_0 corresponding to any convenient values u_0 and v_0, and label with this value, t_0, the point where the t-axis is cut by a straight line joining the points u_0 and v_0. Using this point t_0 as an anchorage, lay off along the t-line the scale determined by $z = m_3 f_3(t)$ where $m_3 = (m_1 m_2)/(m_1 + m_2)$. The third scale is thus completed, and the chart is ready for use.

Note that the units of measurement for x, y, and z (which do not appear on the completed chart) must of course be the same. Note also that to ensure accuracy on the third scale, especially if the modulus m_3 is small, it is well to compute more than one anchorage point, t_0.



$uv^{1.41} = t$

FIG. 18.

The construction is greatly facilitated by the use of previously constructed uniform and logarithmic scales with various moduli.

EXAMPLE (Fig. 18). Let $uv^{1.41} = t$, for a range of values of u and v between 1 and 10. By taking the logarithm of both sides, reduce the equation to the form $\log u + 1.41 \log v = \log t$. Here $f_1(u) = \log u$, $f_2(v) = 1.41 \log v$, $f_3(t) = \log t$. For a height of paper $h = 10$, and a width $k = 5$, we may take $m_1 = 10$ and $m_2 = 10/1.41 = 7.09$; whence $m_3 = 4.15$ and $m_1 k/(m_1 + m_2) = 2.92$. Hence the chart is readily constructed, as shown.

Method 1a. Method 1 may be readily extended to equations of the form

$$f_1(u) + f_2(v) + f_3(w) = f_4(t),$$

involving four variables, t, u, v, w.

Let $f_1(u) + f_2(v) = q$ and chart this equation by Method 1. Then chart the equation $q + f_3(w) = f_4(t)$ by the same method, using as one of the scales the q-scale already drawn. (The q-scale need not be graduated; the position of the q-axis is all that is important.) In reading the completed chart, we use two index lines, one joining points u and v, and cutting the q-axis in an (unlabeled) point q; the other joining points w and t, and cutting the q-axis in the same (unlabeled) point q. Thus when any three of the four variables are given, the fourth can be found.

A further extension to equations of the same form involving five or more variables is obvious.

EXAMPLE (Fig. 19). Let $t = w\sqrt{uv}$ whence $\frac{1}{2}\log u + \frac{1}{2}\log v + \log w = \log t$. Here $f_1(u) = \frac{1}{2}\log u$, $f_2(v) = \frac{1}{2}\log v$, $f_3(w) = \log w$, and $f_4(t) = \log t$.

Method 2. Given, an equation which can be thrown into the form

$$f_1(u) = f_2(v).f_3(t)$$

where $f_1(u)$ is a function of u alone, $f_2(v)$ a function of v alone, etc. First, choose two "moduli" m_1 and m_2 (to fit size of paper) exactly as in Method 1.

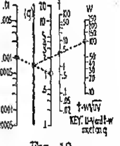


FIG. 19.

Secondly, draw two parallel vertical axes, AX and BY, oppositely directed, the diagonal line AB being of any convenient length k. With A and B as origins, lay off along these axes the distances

$$x = m_1f_1(u) \text{ and } y = m_2f_2(v),$$

for successive values of u and v, respectively, and label each point thus plotted with the corresponding value of u (or v). The u and v scales are thus completed.

Thirdly, on BY select a point F at any convenient distance, l, from B; compute an auxiliary modulus, n, by the formula $n = lm_1/m_2$; and lay off along AX an auxiliary scale, $x' = nf_3(t)$ for successive values of t, marking each point (temporarily) with the corresponding value of t. Then transfer this auxiliary scale to the axis AB by means of projecting lines drawn through the point F, marking each point (permanently) with the corresponding value of t. The t-scale, along the axis AB, is thus completed, and the chart is ready for use.

As a check, note that along the t-axis, the distance z from A to any point labeled t should be given by

$$z = km_1f_3(t)/[m_1f_3(t) + m_2].$$

Indeed the points of the t-scale may be laid down independently by the use of this formula, if desired, stead of by the graphical method above described.

This type of chart is known as a *Z-chart*.

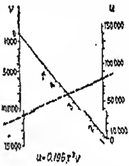


FIG. 20.

EXAMPLE (Fig. 20). Let $u = 0.196t^2v$, where u is to range from 0 to 150,000 and v from 0 to 15,000. The equation may be written $u = (10v)(0.0196t^2)$.

Here $f_1(u) = u$, $f_2(v) = 10v$, $f_3(t) = 0.0196\,t^2$.

The theory underlying Methods 1 and 2 depends only on simple properties of similar triangles. The following methods are based on certain standard equations of the straight line in analytical geometry, and the notation in what follows has been suggested by the form of these equations.

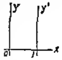
Notation. In each of the equations which follow, U stands for any function of u alone, V for any function of v alone, and $F_1(t)$, $F_2(t)$ for any functions of t alone. Any of these functions may reduce to a constant. The axes of x, y, and y' which are mentioned are of merely temporary use in constructing the diagram, and the letters x, y, y' should not be written on the chart. It is not necessary that the axes be at right angles, provided the x of a point is always measured parallel to the x-axis, and its y parallel to the y-axis.

Method 3. Given, an equation which can be thrown into the form

$$U{\cdot}F_1(t) + V{\cdot}F_2(t) = 1,$$

where, for the given range of values of u and v, the largest variations in U and V are less than a certain number m.

Draw a pair of (temporary) x,y axes (Fig. 21), and through the point $x = 1$ draw a third axis, which may be called the axis of y', parallel to the axis of y. In ordinary cases, the unit of measurement along x should be nearly equal to the full width of the paper. Now choose a unit for y and y' such that m times this unit will about equal the height of the paper, and plot, in the usual way, the points (x,y) given by



$$x = \frac{F_2(t)}{F_1(t) + F_2(t)}, \quad y = \frac{1}{F_1(t) + F_2(t)},$$

FIG. 21.

labeling each point with the value of t to which it corresponds. Connect these points by a smooth curve, which gives the t-scale of the diagram. [If $F_1(t)/F_2(t)$ = a constant, the t-scale will prove to be a straight line parallel to the y-axis.]

Then, using the same units as above, plot along y the points given by $y = U$, labeling each point with the corresponding value of u; and plot along y' the points given by $y' = V$, labeling each of these points with the corresponding value of v. This gives the u- and v-scales of the diagram. The three scales being thus constructed, the x-axis may now be erased, and the diagram is ready for use. Any three points t, u, v which lie in line correspond to three values of t, u, v, which satisfy the given equation. The numbering on each scale should be shown at sufficiently frequent intervals to permit of easy interpolation.

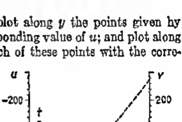


FIG. 22.

EXAMPLE (Fig. 22). Let $v = ut + 16t^2$, which reduces to the form $(-u/16)(1/t) + (v/16)(1/t^2) = 1$. Here $U = -u/16$, $V = v/16$, $F_1(t) = 1/t$, $F_2(t) = 1/t^2$ and $x = 1/(1+t)$, $y = t^2/(1+t)$.

NOTE. If $m = \infty$, values of u and v which give large values of U and V cannot be shown within the limits of the paper. In such cases, the chart may be supplemented by a second chart, made according to Method 4, below.

Method 4. Given, an equation which can be thrown into the form

$$\frac{F_1(t)}{U} + \frac{F_2(t)}{V} = 1,$$

where, for the given range of values of u and v, the largest variation in U is less than a certain number m, and the largest variation in V is less than a certain number n.

Draw a pair of temporary x,y axes, and having chosen a unit for the x-axis equal to about $(1/m)$th of the width of the paper, and a unit for the y-axis equal to about $(1/n)$th of the height, plot the points (x,y) given by

$$x = F_1(t), \quad y = F_2(t),$$

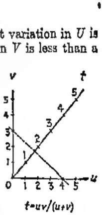


FIG. 23.

labeling each point of this curve with the value of t to which it corresponds. Connect these points by a smooth curve, which gives the t-scale of the diagram. [If $F_1(t)/F_2(t)$ = a constant, the t-scale will be a straight line through the origin.]

Then, using the same units as above, plot along x the values of U, labeling each point with the corresponding value of u; and plot along y the values of V, labeling each point with the corresponding value of v.

Draw axes x, y, and y' as in Method 3, and plot the network of curves given by the equations

$$x = \frac{F_2(r,s)}{F_1(r,s) + F_2(r,s)}, \qquad y = \frac{1}{F_1(r,s) + F_2(r,s)}.$$

[To do this (Fig. 25), find the point (x,y) that corresponds to each given pair of values of r and s, by direct substitution in the equations for x and y. Connect all the points for which $r = 1$ by a curve, and label it $r = 1$; connect all the points for which $r = 2$ by another curve, and label it $r = 2$; etc. This gives the family of r-curves. Similarly, through all the points for which $s = 1$ draw a curve labeled $s = 1$; through all the points for which $s = 2$ draw a curve labeled $s = 2$; etc. This gives the family of s-curves, intersecting the family of r-curves. Note, however, that if it is possible to eliminate s (or r) from the equations that give x and y, the resulting equation in x, y, and r (or x, y, and s) can often be plotted directly for each given value of r (or of s).]

Next, construct the u- and v-scales along the axes of y and y' as in Method 3. [The letters x, y, and y', and the units used in plotting along these axes, should be omitted from the finished diagram, as should also the axis of x.]

In the chart, as thus completed, any three points, (r,s), u, and v which lie in a straight line, correspond to values of r, s, u, v which satisfy the given equation. Hence, when any three of these four values are given, the fourth can be found from the chart.

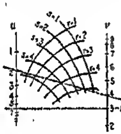


FIG. 25.

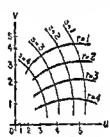


FIG. 26.

Method 4a. Given, an equation of the form

$$\frac{F_1(r,s)}{U} + \frac{F_2(r,s)}{V} = 1.$$

Draw axes of x and y as in Method 4, and plot the network of curves given by

$$x = F_1(r,s), \; y = F_2(r,s).$$

To do this, follow the plan outlined for a similar case under Method 3a, labeling each curve of the r-family (Fig. 26) with the corresponding value of r, and each curve of the s-family with the corresponding value of s. Next, construct the u- and v-scales along the x- and y-axes, precisely as in Method 4. Then any three points, (r,s), u, and v, which lie in a straight line correspond to values of r, s, u, v which satisfy the given equation.

Method 5a. Given, an equation of the form

$$F_2(r,s) = V \cdot F_1(r,s) + U.$$

Draw axes of x and y, as in Method 5, and plot the network of curves given by $x = F_1(r,s)$, $y = F_2(r,s)$, following the plan outlined for a similar case under Method 3a, and labeling each curve of the r-family (or s-family) with the value of r (or s) to which it corresponds. Next, construct the u-scale

Method C. Given, an equation of the form

$$R - S = \frac{V}{U}.$$

In Fig. 30, take a pair of axes, x,y, and through the point $x = 1$ draw a third axis, y', parallel to y. Also, take a second pair of axes, x_2,y_2, parallel to (or coinciding with) the axes of x and y. Having chosen a suitable unit for x and x_2, and a suitable unit for y, y', and y_2, lay off the values of R and

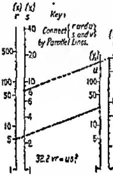


Fig. 28.

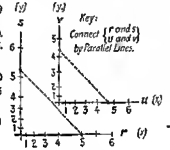


Fig. 29.

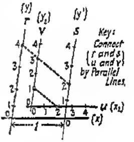


Fig. 30.

S along y and y', respectively, labeling each point with the value of r or s to which it corresponds; and lay off the values of U and V along x_2 and y_2, labeling each point with the value of u or v to which it corresponds. Then if the line joining two points r and s is parallel to the line joining two points u and v, the four values r, s, u, v will satisfy the given equation.

For further examples, see R. C. Strachan, "Nomographic Solutions for Formulas of Various Types," Trans. Am. Soc. Civil Engineers, vol. 78, 1915.

VECTOR ANALYSIS

Many problems involving directed magnitudes can be advantageously treated by the methods of vector analysis. The following is a brief summary of the principal definitions and formulæ.

A set of arrows, each arrow having a given *length* and pointing in a given *direction*, is called a set of **vectors**, provided they combine by addition according to the parallelogram law (see below). Notation: $\mathbf{a}$ or a for a vector; a or $|\mathbf{a}|$ for its length. Two "free" vectors are equal if they have the same length and point in the same direction; two "sliding" vectors are equal if they have the same length and direction, and also lie in the same line.

A **scalar** is any real number, positive, negative, or zero.

Addition of vectors.—If an arrow $\mathbf{a}$ is immediately followed, tip to tail, by a second arrow $\mathbf{b}$, then the arrow which runs from the beginning of $\mathbf{a}$ to the end of $\mathbf{b}$ is called the **sum** of $\mathbf{a}$ and $\mathbf{b}$, denoted by $\mathbf{a} + \mathbf{b}$. Conversely, if $\mathbf{a} + \mathbf{x} = \mathbf{b}$, then $\mathbf{x} = \mathbf{b} - \mathbf{a}$. The laws of operation for $+$ and $-$ are the same as in ordinary algebra (pp. 112, 124). If m is a scalar, then $m\mathbf{a}$ means a vector having the same direction as $\mathbf{a}$, and m times its length.

This gives the u- and v-scales of the diagram. On the chart as thus completed, any three points t, u, v which lie in line correspond to three values of t, u, v which satisfy the given equation.

EXAMPLE (Fig. 23). Let $t = (uv)/(u + v)$, which may be written in the form $t/u + t/v = 1$. Here $U = u$, $V = v$, $F_1(t) = t$, $F_2(t) = t$.

NOTE. If $m = \infty$ and $n = \infty$, values of u and v which give large values of U and V cannot be shown within the limits of the paper. In such cases the chart may be supplemented by a second chart, made according to Method 3, above.

Method 5. Given, an equation which can conveniently be thrown into the form

$$F_2(t) = V \cdot F_1(t) + U,$$

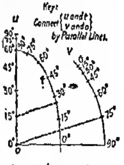
where, for the given range of values of t, the largest variation in $F_1(t)$ is less than a certain number m, and the largest variation in $F_2(t)$ is less than a certain number n.

Draw a pair of temporary x,y axes, and, having chosen a unit for x equal to about $(1/m)$th of the width of the paper and a unit for y equal to about $(1/n)$th of the height, plot the points (x,y) given by

$$x = F_1(t), \quad y = F_2(t),$$

labeling each point of the curve with the value of t to which it corresponds Connect these points by a smooth curve, which forms the t-scale. Next using the same unit for y as above, plot along the y-axis the values of U labeling each point with the corresponding value of u. This gives the u-scale. Finally, with the origin as center, and any convenient radius, draw a circle cutting the x-axis in A. Along this circular arc, starting from A in the counterclockwise direction, lay off the angles whose slopes are equal to V, labeling each point of the arc with the value of v to which it corresponds. This gives the v-scale, which in this case, however, plays a peculiar rôle, since, in using this form of chart, two straight lines are required instead of one. Thus:

In order to determine whether three values, t, u, v, satisfy the given equation, lay one straight line through the points t and u, and another straight line through the point v and the origin; if these lines are parallel, the three values of t, u, v satisfy the equation. It will be noticed that the function of the v-scale here is to measure, in a certain sense, the slope of the line joining t and u. A chart of this type may be called "an alignment chart with a sliding scale for one of the variables."



$\sin u = \sin 60° \sin t - \cos 60° \cos t \cos v$

FIG. 24.

EXAMPLE. Let $\sin u = \sin 60° \sin t - \cos 60° \cos t \cos v$ (Fig. 24), which may be put in the form

$$(\sin 60° \sin t) = \cos v \,(\cos 60° \cos t) + \sin u.$$

Here $F_1(t) = \cos 60° \cos t$, $F_2(t) = \sin 60° \sin t$, $U = \sin u$, $V = \cos v$.

Alignment Charts for Four Variables. The extension of methods 3, 4, and 5 to the case of four variables, say r, s, u, v, consists essentially in replacing the t-scale of the earlier diagram by a network of two scales, one for r and one for s. The point where a curve $r = r_1$ and a curve $s = s_1$ intersect may be spoken of as the point (r_1,s_1). In the following equations, U denotes as before any function of u alone, V any function of v alone; while $F_1(r,s)$ and $F_2(r,s)$ represent any functions of r and s.

Method 3a. Given, an equation of the form

$$U \cdot F_1(r,s) + V \cdot F_2(r,s) = 1.$$

MECHANICS OF RIGID BODIES

BY

H. W. HAYWARD

(Revised by J. P. Den Hartog and A. Haertlein)

KINEMATICS

Motion in a Straight Line—Rectilinear Motion

The position of a moving point at any instant may be stated by giving its distance and direction from some fixed point in its path which is taken as an origin. This distance s can be taken as a function of the time t, giving the equation of position $s = f(t)$.

Velocity. The velocity v of a moving point is the rate at which the distance s changes with respect to the time t, and may be uniform or varying.

$$v = ds/dt; \qquad s = \int v dt.$$

A space-time curve offers a convenient means for the study of the motion of a point. **The slope of the curve** at any point will represent the **velocity** at that time. In Fig. 1(*a*) the slope is constant, as the graph is a straight line; the velocity is therefore uniform. In Fig. 1(*b*) the slope of the curve varies from point to point, so the velocity must vary also. At p and q the slope is zero, therefore the velocity of the point at the corresponding times must also be zero.

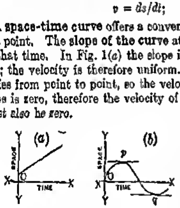


FIG. 1.

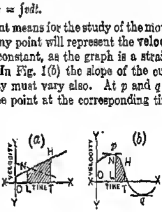


FIG. 2.

The acceleration a of a moving point is the rate at which its velocity v changes with respect to the time t, and may be uniform or varying, positive or negative.

For rectilinear motion, $a = dv/dt = d^2s/dt^2$; $s = \int\int a dt^2 = \int v dt$.

A velocity-time curve *offers a convenient means for the study of acceleration.* **The slope of the curve** at any point will represent the **acceleration** at that time. In Fig. 2(*a*) the slope is constant, so the acceleration must be constant. In the case represented by the full line, the acceleration is positive, so the velocity is increasing. The dotted line shows a negative acceleration and therefore a decreasing velocity. In Fig. 2(*b*) the slope of the curve varies from point to point, so the acceleration must also vary. At p and q the slope is zero, therefore the acceleration of the point at the corresponding times must also be zero and the velocity uniform. The area under the velocity-time curve between any two ordinates such as NL and HT will represent the distance moved in time interval LT. In the case of the uniformly accelerated motion shown by the full line in Fig. 2(*a*), the area $LNHT$ is $\frac{1}{2}(NL + HT) \times (OT - OL)$ = mean velocity multiplied by the time interval = space passed over during this time interval. In Fig. 2(*b*) the mean velocity can be obtained

along the y-axis, and the v-scale along a circular arc, precisely as in Method 5. Then any three points, (r,s), u, and v, which are so related that the line through (r,s) and u is parallel to the line joining v with the origin, will correspond to values of r, s, u, v which satisfy the given equation.

EXAMPLE for Method 5a (Fig. 27). Let $\cot v = \cot r \cos s + \csc r \sin s \cot u$, which may be written $(\cos r \cot s) = \cot v (\sin r \csc s) - \cot u$. Here $U = -\cot u$, $V = \cot v$, $F_1(r,s) = \sin r \csc s$, $F_2(r,s) = \cos r \cot s$, whence $\frac{x^2}{\csc^2 s} + \frac{y^2}{\cot^2 s} = 1$, $\frac{x^2}{\sin^2 r} - \frac{y^2}{\cos^2 r} = 1$, so that the s-curves are ellipses and the r-curves hyperbolas.

Parallel Charts, or Proportional Charts, for Four Variables. In the following methods of representation there are four scales, one for each of the four variables, and the method of using the diagram consists in connecting two pairs of points by parallel lines.

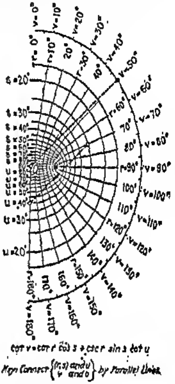


FIG. 27.

Method A. Given, an equation of the form

$$R - S = U - V$$

where R, S, U, V are any functions of the variables r, s, u, v, respectively. [It will be noted that any proportion $R/S = U/V$ can at once be thrown into this form by taking the logarithm of both sides.]

In Fig. 28, draw four vertical axes, y_1, y_2, y'_1, y'_2, such that the distance between y_1 and y'_1 (which may be zero) is equal to the distance between y_2 and y'_2, and so that the four zero points lie in line. Along these axes, using the same unit for all, plot the points given by $y_1 = R$, $y'_1 = S$, $y_2 = U$, $y'_2 = V$, and label each point with the value of r, s, u, or v to which it corresponds. (The letters y_1, y_2, y'_1, y'_2 are temporary, and should not appear on the diagram.) Then if the line joining two points r and u is parallel to the line joining two points s and v, the four values of r, s, u, v will satisfy the given equation. In this and the following methods, a parallel ruler, or a pair of draftsman's triangles, will be useful in reading the chart. A "key" stating which points are to be joined with which, should be clearly given on the diagram.

EXAMPLE (Fig. 28). Let $32.2\, vr = us^2$, or $\log r - 2 \log s = \log u - \log (32.2\, v)$. Here $R = \log r$, $S = 2 \log s$, $U = \log u$, $V = \log (32.2\, v)$.

Method B. Given, an equation of the form

$$\frac{R}{S} = \frac{U}{V}$$

In Fig. 29, draw a pair of axes, x,y, and parallel to them (or coinciding with them) a second pair of axes, x_1,y_1. Using any convenient horizontal unit, plot along x and x_1 the points given by $x = R$, $x_1 = U$, and using any convenient vertical unit, plot along y and y_1 the points given by $y = S$, $y_1 = V$. Label each point with the value of r, s, u, v, to which it corresponds. (The letters x, y, x_1, y_1 should not appear on the diagram.) Then if the line joining two points r and s is parallel to the line joining two points u and v, the four values r, s, u, v will satisfy the given equation.

moves about O its projection moves from O to Y, back to O, then to Y' and back to O again. This cycle of motion may be repeated indefinitely. If ωt is the variable angle XOp, the displacement s of n from O is $r \sin \omega t$, where $r = Op$. Velocity $v = ds/dt$, $\therefore v = \omega r \cos \omega t$. Acceleration $a = dv/dt$, $\therefore a = -\omega^2 r \sin \omega t = -\omega^2 s$. These equations assume that Op starts from position OX. If there is a lead angle $XOp' = B$, or a negative angle known as a lag, the expression will become $s = r \sin (\omega t + B)$, $v = \omega r \cos (\omega t + B)$, $a = -\omega^2 r \sin (\omega t + B) = -\omega^2 s$. When ωt has increased by 2π, *i.e.*, after the time $t = 2\pi/\omega$ has elapsed, s, v, and a regain their original values. t is called the **period of the motion**. The angle $(\omega t + B)$ is the **phase angle.**

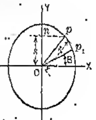


Fig. 5.

Composition and Resolution of Velocities

A velocity can be represented by a **vector**, which is a straight line having an arrow representing the direction of the motion and a length representing its magnitude.

Resultant. A velocity is said to be the resultant of two other velocities when it is represented by a vector that is the geometric sum of the vectors representing the other two velocities. This is the **parallelogram of motion.** In Fig. 6, v_r is the resultant of v_1 and v_2 and is represented by the diagonal of a parallelogram of which v_1 and v_2 are the sides; or it is the third side of a triangle of which v_1 and v_2 are the other two sides.

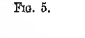


Fig. 6.

Polygon of Motion. The parallelogram of motion may be extended to the polygon of motion. Let v_1, v_2, v_3, v_4 [Fig. 7(a)] show the directions of four velocities imparted in the same plane to point O. If the lines v_1, v_2, v_3, v_4 [Fig. 7(b)] are drawn parallel to and proportional to the velocities imparted to point O, v_r will represent the resultant velocity imparted to O. It will make no difference in what order the velocities are taken in constructing the motion polygon. As long as the arrows showing the direction of the motion follow each other in order about the polygon, the resultant velocity of the point will be represented in magnitude by the closing side of the polygon, but opposite in direction.

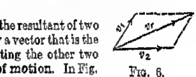


Fig. 7.

Resolution of Velocities. Velocities may be resolved into component velocities in the same plane, as shown by Fig. 8. Let the velocity of point O be v_r. In Fig. 8(a) this velocity is resolved into two components in the same plane as v_r and at right angles to each other.

$$v_r = \sqrt{(v_1)^2 + (v_2)^2}$$

In Fig. 8(b) the components are in same plane as v_r, but are not at right angles to each other. In this case,

$$v_r = \sqrt{(v_1)^2 + (v_2)^2 + 2v_1v_2 \cos B}$$

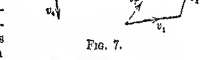


Fig. 8.

Multiplication of vectors is of two kinds, as follows:

The **scalar product**, or dot product, of two vectors $\mathbf{a}$ and $\mathbf{b}$, denoted by $\mathbf{a}\cdot\mathbf{b}$—or sometimes by $S\mathbf{ab}$, or by $(\mathbf{ab})$ in round parentheses—is defined as the scalar quantity $ab \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

EXAMPLE. If $\mathbf{F}$ is a force whose point of application moves along a vector distance $\mathbf{x}$, then $\mathbf{F}\cdot\mathbf{x}$ = *work* done by $\mathbf{F}$ during this displacement.

Peculiarities of scalar products: (1) Since $\mathbf{a}\cdot\mathbf{b}$ is not a vector, expressions like $(\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ will not occur; (2) from $\mathbf{a}\cdot\mathbf{x} = \mathbf{a}\cdot\mathbf{y}$ we cannot infer that $\mathbf{x} = \mathbf{y}$, hence, quotients will not occur; (3) from $\mathbf{a}\cdot\mathbf{b} = 0$, it follows that $\mathbf{a}$ is perpendicular to $\mathbf{b}$ (unless $\mathbf{a}$ or $\mathbf{b}$ is zero).

On the other hand, scalar products are like ordinary products in the following respects: $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$, and $(\mathbf{a} + \mathbf{b})\cdot(\mathbf{c} + \mathbf{d}) = \mathbf{a}\cdot\mathbf{c} + \mathbf{a}\cdot\mathbf{d} + \mathbf{b}\cdot\mathbf{c} + \mathbf{b}\cdot\mathbf{d}$; also, $m(\mathbf{a}\cdot\mathbf{b}) = (m\mathbf{a}\cdot\mathbf{b}) = \mathbf{a}\cdot(m\mathbf{b})$, where m is any scalar.

The **vector product**, or cross product, of two vectors a nd $\mathbf{b}$, denoted by $\mathbf{a}\times\mathbf{b}$—or sometimes by $V\mathbf{ab}$, or by $[\mathbf{ab}]$ in square bracket is defined as the *vector* whose length is $ab \sin \theta$, where θ is the angle betwee and $\mathbf{b}$, and whose direction is perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$ (in such a sense that a right-handed screw advancing along $\mathbf{a}\times\mathbf{b}$ would turn $\mathbf{a}$ toward $\mathbf{b}$).

EXAMPLE. If $\mathbf{F}$ is a force acting on a particle whose radius vector is $\mathbf{r}$, then $\mathbf{r}\times\mathbf{F}$ = the *torque* of $\mathbf{F}$ about the origin.

Peculiarities of vector products: (1) $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$, so that the order of the factors is always important; (2) $\mathbf{a}\times\mathbf{a} = 0$; (3) it is not true that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\times\mathbf{c}$; (4) from $\mathbf{a}\times\mathbf{x} = \mathbf{a}\times\mathbf{y}$ it does not follow that $\mathbf{x} = \mathbf{y}$; hence, quotients will not occur; (5) from $\mathbf{a}\times\mathbf{b} = 0$, it follows that $\mathbf{a}$ and $\mathbf{b}$ are **parallel** (unless $\mathbf{a}$ or $\mathbf{b}$ is zero).

On the other hand, as in ordinary algebra

$$(\mathbf{a} + \mathbf{b})\times(\mathbf{c} + \mathbf{d}) = \mathbf{a}\times\mathbf{c} + \mathbf{a}\times\mathbf{d} + \mathbf{b}\times\mathbf{c} + \mathbf{b}\times\mathbf{d},$$

provided the order of factors in each product is preserved; also, $m(\mathbf{a}\times\mathbf{b}) = (m\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(m\mathbf{b})$, where m is any scalar. Further laws are:

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}); \text{ and } \mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}.$$

Vector Differentiation. If $\mathbf{r} = \mathbf{f}(t)$ gives a vector $\mathbf{r}$ as a function of a scalar t, then $d\mathbf{r}/dt = \lim\{[\mathbf{f}(t + \Delta t) - \mathbf{f}(t)]/\Delta t\}$ as Δt approaches zero.

$$d(\mathbf{a} + \mathbf{b}) = d\mathbf{a} + d\mathbf{b}, \quad d(m\mathbf{a}) = m(d\mathbf{a}) + (dm)\mathbf{a},$$

$$d(\mathbf{a}\cdot\mathbf{b}) = (d\mathbf{a})\cdot\mathbf{b} + \mathbf{a}\cdot(d\mathbf{b}), \quad d(\mathbf{a}\times\mathbf{b}) = (d\mathbf{a})\times\mathbf{b} + \mathbf{a}\times(d\mathbf{b}).$$

EXAMPLE. If $\mathbf{r} = \mathbf{f}(t)$ gives the position-vector of a moving particle as a function of the time t, then $d\mathbf{r}/dt$ = its vector velocity, $\mathbf{v}$, and $d\mathbf{v}/dt$ = its vector acceleration, $\mathbf{a}$. If $\mathbf{m}$ and $\mathbf{n}$ are unit vectors in the direction of the tangent and normal to the path at the time t, then $\mathbf{v} = v\mathbf{m}$, where $v = ds/dt$ = the (scalar) path-velocity, and $d\mathbf{m} = [(ds/R)]\mathbf{n}$, where R = the (scalar) radius of curvature of the path. Then

$$\mathbf{a} = \frac{d(v\mathbf{m})}{dt} = \frac{dv}{dt}\mathbf{m} + v\frac{d\mathbf{m}}{dt} = \frac{dv}{dt}\mathbf{m} + \frac{v^2}{R}\mathbf{n}.$$

Here dv/dt and v^2/R are the familiar expressions for the components of acceleration along the tangent and normal.

Motion of a Particle in a Circular Path. If a point moves in a circle of radius r, its angular velocity being ω, angular acceleration a_a, and linear velocity v, $v = \omega r$, $a_t = a_a r$, $a_n = v^2/r = r\omega^2$.

Uniform Rotary Motion. In the case of uniform rotary motion, equal circular paths are traveled in equal intervals of time. ∴ Angular acceleration $a_a = 0$ and $a_t = a_a r = 0$. $a_n = v^2/r$. The **angular velocity** is usually expressed in **radians per sec**, and when the number (N) of revolutions per min (rpm) is known, the angular velocity is $\omega = 2\pi N/60 = 0.10472N$; linear velocity $v = \omega r$.

Compositions of Motions

A point may have several motions imparted to it at the same time by different means, in which case its motion is the resultant of the different component motions. Each of these components, by what is known as the **principle of independence**, exerts its full influence. The polygon or parallelopiped of motion may be used for the composition of motions.

Many examples of **combined motion** occur, *e.g.*, velocity given by gravity to a body having a horizontal motion; combined rotation and rectilinear motion of a point on a rolling wheel; a point whose motion is governed by the action of two or more cams; the movement of persons or machinery on board trains or boats; movement of a projectile.

Relative Motions. In order that a motion shall be fully determined, it is necessary that the base to which it is referred be carefully stated. The majority of engineering problems assume the earth as the reference base.

Motion of Rigid Bodies

A body is said to be rigid when the distances between all of its particles are invariable. Theoretically, rigid bodies do not exist, but materials used in engineering are practically rigid under their working stresses.

Elementary Motion. If a body moves so that a straight line connecting any two of its particles remains fixed in direction, it is said to have a motion of **translation**. If the motion is indefinitely small—distance $= ds$ and time $= dt$—it may be considered to have **elementary translation**; $ds/dt =$ velocity of translation. The elementary translation is fully determined by a line drawn from any point in the rigid body, representing the velocity in magnitude and direction.

If a body moves so that all its particles describe circles whose planes are at right angles to a straight line XX which contains simultaneously the centers of all the circles, it is said to have a **rotary motion** about the axis XX. If the motion is indefinitely small—angle of rotation $= d\theta$ and time $= dt$—it is called **elementary rotation**. The path ds of a point distant r from the axis of rotation is $ds = r d\theta$, and $d\theta/dt =$ elementary angular velocity of rotation. Elementary rotation is completely determined by a straight line whose length is equal to ω drawn on the axis of rotation with an arrow so placed that when looking in the direction of the arrow rotation takes place in a plane at right angles to that line and in the direction of the hands of a clock.

The motion of a body in space is determined by the motion of three points of the body which do not lie in the same straight line. The **instantaneous axis** is the axis about which a body is revolving at a given instant. If a body is being translated at a given instant, the instantaneous axis is at an infinite distance.

SECTION 3

MECHANICS OF SOLIDS AND LIQUIDS

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center. See Fig. 15, in which the wheel rolls along the track XX. Point A has a linear velocity of v ft per sec; O is the instantaneous center. v/AO = angular velocity of wheel. If v_1 is the velocity of point B, the angular velocity of wheel $= v_1/BO = v/AO$.

For linkages, see p. 750.

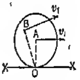


FIG. 15.

PHYSICAL MECHANICS

General Laws

Every particle of matter remains either at rest or moves uniformly in a straight line unless acted upon by some external influence. All variations of velocity, either in magnitude or direction, require the action of force.

Laws of Gravity. A body falling in a vacuum moves with a uniformly accelerated motion, the acceleration of the fall being the same for all bodies at the same place. Common values taken for this acceleration (usually designated as g) are 32.16 ft per sec per sec or 9.81 meters per sec per sec.

$g = 32.1721 - 0.08211 \cos 2a - 0.000003h$, where a is the latitude in degrees, and h the height in feet above sea level.

If a body is suspended by an elastic cord, the latter is elongated by the action of what is known as a force, in this case by the force of gravity. The suspended body is not only acted upon by gravity, but by a force exerted upon it by the cord, which imparts to it the same acceleration upward as gravity does downward, thereby maintaining the body at rest. The two forces are equal in magnitude.

Action is Equal to Reaction. The forces exerted by two bodies upon each other act in the same straight line are equal in magnitude and are opposite in direction.

Relations between Mass, Acceleration and Force. When acted upon by a constant unbalanced force, a body will move with a **uniformly accelerated motion. The accelerations produced in any body by different forces are proportional to the forces,** so that $a_1/p_1 = a_n/p_n$ = constant = $1/m$, where p_1, p_n are the forces applied and a_1, a_n are the resulting accelerations. The constant ratio m is known as the mass of the body.

Fundamental Equation: Force = mass × acceleration.

Mass $= w/g$, where w = weight and g = falling acceleration due to gravity.

$$F = (w/g)a = ma = mdv/dt = md^2s/dt^2.$$

In the case of two bodies at the same place, $w_1 = m_1g$, $w_2 = m_2g$ and $m_1/m_2 = w_1/w_2$; *i.e.*, **the masses of the two bodies are proportional to their weights.**

Example. If an unbalanced force of 5 lb acts upon a body weighing 10 lb, what will be the acceleration of the body? $5 = 10a/g$; $\therefore a = 5g/10 = g/2$.

Forces, like velocities and accelerations, are quantities in which **direction is a factor,** *i.e.*, their complete determination requires not only a statement of their magnitude, but also of their line of action and direction.

Law of the Conservation of Mass. The mass of a body remains unchanged by any ordinary physical or chemical change to which it may be subjected.

Technical Systems of Measurement

In absolute systems, see p. 72, the units of length, mass, and time are arbitrarily taken; all other units, including that of force, are derived.

from the equation of the curve by means of the calculus, or graphically by use of instruments.

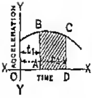


FIG. 3.

An acceleration-time curve (Fig. 3) may be constructed by plotting accelerations as ordinates, and times as abscissæ. The area under this curve between any two ordinates will represent the total increase in velocity during the time interval. The area *ABCD* represents the total increase in velocity between time t_1 and time t_2.

General Expressions Showing the Relations between Space, Time, Velocity and Acceleration, for Rectilinear Motion

Given $s = f(t)$; $v = ds/dt$, and $a = dv/dt = d^2s/dt^2$.

Given $v = f(t)$; $s = s_0 + \int_0^t v dt$, and $a = dv/dt$.

Given $v = f(s)$; $t = \int_{s_0}^{s} ds/v$, and $a = v dv/ds$.

Given $a = f(t)$; $v = v_0 + \int_0^t a dt$, and $s = s_0 + \int_0^t v dt$.

Given $a = f(s)$; $v = \sqrt{v_0^2 + 2\int_{s_0}^{s} a ds}$, and $t = \int_{s_0}^{s} ds/v$.

Given $a = f(v)$; $s = s_0 + \int_{v_0}^{v} \frac{v}{a} dv$, and $t = \int_{v_0}^{v} dv/a$

Special Motions

Uniform Motion. If the velocity is constant the acceleration must be zero, and the point has uniform motion. The space-time curve becomes a straight line inclined toward the time axis [Fig. 1(*a*)]. The velocity-time curve becomes a straight line parallel to the time axis. For this motion $a = 0$, $v =$ constant, and $s = s_0 + vt$.

Uniformly Accelerated or Retarded Motion. If the velocity is not uniform but the acceleration is constant, the point has uniformly accelerated motion; the acceleration may be either positive or negative. The space-time curve becomes a parabola and the velocity-time curve becomes a straight line inclined toward the time axis, Fig. 2(*a*). The acceleration-time curve becomes a straight line parallel to the time axis. For this motion $a =$ constant, $v = v_0 + at$, $s = s_0 + v_0t + \frac{1}{2}at^2$.

If the point starts from rest, $v_0 = 0$. Care should be taken concerning the sign + or − for acceleration.

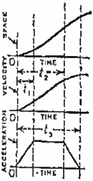


FIG. 4.

Example. Starting motion of a street car or hauling engine—see Fig. 4. During the period of time O to t_1, the acceleration increases from 0 to a maximum approximately in a straight line, the velocity-time curve is a parabola and the space-time curve one of the third degree. From t_1 to t_2 the acceleration is constant, the velocity increases uniformly, and the space-time curve is a parabola. From t_2 to t_3 the acceleration decreases from its maximum to 0 approximately in a straight line, and the space-time and velocity-time curves are like those from O to t_1. This concludes the start and a condition of equilibrium follows, with the acceleration zero, the velocity constant, and the space increasing uniformly.

Periodic Motion (Simple Harmonic Motion). If a point travels uniformly in a circle, the motion of its projection upon any diameter is a simple harmonic one. In Fig. 5, the line *Op* revolves about *O* with a uniform angular velocity ω; *n* is the projection of point *p* upon diameter *YY'*. As *p*

A force R may be resolved into two component forces intersecting anywhere on R and acting in the same plane as R, by the reverse of the operation shown by Figs. 16 and 17; and by repeating the operation with the components, R may be resolved into any number of component forces intersecting R at the same point and in the same plane.

Fig. 16.

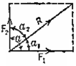


Fig. 17.

Resultant of Any Number of Forces Applied to a Rigid Body at the Same Point. Resolve each of the given forces F into components along three rectangular coordinate axes. If A, B and C are the angles made with XX, YY, and ZZ, respectively, by any force F, the components will be $F \cos A$ along XX, $F \cos B$ along YY, $F \cos C$ along ZZ; add the components of all the forces along each axis algebraically and obtain $\Sigma F \cos A = \Sigma X$ along XX, $\Sigma F \cos B = \Sigma Y$ along YY, and $\Sigma F \cos C = \Sigma Z$ along ZZ.

The resultant $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2 + (\Sigma Z)^2}$. The angles made by the resultant with the three axes are A_r with XX, B_r with YY, C_r with ZZ, where

$$\cos A_r = \Sigma X/R, \qquad \cos B_r = \Sigma Y/R, \qquad \cos C_r = \Sigma Z/R.$$

The direction of the resultant can be determined by plotting the algebraic sums of the components.

If the forces are all in the same plane the components of each of the forces along one of the three axes (say, ZZ) will be 0, that is, angle $C_r = 90°$ and $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$, $\cos A_r = \Sigma X/R$, and $\cos B_r = \Sigma Y/R$.

For equilibrium, it is necessary that $R = 0$; that is to say, ΣX, ΣY, and ΣZ must each be equal to zero.

General Law. In order that a number of forces acting at the same point shall be in equilibrium, the algebraic sum of their components along any *three* co-ordinate axes must each be equal to zero. When the forces all act in the same plane, the algebraic sum of their components along any *two* co-ordinate axes must each equal zero.

When the Forces Form a System in Equilibrium. Three unknown forces can be determined if the lines of action of the forces are *all* known and are in different planes. If the forces are all in the same plane, the lines of action being known, only *two* unknown forces can be determined. If the lines of action of the unknown forces are *not* known, only *one* unknown force can be determined in either case.

Couples and Moments

Couple. Two forces of equal magnitude, Fig. 18, which act in opposite and parallel directions form a couple. **A couple cannot be reduced to a single force.**

Displacement and Change of a Couple. The forces forming a couple may be moved about and their magnitude and direction changed, provided that they always remain parallel to each other, remain either in the original plane or one parallel to it, and provided that the product of one of the forces and the perpendicular distance between the two is constant and the direction of rotation remains the same.

Moment of a Couple. The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between the lines of action of the forces. Fa = moment of couple; a = arm of couple.

If the components v_1 and v_2 and angle B are known, the direction of v_r can be determined. $\sin bOc = (v_1/v_r)\sin B$. $\sin cOa = (v_2/v_r)\sin B$. Where v_1 and v_2 are at right angles to each other, $\sin B = 1$.

Composition and Resolution of Accelerations. Accelerations may be combined and resolved in the same manner as velocities, but in this case the lines or vectors represent accelerations instead of velocities. Velocities and accelerations may be resolved into components *not* in the same plane by what is known as the **parallelopiped of motion.** In this case the **resultant of three motions not in the same plane** is the diagonal of a parallelopiped whose sides are lines whose length and direction represent the motions. See Fig. 9. Oa, Oc and Ob represent three velocities or accelerations; then Od represents the resultant velocity or acceleration. When the velocities or accelerations are at right angles to each other, the angles that the resultant Od makes with these axes are A, B and C, respectively. Then $\cos A = Oa/Od$, $\cos B = Ob/Od$, $\cos C = Oc/Od$, and

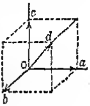


FIG. 9.

$$Od = \sqrt{Oa^2 + Ob^2 + Oc^2}.$$

Curvilinear Motion

The linear velocity $v = ds/dt$ is the same as for rectilinear motion, and its direction is tangent to the path of the point. In Fig. 10(a), let $p_1p_2p_3$ be the path of a moving point, and v_1, v_2, v_3 represent its velocity at points p_1, p_2, p_3, respectively. If O be taken as a pole [Fig. 10(b)] and vectors v_1, v_2, v_3 representing the velocities of the point at p_1, p_2, and p_3 be drawn, the curve connecting the terminal points of these vectors is known as the **hodograph** of the motion. This velocity diagram is applicable only to motions all in the same plane.

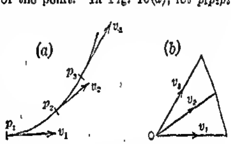


FIG. 10.

Acceleration. Tangents to the curve, Fig. 10(b), indicate the directions of the **momentary accelerations.** The direction of the tangents does not, as a rule, coincide with the direction of the velocities as represented by tangents to the path. If the acceleration a at some point in the path is resolved by means of a parallelogram into components tangent and normal to the path, the normal acceleration $a_n = v^2/r$, where r = radius of curvature of the path at the point in question, and the tangential acceleration $a_t = dv/dt$, where v = velocity tangent to the path at the same point. $a = \sqrt{a_n^2 + a_t^2}$. The normal acceleration is constantly directed toward the center of the path.

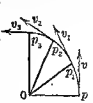


FIG. 11.

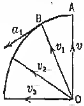


FIG. 12.

Example. Find the acceleration of a point moving in a circle with uniform velocity. In Fig. 11, p, p_1, p_2, p_3 is the circular path of a point. v, v_1, v_2, v_3 represent the uniform velocities at p, p_1, p_2, p_3. Construct the hodograph of the motion, as in Fig. 12. At point p_1 the acceleration is in the direction of the tangent to the hodograph at B, shown by line a_1. Let $pp_1 = s$, $AB = s_1$ and $Op = r$; then $s/r = s_1/v$. $\therefore s_1 = sv/r$. $ds_1/dt = (ds/dt)(v/r) = v^2/r$ and as $ds_1/dt = a_n$, $a_n = v^2/r$ and is in a direction toward the center of the circle, $a_t = 0$.

left of it (shown dotted), making the moment of the resultant about any point on F positive.

To effect a parallel displacement of a single force F over a distance a, a couple whose moment is Fa must be added to the system. The sense of the couple will depend upon which way it is desired to displace force F.

The moment of a force with respect to a point is the product of the force F and the perpendicular distance from the point to the line of action of the force.

The Moment of a Force with Respect to a Straight Line. If the force is resolved into components parallel and perpendicular to the given line, the moment of the force with respect to the line is the product of the magnitude of the perpendicular component and the distance from its line of action to the given line.

Forces with Different Points of Application

Composition of Forces. If each force F is resolved into components parallel to three rectangular coordinate axes XX, YY and ZZ, the magnitude of the resultant is $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2 + (\Sigma Z)^2}$, and its line of action makes angles A_r, B_r and C_r with axes XX, YY and ZZ, where $\cos A_r = \Sigma X/R$, $\cos B_r = \Sigma Y/R$, and $\cos C_r = \Sigma Z/R$; and there are three couples which may be combined by their moment vectors into a single resultant couple having the moment $M_r = \sqrt{(M_x)^2 + (M_y)^2 + (M_z)^2}$, whose moment vector makes angles of A_m, B_m and C_m with axes XX, YY and ZZ, such that $\cos A_m = M_x/M_r$, $\cos B_m = M_y/M_r$, and $\cos C_m = M_z/M_r$. If this single resulting couple is in the same plane as the single resulting force at the origin or a plane parallel to it, the system may be reduced to a single force R acting at a distance from $R = M_r/R$. If the couple and force are not in the same or parallel planes, it is impossible to reduce the system to a single force. If $R = 0$, that is, if ΣX, ΣY and ΣZ all equal zero, the system will reduce to a single couple whose moment is M_r. If $M_r = 0$, that is, if M_x, M_y and M_z all equal zero, the resultant will be a single force R.

When the forces are all in the same plane, cosine of one of the angles A_r, B_r or $C_r = 0$, say, $C_r = 90°$. Then $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$, $M_r = \sqrt{M_x^2 + M_y^2}$ and the final resultant is a force equal and parallel to R, acting at a distance from R equal to M_r/R.

A system of forces in the same plane can always be replaced by either a couple or a single force. If $R = 0$ and $M_r > 0$, the resultant is a couple. If $M_r = 0$ and $R > 0$, the resultant is a single force.

A rigid body is in equilibrium when acted upon by a system of forces whenever $R = 0$ and $M_r = 0$, *i.e.*, when the following six conditions hold true: $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma Z = 0$, $M_x = 0$, $M_y = 0$ and $M_z = 0$. When the system of forces is in the same plane, equilibrium prevails when the following three conditions hold true: $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$.

Forces Applied to Support Rigid Bodies

The external forces in equilibrium acting upon a body may be statically determinate or indeterminate according to the number of unknown forces existing. When the forces are all in the same plane and act at a common point, two unknown forces may be determined if their lines of action are known, one if unknown.

Motion in a Plane

A plane motion is one in which all points of the moving body remain at constant distances from a fixed plane.

Angular Displacement. The angular displacement of any body moving in a plane is the angle described by any line drawn in the body parallel to the plane of the motion.

Angular velocity is the rate at which the angular displacement θ varies with respect to the time t. Angular velocity $\omega = d\theta/dt$.

Angular acceleration is the rate at which the angular velocity ω varies with respect to the time t. Angular acceleration $a_a = d\omega/dt = d^2\theta/dt^2$.

Instantaneous Axis. When the axis about which any body may be considered to rotate changes its position, any one position is known as an instantaneous axis, and the line through all positions of the instantaneous axis as the centrode.

When the motion of two points in the same plane of a rigid body having plane motion is known, the instantaneous axis for the body will be at the intersection of the lines drawn from each point and perpendicular to its line of motion. See Fig. 13, in which A and B are two points on the rod AB, v_1 and v_2 representing their velocities. O is the instantaneous axis for AB; $\therefore$ point C will move as shown in a line perpendicular to OC.

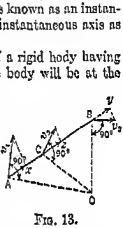


FIG. 13.

Linear velocities of points in a body rotating about an instantaneous axis are proportional to their distances from this axis. In Fig. 13, $v_1:v_2:v_3 = AO:OB:OC$. If the motions of A and B were parallel, the lines OA and OB would also be parallel and there would be no instantaneous axis. The motion of the rod would be translation, and all points would be moving with the same velocity in parallel straight lines.

If a body has plane motion, the components of the velocities of any two points in the body along the straight line joining them must be equal. Ax must be equal to By and Cz in Fig. 13.

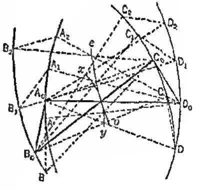


FIG. 14.

Centrode. If the path of the instantaneous center be plotted, the centrode for the motion of the body is obtained. See Fig. 14. Consider A_0B_0 to be fixed and let C_0D_0 rotate about it. Its motion being determined by the links AD and BC, the path of the instantaneous center for CD can be plotted as the curve xy. CD can be considered to be fixed and the centrode for AB found at eo. If the bars AB and CD are fixed to these centrodes and they are rolled upon each other, the relative motion of the two bars A_0B_0 and C_0D_0 will be the same as if they were connected by the links AD and BC. This principle is made use of in the design of **elliptic gears for quick-return motions.** If a wheel rolls along a track, the centrode is the surface of the track.

The angular velocity of a body at any time can be found by dividing the linear velocity of any point by the distance of the point from the instantaneous

on the member or by the member at these two points must be along a line connecting the pins.

If the external forces acting upon a rigid body in equilibrium are all in the same plane, the equations $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$ must be satisfied. When trusses, frames and other structures are under discussion, these equations are usually used as $\Sigma V = 0$, $\Sigma H = 0$, $\Sigma M = 0$, where V and H represent *vertical* and *horizontal* components, respectively.

The **supports** are said to be **determinate statically** when the laws of equilibrium are sufficient for their determination. When the conditions are not sufficient for the determination of the supports or other forces, the structure is said to be **statically indeterminate**; the unknown forces can then be determined from considerations involving the deformation of the *material*.

When several bodies are so connected to one another as to make up a rigid structure, the forces at the points of connection must be considered as internal forces and are not taken into consideration in the determination of the supporting forces for the structure as a whole.

The distortion of any practically rigid structure under its working loads is so small as to be negligible when determining supporting forces. When the forces acting at the different joints in a built-up structure cannot be determined by dividing the structure up into parts, the structure is said to be **statically indeterminate internally.** A structure may be statically *indeterminate internally and still be statically determinate externally.*

Fundamental Problems in Graphical Statics

A force may be represented by a straight line in a determined position, and its magnitude by the length of the straight line. The direction in which it acts may be indicated by an arrow.

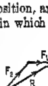


FIG. 25.

Polygon of Forces. The parallelogram of two forces intersecting each other (see Fig. 8) leads directly to the graphic composition by means of the triangle of forces. In Fig. 25, R is called the **closing side**, and represents the resultant of the forces F_1 and F_2 in magnitude and direction. Its position is given by the point of application O. By means of repeated use of the triangle of forces and by omitting the closing sides of the individual triangles, *the magnitude and direction of the resultant R of any number of forces in the same plane and intersecting at a single point can be found.* In Fig. 26 the lines representing the forces start from point O, and in the force polygon, Fig. 27, they are joined in any order, the arrows showing their directions following around the polygon in the same direction. The magnitude of the resultant at the point of application of the forces is represented by the *closing side R* of the force polygon; its direction, as shown by the arrow, is counter to that in the other sides of the polygon.

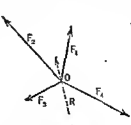


FIG. 26.

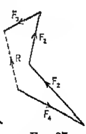


FIG. 27.

If the forces are in equilibrium, R must equal zero, *i.e.*, the force polygon must close.

If in a closed polygon one of the forces is reversed in direction, this force becomes the resultant of all the others.

In **gravitational systems**, the unit of length, force, and time are arbitrarily taken, the other units, including that of mass, being derived. The force unit is defined as the weight of a certain piece of metal, or of a prescribed volume of water, and thus this force unit depends on g and varies slightly with altitude and location.

STATICS OF RIGID BODIES

General Considerations

If the forces acting on a rigid body do not produce any acceleration, they must neutralize each other, that is, form a **system of forces in equilibrium.** Equilibrium is said to be **stable** when the body with the forces acting upon it returns to its original position after being displaced a very small amount from that position; **unstable** when the body tends to move still further from its original position than the very small displacement; and **neutral** when the forces retain their equilibrium when the body is in its new position.

External and Internal Forces. The forces by which the individual particles of a body act on each other are known as internal forces. All other forces are called external forces. If a body is supported by other bodies while subject to the action of forces, deformations and forces will be produced at the points of support or contact and these internal forces will be distributed throughout the body until equilibrium exists and the body is said to be in a state of tension, compression or shear. The forces exerted by the body on the supports are known as **reactions.** They are equal in magnitude and opposite in direction to the forces with which the supports act on the body, known as **supporting forces.** The supporting forces are external forces applied to the body.

In considering a body at a definite section, it will be found that all the internal forces act in pairs, the two forces being equal and opposite. The external forces act singly.

General Law. When a body is at rest, the forces acting externally to it must form an equilibrium system. This law will hold for any part of the body, in which case the forces acting at any section of the body become external forces when the part on either side of the section is considered alone. In the case of a rigid body, any two forces of the same magnitude but acting in opposite directions in any straight line, may be added or removed without change in the action of the forces acting on the body, providing the strength of the body is not affected.

Composition, Resolution and Equilibrium of Forces

(For graphical methods, see p. 202)

The **resultant** of several forces acting at a point is a force which will produce the same effect as all the individual forces acting together.

Forces Acting on a Body at the Same Point. The resultant R of two forces F_1 and F_2 applied to a rigid body at the same point is represented in magnitude and direction by the diagonal of the parallelogram formed by F_1 and F_2. See Figs. 16 and 17.

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos a};\ \sin a_1 = (F_2 \sin a)/R;\ \sin a_2 = (F_1 \sin a)/R$$

When $a = 90$ deg, $R = \sqrt{F_1^2 + F_2^2}$, $\sin a_1 = F_2/R$ and $\sin a_2 = F_1/R$;

$$\left.\begin{array}{l}\text{When } a = 0 \text{ deg, } R = F_1 + F_2 \\ \text{When } a = 180 \text{ deg, } R = F_1 - F_2\end{array}\right\} \text{ Forces act in same straight line.}$$

straight line A_0A_n parallel to OO_1. **This line is the polar axis** of both funicular polygons.

Let the starting point of the force polygon be taken as a pole; then the sides of the funicular polygon will always give the line of action of the resultants of all the forces preceding. The last side will coincide with the resultant of all the forces.

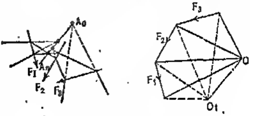


Fig. 31.

To determine the supporting forces for a rigid body when the external forces are all in the same plane and are statically determinate, see Fig. 32. *Let ab and bc be two forces acting* on beam which is supported at M by a roller and at N by a pin. Construct a force polygon, Fig. 32(b), using any convenient point O for a pole, and the funicular polygon shown in Fig. 32(a). This latter must start at N, as this is the only point on the line of action of the unknown force at N that is known. Draw the strings parallel to the corresponding rays of the force polygon and find that the last string intersects the line of action of the left-hand support at P. If the forces are in equilibrium, both force and funicular polygons must close. The closing side of the funicular polygon is PN. OD, Fig. 32(b), is drawn parallel to PN until it meets a line drawn from A parallel to the supporting force at M. The line CD closes the force polygon and gives the magnitude

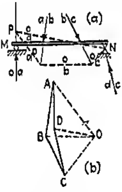


Fig. 32.

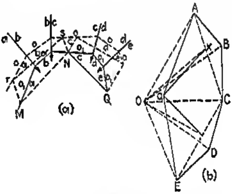


Fig. 33.

and direction of the supporting force at N. It is necessary that either the lines of action of both unknown supports, or the line of action of one and the point of application of the other, be known in order to use this construction.

To draw a **funicular polygon through three points,** see Fig. 33. Given the forces ab, bc, cd, and de, Fig. 33(a), let it be required to draw a funicular polygon so that any three given strings shall pass through any three given points, not on a straight line; for example, string oa through M, oc through N, and oe through Q. Construct a force polygon, Fig. 33(b), using O as a pole, and a corresponding funicular polygon, oa, ob, oc, od, oe Fig. 33(a). Consider the forces between points M and N; if they are to be balanced by two forces each parallel to their resultant and acting through M and N,

If the forces are measured in pounds and the distance a in feet, the **unit of rotation moment** is the foot-pound. If the force is measured in kilograms and the distance in meters, the unit is the meter-kilogram. In the cgs system the unit of rotation moment is 1 cm-dyne.

Rotation moments of couples acting in the same plane are considered to be positive or negative, according to whether they appear to rotate in the direction of the hands of a clock or in the reverse direction. The couple shown in Fig. 18 is positive. The magnitude, direction and sense of rotation of a couple are completely determined by its moment axis, or moment vector, which is a line drawn perpendicular to the plane in which the couple acts, with an arrow indicating the direction from which the couple will appear to have right-handed rotation; the length of the line represents the magnitude of the moment of the couple. See Fig. 19, in which AB represents the magnitude of the moment of the couple. Looking along the line in the direction of the arrow, the couple will have right-handed rotation in any plane perpendicular to the line.

Fig. 18.

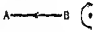


Fig. 19.

Composition of Couples. Couples may be combined by adding their moment vectors geometrically, in accordance with the parallelogram rule, in the same manner in which forces are combined.

Couples lying in the same or parallel planes are added algebraically. Let $+40$, -60, and $+100$ ft-lb be the moments of three couples in the same or parallel planes; their resultant is a single couple lying in the same or in a parallel plane, whose moment is $\Sigma M = +40 - 60 + 100 = +80$ ft-lb.

If the polygon formed by the moment vectors of several couples closes itself, the couples form an equilibrium system. Two couples will balance each other when they lie in the same or parallel planes, and have the same moment in magnitude, but opposite in sign.

Combination of a Couple and a Single Force in the Same Plane. (See Fig. 20.) Given a force $F = 20$ lb acting as shown distant x from YY, and a couple whose moment is -60 ft-lb in the same or a parallel plane, to find the resultant. A couple may be changed to any other couple in the same or a parallel plane having the same moment and same sign. Let the couple consist of two forces of 20 lb each and let the arm be 3 ft. Place the couple in such a manner that one of its forces is opposed to the given force at p. This force of the couple and the given force being of the same magnitude and opposite in direction, will neutralize each other, leaving the other force of the couple acting at a distance of 3 ft from p and parallel and equal to the given force $F = 20$.

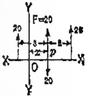


Fig. 20.

General Rule. The resultant of a couple and a single force lying in the same or parallel planes is a single force, equal in magnitude, in the same direction and parallel to the single force, and acting at a distance from the line of action of the single force equal to the moment of the couple divided by the single force. The moment of the resultant force about any point on the line of action of the given single force must be of the same sense as that of the couple, positive if the moment of the couple is positive, and negative if moment of couple is negative. If the moment of the couple in Fig. 20 had been $+60$ instead of -60, the resultant would have been a force of 20 lb acting in the same direction and parallel to F, but at a distance of 3 ft to the

Equilibrium. In order that the whole structure should be in equilibrium, it is necessary that the external forces (loads and supports) shall form a balanced system. Graphical and analytical methods are both of service.

Supporting Forces. When the supporting forces are to be determined, it is not necessary to pay any attention to the make-up of the structure under consideration so long as it is practically rigid; the loads may be taken as they occur, or the resultant of the loads may be used instead. When the stresses in the members of the structure are being determined, the loads *must* be distributed at the joints where they belong.

Method of Joints. When all the external forces have been determined, any joint at which there are not more than two unknown forces may be taken and these unknown forces determined by the methods of the stress polygon, resolution or moments. In Fig. 36, let O be the joint of a structure and F be the only known force; but let $O1$ and $O2$ be two members of the structure joined at O. Then the lines of action of the unknown forces are known and their magnitude may be determined (*a*) by a **stress polygon** which, for equilibrium, must close; (*b*) by resolution into H and V components, using the condition of equilibrium $\Sigma H = 0$, $\Sigma V = 0$; or (*c*) by moments, using any convenient point on the line of action of $O1$ or $O2$ and the condition of equilibrium $\Sigma M = 0$. No more than *two* unknown forces can be determined. In this manner, proceeding from joint to joint, the stresses in all the members of the truss can usually be determined if the structure is statically determinate internally.

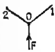


FIG. 36.

The structure may be divided into two parts by passing a section through it cutting some of its members; one part may then be treated as a rigid body and the external forces acting upon it determined. Some of these forces will be the stresses in the members themselves. For example, let xx (Fig. 37) be a section taken through a truss loaded at P_1, P_2, and P_3, and supported on rollers at S. As the whole truss is in equilibrium, any part of it must be also, and consequently the part shown to the left of xx must be in equilibrium under the action of the forces acting externally to it. Three of these forces are the stresses in the members aa, bb, and bc, and are the unknown forces to be determined. They can be determined by applying the condition of equilibrium of forces acting in the same plane but not at the same point. $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$. The three unknown forces can be determined only if they are not parallel or do not pass through the same point; if, however, the forces are parallel or meet in a point, two unknown forces only can be determined. The conditions of equilibrium when using the funicular polygon construction are that both the funicular and force polygons must close. Sections may be passed through a structure cutting members in any convenient manner; as a rule, however, cutting not more than three members.

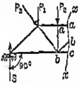


FIG. 37.

For the determination of stresses in framed structures, see p. 226.

Center of a Set of Parallel Forces; Center of Gravity

The center of a set of parallel forces is the point through which the resultant of the forces always passes, no matter how the forces are turned, providing that they always remain parallel and their points of application remain in the same relation to each other.

The resultant R of the two parallel forces F_1 and F_2 (Fig. 38) divides the line connecting the points of application A_1, A_2 into two segments a_1 and a_2

When the forces are all in the same plane and are parallel, two unknown forces may be determined if the lines of action are known, one if unknown.

When the forces are anywhere in the same plane, three unknown forces may be determined if their lines of action are known, if they are not parallel or do not pass through a common point; if the lines of action are unknown, only one unknown force can be determined.

If the forces all act at a common point but are in different planes, three unknown forces can be determined if the lines of action are known, one if unknown.

If the forces act in different planes but are parallel, three unknown forces can be determined if their lines of action are known, one if unknown.

The first step in the solution of problems in statics is the determination of the supporting forces. The following data are required for the complete knowledge of supporting forces: magnitude, direction, and point of application. According to the nature of the problem, none, one, or two of these quantities are known.

Fig. 21.

One Fixed Support. The point of application, direction, and magnitude of the load are known. See Fig. 21. As the body on which the forces act is in equilibrium, the supporting force P must be equal in magnitude and opposite in direction to the resultant of the loads L.

In the case of a **rolling surface,** the point of application of the support is obtained from the center of the connecting bolt A (Fig. 22), both the direction and magnitude being unknown. The point of application and line of action of the support at B are known, being determined by the rollers.

When three forces acting in the same plane on the same rigid body are in equilibrium, their lines of action must pass through the same point O. The load L is known in magnitude and direction. The line of action of the support at B is known on account of the rollers. The point of application of the support at A is known. The three forces are in equilibrium and are in the same plane, and therefore the lines of action must meet at the point O.

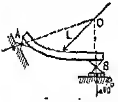


Fig. 22.

Fig. 23.

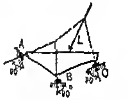


Fig. 24.

In the case of the rolling surfaces shown in Fig. 23, the direction of the support at A is known, the magnitude and point of application unknown. The line of action and point of application of the supporting force at B are known, its magnitude unknown. The lines of action of the three forces must meet in a point, and the supporting force at A must be perpendicular to the plane XX. In the case shown in Fig. 24, the directions and points of application of the supporting forces are known, and the magnitudes unknown. The lines of action of resultant of supports A and B, the support at C and load L must meet at a point. Resolve the resultant of supports at A and B into components at A and B, their direction being determined by the rollers.

If a member of a truss or frame in equilibrium is pinned at two points and loaded at these two points only, the line of action of the forces exerted

Trapezoid, Fig. 42. Cg lies on the line joining the middle points m and n of the parallel sides. The distances h_a and h_b are

$$h_a = h(a + 2b)/3(a + b); \quad h_b = h(2a + b)/3(a + b).$$

Draw $BE = a$ and $CF = b$; EF will then intersect mn at cg

Any Quadrilateral. The cg of any quadrilateral may be determined by the general rule for areas, or graphically by dividing it into two sets of triangles by means of the diagonals. Find the cg of each of the four triangles and connect the cg's of the triangles belonging to the same set. The intersection of these lines will be cg of area. Thus, in Fig. 43, O, O_1, O_2 and O_3 are respectively the centers of gravity of the triangles ABD, ABC, BDC, and ACD. The intersection of O_1O_3 with OO_2 gives cg.

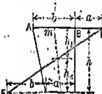


Fig. 42.

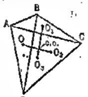


Fig. 43.

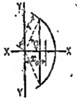


Fig. 44.

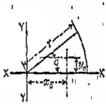


Fig. 45.

Segment of a Circle, Fig. 44: $x_0 = \frac{2}{3} r \sin^3 c/(\text{rad } c - \cos c \sin c)$. A segment may be considered to be a sector from which a triangle is subtracted, and the general rule applied.

Sector of a Circle, Fig. 45: $x_0 = \frac{2}{3} r \sin c/\text{rad } c$; $y_0 = \frac{4}{3} r \sin^2 \frac{1}{2}c/\text{rad } c$.

Semi-circle, $x_0 = \frac{4}{3} r/\pi = 0.4244r$; $y_0 = 0$.

Quadrant (90 deg sector): $x_0 = y_0 = \frac{4}{3} r/\pi = 0.4244r$.

Parabolic Half Segment, Fig. 46, Area ABO: $x_0 = \frac{3}{5} x_1$; $y_0 = \frac{3}{8} y_1$.

Parabolic Spandrel, Fig. 46, Area AOC: $x'_0 = \frac{3}{10} x_1$; $y'_0 = \frac{3}{4} y_1$.

Quadrant of an Ellipse, Fig. 47, Area OAB: $x_0 = \frac{4}{3}(a/\pi)$; $y_0 = \frac{4}{3}(b/\pi)$.

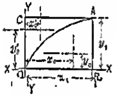


Fig. 46.

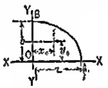


Fig. 47.

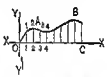


Fig. 48.

The center of gravity of a figure such as that shown in Fig. 48 may be determined as follows: Divide the area $OABC$ into a number of parts by lines drawn perpendicular to the axis XX, *e.g.*, 11, 22, 33, etc. These parts will be approximately either triangles, rectangles or trapezoids. The area of each division may be obtained by taking the product of its mean height and its base. The center of gravity of each area may be obtained as previously shown. The sum of the moments of all the areas about XX and YY, respectively, divided by the sum of the areas will give approximately the distances from the center of gravity of the whole area to the axes XX and YY. The greater the number of areas taken the more nearly exact the result.

Centers of Gravity of Solids

Prism or Cylinder with Parallel Bases. Cg lies in the center of the line connecting the centers of gravity of the bases.

If the forces do **not** all lie **in the same plane,** the diagram becomes a polygon in space. If the projections of the forces be drawn on any two planes, any horizontal and vertical, the closing lines of the projected polygons are the projections of the closing line of the force diagram in space.

Funicular Polygon

Composition of Forces in a Plane. In Fig. 28, to determine the resultant R of the forces ab, bc, cd, and de all in the same plane, construct the force polygon shown at (b). R will represent the magnitude and direction of the resultant force, and its location in (a) may be determined as follows: In Fig. 28(b) assume any convenient point O as a **pole,** draw the straight lines OA, OB, OC, OD and OE which are components of the given forces and in Fig. 28(a) draw the lines oa, ob, oc, od, oe parallel to the similarly lettered lines in Fig. 28(b). The location of oa in Fig. 28(a) is optional, so long as it intersects the line of action of ab. R must pass through the intersection of oa and oe and be parallel to AE in Fig. 28(b).

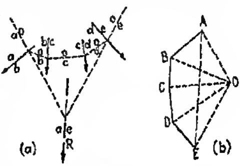


FIG. 28.

The figure $OABCDE$ is called the **polygon of forces** or **force polygon,** the lines OA, OB, OC, OD, OE in Fig. 28(b) are **rays,** and the series of lines oa, ob, oc, od, oe in Fig. 28(a) is called a **string polygon** or **funicular polygon,** oa, ob, oc, od, oe in (a) being known as **strings.** The following three cases may be met:

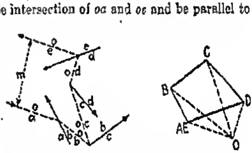


FIG. 29.

(a) The general case, where the starting point of the polygon of forces does not coincide with the terminal point. The resultant is a single force, as shown by Fig. 28. (b) The polygon of forces closes and the end strings of the funicular polygon are parallel. In this case (Fig. 29), the resultant is a couple whose moment is $OA \times m = OE \times m$. (c) The polygon of forces closes and the funicular polygon also closes, the result being **equilibrium.** See Fig. 30. The two coinciding terminal sides of the funicular polygon are called the **closing sides** or **strings.**

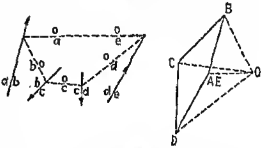


FIG. 30.

Let a polygon of forces and a funicular polygon be drawn for the system of forces shown in Fig. 31 with poles at both O and O_1. The intersections of the corresponding strings of the funicular polygons all lie in the same

The moment of inertia may be expressed in weight units ($I_w = \int y^2 dw$), in which case the moment of inertia in weight units, I_w, is equal to the moment of inertia in mass units, I, multiplied by g.

If $I = k^2 m$, the quantity k is called the **radius of gyration** or the **radius of inertia.**

If a body is considered to be composed of a number of parts, its moment of inertia about an axis is equal to the sum of the moments of inertia of the several parts about the same axis, or $I = I_1 + I_2 + I_3 + \cdots I_n$.

The moment of inertia of an area with respect to a given axis is the limit of the sum of the products of the elementary areas into which the area may be conceived to be divided and the square of their distance (y) from the axis in question. $I = \int y^2 dA = k^2 A$, where k = **radius of gyration.**

Formulae for moments of inertia and radii of gyration of various areas are given on pp. 457 to 460.

Relation between the Moments of Inertia of an Area and a Solid. The moment of inertia of a solid of elementary thickness about an axis is equal to the moment of inertia of the area of one face of the solid about the same axis multiplied by the mass per unit volume of the solid times the elementary thickness of the solid.

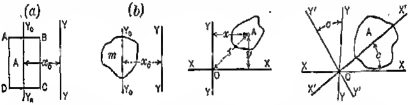


FIG. 52. FIG. 53. FIG. 54.

Moments of Inertia about Parallel Axes. The moment of inertia of an area or solid about any given axis is equal to the moment of inertia about a parallel axis through the center of gravity plus the square of the distance between the two axes times the area or mass.

In Fig. 52(a), the moment of inertia of the area $ABCD$ about axis YY is equal to I_0 (or the moment of inertia about Y_0Y_0 through the center of gravity of the area and parallel to YY) plus x_0^2A, where A = area of $ABCD$. In Fig. 52(b), the moment of inertia of the mass m about $YY = I_0 + x_0^2 m$. Y_0Y_0 passes through the cg of the mass and is parallel to YY.

Polar Moment of Inertia. The polar moment of inertia, Fig. 53, is taken about an axis perpendicular to the plane of the area. Referring to Fig. 53, if I_y and I_x be the moments of inertia of the area A about YY and XX, respectively, then the polar moment of inertia of area about $I_p = I_x + I_y$, or the polar moment of inertia is equal to the sum of the moments of inertia about any two axes at right angles to each other in the plane of the area and intersecting at the pole.

Moment of Deviation, or Product of Inertia. This quantity will be represented by I_{xy} and is $\iint xydydx$, where x and y are the co-ordinates of any elementary part into which the area may be conceived to be divided. I_{xy} may be positive or negative, depending upon the position of the area with respect to the co-ordinate axes XX and XY.

Relation between Moments of Inertia about Axes Inclined to Each Other. Referring to Fig. 54, let I_y and I_x be the moments of inertia of the

these two forces can be determined by drawing lines parallel to AC, Fig. 33(b), through M and N Fig. 33(a). The points where these lines are intersected by the funicular polygon are r and s, the closing side of a funicular polygon for these two forces is rs and the magnitude of the balancing forces can be determined as CX and XA, Fig. 33(b), by drawing from O a line parallel to rs and intersecting AC which is parallel to the forces at r and s. If the funicular polygon had passed through M and N, the same balancing forces for ab and bc parallel to AC must be found so that the locus of the poles of all the funicular polygons for ab and bc that will pass through M and N must lie on a line drawn from x and parallel to MN. In the same manner, the locus of the poles of all the funicular polygons for cd and de can be determined, and the intersection of these two loci will be the pole O_1 required.

Moment of Any System of Forces in a Plane. The resultant moment M of the forces ab, bc, cd (Fig. 34) with reference to point Q can be determined as follows: Fig. 34(b) is a force polygon for ab, bc, cd, using any convenient point O as a pole. Fig. 34(a) shows a corresponding funicular polygon. Through points Q and P of (a) draw lines parallel to the resultant of the forces R in (b); point P is the intersection of the end strings of the funicular polygon, and m and q are the points where the line drawn through point Q intersects the end strings. Consider the shaded triangles; the homologous sides are parallel, the triangles are therefore similar, and $mq/x = R/p$. $\therefore$ $(mq)p = Rx$; but Rx is the moment of the resultant of the forces with respect to Q: $\therefore$ $(mq)p$ must also be the same moment. mq is known as the **intercept** and p is known as the **pole distance.** In the case of parallel forces, the pole distance will be a constant; therefore, the moment at different points will vary as the intercept.

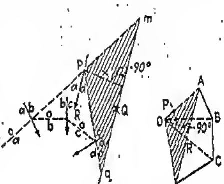


Fig. 34.

Resolution of Forces. Forces may be resolved into components in the same plane by means of the funicular polygon. See Fig. 35.

The funicular polygon is of great service in the solution of **stone arch and dam problems,** in determining the line of resistance at the joints. Bending moments and shearing forces for beams and other structures may also be determined graphically by its use.

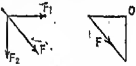


Fig. 35.

Determination of Stresses in Members of a Statically Determinate Plane Structure with Loads at Rest

It will be assumed that the loads are applied at the joints of the structure, *i.e.*, at the points where the different members are connected, and that the connections are pins with no friction. The stresses in the members must then be along lines connecting the pins, unless any member is loaded at more than two points by pin connections. If the members are straight, the forces exerted on them or by them must coincide with the axes of the members. In other words, there shall be no bending stresses in any of the members of the structure.

rectangle *edkg*, and triangles *mfg* and *hkl*, from which the rectangle *oprs* is to be subtracted. Referring to axis XX,

$I_{xx} = \pi 4^4/8$ for semi-circle *abc*; $= (2 \times 11^3)/3$ for rectangle *edkg*;
$= 2[(5 \times 3^3)/36 + 10^2(5 \times 3)/2]$ for the two triangles *mfg* and *hkl*.

From the sum of these there is to be subtracted $I_{xx}^1 = [(2 \times 3^3)/12 + 4^2(2 \times 3)]$ for the rectangle *oprs*.

If the moment of inertia of the whole area is required about an axis parallel to XX, but passing through the center of gravity of the whole area, $I_0 = I_{xx} - x_0^2 A$, where x_0 = distance from XX to center of gravity. The **moments of inertia of built-up sections** used in structural work may be found in the same manner, the moments of inertia of the different rolled sections being given in steel manufacturer's handbooks.

Moments of Inertia of Solids. For moments of inertia of solids about parallel axes, $I_x = I_0 + x_0^2 m$, see p. 210.

Moment of Inertia with Reference to Any Axis. Let a mass particle dm of a body have x, y and z as co-ordinates, XX, YY, and ZZ being the co-ordinate axes and O the origin. Let $X'X'$ be any axis passing through the origin and making angles of A, B and C with XX, YY and ZZ, respectively. The moment of inertia with respect to this axis then becomes =

$$I_{x'} = \cos^2 A\int(y^2 + z^2)dm + \cos^2 B\int(z^2 + x^2)dm + \cos^2 C\int(x^2 + y^2)dm$$
$$-2\cos B\cos C\int yz dm - 2\cos C\cos A\int zx dm - 2\cos A\cos B\int xy dm.$$

Let the moment of inertia about $XX = I_x = \int(y^2 + z^2)dm$, about $YY = I_y = \int(z^2 + x^2)dm$, and about $ZZ = I_z = \int(x^2 + y^2)dm$.

Let the products of inertia about the three co-ordinates axes be

$$I_{yz} = \int yz dm,\ I_{zx} = \int zx dm, \text{ and } I_{xy} = \int xy dm;$$

then the moment of inertia $I_{x'}$ becomes equal to

$$I_x\cos^2 A + I_y\cos^2 B + I_z\cos^2 C - 2I_{yz}\cos B\cos C - 2I_{zx}\cos C\cos A$$
$$-2I_{xy}\cos A\cos B$$

The moment of inertia of any solid may be considered to be made up of the sum or difference of the moments of inertia of simple solids of which the moments of inertia are known.

Moments of Inertia of Important Solids (Homogeneous)

$m = w/g$ = mass per unit of volume of the body.
$M = W/g$ = total mass of body.
r = radius. I = moment of inertia (mass units).
$I_w = I \times g$ = moment of inertia (weight units).

Solid circular cylinder about its axis:

$$I = \pi r^4 ma/2 = Mr^2/2. \quad (a = \text{length of axis of cylinder.})$$

Solid circular cylinder about an axis through the center of gravity and perpendicular to axis of cylinder: $I = M[r^2 + (a^2/3)]/4$.

Hollow circular cylinder about its axis:

$$I = \pi ma(r_1^4 - r_2^4)/2. \quad (r_1 \text{ and } r_2 = \text{outer and inner radii}; a = \text{length.})$$

Thin hollow circular cylinder about its axis: $I = Mr^2$.
Solid sphere about a diameter: $I = 8m\pi r^5/15 = 2Mr^2/5$
Thin hollow sphere about a diameter: $I = 2Mr_1^2/3$.
Thick hollow sphere about a diameter:

$$I = 8m\pi(r_1^5 - r_2^5)/15. \quad (r_1 \text{ and } r_2 \text{ are outer and inner radii.})$$

Rectangular prism about an axis through center of gravity and perpendicular to a face whose dimensions are a and b: $I = M(a^2 + b^2)/12$.

at the point S. $F_1a_1 = F_2a_2$. The point S is independent of the direction of the parallel forces, provided they always remain parallel and applied at A_1 and A_2. The point S, through which the resultant always passes, is called the center of the parallel forces F_1 and F_2.

The static moment of a line, surface or solid with respect to an axis is the length of the line, the area of the surface or the weight of the solid multiplied by the distance from the axis to the center of gravity of the line, surface or solid.

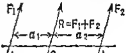


Fig. 38.

Centers of Gravity of Lines. To find the center of gravity of a line, divide it into an indefinitely large number of indefinitely short lengths, to be known as elementary divisions. Multiply the length of each elementary division by its perpendicular distance from some reference axis in a plane containing the line. When the line is all on one side of the axis, add these products (or moments, as they are usually designated), but when the line is on both sides of the axis, subtract the moments of the part on one side from the moments of the part on the other. Divide this sum or difference by the total length of the line, and the quotient will be the perpendicular distance from the axis to the center of gravity of the line and should be measured on the side of the axis that has the greater moment if the line is on both sides of the axis.

Centers of Gravity of Plane Areas. Proceed in the same manner as for lines, dividing the area into elementary areas. If the plane area has an axis of symmetry, the center of gravity lies on this axis.

Centers of Gravity of Technically Important Lines, Areas, and Solids

Centers of Gravity of Lines

Straight Line. The center of gravity is at its middle point.

Circular Arc AB, Fig. 39(a): $x_0 = r \sin c/\text{rad } c$; $y_0 = 2r \sin^2 \frac{1}{2}c/\text{rad } c$. (rad c = angle c measured in radians; see p. 44.)

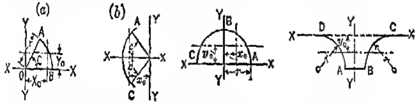


Fig. 39. Fig. 40. Fig. 41.

Circular Arc AC, Fig. 39(b): $x_0 = r \sin c/\text{rad } c$; $y_0 = 0$.

Quadrant, AB, Fig. 40: $x_0 = y_0 = 2r/\pi = 0.6366\ r$.

Semi-circumference, AC, Fig. 40: $y_0 = 2r/\pi = 0.6366r$; $x_0 = 0$.

Combination of Arcs and Straight Line, Fig. 41: AD and BC are two quadrants of radius r. $y_0 = \{(AB)r + 2[0.5\pi r(r - 0.6366r)]\} \div [AB + 2\ (0.5\pi r)]$.

Centers of Gravity of Plane Areas

Triangle. Center of gravity cg lies at the intersection of the lines joining the vertices with the mid-points of the sides, and at a distance from any side equal to one-third of the corresponding altitude.

Parallelogram. Cg lies at the point of intersection of the diagonals.

from XX to the center of gravity of MNP = area $M'N'P' \times d$/area MNP. Also, area $M''N''P'' \times d^2$ = moment of inertia of MNP about XX. The areas $M'N'P'$ and $M''N''P''$ can best be obtained by use of a planimeter.

DYNAMICS OF RIGID BODIES

Motion Under Unbalanced Forces

If a body is acted upon by a system of forces forming a balanced system, it will either remain at rest or move uniformly in a straight line. Only unbalanced forces change the motion of a body.

Uniformly accelerated or retarded motion requires the action of a constant unbalanced force. A common example of **uniformly accelerated motion** is the action of gravity on a falling body. If the action of gravity is wholly unresisted, the velocity v at the end of time t (in sec), starting from rest, will be $v = gt$, where g = acceleration due to gravity. If the body has an initial velocity v_0, $v = v_0 + gt$. An example of **uniformly retarded motion** is that of a body projected into the air vertically with a velocity v_0; its velocity after any time t will be $v = v_0 - gt$. If s = space passed through, $s = \frac{1}{2}gt^2$ for a falling body starting from rest, and $s = v_0t + \frac{1}{2}gt^2$ if starting with an initial velocity v_0. For a body projected vertically upward, $s = v_0t - \frac{1}{2}gt^2$.

General Formulas for the Motion of a Body Under the Action of a Constant Unbalanced Force

s = space in ft, a = acceleration in ft per sec per sec; v = velocity in ft per sec, v_0 = initial velocity in ft per sec, h = height in ft, F = force; m = mass, w = weight, g = acceleration due to gravity.

Initial velocity = 0:	Initial velocity = v_0:
$F = ma = (w/g)a.$	$F = ma = (w/g)a.$
$v = at.$	$v = v_0 + at.$
$s = \frac{1}{2}at^2 = \frac{1}{2}vt.$	$s = v_0t + \frac{1}{2}at^2 = \frac{1}{2}v_0t + \frac{1}{2}vt.$

$v = \sqrt{2as} = \sqrt{2gh}$ (falling freely from rest).

If a body is to be moved in a straight line by a force, the line of action of this force must pass through its center of gravity.

General Rule for the Solution of Problems When the Forces Are Constant in Magnitude and Direction. Resolve all the forces acting on the body into two components, one in the direction of the body's motion and one at right angles to it. Add the components in the direction of the body's motion algebraically and find the **unbalanced force**, if any exists.

Examples. (a) The body in Fig. 63 weighs 100 lb, is subjected to external forces F and F_1, and the coefficient of friction between the body and the inclined plane is 0.1. Required, the velocity of the body at the end of five (5) sec if it starts from rest.

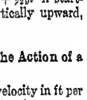


Fig. 63.

First, determine all of the forces acting externally to the body. These are F = 40 lb, F_1 = 80 lb, W = 100 lb, and the force with which the plane reacts upon the body. Resolve the forces into components along the plane and normal to it. The components along the plane are 32 lb and 60 lb acting down, 80 lb acting up, and that component of the plane's resistance which acts against motion. This component of the plane's resistance is the normal component of the pressure between the surfaces multiplied by 0.1, which is the coefficient of friction. The components normal to the plane are 24 lb acting away and 80 lb acting toward the plane. The normal pressure is 80 − 24 = 56 lb, and the friction is 56 × 0.1 = 5.6 lb. The unbalanced force acting on the body along the plane is

Oblique Frustum of a Right Circular Cylinder, Fig. 49. Let 1 2 3 4 be the plane of symmetry. The distance from the base to the cg is $\frac{1}{2}h + (r^2 \tan^2 c)/8h$, where c is the angle of inclination of the oblique section to the base. The distance of the cg from the axis of the cylinder is $r^2 \tan^2 c/4h$.

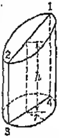


Fig. 49.

Pyramid or Cone. Cg lies in the line connecting the cg of base with the vertex and at a distance of one-fourth of the altitude above the base.

Truncated Pyramid. If h is the height of the truncated pyramid and A and B the areas of its bases, the distance of its cg from the surface of A is

$$h(A + 2\sqrt{AB} + 3B)/4(A + \sqrt{AB} + B).$$

Truncated Circular Cone. If h is the height of the frustum and R and r the radii of the bases, the distance from the surface of the base whose radius is R to the center of gravity is $h(R^2 + 2Rr + 3r^2)/4(R^2 + Rr + r^2)$.

Segment of a sphere, Fig. 50, volume ABC: $x_0 = 3(2r - h)^2/4(3r - h)$.

Hemisphere. $x_0 = 3r/8$.

Hollow Hemisphere. $x_0 = 3(R^4 - r^4)/8(R^3 - r^3)$, where R and r are respectively the outer and inner radii.

Sector of a sphere, Fig. 50, volume $OABCO$: $x'_0 = \frac{3}{8}(2r - h)$.

Ellipsoid, with semi-axes a, b and c. For each octant, distance from center of gravity to each of the bounding planes = $\frac{3}{8}$ × length of semi-axis perpendicular to the plane considered.

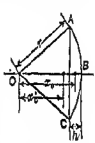


Fig. 50.

The formulas given for the determination of the centers of gravity of lines and areas can be used to determine the areas and volumes of surfaces and solids of revolution, respectively, by employing the theorems of Pappus, p. 111.

Determination of Center of Gravity of a Body by Experiment. The center of gravity may be determined by hanging the body up from different points and plumbing down; the point of intersection of the plumb lines will give the center of gravity. The cg may also be determined as shown in Fig. 51. The body is placed on knife edges which rest on platform scales. The sum of the weights registered on the two scales ($w_1 + w_2$) must equal the weight (w) of the body. Taking a moment axis at either end (say, O), $w_2A/w = x_0$ = distance from O to plane containing the center of gravity.

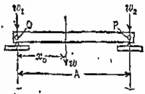


Fig. 51.

Graphical Determination of the Centers of Gravity of Plane Areas. See p. 213 and Fig. 62.

Moment of Inertia

The moment of inertia of a solid body with respect to a given axis is the limit of the sum of the products of the masses of each of the elementary particles into which the body may be conceived to be divided and the square of their distance from the given axis.

If $dm = dw/g$ represents the mass of an elementary particle and y its distance from an axis, the moment of inertia I of the body about this axis will be $I = \int y^2 dm = \int y^2 dw/g$.

Let ω be the angular velocity of point A. Consider that it starts from X' and moves to A in time t; then the angle $B = \omega t$. If $CA = r$, $X'P = r - CP = r - r \cos \omega t = s$. The velocity v of the point P will equal $ds/dt = \omega r \sin \omega t$, and the acceleration $a = dv/dt = -\omega^2 r \cos \omega t$. The unbalanced force acting on the body P at any time $= F = ma = m\omega^2 r \cos \omega t = (w/g)\omega^2 r \cos \omega t = m\omega^2 CP$, showing that the force varies as the distance of the body from the center of its path, being 0 at the center and $(w/g)\ \omega^2 r$ at the ends of the path. The velocity is 0 at the ends and a maximum at the middle, where $v = \omega r$.

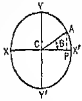


FIG. 66.

When a body is acted upon by varying forces, methods similar to those adopted for constant forces are used, but, in general, more simple solutions can be found by using the principles of work and energy.

Work and Energy

Work. When a body is displaced against resistance or accelerated, work must be done upon it. The work done by a constant force is the force F multiplied by the distance through which the force moves, or work $= Fs$. The work done by a force that varies $= \int Fds$ when the force F is expressed as a function of the space s. In the **work diagram**, Fig. 67, the ordinates are forces at different times and the abscissas are spaces passed over. The area under the curve represents the work done, which is equal to $\int Fds$.

Units of Work. When the force of 1 lb acts through the distance of 1 ft **1 ft-lb** of work is done. The **foot-ton** unit is sometimes used. In countries using the metric system the unit employed is the **meter-kilogram.**

Energy. A body is said to possess energy when it can do work. A body may possess this capacity through its **position** or **condition.** When a body is so held that it can do work, if released, it is said to possess energy of position or **potential energy.** When a body is moving with some velocity, it is said to possess energy of motion or **kinetic energy.** An example of potential energy is a body held suspended by a rope; the position of the body is such that if the rope be removed work can be done by the body.

Energy is expressed in the same units as work. The kinetic energy of a body is expressed by the formula $E = \frac{1}{2}mv^2 = \frac{1}{2}(w/g)v^2$.

If a force which varies acts through a space on a body of mass m, the work done is $\int_{s_1}^{s_2} Fds$, and if the work is all used in giving kinetic energy to the body it is equal to $\frac{1}{2}m(v_2^2 - v_1^2) =$ **change in kinetic energy,** where v_2 and v_1 are the velocities at distances s_2 and s_1 respectively. For the kinetic energy of rotation, see p. 220.

If a force which varies acts for a certain time on a body of mass m, the quantity $\int_{t_0}^{t_1} Fdt = m(v_1 - v_0)$, = the **change in momentum** of the body.

Certain problems in which the velocity of a body at any point in its straightline path when acted upon by varying forces is required, can be easily solved by the use of a **work diagram.**

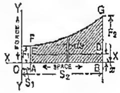


FIG. 67.

In Fig. 67, let a body weighing 100 lb start from rest at A and be acted upon by a force that varies in accordance with the diagram $AFGBA$. Let the resistance to motion be a constant force $= x$. Find the velocity of the body at point B. The area $AFGBA$ represents the work done upon the body and the area $AEDBA$ (= force x × distance AB) represents the work that

area A about YY and XX, respectively, I_y' and I_x' the moments about $Y'Y'$ and $X'X'$, and I_{xy} and I_{xy}' the products of inertia for XX and YY, and $X'X'$ and $Y'Y'$, respectively. Also, let c be the angle between the respective pairs of axes, as shown. Then,

$$I_y' = I_y \cos^2 c + I_x \sin^2 c + I_{xy} \sin 2c$$
$$I_x' = I_x \cos^2 c + I_y \sin^2 c - I_{xy} \sin 2c$$
$$I_{xy}' = \frac{(I_x - I_y)}{2} \sin 2c + I_{xy} \cos 2c$$

Principal Moments of Inertia. In every plane area, a given point being taken as the origin, there is at least one pair of rectangular axes in the plane of the area about one of which the moment of inertia is a maximum, and a minimum about the other. These moments of inertia are called the **principal moments of inertia**, and the axes about which they are taken are the **principal axes of inertia.** One of the conditions for principal moments of inertia is that the product of inertia I_{xy} shall equal zero. **Axes of symmetry** of an area are always principal axes of inertia.

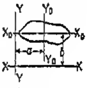


FIG. 55.

Relation between Products of Inertia for Parallel Axes. In Fig. 55, X_0X_0 and Y_0Y_0 pass through the center of gravity of the area parallel to the given axes XX and YY. If I_{xy} be the product of inertia for XX and YY, and $I_{x_0y_0}$ that for X_0X_0 and Y_0Y_0, then $I_{xy} = I_{x_0y_0} + abA$.

Ellipse of Inertia. In Fig. 56, let $I_y = \int x^2dA$ and $I_x = \int y^2dA$ be the moments of inertia about the principal axes XX and YY. The moment of inertia about axis $AA = I_y \sin^2 c + I_x \cos^2 c$, as $I_{xy} = 0$. The polar equation of an ellipse constructed on the semiaxes a and b will be $a^2 \sin^2 c + b^2 \cos^2 c = (a^2b^2)/r^2$, r being the radius vector of any point on the ellipse and c the angle that it makes with XX. Lay off $a = \sqrt{I_y}$ and $b = \sqrt{I_x}$; then the moment of inertia about $AA = (a^2b^2)/r^2 = I_xI_y/r^2$.

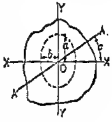


FIG. 56.

The moment of inertia of the area about any axis in the plane and passing through O will be inversely proportional to the square of the semi-diameter cut by that axis from an ellipse constructed with $a = \sqrt{I_y}$ and $b = \sqrt{I_x}$, a and b being the semi-major and semi-minor axes, respectively.

The following **conclusions** may be drawn: If the principal moments of inertia are equal, the ellipse of inertia becomes a circle and the moments of inertia about all axes in the plane and passing through O are the same. If the moments of inertia about more than two axes in the plane and passing through O are equal, the moments of inertia about all axes in the plane and passing through O are the same. There are two axes in the plane making equal and opposite angles with a principal axis and passing through O, about which the moments of inertia are equal.

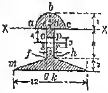


FIG. 57.

The moment of inertia of any area may be considered to be made up of the sum or difference of the known moments of inertia of simple figures. For example, the dimensioned figure shown in Fig. 57 represents the section of a rolled shape with hole *oprs*, and may be divided into the semi-circle *abc*,

the body to the connection is called a **centrifugal force.** The acceleration toward the axis necessary to keep a particle moving in a circle about that axis is v^2/r; therefore the force necessary is $ma = mv^2/r = wv^2/gr = w\pi^2N^2r/900g$, where N = rpm. This force is constantly directed toward the axis.

The centrifugal force of a solid body revolving about an axis is the same as if the whole mass of the body were concentrated at its center of gravity. Centrifugal force $= wv^2/gr = mv^2/r = w\omega^2r/g$, where w and m are the weight and mass of the whole body, r is the distance from the axis about which the body is rotating to the center of gravity of the body, ω the angular velocity of the body about the axis in radians, and v the linear velocity of the center of gravity of the body.

Examples (Uniform angular velocity). (1) In Fig. 69 a homogeneous rod of length l revolves about XX at a speed of N rpm: find the pressure on the axis XX due to centrifugal force (C. F.). As the whole body is deviated toward XX, the force necessary must be C. F. $= (w/g)(2\pi N/60)^2(l/2)$, w being the weight deviated, $2\pi N/60$ = the angular velocity and $l/2$ the distance from axis to center of gravity.

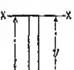


FIG. 69.

(2) Find the tension in the rod at aa, distant y from the axis. The force with which the length y of the rod acts on the length ab is the deviating force necessary to keep $aabba$ constantly accelerated toward the axis. This force $= \dfrac{w'}{g}\left(\dfrac{2\pi N}{60}\right)^2\left[y + \dfrac{(l-y)}{2}\right]$ w' being the weight of $aabba$, and $\left[y + \dfrac{(l-y)}{2}\right]$ being the distance from XX to the center of gravity of $aabba$.

(3) In Fig. 70 find the pressure on the axis YY and maximum tension due to centrifugal force. Determine the center of gravity of the body; let it be r_0 from the axis YY; then the pressure on the axis will be $(w/g)(2\pi N/60)^2r_0$. To determine the maximum tension due to centrifugal force, determine which part of the body on one side of the axis has the greater moment about the axis; the maximum tension will be on this side of the axis, and the pressure on the axis will be the difference between the tensions on the two sides.

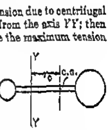


FIG. 70.

The tension in a belt, in lb per sq ft, due to centrifugal force is wv^2/g, where w = weight of material per cu ft, and v = velocity, fps. It should be noted that the *tension is independent of the diameter of the pulley* upon which the belt runs.

A body revolving about an axis passing through its center of gravity exerts no pressure on the axis due to centrifugal force.

Conical Pendulum. In the simple revolving pendulum shown in Fig. 71 a ball weighing w lb revolves about YY at a speed of N rpm; find the angle a. The forces acting upon the mass of the ball are the weight w and the tension T in the cord. As the ball stands out at a constant angle, the vertical component of T must equal w, leaving the horizontal component of T unbalanced. This unbalanced component is the deviating force necessary to keep the weight revolving about YY. This deviating force is the centripetal force.

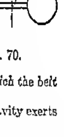


FIG. 71.

$T \sin a$ = C. F. and $T \cos a = w$, but C. F. $= w\omega^2r/g$; $\therefore$ $T \sin a = w\omega^2r/g$, $T = w\omega^2r/g \sin a$, and $\sin a = w\omega^2r/gT$; $r = l \sin a$, and $\omega = 2\pi N/60$. From these formulas the tension and angle can be found if N is known, or the tension and number of revolutions per minute can be determined if the angle is known. It should be noted that ω^2l must be greater than g.

Solid right circular cone about an axis through its apex and perpendicular to its axis:

$I = 3M[(r^2/4) + h^2]/5$. (h = altitude, of cone, r = radius of base.)

Solid right circular cone about its axis of revolution: $I = 3Mr^2/10$.

Ellipsoid with semi-axes a, b, and c: I about diameter $2c$ (z-axis) = $I = 4m\pi abc\,(a^2 + b^2)/15$.

[Equation of ellipsoid: $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$.]

Ring with Circular Section, Fig. 58:

$I_{yy} = \frac{1}{4}m\pi^2Ra^2(4R^2 + 3a^2)$;

$I_{xx} = m\pi^2Ra^2[R^2 + (5a^2/4)]$.

Approximate Moments of Inertia of Solids. In order to determine the moment of inertia of a solid, it is necessary to know all its dimensions. In the case of a rod of mass M, Fig. 59, and length l, with shape and size of the cross-section unknown, making the approximation that the weight is all concentrated along the axis of the rod, the moment of inertia about YY will be $I_{yy} = \int_0^l (M/l)x^2dx = Ml^2/3$.

Fig. 58.

Fig. 59.

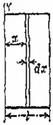


Fig. 60.

A thin plate may be treated in the same way (Fig. 60):

$$I_{yy} = \int_0^l (M/l)x^2dx$$

Here the mass of the plate is assumed as concentrated at its middle layer.

Thin Ring, or Cylinder (Fig. 61). Assume the mass, M, of the ring or cylinder to be concentrated at a distance r from O. The moment of inertia about an axis through O perpendicular to plane of ring or along axis of cylinder will be $I = Mr^2$; this will be greater than the exact moment of inertia, and r is sometimes taken as the distance from O to the center of gravity of the cross-section of the rim.

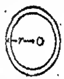


Fig. 61.

Fly-wheel Effect. The moment of inertia of a solid is often called fly-wheel effect in the solution of problems dealing with rotating bodies, and is usually expressed in lb-ft² (I_w).

Graphical Determination of the Centers of Gravity and Moments of Inertia of Plane Areas. Required, to find the center of gravity of the area MNP, Fig. 62, and its moment of inertia about any axis XX.

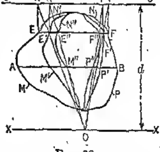


Fig. 62.

Draw any line SS parallel to XX and at a distance d from it. Draw a number of lines such as AB and EF across the figure parallel to XX. From E and F draw ER and FT perpendicular to SS. Select as a pole any point O on XX, preferably the point nearest the area, and draw OR and OT, cutting EF at E' and F'. If the same construction is repeated, using other lines parallel to XX, a number of points will be obtained, which, if connected by a smooth curve, will give the area $M'N'P'$. Project E' and F' on to SS by lines $E'R'$ and $F'T'$. Join R' and T' with O', obtaining E'' and F''; connect the points obtained using other lines parallel to XX and obtain an area $M''N''P''$. The area $M'N'P' \times d$ = moment of area MNP about the line XX, and the distance

Rotation of Solid Bodies about Axes

If a body rotates about an axis under the action of a constant force F, the relations between the angular acceleration a_c, the angular velocity ω, the angle moved through θ, and the kinetic energy of rotation E, are $Fl = a_c I$; $\omega = a_c t$, or $= \omega_0 + a_c t$; $\theta = \frac{1}{2}a_c t^2 = \frac{1}{2}\omega t$, or $= \omega_0 t + \frac{1}{2}a_c t^2$; $E = \omega_1^2 I/2$; where Fl is the unbalanced turning moment, I is the moment of inertia of the body ($\int y^2 dm$) and ω_0 and is the initial angular velocity.

General Rule for Rotating Bodies. Determine all the external forces acting and their moments about the axis of rotation. If these moments are balanced, there will be no change of motion. If the moments are unbalanced, this unbalanced moment or **torque** will cause an angular acceleration about the axis.

Rotation About an Axis Passing Through the Center of Gravity. The rotation of a body about its center of gravity can only be caused or changed by a **couple**. See Fig. 75. If a single force F is applied to the rim of the wheel, the axis immediately acts on the wheel with an equal force to prevent translation, and the result is a couple (moment Fr) acting on the body and causing rotation about its center of gravity. **The work done by the couple causing rotation** is $M\theta$, where M is the moment of the couple and θ is the angular space passed over, in radians. **If there is no unbalanced moment there must exist either rest or uniform rotation.**

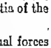


FIG. 75.

In calculations pertaining to **hoisting machinery**, it is sometimes necessary to determine the total weight to be accelerated. This consists of the combined weight of the load to be hoisted, the rope or chain and the cage or skip (if used) *plus* a certain weight w_1 equivalent to that of the hoisting drum if concentrated at a point in its circumference. This latter may be determined as follows: Energy in ft-lb of rotating drum $= E = I\omega^2/2 = w\omega^2r^2/2g$, where r = radius of gyration of drum, in ft; ω = angular velocity, and w = weight in lb. The product wr^2 is sometimes known as the **flywheel effect**. Let R = radius of drum from shaft axis to rope center, in ft. Then, considering the weight concentrated at radius R, for the same quantity of energy, $E = w_1\omega^2R^2/2g$, whence $w_1R^2 = wr^2$, or $w_1 = wr^2/R^2$. For example, in a large electric mine hoist, $w = 16{,}000$ lb, $r = 6.4$ ft and $R = 8$ ft. $\therefore$ equivalent weight at $R = w_1 = (16{,}000 \times 6.4 \times 6.4)/(8 \times 8) = 10{,}240$ lb.

Combined Rotation and Translation. Any body that is revolving about an axis *not* passing through its center of gravity may be considered *to have a motion of translation and a motion of rotation about* its center of gravity combined. The body shown in Fig. 76 is revolving about C with an angular velocity ω, but this motion may be considered to be made up of a translation with the velocity ωl of the center of gravity of the body about C and a revolution about the center of gravity with the same angular velocity (ω) as that with which the body rotates about C.

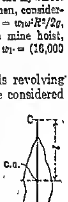


FIG. 76.

The kinetic energy of the body is $\omega^2 I/2$, in which I is the moment of inertia of the body with respect to C. This is the same as $\frac{1}{2}mv^2 + \omega^2 I_0/2$, where $v = \omega l$ and I_0 = moment of inertia of the body with respect to an axis passing through its center of gravity and parallel to the axis at C. The motion of translation requires a single force applied at the center of gravity of the body, and the motion of rotation about the center of gravity requires the action of a couple, therefore the combined motion must be caused by the resultant of a single force and a couple.

The point of application of this resultant is known as the **center of percussion** and may be defined as the point of application of the resultant of

$60 + 32 - 80 - 5.6 = 6.4$ lb and acts downward

$$F = ma = (100/g)a \therefore a = 0.064g$$

The body is acted upon by constant forces and starts from rest, therefore, $v = at$ and $s = \frac{1}{2}at^2$. If $t = 5$, $v = 0.064g \times 5 = 0.32g$, $s = (1/2)0.32g \times 5 = 0.8g$; taking $g = 32.16$, $v = 0.32 \times 32.16 = 10.29$ ft per sec, $s = 0.8 \times 32.16 = 25.73$ ft.

(b) Find the constant force necessary to start from rest a train weighing 100 tons and give it a speed of 30 mph in 1 min; track to be level and straight, train resistance to be constant and equal to 12 lb per ton. The external forces acting upon the train are the resistance of $100 \times 12 = 1200$ lb, the constant force F pulling the train along the level track, the force of gravity equal to weight of train, and the reaction of the track. The forces normal to the track form a balanced system, but, as the train is to be started from rest and given a velocity, the forces acting along the track must form an unbalanced system. A velocity of 30 mph = 44 fps; to acquire this velocity in 60 sec requires an acceleration of 44/60, as $a = v/t$. The unbalanced force to give the train this acceleration is $F = ma = (100 \times 2000/g)(44/60) = 4580$ lb. The total tractive force required is therefore $4580 + 1200 = 5780$ lb. If the train is on a grade, the problem is the same, with the addition of another force acting along the track, that is, the component of the weight along the track.

The so-called laws of inclined planes are simply derived by the above method. In Fig. 64 the weight W slides down the plane without friction. The unbalanced component of the forces acting along the plane is $W \sin B$, $\therefore a = g \sin B$, $v = (g \sin B)t$, and $s = (\frac{1}{2}g \sin B)t^2$ as the body starts from rest. The time necessary to pass over a distance s is $t = \sqrt{2s/g \sin B}$, and $v = at = \sqrt{2gs \sin B} = \sqrt{2gH}$, showing that if the body slides without friction from T to A, its velocity will be the same as if it fell through the same vertical distance TN. With initial velocity v_0, $v = v_0 \pm (g \sin B)t$, and $s = v_0t + (\frac{1}{2}g \sin B)t^2$.

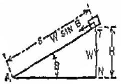


FIG. 64.

Tension in a Rope Connected to a Moving Body. The force with which the rope acts on the body is equal and opposite to the force with which the body acts on the rope, and each is equal to the tension in the rope. In Fig. 65, neglecting the weight of pulley A and the cord, find the tension in the cord. The unbalanced force acting on the two weights is $20 - 5 = 15$ lb and acts on the two weights. $\therefore 15 = (25/g)a$, and $a = 3g/5$. w_1 is accelerated in an upward direction and w_2 downward. $T_1 = w_1 +$ the unbalanced force necessary to give w_1 an upward acceleration of $(3g/5) = 5 + (5/g) \times (3g/5) = 8$ lb. $T_2 = w_2 -$ the unbalanced force necessary to give w_2 a downward acceleration of $(3g/5) = 20 - (20/g) \times (3g/5) = 8$ lb. If a body is moved uniformly upward by a rope, the tension in the rope is equal to the weight of the body, without regard to the velocity.

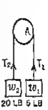


FIG. 65.

If a weight is to be pulled up an inclined plane by means of a rope, the tension in the rope is equal to the component of the weight along the plane plus the normal pressure between the surfaces times the coefficient of friction, plus the unbalanced force necessary to give the weight the acceleration that it has. If the weight is to move uniformly up the plane there will be no acceleration, and the tension in the rope must simply balance all resistance to motion.

Harmonic Motion

Forces Necessary to Cause Harmonic Motion. In Fig. 66, point A moves uniformly in the circle $X'YXY'$, and point P moves to and fro along $X'CX$.

$(w_1 + w_2)a/g$ is the unbalanced force necessary to give the two weights the acceleration a, and $a_a I$ is the unbalanced moment necessary to give the pulley the angular acceleration a_a. If there is no slipping, $a_a = a/r$.

Relation between the Center of Percussion and Radius of Gyration. $l = I/mx_0 = k^2/x_0 \therefore k^2 = x_0 l$; where k = radius of gyration. Therefore the radius of gyration is a mean proportional between the distance from the axis of oscillation to the center of percussion and the distance from the same axis to the center of gravity.

Interchangeability of Center of Percussion and Axis of Oscillation. If a body is suspended from an axis, the center of percussion for that axis can be found. If the body be suspended from this center of percussion as an axis, the original axis of suspension will then become the center of percussion. The center of percussion is sometimes known as the **center of oscillation.**

Time of Oscillation of a Compound Pendulum. The length of an equivalent simple pendulum is the distance from the axis of suspension to the center of percussion of the body in question. To find the **time of oscillation** of a body about a given axis, find the distance $l = I/mx_0$ from that axis to the center of percussion of the swinging body. The length of the simple pendulum that will oscillate in the same time is this distance l. The time of oscillation for the equivalent single pendulum is $t = \pi\sqrt{l/g}$. This is the time necessary for a single swing, not over and back.

Determination of Moment of Inertia by Experiment. To find the moment of inertia of a body, suspend it from some axis not passing through the center of gravity and, by swinging it, determine the time of a single oscillation in seconds. The known values will then be t = time of single oscillation, x_0 = distance from axis to center of gravity, and m = mass of rod. The length of the equivalent simple pendulum is $l = I/mx_0$. Substituting this value of l in $t = \pi\sqrt{l/g}$ gives $t = \pi\sqrt{I/mx_0g}$, from which $t^2 = \pi^2 I/mx_0g$, or $I = mx_0gt^2/\pi^2$.

Impulse and Momentum

Impulse of a force $= F(t_2 - t_1)$ when the force is constant and $t_2 - t_1$ is the time interval during which it acts. If the force is not constant in magnitude but always acts in the same direction, impulse $= \int_{t_1}^{t_2} F dt$.

Impulses may be added algebraically by means of a polygon, or an impulse may be resolved into components by means of a parallelogram. The moment of an impulse may be found in the same manner as the moment of a force by representing the impulse by a line and taking the product of the magnitude of the impulse by the perpendicular distance from the line representing it to the point about which the moment is to be taken.

Momentum. The momentum of a particle is mv, where m = mass of the particle and v = its velocity. Momentums can be represented by lines (vectors) and added and resolved in the same manner as forces by means of polygon and parallelogram. The moment of momentum can be determined by the same methods as used for the moment of a force or moment of an impulse, the momentum being represented by a line. **Angular momentum** is the product of the component of momentum at right angles to the radius, and the radius.

Impact

The straight line perpendicular to the common contact plane of two colliding bodies and passing through the point of contact is called the

must be done to overcome resistance. The difference of these areas, or *EFGDE*, will represent work done in excess of that required to overcome resistance, and consequently is equal to the increase in kinetic energy. Equating the work represented by the area *EFGDE* to $\frac{1}{2}wv^2/g$ and solving for v, will give the required velocity at B. If the body did not start from rest this area would represent the change in kinetic energy, and the velocity could be obtained by the formula: Work $= \frac{1}{2}(w/g)(v_1^2 - v_0^2)$, v_1 being the required velocity.

If the forces acting upon a body are functions of the time, a diagram similar to Fig. 67 may be drawn using *time* as abscissas. In this case the areas represent the impulses of the forces. The area *EFGDE* will then represent the unbalanced impulse and therefore the change in momentum and the velocity v_1 can be determined by placing this area equal to $w/g(v_1 - v_0)$.

General Rule for Rectilinear Motion. Resolve each force acting on the body into components, one of which acts along the line of motion of the body and the other at right angles to the line of motion. Take the sum of all the components acting in the direction of the motion and multiply this sum by the distance moved through for constant forces. (Take the average force times distance for forces that vary.) This product will be the total work done upon the body. If there is no unbalanced component there will be no change in kinetic energy and consequently no change in velocity. If there is an unbalanced component, the change in kinetic energy will be this unbalanced component multiplied by the distance moved through.

Conservation of Energy. The sum of the kinetic and potential energies of a body acted on by gravity alone is constant.

If a body is at a certain height h above the ground and is motionless, its potential energy is equal to the work done in raising it to that height. If the body falls to the ground its velocity v when it strikes will be $\sqrt{2gh}$, and its kinetic energy will be $wv^2/2g$. The potential energy before starting to drop is, therefore, the same as its kinetic energy when it reaches the earth. At any point during its fall its total energy, obtained by adding the kinetic and potential energies, will be the same $wv^2/2g$, where v is the velocity that the body would have if it fell freely to the earth from the original height.

The work done by a system of forces acting on a body is equal to the algebraic sum of the work done by each force taken separately.

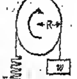


Fig. 68.

Power is the rate at which work is performed, or the number of units of work performed in unit time. The units of power employed by engineers are the **horse power**, or 33,000 ft-lb per min = 550 ft-lb per sec, and the **kilowatt** = 1.341 hp = 737.55 ft-lb per sec.

Friction Brake. In Fig. 68 a pulley revolves under the band and in direction of the arrow, exerting a pull of T on the spring. The friction of the band on the rim of the pulley is $(T - w)$, where w is the weight attached to one end of the band. Let the pulley make N rpm; then the work done per minute against friction by the rim of the pulley is $2\pi RN(T - w)$, and the horse power absorbed by brake $= 2\pi RN(T - w)/33,000$.

Centrifugal Force

Centrifugal and Centripetal Forces. When a body revolves about an axis, some connection must exist capable of applying force enough to the body to constantly deviate it toward the axis. This deviating force is known as **centripetal force.** The equal and opposite resistance offered by

The **strength** or **intensity of the field** at any given point is the relation between a force F acting on a mass m at that point and the mass. Intensity of field $= i = F/m$; $F = mi$.

The **unit of field intensity** is the same as the unit of acceleration, *i.e.*, 1 ft per sec per sec, or 1 m per sec per sec. The intensity of a field of force may be represented by a line (or **vector**).

A field of force is said to be **homogeneous** when the intensity of all points is uniform and in the same direction.

A field of force is called a **central field of force** with a center O, if the direction of the force acting on the mass particle m in every point of the field passes through O and its magnitude is a function only of the distance r from O to m. A line so drawn through the field of force that its direction coincides at every point with that of the force prevailing at that point, is called a **line of force**.

Law of Gravitation. Two particles attract each other with a force F proportional to their masses m_1 and m_2 and inversely proportional to the square of the distance r between them, or $F = km_1m_2/r^2$, in which k is the **gravitation constant**, or the force with which two spheres of unit mass attract each other through unit distance. The value of gravitation constant is $k = 6.66 \times 10^{-8}$ cm^3 g^{-1} sec^{-2} in cgs units, or $k = 3.44 \times 10^{-8}$ ft^4 lb^{-1} sec^{-4} in engineering units. The **density of a body** is its mass per unit of volume. Density $= d =$ mass/volume.

THE GYROSCOPE

By E. V. HUNTINGTON

The elementary facts about gyroscopic forces may be briefly illustrated by the following example. Given, a wheel or disk, of weight w and radius R, spinning rapidly about its axis OA (Fig. 1), and mounted in gimbal rings in such a way that its center is stationary, while its axis is free to point in any

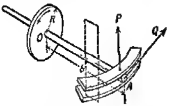


FIG. 1.

FIG. 2.

direction in space. (See Fig. 2.) Let $k =$ the radius of gyration of the wheel about its axis. (If all the weight is in the rim, $k^2 = R^2$; if the wheel is a homogeneous disk, $k^2 = R^2/2$.) Now suppose the axle end A is constrained to move between two smooth fixed guides, forming a slot as in Fig. 1, and suppose that a force Q is applied to pull the point A along the slot. Then, (1) as far as the motion of A along the slot is concerned, it makes no difference whether the disk is rotating or not; in fact, $Qb = (w/g)(k^2/2)(dv/bdt)$, where Qb is the moment of Q about the center O, and dv/dt is the (linear) acceleration of the point A along the slot; but (2) if the wheel is rotating, then the mere motion of A along the slot will call into play a strong lateral pressure P (called a gyroscopic reaction) against one of the guides.

Balancing

A rotating body is said to be in **standing balance** when its center of gravity coincides with the axis upon which it revolves. Standing balance may be obtained by resting the axis carrying the body upon two horizontal plane surfaces, as in Fig. 72. If the center of gravity of the wheel A coincides with the center of the shaft B, there will be no movement, but if the center of gravity does not coincide with the center of the shaft, the shaft will roll until the center of gravity of the wheel comes directly under the center of the shaft. The center of gravity may be brought to the center of the shaft by adding or taking away weight at proper points on the diameter passing through the center of gravity and the center of the shaft. Weights may be added to or subtracted from any part of the wheel so long as its center of gravity is brought to the center of the shaft.

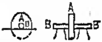


FIG. 72.

A rotating body may be in standing balance and not in **running balance.** In Fig. 73, AA and BB are two disks whose centers of gravity are at o and p, respectively. The shaft and the disks are in standing balance if the disks are of the same weight and the distances of o and p from the center of the shaft are equal, and o and p lie in the same axial plane but on opposite sides of the shaft. Let the weight of each disk be w and the distances of o and p from the center of the shaft each be equal to r. The force exerted on the shaft by AA is equal to $w\omega^2r/g$, where ω is the angular velocity of shaft. Also, the force exerted on shaft by $BB = w\omega^2r/g$. These two equal and opposite parallel forces act at a distance x apart, and constitute a couple with a moment tending to rotate the shaft, as shown by the arrows, of $(w\omega^2r/g)x$. A couple cannot be balanced by a single force, so two forces at least must be added to or subtracted from the system to get running balance.

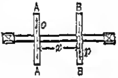


FIG. 73.

Curvilinear Motion

An unresisted projectile has a motion compounded of the vertical motion of a falling body, and of the horizontal motion due to the horizontal component of the velocity of projection. In Fig. 74 the only force acting after the projectile starts is gravity, which causes an acceleration downward. The horizontal component of the original velocity v_0 is not changed by gravity. The projectile will rise until the velocity given to it by gravity is equal to the vertical component of the starting velocity v_0, and the equation $v_0 \sin a = gt$, gives the time t required to reach the highest point in the curve. The same time will be taken in falling if the surface XX is level, and the projectile will therefore be in flight $2t$ sec. The distance $s = v_0 \cos a \times 2t$, and the maximum height of ascent $h = (v_0 \sin a)^2/2g$. The expressions for the co-ordinates of any point on the path of the projectile are: $x = (v_0 \cos a)t$, and $y = (v_0 \sin a)t - \frac{1}{2}gt^2$, giving $y = x \tan a - (gx^2/2v_0^2 \cos^2 a)$ as the equation for the curve of the path. The radius of curvature of the highest point may be found by using the general expression C. F. $= wv^2/gr$ and solving for r, v being taken equal to $v_0 \cos a$.

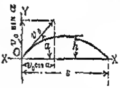


FIG. 74.

Simple Pendulum. The time of a single oscillation in one direction $= t = \pi\sqrt{l/g}$, where l is the length of the pendulum and the length of the swing is not great as compared to l.

STRESSES IN FRAMED STRUCTURES

BY

A. HAERTLEIN

(Originally prepared by H. W. Hayward)

An ideal truss is a framework consisting of straight bars or members connected at their ends by frictionless pins. The external forces are applied only at these pins. Internal forces or stresses in such straight bars are axial, either tension or compression, without bending. Since frictionless pins are impossible and the ends of bars are often riveted or welded, the ideal truss is never realized. For purposes of analysis, the primary stresses, which are always axial, are determined on the assumption that the truss under consideration conforms to the ideal. Secondary stresses are additional stresses, generally flexural or bending, brought about by all the factors that make the actual truss different from the ideal. In the following discussion, primary stresses will be considered.

General Procedure. After all external forces, loads, and reactions have been determined, the internal force or stress in any member is found (1) by

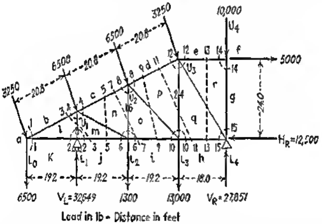


FIG. 1.

taking a section, making an imaginary cut through the members of the truss, including the one whose stress is to be found, so as to separate the truss into two parts; (2) by isolating either of these parts; (3) by replacing each bar cut by a force, representing the stress in the bar or the force required to come to the part of the truss isolated from the other part removed; and (4) by applying the equations of statics to the part isolated.

The various ways in which sections can be taken and the equations used to determine the stresses are illustrated by a solution of the truss (Fig. 1).

Stress in bl and lk. Section 1-1. Isolate part to left [Fig. 2(*a*)]. Assume the unknown stresses in *bl* and *kl* as tension. Since the forces are concurrent and coplanar, only two independent equations of statics are available. These may be either $\Sigma x = 0$ and $\Sigma y = 0$, or $\Sigma M = 0$ taken about two different points selected so that neither point is the intersection of the forces, or so that the line joining the two points is not coincident with either of the unknown internal forces.

Using the first set of equations and taking components of all forces along a horizontal and vertical axis, for example, the horizontal component of the 3,250 lb force is 1,250

all the forces tending to cause a body to rotate about a certain axis. It is the point at which a suspended body may be struck without causing any pressure on the axis passing through the point of suspension.

Center of Percussion. The distance from the axis of suspension to the center of percussion is $l = I/mx_0$, where I = moment of inertia of the body about its axis of suspension, m = total mass of the body and x_0 = distance from the axis of suspension to the center of gravity of the body.

Examples. (1) Find the center of percussion of the homogeneous rod (Fig. 77) of length L and mass m, suspended at XX.

$$l = \frac{I}{mx_0};\quad I(\text{approx}) = \frac{m}{L}\int_0^L x^2dx;\quad x_0 = \frac{L}{2};\quad \therefore l = \frac{2}{L^2}\int_0^L x^2dx = 2L/3.$$

(2) Find the center of percussion of a solid cylinder, of mass m, resting on a horizontal plane. In Fig. 78, the instantaneous center of the cylinder is at A. The center of percussion will therefore be a height above the plane equal to $l = I/mx_0$. Since $I = (mr^2/2) + mr^2$ and $x_0 = r$, $l = 3r/2$.

Fig. 77.

(3) In Fig. 79, a solid cylinder weighing 100 lb is moved along a horizontal plane by a horizontal force of 12 lb applied at the axis. If the cylinder rolls without slipping, how far may it be moved in 10 sec and what must be the friction?

The forces acting are F, gravity, and the reaction of the plane. The vertical component of the reaction of the plane balances gravity, leaving the horizontal component of the reaction of plane and F as the forces causing rotation about the instantaneous center O. The resultant R of the 12 lb force and the friction must be applied at the center of percussion, which is at the distance I/mx_0 from O or 3 ft. The resultant can be found by taking moments about O, $3R = 2 \times 12$; $\therefore R = 8$ lb. The friction must be $12 - 8 = 4$ lb acting to the left, because 8 lb is the resultant of $F = 12$ lb and the friction. This resultant of 8 lb at a distance of 3 ft from O has the same effect as the two forces of 12 lb and 4 lb acting together. This single force of 8 lb must be also the resultant of a couple and a single force at the axis, see Fig. 79(b). The single force of 8 lb acting to the right at the axis causes translation, and the couple whose moment is 8×1 causes rotation about the center of gravity.

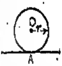


Fig. 78.

The linear acceleration along the plane is found by using the 8 lb force. $F = ma$ $\therefore 8 = 100a/g$, $a = 8g/100$, and $v = (8g/100) \times 10 = (4g/5)$ fps. $S = \frac{1}{2}(80g/100) \times 10$.

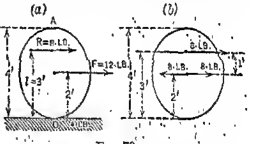


Fig. 79.

Wheel or Cylinder Rolling Down a Plane. In this case the component of the weight along the plane tends to make it roll down, and is treated as a force causing rotation. The forces acting on the body should be resolved into components along the line of motion and perpendicular to it. If the forces are all known, their resultant is at the center of percussion. If one force is to be determined (the exact conditions as regards slipping or not slipping must be known), the center of percussion can be determined and the unknown force found.

When weights act as shown in Fig. 80, where the pulley over which the ropes run has appreciable weight, it is necessary to take into account the moment of the force required to accelerate the pulley itself.

Fig. 80.

In Fig. 80, the unbalanced force $w_1 - w_2$ is accelerating the pulley as well as the two weights. Therefore, $w_1 - w_2 = (w_1 + w_2)(a/g) + a_0I/r$, where

Stress in bars cn and nm. Section 4-4. Cut the members around the pin at U_1 and so use $\Sigma x = 0$ and $\Sigma y = 0$ for concurrent forces on the upper part of the cut isolated [Fig. 2(*d*)]. It will be observed that there cannot be more than two unknown stresses in the section taken around a pin or point in order to have the stresses determined from the equations of statics. Hence the sections cannot be taken at random, for example, section 6-6 could not be taken at this time because the stresses in bar *mn*, *no*, and *oi* are unknown. The numerical work is generally made easier by resolving the forces into components parallel to the axes which may be selected in any convenient direction. For this particular case, the x-axis is taken as horizontal and the y-axis perpendicular to it.

$$\Sigma x = 0 = s_{cn}\ 19.2/20.8 + s_{mn}\ 19.2/20.8 + 2500 - 24{,}700 \times 19.2/20.8$$
$$\Sigma y = 0 = s_{cn}\ 8/20.8 - s_{mn}\ 8/20.8 - 6000 + 32{,}649 - 24{,}700 \times 8/20.8$$

Solving these equations simultaneously, $s_{cn} = 11{,}298$ compression
$s_{mn} = 33{,}290$ tension

By taking section 5-5 and considering the part to the left [Fig. 2(*e*)]

ΣM about $L_0 = 0 = s_{mn} \times 8/20.8 \times 38.4 + 2500 \times 8 - (32{,}649 - 6000)19.2$
ΣM about $U_1 = 0 = -s_{mj} \times 8 - 1250 \times 8 - 9500 \times 19.2$
ΣM about $L_2 = 0 = s_{cn} \times 19.2/20.8 \times 18 + 2500 \times 8 - 9500 \times 38.4 + 26{,}649 \times 19.2$

from which $s_{mn} = 33{,}290$ tension, $s_{mj} = 24{,}050$ compression, and $s_{cn} = 11{,}298$ compression.

Stress in bars no and oi. Section 6-6. Consider the part around the pin L_2 [Fig. 2(*f*)]

$$\Sigma x = 0 = s_{oi} + 24{,}050 - 33{,}290 \times 19.2/20.8;\ s_{oi} = 6{,}679 \text{ tension}$$
$$\Sigma y = 0 = s_{no} + 33{,}290 \times 8/20.8 - 13{,}000;\ s_{no} = 195 \text{ tension}$$

Or by taking section 7-7 and considering the part to the left Fig. 2(*g*)

ΣM about $L_0 = 0 = -s_{no} \times 38.4 + 2500 \times 8 - 26{,}649 \times 19.2 + 13{,}000 \times 38.4$
ΣM about $U_2 = 0 = -s_{oi} \times 18 - 1250 \times 16 - 2500 \times 8 - 9500 \times 38.4 + 26{,}649 \times 19.2$

from which $s_{no} = 196$ tension and $s_{oi} = 6{,}679$ tension.

Stresses in bars dp and op. Section 8-8. Consider the part around the pin U_2 [Fig. 2(*h*)]

$$\Sigma x = 0 = s_{dp} \times 19.2/20.8 + s_{op} \times 19.2/24.99 + 2500 + 11{,}298 \times 19.2/20.8$$
$$\Sigma y = 0 = s_{dp} \times 8/20.8 - s_{op} \times 16.0/24.99 - 6105 + 11{,}298 \times 8/20.8$$

from which $s_{dp} = 7734$ compression and $s_{op} = 7535$ compression, or by taking section 9-9 and considering the section to left [Fig. 2(*i*)]

ΣM about $L_3 = 0 = s_{dp} \times 19.2/20.8 \times 24 + 2500(8 + 16) - 6000(19.2 + 38.4) - 9500 \times 57.6 - 13{,}000 \times 19.2 + 32{,}649 \times 38.4$
ΣM about $L_0 = 0 = s_{op} \times 16/24.99 \times 57.6 + 2500(8 + 16) - 26{,}649 \times 19.2 + 19{,}000 \times 38.4$
ΣM about $U_2 = 0 = -s_{oi} \cdot 16 - 1250 \times 16 - 2500 \times 8 - 9500 \times 38.2 + 26{,}649 \times 19.2$

from which $s_{dp} = 7{,}733$ compression, $s_{op} = 7{,}538$ compression, and $s_{oi} = 6{,}679$ tension.

Stresses in bars pq and qh. Section 10-10. Consider the part around the pin L_3 [Fig. 2(*j*)]

$$\Sigma x = 0 = s_{qh} + 7538 \times 19.22/24.99 - 6679;\ \text{from which } s_{qh} = 881 \text{ tension}$$
$$\Sigma y = 0 = s_{pq} - 13{,}000 - 7538 \times 18/24.99;\ \text{from which } s_{pq} = 17{,}826 \text{ tension}$$

Or taking section 11-11 and considering the portion to the right because it has the fewer forces acting on it [Fig. 21(*k*)].

ΣM about $L_6 = 0 = s_{pq} \times 57.6 + 6250 \times 24 + 3000 \times 57.6 - (27{,}851 - 10{,}000)75.6$
ΣM about $L_3 = 0 = -s_{dp} \times 19.2/20.8 \times 24 + 6250 \times 24 - 17{,}851 \times 18$
ΣM about $U_3 = 0 = s_{qh} \times 24 + 12{,}500 \times 24 - 17{,}851 \times 18$

from which $s_{pq} = 17{,}825$ tension, $s_{dp} = 7{,}733$ compression, and $s_{qh} = 888$ tension.

Stresses in er and rq. Section 12-12. Consider one part around U_3 [Fig. 2(*l*)]

$$\Sigma x = 0 = s_{er} + s_{rq} \times 18/30 + 7733 \times 19.2/20.8 + 1250$$
$$\Sigma y = 0 = -s_{rq} \times 24/30 + 7733 \times 8/20.8 - 20{,}825$$

line of impact. If the centers of gravity of the bodies lie on this line, the impact is called **central impact**; in any other case, **eccentric impact**. If the directions of the motions of the two bodies coincide with the line of impact, the impact is said to be **direct**; in any other case, **oblique**.

Direct Central Impact. When two masses m_1 and m_2 having velocities u_1 and u_2 and moving in the same line meet each other, the relation between the velocities u_1 and u_2 before and v_1 and v_2 after collision is expressed by the equation: $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$. If $v' =$ velocity of m_1, $v'' =$ velocity of m_2 at any time during impact, and $v =$ common velocity at instant of greatest compression, $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 = m_1v' + m_2v''$, and $v = (m_1u_1 + m_2u_2)/(m_1 + m_2)$. **The sum of the momentums of the two masses before, after, and during impact is the same.**

A second relation between the velocities is $e = (v_2 - v_1)/(u_1 - u_2)$, e being called the **coefficient of impact**, or the **coefficient of restitution.** Its value depends on the elastic or plastic properties of the bodies which come in collision, being 0 for inelastic bodies (nearly 0 for soft substances such as putty or lead) and 1 for perfectly elastic bodies.

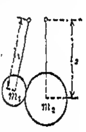


FIG. 81.

To determine the coefficient of restitution between two materials, let m_2 be so large in comparison to m_1 that it may be considered as infinite, *e.g.*, a foundation weighing many tons and a ball weighing a few ounces. Let m_2 be stationary, making u_2 and $v_2 = 0$; the formula $e = (v_2 - v_1)/(u_1 - u_2)$ will then reduce to $e = -v_1/u_1$. Drop the ball from the height H upon m_2 and note the rebound h. Then $u_1 = \sqrt{2gH}$ downward and $v_1 = \sqrt{2gh}$ upward, and $e = -v_1/u_1 = \sqrt{h/H}$. For central impact when the bodies are swinging (see Fig. 81), $v_1 + eu_1 = v_2 + eu_2$, and also if I_1 and $I_2 =$ moments of inertia of m_1 and m_2, $(I_1v_1/l_1^2) + (I_2v_2/l_2^2) = (I_1u_1/l_1^2) + (I_2u_2/l_2^2)$.

Impact of Perfectly Inelastic Bodies. The two masses move after the impact with a common velocity $u = u_1 = u_2 = (m_1v_1 + m_2v_2)/(m_1 + m_2)$, and **the loss of kinetic energy due to impact** $= E = \frac{1}{2}m_1m_2(v_1 - v_2)^2/(m_1 + m_2)$.

If $v_2 = 0$, $u = m_1v_1/(m_1 + m_2)$ and the loss of energy $= \frac{1}{2}m_1m_2v_1^2/(m_1 + m_2)$

Perfectly Elastic Impact. In this case, the loss of energy $E = 0$. The velocities after impact become: $u_1 = v_1 - [2m_2(v_1 - v_2)/(m_1 + m_2)]$, and $u_2 = v_2 + [2m_1(v_1 - v_2)/(m_1 + m_2)]$. For $v_2 = 0$, $u_1 = (m_1 - m_2)v_1/(m_1 + m_2)$, and $u_2 = 2m_1v_1/(m_1 + m_2)$; there is no loss of energy and the body m_1 retraces its path.

Imperfectly Elastic Impact. If the coefficient of impact is e, $u_1 = v_1 - [m_2(1 + e)(v_1 - v_2)/(m_1 + m_2)]$; $u_2 = v_2 + [m_1(1 + e)(v_1 - v_2)/(m_1 + m_2)]$. Loss of energy $= m_1m_2(1 - e^2)(v_1 - v_2)^2/2(m_1 + m_2)$.

When a jet of water strikes a flat plate perpendicularly to its surface, the force exerted by the water on the plate is wv/g, where w is the weight of water striking the plate in a unit of time and v is the velocity. When the jet is inclined to the surface by an angle A, the pressure is (wv/g) cos A.

Fields of Force—Attraction

The space within which the action of a physical force comes into play on bodies lying within its boundaries is called **the field of the force.**

HB is of the same magnitude as for the lower joint but acts in the opposite direction. The stress in the member *kh* is *KH* and is therefore compression.

Problem. The truss shown in Fig. 7 is loaded with a dead load of 21,760 lb. uniformly distributed over the upper chord, together with a wind load of 13,600 lb on the right side. Both ends of the truss are fixed and the horizontal components of the supporting forces are assumed to be equal. The supporting forces may be determined graphically by first assuming a roller under one end of the truss, or by the funicular polygon construction (see p. 203). They are often calculated and the polygon of external forces plotted from the results of the calculation. This is a desirable method, as it offers a check on this most important part of the graphical work.

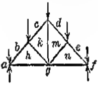


Fig. 5.

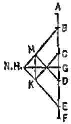


Fig. 6.

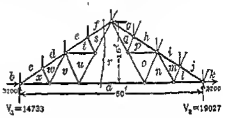


Fig. 7.

The horizontal component of the wind load is $(16/34)13{,}600 = 6400$ lb. The *H* component of each supporting force is assumed to be $6400/2 = 3200$ lb. The vertical component of the wind load is $(30/34)13{,}600 = 12{,}000$ lb. Taking moments about the right end of the truss, $(21{,}760 \times 30) + (13{,}600 \times 17) = 60V_1$. $\therefore V_1 = 14{,}733$; $V_2 = 21{,}760 + 12{,}000 - 14{,}733 = 19{,}027$.

The polygon of external forces can now be constructed, as in Fig. 8. The dotted part of the diagram is the combination of the dead and wind loads, assuming that they

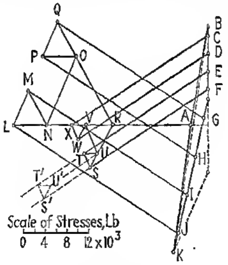


Fig. 8.

are each concentrated. The dotted line *BK* is the resultant of these loads. The supporting forces *KA* and *AB* are determined by plotting to scale their horizontal and vertical components as calculated. The polygon of external forces for the truss is *BCDEFGHIJKAB*, and must check with the polygon shown dotted. The forces *GH*, *HI*, *IJ*, and *JK* are the resultants of the forces acting at the joints on the right side of the truss.

Which one of the two guides the force P will act against depends on the direction of spin, and is best determined by the following rule (E. V. Huntington, *Eng. News*, July 21, 1910): If the applied force Q is thought of as due to the pressure of a flat board against the side of the revolving axle (the board being perpendicular to Q as shown in dotted line in Fig. 1), then the axle will strive to move in the direction in which it would naturally roll if the board were rough. Thus, in Fig. 1, the axle end A will push against the *upper* guide, and would actually move in the direction of the P arrow if the guide were not present.

The magnitude of the gyroscopic force P at any instant is proportional to the spinning velocity, and also to the velocity of A along the slot, being given by

$$Pb = (w/g)k^2(\pi N/30)(v/b),$$

where Pb = the moment of P about the center, v = the linear velocity of the point A along the slot at the instant in question, N = rpm in the spinning velocity of the wheel, and g = 32.17 ft per sec per sec.

It should be noted that the gyroscopic reaction P against the fixed guide does not depend on the magnitude of the accelerating force Q, except in so far as Q is necessary to create the velocity v, in case A starts from rest; when once the velocity v has been established, Q can be made zero, and both v and P will (neglecting friction) continue constant. On the other hand, if the guide is movable, and exerts a pressure $P' = P$ against the axle at A, this force will itself maintain the velocity v (called the precessional velocity) of the point A along the guide. In brief, *the end of the axle of a rotating gyroscope always tends to move at right angles to any force impressed upon it.*

For example, suppose an airplane is driven by a right-handed propeller (turning like a right-handed screw when moving forward); if a gust of wind or other force turns the machine to the *left*, the gyroscopic action of the propeller will make the forward end of the shaft strive to *rise*; if the wing surface is large, this motion will be practically prevented by the resistance of the air, and the gyroscopic forces become effective merely as internal stresses, whose maximum value can be computed by the formula above. Similarly, if the airplane is dipped *downward*, the gyroscopic action will make the forward end of the shaft strive to turn to the *left*.

In the Brennan monorail car, the slot is parallel to the rail, so that a force Q, applied automatically at the axle end when the car starts to tip, will be converted gyroscopically into a strong righting moment Pb, which forces the car back into a position of lateral equilibrium. Numerical case: If w = 3200 lb, k = 1.4 ft, b = 2 ft, and spinning velocity = 3000 rpm, then a force Q of 50 lb acting for ½ sec will produce at the end of that time a precessional velocity of 1 ft per sec along the slot, with a resulting lateral force against the guide of nearly 16,000 lb. The displacement of the point A along the slot will be about 6 deg. An essential feature of the apparatus is a device for bringing the axle back into the ready position after each period of activity.

Other applications are the Whitehead torpedo, the Schlick and the Sperry ship-steadying devices, the gyrocompass and automatic steering mechanism for ships, the artificial horizon and the directional gyro for aircraft, and the "gyropilot," which automatically keeps an airplane on its course. These devices are made by the Sperry Gyroscope Co. For the mathematical theory of the gyroscope, see H. Crabtree, "Spinning Tops and Gyroscopic Motion," Longmans. See also Deimel, "Mechanics of the Gyroscope," Macmillan.

FRICTION

REVISED BY

GEORGE B. KARELITZ

(Previously revised by Guido H. Marx)

REFERENCES: Archbutt and Deeley, "Lubrication and Lubricants," Griffin. Stanton, "Friction," Longmans. Bevan, "Theory of Machines," Longmans. Heck, "Kinematics and Dynamics of Machinery," McGraw-Hill. Nash and Bowen, "The Principles and Practice of Lubrication," Chapman-Hall. "General Discussion on Lubrication and Lubricants," A.S.M.E.

Friction is the resistance that is encountered when two solid surfaces slide or tend to slide over each other. The surfaces may be either dry or lubricated. In the first case, when the surfaces are free from contaminating fluids, the resistance is called **dry friction.** The friction of brake shoes on the rim of a wheel, of a dry grinding stone on the worked metal, and of a pier on its foundation are examples of dry friction.

When the rubbing surfaces are separated from each other by a very thin film of lubricant, the friction is that of **boundary (or greasy) lubrication.** The lubrication depends in this case on the strong adhesion of the lubricant to the material of the rubbing surfaces; the layers of lubricant slip over each other instead of the dry surfaces. A journal turning at very low-speed under a heavy load, or the parallel guides in a machine tool, are representative instances of boundary lubrication.

When the lubrication is arranged so that the rubbing surfaces are separated by a fluid film, and the load on the surfaces is carried entirely by the hydrostatic pressure in the film, the friction is that of **complete** (or **viscous**) **lubrication.** In this case, the frictional losses are due solely to the internal fluid friction in the film. Oil ring bearings, bearings with forced feed of oil, and pivoted-shoe-type thrust bearings operating in an oil bath are instances of complete lubrication.

Incomplete lubrication takes place when the load on the rubbing surfaces is carried partly by a fluid viscous film and partly by areas of boundary lubrication. The friction is intermediate between that of fluid and boundary lubrication. Incomplete lubrication exists in bearings with drop-feed lubrication or on the guides of a crosshead.

Static and Sliding Coefficients of Friction

In the absence of friction, the resultant of the forces between the surfaces of two bodies pressing upon each other is normal to the surface of contact. With friction, the resultant deviates from the normal.

If one body is pressed against another by a force P, as in Fig. 1, the first body will not move provided the angle α_0 included between the line of action of the force and a normal to the surfaces in contact does not exceed a certain value which depends upon the nature of the surfaces. The resultant force R has the same magnitude and line of action as the force P. In Fig. 1, R is resolved into two components: a force N normal to the surfaces in contact and a force F_r parallel to the surfaces in contact. From the above statement it follows that

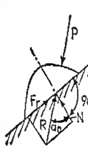


FIG. 1.

$$F_r \leq N \tan \alpha_0 \leq Nf_0$$

and the vertical component is 3,000 lb.

$$\Sigma x = 0 = s_{lk} + s_{bl} \times 19.2/20.8 + 1250$$
$$\Sigma y = 0 = s_{bl} \times 8/20.8 - 9500$$

From these equations, $s_{lk} = -24{,}050$ and $s_{bl} = 24{,}700$ tension.

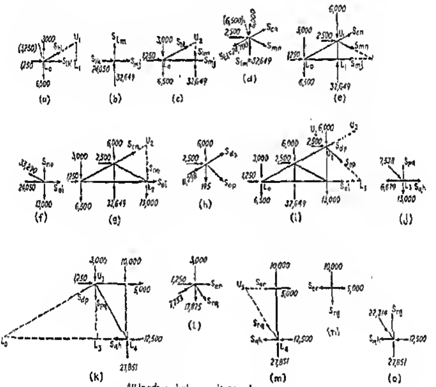
The minus sign indicates that the force in *lk* acts opposite to the assumed direction, *i.e.*, in compression.

Using the $\Sigma M = 0$ twice

ΣM about $U_1 = 0 = -s_{lk} \times 8 - 1250 \times 8 - 9500 \times 19.2$; from which $s_{lk} = -24{,}050$ or 24,050 compression

ΣM about $L_1 = 0 = s_{bl} \times 8/20.8 \times 19.2 - 9500 \times 19.2$; from which $s_{bl} = 24{,}700$ tension

The first method is frequently referred to as the "method of joints" and the second the "method of moments." Each method consists in applying the equations of statics



All loads and stresses in pounds

FIG. 2.

to a part of the truss cut from the whole in which the cut members are replaced by internal forces.

Stress in lm and mj. Sections 2-2. Isolate part below section [Fig. 2(*b*)]. Assume unknown forces as tension. s_{lk} was found to be compression, hence will act toward the pin.

$$\Sigma x = 0 = s_{mj} + 24{,}050; \text{ from which } s_{mj} = -24{,}050 \text{ or } 24{,}050 \text{ compression}$$
$$\Sigma y = 0 = s_{lm} + 32{,}649; \text{ from which } s_{lm} = -32{,}649 \text{ or } 32{,}649 \text{ compression}$$

If section 3-3 is taken, isolate part to left Fig. 2(*c*).

ΣM about $L_0 = 0 = s_{lm} \times 19.2 + 32{,}649 \times 19.2$; from which $s_{lm} = -32{,}649$ or 32,649 compression.

ΣM about $U_1 = 0 = -s_{mj} \times 8 - 1250 \times 8 - 9500 \times 19.2$; from which $s_{mj} = -24{,}050$ or 24,050 compression.

Stress in *bl* could also be found by ΣM about $L_1 = 0$.

Coefficients of Static Friction

Masonry and earth: dry masonry on brickwork, 0.6–0.7; timber on polished stone, 0.40; iron on stone, 0.3–0.7; masonry on dry clay, 0.51; masonry on moist clay, 0.33.
Earth on earth: dry sand, clay, mixed earth, 0.4–0.7; damp clay, 1.0; wet clay, 0.31; shingle and gravel, 0.8–1.1.

Natural cork: on cork, 0.59; on pine with grain, 0.49; on glass, 0.52; on dry steel, 0.45; on wet steel, 0.69; on hot steel, 0.64; on oiled steel, 0.45; water-soaked cork on steel, 0.56; oil-soaked cork on steel, 0.42.

Table 2. Coefficients of Sliding Friction for Clean Dry Surfaces

Brass on mild steel	0.44	According to Sachs: determined with two 12 in. disks rubbing against each other while pressed together with forces up to 100 lb
Brass on cast iron	0.30	
Babbitt on cast iron	0.29	
Babbitt on mild steel	0.33	
Cast iron on mild steel	0.23	
Hard leather on wood	0.36	
Babbitt on wood	0.56	
Hard wood on hard wood	0.54	
Fiber on fiber	0.51	
Glass on glass	0.40	Beare and Bowden: determined with small sphere rubbing against a plate, sliding velocity up to 15 fps
Mild steel on mild steel	0.57	
Hard steel on hard steel	0.43	
Nickel on mild steel	0.64	
Carbon on glass	0.18	
Garnet on mild steel	0.38	
Zinc on cast iron	0.21	Honda and Yamada: determined with a button of the various materials rubbing against cast-iron disk; loads up to 10 lb per sq in. and speeds near 1.5 fps
Magnesium on cast iron	0.25	
Copper on cast iron	0.29	
Tin on cast iron	0.32	
Lead on cast iron	0.43	
Antimony on cast iron	0.29	

Table 3. Coefficients of Friction for Various Materials

(Tomlinson: determined with clean dry surfaces at low velocities, and therefore representative for coefficients of static friction)

	Hard steel	Mild steel	Platinum	Nickel	Copper	Brass	Aluminum	Glass	Tin	Lead
Hard steel	0.39									
Mild steel	0.41	0.41								
Platinum	0.40	0.43	0.45							
Nickel	0.43	0.43	0.39	0.39						
Copper	0.55	0.53	0.50	0.56	0.60					
Brass	0.54	0.51	0.56	0.50	0.62	0.63				
Aluminum	0.65	0.61	0.80	0.75	0.70	0.71	0.94			
Glass	0.61	0.72	0.57	0.78	0.68	0.87	0.85	0.94		
Tin	0.79	0.77	0.86	0.90	0.88	0.75	0.91	0.94	1.11	
Lead	1.96	1.93	2.07	2.15	1.95	2.11	2.00	2.40	2.20	3.30

Compound Sliding. A body sliding across another may be deflected crosswise from its original direction by a small force. This explains the ease with which an automobile may skid on the road or with which a plug gage can be inserted into a hole if it is rotated while being pushed in.

Coefficients of Sliding Friction for Special Cases

Soapy Wood. E. P. Lesley gives for wood on wood, copiously lubricated with tallow, stearine, and soft soap (as used in launching practice), a starting coefficient of

from which $s_{er} = 5{,}000$ tension and $s_{rq} = 22{,}314$ compression, or by taking section 13-13 and considering part to the right [Fig. 2(*m*)]

$$\Sigma M \text{ about } U_3 = 0 = s_{qh} \times 24 - (27{,}851 - 10{,}000)18 + 12{,}500 \times 24$$
$$\Sigma M \text{ about } L_4 = 0 = -s_{er} \times 24 + 5000 \times 24$$
$$\Sigma Y = 0 = s_{rq} \times 24/30 + 27{,}851 - 10{,}000$$

from which $s_{qh} = 888$ tension, $s_{er} = 5{,}000$ tension, and $s_{rq} = 22{,}314$ compression.

Stress in rg. Section 14-14. Consider the part around U_4 [Fig. 2(*n*)]

$$\Sigma x = 0 = -s_{er} + 5000 \text{ from which } s_{er} = 5000 \text{ tension}$$
$$\Sigma y = 0 = -s_{rg} - 10{,}000 \text{ from which } s_{rg} = -10{,}000 \text{ or } 10{,}000 \text{ compression}$$

or taking section 15-15 and considering the part around L_4 [Fig. 2(*o*)]

$$\Sigma x = 0 = -s_{gh} + 22{,}314 \times 18/30 - 12{,}500; \text{ from which } s_{gh} = 888 \text{ tension}$$
$$\Sigma y = 0 = s_{rg} - 22{,}314 \times 24/30 + 27{,}851; \text{ from which } s_{rg} = 10{,}000 \text{ compression}$$

Graphical Solution of Trusses

When the external forces acting upon a truss are all in the same plane and may be assumed to act at the joints of the structure, it is often convenient to make use of graphical constructions to determine the stresses in the members. A series of stress polygons may be constructed for the forces acting at the different joints, using the methods shown on p. 206, but a combination diagram, using what is known as "Bow's Notation," greatly simplifies the work.

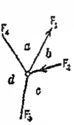


FIG. 3.

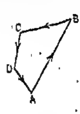


FIG. 4.

The forces F_1, F_2, F_3, and F_4 (Fig. 3) are all in the same plane and form a system in equilibrium. Their lines of action all pass through the same point (*o*). Letters are so placed as to bring each force between two letters, these letters being usually read in a right-handed or clockwise direction around the joint. F_1, F_2, F_3, and F_4 will then be designated *ab*, *bc*, *cd*, and *da*, respectively. A stress polygon (Fig. 4) is then drawn for the forces. Assume that the magnitudes, lines of action, and directions of F_1 and F_2 are known and that the lines of action only of F_3 and F_4 are known. The forces must be taken in order and the letters must be so placed that when the forces are read righthanded about the point *o* (Fig. 3) the sequence of the letters (in Fig. 4) will indicate the direction in which the forces act upon point *o*. The manner of using Bow's notation is illustrated by the following problem

Bow's Notation Applied to a Truss. The truss shown in Fig. 5 is loaded with a uniformly distributed dead load which may be considered to be concentrated at the joints as shown. First plot the polygon of external forces *ABCDEFGA* (Fig. 6). As the forces all act vertically, the sides of the polygon fall in the straight line *AF*. The supporting forces *GA* and *FG* are equal in this case, each being one-half the total load, so that no special construction for their determination is necessary. Start at any joint in the truss where there are not more than two unknown forces, *e.g.*, at the left end of the truss. The stress polygon for this joint is *ABHGA*. Reading the letters in a right-hand direction about the joint, the stress in the upper chord member is *BH*. This sequence of letters in the stress polygon indicates that this member acts downward and to the left on the joint, and therefore is in compression. The stress in the lower chord member is *HG*, and this sequence of letters indicates that this member acts to the right on the joint and therefore is in tension.

At the joint in the middle of the upper chord there are now two unknown stresses. Draw the stress polygon *HBCKH* for the second joint. The stress

Inflation pressure, lb per sq in.	Dry pavement		Wet pavement	
	Static, f_0	Sliding, f	Static, f_0	Sliding f
40	0.90	0.85	0.74	0.69
50	0.88	0.84	0.64	0.58
60	0.80	0.76	0.63	0.56

Tests of the Goodrich Company, on wet brick pavement, with balloon tires of different treads gave the following values of f.

	Coefficients of friction			
	Static (before slipping)		Sliding (after slipping)	
Speed, mph	5	30	5	30
Smooth tire	0.49	0.28	0.43	0.26
Circumferential grooves	0.58	0.42	0.52	0.36
Angular grooves at 60 deg	0.75	0.55	0.70	0.39
Angular grooves at 45 deg	0.77	0.55	0.68	0.44

Sleds. For **unshod wooden runners** on smooth wood or stone surfaces, $f = 0.07$ (0.15) when tallow (dry soap) is used as a lubricant (= 0.38 when not lubricated); on snow and ice, $f = 0.035$. For **runners with metal shoes** on snow and ice, $f = 0.02$. Rennie found for steel on ice, $f = 0.014$.

Rolling Friction

Rolling is substituted frequently for sliding friction, as in the case of wheels under vehicles, balls or rollers in bearings, rollers under skids when moving loads; frictional resistance to the rolling motion is substantially smaller than to sliding motion. The coefficient of rolling friction $f_r = P/L$ where L is the load and P is the frictional resistance.

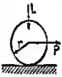


Fig. 2.

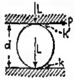


Fig. 3.

The frictional resistance P to the rolling of a cylinder under a load L applied at the center of the roller (Fig. 2) is inversely proportional to the radius r of the roller; $P = \frac{k}{r}L$. If r is in inches, values of k are as follows: hard wood on hard wood, 0.02; iron on iron, steel on steel, 0.002; hard polished steel on hard polished steel, 0.0002 to 0.0004.

Data on rolling friction are scarce. Noonan and Strange give for clean steel rollers on steel plates, for loads varying from light to those causing a permanent set, of the material, the following values of k: surfaces well-finished and clean; 0.0002 to 0.0005; surfaces well-oiled, 0.0005; surfaces covered with silt, 0.0015; surfaces rusty, 0.003.

When supporting forces are to be determined the loads *may* be concentrated at the lines of action of their resultants, but when internal stresses are to be determined the loads *must* be distributed at the various joints. Start with some joint of the truss where there are only two unknown forces and draw the stress diagram, Fig. 8. The magnitudes of the stresses in the different members are determined directly from the lengths of the corresponding lines and the scale used in the construction of the diagram. The nature of the stress (tension or compression) is determined by the use of Bow's notation.

A difficulty often arises that is illustrated by the truss of Fig. 7. It is impossible by the usual graphical procedure to complete the stress diagram, as it will be found that, after obtaining the stresses in the members meeting at the left end and at the next joint on both upper and lower chords, no joint with less than three unknown forces is available. To overcome this difficulty, some unknown stress may be calculated and placed on the diagram before proceeding with the graphical solution.

The stress which should be calculated in this case is that in the middle member of the lower chord. Taking moments about a point at the middle of the upper chord, $16RA = (14{,}733 \times 30) - (3200 \times 16) - [2720(22\frac{1}{2} + 15 + 7\frac{1}{2})] - (1360 \times 30) = 227{,}600 \therefore RA = 14{,}225$ (tension). Place this stress in the diagram and proceed.

The solution may be entirely graphical by noting that T must be on a line through E parallel to *et* and S must be on a line through f parallel to *fs* and also that TS must be parallel to *ts*. Hence the geometrical relationship may be indicated by any points T' and S' on the line through E and F as long as $T'S'$ is parallel to *ts*. Furthermore, TU must be parallel to *tu* and SR parallel to *sr*. If T' and S' are arbitrarily selected as shown in dotted lines, then U' is fixed. However U must lie on a line through V parallel to UV. All conditions can be fulfilled by moving the triangle $T'S'U'$ so that the sides move parallel until U' comes on the line through V parallel to *uv*. This point will then be U, and T and S will be determined.

Taper Keys. In Fig. 5 if the key be moved in the direction of the force P, the force H must be overcome. The supporting reactions K_1, K_2, and K_3 together with the required force P may be obtained by drawing the force polygon (Fig. 6). The friction angles of these faces are a_1, a_2, and a_3, respectively. In Fig. 6, draw AB parallel to H in Fig. 5, and lay it off to scale to represent H. From the point A, draw AC parallel to K_1, that is, making

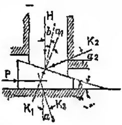


Fig. 5.

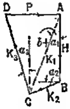


Fig. 6.

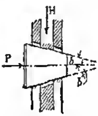


Fig. 7.

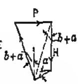


Fig. 8.

the angle $b + a_1$ with AB; from the other extremity of AB, draw BC parallel to K_2 in Fig. 5. AC and CB then give the magnitudes of K_1 and K_2, respectively. Now through C draw CD parallel to K_3 to its intersection with AD which has been drawn through A parallel to P. The magnitudes of K_3 and P are then given by the lengths of CD and DA.

By calculation, $K_1/H = \cos a_2/\cos (b + a_1 + a_2)$

$$P/K_1 = \sin (b + a_1 + a_3)/\cos a_3$$

$$P/H = \cos a_2 \sin (b + a_1 + a_3)/\cos a_3 \cos (b + a_1 + a_2)$$

If $a_1 = a_2 = a_3 = a$, then $P = H \tan (b + 2a)$, and efficiency $e = \tan b/\tan (b + 2a)$. Force required to loosen the key $= P_1 = H \tan (2a - b)$. In order for the key not to slide out when force P is removed, it is necessary that $b < (a_1 + a_3)$, or $b < 2a$.

The forces acting upon the taper key of Fig. 7 may be found in a similar way (see Fig. 8).

$$P = 2H \cos a \sin (b + a)/\cos (b + 2a)$$
$$P = 2H \tan (b + a)/[1 - \tan a \tan (b + a)]$$
$$= 2H \tan (b + a) \text{ approx.}$$

The force to loosen the key is $P_1 = 2H \tan (a - b)$ approx, and the efficiency $e = \tan b/\tan (b + a)$. The key will be self-locking when $b < a$, or, more generally, when $2b < (a_1 + a_3)$.

Screws

Screws with Square Threads (Fig. 9). Let r = mean radius of the thread $= \frac{1}{2}$ (radius at root + outside radius), and l = pitch (or lead of a single-threaded screw), both in inches; b = angle of inclination of thread to a plane at right angles to the axis of screw ($\tan b = l/2\pi r$); and f = coefficient of sliding friction $= \tan a$. Then, for a screw in uniform motion (friction of the root and outside surfaces being neglected) there is required a force P acting at right angles to the axis at the distance r. $P = L \tan (b \pm a) = L(l \pm 2\pi rf)/(2\pi r \mp fl)$, where the upper signs are for motion in a direction opposed to that of L and the lower for motion in the same direction as that of L. When $b \leq a$, the screw will not "overhaul" (or move under the action of the load L).

Fig. 9.

where $f_0 = \tan \alpha_0$ is called the **coefficient of friction of rest** (or of static friction) and α_0 is the **angle of friction of rest** (or angle of repose).

If the normal force N between the surfaces is kept constant, and the tangential force F_r is gradually increased, there will be no motion while $F_r < Nf_0$. A state of *impending motion* is reached when F_r nears the value of Nf_0. If one surface slides over the other, being pressed together by a normal force N, a frictional force F resisting the motion must be overcome. This force is usually smaller than F_r. The force F is commonly expressed as $F = fN$, where f is the **coefficient of sliding friction, or kinetic friction.** In the range of practical velocities of sliding, the coefficients of sliding friction are smaller than the coefficients of static friction. With small velocities of sliding and very clean surfaces, the two coefficients do not differ appreciably.

Under moderate pressures, the frictional force is proportional to the normal load on the rubbing surfaces. It is independent of the pressure per unit area of the surfaces. The coefficient of friction is approximately independent of the rubbing speed, when the speed is sufficiently low so as not to affect the temperature of the surface; at higher velocities, the coefficient of friction decreases as the velocity increases.

The coefficients of friction for dry surfaces (dry friction) depend on the materials sliding over each other and on the finished condition of the surfaces. With greasy (boundary) lubrication, the coefficients depend both on the materials of the surfaces and on the lubricants employed.

Data on the actual values of the coefficients of dry friction are not abundant, and those available depend on the conditions of the tests. Typical values of f and f_0 are given in Tables 1, 2, and 3. It has been generally observed that the sliding friction between harder materials is smaller than that between softer surfaces.

Table 1. Coefficients of Static and Sliding Friction
(Collected from various sources, mostly Morin)

Rubbing surfaces	Static: Dry	Static: Greasy	Sliding: Dry	Sliding: Greasy
Metals:				
Bronze on bronze	....	....	0.20	
Bronze on cast iron	....	0.16	0.21	0.15
Bronze on iron	0.19	....	0.18	0.16
Wrought iron on wrought iron	....	....	0.44	
Steel on steel	0.15			
Other materials:				
Metal on oak	0.62	....	0.50–0.60	0.19
Metal on wet oak	0.65			
Oak on oak: parallel	0.62	....	0.48	
cross-fiber	0.43	....	0.19	0.25 (soap)
Leather on oak, dry	0.43	....	0.33	
wet	0.79	....	0.29	
Hemp on oak, dry	0.50			
wet	0.33			
Leather on metal, dry	....	....	0.56	0.23
water-soaked	0.36			

Kingsbury, from tests on U. S. standard bolts, finds efficiencies for tightening up nuts from 0.06 to 0.12, depending upon the roughness of the contact surfaces and the character of the lubrication.

Toothed and Worm Gearing

The efficiency of **spur and bevel gearing** depends on the material and the workmanship of the gears, and on the lubricant employed. For high-speed gears of good quality the efficiency of the gear transmission is 99 percent, and with slow-speed gears of average workmanship the efficiency of 96 percent is common. On the average, efficiencies of 97 to 98 percent can be considered normal.

In helical gears, where considerable transverse sliding of the meshing teeth on each other takes place, the friction is much greater. If b and c are, respectively, the spiral angles of the teeth of the driving and driven helical gears (*i.e.*, the complements of their angles of inclination), $b + c$ is the shaft angle of the two gears, and $f = \tan a$ is the coefficient of sliding friction of the teeth, the efficiency of the gear transmission is $e = [\cos b \cos (c + a)]/[\cos c \cos (b - a)]$.

In the case of **worm gearing** when the shafts are normal to each other ($b + c = 90$), the efficiency is

$e = \tan c/\tan (c + a) = (1 - pf/2\pi r)/(1 + 2\pi rf/p)$, where c is the spiral angle of the worm wheel, or the lead angle of the worm; p the lead, or pitch of the worm thread; and r the mean radius of the worm. Typical values of f are as follows:

Rubbing speed of worm, fpm.	100	200	300	500	800	1200
Phosphor-bronze wheel, polished-steel worm	0.054	0.045	0.039	0.030	0.024	0.020
Single-threaded cast-iron worm and gear. . .	0.060	0.051	0.047	0.034	0.025	

Journals and Bearings

Friction of Journal Bearings. If P = total load on journal in lb, l = journal length in in., and $2r$ = journal diam in in., then $p = P/2rl$ = mean normal pressure, lb per sq in. of the projected area of the journal. Also, if f_1 be the coefficient of **journal friction**, the **moment of journal friction** for a cylindrical journal is $M = f_1Pr$ in.-lb. The work **expended in friction** at a speed of n rpm is $W_f = 2\pi Mn = 6.283f_1Prn$ in.-lb per min. For the **conical bearing** (Fig. 10) the mean radius $r_m = (r + R)/2$ is to be used.

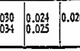


FIG. 10.

Values of Coefficient of Friction. For very low velocities of rotation, (*e.g.*, below 10 rpm), high loads, and with good lubrication, the coefficient of friction approaches the value of greasy friction, 0.07 to 0.15. This is also the "pull out" coefficient of friction *on starting* the journal. With higher velocities, a fluid film is established between the journal and bearing, and the values of the coefficient of friction depend on the speed of rotation, the pressure on the bearing, and the viscosity of the oil. For journals running in complete bearing bushings, with a small clearance, *i.e.*, with the diameter of the bushing slightly larger than the diameter of the journal, the experimental data of McKee give an approximate value of the coefficient of friction (Fig. 11).

friction equal to 0.036, diminishing to an average value of 0.019 for the first 50 ft of motion of the ship. Rennie gives 0.0385 for wood on wood, lubricated with soft soap, under a load of 56 lb per sq in.

Asbestos-fabric Brake Material. The coefficient of sliding friction f of asbestos fabric against a cast-iron drum is 0.25 to 0.30 when at normal temperature. It drops rapidly with a rise of the brake temperature and is 0.16 to 0.20 at a temperature of 200 F. With a further increase in temperature, the value of f may show an increase caused by disruption of the brake surface.

Steel Tires on Steel Rails (Galton).

Speed mph	Start	6.8	13.5	27.3	40.9	54.4	60
Values of f	0.242	0.088	0.072	0.07	0.057	0.038	0.027

Railway Brake Shoes on Steel Tires. Galton and Westinghouse give, for cast-iron brakes, values for f, which decrease rapidly with the speed of the rim; the coefficient f decreases also with time, as the temperature of the shoe increases.

Speed, mph	10	20	30	40	50	60
f, when brakes were applied	0.32	0.21	0.18	0.13	0.10	0.06
f, after 5 sec	0.21	0.17	0.11	0.10	0.07	0.05
f, after 12 sec	....	0.13	0.10	0.08	0.06	0.05

Schmidt and Schrader confirm the marked decrease in the coefficient of friction with the increase of rim speed. They also show an irregular slight decrease in the value of f with higher shoe pressure on the wheel, but they did not find the drop in friction after a prolonged application of the brakes. Their observations give as follows:

Speed, mph	20	30	40	50	60
Coefficient of friction	0.25	0.23	0.19	0.17	0.16

Wood Brake Blocks. According to L. Klein, f is practically constant for velocities from 200 to 4,000 fpm and for pressures from 7 to 142 lb per sq in. The following values of f are for wood on lengthwise fiber brake blocks carefully machined:

	Beech	Oak	Poplar	Elm	Willow
Cast iron	0.29–0.37	0.30–0.34	0.35–0.40	0.36–0.37	0.46–0.47
Wrought iron	0.54	0.61–0.40	0.65–0.60	0.60–0.49	0.63–0.60

The higher values apply for cast iron when the brake wheel is cleaned with gasoline, the lower when it is only wiped clean; the reverse holds for wrought iron.

Hydraulic Hoists. According to H. Lang, for bronze or lignum vitae sliding surfaces on bronze, f is constant for slow reversing motion and for pressures of 30 to 1,500 lb per sq in. For surfaces continuously lubricated, $f = 0.08$; for surfaces wet with water through numerous slots, 0.10; for surfaces running dry and creaking, up to 0.30.

Stuffing boxes packed with hemp, cotton, or leather: f is constant for hydraulic pressures between 15 and 750 lb per sq in. For hemp or cotton packing, loose or woven, soaked in hot tallow, smooth rod, box not set up too tightly so that the packing is still elastic, usual dimensions—even after months of use, $f = 0.06$ to 0.11 (see also *Am. Mach.*, Feb. 3, 1898). When the packing is rendered difficult by unfavorable conditions, $f =$ up to 0.25. For well-made leather packing rings of soft leather $f = 0.03$ to 0.07; if of hard, stiff tanned leather, 0.10 to 0.13; under unfavorable conditions such as rough piston and dirty water, up to 0.20. The coefficient of friction is found to change inversely to the diameter of the cylinder; the depth of the leather does not influence the friction.

Grindstones. The coefficient of friction between coarse-grained sandstone and cast iron is $f = 0.21$ to 0.24; for steel, 0.29; for wrought iron, 0.41 to 0.46, according as the stone is freshly trued or dull; for fine-grained sandstone (wet grinding) $f = 0.72$ for cast iron, 0.94 for steel, and 1.0 for wrought iron.

Honda and Yamada give $f = 0.28$ to 0.50 for carbon steel on emery, depending on the roughness of the wheel.

Rubber Tires on Pavement. Arnoux gives $f = 0.67$ for dry macadam, 0.71 for dry asphalt, 0.81 for wet asphalt, and 0.17 to 0.06 for soft, slippery roads. For a cord tire on a sand-filled brick surface in fair condition, Agg (*Bull. 88, Iowa State College Engineering Experiment Station*, 1928) gives the following values of f depending on the inflation of the tire.

its action, and the equivalent lever is shorter with friction than without friction; the friction throws the line of action toward the center of rotation of link c.

Tension Elements

Frictional Resistance. In Fig. 15, let T_1 and T_2 be the tensions with which a rope, belt, chain, or brake band is strained over a drum, pulley, or sheave, and let the rope or belt be on the point of slipping from T_2 toward T_1 by reason of the difference of tension $T_1 - T_2$. Then $T_1 - T_2$ = circumferential force P transferred by friction must be equal to the frictional resistance W of the belt, rope, or band on the drum or pulley. Also, let a = angle subtending the arc of contact between the drum and tension element, measured in radians. Then, disregarding centrifugal forces,

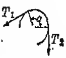


FIG. 15.

$$T_1 = T_2 e^{fa} \text{ and } P = (e^{fa} - 1)T_1/e^{fa} = (e^{fa} - 1)T_2 = W$$

where e = base of the Napierian system of logarithms = 2.718+.

Table 4. Values of e^{fa}

$\frac{a°}{360°}$	f 0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.1	1.06	1.1	1.13	1.17	1.21	1.25	1.29	1.33	1.37
0.2	1.13	1.21	1.29	1.37	1.46	1.55	1.65	1.76	1.87
0.3	1.21	1.32	1.45	1.60	1.76	1.93	2.13	2.34	2.57
0.4	1.29	1.46	1.65	1.87	2.12	2.41	2.73	3.10	3.51
0.425	1.31	1.49	1.70	1.95	2.23	2.55	2.91	3.35	3.80
0.45	1.33	1.53	1.76	2.03	2.34	2.69	3.10	3.57	4.11
0.475	1.35	1.56	1.82	2.11	2.45	2.84	3.30	3.83	4.45
0.5	1.37	1.60	1.87	2.19	2.57	3.00	3.51	4.11	4.81
0.525	1.39	1.64	1.93	2.28	2.69	3.17	3.74	4.41	5.20
0.55	1.41	1.68	2.00	2.37	2.82	3.35	3.98	4.74	5.63
0.6	1.46	1.76	2.13	2.57	3.10	3.74	4.52	5.45	6.59
0.7	1.55	1.93	2.41	3.00	3.74	4.66	5.81	7.24	9.02
0.8	1.65	2.13	2.73	3.51	4.52	5.81	7.47	9.60	12.35
0.9	1.76	2.34	3.10	4.11	5.45	7.24	9.60	12.74	16.90
1.0	1.87	2.57	3.51	4.81	6.59	9.02	12.35	16.90	23.14
1.5	2.57	4.11	6.59	10.55	16.90	27.08	43.38	69.49	111.32
2.0	3.51	6.59	12.35	23.14	43.38	81.31	152.40	285.68	535.49
2.5	4.81	10.55	23.14	50.75	111.32	244 15	535.49	1,174.5	2,575.9
3.0	6.59	16.90	43.38	111.32	285.68	733.14	1,881.5	4,828.5	12,391
3.5	9.02	27.08	81.31	244.15	733.14	2,199.90	6,610.7	19,851	59,608
4.0	12.35	43.38	152.40	535.49	1,881.5	6,610.7	23,227	81,610	286,744

$e^{\pi} = 23.1407$. $\log e^{\pi} = 1.3643764$.

f is the coefficient of friction of repose (f_0) when there is no slip of the belt or band on the drum and the coefficient of sliding friction (f) when slip takes place. In addition to the values given below, see p. 233.

Average values of f_0 for belts, ropes, and brake bands are as follows: For leather belt on slightly greasy wood pulley, 0.47. For leather belt on cast-iron pulley, very greasy, 0.12; slightly greasy, 0.28; moist, 0.38. For hemp rope on cast-iron drum, 0.25; on wooden drum, 0.40; on rough wood, 0.50; on polished wood, 0.33. For iron brake bands on cast-iron pulleys, 0.18.

Effect of Friction and Stiffness in Tension Elements. Let d = the rope diam, the diam of the chain stock, or the diam of the pin in link chains,

If a load L is moved on a roller (Fig. 3) and if k and k' are the respective coefficients of friction for the lower and upper surfaces, the frictional force $P = (k + k')L/d$.

Vehicles on Roads (wheels with iron tires). Values of $f_r = P/L$:

Smooth granite flags	0.006	Macadam, dust covered	0.028
Avg. street railway tracks	0.006–0.008	Poor stone pavement	0.033
Good asphalt	0.010	Macadam, mud covered, rutted	0.035
Very good stone pavement	0.015	Dirt road, very good	0.045
Macadam in best condition	0.010	Macadam of very poor quality	0.050
Good wood block pavement	0.018	Dirt roads, good to poor	0.08–0.16
Good stone pavement	0.020	Loose sand	0.15–0.30
Macadam in good condition	0.023		

For pneumatic rubber tires on smooth roads, f_r is 0.02 to 0.03.

FRICTION OF MACHINE ELEMENTS

Work of Friction—Efficiency. In a simple machine or assemblage of two elements, the work done by an applied force P acting through the distance s is measured by the product Ps. The useful work done is less, and is measured by the product Ll of the resistance L by the distance l through which it acts. The **efficiency** e of the machine is the ratio of the useful work performed to the total work received, or, $e = Ll/Ps$. The **work expended in friction** W_f is the difference between the total work received and the useful work, or $W_f = Ps - Ll$. The lost-work ratio $= V = W_f/Ll$, and $e = 1/(1 + V)$.

If a machine consist of a train of mechanisms having the respective efficiencies, $e_1, e_2, e_3 \ldots e_n$, the combined efficiency of the machine is equal to the product of these efficiencies.

Efficiencies of Machines and Machine Elements. The following values for machine elements are from "Elements of Machine Design," by Kimball and Barr. Those for machines are from Goodman's "Mechanics Applied to Engineering." The quantities given are percentage efficiencies.

Common bearing (singly)	96–98	Belting	96–98
Long lines of shafting	95	Pin-connected chains (bicycle)	95–97
Roller bearings	98	High-grade transmission chains	97–99
Ball bearings	99	Weston pulley block (½ ton)	30–47
Spur gear, including bearings:		Epicycloidal pulley block	40–45
Cast teeth	93	1 ton, steam hoist or windlass	50–70
Cut teeth	96	Hydraulic windlass	60–80
Bevel gear, including bearings:		Hydraulic jack	80–90
Cast teeth	92	Cranes (steam)	60–70
Cut teeth	95	Overhead traveling cranes	30–50
Worm gear: varies with thread angle	..	Locomotives (drawbar hp/ihp)	65–75

Wedges

Sliding in V Guides. If a wedge-shaped slide having an angle $2b$ is pressed into a V guide by a force P (Fig. 4) the force normal to the wedge faces will be $N = P/\sin b$, and the frictional force opposing motion along the axis of the wedge is $F = fN = fP/\sin b = f'P$, where $f' = f/\sin b$ is improperly called the coefficient of friction. In these formulas, the fact that the elasticity of the materials permits an advance of the wedge into the guide under the load P, has been neglected. If it be taken into account, then $F = fP$ $(\sin b + f \cos b)$. The common efficiency for V guides is $e = 0.88$ to 0.90.

FIG. 4.

HYDRAULICS

BY

E. W. SCHODER

REFERENCES: Daugherty, "Hydraulics," McGraw-Hill. Schoder and Dawson, "Hydraulics," McGraw-Hill. Harris, "Hydraulics," Wiley. Vennard, "Elementary Fluid Mechanics," Wiley. O'Brien and Hickox, "Applied Fluid Mechanics," McGraw-Hill. Report of Fluid Meters Committee, A.S.M.E.

General Properties of Liquids

A **liquid** is a substance that has a **definite volume**, but whose particles move relatively to each other so readily that, when unconfined laterally, the action of gravity causes it to flow and seek the lowest possible level. Hence a liquid **conforms to the shape of the containing vessel** or reservoir and, when at rest, presents a level upper surface unless restrained by the rigidity of the walls of a completely filled container.

Liquids and gases at rest may be regarded as having perfect fluidity. With **flowing fluids**, the existence of internal friction arising from viscosity must be considered and allowance must be made for its effects. Viscous resistance is a function of the time rate of internal slipping or distortion. True fluids have no rigidity or **form elasticity** but possess **bulk elasticity**.

Viscosity μ is the resistance offered by a fluid to relative motion of its parts. The absolute cgs unit of viscosity, the *poise*, is the resistance (in dynes per square centimeter of its surface) offered by a layer of the fluid to the motion, parallel to that layer, of another layer of the fluid at a distance of 1 cm from it, with a relative velocity of 1 cm per sec. The dimensions of the poise are (gm mass)/(sec)(cm) or (dynes)(sec)/(cm²). A customary unit is the **centipoise** (= poise/100), which happens to be the viscosity of water at 68.4 F. To convert centipoises into other units, multiply the centipoises by the following constants:

0.000672 for viscosity in (lb mass)/(sec)(ft) units
2.42 for viscosity in (lb mass)/(hr)(ft) units
3.60 for viscosity in (kg mass)/(hr)(m) units
5.60×10^{-5} for viscosity in (lb mass)/(sec)(in.) units
1.45×10^{-7} for viscosity in (lb force)(sec)/(in.²) units

Viscometers. Viscosity is measured by observing the time required for a certain volume of the liquid to flow, under stated conditions as to head, through a short tube of small bore. The **Saybolt Universal viscometer** has a vertical tube 0.483 ± 0.004 in. long and 0.0695 ± 0.0006 in. diam. For heavy oils the **Saybolt-Furol viscometer** is used; it differs from the Saybolt in having a tube diameter of 0.1240 ± 0.0008 in. The time of efflux of 60 cc in seconds is the viscosity in seconds Saybolt. The time of efflux of the Furol is approximately $1/10$ that of the Universal.

Viscosities of lubricants are measured at 100 and 210 F, of fuel oils at 77 and 122 F.

Kinematic viscosity ν is viscosity divided by mass density, or $\nu = \mu/\rho$. The unit in cgs units is called the **stoke**; a customary unit is the **centistoke** (= stoke/100).

The value of the kinematic viscosity [in (cm²)/(sec) units] can be obtained from the indications in seconds t of various viscometers by the following equations:

Saybolt Universal, when $32 < t < 100$, $\nu = 0.00226t - 1.95/t$
when $t > 100$, $\nu = 0.00220t - 1.35/t$

The efficiency for motion opposed to direction in which L acts $= e = \tan b/\tan (b + a)$; for motion in the same direction in which L acts, $e = \tan (b - a)/\tan b$.

The value of e is a maximum when $b = 45$ deg $- \frac{1}{2}a$; for example, $e_{max} = 0.81$ for $b = 42$ deg and $f = 0.1$. Since e increases rapidly for values of b up to 20 deg, this angle is generally not exceeded; for $b = 20$ deg, and $f_1 = 0.10$, $e = 0.74$. In presses, where the mechanical advantage is required to be great, b is taken down to 3 deg, for which value $e = 0.34$.

Kingsbury found for square-threaded screws running in loose-fitting nuts, the following coefficients of friction: lard oil, 0.09 to 0.25; heavy mineral oil, 0.11 to 0.19; heavy oil with graphite, 0.03 to 0.15.

Ham and Ryan give for screws the following values of coefficients of friction, with medium machine oil: High-grade materials and workmanship, 0.10; average quality materials and workmanship, 0.12; poor workmanship, 0.15.

The use of castor oil as a lubricant lowered f from 0.10 to 0.066.

The coefficients of friction of a **plain collar thrust bearing** used with a power screw, were: soft steel on cast iron, 0.12; hardened steel on cast iron, 0.09; soft steel on bronze, 0.08; hardened steel on bronze, 0.6.

The coefficients of static friction (at starting) were 30 percent higher.

Screws with V Threads. Let c = half the angle between the faces of a thread. Then, using the same notation as for square-threaded screws, for a screw in motion (neglecting friction of root and outside surfaces),

$$P = L(l \pm 2\pi rf \sec d)/(2\pi r \mp lf \sec d)$$

d is the angle between a plane normal to the axis of the screw through the point of the resultant thread friction, and a plane which is tangent to the surface of the thread at the same point (see Groat, *Proc. Engs. Soc. West, Penn*, vol. 34). Sec $d = \sec c\sqrt{1 - (\sin b \sin c)^2}$. For small values of b this reduces practically to $\sec d = \sec c$, and, for all cases the approximation, $P = L(l \pm 2\pi rf \sec c)/(2\pi r \mp lf \sec c)$ is within the limits of probable error in estimating values to be used for f.

The **efficiencies** are: $e = \tan b(1 - f \tan b \sec d)/(\tan b + f \sec d)$ for motion opposed to L, and $e = (\tan b - f \sec d)/\tan b(1 + f \tan b \sec d)$ for motion with L. If we let $\tan d' = f \sec d$, these equations reduce, respectively, to $e = \tan b/\tan (b + d')$, and $e = \tan (b - d')/\tan b$. Negative values in the latter case merely mean that the thread will not overhaul. Subtract the values from unity for actual efficiency considering the external moment and not the load L as being the driver. The efficiency of a V thread is lower than that of a square thread of the same helix angle, since $d' > a$.

For a **V-threaded screw and nut**, let r_1 = outside radius of thread, r_2 = radius at root of thread, $r = (r_1 + r_2)/2$, $\tan d' = f \sec d$, r_0 = mean radius of nut seat $= 1.5r$ (approx) and f' = coefficient of friction between nut and seat.

To tighten up the nut the turning moment required is $M = Pr + Lr_0f = Lr[\tan (d' + b) + 1.5f']$. **To loosen,** $M = Lr[\tan (d' - b) + 1.5f']$.

The total tension in a bolt due to tightening up with a moment M is $T = 2\pi M/(l + fl \sec b \sec d \operatorname{cosec} b + f'3\pi r)$. $T \div$ area at root gives unit pure tensile stress induced, S_t. There is also a unit torsional stress: $S_s = 2(M - 1.5rf'T)/\pi r_2^3$. The equivalent combined stress is[1] $S = 0.35S_t + 0.65\sqrt{S_t^2 + 4S_s^2}$.

position and the surfaces of division between the liquids are level. (But see below for capillarity, cohesion, adhesion, and surface tension as affecting glass-column gages.) In any continuous body of liquid at rest or moving at a uniform rate in a straight line the pressures at all points in a horizontal plane are equal.

Compressibility and Elasticity of Liquids. A pressure of 1 lb per sq in. compresses liquids in volume as follows: Water, 1 part in about 300,000 (see p. 534); mercury, 1 part in about 4,700,000; ether, 1 part in about 120,000. For water in an iron pipe, this corresponds to a compression of 2 in. per mile length for a pressure of 10 lb per sq in. Because of this small compressibility, liquids are said to be practically incompressible. Liquids are perfectly elastic, *i.e.*, they quickly regain their former volume upon removal of the pressure. As a consequence of the small compressibility and decided elasticity of liquids, pressure waves, exactly analogous to sound waves in air, are transmitted through liquids with high velocities—for water, over 4 times as fast as sound in air.

Surface Tension. The surface between two immiscible fluids tends to assume the minimum area consistent with the volume of fluid enclosed and with the external forces acting on the fluids. Familiar illustrations are rain drops and soap bubbles, also bubbles of gas in a liquid. When one of the fluids is a gas, changes in its pressure and temperature usually cause little change in the interfacial tension, and consequently the latter is commonly treated as the surface tension of the liquid alone. Values of surface tension are given in Table 2.

Table 2. Surface Tension of Liquids in Contact with Air

Liquid	Temperature, deg F	Dynes per cm
Water	68	72.8
Mercury	59	487
Bisulphide of carbon	68	31.4
Chloroform	68	27.1
Ethyl alcohol (absolute)	77	22.0
Olive oil	68	34.7
Turpentine	70	28.5
Petroleum	68	25.9
Hydrochloric acid	68	66

Capillary Attraction. Cohesion, Adhesion, and Surface Tension. The exception to the otherwise general statement that the upper surface of a free body of liquid at rest is level, consists in the condition at the edges of the surface area, close to a bounding solid. If the liquid wets the solid (*e.g.*, water and clean glass), it is because there is a greater attraction between the liquid and the solid than between particles of the liquid, or adhesion is stronger than cohesion, and conditions are as shown by Fig. 1 (*a*). On the other hand, if the liquid does not wet the solid (*e.g.*, mercury and glass), cohesion is stronger than adhesion and conditions are as shown by (*b*) Fig. 1. The curved upper surface is called a **meniscus**.

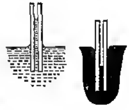

(a) Water (b) Mercury

FIG. 1.—Capillary Attraction.

If small-bore glass tubes are used in gages, the effects of capillarity will cause water to stand higher and mercury lower than with large glass tubes, and even with the

If d_1 is the diameter of the bushing in inches, d the diameter of the journal in inches, then $(d_1 - d)$ is the diametral clearance and $m = (d_1 - d)/d$ is the clearance ratio. The diagram of McKee (Fig. 11) gives the coefficient of friction as a function of the characteristic number ZN/p, where N is the rpm, $p = P/dl$ the average pressure, lb per sq in., on the projected area of the bearing, where P is the load, lb; l the length of bushing, in.; Z the absolute viscosity of the oil in centipoises (see p. 244). Approximate values of Z at 100 (130) F are as follows: light machine oil, 30 (16); medium machine oil, 60 (25); medium-heavy machine oil, 120 (40); heavy machine oil, 160 (60).

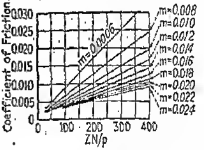


FIG. 11.—Coefficients of Friction of Journals.

For purposes of design of ordinary machinery with bearing pressures from 50 to 300 lb per sq in. and speeds of 100 to 3,000 rpm, values for the coefficient of journal friction can be taken from 0.008 to 0.020.

Thrust Bearings

Frictional Resistance. Step bearings or pivots may be used to resist the end thrust of shafts. Let L = total load, lb in the direction of the shaft axis; dA = an elementary area of the thrust-bearing surface, sq in.; y = distance of the area dA from its axis of revolution, in.; p = pressure on dA due to load L, lb per sq in.; and f = coefficient of sliding friction. Then, **moment of thrust friction** $= M = fp\int y\,dA$ in in.-lb; and the **work expended in friction** per min at n rpm. $= W_f = 2\pi Mn$ in in.-lb.

For a **ring-shaped flat step bearing** such as that shown in Fig. 12 (or a **collar bearing**), $M = \frac{1}{3}fL(D^3 - d^3)/(D^2 - d^2)$, where D and d are in in. For a **flat circular step bearing**, $d = 0$, and $M = \frac{1}{3}fLD$.

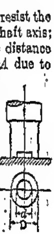


FIG. 12.

The value of the coefficient of sliding friction is 0.08 to 0.15 when the speed of rotation is very slow. At higher velocities when a collar or step bearing is used, f = 0.04 to 0.06. If the design provides for the formation of a load carrying oil film, as in the case of the Kingsbury thrust bearing, the coefficient of friction has values f = 0.001 to 0.0025.

Frictional Forces in Pin Joints of Mechanisms

In the absence of friction, or when the effect of friction is negligible, the force transmitted by the link b from the driver a to the driven link c (Figs. 13 and 14) acts through the center line OO of the pins connecting the link b with links a and c. With friction, this line of action shifts to the line AA, tangent to small circles of diameter d. The diameter d of the circle, called the **friction circle**, for each individual joint, is equal to fD, where D is the diameter of the pin and f is the coefficient of friction between the pin and the link. The choice of the proper disposition of the tangent AA with respect to the two friction circles is dictated by the consideration that friction always opposes the action of the linkage. The force F opposes the motion of a, therefore with friction it acts on a longer lever than without friction (Figs. 13 and 14). On the other hand, the force F drives the link c; friction hinders

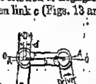


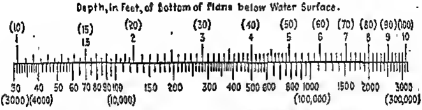
FIG. 13.

FIG. 14.

3. Liquid pressure is exerted with equal intensity in all directions.

4. **Liquid pressure acts perpendicularly to surfaces** in contact with the liquid. *For curved surfaces, the pressure at any point acts in the direction of the normal to the surface at that point.*

5. **The total liquid pressure against a submerged plane area** equals the product of the average intensity of pressure by the area, *i.e.*, the product of the intensity of pressure at the center of gravity of the area by the area,



Total Horizontal Pressure, in lbs., on Planes Extending from Water Surface to Depths Indicated on Upper Half of Scale.

FIG. 3.

or, as often stated, it equals the weight of a liquid column with a base equal to the area and height equal to the head on the center of gravity of the area. For a vertical rectangular plane extending to surface of liquid, with depth h and width b, the formula becomes: Total pressure $= P = \frac{1}{2}wbh^2$, where w is the weight of a unit volume of liquid. See **special calculating scale** for total pressures of water against vertical planes 1 ft wide and 1 to 100 ft deep (Fig. 3).

6. **The component in any direction** of the total liquid pressure on any submerged plane surface equals the product of the area of the projection of the surface on a plane perpendicular to the direction by the intensity of pressure at the center of gravity of the surface. For curved or non-planar surfaces, the principle may be applied to small portions of the surface, each regarded as plane, and the summation taken. Such cases, however, as **hollowed-out pump plungers and dished tank ends** have the same total pressures in the direction of the axis of the cylinder or tank as if the plunger or end were plane, *i.e.*, $P = Ap$, where A is the cross-sectional area of tank and p the average pressure. For submerged and floating bodies, see Buoyancy and Flotation, p. 249.

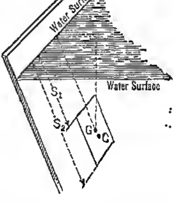


FIG. 4.

7. The **center of pressure** of a plane area subject to liquid pressure, *i.e.*, the point where the application of a single support will balance the liquid pressure, is located by the formula

$$e = \text{moment of inertia of area/area} \times x_{cg}$$

where e is the distance, in the plane, of the center of pressure below the center of gravity, x_{cg} is the distance from the water surface to the center of gravity of the area, and the moment of inertia is referred to the line where the plane cuts the water surface. In Fig. 4, the point G represents the center of gravity of the area and point C the center of pressure. C is always below G, but with relatively small areas submerged under large heads they are very close together and for most purposes may be considered coincident. For common shapes, calling the distance to the upper edge s_1 and to the lower edge s_2 (see Fig. 4), the values of e are

$$\text{Rectangle, } (s_2 - s_1)^2/12x_{cg}; \text{ Triangle, } (s_2 - s_1)^2/18x_{cg}; \text{ Circle, } d^2/16x_{cg}.$$

and R = the radius of the pitch circle in which the tension element travels, both in inches. The internal friction rigidity of the tension element causes a shortening at the driving end T_1 of the lever arm R an amount h_1 in inches, and at the following end a lengthening of the lever arm R an amount $= h_2$ in. Then, for simultaneous winding on and off, $T_1(R - h_1) = T_2(R + h_2)$. Approximately, $h_1 = h_2 = h$, whence $h_1 + h_2 = 2h$ (approx) and $T_1 = [1 + (2h/R)]T_2$.

This results in a frictional loss, and the efficiency of transmission is $e = 1 - (2h/R)$. If the tension element is only wound on the drum, $h_1 = 0$ and $T_1 = [1 + (h/R)]T_2$, with the corresponding efficiency $e = 1 - h/R$.

For **chains** the coefficient of friction between the link faces or pivots is $f = 0.2$ to 0.3; and when d = the diam of the link pin, $h = fd/2$.

For **hemp ropes,** $h = 0.03d^2$ to $0.09d^2$ according to the construction, material, and condition of the rope. In the absence of reliable values for wire ropes, those for chains may be tentatively used.

The **elastic rigidity** of the material is not a factor in simultaneous winding on and off, since the lever arm R is increased equally at the points of winding on and off; the work expended in bending the tension element as it is wound on is recovered as it straightens out in unwinding. But if there is only winding on, then the effect of the bending is to be taken into account.

Efficiency of rope and chain sheaves at low speeds, including journal friction (180 deg contact):

For fixed sheaves, chain, and wire rope, e = 0.94 to 0.96.
For floating sheaves, chain, and wire rope, e = 0.97.
Hemp rope sheaves:

Rope diam, in.	3/4	1	1 1/2	1 3/4	2
Fixed sheaves: e =	0.95–0.96	0.91–0.96	0.89–0.93	0.84–0.92	0.85–0.91
Floating sheaves: e =	0.97	0.96	0.95	0.94	0.93

For submerged bodies, the "weight in water" (or other liquid) $= (W - V_w)$, where V_w is the "loss of weight." The so-called "loss of weight" is not a real loss, but only apparent by reason of the buoyant force of the liquid. Thus, a block of granite weighing 170 lb per cu ft can be supported under water by an upward force of $(170 - 62.4) = 107.6$ lb per cu ft of granite. The difference, 62.4 lb per cu ft, is borne by the water, and whatever supports the water supports the difference also. Thus, when a ship moves into a drydock and displaces water from the dock, the bottom of the dock is not relieved of any weight at all.

The Hydrometer. A floating body rides higher in a heavy liquid than in a light one. The density and specific gravity, therefore, may be found by noting the depth to which a specially prepared float sinks. Such a device is called a **hydrometer.** The most common type is made of glass, consisting of a graduated stem above a hollow bulb, below *which is a smaller bulb containing mercury to make the whole instrument float upright.* By properly proportioning the weight and volume, any desired degree of sensitiveness may be obtained, subject to the increasing influence of surface tension on the stem as its size decreases. Hydrometers to suit various needs are obtainable in the market. The scale is graduated to read either specific gravity (referred to water) directly or according to some arbitrary scale. Of the latter the most widely used is the **Baumé scale** (see p. 86 for equivalents). **Twaddell's hydrometer** is used in England for liquids denser than water. See p. 85 for special hydrometer scales. Fahrenheit's and Nicholson's hydrometers are arranged so that weights may be added to sink them to a standard mark on the stem. For details, see textbooks on physics. For precise calculations, the temperature of the liquid should be measured and corrections made to the hydrometer indications. Nearly all liquids expand and contract much more than water for equal temperature changes near 60 F.

Determination of Specific Gravity and Volume of Solids by Immersion. By weighing an insoluble solid heavier than water in air and then in water, the specific gravity referred to water may be readily found, the specific gravity being (weight in air)/(loss of weight). Also, the volume of the solid = (loss of weight)/(weight of unit volume of water). The method is at once a convenient field method and one of precision when weights are accurately measured.

THE FLOW OF FLUIDS

General Considerations Regarding Flow

In the consideration of the flow of fluids, considerable advances are being made toward fuller logical interpretations of experimentally found facts and toward better guidance for future analytical and experimental undertakings.

Dimensional analysis (see p. 284) starts with the assumption that motion of a fluid may be influenced by any or all of a number of obvious variables such as the net force per unit area or pressure gradient p, the prevailing mean velocity V, and by the several properties of the fluid, such as weight per unit volume w, density or mass per unit volume $\rho(= w/g)$, viscosity μ, bulk modulus of elasticity e, surface tension σ; also by size, area, distances from and to boundaries, etc., expressible as several linear dimensions a, b, c, d. A grouping of terms embodying the essentials of the flow (but dimensionless and therefore of general application) can be written equal to a constant numerical factor times a (dimensionally independent) function of the dimensionless ratios d/a, d/b, d/c and of certain dimensionless groupings or numbers such as $Vd\rho/\mu$ and $V/\sqrt{dg}$. These last are known as the Reynolds and Froude numbers.

Saybolt Furol, when $25 < t < 40$,	$\nu = 0.0224t - 1.84/t$
when $t > 40$,	$\nu = 0.0216t - 0.60/t$
Redwood No. 1 (English), when $34 < t < 100$,	$\nu = 0.00260t - 1.79/t$
when $t > 100$	$\nu = 0.00247t - 0.50/t$
Redwood Admiralty (English),	$\nu = 0.027t - 20/t$
Engler (German),	$\nu = 0.00147t - 3.74/t$

For kinematic viscosity conversion table see p. 1905.

To convert kinematic viscosity to units of ft^2 per sec multiply by 1.075×10^{-3}. See also p. 1905.

Table 1. Viscosities of Fluids in Centipoises

Fluid	Temp, deg F	Viscosity	Fluid	Temp, deg F	Viscosity
Water	32	1.792	Spindle oil, sp gr = 0.885	60	45
	50	1.308		150	6.4
	68.4	1.000	Glycerin, 50% sol.	60	7.0
	100	0.679		150	1.5
	150	0.432	100%	60	1400
	212	0.284		150	70
Hydrogen	32	0.0084	Lubricating oils:		
	150	0.0098	Eastern S.A.E. 10	60	100
Air	32	0.0171		150	10
	150	0.0201	S.A.E. 30	60	400
Liquid ammonia	−10	0.20		150	27
	32	0.12	Light machy., sp gr 0.907	60	114
Mercury	32	1.70		150	12
	150	1.35	Heavy machy.	60	661
Gasoline, sp gr, 0.70	32	0.50		150	35
	150	0.25	Calif. crude, light	60	48
sp gr 0.75	32	0.95		150	9.0
	150	0.40	Heavy	60	3500
Sperm oil	60	42		150	70
	150	9.0			

Viscosity changes with temperature; for liquids it decreases as temperature increases, for gases it increases. Values of viscosity for various fluids at atmospheric pressure are given in Table 1. The viscosity of a gas does not vary with pressure for moderate ranges; for the "permanent" gases this is true up to 100 atmospheres.

The viscosity of glycerin and of castor oil is about 1,000 times, and that of machine oils 100 to 300 times, as great as that of water at ordinary room temperatures. The viscosity of oils changes much more with change of temperature than does the viscosity of water; water changes from 0.0179 poise at 32 to 0.0131 at 50 deg, 0.0100 at 68.4 deg, 0.0068 at 100 deg, and 0.0040 at 160 F.

The change in viscosity with temperature of typical American mineral lubricating oils is as follows:

Saybolt viscosity at 100 F	380	700	1,200	3,300	7,500
Saybolt viscosity at 212 F	68	90	120	210	300

Investigations published by the British Department of Scientific and Industrial Research show that the viscosity of water increases but slightly with increase of pressure; for animal and vegetable oils at 5,000 (10,000) [15,000] lb per sq in. pressure the percentage increase of viscosity is about 75 (200) [350] percent; for mineral oils at the same pressures the increase is 125 (550) [1,600] percent.

When two or more liquids that do not mix (*e.g.*, water, oil and mercury) are contained in the same vessel, the heaviest liquid occupies the lowest

such that $h = V^2/2g$ or $V = \sqrt{2gh}$; *i.e.*, the Pitot tube indicates the velocity head of the flowing water. If a tube of this sort is inserted in a pipe with water flowing under pressure, see Fig. 7, the water rises to a height h_v above the height in an open water column attached to a hole in the wall of the pipe; *i.e.*, the Pitot tube shows the sum of the pressure head and the velocity head of the water at the point of the tube, demonstrating that both sorts of head co-exist. The **Venturi meter** also illustrates the relation between pressure and velocity heads. This device (Fig. 8) consists of a conical nozzle-like reducer followed by a more gradual enlargement to the original size, which is that of the pipe line in which the meter is laid. At 1, 2, and 3 the pressure heads in the pipe are shown as they appear when measured by open liquid columns. Experiment shows that h_2 is less than h_1 by very nearly the difference in velocity heads $[(V_2^2/2g) - (V_1^2/2g)]$, and that at 3, where the mean

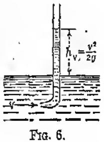


FIG. 6.

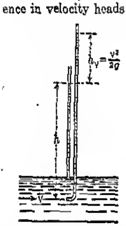


FIG. 7.—Pitot Tube.

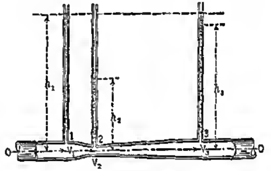


FIG. 8.—Venturi Meter.

velocity is again V_1, the pressure head is nearly h_1 again, the diminution being accounted for by loss due to friction of the flowing fluid. If the pressure in the pipe is changed, but the rate of flow kept the same, *e.g.*, by manipulating valves to the left of 1 and right of 3, all the pressure heads rise or fall equal amounts, *i.e.*, the **differences** in heads remain **unchanged**. If the pipe is not level, Fig. 9, allowance is made as follows: All points are referred to any convenient reference level, *O-O*. The difference in level of the tops of the open liquid columns is $H_1 - H_2 = (h_1 + Z_1) - (h_2 + Z_2)$, where Z_1 and Z_2 are the heights of 1 and 2 above the datum level, and are called the **potential** heads of these points. With no flow, liquid would stand at the same level in columns 1, 2, and 3, for both the level and sloping pipes. Hence the differences $h_1 - h_2$ in Fig. 8, and $H_1 - H_2$, in Fig. 9, are due solely to the flow of the fluid. It is to be noted that the difference in pressure heads of

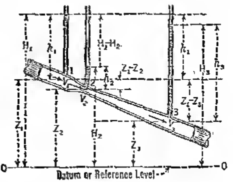


FIG. 9.

large tubes there is a curved surface where the liquid touches the glass. For water, the extra height is about $0.046/d$ in., where d is the internal diameter of the tube in inches. Thus, if a differential water-air gage (as in Fig. 7) has two glass columns nominally both ⅛ in. in internal diam, but actually one 0.01 in. larger and the other 0.01 in. smaller, the water will stand about $\frac{1}{16}$ in. higher in the smaller tube than in the larger, when a common hydrostatic pressure would lead one to expect them to stand at exactly the same level. In experimental work, this possible difficulty may usually be avoided by using larger glass tubes, say of ⅜ in. internal diam. **All readings should be taken at the level of the middle of the meniscus,** *i.e.*, the bottom of the curve for water and the top for mercury. These positions are away from the maximum effects of capillary attraction and are nearest to the proper level.

With many liquids such as water, ethyl alcohol, chloroform, benzene, turpentine, and olive oil the angle of contact between liquid and glass is 0 deg and in capillary tubes the meniscus is approximately hemispherical. In such a case the capillary rise = $2\gamma/(d_l - d_g)r$ where γ is the surface tension, d_l and d_g are the weight densities of the liquid and the gas (or vapor), and r is the internal radius of the capillary tube.

HYDROSTATICS

Pressure of a fluid usually means **intensity of pressure** above the local atmospheric pressure.

Gage pressure is the pressure indicated by a gage and shows pressure above the local atmospheric pressure. **Absolute pressure** commonly means **intensity of pressure referred to vacuum as zero.** Pressures less than the local atmospheric are always referred to by some distinct phrase, *e.g.*, "**10 lb suction,**" "20 in. (mercury) **vacuum,**" "5 lb per sq in. **absolute** pressure."

Pressure Head. Liquid pressure is often caused by the weight of the overlying liquid, and all liquid pressures can be thought of as being so caused. Special names are given to this real or imaginary height, as "**pressure head,**" "**static head,**" or, simply, "**head.**"

General Considerations Regarding Liquid Pressure

1. Pressure head is the height to the real or imaginary free liquid surface corresponding to the pressure of the liquid.

2. In any continuous body of liquid at rest, the **intensity of pressure** increases directly with the depth of the liquid. $p = hw$, or $h = p/w$, where p is the increase of pressure per unit area, h the increase of depth (or head), and w the weight per unit volume of the liquid. This law is true whether the liquid has a free upper surface open to the atmosphere or is confined and subjected to mechanical, gaseous, or other pressure.

For a liquid of sp gr s, $p = 62.4sh$ lb per sq ft $= 0.433sh$ lb per sq in. where h is the head, ft. Vacuum $= 1.13H/s$ ft $= 0.49H$ lb per sq in., where H is vacuum, in. of mercury.

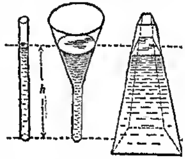


FIG. 2.

Hydrostatic Paradox. In the cases shown in Fig. 2, the intensity of pressure in pounds per square inch is the same at the bottoms of the variously shaped liquid containers. But the total liquid pressures against the various bottoms are proportional to the areas of the bottoms. The amount of liquid or its total weight makes no difference in either the intensity of pressure or the total pressure as long as the head h is the same. The fact that the total liquid pressure (bursting pressure) against the bottom may be many times greater or less than the total weight of the liquid is termed the **hydrostatic paradox.**

velocity of all the water flowing out, but to more than the average, *i.e.*, the actual discharge is less than that shown by the formula. (For theoretical principle, see Weirs, p. 259.) When the head above the center is equal to the vertical dimension of the orifice, the discharge is only about 1 per cent less, and when the head is twice the vertical dimension the diminution is

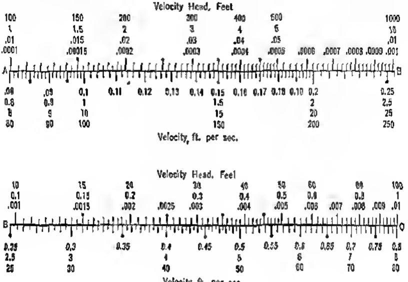


FIG. 11.—Velocity-velocity Head Conversion Scale.

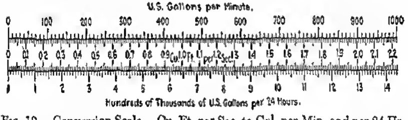


FIG. 12.—Conversion Scale. Cu. Ft. per Sec. to Gal. per Min. and per 24 Hr.

negligible, except for the most precise sort of investigations, for which a special calibration should be made to determine the exact coefficient. For lower heads, if h_c represents the head above the center and O the height of opening, Table 3 gives the percentage reduction in discharge and the corrected discharge coefficient for circular and rectangular orifices. The value

Table 3. Corrections for Rectangular and Circular Orifices

$\frac{h_c}{O}$	Rectangular Orifices: Reduction in discharge, percent	Rectangular Orifices: Corrected coefficient of discharge	$\frac{h_c}{O}$	Circular Orifices: Reduction in discharge, percent	Circular Orifices: Corrected coefficient of discharge
0.50	5.7	0.575	0.50	4.0	0.585
0.62	3.2	0.590	0.55	3.2	0.590
0.87	1.6	0.600	0.73	1.6	0.600
1.00	1.1	0.603	1.00	0.8	0.605

If the upper edge or top is at the surface, the distance of the center of pressure below the top is

Rectangle, $2s/3$; Triangle: vertex up, $3s/4$; vertex down, $s/2$; Circle, $5r/4$.

The maximum bending moment on a vertical rectangular beam subjected throughout its full length to liquid pressure, supported at top and bottom, and with the top support at the surface, occurs $0.09s$ above the center of pressure, or $\frac{1}{11}$ the span below the middle.

Example. Required the total pressure against a vertical gate 3 ft wide and 5 ft high at the bottom of a tank with water 12 ft deep; also the position of the center of pressure. The center of gravity of the gate face is $(7 + \frac{5}{2}) = 9\frac{1}{2}$ ft below the surface, and according to (5), total pressure against gate $= 9\frac{1}{2} \times 62.4 \times (5 \times 3) =$ 8,892, say, 8,900 lb. The total pressure scale (Fig. 3) gives 4,500 and 1,530 lb, respectively, for depths of 12 and 7 ft. The difference, 2,970 lb, is the pressure per foot of width, or 8,910 lb total. The center of pressure for the submerged rectangle is $(12 - 7)^2/12(7 + \frac{5}{2}) = 0.219$ ft below the cg, or 9.72 ft below the surface. Graphically, the center of pressure is the point where the area is cut by a normal through the center of gravity of the trapezoidal "pressure prism."

Buoyancy and Flotation. The total upward pressure of a liquid against a floating or a submerged body is the same as if the portion of the body below the surface were replaced by the liquid. In the latter case there would be simply a mass of liquid at rest, *i.e.*, with the net upward pressure from the surrounding liquid just **equal to the weight of the liquid** volume in question, which is the **volume displaced** by the floating or submerged body. Obviously, for a floating body, this weight and upward pressure equals the weight of the body. Hence, for any floating body, $P_u = W = Vw$, where P_u is the total upward pressure, W the weight of the body, V the volume of the displaced liquid (= "the displacement"), and w the weight of a unit volume of the liquid.

The draft or depth of flotation may be calculated when the weight, size, and shape of the floating body are known, for in all cases $W = Vw$.

Example. (Float control of a mechanism.) It is desired to obtain a force of 100 lb from a float when the water surface rises or falls 1 in. Required to find size of float necessary. From $P_u = Vw$, $100 = Vw =$ water-line area $\times \frac{1}{12} \times 62.4$, or area $=$ 19.25 sq ft, calling for, *e.g.*, a circular float 4.95 ft diam.

The center of buoyancy is at the center of gravity of the displaced volume of liquid. It shifts its position, both vertically and sidewise, as the floating body tips. The total upward pressure of the liquid may be regarded as concentrated through the center of buoyancy, about which it balances.

Stability, or tendency to float upright, demands that, when the floating body is tipped, the vertical through the new center of buoyancy shall pass the center line above the center of gravity of the body. The center line is the vertical line passing through the cg when the body is in equilibrium. When it does so pass, both the weight and buoyancy tend to right the floating body. When it passes below the center of gravity, both the weight and buoyancy act to overturn it. It is not necessary for stability that the center of gravity be below the center of buoyancy. In ships it is usually above. For submerged submarine boats, the center of gravity must be below the center of buoyancy for stability, because in this case the shape of the displaced volume of liquid does not change when the boat tips and consequently the center of buoyancy does not move relatively to the boat. The point where the vertical through the new center of buoyancy cuts the original vertical (now inclined), through the center of gravity, is called the **metacenter.** If, then, the metacenter is above the center of gravity, there is stability; if below, instability. The more it is above, the greater the stability.

suction and making the effective head greater than h. Values of the discharge coefficient as high as 1.55 have been found by experiment. For the downstream end of the tube, the coefficient is always less than 1. With high heads the jet may jump free and discharge as (*b*).

Rounding or beveling the sharp upstream edge, even slightly, increases the discharge of an orifice. Experiments at Cornell University on circular and square orifices, all free discharge, with various degrees of rounding of the upstream edge, show that the per cent increase in Q, over the sharp-edged Q, is 3.1 times the percent that the radius of the rounding is of the diameter of the orifice. This applies for roundings up to (but not beyond) 10 percent of the diameter of the orifice, where the increase is 31 percent, the coefficient being 0.80. Placing a sharp-edged orifice close to a side wall or a corner "suppresses" the contraction and increases the discharge. For each side of a rectangular orifice on which contraction is suppressed, the discharge is increased about 3 percent provided the side walls or the bottom do not extend beyond the plane of the orifice. If they do so extend, the coefficient remains the same as for fully contracted flow.

Attainable Precision in estimating the flow through a well-made sharp-edged or rounded-approach orifice depends on several factors. The orifice should be located at least 3 diameters away from any side wall and should be screened from the eddying effects of in-rushing water. If the head is low, guide vanes may be necessary to prevent whirlpools or vortices that obstruct the discharge. The size of the opening must be accurately measured. An error or uncertainty of a small percentage in the diameter makes twice that error in the discharging area; and a small percentage change in the head affects the discharge by half that percentage. **With accurate measurements** made with a knowledge of the various effects above mentioned, the engineer may feel assured that **the discharge will be within 3 percent of the calculated value.** By calibrating an orifice under the exact conditions of use, a future precision of 0.5 percent may be assured. Such an accuracy is seldom necessary except for precise testing or investigation. For an assured **precision within 1 percent**, maintenance of the orifice in "sharp-edge" condition is essential. A very slight dulling of the edge increases the discharge as much as 1 to 2 percent. As to the effects of a little corrosion, the nature and extent of the change in the coefficient depend on those of the defect. If a corroded edge is tuberculated, the discharge is decreased. If the rust accumulation is brushed or washed off, the effect is the same as dulling the edge. Published tables that give **coefficients to three places**, *e.g.*, 0.615, are deceptive, in that they suggest a precision of 1 in 600, when 1 in 60 would be nearer to the accuracy expected by an experienced engineer for routine cases.

Submergence, or Discharge under Water (*e.g.*, headgates). For openings in a "thin plate" or in a thin wall, the coefficients already given apply, but the value of h in the formula is the difference in water levels upstream and downstream from the opening. In the case of headgates there is liable to be uncertainty as to the effective area of the opening and of the amount of contraction. An uncertainty of from 5 to 10 percent is to be expected.

If the **submerged opening is in a thick wall**, even if the thickness is as slight as ½ the diameter of the orifice, the discharge is greater than for the opening in a thin wall. In this respect submerged discharge differs from discharge into the atmosphere, where the tube must be several diameters long before the coefficient increases. For a submerged tube 1 diameter long, square-edged entry, the discharge coefficient is about 0.76, as compared to 0.60 for the same tube with free discharge into the atmosphere. Rounding the entry further increases the discharge.

Investigations on **submerged square tubes** at the University of Wisconsin (*U. of W. Bull.* 216) and at Cornell University (*Eng. News.*, Nov. 2, 1916) show that the discharge coefficient under low heads becomes practically constant for a tube length greater than 1.5 diameters and that the value of the constant coefficient is a function of R/d, where R is the radius of the quad-

In practical application to particular cases of flow, one of these fluid properties is often dominant for a certain range of values of the pertinent dimensionless number, thus simplifying analytical treatment and study of experimental results. There should be recognition, however, of the existence of the other influences which may sometimes greatly affect the flow.

Steady, Unsteady, Uniform and Non-uniform Flows Defined. Flow is called **steady** when conditions in the stream (*i.e.*, in any cross-section of the flowing fluid) remain unchanged, especially the mass flowing per second, the mean velocity, and the size of the stream. If, also, the size and shape of the stream remain nearly constant from point to point along the course, the flow is called **uniform**. If the size of the stream and the velocity change from point to point along the course, even though the flow be steady, it is called **non-uniform**.

Examples of steady **uniform flow** are the common flows in pipes, hose, canals, and rivers. This sort of flow is seen to be associated with long conduits, and is controlled by the laws of fluid friction. **Pressure and head** are consumed (hence "lost") in forcing the fluid along against the resistances caused by the portions of the conduit in contact with the liquid.

Steady non-uniform flow is exemplified by the flow through orifices, nozzles, Venturi meters, revolving water turbines, weirs, etc. It is seen to be associated with cases of changing velocity, where flow starts from an early quiet body of fluid or where a flowing stream changes its size and its velocity. For these cases **pressure and head** are not consumed and lost (except to a small extent), but are **converted into moving energy**.

Unsteady flow proceeds in waves, pronounced pulsations or surges, or under rapidly changing conditions, *e.g.*, breaking ocean waves, pressure and velocity surges in a pipe line supplying a hydraulic ram, etc. No flow is steady or uniform when started. A short time elapses, of necessity, before steady conditions can be established. No ordinary flow is ever perfectly steady, there always being some pulsations and tremblings in velocity and pressure. The flow through a long pipe line carrying the discharge from a good double-acting plunger or piston pump with air chambers in good order is considered practically steady as far as flow calculations are concerned, although a pressure gage shows fluctuations of several percent.

Interrelations of Heads. Potential, Pressure and Velocity Heads. When water or any liquid of low viscosity spouts through an orifice in the side of a tank (Fig. 5), the jet has a velocity $V = \sqrt{2gh}$, nearly, where h is the head at the level of the orifice in the body of liquid well away from the orifice, and g is the acceleration constant of gravity. As a result of liquid friction at the orifice the actual velocity is 1 to 2 percent less than that given by the above formula. A jet issues in any direction according to the same law. If issuing nearly vertically, as in Fig. 5, the jet will rise nearly to the height of h ft above the orifice. For great heights the friction of the air and the breaking-up of the jet reduce the height considerably. See p. 274. For a fluid flowing with a velocity V, the name **velocity head** is given to the quantity $V^2/2g$, which corresponds to the static or pressure head h that could cause or could be caused by the velocity V.

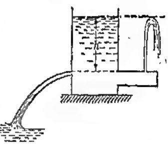


FIG. 5.

Figure 6 shows the water surface of an open stream flowing with velocity V, and against the current of which is directed the opening of a **Pitot tube**. The water rises in the tube to a height of h ft above the surface of the stream,

of the jet, the amount depending on the shape. A plain conical nozzle (called a **smooth nozzle**) causes no contraction. The velocity coefficient of a good nozzle is from 0.97 to 0.99, and this is also the discharge coefficient for smooth nozzles. A **square-ring nozzle** is really a sharp-edged orifice with high velocity of approach. The discharge coefficient depends on the proportion of the area taken up by the ring (see curve in Fig. 16, determined by J. R. Freeman from his experiments). The use of ring and undercut-ring nozzles arose from the mistaken notion that a better fire stream could be thrown from these than from a smooth nozzle. **Undercut-ring nozzles** have discharge coefficients of from 0.58 to 0.72, varying as for square-ring nozzles. The discharge coefficient of a well-finished **Doble needle nozzle** (Fig. 15) referred to the smallest area of passageway (surface of frustum of cone) between needle and nozzle proper, is from about 0.82 to 0.95, depending on the "opening" or position of the "needle." When nearly closed the coefficient is lowest; it is highest at about a ¾-open or full-load operating position, for which the curves are especially designed.

The nozzle formula is:

$$Q = CA_2\sqrt{\frac{1}{1-(A_2/A_1)^2}}\sqrt{2gh} = CA_2\sqrt{\frac{1}{1-(d_2/d_1)^4}}\sqrt{2gh}$$

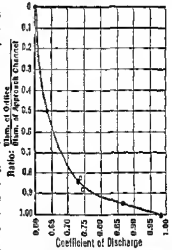


Fig. 16.—Discharge Coefficients for Square-ring Nozzles.

where A_1 and A_2 are the cross-sectional areas at the base of the nozzle (*i.e.*, where the pressure head is measured) and at the discharge tip, respectively, and d_1 and d_2 the corresponding diameters. C is the discharge coefficient and h is the pressure head in the pipe or hose just upstream from the convergence, referred to the level of the center of the discharge tip. (For ordinary purposes, h is usually measured by a spring steam-gage, or, more precisely, by an open water-column gage for low heads and a mercury gage for high heads. h is sometimes called the gage **pressure head**, and $[h + (V_1^2/2g)]$ the **theoretical or total head.**

For values of the **velocity-of-approach factor**, see p. 2008.

Range of Coefficients. Certainty. For water in smooth fire-hose nozzles, J. R. Freeman found an average coefficient of 0.977, with variations in averages for particular nozzles of 0.5 percent each way. The nozzles were attached to 2½ in. hose and had tip diameters of from ¾ to 1⅜ in. Several other experimenters have checked this value closely. For 1¾, 2 and 2½ in. tip diameters with convergence angles of from 10 to 15 deg, Mr. Freeman found an average coefficient of 0.995, with a variation (in averages for individual nozzles) of from 0.4 percent above to 0.8 percent below. The writer designed three nozzles for 4, 6 and 8 in. pipes, the tips being respectively 1.78, 2.74 and 3.57 in. in diameter; a common convergence angle of 25 deg was used, and all changes in direction were made by easy curves of 9 in. radius. The average discharge coefficient was found to be 0.986, with variations in averages within 0.3 percent and extreme variations in individual experiments (heads from 2 to 60 ft) of about 1 percent. **A coefficient of 0.98** may be considered reliable for good smooth nozzles within 1½ percent, without

the fluid in the pipe is $h_1 - h_2$ in both Figs. 8 and 9, but in Fig. 9 this difference is due partly to the hydrostatic effect of the difference of level. Consequently it is seen that a difference in pressure heads at two points along a stream (the velocity being different at these two points), may be due not only to a difference of velocity heads, but also in part to a difference in potential heads according to the hydrostatic law. Hence,

$$(h_1 + Z_1) - (h_2 + Z_2) = (V_2^2/2g) - (V_1^2/2g);$$
$$\text{that is, } h_1 + Z_1 + (V_1^2/2g) = h_2 + Z_2 + (V_2^2/2g)$$

This formula is the mathematical expression of **Bernoulli's Theorem.**

To summarize:

1. The total head present in a mass of flowing liquid is made up of potential head, pressure head and velocity head, mutually convertible, each into the other's form.

2. **Bernoulli's Theorem.** The total head in a particle of a continuous mass of flowing liquid at any one point in its stream line (*i.e.*, the path along which the liquid flows) is equal to the total head at any other position of its stream line, *provided* that there is no loss between the two positions due to friction, the giving up of work, etc., and no gain due to the application of outside work.

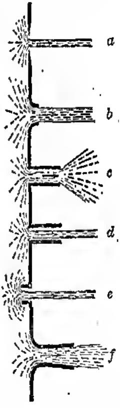


Fig. 10.—Types of Orifices.

Flow through Orifices

Flow of Liquids through Orifices. The general law of flow is $V = \sqrt{2gh}$, in which V is the velocity of the jet at its smallest section near or in the plane of the opening, h is the head referred to the level of the center of the stream cross section where the filaments of the issuing jet first become parallel.

The Standard Orifice for measuring or regulating purposes is the **sharp-edged orifice,** also called "orifice in thin plate." The jet contracts in size (Fig. 10*a*) just after coming through the opening of such an orifice. The area of the cross section of the jet, at a distance out from the opening about one-half its diameter, is about 0.62 of the area of the opening. The average velocity past this contracted section is about 0.98 to 0.99 of $\sqrt{2gh}$. Hence, calling A the area of the opening, the **discharge of a standard** sharp-edged orifice is $Q = 0.61A\sqrt{2gh}$, Q being in cu ft per sec when A is in sq ft, h in ft, and g in ft per sec per sec. The value of 0.62 is called the **contraction coefficient**; 0.98 to 0.99 is the **velocity coefficient,** and their product, or 0.61, is the **discharge coefficient.**

For Values of $\sqrt{2gh}$ for any head, see Velocity Head Scale, Fig. 11, also Conversion Scales (cu ft per sec to gal per min and per 24 hr), Fig. 12.

Example. For a circular sharp-edged orifice, 1 in. in diam, under 50 ft head, $Q = 0.189$ cu ft per sec = 85 gal per min.

Large Orifices with Low Heads, *e.g.*, sluice gates (see also Submergence). Unless the orifice is in a horizontal plane, *e.g.*, in the bottom of a tank, the top may be so much nearer the surface of the liquid than the bottom that the head at the level of center of orifice does not correspond to the real average

see under broad flat crests p. 263), and is frequently made of a steel plate or angle planed off and sometimes beveled on the downstream side to give a top flat portion from $\frac{1}{16}$ to $\frac{1}{8}$ in. wide.

Accuracy of Francis Formula. This formula is very widely used by engineers. It is reliable within from 1 to 3 percent for heads above 0.3 ft, provided that (1) the weir bulkhead has a vertical upstream face and occupies the full width of the channel, (2) the crest is level, (3) the channel of approach is deep (*i.e.*, the weir bulkhead is high, so that the velocity of approach is small—see below for corrections), (4) there is free access of the air to the space below the falling sheet of water (*i.e.*, between it and the downstream face of the weir), (5) the head is measured a distance upstream from

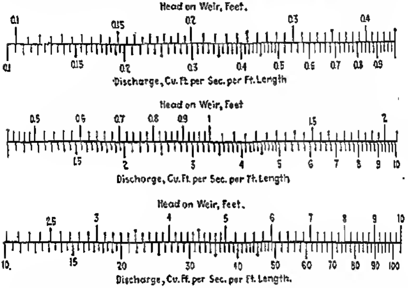


Fig. 17.—Weir Discharge Scale. Francis Formula, $Q = 3.33\,h^{3/2}$.

the weir of at least 4 times the head, (6) the side walls extend downstream from the weir (above the crest level) to prevent a lateral spreading as the water passes over the crest, and (7) due precautions in measuring are taken.

For very low heads, Francis formula results for discharge must be increased 1(3) percent when $h = 0.2$ ft, and 4(7) percent when $h = 0.1$ ft, the low values for a strictly sharp crest-piece with extremely smooth upstream face, the high values for ordinary iron plate with machine-planed top but somewhat corroded.

Effects of Errors or Uncertainties. A small percentage error in head causes $1\frac{1}{2}$ times that error in discharge. Conversely, the allowable percentage error in head is $\frac{2}{3}$ that in discharge. Considerable errors in the measured head may be made by careless methods of referencing the crest level to the head gage scale, and by assuming that a non-level crest is level without knowing the average level. The mean level of the water surface must be measured, not the crests of the small waves or surges existing in all flowing water. Rounding the upstream corner of the crest increases the discharge especially at low heads. Roughness of the upstream vertical face of the weir has the same effect.

For a **rectangular notch** with vertical sharp-edged ends, contraction occurs at the ends as well as at the crest, and the discharge is reduced approximately as though the length of the weir were decreased by $0.1h$ for each end contraction. Hence, the **Francis formula for a sharp-crested weir with**

of the coefficient, however, is not certain within from 1 to 3 percent without special calibration of the particular orifice.

Hamilton Smith's Coefficients. Relying chiefly on his own precise experiments of 1885 and earlier, and on those of Poncelet and Lesbros, 1827–1835, Smith published tables of coefficients for flow of water at ordinary low temperatures through sharp-edged orifices with square and round openings. Figure 13 shows the variation with head and diameter of these coefficients. Schoder has found corresponding variations with small circular sharp-edged orifices ½ to 1 in. in diameter. It will be noted that there is little change in the coefficient for heads above 2 ft and diameters greater than ½ in. Tests by Medaugh and Johnson (*Civil Eng.*, July, 1940) on very precisely made circular orifices agree with Smith within ½ of 1 percent.

Variously Shaped Orifices. Typical shapes are shown in Fig. 10. (*a*) is a sharp-edged orifice; (*b*) is a rounded-approach orifice; (*c*) is a short pipe or orifice in thick wall (length = 2.5 to 3 × diam), flowing full; (*d*) is a short pipe flowing like a sharp-edged orifice (see below); (*e*) is a re-entrant short pipe, or a Borda mouthpiece (when length < diam); (*f*) is a Venturi orifice.

For (*b*), when the curve is not too abrupt and there is a short length (= ¼ to ½ diam) of uniform bore where the jet issues, there is no contraction and the coefficients of discharge and of velocity are equal. The value for an average good, smooth, rounded orifice is 0.97; for the very best, 0.99; but with poor curvature, causing contraction and cross currents, the coefficient may be as low as 0.90.

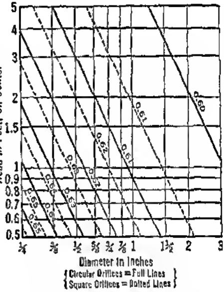


FIG. 13.—Coefficients for Sharp-edged Orifices. (Hamilton Smith.)

For (*c*), at low heads, the tube flows full at the downstream end and the discharge is "broomy," *i.e.*, agitated and divergent, while at high heads, above 40 to 50 ft, the jet jumps free at the upstream corner as from a sharp-edged orifice, as shown in Fig. 10(*d*). If the jet is once made to jump free by a high head, the head may be lowered to only a few times the orifice diameter without bringing back the "broomy" flow conditions. For "broomy" flow, the discharge coefficient is about 0.82; for clear flow it is 0.61, as for the sharp-edge. If the interior of the tube is greasy, the broomy flow will not occur, even at a low head. Obviously, this is an uncertain measuring device unless conditions are known.

For (*e*), when the length is 2.5 to 3 times the diameter, the above statements for (*c*) and (*d*) apply. The discharge coefficient is about 0.72 for "broomy" and 0.53 for clear discharge. For Borda's mouthpiece the coefficient is 0.53.

For (*f*), if the divergence angle is not greater than 7½ deg nor the length more than 10 nor less than 5 times the smallest diameter, the tube will flow full, and the velocity through the narrow "throat" will be greater than $\sqrt{2gh}$, because the pressure is less than atmospheric at that point, causing a

Narrow Rectangular and Trapezoidal Sharp-edged Notches with Crest Length Less than $2 \times h$. Experiments on narrow rectangular notches beyond the applicability of the Francis formula, covering heads up to 3 ft and widths down to 2.7 in., have shown that the formula $Q = 0.56(\frac{2}{3}lh\sqrt{2gh})$, *i.e.*, $Q = 3.00lh^{3/2}$, is reliable when h is as great as or greater than l. Hence for such cases we may take 90 percent of the discharge given by Fig. 17. For notches with l between h and $2h$ the coefficient increases and merges into values given by the Francis formula for two end contractions. For narrow trapezoidal notches the discharge may be expressed as the sum of that through a central rectangle and two triangles at the ends (see above formulas). A discharge coefficient of from 0.58 to 0.60 has been found to apply.

The inverted notch, with discharge proportional to head, allows very simple regulating and recording devices to be used with it. It is constructed with curved sides such that the width of notch above the straight level crest decreases just enough to keep the discharge proportional to the head. The rate of flow is given by the formula, $Q = C \times 1.57\sqrt{2g}(l\sqrt{h})h$, where Q is in cu ft per sec, l and h are notch width and head, respectively, in ft. With $C = 0.60$, this becomes $Q = 7.55(l\sqrt{h})h$. The product $l\sqrt{h}$ is constant, and this gives the relation for the curves of the sides. Starting with some desired or convenient head for a value of the discharge, say near the probable maximum to be expected, the value of the notch width at that height above the crest is calculated from the above formula for Q. These values of width and head give the constant value $l\sqrt{h}$. It is not necessary to continue the side curves of the notch closer to the crest than about $\frac{1}{32}$ to $\frac{1}{4}$ in. (depending on the range of heads); they may terminate in short verticals to the crest. The ratio of the discharge shut off by reason of such an abrupt termination of the curves to the full opening discharge (if it could be secured) is $0.64\sqrt{h_1/h}$, where h_1 is the height of the end verticals and h is the head of water. For values of $h/h_1 = 400, 100, 25$ and 10, respectively, the percentage reductions in discharge are 3, 6, 13 and 20. Unless the notch curves are accurately constructed, large errors in estimated discharge are to be expected. For results of tests see *Eng. News*, Nov. 25, 1915.

Velocity-of-approach effects, causing a greater discharge than from a deep quiet pool with the same head, have been variously expressed in the formulas adopted by different engineers. The Bazin and Rehbock formulas avoid the direct use of $v^2/2g$, where v is the mean velocity through the cross-section of the channel of approach upstream from the weir, where the head is measured. They provide for the changes in the discharge coefficient by expressions involving both the head and height of weir, but on the assumption that the distribution of velocities in the channel of approach is fairly uniform. The Fteley and Stearns and the Hamilton Smith formulas add $a(v^2/2g)$ to the observed (static) head, each using a constant value for a, (1.5 and $1\frac{1}{3}$ respectively, for weirs without end contractions), thus also assuming fixed natural velocity distribution, yet recognizing that the flow-assisting velocities near the surface of the channel of approach are usually greater than those near the bottom.

Recent experimental evidence favors the **Rehbock Formula** as probably the best of the rigid type of weir formulas for normal, or fairly uniform, velocity distribution in the upstream channel. This is for sharp-crested weirs without end contraction:

$$Q = \left(0.605 + \frac{1}{320h - 3} + \frac{0.08h}{d_0}\right)\frac{2}{3}Lh\sqrt{2gh}, \text{ (ft-sec units)},$$

$$= \left(3.234 + \frac{5.347}{320h - 3} + 0.428\frac{h}{d_0}\right)Lh^{3/2}$$

where d_0 is the height of weir or the depth of water at zero head. (See "Precise Weir Measurements" and discussions, *Trans. A.S.C.E.*, Vol. 93, 1929.) Although this is not strictly correct dimensionally, it gives values

rant rounding of the entry edges and d is the diameter of the orifice or the side of the square. Values of this constant are given below:

R/d, percent	33.3	16.7	8.3	6.2	4.2	3.1	2.1	1.0	0.0
Discharge coefficient	0.95	0.94	0.92	0.90	0.88	0.85	0.84	0.82	0.80

Several openings side by side (*e.g.*, headgates, intake screens, and baffles) cause an increase in the discharge, if submerged on the downstream side. For openings close together, increases of from 5 to 10 percent have been measured.

Velocity of Approach. If, upstream from the orifice, there is a high velocity of approach due to a small approach channel or passageway or due to peculiar flow conditions, the **velocity head**, $V^2/2g$, should be added to the static head to obtain h for the formula. The percentage effect is seldom large unless the channel is so small as to interfere with the flow by reducing contraction, etc., and then, of course, there will be considerable uncertainty. (See Nozzles, below.)

Pipe orifices (or orifice meters), *i.e.*, orifices in diaphragms inserted between pairs of flanges in straight portions of pipe lines, and **orifices in end-caps** on pipes, have been found to be reliable measuring devices. For a discussion of such orifices, see Head Meters, p. 1803.

The Miner's Inch. In some of the western states, water for mining and irrigation is sold by the **Miner's Inch**, which is the quantity of water that will flow through 1 sq in. of an opening in a plank under a head of (usually) 6 in. The arrangement of the opening (from 1 to 4 in. high), the thickness of the plank (from 1 to 3 in.), the datum

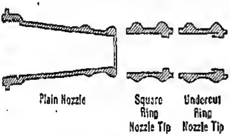


FIG. 14.—Types of Nozzles.

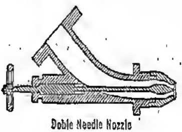


FIG. 15.

level for the measurement of the head (the top, center, or bottom of opening, as well as the head itself (from 3 to 9 in.), are largely matters of local custom and of state law. In Cal. and Mont., 40; in Colo., 38.4; and in Ariz., Idaho, Nev. and Utah, 50 miner's inches = 1 cu ft per sec. In some old deeds in the eastern states, water rights were sold by inches, but this commonly meant the discharge through 1 sq in. of some portion of an old-fashioned water wheel under the head at the particular power site, and, of course, is an extremely variable unit, the subject of many controversies in courts.

Oils, if not too tarry or viscous, and mercury have been found to have practically the same coefficients as water.

Time Required to Empty Tanks. An open tank whose cross-sectional area is uniform throughout its depth (*i.e.*, a vertical cylindrical or prismatic tank) will empty itself through an orifice, or a pipe (the discharge point being at the same level as the bottom of the tank), if there is no inflow, in just twice the time required to discharge the same amount of water under the initial head. For other shapes of reservoirs, the time may be computed by dividing the volume into five to ten horizontal layers and computing the time for each layer to discharge under the average head for that layer.

Nozzles. Figure 14 shows typical forms of nozzles. For **rounded-entrance flow nozzles** for the measurement of flow, see p. 1805. The **orifice formula**, modified to include velocity of approach, applies to the flow from nozzles. **Ring and needle nozzles** (Figs. 14 and 15) cause contraction

with the unsubmerged condition, until the downstream head, or backwater, is nearly one-half the upstream head above the crest. This is especially true for broad-crested dams, whether flat or rounded or with easy upstream sloping faces, since in these cases there is a considerable surface drop to the water and a decided increase in velocity even before the water leaves the crest. This condition remains practically unchanged with submergence until the downstream backwater head is over half the upstream head, *i.e.*, high enough to smother the flow.

For experimental results on weirs, see "Weir Experiments, Coefficients and Formulas," by R. E. Horton (U. S. Geol. Survey *W. S. & I. Paper* No. 200)

Flow of Fluids in Pipes

In any pipe or conduit through which a fluid is flowing, there is a continuous **loss of head** and of pressure. Figure 20 shows a portion of a pipe line through which fluid is being forced uphill from A toward B. With no flow, *i.e.*, only static fluid pressure, the head at B should be h ft less than at A. But, owing to the flow, there is loss of head along the pipe and the head at B is $(h + h_f)$ ft less than at A, where h_f designates the friction head between A and B. At the same time, the heads at both A and B are less than with the no-flow condition because of the loss of head upstream from A.

The loss of head or **friction head** depends on (1) the kind of fluid flowing, especially its viscosity; (2) the velocity of the fluid; (3) the size (*e.g.*, hydraulic radius) of the pipe or conduit; (4) the roughness of the interior surface; and (5) the length. Pressure has practically no effect on the loss of head because the viscosity of liquids and gases varies only slightly with pressure within the range ordinarily occurring in practice. The *pressure drop* due to friction $p_f = h_f/w$ depends on the density and therefore, for gases, on the pressure in the pipe.

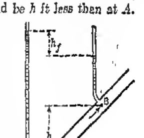


FIG. 20.

Viscous, Laminar, or Streamline Flow. A forced movement of fluid through a filled pipe is resisted by molecular forces at the inside wall, and slipping of the adjacent layers of fluid occurs, the result being a sort of telescopic sliding motion. If the rate of flow is not forced beyond the critical velocity, this viscous flow prevails throughout the entire cross section; in a circular pipe, the central core moves at twice the mean velocity. **Poiseuille's formula,** originally found experimentally in 1842 for capillary tubes, is now known to be applicable for the viscous flow of liquids and gases in all sizes of pipes. It is $h_f/L = 0.0326(\mu/s)(V/D^2)$, cgs units; where h_f is the loss of head in length L both in the same units; μ the viscosity, poises; s the specific gravity (weight, grams per cc); V the velocity, cm per sec; and D the diameter, cm. For V in fps and D in in., $h_f/L = 0.154(\mu/s)(V/D^2)$. For Q in cfs and D in in., $h_f/L = 28.3(\mu/s)(Q/D^4)$. The loss of pressure p in pounds per square inch per 1,000 ft of pipe is given by $p = 27.3\mu G/D^4$, where G is the flow, in U. S. gal per min; D the diameter, in.; and μ the viscosity, poises. Using B for barrels per hour (1 bbl = 42 U. S. gal), $p = 19.1\mu B/D^4$. The roughness of the pipe has practically no effect on true viscous flow except as the effective cross-sectional area may be less than for a clean pipe, and consequently a smaller value for D appropriate.

The Critical Velocity and Turbulent or Eddy Flow. When the rate of flow is forced beyond the critical velocity, turbulent or eddy flow sets in.

necessity of special tests. The writer tested, as a nozzle on a 6 in. W. I. pipe, a 6 × 3 in. rusted, riveted-steel reducer, 24 in. long, with projecting flat rivet heads at both ends and along a longitudinal seam. The discharge coefficient was 0.91 under heads of from 2 to 50 ft, allowance being made for rivet heads reducing the tip area. This may be taken as the low limit for an **extra rough conical nozzle.** In case of such a nozzle the coefficient 0.91 means that 17.2 percent of the head is "lost," *i.e.*, used up in overcoming friction.

The Venturi Meter. This device was invented by Clemens Herschel in 1887, and named after Venturi, who observed the principle in 1791. The general formula of this useful flow meter is discussed on p. 252. Referring to Fig. 9, the practical formula connecting the measured difference of pressure heads, $H_1 - H_2 = h$, with the mean velocity of flow in the throat is the same as the nozzle formula on p. 258. For coefficients, see p. 1806.

The nominal size of a Venturi meter is that of the pipe line in which it is to be placed. The throat diameter usually is made some size between 0.25 and 0.75 of the upstream diameter, depending on the rate of flow to be expected and on the pressure in the line. The throat velocity with the low rates of flow should be great enough so that the difference of pressure heads (often called the **Venturi difference**) will cause a measurable indication on the register or gage, but should not be so great as to cause the throat pressure to drop below atmospheric, lest there be trouble with air leaking into the gage and register connections. The upstream convergence angle of a Venturi meter may be from 25 to 30 deg, but the downstream divergence angle should not be greater than 7½ deg. The pressure connections usually are made to **ring piezometers**, consisting of circumferential passageways communicating to the interior of the meter by four or more small holes equally spaced around the pipe. The object is to assure the obtaining of the average pressure by avoiding dependence on a single piezometer hole subject to possible local disturbances. The ring piezometers are cast as part of the meter tube. The meters are usually made of cast iron and in several sections, bolted together. The throat is bronze-lined, very accurately bored to size and very smoothly finished. Venturi meters have been made of concrete and wood staves, the throats being lathe-finished bronze castings.

For the **loss of head** caused by the presence of a Venturi meter in a pipe line, see p. 1806.

Flow over Dams and Weirs

Weirs. A weir is a **bulkhead or dam** over which water flows, or a **notch** in the top of such a structure through which water flows. An orifice becomes a weir if the water surface upstream falls below its top edge. The velocity of approach is sometimes considerable, as for a low dam in a river at flood time, or it may be practically negligible, as for a small notch in the side of a large tank.

The theoretical discharge over a weir of width l is $Q = \frac{2}{3}lh\sqrt{2gh}$. This is two-thirds of the theoretical discharge through a rectangular orifice l ft long and h ft high under a head h. Actually, for a sharp-crested weir, contraction occurs at the crest. The under surface of the falling sheet of water rises as it leaves the sharp crest, just as for an orifice. The discharge coefficient is about 0.62, *i.e.*, $Q = 0.62(\frac{2}{3}lh\sqrt{2gh})$. This formula, for which the product of the constants = 3.33, may be expressed as $Q = 3.33lh^{3/2}$, and as such is known as the **Francis formula** for sharp-crested weirs. Q is in cubic foot per second and l and h are in feet. (See Weir Discharge Scale, Fig. 17.)

By a **sharp crest** is meant one with a sharp upstream corner from which the water springs. The crest itself may be of considerable thickness (but

(= cross-sectional area/wetted perimeter). In calculating the value of the Reynolds number, which is dimensionless, any consistent system of units

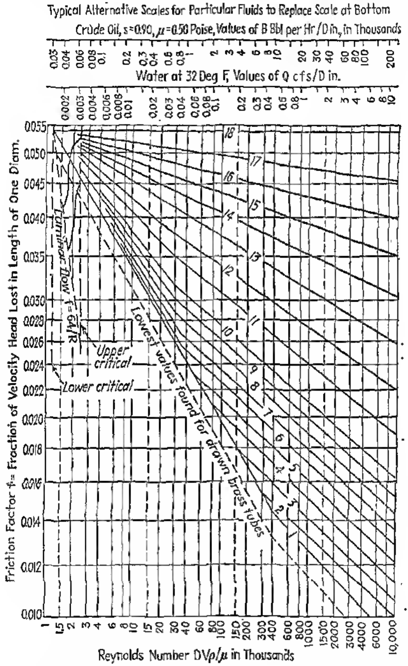


FIG. 21.—Friction Factors for Flow of Fluids in Pipes (to be used in conjunction with Table 4).

may be used. It is frequently convenient to use an equivalent of the Reynolds number R in which volume or mass flow replace velocity and in which

two end contractions is

$$Q = 3.33(l - 0.2h)h^{3/2}$$

This formula must not be used unless l is at least $2 \times h$.

See "Flow through Weir Notches [rectangular, triangular, Cipolletti, circular] with Thin Edges and Full Contractions," by V. M. Cone, *Journal of Agricultural Research*, U. S. Department of Agriculture, Vol. 5, No. 23 (1916); and a variety of weir-notch data by W. S. Pardoe in his discussion of "Precise Weir Measurements," *Trans. A.S.C.E.*, Vol. 93 (1929). See also Narrow Rectangular Notches, p. 262.

Cipolletti Trapezoidal Weir. Cipolletti found that a trapezoidal notch with end slopes 1 horizontal to 4 vertical (Fig. 18) just compensated for the reduction due to end contraction in a rectangular notch of the same crest length, and, accordingly, the simple Francis formula, $Q = 3.33lh^{3/2}$, may be used for such a trapezoidal notch. The crest length l must be at least $2 \times h$, and the crest should be 2 or $3 \times h$ above the bottom of the channel of approach to avoid a velocity-of-approach correction (see Narrow Trapezoidal Notches below). The Cipolletti weir has become popular in the western states for measuring irrigation water.

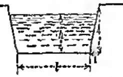


FIG. 18.—Cipolletti Weir.

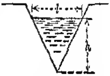


FIG. 19.—Triangular Notch Weir.

Triangular Notch. There are certain advantages in the triangular notch for measuring discharges that vary from a moderate maximum to a very small minimum, *e.g.*, 1 gal per min, and where about the same degree of precision is desired whether the discharge is high or low. The formula for a sharp-edged notch (Fig. 19) is $Q = C(\frac{4}{15}lh\sqrt{2gh})$, where l is the width of the notch a distance h above the vertex, and C is a coefficient with a value of from 0.58 to 0.59, based on experiments at Cornell University with heads up to 3 ft. and vertex angles of 28, 60, 90 and 120 deg. The average coefficient 0.58 was found for the 60 and 90 deg weirs, and 0.59 for the 28 and 120 deg weirs. These results are close to those of Cone (*loc. cit.*). Prof. Thomson found for 90 deg, 0.60, and for 127 deg, 0.62, heads 0.2 to 0.8 ft. Hence the discharge through a triangular notch $\left(\text{for which } l = 2h \tan \frac{\text{notch angle}}{2}\right)$ is from 38 to 40 percent of that given by the discharge scale (Fig. 17).

Very precise experiments on 90 deg V notches, with heads up to 10 in., have been made by James Barr (London *Engineering*, Apr. 8 and 15, and Oct. 28, 1910). With smooth upstream face and wide and deep tank, the coefficients at the same heads on several weirs were found constant within $\frac{1}{3}$ of 1 percent for heads ranging from 3 to 10 in., and within 1 percent for heads as low as $1\frac{1}{4}$ in. The tests show noticeable changes in discharge due to variations in roughness of upstream face of the notch plate. A surface like coarse emery increases the discharge over that for smooth brass some 2 percent (for a head of 0.3 ft). It was found also that the tank upstream from the notch should be seven to eight times the head in width and three to four times the head in depth below the vertex to avoid, respectively, increases and decreases up to 1 to 2 percent in discharge with narrow and shallow tanks. As above, $Q = C(\frac{4}{15}lh\sqrt{2gh})$. For a 90 deg notch $l = 2h$ and $Q = C(\frac{8}{15}\sqrt{2g})h^{5/2} = Kh^{5/2}$. With sharp-edged notches in smooth brass plates the formula $K = (0.2907 + 0.028/\sqrt{h})$, proposed by T. P. Strickland, applies. This is for Q in cu ft per min and h in in. For Q in cu ft per sec and h in ft, $K = (2.42 + 0.067/\sqrt{h})$ and $C = (0.565 + 0.016/\sqrt{h})$.

nolds number which may be calculated from any of the following expressions:

$$R = 77.4\frac{DV}{\nu} = 14{,}200\frac{Q}{\nu D} = 31.6\frac{G}{\nu D} = 22.2\frac{B}{\nu D} = 3.94\frac{(W/w)}{\nu D}$$

where ν is the kinematic viscosity ($= \mu/\rho$, where ρ is density, g per cc, or, very closely, $= \mu/s$), w the weight density, lb per cu ft, and the other quantities are in the units given above.

For non-circular conduits or passageways, substitute $4m$ for D in the expressions for the Reynolds number, where m is the hydraulic radius, in. (= cross-sectional area, sq in. divided by wetted perimeter, in.)

Examples. *Plain Pipe Problem.* A reservoir is to be supplied with cold water by a pump through a cast-iron pipe line 2,800 ft long containing several elbows, T's, etc, and making the total equivalent length (see p. 271) 3,000 ft. The difference of elevation is 100 ft, and the rate of pumping is to be 200 gpm. What size of pipe should be laid, and for what working pressure should the pump be specified? *Solution:* The pump must force water against a head of 100 ft plus the friction loss of head. The practical problem is to avoid either too small a pipe with its large friction head and costly operation or one too large with unjustifiable first cost in view of the saving of head. Using the formula for the discharge units given in p. 265, $h_f = fLG^2/32.1D^5 = (3000 \times 40{,}000/32.1)(f/D^5) = 3{,}740{,}000f/D^5$. For the value of f, from Fig. 21, first compute Reynolds number (see above). Assuming water at 32 F, $\mu = .0179$ poise. $R = 31.6\ G\rho/\mu D = 31.6 \times 200 \times 1/0.0179D = 353{,}000/D$. Approximate calculations suffice for the value of Reynolds number for different assumed diameters. Trying $D = 4$ in., $R = 353{,}000/4 \approx 90{,}000$, and Fig. 21 shows, for category D, line 7, $f \approx 0.024$. Then $h_f = 3{,}740{,}000 \times 0.024/1{,}024 = 88$ ft. A 5 in. pipe, with $f = 0.025$, gives $h_f = 30$ ft; a 6 in. pipe, $h_f = 12$ ft; an 8 in., 3 ft; a 10 in., 1 ft; a 12 in., 0.4 ft, etc. The heads against which the pump must operate would be, for the 4, 5, 6, 8, 10, and 12 in. pipes, respectively, 188, 130, 112, 103, 101, 100.4 ft. The choice would be, probably, between the 6 in. pipe with a total head of 112 ft and an 8 in. pipe with 103 ft. If there were likelihood of some increase in demand beyond the specified 200 gpm, the 8 in. pipe would be preferable.

To Find Frictional Loss of Pressure. It is required to pump 100 bbl per hr of an oil of 20 deg Baumé gravity and 250 sec Saybolt viscosity through a 3 in. pipe. What pump pressure per 1,000 ft of pipe must be provided? *Solution:* The specific gravity for 20 deg Baumé is 0.933, and by the formula on p. 244 the absolute viscosity is 0.51. Reynolds number (see above) $= 22.2 \times 200 \times 0.933/0.51 \times 3 = 2710$, for which $f = .045$ (from line 6, Fig. 21, and category B, Table 4). Then $p = 0.0065 \times 0.045 \times 1000 \times 0.933 \times 10{,}000/243 = 11.4$ lb per sq in per 1,000 ft, or 60 lb per mile. The actual inside diameter of standard weight 3 in. pipe is 3.07 in. In the above example the use of 3.07 instead of 3 in. changes the result to 10.2 lb per sq in. This emphasizes the effect of small changes in diameter (see Percentage Effects, p. 270), and indicates the undesirability of too precise computations in cases where there are likely to be deposits of paraffin or of asphalt on the inside of the pipe.

To Find Proper Diameter. In pumping 1,300 bbl per hr of an oil of sp gr 0.9 and absolute viscosity 0.7, what is the minimum diameter of pipe permissible, if a friction loss of pressure of 160 lb per sq in. per mile is not to be exceeded? *Solution:* In practice, one of the commercial sizes available must be chosen. Proceeding by trial, assume $D = 6$ in., making the value of $R = 22.2B\rho/\mu D = 22.2 \times 1300 \times 0.9/0.70 \times 6 = 6200$, for which curve 4 of Fig. 21 shows $f = 0.037$, making $p = 252$ lb per sq in. per mile. This is nearly 1.6 times the specified 160 lb, thus indicating that a larger pipe is needed. But a relatively small increase in diameter causes a large decrease in friction, since D^5 is involved. Trying 7 in., R is $6/7$ of the former value, 6,200, or 5,300, and $f = 0.038$, making $p = 120$ lb per sq in. per mile. This is smaller than the specified 160 lb, and hence a 7 in. pipe is amply large, but, being a size not in common use, it would probably be preferable to select an 8 in. pipe which demands only 65 lb per sq in. per mile.

If the oil were quite viscous, say 1620 sec, Saybolt Furol ($\mu = 34$), sp gr $= 0.97$ (= 14 deg Baumé), a much larger pipe would be needed. For 12 in. pipe, R is 63.6. This indicates viscous flow, for which (see p. 264), $p = 19.1\mu B/D^4$, or $160/5.28 =$

agreeing within less than 1 percent with experiments on weirs from 0.2 to 4 ft in height with heads from 0.1 to 2 ft subject to the ratio h/d_0 not exceeding 4. For any particular weir and head, the discharge by this formula may be found readily by multiplying the discharge read from Fig. 17 by the ratio of the parenthesis value to the 0.623 corresponding to the Francis formula (p. 259). Thus for a head of 1.605 ft on a weir 3 ft high the parenthesis value is $0.605 + .0020 + .0428 = 0.650$. This is 4.3 percent greater than 0.623, and the 6.77 cfs per ft read from Fig. 17 for the head 1.605 ft is to be increased by this percentage, making it 7.06 cfs per ft of weir crest. Rehbock has suggested a dimensionally correct formula yielding results approximating closely to those of the regular Rehbock formula, but not quite so close to experimental data, *viz.*

$$Q = L\left(3.218 + 0.445\frac{h}{d_0}\right)(h + 0.0041)^{3/2}$$

The added value, 0.0041 ft, is thought to be due to capillarity at the crest.

A more general formula providing precisely for a wide variety in distributions of velocity in the approach channel (*op. cit.*), is

$$Q = 3.33L\left[\left(h + \frac{v_a^2}{2g}\right)^{3/2} + \frac{h}{3.33}\frac{v_b^2}{2g}\right], \text{ or the practical equivalent}$$

$$Q = 3.33Lh^{3/2}\left[1 + \frac{3}{2}\frac{v_a^2}{2gh} + \frac{3}{8}\left(\frac{v_a^2}{2gh}\right)^2 + \frac{v_b^2}{6.66g\sqrt{h}}\right]$$

the value in brackets in the last formula being the multiplier for the discharge by the simple Francis formula (Fig. 17). Here v_a and v_b are, respectively, the mean velocities of approach above and below the crest level in the channel upstream from the weir, through the cross-section where h is measured. This formula can be used only when measurements have been made of not only the head but also the approach velocities, by current meter, Pitot tube, or floats, etc. For precision it is necessary to have certain knowledge of the upstream conditions.

Flow of water over dams varies from about 20 percent less to 20 percent more than for a sharp crest of the same length and with the same head. For **broad flat-crested dams,** the flat top wider than the head, the coefficient 2.64, instead of Francis's 3.33, applies, or, with sufficient accuracy, the discharge is 80 percent of that given by the Francis formula. If the upstream corner is rounded, the discharge may be greater. **Dams with steeply sloping upstream faces** (about 1 to 1) may have coefficients nearly as high as 4, as will also a **thin vertical bulkhead with a rounded upstream corner** (radius = 2 to 3 in. = thickness of bulkhead). **A very gradually sloping approach,** *e.g.*, 5 horizontal to 1 vertical, or a **rounding crest of large radius** introduces an appreciable friction effect, and the discharge may be no greater than for a sharp-crested weir. Non-vertical upstream faces on sharp-crested dams increase the discharge if inclined downstream and decrease it if inclined upstream, the coefficient being 3.10 for a 1 to 1 upstream and 3.73 for a 2 horiz to 1 vert downstream inclination, coefficients for intermediate inclinations being between these values. If **air is not allowed free access under the falling sheet** of water at the crest, the discharge over any **narrow-top** weir or dam is increased, but is also made less certain due to the tendency of the partial vacuum so formed to break at intervals and cause pulsating flow.

Submerged weirs and dams, where the water surface downstream from the dam is at a higher level than the crest, do not show much reduction in discharge, as compared

balance of discharges, as specified, might not result, but there would be valves in the pipes going to the reservoirs to adjust the discharges from time to time according to the needs, this being a matter for which the design of the pipe sizes, alone, does not provide. If it is desired to make the pipes in *A-B* and *B-C* of the same diameter, more loss of head must be allowed from *A* to *B*, say 8 ft. This gives about a 33 in. pipe for both *A-B* and *B-C* and a 15 in. pipe for *B-D*. If it is planned to limit the diameter for *B-D* to 12 in., the loss of head in *B-D* for this diameter and 5 cfs is first investigated. It is about 60 ft, and since there is only a total head of 30 ft available, a 12 in. pipe is not large enough.

Looping Pipe Problem. When a pipe branches into two or more pipes which again come together or are cross-connected (Fig. 23), the arrangement is known as a "loop." In such a case, there is the same total loss of head for each branch. For example, for fire-protection purposes it is desired to have available at *B* a supply of 2,500 gpm and 100 lb per sq in. hydrant pressure with the water flowing. What pressure must be maintained at *A* with this volume of flow and pressure at *B*? *Solution:* For a fixed h_f, common to the two branches, Q varies nearly as $\sqrt{D^5/L}$. Hence the ratio of the Q's is about $\sqrt{10^5/1000}/\sqrt{8^5/1800} \approx 2.35$. Dividing the 2,500 gpm into two parts, one 2.35 times the other, the 10 in. pipe has 1,755 and the 8 in. pipe 745 gpm. The friction head in each of the two branches is now calculated and compared for the necessary equality with the other branch. The formula is $h_f = fLG^2/32.1\ D^5$, with f corresponding (in Fig. 21) to $R = 31.6\,G\rho/\mu D = 310{,}000$ (for cold water) for the 10 in. and 132,000 for the 8 in. pipe. Figure 21, line 6, (category *D*) gives $f = 0.020$ and 0.022 for the 10 and 8 in. pipes, and the respective h_f values are $0.020 \times 1000 \times 1755^2/32.1 \times 10^5 = 19.2$ ft and $0.022 \times 1800 \times 745^2/32.1 \times 8^5 = 21$ ft. This is a fair check, and the loss of head, *A* to *B*, may be taken as 20 ft, or, say, 9 lb per sq in. drop in pressure, *i.e.*, the pressure at *A* must be 109 lb per sq in. If the whole discharge flowed through the 10 in. pipe alone, the loss of head would be 37 ft, or, 16 lb per sq in. drop in pressure. (For losses in elbows, T's, and hydrant and slight corrections to above, see p. 271.)

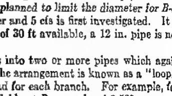


FIG. 23.

Useful Relations. From the formula $h_f = f(L/D)(V^2/2g)$, it is seen that h_f varies as $(L/D)V^2$ if f be assumed constant, which, for a moderate range of values, is approximately true. Introducing into this relation the discharge, $Q(=\pi D^2V/4)$, it is apparent that h_f varies as $(L/D^5)Q^2$. Also, Q varies as $D^{5/2}\sqrt{h_f/L}$, D varies as $Q^{2/5}/(h_f/L)^{1/5}$, and L varies as h_fD^5/Q^2. These relations enable one, at first trial, to make close estimates in cases where it is necessary to compare the (unknown) values of Q, h_f, D or L for two or more pipes in terms of the other (known or assumed) quantities.

Percentage effects due to a change, uncertainty, or error in any one factor: The above relations show that, for any pipe line under consideration, the other factors remaining the same, a small percentage change in available friction head causes ½ that percentage change in the discharge Q, or corresponds to ⅕ that percentage change in diam (in opposite way). A small change in Q corresponds to twice that percentage change in h_f, or ⅖ that in D; in D corresponds to 2½ times that percentage change in Q, or an opposite change of 5 times that percentage in h_f; in L corresponds to ⅕ that percentage change in D, or an opposite change of ½ that percentage in Q.

The considerable effect of small variations in diameter on the discharge and friction head are thus emphasized. To illustrate, if the diameters of two nominal 6 in. pipe lines are actually 5⅞ and 6⅛ in., a difference of 4 percent, the discharge of the latter, *with the same* h_f, *will be 10 percent more and the friction head, with the same* Q, *will* be 20 percent less than the former, due solely to the difference in diameters. If the smaller diameter is in whole or part the result of rusting, tuberculation, or incrustation, the increased roughness will still further decrease the discharge or increase the friction head if the same Q is forced through the pipe.

The loss of head along the pipe for such turbulent flow varies as $V^{1.75}$ for very smooth pipes at ordinary velocities and up to V^2 for quite rough pipes at ordinary velocities or for the smoothest pipes at velocities over 100 ft per sec; also it no longer varies inversely as the cross-sectional area, as for streamline flow, but more nearly inversely as the circumference (or the rubbing perimeter), actually as about $1/(\pi D)^{1.25 \text{ to } 1.16}$; also it now varies as a power of the viscosity much less than 1, probably about 0.25 to 0.15.

The formula $h_f = f(L/D)(V^2/2g)$ may be used generally for the turbulent flow of liquids and gases in long pipes. The friction factor f is a dimensionless number, L is the length of the pipe, D its diameter, V the mean velocity, and g the gravity acceleration. By extension, the same formula serves for *viscous* flow, when $f = 64/R$ and R is the Reynolds number. The values of f in Fig. 21 (a modification of Pigott's diagram) cover turbulent flow, and the formula is applicable to the flow of both liquids and gases in pipes. It may also be written

$$h_f = 6270fLQ^2/D^5 = fLG^2/32.1D^5 = fLB^2/65.6D^5,$$

where h_f and L are in the same units, D is in in., Q in cfs, G in U.S. gal per min, and B in bbl per hr. For other forms of this formula, see General Formulas for Flow of Fluids in Pipes, p. 267. For Flow of Gases see p. 272.

In arriving at a reasonably reliable formula for the ordinary pipes of engineering practice, there is considerable difficulty, chiefly for the following reasons: (1) There exists no standard of roughness; (2) the degree of roughness of a pipe's interior surface does not remain constant in service. The experimental data do not indicate precise coefficients but show a rather wide range of values from which the investigator must select limits and averages according to his judgment. (See Pigott, *Mech. Eng.*, Aug., 1933, and Kemler, *Trans. A.S.M.E.*, 55, 1933.)

In one case a few months' rusting of a 4 in. wrought-iron pipe increased the friction 20 percent. In another, incrustations due to 17 years' service increased the friction of a 48 in. cast-iron pipe 70 percent. An unlined linen hose causes over twice the resistance of a high-grade rubber-lined hose. One year's growth of slime in a large aqueduct increased the friction 20 percent. In these cases the discharges were reduced about half as much as the above-stated percentages for the same head.

Attainable Precision. A pipe, when new, may possibly discharge from 1 to 10 percent more or less than did some other pipe line apparently exactly like it. This difference in performance may be expected to change appreciably after water has been flowing and standing in the pipes for several months, and quite pronouncedly after several years, especially with different waters and in different climates. Nevertheless, data are available that serve to indicate approximately how much such service effects as "a slight layer of rust," "a thin deposit of lime," "a growth of slime," "considerable tuberculation," "heavy incrustation," etc., may be expected to reduce the discharge through a pipe line or increase the loss of head if the same discharge is maintained. The formulas and diagrams given herewith make due allowances for such of these effects as are stated with them.

Reynolds Number and Critical Velocity. Whether the flow in a certain pipe or other conduit is turbulent or viscous, i.e., whether it takes place above or below the critical velocity, is determined by the corresponding critical value of Reynolds number $VD\rho/\mu$, where V is the mean velocity, D the diameter, ρ the density (mass per unit volume), and μ the absolute viscosity of the fluid in the pipe. There is viscous or streamline flow if this number is less than about 2,000, or if $Q\rho/D\mu (= (W/w)\rho/D\mu)$ is less than about 1,570, where Q is the volume per second, W the total weight per second flowing, and w the weight per unit volume. For non-circular conduits or passageways, $4m$ is to be substituted for D, where m is the hydraulic radius

for a moderate range of diameters such as is found in any one class of work, *e.g.*, house-service pipes, street mains, power conduits. Actually, these resistances are not entirely concentrated in the fitting itself. They create a disturbed abnormal flow that does not return to normal until some distance downstream. The values given in Table 5 are for the total extra loss. The second column is based on $L/D = 40$ for $h_f = V^2/2g$ or, $f = 0.025$.

Problem of Short Pipe with Several Bends. A fairly smooth wrought-iron pipe line 150 ft long is to connect two tanks and must have a capacity of 15 cfs with a 20 ft head. There is a square-edged entry and there are four 90 deg short-turn elbows and two tees (with 90 deg deflections for the main line). What size pipe is proper? **Solution.** The total head is used up in supplying the loss at entry, the friction loss along the pipe, the losses in the elbows and tees, and finally the terminal velocity-head. (See Table 5.) Then $20 = (0.5 + f(L/D) + 4 \times 0.75 + 2 \times 1.50 + 1)V^2/2g$, or $20 = (7.5 + 150f/D)V^2/2g$. Figure 21 shows that f does not change much as D changes, since for a fixed Q the smaller D the larger is V. A value 0.017 seems to be about right for fairly smooth pipes from 12 to 20 in. with a velocity = about 10 fps. Substituting from $V = Q/A = Q/(\pi/4)D^2$, whence $V^2/2g = Q^2/[(\pi/4)^2 2gD^4]$, $20 = 42.4/D^4 + 14.4/D^5$. By trial, $D = 1.28$ ft, say 16 in. For this value of D, $V = Q/(\pi/4)D^2 = 11.6$ fps.

Flow of Gases in Long Pipes. For steady isothermal flow of nearly perfect gases through long pipes, three cases are readily distinguishable: (1) where the difference between the absolute pressures at the upstream and downstream ends of the pipe is small, say less than 10 or 15 percent; (2) where the difference is large but where the change in inertia due to change in velocity is negligible in comparison with the friction; (3) where both the difference between the end absolute pressures and the change in kinetic energy are relatively large.

For case (1), formulas of the type shown on p. 267 are applicable, $p_1 - p_2 = 2720fLsQ^2/D^5$, or $= fLW^2/(1.434D^5s)$. Here $p_1 - p_2$ is the drop in pressure in pounds per square inch between the upstream and downstream ends, Q and W are, respectively, the volume in cfs and the weight in pounds per second flowing; other symbols as on p. 267. For computing s (the specific gravity of the gas relative to water at max density), the mean of the terminal absolute pressures and temperatures may be used. For case (3), the formula is $p_1^2 - p_2^2 = \frac{W^2}{A^2 g}\frac{p_2}{w_2}\left[\frac{fL}{D} + 2\log_e\left(\frac{p_1}{p_2}\right)\right]$, and for case (2) the second term in the brackets may be omitted. For example, if the pipe is 5,000 diam long with $f = 0.020$, the first term $fL/D = 100$; and for $p_1/p_2 = 2$, the second term $2\log_e(p_1/p_2) = 1.39$ which is practically negligible in view of the magnitude of the first term and of the uncertainties in the proper value for f and other factors. But if $L/D = 1{,}000$ and $p_1/p_2 = 25$, the two terms become 20 and 6.44, and the latter cannot be ignored. For case (3), it may be expedient to neglect the second term for a first-trial solution. In place of the ratio p_1/p_2, the equal ratios w_2/w_1 or V_1/V_2 may be substituted because steady flow in a pipe of uniform diameter is assumed.

Flow of water in very small tubes is viscous flow. The formulas on p. 265, with values of f from Fig. 21 may be applied. For viscous flow of water in tubes 0.001 to 1 in. diam, the formula $V = 500(h_f/L)D^2(t + 10)/60$ gives close agreement with Poiseuille's formula between 40 and 100 F. Values of h_f and L must be in the same units, t in deg F; D in in., V in fps.

Flow of Light Crude Oils in Pipes. The resistance to the flow of oils lighter than about 30 deg Baumé (sp gr, 0.875) is not much different from that of water. The value 96 for C in the Chézy formula ($V = C\sqrt{mS}$, see p. 276) is sometimes used in designing

the units are not consistent. If Q is the volume flowing in cfs, s is the sp gr referred to water at max density, D is in in., G is U. S. gal per min, B is bbl per hr, W is weight flow, lb per hr, w is weight density, lb per cu ft, and μ is in poises, then the condition corresponding to the critical value $R = 2000$ is given by

$$Qs/D\mu = 0.141; \quad Gs/D\mu = 63; \quad Bs/D\mu = 90; \quad Ws/D\mu w = 508$$

These values indicate where the flow, if gradually decreased, ceases to be turbulent and true viscous flow begins. Turbulent flow may prevail with the value of $VD\rho/\mu$ as small as 1,200, and an intermediate or an alternating type of flow may occur under some conditions in a particular pipe for values between 1,500 and 3,000; in fact, under extraordinarily quiet conditions fully viscous flow may prevail for much higher Reynolds numbers.

General Formulas for Flow of Fluids in Pipes. If Δp is the pressure drop, lb per sq in., in a length of L ft of pipe of diam D in. discharging G g (U. S.) pm. or B bbl per hr or W lb per hr of a fluid of sp gr s referred to water at max density and of absolute viscosity μ poises, and if V is the mean velocity of flow in the pipe, fps, then the pipe formula may be written

$$\Delta p = \frac{0.081fLsV^2}{D} = 2720\frac{fLsQ^2}{D^5} = \frac{0.0134fLsG^2}{D^5} = \frac{0.0066fLsB^2}{D^5} = \frac{fLW^2}{1858 \times 10^4 D^5 s}$$

The value of f may be taken from Fig. 21 and Table 4 for a value of the Rey-

Table 4. Roughness Classification of Pipes by Material and Diameter, for Selection of Friction Factor f in Fig. 21

Curve No. to be used in Fig. 21	Diameter, in.: for (*A*), actual inside diam; for (*D*), (*E*), (*F*), nominal inside diam; for (*B*) and (*C*), nominal size of standard weight. [For extra- and double-extra-strong, use standard pipe of approx same inside diam, *e.g.*, ½ in. double-extra-strong corresponds to ⅛ in. standard weight]					
	(*A*) Drawn tubing, brass, tin, lead, glass, [uncorroded, free from thick deposits—otherwise use (*B*) or (*C*)]	(*B*) Clean steel and wrought iron. [If deteriorated, use (*D*) or (*E*)]	(*C*) Clean galvanized. [If deteriorated, use (*D*) or (*E*)]	(*D*) Best cast iron (coated), cement, light riveted sheet ducts	(*E*) Average cast iron, rough-formed concrete	(*F*) First class brick, heavy riveted, spiral riveted
1	0.35 up	72				
2	......	48-66				
3	......	14-42	30	48-96	96	220
4	......	6-12	10-24	20-48	42-90	84-204
5	......	4- 5	6- 8	12-16	24-36	48- 72
6	......	2- 3	3- 5	5-10	10-20	20- 42
7	......	1½	2½	3- 4	6- 8	16- 18
8	......	1-1¼	1½-2	2-2½	4- 5	10- 14
9	......	¾	1¼	1½	3	8
10	......	½	1	1¼	.....	5
11	......	⅜	¾	1	.....	4
12	......	¼	½	.....	.....	3
13	......	⅛				
14	0.125	.....	⅜			
15						
16	......	.....	¼			
17						
18	0.0625	...	⅛			

Such a stream (see Figs. 26 and 27) will be effective as a good fire stream for a height of 60 ft above the nozzle or for a horizontal distance of about 57 ft, the nozzle being pointed at angles above the horizontal of about 75 deg and 32 deg respectively, for farthest throw.

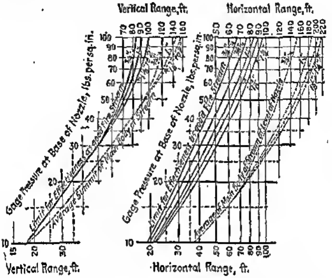


FIG. 27.—Vertical and Horizontal Ranges of Fire Streams.

Extra Losses of Pressure. With as many streams flowing as there are connections on the hydrant, the loss of pressure ranges from about 2 to 10 lb per sq in., but the total loss from street main to hose connection need not exceed 5 lb with ample-size waterways in the pipe connections and a well-designed hydrant. **Projecting washers** at the hose couplings cause an extra loss of head; *e.g.*, a single washer 3/16 in. thick with a hole 2.28 in. diam used in a 2½ in. hose was found to cause an average loss of 0.56 lb per sq in., and a 2 in. washer caused a loss of 3.11 lb per sq in., the rate of flow being 240 gpm or a velocity of about 16 fps. From this an idea may be gained of the resistance caused in small wrought-iron pipes by failure to remove inside burrs due to cutting off with wheel cutters. Failure to open fully hydrant valves causes much loss of pressure.

Practical Recommendations for Fire-protection Installations. Hydrants should be located at such intervals that not over 300 ft of hose need be laid in any one line, and so that about ten first-class fire streams can be concentrated on any large building. The drop will amount to 14 lb per sq in. for each 100 ft of the very best 2½ in. hose, and as high as 30 lb for a rough quality hose (when 240 gpm are flowing). A main pressure of 100 lb per sq in. during a fire is desirable, but, if the hose lines are rough and 500 ft long, fire protection cannot be assured unless a pressure of 200 lb is maintained at the hydrant. For important buildings the mains and laterals to hydrants should be at least 6 in. in diameter and as short as possible.

Water Hammer

Water hammer is the series of shocks, sounding like hammer blows, produced by suddenly checking the flow of water in a pipe. If a valve, turbine-gate, or faucet is suddenly closed, the kinetic energy of the arrested column of water is expended, if no relief devices are provided, in compressing the water and in stretching the pipe walls. Starting at the suddenly closed valve,

$19.1 \times 34 \times 1300/D^4$, whence $D = 12.9$ in. For a 14 in. pipe, $p = 116$ lb per sq in. per mile. As a check, $f = 64/R$ for viscous flow, and $p = 0.0066\, fLsB^2/D^5$.

To Find Discharging Capacity. For 15 miles between pumping stations with working pressures of 900 lb per sq in., what is the rate of flow for an 8 in. pipe, for an oil of specific gravity 0.92 and absolute viscosity 2.0? *Solution:* To get an approximate value for f, assume a value for B, say 1,000 bbl per hr, making $R = 1285$ and $f = 0.054$. Then $900 = 0.0066 \times 0.054 \times 79{,}200 \times 0.92B^2/8^5$, whence $B = 1{,}065$ bbl per hr. Checking back on R and f with this value of B, $R = 1{,}360$ and $f = 0.053$, close enough to the 0.054 used. Hence $B = 1{,}065$ bbl per hr may be taken as correct.

If the oil were very viscous, with $\mu = 40$ and $s = 0.96$, then an assumed 1,000 bbl per hr makes $R = 66.6$, indicating fully viscous flow. Then $900 = 13.1 \times 79.2 \times 40B/8^4$, whence $B = 610$ bbl per hr.

In cases of *turbulent flow*, when adjusting incorrectly assumed values of Q or D, it should be remembered that, for the same kind and length of pipe, p varies roughly as Q^2/D^5 (see Useful Relations, p. 270), or more exactly as $Q^{1.8}/D^{4.8}$ as an average for clean pipes.

Compound Pipe Problem. A pipe line is to be made up of stretches of several different diameters in series, the same Q flowing through all. Required to find the discharge Q for a given total loss of head. *Solution:* Apportion the head properly among the several pipes, and then check to find if the Q's are equal. Approximate results may be obtained quickly by noting that, for a fixed Q, h_f varies nearly as L/D^5, where L is the length of pipe of a certain diameter. Thus, if the total available friction head is 80 ft, and there are 3,000 ft of 12 in. and 2,000 ft of 8 in. pipe, the ratio of the two friction heads is $(3000/12^5)/(2000/8^5) = 0.198$, i.e., about $\frac{1}{5}$ as much loss of head in the 12 in. pipe as in the 8 in. pipe. Hence the available 80 ft for friction head is divided into 13.3 and 66.7 ft, or, per 1,000 ft, into 4.4 and 33.4 ft for the 12 and 8 in. pipes, respectively. With this apportionment of the friction head, the Q's are computed in turn for the 12 and 8 in. pipes, by the formula $h_f = 6270fLQ^2/D^5$, for Q in cfs. The value of f must correspond to the appropriate Reynolds number, $R = 14{,}200Q\rho/\mu D$. Not knowing Q in advance, a rough guess is made in order to get a first approximation to f, say $Q = 1$ cfs. Then R (for the particular fluid involved, *e.g.*, water at 32 F) $= 14{,}200 \times 1 \times 1/0.0179 \times 12 = 66{,}000$ for the 12 in. pipe. Figure 21 line 4, for category B, shows $f =$ about 0.022. Then $4.4 = 6270 \times 0.022 \times 1000 \times Q^2/12^5$, and $Q = 2.8$ cfs. Using this value in place of the assumed 1 cfs, $R = 185{,}000$. This changes f to 0.019, and Q becomes 3.0 cfs. For the 8 in. pipe $R = 300{,}000$, $f =$ about 0.018, and $33.4 = 6270 \times 0.018 \times 1000 \times Q^2/8^5$, or $Q = 3.1$ cfs, a fairly close check on the 3.0 cfs for the 12 in. pipe.

Branching Pipe Problem. It is sometimes necessary to design several branching pipes, fed by a single pipe, to discharge certain definite quantities. Such problems are usually solved by trial. Approximate results are readily obtainable in many cases by assuming a single value of f for all the pipes. The pressure head where the branching occurs is assumed (or else the loss of head to the branching), and the discharges for the several branches are calculated. The sum of these discharges must, of course, equal the discharge of the main pipe. If it does not, a second trial is made. In Fig. 22, the reservoir A is to supply reservoirs C and D, respectively, 10 and 30 ft lower than A.

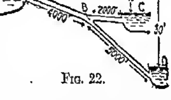


Fig. 22.

A-B = 4,000 ft, B-C = 2,000 ft, and B-D = 5,000 ft. (The nature of the problem is unchanged if A is a pump at a low elevation pumping into two reservoirs or into two districts.) What sizes of pipe are necessary to supply 20 cfs to C and 5 cfs to D? As a first trial, assume 5 ft loss from A to B, leaving 5 ft for B-C and 25 ft for B-D. The discharge Q for A-B is $20 + 5 = 25$ cfs. Now, guessing a medium $f = 0.020$, we have, for A-B, $5 = 6270 \times 0.020 \times 4000 \times 25^2/D^5$, whence $D \approx 36.5$ in. For B-C, $10 - 5 = 6270 \times 0.020 \times 2000 \times 20^2/D^5$, and $D \approx 29$ in. For B-D, $30 - 5 = 6270 \times 0.020 \times 5000 \times 5^2/D^5$, and $D \approx 14.5$ in. These approximate diameters may now be used to obtain practically correct values of Reynolds number and then f from Fig. 21, and the diameters are then recalculated. The nearest market sizes to these are the desired sizes. An exact

Relief Devices. Adequately proportioned air chambers on pumping mains and surge tanks on water-power supply pipes serve to absorb almost entirely the shock of water hammer. Means must be provided for replacing the compressed-air in the air chambers as this is soon absorbed by the water. Relief or safety valves with adjustable springs are not so good for water-hammer shocks, and are more likely to be out of order.

Flow in Open Channels

Flow of Water in Open Channels. Open channels may include canals, flumes, rivers, etc., and also any closed conduit, *e.g.*, a large pipe or a tunnel, when it flows only partly full. For open channels, instead of a loss of pressure head in a unit of length, its equivalent, or the surface slope or surface fall of the stream, is taken. The flow in an open channel, if long and of uniform cross-section, tends to adjust itself to a steady uniform flow, so that the surface is parallel to the bottom.

The Manning Formula. Because of the multitude of different shapes, the dimensions of the cross-section do not appear directly in the general formula, but indirectly in the **hydraulic radius**, or "hydraulic mean depth," m ($= A/w$ *i.e.*, cross-sectional area ÷ wetted perimeter). The Manning formula has been widely accepted, $V = 1.486m^{2/3}S^{1/2}/n$ (ft-sec), or $V = m^{2/3}S^{1/2}/n$ (metric). It is satisfactory for the steady uniform flow of water in open channels and is applicable also to pipes for the ordinary turbulent flow of water at usual low temperatures. Here V is the mean velocity, n the coefficient of roughness (practically equal to the old Kutter n, see Table 6), m the hydraulic radius, and S the hydraulic slope = surface fall/length. This may be regarded as a special form of the old **Chézy formula,** $V = C\sqrt{mS}$, with $C = 1.486m^{1/6}/n$. In terms of the pipe formula (see p. 265), Manning's $n = 0.0926\ m^{1/6}f^{1/2}$ (ft sec units).

Table 6. Values of Coefficient of Roughness, n, for Open Channels

Type of channel	n
ARTIFICIAL CHANNELS OF UNIFORM CROSS SECTION:	
Sides and bottom lined with well-planed timber evenly laid	0.009
Neat cement plaster, smoothest pipes	0.010
Cement plaster (3 cement to 1 sand), smooth iron pipes	0.011
Unplaned timber evenly laid, ordinary iron pipes	0.012
Ashlar masonry, best brickwork, well-laid new sewer pipe	0.013
(This last value should be used for the previous categories, if in doubt as to the excellence of construction and the maintenance free from slime, rust, or other growths and deposits.)	
Average brickwork, foul planks, foul iron pipes, ordinary sewer pipes after average uneven settlement and average fouling	0.015
Good rubble masonry, concrete laid in rough forms, poor brickwork; heavily incrusted iron pipes	0.017
CHANNELS SUBJECT TO NON-UNIFORMITY OF CROSS SECTION:	
Excellent clean canals in firm gravel, of fairly uniform section; rough rubble, "dry paving"	0.020
Ordinary earth canals and rivers in good order, free from large stones and heavy weeds	0.025
Canals and rivers with many stones and weeds	0.03–0.04

Degree of Roughness. Certainty. Open concrete, masonry, metal, or timber flumes can be cleaned periodically, and the removal of slime, weeds, silt deposits, etc., considerably increases their capacity. (See *Eng. News*, Aug. 29, 1912, p. 416.) For earth channels this is not readily done and for pipes it is more difficult and expensive—often impracticable. Open flumes are subject to rapid ice formation in winter in northern latitudes, to upstream retarding wind action, and to very rapid vegetable growth.

Siphons are arrangements of pipe or hose to cause liquids to flow from one level, *e.g.*, A in Fig. 24, to a lower level C, over an intermediate summit B. Before flow will start the pipe must be filled with the liquid, either by pumping from the end or by pouring in at the summit B. Valves are located, usually, at A and C. B cannot be more than 34 ft above A (at sea level) *less* the friction head from A to B *less* the velocity head in the pipe. With this limitation, calculations are made as for any pipe, the head being the difference in level between A and C. If the pressure at B is low, the dissolved air in the water is liberated and may accumulate in sufficient quantity to stop the flow, unless there is provision for removing it. A low summit, a high velocity and a small pipe tend to counteract the accumulation of air.

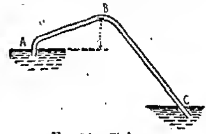


FIG. 24.—Siphon.

The loss of head due to curves, elbows, meters, etc., may be expressed either in terms of the velocity head of the flowing water or as equal to the loss in certain additional lengths of straight pipe. When these lengths are expressed in **pipe diameters** instead of in feet, it has been found that the value is nearly enough constant for a certain source of resistance so that a single average value may be used as sufficiently exact for designing purposes

Table 5. Loss of Head in Flow of Water through Elbows, Meters, Etc.
(Also expressed in equivalent lengths of straight pipe to cause equal loss)

Nature of resistance	Loss of head as a decimal or multiple of $V^2/2g$ (safe avg values)	Equivalent length of straight pipe expressed in pipe diameters
Square-edged entry. Upstream end of pipe flush with inside face of reservoir wall......	0.50	20
Entry like Borda's mouthpiece (see Orifices)..	1.00	40
Rounded entry or very large radius bends....	0–0.05	Zero
90 deg curves, smooth,* same inside diam as pipe:		
Center-line radius = diam of pipe.......	0.50	20
Center-line radius = 2 to 8 diam........	0.25	10
90 deg elbows, common screw end, short turn (experiments on ⅜ to 6 in. ells)	0.75	30
Tees, common screw end, full size branch (experiments on 1 to 4 in. tees)............	1.50	60
Square elbow (intersection of two cylinders)..	1.25	50
Obtuse-angled elbows, deflection in pipe = a deg (less than 90 deg)	$1.25 \times (a/90)^2$	$50 \times (a/90)^2$
Water meters†		
Disk or wobble type.................	3.4 to 10	135–400
Rotary (disk of star or cog-wheel shape as piston)........................	10	400
Reciprocating piston (like a piston pump).	15	600
Turbine wheel type (double flow, balanced)	5–7.5	200–300

For Venturi water meters, see p. 1806, and for hydrants for fire hose, and for inside projecting coupling-washers, see p. 274.
For pipe orifices in diaphragms see p. 1803.
* *Trans. Am. Soc. C. E.*, vol. 62, 1909, pp. 67–112. Also *Bull.* 403, Univ. of Wis., Madison, Wis.
† Different makes of meters and different sizes of the same make vary considerably.

the path of the float is obtainable. By properly distributing floats across the stream, a fair estimate of the total discharge is obtainable if the width and depths have been accurately measured. The use of float measurements is commonly restricted to **surface floats** for rough approximations in reconnaissance, checking other measurements, and in times of high floods when there is too much ice or débris to allow the use of a current meter, and to **rod float measurements** in channels of fairly uniform depth where current meters or weirs are not available. For the surface floats any small pieces of wood will serve for one or two measurements. For more systematic work use $2 \times 2 \times 12$ in. wooden rods, weighted with iron or lead so they will float upright with an inch or so above water. Long rods reaching nearly to the bottom of the stream may be made in the same way, or 2 in. tin cylinders loaded with sand.

For surface floats the common assumption, if nothing is known as to the distribution of velocities from surface to bottom, is that the **mean velocity** is **about 0.8** *of the* **surface velocity**, *but there are frequent cases where the* mean velocity actually is from 90 to 95 percent of the surface, and occasionally the mean is greater than the surface. Hence, errors of 10 to 20 percent *are to be expected on a surface float measurement standing by itself. For long* rod floats, used in uniform-depth channels and adjusted to sink within a few inches of the bottom, careful work will give the discharge within 5 percent of the truth.

Current Meters. The best method of obtaining the discharge of a stream where no weir or dam exists and where the water cannot be measured by volume, is by using a current meter. This instrument is essentially a small wheel like a fan, propeller, or anemometer that revolves when held in flowing water, the speed of revolution depending on the velocity of the water. The meter is commonly calibrated by dragging through still water at different speeds, noting the time and distance for a whole number of revolutions indicated by a telephone receiver, a counter, or a chronograph. Current meters are variously arranged to be lowered into the water by cable, wire or rod. A "tail" like that of a weather vane serves to keep the wheel pointed against the current. When suspended from a wire or cable, a weight below the wheel assists in preventing the meter from being swept downstream. In deep and swift streams, or where the meter is held from high above the surface, a guy wire is used, extending upstream from just above the meter. When the meter *is attached to a rod,* the weight and tail are unnecessary. In order to hold the current meter in various parts of the cross-section of a stream, an overhead bridge, or a boat with anchors or guy lines, may be utilized. *(For stream gaging in winter, see Eng. News,* Sept. 12, 1912.)

Careful observations make it possible to determine the **discharge within 5 percent** if the location has been chosen where the currents are everywhere *fairly parallel to the banks, and if the bed of the stream is not too rough* with large boulders, or too indefinite because of soft mud, for accurate depth measurements as well as velocity determinations. **In canals,** with all conditions favorable, results **within 2 percent** of truth may be obtained by experienced observers.

The **Price current meter,** a cup-wheel meter with a vertical shaft (W. & L. E. Gurley, Troy, N. Y.) and the **Haskell current meter,** a propeller-type instrument having a horizontal shaft (E. S. Ritchie & Sons, Boston, Mass.), are arranged commonly so that the revolutions of the wheel operate a make-and-break in an electric circuit, indicating by a telephone receiver or by a counter register.

pipe lines for crude oil (42–43 deg Baumé, sp gr 0.814 to 0.809). One percent is added for each 3 deg Baumé, *i.e.*, lighter oils flow more easily. Diagrams based on the more practical form of the formula, *viz.*, $B = 1.125D^{2.5}\sqrt{p/L}$, are used, in which B is expressed in barrels (of 42 U. S. gal) per hour and D in inches; p is the necessary pump pressure in pounds per square inch and L the length of line in miles.

Fire Hose and Fire-extinguishing Streams. The roughest sort of hose is the unlined woven linen or cotton, and the smoothest is the heavy rubber-lined hose with the fabric interstices so well filled with rubber that the inside remains smooth even under high pressures. A poor grade of rubber lining or a poor fabric, or both, allows the rubber to be forced into the fabric, when under water pressure, and the interior of the hose may become nearly as rough as if there were no rubber lining at all. In purchasing what purports to be high-grade hose, it should be tested not only for strength to resist bursting but also for loss of head. The diagram in Fig. 25 shows the loss of pressure per 100 ft of length for various rates of flow in various grades of 2, $2\frac{1}{2}$ and 3 in. internal diameter hose. Figure 26 shows the discharge from various sizes of smooth nozzles for various pressures in the hose close to the nozzle. These diagrams are based on J. R. Freeman's experiments, *Trans. A.S.C.E.*, 1889. Figure 27 is a diagram showing the ranges, horizontal and vertical, of "good fire streams" and of "average of main body of stream," for various pressures in the hose line at nozzle. By a good fire stream is meant one that, at the range in question, has not broken into spray, and that projects about three-fourths of the water through a 10 in. circle and nine-tenths of it through a 15 in. circle. The "main body of stream" and the "extreme drops" carry much farther, but the water is too much spread out to be able to penetrate a hot fire and reach the burning materials before evaporating. A first-class fire stream is one from a nozzle of at least $1\frac{1}{8}$ in. tip diameter with at least 40 lb per sq in. pressure in the hose close to the nozzle. For $1\frac{1}{8}$ in. and 40 lb the discharge from a smooth nozzle is 240 gpm.

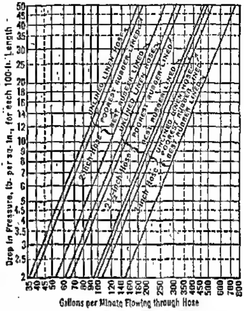


FIG. 25.—Loss of Pressure in 100 Ft. of Fire Hose at Various Rates of Flow.

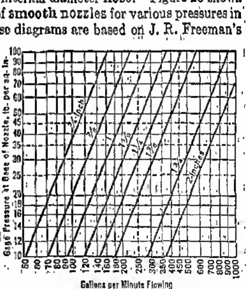


FIG. 26.—Discharge from Fire-hose Nozzles at Various Pressures.

opening projecting a little upstream from the body of the tube (see Fig. 36). The pressure holes in the pipe should be about 1 in. upstream from the impact point of the tube, so that the flow past them will be undisturbed by the presence of the tube in the pipe.

For abnormal flow in curved passages, or wherever a wall opening is subject to centrifugal action due to the flow not proceeding parallel to the wall, a two-opening tube is most convenient, as also where many tests in different pipes are to be made.

A sheath, or elongated stuffing box (Fig. 36), into which the point of the tube can be drawn back out of the pipe, is very convenient and is *practically essential for high pressures* where the flow in the pipe cannot be shut off. A corporation cock (½, ¾, 1 in. or larger, to suit the tube) may be permanently tapped into the pipe. The sheath should have a union coupling adapted for connection to the corporation cock. This gives a shut-off that allows a ready insertion of the tube at any future time. A ¼ in. tap and a gage cock are suitable for the pressure holes when a single-opening tube is used.

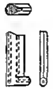


FIG. 34.

FIG. 35.

FIG. 36.

FIGS. 34–36.—Pitot Tubes for Insertion in Pipes.

The float gage *(Fig. 37) is used for determining* the level of a fluid surface and consists of a hollow float of noncorrodible metal or other substance, either spherical or cylindrical with convex or conical top and bottom. A light, rigid stem extends vertically upward, having an index mark that is guided along the edge of a graduated scale or is attached to an automatic recording mechanism. For cases of **great range of levels,** a graduated tape attached to the float and passing over a pulley or system of pulleys, and with a suitable counterweight, makes a sensitive indicator. The tape need be marked only every foot, if it passes the edge of a fixed auxiliary scale with subdivisions.

For precise work the float should be confined against lateral movement by a vertical pipe with internal diameter ¼ or ½ in. larger than the float. For the **greatest accuracy,** where errors of 0.001 to 0.004 ft are inadmissible, the float should be made at least *1 in. smaller than the pipe and should be loosely guided* (at the stem or by rounded radial fins on the float itself) to keep it away from the pipe so as to avoid **capillary lifting.** To lessen oscillations due to surges and waves, the lower end of the float pipe is capped and one or more small openings are made near the bottom to allow equalization of levels inside and outside. The area of the openings may be some 1 to 3 percent of the float-pipe section. For standing liquids in open tanks or reservoirs, the float pipe may be placed either inside or outside the container, as is most convenient. For flowing water in open channels it is placed outside the channel and communication is made by a small pipe, which must end perpendicularly to the inside wall of the channel and without projecting into the channel: *i.e.,* its end should be flush with the inside wall, so as to avoid suction effect (see Fig. 37).

Hook Gage, Point Gage, Plumb-bob Gage (Fig. 37). These are devices for locating the height of a liquid surface by observing its contact with the "point" attached to the movable graduated scale of the gage. The point gage has a sharpened point directed downward in place of the hook of the hook gage. For these gages the scale usually is engraved on a square brass rod. A steel or bronze tape is used for the **plumb-bob gage.** The

a wave of increased pressure is transmitted back through the pipe with constant velocity and intensity. The shock pressure is not concentrated at the valve, but if a bursting pressure is produced, it may show its effects near the valve simply because it acts there first. The velocity of the pressure wave for ordinary cast-iron pipe, 2 to 6 in. in diameter, is about 4,200 fps; for a 24 in. pipe it is about 3,300 fps; it depends on the elasticity of the metal and upon the ratio of its thickness to the diameter of the pipe. If the pipe were perfectly rigid the velocity would be that of sound through water, about 4,700 fps.

The increase of pressure is proportional to the destroyed velocity of flow and to the speed of propagation of the pressure wave. This increase is about 60 lb per sq in. for each ft per sec of extinguished velocity for 2 to 6 in. pipes, and about 45 lb per sq in. for each ft per sec for 24 in. cast-iron pipe. These increases of pressure will be attained only in case the valve is closed in less time than one round trip of the pressure wave.

When the pressure wave has travelled upstream to the end of the pipe where there is a reservoir or a larger main (the whole pipe then being under increased pressure with checked flow throughout), the elasticity of the compressed water and that of the distended pipe reverse the flow at that end of the pipe, and a wave of normal pressure (that of the reservoir or main) travels downstream, the flow being progressively reversed as the compressed water expands. When this wave of normal pressure reaches the valve, the kinetic energy of the column of water with reversed flow tends to create a vacuum at the valve. There the reversed flow is checked and the checking proceeds progressively upstream accompanied by a wave of subnormal pressure. When this wave reaches the upstream end (the whole pipe then being under subnormal pressure), the greater normal pressure in the reservoir or large main starts flow into the pipe, and a wave of normal pressure and forward flow travels downstream. When this wave reaches the valve there is forward flow throughout the pipe, the conditions being the same as when the valve was suddenly closed, and a wave of increased pressure and of checked flow again starts upstream. A complete cycle of pressure waves and reversals of flow occupies the time required for two round trips. The amplitude of the pressure vibrations becomes less with succeeding cycles because of friction, but the time interval remains constant.

If a high-pressure wave, in its travel through the pipe, enters a branch pipe with a closed, or "dead," end, there will be almost a doubling in the increase of pressure when the wave strikes the closed end. In some pipe systems dangerous water-hammer pressures are built up, for, if the back wave from a branch pipe with dead end has access to another branch the high pressure may receive further augmentation.

As the intensity of the excess pressure in the "hammer" wave depends on the amount of "extinguished" velocity, the same excess pressure is produced by suddenly reducing the velocity from 7 to 4 fps as by entirely stopping a velocity of 3 fps. If the flow is not checked rapidly, so that the wave from the first movement of the gate has time to travel upstream to the end and back again several times while the checking is in progress, the excess pressure is very much reduced. Hence, the wisdom of using slow-closing valves on long pipe lines. (See Relief Devices below.)

The excess pressure and the speed of the pressure waves are given by the formulas: $p = V\sqrt{Ew/g}$; $S = \sqrt{Eg/w}$, and also $p = V\sqrt{(w/g)\,(EE't)/(tE' + DE)}$; $S = \sqrt{(g/w)(EE't)/(tE' + DE)}$. [See Church's "Hydraulic Motors," Wiley, and *Proc. Am. Water Works Assn.*, 1904; also "Symposium on Water Hammer," 1933, Joint A.S.M.E. and A.S.C.E.] In these formulas p is the excess pressure intensity and S the speed of transmission of the pressure wave through the water in the pipe. The first two simpler formulas consider the pipe as perfectly inelastic. The last two formulas take into account the elasticity of the metal of the pipe. V is the extinguished velocity, fps, w the weight of 1 cu ft of water, $g = 32.2$, E the bulk modulus of elasticity of water = about 300,000 lb per sq in., E' the linear modulus of the pipe metal = about 30,000,000 lb per sq in. for steel, t the thickness of the pipe metal, and D internal diameter of the pipe. The same system of units should be used throughout. If the ft-lb sec system is used, the above values for E and E' must be multiplied by 144.

DIMENSIONAL ANALYSIS

BY

P. W. BRIDGMAN

REFERENCES: Bridgman, "Dimensional Analysis," Yale University Press. Campbell, "An Account of the Principles of Measurements and Calculation," Chap. XIII, Longmans.

Dimensional analysis is a method by which a partial knowledge of a physical situation may be capitalized and put into available form. The kind of partial knowledge necessary is a knowledge of the general nature of the fundamental equations which govern the system (that is, we must know whether the system is a mechanical, or electrical, or a thermodynamic system), and in addition, the nature of the boundary conditions which, together with the equations, determine the detailed solution in any special case. It is not required that the equations should be actually written out in detail; in fact the utility of the method is largely in its application to problems so complicated that the fundamental equations could not be actually written down, as, for example, in most practical problems of hydraulics.

The method depends on a property of all fundamental equations as ordinarily written, namely that the **form of such equations is independent of the size of the various units.** For example, the fundamental equation of mechanics, force = mass × acceleration, remains true no matter what the size of the units of mass, length, and time. It follows that any result that might be obtained by the solution of such equations must also be independent

Common Quantities and Their Dimensions

MECHANICAL QUANTITIES

Quantity	Dimension	Quantity	Dimension
Area	L^2	Moment of couple, Torque	ML^2T^{-2}
Volume	L^3	Work, energy	ML^2T^{-2}
Frequency	T^{-1}	Power	ML^2T^{-3}
Velocity	LT^{-1}	Action	$ML^{-2}T^{-1}$
Acceleration	LT^{-2}	Intensity of stress, Pressure	$ML^{-1}T^{-2}$
Density	ML^{-3}	Strain	0
Angle	0	Elastic modulus	$ML^{-1}T^{-2}$
Angular velocity	T^{-1}	Compressibility	$M^{-1}LT^2$
Angular acceleration	T^{-2}	Viscosity	$ML^{-1}T^{-1}$
Momentum	MLT^{-1}	Kinematic viscosity	L^2T^{-1}
Moment of momentum	ML^2T^{-1}	Capillary constant	MT^{-2}
Moment of inertia	ML^2		
Angular momentum	ML^2T^{-1}		
Force	MLT^{-2}		

THERMAL QUANTITIES

Quantity	Dimension	
Temperature	t	t
Quantity of heat	H	ML^2T^{-2}
Heat capacity, per unit volume	$Ht^{-1}L^{-3}$	$ML^{-1}T^{-2}t^{-1}$
Heat capacity, per unit mass	$Ht^{-1}M^{-1}$	$L^2T^{-2}t^{-1}$
Temperature gradient	tL^{-1}	tL^{-1}
Thermal conductivity	$HL^{-1}T^{-1}t^{-1}$	$MLT^{-3}t^{-1}$
Entropy	Ht^{-1}	$ML^2T^{-2}t^{-1}$
Enthalpy	H	ML^2T^{-2}

Two sets of dimensions are given for thermal quantities; the first set may be used in all problems in which *only phenomena of transfer enter, and the conversion of work into* heat does not play an essential part; the second set of dimensions must be used in connection with phenomena of conversion.

in many localities. These cause a rapid drop in discharging capacity, and should be allowed for by increased cross section or slope, or both. Channels in earth are also subject to loss of water by seepage, sometimes so considerable as to require special investigation. The state of uniformity of cross section is as important as mere local roughness. A channel with frequent minute changes of cross section must be regarded as a rough channel even if the sides and bottom are smooth in appearance. A circular section flowing only one-third or less full usually has a larger value of n than the same section flowing more than half full.

Erosion. All canals should be designed to have velocities high enough to prevent silt deposits and to retard vegetable growths. Earth channels, however, must not have such high velocities as to scour loose the material of the sides and bottom. Roughly approximate values of eroding velocities, in feet per second, are: for clay, ⅓; fine sand, ½; medium sand, 1; coarse sand and fine gravel, 2; gravel, 2 to 4.

Table 7. Values of the Hydraulic Radius, m, for Various Cross-sections

(m = cross-sectional area ÷ wetted perimeter)

Form of cross section	Value of m
Circle, flowing full, also half full	$d/4$
Square, no top (depth = width = d)	$d/3$
Half square (width = $2d$, depth = d)	$d/2$
Trapezoidal Channels (bottom width = b; depth = d):	
Half regular hexagon, side slopes 60 deg	$d/2$
Channel with 45 deg side slopes	$(bd + d^2)/(b + 2.83d)$
Channel with side slopes 1½ hor to 1 vert	$(bd + 1.5d^2)/(b + 3.61d)$
Channel with side slopes 2 hor to 1 vert	$(bd + 2d^2)/(b + 4.47d)$
Wide, shallow stream[1]	d (approx)

[1] Hence the term "hydraulic mean depth," sometimes used for m.

Open Channel Problem. It is necessary to carry 150 cu ft of water per sec in a rectangular unplaned timber flume whose width is to be twice the depth of water ($m = d/2$). What are the required dimensions for various slopes of the flume? *Solution:* For unplaned timber a value of $n = 0.013$ is safe. For assumed values of d = 3, 4, 5, and 6 ft, the cross-sectional areas of stream are 18, 32, 50 and 72 sq ft and the mean velocities for 150 cfs are 8.3, 4.7, 3.0 and 2.1 fps, respectively. By the Manning formula, reversed, $S = \dfrac{n^2}{2.208}\dfrac{V^2}{m^{4/3}}$ and, for the corresponding values of m, (1.5, 2, 2.5, and 3 ft) the slopes are found to be 3.1, 0.67, 0.20 and 0.077 ft per 1,000 ft, respectively, for the above assumed sizes. If it is required to find the dimensions of a canal with some desired depth or width for a given discharge and fixed slope, the procedure is to assume trial dimensions, calculate the mean velocity (= Q/area) and see if the value of m calculated by the Manning formula agrees closely with the m for the assumed dimensions. If not, new trials are made until there is close agreement. If the *shape* is fixed, *e.g.*, a semicircle or a 90 deg V flume, so that V = given Q ÷ (const × d^2), and m = const × d, the value of d may be calculated directly by the Manning formula.

Measurements of Flow in Open Channels. Where a weir or a dam does not exist, the velocity of flow is generally determined by the use of **floats** or by a **current meter** (See Hoyt and Grover, "River Discharge," Wiley). **Headgates and water wheels,** considered as orifices, are sometimes used, and are fairly accurate if calibrated for various gate openings by weir, floats or current meter. **Calculation based on open channel flow formulas is sometimes resorted to in cases such as extreme flood flows, where only high-water marks are left, in backwater problems, etc., this method being practically the only one available.**

Floats. By observing the time required for floating objects in a stream to pass over a measured distance, a good estimate of the velocity of the water in

of motion. There is obviously only one of these latter quantities, namely, g, the acceleration of gravity. From the detailed solution of the problem there should be found a connection between these quantities and the time of swing. The list of the variables and their dimensions is as shown in the table on page 282.

There are here 5 variables, expressed in terms of three fundamental units. Hence there are in general two independent dimensionless products. One of these is obviously w, the angle. The other is some product of powers of the remaining quantities. Values must be found of a, b, c, and d so that $l^a m^b g^c t^d$ is dimensionless. It is at once obvious that $b = 0$, because M enters only into m, and if b were not zero, there would be no way by which M could disappear from the product. It is also obvious that $a = -c$, in order that L should cancel, and $d = 2c = -2a$, in order that T should cancel. Hence the second dimensionless product is $(l/gt^2)^a$, or simply l/gt^2, since any power of a dimensionless quantity is itself dimensionless. Our general theorem now states that $f_1\{(l/gt^2), w\} =$ const., where f_1 is arbitrary. This relation may at once be put in another form, namely: $l/gt^2 = f_2(w)$; or finally, $t = \sqrt{l/g}\, f_3(w)$, where $f_3(w)$ is an arbitrary function of the angle. Dimensional analysis permits therefore the statement that the time of swing of a pendulum is independent of its mass, directly proportional to the square root of its length, and inversely proportional to the square root of gravity, but it does not give information about the way in which it depends on the angle. This latter information can only be obtained by actual detailed solution of the equation of motion (which is not here even written down); it is known that as a matter of fact $f_3(w)$ is approximately equal, for small angles, to π.

(2) *The problem of* **steady flow of liquid** *in a pipe of circular section under a pressure head.* The equations which govern are the equations of hydraulics; these equations contain the viscosity of the liquid, but not its density. The boundary conditions contain the diameter and length of the pipe and the difference of pressure between the two ends. From a complete solution of the problem the delivery through the pipe in unit time could be found, *and this would be expressed in terms of the quantities just mentioned.* The list of quantities and dimensions is:

Quantity	Symbol	Dimensions
Diameter of pipe	r	L
Length of pipe	l	L
Pressure difference	P	$ML^{-1}T^{-2}$
Viscosity	v	$ML^{-1}T^{-1}$
Delivery per unit time	D	L^3T^{-1}

Next, form the two dimensionless products. One of these is evidently r/l, and the second may be formed from r, P, v, and D, and is $r^aP^bv^cD^d$, or expressed in dimensional form, $L^a(ML^{-1}T^{-2})^b(ML^{-1}T^{-1})^c(L^3T^{-1})^d$. This must be dimensionless in L, M, and T, which gives the three equations:

$$\begin{aligned} a - b - c + 3d &= 0, \text{ condition on exponent of } L \\ b + c \phantom{{}+3d} &= 0, \text{ condition on exponent of } M \\ -2b - c - d &= 0, \text{ condition on exponent of } T \end{aligned}$$

The solution is: $d = -(a/3)$; $b = (a/3)$; $c = -(a/3)$, where a is arbitrary. Hence the dimensionless product is $(r^3Pv^{-1}D^{-1})^a$, or simply $r^3Pv^{-1}D^{-1}$.

Pressure Due to Deviated Flow

Force or Pressure Due to Deviated Flow. Pressure of a Stream against Stationary Solids. When a flowing liquid is turned from its course by a body such as a curved vane or a pipe bend, a pressure is exerted on that body. The total force exerted in any direction is equal to the mass of water being deviated, multiplied by the change of velocity in that direction. If the angle turned is B (see Figs. 28 and 29), the pressure or thrust in the direction before the turning is $P_x = (QwV/g)(1 - \cos B)$, where Q is the quantity flowing per second, w the weight of unit volume of liquid, V the velocity of the liquid, and g the gravity acceleration constant. If A is the cross section of the stream before turning, the equation may be written $P_x = 2Aw(1 - \cos B)V^2/2g$, thus introducing the velocity head in the flowing liquid. Taking Q in cu ft per sec and V in ft per sec the formula becomes, for water, $P_x = 1.94QV(1 - \cos B)$. For $B = 90$ deg and 180 deg respectively (Figs. 31 and 32), it becomes $P_x = 1.94QV$ and $3.88QV$. The pressure in a direction at right angles to the original direction is $P_y = (QwV/g) \sin B$, or $2Aw \sin BV^2/2g$, or, for water and ft-lb-sec units, $1.94QV \sin B$. With solids shaped as in Figs. 29 and 33, the components of the pressure in directions other than the original direction neutralize each other due to the divided stream. The **total resultant pressure** due to turning the water aside (P in Fig. 28) is $P = (QwV/g)\sqrt{2(1 - \cos B)}$ and its direction is 90 deg from midway between the original and final directions of the flowing liquid, *i.e.*, angle C in Fig. 28 = 90 deg − ½B. If a stream impinges squarely against a wall, as in Fig. 30, the effect is the same as it would be in Fig. 29 with $B = 90$ deg.

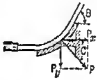


FIG. 28.

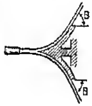


FIG. 29.

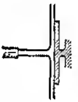


FIG. 30.

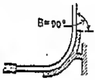


FIG. 31.

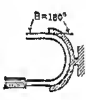


FIG. 32.

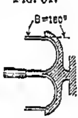


FIG. 33.

Gages for Measuring Pressures, Velocities, and Levels

The Pitot Tube. The Pitot tube shown on p. 252 is not readily adapted for insertion through a small hole in the wall of a pipe. Figures 34 and 35 illustrate representative tubes of more compact type. Such tubes have coefficients less than 1. For tubes as in Fig. 35, $V = 0.84\sqrt{2gh}$, *i.e.*, the indicated water column difference h is about $(1/0.84)^2$ or 42 percent greater than the h_v of Fig. 7. Very slight changes in the shape of a compact-type side-pressure-opening tube, as in Fig. 34, or in the location of the pressure openings, cause a change in the coefficient of the tube. For Fig. 34, V = 0.84 to $0.88\sqrt{2gh}$, depending on location of side openings and shape of tube; for precise work such tubes should be rated. A single-opening tube in connection with a "wall piezometer" (*i.e.*, either a single hole as Fig. 7, a pair of diametrically opposite holes, or a ring piezometer, see p. 259) is the simplest arrangement and least liable to errors resulting from imperfections in construction. The coefficient is unity, *i.e.*, $V = \sqrt{2gh}$, for a tube that has its impact

The solution is $b = -2a$, $c = -a$, $d = -a$, $e = -a$.
Whence $hl^{-2}v^{-1}t^{-1}q^{-1}$ is dimensionless.

For the variables for the other product there should be chosen a set not containing h, or l, v, t, q, and k, and it is required that $l^a v^b t^c q^d k^e$ be dimensionless. Work similar to that just carried through shows that $lvqk^{-1}$ is dimensionless, t dropping out, since $c = 0$. Hence the result is:

$$f_1\{(hl^{-2}v^{-1}t^{-1}q^{-1}),\ (lvqk^{-1})\} = \text{const},$$

which may be rewritten in the form:

$$h = l^2tvq.f(lvq/k).$$

This shows that the rate of heat transfer is proportional to the temperature difference, as was to be expected, and also that the heat transfer depends only on the product of velocity and heat capacity. *A single set of experiments* in which the way in which heat transfer varies with l is determined, would evidently now be sufficient to fix the form of f, and hence completely determine the solution.

"point" should not be a "needle point," but a conical point with a vertex angle of 90 to 120 deg, so as to produce a maximum of optical effect for slight variations from perfect contact of the point with the liquid surface. The use of a stilling box, such as the float pipe above described, is desirable especially for the hook gage (see Fig. 37).

In use, the hook-gage point is moved up from beneath until it just pierces the liquid surface. The point gage and plumb-bob gage are manipulated by lowering from above until a "bubble" shows contact with the liquid surface. If used for flowing water or a large surface of a tank or reservoir, where there are waves or surges, and where

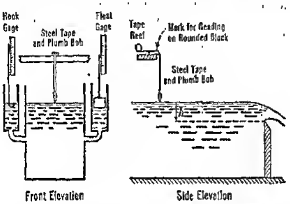


Fig. 37.—Hook, Plumb-bob and Float Gages.

a stilling box is not available, the gage should be set so that the average time durations during which the point is alternately submerged and exposed shall be as nearly equal as the observer can estimate. Even with a stilling box there may be surges of small amplitude, and several sets of observations may be necessary to assure a determination within 0.001 ft.

The float gage is the best of the gages mentioned for sensitiveness and accuracy. It automatically shows the small variations of water surface level, and the observer needs merely to watch the index on the float-stem. Any desired degree of damping out of minor or temporary fluctuations of water level may be accomplished by changing the size of the opening or by regulating a shut-off cock in the connecting pipe.

Whenever there is a perceptible vibration of the water surface, maximum and minimum readings should be taken, corresponding to the crests and troughs of the waves. The average will give the mean level in most cases.

Flow of a Viscous Fluid.

Let u, v, w = components of velocities along the x-, y-, z-axes

$$\rho = \text{fluid density} = \frac{\text{mass}}{\text{volume}}$$

μ = coefficient of viscosity

t = time

The differential equations of this problem are of the type

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

Assume $V = k_V V_m$ where k_V is the scale factor for all velocities.
$l = k_l\, l_m$ where k_l is the scale factor for all lengths.
$\rho = k_\rho\, \rho_m$ where k_ρ is the scale factor for the fluid density. (6)
$\mu = k_\mu\, \mu_m$ where k_μ is the scale factor for the fluid viscosity.
$t = k_t\, t_m$ where k_t is the scale factor for all time.

The scale factor k_t is obviously not independent of the others. Since

$$V = \frac{l}{t} \quad\text{and}\quad V_m = \frac{l_m}{t_m}, \quad k_V V_m = \frac{k_l\, l_m}{k_t\, t_m}, \quad\text{or}\quad k_t = \frac{k_l}{k_V}$$

Upon substituting (6), equation (5) becomes

$$\frac{k_V}{k_t}\cdot\frac{\partial u_m}{\partial t_m} + \frac{k_V^2}{k_l}\cdot\left(u_m\frac{\partial u_m}{\partial x_m} + v_m\cdot\frac{\partial u_m}{\partial y_m} + w_m\cdot\frac{\partial u_m}{\partial z_m}\right) = \frac{k_\mu\cdot k_V}{k_\rho\cdot k_l^2}\cdot\left(\frac{\partial^2 u_m}{\partial x_m^2} + \frac{\partial^2 u_m}{\partial y_m^2} + \frac{\partial^2 u_m}{\partial z_m^2}\right)$$

The similarity conditions in this case are

$$\frac{k_V}{k_t} = \frac{k_V^2}{k_l} = \frac{k_\mu k_V}{k_\rho k_l^2}$$

The relation $\frac{k_V}{k_t} = \frac{k_V^2}{k_l}$ is automatically satisfied since $k_V = \frac{k_l}{k_t}$. The other condition is

$$\frac{k_V^2}{k_l} = \frac{k_\mu k_V}{k_\rho k_l^2} \quad\text{or}\quad \frac{k_\rho}{k_\mu}\cdot k_V k_l = 1$$

This condition states that

$$\frac{\frac{\rho}{\rho_m}\cdot\frac{V}{V_m}\cdot\frac{l}{l_m}}{\frac{\mu}{\mu_m}} = 1 \quad\text{or}\quad \frac{\rho V l}{\mu} = \frac{\rho_m V_m l_m}{\mu_m}$$

This quantity $R = \frac{\rho V l}{\mu}$ is the **Reynolds number.** In order to have similar conditions of flow on model and full scale, it is necessary that the Reynolds number be the same for each.

Whenever the equations of a problem are known, the above method of analysis is applicable. The method can also be applied if only the general nature of the factors involved in a problem are known.

Consider, for instance, the problem of determining the resistance of ships. In order for the paths of flow about the ship to be similar, it is necessary that the scale factor for the forces $k_F = F/F_m$ be the same for all the types of forces acting on a small fluid element. Assume that the wave

of the size of the units. But the condition that this be true involves the way in which the numerical magnitude of the various quantities which enter the equations vary with the size of the fundamental units, and this in turn is determined by the dimensions of these quantities.

In applying the method, the first step, therefore, is to write out the dimensions of all the quantities which enter. The dimensions of a quantity are determined by its definition in terms of the quantities which are selected as fundamental. (For example, the dimensions of velocity are LT^{-1}, because velocity is defined as distance travelled in unit time.) The choice of fundamental quantities is in large part a matter of convenience, and there is, therefore, nothing absolute about the dimensions of any quantity. In most engineering problems, the ordinary mechanical quantities, mass (M), time (T), and length (L), may most conveniently be taken as fundamental, with the addition of temperature (t), in problems involving temperature effects. *In the table on page 282 are given the dimensions of some of the most usual* quantities.

The theorems on which dimensional analysis is based are the following. (1) Any equation which continues to hold when the size of the fundamental units changes can be thrown into such a form that the variables enter the equations only through certain combinations which are dimensionless. To determine the possible number of such dimensionless combinations we have the π theorem (2) which states that the number of such combinations is in general equal to the difference between the number of variables and the number of fundamental units, but in exceptional cases may be greater than the difference. There is also another theorem (3) by which it can be proved that any dimensionless combination must be expressible as a product of powers of the variables.

An application of these theorems results in the statement of the following important result of dimensionless analysis: *Any general connection between the variables can be expressed in the form of an arbitrary function of all the independent dimensionless products of the variables set equal to a constant.*

The information contained in expressing the connection in this form is most useful, in spite of the arbitrary character of the function, because of the reduction in the number of the arguments from the large number of independent variables to the smaller number of dimensionless products. The whole theory of models is contained here, in which, by making a change in a comparatively small number of physical variables, the effect of changes in all the variables can be told.

Examples. (1) Beginning with the classical pendulum problem, what is known about the time of swing of a pendulum? This is evidently a problem in mechanics. The solution of the equation of motion will contain the parameters needed to describe the pendulum, namely its mass, length, and initial angular displacement, and the solution will also contain any quantities with dimensions which are necessary in writing down the equations

Quantity	Symbol	Dimensions
Length of pendulum	l	L
Mass of pendulum	m	M
Acceleration of gravity	g	LT^{-2}
Initial angular displacement	w	0
Time of swing	t	T

The general theorem now states that an arbitrary function of these two products is to be set equal to a constant, or $f_1\{(r^3Pv^{-1}D^{-1}), (r/l)\} = \text{const}$; or, $r^3Pv^{-1}D^{-1} = f_2(r/l)$; or, $D = (r^3P/v)f_3(r/l)$; where f_1, f_2 and f_3 are arbitrary functions.

Hence the delivery varies as the pressure difference, and inversely as the viscosity. Suppose now that it is known in addition, as very simple considerations show, that the flow is proportional to the pressure gradient, that is, to P/l. Then the formula shows that f_3 must be proportional to r/l, so that: $D = \text{const.} \times (r^4P/vl)$, and the additional and important result is obtained that the delivery is directly proportional to the fourth power of the radius.

If the condition that the section of the pipe be circular is given up and it is assumed to be elliptical, with major and minor axes, r_1 and r_2, then there is an additional dimensionless product, r_1/r_2, and the result may be put in the form:

$$D = (r_1^4P/vl)f(r_1/r_2),$$

where f is an unknown function, but as before, the result is obtained that the delivery through a pipe with a section of given geometrical shape is proportional to the fourth power of the linear dimensions of the section.

(3) Rayleigh's Problem of Heat Transfer. A solid body, of definite geometrical shape, but variable absolute dimensions, is fixed in a stream of liquid, and maintained at a definite temperature higher than the temperature of the liquid at points remote from the body. Required to find the rate at which heat is transferred from the body to the liquid.

In writing the fundamental equations which govern this problem, the fact that heat and mechanical energy are convertible will evidently not be made use of. This fact may therefore be ignored in framing the fundamental definitions which determine the dimensions, and the quantity of heat, H, may be used as one of the fundamental quantities. It is furthermore evident, since the mechanics of the situation is not involved, and the lines of flow in the liquid are determined only by the geometry, that the viscosity of the liquid will not enter those equations which determine heat transfer. The quantities and dimensions are:

Quantity	Symbol	Dimensions
Rate of heat transfer	h	HT^{-1}
Linear dimensions of body	l	L
Velocity	v	LT^{-1}
Temperature difference	t	t
Heat capacity of liquid per unit volume	q	$HL^{-3}t^{-1}$
Thermal conductivity of liquid	k	$HL^{-1}T^{-1}t^{-1}$

There are 6 variables and 4 fundamental quantities, and hence two dimensionless products. In general, these two products may be composed of any two sets of four chosen from the six variables. Since the problem concerns h, one of the products is chosen so as to contain h. It is required that $h^al^bv^ct^dq^e$, or $(HT^{-1})^aL^b(LT^{-1})^ct^d(HL^{-3}t^{-1})^e$ be dimensionless. This gives the equations:

$$\begin{array}{lllll} a & & & + e & = 0, \text{ condition on exponent of } H \\ -a & & - c & - & = 0, \text{ condition on exponent of } T \\ & b & + c & - 3e & = 0, \text{ condition on exponent of } L \\ & & d & - e & = 0, \text{ condition on exponent of } t \end{array}$$

THERMAL PROPERTIES OF BODIES AND THERMODYNAMICS

BY

H. C. WEBER

(Originally prepared by G. A. Goodenough)

REFERENCES: Preston, "Theory of Heat," Macmillan. Zeuner, "Technical Thermodynamics," Van Nostrand. Goodenough, "Principles of Thermodynamics," Holt. Bryan, "Thermodynamics," Teubner. Lucke, "Engineering Thermodynamics," McGraw-Hill. Glazebrook, "Dictionary of Applied Physics," Vol. I, Macmillan. Ewing, "The Mechanical Production of Cold," Cambridge University Press. *Univ. Ill., Eng. Exp. Sta., Bull.* 30, 40, 66, 75, 139. "International Critical Tables," McGraw-Hill. Roberts, "Heat and Thermodynamics," Blackie. Keenan and Keyes, "Thermodynamic Properties of Steam," Wiley. Kiefer and Stuart, "Principles of Engineering Thermodynamics," Wiley. Weber, "Thermodynamics for Chemical Engineers," Wiley.

THERMAL PROPERTIES OF BODIES

The Measurement of Temperature

Thermometers. The scale of the constant-volume **hydrogen thermometer** is taken as the standard temperature scale. Within the limits of ordinary use, the scale of the mercury thermometer agrees closely with the standard scale, but above 500 F the divergence between the two scales may be appreciable.

The ordinary **mercury thermometer** may be used to about 600 F; this limit may be extended to 1000 F if the capillary tube above the mercury is filled with nitrogen or carbon dioxide under high pressure. The lower temperature limit for the mercury thermometer is −39 F. For lower temperatures, alcohol, pentane, or petroleum ether may be used as the thermometric substance. For stem exposure corrections, see Temperature Measurements, p. 1783. Very high temperatures are measured by various forms of **pyrometers** (see pp. 1785–1790).

Thermometer Scales. Let F and C denote the readings on the Fahrenheit and centigrade (or Celsius) scales, respectively, for the same temperature; then

$$C = \tfrac{5}{9}(F - 32), \qquad F = \tfrac{9}{5}C + 32$$

Table 1 gives corresponding readings on the two scales.

THEORY OF MODELS

BY

W. BOLLAY

REFERENCES: Prandtl and Tietjens, "Applied Hydro- and Aeromechanics," McGraw-Hill. Stodola-Loewenstein, "Steam and Gas Turbines," McGraw-Hill, Vol. 2, p. 1016.

Engineering problems that are too complicated for mathematical or graphical solution are generally studied by model tests. This procedure is used extensively in the design of ships, airplanes, hydraulic pumps, and turbines as well as in problems dealing with heat flow and complex elastic structures. In each case, a model is constructed which is geometrically similar to the full-size project. Measurements are made of the performance of the model including velocities, powers, temperatures, or stresses as the case may be. The problem of the theory of models is to determine the requisite conditions under which the performance of the model is similar to that of the full-size construction and to predict the behavior of the latter.

Consider for instance the **buckling of a column.** For a buckled column, the radius of curvature R is expressed in terms of the bending moment $M = Py$ and I, the moment of inertia of the cross section, as

$$\frac{1}{R} = \frac{M}{EI} = \frac{Py}{EI} \tag{1}$$

where P is the buckling load and y the sideward deflection of the column. The idea of the theory of models is that it makes no difference what the absolute scale is, by means of which the length l, the load P, or the elastic modulus E is measured. Thus assume that the relation between full-scale and model condition is expressed as

$$l = k_l l_m; \quad R = k_l R_m; \quad y = k_l y_m; \quad P = k_P P_m; \quad E = k_E E_m \tag{2}$$

where the subscript m refers to conditions on the model. The ratio k_l expresses the ratio between all corresponding length dimensions of model and full scale. If the model conditions are to be similar to full size, then equation (1) must also be satisfied in terms of the model scales, *i.e.*,

$$\frac{1}{R_m} = \frac{P_m y_m}{E_m I_m} \tag{3}$$

But if into (1) we substitute the relations (2), we get

$$\frac{1}{k_l R_m} = \frac{(k_P P_m)(k_l y_m)}{(k_E E_m)(k_l^4 I_m)}$$

or

$$\frac{1}{R_m} = \frac{k_P}{k_E k_l^2} \cdot \frac{P_m y_m}{E_m I_m} \tag{4}$$

In order for equation (4) to agree with (3), it is necessary that $\frac{k_P}{k_E k_l^2} = 1$, *i.e.*, $k_P = k_E k_l^2$. Thus if the buckling load is 100 lb on a quarter-scale model constructed out of a plastic with $\frac{1}{30}$ of the modulus of elasticity of the full-scale model, then the full-scale structure will carry $100 \times 30 \times (4)^2 = 48{,}000$ lb.

In dealing with problems in dynamics or heat flow, the fundamental equations describing the problem are generally differential equations. The procedure in these cases is analogous.

Degrees Fahrenheit to Degrees Centigrade*

F	C	F	C	F	C	F	C	F	C
−40	−40.00	+30	−1.11	+80	+26.67	+250	+121.11	+500	+260.00	+900	+482.22
−38	−38.89	31	−0.56	81	27.22	255	123.89	505	262.78	910	487.78
−36	−37.78	32	0.00	82	27.78	260	126.67	510	265.56	920	493.33
−34	−36.67	33	+0.56	83	28.33	265	129.44	515	268.33	930	498.89
−32	−35.56	34	1.11	84	28.89	270	132.22	520	271.11	940	504.44
−30	−34.44	35	1.67	85	29.44	275	135.00	525	273.89	950	510.00
−28	−33.33	36	2.22	86	30.00	280	137.78	530	276.67	960	515.56
−26	−32.22	37	2.78	87	30.56	285	140.55	535	279.44	970	521.11
−24	−31.11	38	3.33	88	31.11	290	143.33	540	282.22	980	526.67
−22	−30.00	39	3.89	89	31.67	295	146.11	545	285.00	990	532.22
−20	−28.89	40	4.44	90	32.22	300	148.89	550	287.78	1000	537.78
−18	−27.78	41	5.00	91	32.78	305	151.67	555	290.55	1050	565.56
−16	−26.67	42	5.56	92	33.33	310	154.44	560	293.33	1100	593.33
−14	−25.56	43	6.11	93	33.89	315	157.22	565	296.11	1150	621.11
−12	−24.44	44	6.67	94	34.44	320	160.00	570	298.89	1200	648.89
−10	−23.33	45	7.22	95	35.00	325	162.78	575	301.67	1250	676.67
− 8	−22.22	46	7.78	96	35.56	330	165.56	580	304.44	1300	704.44
− 6	−21.11	47	8.33	97	36.11	335	168.33	585	307.22	1350	732.22
− 4	−20.00	48	8.89	98	36.67	340	171.11	590	310.00	1400	760.00
− 2	−18.89	49	9.44	99	37.22	345	173.89	595	312.78	1450	787.78
0	−17.78	50	10.00	100	37.78	350	176.67	600	315.56	1500	815.56
+ 1	−17.22	51	10.56	105	40.55	355	179.44	610	321.11	1550	843.33
2	−16.67	52	11.11	110	43.33	360	182.22	620	326.67	1600	871.11
3	−16.11	53	11.67	115	46.11	365	185.00	630	332.22	1650	898.89
4	−15.56	54	12.22	128	48.89	370	187.78	640	337.78	1700	926.67
5	−15.00	55	12.78	125	51.67	375	190.55	650	343.33	1750	954.44
6	−14.44	56	13.33	130	54.44	380	193.33	660	348.89	1800	982.22
7	−13.89	57	13.89	135	57.22	385	196.11	670	354.44	1850	1010.00
8	−13.33	58	14.44	140	60.00	390	198.89	680	360.00	1900	1037.78
9	−12.78	59	15.00	145	62.78	395	201.67	690	365.56	1950	1065.56
10	−12.22	60	15.56	150	65.56	400	204.44	700	371.11	2000	1093.33
11	−11.67	61	16.11	155	68.33	405	207.22	710	376.67	2050	1121.11
12	−11.11	62	16.67	160	71.11	410	210.00	720	382.22	2100	1148.89
13	−10.56	63	17.22	165	73.89	415	212.78	730	387.78	2150	1176.67
14	−10.00	64	17.78	170	76.67	420	215.56	740	393.33	2200	1204.44
15	− 9.44	65	18.33	175	79.44	425	218.33	750	398.89	2250	1232.22
16	− 8.89	66	18.89	180	82.22	430	221.11	760	404.44	2300	1260.00
17	− 8.33	67	19.44	185	85.00	435	223.89	770	410.00	2350	1287.78
18	− 7.78	68	20.00	190	87.78	440	226.67	780	415.56	2400	1315.56
19	− 7.22	69	20.56	195	90.55	445	229.44	790	421.11	2450	1343.33
20	− 6.67	70	21.11	200	93.33	450	232.22	800	426.67	2500	1371.11
21	− 6.11	71	21.67	205	96.11	455	235.00	810	432.22	2550	1398.89
22	− 5.56	72	22.22	210	98.89	460	237.78	820	437.78	2600	1426.67
23	− 5.00	73	22.78	215	101.67	465	240.55	830	443.33	2650	1454.44
24	− 4.44	74	23.33	220	104.44	470	243.33	840	448.89	2700	1482.22
25	− 3.89	75	23.89	225	107.22	475	246.11	850	454.44	2750	1510.00
26	− 3.33	76	24.44	230	110.00	480	248.89	860	460.00	2800	1537.78
27	− 2.78	77	25.00	235	112.78	485	251.67	870	465.56	2850	1565.56
28	− 2.22	78	25.56	240	115.56	490	254.44	880	471.11	2900	1593.33
29	− 1.67	79	26.11	245	118.33	495	257.22	890	476.67	2950	1621.11

Table of Values for Interpolation in the Above Table

Degrees Fahrenheit......	1	2	3	4	5	6	7	8	9
Degrees centigrade*......	0.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00

* All decimals in the table are repeating decimals; 37.78 is really 37.777 . . . . . .

motion is principally the result of the interaction between gravity forces and the inertia forces of the fluid. The gravity forces on a fluid element of length l are proportional to $\rho g l^3$, the inertia forces to mass $\times$ acceleration $=$ $(\rho l^3) \cdot \frac{l}{t^2} = (\rho l^3) \cdot \frac{V^2}{l} = \rho V^2 \cdot l^2$ since $V = \frac{l}{t}$. By the criterion for similar flow,

$$\rho g l^3 = k_F \cdot \rho_m g_m l_m^3$$
$$\rho V^2 l^2 = k_F \cdot \rho_m V_m^2 l_m^2$$

Therefore,

$$k_F = \frac{\rho}{\rho_m} \cdot \frac{g}{g_m} \cdot \frac{l^3}{l_m^3} = \frac{\rho}{\rho_m} \cdot \frac{V^2}{V_m^2} \cdot \frac{l^2}{l_m^2} \qquad \text{or} \qquad \frac{V^2}{gl} = \frac{V_m^2}{g_m l_m}$$

The quantity $F = \frac{V}{\sqrt{gl}}$ is called **Froude's number.** In making a test of a ship model in a towing tank, this quantity must be kept the same as full scale.

In the problem of **vibrations of an elastic body,** the important forces are the inertia forces and the elastic forces. The inertia force $\propto$ mass $\times$ acceleration $= (\rho l^3) \cdot \frac{l}{t^2} = \frac{\rho l^4}{t^2}$. The elastic force $\propto$ stress $\times$ area $= \sigma \cdot l^2 = \epsilon E \cdot l^2$ which is dimensionally the same as El^2 since the strain ϵ is dimensionless. The criterion for similarity is that

$$k_F = \frac{\frac{\rho l^4}{t^2}}{\frac{\rho_m l_m^4}{t^2}} = \frac{El^2}{E_m l_m^2} \qquad \text{or} \qquad \frac{\rho \frac{l^2}{t^2}}{E} = \frac{\rho_m \frac{l_m^2}{t_m^2}}{E_m} = C$$

where $C =$ **Cauchy's number.**

The problem of **heat conduction** is governed by the equation

$$\frac{\partial \theta}{\partial t} = a \left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right]$$

where $\theta =$ temperature, $t =$ time, $a = \frac{k}{\gamma c}$, $k =$ coefficient of heat conduction, $\gamma c =$ specific heat per unit volume, $\gamma =$ weight per unit volume, $x, y, z =$ rectangular coordinates. The similarity conditions lead to

$$\frac{k_\theta}{k_t} = k_a \cdot \frac{k_\theta}{k_l^2}$$

where $k_\theta = \frac{\theta}{\theta_m}$ etc. $\qquad$ or $\qquad \frac{k_a \cdot k_t}{k_l^2} = 1 \qquad$ i.e. $\qquad \frac{at}{l^2} = \frac{a_m t_m}{l_m^2} = F_0$

where F_0 is the **Fourier number.**

Table 4. Freezing Points of Liquids at Atmospheric Pressure
(Deg F)

Ammonia	−107.8	Calcium chloride (sat. sol.)	−40	Methyl alcohol	−144.2
Aniline	20.8	Ether	−180	Rapeseed oil	25.7
Benzol	41.9	Ethyl alcohol	−174.6	Turpentine	14.0
Carbon bisulphide	−168.1	Glycerin	64	Sulphuric acid	−105
Carbon dioxide	−*110.2*	Naphthalene	176	*Salt (NaCl) sol, sat.*	−0.4
Chloroform	−82.3	Linseed oil	−4	Sea water	27.5
		Mercury	−38	Toluene	−149

Mixtures of glycerin and water (Bolley)			Mixtures of ethyl alcohol and water (F. Beilstein)			
Percent by weight of glycerin	Specific gravity	Freezing point, deg F	Percent by weight of alcohol	Freezing point, deg F	Percent by weight of alcohol	Freezing point, deg F
10	1.0245	30.2	2.58	30.2	21.7	10.4
20	1.0498	27.5	5.22	28.4	23.8	6.8
30	1.0771	20.8	7.36	26.6	26.0	3.2
40	1.1045	1.0	9.58	24.8	28.0	−0.4
45	1.1183	−15.2	11.50	23.0	30.0	−4.0
50	1.1320	−25.6	13.27	21.2	33.5	−11.2
60	1.1582	Below −31.0	16.53	17.6	37.3	−18.4
			19.09	14.0	41.2	−25.6

Table 5. Boiling Points (Deg F) at Atmospheric Pressure

Zinc	1665	Glycerin	554	Turpentine	320.0
Sulphur	832	Phosphorus	536	Toluene	231.0
Mercury	675	Naphthalene	424	Sodium chloride (sat sol)	226.4
Linseed oil	549	Aniline	364		
Paraffin	572	Calcium chloride (sat sol)	356	Helium	−452.0

Table 6. Boiling Points of Hydrocarbon Compounds, Deg F

Paraffin Series		Olefin Series		Aromatic Series	
C_3H_8	−48.2	C_5H_{10}	98.2	C_6H_6	175.3
C_4H_{10}	+31.1	C_6H_{12}	142.4	C_7H_8	230.9
C_5H_{12}	97.2	C_7H_{14}	244.6	C_8H_{10}	281.2
C_6H_{14}	156.2	C_8H_{16}	253.4		
C_7H_{16}	209.1	C_9H_{18}	301.8	Acetylene Series	
C_8H_{18}	256.3	$C_{10}H_{20}$	341.6	C_5H_8	104
C_9H_{20}	303.1	$C_{11}H_{22}$	370.4	C_6H_{10}	160.7
$C_{10}H_{22}$	345.2	$C_{12}H_{24}$	414–417	C_7H_{12}	231
$C_{11}H_{24}$	386.6	$C_{13}H_{26}$	450	C_8H_{14}	257
$C_{12}H_{26}$	420.8				
$C_{13}H_{28}$	453.2				
$C_{14}H_{30}$	486.5				
$C_{15}H_{32}$	518.9				

Relation of Color to Temperature of Iron or Steel

	Deg F		Deg F
Dark blood red, black red	990	Orange, free scaling heat	1650
Dark red, blood red; low red	1050	Light orange	1725
Dark cherry red	1175	Yellow	1825
Medium cherry red	1250	Light yellow	1975
Cherry, full red	1375	White	2200
Light cherry, light red	1550		

Expansion of Bodies by Heat

Coefficients of Expansion. The coefficient of linear expansion of a solid is defined as the increment of length in a unit of length for a rise in

SECTION 4

HEAT

BY

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

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Table 8. Coefficients of Expansion

(For pure metals, see p. 608)

Coefficients of Linear Expansion
(Mean values of 10,000a' between 32 and 212 F)

METALS		Type metal	0.108	Masonry	0.025 to 0.050
Aluminum bronze	0.094	OTHER MATERIALS		Paraffin	
Brass, cast	0.104	Bakelite, bleached	0.122	32 F–61 F	0.592
Brass, wire	0.107	Brick	0.031	61 F–100 F	0.724
Bronze	0.100	Caoutchouc	0.372	100 F–120 F	2.612
Constantan (60 Cu, 40		Carbon—coke	0.030	Porcelain	0.02
Ni)	0.095	Cement, neat	0.060	Quartz	
German silver	0.102	Concrete	0.080	Parallel to axis	0.044
Iron:		Ebonite	0.468	Perpend. to axis	0.029
Cast	0.059	Glass:		Quartz glass	0.0032
Soft forged	0.063	Thermometer	0.045	Rubber	0.428
Wire	0.080	Hard	0.033	Vulcanite	0.400
Magnalium (86 Al, 13		Plate and crown	0.050	Wood (∥ to fiber):	
Mg)	0.133	Flint	0.044	Ash	0.053
Phosphor bronze	0.094	Pyrex	0.018	Chestnut and maple	0.036
Solder	0.134	Granite	0.04 to 0.05	Oak	0.027
Speculum metal	*0.107*	*Graphite*	*0.044*	*Pine*	*0.030*
Steel:		Gutta percha	0.875	Across the fiber:	
Bessemer, rolled hard	0.056	Ice	0.283	Chestnut and pine	0.019
Bessemer, rolled soft	0.063	Limestone	0.023 to 0.05	Maple	0.027
Nickel (10% Ni)	0.073	Marble	0.02 to 0.09	Oak	0.030

Coefficients of Cubical Expansion
(Mean values of 1000a''' at ordinary room temperatures)

LIQUIDS		Hydrochloric acid,		Sulphuric acid, 50%	
Acetic acid	0.80	50% solution	0.52	solution	0.45
Alcohol (ethyl)	0.61	Mercury	0.10	Turpentine	0.54
Alcohol (methyl)	*0.80*	*Olive oil*	*0.41*	*Water*	*0.115*
Benzene	0.77	Petroleum, Pennsyl-		SOLIDS	
Benzol	0.70	vania	0.50	Fluor spar	0.035
Calcium chloride		Petroleum, California	0.43	Ice (4 to 30 F)	0.62
($CaCl_2$); 5 to 50%		Petroleum, Texas	0.42	Paraffin wax	0.61
solution	0.28	Phenol (C_6H_6O)	0.50	Rock salt	0.67
Chloroform	0.77	Rapeseed oil	0.50	Sulphur	0.40
Ether	0.92	Salt 1.6% solution	0.60	Wood (beech)	0.016
Glycerin	0.28	Salt 26% solution	0.24	Wood (pine)	0.028
Hydrochloric acid	0.27	Sulphuric acid	0.31		

The **linear shrinkage of castings** is approximately as follows:

Bar iron, rolled	1:55	Cast iron	1:96	Steel, puddled	1:72
Bell metal	1:65	Gun metal	1:134	Steel, wrought	1:64
Bismuth	1:265	Iron, fine grained	1:72	Tin	1:128
Brass	1:65	Lead	1:92	Zinc, cast	1:62
Bronze	1:63	Steel castings	1:50	8 Cu + 1Sn (by wt)	1:134

The coefficients of cubical expansion for different gases at ordinary temperatures are about the same. From 0 to 212 F and at atmospheric pressure, the values multiplied by 1,000 are as follows: for NH_3, 2.11; CO, 2.04; CO_2, 2.07; H_2, 2.03; NO, 2.07.

Measurement of Heat

Units of Heat. Many units of heat have been dependent on the experimentally determined properties of some substance. To eliminate experimental variations, the unit of heat may be defined in terms of fundamental units. The International Steam Table Conference (London, 1929) defines

Table 1. Conversion of Thermometer Readings

DEGREES CENTIGRADE TO DEGREES FAHRENHEIT

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
−40	−40.0	+5	+41.0	+40	+104.0	+175	+347	+350	+662	+750	+1382
−38	−36.4	6	42.8	41	105.8	180	356	355	671	800	1472
−36	−32.8	7	44.6	42	107.6	185	365	360	680	850	1562
−34	−29.2	8	46.4	43	109.4	190	374	365	689	900	1652
−32	−25.6	9	48.2	44	111.2	195	383	370	698	950	1742
−30	−22.0	10	50.0	45	113.0	200	392	375	707	1000	1832
−28	−18.4	11	51.8	46	114.8	205	401	380	716	1050	1922
−26	−14.8	12	53.6	47	116.6	210	410	385	725	1100	2012
−24	−11.2	13	55.4	48	118.4	215	419	390	734	1150	2102
−22	− 7.6	14	57.2	49	120.2	220	428	395	743	1200	2192
−20	− 4.0	15	59.0	50	122.0	225	437	400	752	1250	2282
−19	− 2.2	16	60.8	55	131.0	230	446	405	761	1300	2372
−18	− 0.4	17	62.6	60	140.0	235	455	410	770	1350	2462
−17	+ 1.4	18	64.4	65	149.0	240	464	415	779	1400	2552
−16	3.2	19	66.2	70	158.0	245	473	420	788	1450	2642
−15	5.0	20	68.0	75	167.0	250	482	425	797	1500	2732
−14	6.8	21	69.8	80	176.0	255	491	430	806	1550	2822
−13	8.6	22	71.6	85	185.0	260	500	435	815	1600	2912
−12	10.4	23	73.4	90	194.0	265	509	440	824	1650	3002
−11	12.2	24	75.2	95	203.0	270	518	445	833	1700	3092
−10	14.0	25	77.0	100	212.0	275	527	450	842	1750	3182
− 9	15.8	26	78.8	105	221.0	280	536	455	851	1800	3272
− 8	17.6	27	80.6	110	230.0	285	545	460	860	1850	3362
− 7	19.4	28	82.4	115	239.0	290	554	465	869	1900	3452
− 6	21.2	29	84.2	120	248.0	295	563	470	878	1950	3542
− 5	23.0	30	86.0	125	257.0	300	572	475	887	2000	3632
− 4	24.8	31	87.8	130	266.0	305	581	480	896	2050	3722
− 3	26.6	32	89.6	135	275.0	310	590	485	905	2100	3812
− 2	28.4	33	91.4	140	284.0	315	599	490	914	2150	3902
− 1	30.2	34	93.2	145	293.0	320	608	495	923	2200	3992
0	32.0	35	95.0	150	302.0	325	617	500	932	2250	4082
+1	33.8	36	96.8	155	311.0	330	626	550	1022	2300	4172
2	35.6	37	98.6	160	320.0	335	635	600	1112	2350	4262
3	37.4	38	100.4	165	329.0	340	644	650	1202	2400	4352
4	39.2	39	102.2	170	338.0	345	653	700	1292	2450	4442

TABLE OF VALUES FOR INTERPOLATION IN THE ABOVE TABLE

Degrees centigrade	1	2	3	4	5	6	7	8	9
Degrees fahrenheit	1.8	3.6	5.4	7.2	9.0	10.8	12.0	14.4	16.2

compound at room temperature is equal to the sum of the specific heats of the atoms forming the compound. The values given in the following table are to be used in connection with Kopp's law.

Constants for Kopp's Law

Elements	Specific Heats of the Atoms	Elements	Specific Heats of the Atoms
Heavy elements	6.4	Oxygen	4.0
Boron	2.7	Phosphorus	5.4
Carbon	1.8	Silicon	3.5
Fluorine	5.0	Sulphur	5.4
Hydrogen	2.3		

Example. Specific heat of the atoms of $Na_2SO_4 = 2(6.4) + 5.4 + 4(4) = 34.2$ Molecular weight $Na_2SO_4 = 142$. Specific heat $= 34.2/142 = 0.24$ (I.C.T. value = 0.20). Errors of 20 percent are not uncommon using Kopp's law.

Table 9. Mean Specific Heats of Various Solids and Liquids between 32 and 212 F

(For gases, see p. 310 and Fig. 1; for pure metals, p. 608; for refractories, p. 731)

SOLIDS	
Alloys:	
Bismuth-tin	0.040-0.045
Bell metal	0.086
Brass, yellow	0.0883
Brass, red	0.090
Bronze	0.104
Constantan	0.098
D'Arcet's metal	0.050
German silver	0.095
Lipowitz's metal	0.040
Nickel steel	0.109
Rose's metal	0.050
Solders (Pb and Sn)	0.040-0.045
Type metal	0.0388
Wood's metal	0.040
40 Pb + 60 Bi	0.0317
25 Pb + 75 Bi	0.030
Asbestos	0.20
Ashes	0.20
Bakelite	0.3-0.4
Basalt (lava)	0.20
Borax	0.229
Brick	0.22
Carbon-coke	0.203
Chalk	0.215
Charcoal	0.20
Cinders	0.18
Coal	0.3
Concrete	0.156
Cork	0.485
Corundum	0.198
Dolomite	0.222
Ebonite	0.33
Glass:	
Normal	0.199
Crown	0.16
Flint	0.12
Gneiss	0.18
Granite	0.195
Graphite	0.201
Gypsum	0.259
Hornblende	0.195
Humus (soil)	0.44
Ice:	
−4 F	0.465
32 F	0.487
India rubber (Para)	0.27-0.48
Kaolin	0.224
Limestone	0.217
Marble	0.210
Oxides:	
Alumina (Al_2O_3)	0.183
Cu_2O	0.111
Lead oxide (PbO)	0.055
Lodestone	0.156
Magnesia	0.222
Magnetite (Fe_3O_4)	0.168
Silica	0.191
Soda	0.231
Zinc oxide (ZnO)	0.125
Paraffin wax	0.69
Porcelain	0.22
Quartz	0.17-0.28
Quicklime	0.217
Salt, rock	0.21
Sand	0.195
Sandstone	0.22
Serpentine	0.25
Sulphur	0.180
Talc	0.209
Tufa	0.33
Vulcanite	0.331
Wood:	
Fir	0.65
Oak	0.57
Pine	0.67
LIQUIDS	
Acetic acid	0.51
Alcohol (absolute)	0.58
Aniline	0.49
Benzol	0.40
Chloroform	0.23
Ether	0.54
Fusel oil	0.56
Gasoline	0.50
Glycerin	0.58
Hydrochloric acid	0.60
Kerosene	0.50
Naphthalene	0.31
Machine oil	0.40
Mercury	0.033
Olive oil	0.40
Paraffin oil	0.52
Petroleum	0.50
Sulphuric acid	0.336
Sea water	0.94
Toluene	0.40
Turpentine	0.42
Molten metals:	
Bismuth (535-725 F)	0.036
Lead (590-680 F)	0.041
Sulphur (246-297 F)	0.235
Tin (460-660 F)	0.058

Mean Specific Heat of Iron (c_m) between 32 and t F (Oberhoffer)

t	600	800	1000	1200	1400	1600	1800	2000	2250	2500
c_m	0.127	0.133	0.139	0.148	0.167	0.170	0.169	0.168	0.167	0.167

Specific Heats of Gases. For monatomic gases, the specific heats do not vary with temperature, and k, the value of c_p/c_v, is 1.66. For diatomic

If the pressure readings of a constant-volume hydrogen thermometer are extrapolated to zero pressure, it is found that the corresponding temperature is −273.16 C, or −459.69 F. It is convenient to have a so-called **absolute temperature scale** on which zero corresponds with zero pressure on the hydrogen thermometer. Such a scale has fundamental thermodynamic significance and agrees very closely with the thermodynamic temperature scale. The absolute scales in use are:

Degrees Kelvin (K) = degrees centigrade + 273.16.

Degrees Rankine (R) = degrees Fahrenheit + 459.69.

Fixed Temperatures. When calibrating thermometers, it is convenient to have fixed temperatures. The following are recommended by the *U. S. Bureau of Standards:*

Substance	Deg F	Deg C	Substance	Deg F	Deg C
Liquid naphthalene boils at	424.2	217.9	Liquid copper-silver eutectic solidifies at	1434	779.0
Liquid tin solidifies at	449	232	Solid silver melts at	1761	960.5
Liquid benzophenone boils at	582.6	305.9	Solid gold melts at	1945.5	1063
Liquid lead solidifies at	621	327	Liquid copper solidifies at	1981	1083
Liquid zinc solidifies at	787	419.4	Solid nickel melts at	2646	1452
Liquid sulphur boils at	832.5	444.7	Solid palladium melts at	2822	1550
Liquid antimony solidifies at	1166	630	Solid platinum melts at	3191	1755
Liquid aluminum (97.7 per cent pure) solidifies at	1218	659	Solid alumina melts at	3722	2050
			Solid tungsten melts at	6134	3390

Table 2. Melting Points of Non-metallic Elements, Deg F

Helium	−456*	Argon	−309	Phosphorus	111
Hydrogen	−434	Krypton	−272	Iodine	236.3
Neon	−416	Xenon	−220	Sulphur	235
Fluorine	−367	Chlorine	−150.9	Silicon	2588
Oxygen	−360	Bromine	+ 19	Carbon	>6500
Nitrogen	−346				

* At 25 atm.

Table 3. Melting Points of Various Solids, Deg F

(For pure metals, see p. 608; for refractories see p. 731).

Alloys:		18 Bi + 36 Pb + 46 Sn	305	Zinc	689
Bismuth solder	200-262	10 Bi + 40 Pb + 50 Sn	324	Enamel colors	1760
Brass and bronze (about)	1650	Tin solder	275-350	India rubber	257
80 Cu + 20 Zn	1845	Blast-furnace slag	2370-2600	Paraffin	129
50 Cu + 50 Zn	1615	Borax	1040	Phosphorus	111
20 Cu + 80 Zn	1300	Cast iron, gray	2460-2550	Porcelain	2820
Delta metal	1742	Cast iron, white	1920-2010	Potassium	143.6
20 Sn + 80 Pb	530			Sodium	207.5
50 Sn + 50 Pb	400	Chlorides:		Spermaceti	120
80 Sn + 20 Pb	388	Calcium	1422	Stearine	122
Fusible alloys:		Potassium	1474	Steel	2370-2550
33 Bi + 33 Pb + 33 Sn	250	Sodium	1479	Wrought iron	2460-2640

On a molal basis, the average values of specific heat for some of the more common gases are given by Fig. 1. These values have been corrected to zero pressure but they are practically the same at 1 atm. Table 10 gives equations for specific heats for several gases at a constant pressure of 1 atm.

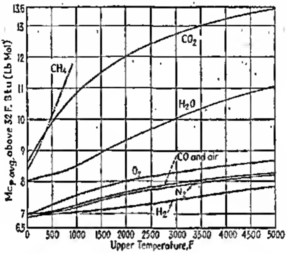


FIG. 1.—Values of the Mean Molal Specific Heat at Constant Pressure, above 32 F.

Specific Heat of Mixtures. If w_1 lb of a substance at temperature t_1 and with specific heat c_1 is mixed with w_2 lb of a second substance at temperature

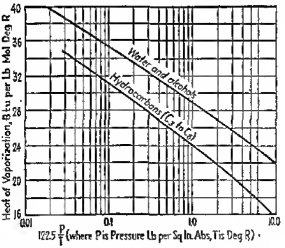


FIG. 2.—Hildebrand Function for Enthalpy of Vaporization

t_2 and with specific heat c_2, provided chemical reaction, heat evolution, or heat absorption do not occur, the specific heat of the mixture is

$$c_m = (w_1c_1 + w_2c_2)/(w_1 + w_2)$$

and the temperature of the mixture is

$$t_m = (w_1c_1t_1 + w_2c_2t_2)/(w_1c_1 + w_2c_2)$$

In general, $t_m = \Sigma wct/\Sigma wc$.

temperature of 1 deg. Likewise, the **coefficient of cubical expansion** of a solid, liquid, or gas is the increment of volume of a unit volume for a rise of temperature of 1 deg. Denoting these coefficients by a' and a''', respectively,

$$a' = \frac{1}{l}\frac{dl}{dt} \qquad a''' = \frac{1}{V}\frac{dV}{dt}$$

in which l denotes length, V volume, and t temperature. For homogeneous solids $a''' = 3a'$ and the **coefficient of superficial expansion** $a'' = 2a'$.

The coefficients of expansion are, in general, dependent upon the temperature, but for ordinary ranges of temperature, constant mean values may be taken. If lengths, areas, and volumes at 32 F (0 C) be taken as standard, then these magnitudes at other temperatures t_1 and t_2 are related as follows:

$$\frac{l_1}{l_2} = \frac{1 + a't_1}{1 + a't_2} \qquad \frac{A_1}{A_2} = \frac{1 + a''t_1}{1 + a''t_2} \qquad \frac{V_1}{V_2} = \frac{1 + a'''t_1}{1 + a'''t_2}$$

Since for solids and liquids the expansion is small, the preceding formulas for these bodies become approximately

$$l_2 - l_1 = a'l_1(t_2 - t_1). \quad A_2 - A_1 = a''A_1(t_2 - t_1). \quad V_2 - V_1 = a'''V_1(t_2 - t_1).$$

For certain metals, the variation of the coefficient of expansion with temperature is given by an equation in which, denoting by l_0 the length at 32 F, and by l the length at temperature t, the following relation is obtained:

$$l = l_0\left[1 + a\left(\frac{t - 32}{1000}\right) + b\left(\frac{t - 32}{1000}\right)^2\right]$$

The following table gives values of the constants.

Metal	1000 a	1000 b	Temperature range, deg F
Aluminum	12.58	3.0	32–1130
Cast iron	5.441	1.747	32–1160
Ingot iron	6.375	1.636	32–1380
Malleable iron	6.503	1.622	32–930
Ingot steel	6.212	1.623	32–1380
Copper	9.278	1.244	32–1160
Nickel	7.652	1.023	32–1830

Gruneisen finds that a' varies directly as the specific heat.

Table 7. Compressibility of Water $(v_f - v) \times 10^5$.
(Abstracted from Keenan and Keyes, "Thermodynamic Properties of Steam")

Pressure, lb per sq in. abs		Temperature, deg F								
		32	100	200	300	400	500	600	700	705.4
Saturated liquid	p, lb per sq in.	0.08854	0.9492	11.526	67.013	247.31	680.8	1542.9	3093.7	3206.2
	100 v_f, cu ft per lb	1.6022	1.6132	1.6634	1.7449	1.8369	2.0432	2.3629	3.692	5.030
1,000		5.7	5.1	5.4	6.9	8.7	16.4			
1,500		8.4	7.5	8.1	10.4	14.1	17.3			
2,000		11.0	9.9	10.8	13.8	19.5	27.8	32.6		
2,500		13.7	12.3	13.4	17.2	24.8	37.7	61.9		
3,000		16.3	14.7	16.0	20.7	30.0	47.1	87.9		
3,206.2		17.5	15.7	17.1	22.2	32.1	51.0	98.0	354	[illegible]
4,000		21.5	19.2	21.0	27.5	40.0	64.5	132.2	821	2079
5,000		26.7	23.6	26.0	34.0	49.6	80.5	169.3	1017	2309
6,000		31.7	27.8	30.8	40.5	58.7	96.1	202.9		

Table 13. Vapor Pressures of Certain Liquids at Pressure up to 1 Atm
(See also special tables, pp. 328 to 344 for other liquids and for vapor pressures at higher pressures)

Substance	Pressures, mm of mercury					
	10	100	200	400	600	760
	Temperature, deg F					
Acetone	−25.6	45.1	71.2	101.7	121.1	133.1
Benzene	−9.5	71.2	109.4	141.9	163.7	176.2
Carbon tetrachloride	−4.0	72.2	102.3	134.6	156.1	170.2
Chloroform	−22.0	49.1	77.3	108	128.8	142.0
Diphenyl	246.0	357.5	400.5	441.1	471.8	493.0
Ethyl alcohol	26.6	94.8	118.8	145.5	162.4	173.0
Ethyl ether	−56.2	17.6	35.4	63.5	83.1	94.2
Ethylene glycol	190.5	280.5	313.0	348.8	372.0	384.7
Methyl alcohol	3.74	69.6	93.7	121.0	138.2	148.4
Mercury	363	501	555	613.5	650.5	772.5
Water	52.3	123.8	141.2	176.5	200.4	212.0

General Principles of Thermodynamics

Thermodynamics is the study of changes in which energy is involved. The quantity of matter under consideration is called the **system**, and everything else is spoken of as the **surroundings**. Any change the system may undergo is known as a **process**. Any process or series of processes that returns the system to its original condition or state is called a **cycle**.

Heat is energy **in transit** from one mass to another because of a temperature difference between the two. Whenever a force of any kind acts through a distance, **work** is done. Like heat, work is also energy in transit. Work is to be differentiated from the capacity of a quantity of energy to do work.

Notation.

$A = 1/J$ = reciprocal of mechanical equivalent.
B = availability (by definition, $B = H - T_oS$).
c_p = specific heat at constant pressure.
c_v = specific heat at constant volume.
E = total energy associated with a system.
H, h = enthalpy, Btu (by definition $h = u + Apv$).
J = mechanical equivalent of heat = 778.26 ft-lb per Btu = 4.1861 joules per cal.
$k = c_p/c_v$.
M = molecular weight.
p = absolute pressure, lb per sq ft.
Q, q = quantity of heat absorbed by the system from the surroundings, Btu.
R = perfect gas constant.
S, s = entropy.
t = temperature, deg F.
$T = t + 459$ [illegible] erature = deg R.
T_o = sink or [illegible]
U, u = internal energy, Btu.
V = linear velocity, fps (or total volume).

the Steam Table (IT) calorie as $\frac{1}{860}$ of a watthour. One British thermal unit (Btu) is defined as 251.996 IT cal, or 778.26 ft-lb.

Previously, the Btu was defined as the heat necessary to raise one pound of water one degree Fahrenheit at some arbitrarily chosen temperature level. Similarly, the calorie was defined as the heat required to heat one gram of water one degree centigrade at 15 C (or at 17.5 C). These units are roughly the same in value as those mentioned above.

Heat Capacity and Specific Heat. The amount of heat required to raise unit mass of a material 1 deg in temperature is called the **heat capacity** of that material. The ratio of the amount of heat required to raise unit mass of a material 1 deg to that required to raise unit mass of water 1 deg at some specified temperature is the **specific heat** of the material. For most engineering purposes, heat capacities may be assumed numerically equal to specific heats. Two heat capacities are generally used, that at constant pressure c_p and that at constant volume c_v. For unit mass, the instantaneous heat capacities are defined as

$$\left(\frac{\partial Q}{\partial t}\right)_p = c_p \qquad \left(\frac{\partial Q}{\partial t}\right)_v = c_v$$

Over a range in temperature, the mean heat capacities are given by

$$c_{p_m} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} c_p dt, \qquad c_{v_m} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} c_v dt$$

Denoting by c the heat capacity, the heat required to raise the temperature of w lb of a substance from t_1 to t_2 is $Q = wc(t_2 - t_1)$, provided c is a constant.

In general, c varies with the temperature, though for moderate temperature ranges a constant mean value may be taken. If, however, c is taken as variable, then $Q = w\int_{t_1}^{t_2} c dt$. The *mean* heat capacity from 0 to t deg is given by $c_m = \frac{1}{t}\int_0^t c dt$. If $c = a_1 + a_2 t + a_3 t^2 + \cdots$

$$c_m = a_1 + \tfrac{1}{2}a_2 t + \tfrac{1}{3}a_3 t^2 + \cdots$$

Specific Heat of Water at Constant Pressure
(Interpolated from Keenan and Keyes Tables)

Temp, deg F	Pressure, lb per sq in.			
...	1,000	2,000	4,000	6,000
200	1.0	1.0	0.99	0.98
400	1.07	1.06	1.05	1.04
600	....	1.45	1.29	1.21
680	....	....	1.77	1.45

Specific Heat of Water at 1 Atm (I.C.T.)

Temp, deg F	32	50	100	150	212
c_p	1.001	1.002	1.004	1.009	1.021

Specific Heat of Solids. For elements near room temperature, the specific heat may be approximated by the rule of Dulong and Petit, that the specific heat at constant volume for one atomic weight of any solid element is 6.4. For solid compounds at about room temperature, Kopp's approximation is often useful. This states that the specific heat of a solid

starts and finishes with the system in the same state is called a **reversible cycle.**

Flow Processes. With **steady flow,** the conditions at any point in an apparatus through which a fluid is flowing do not change progressively with time. Steady-flow processes involving only mechanical effects are equivalent to similar non-flow processes occurring between two weightless frictionless diaphragms or pistons moving at constant pressure with the system as a whole in motion. Under these circumstances, the total work done by or on a unit amount of fluid is made up of that done on the two diaphragms $p_2v_2 - p_1v_1$ and that done on the rest of the surroundings $\int pdv - p_2v_2 + p_1v_1$. Differentiating, $pdv - d(pv) = -vdp$. The net, useful flow work done on the surroundings is $-\int vdp$. This is often called the shaft work. The net, useful or shaft work differs from the total work by $p_2v_2 - p_1v_1$. The first-law equation may be written to indicate this result

$$Jq - W_{\text{net}} = Ju_{12} + p_2v_2 - p_1v_1 + \frac{1}{2g}(V_2^2 - V_1^2) + X_2 - X_1$$

or since by definition

$$Ju + pv = Jh$$

$$Jq - W_{\text{net}} = Jh_{12} + \left(\frac{V^2}{2g}\right)_{12} + X_{12}$$

If all net work effects are mechanical,

$$Jq + \int vdp = Jh_{12} + \left(\frac{V^2}{2g}\right)_{12} + X_{12}$$

Since in evaluating $\int vdp$ the pressure is that *effectively* applied to the surroundings, the integration cannot usually be performed except for reversible processes.

If a fluid is passed adiabatically through a conduit (*i.e.*, without heat exchange with the conduit), without doing any net or useful work, and if velocity and potential effects are negligible, $h_{12} = 0$. A process of the kind indicated is the Joule-Thomson flow (see p. 361), and the ratio $(\partial T/\partial p)$ for such a flow is the Joule-Thomson coefficient.

If a fluid is passed through a non-adiabatic conduit without doing any net or useful work and if velocity and potential effects are negligible, $Q = wh_{12}$. This equation is important in the calculation of heat balances on flow apparatus, *e.g.*, condensers, heat exchangers, and coolers.

The second law of thermodynamics (p. 305) is a statement that conversion of heat to work is limited by the temperature at which conversion occurs. It may be shown that

1. No cycle can be more efficient than a reversible cycle operating between given temperature limits.

2. The efficiency of all reversible cycles absorbing heat only at a single constant higher temperature T_1 and rejecting heat only at a single constant lower temperature T_2 must be the same.

3. For all such cycles, the efficiency is

$$e = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$$

This is usually called the **Carnot cycle efficiency.** By the first law $W = Q_1 + Q_2$,

$$(Q_1 + Q_2)/Q_1 = (T_1 - T_2)/T_1$$

gases (oxygen, nitrogen, etc.), the specific heats vary with temperature but for many purposes may be assumed constant over considerable ranges of temperature. For diatomic gases, k is approximately 1.40. For more complex gases, generalizations are not possible. Specific heat increases with molecular complexity, and the value of k decreases (see also Table 15).

Properties of gases are, usually, most readily correlated on the mol basis. A pound mol is the weight in pounds equal to the molecular weight. Thus 1 pound mol of oxygen weighs 32 lb. At the same pressure and temperature, the volume of one mol is the same for all perfect gases, *i.e.*, following the gas laws. For perfect gases, $Mc_p - Mc_v = AMR = 1.987$.

$$c_v = AR/(k-1); \qquad c_p = ARk/(k-1)$$

Table 10. Specific Heats of Gases at 1 Atm

Gas	Symbol	Equation for C_p in Btu per mol	Temp range, deg R	Source
Oxygen	O_2	$11.515 - \left(\frac{172}{\sqrt{T}}\right) + \left(\frac{1530}{T}\right)$	540–5000	a
		$11.515 - \left(\frac{172}{\sqrt{T}}\right) + \left(\frac{1530}{T}\right) + \left(\frac{0.05(T-4000)}{1000}\right)$	5000–9000	a
Nitrogen	N_2	$9.47 - \left(\frac{3.47 \times 10^3}{T}\right) + \left(\frac{1.16 \times 10^6}{T^2}\right)$	540–5000	a
Carbon monoxide	CO	$9.46 - \left(\frac{3.29 \times 10^3}{T}\right) + \left(\frac{1.07 \times 10^6}{T^2}\right)$	540–5000	a
Hydrogen	H_2	$5.76 + \left(\frac{0.578T}{1000}\right) + \left(\frac{20}{\sqrt{T}}\right)$	540–4000	a
		$5.76 + \left(\frac{0.578T}{1000}\right) + \left(\frac{20}{\sqrt{T}}\right) - \left(\frac{0.33(T-4000)}{1000}\right)$	4000–9000	a
Water	H_2O	$19.86 - \left(\frac{597}{\sqrt{T}}\right) + \left(\frac{7500}{T}\right)$	540–5000	a
Carbon dioxide	CO_2	$16.2 - \left(\frac{6.53 \times 10^3}{T}\right) + \left(\frac{1.41 \times 10^6}{T^2}\right)$	540–6300	a
Methane	CH_4	$4.22 + 8.211 \times 10^{-3}T$	492–1800	b
		$27.0 - \frac{14{,}400}{T}$	1800–5940	b
Ethylene	C_2H_4	$6.0 + 8.33 \times 10^{-3}T$	720–1440	c
Ethane	C_2H_6	$6.6 + 13.33 \times 10^{-3}T$	720–1440	c
Ethyl alcohol	C_2H_6O	$4.5 + 21.1 \times 10^{-3}T$	680–1120	c
Methyl alcohol	CH_4O	$2.0 + 16.67 \times 10^{-3}T$	680–1100	c
Benzene	C_6H_6	$6.5 + 28.9 \times 10^{-3}T$	520–1120	c
Octane	C_8H_{18}	$14.4 + 53.3 \times 10^{-3}T$	720–1440	c
Dodecane	$C_{12}H_{26}$	$19.6 + 80.0 \times 10^{-3}T$	720–1440	c

[a] Sweigert and Beardsley, "Empirical Specific Heat Equations Based upon Spectroscopic Data," *Ga. School Tech., State Eng. Expt. Sta. Bull.* 2, 1938.

[b] Schwarz, "Die Spezifischen Wärmen der Gase als Hilfswerte zur Berechung von Gleichgewichten," *Arch. Eisenhüttenw.*, 9, 1936, p. 389.

[c] Parks and Huffman, *Am. Chem. Soc. Mon.* 60, 1932.

This function is of particular importance in processes where chemical changes occur. For reversible isothermal steady-flow processes, or for reversible constant-pressure isothermal non-flow processes, change in free energy is equal to net work.

Helmholtz free energy, $\psi = U - TS$, is equal to the work during a constant-volume isothermal reversible non-flow process.

All three of these functions B, Z, and ψ are point functions, and like E, H, and S their differentials are complete or perfect.

Perfect Differentials. Maxwell Relations. If z is some function of x and y, in general

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

Substituting M for $\left(\frac{\partial z}{\partial x}\right)_y$ and N for $\left(\frac{\partial z}{\partial y}\right)_x$

$$dz = M\,dx + N\,dy$$

But $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$ or $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. This is Euler's criterion for integrability. A perfect differential has the characteristics of dz stated above. Many important thermodynamic relations may be derived from the appropriate point function by the use of this relation. Some of these relations follow:

Table 14. Maxwell Relations

Function	Differential	Maxwell relation
$\Delta u = q - W$	$du = T\,ds - Ap\,dv$	$\left(\frac{\partial T}{\partial v}\right)_s = -A\left(\frac{\partial p}{\partial s}\right)_v$
$h = u + Apv$	$dh = T\,ds + Av\,dp$	$\left(\frac{\partial T}{\partial p}\right)_s = A\left(\frac{\partial v}{\partial s}\right)_p$
$\psi = u - Ts$	$d\psi = -s\,dT - Ap\,dv$	$\left(\frac{\partial s}{\partial v}\right)_T = A\left(\frac{\partial p}{\partial T}\right)_v$
$z = h - Ts$	$dz = -s\,dT + Av\,dp$	$\left(\frac{\partial s}{\partial p}\right)_T = -A\left(\frac{\partial v}{\partial T}\right)_p$

By holding certain variables constant, a second set of relations is obtained.

Differential	Independent variable held constant	Relation
$du = T\,ds - Ap\,dv$	s	$\left(\frac{\partial u}{\partial v}\right)_s = -Ap$
	v	$\left(\frac{\partial u}{\partial s}\right)_v = T$
$dh = T\,ds + Av\,dp$	s	$\left(\frac{\partial h}{\partial p}\right)_s = Av$
	p	$\left(\frac{\partial h}{\partial s}\right)_p = T$
$d\psi = -s\,dT - Ap\,dv$	T	$\left(\frac{\partial \psi}{\partial v}\right)_T = -Ap$
	v	$\left(\frac{\partial \psi}{\partial T}\right)_v = -s$
$dz = -s\,dT + Av\,dp$	T	$\left(\frac{\partial z}{\partial p}\right)_T = Av$
	p	$\left(\frac{\partial z}{\partial T}\right)_p = -s$

To raise the temperature of w_1 lb of a substance having a specific heat c_1 from t_1 to t_m, the weight w_2 of a second substance required is

$$w_2 = w_1c_1(t_m - t_1)/c_2(t_2 - t_m)$$

For mixing two bodies of the same (perfect) gas at constant pressure,

$$t_m = [(V_1 + V_2)/(V_1/T_1 + V_2/T_2)] - 459.69$$

Specific Heat of Solutions. For aqueous solutions of salts, the specific heat may be estimated by assuming the specific heat of the solution equal to that of the water alone. Thus, for a 20 percent by weight solution of sodium chloride in water, the specific heat would be approximately 0.8.

Latent Heats. For pure substances, the heat effects accompanying changes in state at constant pressure are known as latent effects, because no temperature change is evident. Heat of fusion, vaporization, sublimation, and change in crystal form are examples. Heats of vaporization at low pressures for pure liquids of similar chemical characteristics are well correlated by the methods proposed by Hildebrand. Such a correlation is given in Fig. 2.

Example. For water at 25 lb per sq in. abs and 240 F, the heat of vaporization is 952 Btu. Referring to Fig. 2, $122.5\dfrac{25}{240 + 460} = 4.4$, and the corresponding value of the molal heat of vaporization is $24.6(240 + 460) = 17{,}200$ Btu per lb mol or 958 Btu per lb.

Table 11. Heat of Fusion, Btu per Lb

Aluminum	167.5	Potassium	26.2	D'Arcet's metal	10.4
Bismuth	22.4	Silver	43.9	Lipowitz's metal	12.3
Blast-furnace slag	90.0	Sodium	49.5	Rose's metal	12.3
Cadmium	23.8	Tin	25.4	Wood's metal	14.0
Chromium	126.0	Zinc	46.8	Zinc and bismuth	20–23
Cobalt	115.2	Alloys:		Benzol (C_6H_6)	55.0
Copper	78	30.5 Pb + 69.5 Sn	30.6	Calcium chloride sol. ($CaCl_2 + 6H_2O$)	72.5
Gold	28.7	36.9 Pb + 61.3 Sn	28.0	Glycerin	76.5
Iron, gray cast	40.0	63.7 Pb + 36.3 Sn	11.6	Ice	144.0
Iron, white	60.0	77.8 Pb + 22.2 Sn	17.0	Naphthalene ($C_{10}H_8$)	64.1
Lead	9.8	78.4 Sn + 21.6 Zn	42.3	Phosphorus	9.05
Mercury	5.0	93.56 Sn + 6.44 Zn	31.8	Phenol (C_6H_6O)	45.0
Nickel	133	97.32 Sn + 2.68 Zn	27.2	Sulphur	15.8
Palladium	64.6	Britannia metal (9 Sn + 1 Pb)	50.4		
Platinum	48.4				

Table 12. Latent Heat of Vaporization at Atmospheric Pressure, Btu per Lb

Ethyl alcohol (98 percent)	406	Hexane	156	Hydrogen	194
Methyl alcohol	482	Heptane	133	Nitrogen	86
Aniline	198	Octane	128	Oxygen	92
Benzene	172	Decane	110	Chlorine	121
Toluene	151	Gasoline	133–145	Sulphur	120
Chloroform	110	Kerosene	105–110	Acetone	239
Ether	162	Turpentine	126	Carbon bisulphide	152
				Carbon tetrachloride	835

Vapor Pressures. At a specified temperature, a pure liquid can exist in equilibrium contact with its vapor at but one pressure, its vapor pressure. A plot of these pressures against the corresponding temperatures is known as a vapor-pressure curve.

Presentation of Thermal Properties. For calculation of numerical results, experimentally determined properties of substances must be available. These properties may be presented in several different ways.

1. As equations of state, *e.g.*, the perfect gas laws and the van der Waals equation.
2. As charts or graphs.
3. As tables.
4. As approximations which may be useful when more reliable data are not available.

The Perfect Gas Laws. At low pressures and high enough temperatures, in the absence of chemical reaction, all gases approach a condition such that their *P-V-T* properties may be expressed by the simple relation

$$pv = RT$$

in which R is a constant. If v is expressed as volume per unit weight, the value of the constant R will be different for different gases. If v is expressed as the volume of one molecular weight of gas, then R is the same for all gases in any chosen system of units.

In general, for any amount of gas, the perfect gas equation becomes

$$pV = NMRT$$

Table 15. Properties of Gases

Gas	Chemical symbol	Approx molecular weight[a]	Weight in lb of 1 cu ft at standard atmos pressure and 68 F[b]	Density relative to air	Gas constant, R	Specific heat per lb at room temperatures		Specific heat per cu ft at standard atmos pressure and 68 F[b]		$k = c_p/c_v$
						c_p	c_v	c_p	c_v	
Helium	He	4.0	0.01039	0.138	386.3	1.25	0.754	0.0130	0.0078	1.66
Argon	A	40.0	0.1037	1.377	38.70	0.124	0.0743	0.0129	0.0077	1.67
Air	......	29.0	0.07528	1.000	53.30	0.241	0.1725	0.0181	0.0130	1.40
Oxygen	O_2	32.0	0.08305	1.103	48.31	0.217	0.1549	0.0180	0.0129	1.40
Nitrogen	N_2	28.0	0.07274	0.966	55.16	0.247	0.1761	0.0179	0.0128	1.40
Hydrogen	H_2	2.0	0.005234	0.0695	766.8	3.42	2.435	0.0179	0.0127	1.40
Nitric oxide	NO	38.0	0.07788	1.034	51.52	0.231	0.1648	0.0180	0.0128	1.40
Carbon monoxide	CO	28.0	0.07269	0.965	55.19	0.243	0.1721	0.0177	0.0125	1.41
Hydrochloric acid	HCl	36.5	0.09460	1.256	42.41	0.191	0.1365	0.0181	0.0129	1.40
Steam[c]	H_2O	18	....	0.623	85.81	0.46	0.36	.....	.....	1.28
Carbon dioxide	CO_2	44.0	0.1142	1.516	35.13	0.205	0.1599	0.0234	0.0183	1.28
Nitrous oxide	N_2O	44.0	0.1143	1.518	35.12	0.221	0.1759	0.0253	0.0201	1.26
Sulphur dioxide	SO_2	64.0	0.1663	2.208	24.13	0.154	0.1230	0.0256	0.0204	1.25
Ammonia	NH_3	17.0	0.04420	0.587	90.77	0.523	0.4064	0.0231	0.0179	1.29
Acetylene	C_2H_2	26.0	0.06754	0.897	59.40	0.350	0.2737	0.0236	0.0185	1.28
Methyl chloride	CH_3Cl	50.5	0.1309	1.738	30.62	0.24	0.2006	0.0314	0.0263	1.20
Methane	CH_4	16.0	0.04163	0.553	96.37	0.593	0.4692	0.0247	0.0195	1.26
Ethylene	C_2H_4	28.0	0.07280	0.967	55.11	0.40	0.3292	0.0291	0.0240	1.22

For more accurate values of specific heats, see pp. 301, 302.

[a] For accurate values of atomic weights, see p. 530.

[b] For values at 60 F, multiply by 1.0154.

[c] Very rough values applying to low pressures and temperatures only. See steam tables, pp. 328 to 335, for more exact values.

v = volume, cu ft.
w = weight of substance under consideration, lb.
W = external work performed on surroundings during a change of state.
X = distance above or below a chosen datum.
$Y = \left(\frac{p_1}{p_2}\right)^{(k-1)/k} - 1$
Z = free energy (by definition, $Z = H - TS$).
ψ = Helmholtz free energy (by definition, $\psi = U - TS$).

In this notation, small letters denote magnitudes referred to unit weight of the substance, capital letters corresponding magnitudes referred to w units of weight. Thus, v denotes the volume of 1 lb, $V = wv$, the volume of w lb. Similarly, $U = wu$, $S = ws$, etc. Subscripts are used to indicate different states; thus, p_1, v_1, T_1, u_1, s_1 refer to state 1, p_2, v_2, T_2, u_2, s_2 refer to state 2. Q_{12} is used to denote the heat absorbed by a body during the change from state 1 to state 2, and W_{12} denotes the external work done during the same change.

The two fundamental and general laws of thermodynamics are: (1) Energy may be neither created nor destroyed. (2) It is impossible to bring about any change or series of changes the sole net result of which is transfer of energy as heat from a low temperature to a high temperature, or, in other words, heat will not of itself flow from low to high temperatures.

The first law of thermodynamics, the statement that energy can be neither created nor destroyed, may be stated mathematically

$$JQ - W = JE_{12}$$

In this equation, W and Q are the heat and work effects as the system passes from the initial to the final state. Care must be exercised in giving the correct signs to Q and W. Heat added to the system or withdrawn from the surroundings is positive, and work done by the system is positive. All terms in the first-law equation must be expressed in the same energy units. The total energy associated with a system E is often called its **internal or intrinsic energy.** Sometimes parts of this may be separated; *e.g.*, the energy due to motion or the kinetic energy $wV^2/2g$, which is important only when flow velocities are considerable, or that due to position above or below a chosen datum, the potential energy which is nearly always negligible. The residual energy of the system is U. The first law might therefore be written

$$JQ - W = JU_{12} + \frac{w}{2g}(V_2^2 - V_1^2) + w(X_2 - X_1)$$

Since as stated above, the last two terms are often negligible they will be omitted for simplicity except when such omission would introduce appreciable error.

Work done in overcoming a fluid pressure is measured by $W = \int p\,dv$, where p is the pressure *effectively* applied to the surroundings for doing work and dv represents the change in volume of the system.

Reversible and Irreversible Processes. A reversible process is one in which both the system and surroundings may be returned to their original states. After an irreversible process, this is not possible. No process involving friction or an unbalanced potential can be reversible. No loss in ability to do work is suffered because of a reversible process but there is always a loss in ability to do work because of an irreversible process. All actual processes are irreversible. Any series of reversible processes that

No entirely satisfactory method for calculation for gaseous mixtures has been developed, but the use of average critical constants as proposed by Kay, (*Ind. Eng. Chem.*, **28**, 1936, p. 1014) is easy and gives satisfactory results under conditions considerably removed from the critical. He assumes the

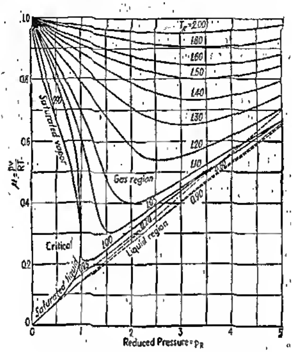


FIG. 3.—The μ Chart (Low Range).

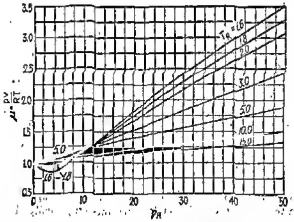


FIG. 4.—The μ Chart (High Range).

gaseous mixture can be treated as if it were a single pure gas with a pseudo-critical pressure and temperature estimated by a method of molal averaging.

$$(T_c)_{\text{mixture}} = (T_c)_a y_a + (T_c)_b y_b + (T_c)_c y_c + \cdots$$
$$(p_c)_{\text{mixture}} = (p_c)_a y_a + (p_c)_b y_b + (p_c)_c y_c + \cdots$$

where $(T_c)_a$ is the critical temperature of pure a, etc.; $(p_c)_a$ is the critical pressure of pure a, etc.; and y_a is the mol fraction of a, etc. For a gaseous

By algebraic rearrangement,

$$(Q_1/T_1) + (Q_2/T_2) = 0$$

Clapeyron Equation.

$$\frac{dp}{dT} = \frac{Q}{ATV_{12}}$$

This important relation is useful in calculations relating to constant-pressure evaporation of pure substances. In that case the equation may be written

$$v_{fg} = \frac{h_{fg}}{AT}\frac{1}{(dp/dT)}$$

where the symbols are as on p. 321.

Entropy. For reversible cyclical processes in which the temperature varies during heat absorption and rejection, *i.e.*, for any *reversible cycle*, $\int \frac{dQ}{T} = 0$. Consequently, for any *reversible process* $\int \frac{dQ}{T}$ is not a function of the particular *reversible path* followed. This integral is called the entropy change, or $\int_1^2 \frac{dQ_{rev}}{T} = S_2 - S_1 = S_{12}$. The entropy of a substance is dependent only on its state or condition. Mathematically, dS is a complete or perfect differential and S is a point function in contrast with Q and W which are path functions. For any reversible process, the change in entropy of the system and surroundings is zero, whereas for any irreversible process, the net entropy change is positive.

All actual processes are irreversible and therefore occur with a decrease in the amount of energy available for doing work, *i.e.*, with an increase in unavailable energy. The increase in unavailable energy is the product of two factors, T_0 the lowest available temperature for heat discard (practically always the temperature of the atmosphere) and the net change in entropy. The increase in unavailable energy is $T_0\,\Delta S_{net}$. Any process that occurs of itself (any spontaneous process) will proceed in such a direction as to result in a net increase in entropy. This is an important concept in the application of thermodynamics to chemical processes.

Availability of a system or quantity of energy is defined as $B = H - T_0S$. In this equation, all quantities except T_0 refer to the system irrespective of the state of the surroundings. T_0 is the lowest temperature available for heat discard. The preceding definition assumes the absence of velocity, potential, and similar effects. When these are not negligible, proper allowance must be made, *e.g.*, $B = H - T_0S + \frac{wV^2}{2Jg} + \frac{wX}{J}$. By substitution of $Q = T_0S_{12}$ in the appropriate first-law expressions, it may be shown that for any steady-flow process, or for any constant-pressure non-flow process, decrease in availability is equal to the maximum possible (reversible) net work effect with a sink for heat discard at T_0.

The availability function B is of particular value in the thermodynamic analysis of changes occurring in the stages of a turbine and is of general utility in determining thermodynamic efficiencies, *i.e.*, the ratio of actual work performed during a process to that which theoretically should have been performed.

Free energy, the Gibbs function, is defined as

$$Z = H - TS$$

3. Isothermal. (Constant Temperature): $p_2/p_1 = V_1/V_2$.

$U_2 - U_1 = 0$. $W_{12} = wRT \log_e(V_2/V_1) = p_1V_1 \log_e(V_2/V_1)$.

$Q_{12} = AW_{12}$. $s_2 - s_1 = Q_{12}/T = wAR \log_e(V_2/V_1)$.

Table 16. Polytropic Expansions

(See also p. 1636 for a chart of p, V, T, n relations for air)

$\frac{p_1}{p_2}$	n				n			
	1.4	1.3	1.2	1.1	1.4	1.3	1.2	1.1
	$V_2/V_1 = (p_1/p_2)^{1/n}$				$T_1/T_2 = (p_1/p_2)^{(n-1)/n}$			
1.1	1.070	1.076	1.083	1.090	1.028	1.022	1.016	1.009
1.2	1.139	1.151	1.164	1.180	1.053	1.043	1.031	1.017
1.3	1.206	1.224	1.244	1.269	1.078	1.062	1.045	1.024
1.4	1.271	1.295	1.323	1.358	1.101	1.081	1.058	1.031
1.5	1.336	1.366	1.401	1.445	1.123	1.098	1.070	1.038
1.6	1.399	1.436	1.479	1.533	1.144	1.115	1.081	1.044
1.7	1.461	1.504	1.557	1.620	1.164	1.130	1.092	1.050
1.8	1.522	1.571	1.633	1.706	1.183	1.145	1.103	1.055
1.9	1.581	1.638	1.706	1.791	1.201	1.160	1.113	1.060
2.0	1.641	1.705	1.782	1.879	1.219	1.174	1.123	1.065
2.5	1.924	2.023	2.145	2.300	1.299	1.235	1.165	1.087
3.0	2.193	2.330	2.498	2.715	1.369	1.289	1.201	1.105
3.5	2.449	2.624	2.842	3.126	1.431	1.336	1.232	1.121
4.0	2.692	2.907	3.117	3.505	1.487	1.378	1.260	1.134
4.5	2.926	3.187	3.500	3.925	1.526	1.415	1.285	1.147
5.0	3.156	3.449	3.824	4.320	1.583	1.449	1.307	1.157
5.5	3.378	3.712	4.142	4.710	1.627	1.482	1.328	1.167
6.0	3.598	3.970	4.447	5.100	1.668	1.512	1.348	1.177
6.5	3.809	4.218	4.760	5.483	1.707	1.540	1.366	1.186
7.0	4.012	4.467	5.058	5.861	1.742	1.566	1.383	1.194
7.5	4.217	4.710	5.360	6.250	1.778	1.591	1.399	1.201
8.0	4.415	4.950	5.650	6.620	1.811	1.616	1.414	1.208
9.0	4.800	5.420	6.240	7.370	1.873	1.660	1.442	1.221
10.0	5.188	5.885	6.820	8.120	1.931	1.701	1.468	1.233
11.0	5.544	6.325	7.376	8.845	1.984	1.739	1.491	1.244
12.0	5.900	6.763	7.931	9.574	2.034	1.774	1.513	1.253
13.0	6.247	7.193	8.478	10.30	2.081	1.807	1.533	1.263
14.0	6.587	7.614	9.018	11.01	2.126	1.839	1.549	1.271
15.0	6.919	8.030	9.551	11.73	2.168	1.868	1.570	1.279
16.0	7.246	8.438	10.08	12.44	2.208	1.896	1.587	1.287
17.0	7.566	8.841	10.60	13.14	2.247	1.923	1.604	1.294
18.0	7.882	9.238	11.12	13.84	2.284	1.948	1.619	1.301
19.0	8.192	9.631	11.63	14.54	2.319	1.973	1.633	1.307
20.0	8.498	10.02	12.14	15.23	2.354	1.996	1.648	1.313
22.0	9.097	10.78	13.14	16.61	2.418	2.041	1.674	1.324
24.0	9.680	11.53	14.13	17.97	2.479	2.082	1.698	1.335
26.0	10.25	12.26	15.10	19.34	2.537	2.121	1.721	1.345
28.0	10.81	12.98	16.07	20.68	2.591	2.158	1.743	1.354
30.0	11.35	13.68	17.02	22.02	2.643	2.192	1.763	1.362
32.0	11.89	14.38	17.96	23.35	2.692	2.225	1.782	1.370
34.0	12.42	15.06	18.89	24.68	2.739	2.256	1.800	1.378
36.0	12.93	15.74	19.81	25.99	2.784	2.287	1.817	1.385
38.0	13.44	16.41	20.72	27.30	2.827	2.315	1.834	1.392
40.0	13.94	17.07	21.63	28.68	2.869	2.343	1.850	1.398

By equating these various terms in the third column which are equal, one may obtain

$$\left(\frac{\partial u}{\partial s}\right)_v = \left(\frac{\partial h}{\partial s}\right)_p \qquad \left(\frac{\partial u}{\partial v}\right)_s = \left(\frac{\partial \psi}{\partial v}\right)_T$$

$$\left(\frac{\partial h}{\partial p}\right)_s = \left(\frac{\partial z}{\partial p}\right)_T \qquad \left(\frac{\partial z}{\partial T}\right)_p = \left(\frac{\partial \psi}{\partial T}\right)_v$$

By mathematical manipulation of equations previously given, the following important relations may be formulated:

$$c_v = \left(\frac{\partial q}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v = \left(\frac{\partial u}{\partial T}\right)_v$$

$$c_p = \left(\frac{\partial q}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial h}{\partial T}\right)_p$$

$$c_p - c_v = AT\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_v$$

$$\left(\frac{\partial c_v}{\partial v}\right)_T = AT\left(\frac{\partial^2 p}{\partial T^2}\right)_v \qquad \left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

Relations involving q, u, h, and s:

$$dq = c_v dT + AT\left(\frac{\partial p}{\partial T}\right)_v dv = c_p dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$du = c_v dt + A\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$

$$dh = c_p dT - A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]dp$$

$$ds = c_v\frac{dT}{T} + A\left(\frac{\partial p}{\partial T}\right)_v dv = c_p\frac{dT}{T} - A\left(\frac{\partial v}{\partial T}\right)_p dp$$

Since $q - AW = u_{12}$ and $h = u + Apv$, for reversible processes

$$du = T\,ds - Apdv \qquad \text{and} \qquad dh = du + Apdv + Avdp$$

it follows that

$$Av = -T\left(\frac{\partial s}{\partial p}\right)_T + \left(\frac{\partial h}{\partial p}\right)_T$$

But from Table 14, $A\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$

Therefore, $\left(\frac{\partial h}{\partial p}\right)_T = A\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]$

Similarly,

$$\left(\frac{\partial u}{\partial v}\right)_T = -A\left[p - T\left(\frac{\partial p}{\partial T}\right)_v\right]$$

These last two equations give in terms of p, v, and T the necessary relations that must hold for any system, however complex. An equation in p, v, and T for the properties of a substance is called an equation of state. These two equations applicable to any substance or system are known as **thermodynamic equations of state.**

from Moss and Smith, "Engineering Computations for Air and Gases," *Trans. A.S.M.E.*, 1929) which is calculated for $(k - 1)/k = 0.29$, or $k = 1.3947$. These values apply to normal air with 36 percent humidity and to perfect diatomic gases. For values of (p_2/p_1) greater than 10, the entries in Table 16 for $n = 1.4$ are available; these give the quantity $Y + 1$.

Determination of Exponent n. Lay off successive values of p and V, measured at chosen points on the curve under investigation, on logarithmic cross-section paper; or, lay off values of $\log p$ and $\log V$ on ordinary cross-section paper. If n is a constant, the points will lie in a straight line, and the slope of the line gives the value of n.

If two representative points (p_1, V_1 and p_2, V_2) be chosen, then

$$n = (\log p_1 - \log p_2)/(\log V_2 - \log V_1)$$

Several pairs of points should be used to test the constancy of n.

Changes of State with Variable Specific Heat. In case of a considerable range of temperature, the assumption of constant specific heat is not permissible, and the equations referring to changes of state must be suitably modified. Experiments on the specific heat of various gases show that the specific heat may sometimes be taken as a linear function of the temperature; thus, $c_v = a + bT$; $c_p = a' + bT$. In that case, the following expressions apply for the change of internal energy and entropy, respectively:

$$U_2 - U_1 = w[a(T_2 - T_1) + 0.5b(T_2^2 - T_1^2)]$$

$$S_2 - S_1 = w[a \log_e (T_2/T_1) + b(T_2 - T_1) + AR \log_e (V_2/V_1)]$$

and for an isentropic change,

$$W_{12} = J(U_1 - U_2)$$

$$AR \log_e (p_2/p_1) = (a + AR) \log_e (T_2/T_1) + b(T_2 - T_1)$$

$$AR \log_e (V_1/V_2) = a \log_e (T_2/T_1) + b(T_2 - T_1)$$

Graphical Representation. The change of state of a substance may be shown graphically by taking any two of the six variables p, V, T, S, U, H as independent coordinates and drawing a curve to represent the successive values of these two variables as the change proceeds. While any pair may be chosen, there are three systems of graphical representation that are specially useful.

1. p and V. The curve (Fig. 5) represents the simultaneous values of p and V during the change (reversible) from state 1 to state 2. The area between the curve and the axis OV is given by the integral $\int_{V_1}^{V_2} p dV$ and therefore represents the external work W_{12} done by the gas during the change. The area included by a closed cycle represents the work of the cycle (as in the indicator diagram of the steam engine).

2. T and S (Fig. 6). The absolute temperature T is taken as the ordinate; the entropy S as the abscissa. The area between the curve of change of state and the S-axis is given by the integral $\int_{S_1}^{S_2} T dS$, and it therefore represents the heat Q_{12} absorbed by the substance from external sources provided there are no irreversible effects. On the T-S diagram, an isothermal is a straight line, as AB, parallel to the S-axis; a *reversible* adiabatic is a straight line, as CD, parallel to the T-axis.

where V is now the total gas volume and N is the number of molecular weights of gas in the volume V, while M is the molecular weight and MR the universal gas constant. If the amount of gas is expressed as w lb, the preceding equation becomes

$$pV = wRT$$

For all perfect gases, MR in lb-ft is 1546. One pound mol of any perfect gas occupies a volume of 359 cu ft at 32 F and 1 atm.

For many engineering purposes, use of the gas laws is permissible up to pressures of 100 to 200 lb per sq in. if the absolute temperatures are at least twice the critical temperatures. Below the critical temperature, errors introduced by use of the gas laws may usually be neglected up to 15 lb per sq in. pressure although errors of 5 percent are often met when dealing with saturated vapors.

The **van der Waals equation of state,** $p = [BT/(v - b)] - a/v^2$, is a modification of the perfect gas law which is sometimes useful at high pressures. The quantities B, a, and b are constants.

Approximate P-V-T Relations. For most gases, suitable P-V-T data are not available. An approximation useful under such circumstances is based on the observation of van der Waals that in terms of reduced properties most gases approximate a common **reduced equation of state.** The reduced quantities are the actual ones divided by the corresponding critical quantities, *e.g.*, the reduced temperature $T_R = T_{\text{actual}}/T_{\text{critical}}$, the reduced volume $v_R = v_{\text{actual}}/v_{\text{critical}}$, the reduced pressure $p_R = p_{\text{actual}}/p_{\text{critical}}$. The gas laws may be made to apply to any non-perfect gas by the introduction of a correction factor μ

$$pV = \mu NMRT$$

When the gas laws apply, $\mu = 1$ and on a molal basis $\mu = pV/MRT$. If on a plot of μ versus p_R lines of constant T_R are drawn, for different substances these are found to fall in narrow bands. Single T_R lines may be drawn to approximately represent the various bands. This has been done in Figs. 3 and 4. The first chart is for low ranges of p_R and T_R and is based on data for hydrocarbons above methane. The second for high ranges of p_R and T_R is based mainly on hydrogen, nitrogen, and oxygen. To use the charts, only the critical pressure and temperature of the gas need be known.

Example. Find the volume of 1 lb of steam at 5,500 lb abs and 1200 F (by steam tables, $v = 0.1516$ cu ft per lb).

For water, critical temperature = 705.4 F; critical pressure = 3,206.4 lb per sq in. abs; reduced temp = 1660/1165 = 1.43; reduced pressure = 5,500/3206.4 = 1.72 μ (see Fig 3) = 0.83. $v = 0.83 \dfrac{(1546)(1660)}{(18)(5500)(144)} = 0.140$ cu ft. Error = 100(0.152 − 0.149)/0.152 ≐ 1.7 percent. If the gas laws had been used, the error would have been 17 percent.

For gases not conforming to these charts, satisfactory approximations may be obtained by using so-called **pseudo constants** empirically chosen.

Pseudo-critical Constants

Gas	T_c', deg K	p_c' atm
Helium	8.3	10.3
Hydrogen	41.3	20.8
Neon	52.3	33.9

If the cycle is traversed in the reverse sense, $Q_{43} = wART_0 \log_e (V_3/V_4)$ is the heat absorbed from the cold body (brine), and the ratio $Q_{43}: A(W) = T_0:(T - T_0)$ is the **coefficient of performance** of the refrigerating machine.

Stirling and Ericsson Cycles. In the Stirling air engine (Fig. 9), the adiabatics of the Carnot cycle are replaced by constant-volume curves; in the Ericsson engine, by constant-pressure curves. By the use of a regenerator, the heat Q_{23} rejected during the operation 23 is stored and is given back to the medium during the operation 41. In the ideal case, $Q_{23} = Q_{41}$, hence the heat absorbed from the *source is* $Q_{12} = wART \log_e V_2/V_1$, *as in the Carnot* cycle, and the efficiency is identical with that of the Carnot cycle.

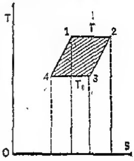


Fig. 9.—Stirling Cycle.

Air engines of the Stirling and Ericsson type, in which the medium is separated from the furnace by a metal wall, have been failures, and have been replaced by the internal-combustion type, in which *the air comes into direct contact with the fuel inside* of the working cylinder. The rapid chemical action supported by the medium itself makes possible the rapid heating of large quantities of gas to a very high temperature. By proper cooling of the outside surface of the metal walls, the deterioration of the metal is prevented even at high temperatures.

The ideal cycles usually employed for internal-combustion engines may be classified in two groups: (1) Explosive—*Otto.* (2) Non-explosive—*Diesel, Joule.*

The Otto Cycle (Fig. 10). Isentropic compression 12 is followed by ignition and *rapid heating at constant volume,* 23. This is followed by isentropic expansion, 34. Assuming constant specific heats the following relations hold:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{p_3}{p_4}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$Q_{23} = wc_v(T_3 - T_2)$$

$$(W) = JQ_{23}[1 - (T_1/T_2)] = Jwc_v(T_3 - T_4 - T_2 + T_1)$$

$$\text{Efficiency} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{k-1} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}$$

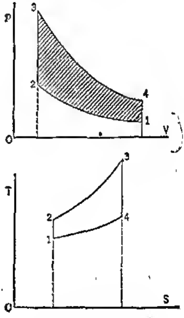


Fig. 10.—Otto Cycle.

If the compression and expansion curves are polytropics with the same value of n, replace k by n in the first relation above. In this case,

$$(W) = [(p_3V_3 - p_4V_4) - (p_2V_2 - p_1V_1)]/(n-1) = wR(T_3 - T_4 - T_2 + T_1)/(n-1)$$

The **mean effective pressure** of the diagram is given by

$$p_m = ap_1[(p_3/p_2) - 1]$$

where a has the values given in the following table.

mixture made up of gases a, b, c, etc., the pseudo-critical constants having been determined, the gaseous mixture is handled on the μ charts as if it were a single pure gas.

Perfect Gas Mixtures. Let V denote the total volume of the mixture, w_1, w_2, w_3 . . . the weights of the constituent gases, R_1, R_2, R_3 . . . the corresponding gas constants, and R_m the constant for the mixture. The partial pressures of the constituents, *i.e.*, the pressures that the constituents would have if occupying the total volume V, are $p_1 = w_1R_1T/V$, $p_2 = w_2R_2T/V$, etc.

According to Dalton's law, the **total pressure** p **of the mixture** is the sum of the partial pressures; *i.e.*, $p = p_1 + p_2 + p_3 + \cdots$. Let $w = w_1 + w_2 + w_3 + \cdots$ denote the total weight of the mixture; then $pV = wR_mT$ and $R_m = \Sigma(w_iR_i)/w$. Also $p_1/p = w_1R_1/wR_m$, $p_2/p = w_2R_2/wR_m$, etc.

Let V_1, V_2, V_3 . . . denote the volumes that would be occupied by the constituents at pressure p and temperature T (these are given by the volume composition of the gas). Then $V = V_1 + V_2 + V_3 + \cdots$ and the apparent molecular weight m_m of the mixture is $m_m = \Sigma(w_iV_i)/V$. Then $R_m = 1546/m_m$. The subscript i denotes an individual constituent.

Volume of 1 lb at 32 F and atm pressure $= 359/m_m$.

Weight of 1 cu ft at 32 F and atm pressure $= 0.002788m_m$.

The specific heats of the mixture are, respectively,

$$c_p = \Sigma(w_ic_{pi})/w, \qquad c_v = \Sigma(w_ic_{vi})/w$$

Internal Energy, Enthalpy, and Entropy. If a perfect gas with constant specific heats changes from an initial state p_1, V_1, T_1, to a final state p_2, V_2, T_2, the following equations hold:

$$U_2 - U_1 = wc_v(T_2 - T_1) = A(p_2V_2 - p_1V_1)/(k - 1)$$

$$H_2 - H_1 = wc_p(T_2 - T_1) = Ak(p_2V_2 - p_1V_1)/(k - 1)$$

$$S_2 - S_1 = w\left(c_v \log_e \frac{T_2}{T_1} + AR \log_e \frac{V_2}{V_1}\right) = w\left(c_p \log_e \frac{T_2}{T_1} - AR \log_e \frac{p_2}{p_1}\right)$$

$$= w\left(c_p \log_e \frac{V_2}{V_1} + c_v \log_e \frac{p_2}{p_1}\right)$$

In general, the energy per unit weight is $u = c_vT + u_0$,

the enthalpy is $h = c_pT + h_0$,

and the entropy is $s = c_v \log_e T + AR \log_e v + s_0$

$= c_p \log_e T - AR \log_e p + s'_0 = c_p \log_e v + c_v \log_e p + s''_0$

The two fundamental equations for perfect gases are

$$dq = c_vdT + Apdv, \qquad dq = c_pdT - Avdp$$

Special Changes of State for Perfect Gases

(Specific heats assumed constant)

In the following formulas, the subscripts 1 and 2 refer to the initial and final states, respectively.

1. Constant Volume: $p_2/p_1 = T_2/T_1$.

$$Q_{12} = U_2 - U_1 = wc_v(t_2 - t_1) = AV(p_2 - p_1)/(k - 1).$$

$$W_{12} = 0. \qquad s_2 - s_1 = wc_v \log_e(T_2/T_1).$$

2. Constant Pressure: $V_2/V_1 = T_2/T_1$.

$$W_{12} = p(V_2 - V_1) = wR(t_2 - t_1).$$

$$Q_{12} = wc_p(t_2 - t_1) = AkW_{12}/(k - 1).$$

$$s_2 - s_1 = wc_p \log_e(T_2/T_1).$$

effected is best shown on the T-S plane (Fig. 14). With single-stage compression, 12 represents the compression from p_1 to p_2, and if the constant-pressure line 23 is drawn cutting the isothermal through point 1 in point 3, the area $1'1233'$ represents the work W. When two stages are used, 14 represents the compression from p_1 to an intermediate pressure p'; 45 cooling at constant pressure in the intercooler between the cylinders, and 56 the compression in the second stage. The area under 14563 represents the work of the two stages and the area 2456 the saving effected by compounding. This saving is a

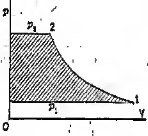


FIG. 13.

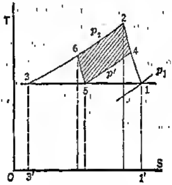


FIG. 14.

Air-compressor Cycle.

maximum when $T_4 = T_6$, and this is the case when the intermediate pressure p' is given by $p' = \sqrt{p_1 p_2}$. See p. 1649

The total work in two-stage compression is

$$-np_1V_1[(p'/p_1)^{(n-1)/n} + (p_2/p')^{(n-1)/n} - 2]/(n-1).$$

Vapors

General Characteristics of Vapors. Let a gas be compressed at constant temperature; then, provided this temperature does not exceed a certain critical value, the gas begins to liquefy at a definite pressure, which depends upon the temperature. At the beginning of liquefaction, a unit weight of gas will also have a definite volume v_g, depending on the temperature. In Fig. 15, AB represents the compression and the point B gives the **saturation** pressure and volume. If the compression is continued, the pressure remains constant with the temperature, as indicated by BC, until at C the substance is in the liquid state, with the volume v_f.

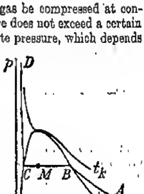


FIG. 15.

The curves v_f and v_g, giving the volumes for various temperatures at the end and beginning of liquefaction, respectively, may be called the **limit curves**. A point B on curve v_g represents the state of **saturated vapor**; a point C on the curve v_f represents the liquid state; and a point M between B and C represents a mixture of vapor and liquid of which the part $x = MC/BC$ is vapor and the part $1 - x = BM/BC$ is liquid. The ratio x is called the **quality of the mixture.** The region between the curves v_f and v_g is thus the region of liquid and vapor mixtures. The region to the right of

4. **Reversible Adiabatic. Isentropic:** $p_1V_1^k = p_2V_2^k$.

$T_2/T_1 = (V_1/V_2)^{k-1} = (p_2/p_1)^{(k-1)/k} = 1/(Y+1)$.

$$AW_{12} = U_1 - U_2 = wc_v(t_1 - t_2). \qquad Q_{12} = 0. \qquad s_2 - s_1 = 0.$$

$$W_{12} = (p_1V_1 - p_2V_2)/(k-1) = p_1V_1[1 - (p_2/p_1)^{(k-1)/k}]/(k-1)$$

$$= p_1V_1\frac{Y}{Y+1}/(k-1) = p_2V_2Y/(k-1)$$

For values of Y, see Table 17.

5. **Polytropic.** This name is given to the change of state which is represented by the equation $pV^n = \text{const.}$ A polytropic curve usually represents actual expansion and compression curves in motors and air compressors for pressures up to a few hundred pounds. By giving n different values and assuming specific heats constant, the preceding changes may be made special cases of the polytropic change, thus,

for $n = 1$,	$pv = \text{const.}$	isothermal
$n = k$,	$pv^k = \text{const.}$	isentropic
$n = 0$,	$p = \text{const.}$	constant pressure
$n = \infty$,	$v = \text{const.}$	constant volume

For a polytropic change of a perfect gas (for which c_v is constant), the specific heat is given by the relation $c_n = c_v(n-k)/(n-1)$; hence for $1 < n < k$, c_n is negative. This is approximately the case in air compression up to a few hundred pounds pressure. The following are the principal formulas: $p_1V_1^n = p_2V_2^n$. $\quad T_2/T_1 = (V_1/V_2)^{n-1} = (p_2/p_1)^{(n-1)/n}$.

$$W_{12} = (p_1V_1 - p_2V_2)/(n-1) = p_1V_1[1 - (p_2/p_1)^{(n-1)/n}]/(n-1).$$

$$Q_{12} = wc_n(t_2 - t_1).$$

$$AW_{12}\!:U_2 - U_1\!:Q_{12} = k - 1\!:1 - n\!:k - n$$

The quantity $Y = (p_1/p_2)^{(k-1)/k} - 1$ occurs frequently in calculations for perfect gases. Values of this quantity are given in Table 17 (condensed

Table 17. Values of Y for Normal Air and Perfect Diatomic Gases

$Y = (p_1/p_2)^{0.283} - 1$

p_1/p_2	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0028	0056	0084	0112	0139	0166	0193	0220	0247
1.1	0273	0300	0326	0352	0378	0404	0429	0454	0480	0505
1.2	0530	0554	0579	0603	0628	0652	0676	0700	0724	0747
1.3	0771	0794	0817	0841	0864	0886	0909	0932	0954	0977
1.4	0999	1021	1043	1065	1087	1109	1130	1152	1173	1195
1.5	0.1216	1237	1258	1279	1300	1321	1341	1362	1382	1402
1.6	1423	1443	1463	1483	1503	1523	1542	1562	1581	1601
1.7	1620	1640	1659	1678	1697	1716	1735	1754	1773	1791
1.8	1810	1828	1847	1865	1884	1902	1920	1938	1956	1974
1.9	1992	2010	2028	2045	2063	2080	2098	2115	2133	2150
2	0.2167	2336	2500	2658	2812	2960	3105	3246	3383	3516
3	3647	3774	3898	4020	4139	4255	4369	4481	4591	4698
4	4804	4908	5010	5110	5209	5306	5401	5495	5588	5679
5	5769	5858	5945	6031	6116	6200	6283	6365	6446	6525
6	6604	6682	6759	6835	6910	6985	7058	7131	7203	7274
7	0.7345	7414	7483	7552	7620	7687	7753	7819	7884	7949
8	8013	8076	8139	8201	8263	8324	8385	8445	8505	8564
9	8623	8681	8739	8797	8854	8910	8966	9022	9077	9132
10	9187	9241	9295	9348	9401	9453	9506	9558	9609	9660
11	9711	9762	9812	9862	9912	9961	1.0010	1.0058	1.0107	1.0155

$u_{fg} = u_g - u_f$ = increase of internal energy during vaporization.
$s_{fg} = s_g - s_f = h_{fg}/T$ = increase of entropy during vaporization.
Apv_{fg} = work performed during vaporization.

The energy equation applied to the vaporization process is

$$h_{fg} = u_{fg} + Apv_{fg}$$

The properties of a unit weight of a mixture of liquid and vapor of quality x are given by the following expressions.

$$v = v_f + xv_{fg} \qquad u = u_f + xu_{fg}$$
$$h = h_f + xh_{fg} \qquad s = s_f + xs_{fg}$$

Tables of superheated vapor usually give values of v, h, and s per unit weight. The internal energy u per unit weight can be found from the equation

$$u = h - Apv$$

or with p in lb per sq in.

$$u = h - 0.1852pv$$

Charts for Saturated and Superheated Vapors

Certain properties of vapor mixtures and superheated vapors may be shown graphically by means of charts. Such charts show the behavior of vapors and have a practical application in the solution of certain problems.

1. **Temperature-entropy Chart.** Figure 16 shows the temperature-entropy chart for water vapor. The liquid curve is obtained by plotting corresponding values of T and s_f, and the saturation curve by plotting values of T and s_g. The values are taken from Tables 20 and 21. The two curves merge into each other at the critical temperature T = 1165.1 R. Between these two curves, constant pressure lines are also lines of constant temperature; but at the saturation curve the constant pressure lines show a sharp break with rising temperature. The constant quality lines x = 0.2, 0.4, etc., are equally spaced between the liquid and saturation curves.

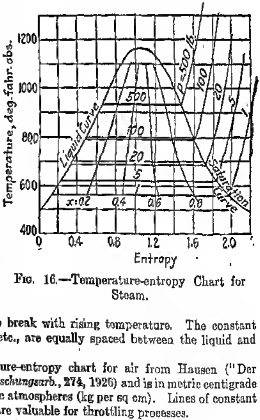


FIG. 16.—Temperature-entropy Chart for Steam.

Figure 17 is a temperature-entropy chart for air from Hausen ("Der Thomson-Joule-Effekt," *Forschungsarb.*, **274**, 1926) and is in metric centigrade units; pressures are in metric atmospheres (kg per sq cm). Lines of constant enthalpy are included and are valuable for throttling processes.

Figure 18 shows a temperature-entropy chart for isobutane. The form of the saturation curve is worthy of note. In the case of water vapor, this curve has a negative slope throughout; but in the case of isobutane, the curve has a positive slope except near the critical temperature. In the case of water

In the case of internal generation of heat through friction, as in steam turbines, the increase of entropy is given by $\int_{T_1}^{T_2} \frac{dQ'}{T}$ (see p. 307) and the area under the curve represents the heat Q' thus generated. In this case, an adiabatic is *not* a straight line parallel to the T-axis.

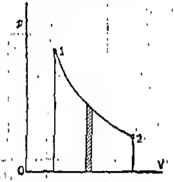


FIG. 5.

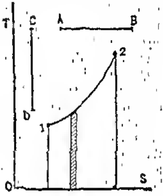


FIG. 6.

3. H and S. In the system of representation devised by Dr. Mollier, the enthalpy H is taken as the ordinate and the entropy S as the abscissa. If on this diagram (Fig. 7) a line of constant pressure, as 12, be drawn, the heat absorbed during the change at constant pressure is given by $Q_{12} = H_2 - H_1$, and this is represented by the line segment 23. The **Mollier diagram** is specially useful in problems that involve the flow of fluids, throttling, and the action of steam in turbines. A Mollier diagram for steam is given on p. 326, and for ammonia on p. 340.

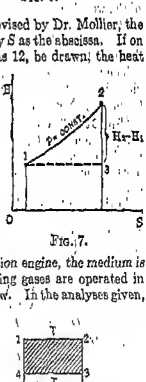


FIG. 7.

Ideal Cycles with Perfect Gases

Gases are used as heat mediums in several important types of motors. In air compressors, air engines, and air refrigerating machines, atmospheric air is the medium. *In the internal-combustion engine, the medium is* a mixture of products of combustion. Motors using gases are operated in certain well-defined cycles, which are described below. In the analyses given, ideal conditions that cannot be attained by actual motors are assumed. However, conclusions derived from such analyses are usually approximately valid for the modified actual cycle.

In the following, the subscripts 1, 2, 3, etc., refer to corresponding points shown in the figures. The work of the cycle is denoted by (W) and the net heat absorbed by (Q).

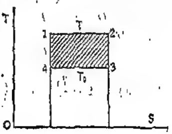


FIG. 8.—Carnot Cycle.

Carnot Cycle. The Carnot cycle (Fig. 8) is of historic interest. It consists of two isothermals and two isentropics. The heat absorbed along the upper isothermal 12 is $Q_{12} = wART \log_e (V_2/V_1)$, and the heat transformed into work, represented by the cycle area, is $A(W) = Q_{12}[1 - (T_0/T)]$.

Hence,
$$(W) = wR(T - T_0) \log_e (V_2/V_1)$$

Following a constant entropy line from the point 240 lb, 580 F, it is found that this line intersects the line $p = 60$ lb on the saturation curve and that the value of h at the point of intersection is 1177.6 Btu. Hence the steam in the second state is just saturated, and the decrease in enthalpy is $1308.5 - 1177.6 = 130.9$ Btu.

3. **Pressure-enthalpy Chart.** For refrigeration media it is convenient to use pressure and enthalpy as the variables to be plotted. Such a chart for ammonia is shown in Fig. 21, p. 340.

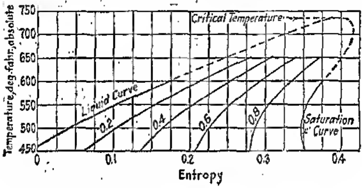


FIG. 18.—Temperature-entropy Chart for Isobutane.

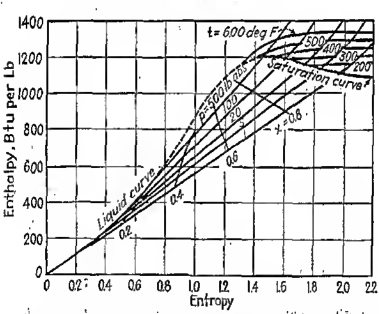


FIG. 19.—Enthalpy-entropy Chart for Steam.

Changes of State. Superheated Vapors and Mixtures of Liquid and Vapor

Isothermal. In the only important cases, the fluid is a mixture of liquid and vapor in both initial and final states.

$t =$ const. $\quad p =$ const. $\quad x_1, x_2 =$ initial and final qualities

$$Q_{12} = wh_{fg}(x_2 - x_1). \qquad U_2 - U_1 = wu_{fg}(x_2 - x_1)$$

$$W_{12} = wpv_{fg}(x_2 - x_1). \qquad S_2 - S_1 = Q_{12}/T$$

Constant Pressure. If the fluid is a mixture at the beginning and end of the change, the constant pressure change is also isothermal. If the initial

	$p_2/p_1 = 3$	4	5	6	8	10	12	14	16
($n = 1.4$)...........	$a = 1.70$	1.94	2.13	2.31	2.02	2.88	3.10	3.31	3.50
($n = 1.3$)...........	$a = 1.69$	1.92	2.11	2.28	2.57	2.81	3.03	3.22	3.39
($n = 1.2$)...........	$a = 1.68$	1.90	2.08	2.25	2.51	2.74	2.94	3.12	3.27

The Diesel Cycle. In the Diesel oil engine, air is compressed to a high pressure. Fuel is then injected into the air and, as the temperature is above the ignition point, burns at nearly constant pressure (23, in Fig. 11). Isentropic expansion of the products of combustion is followed by exhaust and suction of fresh air, as in the Otto cycle.

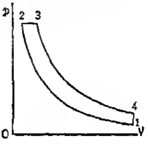


FIG 11.—Diesel Cycle.

The work obtained is

$$(W) = Jw[c_p(T_3 - T_2) - c_v(T_4 - T_1)]$$

and the efficiency of the ideal cycle is

$$1 - [(T_4 - T_1)/k(T_3 - T_2)]$$

The Joule cycle (Fig. 12) consists of two isentropics and two constant-pressure lines. The following relations hold:

$$V_3/V_2 = V_4/V_1 = T_3/T_2 = T_4/T_1$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{V_4}{V_3}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$(W) = Jwc_p(T_3 - T_2 - T_4 + T_1)$. Efficiency $= (W)/JQ_{23} = 1 - (T_1/T_2)$

For the use of the Joule cycle in refrigeration, see p. 349.

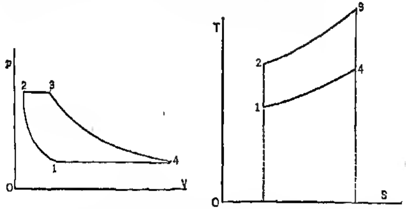


FIG. 12.—Joule Cycle.

Air Compression. It is assumed that the compressor works under ideal reversible conditions without clearance and without friction losses and that the changes are over ranges where the gas laws are applicable. Where the gas laws cannot be used, analysis in terms of μ charts (p. 311) is convenient. If the compression from p_1 to p_2 (Fig. 13) follows the law pV^n = const., the work represented by the indicator diagram is

$$-W = n(p_2V_2 - p_1V_1)/(n - 1) = np_1V_1[(p_2/p_1)^{(n-1)/n} - 1]/(n - 1).$$

The temperature at the end of compression is given by $T_2/T_1 = (p_2/p_1)^{(n-1)/n}$. The work W is smaller the smaller the value of n, and the purpose of the water jacket is to reduce n from the isentropic value 1.4. Under usual working conditions, n is about 1.3.

When the pressure p_2 is high, it is advantageous to divide the process into two or more stages and cool the air between the cylinders. The saving

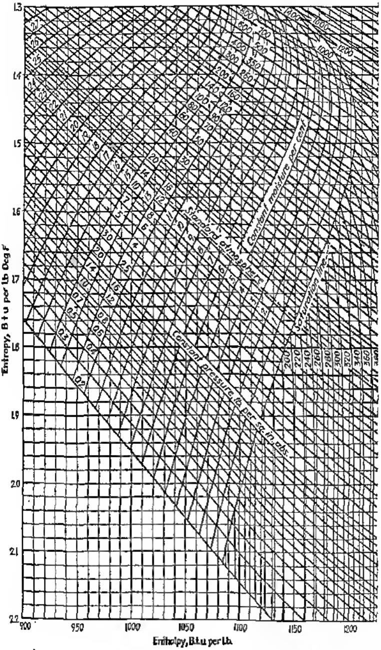


Fig. 20.—Enthalpy-

curve v_g is the region of **superheated vapor**. The curve v_g dividing these regions represents the so-called **saturated vapor**.

For saturated vapor or a **mixture of vapor and liquid**, the pressure is a function of the temperature **only**, and the **volume** of the mixture depends upon the temperature and quality x. That is, $p = f(t)$, $v = F(t, x)$.

For the vapor in the superheated state, the volume depends on pressure and temperature [$v = F_1(p, t)$], and these may be varied independently.

Critical State. If the temperature of the gas lies above a definite temperature t_c called the **critical temperature**, the gas cannot be liquefied by compression alone. The saturation pressure corresponding to t_c is the **critical pressure** and is denoted by p_c. At the critical state, the limit curves v_f and v_g merge; hence for temperatures above t_c, it is impossible to have a mixture of vapor and liquid. Table 18 gives the critical data for various gases; also the boiling temperature t_b corresponding to atmospheric pressure.

Table 18. Critical Data for Various Gases
(Condensed from the International Critical Tables)

Substance	t_b deg F	t_c deg F	p_c atm	v_c cu ft per lb	Substance	t_b deg F	t_c deg F	p_c atm	v_c cu ft per lb
Air.......	−317.6	−220.3	37.2	0.0457	C_2H_6...	−127.5	90.0	48.2	0.079
Helium.....	−452.0	−450.2	2.26	0.231	C_3H_8...	− 48.1	206.26	42.01	0.071
Hydrogen..	−422.9	−399.8	12.8	0.516	C_4H_{10}...	31.5	307.4	37.48	0.071
Argon.....	−302.3	−187.7	48.0	0.03	C_5H_{12}...	97.0	387.0	33	0.069
Nitrogen..	−320.4	−232.8	33.5	0.053	C_6H_{14}...	156.1	454.6	29.5	0.0685
Oxygen....	−297.2	−181.8	49.7	0.037	C_7H_{16}...	209.2	517.1	27.65	0.0685
Bromine..	137.8	575.6	....	....	C_8H_{18}...	259.2	565.7	24.8	0.0685
Chlorine..	− 30.3	291.3	76.1	0.028	C_2H_2...	−118.5	96.3	62	0.0693
HCl......	−121	124.5	81.6	0.038	C_2H_4...	−155.0	49.3	50.9	0.073
H_2S......	− 74.9	212.7	88.9	....	C_6H_6...	176	551.4	47.7	0.0526
NO.......	−239.8	−136.7	65.0	0.031	C_7H_8...	231.3	609.1	41.6	0.055
N_2O......	−129.1	97.7	71.7	0.036	CH_4O..	147.3	464.0	78.7	0.059
NH_3......	− 28	270.3	111.5	0.068	C_2H_6O..	173.0	469.6	63.1	0.058
H_2O......	212	705.45	218.53	0.0503	C_3H_6O..	133	455	47	0.060
SO_2......	14	315.0	77.7	0.031	$C_4H_{10}O$.	101.8	380.8	35.5	0.061
CO.......	−313.6	−220.33	34.53	0.053	$CHCl_3$..	142.2	505.4	....	0.031
CO_2......	−109.3	88.0	73.0	0.035	CH_3Cl..	− 10.25	289.6	65.8	0.043
CS_2......	115.3	523.4	76.0	....	C_2H_5Cl.	54.9	369.0	52	0.0485
CH_4......	−258.5	−116.5	45.8	0.099					

Thermal Properties of Saturated Vapors and of Vapor and Liquid Mixtures

Notation:

v_f = volume in cu ft of 1 lb of saturated liquid.
v_g = volume in cu ft of 1 lb of saturated vapor.
c_f = specific heat of saturated liquid.
c_g = specific heat of saturated vapor.
h_f, h_g = specific enthalpy of saturated liquid and vapor, respectively.
u_f, u_g = specific internal energy of saturated liquid and vapor, respectively.
s_f = specific entropy of saturated liquid.
s_g = specific entropy of saturated vapor.
$v_{fg} = v_g - v_f$ = increase of volume during vaporization.
$h_{fg} = h_g - h_f$ = heat of vaporization, or heat required to vaporize 1 lb of liquid at constant pressure and temperature.

r may be used for h_{fg} when several heats of vaporization (as r_1, r_2, r_3, etc.) are under consideration.

Table 20. Properties of Saturated Steam

(From Keenan and Keyes, "Thermodynamic Properties of Steam")

Abs press., lb per sq in.	Temp, deg F	Specific volume		Enthalpy			Entropy			Internal energy
		Liquid	Vapor	Liquid	Evap	Vapor	Liquid	Evap	Vapor	Evap
1.0	101.74	0.01614	333.6	69.70	1036.3	1106.0	0.1326	1.8456	1.9782	974.6
1.2	107.92	0.01616	280.9	75.87	1032.7	1108.6	0.1435	1.8193	1.9628	970.3
1.4	113.26	0.01618	243.0	81.20	1029.6	1110.8	0.1528	1.7971	1.9498	966.7
1.6	117.99	0.01620	214.3	85.91	1026.9	1112.8	0.1610	1.7776	1.9386	963.5
1.8	122.23	0.01621	191.8	90.14	1024.5	1114.6	0.1683	1.7605	1.9288	960.6
2.0	126.08	0.01623	173.73	93.99	1022.2	1116.2	0.1749	1.7451	1.9200	957.9
2.2	129.62	0.01624	158.85	97.52	1020.2	1117.7	0.1809	1.7311	1.9120	955.5
2.4	132.89	0.01626	146.38	100.79	1018.3	1119.1	0.1864	1.7183	1.9047	953.3
2.6	135.94	0.01627	135.78	103.83	1016.5	1120.3	0.1916	1.7065	1.8981	951.2
2.8	138.79	0.01629	126.65	106.68	1014.8	1121.5	0.1963	1.6957	1.8920	949.2
3.0	141.48	0.01630	118.71	109.37	1013.2	1122.6	0.2008	1.6855	1.8863	947.3
4.0	152.97	0.01636	90.63	120.86	1006.4	1127.3	0.2198	1.6427	1.8625	939.3
5.0	162.24	0.01640	73.52	130.13	1001.0	1131.1	0.2347	1.6094	1.8441	933.0
6.0	170.06	0.01645	61.98	137.96	996.2	1134.2	0.2472	1.5820	1.8292	927.5
7.0	176.85	0.01649	53.64	144.76	992.1	1136.9	0.2581	1.5586	1.8167	922.7
8.0	182.86	0.01653	47.34	150.79	988.5	1139.3	0.2674	1.5383	1.8057	918.4
9.0	188.28	0.01656	42.40	156.22	985.2	1141.4	0.2759	1.5203	1.7962	914.6
10	193.21	0.01659	38.42	161.17	982.1	1143.3	0.2835	1.5041	1.7876	911.1
11	197.75	0.01662	35.14	165.73	979.3	1145.0	0.2903	1.4897	1.7800	907.8
12	201.96	0.01665	32.40	169.96	976.6	1146.6	0.2967	1.4763	1.7730	904.8
13	205.88	0.01667	30.06	173.91	974.2	1148.1	0.3027	1.4638	1.7665	901.9
14	209.56	0.01670	28.04	177.61	971.9	1149.5	0.3083	1.4522	1.7605	899.3
14.696	212.00	0.01672	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	897.5
15	213.03	0.01672	26.29	181.11	969.7	1150.8	0.3135	1.4415	1.7549	896.7
16	216.32	0.01674	24.75	184.42	967.6	1152.0	0.3184	1.4313	1.7497	894.3
17	219.44	0.01677	23.39	187.56	965.5	1153.1	0.3231	1.4218	1.7449	892.0
18	222.41	0.01679	22.17	190.56	963.6	1154.2	0.3275	1.4128	1.7403	889.9
19	225.24	0.01681	21.08	193.42	961.9	1155.3	0.3317	1.4043	1.7360	887.8
20	227.96	0.01683	20.[illegible]	196.16	960.[illegible]	1156.3	0.3356	1.3962	1.73[illegible]	885.8
21	230.57	0.01685	19.192	198.79	958.4	1157.2	0.3395	1.3885	1.7280	883.9
22	233.07	0.01687	18.375	201.33	956.8	1158.1	0.3431	1.3811	1.7242	882.0
23	235.49	0.01689	17.627	203.78	955.2	1159.0	0.3466	1.3740	1.7206	880.2
24	237.82	0.01691	16.938	206.14	953.7	1159.8	0.3500	1.3672	1.7172	878.5
25	240.07	0.01692	16.303	208.42	952.1	1160.6	0.3533	1.3606	1.7139	876.8
26	242.25	0.01694	15.715	210.62	950.7	1161.3	0.3564	1.3544	1.7108	875.2
27	244.36	0.01696	15.170	212.75	949.3	1162.0	0.3594	1.3484	1.7078	873.6
28	246.41	0.01698	14.663	214.83	947.9	1162.7	0.3623	1.3425	1.7048	872.1
29	248.40	0.01699	14.189	216.86	946.5	1163.4	0.3652	1.3368	1.7020	870.5
30	250.33	0.01701	13.746	218.82	945.3	1164.1	0.3680	1.3313	1.6993	869.1
31	252.22	0.01702	13.330	220.73	944.0	1164.7	0.3707	1.3260	1.6967	867.7
32	254.05	0.01704	12.940	222.59	942.8	1165.4	0.3733	1.3209	1.6941	866.3
33	255.84	0.01705	12.572	224.41	941.6	1166.0	0.3758	1.3159	1.6917	864.9
34	257.58	0.01707	12.226	226.18	940.3	1166.5	0.3783	1.3110	1.6893	863.5
35	259.28	0.01708	11.898	227.91	939.2	1167.1	0.3807	1.3063	1.6870	862.3
36	260.95	0.01709	11.588	229.60	938.0	1167.6	0.3831	1.3017	1.6848	861.0
37	262.57	0.01711	11.294	231.26	936.9	1168.2	0.3854	1.2972	1.6826	859.8
38	264.16	0.01712	11.01[illegible]	232.89	935.8	1168.7	0.3876	1.2929	1.6805	858.5
39	265.72	0.01714	10.758	234.48	934.7	1169.2	0.3898	1.2886	1.6784	857.2

vapor, therefore, isentropic expansion of dry vapor (s = const) will be accompanied by condensation, isentropic compression by superheating. For isobutane, in the region where the saturation curve has the positive slope, the conditions are reversed; isentropic expansion is accompanied by superheating, isentropic compression by condensation.

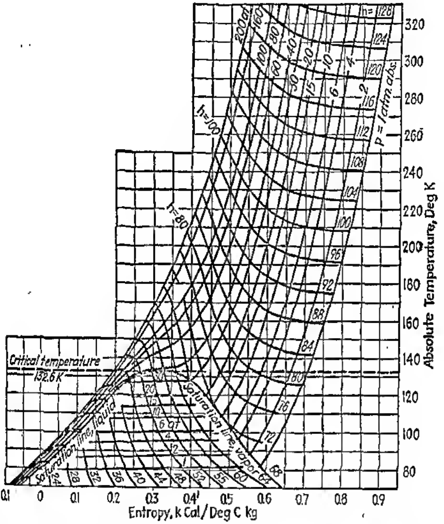


FIG. 17.—Temperature-entropy Chart for Air. (From Hausen, "Der Thomson-Joule Effekt," *Forschungsarbeiten*, 274, 1926.)

2. **Enthalpy-entropy Chart (Mollier Chart).** In this chart, the enthalpy h is taken as the ordinate and entropy as the abscissa. Figure 19 shows a Mollier chart for water vapor. A large-scale chart (Fig. 20) covering only the region near the saturation curve is shown on pp. 326 and 327.

The following examples illustrate the use of the Mollier chart.

1. Steam enters a superheater at a pressure of 240 lb containing 2 percent water. It leaves the superheater at a temperature of 580 F. Required the heat per pound of steam to effect this change.

The initial and final points are located on the Mollier chart. From the enthalpy scale, $h_1 = 1184.2$ and $h_2 = 1308.5$; therefore $q_{12} = 1308.5 - 1184.2 = 124.3$ B.t.u.

2. Steam at $p = 240$ lb, $t = 580$ F expands isentropically to a pressure of 60 lb. Required the final condition of the steam and the decrease in enthalpy.

Table 20. Properties of Saturated Steam.—(*Continued*)

Abs press, lb per sq in.	Temp, deg F	Specific volume		Enthalpy			Entropy			Internal energy
		Liquid	Vapor	Liquid	Evap	Vapor	Liquid	Evap	Vapor	Evap
90	320.27	0.01766	4.896	290.56	894.7	1185.3	0.4641	1.1471	1.6112	813.4
91	321.06	0.01767	4.845	291.38	894.1	1185.5	0.4651	1.1452	1.6103	812.8
92	321.83	0.01768	4.796	292.18	893.5	1185.7	0.4661	1.1433	1.6094	812.2
93	322.60	0.01768	4.747	292.98	892.9	1185.9	0.4672	1.1413	1.6085	811.5
94	323.36	0.01769	4.699	293.78	892.3	1186.1	0.4682	1.1394	1.607[illegible]	810.9
95	324.12	0.01770	4.652	294.56	891.7	1186.2	0.4692	1.1376	1.6068	810.2
96	324.87	0.01771	4.606	295.34	891.1	1186.4	0.4702	1.1358	1.6060	809.6
97	325.61	0.01772	4.561	296.12	890.5	1186.6	0.4711	1.1340	1.6051	808.9
98	326.35	0.01772	4.517	296.09	889.9	1186.8	0.4721	1.1322	1.6043	808.3
99	327.08	0.01773	4.474	297.65	889.4	1187.0	0.4731	1.1304	1.6035	807.7
100	327.81	0.01774	4.432	298.40	888.8	1187.2	0.4740	1.1286	1.6026	807.1
102	329.25	0.01775	4.350	299.90	887.6	1187.5	0.4759	1.1251	1.6010	805.9
104	330.66	0.01777	4.271	301.37	886.5	1187.9	0.4778	1.1216	1.5994	804.7
106	332.05	0.01778	4.194	302.82	865.4	1188.2	0.4796	1.1182	1.5978	803.5
108	333.42	0.01780	4.120	304.26	884.3	1188.6	0.4814	1.1149	1.5963	802.4
110	334.77	0.01782	4.049	305.66	883.2	1188.9	0.4832	1.1117	1.5948	801.2
112	336.11	0.01783	3.981	307.06	882.1	1189.2	0.4849	1.1085	1.5934	800.0
114	337.42	0.01784	3.914	308.43	881.1	1189.5	0.4866	1.1053	1.5919	798.9
116	338.72	0.01786	3.850	309.79	880.0	1189.8	0.4883	1.1022	1.5905	797.8
118	339.99	0.01787	3.788	311.12	879.0	1190.1	0.4900	1.0992	1.5891	796.7
120	341.25	0.01789	3.728	312.44	877.9	1190.4	0.4916	1.0962	1.5878	795.6
122	342.50	0.01791	3.670	313.75	876.9	1190.7	0.4932	1.0933	1.5865	794.5
124	343.72	0.01792	3.614	315.04	875.9	1190.9	0.4948	1.0903	1.5851	793.4
126	344.94	0.01793	3.560	316.31	874.9	1191.2	0.4964	1.0874	1.5838	792.3
128	346.13	0.01794	3.507	317.57	873.9	1191.5	0.4980	1.0845	1.5825	791.3
130	347.32	0.01796	3.455	318.81	872.9	1191.7	0.4995	1.0817	1.5812	790.2
132	348.48	0.01797	3.405	320.04	872.0	1192.0	0.5010	1.0790	1.5800	789.2
134	349.64	0.01799	3.357	321.25	871.0	1192.2	0.5025	1.0762	1.5787	788.2
136	350.78	0.01800	3.310	322.45	870.1	1192.5	0.5040	1.0735	1.5775	787.2
138	351.91	0.01801	3.264	323.64	869.1	1192.7	0.5054	1.0709	1.5763	786.2
140	353.02	0.01802	3.220	324.82	868.2	1193.0	0.5069	1.0682	1.5751	785.2
142	354.12	0.01804	3.177	325.98	067.2	1193.2	0.5083	1.0657	1.5740	784.3
144	355.21	0.01805	3.134	327.13	866.3	1193.4	0.5097	1.0631	1.5728	783.3
146	356.29	0.01806	3.094	328.27	865.3	1193.6	0.5111	1.0605	1.5716	782.3
148	357.36	0.01808	3.054	329.39	864.5	1193.9	0.5124	1.0580	1.5705	781.4
150	358.42	0.01809	3.015	330.51	863.6	1194.1	0.5138	1.0556	1.5694	780.5
152	359.46	0.01810	2.977	331.61	862.7	1194.3	0.5151	1.0532	1.5683	779.5
154	360.49	0.01812	2.940	332.70	861.8	1194.5	0.5165	1.0507	1.5672	778.5
156	361.52	0.01813	2.904	333.79	860.9	1194.7	0.5178	1.0483	1.5661	777.6
158	362.53	0.01814	2.869	334.86	860.0	1194.9	0.5191	1.0459	1.5650	776.8
160	363.53	0.01815	2.834	335.93	859.2	1195.1	0.5204	1.0436	1.5640	775.8
162	364.53	0.01817	2.801	336.98	858.3	1195.3	0.5216	1.0414	1.5630	775.0
164	365.51	0.01818	2.768	338.02	857.5	1195.5	0.5229	1.0391	1.5620	774.1
166	366.48	0.01819	2.736	339.05	856.6	1195.7	0.5241	1.0369	1.5610	773.2
168	367.45	0.01820	2.705	340.07	855.7	1195.8	0.5254	1.0346	1.5600	772.3
170	368.41	0.01822	2.675	341:09	854.9	1196.0	0.5266	1.0324	1.5590	771.4
172	369.35	0.01823	2.645	342.10	854.1	1196.2	0.5278	1.0302	1.5580	770.5
174	370.29	0.01824	2.616	343.10	853.3	1196.4	0.5290	1.0280	1.5570	769.7
176	371.22	0.01825	2.587	344.09	052.4	1196.5	0.5302	1.0259	1.5561	768.8
178	372.14	0.01826	2.559	345.06	851.6	1196.7	0.5313	1.0238	1.5551	767.9

state is in the mixture region and the final state is that of a superheated vapor, the following are the equations for Q_{12}, etc. Let h_2, u_2, v_2, and s_2 be the properties of 1 lb of superheated vapor in the final state 2: then

$$Q_{12} = w(h_2 - h_1), \qquad h_1 = h_{f1} + x_1 h_{fg1}$$
$$U_2 - U_1 = w(u_2 - u_1), \qquad u_1 = u_{f1} + x_1 u_{fg1}$$
$$S_2 - S_1 = w(s_2 - s_1), \qquad s_1 = s_{f1} + x_1 s_{fg1}$$
$$W_{12} = wp(v_2 - v_1), \qquad v_1 = v_{f1} + x_1 v_{fg1}$$

Constant Volume. Since v_f the liquid volume is nearly constant,

$$x_1 v_{fg1} = x_2 v_{fg2}$$
$$x_2 = x_1 v_{fg1}/v_{fg2}, \qquad \text{or} \qquad x_2 = x_1 v_{g1}/v_{g2} \text{ approx}$$
$$Q_{12} = U_2 - U_1 = w(u_2 - u_1). \qquad W_{12} = 0$$

Isentropic. $s = \text{const.}$

If the fluid is a mixture in the initial and final states,

$$s_{f1} + x_1 s_{fg1} = s_{f2} + x_2 s_{fg2}$$

If the initial state is that of superheated vapor,

$$s_1 = s_{f2} + x_2 s_{fg2}$$

in which s_1 is read from the table of superheated vapor. The final value x_2 is determined from one of these equations, and the final energy u_2 is then $u_{f2} + x_2 u_{fg2}$. $\quad Q_{12} = 0. \quad W_{12} = J(U_1 - U_2) = Jw(u_1 - u_2)$.

For water vapor, the relation between p and v during an isentropic change may be represented approximately by the equation $pv^n = \text{const}$. The exponent n is not constant, but varies with the initial quality and initial pressure, as shown in Table 19.

Table 19. Values of n (Water Vapor)

Initial quality	Initial pressure, lb per sq in. abs											
	20	40	60	80	100	120	140	160	160	200	220	240
1.00	1.131	1.132	1.133	1.134	1.136	1.137	1.138	1.139	1.141	1.142	1.143	1.145
0.95	1.127	1.128	1.129	1.130	1.131	1.131	1.132	1.133	1.134	1.135	1.136	1.137
0.90	1.123	1.123	1.124	1.124	1.125	1.125	1.126	1.126	1.127	1.127	1.128	1.129
0.85	1.119	1.119	1.119	1.119	1.120	1.120	1.120	1.120	1.120	1.120	1.120	1.121
0.80	1.115	1.115	1.114	1.114	1.114	1.114	1.113	1.113	1.113	1.113	1.112	1.112
0.75	1.111	1.110	1.110	1.109	1.109	1.108	1.107	1.106	1.106	1.105	1.104	1.104

The isentropic expansion of superheated steam is fairly represented by $pv^n = \text{const}$, with $n = 1.315$.

The volume at the end of expansion (or compression) is $V_2 = V_1(p_1/p_2)^{1/n}$, and the external work is

$$W_{12} = (p_1V_1 - p_2V_2)/(n - 1) = p_1V_1[1 - (p_2/p_1)^{(n-1)/n}]/(n - 1).$$

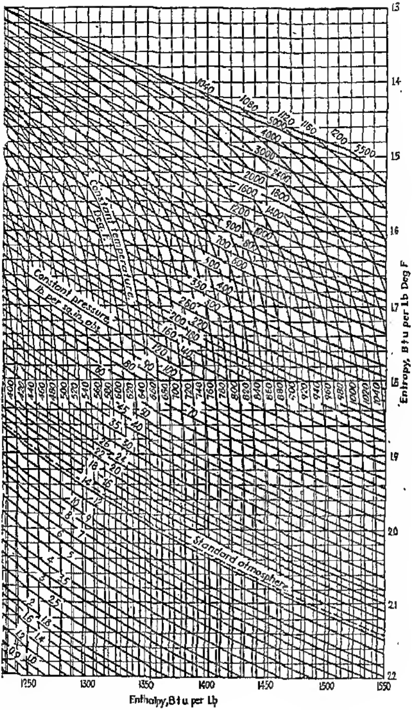
If the initial state is in the region of superheat and the final state in the mixture region, two values of n must be used: $n = 1.315$ for the expansion to the state of saturation, and the appropriate value from the first row of Table 19 for the expansion of the mixture.

Tables of Thermal Properties of Vapors

Tables 20, 21, and 22, abstracted from Keenan and Keyes, "Thermodynamic Properties of Steam," give internationally accepted values which are believed to be correct within very narrow tolerance. They are a representation of the results obtained from an organized body of research and agreed to in a series of international conferences.

Table 20. Properties of Saturated Steam.—(*Continued*)

Abs press, lb per sq in.	Temp, deg F	Specific volume		Enthalpy			Entropy			Internal energy
		Liquid	Vapor	Liquid	Evap	Vapor	Liquid	Evap	Vapor	Vapor
500	467.01	0.0197	0.9278	449.4	755.0	1204.4	0.6487	0.8147	1.4634	1118.6
520	471.07	0.0198	0.8915	454.1	750.1	1204.2	0.6536	0.8060	1.4596	1118.4
540	475.01	0.0199	0.8578	458.6	745.4	1204.0	0.6584	0.7976	1.4560	1118.3
560	478.85	0.0200	0.8265	463.0	740.8	1203.8	0.6631	0.7893	1.4524	1118.2
580	482.58	0.0201	0.7973	467.4	736.1	1203.5	0.6676	0.7813	1.4489	1118.0
600	486.21	0.0201	0.7698	471.6	731.6	1203.2	0.6720	0.7734	1.4454	1117.7
620	489.75	0.0202	0.7440	475.7	727.2	1202.9	0.6763	0.7658	1.4421	1117.5
640	493.21	0.0203	0.7198	479.8	722.7	1202.5	0.6805	0.7584	1.4389	1117.3
660	496.58	0.0204	0.6971	483.8	718.3	1202.1	0.6846	0.7512	1.4358	1117.0
680	499.88	0.0204	0.6757	487.7	714.0	1201.7	0.6886	0.7441	1.4327	1116.7
700	503.10	0.0205	0.6554	491.5	709.7	1201.2	0.6925	0.7371	1.4296	1116.3
720	506.25	0.0206	0.6362	495.3	705.4	1200.7	0.6963	0.7303	1.4266	1116.0
740	509.34	0.0207	0.6180	499.8	701.2	1200.2	0.7001	0.7237	1.4237	1115.6
760	512.36	0.0207	0.6007	502.6	697.1	1199.7	0.7037	0.7172	1.4209	1115.2
780	515.33	0.0208	0.5843	506.2	692.9	1199.1	0.7073	0.7108	1.4181	1114.8
800	518.23	0.0209	0.5687	509.7	688.9	1198.6	0.7108	0.7045	1.4153	1114.4
820	521.08	0.0209	0.5538	513.2	684.8	1198.0	0.7143	0.6983	1.4126	1114.0
840	523.88	0.0218	0.5396	516.6	680.8	1197.4	0.7177	0.6922	1.4099	1113.6
860	526.63	0.0211	0.5260	520.8	626.8	1196.8	0.7210	0.6862	1.4072	1113.1
880	529.33	0.0212	0.5130	523.3	672.8	1196.1	0.7243	0.6803	1.4046	1112.6
900	531.98	0.0212	0.5006	526.6	668.8	1195.4	0.7275	0.6744	1.4020	1112.1
920	534.59	0.0213	0.4886	529.8	664.9	1194.7	0.7307	0.6687	1.3995	1111.5
940	537.16	0.0214	0.4772	533.0	661.8	1194.0	0.7339	0.6631	1.3970	1111.0
960	539.68	0.0214	0.4663	536.2	657.1	1193.3	0.7370	0.6576	1.3945	1110.5
980	542.17	0.0215	0.4557	539.3	653.3	1192.6	0.7400	0.6521	1.3921	1110.8
1,000	544.61	0.0216	0.4456	542.4	649.4	1191.8	0.7430	0.6467	1.3897	1109.4
1,050	550.57	0.0218	0.4218	550.8	639.9	1189.9	0.7504	0.6334	1.3838	1108.0
1,100	556.31	0.0220	0.4001	557.4	630.4	1187.8	0.7575	0.6205	1.3780	1106.4
1,150	561.86	0.0221	0.3802	564.6	621.0	1185.6	0.7644	0.6079	1.372[illegible]	1104.7
1,200	567.22	0.0223	0.3619	571.7	611.7	1183.4	0.7711	0.5956	1.3667	1103.0
1,250	572.42	0.0225	0.3450	578.6	602.4	1181.0	0.7776	0.5836	1.3612	1101.2
1,300	577.46	0.0227	0.3293	505.4	593.2	1178.6	0.7840	0.5719	1.3559	1099.4
1,350	582.35	0.0229	0.3148	592.1	584.0	1176.1	0.7902	0.5604	1.3506	1097.5
1,400	587.10	0.0231	0.3012	598.7	574.7	1173.4	0.7963	0.5491	1.3454	1095.4
1,450	591.73	0.0233	0.2884	605.2	565.5	1170.7	0.8023	0.5379	1.3402	1093.3
1,500	596.23	0.0235	0.2765	611.6	556.3	1167.9	0.8082	0.5269	1.3351	1091.2
1,600	604.90	0.0239	0.2548	624.1	538.0	1162.1	0.8196	0.5053	1.3249	1086.7
1,700	613.15	0.0243	0.2354	636.3	519.6	1155.9	0.8306	0.4843	1.3149	1081.8
1,800	621.03	0.0247	0.2179	648.3	501.1	1149.4	0.8412	0.4637	1.3049	1076.8
1,900	628.58	0.0252	0.2021	660.1	482.4	1142.4	0.8516	0.4433	1.2949	1071.4
2,000	635.82	0.0257	0.1878	671.7	463.4	1135.1	0.8619	0.4230	1.2849	1065.6
2,200	649.46	0.0268	0.1625	694.8	424.4	1119.2	0.8820	0.3826	1.2646	1053.1
2,400	662.12	0.0280	0.1407	718.4	382.7	1101.1	0.9023	0.3411	1.2434	1038.6
2,600	673.94	0.0295	0.121[illegible]	743.0	337.2	1080.2	0.9232	0.2973	1.2205	1021.9
2,800	684.99	0.0315	0.1035	770.1	284.7	1054.8	0.9459	0.2487	1.1946	1001.2
3,000	695.36	0.0346	0.085[illegible]	802.5	217.8	1020.3	0.9731	0.1885	1.1615	972.7
3,200	705.11	0.0444	0.0580	872.4	62.0	934.4	1.0320	0.0532	1.0852	898.4
3,206.2	705.40	0.0503	0.0503	902.7	0	902.7	1.0580	0	1.0580	872.9



entropy Chart for Steam.

Table 21. Superheated Steam Tables.—*(Continued)*

Pressure, lb per sq in. abs (Saturation temp, deg F)		Temperature of steam, deg F								
		500	550	600	650	700	750	800	900	1000
450 (456.28)	v	1.1231	1.2154	1.3005	1.3810	1.4584	1.5337	1.6074	1.7516	1.8928
	h	1238.4	1272.0	1302.8	1331.9	1359.9	1387.3	1414.3	1467.7	1521.0
	s	1.5095	1.5437	1.5735	1.6003	1.6250	1.6481	1.6699	1.7108	1.7486
500 (467.01)	v	0.9927	1.0798	1.1591	1.2333	1.3044	1.3732	1.4405	1.5715	1.6996
	h	1231.3	1266.7	1298.6	1328.4	1357.0	1384.8	1412.1	1466.0	1519.6
	s	1.4919	1.5279	1.5588	1.5863	1.6115	1.6350	1.6571	1.6982	1.7363
550 (476.94)	v	0.8652	0.9686	1.0431	1.1124	1.1783	1.2419	1.3038	1.4241	1.5414
	h	1223.7	1261.2	1294.3	1324.9	1354.0	1382.3	1409.9	1464.3	1518.2
	s	1.4751	1.5131	1.5451	1.5734	1.5991	1.6228	1.6452	1.6868	1.7250
600 (486.21)	v	0.7947	0.8753	0.9463	1.0115	1.0732	1.1324	1.1899	1.3013	1.4096
	h	1215.7	1255.5	1289.9	1321.3	1351.1	1379.7	1407.7	1462.5	1516.7
	s	1.4586	1.4990	1.5323	1.5613	1.5875	1.6117	1.6343	1.6762	1.7147
700 (503.10)	v	......	0.7277	0.7934	0.8525	0.9077	0.9601	1.0108	1.1082	1.2024
	h	......	1243.2	1280.6	1313.9	1345.0	1374.5	1403.2	1459.0	1513.9
	s	......	1.4722	1.5084	1.5391	1.5665	1.5914	1.6147	1.6573	1.6963
800 (518.23)	v	......	0.6154	0.6779	0.7328	0.7833	0.8308	0.8763	0.9633	1.0470
	h	......	1229.8	1270.7	1306.2	1338.6	1369.2	1398.6	1455.4	1511.0
	s	......	1.4467	1.4863	1.5190	1.5476	1.5734	1.5972	1.6407	1.6801
900 (531.98)	v	......	0.5264	0.5873	0.6393	0.6863	0.7300	0.7716	0.8506	0.9262
	h	......	1215.0	1260.1	1298.0	1332.1	1363.7	1393.9	1451.8	1508.1
	s	......	1.4216	1.4653	1.5002	1.5303	1.5570	1.5814	1.6257	1.6656
1,000 (544.61)	v	......	0.4533	0.5140	0.5640	0.6084	0.6492	0.6878	0.7604	0.8294
	h	......	1198.3	1248.8	1289.5	1325.3	1358.1	1389.2	1448.2	1505.1
	s	......	1.3961	1.4450	1.4825	1.5141	1.5418	1.5670	1.6121	1.6525
1,100 (556.31)	v	......	......	0.4532	0.5020	0.5445	0.5830	0.6191	0.6866	0.7503
	h	......	......	1236.7	1280.5	1318.3	1352.4	1384.3	1444.5	1502.2
	s	......	......	1.4251	1.4658	1.4989	1.5276	1.5535	1.5995	1.6405
1,200 (567.22)	v	......	......	0.4016	0.4498	0.4909	0.5277	0.5617	0.6250	0.6843
	h	......	......	1223.5	1271.0	1311.0	1346.4	1379.3	1440.7	1499.2
	s	......	......	1.4052	1.4491	1.4843	1.5142	1.5409	1.5879	1.6293
1,400 (587.10)	v	......	......	0.3174	0.3668	0.4062	0.4403	0.4714	0.5281	0.5805
	h	......	......	1193.0	1250.6	1295.5	1334.0	1369.1	1433.1	1493.2
	s	......	......	1.3639	1.4171	1.4567	1.4893	1.5177	1.5666	1.6093
1,600 (604.90)	v	......	......	......	0.3027	0.3417	0.3743	0.4034	0.4553	0.5027
	h	......	......	......	1227.3	1278.7	1320.9	1358.4	1425.3	1487.0
	s	......	......	......	1.3850	1.4303	1.4660	1.4964	1.5476	1.5914
1,800 (621.03)	v	......	......	......	0.2506	0.2907	0.3225	0.3502	0.3986	0.4421
	h	......	......	......	1200.3	1260.3	1307.0	1347.2	1417.4	1480.8
	s	......	......	......	1.3515	1.4044	1.4438	1.4765	1.5301	1.5752
2,000 (635.82)	v	......	......	......	0.2058	0.2489	0.2806	0.3074	0.3532	0.3935
	h	......	......	......	1167.0	1240.0	1292.0	1335.5	1409.2	1474.5
	s	......	......	......	1.3139	1.3783	1.4223	1.4576	1.5139	1.5380
2,200 (649.46)	v	......	......	......	0.1633	0.2135	0.2457	0.2721	0.3159	0.3538
	h	......	......	......	1121.0	1217.4	1276.0	1323.3	1400.8	1468.2
	s	......	......	......	1.2665	1.3515	1.4010	1.4393	1.4986	1.5465

Table 20. Properties of Saturated Steam.—(*Continued*)

Abs press, lb per sq in.	Temp, deg F	Specific volume		Enthalpy			Entropy			Internal energy
		Liquid	Vapor	Liquid	Evap	Vapor	Liquid	Evap	Vapor	Evap
40	267.25	0.01715	10.498	236.03	933.7	1169.7	0.3919	1.2844	1.6763	856.1
41	268.74	0.01716	10.258	237.55	932.6	1170.2	0.3940	1.2803	1.6743	855.0
42	270.21	0.01717	10.029	239.04	931.6	1170.7	0.3960	1.2764	1.6724	853.8
43	271.64	0.01719	9.810	240.51	930.6	1171.1	0.3980	1.2726	1.6706	852.7
44	273.05	0.01720	9.601	241.95	929.6	1171.6	0.4000	1.2687	1.6687	851.6
45	274.44	0.01721	9.401	243.36	928.6	1172.0	0.4019	1.2650	1.6669	850.5
46	275.80	0.01722	9.209	244.75	927.7	1172.4	0.4038	1.2613	1.6652	849.5
47	277.13	0.01723	9.025	246.12	926.7	1172.9	0.4057	1.2577	1.6634	848.4
48	278.45	0.01725	8.848	247.47	925.8	1173.3	0.4075	1.2542	1.6617	847.4
49	279.74	0.01726	8.678	248.79	924.9	1123.7	0.4093	1.2508	1.6601	846.4
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	845.4
51	282.26	0.01728	8.359	251.37	923.0	1174.4	0.4127	1.2442	1.6569	844.3
52	283.49	0.01729	8.208	252.63	922.2	1174.8	0.4144	1.2409	1.6553	843.3
53	284.70	0.01730	8.062	253.87	921.3	1175.2	0.4161	1.2377	1.6538	842.4
54	285.90	0.01731	7.922	255.09	920.5	1175.6	0.4177	1.2346	1.6523	841.5
55	287.07	0.01732	7.787	256.30	919.6	1175.9	0.4193	1.2316	1.6509	840.6
56	288.23	0.01733	7.656	257.50	918.8	1176.3	0.4209	1.2285	1.6494	839.7
57	289.37	0.01734	7.529	258.67	917.9	1176.6	0.4225	1.2255	1.6480	838.7
58	290.50	0.01736	7.407	259.82	917.1	1176.9	0.4240	1.2226	1.6466	837.8
59	291.61	0.01737	7.289	260.96	916.3	1177.3	0.4255	1.2197	1.6452	836.9
60	292.71	0.01738	7.175	262.09	915.5	1177.6	0.4270	1.2168	1.6438	836.0
61	293.79	0.01739	7.064	263.20	914.7	1177.9	0.4285	1.2140	1.6425	835.2
62	294.85	0.01740	6.957	264.30	913.9	1178.2	0.4300	1.2112	1.6412	834.3
63	295.90	0.01741	6.853	265.38	913.1	1178.5	0.4314	1.2085	1.6399	833.4
64	296.94	0.01742	6.752	266.45	912.3	1178.8	0.4328	1.2059	1.6387	832.6
65	297.97	0.01743	6.655	267.50	911.6	1179.1	0.4342	1.2032	1.6374	831.8
66	298.99	0.01744	6.560	268.55	910.8	1179.4	0.4356	1.2006	1.6362	831.0
67	299.99	0.01745	6.468	269.58	910.1	1179.7	0.4369	1.1981	1.6350	830.2
68	300.98	0.01746	6.378	270.60	909.4	1180.0	0.4383	1.1955	1.6338	829.4
69	301.96	0.01747	6.291	271.61	908.7	1180.3	0.4396	1.1930	1.6326	828.6
70	302.92	0.01748	6.206	272.61	907.9	1180.6	0.4409	1.1906	1.6315	827.8
71	303.88	0.01749	6.124	273.60	907.2	1180.8	0.4422	1.1881	1.6303	827.0
72	304.83	0.01750	6.044	274.57	906.5	1181.1	0.4435	1.1857	1.6292	826.3
73	305.76	0.01751	5.966	275.54	905.8	1181.3	0.4447	1.1834	1.6281	825.5
74	306.68	0.01752	5.890	276.49	905.1	1181.6	0.4460	1.1810	1.6270	824.7
75	307.60	0.01753	5.816	277.43	904.5	1181.9	0.4472	1.1787	1.6259	824.0
76	308.50	0.01754	5.743	278.37	903.7	1182.1	0.4484	1.1764	1.6248	823.3
77	309.40	0.01754	5.673	279.30	903.1	1182.4	0.4496	1.1742	1.6238	822.5
78	310.29	0.01755	5.604	280.21	902.4	1182.6	0.4508	1.1720	1.6228	821.7
79	311.16	0.01756	5.537	281.12	901.7	1182.8	0.4520	1.1698	1.6217	821.0
80	312.03	0.01757	5.472	282.02	901.1	1183.1	0.4531	1.1676	1.6207	820.3
81	312.89	0.01758	5.408	282.91	900.4	1183.3	0.4543	1.1654	1.6197	819.6
82	313.74	0.01759	5.346	283.79	899.7	1183.5	0.4554	1.1633	1.6187	818.9
83	314.59	0.01760	5.285	284.66	899.1	1183.8	0.4565	1.1612	1.6177	818.2
84	315.42	0.01761	5.226	285.53	898.5	1184.0	0.4576	1.1592	1.6168	817.5
85	316.25	0.01761	5.168	286.39	897.8	1184.2	0.4587	1.1571	1.6158	816.8
86	317.07	0.01762	5.111	287.24	897.2	1184.4	0.4598	1.1551	1.6149	816.1
87	317.88	0.01763	5.055	288.08	896.5	1184.6	0.4609	1.1530	1.6139	815.4
88	318.68	0.01764	5.001	288.91	895.9	1184.8	0.4620	1.1510	1.6130	814.8
89	319.48	0.01765	4.948	289.74	895.3	1185.1	0.4630	1.1491	1.6121	814.1

Table 23. Properties of Mercury Vapor
(h and s are measured from 32 F)
By L. A. Sheldon, General Electric Co.

Pressure, lb per sq in. abs, p	Temp deg F t	Specific vol, cu ft, per lb, v_g	Enthalpy, Btu			Entropy		
			Sat liquid, h_f	Vaporization, h_{fg}	Sat vapor, h_g	Sat liquid, s_f	Vaporization, s_{fg}	Sat vapor, s_g
0.4	402.3	114.5	13.81	128.1	141.9	0.02094	0.1486	0.1696
0.6	426.1	78.23	14.70	127.6	142.3	0.02195	0.1441	0.1660
0.8	443.8	59.71	15.36	127.2	142.6	0.02269	0.1408	0.1635
1.0	458.1	48.45	15.89	126.9	142.8	0.02328	0.1382	0.1615
1.5	485.1	33.14	16.90	126.3	143.2	0.02436	0.1337	0.1580
2	505.2	25.31	17.65	125.8	143.5	0.02514	0.1304	0.1556
3	535.4	17.34	18.70	125.2	144.0	0.02629	0.1258	0.1521
4	558.0	13.26	19.62	124.7	144.3	0.02714	0.1225	0.1497
5	576.2	10.77	20.30	124.3	144.6	0.02780	0.1200	0.1478
6	591.4	9.096	20.87	123.9	144.0	0.02834	0.1179	0.1462
7	605.0	7.882	21.37	123.6	145.0	0.02882	0.1161	0.1450
8	616.8	6.963	21.01	123.4	145.2	0.02923	1.1146	0.1439
9	627.5	6.244	22.21	123.2	145.4	0.02960	0.1133	0.1429
10	637.3	5.661	22.58	122.9	145.5	0.02993	0.1121	0.1420
15	676.5	3.892	24.04	122.1	146.1	0.03124	0.1074	0.1387
20	706.2	2.983	25.15	121.4	146.6	0.03220	0.1041	0.1363
25	730.4	2.429	26.05	120.9	146.9	0.03297	0.1016	0.1345
30	750.9	2.053	26.81	120.4	147.2	0.03360	0.09953	0.1331
35	769.0	1.781	27.49	120.0	147.5	0.03416	0.09774	0.1319
40	784.0	1.576	28.08	119.7	147.8	0.03464	0.09621	0.1308
45	799.3	1.414	28.62	119.4	148.0	0.03507	0.09486	0.1299
50	812.5	1.284	29.11	119.1	148.2	0.03546	0.09364	0.1291
60	836.1	1.086	29.99	118.6	148.6	0.03614	0.09154	0.1276
70	856.6	0.9436	30.75	118.1	140.9	0.03672	0.08976	0.1264
80	874.8	0.8349	31.43	117.7	149.1	0.03725	0.08824	0.1254
90	891.6	0.7497	32.06	117.3	149.4	0.03771	0.08687	0.1245
100	906.9	0.6811	32.63	117.0	149.6	0.03813	0.08565	0.1237
120	934.4	0.5767	33.60	116.4	150.1	0.03887	0.08353	0.1224
140	958.3	0.5012	34.55	115.9	150.4	0.03951	0.08175	0.1212
160	979.9	0.4438	35.35	115.4	150.0	0.04007	0.08019	0.1202
180	999.6	0.3990	36.09	115.0	151.1	0.04058	0.07881	0.1193

Diphenyl $(C_6H_5)_2$ has the following properties (Chipman and Peltier, *Ind. Chem. Eng.*, Nov., 1929, p. 1106): boiling point, 491.5 F; density of the liquid 53 lb per cu ft; density of saturated vapor, 0.242 lb per cu ft; heat of vaporization, 134 Btu per lb: all measured at the boiling point. The vapor pressures (p, lb per sq in. abs) at various temperatures (t, deg F) are as follows:

t	200	250	300	350	400	450	500
p	0.060	0.227	0.701	1.832	4.117	8.638	16.29
t	550	600	650	700	750	800	
p	28.54	47.01	73.55	110.1	158.6	221.0	

Dowtherm is a mixture of diphenyl oxide and diphenyl. It is used as a liquid heating medium at elevated temperatures. Its low vapor pressure permits high temperature without attendant high pressures. Table 24 (Badger, *Ind. Chem.*, Sept., 1937) gives properties of a mixture containing 73.5 percent of diphenyl oxide and 26.5 percent of diphenyl which melts at 53.6 F.

Table 20. Properties of Saturated Steam.—(*Continued*)

Abs press, lb per sq in.	Temp, deg F	Specific volume		Enthalpy			Entropy			Internal energy
		Liquid	Vapor	Liquid	Evap	Vapor	Liquid	Evap	Vapor	Evap
180	373.06	0.01827	2.532	346.03	850.8	1196.9	0.5325	1.0217	1.5542	767.1
182	373.96	0.01829	2.505	347.00	850.0	1197.0	0.5336	1.0196	1.5532	766.2
184	374.86	0.01830	2.479	347.96	849.2	1197.2	0.5348	1.0175	1.5523	765.4
186	375.75	0.01831	2.454	348.92	848.4	1197.3	0.5359	1.0155	1.5514	764.6
188	376.64	0.01832	2.429	349.86	847.6	1197.5	0.5370	1.0136	1.5506	763.8
190	377.51	0.01833	2.404	350.79	846.8	1197.6	0.5381	1.0116	1.5497	763.0
192	378.38	0.01834	2.380	351.72	846.1	1197.8	0.5392	1.0096	1.5488	762.1
194	379.24	0.01835	2.356	352.64	845.3	1197.9	0.5403	1.0076	1.5479	761.3
196	380.10	0.01836	2.333	353.55	844.5	1198.1	0.5414	1.0056	1.5470	760.6
198	380.95	0.01838	2.310	354.46	843.7	1198.2	0.5425	1.0037	1.5462	759.8
200	381.79	0.01839	2.288	355.36	843.0	1198.4	0.5435	1.0018	1.5453	759.0
205	383.86	0.01842	2.234	357.58	841.1	1198.7	0.5461	0.9971	1.5432	757.1
210	385.90	0.01844	2.183	359.77	839.2	1199.0	0.5487	0.9925	1.5412	755.2
215	387.89	0.01847	2.134	361.91	837.4	1199.3	0.5512	0.9880	1.5392	753.2
220	389.86	0.01850	2.087	364.02	835.6	1199.6	0.5537	0.9835	1.5372	751.3
225	391.79	0.01852	2.0422	366.09	833.8	1199.9	0.5561	0.9792	1.5353	749.5
230	393.68	0.01854	1.9992	368.13	832.0	1200.1	0.5585	0.9750	1.5334	747.7
235	395.54	0.01857	1.9579	370.14	830.3	1200.4	0.5608	0.9708	1.5316	745.9
240	397.37	0.01860	1.9183	372.12	828.5	1200.6	0.5631	0.9667	1.5298	744.1
245	399.18	0.01863	1.8803	374.08	826.8	1200.9	0.5653	0.9627	1.5280	742.4
250	400.95	0.01865	1.8438	376.00	825.1	1201.1	0.5675	0.9588	1.5263	740.7
260	404.42	0.01870	1.7748	379.76	821.8	1201.5	0.5719	0.9510	1.5229	737.3
270	407.78	0.01875	1.7107	383.42	818.5	1201.9	0.5760	0.9436	1.5196	733.9
280	411.05	0.01880	1.6511	386.98	815.3	1202.3	0.5801	0.9363	1.5164	730.7
290	414.23	0.01885	1.5954	390.46	812.1	1202.6	0.5841	0.9292	1.5133	727.5
300	417.33	0.01890	1.5433	393.84	809.0	1202.8	0.5879	0.9225	1.5104	724.3
320	423.29	0.01899	1.4485	400.39	803.0	1203.4	0.5952	0.9094	1.5046	718.3
340	428.97	0.01908	1.3645	406.66	797.1	1203.7	0.6022	0.8970	1.4992	712.4
360	434.40	0.01917	1.2895	412.67	791.4	1204.1	0.6090	0.8851	1.4941	706.8
380	439.60	0.01925	1.222	418.45	785.8	1204.3	0.6153	0.8738	1.4891	701.3
400	444.59	0.0193	1.1613	424.0	780.5	1204.5	0.6214	0.8630	1.4844	695.9
420	449.39	0.0194	1.1061	429.4	775.2	1204.6	0.6272	0.8527	1.4799	690.8
440	454.02	0.0195	1.0556	434.6	770.0	1204.6	0.6329	0.8426	1.4755	685.7
460	458.50	0.0196	1.0094	439.7	764.9	1204.[illegible]	0.6383	0.8330	1.4713	680.7
480	462.82	0.0197	0.9670	444.6	759.9	1204.5	0.6436	0.8237	1.4673	675.7

Ammonia Vapor. The properties of saturated and superheated ammonia vapor have been determined accurately by the Bureau of Standards (*Circ.*, 142, 1923). The principal properties are given in Tables 25 and 26 and Fig. 21.

In the Bureau of Standards table, the entropy s_f and the heat of the liquid h_f are taken as zero at −40 F instead of at 32 F, as is customary in most tables.

Table 25. Properties of Saturated Ammonia

(h and s are measured from −40 F)

Temp, deg F	Pressure, lb per sq in. abs	Specific volume, cu ft per lb		Enthalpy, Btu			Entropy		
		Sat liquid	Sat vapor	Sat liquid	Vaporization	Sat vapor	Sat liquid	Vaporization	Sat vapor
t	p	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
−40	10.41	0.02322	24.86	0.0	597.6	597.6	0.000	1.4242	1.4242
−38	11.04	0.02326	23.53	2.1	596.2	598.3	0.0051	1.4142	1.4193
−36	11.71	0.02331	22.27	4.3	594.8	599.1	0.0101	1.4043	1.4144
−34	12.41	0.02335	21.10	6.4	593.5	599.9	0.0151	1.3945	1.4096
−32	13.14	0.02340	20.00	8.5	592.1	600.6	0.0201	1.3847	1.4048
−30	13.90	0.02345	18.97	10.7	590.7	601.4	0.0250	1.3751	1.4001
−28	14.71	0.02349	18.00	12.8	589.3	602.1	0.0300	1.3655	1.3955
−26	15.55	0.02354	17.09	14.9	587.9	602.8	0.0350	1.3559	1.3909
−24	16.42	0.02359	16.24	17.1	586.5	603.6	0.0399	1.3464	1.3863
−22	17.34	0.02364	15.43	19.2	585.1	604.3	0.0448	1.3370	1.3818
−20	18.30	0.02369	14.68	21.4	583.6	605.0	0.0497	1.3277	1.3774
−18	19.30	0.02374	13.97	23.5	582.2	605.7	0.0545	1.3184	1.3729
−16	20.34	0.02378	13.29	25.6	580.8	606.4	0.0594	1.3092	1.3686
−14	21.43	0.02383	12.66	27.8	579.3	607.1	0.0642	1.3001	1.3643
−12	22.56	0.02384	12.06	30.0	577.8	607.8	0.0690	1.2910	1.3600
−10	23.74	0.02393	11.50	32.1	576.4	608.5	0.0738	1.2820	1.3558
−8	24.97	0.02399	10.97	34.3	574.9	609.2	0.0786	1.2730	1.3516
−6	26.26	0.02404	10.47	36.4	573.4	609.8	0.0833	1.2641	1.3474
−4	27.59	0.02409	9.991	38.6	571.9	610.5	0.0880	1.2553	1.3433
−2	28.98	0.02414	9.541	40.7	570.4	611.1	0.0928	1.2465	1.3393
0	30.42	0.02419	9.116	42.9	568.9	611.8	0.0975	1.2377	1.3352
2	31.92	0.02424	8.714	45.1	567.3	612.4	0.1022	1.2290	1.3312
4	33.47	0.02430	8.333	47.2	565.8	613.0	0.1069	1.2204	1.3273
6	35.89	0.02435	7.971	49.4	564.2	613.6	0.1115	1.2119	1.3234
8	36.77	0.02440	7.629	51.6	562.7	614.3	0.1162	1.2033	1.3195

Table 21. Superheated Steam Tables

(Abstracted from Keenan and Keyes, "Thermodynamic Properties of Steam")
(v = specific volume, cu ft per lb; h = enthalpy, Btu per lb; s = entropy)

Pressure, lb per sq in. abs (Saturation temp, deg F)		Temperature of steam, deg F								
		340	380	420	460	500	550	600	650	700
20 (227.96)	v	23.60	24.82	26.04	27.25	28.46	29.97	31.47	32.97	34.47
	h	1210.8	1229.7	1248.7	1267.6	1286.6	1310.5	1334.4	1358.6	1382.9
	s	1.8053	1.8285	1.8505	1.8716	1.8918	1.9160	1.9392	1.9671	1.9829
40 (267.25)	v	11.684	12.315	12.938	13.555	14.168	14.930	15.688	16.444	17.198
	h	1207.0	1226.7	1246.2	1265.5	1284.8	1309.0	1333.1	1357.4	1381.9
	s	1.7252	1.7493	1.7719	1.7934	1.8140	1.8385	1.8619	1.8843	1.9058
60 (292.71)	v	7.708	8.143	8.569	8.988	9.403	9.917	10.427	10.935	11.441
	h	1203.0	1223.6	1243.6	1263.4	1283.0	1307.4	1331.8	1356.3	1380.9
	s	1.6766	1.7135	1.7250	1.7478	1.7678	1.7927	1.8162	1.8388	1.8605
80 (312.03)	v	5.718	6.055	6.383	6.704	7.020	7.410	7.797	8.180	8.562
	h	1198.8	1220.3	1240.9	1261.1	1281.1	1305.8	1330.5	1355.1	1379.9
	s	1.6407	1.6669	1.6909	1.7134	1.7346	1.7598	1.7836	1.8063	1.8281
100 (327.81)	v	4.521	4.801	5.071	5.333	5.589	5.906	6.218	6.527	6.835
	h	1194.3	1216.8	1238.1	1258.8	1279.1	1304.2	1329.1	1354.0	1378.9
	s	1.6117	1.6391	1.6639	1.6869	1.7085	1.7340	1.7581	1.7810	1.8029
120 (341.25)	v	......	3.964	4.195	4.418	4.636	4.902	5.165	5.426	5.683
	h	......	1213.2	1235.3	1256.5	1277.2	1302.6	1327.7	1352.8	1377.8
	s	......	1.6156	1.6413	1.6649	1.6869	1.7127	1.7370	1.7601	1.7822
140 (353.02)	v	......	3.365	3.569	3.764	3.954	4.186	4.413	4.638	4.861
	h	......	1209.4	1232.3	1254.1	1275.2	1300.9	1326.4	1351.6	1376.8
	s	......	1.5950	1.6217	1.6458	1.6683	1.6945	1.7190	1.7423	1.7645
160 (363.53)	v	......	2.914	3.098	3.273	3.443	3.648	3.849	4.048	4.244
	h	......	1205.5	1229.3	1251.6	1273.1	1299.3	1325.0	1350.4	1375.7
	s	......	1.5766	1.6042	1.6291	1.6519	1.6785	1.7033	1.7268	1.7491
180 (373.06)	v	......	2.563	2.732	2.891	3.044	3.230	3.411	3.588	3.764
	h	......	1201.4	1226.1	1249.1	1271.0	1297.6	1323.5	1349.2	1374.7
	s	......	1.5596	1.5884	1.6139	1.6373	1.6642	1.6894	1.7130	1.7355
200 (381.79)	v	......	......	2.438	2.585	2.726	2.895	3.060	3.221	3.380
	h	......	......	1222.9	1246.5	1268.9	1295.8	1322.1	1348.0	1373.6
	s	......	......	1.5738	1.6001	1.6240	1.6513	1.6767	1.7006	1.7232
220 (389.86)	v	......	......	2.198	2.335	2.465	2.621	2.772	2.920	3.066
	h	......	......	1219.5	1243.8	1266.7	1294.1	1320.7	1346.8	1372.6
	s	......	......	1.5603	1.5874	1.6117	1.6395	1.6652	1.6892	1.7120
260 (404.42)	v	......	......	1.8257	1.9483	2.063	2.199	2.330	2.457	2.582
	h	......	......	1212.4	1238.3	1262.3	1290.5	1317.7	1344.3	1370.4
	s	......	......	1.5354	1.5642	1.5897	1.6184	1.6447	1.6692	1.6922
300 (417.33)	v	......	......	1.5513	1.6638	1.7675	1.8891	2.005	2.118	2.227
	h	......	......	1204.8	1232.5	1257.6	1286.8	1314.7	1341.8	1368.3
	s	......	......	1.5126	1.5434	1.5701	1.5998	1.6268	1.6517	1.6751
350 (431.72)	v	......	......	......	1.3984	1.4923	1.6010	1.7036	1.8021	1.8980
	h	......	......	......	1224.8	1251.5	1282.1	1310.9	1338.5	1365.5
	s	......	......	......	1.5197	1.5481	1.5792	1.6070	1.6325	1.6563
400 (444.59)	v	......	......	......	1.1978	1.2051	1.3843	1.4770	1.5654	1.6508
	h	......	......	......	1216.5	1245.1	1277.2	1306.9	1335.2	1362.7
	s	......	......	......	1.4977	1.5281	1.5607	1.5894	1.6155	1.6398

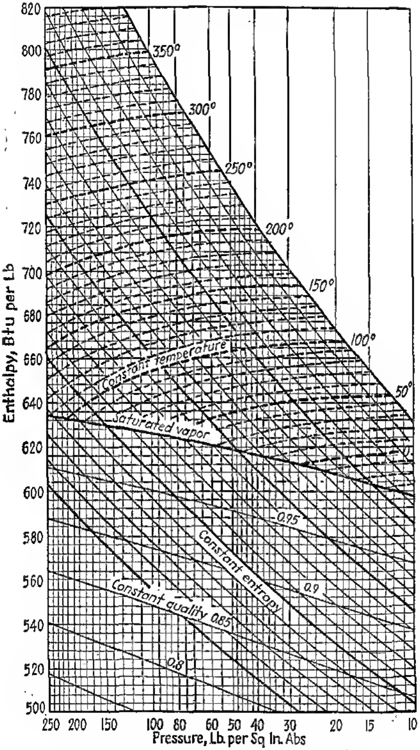


FIG. 21.—Enthalpy-pressure Chart for Ammonia.

Table 22. Steam Table for Use in Condenser Calculations
(Abstracted from Keenan and Keyes, "Thermodynamic Properties of Steam")

Temp, deg F t	Abs pressure p: Lb per sq in.	Abs pressure p: In. hg	Specific volume: Sat vapor v_g	Enthalpy: Sat liquid h_f	Enthalpy: Evap h_{fg}	Enthalpy: Sat vapor h_g	Entropy: Sat liquid s_f	Entropy: Sat vapor s_g
50	0.17811	0.3626	1703.2	18.07	1065.6	1083.7	0.0361	2.1264
52	0.19182	0.3906	1587.6	20.07	1064.4	1084.5	0.0400	2.1199
54	0.20642	0.4203	1481.0	22.07	1063.3	1085.4	0.0439	2.1136
56	0.2220	0.4520	1382.4	24.06	1062.2	1086.3	0.0478	2.1072
58	0.2386	0.4858	1291.1	26.06	1061.0	1087.1	0.0517	2.1010
60	0.2563	0.5218	1206.7	28.06	1059.9	1088.0	0.0555	2.0948
62	0.2751	0.5601	1128.4	30.05	1058.8	1088.9	0.0593	2.0886
64	0.2951	0.6009	1055.7	32.05	1057.6	1089.7	0.0632	2.0826
66	0.3164	0.6442	988.4	34.05	1056.5	1090.6	0.0670	2.0766
68	0.3390	0.6903	925.9	36.04	1055.5	1091.5	0.0708	2.0706
70	0.3631	0.7392	867.9	38.04	1054.3	1092.3	0.0745	2.0647
72	0.3886	0.7912	813.9	40.04	1053.2	1093.2	0.0783	2.0588
74	0.4156	0.8462	763.8	42.03	1052.1	1094.1	0.0820	2.0530
76	0.4443	0.9046	717.1	44.03	1050.9	1094.9	0.0858	2.0473
78	0.4747	0.9666	673.6	46.02	1049.8	1095.8	0.0895	2.0416
80	0.5069	1.0321	633.1	48.02	1048.6	1096.6	0.0932	2.0360
82	0.5410	1.1016	595.3	50.01	1047.5	1097.5	0.0969	2.0304
84	0.5771	1.1750	560.2	52.01	1046.4	1098.4	0.1005	2.0249
86	0.6152	1.2527	527.3	54.00	1045.2	1099.2	0.1042	2.0195
88	0.6556	1.3347	496.7	56.00	1044.1	1100.1	0.1079	2.0141
90	0.6982	1.4215	468.0	57.99	1042.9	1100.9	0.1115	2.0087
92	0.7432	1.5131	441.3	59.99	1041.8	1101.8	0.1151	2.0034
94	0.7906	1.6097	416.2	61.98	1040.7	1102.6	0.1187	1.9981
96	0.8407	1.7117	392.8	63.98	1039.5	1103.5	0.1223	1.9929
98	0.8935	1.8192	370.9	65.97	1038.4	1104.4	0.1259	1.9877
100	0.9492	1.9325	350.4	67.97	1037.2	1105.2	0.1295	1.9826
102	1.0078	2.0519	331.1	69.96	1036.1	1106.1	0.1330	1.9775
104	1.0695	2.1775	313.1	71.96	1034.9	1106.9	0.1366	1.9725
106	1.1345	2.3099	296.2	73.95	1033.8	1107.8	0.1401	1.9675
108	1.2029	2.4491	280.3	75.95	1032.7	1108.6	0.1436	1.9626
110	1.2748	2.5955	265.4	77.94	1031.6	1109.5	0.1471	1.9577
112	1.3504	2.7494	251.4	79.94	1030.4	1110.3	0.1506	1.9529
114	1.4298	2.9111	238.2	81.93	1029.2	1111.1	0.1541	1.9481
116	1.5130	3.0806	225.8	83.93	1028.1	1112.0	0.1576	1.9433
118	1.6006	3.2589	214.2	85.92	1026.9	1112.8	0.1610	1.9386
120	1.6924	3.4458	203.27	87.92	1025.8	1113.7	0.1645	1.9339
122	1.7888	3.6420	192.95	89.92	1024.6	1114.5	0.1679	1.9293
124	1.8897	3.8475	183.25	91.91	1023.4	1115.3	0.1714	1.9247
126	1.9955	4.0629	174.10	93.91	1022.3	1116.2	0.1748	1.9202
128	2.1064	4.2887	165.47	95.91	1021.1	1117.0	0.1782	1.9156

Properties of Other Refrigerants. Complete and consistent data are not available on most of the working fluids used for refrigeration. The data presented in this section on refrigerating fluids other than ammonia are, in many cases, not of as high order of accuracy as the values given for steam (see also pp. 1854 to 1857).

Table 27. Properties of Saturated Sulphur Dioxide

(h and s are measured from −40 F)

Temp, deg F	Pressure, lb per sq in. abs	Specific volume cu ft per lb		Enthalpy, Btu			Entropy		
		Sat liquid	Sat vapor	Sat liquid	Vaporization	Sat vapor	Sat liquid	Vaporization	Sat vapor
t	p	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
−40	3.136	0.01044	22.42	0.00	178.6	178.6	0.0000	0.4256	0.4256
−30	4.331	0.01053	16.56	2.93	177.0	179.9	0.00674	0.4119	0.4186
−20	5.883	0.01062	12.42	5.98	175.1	181.1	0.01366	0.3983	0.4119
−10	7.863	0.01072	9.44	9.16	173.0	182.1	0.02075	0.3847	0.4054
0	10.35	0.01082	7.28	12.44	170.6	183.1	0.02795	0.3712	0.3992
10	13.42	0.01092	5.682	15.80	168.1	183.9	0.03519	0.3579	0.3931
20	17.18	0.01103	4.487	19.20	165.3	184.5	0.04241	0.3447	0.3871
30	21.70	0.01114	3.581	22.64	162.4	185.0	0.04956	0.3316	0.3812
40	27.10	0.01126	2.887	26.12	159.3	185.4	0.05668	0.3187	0.3754
50	33.45	0.01138	2.348	29.61	156.0	185.6	0.06370	0.3060	0.3697
60	40.93	0.01150	1.926	33.10	152.5	185.6	0.07060	0.2935	0.3641
70	49.62	0.01163	1.590	36.58	148.9	165.5	0.07736	0.2811	0.3585
80	59.68	0.01176	1.321	40.05	145.1	185.2	0.08399	0.2690	0.3529
90	71.25	0.01190	1.104	43.50	141.2	184.7	0.09038	0.2569	0.3473
100	84.52	0.01204	0.9262	46.90	137.2	184.1	0.09657	0.2452	0.3417
110	99.76	0.01219	0.7804	50.26	133.1	183.3	0.1025	0.2336	0.3361
120	120.9	0.01236	0.6598	53.50	128.0	182.4	0.1083	0.2222	0.3305
130	136.5	0.01253	0.5595	56.05	124.4	181.2	0.1138	0.2110	0.3247
140	158.6	0.01272	0.4758	60.04	119.9	179.9	0.1189	0.1999	0.3189

Table 24. Properties of Saturated Dowtherm Vapor

Temp, deg F	Pressure, lb per sq in. abs	Enthalpy, Btu per lb, above 53.6 F: Sat liquid, h_f	Enthalpy, Btu per lb, above 53.6 F: Vaporization, h_{fg}	Enthalpy, Btu per lb, above 53.6 F: Sat vapor, h_g	Specific heat, liquid	Density, lb per cu ft: Liquid	Density, lb per cu ft: Vapor
500.0	14.7	222.0	123	345	0.63	54.1	0.28
510.0	18.1	228.0	121	349	0.63	53.7	0.32
520.0	20.4	234.0	120	354	0.64	53.2	0.36
530.0	22.7	240.0	119	359	0.64	53.0	0.40
540.0	25.1	247.0	118	365	0.65	52.7	0.44
550.0	27.0	253.0	117	370	0.65	52.3	0.48
560.0	30.8	260.0	115	375	0.65	51.9	0.54
570.0	34.6	267.0	114	381	0.66	51.6	0.60
580.0	36.6	274.0	112	386	0.66	51.2	0.67
590.0	41.4	281.0	111	392	0.66	50.8	0.75
600.0	44.3	288.0	110	398	0.66	50.4	0.88
610.0	46.2	295.0	109	404	0.67	50.1	1.00
620.0	53.0	302.0	107	409	0.67	49.8	1.10
630.0	57.6	309.0	106	415	0.67	49.3	1.17
640.0	63.6	316.0	105	421	0.67	49.1	1.24
650.0	68.4	323.0	104	427	0.67	48.6	1.29
660.0	74.2	330.0	102	432	0.68	48.4	1.34
670.0	80.8	337.0	101	438	0.68	47.9	1.40
680.0	87.7	344.0	99	443	0.68	47.5	1.5
690.0	95.4	351.0	98	449	0.68	47.2	1.6
700.0	104.0	358.0	97	455	0.68	46.9	1.7
710.0	113.0	365.0	95	460	0.68	46.3	1.8
720.0	119.0	372.0	93	465	0.68	45.9	1.9
730.0	131.0	379.0	92	471	0.68	45.5	2.1
740.0	142.0	386.0	90	476	0.68	44.9	2.3
750.0	150.0	393.0	89	482	0.68	44.4	2.5

Table 29. Properties of Carbon Dioxide

(h and s are measured from 32 F)

Temperature, deg F t	Pressure, lb per sq in. p	Density lb per cu ft: Sat liquid v_f	Density lb per cu ft: Sat vapor v_g	Enthalpy, Btu: Sat liquid h_f	Enthalpy, Btu: Vaporization h_{fg}	Enthalpy, Btu: Sat vapor h_g	Entropy: Sat liquid s_f	Entropy: Sat vapor s_g
−40	145.87	69.8	1.64	−38.5	136.5	98.0	−0.0850	0.2400
−35	161.33	69.1	1.83	−35.8	134.3	98.5	−0.0793	0.2367
−30	177.97	68.3	2.02	−33.1	132.1	99.0	−0.0735	0.2336
−25	195.85	67.6	2.23	−30.4	129.8	99.4	−0.0676	0.2306
−20	215.02	66.9	2.44	−27.7	127.5	99.8	−0.0619	0.2277
−15	235.53	66.1	2.66	−24.9	125.0	100.1	−0.0560	0.2250
−10	257.46	65.3	2.91	−22.1	122.4	100.3	−0.0500	0.2220
−5	280.85	64.5	3.17	−19.4	120.0	100.6	−0.0440	0.2198
0	305.76	63.6	3.46	−16.7	117.5	100.8	−0.0381	0.2173
5	332.2	62.8	3.77	−14.0	115.0	101.0	−0.0322	0.2151
10	360.4	61.9	4.12	−11.2	112.2	101.0	−0.0264	0.2124
15	390.2	61.0	4.49	−8.4	109.4	101.0	−0.0204	0.2100
20	421.8	60.0	4.89	−5.5	106.3	100.8	−0.0144	0.2071
25	455.3	59.0	5.33	−2.5	103.1	100.6	−0.0083	0.2043
30	490.6	58.0	5.81	+0.4	99.7	100.1	−0.0021	0.2012
35	528.0	57.0	6.35	3.5	95.8	99.3	+0.0039	0.1975
40	567.3	55.9	6.91	6.6	91.8	98.4	0.0099	0.1934
45	608.9	54.7	7.60	9.8	87.5	97.3	0.0160	0.1892
50	652.7	53.4	8.37	12.9	83.2	96.1	0.0220	0.1852
55	698.8	52.1	9.27	16.1	78.7	94.8	0.0282	0.1809
60	747.4	50.7	10.2	19.4	74.0	93.4	0.0345	0.1767
65	798.6	49.1	11.3	22.9	68.9	91.8	0.0412	0.1724
70	852.4	47.3	12.6	26.6	62.7	89.3	0.0482	0.1665
75	909.3	45.1	14.2	30.9	54.8	85.7	0.0562	0.1587
80	969.3	42.4	16.2	35.6	44.8	79.6	0.0649	0.1464
85	1032.7	38.2	19.1	41.7	27.5	69.2	0.0761	0.1265
88	1072.1	32.9	25.4	Critical Point at 88.43 F				

Table 30. Properties of Ethyl Chloride and of Methyl Chloride

(h_f and s_f are measured from 32 F)

Temp, deg F t	Ethyl chloride: Pressure, lb per sq in. abs p	Ethyl chloride: Specific vol of vapor, cu ft per lb v_g	Ethyl chloride: Enthalpy, Btu: Sat liquid h_f	Ethyl chloride: Enthalpy, Btu: Vaporization h_{fg}	Ethyl chloride: Enthalpy, Btu: Sat vapor h_g	Methyl chloride: Pressure, lb per sq in. abs p	Methyl chloride: Specific vol of vapor, cu ft per lb v_g	Methyl chloride: Enthalpy, Btu: Sat liquid h_f	Methyl chloride: Enthalpy, Btu: Vaporization h_{fg}	Methyl chloride: Enthalpy, Btu: Sat vapor h_g
−20	2.16	29.54	−19.0	177.6	158.6	11.75	8.09	−19.0	186.4	167.4
−15	2.53	26.07	−17.2	176.8	159.6	13.43	7.22	−17.19	185.3	168.1
−10	2.94	22.95	−15.4	175.9	160.5	15.00	6.46	−15.38	184.2	168.8
−5	3.41	20.28	−13.5	175.1	161.6	16.79	5.80	−13.58	183.1	169.5
0	3.93	18.04	−11.7	174.2	162.5	18.80	5.18	−11.75	182.0	170.2
+5	4.50	16.10	−9.8	173.4	163.6	21.00	4.65	−9.93	180.8	170.9
10	5.13	14.36	−8.1	172.6	164.5	23.30	4.18	−8.06	179.6	171.6
15	5.86	12.86	−6.3	171.8	165.5	25.92	3.78	−6.24	178.5	172.2
20	6.65	11.56	−4.4	170.9	166.5	28.8	3.41	−4.32	177.3	172.9
25	7.48	10.35	−2.5	170.1	167.6	31.9	3.09	−2.48	176.1	173.6
30	8.40	9.22	−0.6	169.2	168.6	35.2	2.81	−0.62	174.9	174.3
35	9.42	8.27	+1.3	168.4	169.6	38.7	2.54	+1.25	173.7	174.9
40	10.53	7.43	3.2	167.5	170.7	42.6	2.31	3.15	172.4	175.6
45	11.77	6.70	5.1	166.6	171.7	47.0	2.10	5.04	171.2	176.2
50	13.20	6.04	6.9	165.6	172.5	51.5	1.93	6.88	169.9	176.8
55	14.67	5.42	8.9	164.6	173.5	56.4	1.76	8.80	168.6	177.4
60	16.45	4.84	10.8	163.6	174.4	61.6	1.61	10.70	167.3	178.0
65	18.17	4.38	12.8	162.6	175.4	67.3	1.48	12.63	166.0	178.6
70	20.03	4.00	14.7	161.6	176.3	73.3	1.34	14.52	164.6	179.2
75	22.11	3.65	16.6	160.6	177.2	79.2	1.24	16.46	163.3	179.7
80	24.33	3.39	18.6	159.5	178.1	85.3	1.14	18.36	161.9	180.2
85	26.67	3.17	20.6	158.5	179.1	94.1	1.05	20.26	160.5	180.7
90	29.24	2.99	22.6	157.3	179.9	102.1	0.98	22.13	159.1	181.2
95	32.03	2.83	24.6	156.1	180.6	110.3	0.906	24.07	157.7	181.8
100	34.93	2.70	26.6	154.9	181.5	118.8	0.85	26.06	156.3	182.3
105	37.97	2.61	28.6	153.0	182.4	128.1	0.804	28.02	154.9	182.9
110	41.10	2.51	30.5	152.6	183.1	137.6	0.765	30.03	153.4	183.5

Table 25. Properties of Saturated Ammonia.—(*Continued*)

Temp, deg F t	Pressure, lb per sq in. abs p	Specific volume, cu ft per lb: Sat liquid v_f	Specific volume, cu ft per lb: Sat vapor v_g	Enthalpy, Btu: Sat liquid h_f	Enthalpy, Btu: Vaporization h_{fg}	Enthalpy, Btu: Sat vapor h_g	Entropy: Sat liquid s_f	Entropy: Vaporization s_{fg}	Entropy: Sat vapor s_g
10	38.51	0.02446	7.304	53.8	561.1	614.9	0.1208	1.1949	1.3157
12	40.31	0.02451	6.996	56.0	559.5	615.5	0.1254	1.1864	1.3118
14	42.18	0.02457	6.703	58.2	557.9	616.1	0.1300	1.1781	1.3081
16	44.12	0.02462	6.425	60.3	556.3	616.6	0.1346	1.1697	1.3043
18	46.13	0.02468	6.161	62.5	554.7	617.2	0.1392	1.1614	1.3006
20	48.21	0.02474	5.910	64.7	553.1	617.8	0.1437	1.1532	1.2969
22	50.36	0.02479	5.671	66.9	551.4	618.3	0.1483	1.1450	1.2933
24	52.59	0.02485	5.443	69.1	549.8	618.9	0.1528	1.1369	1.2897
26	54.90	0.02491	5.227	71.3	548.1	619.4	0.1573	1.1288	1.2861
28	57.28	0.02497	5.021	73.5	546.4	619.9	0.1618	1.1207	1.2825
30	59.74	0.02503	4.825	75.7	544.8	620.5	0.1663	1.1127	1.2790
32	62.29	0.02508	4.637	77.9	543.1	621.0	0.1708	1.1047	1.2755
34	64.91	0.02514	4.459	80.1	541.4	621.5	0.1753	1.0968	1.2721
36	67.63	0.02521	4.289	82.3	539.7	622.0	0.1797	1.0889	1.2686
38	70.43	0.02527	4.126	84.6	537.9	622.5	0.1841	1.0811	1.2652
40	73.32	0.02533	3.971	86.8	536.2	623.0	0.1885	1.0733	1.2618
42	76.31	0.02539	3.823	89.0	534.4	623.4	0.1930	1.0655	1.2585
44	79.38	0.02545	3.682	91.2	532.7	623.9	0.1974	1.0578	1.2552
46	82.55	0.02551	3.547	93.5	530.9	624.4	0.2018	1.0501	1.2519
48	85.82	0.02558	3.418	95.7	529.1	624.8	0.2062	1.0424	1.2486
50	89.19	0.02564	3.294	97.9	527.3	625.2	0.2105	1.0348	1.2453
52	92.66	0.02571	3.176	100.2	525.5	625.7	0.2149	1.0272	1.2421
54	96.23	0.02577	3.063	102.4	523.7	626.1	0.2192	1.0197	1.2389
56	99.91	0.02584	2.954	104.7	521.8	626.5	0.2236	1.0121	1.2357
58	103.7	0.02590	2.851	106.9	520.0	626.9	0.2279	1.0046	1.2325
60	107.6	0.02597	2.751	109.2	518.1	627.3	0.2322	0.9972	1.2294
62	111.6	0.02604	2.656	111.5	516.2	627.7	0.2365	0.9897	1.2262
64	115.7	0.02611	2.565	113.7	514.3	628.0	0.2408	0.9823	1.2231
66	120.0	0.02618	2.477	116.0	512.4	628.4	0.2451	0.9750	1.2201
68	124.3	0.02625	2.393	118.3	510.5	628.8	0.2494	0.9676	1.2170
70	128.8	0.02632	2.312	120.5	508.6	629.1	0.2537	0.9603	1.2140
72	133.4	0.02639	2.235	122.8	506.6	629.4	0.2579	0.9531	1.2110
74	138.1	0.02646	2.161	125.1	504.7	629.8	0.2622	0.9458	1.2080
76	143.0	0.02653	2.089	127.4	502.7	630.1	0.2664	0.9386	1.2050
78	147.9	0.02661	2.021	129.7	500.7	630.4	0.2706	0.9314	1.2020
80	153.0	0.02668	1.955	132.0	498.7	630.7	0.2749	0.9242	1.1991
82	158.3	0.02675	1.892	134.3	496.7	631.0	0.2791	0.9171	1.1962
84	163.7	0.02684	1.831	136.6	494.7	631.3	0.2833	0.9100	1.1933
86	169.2	0.02691	1.772	138.9	492.6	631.5	0.2875	0.9029	1.1904
88	174.8	0.02699	1.716	141.2	490.6	631.8	0.2917	0.8958	1.1875
90	180.6	0.02707	1.661	143.5	488.5	632.0	0.2958	0.8888	1.1846
92	186.6	0.02715	1.609	145.8	486.4	632.2	0.3000	0.8818	1.1818
94	192.7	0.02723	1.559	148.2	484.3	632.5	0.3041	0.8748	1.1789
96	198.9	0.02731	1.510	150.5	482.1	632.6	0.3083	0.8678	1.1761
98	205.3	0.02739	1.464	152.9	480.0	632.9	0.3125	0.8608	1.1733
100	211.9	0.02747	1.419	155.2	477.8	633.0	0.3166	0.8539	1.1705
105	228.9	0.02769	1.313	161.1	472.3	633.4	0.3269	0.8366	1.1635
110	247.0	0.02790	1.217	167.0	466.7	633.7	0.3372	0.8194	1.1566
115	266.2	0.02813	1.128	173.0	460.9	633.9	0.3474	0.8023	1.1497
120	286.4	0.02836	1.047	179.0	455.0	634.0	0.3576	0.7851	1.1427

For the second vessel,

$$w_1(h_1 - t_2 + 32) = w_2r_2 - (w - w_1)(t_1 - t_2)$$

For the third vessel,

$$w_2(h_2 - t_3 + 32) = w_3r_3 - (w - w_1 - w_2)(t_2 - t_3)$$

For a triple-effect system, $w - w_1 - w_2 = w_0$; hence with w and w_0 known, w_1, w_2, and w_3 can be calculated provided the temperatures and pressures within the vessels are known.

The rate q of heat transfer from steam to boiling fluid in a vessel is given by $q = UA\Delta_0$, in which A is the heating surface in square feet, Δ_0 the difference in temperature between the steam and boiling fluid, and U the over-all coefficient of heat transmission in Btu per sq ft per hr per deg F difference in temperature. The value of U is not the same for the different vessels of a multiple-effect system. From the experiments of Claassen, the ratios for different vessels may be taken as follows:

	1st	2d	3d	4th
Double-effect	1	0.66		
Triple-effect	1	0.70	0.33	
Quadruple-effect	1	0.91	0.75	0.55

The value of U depends on many conditions and is difficult to ascertain with accuracy. Creighton ("Steam Engines," p. 524) suggests the following values for cane-sugar evaporators: first vessel, $U = 385$; second, $U = 300$; third, $U = 205$; fourth, $U = 110$. If the fall of temperature in each effect is the same, the heating surfaces must be made larger in the successive vessels. It is usually considered better practice to keep the heating surface practically the same and make the temperature drops larger in the successive vessels.

The water evaporated per hour per square foot of surface varies considerably with the conditions of operation. Hausbrand gives the following figures as the result of practical experience: Ordinary vertical evaporators, with brass heating tubes 3 ft long and over, evaporate from aqueous solutions with single-effect (double-effect) [triple-effect] {quadruple-effect}, 14.0–16.0 (6.0–7.2) [4.0–5.0] {3.5–4.2} lb per sq ft per hr. With steel tubes, the evaporation is decreased 10 to 15 percent.

Steam Cycles

The Rankine Cycle. The ideal Rankine cycle is generally employed by engineers as a standard of reference for comparing the performance of actual steam engines and steam turbines. Figure 22 shows this cycle on the T-S and p-V planes. AB represents the heating of the water in the boiler, BC represents evaporation (and superheating if there is any), CD the assumed isentropic expansion in the engine cylinder, and DA condensation in the condenser.

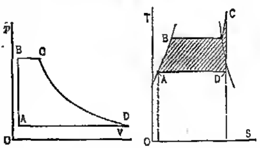


FIG. 22.—Rankine Cycle.

Let h_a, h_b, h_c, h_d represent the enthalpy per pound of steam in the four states A, B, C, and D, respectively. Then the energy transformed into work, represented by the area $ABCD$, is $h_c - h_d$.

Table 26. Properties of Superheated Ammonia

(Condensed from *U. S. B. of S., Circ.* 142, 1923)

v = specific volume in cu ft per lb; h = enthalpy in Btu per lb; s = entropy.
(h and s are measured from −40 F)

Pressure lb per sq in. abs	Temp of saturated vapor, F		Temperature of superheated vapor, deg F								
			−30	−20	−10	0	10	20	30	40	50
10	−41.34	v	26.58	27.26	27.92	28.58	29.24	29.90	30.55	312.0	31.85
		h	603.2	608.5	613.7	618.9	624.0	629.1	634.2	639.3	644.4
		s	1.4420	1.4542	1.4659	1.4773	1.4884	1.4992	1.509[illegible]	1.5200	1.5301
20	−16.64	v	......	......	13.74	14.09	14.44	14.78	15.11	15.45	15.78
		h	......	......	610.0	615.5	621.0	626.4	631.7	637.0	642.3
		s	......	......	1.3784	1.3907	1.4025	1.4138	1.424	1.4356	1.4460
30	− 0.57	v	......	......	......	9.250	9.492	9.731	9.966	10.20	10.43
		h	......	......	......	611.9	617.8	623.5	629.1	634.6	640.1
		s	......	......	......	1.3371	1.3497	1.3618	1.3733	1.3845	1.3953
40	11.66	v	......	......	......	......	......	7.203	7.387	7.568	7.746
		h	......	......	......	......	......	620.4	626.5	632.1	637.8
		s	......	......	......	......	......	1.3231	1.3353	1.3470	1.3583
50	21.67	v	......	......	......	......	......	......	5.838	5.988	6.135
		h	......	......	......	......	......	......	623.4	629.5	635.4
		s	......	......	......	......	......	......	1.304[illegible]	1.31[illegible]	1.3286
			100	120	140	160	180	200	240	280	320
80	44.40	v	4.190	4.371	4.548	4.722	4.893	5.063	5.398	5.730	
		h	658.7	670.4	681.8	693.2	704.4	715.6	738.1	760.7	
		s	1.3199	1.3404	1.3598	1.3784	1.3963	1.4136	1.4467	1.4781	
100	56.05	v	3.304	3.454	3.600	3.743	3.883	4.021	4.294	4.562	
		h	655.2	667.3	679.2	690.8	702.3	713.7	736.5	759.4	
		s	1.2891	1.3104	1.3305	1.3495	1.3678	1.3854	1.4190	1.4507	
120	66.02	v	2.712	2.842	2.967	3.089	3.029	3.326	3.557	3.783	
		h	651.6	664.2	676.5	688.5	700.2	711.8	734.9	758.0	
		s	1.2628	1.2850	1.3058	1.3254	1.3441	1.3620	1.3960	1.4281	
140	74.79	v	2.288	2.404	2.515	2.622	2.727	2.830	3.030	3.227	3.420
		h	[illegible]7.8	661.1	673.7	686.0	698.0	709.9	733.3	756.7	780.0
		s	1.2396	1.2628	1.2843	1.3045	1.3236	1.3418	1.3763	1.4088	1.4395
160	82.64	v	1.969	2.075	2.175	2.272	2.365	2.457	2.635	2.809	2.980
		h	643.9	657.8	670.9	683.5	695.8	707.9	731.7	755.3	778.9
		s	1.2186	1.2429	1.2652	1.2859	1.3054	1.3240	1.3591	1.3919	1.4229
180	89.78	v	1.720	1.818	1.910	1.999	2.084	2.167	2.328	2.484	2.637
		h	639.9	654.4	668.0	681.0	693.6	705.9	730.1	753.9	777.7
		s	1.1992	1.2247	1.2477	1.2691	1.2891	1.3081	1.3436	1.3768	1.4081
200	96.34	v	1.520	1.612	1.698	1.780	1.859	1.935	2.082	2.225	2.364
		h	635.6	650.9	665.0	678.4	691.3	703.9	728.4	752.5	776.5
		s	1.1809	1.2077	1.2317	1.2537	1.2742	1.2935	1.3296	1.3631	1.3947
220	102.42	v	......	1.443	1.525	1.601	1.675	1.745	1.881	2.012	2.140
		h	......	647.3	662.0	675.8	689.1	701.9	726.8	751.1	775.3
		s	......	1.1917	1.2167	1.2394	1.2604	1.2801	1.3168	1.3507	1.3825
240	108.09	v	......	1.302	1.380	1.452	1.521	1.587	1.714	1.835	1.954
		h	......	643.5	658.8	673.1	686.7	699.8	725.1	749.8	774.1
		s	......	1.1764	1.2[illegible]	1.2259	1.2475	1.26[illegible]	1.3049	1.3392	1.3712
260	113.42	v	......	1.182	1.257	1.326	1.391	1.453	1.572	1.686	1.796
		h	......	639.5	655.6	670.4	684.4	697.7	723.4	748.4	772.9
		s	......	1.1612	1.1[illegible]	1.2132	1.2354	1.25[illegible]	1.2938	1.3285	1.3608

interchange of heat in the heater is

$$w(h_2 - h') = h' - h_{f3}$$

The work done by the bled steam is $w(h_1 - h_2)$ and that by the 1 lb of steam going completely through the turbine is $h_1 - h_3$. Total work $= w(h_1 - h_2) + (h_1 - h_3)$. The heat supplied between feed-water heater and turbine is $(1 + w)(h_1 - h')$. Hence the ideal efficiency of the cycle is

$$e_t = \frac{w(h_1 - h_2) + h_1 - h_3}{(1 + w)(h_1 - h')}$$

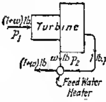


FIG. 24.—Regenerative Feed-water Heating.

For a full discussion of reheating and regenerative cycles, see *Trans. A.S.M.E.*, vol. 45, articles by Wohlenberg, and by Hirschfield and Ellenwood. See also Stodola "Steam and Gas Turbines," Loewenstein's translation, 1927, p. 1310.

Steam Accumulators

A steam accumulator is a device for storing the energy of steam. A large mass of water in a suitable vessel is heated by mixture with steam. When there is an excess of steam, this excess is condensed in the accumulator with a consequent rise of its temperature and pressure. Then during intervals of deficiency of steam, the deficit is drawn from the accumulator. As the pressure in the accumulator drops, the hot water is vaporized at the expense of the stored energy.

The **Rateau** accumulator is principally used to furnish steam to low-pressure turbines. It takes the exhaust from engines working intermittently, as hoisting engines or rolling mill engines, and with sufficient water capacity furnishes a steady supply of steam to the turbine. This accumulator operates at low pressure, and the pressure variation is necessarily small.

The **Ruths** accumulator has its special field in plants that are required to furnish steam for process work; such as plants for sugar factories, pulp and paper mills, textile and chemical industries. It may also be used in plants for the generation of power, solely, and in power and heating stations. The use of the accumulator permits the operation of the boilers under steady conditions at maximum efficiency. The fluctuations in the steam demand due to changes of load are taken care of by the accumulator, leaving the boilers unaffected. In the Ruths accumulator, the pressure may be relatively high, and the change of pressure during operation may be considerable.

For a comprehensive discussion of accumulators, see Stodola "Steam and Gas Turbines," Loewenstein's translation, 1927, p. 863: Taylor and Wettstein, *Mech. Eng.*, Aug. 1925; and Christie, "The Peak-load Problems in Steam Power Stations," *A.S.M.E.*, Dec., 1928.

Accumulator Capacity. The amount of water contained must be sufficient to give the weight of steam required during the discharge period with a predetermined drop of pressure.

Let h_{fg} denote the enthalpy of vaporization, h' the enthalpy of water, W the total weight of water, and p_1, p_2 the initial and final pressures. To evaporate a weight dw, requires an amount of heat $h_{fg}\,dw$ and this is furnished by a drop in temperature of the water. Hence the equation

$$h_{fg}\,dw = W dh'$$

If during the evaporation the pressure drops by dp, then

Table 28. Properties of Superheated Sulphur Dioxide

(v = specific volume in cu ft per lb; h = enthalpy in Btu per lb; s = entropy)
(h and s are measured from −40 F)

Pressure, lb per sq in. abs	Temp of saturated vapor, deg F		Temperature of superheated vapor, deg F								
			0	20	40	60	80	100	120	140	160
6	−19.37	v	12.75	13.34	13.93	14.52	15.11	15.69	16.26	16.82	17.35
		h	184.3	187.5	190.7	193.9	197.2	200.5	203.8	207.1	210.4
		s	0.4185	0.4254	0.4320	0.4383	0.4444	0.4504	0.4561	0.4618	0.4672
10	− 1.34	v	7.545	7.939	8.316	8.681	9.038	9.389	9.736	10.08	10.42
		h	183.2	186.7	190.1	193.5	196.9	200.3	203.7	207.1	210.5
		s	0.4005	0.4080	0.4151	0.4216	0.4280	0.4341	0.4400	0.4457	0.4512
15	14.43	v	....	5.192	5.470	5.734	5.988	6.233	6.471	6.705	6.937
		h	....	185.4	189.2	192.8	196.4	199.9	203.3	206.7	210.1
		s	....	0.3927	0.4005	0.4078	0.4144	0.4208	0.4268	0.4326	0.4383
20	26.44	v	....	....	4.035	4.251	4.454	4.648	4.834	5.015	5.193
		h	....	....	187.8	191.8	195.6	199.3	202.9	206.5	209.9
		s	....	....	0.3896	0.3972	0.4043	0.4109	0.4173	0.4232	0.4290
25	36.33	v	....	....	3.181	3.363	3.536	3.696	3.848	3.998	4.145
		h	....	....	186.1	190.6	194.7	198.6	202.4	206.0	209.6
		s	....	....	0.3793	0.3880	0.3958	0.4029	0.4095	0.4157	0.4216
			60	80	100	120	140	160	180	200	220
30	44.76	v	2.747	2.907	3.052	3.189	3.318	3.443	3.565	3.685	3.803
		h	189.3	193.8	197.9	201.8	205.6	209.3	212.9	216.5	220.1
		s	0.3797	0.3885	0.3960	0.4029	0.4094	0.4154	0.4211	0.4266	0.4318
40	58.83	v	1.980	2.121	2.246	2.360	2.465	2.565	2.662	2.755	2.845
		h	185.9	191.3	196.1	200.4	204.6	208.5	212.3	216.0	291.7
		s	0.3654	0.3754	0.3842	0.3918	0.3988	0.4153	0.4113	0.4169	0.4223
60	80.29	v	....	....	1.288	1.403	1.514	1.608	1.689	1.751	1.819
		h	....	....	191.4	197.0	201.9	206.5	210.7	214.8	218.7
		s	....	....	0.3640	0.3738	0.3822	0.3896	0.3964	0.4026	0.4084
80	96.88	v	....	....	0.993	1.084	1.163	1.232	1.292	1.347	1.400
		h	....	....	185.6	192.5	198.6	203.9	208.7	213.3	217.5
		s	....	....	0.3457	0.3580	0.3682	0.3769	0.3846	0.3915	0.3978

$T_b/T_a = T_c/T_d = (p_2/p_1)^{(k-1)/k}$. $w = Q/c_p(T_a - T_d)$.

$W = 778Q(T_b - T_a)/T_a$. Horsepower $= W/33{,}000$.

$V_c = wRT_a/144p_1n$. $V_2 = wRT_d/144p_1n$.

For values of $(p_2/p_1)^{(k-1)/k}$ see Table 17.

Vapor-compression Machines. The essential parts of a vapor-compression system are the same as in the system using air, except that the expansion cylinder is replaced by an expansion valve through which the liquefied medium flows from the high-pressure condensing coils to the low-pressure brine coils. The cycle of operation is best shown on the T-S plane (Fig. 26). The point B represents the state of the refrigerating medium leaving the brine coils and entering the compressor. Usually in this state the fluid is nearly dry saturated vapor, *i.e.*, point B is near the saturation curve S_g. BC represents the assumed reversible adiabatic compression, during which the fluid is usually superheated. In the state C, the superheated vapor passes into the cooling coils and is cooled at constant pressure, as indicated by CD, and then condensed at temperature T_2, as shown by DE. The liquid now flows through the expansion valve into the brine coils. This is a throttling process, and the final-state point A is located on the T_1-line in such a position as to make the enthalpy for state A (= area $OHGAA_1$) equal to the enthalpy at E (= area $OHEE_1$). The mixture of liquid and vapor now absorbs heat from the brine and vaporizes, as indicated by AB.

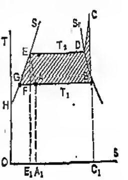


FIG. 26.—Vapor Compression Refrigeration Cycle.

The heat absorbed from the brine, represented by area A_1ABC_1, is

$$Q_1 = h_b - h_a = h_b - h_e$$

The heat rejected to the cooling water, represented by area C_1CDEE_1,

$$Q_2 = h_c - h_e = \text{approximately } c_p(T_c - T_d) + r_2$$

where r_2 denotes the enthalpy of vaporization at the upper temperature T and c_p the specific heat of the superheated vapor. The work that must b supplied per pound of fluid circulated is $W = J(Q_2 - Q_1) = J(h_c - h_b)$. The ratio $JQ_1/W = (h_b - h_e)/(h_c - h_b)$ is sometimes called the **coefficien of performance.**

If Q denotes the heat to be absorbed from the brine per hour, then th weight of fluid circulated per hour is $w = Q/(h_b - h_a)$; or, if B is take on the saturation curve, $w = Q/(h_{g1} - h_{f2})$.

The work per hour is $W = Jw(h_c - h_{g1}) = JQ(h_c - h_{g1})/(h_{g1} - h_{f2})$ ft-lb and the **horsepower required** is $H = Q(h_c - h_{g1})/2544(h_{g1} - h_{f2})$.

If v_{g1} is the volume of the saturated vapor at the temperature T_1 in the brin coils, and n the number of working strokes per minute, the **displacemen volume of the compressor cylinder** is $V = wv_{g1}/60n$.

The values of h_{g1}, h_{f2} in the preceding formulas are found in the tables c saturated vapors. The **enthalpy** h_c of the superheated vapor may be deter mined in the case of ammonia from Table 26 for superheated ammonia.

The carbon dioxide cycle may be investigated by the use of the accom panying chart (Fig. 27). The curves of isentropic compression in the super heat region are bcj, ghm, etc. The boundaries $abcda$ and $aghda$ represent th cycle of the refrigerating machine, the second for the case in which the vapo is superheated at the beginning of compression. Cycles $lbjkl$ and $lgmkl$ appl

Evaporation

Vacuum evaporation, especially of water from solutions of non-volatile solids, is often carried out under less than atmospheric pressure. By this procedure, boiling is obtained at lower temperatures and exhaust steam in closed coils may be used as the heating medium. For many materials, such as milk and sugar, the decomposition and deterioration that occur at elevated temperatures are prevented. For economy, **multiple-effect evaporators** are used. In these, the steam from the boiling solution in the first effect and, in some instances, the condensed steam used in the heating element of the first effect serve as the source of heat to the second effect. The second effect is under a lower pressure than the first and the solution boils at a lower temperature than that prevailing in the first effect. Condensate from the heating coils and steam arising from the solution in the second effect may go to the heating surface in the third effect, and so on through as many as four or more effects. Usually, only three or four effects are used. The solution to be evaporated is usually, but not always, passed through the apparatus concurrent with the passage of the heating medium, *i.e.*, entering at the first and emerging from the last effect.

For details see Hausbrand, "Evaporating, Condensing and Cooling Apparatus" (Scott Greenwood) or Walker, Lewis, McAdams and Gilliland, "Principles of Chemical Engineering" (McGraw-Hill).

In **recompression evaporators** (Prache and Bouillon method), the steam arising from the solution is compressed mechanically and refed to the heating coil. This is a method of restoring to the steam the availability it lost in passage through the evaporator. Most recompression evaporators are single effect (see *Engineering*, Dec., 1916).

Assuming adiabatic operation, enthalpy and material balances may be set up for each effect and for the over-all unit. For simplicity, average boiling temperatures of the solutions passing through each effect are used, and usually it is permissible to assume the enthalpy of the solution at every point equal to the enthalpy of the water plus the solid from which the solution is formed, *i.e.*, no heat effects on mixing. With such simplification, for each effect, the enthalpy of the steam formed in the preceding effect + the enthalpy of the condensate entering from the preceding effect − the enthalpy of the condensate from the heating coil of the effect under consideration = the enthalpy of the steam formed from solution in that effect + the change in enthalpy of the solution passing through that effect. For *dilute* aqueous solutions, the specific heat may be assumed equal to that of water, and the vapor arising from the solution may be assumed saturated instead of superheated as is actually the case. With these additional assumptions, the following approximate energy balances may be formulated.

Let w = weight of liquid introduced in first vessel.

w_1, w_2, w_3, etc. = water evaporated from liquid in first, second, third, etc., vessels.

w_0 = weight of liquid withdrawn from last vessel.

G = weight of steam introduced into first vessel.

t_0 = temperature of liquid entering first vessel.

t_1, t_2, t_3, etc. = mean temperatures of boiling liquid in first, second, third, etc., vessels.

h = enthalpy of steam G, r = latent heat of same.

h_1, h_2, h_3, etc. = enthalpy of vapor in successive vessels.

r_1, r_2, r_3, etc. = latent heats of vapor in successive vessels.

For the first vessel, the energy equation is

$$G(h - t_1 + 32) = w(t_1 - t_0) + w_1r_1$$

when the temperature of the cooling water is so high that the condensing pressure is greater than the critical pressure. In *abcda*, the segment $ab = h_b - h_a$ represents the heat absorbed from the brine and the segment $cd = h_c - h_d$ represents the heat rejected to the cooling water. These quantities may be read directly from the chart.

Figure 28 is a similar chart for Freon.

Hampson-Linde Regenerative Process. The Joule-Thomson effect (see p. 361) has been used by Hampson, Linde, and others for the liquefaction of air and other gases. In the Hampson process (Fig. 29), air is compressed to a pressure of 150 to 200 atm and discharged into a pipe leading to the chamber *c*. The air in its passage to *c* is cooled by water. From *c*, the air passes through an expansion valve into a vessel *d*, in which the pressure is only slightly above atmospheric. The throttling in the expansion valve causes a marked drop in temperature.

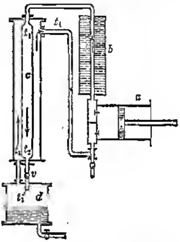


Fig. 29.—Regenerative Refrigeration Process.

The air now passes back to the compressor and in so doing cools the air in *c*. In the actual machine, the countercurrent apparatus, which in the figure is shown as the chamber *c* and the space surrounding it, consists of one or more pipes wound in a succession of flat coils one above the other, over which the return stream flows. By the cooling action of the return current of air, the temperature t_2 gradually sinks, and the temperature t_3 sinks more rapidly. Ultimately t_3 becomes lower than the critical temperature of air (−220 F), and liquefaction begins.

A small Linde liquid-air machine with an output of 20 to 25 lb of liquid air per hour consumes 30 to 40 hp, compresses 3,000 cu ft of air per hour, and has an efficiency of about 8 percent. A laboratory size Hampson machine delivers 2 lb of liquid air per hour and consumes 10 to 12 hp. With a plant of a capacity of at least 30 lb of liquid air per hour, Linde states (*Mech. Eng.*, 46, p. 582) that 1 lb of liquid air can be produced with from 0.9 to 1.6 hp-hr depending on whether double- or single-stage expansion is used.

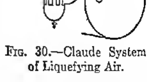


Fig. 30.—Claude System of Liquefying Air.

By the use of the regenerative process, liquid air or other gas may be produced continuously and in relatively large quantities by the simple mechanical process of compression. The necessary lowering of temperature is effected by repeated throttling of the gas, and a second refrigerating medium is not required.

Claude's System of Liquefying Air. Claude has succeeded in liquefying air by direct expansion of the air in a cylinder. A diagram of the apparatus in its simplest form is shown in Fig. 30. Air at a pressure of 25 to 30 atm enters through the central pipe of the interchanger *M*. Part of the air passes into the cylinder

The energy expended on the fluid is $h_c - h_a$; hence the Rankine cycle efficiency is $e_t = (h_c - h_d)/(h_c - h_a)$.

The **steam consumption** of the ideal Rankine engine in pounds per horsepower hour is $N_r = 2544/(h_c - h_d)$. Expressed in pounds per kilowatt-hour, the steam consumption of the ideal Rankine cycle is $3{,}412.7/(h_c - h_d)$.

The performance of an engine is frequently stated in terms of the heat used per hp-hr. For the ideal Rankine engine, this is

$$Q_r = 2544/e_t = 2544(h_c - h_a)/(h_c - h_d)$$

Efficiency of the Actual Engine. Let Q denote the heat transformed into work per pound of steam by the actual engine; then if Q_1 is the heat furnished by the boiler per pound of steam, the thermal efficiency of the engine is $e_t = Q/Q_1$.

The efficiency thus defined is misleading, as it takes no account of the conditions of boiler and condenser pressure, superheat, or quality of steam. It is customary therefore to define the efficiency as the ratio Q/Q_a, where Q_a is the **available heat**, or the heat that could be transformed under ideal conditions. For steam engines and turbines, the Rankine cycle is usually taken as the ideal, and the quantity $Q/Q_a = Q/(h_c - h_d)$ is called the **engine efficiency.** For engines and turbines, this efficiency ranges from 0.50 to 0.85. The engine efficiency e may also be expressed in terms of steam consumed; thus, if N_a is the steam consumption of the actual engine and N_r is the steam consumption of the ideal Rankine engine under similar conditions, then $e = N_r/N_a$.

Example. Suppose the boiler pressure to be 180 lb per sq in. abs, superheat 150 deg, and the condenser pressure 3 in. of mercury. From the steam tables or diagram, the following values are found: $h_c = 1283.3$, $h_d = 942$, $h_a = 82.99$. The available heat is $Q_a = 1283.3 - 942 = 341.3$ Btu, and the thermodynamic efficiency of the cycle is $341.3/(1283.3 - 82.99) = 0.284$. The steam consumption per horsepower-hour is $2546/341.3 = 7.46$ lb, and the heat used per horsepower-hour is $2546/0.284 = 8960$ Btu. If an actual engine working under the same conditions has a steam consumption of 11.4 lb. per hp-hr, its efficiency is $7.46/11.4 = 0.655$, and its heat consumption per horsepower-hour is $8960/0.655 = 13{,}680$ Btu.

Reheating Cycle. Let the steam after expansion from p_1 to an intermediate pressure p_2 (cd, Fig. 23) be reheated at constant pressure p_2, as indicated by de. Then follows the isentropic expansion to pressure p_3, represented by ef.

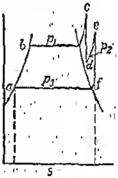


Fig. 23.—Reheating Cycle.

The energy absorbed by 1 lb of steam is $h_c - h_a$ from the boiler, and $h_e - h_d$ from the reheating. The work done is $h_c - h_d + h_e - h_f$. Hence the efficiency of the cycle is

$$e_t = \frac{h_c - h_d + h_e - h_f}{h_c - h_a + h_e - h_d}$$

Bleeding Cycle. In the **regenerative** or bleeding cycle, steam is drawn from the turbine at one or more stages and used to heat the feed water. Figure 24 shows a diagrammatic arrangement for bleeding at one stage. Entering the turbine is $1 + w$ lb, of steam at p_1, t_1, and enthalpy h_1. At the bleeding point w lb at p_2, t_2, h_2 enters the feed-water heater. The remaining 1 lb passes through the turbine and condenser and enters the feed-water heater as water at temperature t_3. Let t' denote the temperature of the water leaving the heater, and h' the corresponding enthalpy of the liquid. Then the equation for the

1. The continuity equation, or material balance,

$$w = \frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2} \quad \text{or} \quad \frac{dv}{v} = \frac{dA}{A} + \frac{dV}{V}$$

2. The first law or energy balance for steady flow

$$Jq_{12} = Jh_{12} + \frac{V_2^2 - V_1^2}{2g} + X_2 - X_1$$

3. The available energy balance for a steady-flow process

$$v\,dp + \frac{V\,dV}{g} + dF + dX = 0$$

In the processes here discussed, no net external or shaft work is performed. For most actual processes, the third equation cannot be integrated because the actual path is not known. Usually, adiabatic flow is assumed but occasionally the assumption of isothermal conditions may be more nearly correct.

Flow through Orifices and Nozzles. As a compressible fluid passes through a nozzle, drop in pressure and simultaneous increase in velocity result. By assuming the type of flow, *e.g.*, adiabatic, it is possible to calculate from the properties of the fluid the required area for the cross section of the nozzle at any point in order that the flowing fluid may just fill the provided space. From this calculation, it is found that for all compressible fluids the nozzle form must first be converging but eventually, if the pressure drops sufficiently, a place is reached where to accommodate the increased volume due to the expansion the nozzle must become diverging in form. The smallest cross section of the nozzle is called the throat, and the pressure at the throat is the **critical flow pressure** (not to be confused with the previously mentioned critical pressure, p. 321). If the nozzle is cut off at the throat with no diverging section and the pressure at the discharge end is progressively decreased, with fixed inlet pressure, the amount of fluid passing increases until the discharge pressure equals the critical, but further decrease in discharge pressure does not result in increased flow. A similar phenomenon is found for orifices. For any particular gas, the ratio of critical to inlet pressure is approximately constant. For gases, $p_m/p_1 = 0.53$ approximately; for saturated steam, the ratio is about 0.575; and for moderately superheated steam, about 0.55.

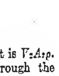


FIG. 32.

Formulas for Orifice Computations. The general fundamental relation is given by the energy balance $(V_2^2 - V_1^2)/2g = -Jh_{12}$. Referring to Fig. 32, let section 2 be taken at the orifice, section 3 is somewhat beyond the orifice on the downstream side, and section 1 is before the orifice on the upstream side. Then

$$V_2 = \frac{C\sqrt{2gJ(h_1 - h_2)}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2\left(\frac{v_1}{v_2}\right)^2}}$$

The coefficient of discharge factor C is discussed on p. 1804.

The volume of gas passing is V_2A_2 cu ft per sec, and the weight is $V_2A_2\rho$. For perfect gases, assuming reversible adiabatic expansion through the orifice,

$$\frac{dw}{dp} = \frac{W}{h_{fg}}\frac{dh'}{dp} \qquad \text{or} \qquad w = \int_{p_2}^{p_1} \frac{W}{h_{fg}}\frac{dh'}{dp}dp$$

If in this equation W is taken as the weight of a cubic foot of water, then w obtained from the integral will be the weight of steam generated per cubic foot of water for a drop in pressure p_1 to p_2. Since W, h_{fg}, and h' vary with the temperature (and therefore with the pressure), to perform the indicated integration the expression $(W/h_{fg})(dh'/dp)$ must be expressed as a function of p. The result is the following approximate expression for w, the pressures being in pounds per square inch absolute:

$$w = \tfrac{1}{2}[10 \log (p_1/p_2) + p_1^{1/2} - p_2^{1/2}]$$

For example, let $p_1 = 100$, $p_2 = 80$ lb per sq in. Then

$$w = \tfrac{1}{2}[10 \times 0.09691 + 10 - 8.944] = 1.013 \text{ lb}$$

REFRIGERATION

(See also pp. 1851 to 1881)

Air Refrigeration. When air is used as a medium for refrigeration, the reversed Joule cycle is employed (see p. 319). The refrigerating machine has four essential parts: (1) a compressor in which the air is compressed; (2) a cooling coil surrounded by water; (3) an expansion cylinder; (4) a brine coil. Figure 25 represents ideal p-V and T-S diagrams. Point A represents the state of the air entering the compressor from the brine coil or cold room, AB represents the compression, BC the cooling of the air at constant pressure, CD the expansion of air in the expansion cylinder, and DA the absorption of heat by the air during the passage through the brine coils. In the p-V diagram, $ABEF$ represents the indicator diagram of the compressor, $ECDF$ that of the expansion cylinder. The difference, area $ABCD$, represents the work that must be furnished from external sources. In the T-S diagram, area B_1BCC_1 represents the heat absorbed from the air by the cooling water, area C_1DAB_1, the heat absorbed by the air from the brine, and area $ABCD$ the heat equivalent of the work required to drive the machine. The temperature at point A must be somewhat lower than the brine temperature, the temperature at C somewhat above the temperature of the cooling water. With the open-cycle machine, the lower pressure p_1 is atmospheric pressure; with a closed-cycle machine, as the Allen dense-air machine, p_1 may be 40 to 60 lb and p_2 as high as 200 lb per sq in.

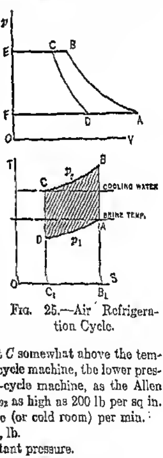


Fig. 25.—Air Refrigeration Cycle.

Let Q = heat in Btu absorbed from brine (or cold room) per min.
- w = weight of air circulated per min, lb.
- c_p = 0.24, specific heat of air at constant pressure.
- W = work required per min, ft-lb.
- V_c = displacement volume of compressor cylinder, cu ft.
- V_e = displacement volume of expansion cylinder, cu ft.
- n = number of working strokes per min.

By assuming (1) the gas laws, (2) constant specific heats, (3) reversible operation, and (4) isentropic compression and expansion, it is found that

express, approximately, the discharge w of saturated steam in terms of A_2 and p_1 as follows:

1. Napier's equation, $w = A_2p_1/70$.
2. Grashof's formula, $w = 0.0165A_2p_1^{0.97}$.
3. Rateau's formula, $w \doteq A_2p_1(16.367 - 0.96 \log p_1)/1000$.

In these formulas, A_2 is to be taken in square inches, p_1 in lb per sq in. Napier's formula is merely convenient as a rough check. Formulas 2 and 3 are applicable to well-rounded convergent orifices, in which case the coefficient of discharge may be taken as 1; *i.e.*, no correction is required. For flow of superheated steam, see also p. 1186.

When the back pressure p_2 is greater than the critical pressure p_m, the velocity and discharge are found most conveniently from the general formulas of flow. From the steam tables (p. 328) or from the Mollier chart (p. 326), find the initial enthalpy h_1 and the enthalpy h_2 after isentropic expansion; also the specific volume v_2 (see Fig. 33). Then

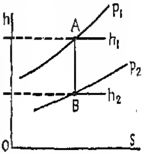


FIG. 33.

$$V_2 = 223.7\sqrt{h_1 - h_2} \qquad \text{and} \qquad w = A_2V_2/v_2$$

The same method is used in the case of steam initially superheated.

Example. Required the discharge through an orifice ½ in. diam of steam at 140 lb per sq in. superheated 110 F, back pressure, 90 lb per sq in.

From the Mollier chart, $h_1 = 1255.7$, and $h_2 = 1214$. Also $v_2 = 5.30$ cu ft

$$V_2 = 223.7\sqrt{1255.7 - 1214} = 1445$$
$$A_2 = 0.1964 \text{ sq in.} = (0.1964/144) \text{ sq ft}$$
$$w = A_2V_2/v_2 = (0.1964/144) \times (1445/5.30) = 0.372 \text{ lb per sec}$$

This calculation assumes ideal conditions, and the results must be multiplied by the correct coefficient of discharge to get actual results.

Flow through Converging-diverging Nozzles. At the throat, or smallest cross section of the nozzle (Fig. 34), the pressure of saturated steam takes the value $p_m = 0.57p_1$. The weight discharged is fixed by the area A_2 of the throat and the initial pressure p_1. For saturated steam, Grashof's or Rateau's formula (see above) may be used. The diverging part of the nozzle permits further expansion to the back pressure p_3, the velocity of the jet meanwhile increasing from $V_m (= V_2)$, the critical velocity at the throat, to V_3 given by the fundamental equation $V_3 = 223.7\sqrt{h_1 - h_3}$.

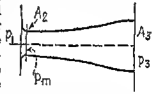


FIG. 34.

The frictional resistances in the nozzle have the effect of decreasing the jet energy $V_3^2/2g$ and correspondingly increasing the enthalpy of the flowing fluid. Thus, if h_3 is the enthalpy in the final state with frictionless expansion, $h'_3(>h_3)$ is the enthalpy when friction is taken into account; hence $(V'_3)^2/2g = J(h_1 - h'_3)$ is less than $V_3^2/2g = J(h_1 - h_3)$. The loss of kinetic energy, in Btu, is $h'_3 - h_3$, and the ratio of this loss to the available kinetic energy, *i.e.* $(h'_3 - h_3)/(h_1 - h_3)$, is denoted by y. For values of y, see p. 358.

The **design of a nozzle for a given discharge** w with pressures p_1 and p_3 is most conveniently effected with the aid of the Mollier chart. Determine p_m, the critical pressure, and h_1, h_m, h_3, assuming frictionless flow. Then

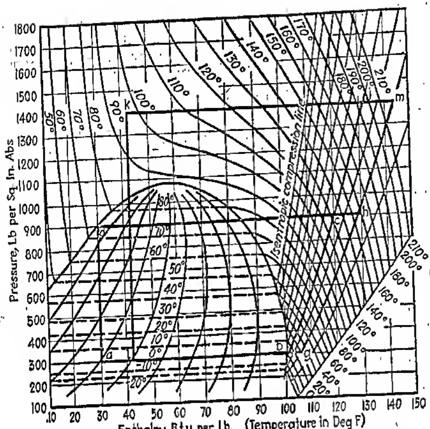


FIG. 27.—Pressure-enthalpy Chart for Carbon Dioxide.

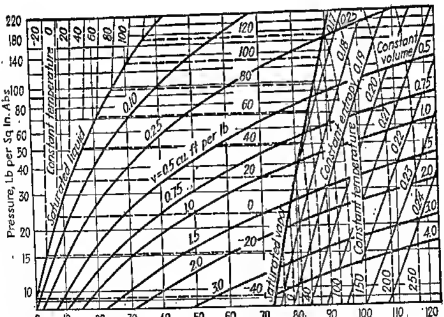


FIG. 28.—Pressure-enthalpy Chart for Freon.

pv^n = const throughout the expansion. With a value of $n = 1.315$, which holds good in the superheat region, the following results are found.

1. Equilibrium expansion	2. Expansion with supersaturation
$t_2 = 228$ F	173.4 F
$v_2 = 19.46$ cu ft	18.245 cu ft
$V = 1612$ fps	1597 fps
$V/v_2 = 82.8$	87.53

In the first case, the velocity is obtained from the usual equation $V = 223.7\sqrt{h_1 - h_2}$, in the second case, from the general equation $V^2/2g = n(p_1v_1 - p_2v_2)/(n - 1)$. The discharge is proportional to the ratio V/v_2; and since $87.53/82.8 = 1.056$, the discharge with supersaturation is 5.4 percent greater than with equilibrium expansion. The difference between the two values of t_2, *viz.*, $228 - 173.4 = 54.6$ deg, is the **undercooling** of the supersaturated steam. The pressure of saturated steam at $t_2 = 173.4$ F is 6.48 lb per sq in. The actual pressure of the undercooled steam at this temperature is 20 lb per sq in. The ratio of these pressures $20/6.48 = 3.09$ is the **degree of supersaturation.**

The effect of supersaturation in turbines is a loss of energy, the amount of which may be 1.5 to 3 percent of the available energy of the steam.

Flow of Wet Steam. When the steam entering a nozzle is wet, the speed of the water particles at exit is not the same as the speed of the stream. Denoting by V the speed of the steam, the speed of the water drops is fV, and f may vary perhaps 0.20 to 0.05 or less, depending on the pressure. The actual velocity V of the steam is greater than the velocity V_0 calculated on the usual assumption that steam and water have the same velocity. If x is the quality of the steam, the ratio of these velocities is

$$V/V_0 = 1/\sqrt{x + f^2(1 - x)}$$

Thus with $x = 0.92$, $f = 0.15$, $V/V_0 = 1.036$. Since the discharge is practically proportional to the steam velocity, the actual discharge in this case is 3.6 percent greater than the discharge computed on the usual assumptions.

Velocity Coefficients. Loss of Energy y. On account of friction losses, the actual velocity V attained by the jet is less than the velocity V_0 calculated under ideal conditions. That is, $V = xV_0$, where x (<1) is a velocity coefficient. The coefficient x is connected with the coefficient y, giving the loss of energy, by the relation, $y = 1 - x^2$.

The elaborate and accurate experiments of the General Electric Co. on turbine nozzles (Warren and Keenan, *Trans. A.S.M.E.*, 48, p. 33) give for convergent nozzles values of x in excess of 0.98, with a corresponding loss of energy $y = 0.025$ to 0.04. For similar nozzles, the experiments of the Steam Nozzles Research Committee (of England) by a different method give values of x around 0.96, or $y = 0.08$. In the case of divergent nozzles, the velocity coefficient may be somewhat lower.

Flow of Fluids in Circular Pipes

(See pp. 264 to 272)

The fundamental equation as previously given, assuming the pipe horizontal, is

$$(VdV/g) + vdp + dF = 0$$

The friction term dF includes not only losses due to frictional flow along the pipe but also those due to fittings, valves, etc., as well as losses occasioned by any enlargement or contraction of the pipe as, for instance, the loss occurring when a fluid passes from a pipe into a tank. For long straight pipes of

D where it expands doing work, and its temperature is thereby lowered. It is then discharged into the liquefier L where it cools the remainder of the compressed air which has entered the tubes of L.

With an expansion cylinder, 1 lb of liquid oxygen of 88 percent purity can be obtained from 0.77 hp-hr of work (*Mech. Eng.*, 46, p. 582).

For a complete account of the liquefaction of gases, see Claude, "Liquid Air, Oxygen and Nitrogen."

FLOW OF COMPRESSIBLE FLUIDS

REFERENCES: Stodola, "Steam and Gas Turbines," Loewenstein's translation, McGraw-Hill. Goodenough, "Principles of Thermodynamics," Van Nostrand. Lucke, "Engineering Thermodynamics," McGraw-Hill. Berry, *Mech. Eng.*, Nov., 1929. Gutermuth, *Z.V.d.I.*, 48. Buchner, *Z.V.d.I.*, 49. Walker, Lewis, McAdams, and Gilliland, "Principles of Chemical Engineering," McGraw-Hill.

Important examples of the flow of compressible fluids are the following: (1) The flow of air and steam through orifices and short tubes or nozzles, as in the steam turbine. (2) The flow of compressed air, steam, and illuminating gas in long mains. (3) The flow of low-pressure gases, as furnace gases in ducts and chimneys or air in ventilating ducts. (4) The flow of gases in moving channels, as in the centrifugal fan.

Notation.

Let A = area of section, sq ft.
C = empirically determined coefficient of discharge.
D = inside diameter of pipe, ft.
$d = 12D$ = inside diameter of pipe, in.
F_{12} = energy expended in overcoming internal and external friction between sections A_1 and A_2.
F' = energy used in overcoming friction, ft-lb per lb of fluid flowing.
f = friction factor.
$g = 32.2$
h = enthalpy, Btu per lb.
$J = 778.3$
$k = c_p/c_v$.
L = equivalent length of pipe, ft.
μ = viscosity, centipoises.
p = pressure of fluid at given section, lb per sq ft abs.
p_m = critical flow pressure.
Q_{12} = heat entering the flowing fluid between sections A_1 and A_2.
R = perfect gas constant.
ρ = density, lb per cu ft.
T = temperature R.
V = mean velocity in ft per sec at the given section.
v = volume per unit weight, cu ft per lb.
w = weight in lb of fluid flowing past a given section per sec.
X = height from center of gravity of flow to a fixed base level, ft.

The cross sections of the tube or channel are denoted by A_1, A_2, etc. (Fig. 31), and the various magnitudes pertaining to these sections are denoted by corresponding subscripts. Thus, at section A_1, the velocity, volume, and pressure are, respectively, V_1, v_1, p_1; at section A_2, they are V_2, v_2, p_2.

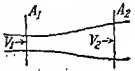


FIG. 31.

Fundamental Equations. In the interpretation of fluid-flow phenomena, three fundamental equations are of importance.

to 0.0054. Babcock has suggested the approximation $f = 0.0027(1 + 3.6/d)$ for steam.

Values of f as a function of pipe surface are given by Fig. 21, p. 266.

For predicting the capacity of a given pipe operating on a chosen fluid with fixed pressure drop, the use of Fig. 37 eliminates the trial-and-error methods usually involved. Figure 38 gives the viscosity of various gases.

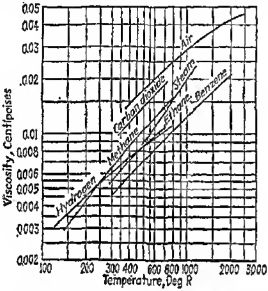


FIG. 38.—Viscosity of Gases.

Example. Air at 68 F is flowing isothermally through a straight standard 1 in. pipe (I.D. = 1.049 in.). The pipe is 200 ft long, the pressure at the pipe inlet is 60 lb per sq in. gage, and the pressure drop through the pipe is 5 lb per sq in. The average density of the air in the pipe is

$$\rho_{avg} = 0.0753\,[(60 + 14.7) + (55 + 14.7)]/2 \times 14.7 = 0.370 \text{ lb per cu ft}$$

$$F' = \frac{(5.0)(144)}{0.370} = 1945 \text{ ft lb/lb}$$

$$D = \frac{1.049}{12} = 0.0875 \text{ ft}$$

$$x = \sqrt{\frac{2(32.2)(0.0875)(0.370)^2(1.945)}{200}} = 2.74$$

μ = 0.018 centipoise (Fig. 38), or (0.018)(0.000672) = 0.0000121 in lb ft sec units.

$$\frac{Dx}{\mu} = \frac{(0.0875)(2.74)}{0.0000121} = 19{,}800 \text{ and from Fig. 37 the corresponding value of } \frac{\rho V}{x} \text{ is } 7.05.$$

$$V = \frac{7.05(2.74)}{0.370} = 52.2 \text{ ft per sec}$$

$$w = AV\rho = \frac{(3.1416)(0.0875)^2}{4}(52.2)(0.370) = 0.116 \text{ lb per sec}$$

Resistances due to fittings, expressed in terms of L/D, are as follows: 90 deg elbows, 1–2½ (3–6) [7–10] in., 30 (40) [50]; 90 deg curves, radius of center line of curve 2–8 pipe diameters, 10; globe valves, 1–2½ (3–6) [7–10] in., 45 (60) [75]; tees, 1–4 in., 60. The resistance in energy units, due to sudden enlargement in a pipe, is approximately $(V_1 - V_2)^2/2g$. For sudden contraction it is $1.5(1 - r)V_2^2/2g(3 - r)$, where $r = A_2/A_1$.

$$V_2 = \frac{C\sqrt{2gp_1v_1\frac{k}{k-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}}}}$$

$$w = \frac{CA_2p_2\sqrt{\frac{2g}{RT_1}\frac{k}{k-1}\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}\left[\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}-1\right]}}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}}}}$$

V_1 is often small compared with V_2, and under these conditions the denominators in the preceding equations become approximately equal to unity. For air assuming $R = 53.3$, $k = 1.40$, and V_1 negligible,

$$w = 2.05CA_2p_2\sqrt{(1/T_1)(p_1/p_2)^{0.283}[(p_1/p_2)^{0.283}-1]}$$
$$= 2.05CA_2p_2\sqrt{(1/T_1)(1+Y)Y}$$

Values of Y are given on p. 315.

Although the preceding formulas are generally applicable under the assumed conditions, it must be remembered that irrespective of the value of p_1, p_2 cannot become less than p_m. When p_2 is less than p_m, the flow rate becomes independent of the downstream pressure; for perfect gases,

$$w = CA_2p_1\sqrt{\frac{g}{RT_1}k\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}$$

or for air

$$w = 0.53Cp_1\frac{A_2}{\sqrt{T_1}}$$

Where the pressure drop through the orifice is small, the hydraulic formulas (pp. 253, 1803) applicable to incompressible fluids may be employed for gases and other compressible fluids.

In general, the formulas of the preceding section are applicable to nozzles. When so used however, the proper value of the discharge coefficient must be employed. For steam nozzles, this may be as high as 0.94 to 0.96, although for many orifice installations it is as low as 0.50 to 0.60. Steam nozzles constitute a most important type, and calculations for these are best carried out with the aid of a Mollier or similar chart.

Formulas for Discharge of Steam. When the back pressure p_2 is less than the critical pressure p_m, the discharge depends upon the area of orifice A_2 and reservoir pressure p_1. There are three formulas widely used to

Table 31. Values of $p^{0.97}$ for Use in Grashof's Formula

p	$p^{0.97}$	p	$p^{0.97}$	p	$p^{0.97}$	p	$p^{0.97}$
15	13.8	50	44.5	110	95.5	225	191.2
20	18.3	55	48.8	120	104.0	250	212.0
25	22.7	60	53.1	130	112.4	275	232.0
30	27.1	70	61.6	140	120.7	300	253.0
35	31.5	80	70.1	150	129.1		
40	35.8	90	78.6	175	150.0		
45	40.1	100	87.1	200	170.6		

as a product, carbon dioxide, CO_2; the combustion of hydrogen gives water, H_2O.

Combustion of Gaseous and Liquid Fuels

Combustion Equations. The approximate molecular weights of the important elements and compounds entering into combustion calculations are:

Gas	H_2	O_2	N_2	CO	CO_2	H_2O	CH_4	C_2H_4	C_2H_6O
Molecular weight	2	32	28	28	44	18	16	28	46

For the elements C and H, the equations of complete combustion are

$$\begin{cases} C + O_2 = CO_2 \\ 12\text{ lb} + 32\text{ lb} = 44\text{ lb} \end{cases} \qquad \begin{matrix} H_2 + \tfrac{1}{2}O_2 = H_2O \\ 2\text{ lb} + 16\text{ lb} = 18\text{ lb} \end{matrix}$$

For a combustible compound, as CH_4, the equation may be written

$$CH_4 + x\cdot O_2 = y\cdot CO_2 + z\cdot H_2O$$

Taking, as a basis, 1 molecule of CH_4 and making a balance of the atoms on the two sides of the equation, it is seen that

$$y = 1, \quad z = 2; \quad 2x = 2y + z, \quad \text{or } x = 2$$

Hence
$$\begin{cases} CH_4 + 2O_2 = CO_2 + 2H_2O \\ 16\text{ lb} + 64\text{ lb} = 44\text{ lb} + 36\text{ lb} \end{cases}$$

The coefficients in the combustion equation give the combining volumes of the gaseous components. Thus, in the last equation 1 cu ft of CH_4 requires for combustion 2 cu ft of oxygen and the resulting gaseous products of combustion are 1 cu ft of CO_2 and 2 cu ft of H_2O. The coefficients multiplied by the corresponding molecular weights give the combining weights. These are conveniently referred to 1 lb of the fuel. In the combustion of CH_4, for example, 1 lb of CH_4 requires $64/16 = 4$ lb of oxygen for complete combustion and the products are $44/16 = 2.75$ lb of CO_2 and $36/16 = 2.25$ lb of H_2O.

Air Required for Combustion. The composition of air is approximately 0.232 O_2 and 0.768 N_2 by weight, or 0.21 O_2 and 0.79 N_2 by volume. For exact analyses, it may be necessary sometimes to take account of the water vapor mixed with the air, but ordinarily this may be neglected.

The minimum weight of air required for the combustion of 1 lb of a fuel is the weight of oxygen required, as found from the combustion equation, divided by 0.232. Likewise, the minimum volume of air required for the combustion of 1 cu ft of a fuel gas is the volume of oxygen divided by 0.21. For example, in the combustion of CH_4 the weight of air required per pound of CH_4 is $4/0.232 = 17.24$ lb and the volume of air per cubic foot of CH_4 is $2/0.21 = 9.52$ cu ft. Ordinarily, more air is provided than is required for complete combustion. Let a denote the minimum weight required and xa the weight of air admitted; then $x - 1$ is the **excess coefficient.**

Products of Combustion. The products arising from the complete combustion of a fuel are CO_2, H_2O, and, if sulphur is present, SO_2. Accompanying these are the nitrogen brought in with the air and the oxygen in the excess of air. Hence the products of complete combustion are principally CO_2, H_2O, N_2, and O_2. The presence of CO indicates incomplete combustion. The composition of the products of combustion is readily calculated from the combustion equations, as shown by the following illustrative example.

Example. A producer gas having the volume composition given is burned with 20 percent excess of air; required the volume composition of the exhaust gases.

$$V_m = 223.7\sqrt{h_1 - h_m}, \quad \text{and} \quad V'_3 = 223.7\sqrt{(1-y)(h_1 - h_3)}$$

Next find v_m and v'_3. Then, from the equation of continuity,

$$A_m = wv_m/V_m \quad \text{and} \quad A'_3 = wv'_3/V'_3$$

The following example illustrates the method.

Example. Required the throat and end sections of a nozzle to deliver 0.7 lb of steam per sec. The initial pressure is 160 lb, the back pressure 15 lb, and the steam is initially superheated 100 F; $y = 0.15$.

The critical pressure is $160 \times 0.55 = 88$ lb. On the Mollier chart (Fig. 35), the point A representing the initial state is located, and line of constant entropy (a frictionless adiabatic) is drawn from A. This cuts the curves $p = 88$ and $p = 15$ in the points B and C, respectively. The three values of h are found to be $h_1 = 1253$, $h_m = 1199$, $h_3 = 1067$. Of the available drop in enthalpy, $h_1 - h_3 = 185.5$ Btu, 15 per cent or 27.9 Btu is lost through friction. Hence, $CD = 27.9$ is laid off and D is projected horizontally to point C' on the curve $p = 15$. Then C' represents the final state of the steam, and the quality is found to be $x = 0.943$. The specific volume in the state C' is $26.29 \times 0.943 = 24.8$ cu ft. Likewise, the specific volume for the state B is found to be 5.29 cu ft.

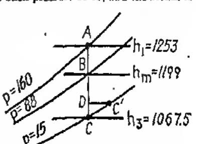


FIG. 35.

For the velocities at throat and end sections,

$$V_m = 223.7\sqrt{1253 - 1199} = 1643 \text{ fps}$$
$$V_3 = 223.7\sqrt{185.5 - 27.9} = 2813 \text{ fps}$$
$$A_m = (0.7 \times 5.29)/1643 = 0.00225 \text{ sq ft} = 0.324 \text{ sq in.}$$
$$A_3 = (0.7 \times 24.8)/2813 = 0.00617 \text{ sq ft} = 0.89 \text{ sq in.}$$

The diameters are $d_m = 0.643$ in. and $d_3 = 1.064$ in.

Divergence of Nozzles. The diagram (Fig. 36) gives, for various ratios of expansion, the required "divergence" of the nozzle, *i.e.*, the ratio of the area of any section to the throat area. Thus in the case of saturated steam, if the final pressure is $\frac{1}{15}$ of the initial pressure the ratio of the areas is 3.25. The curves apply to frictionless flow; the effect of friction is to increase the divergence.

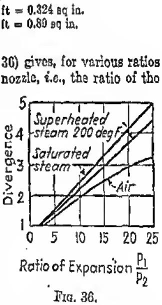


FIG. 36.

Theory of Supersaturation. Certain discrepancies between the discharge of saturated steam through an orifice as calculated from the preceding theory and the discharge actually observed are explained by a hypothesis first advanced by Martin, *viz.*, that steam when expanded rapidly, as in turbine nozzles, becomes supersaturated; in other words, the condensation required by the ordinary theory of adiabatic expansion does not occur on account of the rapidity of the expansion (see Goodenough, *Power*, Sept. 27, Oct. 4, 1927).

The following example from the article cited illustrates the effect of supersaturation on the flow of steam. Let the initial condition of the steam be $p_1 = 40$ lb per sq in. abs, $t_1 = 283.2$ F (16 deg superheat), $v_1 = 10.76$ cu ft per lb, $s_1 = 1.6877$. The steam expands adiabatically in a nozzle to a pressure of 20 lb per sq in. Two calculations are made. (1) It is assumed that equilibrium conditions are maintained and that after the saturation state is reached the expansion is accompanied by condensation. (2) It is assumed that there is no condensation and that the steam follows the gas law

lent of the work done on the gas during the contraction. For example, in the burning of CO according to the equation $CO + \frac{1}{2}O_2 = CO_2$, there is a contraction of $\frac{1}{2}$ volume. Taking 62 F as the temperature, the volume of 1 lb CO at atmospheric pressure is 13.6 cu ft; hence the equivalent of the work done at atmospheric pressure is $\frac{1}{2} \times 13.6 \times 2116/778 = 18.5$ Btu, which is about 0.4 percent of the heat value of CO. Since the difference between H_p and H_v is small in most fuels, it is usually neglected.

It is also to be noted that heat values vary with the initial temperature (which is also the final temperature), but the variation is usually negligible.

Heat Value per Unit Volume. Since the consumption of a fuel gas is more easily measured by volume than by weight, it is convenient to express heat values in terms of volumes. For this purpose, a standard temperature and pressure must be assumed. It is customary to take atmospheric pressure (14.70 lb per sq in.) as standard, but there is diversity of practice in the matter of a standard temperature. The temperature of 68 F (20 C) is generally accepted in metric countries and has been recommended by the American delegates to the meeting of the International Committee of Weights and Measures and also by the A.S.M.E. Power Test Codes Committee. The American Gas Assoc. uses 60 F as the standard temperature of reference. Conversion of density and heat values from 68 to 60 F of dry (saturated) gas is obtained by multiplying by the factor 1.0154 (1.0212). Conversion of specific volumes of dry (saturated) gas is obtained by multiplying by the factor 0.9848 (0.9792).

If the gas is at some other pressure and temperature, say p_1 lb per sq in. abs and T_1 deg R, the heat value per cubic foot is found by multiplying the heat value per cubic foot under standard conditions by $35.9p_1/T_1$.

Table 32. Heats of Combustion

Fuel	Chemical symbol	High heat value, Btu		Low heat value, Btu	
		Per lb	Per cu ft[a]	Per lb	Per cu ft[a]
Carbon to CO_2	C	14,520			
Carbon to CO	C	4,340			
CO to CO_2	CO	4,345	315.8		
Sulphur	S	3,982			
Hydrogen	H_2	61,045	319.5	51,608	270.1
Methane	CH_4	23,910	995.4	21,529	896.2
Ethane	C_2H_6	22,330	1,739	20,425	1,591
Propane	C_3H_8	21,670	2,475	19,940	2,278
Butane	C_4H_{10}	21,320	3,211	19,670	2,960
Pentane	C_5H_{12}	21,100	3,944	19,514	3,647
Hexane	C_6H_{14}	20,950	4,676	19,400	4,330
Octane	C_8H_{18}	20,750	6,141	19,245	5,694
Decane	$C_{10}H_{22}$	20,700	7,630	19,350	7,132
Ethylene	C_2H_4	21,660	1,577	20,300	1,478
Propylene	C_3H_6	21,330	2,325	19,970	2,177
Acetylene	C_2H_2	21,570	1,457	20,840	1,407
Benzene	C_6H_6	18,160	3,677	17,190	3,481
Toluene	C_7H_8	18,300	4,370	17,460	4,169
Methyl alcohol	CH_4O	10,270	853.1	9,078	754.1
Ethyl alcohol	C_2H_6O	13,120	1,573	11,930	1,424
Naphthalene	$C_{10}H_8$	17,300	5,749	16,700	5,549

[a] Measured as a gas at 68 F and 14.70 lb per sq in. abs. Multiply by 1.0154 for 60 F and 14.70 lb.

uniform diameter, dF is approximately equal to $2f\frac{V^2dL}{gD}$. It is usual to express friction due to fittings, etc., in terms of additional length of pipe, adding this to the actual pipe length to get the equivalent pipe length.

Integration of the fundamental equation leads to two sets of formulas.

1. For pressure drops, small relative to the initial pressure, the specific volume v and the velocity V may be assumed constant. Then approximately

$$p_1 - p_2 = 2fV^2L/vgD$$

Expressing pressure in pounds per square inch, p', the diameter in inches,

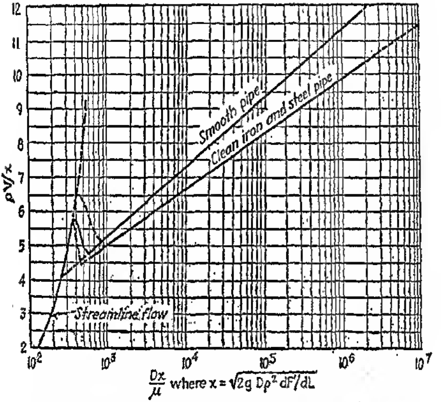


FIG. 37.—Chart for Estimating Rate of Flow from the Pressure Gradient.

and V as a function of wv/D^2, this equation becomes

$$p'_1 - p'_2 = 174.2fw^2vL/d^5$$

2. For considerable pressure drops, when dealing with approximately isothermal flow of gases and vapors to which the gas laws are applicable, the fundamental equation may be integrated to give

$$p_1^2 - p_2^2 = \frac{2w^2RT}{gA^2}\log_e\frac{v_2}{v_1} + \frac{4fRTw^2L}{gA^2D}$$

Coefficients of Friction. The coefficient of friction f is not a constant but is a function of the dimensionless expression $\mu/\rho Vd$ or $\mu v/Vd$, which is the reciprocal of the Reynolds number. McAdams and Sherwood (*Mech. Eng.*, Oct., 1926) formulate the expression

$$f = 0.0054 + 0.375(\mu v/Vd).$$

This formula is applicable to water and other fluids. For high-pressure steam, the second term in the expression is small and f is approximately equal

Low and High Heat Values. Any fuel containing hydrogen yields water as one product of combustion. At atmospheric pressure, the partial pressure of the water vapor in the resulting combustion gas mixture will usually be sufficiently high to cause water to condense out if the temperature is allowed to fall below 120–140 F. This causes liberation of the heat of vaporization of any water condensed. The low heat value is evaluated assuming no water vapor condensed, whereas the high heat value is calculated assuming all water vapor condensed.

To facilitate calculations of the temperature attained by combustion, it is desirable to make use of the low heat value. The necessity of taking into account the heat of vaporization of the water vapor and the difference between the specific heats of liquid water and of water vapor is thus avoided. The high heat of combustion exceeds the low heat of combustion by the difference between the heat actually given up on cooling the products to the initial temperature and that which would have been given up if the products had remained in the gaseous state. A bomb calorimeter (constant volume) gives practically correct values of the high heat value; a gas calorimeter (constant pressure) gives values which, for the usual fuels, may be incorrect by a fraction of 1 percent. The quantity to be subtracted from the high heat value to obtain the low heat value will vary with the composition of the fuel; an approximate value is 1050 m, where m is the number of pounds of H_2O formed per pound of fuel burned.

In Germany, the low heat value of the fuel is used in calculating efficiencies of internal-combustion engines. In the United States, the high value is specified by the A.S.M.E. Power Test Codes.

Heat of Formation. The change in enthalpy resulting when a compound is formed from its elements isothermally and at constant pressure is called the heat of formation. It is equal to the difference between the heats of combustion of the constituents forming the compound and the heat of combustion of the compound itself. The following values for heats of formation are in Btu per lb of the compound. The elements before the change and the compounds formed are assumed in their ordinary stable states at 65 F and 1 atm. A plus sign indicates heat evolved on forming the compound, a minus sign heat absorbed from the surroundings.

Fuels. Methane, CH_4, (gas) 2048; ethane, C_2H_6, (gas) 1258; propane, C_3H_8, (vapor) 1440; acetylene, C_2H_2, (gas) −3730; ethylene, C_2H_4, (gas) −707.3; benzene, C_6H_6, (vapor) −288; toluene, C_7H_8, (vapor) −68; methyl alcohol, CH_3OH, (liquid) 3234; ethyl alcohol, C_2H_5OH, (liquid) 2625.

Inorganic Compounds. Al_2O_3, 9234; CaO, 7585; $CaCO_3$, 5211; FeO, 1608; Fe_2O_3, 2243; Fe_3O_4, 2071; FeS_2, 532.5; HCl (gas), 1088; HNO_3 (liquid), 1191; H_2O (liquid), 6837; H_2S, 280.4; H_2SO_4 (liquid), 3365; K_2O, 1646; MgO, 6531; MnO, 2441; NO, −1296; N_2O, −803.7; Na_2O, 2884; NH_3, 1165; NH_4Cl, 2518; NiO, 1407; P_2O_5 (solid), 4572; PbO, 424.9; PbO_2, 488.8; SO_2, 1993; SnO, 907.2; ZnO, 1845.

Internal Energy and Enthalpy of Gases. Table 34 gives the internal energy of various common gases in Btu per lb mol measured above 520 R (60 F). The corresponding values of the enthalpy are obtained by adding the value of Apv from the last column.

Temperature Attained by Combustion. Excluding the effect of dissociation, the temperature attained at the end of combustion may be calculated by a simple energy balance. The heat of combustion less the heat

Throttling

Throttling or Wiredrawing. When a fluid flows from a region of higher pressure into a region of lower pressure through a valve or constricted passage, it is said to be throttled or wiredrawn. Examples are seen in the passage of steam through pressure-reducing valves, in the flow through ports and passages in the steam engine, and in the expansion valve of the refrigerating machine.

The general equation applicable to throttling processes is

$$(V_2^2 - V_1^2)/2g = (h_1 - h_2)J$$

The velocities V_2 and V_1 are practically equal, and it follows that $h_1 = h_2$; *i.e.*, in a throttling process there is no change in enthalpy.

For a mixture of liquid and vapor, $h = h_f + xh_{fg}$; hence the equation of throttling is $h_{f1} + x_1h_{fg1} = h_{f2} + x_2h_{fg2}$. In the case of a perfect gas, $h = c_pT + h_0$; hence the equation of throttling is $c_pT_1 + h_0 = c_pT_2 + h_0$, or $T_1 = T_2$.

Joule-Thomson Effect. The investigations of Joule and Lord Kelvin showed that a gas drops in temperature when throttled. This is not universally true. For some gases, notably hydrogen, the temperature rises for throttling processes over ordinary ranges of temperature and pressure. Whether there is a rise or fall in temperature depends on the particular range of pressure and temperature over which the change occurs. For every gas, there is one temperature at which no temperature change occurs during a Joule-Thomson expansion; this is called the **inversion temperature.** Below this temperature, a gas cools on throttling; above this temperature, its temperature rises. The ratio of the observed drop in temperature to the drop in pressure, *i.e.*, dT/dp, is the **Joule-Thomson coefficient.**

The variations of the Joule-Thomson coefficient, for air, with both temperature and pressure, can be determined from the constant enthalpy curves of Fig. 17. Values of the coefficient for steam vary from 0.465 at 300 deg to 0.165 at 530 F, the unit being deg per lb per sq in.

The cooling effect produced by throttling has been applied to the liquefaction of gases (see p. 352).

Loss Due to Throttling. A throttling process in a cycle of operations always introduces a loss of efficiency. If T_0 is the temperature corresponding to the back pressure, the loss of available energy is the product of T_0 and the increase of entropy during the throttling process. The following example illustrates the calculation in the case of ammonia passing through the expansion valve of a refrigerating machine.

Example. The liquid ammonia at a temperature of 70 F passes through the valve into the brine coil in which the temperature is 20 deg and the pressure is 48.21 lb per sq in. The initial enthalpy of the liquid ammonia (see Table 25) is $h_{f1} = 120.5$, and therefore the final enthalpy is $h_{f2} + x_2h_{fg2} = 64.7 + 553.1x_2 = 120.5$, whence $x_2 = 0.101$. From the table, the initial entropy is $s_{f1} = 0.254$. The final entropy is $s_{f2} + (x_2h_{fg2}/T_2) = 0.144 + 0.101 \times 1.153 = 0.260$. $T_0 = 20 + 460 = 480$; hence the loss of refrigerating effect is $480 \times (0.260 - 0.254) = 2.9$ Btu.

COMBUSTION

Fuels. For special properties of various fuels, see pp. 1006 and 1014. In general, fuels may be classed under three heads: (1) gaseous fuels, (2) liquid fuels, (3) solid fuels.

The combustible elements that characterize fuels are carbon, hydrogen, and, in some cases, sulphur. The complete combustion of carbon gives,

lost by conduction and radiation during the process is equal to the increase in internal energy of the products mixture if the combustion is at constant volume; or, if the combustion is at constant pressure, the difference is equal *to the increase in enthalpy of the products mixture.*

As an **example**, the temperature of combustion of a fuel gas having the composition $H_2 = 0.50$, $CO = 0.40$, $CO_2 = 0.04$ is calculated. The gas is burned with 15 percent excess air at constant volume, and the initial temperature in 62 F, *i.e.* $T = 522$.

The volume composition of the initial mixture of fuel gas and air and the mixture of products are, respectively:

Initial: H_2, 0.50; CO, 0.46; CO_2, 0.04; O_2, 0.552; N_2, 2.098

Products: H_2O, 0.50; CO_2, 0.50; O_2, 0.072; N_2, 2.098

Since a volume composition is also a mol composition, the products mixture may be regarded as made up of 0.5 mol each of H_2O and CO_2, 0.072 mols of O_2, and 2.098 mols of N_2. If values are taken from Table 32, the heat generated by combustion of the fuel mixture in $0.50 \times 2 \times 51{,}608 + 0.46 \times 28 \times 4345 = 107{,}584$ Btu. The internal energy u of the products mixture at $T = 522$ is now calculated. For 0.5 mol H_2O + 0.5 mol CO_2 + 0.072 mol O_2 + 2.098 mol N_2 this is $3.05 + 3.48 + 0.36 + 10.17 = 17.06$ Btu.

The *energy u_2 of the mixture is next calculated for various assumed temperatures*, the proper values being taken from Table 34.

T_2 assumed	4700	4800	4900	5000	5200
Energy 0.5 mol H_2O	18,308	18,851	10,391	19,943	20,492
Energy 0.5 mol CO_2	23,742	24,358	25,035	25,682	26,331
Energy 0.072 mol O_2	1,976	2,026	2,080	2,135	2,185
Energy 2.098 mol N_2	53,500	55,017	56,446	57,882	59,321
$u_2 =$	97,526	100,282	102,952	105,642	108,329
$u_1 =$	17	17	17	17	17
	97,509	100,265	102,935	105,625	108,312

If the heat of combustion, 107,584 Btu, *is entirely used in the increase of energy*, the temperature attained lies somewhere between 5000 and 5200; by interpolation, the value 5140 deg is obtained.

Loss of heat during combustion may readily be taken into account; then if 10 percent *of the heat of combustion is lost, the amount available for increasing the energy of the* products is $107{,}584 \times 0.90 = 96{,}827$ Btu, and this increase gives $T_2 = 4670$ deg. If the fuel is burned at constant pressure, H_p is used instead of H_v and values of h are determined from Table 34 instead of values of u.

Effect of Dissociation. The maximum temperature that can be obtained by the combustion of any fuel is limited by the dissociation of the products formed. Thus CO_2 dissociates into CO and $\frac{1}{2}O_2$, whereas H_2O goes to $H_2 + \frac{1}{2}O_2$. The amount of such dissociation depends upon the temperature and also upon the pressure; the higher the pressure, the less the dissociation. The percentage dissociation, according to Tizard and Pye (*Automobile Eng.*, Feb. 1921) is given in Table 35.

Table 35. Dissociation of H_2O and CO_2
(Dissociation, percent)

Temperature, deg F	Pressure, atm							
	0.1	1.0	10	100	0.1	1.0	10	100
	H_2O				CO_2			
2730	0.043	0.02	0.009	0.004	0.104	0.048	0.0224	0.01
3630	1.25	0.58	0.27	0.125	4.35	2.05	0.96	0.445
4530	8.84	4.21	1.98	0.927	33.5	17.6	8.63	4.09
5430	28.4	14.4	7.04	3.33	77.1	54.8	32.2	16.9

	V	Coefficients in reaction equations O_2	CO_2	H_2O	Coefficients multiplied by V O_2	CO_2	H_2O
H_2	0.08	0.5	0	1	0.04	0	0.08
CO	0.22	0.5	1	0	0.11	0.22	0
CH_4	0.024	2	1	2	0.048	0.024	0.048
CO_2	0.066	0	1	0	0	0.066	0
N_2	0.61	0	0	0	0	0	0
	1.0				0.198	0.31	0.128

For 1 cu ft of the producer gas, 0.198 cu ft of O_2 is required for complete combustion. The minimum volume of air required is $0.198/0.21 = 0.943$ cu ft and with 20 percent excess the air supplied is $0.943 \times 1.2 = 1.132$ cu ft. Of this, 0.238 cu ft is oxygen and 0.894 cu ft is N_2. Consequently, for 1 cu ft of the fuel gas, the exhaust gas contains

CO_2	0.31 cu ft	or	CO_2	15.7 percent
H_2O	0.128 cu ft		H_2O	6.5 percent
N_2 ..... $0.61 + 0.894 =$	1.504 cu ft		N_2	75.8 percent
O_2 (excess) ... $0.238 - 0.198 =$	0.040 cu ft		O_2	2.0 percent
	1.982 cu ft			100.0 percent

Volume Contraction. As a result of chemical action, there is often a change of volume; for example, in the reaction $2H_2 + O_2 = 2H_2O$, three volumes (two of H_2 and one of O_2) contract to two volumes of water vapor. In the example just given, the volume of producer gas and air supplied is 1 cu ft gas + 1.132 cu ft air = 2.132 cu ft, and the corresponding volume of the exhaust gas is 1.982 cu ft, showing a contraction of about 7 percent. For a hydrocarbon having the composition C_mH_n, the relative volume contraction is $[1 - (n/4)]$; thus for CH_4 and C_2H_4 there is no change of volume, for C_2H_2 the contraction is ½ volume, and for C_2H_6 there is an increase of ½ in volume.

The change of volume accompanying a chemical reaction, such as a combustion, causes a corresponding change in the gas constant R. Let R' denote the constant for the mixture of gas and air (1 lb of gas and xa lb of air) before combustion, and R'' the constant of the mixture of resulting products of combustion. Then, if y is the resulting contraction of volume, $R''/R' = (1 + xa - y)/(1 + xa)$.

Heat of Combustion. Usually, a chemical change is accompanied by the generation or absorption of heat. The union of a combustible with oxygen produces heat, and the heat thus generated when 1 lb of combustible is completely burned is called the **heat of combustion** or the **heat value** of the combustible. Heat values are determined experimentally by calorimeters in which the products of combustion are cooled to the initial temperature and the heat absorbed by the cooling medium is measured. This is called the high heat value.

An accurate definition of the heat value requires specification of the conditions under which the combustion proceeds. Two heat values may thus be distinguished:

1. The **heat value at constant volume** (H_v) is the quantity of heat rejected, from a calorimeter, to external surroundings when the temperature and volume of the combustion products are brought to the temperature and volume, respectively, of the gaseous mixture before burning. Numerically, this is equal to the change in internal energy.

2. The **heat value at constant pressure** (H_p) is the quantity of heat rejected, from a calorimeter, when the temperature and pressure of the products are brought back to the temperature and pressure, respectively, of the gaseous mixture before burning. This is equal to the change in enthalpy.

If there is no change of volume due to the combustion, the heat values H_p and H_v are the same. When there is a contraction of volume, H_p exceeds H_v by the heat equiva-

pheric pressure. This was compressed to $\frac{1}{6}$ of its volume and exploded. It was assumed that no heat was lost during explosion. With no dissociation, the maximum temperature reached is 5473 F and the maximum pressure 658 lb per sq in. With dissociation, the maximum temperature is 4811 F and the maximum pressure 600.7 lb per sq in. Similar results are found for other liquid fuels, as alcohol, kerosene, etc.

As the temperature falls after reaching the maximum, the uncombined CO and H_2 burn with the oxygen present, and when the temperature has reached about 3000 F the combustion is practically complete. Thus in the internal-combustion engine dissociation has the effect of reducing the explosion temperature and pressure and afterward of raising the expansion curve. The effect on the efficiency is not large. For an engine using benzene according to the conditions just noted, the ideal efficiency with no heat loss and without dissociation was found by Tizard and Pye to be 35.9 percent; with dissociation and subsequent recombination, it was found to be 33.8 percent.

Combustion of Solid Fuels

For properties of coals, heat values, etc., see p. 1014.

Air Required for Combustion. Let c, h, and o, denote, respectively, the parts by weight of carbon, hydrogen, and oxygen in 1 lb of the fuel. Then the **minimum weight of oxygen** required for complete combustion is $2.67c + 8h - o$ lb, and the **minimum weight of air** required is $a = (2.67c + 8h - o)/0.23 = 11.6[c + 3(h - o/8)]$ lb.

With air at 62 F and at atmospheric pressure, the minimum volume of air required is $v_m = 147[c + 3(h - o/8)]$ cu ft. In practice, an excess of air over that required for combustion is admitted to the furnace. The actual weight admitted per pound of fuel may be denoted by xa. Then x = weight admitted ÷ minimum weight.

Combustion Products. If v_m is the minimum volume of air required for complete combustion and xv_m the actual volume supplied, then the products will contain per pound of fuel, $O_2 = 0.21v_m(x - 1)$ cu ft, $N_2 = 0.79\,xv_m$ cu ft.

From the reaction equation $C + O_2 = CO_2$, the volume of CO_2 formed is equal to the volume of oxygen required for the carbon constituent alone; hence volume of $CO_2 = 0.21v_mc/[c + 3(h - 0.125o)]$.

Of the *dry* gaseous products (i.e., without water), the CO_2 content by volume is therefore given by the expression

$$CO_2 = 0.21c/[xc + (x - 0.21)3(h - 0.125o)]$$

The combined CO_2 and O_2 content is

$$CO_2 + O_2 = 0.21\{1 - 0.79/[x + cx/3(h - 0.125o) - 0.21]\}$$

If the fuel is all carbon, the combined CO_2 and O_2 is by volume 21 percent of the gaseous products. The more hydrogen contained in the fuel, the smaller is the $CO_2 + O_2$ content. The CO_2 content depends in the first instance on the excess of air. Thus, for pure carbon, it is $CO_2 = 0.21/x$.

The excess of air may be calculated from the composition of the gases and that of the fuel. Thus

$$x = 0.21\left[\frac{c}{[CO_2]} + 3(h - 0.125o)\right] \Big/ [c + 3(h - 0.125o)],$$

in which $[CO_2]$ denotes the percent by volume of the CO_2 in the dry gas.

Table 33. Products of Combustion
(From Marks, "The Airplane Engine")

Fuel	Chemical formula	Molecular weight $O_2 = 32$	Specific weight, lb per cu ft at 68 F and 14.70 lb per sq in.	Volume of air necessary for combustion of unit volume of fuel at same temperature and pressure	Products of combustion of 1 cu ft of fuel in theoretical amount of air, cu ft			Weight of air necessary for combustion of unit weight of fuel	Products of combustion of 1 lb of fuel in theoretical amount of air, lb		
					CO_2	H_2O	N_2		CO_2	H_2O	N_2
Oxygen	O_2	32	0.0831								
Nitrogen	N_2	28.08	0.0727								
Air	....	....	0.0753								
Hydrogen	H_2	2.016	0.0052	2.39	0	1	1.89	34.2	0.0	8.94	26.28
Steam	H_2O	18.016									
Carbon monoxide	CO	28.00	0.0727	2.39							
Carbon dioxide	CO_2	44.00	0.1142								
Methane	CH_4	16.03	0.0416	9.55	1	2	7.55	17.21	2.75	2.248	13.22
Ethane	C_2H_6	30.05	0.0779	16.71	2	3	13.21	16.07	2.93	1.799	12.34
Propane	C_3H_8	44.06	0.1142	23.87	3	4	18.87	15.65	3.00	1.635	12.02
Butane	C_4H_{10}	58.1	0.1506	30.94	4	5	24.53	15.44	3.03	1.551	11.86
Pentane	C_5H_{12}	72.1	0.1869	38.08	5	6	30.2	15.31	3.05	1.499	11.76
Hexane	C_6H_{14}	86.1	0.2232	45.3	6	7	35.8	15.22	3.07	1.465	11.69
Heptane	C_7H_{16}	100.1	0.2596	52.5	7	8	41.5	15.15	3.08	1.439	11.64
Octane	C_8H_{18}	114.1	0.2959	59.7	8	9	47.2	15.11	3.08	1.421	11.60
Nonane	C_9H_{20}	128.2	0.3323	66.8	9	10	52.8	15.07	3.09	1.406	11.57
Benzene	C_6H_6	78.0	0.2025	35.8	6	3	28.3	13.26	3.38	0.693	10.18
Toluene	C_7H_8	92.1	0.2388	42.9	7	4	34.0	13.50	3.35	0.783	10.36
Xylene	C_8H_{10}	106.2	0.2752	50.1	8	5	39.6	13.57	3.31	0.845	10.42
Cyclohexane	C_6H_{12}	84.0	0.2180	43.0	6	6	34.0	14.76	3.14	1.285	11.34
Ethylene	C_2H_4	28.03	0.0728	14.32	2	2	11.32	14.76	3.14	1.285	11.34
Propylene	C_3H_6	42.0	0.1090	21.48	3	3	16.98	14.76	3.14	1.285	11.34
Butylene	C_4H_8	64.1	0.1454	28.64	4	4	22.64	14.76	3.14	1.285	11.34
Acetylene	C_2H_2	26.02	0.0675	11.93	2	1	9.43	13.26	3.38	0.693	10.18
Allylene	C_3H_4	40.0	0.1038	19.09	3	2	15.09	13.78	3.30	0.900	10.59
Napthalene	$C_{10}H_8$	128.1	0.3322	57.3	10	4	45.28	12.93	3.44	0.563	9.93
Methyl alcohol	CH_4O	32.0	0.0830	7.16	1	2	5.66	6.46	1.37	1.125	4.96
Ethyl alcohol	C_2H_6O	46.0	0.1194	14.32	2	3	11.32	8.99	1.91	1.174	6.90

Heat Value per Unit Volume of Mixture. Let a denote the volume of air required for the combustion of 1 cu ft of fuel gas and xa the value of air actually admitted, $x - 1$ being therefore the excess. Then the volume of the mixture of fuel gas and air is $1 + xa$, and the quotient $H/(1 + xa)$ may be called the heat value per cubic foot of mixture. This magnitude is useful in comparing the relative volumes of mixture required with different fuel gases. Thus a lean gas, as blast-furnace gas or producer gas, has a low heat value H, but the value of a is correspondingly low. On the other hand, a rich gas, like natural gas, has a high heat value but requires a large volume of air for combustion.

The ratio of air supplied per pound of combustible to that theoretically required is $N/[N - 3.782(O - \frac{1}{2}CO)]$, on the assumption that all the nitrogen in the flue gas comes from the air supplied. Figure 39 gives the value of this ratio for varying flue-gas analyses where there is no CO present.

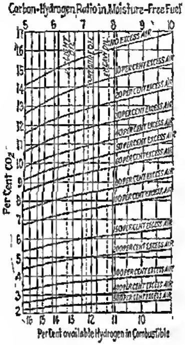


FIG. 40.—Relation of CO_2 to Excess Air for Oil Fuels.

For petroleum fuels with hydrogen content from 9 to 16 percent, the excess air can be determined from the CO_2 content of the flue gases (with no CO present) by the use of Fig. 40. The curves are based on the assumption of 0.4 percent sulphur in the oil.

Table 34. Internal Energy of Gases

(Btu per lb mol above 520 R)

(From L. C. Lichty, "Internal Combustion Engines," 1939, p. 582, derived from data given by Hershey, Eberhardt and Hottel, *Trans. S.A.E.*, 31, 1936, p. 409)

Temp, R	O_2	N_2	Air	CO_2	H_2O	H_2	CO	Apv
520	0	0	0	0	0	0	0	1,033
540	100	97	97	139	122	96	97	1,072
560	200	196	196	280	244	193	196	1,112
580	301	295	295	424	357	291	295	1,152
600	402	395	395	570	490	390	396	1,192
700	920	896	897	1,320	1,110	887	896	1,390
800	1,449	1,399	1,403	2,120	1,734	1,386	1,402	1,589
900	1,989	1,905	1,915	2,965	2,366	1,886	1,913	1,787
1000	2,539	2,416	2,431	3,852	3,009	2,387	2,430	1,986
1100	3,101	2,934	2,957	4,778	3,666	2,889	2,954	2,185
1200	3,675	3,461	3,492	5,736	4,339	3,393	3,485	2,383
1300	4,262	3,996	4,036	6,721	5,030	3,899	4,026	2,582
1400	4,861	4,539	4,587	7,731	5,740	4,406	4,580	2,780
1500	5,472	5,091	5,149	8,764	6,468	4,916	5,145	2,979
1600	6,092	5,652	5,720	9,819	7,212	5,429	5,720	3,178
1700	6,718	6,224	6,301	10,896	7,970	5,945	6,305	3,376
1800	7,349	6,805	6,889	11,993	8,741	6,464	6,899	3,575
1900	7,985	7,393	7,485	13,105	9,526	6,988	7,501	3,773
2000	8,629	7,989	8,087	14,230	10,327	7,517	8,109	3,972
2100	9,279	8,592	8,698	15,368	11,146	8,053	8,722	4,171
2200	9,934	9,203	9,314	16,518	11,983	8,597	9,339	4,369
2300	10,592	9,817	9,934	17,680	12,835	9,147	9,961	4,568
2400	11,252	10,435	10,558	18,852	13,700	9,703	10,588	4,766
2500	11,916	11,056	11,185	20,033	14,578	10,263	11,220	4,965
2600	12,584	11,682	11,817	21,222	15,469	10,827	11,857	5,164
2700	13,257	12,313	12,453	22,419	16,372	11,396	12,499	5,362
2800	13,937	12,949	13,095	23,624	17,288	11,970	13,144	5,561
2900	14,622	13,590	13,742	24,836	18,217	12,549	13,792	5,759
3000	15,309	14,236	14,394	26,055	19,160	13,133	14,443	5,958
3100	16,001	14,888	15,051	27,281	20,117	13,723	15,097	6,157
3200	16,693	15,543	15,710	28,513	21,086	14,319	15,754	6,355
3300	17,386	16,199	16,369	29,750	22,066	14,921	16,414	6,554
3400	18,080	16,855	17,030	30,991	23,057	15,529	17,078	6,752
3500	18,776	17,512	17,692	32,237	24,057	16,143	17,744	6,951
3600	19,475	18,171	18,356	33,487	25,067	16,762	18,412	7,150
3700	20,179	18,833	19,022	34,741	26,085	17,385	19,082	7,348
3800	20,887	19,496	19,691	35,998	27,110	18,011	19,755	7,547
3900	21,598	20,162	20,363	37,258	28,141	18,641	20,430	7,745
4000	22,314	20,830	21,037	38,522	29,178	19,274	21,107	7,944
4100	23,034	21,500	21,714	39,791	30,221	19,911	21,784	8,143
4200	23,757	22,172	22,393	41,064	31,270	20,552	22,462	8,341
4300	24,482	22,845	23,073	42,341	32,326	21,197	23,143	8,540
4400	25,209	23,519	23,755	43,622	33,389	21,845	23,823	8,738
4500	25,938	24,194	24,437	44,906	34,459	22,497	24,503	8,937
4600	26,668	24,869	25,120	46,193	35,535	23,154	25,186	9,136
4700	27,401	25,546	25,805	47,483	36,616	23,816	25,868	9,334
4800	28,136	26,224	26,491	48,775	37,701	24,480	26,533	9,533
4900	28,874	26,905	27,180	50,069	38,791	25,418	27,219	9,731
5000	29,616	27,589	27,872	51,365	39,885	25,819	27,907	9,930
5100	30,361	28,275	28,566	52,663	40,983	26,492	28,597	10,129
5200	31,308	28,961	29,262	53,963	42,084	27,166	29,288	10,327
5300	31,857	29,648	29,958	55,265	43,187	27,842	29,980	10,526
5400	32,607	30,337	30,655	56,569	44,293	28,519	30,674	10,724

precise work, it requires frequent calibration. (3) The wet- and dry-bulb psychrometer is most widely used. For precise work, radiation to the wet bulb must be considered. The velocity of atmospheric movement past the wet bulb affects the temperature attained; at about 1,200 fpm, the maximum wet-bulb depression is reached. This velocity is set up by a fan blowing over the wet bulb or by swinging the wet bulb vigorously. An unventilated wet bulb is unreliable. At the wet-bulb temperature, the rate of evaporation and the diffusion of water vapor (dependent upon the excess of p_w over p_v) are balanced by the rate of heat transfer from the surrounding atmosphere to the wet bulb (dependent upon the wet-bulb depression). A constant is determined by calibration.

The electric hygrometer is a device whose electrical resistance or capacity varies with the relative humidity of the surrounding air. For details see Dunmore, "An Improved Electric Hygrometer," *Jour. Research, B. of S.*, *23, p. 701, Dec., 1939; also Research Paper 1265.*

Table 1 gives the pressure-temperature relations for saturated water vapor over water or ice.

Table 1. Vapor Pressure of Water in Inches of Hg at 32 F
(Ice below 32 F)
(From Keenan and Keyes, "Thermodynamic Properties of Steam")

t	0	1	2	3	4	5	6	7	8	9
−40	0.0039									
−30	0.0071	0.0067	0.0062	0.0058	0.0055	0.0051	0.0048	0.0045	0.0043	0.0041
−20	0.0126	0.0119	0.0112	0.0106	0.0100	0.0094	0.0089	0.0084	0.0079	0.0075
−10	0.0220	0.0200	0.0197	0.0187	0.0176	0.0167	0.0158	0.0149	0.0141	0.0133
− 0	......	0.0358	0.0339	0.0322	0.0305	0.0289	0.0274	0.0259	0.0246	0.0233
+ 0	0.0377	0.0397	0.0419	0.0441	0.0465	0.0489	0.0514	0.0541	0.0569	0.0598
10	0.0629	0.0661	0.0695	0.0730	0.0767	0.0806	0.0847	0.0889	0.0933	0.0980
20	0.103	0.108	0.113	0.119	0.124	0.130	0.137	0.143	0.150	0.157
30	0.165	0.172	0.180	0.188	0.196	0.204	0.212	0.220	0.229	0.238
40	0.248	0.258	0.268	0.278	0.289	0.300	0.312	0.324	0.336	0.349
50	0.363	0.376	0.391	0.405	0.420	0.436	0.452	0.469	0.486	0.504
60	0.522	0.541	0.560	0.580	0.601	0.622	0.644	0.667	0.690	0.714
70	0.739	0.765	0.791	0.818	0.846	0.875	0.905	0.935	0.967	0.999
80	1.032	1.066	1.102	1.138	1.175	1.213	1.253	1.293	1.335	1.378
90	1.422	1.467	1.513	1.561	1.610	1.660	1.712	1.765	1.819	1.875
100	1.933	1.992	2.052	2.114	2.178	2.243	2.310	2.379	2.449	2.521

The following equations give various quantities in terms of vapor pressure and observed quantities. Equation (1) (Apjohn, 1835) is sufficiently accurate for most purposes, with a modified value of the constant.

The suggested degree of accuracy for each equation corresponds to temperature readings to the nearest degree F. An additional significant digit demands temperature readings accurate to at least 0.1 F (Berry, *loc. cit.*).

Vapor pressure: $$p_v = p_w - B(t_d - t_w)/2700 \qquad (1)$$

Relative humidity: $$r = p_v/p_d \qquad (2)$$

Molal humidity: $$m = p_v/(B - p_v) \quad \text{or} \quad p_v = mB/(1 + m) \qquad (3)$$

Specific humidity: $$s = m/1.61 = p_v/1.61(B - p_v) \qquad (4)$$

Air density: $$\rho_a = (B - p_v)/RT_d \qquad (5)$$

(R = 0.3704 for pressures in lb per sq in. = 0.7541 in in. of Hg)

Vapor density: $$\rho_v = \rho_a s \qquad (6)$$

Mixture density: $$\rho_m = \rho_a + \rho_v = \rho_a(1 + s) = (B - 0.38p_v)/RT_d \qquad (7)$$

By means of the equilibrium equations for the H_2 and CO reactions, it is possible to calculate the maximum temperature when equilibrium is attained, and also the amount of dissociation of H_2O and CO_2.

To show the extent of the dissociation, Table 36 (calculated by G. T. Felbeck), is given. The values are based on the following assumptions:

1. The mixture of fuel and air has an initial pressure of 14.7 lb per sq in. and an initial temperature of 60 F.
2. The combustion is at constant volume as in the ideal Otto cycle.
3. The loss of heat is taken as 10 percent of the heat of combustion.

Table 36. Percentage Dissociation and Explosion Temperatures of Various Fuel Mixtures

(x = dissociation of CO_2, percent; y = dissociation of H_2O, percent; T = temperature deg F abs)

Fuel		Percent excess air 0	10	20	30	40
Carbon monoxide	T	4710.0	4640.0	4560.0	4460.0	4360.0
	x	15.4	11.0	0.7	6.2	4.3
Hydrogen	T	4700.0	4570.0	4420.0	4260.0	4100.0
	y	4.0	1.8	0.9	0.5	0.3
Carbureted water gas	T	4580.0	4460.0	4330.0	4180.0	4020.0
	x	15.3	8.6	4.7	2.7	1.5
	y	2.2	1.2	0.7	0.4	0.1
Cambridge coal gas	T	4530.0	4410.0	4260.0	4100.0	3950.0
	x	15.2	7.8	4.2	2.3	1.1
	y	2.1	1.1	0.5	0.3	0.1
Natural gas	T	4450.0	4330.0	4170.0	4000.0	3840.0
	x	13.1	5.9	2.9	1.5	0.6
	y	1.9	0.9	0.4	0	0
Producer gas	T	3810.0	3720.0	3620.0	3510.0	3410.0
	x	2.9	1.5	0.4	0	0
	y	0.5	0.2	0.1	0	0
Blast furnace gas	T	3560.0	3500.0	3420.0	3340.0	3250.0
	x	1.4	0.4	0	0	0
	y	0.2	0	0	0	0

Inspection of Table 36 shows (1) that the dissociation of H_2O is negligible for temperatures below 4700 deg abs; (2) that the dissociation of CO_2 may reach an appreciable magnitude if the absolute temperature exceeds 4300 F, (3) that an excess of air by reducing the temperature reduces the dissociation; (4) that dissociation is less for the lean gases than for the rich gases.

The maximum temperature attainable by the combustion of hydrogen with oxygen at atmospheric pressure is about 5270 F. In the oxyacetylene flame, the temperature may be as high as 6020 F.

In the case of **explosion in the internal-combustion engine**, the figures in the table will be somewhat changed. The effect of compression is to increase both the initial temperature and the initial pressure. The resulting increase in the explosion temperature will tend to increase the dissociation, the increase of pressure will tend to reduce it. The net effect will be a small reduction.

Combustion of Liquid Fuels. For properties of fuel oils, heat values, etc., see page 1006. Calculations for the burning of liquid fuels are fundamentally the same as for gaseous fuels. Liquid fuels are almost always gasified before or during actual combustion.

Tizard and Pye (*Automobile Eng.*, Feb., 1921) give the following result on a theoretically correct mixture of benzene vapor and air at 212 F and atmos-

$$\Sigma = h_m - s \times (t_w - 32) = 0.240t_d + s \times (h_v - t_w + 32) \text{ Btu per lb air} \quad (10)$$

An advantage of the sigma function is that it is determined by the barometric pressure and the wet-bulb temperature; its value is independent of the dry-bulb temperature. It equals the enthalpy of dry air at the given wet-bulb temperature; at zero relative humidity, the sigma function and the enthalpy are identical. Values are given in Table 2. The sigma function has a shifting datum depending upon the wet-bulb temperature; in any change of state during which the wet-bulb temperature changes, correction must be made for this.

Examples. 1. To Find p_v from Psychrometer Data. $B = 24$ in. Hg; $t_d = 76$ F; $t_w = 62$ F. From Table 1, $p_w = 0.56$ in. Hg. From Eq. (1), $p_v = 0.56 - [24(76 - 62)/2700] = 0.44$ in. Hg.

2. To Find p_v from Temperature and Relative Humidity. $t_d = 72$ F, $r = 0.73$. From Table 1, $p_d = 0.79$ in. Hg. From Eq. (2), $p_v = r \times p_d = 0.73 \times 0.79 = 0.58$ in. Hg.

3. To Find Composition, Density, Etc.

$$B = 28 \text{ in. Hg}; \qquad t_d = 82 \text{ F}; \qquad t_w = 70 \text{ F}; \qquad p_v = 0.62 \text{ in. Hg}.$$

Relative humidity: from Table 1, $p_d = 1.10$ in. Hg; from Eq. (2), $r = 0.62/1.10 = 0.56$ or 56 percent.

Molal humidity: from Eq. (3), $m = 0.62/(28 - 0.62) = 0.0226$ mols vapor per mol air.

Specific humidity: from Eq. (4), $s = 0.62/1.61 \times (28 - 0.62) = 0.0141$ lb vapor per lb air (7,000 grains = 1 lb avdp.) = 99 grains vapor per lb air.

Air density: from Eq. (5), $\rho_a = (28 - 0.62)/0.754 \times 542 = 0.0670$ lb air per cu ft.

Mixture density: from Eq. (7), $\rho_m = (28 - 0.38 \times 0.62)/0.754 \times 542 = 0.0680$ lb air + vapor per cu ft.

Enthalpy: from Eq. (8), $h_v = 1062 + 0.44 \times 82 = 1098$ Btu per lb vapor; from Eq. (9), $h_m = 0.240 \times 82 + 0.0141 \times 1098 = 35.2$ Btu per lb air.

Sigma-function: from Eq. (10) $\Sigma = 0.240 \times 82 + 0.0141 \times (1098 - 70 + 32) = 34.8$ Btu per lb air.

Psychrometric Charts. For occasional use, algebraic equations are less confusing and more reliable; for frequent use, a psychrometric chart may be preferable. A disadvantage of charts is that each applies for only one value of barometric pressure, usually 760 mm or 30 in. of mercury. *Correction to other barometric readings is not simple.* The equations have the advantage that the actual barometric pressure is taken into account. The equations are often more convenient for equal accuracy or more accurate for equal convenience.

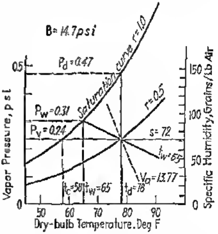


FIG. 1.—Skeleton Humidity Chart.

Psychrometric charts are usually plotted, as indicated by Fig. 1, with dry-bulb temperature as abscissa and specific humidity as ordinate. Since the specific humidity is determined by the vapor pressure and the barometric pressure (which is constant for a given chart), and is nearly proportional to the vapor pressure, a second ordinate scale, departing slightly from uniform

The temperature of combustion is calculated by the same method as for gaseous fuels.

Loss Due to Incomplete Combustion. The loss due to incomplete combustion of the carbon in the fuel, in Btu per lb of fuel, is

$$L = 10{,}180C \times CO/(CO + CO_2)$$

where 10,180 = difference in heat evolved in burning 1 lb of carbon to CO_2 and to CO; CO and CO_2 = percentages by volume of carbon monoxide and carbon dioxide as found by analysis; and C = fraction by weight of carbon in the fuel which is actually burned and passes up the stack, either as CO or CO_2. The presence of 1 percent of CO in the flue gases will represent a decrease in the boiler efficiency of 4.5 percent. An additional loss is caused by passage through the grate to the ashpit of any unburned or partly burned fuel.

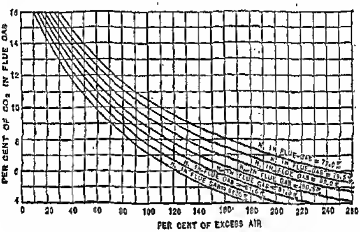


FIG. 39.—Ratio of Air Supplied per Pound of Combustible to That Theoretically Required.

It is generally assumed that high CO_2 readings are indicative of good combustion and, hence, of high efficiencies. Such readings are not satisfactory when considered apart from the CO determination. The best percentage of CO_2 to maintain varies with different fuels and is lower for those with a high hydrogen content than for fuel mainly composed of carbon.

Hydrogen in a fuel increases the nitrogen content of the flue gases. This is due to the fact that the water vapor formed by the combustion of hydrogen will condense at the temperature at which the analysis is made, while the nitrogen which accompanied the oxygen maintains its gaseous form and passes in that form into the sampling apparatus. For this reason, where highly volatile coals containing considerable hydrogen are burned, the flue gas contains an apparently increased amount of nitrogen. The effect is even more pronounced when burning gaseous or liquid hydrocarbon fuels.

The weight of flue gases per pound of fuel, including moisture formed by the hydrogen component, is approximately $= 3.02\,[N/(CO_2 + CO)]C + (1 - A)$, where A = percent of ash as found in test. The weight of dry flue gases per pound of fuel may be approximated from the formula: $W_2 = C[11CO_2 + 8O + 7(CO + N)]/3(CO_2 + CO)$. In these formulas, the weight of gas is per pound of dry or moist fuel as the percentage of C is referred to a dry or moist basis.

The ratio of air supplied per pound of fuel to the air theoretically required is $W_1/W = 3.02C\left(\dfrac{N}{CO_2 + CO}\right)\Big/34.56\left(\dfrac{C}{3} + H - \dfrac{O}{8}\right)$.

graduations, will give the vapor pressure. The saturation curve ($r = 1.0$) gives the specific humidity and vapor pressure for a mixture of air and saturated vapor. Similar curves below it give results for various values of relative humidity. Inclined lines of one set carry fixed values of the wet-bulb temperature, and those of another set carry fixed values of v_a, cubic feet per pound of air, the reciprocal of the air density given by Eq. (5).

Any two values will locate the point representing the state of the atmosphere, and the desired values can be read directly.

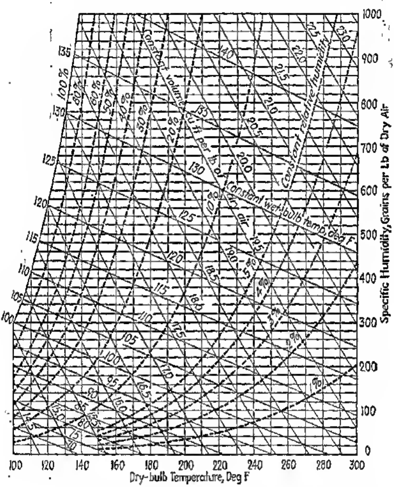


FIG. 3.—Humidity Chart for Medium Temperatures. (From Ellenwood and Mackey, "Vapor Charts," Wiley.)

Figures 2 and 3 are psychrometric charts from Ellenwood and Mackey, ("Vapor Charts," Wiley) covering a dry-bulb temperature range from 32 to 300 F. They are accurate only for a barometric pressure of 29.92 in. Hg.

Air-conditioning processes alter the temperature and specific humidity of the atmosphere. The weight of air remains constant and consequently computations are best based upon 1 lb of air.

Liquid water may enter or leave the apparatus. Its weight W_f lb per lb of air is often merely the difference between the specific humidities of the

MIXTURES OF GASES AND VAPORS

BY

C. H. BERRY

(See Berry, "Humidity Computations," *Combustion*, Aug., Sept., Oct., 1934)

Atmospheric Humidity. The atmosphere is a mixture of air and water vapor. Dalton's law of partial pressures (for the mixture) and the ideal gas law (for each constituent) may safely be assumed to apply. The **total pressure** B (barometric pressure) is the sum of the **vapor pressure** p_v and the **air pressure** p_a.

The temperature of the atmosphere, as indicated by an ordinary thermometer, is the **dry-bulb temperature** t_d. If the atmosphere is cooled under constant total pressure, the partial pressures remain constant until a temperature is reached at which condensation of vapor begins. This temperature is the **dew-point** t_c (condensation temperature), and is the saturation temperature, or boiling point, corresponding to the actual vapor pressure p_v. If a thermometer bulb is covered with absorbent material, *e.g.*, linen, wet with distilled water and exposed to the atmosphere, evaporation will cool the water and the thermometer bulb to the **wet-bulb temperature** t_w. This is the temperature given by a psychrometer (see p. 374). The wet-bulb temperature lies between the dry-bulb temperature and the dew point. These three temperatures are distinct except for a saturated atmosphere, for which they are identical. For each of these temperatures, there is a corresponding vapor pressure. The actual vapor pressure p_v corresponds with the dew point t_c. The vapor pressures p_d and p_w, corresponding with t_d and t_w, do not represent pressures actually appearing in the atmosphere, but are used in computations.

Relative humidity r is the ratio of the actual vapor pressure to the pressure of saturated vapor at the prevailing dry-bulb temperature $r = p_v/p_d$. Within the limits of usual accuracy, this equals the ratio of actual vapor density to the density of saturated vapor at dry-bulb temperature, $r = \rho_v/\rho_d$.

Specific humidity s is the weight of water vapor (pounds or grains) per pound of air.

Molal humidity m is the weight of water vapor in mols per one mol of air. The molecular weight of water is 18, and the equivalent molecular weight of air is 28.95. The ratio $28.95/18 = 1.608$, or 1.61, with ample accuracy. The laws of Dalton and Avogadro state that the molal composition of a mixture is proportional to the distribution of partial pressures, wherefore $m = p_v/p_a = p_v/(B - p_v)$.

Air density ρ_a is the pounds of air in one cubic foot. **Vapor density** ρ_v is the pounds of vapor in one cubic foot. **Mixture density** ρ_m is the sum of these, *i.e.* the pounds of air plus vapor in one cubic foot.

Notation. The subscripts a, v, m, and f apply to air, vapor, mixture, and liquid water respectively. The subscripts d and w apply to conditions pertaining to the dry- and wet-bulb temperature, respectively.

Humidity Measurements. Many methods are in use; three are common. (1) The **dew-point** method measures the temperature at which condensation begins, whence vapor pressure is found from steam tables or means derived from them. Dew-point instruments are used in special cases, principally to calibrate other instruments. (2) The **hair hygrometer** measures relative humidity with accuracy sufficient for some purposes. For

Example. Initial conditions: $B = 29$ in. Hg; $t_d = 75$ F; $t_w = 65$ F; $p_v = 0.52$ in. Hg; $V = 1{,}500$ cu. ft.

Final conditions: $t_d = 45$ F.

Initial computed values: $r = 0.58$; $s = 0.0113$ lb vapor per lb air; $\rho_a = 0.0706$ lb air per cu ft; $W_a = 1500 \times 0.0706 = 106.0$ lb air; $h_m = 29.7$ Btu per lb air; $t_c = 60$ F.

Final computed values: $t_d = 45$ F; $p_v = 0.30$ in. Hg; $r = 1.0$; $s = 0.0065$ lb vapor per lb air; $\rho_a = 0.0754$ lb air per cu ft; $V = 106.0/0.0754 = 1{,}406$ cu ft; $h_m = 17.8$ Btu per lb air.

Liquid formed: $W_f = s_1 - s_2 = 0.0113 - 0.0065 = 0.0048$ lb liquid per lb air; $h_f = 50 - 32 = 18$ Btu per lb liquid (assuming that the liquid is drained out at an average temperature $t_f = 50$ F).

Heat abstracted: $q = h_{m1} - h_{m2} - W_f h_f = 29.7 - 17.8 - 0.0048 \times 18 = 11.8$ Btu per lb air; $Q = q \times W_a = 11.8 \times 106.0 = 1{,}250$ Btu.

Solution of the same problem *by chart* yields:

Initial readings: $r = 0.59$; $s = 76$ grains vapor per lb air $= 0.0108$ lb vapor per lb air; $v_a = 13.71$ cu ft per lb air; $\rho_a = 0.0729$; $W_a = 1500/13.71 = 109.4$ lb air; $\Sigma = 29.5$ Btu per lb air.

Final readings: $r = 1.0$; $s = 44$ grains vapor per lb air; $v_a = 12.84$ cu ft per lb air; $\rho_a = 0.0779$; $V = 109.4 \times 12.84 = 1{,}405$ cu ft; $\Sigma = 17.5$ Btu per lb air.

Liquid formed: $W_f = 76 - 44 = 32$ grains liquid per lb air $= 0.0046$ lb liquid per lb air; $h_f = 18$ (as above).

Heat abstracted: $q = \Sigma_1 - \Sigma_2 + s_1 h_{fw1} - s_2 h_{fw2} - W_f h_f = 29.5 - 17.5 + \frac{76}{7000}(65 - 32) - \frac{44}{7000}(45 - 32) - 0.0046 \times 18 = 12.2$ Btu per lb air; $Q = q \times W_a = 12.2 \times 109.4 = 1335$ Btu.

It is seen that the chart for $B = 29.92$ does not give correct results for a process conducted at $B = 29$ in. Hg.

Dehumidification may be accomplished in a **surface cooler**, in which the air passes over tubes cooled by brine or refrigerant flowing through them. The solution of this type of problem is most easily handled on the chart (see Fig. 6). Locate the point representing the state of the entering atmosphere, and draw a straight line to a point on the saturation curve ($r = 1.0$) at the temperature of the cooling surface. The final state of the issuing atmosphere is represented by a point on this line whose position on the line is determined by the heat abstracted by the cooling medium. This depends upon the extent of surface and the coefficient of heat transfer.

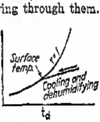


FIG. 6.—Cooling and Dehumidifying.

Adiabatic saturation, humidification, is conducted in a spray chamber through which atmosphere flows. A large excess of water is recirculated through spray nozzles, and evaporation is made up by a suitable water supply. After the process has been operating for some time, the water in the spray chamber will have been cooled to the temperature of adiabatic saturation, which differs from the wet-bulb temperature only because of radiation and velocity errors that affect the wet-bulb thermometer. No heat is added or abstracted; the process is adiabatic. The heat of vaporization for the water that is evaporated is supplied by the cooling of the air passing through the chamber. The wet-bulb temperature of the atmosphere is constant throughout the chamber (Fig. 7). If the chamber is sufficiently large, the issuing atmosphere will be saturated at the wet-bulb temperature of the entering atmosphere. That is, as the atmosphere passes through the chamber, t_w remains constant, t_d is reduced from its initial value to t_w, and t_c is increased from its initial value to t_w. In a chamber of commercial size, the action may

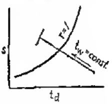


FIG. 7.—Evaporative Cooling. Adiabatic Saturation.

In Eqs. (1) and (2), two significant figures are ordinarily justifiable; in Eqs. (5) and (7), three significant figures; in Eqs. (3), (4), and (6), four decimal places.

Thermal results may be computed by the use of the enthalpy or of the sigma function. The **specific enthalpy** of air (above 0 F) is $h_a = 0.240t_d$ (up to 130 F the specific heat of air is 0.240; at higher temperatures it is larger). The specific enthalpy of low-pressure water vapor (saturated or superheated) is virtually independent of the vapor pressure and is a function of the temperature t_d. Values of h_v can be taken (to the nearest Btu per lb) from Table 1 of Keenan and Keyes, or may be computed to four significant figures by the following equation for the range of temperatures from −40 to 250 F.

$$h_v = 1062 + 0.44t_d, \text{ Btu per lb} = 0.1517 + 0.000063t_d \text{ Btu per grain} \quad (8)$$

The enthalpy of a mixture of air and vapor, in Btu per pound of air, is the sum of the enthalpy of 1 lb of air plus the enthalpy of the specific humidity (pounds or grains of vapor per pound of air)

$$h_m = h_a + s \times h_v = 0.240t_d + s \times h_v \quad (9)$$

The **sigma function** Σ has long been used under the name total heat. It differs from the enthalpy (see Carrier and Mackey, *Trans. A.S.M.E.*, Jan., 1937). The sigma function is the enthalpy of the mixture, Btu per lb air, minus the enthalpy at t_w of a weight of saturated liquid water equal to the specific humidity. The specific enthalpy of saturated liquid may be taken as $(t_w - 32)$ with ample accuracy, so that

Table 2. Sigma Function, Btu per Lb Air, as a Function of t_w, Deg F
(B = 29.921 in Hg at 32 F)

t_w, F	0	1	2	3	4	5	6	7	8	9
					Over Ice					
−40	− 9.5									
−30	− 7.0	− 7.3	− 7.5	− 7.8	− 8.0	− 8.3	− 8.5	− 8.8	− 9.0	− 9.3
−20	− 4.5	− 4.7	− 5.0	− 5.3	− 5.5	− 5.8	− 6.0	− 6.3	− 6.5	− 6.8
−10	− 1.8	− 2.1	− 2.4	− 2.7	− 2.9	− 3.2	− 3.4	− 3.7	− 4.0	− 4.2
− 0	+ 1.0	+ 0.7	+ 0.4	+ 0.1	− 0.3	− 0.5	− 0.7	− 1.0	− 1.3	− 1.6
+ 0	1.0	1.3	1.6	1.8	2.1	2.4	2.7	3.2	3.5	3.8
+10	4.0	4.3	4.6	5.0	5.3	5.7	6.0	6.3	6.7	7.1
+20	7.4	7.8	8.2	8.5	8.9	9.3	9.7	10.2	10.6	11.0
+30	11.4	11.8	12.3							
					Over Water					
+30	......	......	11.7	12.2	12.6	13.0	13.4	13.8	14.3	14.7
40	15.2	15.6	16.1	16.6	17.1	17.5	18.0	18.6	19.1	19.6
50	20.1	20.7	21.2	21.8	22.4	23.0	23.6	24.2	24.8	25.5
60	26.1	26.8	27.4	28.1	28.8	29.6	30.3	31.1	31.8	32.6
70	33.4	34.2	35.1	35.9	36.8	37.7	38.6	39.6	40.5	41.5
80	42.5	43.5	44.6	45.7	46.8	47.9	49.1	50.3	51.5	52.7
90	54.0	55.3	56.6	58.0	59.4	60.8	62.3	63.8	65.4	67.0
100	68.6	70.2	71.9	73.7	75.5	77.3	79.1	81.2	83.2	85.2
110	87.3	89.5	91.7	94.0	96.3	98.7	101.2	103.8	106.4	109.1
120	111.8	114.7	117.6	120.6	123.7	126.8	130.1	133.5	137.0	140.5
130	144.2	148.0	151.9	155.9	160.1	164.4	168.8	173.3	178.0	182.9
140	187.9									

This method is not exact; results may be in error 1 or 2 deg F.

Example. Two atmospheres are to be mixed:

Atmosphere 1:

$t_d = 120$ F; $r = 0.90$; $V = 1$ cu ft; $B = 30$ in. Hg

Atmosphere 2:

$t_d = 0$ F; $r = 0.50$; $V = 1.5$ cu ft

By computation, $s_1 = 0.0726$ and $s_2 = 0.00041$ lb vapor per lb air; $\rho_{a1} = 0.0615$ and $\rho_{a2} = 0.0853$ lb air per cu ft.

Since volume times density equals total weight,

$$W_{a1} = 1 \times 0.0615 = 0.0615 \text{ lb air}; \quad W_{a2} = 1.5 \times 0.0853 = 0.1278 \text{ lb air};$$
$$W_{a3} = 0.0615 + 0.1278 = 0.1893 \text{ lb air};$$

$$s_3 = \frac{0.0615 \times 0.0726 + 0.1278 \times 0.00041}{0.1893} = 0.0238 \text{ lb vapor per lb air}$$
$$= 167 \text{ grains vapor per lb air}$$
$$t_{d3} = \frac{0.0615 \times 120 + 0.1278 \times 0}{0.1893} = 39 \text{ F}$$

By means of a draftsman's triangle, locate the point on the chart for which $t_d = 39$ and $s = 167$ grains of vapor per pound of air. This point falls far above the saturation curve. With a second triangle, a line is located passing through this point and running parallel to the lines of constant wet-bulb temperature; this line intersects the saturation curve at $t_d = 71$, which is the true final temperature of the mixture, and the specific humidity is 114 grains of vapor per pound of air. The difference in specific humidities is the weight of condensate, $167 - 114 = 53$ grains of condensate per pound of air.

If the mixing proportions are not given, but the final temperature is prescribed, the line is drawn between the two initial points, and its intersection with the desired final temperature is noted. A reversal of the foregoing method of computing will yield the mixing proportions.

The **cooling tower** is a chamber in which outdoor atmosphere flows through a spray of entering hot water, which is to be cooled. The temperature of the water is reduced in part by the warming of the air, and in greater part by the evaporation of a portion of the water. The atmosphere enters at given conditions and emerges at a higher temperature and usually saturated ($r = 1$). It is commonly possible to cool the water below the temperature of the entering air, often to about halfway between t_d and t_w. The volume of atmosphere per pound of entering water and the weight of water evaporated are to be computed.

Example. A cooling tower is to receive water at 120 F and atmosphere at $t_d = 90$, $t_w = 80$, whence $p_v = 0.92$, $s = 0.0196$ lb vapor per lb air, $\rho_a = 0.0702$ lb air per cu ft, and $h_m = 43.2$. The water is to be cooled to 85 F. What volume of atmosphere must be passed through the tower, and what weight of water will be lost by evaporation?

The issuing atmosphere will be assumed to be saturated at 115 F. Then $t_d = 115$ F, $p_v = 3.0$ in. Hg, $s = 0.0690$ lb vapor per lb air, $\rho_a = 0.0623$ lb air per cu ft, and $h_m = 104.4$ Btu per lb air.

The two unknowns are the weight of air to be passed through the tower and the weight of water to be evaporated. The two equations are the water-weight balance and the enthalpy balance (the steady-flow equation for zero heat transfer to or from outside). Assume that 1 lb water enters, of which x lb are evaporated. The water-weight balance $1 + W_a s_1 = 1 - x + W_a s_2$, becomes, $x = W_a(s_2 - s_1) = W_a(0.0690 - 0.0196) = 0.0494W_a$. The enthalpy balance $1 \times (120 - 32) + W_a h_{m1} = (1 - x)(85 - 32) + W_a h_{m2}$ becomes, $88 + 43.2W_a = 53(1 - x) + 104.4W_a$; whence, $53x = 53 - 88 + W_a(104.4 - 43.2) = -35 + 61.2W_a$.

Solving these simultaneous equations, $x = 0.0295$ lb water evaporated per pound of water entering and $W_a = 0.597$ lb air per pound water entering.

In an **evaporative condenser**, vapor is condensed within tubes that are cooled by the evaporation of water flowing over the outside of the tubes;

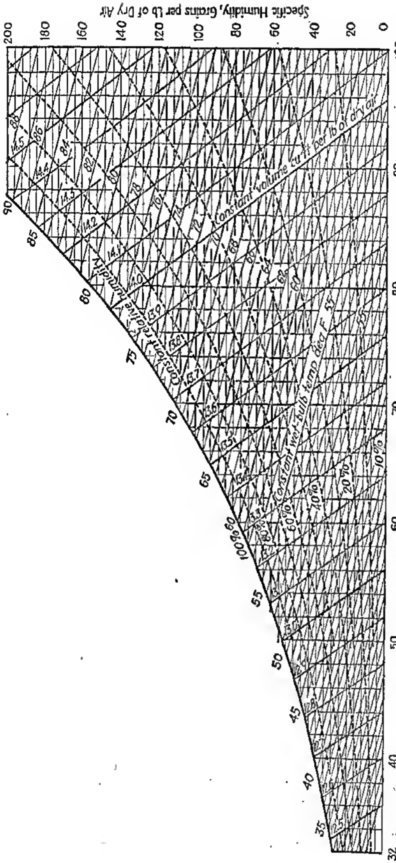


FIG. 2.—Humidity Chart for Low Temperatures. (From Ellenwood and Mackey, "Vapor Charts," Wiley.)

TRANSMISSION OF HEAT BY CONDUCTION AND CONVECTION

BY

W. H. McADAMS

REFERENCES: Royds, "Heat Transmission by Radiation, Conduction, and Convection," Constable. McAdams, "Heat Transmission," McGraw-Hill. Schack, "Industrial Heat Transfer," Wiley. Walker, Lewis, McAdams and Gilliland, "Principles of Chemical Engineering," McGraw-Hill. Perry, "Chemical Engineers' Handbook," McGraw-Hill.

Nomenclature and Units

The units are based on feet, pounds, hours, degrees Fahrenheit, and Btu. Any other consistent set may be used in the dimensionless relations given, but for the dimensional equations the units of this table must be used.

A = area of heat-transfer surface, sq ft.
A_i = inside area.
A_o = outside area.
A_m = average value of A, sq ft.
a = an empirical dimensionless constant.
a' = cost of work delivered to fluid, dollars per foot-pound.
B = ratio of equivalent frictional length to that of the straight tubes, dimensionless.
b = fixed charges on insulation, dollars per year per sq ft per in. thickness.
b' = fixed charges on exchanger, dollars per hr per sq ft of inside surface.
C_p = specific heats at constant pressure, Btu per lb per deg F.
D = diameter, ft.
D_o = outside diam, ft.
D_i = inside diam, ft.
D' = diameter, in.
D'_o = outside diam, in.
D'_i = inside diam, in.
e = base of natural logarithms, 2.718.
G = mass velocity, equals w/S, lb per hr per sq ft of cross section occupied by fluid.
G_{max} = mass velocity through minimum free area in a row of pipes normal to the fluid stream, lb per hr per sq ft.
g_c = conversion factor, equal to 4.18×10^8 (mass lb)(ft)/(force lb)(hr)^2.
g_L = acceleration due to gravity, 4.18×10^8 ft per hr per hr.
h = local individual coefficient of heat transfer, equals $dq/dA\Delta$, Btu per hr per sq ft per deg F diff.
$h_c + h_r$ = combined coefficient by conduction, convection, and radiation between surface and surroundings.
h_m = mean value of h for entire surface, based on Δ_m.
$h_{a.m.}$ = average h, arbitrarily based on arithmetic-mean temperature difference.
h_s = heat transfer coefficient through scale deposits.
k = thermal conductivity, Btu per hr per sq ft per unit temperature gradient, deg F per ft.

entering and leaving atmospheres. Its specific enthalpy at the observed or assumed temperature of supply or removal t_f is

$$h_f = t_f - 32 \text{ Btu per lb of liquid} \tag{11}$$

Since all air-conditioning apparatus involves steady-flow processes, thermal results are computed by the steady-flow equation, written for 1 lb of air. No work term appears, for the compression work done by a fan is usually negligible. Using subscript 1 for entering atmosphere and liquid water, and for heat supplied; and 2 for departing atmosphere and water, and for heat abstracted; the equation becomes

$$h_{m1} + W_{f1}h_{f1} + q_1 = h_{m2} + W_{f2}h_{f2} + q_2 \text{ Btu per lb air} \tag{12}$$

Usually either or both values of W_f and q will be zero.

In terms of the sigma function, the steady-flow equation becomes

$$\Sigma_1 + s_1(t_{w1} - 32) + W_{f1}h_{f1} + q_1 = \Sigma_2 + s_2(t_{w2} - 32) + W_{f2}h_{f2} + q_2 \text{ Btu per lb air} \tag{13}$$

Unit processes involved in air conditioning include heating and cooling an atmosphere above its dew point, cooling below the dew point, adiabatic saturation, and mixing of two atmospheres. These, in various sequences, make it possible to start with any given atmosphere and produce an atmosphere of any required characteristics.

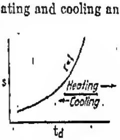


FIG. 4.—Heating or Cooling above the Dew Point.

Heating and cooling above the dew point entails no condensation of vapor. Barometric pressure and composition being unaltered, partial pressures remain constant. The process is represented in Fig. 4.

Example. Initial conditions: $B = 28$ in. Hg; $t_d = 60$ F; $t_w = 50$ F; $p_v = 0.26$ in. Hg; $V = 1{,}200$ cu ft.

Final conditions: $t_d = 82$ F.

Initial computed values: $r = 0.50$; $s = 0.0058$ lb vapor per lb air; $\rho_a = 0.0707$ lb air per cu ft; $W_a = V \times \rho_a = 1200 \times 0.0707 = 84.9$ lb air; $h_m = 20.7$ Btu per lb air.

Final computed values: p_v, s, and W_a are unaltered; $r = 0.24$; $\rho_a = 0.0879$ lb air per cu ft; $V = W_a/\rho_a = 84.9/0.0670 = 1250$ cu ft; $h_m = 20.1$ Btu per lb air.

Heat added: $q = h_{m2} - h_{m1} = 26.1 - 20.7 = 5.4$ Btu per lb air; $Q = q \times W_a = 5.4 \times 84.9 = 458$ Btu.

The *solution* of this same problem *from chart readings* will indicate the discrepancy to be expected from the departure of the barometric pressure from that for which the chart is constructed (29.92 in. Hg).

Initial readings: $r = 0.49$; $s = 38$ grains vapor per lb air $= 0.0054$ lb vapor per lb air; $v_a = 13.21$ cu ft per lb air; $\rho_a = 0.0757$ lb air per cu ft; $W_a = V/v_a = 1200/13.21 = 90.8$ lb air; $\Sigma = 20$ Btu per lb air.

Final readings: $r = 0.24$; $v_a = 13.77$; $\rho_a = 0.0726$; $V = 90.8 \times 13.77 = 1{,}250$ cu ft; $t_w = 59$ F; $\Sigma = 25.4$ Btu per lb air.

Heat added: $q = \Sigma_2 - \Sigma_1 + s \times (t_{w2} - t_{w1}) = 25.4 - 20.0 + 38(59 - 50)/7000 = 5.4$ Btu per lb air; $Q = q \times W_a = 5.4 \times 90.8 = 490$ Btu.

It is evident that the chart gives substantially correct values for the final relative humidity and total volume and for the heat added per pound of air. The weight of air, however, is in error by about 7 percent, *i.e.*, almost in proportion to the barometric pressure, and in consequence the aggregate heat added is in error to this same extent.

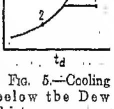


FIG. 5.—Cooling below the Dew Point.

Cooling below the dew point or dehumidification, entails condensation of vapor; the final atmosphere will be saturated, liquid will appear; see Fig. 5.

μ_w = viscosity at wall temperature, lb per hr per ft.
ρ = density, lb per cu ft.
ρ_∞ = density of stream of great depth, lb per cu ft.

Preliminary Statements. There are three methods by which heat may be propagated or conveyed from one place to another.

1. By **conduction.** Heat passing from one part of a body to another part of the same body, or from one body to another in physical contact with it, without displacement of the particles of the body, is said to flow by conduction.

2. By **convection.** Convection is the transfer of heat from one place to another within a fluid (gas or liquid) by the mixing of one portion of the fluid with another.

3. By **radiation.** All bodies give off heat in the form of radiant energy, which is propagated in all directions as a wave motion in the ether. Radiation falling upon a body is absorbed by it either wholly or in part. If two bodies, one hotter than the other, are placed within an enclosure, there is a continual interchange of energy between them. The hotter body radiates more energy than it absorbs, the colder body absorbs more than it radiates. Even after equilibrium of temperature is established the process continues, each body radiating and absorbing energy.

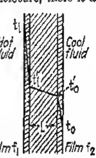


FIG. 1.—Temperature Gradients in Heat Flow through a Wall.

Phenomena of Heat Transmission. In the cases of heat transmission that usually occur in practice—in boilers, condensers, the cooling of engine cylinders, etc. —heat is transmitted from one fluid to another through a wall separating the two. The character of the process is shown in Fig. 1. At the surface of the plate, there is a film of the hot fluid of indefinite thickness. This film offers a considerable resistance to the transmission, as shown by the temperature difference $t_i - t'_i$ through it. A corresponding film on the other side of the plate offers resistance measured by the temperature drop $t'_o - t_o$. Let dq denote the rate of heat transmission in Btu per hr through the surface area dA_i sq ft. Then, for film f_1, $dq = h_i\, dA_i(t_i - t'_i)$; for the plate, $dq = (k/L)dA'(t'_i - t'_o)$; for the film f_2, $dq = h_o\, dA_o(t'_o - t_o)$.

The terms h_i and h_o are the **film coefficients** of the films f_1 and f_2, respectively, and k is the thermal conductivity of the plate; $h_o\, dA_o$ and $h_i\, dA_i$ are the **thermal conductance** of the films and $k\, dA'/L$ is the conductance of the plate. Eliminating the surface temperatures t_i' and t_o' from the equations, the equation is obtained for steady flow through several resistances in series:

$$dq = \frac{t_i - t_o}{(1/h_i\, dA_i) + (L/k\, dA') + (1/h_o\, dA_o)} \cdots \tag{1}$$

The terms $1/h_i\, dA_i$ and $1/h_o\, dA_o$ are the **thermal resistances** of the films, and $L/k\, dA'$ is the resistance of the plate. In the expression $dq = U dA(t_i - t_o)$, the total thermal resistance $1/U dA$ to the flow is the sum of the separate resistances.

$$1/U dA = (1/h_i\, dA_i) + (L/k\, dA') + (1/h_o\, dA_o) + (1/h_s\, dA) \cdots \tag{2}$$

$U dA$ is the over-all conductance of the two films, a scale deposit, and the plate, and U is the corresponding over-all coefficient.

terminate somewhat short of this, the precise end point being determined by the duration and effectiveness of contact between air and spray water. In any case, the weight of water evaporated equals the increase in the specific humidity of the atmosphere.

Example. Initial conditions: $B = 30$ in. Hg; $t_d = 78$ F; $t_w = 55$ F; $r = 0.20$; $s = 28$ grains vapor per lb air.
Final condition: $t_d = t_w = 55$ F; $r = 1.0$; $s = 64$ grains vapor per lb air.
Water evaporated: $s_2 - s_1 = 64 - 28 = 36$ grains water per lb air.

The design of the spray chamber to produce this result is necessarily based upon experience with like apparatus previously built.

In practice, the spray chamber is preceded and followed by heating coils, the first to warm the entering atmosphere to the desired value of t_w, determined by the prescribed final specific humidity, the second to warm the issuing atmosphere to the desired temperature, and simultaneously to reduce its relative humidity to the desired value.

The spray chamber that is used for adiabatic saturation, humidification, in winter may be used for dehumidification in summer by supplying the spray nozzles with refrigerated water instead of recirculated water. In this case, the issuing atmosphere will be saturated at the temperature of the spray water, which will be held at the desired dew point. Subsequent heating of the atmosphere to an acceptable temperature will simultaneously reduce the relative humidity to the desired value.

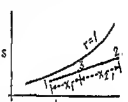


FIG. 8.—Mixing Two Atmospheres. (Final Point below the Saturation Curve.)

Mixing Two Atmospheres. In recirculating ventilation systems, two atmospheres (1 and 2) are mixed to form a third (3). The state of the final atmosphere is readily found graphically on the psychrometric chart (see Fig. 8). Locate the points 1 and 2 representing the states of the initial atmospheres. Connect these points by a straight line. Locate a point that divides this line into segments inversely proportional to the weights of air in the respective atmospheres. The division point represents the state of the final mixture, so long as it falls below the saturation curve ($r = 1$). If the final point falls above the saturation curve, as in Fig. 9, condensation will ensue, and the true final point 4 is found by drawing a line from the apparent point 3, parallel to the lines of constant wet-bulb temperature, to its intersection with the saturation curve. From all the points involved, readings of specific humidity may be taken, including point 3 when it falls above the saturation curve, and in this case the difference between s_3 and s_4 will be the weight of condensate, pounds per pound air.

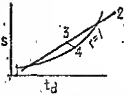


FIG. 9.—Mixing Two Atmospheres. (Final Point above the Saturation Curve.)

If the chart is sectional and the two points do not fall in the same section, or in any case in which it is preferred, the same method may be carried out arithmetically. The state of the final atmosphere is characterized by a specific humidity and a dry-bulb temperature which are the weighted averages of the like quantities for the original atmospheres, the weighting factors being the respective weights of air.

$$W_{a3} = W_{a1} + W_{a2} \tag{14}$$
$$W_{a3}s_3 = W_{a1}s_1 + W_{a2}s_2 . \tag{15}$$
$$W_{a3}t_{d3} = W_{a1}t_{d1} + W_{a2}t_{d2} \tag{16}$$

CONDUCTION

Thermal Conductivity. The basic Fourier conduction law is $dq = -k\,dA(dt/dL)$, which states that the steady rate dq of heat conduction is proportional to the cross-sectional area dA normal to the direction of flow, and to the temperature gradient $-dt/dL$ along the conduction path. The number k in this expression is called the "true" thermal conductivity of the material of the plate. The thermal conductivity of a substance may be

Table 3. Thermal Conductivities of Miscellaneous Solid Substances
(The values of k in the table below are to be regarded as rough average values for the temperature range indicated)

Material	Apparent density, lb per cu ft	Temp, deg F	k
Air spaces (¾ in.) faced with aluminum foil	……	100	0.025
Asbestos	36.0	500	0.122
Asbestos mill board	60.5	86	0.07
Ashes, wood	……	32–212	0.041
Brick masonry	……	70	0.33–0.42
Carborundum brick	129	1112	10.7
Cardboard, corrugated	……	……	0.1–0.2
Celluloid	87.3	86	0.12
Cement	……	……	0.17
Chalk	……	……	0.48
Charcoal flakes	11.9	176	0.043
Charcoal flakes	15.0	172	0.051
Clinker, granular	……	32–1300	0.27
Coke, petroleum	……	212	3.4
Coke, powdered	……	32–212	0.106
Concrete, cinder	……	……	0.167–0.42
Concrete, stone	……	……	0.5–0.75
Cotton wool	5.06	32	0.024
Ebonite	……	……	0.10
Felt, wool	……	……	0.03
Fiber, red	80.5	68	0.27
(With binder, baked)	……	70–208	0.097
Enamel, silicate	……	100	0.5–0.75
Firebrick	……	2000	0.7–1.0
Gas carbon	……	32–212	2.4
Glass	……	……	0.333–0.5
Granite	……	……	1.0–2.32
Graphite, powdered (through 100 mesh)	30.0	104	0.104
Graphite, solid	93.5	122	87
Gypsum, moulded and dry	78.0	68	0.25
Hair felt	17	86	0.021
Ice	57.5	……	1.26
Infusorial earth	20	32–1200	0.06
Kapok	0.88	86	0.020
Lampblack	10.0	104	0.038
Lava	……	……	0.49
Leather, sole	62.4	……	0.092
Limestone	……	……	0.3–0.75
Linen	……	……	0.05
Magnesia, light carbonate	19	70	0.04
Magnesite brick	158	400	2.2
Mica	……	……	0.34
Marble	……	……	1.2–1.7
Mill shavings	……	……	0.033–0.05
Mineral wool	9.4	86	0.023
Paper	……	……	0.075
Paraffin wax	55.6	86	0.145
Plaster	……	……	0.25–0.5
Porcelain	……	……	0.6
Pumice stone	……	70–150	0.135
Pyrex	139	127	0.63
Quartz, parallel to axis	……	……	7.25
Quartz, perpendicular to axis	……	……	3.90
Rubber, hard	74.3	100	0.092
Rubber, soft, vulcanized	68.6	86	0.08
Sand, dry	94.8	68	0.188
Sandstone	140	104	1.1
Sawdust, dry	13.4	68	0.042
Silica brick	……	2000	0.85–1.0
Silk	6.3	……	0.026
Slag, blast-furnace	……	75–260	0.064
Slate	……	201	0.86
Woods: (across grain except as otherwise noted)			
Balsa	8.8	86	0.03
Balsa	20.0	86	0.048
Cypress	28.7	86	0.056
Fir, white	28.1	140	0.062
Fir (along grain)	34.4	77	0.215
Maple	44.3	86	0.11
Oak	51.5	……	0.12
White pine	34	59	0.087
Yellow pine	……	……	0.085
Wool, pure	6.9	86	0.021

the water evaporates into the atmosphere. The computation of results is similar to that for the cooling tower.

Combustion Computations. The total weight of water vapor in the flue-gas mixture is the sum of that arriving with the air and that resulting from the combustion of hydrogen and hydrocarbons. The weight of dry flue gases is computed from the chemical equations. From Eq. (4), the vapor pressure in the flue gas can be computed, and from this the dew point of the flue gas can be estimated on the assumption that the dry gases do not influence the vapor pressure. If the fuel contains an appreciable amount of sulphur, the flue gas will carry enough sulphur oxides to change the result markedly. The vapor pressure of sulphurous acid (SO_2 dissolved in water) is less than the vapor pressure of pure water; that of sulphuric acid (SO_3 dissolved in water) is much more so. Accordingly, the dew point will be higher. The dew point of flue gas, neglecting sulphur oxides, often falls not far from 120 F. Actual dew points as high as 250 F have been observed in flue gas arising from high-sulphur coal.

Table 5. Thermal Conductivities of Insulating Materials at High Temperatures

Material	For temperatures, deg F up to	Mean temperature, deg F									
		100	200	300	400	500	600	800	1000	1500	2000
Laminated asbestos felt (approx 40 laminations per in.)	700	0.033	0.037	0.040	0.044	0.048					
Laminated asbestos felt (approx 20 laminations per in.)	500	0.045	0.050	0.055	0.060	0.065					
Corrugated asbestos (4 plies per in.)	300	0.050	0.058	0.069							
85 percent magnesia	600	0.039	0.041	0.043	0.046						
Diatomaceous silica, asbestos and bonding material	1600	0.045	0.047	0.049	0.050	0.053	0.055	0.060	0.065		
Diatomaceous silica brick	1600	0.054	0.056	0.058	0.060	0.063	0.065	0.069	0.073		
Diatomaceous silica brick	2000	0.127	0.130	0.133	0.137	0.140	0.143	0.150	0.158	0.176	
Diatomaceous silica brick	2500	0.128	0.131	0.135	0.139	0.143	0.148	0.155	0.163	0.183	0.203
Diatomaceous silica powder (density, 18 lb per cu ft)	....	0.039	0.042	0.044	0.048	0.051	0.054	0.061	0.068		
Rock wool	....	0.030	0.034	0.039	0.044	0.050	0.057				

Asbestos cement, 0.1; 85 percent magnesia cement, 0.05; asbestos and rock-wool cement, 0.075 approx.

The conductivity of air as a function of temperature is given by the relation,

$$k_T = 0.0129[717/(T + 225)](T/492)^{3/2}$$

where T is temperature in deg F absolute. This relation is valid from −312 to 212 F. Values of the conductivities of air and steam at a series of temperatures are given below, those at 300 F being extrapolated.

For t (deg F)	32	50	100	200	300
k (for air)	0.0129	0.0132	0.0143	0.0162	0.0180
k (for steam)	0.0086	0.0090	0.0101	0.0122	0.0144

Table 6. Thermal Conductivities of Insulating Materials at Moderate Temperatures (Nusselt)

Material	Weight, lb per cu ft	Temperatures, deg F						
		32	100	200	300	400	600	800
Asbestos	36.0	0.087	0.097	0.110	0.117	0.121	0.125	0.130
Burnt infusorial earth for pipe coverings	12.5	0.043	0.046	0.052	0.057	0.062	0.073	0.085
Insulating composition (loose)	25.0	0.040	0.046	0.050	0.053	0.055		
Cotton	5.0	0.032	0.035	0.039				
Silk hair	9.1	0.026	0.030	0.034				
Silk	6.3	0.025	0.028	0.034				
Wool	0.5	0.022	0.027	0.033				
Pulverized cork	10.0	0.021	0.026	0.032				
Infusorial earth (loose)	22.0	0.035	0.039	0.045	0.047	0.050	0.053	

$k_m = \frac{1}{t_1 - t_2}\int_1^2 k\, dt.$

k_f = k at the "film" temperature, $t_f = (t + t_w)/2$.
L = thickness of conductor, ft.
N = length of heat-transfer surface, heated length, ft.
n = number of rows in a vertical plane.
Q = quantity of heat, Btu.
q = rate of heat flow, Btu per hr.
R = thermal resistances, $1/UA$, $1/hA$, $1/(h_c + h_r)A_o$.
r = radius, ft.
S = cross section, filled by fluid, in plane normal to direction of fluid flow, sq ft.
T = temperature, deg $R = t + 460$.
T_1, T_2 = inlet and outlet bulk temperatures, respectively, of warmer fluid, deg F.
t = bulk temperature (based on heat balance), deg F.
t_w = wall temperature, deg F.
t_1, t_2 = inlet and outlet bulk temperatures of colder fluid, deg F.
t_∞ = temperature of stream of great depth, ambient temperature, F.
t_i, t_o = temperatures of fluid inside and outside, deg F.
t_f = $(t + t_w)/2$.
U = over-all coefficient of heat transfer, Btu per hr per sq ft per deg F; U_i, U_o based on inside and outside surfaces, respectively.
V_s = average velocity, volumetric rate divided by cross section filled by fluid, fps.
V_{sm} = maximum velocity, through minimum cross section, fps.
V_∞ = velocity of stream of great depth, ft per hr.
w = mass rate of flow per tube, lb per hr per tube.
Z = $(T_1 - T_2)/(t_2 - t_1)$.
α = a dimensional constant.
β = volumetric coefficient of thermal expansion, having units of reciprocal of Fahrenheit absolute temperature.
Γ = mass rate of flow, lb/(hr)(ft of wetted periphery measured on a plane normal to direction of fluid flow); $= w/\pi D$ for a vertical and $w/2N$ for a horizontal tube.
Δ = temp diff, deg F.
$\Delta_{a.m.}, \Delta_{l.m.}$ = arithmetic and logarithmic means of terminal temperature differences, respectively, deg F.
Δ_m = true mean value of the terminal temperature differences, deg F.
Δ_o = over-all temp diff, deg F.
Δ_s = temperature difference between surface and surroundings, deg F.
θ = time, hr.
λ = latent heat (enthalpy) of vaporization, Btu per lb.
μ = viscosity at bulk temperature, lb per hr per ft; equals 2.42 times centipoises; equals 106,000 times viscosity in (lb force)(sec)/sq ft.
μ_f = viscosity, lb per hr per ft, at arithmetic mean of wall and fluid temperatures.

$$\frac{1}{UA} = \frac{1}{h_iA_i} + \frac{L}{kA_m} + \frac{1}{h_sA_s} + \frac{1}{h_oA_o} \tag{5a}$$

and the values of h are obtained from the following pages. For the more complex cases of multipass and cross flow, Δ_m is given by Fig. 2. The shell-side fluid is assumed to be well-mixed by suitable baffles.

Example. Assume an exchanger in which the hot fluid enters at 400 F and leaves at 200 F; the cold fluid enters at 100 F and leaves at 200 F. Assuming U independent of temperature, what will be the true mean temperature difference from hot to cold fluid, (1) for counterflow and (2) for a reversed current apparatus with one well-baffled pass in the shell and two passes in the tubes?

1. With counterflow, the terminal differences are $400 - 200 = 200$ F and $200 - 100 = 100$ F; the logarithmic mean difference is $100/(2.3)(0.301) = 144$ F.

2. $Z = (400 - 200)/(200 - 100) = 2$, $X = (200 - 100)/(400 - 100) = \frac{1}{3}$; from section A of Fig. 2, $Y = 0.80 = \Delta_m/144$; $\Delta_m = 115$ deg.

If one of the temperatures remains constant, as in a condenser or in an evaporative cooler, Eq. (5) applies for parallel flow, counterflow, reversed current, and cross flow.

If U varies considerably with temperature, the apparatus should be considered to be divided into stages, in each of which variation of U with temperature or temperature difference is linear. Then for parallel or counterflow operation, the following relation may be applied to each stage:

$$q = A(\Delta_{o1}U_2 - \Delta_{o2}U_1)/\log_e(\Delta_{o1}U_2/\Delta_{o2}U_1) \tag{5b}$$

FILM COEFFICIENTS

The important physical properties, which affect film coefficients (see p. 386), are thermal conductivity, viscosity, density, and specific heat. Obviously, these factors depend upon the temperature level. Factors within the control of the designer include velocity, shape, and arrangement of the heating surface. With forced flow of gases or water, under the conditions usually met in practice, the flow is turbulent in character (see p. 264), and under these conditions the film coefficient can be greatly increased by increasing the velocity of the fluid at the expense of a greater power requirement. For a given velocity and fluid, the film coefficient depends upon the direction of flow of fluid relative to the heating surface. With free or natural convection, for a given fluid and arrangement of surface, the film coefficient depends upon physical properties such as viscosity, density, thermal conductivity, and coefficient of expansion and upon temperature difference, but the convection currents ordinarily are not expressed in terms of definite fluid velocities. With forced convection at low rates of flow, particularly with viscous fluids such as oils, laminar motion may prevail and the film coefficient is almost independent of fluid velocity, being determined primarily by free convection factors. In any event, the film coefficients h are correlated in terms of dimensionless groups of the controlling factors.

The following simplified dimensional equations for film coefficients are recommended as approximations, and require the use of the units given on p. 384:

Turbulent Flow of Gases inside Clean Tubes.

$$h_m = 0.024C_pG^{0.8}/(D'_i)^{0.2} \tag{6a}$$

Table 1. Thermal Conductivities of Metals

Substance	Temp, deg F	k	Substance	Temp, deg F	k
Metals:					
Aluminum	64	117	Mercury	32	4.8
Aluminum	212	119	Nickel	64	36
Antimony	32	10.6	Nickel	212	34
Antimony	212	9.7	Platinum	64	40.2
Bismuth	64	4.7	Platinum	212	41.9
Bismuth	212	3.9	Silver	64	242
Cadmium	64	53.7	Silver	212	238
Cadmium	212	52.2	Tantalum	64	32
Copper	64	224	Tin	64	36
Copper	212	218	Tin	212	34
Gold	64	169	Zinc	64	65
Gold	212	170	Zinc	212	64
Iron, pure	64	39	Alloys		
Iron, pure	212	36.6	Brass, yellow	32	49.4
Iron, wrought	64	34.9	Brass, yellow	212	61.5
Iron, wrought	212	34.6	Brass, red	32	59.5
Iron, cast	129	27.6	Brass, red	212	68.3
Iron, cast	216	26.8	Constantan (60 Cu, 40 Ni)	64	13.1
Steel (1 percent C)	64	26.2	Constantan (60 Cu, 40 Ni)	212	15.5
Steel (1 percent C)	212	25.9	Nickel silver	32	16.9
Steel (18 Cr, 0.2 Ni)	626	19.3	Nickel silver	212	21.5
Steel (18 Cr, 8 Ni)	626	14.8	Manganin (84 Cu, 4 Ni, 12 Mn)	64	12.8
Steel (23 Cr, 12 Ni)	626	13.8			
Lead	64	20.1		212	15.2
Lead	212	19.8	Platinoid	64	14.5
Magnesium	32-212	92.0			

Table 2. Thermal Conductivities of Liquids and Gases

Substance	Temp, deg F	k	Substance	Temp, deg F	k
Liquids			Gases		
Acetone	68	0.103	Air (see also p. 395)	32	0.0129
Ammonia	45	0.29	Ammonia, vapor	32	0.0115
Aniline	32	0.104	Ammonia	212	0.0175
Benzol	86	0.089	Argon	32	0.00912
Carbon bisulphide	68	0.0931	Carbon dioxide	32	0.00791
Ethyl alcohol	68	0.105	Carbon dioxide	212	0.0122
Ether	68	0.0798	Carbon monoxide	32	0.0124
Glycerin, U.S.P., 95 %	68	0.165	Chlorine	32	0.00414
Kerosene	68	0.086	Ethane	32	0.0104
Methyl alcohol	68	0.121	Ethylene	32	0.00947
n-Pentane	68	0.0787	Helium	32	0.0802
Petroleum ether	68	0.0758	Hexane, n.	32	0.0060
Toluene	86	0.088	Hydrogen	32	0.0917
Water	32	0.337	Hydrogen	212	0.115
Water	140	0.385	Methane	32	0.0170
Oil, castor	39	0.104	Neon	32	0.00256
Oil, olive	39	0.101	Nitrogen	32	0.0131
Oil, turpentine	54	0.0734	Nitrous oxide	32	0.0080
Vaseline	59	0.106	Nitrous oxide	212	0.0090
			Nitric oxide	32	0.0120
			Oxygen	32	0.0134
			Pentane, n.	32	0.0067
			Sulphur dioxide	32	0.00443

Natural Convection. For heat loss from surfaces by conduction and convection to air at atmospheric pressure and ordinary temperatures, the coefficients are given by the following equations (using the units of p. 384).

Horizontal pipes, long vertical pipes:

$$h_m = 0.42(\Delta_s/D')^{0.25} \qquad (11a)$$

Vertical plates less than 2 ft high:

$$h_m = 0.28(\Delta_s/N)^{0.25} \qquad (11b)$$

Vertical plates more than 3 ft high:

$$h_m = 0.3\Delta_s^{0.25} \qquad (11c)$$

Horizontal plates, facing upward:

$$h_m = 0.38\Delta_s^{0.25} \qquad (11d)$$

Horizontal plates, facing downward:

$$h_m = 0.2\Delta_s^{0.25} \qquad (11e)$$

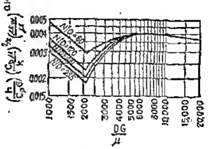


FIG. 3.—Heating and Cooling of Viscous Oils Flowing inside Tubes. (The Curves for DG/μ below 2100 Are Based on Eq. 10.)

For heat transfer by natural convection between liquids and submerged heating coils, data are scarce. As a conservative approximation, the following dimensionless equation, based on heat transfer from single horizontal pipes to liquids or gases, may be used:

$$h_m D/k_f = 0.47[(D^3\rho^2{}_f g_L/\mu^2{}_f)(\beta\Delta_s)(C_p\mu_f/k_f)]^{0.25} \qquad (12)$$

For a given film temperature, this reduces to $h_m = a(\Delta_s/D')^{0.25}$. For $t_f = 150$ F, a is 25 for water and 16 for aniline or 98 percent sulphuric acid.

Economic Velocity. In many heat exchangers, the over-all coefficient follows the relation $U = \alpha G^n$ and the friction factor f (see p. 359) follows the rule $f = a(\mu/DG)^m$. As shown by Drew, Hottel, and McAdams (*Trans. Am. Inst. Chem. Eng.*, 32, 1936, pp. 289–291) the total costs (which vary with G) are a minimum when the optimum velocity is employed:

$$G_{opt} = [2n/(3 - m - n)][(g_c\rho^2 b'/a'fB)]^{1/3} \qquad (12a)$$

The corresponding optimum ratio $(x_p/x_{f.c.})_{opt}$ of power cost to fixed charges on the exchanger is given in terms of the exponents by

$$(x_p/x_{f.c.})_{opt} = n/(3 - m - n) \qquad (12b)$$

The corresponding optimum total costs, $x_p + x_{f.c.}$ except for the cost of the heat itself, is given by

$$(x_p + x_{f.c.})_{opt} = \frac{(3 - m)b'}{(3 - m - n)U_{opt}\Delta_e} \qquad (12c)$$

For the **optimum temperature difference** for recovering waste heat see Lewis, Ward, and Voss (*Ind. Eng. Chem.*, 16, 1924, pp. 467–468).

Condensing Vapors. When condensing a single pure vapor, saturated or superheated, the condensate wets the tube and film-type condensation is obtained. The rate of heat transfer q equals $h_m A\Delta_m$ where Δ_m is the mean difference between the saturation temperature and the temperature of the surface of the tube. So long as the condensate flows in streamline motion ($4\Gamma/\mu < 2100$) the following dimensionless equations may be used:

For **horizontal tubes**;

$$h_m D/k = 0.73(D^3\rho^2\lambda g_L/k\mu_f n\Delta)^{0.25} = 0.76(D^3\rho^2 g_L/\mu_f\Gamma)^{1/3} \qquad (13)$$

defined *as the quantity of heat* (Btu) *that flows in a unit of time* (1 hr) through unit area of plate (1 sq ft) of unit thickness (1 ft) having unit difference of temperature (1 deg F) between its faces. The mean thermal conductivity k_m is given the relation, for steady flow of q Btu per hr through a wall having mean surface area A_m sq ft and a thickness of L ft:

$$q = k_m A_m (t'_i - t'_o)/L, \quad \text{or} \quad k_m = qL/A_m(t'_i - t'_o) \tag{3}$$

It will be observed that the units of the conductivity k contain the element of thickness, while the coefficient h does not. The thermal conductivity of a given material varies with temperature and

$$k_m(t'_i - t'_o) = -\int_{t'_i}^{t'_o} k\, dt$$

Over moderate ranges, k varies linearly with t', and hence k_m may be evaluated at the arithmetic mean of t'_i and t'_o. The numerical value of the mean area

Table 4. Thermal Conductivities of Material for Refrigeration and Building Insulation

Material	Apparent density, lb per cu ft	Mean temp	k	Material	Apparent density, lb per cu ft	Mean temp	k
Balsa wool	2.2	86	0.023	Insulex or pyrocell	8.0	86	0.029
Cabots quilt	4.0	86	0.022	Insulex or pyrocell	12.0	86	0.037
Cork board	10.0	86	0.025	Insulex or pyrocell	18.0	86	0.049
Cork, granulated	7.3	24	0.028	Insulex or pyrocell	24.0	86	0.064
Cork, regranulated, (baked)	8.1	86	0.026	Insulex or pyrocell	30.0	86	0.083
Dry zero (Kapok)	1.0	86	0.020	Linofelt	11.2	86	0.025
Flexlinum and fiberfelt	11.2	86	0.028	Lith	14.3	86	0.033
Gypsum, molded and dried	78.0	68	0.25	Rock cork	16.0	86	0.028
Hair felt	11.0	86	0.022	Rock wool	14.0	86	0.023
Hairinsul	6.3	86	0.023	Sil-O-Cel	10.6	86	0.026
Insulating boards, Insulite, Celotex, etc.	16.0	86	0.028	Thermofil	34.0	86	0.050
				Vermiculite (expanded)	6.2	42	0.027
				Wool felt	20.6	86	0.030

A_m, in $qL/A_m = q\int dL/A$, depends on the shape of the conductor. For flat plates, $A_m = A_i = A_o$; for the hollow cylinder,

$$A_m = (A_i - A_o)/\log_e (A_i/A_o) \tag{4}$$

and for the hollow sphere, $A_m = \sqrt{A_i A_o}$. For more complex shapes, a shape factor may be evaluated by a graphical procedure (Awbery and Schofield, *Proc. 5th Intern. Congr. Refrig.*, 3, 1929, pp. 591–610).

The thermal conductivity of different materials varies greatly. For metals and alloys k is high, while for certain insulating materials, as asbestos, cork, and kapok, k is very low. In general, **k varies with the temperature.** In the case of metals, k usually decreases with rising temperature, while for most other substances the reverse is true. Tables 1 to 7, collected from various sources, give values of thermal conductivity. For refractories, see p. 731; for pipe coverings, p. 953; for ship construction materials, p. 1524. Conversion factors for various units are given on p. 81.

thermal conductivity which forms on the surface. For a given liquid and boiling pressure, the nature of the surface may substantially influence the flux at a given Δ_o, Table 9. These data may be used as rough approximations for a bank of submerged tubes. Film coefficients for scale deposits are given in Table 10.

Table 9. Maximum Flux and Corresponding Over-all Temperature Difference for Liquids Boiled at 1 Atm with a Submerged Horizontal Steam-heated Tube

Liquid	Aluminum		Copper		Chromium-plated copper		Steel	
	$\frac{q/A}{1000}$	Δ_o	$\frac{q/A}{1000}$	Δ_o	$\frac{q/A}{1000}$	Δ_o	$\frac{q/A}{1000}$	Δ_o
Ethyl acetate	41	70	61	55	77	55		
Benzene	51	80	58	70	73	100	82	100
Ethyl alcohol	55	80	85	65	124	65		
Methyl alcohol	..	..	100	95	110	110	155	110
Distilled water	..	..	230	85	350	75	410	150

The effect of reducing the boiling temperature by use of vacuum is illustrated by Fig. 5.

For **forced-circulation evaporators**, vapor binding is also encountered. Thus with liquid benzene entering a 4-pass steam-jacketed pipe at 0.9 fps, up to the point where 60 percent by weight was vaporized, the maximum flux of 60,000 Btu per hr per sq ft was obtained at an over-all temperature difference of 60 F; beyond this point, the coefficient and flux decreased rapidly,

Table 10. Heat Transfer Coefficients (h_s) for Scale Deposits from Water[a]

[For use in Eqs. (2) and (5a)]

Temp of heating medium	Up to 240 F		240 to 400 F	
Temp of water	125 F or less		Above 125 F	
Water velocity, fps	3 and less	Over 3	3 and less	Over 3
Distilled	2,000	2,000	2,000	2,000
Sea water	2,000	2,000	1,000	1,000
Treated boiler feed water	1,000	2,000	500	1,000
Treated make-up for cooling tower	1,000	1,000	500	500
City, well, Great Lakes	1,000	1,000	500	500
Brackish, clean river water	500	1,000	330	500
River water, muddy, silty[b]	330	500	250	330
Hard (over 15 grains per gal)	330	330	200	200
Chicago Sanitary Canal	130	170	100	130

Miscellaneous cases: Refrigerating liquids, brine, clean petroleum distillates, organic vapors, 1000; refrigerant vapor, 500; vegetable oils, 330; fuel oil (topped crude), 200.

[a] From Standards of Heat Exchange Institute.

[b] Delaware, East River (N. Y.), Mississippi, Schuylkill, and New York Bay.

Table 7. Thermal Conductivities of Insulating Materials at Low Temperatures (Gröber)

Material	Weight, lb per cu ft	Temperatures, deg F				
		32	−50	−100	−200	−300
Asbestos	44.0	0.135	0.132	0.130	0.125	0.100
Asbestos	29.0	0.0894	0.0860	0.0820	0.0720	0.0545
Cotton	5.0	0.0325	0.0302	0.0276	0.0235	0.0198
Silk	6.3	0.0290	0.0256	0.0235	0.0196	0.0155

CONDUCTION AND CONVECTION

Mean Temperature Difference. The basic equation for any steadily operated heat exchanger is $dq = U\Delta_0\, dA$, in which U is the over-all coefficient (p. 386), Δ_0 is the over-all temperature difference between hot and cold fluids, and dq/dA is the local rate of flow per unit surface. In order to apply this relation to a finite exchanger, it is necessary to integrate it. The assumptions usually made are constant U, constant mass rates of flow, no changes in phase, constant specific heats, and negligible heat losses. The resulting equation for parallel or countercurrent flow of fluids is

$$q = UA\Delta_m = UA(\Delta_{01} - \Delta_{02})/\log_e (\Delta_{01}/\Delta_{02}) \tag{5}$$

in which Δ_m is the logarithmic mean of the terminal temperature differences, Δ_{01} and Δ_{02}, between hot and cold fluid. The value of UA is evaluated from the resistance concept

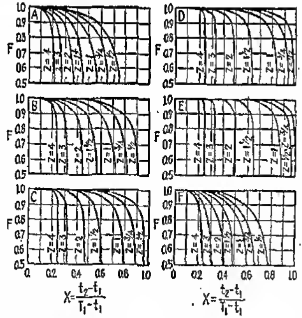


FIG. 2.—*A*, One Shell Pass and Two Tube Passes; *B*, Two Shell Passes and Four Tube Passes; *C*, Three Shell Passes and Six Tube Passes; *D*, Four Shell Passes and Eight Tube Passes; *E*, Six Shell Passes and Twelve Tube Passes; *F*, Cross flow, One Shell Pass and One Tube Pass.

$$F = \text{ordinate} = \frac{\text{true mean temperature difference}}{\text{logarithmic-mean temperature difference for counterflow}}$$

For the other symbols see p. 384.

Examples of Heat Transmission

Building Constructions (see p. 1524). The coefficients given by Harding and Willard are used as the basis of the following figures. As an example of the method of computation, consider a brick wall 13 in. in thickness. The coefficients are $h_1 = 1.4$; $h_2 = 4.2$, $k = 5/12$; hence $R = (1/1.4) + (13/5) + (1/4.2) = 3.6$ and $U = 1/3.6 = 0.28$ Btu per sq ft per hr per deg F.

Table 12. Transmission Coefficients for Various Building Constructions

Construction	Thickness, in.	U	Construction	Thickness, in.	U
Plain brick wall: $h_1 = 1.4$, $h_2 = 4.2$, $k = 5/12$.	9	0.36	Concrete[a]: $h_1 = 1.3$, $h_2 = 3.9$, $k = 5/12$.	2	0.78
	13	0.28		3	0.71
	18	0.22		4	0.66
	24	0.17		6	0.56
Brick wall + air space....	9	0.22	Windows: $h_1 = 1.5$, $h_2 = 4.5$.	Single	1.13
Brick furred and plastered.	13	0.19		Double	0.45
	18	0.16		Triple	0.28
	24	0.13	Solid wood............	1¾	0.37
Wood wall or floor.......	⅜	0.55		2⅝	0.28
			Wood + air space.......	....	0.20

[a] For 3 in. concrete covered with slag roofing, deduct 10 percent.

Heat Transmission through Pipe Insulation. (McMillan, *Trans. A.S.M.E.*, 1915.) For any number of layers of insulation on any size of pipe, Eqs. (4), (5), and (14) combine to give

$$\frac{q_o}{A_o} = \frac{\Delta_o}{\dfrac{r_o}{k_1}\log_e\dfrac{r_2}{r_1} + \dfrac{r_o}{k_2}\log_e\dfrac{r_3}{r_2} + \cdots + \dfrac{1}{h_c + h_r}} \tag{15}$$

where q_o/A_o is the Btu lost per hr per sq ft of outer surface of the last layer; Δ_o is the over-all temperature difference (deg F) between pipe and air; r_o

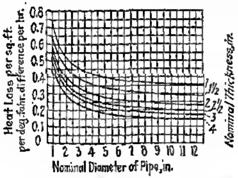


FIG. 6.—Variation with Pipe Size of Coefficient of Heat Transmission through a Given Thickness of Insulation.

is the radius, ft, of the outer surface; r_1 is the outside radius, ft, of the pipe, $r_2 = r_1 +$ thickness of first layer of insulation, ft; $r_3 = r_2$ plus the thickness of second layer, etc.; and k_1, k_2, k_3, etc., are the conductivities of the respective layers. Values of $(h_c + h_r)$ are given on p. 397 for indoor conditions and for various wind velocities. For average indoor conditions, $h_c + h_r$ is often taken

Turbulent Flow of Water inside Clean Tubes.

$$h_m = 160(1 + 0.012t_f)V_s^{0.8}/(D'_i)^{0.2} \tag{6b}$$

Turbulent Flow in Annular Spaces. Use Eqs. (6a) or (6b) with D' taken as the clearance, inches, and base the heat transfer area on that actually heated or cooled.

Water in Coiled Pipes. Multiply h_m for the straight pipe, Eq. (6b), by the factor $1 + (3.5D_i/D_c)$, where D_i is the inside diameter of the pipe and D_c is that of the coil.

Gas Flow Normal to a Single Tube (when $DG/\mu_f > 1000$).

$$h_m = 0.37C_pG^{0.56}/(D'_o)^{0.44} \tag{7}$$

Gas Flow Normal to a Bank of Staggered Tubes.

$$h_m = 0.031C_pT^{0.3}G_m^{2/3}/(D'_o)^{1/3} \tag{8}$$

Water Flow Normal to a Bank of Staggered Tubes.

$$h_m = 370(1 + 0.0067t_f)V_{sm}/(D'_o)^{0.4} \tag{8a}$$

For baffled exchangers, to allow for leakage of fluids around the baffles, use 60 percent of the values of h_m from Eqs. (8) and (8a); for tubes in line, deduct 25 percent from the values of h_m given by Eqs. (8) and (8a).

Water Flow in Layer Form over Horizontal Tubes.

$$h_{a.m.} = 150(\Gamma/D'_o)^{1/3} \tag{9}$$

for Γ ranging from 100 to 1,000 lb of water per hr per ft.

Water Flow in Layer Form down Vertical Tubes.

$$h_m = 120\Gamma^{1/3} \tag{9a}$$

Table 8. Values of h_m for Heating and Cooling, Forced Convection
(D'_o = 1.31 in., D'_i = 1.05 in.)

Fluid and arrangement	t_f, deg F	Velocity		Btu per hr per sq ft per deg F	Eq. No.
		Fps[a]	Lb per hr per sq ft		
Air inside tubes	...	$V_s = 31.8$,	$G = 8600$	8.0	6a
Air normal to staggered tubes	170	$V_s = 7.42$,	$G_m = 2000$	7.5	8
Water inside tubes	100	$V_s = 5.0$,	$G = 1.12 \times 10^6$	1260	6b
Water normal to staggered tubes	100	$V_s = 2.0$,	$G_m = 0.448 \times 10^6$	800	8a
Trickle cooler, water	...	Γ = 100 lb per hr per ft		640	9
Falling water film, vertical tube	...	Γ = 1,000 lb per hr per ft		1000	9a

[a] Velocity in fps at 70 F and 1 atm = $G/3600\ \rho$.

Coefficients for scale deposits are given in Table 10.

Streamline Flow. With liquids of high viscosity, such as viscous oils flowing at values of DG/μ below 2100, streamline flow ensues, and the coefficient of heat transfer is given by the dimensionless equation

$$h_{a.m.}D/k = 2.0(wC_p/kN)^{1/3}(\mu/\mu_w)^{0.14} \tag{10}$$

or

$$(h_{a.m.}/C_pG)(C_p\mu/k)^{2/3}(\mu_w/\mu)^{0.14} = 1.85(D/N)^{1/3}(DG/\mu)^{-2/3} \tag{10a}$$

As the value of DG/μ increases from 2100 to 7000, the effect of heated length upon h, as shown in Fig. 3, diminishes and finally becomes zero.

RADIANT-HEAT TRANSMISSION

BY

HOYT C. HOTTEL

A heated body loses energy continuously by radiation. The quantity emitted depends on the shape, size, and, particularly, the temperature of the body and is independent of the nature of its surroundings. This emitted radiation is capable of passage to a distant body where it may be absorbed, reflected, scattered, or transmitted. Most problems of energy transfer by thermal radiation may conveniently be discussed under one of the following heads: radiant-heat exchange between the surfaces of solids separated by nonabsorbing mediums; radiation from flames and gases; radiation from clouds of particles; the combined action of all these mechanisms in a furnace chamber.

Radiant-heat Exchange between the Surfaces of Solids

Thermal radiation from the surface of a solid is, for engineering purposes, best expressed as a ratio to the radiation from a so-called ideal **radiator** or **black body**. The characteristic properties of a black body are that it absorbs all the radiation incident on its surface and reflects, transmits, or scatters none; that the quality and quantity of the radiation that it emits are completely determined by its temperature, an increase in temperature producing an increase both in total radiation and in the fraction emitted in the short-wave region of the spectrum. The total radiation q from a black surface of area A and absolute temperature T is given by the **Stefan-Boltzmann law**: $q = \sigma AT^4$. The constant σ is known as the Stefan-Boltzmann constant and has the value 0.173×10^{-8} Btu/(sq ft)(hr)(deg R)^4 or 5.71×10^{-5} ergs/(sq cm)(sec)(deg K)^4.

The ratio of the total radiating power of a non-black surface to that of a black surface at the same temperature is called the **emissivity** of the surface and is designated by ϵ. More properly, the term is **total hemispherical emissivity** to differentiate it from (1) monochromatic emissivity ϵ_λ, the ratio of radiating powers at the wave length λ, and from (2) directional emissivity ϵ_θ, the ratio of radiating powers in a direction making the angle θ with the normal to the surface. The emissivity of a surface varies with its temperature, its degree of roughness, and, if a metal, its degree of oxidation. Table 1 gives the emissivities of various surfaces and emphasizes the large variation possible in a single material. Although the values in the table apply strictly to normal radiation from the surface (with a few exceptions), they may be used with negligible error for hemispherical emissivity except in the case of polished metal surfaces, for which the hemispherical emissivity is 15 to 20 percent higher than the normal emissivity.

The **absorptivity** α of a surface—the fraction of the impinging radiation which is directly absorbed—depends on the same factors that affect emissivity and, in addition, on the quality of the incident radiation (completely measured by the temperature of the radiator if the latter is black). One may assign two subscripts to α, the first to indicate the temperature of the receiver and the second that of the incident radiation. In general, α_{12} varies more with T_2 than with T_1. **Kirchhoff's law** states that the emissivity of a surface at temperature T_1 is equal to the absorptivity α_{11} which the surface exhibits for radiation from a source at the same temperature; *i.e.*, a surface of relatively low radiating power is also a poor absorber (or good reflector) of radiation from a source at its own temperature.

For vertical tubes,

$$h_m N/k = 0.94(N^3\rho^2\lambda g_L/k\mu_f\Delta)^{0.25} = 0.93(N^3\rho^2 g_L/\mu_f\Gamma)^{1/3} \qquad (13a)$$

The equations show that a tube of given dimensions, for the usual case where N/Dn is greater than 2.76, is more effective in a horizontal than in a vertical position. Thus for $N/Dn = 100$, a horizontal tube gives an average h which is 2.5 times that for a vertical tube. Since there is but little variation in the thermal conductivity or viscosity of the condensate at the condensing temperature at 1 atm, there is little variation in h_m. Thus with horizontal tubes, h_m may be taken as 200 to 400 for the following vapors condensing at atmospheric pressure: benzene, carbon tetrachloride, dichlormethane, dichlordifluoromethane, diphenyl, ethyl alcohol, heptane, hexane, methyl alcohol, octane, toluene, and xylene. Ammonia gives h_m of 1000, and mixtures of steam and organic vapors, forming immiscible condensates, give h_m ranging from 250 to 750, increasing with increasing proportion of steam. With film-type condensation of clean steam on horizontal tubes, h_m ranges from 1000 to 3000, see Eq. (13). With vertical tubes 10 to 20 ft long ripples form in the film; values of h_m from Eq. (13a) should be increased 20 percent.

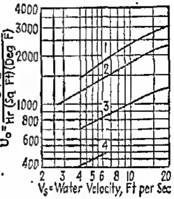


FIG. 4.—Over-all Coefficients between Condensing Steam and Water.

Curve 1. Chrome-plated copper, oleic acid.
Curve 2. Copper, benzyl mercaptan.
Curve 3. Copper, oleic acid.
Curve 4. Admiralty metal, no promoter.

For long vertical tubes, $4\Gamma/\mu$ may exceed 2100; in that case, the value of h_m for $4\Gamma/\mu = 2100$ is conservative. The presence of non-condensible gas, such as air, seriously reduces h, and consequently all vapor-heated apparatus should be well-vented.

With steam, small traces of certain promoters (Nagle, U. S. Patent 1995361) such as oleic acid and benzyl mercaptan become adsorbed in a very thin layer on the surface of the tubes, preventing the condensate from wetting the metal and inducing dropwise condensation, which gives much higher values of h (7000 to 14,000) than film-type condensation. However, with dirty or corroded surfaces, it is difficult to maintain dropwise condensation. Figure 4 shows over-all coefficients U_o for condensing steam at 1 atm on a vertical 10 ft length of copper tube, 5/8 in. O.D., 0.049 in. wall, at various water velocities.

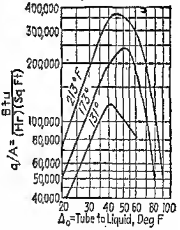


FIG. 5.—Heat Transfer to Boiling Liquid. (Distilled Water on Nickel-plated Copper.)

Boiling Liquids. In natural-convection evaporators, the heat is often supplied by submerged tubes, usually heated internally by condensing vapor. As the temperature difference between tube and liquid is increased, the rate of heat flux q/A increases, owing to better agitation due to more rapid boiling, goes through a maximum at a "critical" temperature difference (Fig. 5); and then decreases rapidly owing to a vapor film of low

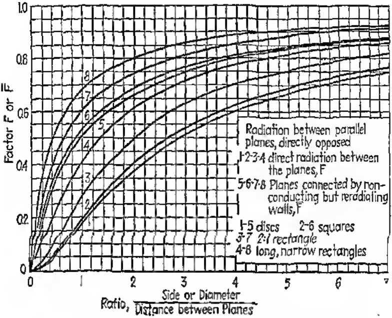


FIG. 1.—Values of the Factor F or $\bar{F}$.

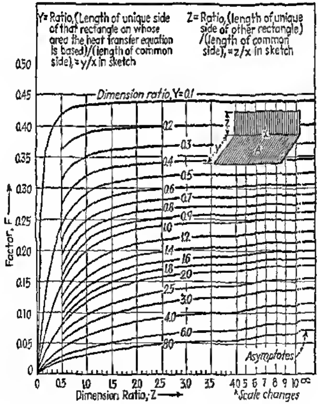


FIG. 2.—Values of the Factor F.

approaching the values obtained in superheating vapor, see Eq. (6a). For comparison, in a natural convection evaporator, a maximum flux of 73,000 Btu per hr per sq ft was obtained at Δ_o of 100 F.

Combined Convection and Radiation Coefficients. In some cases of heat loss, such as that from bare and insulated pipes, where loss is by convection to the air and radiation to the walls of the enclosing space, it is convenient to use a combined convection and radiation coefficient $(h_c + h_r)$. The rate of heat loss thus becomes

$$q = (h_c + h_r)A\Delta_s \qquad (14)$$

where Δ_s is the temperature difference, deg F, between the surface of the hot body and the walls of the space. In evaluating $(h_c + h_r)$, h_c should be calculated by the appropriate convection formula [see Eqs. (11a) to (11e)] and h_r from the equation

$$h_r = 0.692\epsilon(T_{av}/100)^3$$

where ϵ is the black body coefficient of the radiating surface, p. 400, T_{av} is the average temperature of the surface and the enclosing walls, deg R. For oxidized bare steel pipe, the sum $h_c + h_r$ may be taken directly from Table 11.

Table 11. Values of $(h_c + h_r)$
(For horizontal bare or insulated standard steel pipe of various sizes in a room at 80 F)

Pipe diameter, in.	Temperature difference, deg F, from surface to room													
	50	100	150	200	250	300	350	400	450	500	550	600	650	700
1	....	2.50	2.73	3.00	3.29	3.68	3.95	4.34	4.73	5.16	5.60	6.05	6.51	6.98
3	....	2.25	2.47	2.73	3.00	3.31	3.65	4.03	4.43	4.85	5.26	5.71	6.19	6.66
5	1.95	2.15	2.40	2.67	2.95	3.26	3.63	4.04						
10	....	2.07	2.29	2.54	2.82	3.12	3.47	3.84						

The combined coefficient $(h_c + h_r)$ for still air in contact with various materials, according to tests conducted by Lichty, University of Illinois, 1915, has the following values:

Cement plaster finish, 0.9; concrete or corkboard, 1.3; brickwork, sheet asbestos, or wood (finished surface), 1.4; glass window or magnesia board, 1.5.

The combined coefficient $(h_c + h_r)$ for the external surface is obtained by multiplying the coefficient for still air by a factor depending on the air velocity.

Velocity, mph	5	10	15	20
Factor for brickwork	2.4	3	3.8	4.2
Factor for wood	2.2	2	2.9	3.0

Harding and Willard ("Mechanical Equipment of Buildings," vol. 1, p. 56) recommend the approximation that the value of $(h_c + h_r)$ for the outside wall surface be taken as 3 times the value for the inside wall surface.

Loss of heat from the surface of a rotating cylinder to air, as in flywheels, armatures, etc., according to the experiments of Hinlein.

Smooth surfaces (polished copper).

Velocity V, fps	0	20	40	60	80	100
Combined coefficient $(h_c + h_r)$	0.47	1.97	2.79	3.14	3.30	3.38

Rough surfaces (dull black, varnished).

$V =$	0	20	40	60	80	100
$(h_c + h_r) =$	0.47	2.49	3.32	3.95	4.50	5.10

The factor $\bar{F}$ is known exactly for a few geometrically simple cases, and may be approximated for others. If A_1 and A_2 are equal parallel disks, squares, or rectangles, connected by non-conducting but reradiating refractory walls, then $\bar{F}$ is given by Fig. 1, lines 5 to 8. If A_1 represents an infinite plane and A_2 is one or two rows of infinite parallel tubes in a parallel plane, and if the only other surface is a refractory surface behind the tubes, $\bar{F}_{12}$ is given by line 5 or 6 of Fig. 3. If an enclosure may be divided into several radiant-heat sources or sinks A_1, A_2, etc., and the rest of the enclosure (reradiating refractory surface) may be lumped together as A_R at a uniform temperature T_R, then the factor $\bar{F}_{12}$ is given in terms of the simpler geometrical factors F by the expression

$$\bar{F}_{12} = F_{12} + \frac{F_{1R}F_{R2}}{(1 - F_{RR})} \tag{4}$$

If this case is further simplified by considering that the only non-refractory surfaces present in the system are heat source A_1 and heat sink A_2 and that each of these can "see" none of itself (i.e., has no negative curvature), the foregoing expression reduces to

$$\bar{F}_{12} = (A_2 - A_1F_{12}^2)/(A_1 + A_2 - 2A_1F_{12}) \tag{5}$$

which necessitates the evaluation of but one geometrical factor F. This case covers a major fraction of problems of radiant-heat interchange between source and sink in a furnace enclosure, and is in error only to the extent to which the assumption of uniform refractory temperature is not permissible. More complicated expressions are available, permitting approach to the exact answer to any desired degree of accuracy depending on the number of zones into which the refractory is divided.

It is sometimes desirable to find the equilibrium value of refractory surface temperature. For the conditions for which Eq. (5) is valid, the refractory surface temperature is given by

$$T_R = \sqrt[4]{\frac{(A_1 - A_1F_{12})T_1^4 + (A_2 - A_1F_{12})T_2^4}{(A_1 - A_1F_{12}) + (A_2 - A_1F_{12})}}$$

Allowance for Non-black Surfaces. The Factor $\mathfrak{F}$. Precise allowance for the departure of surfaces from black or ideal radiating characteristics is in general too complicated for engineering use. However, if the assumption that all surfaces are gray is permitted, a simple and adequate treatment is possible. If nomenclature is as for $\bar{F}$, except that A_1, A_2, etc., are now surfaces having emissivities (and absorptivities) ϵ_1, ϵ_2, etc., it is found that the net radiant interchange between A_1 and A_2, due now to the combined mechanisms of direct radiation, reradiation from refractory surfaces, and multiple reflection inside the enclosure, may be expressed in the form

$$q_{1\rightleftarrows 2} = A_1\mathfrak{F}_{12}\sigma(T_1^4 - T_2^4) = A_2\mathfrak{F}_{21}\sigma(T_1^4 - T_2^4) \tag{6}$$

Just as the factor $\bar{F}$ could be evaluated from F, so the factor $\mathfrak{F}$ can be evaluated from $\bar{F}$. For the case of two non-refractory surfaces A_1 and A_2 and any number of refractory zones,

$$\mathfrak{F}_{12} = 1/[(1/\bar{F}_{12}) + (1/\epsilon_1 - 1) + (A_1/A_2)(1/\epsilon_2 - 1)] \tag{7}$$

It is to be noted that the emissivity of the refractory surfaces forming the system is not a factor, *i.e.*, that whether a refractory surface maintains its equilibrium by complete absorption and black-body reradiation or by complete reflection and no radiation is immaterial.

as 2 as an approximation, since a substantial error in $h_c + h_r$ will have but little effect on the over-all loss of heat. Figure 6 shows the variation in U_o with pipe size and thickness of insulation for 85 percent magnesia for pipe and air temperatures of 375 and 75 F, respectively.

Economical Thickness of Insulation (McMillan, *Trans. A.S.M.E.*, 1926, and *Mech. Eng.*, May, 1929). When it has been determined that the physical properties of a given insulating material are suited to the requirements, the thickness of material for most economical results on flat surfaces is given by the equation $x = \sqrt{12ak/b} - 12Rk$, in which x is the most economical thickness, in., k is the conductivity, b is the cost of insulation per sq ft per in. thickness per year (first cost per sq ft per in. thickness $\times$ fractional annual fixed charges), R is the sum of the resistances of all other elements in the construction, including surface resistance, and $a = Y(t_i - t_a)M/1{,}000{,}000$, in which Y is hours operation per year, t_i is inside temperature, t_a is temperature of surrounding air, and M is the value of heat in dollars per 1,000,000 Btu. For cylindrical surfaces, Fig. 7 may be used; it is based on $12Rk = 0.3$.

These equations and the chart apply to all materials whose cost is proportional to the product of external area times thickness. They may be used for materials whose cost is figured per "board foot." They apply to cork board and to almost all other insulating materials but not to cork pipe covering.

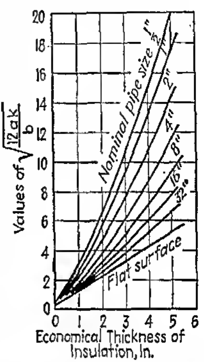


FIG. 7.—Economical Thickness of Insulation on Flat and Cylindrical Surfaces.

Example. If $t_i = 530$, $t_a = 80$, value of heat is \$0.40 per 1,000,000 Btu, hours of operation per year = 7,200, $k = 0.046$, surface resistance = 0.5, cost of insulation = \$0.15 per sq ft per 1 in. thick, and annual fixed charges = 20 percent, then, $a = 7{,}200\ (530 - 80)0.40/1{,}000{,}000 = 1.296$, and $b = 0.15 \times 0.20 = 0.03$.

The economical thickness in flat surface $= \sqrt{1.296 \times 12(0.046)/0.03} - 0.5 \times 12(0.046) = 4.88 - 0.28 = 4.6$ in. (say $4\frac{1}{2}$ in.). The economical thickness for 2 in. pipe (Fig. 7) is 2.1 in. (say 2 in.).

Transmission in Condensers. A valuable graphical method of interpreting the results of tests on surface condensers, proposed by E. E. Wilson, consists in plotting $1/U_o$ as ordinates vs. $1/V_s^{0.8}$, and drawing the best straight line. The value of $1/U_o$, for $1/V_s^{0.8}$ = zero, represents the sum of the resistances of the vapor side R_v, tube wall R_t, and scale deposit R_s. The reciprocal of the slope is the value of B in the expression for the over-all resistance. B is the water-side coefficient at a velocity of 1 fps.

$$1/U_o = R_v + R_t + R_s + 1/BV_s^{0.8}$$

Detailed examples are given in *Trans. A.S.M.E.*, 48, 1926, pp. 1233–1268.

0.208, which means that the net radiation from a tube to the stock is 20.8 percent a much as if the tube were black and completely surrounded by black stock.

Radiation from Flames and Gases

The radiation from a flame is due to radiation from burning soot particle of microscopic and submicroscopic dimensions, from suspended larger pa ticles of coal or ash or both, and from the hot gaseous combustion product

The first of these, soot luminosity, is important where combustion occur under such conditions that hydrocarbon gases in the flame are subjected t heat in the absence of sufficient air, producing thermal decompositio There is at present no method of predicting the luminosity of a flame analyt ically; reliance must be put on experimental measurement of flames simila to that of interest. It is to be noted that visual observation is a poor basi of judgment; a flame so bright that nothing on the other side of it can be see may nevertheless be far from a black body in its radiating characteristic Radiation from such a flame may be estimated from measurements, with a two-color optical pyrometer, of the red and green brightness temperatures of the flame (*Ind. Eng. Chem., Anal. Ed.*, 4, 1932, p. 166).

Radiation from a cloud of microscopic particles (such as a pulverized coal flame) may be calculated if the size, concentration, and temperature of the particles are known, but such an analysis involves so many questionable assumptions as to make the result of limited value.

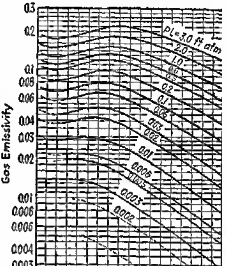


FIG. 5.—Emissivity of Carbon Dioxide.

Of greatest importance in radiant-heat transmission from flames is the infrared radiation from the combustion products, water vapor and carbon dioxide, which overshadows convection at furnace temperatures. If black-body radiation passes through either of these gases, absorption occurs at certain wave lengths. Con versely, if these gases are heated, they emit radiation at those same wav lengths. Consider a hemispherical gas mass of radius L containing carbo dioxide of partial pressure P_c, and let the problem be the evaluation of radiant heat interchange between the gas at temperature T_G and a small element o surface at temperature T_S, located on the base of the hemisphere at its center Per unit of surface, the emission of the gas to the surface is $\sigma T^4{}_G \cdot \epsilon_G$ where ϵ denotes gas emissivity. For carbon dioxide, ϵ_G depends on T_G, the total pres sure, and the product term P_cL, and is given in Fig. 5 which applies for the usua case of total pressure constant at 1 atm. The absorption, by the gas, of radia tion from the surface is $\sigma T^4{}_S \cdot \alpha_G$ where α_G is the absorptivity of the gas fo black-body radiation from the surface. Approximately, α_G is obtained fron the gas-emissivity chart at the same value of P_cL as before but at the tempera ture T_S instead of T_G. Such an approximation is adequate if the gas is hotte

Table 1. Emissivity of Surfaces

Surface	Temp,[a] deg F	Emissivity[a]
METALS AND THEIR OXIDES		
Aluminum:		
Highly polished...	440–1070	0.039–0.057
Polished..........	73	0.040
Rough plate......	78	0.055
Oxidized at 1110 F	390–1110	0.11–0.19
Brass:		
Highly polished .	494–710	0.033–0.037
Rolled plate, natural.........	72	0.06
Rubbed with coarse emery....	72	0.20
Oxidized at 1110 F	390–1110	0.61–0.59
Chromium..........	100–1000	0.08–0.26
Copper:		
Carefully polished.	176	0.018
Commercial polish	66	0.030
Heated at 1110 F..	390–1110	0.57–0.57
Thick oxide coating	77	0.78
Cuprous oxide....	1470–2010	0.66–0.54
Molten copper....	1970–2330	0.16–0.13
Gold, highly polished	440–1160	0.018–0.035
Iron and Steel:		
Highly polished pure Fe........	300–1800	0.05–0.37
Freshly emeried Fe	68	0.24
Ground sheet steel.	1400–2000	0.52–0.60
Turned cast iron..	70–1800	0.43–0.70
Rolled sheet steel..	70	0.65 to 0.82
Well oxidized, smooth.........	70–2000	0.80–0.90
Molten cast iron...	2370–2550	0.29–0.29
Molten mild steel..	2910–3270	0.28–0.28
Lead:		
Pure unoxidized...	260–440	0.057–0.075
Gray oxidized.....	75	0.28
Oxidized at 390 F..	390	0.63
Mercury, pure clean.	32–212	0.09–0.12
Molybdenum filament............	1340–4700	0.096–0.292
Monel metal, oxidized at 1110 F....	390–1110	0.41–0.46
Nickel:		
Pure polished.....	70–700	0.045–0.087
Electroplated, not polished........	68	0.11
Wire.............	368–1844	0.096–0.186
Plate, oxidized at 1110 F.........	390–1110	0.37–0.48
Nickel oxide......	1200–2290	0.59–0.86
Nickel alloys......		
"Chromnickel"...	125–1894	0.64–0.76
Nickelin, gray oxid	70	0.26
KA-28 alloy steel (8Ni, 18Cr; light silvery rough; brown after heating)........	420–914	0.44–0.36
Same, after 24 hr at 980 F........	420–980	0.62–0.73
NCT-3 (20Ni, 25-Cr); brown, splotched, oxidized from service.............	420–980	0.90–0.97
NCT-6 (60Ni, 12-Cr); smooth black firm adhesive oxide coat from service....	520–1045	0.89–0.82
Platinum, polished plate...........	440–2960	0.054–0.17
Platinum filament.	80–2240	0.036–0.192
Silver, pure polished.	440–1160	0.20–0.032
Tantalum filament...	2420–5430	0.194–0.33
Tin, bright.........	76	0.043–0.064
Tungsten, aged filament...........	80–6000	0.032–0.35 0.018
Zinc:		
Comm'l polished..	440–620	0.045–0.053
Oxid. at 750 F....	750	0.11
Galv. iron, fairly bright..........	82	0.23
Galv. iron, gray oxidized........	75	0.28
REFRACTORIES, BUILDING MATERIALS, PAINTS, MISC.		
Aluminum paints (vary with am't of lacquer body and age)..............	212	0.27–0.67
Asbestos...........	100–700	0.93–0.95
Candle soot; lamp-black-water glass..	70–700	0.95 ± 0.01
Lubricating oil, layer 0.01 in. thick:....	68	0.82
Linseed oil, 1 and 2 coats on Al foil....	68	0.56 and 0.57
Rubber, soft gray reclaimed:.........	76	0.86
Misc. I: shiny black lacquer, planed oak, white enamel, serpentine, gypsum, white enamel paint, roofing paper, lime plaster, black matte shellac	70	0.87–0.91
Misc. II: glazed porcelain, white paper, fused quartz, polished marble, rough red brick, smooth glass, hard glossy rubber, flat black lacquer, water, electrographite....	70	0.92–0.96

[a] When two temperatures and two emissivities are given they correspond, first to first and second to second, and linear interpolation is permissible.

absorptivities from Figs. 5 and 6. As PL approaches zero, the mean beam length approaches, as a limit, the value, 4 × (ratio of gas volume to bounding area). For the range of PL encountered in practice, L is always less; 85 percent of the limiting value is generally a satisfactory approximation (F. J. Port, M.I.T. D.Sc. Thesis, 1939). Table 2 summarizes the recommendations.

If gas radiation occurs in a space in which there is a continuous change in temperature of the gas and the surface from one end to the other of the interchanger, exact allowance can be made by conventional graphical integration. A fair approximation may be obtained by using a mean surface temperature equal to the arithmetic mean, and a mean gas temperature equal to the mean surface temperature plus the logarithmic mean of the temperature difference, gas to surface, at the two ends.

$$t_{S_{\text{avg}}} = (t_{S_1} + t_{S_2})/2$$

$$t_{G_{\text{avg}}} = t_{S_{\text{avg}}} + \frac{(t_{G_1} - t_{S_1}) - (t_{G_2} - t_{S_2})}{2.3 \log [(t_{G_1} - t_{S_1})/(t_{G_2} - t_{S_2})]}$$

Table 2. Beam Lengths for Gas Radiation

Shape	Characterizing dimension, D	Factor by which D is multiplied to obtain mean beam length, L	
		When $PL = 0$	For average values of PL
Sphere	Diameter	$\frac{2}{3}$	0.60
Infinite cylinder	Diameter	1	0.90
Space between infinite parallel planes	Distance between planes	2	1.8
Cube	Edge	$\frac{2}{3}$	0.60
1 × 2 × 6 rectangular parallelepiped, radiating to	Shortest edge		
2 × 6 face	....	1.16	1.06
1 × 6 face	....	1.24	
1 × 2 face	....	1.18	
All faces	....	1.20	
Space outside infinite bank of tubes with centers on equilateral triangles; tube diameter = clearance	Clearance	3.4	2.8
Same as preceding, except tube diameter = one-half clearance	Clearance	4.45	3.8

Example. Flue gas containing 9.5 percent CO_2 and 7.1 percent H_2O, wet basis, flows through a bank of tubes of 1.5 in. on equilateral triangular centers 4.5 in. apart. In a section in which the gas temperature drops from 1900 to 1515 F while the tube surface changes from 1050 to 950 F, what is the heat-transfer rate per square foot of tube area, due to gas radiation only? Tube surface emissivity = 0.8.

$$t_{S_{\text{avg}}} = (1050 + 950)/2 = 1000 \text{ F}$$

$$t_{G_{\text{avg}}} = 1000 + \frac{(1900 - 1050) - (1515 - 950)}{2.3 \log (850/565)} = 1700 \text{ F}$$

$$P_cL = 0.095(3.8 \times 4.5/12) = 0.135 \text{ ft atm}$$

$$P_c'L = 0.135(1460/2160) = 0.091 \text{ ft atm at } 1000 \text{ F}$$

$$P_wL = 0.071(3.8 \times 4.5/12) = 0.102 \text{ ft atm}$$

From Fig. 5, for CO_2:

$$\epsilon_G \text{ (at } t_G = 1700, P_cL = 0.135) = 0.080$$

$$\alpha_G \text{ (at } t_S = 1000, P_c'L = 0.091) = 0.075 \times (2160/1460)^{0.65}$$

The net loss of energy by radiation from a body at temperature T_1 in black surroundings at T_2 is given by

$$q_{1,\text{net}} = 0.173A_1[\epsilon_1(T_1/100)^4 - \alpha_{12}(T_2/100)^4] \quad (1)$$

where the units are Btu, ft, hr, deg R.

If the temperatures are so close together that $\alpha_{12} = \alpha_{11} = \epsilon_1$, or if A_1 is a gray surface (monochromatic emissivity the same at all wave lengths), the preceding relation simplifies to

$$q_{1,\text{net}} = 0.173A_1\epsilon_1[(T_1/100)^4 - (T_2/100)^4] \quad (2)$$

The more complicated but important case of radiation interchange in a system of several surfaces at different temperatures and emissivities involves evaluation of a geometrical factor F. F_{12} is defined as the fraction of the radiation leaving surface A_1 in all directions which is intercepted by surface A_2. Values of F have been calculated for various surface arrangements on the assumption that emissivity ϵ_θ is constant, independent of θ. This is exact for black surfaces and quite good for most non-metallic, tarnished, or rough metal surfaces. Values of F for opposed parallel rectangles and disks of equal size appear as lines 1 to 4 of Fig. 1, for perpendicular adjacent rectangles in Fig. 2, for an infinite plane parallel to a system of rows of parallel tubes as lines 1 and 3 of Fig. 3. Other cases are treated in the literature (*Mech. Eng.*, 52, 1932, p. 699). Important and useful concepts in evaluating F's are that $A_1F_{12} = A_2F_{21}$ (since otherwise there would be a net heat flux between A_1 and A_2 even when at the same temperature); that $F_{11} + F_{12} + F_{13} + \cdots = 1$; that of course $F_{11} = 0$ when A_1 can "see" no part of itself.

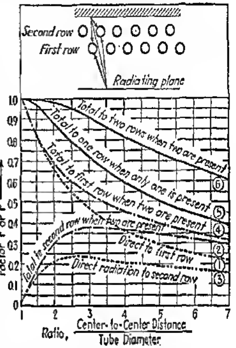


FIG. 3.—Values of F or $\bar{F}$.

In an enclosure of black surfaces, the net heat flux from A_1 is then given by

$$\begin{aligned} q_{1,\text{net}} &= (A_1F_{12}\sigma T_1^4 - A_2F_{21}\sigma T_2^4) + (A_1F_{13}\sigma T_1^4 - A_3F_{31}\sigma T_3^4) + \cdots \\ &\equiv A_1F_{12}\sigma(T_1^4 - T_2^4) + A_1F_{13}\sigma(T_1^4 - T_3^4) + \cdots \\ &\equiv A_1\sigma T_1^4 - (A_1F_{11}\sigma T_1^4 + A_2F_{21}\sigma T_2^4 + A_3F_{31}\sigma T_3^4 + \cdots) \end{aligned} \quad (3)$$

Allowance for Refractory Surfaces. The Factor $\bar{F}$. Consider an enclosure consisting in part of black heat sources and sinks A_1, A_2, A_3 . . . and in part of refractory surfaces A_R, A_S . . . from which there is no *net* radiant-heat flux (fulfilled by the average refractory wall where difference between internal convection and external loss is minute compared with incident radiation). The unknown refractory surface temperatures may be eliminated by heat balances, yielding an equation that expresses the net flux $q_{1\rightleftharpoons2}$ from A_1 to A_2 by the combined mechanisms of direct radiation plus reradiation from the refractory surfaces:

$$q_{1\rightleftharpoons2} = A_1\bar{F}_{12}\sigma(T_1^4 - T_2^4) \equiv A_2\bar{F}_{21}\sigma(T_1^4 - T_2^4)$$

$$q_F = \sigma(T^4_F - T^4_C)A_C\left[\frac{1}{\frac{1}{\epsilon_C} + \frac{A_C}{A_C + A_R}\left(\frac{1}{\epsilon_F} - 1\right)}\right] + h_C A'_C(T_F - T_C) + U_R A_R(T_F - T_O) \quad (12)$$

From this simplified form it may be seen that increasing the flame emissivity increases the heat transmission, but not proportionately; that decreasing surface emissivity (and absorptivity) from unity when the flame is very transparent produces almost no effect on the heat transmission; but that decreasing ϵ_C from unity when the flame is substantially opaque ($\epsilon_F = 1$) produces a proportional decrease in heat transmission.

The derivation of Eqs. (9) and (12) was based on the assumption that A_C was composed of plane areas. If, instead, it is a row of tubes mounted in front of a refractory wall, the value of A_C in the radiation term of the preceding equations is the continuous plane A_p in which the tubes are located, multiplied by the proper factor $\bar{F}$ for tubes with a refractory background (see Fig. 3); the refractory surface A_R should be increased by the amount $(1 - \bar{F})A_p$.

Equations (9) and (12) express a relation between two unknowns T_F and q_F, and a second relation is necessary if a solution is to be obtained. The other relation is an energy balance. If one assumes turbulence to be so great that the mean flame temperature T_F used for calculation of radiation is the same as the temperature T_G of the gas leaving the chamber, then

$$q_F = H - N(T_F - T_O) \quad (13)$$

where H represents the enthalpy or heat content of the entering fuel, air, and recirculated flue gas if any, above a base temperature T_O (water as vapor); and N represents the hourly mean heat capacity (evaluated between T_F and T_O) of the gas leaving the chamber. Equations (9) [or (12)] and (13) may be solved by trial-and-error or by graphical methods involving superimposed plots. Better agreement between predicted and experimental results is obtained on some furnaces when the assumption is made that flame temperature and exit gas temperature are not the same, but differ by a constant amount. In a number of furnace tests, the difference was about 300 F.

Simplified Treatment of Combustion-chamber Heat Transmission. Equation (9) or its equivalent has been used as a basis for deriving various simplified relations, easier to use but restricted in applicability in proportion to the degree of simplification. Several of these follow.

Billet-reheating Furnaces. For continuous billet-reheating furnaces, Eq. (9) may be modified as follows: (1) q is heat transferred to the stock, not from the flame; (2) convection terms have been omitted; (3) to compensate therefor and to allow for steadiness of furnace operation, $\mathfrak{F}_{CF}$ is evaluated using $1.2f \cdot \epsilon_F$ instead of ϵ_F, where f is the ratio of average billet-pushing rate over a period of several hours to pushing rate during periods of steady operation; (4) ϵ_F is flame emissivity due to CO_2 and H_2O only, as calculated in the previous example on gas radiation; (5) $F_{RC} = A_C F_{CR}/A_R = A_C/A_R$; (6) an average value of $(T^4_F - T^4_C)$ is used, equal to the geometric mean of its value at the two ends of the furnace; and at the hot end T_F is taken as the calculated "theoretical" flame temperature, or adiabatic combustion temperature. The equation has been tested on reheating furnaces of various types and found satisfactory (*A.S.M.E.*, **58**, 1936, p. 185; *Heat Treating Forging*, **22**, 1936, pp. 144–149, 193–198).

Petroleum Heaters. For cracking-coil and tube-still furnaces, Eq. (9) may be modified as follows: (1) by omitting the last term, q becomes heat

As in the case of $\bar{F}$, $\mathfrak{F}$ may be evaluated to any desired degree of accuracy by dividing the system into a sufficient number of zones; but most furnace problems do not justify going beyond the expression given above.

Example. A furnace chamber of rectangular parallelopipedal form is heated by the combustion of gas inside vertical radiant tubes lining the side walls. The tubes are 5 in. O.D., on 12 in. centers. The stock forms a continuous plane on the hearth. Roof and end walls are refractory. Dimensions are shown in Fig. 4. The radiant tubes and stock are "gray" bodies having emissivities 0.8 and 0.9, respectively. What is the net rate of heat transmission to the stock by radiation when the mean temperature of the tube surface is 1500 F and that of the stock is 1200 F?

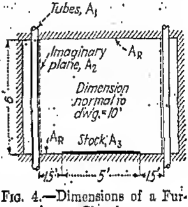


FIG. 4.—Dimensions of a Furnace Chamber.

This problem must be broken up into two parts, first considering the walls with their refractory-backed tubes. To imaginary planes A_2 of area 6×10 ft and located parallel to and inside the rows of radiant tubes, the tubes emit radiation $\sigma T_1^4 A_1 \mathfrak{F}_{12}$, which equals $\sigma T_1^4 A_2 \mathfrak{F}_{21}$. To find $\mathfrak{F}_{21}$ use Fig. 3, line 5, from which $\bar{F}_{21} = 0.81$. Then from Eq. (7),

$$\mathfrak{F}_{21} = 1/[(1/0.81) + (1/1 - 1) + (12/5\pi)(1/0.8 - 1)] = 0.702$$

This amounts to saying that the system of refractory-backed tubes is equal in radiating power to a continuous plane A_2 replacing the tubes and refractory back of them, having a temperature equal to that of the tubes and an equivalent or effective emissivity of 0.702.

The new simplified furnace now consists of an enclosure formed by two 6×10 ft radiating side walls (area A_2, of emissivity 0.702), a 5×10 ft receiving plane on the floor (A_3), and refractory surfaces (A_R) to complete the enclosure (ends, roof, and floor side strips); the desired heat transfer is $q_{2\rightleftarrows 3}$.

$$q_{2\rightleftarrows 3} = \sigma(T_1^4 - T_3^4)A_2\mathfrak{F}_{23}$$

To evaluate $\mathfrak{F}_{23}$, start with the direct interchange factor F_{23}. $F_{23} = F$ from (A_2) to $(A_3 +$ a strip of A_R alongside A_3 which has a common edge with $A_2)$ minus F from (A_2) to (the strip only). These two F's may be evaluated from Fig. 2. For the first F, $Y = 8/10$, $Z = 6.5/10$, $F = 0.239$; for the second F, $Y = 8/10$, $Z = 1.5/10$, $F = 0.100$. Then $F_{23} = 0.239 - 0.10 = 0.139$. Now $\bar{F}$ may be evaluated.

$$\bar{F}_{23} = F_{23} + \frac{1}{(1/F_{2R}) + (A_2/A_3)(1/F_{3R})}$$

Since A_2 "sees" A_R, A_3, and some of itself (the plane opposite), $F_{2R} \doteq 1 - F_{22} - F_{23}$. F_{22}, the direct interchange factor between parallel 6×10 ft rectangles separated by 8 ft, may be taken as the geometric mean of the factors for 6 ft squares separated by 8 ft, and 10 ft squares separated by 8 ft. These come from Fig. 1, line 2, according to which $F_{22} \doteq \sqrt{0.13 \times 0.255} = 0.182$. Then $F_{2R} = 1 - 0.182 - 0.139 = 0.679$. The other required direct factor is $F_{3R} = 1 - F_{32} = 1 - F_{23}A_2/A_3 = 1 - 0.139 \times 120/50 = 0.666$. Then $\bar{F}_{23} = 0.139 + \dfrac{1}{(1/0.679) + 120/50)\,(1/0.666)} = 0.336$. Having $\bar{F}_{23}$, we may now evaluate the factor $\mathfrak{F}_{23}$.

$$\mathfrak{F}_{23} = \frac{1}{(1/0.336) + [(1/0.702) - 1] + (120/50)[(1/0.9) - 1]} = 0.273$$

$$q_{net} = \sigma(T_1^4 - T_3^4)A_2\mathfrak{F}_{23} = 0.173(19.6^4 - 16.6^4)(120)(0.273)$$
$$= 408{,}000 \text{ Btu per hr}$$

A result of interest is obtained by dividing the term $A_2\mathfrak{F}_{23}$ (120×0.273, or 32.7) by the actual area A_1 of the radiating tubes $\left(\frac{5\pi}{12} \times 60 \times 2 = 157 \text{ sq ft}\right)$. This is $32.7/157 =$

than the surface and the absorption term consequently of secondary importance. If the reverse is the case, an accurate value of α_G may be obtained if one reads an emissivity from Fig. 5 at T_S as before but at $PL(T_S/T_G)$ instead of PL and then multiplies the result by $(T_G/T_S)^{0.65}$.

The net radiant-heat interchange is then

$$(\sigma T^4{}_G\, \epsilon_G - \sigma T^4{}_S \,.\, \alpha_G)\epsilon_s$$

where the term ϵ_s allows for the non-black character of the surface.

For water vapor, the gas emissivity ϵ_G is similarly determined by T_G and the product P_wL, and is given in Fig. 6. The absorptivity is given with adequate accuracy by use of a gas emissivity corresponding to P_wL and T_S. There is good evidence (*Ver. deut. Ing. Forschungsheft* 387) that for water vapor, in contrast to carbon dioxide, P_w and L enter as separate variables and not as a single product. However, unpublished experimental data believed to be of high accuracy are in much better agreement with predictions from the charts given here than with those in which allowance is made for the effect of partial pressure P_w in addition to P_wL. Consequently, the simpler procedure is recommended until more data are obtained. The present water-vapor chart is probably conservative at low L's and high P_w's.

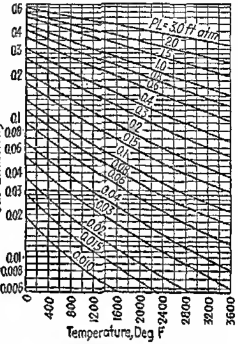


FIG. 6.—Emissivity of Water Vapor.

When carbon dioxide and water vapor are present together, the total radiation due to both is somewhat less than the sum of the separately calculated effects because each gas is somewhat opaque to radiation from the other. A rough but adequate correction for this effect may be read from Fig. 7 in the form of a correction factor K by which the total radiation is reduced. The final formulation of interchange between a gas and its bounding surface is then

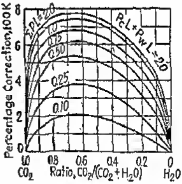


FIG. 7.—Values of Correction Factor, K.

$$q/A = \epsilon_s[\sigma T^4{}_G(\epsilon_{G,CO_2} + \epsilon_{G,H_2O}) - \sigma T^4{}_S(\alpha_{G,CO_2} + \alpha_{G,H_2O})](1 - K) \quad (8)$$

The preceding expression was formulated for the case of interchange between a gas hemisphere and a spot on its base, *i.e.*, for the case in which the length of path L of the radiant beam is the same in all directions. For gas shapes of industrial importance, it is found that any shape is approximately representable by an "equivalent" hemisphere of proper radius, or that there is a mean beam length which can be used in evaluating gas emissivities and

MECHANICAL PROPERTIES OF MATERIALS

BY

ROBERT W. VOSE

REFERENCES: Cowdrey and Adams, "Materials Testing," Wiley. Mills-Hayward-Rader, "Materials of Construction," Wiley. Smith, "Testing of Materials," Van Nostrand. Timoshenko, "Strength of Materials," Part II, Van Nostrand. Moore and Kommers, "Fatigue of Metals," McGraw-Hill. Moore, "Materials of Engineering," McGraw-Hill. Hatt and Scofield, "Laboratory Manual of Testing Materials," McGraw-Hill. Nadai, "Plasticity," McGraw-Hill. White, "Engineering Materials," McGraw-Hill. "A.S.T.M. Standards," A.S.T.M. "Creep Data," A.S.M.E. Gibbons, "Materials Testing Machines," Instruments Publishing Co.

Failure of a material or of a structural part may occur by **fracture** (such as the shattering of glass), **yielding**, **wear**, **corrosion**, and other causes. These failures are failures of the material. **Buckling** may cause failure of the part without any failure of the material.

As load is applied, **deformation** takes place before any final fracture occurs. With all solid materials, some deformation may be sustained without permanent deformation, *i.e.*, the material behaves **elastically**. Beyond the **elastic limit**, the elastic deformation is accompanied by varying amounts of **plastic**, or permanent, deformation. If a material sustains large amounts of plastic deformation before final fracture, it is classed as a **ductile** material, and if fracture occurs with little or no plastic deformation, the material is classed as **brittle**.

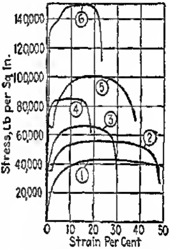


FIG. 1.—Comparative Stress-strain Diagrams. (1) Soft Brass. (2) Low-carbon Steel. (3) Hard Bronze. (4) Cold-rolled Steel. (5) Medium-carbon Steel, Annealed. (6) Medium-carbon Steel, Heat-treated.

Stress-strain Diagrams. Tests of materials may be made in many different ways, such as torsion, compression, and shear, according to the service to which the materials are to be put, but the tension test is the most common and is qualitatively characteristic of all the other types of tests except those made under extremely high hydrostatic pressure. The action of a material under the gradually increasing extension of the tension test is usually represented by plotting **apparent stress** (the total load divided by the original cross-sectional area of the test piece) as ordinates against the **apparent strain** (elongation between two gage points marked on the test piece divided by the original gage length) as abscissas. Typical curves for metals are shown in Figs. 1, 2, and 3. In these, the elastic deformation is, approximately, a straight line, as called for by **Hooke's law**, and the slope of this line, or the ratio of stress to strain within the elastic range, is the **modulus of elasticity** E, sometimes called **Young's modulus**. Beyond the elastic limit, permanent or plastic strain occurs (Fig. 3). If the stress

From Fig. 6, for H_2O:

$$\epsilon_G \text{ (at } t_G = 1700, P_wL = 0.102) = 0.048$$
$$\alpha_G \text{ (at } t_G = 1000, P_wL = 0.102) = 0.072$$

From Fig. 7, the correction due to superimposed radiation is 3.5 percent. Then substituting in Eq. (8):

$$q/A = 0.8[0.173 \times 21.6^4(0.080 + 0.048) - 0.173 \times 14.6^4(0.097 + 0.072)](1 - 0.035) = 2700 \text{ Btu per sq ft per hr,}$$

equivalent to a convection coefficient of 2700/700, or 3.8.

One may define the over-all emissivity ϵ_F of an equivalent "gray" flame by the relation

$$q/A = 0.8\sigma(21.6^4 - 14.6^4)\epsilon_F = 23{,}700\epsilon_F$$

from which ϵ_F is 2700/23,700, or 0.113.

Heat Transmission in the Combustion Chamber of a Furnace

The so-called radiant section of a furnace presents a heat-transfer problem in which there enters the combined action of direct radiation from the flame to the stock or heat sink, radiation from flame to refractory surfaces thence back through the flame (with partial absorption) to the sink, convection, and external losses. A solution of the problem is possible if the following assumptions are accepted: (1) external losses from refractory walls equal convection from flame to refractory; (2) the flame is gray and has an emissivity ϵ_F (defined as in the previous example, with suitable additions for soot luminosity, etc.); (3) all refractory surfaces have a common average (but unknown) temperature; (4) a mean temperature T_F is assignable to the flame and combustion products in the chamber; (5) the heat sink or ultimate receiver has a uniform surface temperature T_C and is gray, with emissivity ϵ_C and area A_C. The solution of the problem, giving the net rate of heat transfer q_F from the flame by all mechanisms, is

$$q_F = \underbrace{\sigma(T^4_F - T^4_C)A_C\mathfrak{F}_{CF}}_{\text{radiation to sink}} + \underbrace{h_CA'_C(T_F - T_C)}_{\text{convection to sink}} + \underbrace{U_RA_R(T_F - T_O)}_{\text{external loss}} \tag{9}$$

in which

$$\mathfrak{F}_{CF} = \frac{1}{(1/\bar{F}_{CF}) + (1/\epsilon_C) - 1} \tag{10}$$

$$\bar{F}_{CF} = \epsilon_F\left[1 + \frac{A_R/A_C}{1 + \epsilon_F/(1 - \epsilon_F)(1/F_{RC})}\right] \tag{11}$$

$$U_R = \frac{1}{(1/h_R) + (L/k) + (1/h_O)}$$

In these equations h_C, h_R, and h_O represent convection coefficients at the sink, inside refractory, and outside refractory surfaces, respectively; L and k are wall thickness and thermal conductivity of the refractory; T_O is outside air temperature; A'_C differs from A_C in excluding that cold surface or ultimate-receiver area which, though in view of the flame and receiving radiation, does not receive heat by convection from the gases until they leave the chamber.

It is to be noted that, as in the case of radiation in an enclosure containing no radiating or absorbing gas, F is built up from $\bar{F}$ and ϵ_C, and $\bar{F}$ from F; but here the flame emissivity ϵ_F is involved. Some simplification is possible if the geometrical factor F_{RC} (the fraction of the radiation leaving refractory surfaces which is directed towards the "cold" surface or heat sink) is replaced by $A_C/(A_R + A_C)$—a fair approximation when the refractory and cold surfaces are not completely segregated from each other. Then

reached in this region is called the **yield point** (YP) and is of great importance in design as well as one that is easily measured. Because certain factors such as surface finish and speed of loading have been found to influence the yield point, latest practice designates this point as the **upper yield point** and the lower part of the yielding region as the **lower yield point**, the latter being taken as the significant and reproducible value. In the harder and stronger steels, and under certain conditions of temperature, the yielding phenomenon is much less prominent and is correspondingly harder to measure.

In materials that do not exhibit a marked yield point, it is customary to measure a **yield strength**. This is arbitrarily defined as the stress at which the material has a specified permanent set (the value of 0.2 percent is widely accepted) and may be determined by measuring off the required set from the origin of the plot and running a line from the point thus found parallel to the original elastic line. The intersection of this line with the curve will give the required yield strength, as shown in Fig. 3 by the point X. Similar arbitrary rules are followed with regard to the elastic limit in commercial practice. Instead of determining the stress up to which there is no permanent set, as required by definition, it is customary to designate the end of the straight portion of the curve (by definition the **proportional limit**) as the elastic limit. Careful practice qualifies this by designating it the **proportional elastic limit**. Although in most materials elasticity and the linearity of the stress strain are closely associated, there are materials that show a non-linear stress-strain relation and yet behave at least approximately elastically. Among these are cast iron, rubber, and, to a certain extent, concrete. These show increasing deformation rates with increasing stress, but a few composite structures such as rope show the reverse action and stiffen at higher stresses. There are also a number of materials such as moist wood that show almost perfect linearity of stress with respect to strain and yet are decidedly inelastic at all loads.

Work hardening, or cold work, is the plastic deformation of a metal, as in a tensile, compressive, or shear test. Beyond the yield range in steel, or beyond the proportional elastic limit in materials that have no definite yielding, cold work increases the proportional elastic limit and the yield strength (or yield point). A material cold-worked to X in Fig. 3 and then released would show a new proportional elastic limit just below X and a correspondingly higher yield strength if retested starting at X'. If a material that has been thus strengthened in tension by tensile cold work is tested in compression, a much smaller strengthening may be found and in many cases an actual weakening of the compressive proportional elastic limit and yield strength will occur, so that cold working introduces definitely **directional properties**. There is a loss of ductility in cold working which corresponds directly to the amount of plastic deformation used in the working process. Heat-treatment after cold working is often extremely beneficial, particularly in removing the directional effect.

Ductility. The ability of materials to suffer large amounts of plastic deformation without fracture is known as **ductility** if the deformation is tensile and as **malleability** if the deformation is compressive, the two properties being similar but not identical and not present to similar degrees in different materials. Ductility is measured commercially by the **ultimate elongation**, sometimes called the **percent elongation** (expressed as a per-

transferred to oil instead of heat lost by flame; (2) $h_C A_C'$ has for simplification been assigned an average value equal to $7A_C\mathfrak{F}_{CF}$ (the term is unimportant relative to the radiation term). The relation is then

$$q_C = [\sigma(T^4_F - T^4_C) + 7(T_F - T_C)]A_C\mathfrak{F}_{CF} \tag{14}$$

Comments on p. 415 concerning the proper values of A_C and A_R for the case of tubes mounted on a wall apply. In evaluating $\mathfrak{F}_{CF}$, ϵ_F is calculated allowing for gas radiation only; $\epsilon_C = 0.9$. F_{RC} is found to be represented adequately by $A_C/(A_C + A_R)$ for values of A_R/A_C from 0 to 1, by A_C/A_R for values of A_R/A_C from 3 to 6.5. Since Eq. (14) involves heat received by oil rather than heat lost by the flame, when it is combined with the energy balance represented by Eq. (13), the latter must be modified. H is replaced by $H(1 - \beta)$, where βH is the external heat loss from the combustion chamber. A simplified graphical treatment of the solution of Eqs. (14) and (13) is available in the literature (*Am. Inst. Chem. Eng.*, 35, 1939, p. 743), together with a comparison of results with 85 tests on 19 furnaces of widely different types and excess air, burning fuel oil or refinery gas; the average deviation was 5.3 percent; excluding tests almost certainly bad, the average deviation was less than 4 percent.

A relation for petroleum heaters, somewhat simpler and quicker to use than Eq. (14) but not so safe, is obtainable by assuming certain terms in Eq. (9) constant, combining with Eq. (13) to eliminate T_F, and finding an expression different in form but numerically similar over the range of interest. The relation is

$$\mu = 1\Bigg/\left[1 + \sqrt{H/A_C\mathfrak{F}_{CF}}/1.4\left(\frac{H/N}{100}\right)^{1.4}\right] \tag{15}$$

where μ is the ratio of heat transferred to oil to the enthalpy of the entering air and fuel (net value). Other equations applicable in this field are available (*Ind. Eng. Chem.*, 24, 1932, p. 486; *Nat. Petr. News*, July 27, 1938).

Steam-boiler Furnaces. For calculating heat transmission in the radiant sections of steam-boiler furnace settings, many empirical relations are available. One of the simplest is the Orrok-Hudson equation

$$\mu = 1/(1 + (G\sqrt{C_0}/27)) \tag{16}$$

in which G is the weight ratio of air to fuel, C_0 is the firing rate expressed as pounds of equivalent good bituminous coal per hour per square foot of exposed tube area (complete circumference if not buried in wall).

Mullikin (*A.S.M.E.*, 57, 1935, p. 517) assumes that the flame emissivity ϵ_F is unity for large pulverized coal-, oil-, or gas-fired furnaces and that compensation for this high value may be made by using the same value for gas temperature in Eqs. (9) and (13). When ϵ_F is unity, the term $A_C\mathfrak{F}_{CF}$ of Eq. (9) becomes simply $A_C\epsilon_C$ (see p. 415 for the proper evaluation of A_C). Mullikin introduces additional multiplying factors on A_C to allow for resistance of overlying slag or refractory facing on metal-block walls. These are 0.7 for bare-faced metal blocks on tubes, 0.35 for refractory-faced metal blocks on tubes. The simplification suggested is unsafe to use on small furnaces where ϵ_F is certainly not unity.

Wohlenberg (*A.S.M.E.*, 47, 1925, p. 127; 57, 1935, p. 531) uses a relation intrinsically similar to Eq. (12), together with a heat balance involving the assumption of equality of flame and exit gas temperatures; he presents the relation for μ in the form of the product of a number of quantities each making separate allowance for one of the variables under control.

Reduction of area is somewhat related to malleability, but is not a direct measure of that property. With certain of the stronger metals which show a low ultimate elongation, the reduction of area may be an important criterion of the value of the material. Ductility and malleability are of importance both in the cold-working operation in manufacturing and as safety factors when a structural part is subjected to shock loading.

Elasticity. The ability of a material to return to its original dimensions after removal of stresses is known as **elasticity**. In metals, the amount of elastic deformation, or strain, is microscopic in amount, but it controls the distribution of stress in all structural parts within their working loads. In almost all metals, the elastic action is associated with the early straight portion of the stress-strain curve and with the straight lines obtained on release of stresses beyond the proportional elastic limit. The value of the modulus of elasticity E is the same in compression and in tension for most metals, the notable exception being cast iron. In metals that have a very low proportional elastic limit, the determination of E should be made from the slope of a line tangent to the stress-strain curve at zero stress, or equally well from the slope of the lines obtained on release of stresses above the proportional elastic limit, and the value so obtained is known as the **tangent modulus of elasticity**.

Shear. Stress-strain diagrams for shear loading of a material are similar to those for tension and compression, but are difficult to obtain since it is almost impossible to load a test piece in pure shear. Common tests involving shear include the **torsion test**, as a shaft, and the so-called **direct shear test**, as a rivet. Neither of these is a fundamental test, but satisfactory strengths for the design of shafts and rivets may be obtained from them. Ordinarily, only the maximum load is of interest in the rivet test and the

Table 2. Elastic Constants of Metals
(From tests of R. W. Vose)

Metal	E	G	K	μ
	Modulus of elasticity (Young's modulus) 1,000,000 lb per sq in.	Modulus of rigidity (shearing modulus) 1,000,000 lb per sq in.	Bulk modulus 1,000,000 lb per sq in.	Poisson's ratio
Cast steel	28.5	11.3	20.2	0.265
Cold-rolled steel	29.5	11.5	23.1	0.287
Stainless steel, 18-8	27.6	10.6	23.6	0.305
All other steels, including high carbon, heat-treated	28.6-30.3	11.0-11.9	22.6-24.0	0.283-0.292
Cast iron	13.5-21.0	5.2-8.2	8.4-15.5	0.211-0.299
Malleable iron	23.6	9.3	17.2	0.271
Copper	15.6	5.8	17.9	0.355
Brass, 70-30	15.9	6.0	15.7	0.331
Cast brass	14.5	5.3	16.8	0.357
Tobin bronze	13.8	5.1	16.3	0.359
Phosphor bronze	15.9	5.9	17.8	0.350
Aluminum alloys, various	9.9-10.3	3.7-3.9	9.9-10.2	0.330-0.334
Monel metal	25.0	9.5	22.5	0.315
Elektron (magnesium alloy)	6.3	2.5	4.8	0.281

For the modulus of rigidity, the bulk modulus, and Poisson's ratio, see pp. 437, 440.

SECTION 5

STRENGTH OF MATERIALS

BY

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

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The color, grain, and shear of the fracture are significant. Forms of fracture are shown in Fig. 4. Terms describing fracture are: silky, dull, granular, crystalline, fibrous.

Failure under compression (Fig. 5) depends on material, slenderness of specimen, and restraint at ends or sides. **Short blocks of brittle materials** like cast iron, stone, cement, when unconfined at the sides, fail in compression, by sliding along inclined planes. The angle of these planes is a function of the shearing stress and of the coefficient of friction of the material. Internal cones (with their bases at the pressure heads of the testing machines) and pyramids form in cylindrical and rectangular specimens, respectively.

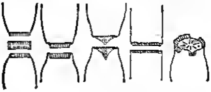


Fig. 4.—Typical Metal Fractures in Tension.

Stress Concentration. In a structural part having any sort of notch or groove, or any abrupt change in cross section, the maximum stress in that region will occur immediately at the notch, groove, or change in section and will be higher than the stress calculated on simple assumptions of stress distribution. The ratio of this maximum stress to the nominal stress (by

Fig. 5.—Typical Compression Fractures.

the simple assumption) is the **stress concentration factor** K for the particular shape and is a constant, independent of the material, except for non-isotropic materials such as wood. One method of determining stress-concentration factors is by the use of photoelasticity, in which transparent models are examined under polarized light, and from this and other sources Figs. 6 to 13 have been compiled. With brittle materials such as glass, this concentration is serious and weakens the piece directly, but with more ductile materials, under ordinary static loading, the material flows sufficiently to eliminate in part or in whole the weakening due to the concentration.

Impact. When a structure is loaded in *impact*, *i.e.*, is struck a blow, the stresses may rise sufficiently high to cause plastic flow. If this flow is distributed through a considerable volume of material, it will not result in fracture unless the blows are continued, and hence it is not harmful unless the exact shape of the part is of importance, as in gear teeth. On the other hand, if a region of stress concentration is present, either in the notch form or in the form of internal weaknesses in the microstructure of the material, then the plastic flow will be correspondingly concentrated and the velocity of flow speeded up, and there may result a brittle type of fracture originating at the concentration and known as **impact failure.** The resistance of different materials to this type of failure is naturally greater with those having ductility than with the brittle materials, but beyond this there is no correlation between **impact strength** and the other properties. Impact tests may be made by measuring the energy required to fracture a standard notched specimen such as the Charpy or Izod (see Fig. 25) either in tension or in bending, but the values so obtained are merely comparative between different materials tested by the same method and cannot be used directly in design. Table 4

is released from point X, the material will contract along XX', generally nearly straight and parallel to the original elastic line, leaving a **permanent set** OX'. The parallelism of these two lines is generally assumed, and the permanent set may be determined by measuring $X''X$ without releasing the load or stopping the test. On reloading from the point X', the line $X'X$ is followed until the original curve is nearly reached at X, where a new elastic limit occurs and the curve then follows a continuation of the original curve just as if the release of stress had not been made. As extension is continued, the material becomes stronger causing a rise of the curve, but at the same time the cross-sectional area of the specimen becomes less as it is drawn out. This loss of area weakens the specimen so that the curve reaches a maximum and then falls off until final fracture occurs (Fig. 1). The stress at the maximum point is called the **tensile strength** (TS) of the material and is its most often quoted property.

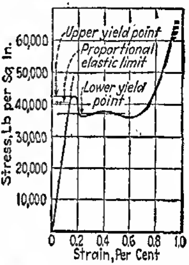


FIG. 2.—Yielding of Annealed Steel.

In a **compression test**, most metals in their annealed condition show a graph which in its early part is exactly the same as for a tension test, but in the part where the change of area exerts a prominent effect the curve rises instead of falling because the area becomes greater. Thus a compression test does not reach a maximum in the same way that a tension test does, and unless shearing, splitting, or crumbling occurs there may not be any fracture and consequently no definite **compressive strength** may be quoted. It is customary to use the tensile strength for the compressive strength when necessary in such cases, although there is little fundamental justification for the practice.

The usual curves for both tension and compression do not give a true picture of the strength of the material itself since they both include the effect of the change of area, *i.e.*, they represent the specimen rather than the material. The **true stress-strain** diagram may be plotted by dividing each load by the actual area of the specimen at the time and by treating strain in a similar manner. When this is done, the compressive and tensile curves become identical for most annealed metals, but little commercial use is made of the true stress-strain relationships except in some of the cold-working operations in manufacturing.

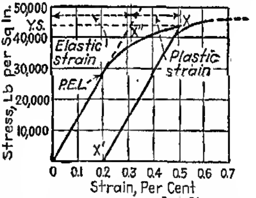


FIG. 3.—General Stress-strain Diagram.

In steels, a curious phenomenon occurs soon after the elastic limit, known as **yielding**. This gives rise to a dip in the general curve (Fig. 2) followed by a period of deformation at approximately constant load before the curve begins to rise again with strengthening of the material. The maximum stress

gives representative impact strengths. The impact strength of a material, as here treated, has to do with highly localized, and generally indeterminate, conditions.

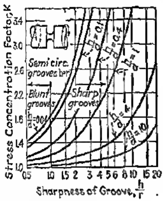


Fig. 12.—Stress Concentration Factors for Grooved Shafts in Torsion.

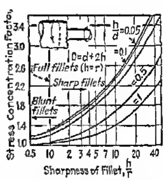


Fig. 13.—Stress Concentration Factors for Filleted Shaft in Torsion.

Table 4. Impact Strengths

(Random values)

	Charpy test, ft-lb		Charpy test, ft-lb
Commercially pure iron..........	20	S.A.E. 1020 cold-rolled, with grain	3–35
Low-alloy structural steel.........	25–50	S.A.E. 1020 cold-rolled, cross grain	3–15
Stainless steels, various..........	35–65	Aluminum die castings.........	0.6–3.5
Manganese steel.................	10–35	Zinc die castings.............	6–20
S.A.E. 1020 annealed............	3–20	Monel metal.................	150–230

Fatigue

When a body containing a region of stress concentration is loaded, the material may flow locally without failure. If the load is reversed, the plastic flow is likewise reversed, and with many repetitions of this cycle a crack may develop in the flowing material. This crack then tends to progress, or propagate, by the same action that originated it, and eventually the member may fail by **fatigue**. Fatigue cracks originate at obvious surface notches and defects, but they may also start at local weaknesses in the microstructure of the material. Fatigue failure may occur at reversed stresses which are considerably below the elastic limit as determined in an ordinary static test.

In fatigue testing, the common loading is alternate tension and compression, of equal numerical value, obtained by rotating a smooth cylindrical specimen while under bending load and noting the number of cycles required to produce failure. Other specimens of the same material are then tested at higher and lower stresses. It is found that if the results are plotted with maximum stress as ordinates and the logarithm of the number of cycles to failure as abscissas, there result characteristic curves called S-N curves (Fig. 14). In steels, these curves are found to be approximately straight up to about 10,000,000 cycles and at about this point are found to level off so that below this stress no fatigue failure can be produced even with an indefinitely large number of cycles. Hence in testing steels, it is only necessary to establish

centage of the gage length), which occurs up to the fracture point in the tensile test. For comparable results, the ultimate elongation must be measured in a definite gage length and on a definite size of specimen, although specimens of geometrically similar shape but of different sizes will give commercially comparable results. For specimens not geometrically similar, there is no satisfactory general rule by which their ductility can be compared and it is *necessary in all specifications that the shape and preferably the size of the specimen should be stated.*

In addition to the ultimate elongation, it is customary to measure the reduction of area of the cross section of the tensile specimen at the fracture. This is expressed in percent and is merely the loss of area expressed in terms of the original area. For round sections, the fracture is usually so nearly round as to be readily measured directly, but with specimens of other cross section the fracture is a distorted section which cannot be accurately measured, and in such cases reduction of area is ordinarily not reported.

Table 1. Strengths of Metals

Metal	Tensile strength, 1,000 lb per sq in.	Yield strength, 1,000 lb per sq in.	Ultimate elongation, percent	Reduction of area, percent	Brinell No.
Cast iron	18-60	8-40	0	0	100-300
Wrought iron	45-55	25-35	35-25	55-30	100
Commercially pure iron, annealed	42	19	48	85	70
Hot rolled	48	30	30	75	90
Cold rolled	100	95	.....	.....	200
Structural steel, ordinary	50-65	30-40	40-30	.....	120
Low alloy, high strength	65-90	40-80	30-15	70-40	150
Steel, S.A.E. 1330, annealed	70	40	26	70	150
Quenched, drawn 1300 F	100	80	24	65	200
Drawn 1000 F	130	110	20	60	260
Drawn 700 F	200	180	14	45	400
Drawn 400 F	240	210	10	30	480
Steel, S.A.E. 4340, annealed	80	45	25	70	170
Quenched, drawn 1300 F	130	110	20	60	270
Drawn 1000 F	190	170	14	50	395
Drawn 700 F	240	215	12	48	480
Drawn 400 F	290	260	10	44	580
Cold-rolled steel, S.A.E. 1112	84	76	18	45	160
Stainless steel, 18-8	85-95	30-35	60-55	75-65	145-160
Steel castings, heat-treated	60-125	30-90	33-14	65-20	120-250
Aluminum, pure, rolled	13-24	5-21	35-5	.....	23-44
Aluminum-copper alloys, cast	19-23	12-16	4-0	.....	50-80
Wrought, heat-treated	30-60	10-50	33-15	.....	50-120
Aluminum die castings	30	.....	2		
Aluminum alloy 17ST	56	34	26	39	100
Aluminum alloy 51ST	48	40	20	35	105
Copper, annealed	32	5	58	73	45
Copper, hard drawn	68	60	4	55	100
Brasses, various	40-120	8-80	60-3	.....	50-170
Phosphor bronze	40-130	.....	55-5	.....	50-200
Tobin bronze, rolled	63	41	40	52	120
Magnesium alloys, various	21-45	11-30	17-.5	.....	47-78
Monel metal, 70Ni, 30Cu	100	50	35	.....	170

Compressive strength of cast iron, 80,000 to 150,000 lb per sq in.

Compressive yield strength of all metals, except those cold worked, = tensile yield strength.

Gough, "Contact Corrosion under Pressure," *Nat. Phys. Lab. Rept., Dept. Ind. Sci. Research*, 1935, p. 150). Although any corroding agent is productive of severe corrosion fatigue, there is so much difference between the effects of "sea water" or of "tap water" from different localities that numerical values are not of much value.

In many structural elements, completely reversed stresses, as are used in determining the endurance limit, do not occur, and other limits of stress variation are encountered. Under such conditions, the **allowable range** may be determined approximately from the empirical graph shown in Fig. 15. Here the absolute values of the extreme stresses are plotted as ordinates and the average value of the two extremes as abscissas. The endurance limit is plotted above and below the origin, and the tensile strength is plotted as a single point whose ordinate and abscissa both equal the tensile strength. Straight lines are then drawn connecting the two endurance limit points with the tensile strength point, and any range between these two lines is allowable. Since the yield strength should not ordinarily be exceeded in design, the graph is usually cut off at this value. The compressive side of the graph is usually taken as being similar to the tensile side.

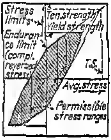


FIG. 15.—Permissible Stress Ranges in Fatigue.

Fatigue strength, as measured by the endurance limit in reversed bending, is not fundamentally related to any other property. However, for materials of a given type, there may be a relationship between the endurance limit and the tensile strength or between the endurance limit and hardness. For the plain carbon steels, annealed, the endurance limit ranges from 0.45 to 0.55 times the tensile strength and for some of the alloy steels and for the nonferrous materials, in general, the ratio is much lower. In direct tension-compression loading, as compared with the reversed bending tests described, the endurance limit is lower, ranging from 0.75 to 0.90 times the endurance limit in reversed bending. The endurance limit in reversed shear ranges from 0.50 to 0.60 times the endurance limit in reversed bending. For partly reversed stresses of any type, the construction shown in Fig. 15 may be used, taking the tensile strength as the upper point in the cases of bending and tension and ½ the tensile strength in the case of shear. The diagram for shear should be limited also by the value ½ times the yield strength.

Fatigue failure is particularly dangerous since the incipient cracks are often invisible, although they invariably originate at a surface, and the final failure may occur with disastrous suddenness in high-speed machinery or vehicles. However, the progress of the crack in its early stages is extremely slow (deForest, "The Rate of Growth of Fatigue Cracks," *Trans. A.S.M.E.*, 58, 1936, p. A23), and losses attendant on failure in service may be largely eliminated by periodic inspection by special means and rejection or repair of parts in which cracks have started. One of the best means for this inspection is the **Magnaflux method**, in which the part (necessarily ferromagnetic) is magnetized and covered with a fine magnetic powder. In the vicinity of a crack there is a disturbance of the magnetic flux and the magnetic powder gathers there, effectively marking the crack. With non-magnetic materials, other methods must be used, such as whitewashing the part and depending on the discoloration of oil seeping out of a crack to reveal such cracks, but none of these approaches the efficacy of the Magnaflux method on steel. Periodic inspection of all vital parts, primarily by the Magnaflux method, is rapidly

proportional elastic limit and the maximum torque in the torsion test. (For a method of obtaining stress-strain diagrams for shear, see Nadai, "Plasticity," McGraw-Hill, p. 126.) With most of the ordinary *ductile* steels, brasses, and aluminum alloys, the direct shear strength is in the vicinity of 0.6 to 0.7 of the tensile strength, and the apparent strength in torsion is 0.8 to 0.9 of the tensile strength.

Table 3. Mechanical Properties of Stone and Brick*

	Specific gravity	Compressive strength, lb per sq in.	Modulus of elasticity, lb per sq in.	Absorption of water, parts by weight	Coefficient of expansion per deg F	Modulus of rupture, per sq in.
Granite........................	2.67	19,400	7,300,000	1/150	0.0000040	1,850
Limestone......................	2.53	9,500	8,460,000	1/38	0.0000045	1,400
Limestone, oolithic............	2.48	6,700	7,000,000	1/23	0.0000045	....
Marble.........................	2.72	12,700	8,000,000	1/500	0.0000045	1,400
Sandstone......................	2.22	9,300	3,000,000	1/24	0.0000055	1,400
Trap...........................	2.96	20,000	12,000,000	....	....	....
Slate..........................	2.77	14,000	14,000,000	1/430	0.0000058	7,700
Brick						
Common.......................	2.00	4,000	2,000,000	1/5	....	....
Hard-burned..................	2.10	0,000	4,000,000	1/6	....	....
Paving.......................	2.42	10,000	7,000,000	1/100	....	....
Sand-lime....................	1.85	3,500	1,000,000	1/10	....	....
Brick masonry						
In lime mortar...............		0.14 × compressive strength of brick				
In cement mortar.............		0.23 × compressive strength of brick				

* Other Strength Functions. Shearing strength of brick and stone is from 10 to 20 percent of the compressive strength; tensile strength is 4 percent of compressive strength; modulus of rupture is 15 percent of compressive strength. Poisson's ratio is 1/4.

Velocity of Testing. Within the usual speeds of commercial testing machines, the speed of testing does not cause any variation of the strengths of metals which is of commercial significance, except possibly in the yield point. Non-metallic materials, such as cloth, wood, rope, and plastics, are in general much more sensitive to testing speed, and attention must be paid to this factor and to the time of loading in service. With very slow loadings of metals, extending over a period of years, some slight reduction of strength may be expected, and under elevated temperatures or with certain metals *creep* (see p. 428) may occur. With extremely high loading speeds, obtained in machines specially constructed for the purpose or in shock and impact loadings of a structure, strengths may increase greatly over the usual values, and entirely new types of failure may occur (see p. 420).

Effect of Size. The strengths shown by large specimens are generally lower than those quoted for the usual 1/2 in. size as a result of the difficulties in heat-treatment and working of sections of large size, and for other, more obscure, reasons. Although no definite rules can be made, a reduction of 10 to 25 percent may be expected for the strength of a 3 in. piece as compared with a 1/2 in. piece. Very small sections, such as wires, often show much higher strengths than conventional sections, and in the case of glass fibers the strength of a 0.0001 in. diam filament has been found to be 20 times that of a 0.03 in. diam rod.

The character of the fracture under tension indicates the quality of the material, but is influenced by the speed and the method of producing fracture and by the shape of the test piece. A metal that is tough and fibrous may appear crystalline if broken quickly at a nicked section. Contraction is greatest in tough and ductile, and least in brittle, materials. The shape of fracture is usually a central flat surface of failure in tension, surrounded by a rim on which the metal shears. The extent of the rim is more pronounced when the ratio of shearing to tensile strength is less, is more developed in soft steels, becomes a complete cone in very soft materials, and vanishes in cast iron,

Table 6. Tensile Tests on Metals and Alloys Subjected to High Temperatures—(*continued*)

Materials	Temperatures, deg F							
	70	300	450	500	600	750	950	1000
CAST MATERIALS								
Malleable Iron:							(822°)	(950°)
Tensile strength	37,625	33,505	33,280	......	34,000	34,055	31,830	27,110
Elongation	63.0	......	......	......	6.3	6.3	......	
Cast Steel:						(720°)	(750°)	
Tensile strength	73,325	76,570	81,167	......	67,366	58,713	41,388	17,568
Elastic limit	39,817	34,020	34,267	......	29,422	33,313	27,223	9,650
Elongation	27.0	15.6	19.8	......	23.1	22.7	2.5	31.3
Reduction of area	35.0	24.5	23.4	......	34.6	34.6	45.4	57.5
[7] Cast Nickel:							(900°)	
Tensile strength	38,029	40,850	36,667	......	35,896	36,536	27,800	16,775
Elastic limit	23,798	25,123	25,137	......	22,875	22,570	14,150	14,740
Elongation	5.7	9.6	6.5	......	6.2	7.2	6.3	3.9
Reduction of area	6.1	14.6	10.3	......	7.9	9.2	11.6	4.8
Monel Metal (cast):		(285°)	(435°)	(450°)				(1030°)
Tensile strength	52,870	53,130	54,100	47,200	39,450	41,787	......	26,400
Elastic limit	30,088	24,910	23,850	22,350	21,531	21,700	......	23,480
Elongation	16.6	20.3	25.8	15.6	18.2	14.1	......	6.3
Reduction of area	19.8	19.9	23.6	29.5	22.0	16.9	......	6.8
[8] U. S. Navy Brass:								
Tensile strength	28,930	27,000	25,600	20,900	13,130	9,025	......	
Elastic limit	16,130	13,800	10,450	12,075	11,350	9,025	......	
Elongation	19.3	18.8	19.5	17.2	1.0	0.0	......	
Reduction of area	23.2	20.1	18.6	16.0	3.1	0.6	......	
[9] U. S. N. Gun Bronze:								
Tensile strength	34,170	36,025	33,050	......	21,380	19,640	9,650	
Elastic limit	25,650	21,900	19,650	......	18,780	18,140	9,150	
Elongation	8.0	8.6	8.3	......	3.7	0.0	0.0	
Reduction of area	7.3	7.1	10.0	......	2.9	0.7	0.0	
[10] U. S. N. Valve Bronze:							(900°)	
Tensile strength	35,345	34,260	27,630	28,160	16,100	13,000	9,530	6,400
Elastic limit	18,740	13,210	15,220	14,520	15,225	12,275	9,530	6,400
Elongation	23.2	26.2	17.2	12.8	3.8	0.6	0.0	0.0
Reduction of area	24.9	26.0	21.0	17.4	4.3	1.1	1.7	0.0
ROLLED MATERIALS								
[11] Rod Brass:							(900°)	
Tensile strength	54,450	52,700	49,000	......	35,050	18,740	10,170	
Elastic limit	45,000	43,200	39,100	......	23,735	15,050	8,060	
Elongation	16.4	26.6	21.9	......	14.9	17.2	21.9	
Reduction of area	18.0	34.2	26.0	......	17.8	21.3	26.0	
[12] Nickel Steel:				(525°)				(1030°)
Tensile strength	99,498	97,000	84,950	83,000	69,576	45,650	......	36,350
Elastic limit	39,850	31,700	32,250	26,200	25,650	21,[illegible]	......	15,500
Elongation	51.2	64.1	62.5	59.4	56.3	43.0	......	37.5
Reduction of area	59.8	65.0	65.0	66.8	72.6	59.4	......	55.7
Monel Metal:				(525°)				(1030°)
Tensile strength	104,900	97,400	97,800	96,400	89,600	67,600	......	47,200
Elastic limit	78,350	58,500	58,600	58,400	57,950	42,550	......	26,800
Elongation	31.3	29.7	29.7	32.8	32.8	28.1	......	28.1
Reduction of area	61.7	57.8	51.0	51.5	59.5	58.1	......	60.7
[13] Cold-rolled Shafting:				(525°)		(795°)		
Tensile strength	82,800	91,850	96,083	96,250	88,525	59,500	39,250	
Elastic limit	76,800	77,100	72,850	75,300	54,275	53,200	30,400	
Elongation	21.9	21.9	21.9	18.8	25.0	25.0	35.2	
Reduction of area	49.5	39.1	38.7	37.5	44.2	58.5	78.0	

[1] A strong copper-tin bronze. [2] Containing 5 percent of aluminum. [3] Highly resistant to acids; elastic limit not determined. [4] Containing 10 percent of aluminum; elastic limit not determined. [5] At 550 F: 32,050, 19,300, 23.4, and 31.6. [6] Ferrosteel, used in extra-heavy valves over 7 in. [7] Rendered malleable by special process and used for rings, disks, and trimmings for iron and steel fittings for high temperatures. [8] For screw pipe fittings (Cu, 77–80; Sn, 4; Pb, 3; Zn, 13–19). [9] (Cu, 88; Sn, 10; Zn, 2.) [10] (Cu, 87; Sn, 7; Pb, 1; Zn, 5.) At 550 F: 17,200, 14,025, 4.4, and 6.2. [11] (Cu, 62.5; Pb, 2.5; Zn, 35.) [12] Rods, 30 percent nickel. [13] Bessemer steel, for valve stems.

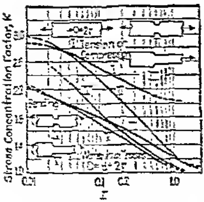


FIG. 6.—Flat Plate with Semicircular Fillets and Grooves or with Holes, in Tension or Compression.

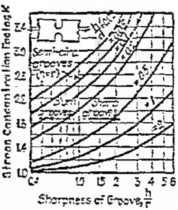


FIG. 7.—Flat Plate with Grooves, in Tension.

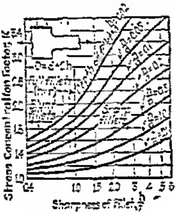


FIG. 8.—Flat Plate with Fillets, in Tension.

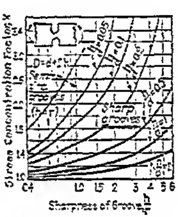


FIG. 9.—Flat Plate with Grooves, in Bending.

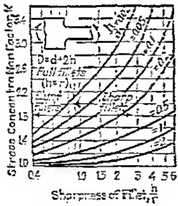


FIG. 10.—Flat Plate with Fillets, in Bending.

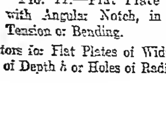


FIG. 11.—Flat Plate with Angular Notch, in Tension or Bending.

FIGS. 6 to 11.—Stress Concentration Factors for Flat Plates of Width D Reduced to Width d by Grooves or Fillets of Depth h or Holes of Radius r.

ferrous metals, are exceptions to the general rule in that they show low malleability and ductility and slightly higher strength through certain temperature ranges and, hence, should not be hot-worked at such temperatures (see p. 610). Tests by the Crane Co. on a wide range of metals and alloys, as reported by Bregowsky and Spring, in *The Valve World*, Jan., 1913, are given in Tables 6 and 7. See also Rudeloff, International Assoc. for Testing Materials, 1909, and "Symposium on Corrosion-resistant, Heat-resistant, and Electrical-resistance Alloys," *Proc. A.S.T.M.*, 24, Part II. For strengths at low temperatures, see p. 630.

Creep and Relaxation

Although the change in the static strengths of materials at elevated temperatures may occasionally be of design significance, there occurs at these temperatures, in most metals (and at room temperature for some metals,

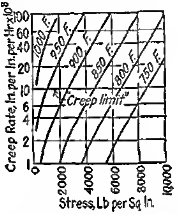


FIG. 16.—Creep Rates for 0.35 Carbon Steel.

such as lead), a slow deformation under stress, called **creep**, which may lead to excessive deformations or to fracture. Creep is generally assumed to proceed at a constant rate at any given temperature and stress; the total deformation at the end of any given time can then be calculated. The working stress should be so chosen that this total deformation during the desired life of the structure will not be excessive. A **creep limit**, which is defined as the stress which at a given temperature will result in 1 percent deformation in 10 years, or in 100,000 hr., has received a certain amount of recognition, but it is advisable to determine the proper stress for each individual case from diagrams of stress vs. creep rate, as shown in Fig. 16 (see Creep Data published by the A.S.T.M. and A.S.M.E., 1938).

Associated with creep, which deals with deformation under constant stress, is **relaxation**, which deals with the release of stress in structural elements that are deformed a predetermined amount and are expected to carry a load at this deformation, such as flange bolts in high-temperature steam service. Direct numerical correlation between relaxation and creep has not yet been generalized (Davenport, "Correlation of Creep and Relaxation Properties of Copper," *Trans. A.S.M.E.*, 40, 1938, p. A55), and relaxation design is not yet on a sound basis.

NOTES ON TABLE 7

[1] Bessemer steel, 0.093 percent carbon. [2] Cumberland cold-rolled shafting, 0.083 percent carbon. [3] Open-hearth steel, 0.084 percent carbon. [4] (Ni, 3.25; V, 0.45; C, 0.365), oil tempered. [5] 25 percent nickel. [6] 30 percent nickel; turned from 1 in. to 0.8 in. diam. [7] Turned to 0.75 in. diam from a 7/8 in. bar. [8] Cumberland shafting, 0.375 percent carbon. [9] Annealed (Cr, 0.49; V, 0.145; C, 0.722). [10] Rolled rod (Cu, 62.5; Zn, 35, Pb, 2.5). [11] Elephant (phosphor) bronze (Cu, 95.5; Sn, 4; P, 0.31).

this break in the curve in the vicinity of the 10,000,000 cycle point, and the corresponding stress is termed the **endurance limit**, or, less commonly, the **fatigue limit**. With the nonferrous metals, the curves exhibit no such definite break as in the case of the steels, and a true endurance limit should not be quoted. Values currently reported are usually obtained from tests running well over 10,000,000 cycles, and in careful practice as many as 500,000,000 cycles are used (see Templin, "Fatigue of Light Metal Alloys," *Metals Alloys*, 10, No. 8, p. 243).

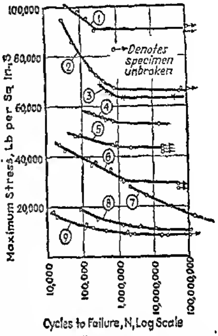


FIG. 14.—S-N Diagrams from Endurance Tests.

(1) 1.20 C steel, quenched, drawn 860 F. (2) S.A.E. 3420, quenched, drawn 1200 F. (3) Alloy structural steel. (4) S.A.E. 1050, quenched, drawn 1200 F. (5) S.A.E. 4130, normalized, annealed. (6) Ordinary structural steel. (7) Duralumin. (8) Copper, annealed. (9) Cast iron.

Surface defects, such as roughness or scratches, and notches of shoulders, all reduce the fatigue strength of a part. With a notch of definite geometric form and known concentration factor, the reduction in strength is materially less than would be called for by the concentration factor itself, but the various metals differ widely in their susceptibility to the effect of roughness and concentrations, or **notch sensitivity**. Further, notch sensitivity seems to be higher, and ordinary fatigue strength lower, in large specimens, necessitating full-size tests in many instances (see Peterson, "Stress Concentration Phenomena in Fatigue of Metals," *Trans. A.S.M.E.*, 55, 1933, p. 157; and Buckwalter and Horger, "Investigation of Fatigue Strength of Axles, Press Fits, Surface Rolling and Effect of Size," *Trans. A.S.M.*, 25, Mar., 1937, p. 229). In addition to other factors, **corrosion** and **galling** (due to rubbing of mating surfaces) each cause great reduction of fatigue strengths, sometimes amounting to as much as 90 percent of the original endurance limit (see

Table 5. Representative Endurance Limits for Reversed Bending

Metal	Tensile strength, 1,000 lb per sq in.	Endurance limit, 1,000 lb per sq in.	Metal	Tensile strength, 1,000 lb per sq in.	Endurance limit, 1,000 lb per sq in.
Cast iron	20–50	6–18	Copper	32–50	12–17
Malleable iron	50	24	Monel	70–120	20–50
Cast steel	60–80	24–32	Phosphor bronze	55	12
Armco iron	44	24	Tobin bronze, hard	65	21
Plain carbon steels	60–150	25–75	Cast aluminum alloys	18–40	6–11
S.A.E. 6150 heat-treated	200	80	Wrought aluminum alloys	25–70	8–18
Nitralloy	125	80	Magnesium alloys	20–45	7–17
Brasses, various	25–75	7–20			

Phosphor bronze. Rate 0.1, temp 400 (550) [700] {800}, stress 15 (6) [4] {1}; rate 0.01, temp 400 (550) [700], stress 8 (4) [2].

Nickel. Rate 0.1, temp 800 (1000), stress 20 (10).

70 *Cu*, 30 *Ni.* Rate 0.1, temp 600 (750), stress 28 (13–18); rate 0.01, temp 600 (750), stress 14 (8–9).

Aluminum alloy #17S (*Duralumin*). Rate 0.1, temp 300 (500) [600], stress 22 (5) [1.5].

Lead, pure (commercial) [0.03 percent Ca], at 110 F, for rate 0.1 percent the stress range, lb per sq in., is 150–180 (60–140) [200–220]; for rate of 0.01 percent, 50–90 (10–50) [110–150].

Hardness

Hardness has been variously defined as resistance to local penetration, to scratching, to machining, to wear or abrasion, and to yielding. The multiplicity of definitions, and corresponding multiplicity of hardness measuring instruments, together with the lack of a fundamental definition, indicates that hardness may not be a fundamental property of a material but rather a composite one including yield strength, work hardening, true tensile strength, modulus of elasticity, and others.

Scratch hardness is measured by Mohs scale of minerals (see p. 86) which is so arranged that each mineral will scratch the mineral of the next lower number. In recent mineralogical work and in certain microscopic metallurgical work, jeweled scratching points either with a set load or else loaded to give a set width of scratch have been used (see Talmadge, "Quantitative Standards for Hardness of the Ore Minerals," *Econ. Geol.*, **20**, 1925, p. 531; and Bierbaum, "The Microcharacter," *Trans. A.S.S.T.*, **18**, 1930, p. 1009). Hardness in its relation to machinability and to wear and abrasion is generally dealt with in direct machining or wear tests, and little attempt is made to separate hardness itself, as a numerically expressed quantity, from the results of such tests.

The resistance to localized penetration, or **indentation hardness**, is widely used industrially as a measure of hardness, and indirectly as an indicator of other desired properties in a manufactured product. The indentation tests described below are essentially non-destructive, and in most applications may be considered non-marring, so that they may be applied to each piece produced, and through the empirical relationships of hardness to such properties as tensile strength, fatigue strength, and impact strength, pieces likely to be deficient in these latter properties may be detected and rejected.

Brinell hardness is determined by forcing a hardened sphere under a known load into the surface of a material and measuring the diameter of the indentation left after the test. The **Brinell hardness number**, or simply the **Brinell number**, is obtained by dividing the load used, in kilograms, by the actual surface area of the indentation, in square millimeters. The result is a pressure, but the units are rarely stated.

$$B_r = P \Big/ \frac{\pi D}{2}(D - \sqrt{D^2 - d^2})$$

where B_r is the Brinell hardness number; P the imposed load, kg; D the diameter of the spherical indentor, mm; and d the diameter of the resulting impression, mm.

Hardened steel bearing balls may be used for hardness up to 500, but beyond this hardness especially treated steel balls or jewels should be used to

becoming standard practice in the transportation and high-speed machinery fields.

Surface fatigue occurs under highly concentrated compressive loading between two rolling members, such as in ball bearings, and results in a flaking off, or **spalling**, of the material of one or both surfaces. Aside from bearings, the only important case of surface fatigue occurs in gear teeth. Both these cases are covered by the specialized design rules of the respective industries (see Buckingham, "Dynamic Loads on Gear Teeth," publication of A.S.M.E.; Almen, "Lubricants and False Brinelling of Ball and Roller Bearings," *Mech. Eng.*, June, 1937; Stribeck, "Kugellager für beliebige Belastung," translated by Hess, *Trans. A.S.M.E.*, **29**, 1907, p. 367).

Strengths at Elevated Temperatures

In general, the strengths, such as elastic limit, yield point or yield strength, and tensile strength, of metals decrease with increasing temperature, whereas the ductility and malleability increase. In steel, the modulus of elasticity E decreases with increasing temperature. In most structural design, these strengths are of little significance since creep is likely to occur below the elastic limit, but where the loads are of short duration only they may be used in the ordinary manner. A few steels, and a number of non-

Table 6. Tensile Tests on Metals and Alloys Subjected to High Temperatures

[Tensile strength and elastic limit in lb per sq in.; elongation (in 2 in.) and reduction of area in percent.]

Materials	Temperatures, deg F							
	70	300	450	500	600	750	950	1000
CAST MATERIALS								
[1] Hard Metal:								
Tensile strength	33,735	34,280	31,810	......	23,150	19,170	10,825	5,710
Elastic limit	25,035	19,630	20,845	......	17,160	15,925	8,775	5,500
Elongation	7.4	10.7	7.8	......	2.8	2.9	0.0	0.0
Reduction of area	9.4	12.8	9.2	......	4.1	3.5	0.5	0.0
[2] Aluminum Bronze:								
Tensile strength	35,205	32,330	35,815	......	24,265	6,425	......	6,230
Elastic limit	27,740	......	15,645	......	18,420	6,425	......	6,230
Elongation	56.3	78.1	65.7	......	45.3	0.0	......	0.0
Reduction of area	46.9	31.4	61.7	......	38.4	0.0	......	0.0
[3] Phosphor Bronze:								
Tensile strength	33,107	33,640	26,376	......	16,537	15,775	......	
Elongation	12.5	12.5	10.9	......	4.7	0.0	......	
Reduction of area	11.9	16.5	12.3	......	0.8	0.0	......	
Steam Metal:								
Tensile strength	31,780	26,370	21,900	20,260	12,180	10,280	6,630	(550°) 12,230
Elastic limit	16,900	12,770	13,130	12,410	11,370	10,280	6,630	11,230
Elongation	21.1	18.8	10.9	8.9	1.1	0.0	0.0	0.0
Reduction of area	23.8	18.2	14.5	10.9	1.2	0.4	0.0	0.7
[4] Aluminum Bronze:								
Tensile strength	41,360	......	42,308	......	38,425	26,800	25,960	
Elongation	9.4	......	12.5	......	11.5	1.6	3.1	
Reduction of area	13.1	......	4.8	......	11.6	2.9	6.7	
[5] Cast Manganese Bronze:								
Tensile strength	56,350	47,775	40,870	37,200	21,450	7,350	(900°) 2,365	(925°) 1,625
Elastic limit	31,875	26,900	25,580	22,200	13,525	5,230	1,500	1,625
Elongation	10.9	12.5	21.4	24.5	32.1	39.6	40.2	48.4
Reduction of area	15.8	17.1	25.3	33.7	41.0	54.0	61.3	63.0
Soft Cast Iron:								
Tensile strength	22,060	23,260	20,730	......	21,240	21,925	(860°) 21,590	19,820
[6] Strong Cast Iron:								
Tensile strength	32,[illegible]	33,290	33,400	......	33,110	32,860	(900°) 25,780	27,310

impression is measured by means of a medium-power compound microscope.

$$V = P/0.5393d^2$$

where V is the Vickers hardness number; P the imposed load, kg; and d the diagonal of indentation, mm. The Vickers method is more flexible and is considered to be more accurate than either the Brinell or the Rockwell, but the equipment is more expensive than either of the others (approx $1,500 against $400) and the Rockwell is somewhat faster in production work.

Among the other hardness methods may be mentioned the **Scleroscope**, in which a diamond-tipped "hammer" is dropped on the surface and the rebound taken as an index of hardness. This type of apparatus is seriously affected by the resilience as well as the hardness of the material and has largely been superseded by other methods. In the **Monotron** method, a penetrator is forced into the material to a predetermined depth and the load required is taken as the indirect measure of the hardness. This is the reverse of the Rockwell method in principle, but the loads and indentations are smaller than those of the latter. In the **Herbert pendulum**, a 1 mm steel or jewel ball resting on the surface to be tested acts as the fulcrum for a 4 kg compound pendulum of 10 sec period. The swinging of the pendulum causes a rolling indentation in the material, and from the behavior of the pendulum several factors in hardness, such as **work hardenability**, may be determined which are not revealed by other methods. Although the Herbert results are of considerable significance, the instrument is suitable for laboratory use only (see Herbert, "The Pendulum Hardness Tester," and "Some Recent Developments in Hardness Testing," *Engineer*, 135, 1923, pp. 390 and 686). In the **Herbert cloudburst** test, a shower of steel balls, dropped from a predetermined height, dulls the surface of a hardened part in proportion to its softness and thus reveals defective areas. A variety of **mutual indentation methods** (Cowdrey, "Hardness by Mutual Indentation," *Proc. A.S.T.M.*, 30, II, 1930, p. 559) in which crossed cylinders or prisms of the material to be tested are forced together, give results comparable with the Brinell test. These are particularly useful on wires and on materials at high temperatures.

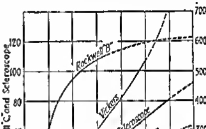


FIG. 17.—Hardness Scales.

The relation among the scales of the various hardness methods is not exact, since no two measure exactly the same sort of hardness and a relationship determined on steels of different hardnesses will be found only approximately true with other materials. The **Vickers-Brinell relation** is nearly linear up to at least 400, with the Vickers approximately 5 percent higher than the Brinell (actual values run from +2 to +11 percent) and nearly independent of the material. Beyond 500, the values become more widely divergent owing to the flattening of the Brinell ball. The **Brinell-Rockwell relation** is fairly satisfactory and is shown in Fig. 17. Approximate relations for the Shore Scleroscope are also given on the same plot.

The **hardness of wood** is defined by the A.S.T.M. as the load in pounds required to force a ball 0.444 in. diam into the wood to a depth of 0.222 in., the speed of penetration being ¼ in. per min. For a summary of the work in hardness to 1929, see Hankins, "Synopsis of Knowledge of Hardness Testing," *Proc. Inst. M.E.*, 1929, Part I, p. 317.

Testing of Materials

Testing Machines. Machines for the mechanical testing of materials usually contain elements (1) for gripping the specimen, (2) for deforming it, and (3) for measuring the load required in performing the deformation.

Table 7. Torsional Tests on Metals and Alloys Subjected to High Temperatures

(Torsional strength and elastic limit in in.-lb per sq in. Total twist in number of turns, with number of degrees excess. Test bars turned to 0.855 in. diam from 1¼ in. rods.)

Materials	Temperatures, deg F			
	70	385	600	800
[1] Cold-rolled Shafting:				
Torsional strength	72,400	82,350	41,175	12,350
Elastic limit	41,050	32,800	26,350	6,550
Twist	3 and 152°	1 and 265°	16	8 and 132°
[2] Cold-rolled Shafting:				
Torsional strength	68,990	77,210	33,680	17,250
Elastic limit	42,710	29,430	25,460	9,034
Twist	3 and 180°	1 and 345°	9 and 115°	12
[3] Machinery Steel:				
Torsional strength	59,590	49,260	33,061	26,530
Elastic limit	24,530	9,830	7,310	2,450
Twist	7 and 130°	3 and 190°	12 and 120°	61 and 260°
[4] Nickel-vanadium Steel:				
Torsional strength	101,200	82,500	19,590	14,340
Elastic limit	57,200	36,200	13,060	6,560
Twist	3 and 140°	3 and 10°	3 and 110°	8 and 35°
[5] Nickel Steel:				
Torsional strength	91,900	64,490	41,300	....
Elastic limit	17,250	8,150	6,560	6,560
Twist	9 and 30°	8 and 115°	8 and 40°	8 and 132°
[6] Nickel Steel:				
Torsional strength	104,800	78,550	53,170	25,350
Elastic limit	21,800	15,700	10,900	6,040
Twist	11 and 100°	6 and 330°	6 and 195°	9 and 210°
Rolled Monel Metal:				
Torsional strength	91,990	78,030	54,210	38,600
Elastic limit	37,780	36,140	19,800	10,680
Twist	11 and 150°	4 and 320°	4 and 90°	7 and 50°
[7] Rolled Monel Metal:				
Torsional strength	94,610	83,030	72,290	40,610
Elastic limit	45,510	33,940	31,300	10,910
Twist	12 and 150°	5	5 and 205°	6 and 240°
[8] Cold-rolled Shafting:				
Torsional strength	83,840	76,650	15,920	7,180
Elastic limit	42,540	36,800	2,040	1,630
Twist	2 and 280°	2 and 30°	8 and 230°	39 and 195°
[9] Vanadium Tool Steel:				
Torsional strength	137,295	124,080	67,710	
Elastic limit	54,100	37,750	17,820	
Twist	345°	1 and 250°	320°	
[10] Rod Brass:				
Torsional strength	51,200	43,780	14,190	
Elastic limit	32,600	22,180	4,100	
Twist	2 and 225°	2 and 185°	5 and 40°	
Tobin Bronze:				
Torsional strength	61,200	36,560	8,860	
Elastic limit	26,850	9,800	1,630	
Twist	2 and 155°	4	3 and 255°	
[11] Phosphor Bronze:				
Torsional strength	70,000	51,020	19,920	
Elastic limit	34,250	21,240	6,560	
Twist	12 and 95°	1 and 215°	15 and 200°	
Delta Metal:				
Torsional strength	61,630	42,860	3,265	
Elastic limit	26,940	14,600	1,360	
Twist	180°	3 and 235°	4 and 300°	
Manganese Bronze:				
Torsional strength	61,630	37,630	8,980	
Elastic limit	22,040	13,060	3,260	
Twist	2 and 5°	3 and 110°	2 and 10°	

For notes see page 428.

Serrated grips may be used to hold ductile materials or the shanks of other holders in tension; a taper of 1 in 6 on the wedge faces gives a self-tightening action without excessive jamming. Ropes are ordinarily held by wet eye splices, but braided ropes or small cords may be given several turns over a fixed pin and then clamped. Wire ropes should be zincked into forged sockets (solder and lead have insufficient strength).

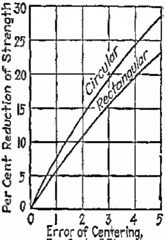


FIG. 18.—Effect of Centering Errors on Brittle Test Specimens.

Accuracy and Calibration. The A.S.T.M. (specification E4-36) requires that commercial machines shall have errors of less than 1 percent within the "loading range" when checked against acceptable standards of comparison at at least five suitably spaced loads. The "loading range" may be any range through which the preceding requirements for accuracy are satisfied, except that it shall not extend below 100 times the least load to which the machine will respond or which can be read on the indicator. Brinell-hardness machines are permitted errors up to 3 percent. The use of calibration plots or tables to correct the results of an otherwise inaccurate machine is not permitted under any circumstances. Machines with errors less than 0.1 percent have recently become commercially available (Tate-Emery, and others), and somewhat greater accuracy is possible in the most refined research apparatus.

Dead loads may be used to check machines of low capacity; accurately calibrated proving levers may be used to extend the range of available weights. Various elastic devices (such as the Morehouse proving ring) made of specially treated steel, with sensitive distortion measuring devices, and calibrated by dead weights at the Bureau of Standards, are made in capacities from 1,000 to 100,000 lb and are the most satisfactory means of checking the higher loads. In an emergency, a number of specimens may be cut from a uniform bar and half the number tested in the machine in question while the remainder are tested in a machine of known accuracy.

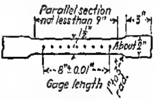


FIG. 19.—Tension Specimen for Plate Stock (A.S.T.M. E8-36).

In general, the **testing speed** should be such that the weighing mechanism will accurately follow the load on the specimen. Specified maximum testing speeds (A.S.T.M.) are: yield strength, ⅛ in. per min; tensile strength on ductile materials, 1½ in. per min; compression, 1 in. specimens, 0.05 in. per min; specimens 3 in. and over, 0.10 in. per min; cement-mortar briquettes, 575 lb per min min, 625 lb per min max; concrete and brick, 0.05 in. per min; wood beams 2 × 2 × 30, 0.10 in. per min; wood compression 2 × 2 × 8, 0.024 in. per min.

Standard **forms of test specimens** (A.S.T.M.) are shown in Figs. 19 to 25. In wrought materials, and particularly in those which have been cold-worked, different properties may be expected in different directions with respect to the direction of the applied work, and the test specimen should be cut from the

Table 8. Stresses for Given Creep Rates and Temperatures
(Compiled from A.S.M.E. and A.S.T.M. "Creep Data")
Based on 1,000 hr tests. Stresses in 1,000 lb per sq in.

Material	Creep rate, 0.1 percent per 1,000 hr					Creep rate 0.01 percent per 1,000 hr				
Temperature, deg F	800	900	1000	1100	1200	800	900	1000	1100	1200
Wrought steels:										
S.A.E. 1015	17–27	11–18	3–12	2–7	1	10–18	6–14	3–8	1	
0.20 C, 0.50 Mo	26–33	18–25	9–16	2–6	1–2	16–24	11–22	4–12	2	1
0.10–.25 C, 4–6 Cr + Mo	22	15–18	9–11	3–6	2–3	14–17	11–15	4–7	2–3	1–2
S.A.E. 4140	27–33	20–25	7–15	4–7	1–2	19–28	12–19	3–8	2–4	1
S.A.E. 1030–1045	8–25	5–15	5	2	1	5–15	3–7	2–4	1	
Commercially pure iron	7	.....	4	...	3	5	.....	2		
0.15 C, 1–2.5 Cr, 0.50 Mo	25–35	18–28	8–20	6–8	3–4	20–30	12–18	3–12	2–5	1–2
S.A.E. 4340	20–40	15–30	2–12	1–3	...	8–20	.....	1–6		
S.A.E. X3140	7–10	.....	.5–4	...	...	3–8	.....	1–2		
0.20 C, 4–6 Cr	30	10–20	7–10	1	...	.....	.....	3–5		
0.25 C, 4–6 Cr + W	30	10–15	4–10	2–8	...	.....	6–11	2–7		
0.16 C, 1.2 Cu	.....	18	18–15	3	1	.....	10–18	7–12		
0.20 C, 1 Mo	35	27	12	...	...	25	12	6		
0.10–0.40 C, 0.2–0.5 Mo, 1–2 Mn	30–40	12–20	4–14	...	...	25–28	8–15	2–8	...	0.5
S.A.E. 2340	7–12	5	2							
S.A.E. 6140	30	12	4	...	...	7	6	1		
S.A.E. 7240	30	21	6–15	2	...	30	11	3–9	1	
Cr + Va + W, various	20–70	14–30	5–25	...	...	18–50	8–18	2–13		
Temperature, deg F	1100	1200	1300	1400	1500	1000	1100	1200	1300	1400
Wrought Chrome-nickel Steels:										
"18-8"[a]	18–18	5–11	3–10	2–5	2.5	11–16	5–12	2–10	...	1–2
10–25 Cr, 10–30 Ni[b]	18–20	5–15	3–10	2–5	...	.....	6–15	3–10	2–8	1–3
Temperature, deg F	800	900	1000	1100	1200	800	900	1000	1100	1200
Cast Steels:										
0.20–0.40 C	10–20	5–10	3	...	...	8–15	.....	1		
0.10–0.30 C, 0.5–1 Mo	28	20–30	6–12	2	...	20	10–15	2–5		
0.15–0.30 C, 4–6 Cr + Mo	25–30	15–25	8–15	8	...	20–25	9–15	2–7	2	
"18-8"[c]	.....	.....	20–25	15	10	.....	.....	20	15	8
Cast iron	20	8	4	...	...	10	.....	2		
Cr Ni cast iron	.....	.....	9	...	...	.....	.....	3		

[a] Additional data. At creep rate **0.1** percent and 1000 (1600) F the stress is 18–25 (1); at creep rate 0.01 percent and 1500 F, the stress is 0.5.
[b] Additional data. At creep rate 0.1 percent and 1000 (1600) F the stress is 20–30 (1).
[c] Additional data. At creep rate 0.1 percent and 1600 F the stress is 3; at creep rate 0.01 and 1500 F, the stress is 2–3.

Additional temperatures (deg F) and stresses (in 1,000 lb per sq in.) for stated creep rates (percent per 1,000 hr) for wrought non-ferrous metals are as follows:

60–40 *brass*. Rate 0.1, temp. 350 (400), stress 8 (2); rate 0.01 temp 300 (350) [400], stress 10 (3) [1].

with a simple lever, as in the **Berry strain gage** (F. F. Metzger & Son, Philadelphia), the dial gage reads reliably to 0.0001 in. over gage lengths of 2 to 20 in. The **Huggenberger Tensometer** (Baldwin-Southwark Corp.) is a very light instrument employing a compound lever system magnifying about 1,000 times and giving readings to 0.00001 in. or better over gage lengths of 0.4 to 8 in. (see Vose, "Characteristics of the Huggenberger Tensometer," *Proc. A.S.T.M.*, 34, 1934, p. 862).

Optical gages have sometimes used microscopes, but their weight renders them extremely inconvenient, whereas the "optical lever" (light beam reflected from a tilting mirror) furnishes an extremely light and sensitive instrument. Tilting mirrors may be easily constructed for any use and can be arranged for any desired magnification, the most refined commercial type being the **Tuckerman gage** (American Instrument Co., Washington, D. C.). Interferometers are sensitive to one millionth inch, are self-calibrating, and can be arranged in numerous ways, but their reading is somewhat difficult (see Tuckerman, "Optical Strain Gages and Extensometers," *Proc. A.S.T.M.*, 23, 1923, p. 602; and Vose, "An Application of the Interferometer Strain Gage in Photoelasticity," *Trans. A.S.M.E.*, 57, 1935, p. A99). In the **deForest Scratch Gage** (Baldwin-Southwark Corp.), a sharp scriber is dragged over a polished target by the deformation, and the inscribed record is later observed under a microscope. Although the deformations recorded on the scratch gage cannot be read to much closer than 0.0001 in., it is the only mechanical device suitable for vibratory stresses and, in addition, is extremely light and cheap and furnishes a permanent record. Some of the more rugged tilting-mirror gages may be used for vibratory stresses at low frequencies.

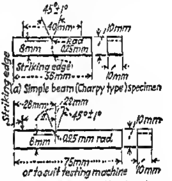


FIG. 25.—Impact Test Specimens (A.S.T.M. E23-34T).

The **electrical type of strain gage** is particularly adapted to vibratory stresses (see p. 525); some of the best types retain their calibration sufficiently well to be useful in measuring static stresses. Change in resistance is almost always employed, with a sensitive element similar to a carbon microphone in the older types (McCollum-Peters Telemeter, Baldwin-Southwark Corp.) and carbon or carbon-bearing strip (Ess Strip, Baldwin-Southwark) or special wire (Ruge-deForest, Baldwin-Southwark, and Davis-Carlson) in more recent types. The Ess Strip may be cemented directly to the surface to be investigated; the wire types may be made extremely sensitive, and it is claimed that as low as 3 lb per sq in. in steel may be detected with them. Changes in magnetic reluctance and in electrical-condenser capacity have also been used in strain gages, but their most satisfactory function seems to be as a null indicator rather than as a measuring device.

For a general indication of strain distribution, a "brittle varnish" ("Stresscoat," Magnaflux Corp., Chicago) may be spread into otherwise inaccessible places and used to indicate tensile stresses and their directions through the cracks that appear when the base material is deformed.

avoid flattening the indentor. The standard size ball is 10 mm and the standard loads 3,000 kg for steels and 500 kg for the softer non-ferrous metals, with 100 kg occasionally being used for very soft materials. If for special reasons any other size of ball is used, the load should be adjusted approximately as follows: for iron and steel, $P = 30D^2$; for brass, bronze, and other soft metals, $P = 5D^2$; for extremely soft metals, $P = D^2$ (see "Methods of Brinell Hardness Testing," A.S.T.M. Specification E10-27). Readings obtained with other than the standard ball and loadings should have the load and ball size appended, as such readings are only approximately equal to those obtained under standard conditions.

The size of the specimen should be sufficient to ensure that no part of the plastic flow around the impression reaches a free surface, and in no case should the thickness be less than 10 times the depth of the impression or the width less than 3 times the diameter of the impression. The load should be applied steadily and should remain on for at least 10 sec in the case of ferrous materials and 30 sec in the case of most non-ferrous materials. Longer periods may be necessary on certain soft materials that exhibit creep at room temperature. In testing thin materials, it is not permissible to pile up several thicknesses of material under the indentor, as the readings so obtained will invariably be lower than the true readings. With such materials, smaller indentors and loads, or different methods of hardness testing, are necessary.

In the standard Brinell test, the diameter of the impression is measured with a low-power hand microscope, but for production work there are available several testing machines which automatically measure the depth of the impression and from this give readings of hardness. Such machines should be frequently calibrated on test blocks of known hardness.

In the **Rockwell method** of hardness testing, the depth of penetration of an indentor under certain arbitrary conditions of test is determined. The indentor may be either a steel ball of some specified diameter, or a spherical-tipped conical diamond of 120 deg angle and 0.2 mm tip radius, called a "Brale." A *minor load* of 10 kg is first applied which causes an initial penetration and holds the indentor in place. Under this condition, the depth-measuring scale is set to its arbitrary maximum value of 130 if any of the balls are used, or to 100 if the Brale is used. A *major load* of 60, 100, or 150 kg is then applied under dashpot control and then removed, returning to the minor load of 10 kg. The hardness number may then be read directly from the scale which measures penetration, and this scale is so arranged that soft materials with deep penetrations give low hardness numbers.

A variety of combinations of indentor and major load are possible; the most commonly used are R_B using as indentor a $\frac{1}{16}$ in. ball and a major load of 100 kg and R_C using a Brale as indentor and a major load of 150 kg (see "Rockwell Hardness Testing of Metallic Materials," A.S.T.M. Specification E18-36).

As compared with the Brinell test, the Rockwell method makes a smaller indentation, may be used on thinner material, and is much more rapid since hardness numbers are read directly and need not be calculated. However, the Brinell test may be made without special apparatus and is somewhat more widely recognized for laboratory use. There is also a **Rockwell superficial hardness** test similar to the regular Rockwell, except that the indentation is much shallower.

The **Vickers** method of hardness testing is similar in principle to the Brinell in that it expresses the result in terms of the pressure under the indentor and uses the same units, kilograms per square millimeter. The indentor is a diamond in the form of a square pyramid with an apical angle of 136 deg, the loads are much lighter, varying between 2 and 120 kg, and the

Stress is an internal distributed force; it is the internal mechanical reaction of the material accompanying deformation. Stresses always occur in pairs. Stresses are **normal** [tensile stress (+) and compressive stress (−)]; and **tangential, or shearing.**

Intensity of stress, or unit stress, S (lb per sq in.) is the amount of force per unit of area of surface (Fig. 3). P is the load acting through the center of gravity of the area. The uniformly distributed normal stress is

$$S = P/A$$

When the stress is not uniformly distributed, $S = dP/dA$.

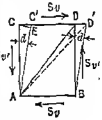


FIG. 2.

A long rod will stretch under its own weight G and a terminal load P (see Fig. 4). The total elongation e is that due to terminal load plus that due to one-half the weight of the rod considered as acting at the end.

$$e = [Pl + (Gl/2)]/AE$$

The maximum stress is at the upper end.

When a load is carried by several paths to a support, the different paths take portions of the load in proportion to their stiffness, which is controlled by material (E) and by design.

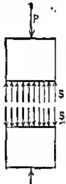


FIG. 3.

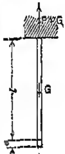


FIG. 4.

Example. Two pairs of bars rigidly connected (with the same elongation) carry a load P_0 (Fig. 5). A_1, A_2 and E_1, E_2 and P_1, P_2 and S_1, S_2 are cross sections, moduli of elasticity, loads and stresses of the bars, respectively; e = elongation.

$e = P_1l/(E_1A_1) = P_2l/(E_2A_2)$; $P_0 = 2P_1 + 2P_2$;
$S_2 = P_2/A_2 = \frac{1}{2}[P_0E_2/(E_1A_1 + E_2A_2)]$ and $S_1 = \frac{1}{2}[P_0E_1/(E_1A_1 + E_2A_2)]$.

Temperature Stresses. When the deformation arising from change of temperature is prevented, temperature stresses arise that are proportional to the amount of deformation that is prevented. Let a = coefficient of expansion per degree of temperature, l_1 = length of bar at temperature t_1, and l_2 = length at temperature t_2. Then

$$l_2 = l_1[1 + a(t_2 - t_1)]$$

If, subsequently, the bar is cooled to a temperature t_1, the proportionate deformation is $s = a(t_2 - t_1)$ and the corresponding unit stress $S = Ea \times (t_2 - t_1)$. For **coefficients of expansion**, see p. 298. In the case of steel, a change of temperature of 12 F will cause a unit stress of 2,340 lb per sq in.

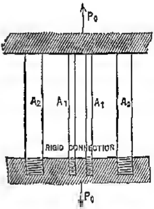


FIG. 5.

Shearing stresses (Fig. 2) act tangentially to the surface of contact and do not change length of sides of elementary volume; they **change the angle between faces** and the **length of diagonal.** Two pairs of shearing stresses must act together. **Shearing stress intensities are of equal magnitude on all four faces of an element.** $S_v = S_v'$ (Fig. 6).

Some machines (ductility testers) omit the measurement of load and substitute a measurement of deformation, whereas other machines include the measurement of both load and deformation through apparatus either integral with the testing machine (stress-strain recorders) or auxiliary to it (strain gages). In most general-purpose testing machines, the deformation is controlled as the independent variable and the resulting load measured, and in many special-purpose machines, particularly those for light loads, the load is controlled and the resulting deformation is measured. Special features may include those for constant rate of loading (pacing disks), for constant rate of straining, for constant load maintenance, and for cyclical load variation (fatigue).

A nut and screw combination, driven through clutches and change gears, was formerly common for **producing deformation**, but this arrangement has been largely superseded by a pump and hydraulic cylinder. The hydraulic arrangement is quieter, more flexible in control, and more durable, but cannot maintain constant deformation for any length of time on account of leakage. In **weighing light loads**, direct weights are accurate, but unless lead shot or water is used small variations in load are inconvenient. Ordinary springs used for this purpose are not sufficiently accurate, but the recently developed "Iso-Elastic" spring (John Chatillon & Sons, New York) has negligible errors. For larger loads, a reducing device with a **hydraulic weighing system** has largely displaced the lever system. Accurately lapped pistons may be made nearly frictionless; in some systems, an oscillatory rotation imposed on either the piston or the cylinder is used to remove any remaining friction. In the Emery hydraulic support, the load is taken by liquid pressure and is applied through a piston of considerable clearance sealed to the cylinder wall by a flexible flat metal diaphragm; although only minute displacement is possible, this system possesses great accuracy and sensitivity. For weighing the reduced load of a lever system, there may be used dead weights (lead shot), gravity pendulums, scale-and-rider combinations, or Iso-Elastic springs. In hydraulic systems, the liquid pressure is occasionally balanced by hydrostatic head, but may be transferred to a mechanical force through small pistons or Emery supports and balanced as for lever systems. Specially accurate pressure gages may be used (Emery-Tatnall system, Baldwin-Southwark Corp., Philadelphia); in the Tate-Emery system (Baldwin-Southwark), the pressure gage element is used as a null indicator, being balanced by Iso-Elastic springs, and gives particularly accurate and sensitive readings. Many systems have included means of **recording automatically** the load and deformation of the test piece, but few have been satisfactory until recent years (Emery-Tatnall and Tate-Emery). (For further information, see Gibbons, "Materials Testing Machines," Instruments Publishing Co., also in *Instruments*, 7, 8, 1934–1935, and *Bull. A.S.T.M.*, Oct., 1939.)

Grips should not only hold the test specimen against slippage but should also apply the load in the desired manner. Centering of the load is of great importance in compression testing, and should not be neglected in tension testing if the material is brittle. Figure 18 shows the theoretical errors due to off-center loading; the results are directly applicable to compression tests using swivel loading blocks. Swivel (ball-and-socket) holders or compression blocks should be used with all except the most ductile materials, and, in compression testing of brittle materials (concrete, stone, brick), any rough faces should be smoothly capped with plaster of Paris or, for greater strength, a mixture of two-thirds plaster of Paris and one-third Portland cement.

For an I-shaped cross section (Fig. 10*d*),

$$S_s \text{ (max)} = \frac{3}{4}\frac{Q}{a}\frac{bc^2 - (b-a)f^2}{bc^3 - (b-a)f^3} \text{ for } y = 0.$$

ELASTICITY

Elasticity is the ability of a material to return to its original dimensions after the removal of stresses. The **elastic limit** S_p is the limit of stress within which the deformation completely disappears after the removal of stress; *i.e.*, no set remains (see p. 414).

Hooke's law states that, within the elastic limit, deformation produced is proportional to the stress. Unless modified, the deduced formulas of mechanics apply only within the elastic limit. Beyond this, they are modified by experimental coefficients, as, for instance, the modulus of rupture.

The modulus of elasticity (pounds per square inch) is the ratio of the increment of unit stress to increment of unit deformation within the elastic limit.

The modulus of elasticity in tension, or Young's modulus,

$$E = \text{unit stress/unit deformation} = Pl/Ae.$$

The modulus of elasticity in compression is similarly measured.

The modulus of elasticity in shear or coefficient of rigidity, $G = S_s/d$ where d is expressed in radians (see Fig. 2).

The bulk modulus of elasticity K is the ratio of normal stress, applied to all six faces of a cube, to the change of volume.

Change of volume under normal stress. Let l, d, and b represent length, width, and thickness; μ = Poisson's ratio; s = unit deformation. Then deformed volume $= (1 + s)l(1 - \mu s)b(1 - \mu s)d \approx (1 + s - 2\mu s)lbd$. Fractional change of volume $\approx (1 - 2\mu)s$. When μ is less than $\frac{1}{2}$, the volume is increased in tension and decreased in compression. For steel ($\mu = \frac{1}{3}$), change of volume is about 1/3,000 part at the elastic limit.

The following relationships exist between the modulus of elasticity in tension or compression E, modulus of elasticity in shear G, bulk modulus of elasticity, K, and Poisson's ratio μ:

$$E = 2G(1 + \mu)$$
$$G = E/2(1 + \mu)$$
$$\mu = (E - 2G)/2G$$
$$K = E/3(1 - 2\mu)$$
$$\mu = (3K - E)/6K$$

Resilience U (in.-lb) is the potential energy stored up in a deformed body. The amount of resilience is equal to the work required to deform the body from zero stress to stress S, when S does not exceed the elastic limit. For normal stress, resilience = work of deformation = average force times deformation $= \frac{1}{2}Pe = \frac{1}{2}AS \times Sl/E = \frac{1}{2}S^2V/E$.

Modulus of resilience U_p (in.-lb per cu in.), or **unit resilience,** is the elastic energy stored up in a cubic inch of material at the elastic limit. For normal stress,

$$U_p = \tfrac{1}{2}S_p^2/E$$

The unit resilience for any other kind of stress, as shearing, bending, torsion, is a constant times one-half the square of the stress divided by the appropriate modulus of elasticity. For values, see Table 1.

parent material in such a way as to give the strength in the desired direction. With the exception of fatigue specimens and specimens of extremely brittle materials, surface finish is of little practical importance, although extreme roughness tends to decrease the ultimate elongation.

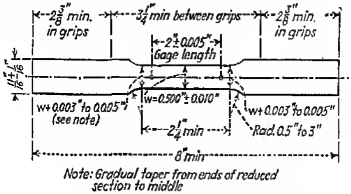


FIG. 20.—Tension Specimen for Sheet Metal (A.S.T.M. E8-36).

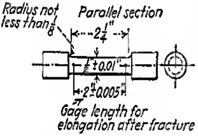


FIG. 21.—Test Specimen, 2 In. by ½ In. in Diameter (A.S.T.M. E8-36).

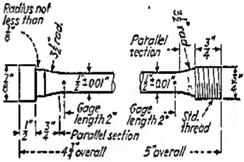


FIG. 22.—Ends for 2 In. by ½ In. Diameter Test Specimen. (A.S.T.M. E8-36).

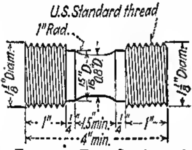


FIG. 23.—Test Specimen for Cast Iron (A.S.T.M. A48-36). Other test pieces 0.505 and 1.26 in. diam.

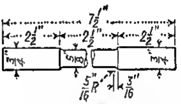


FIG. 24.—Test Specimen for Malleable Iron (A.S.T.M. A197-39).

Strain Gages

For the measurement of deformations of less than 0.01 in., special strain gages should be attached to gage points, on the surface of the specimen, remote from the grips. These gages magnify the deformations by mechanical, optical, or electrical means and indicate visually or on a recording drum or oscillograph.

Mechanical gages include the micrometer screw and, more recently, the dial gage which can be obtained accurate to 0.0001 in. (Ames). Combined

structures; the Forest Products Laboratory of the U. S. Department of Agriculture has issued data for the design of timber structures. Manufacturers and engineers have also formulated allowable unit stresses for the guidance of designers. These specifications are undergoing constant revision with the gain of more accurate knowledge of the factors on which the factor of safety is based.

Consideration having been given to the several factors mentioned above, the designer may be guided by the following procedures for determining allowable unit stresses at normal temperature.

Static Loading. For ductile materials, an allowable unit stress equal to one-third to two-thirds (commonly one-half) of the unit stress at the yield point, and, for brittle materials, an allowable unit stress equal to one-sixth the ultimate unit stress will ordinarily provide for conservative designs. Thus for medium carbon steel, the allowable unit stress would be $35{,}000/2 = 17{,}500$ lb per sq in. and for cast iron it would be $24{,}000/6 = 4{,}000$ lb per sq in.

Dynamic Loading. For cases where the stress varies from a positive maximum to a negative maximum of the same amount, the allowable unit stress of one-third of the endurance limit is recommended for ordinary engineering materials.

Where **static and variable stresses** are **superimposed**, Soderberg recommends that the factor of safety be found from $1/n = s_0/s_y + Ks_r/s_e$ where n = factor of safety, s_0 = static loading, allowable unit stress, s_y = stress at the yield point, K = a factor to allow for stress concentrations or the effect of discontinuities (see p. 421), s_r = allowable unit stress for dynamic loading, and s_e = endurance limit.

IMPACT ON BARS

A **static load** is one at rest. **Dynamic loads** arise from impact. A **suddenly applied load** arises when a load that is just touching a bar is suddenly released. The velocity of approach is zero, the load is constant throughout the entire deformation, and the internal force in the bar increases from zero to some value AS. The stresses and deformations are double those due to an equal static load. G = weight producing a suddenly applied load. e = maximum elongation. P = internal force accompanying e. Work of weight Ge equals internal resilience $\frac{1}{2}Pe$. Therefore, $P = 2G$. The bar springs back and oscillates.

An **impact loading** occurs when a moving weight strikes the end of a bar. The velocity of impact is v. If the elastic limit is not exceeded, the external work of the moving weight falling from height h equals Gh, and also equals the resilience developed. P, S, and e are, respectively, dynamic force, stress, and elongation in bar. $G(h + e) = \frac{1}{2}Pe$. Let e' and S' be the elongation and stress which would accompany G if it acted as a static load. $G/e' = P/e$. Then

$$S = S' + S'[1 + (2h/e')]^{\frac{1}{2}}$$
$$e = e' + e'[1 + (2h/e')]^{\frac{1}{2}}$$

The value of e' is small, and dynamic stresses and deformations are usually large and may exceed the elastic limit. It is here assumed that the entire energy of the weight produces elastic deformation in the bar only, *i.e.*, that the supports are rigid and there is no friction.

Materials show a higher elastic limit of deformation under impact than under static loads. E appears to be unchanged (*Proc. Roy. Soc.*, Feb., 1905).

MECHANICS OF MATERIALS

BY

WILLIAM KENDRICK HATT

Revised by A. Haertlein

REFERENCES: Case, "Strength of Materials," Longmans. Moore, "Textbook of the Materials of Engineering," McGraw-Hill. Morley, "Strength of Materials," Longmans. Seely, "Resistance of Materials," Wiley. Swain, "Strength of Materials," McGraw-Hill. Timoshenko, "Strength of Materials," Van Nostrand. White, "Engineering Materials," McGraw-Hill.

Main Symbols

UNIT STRESS		Bulk	K
Stress, apparent	S	Modulus of resilience	U_p
Pure shearing	S_s	Ultimate resilience	U_R
True (ideal) stress	T	GEOMETRICAL	
Proportional elastic limit	S_p	Length	l
Yield point	S_y	Area	A
Ultimate strength, tension	S_M	Volume	V
Ultimate compression	S_c	Velocity	v
Vertical shear in beams	S_v	Radius of gyration	r
Modulus of rupture	S_R	Rectangular moment of inertia	I
MOMENT		Polar moment of inertia	I_P
Bending	M	DEFORMATION	
Torsion	M_t	Gross, longitudinal	e
EXTERNAL ACTION		Unit, longitudinal	s
Force	P	Angular	d
Weight of body	G	Lateral	s'
Weight of load	W	Poisson's ratio	μ
External shear	V	Reciprocal of Poisson's ratio	n
MODULUS OF ELASTICITY		Radius	r
Longitudinal	E	Deflection	f
Shearing	G		

DEFINITIONS AND SIMPLE STRESSES

Deformations are changes in form produced by external forces or loads that act on non-rigid bodies. Deformations are **longitudinal**, e, a lengthening (+) or shortening (−) of the body; and **angular**, d, a change of angle between the faces.

Unit deformation (dimensionless number) is the deformation in unit distance. Unit longitudinal deformation $s = e/l$ (Fig. 1). Unit angular deformation tan d equals d approx (Fig. 2).

Accompanying a longitudinal deformation e is a **lateral** deformation e' (Fig. 1). The ratio of s'/s is **Poisson's ratio** μ. Values of μ are: glass, 0.244; brass, 0.333; copper, 0.333; cast iron, 0.270; wrought iron, 0.278; steel, 0.303; lead, 0.430; concrete, 0.10 to 0.20 at working stresses and 0.25 at higher stresses.

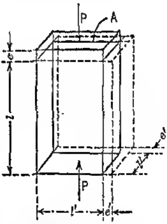


FIG. 1.

$$N = S\cos^2 a + S'\sin^2 a \quad (1)$$
$$T = (S - S')\sin a\cos a \quad (2)$$

$N_{max} = S$ or S' and $T_{max} = \frac{1}{2}(S - S')$ when $a = 45$ deg. If $a + a' = 90$ deg, $N + N' = S + S'$ and $T = T'$. The resultant stress R is the resultant of N and T. $R = \sqrt{N^2 + T^2}$. The direction angle of R is b, where $\sin b = \sin 2a\,(S - S')/2R$.

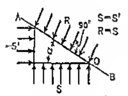


FIG. 13.

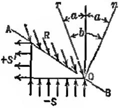


FIG. 14.

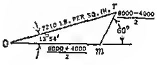


FIG. 15.

Special Cases under Case II. (a) If $+S = +S'$ (Fig. 13), $T = 0$, $R = S = S'$, and direction of R is normal. (b) If $+S = -S'$ (Fig. 14), $R = S = S'$, and *direction angle of $R = 2a$. S bisects the angle nOr.*

Example. The metal in a steam boiler is subjected to a unit stress of 8,000 lb per sq in. in a circumferential direction and 4,000 lb per sq in. in a longitudinal direction. Find the stresses on a plane the normal to which makes an angle of 30 deg with the direction of the 8,000 lb stress. From (1) and (2).

Normal stress $N = 8000\cos^2 30° + 4000\sin^2 30° = 7{,}000$ lb per sq in.
Tangential stress $T = (8000 - 4000)\sin 30°\cos 30° = 1{,}730$ lb per sq in.
Resultant stress $R = \sqrt{7000^2 + 1730^2} = 7{,}210$ lb per sq in.

$\sin b = \sin(2 \times 30°)(8000 - 4000)/(2 \times 7210) = 0.24023$, whence angle b, or the direction angle of R, is 13° 54'. For graphical solution, see Fig. 15.

A block (Fig. 16) is acted upon by three pairs of normal stresses S_x, S_y, and S_z. The deformation e_x along axis X is due to S_x modified by the lateral effects of S_y and S_z (see p. 437). When the stresses are all of the same sign, $e_x = (S_x/E) - [(S_y + S_z)/nE]$, where n = reciprocal of Poisson's ratio, whence

$$Ee_x = S_x - [(S_y + S_z)/n]$$
$$Ee_y = S_y - [(S_x + S_z)/n]$$
$$Ee_z = S_z - [(S_x + S_y)/n]$$

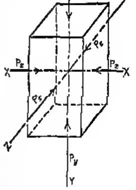


FIG. 16.

When the stresses are of different sign, corresponding changes occur in the formulas. The product Ee is regarded as a stress. The **apparent stress** is S, but the **true stress** $T = Ee$.

Elastic strength under compound stress is fixed, by three separate theories, as a certain value of either (1) the maximum principal apparent stress; (2) the true stress accompanying maximum principal deformation; or (3) the maximum shearing stress.

For example, metal in a boiler shell is under a hoop tension S_x and a longitudinal tension $S_y = \frac{1}{2}S_x$. According to (1), metal will begin to fail when S_x equals the elastic limit of the metal, regardless of the presence of S_y. According to (2), the necessary stress at failure is the true stress $T_x = Ee_x = S_x - S_y/n$. When $n = 3$, $T_x = \frac{5}{6}S_x$. That is, a hoop tension one-fifth greater than the elastic limit under simple stress, as

In the presence of **pure shear on external faces** (Fig. 6), the **resultant stress** S on one diagonal plane at **45 deg** is pure tension and on the other diagonal plane pure compression; $S = S_v = S'_v$. S on diagonal plane is called "diagonal tension" by writers on reinforced concrete. Failure under

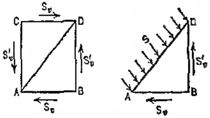


FIG. 6.

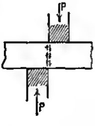


FIG. 7.

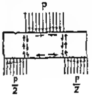


FIG. 8.

pure shear is difficult to produce experimentally, except under torsion and certain special cases. Figure 7 shows an ideal case and Fig. 8 a common form of test piece that introduces bending stresses.

Let Fig. 9 represent the section of area A on which a shearing force Q acts. Then, if pure shear should exist, $S_v = Q/A$. This would be uniformly distributed over the area A. When shear **is accompanied by bending (horizontal shear in beams)**, the unit shear S_v increases from the extreme fiber to the neutral axis OX. The unit shear parallel to axis OX at any point y distant from the neutral axis as at P (Fig. 9) is

$$S_v = \left(Q\int_y^e yz\,dy\right)/Iz$$

where I is the moment of inertia of the cross section about OX.

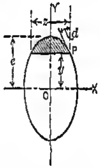


FIG. 9.

For a rectangular cross section (Fig. 10a),

$$S_v = \frac{3}{2}\frac{Q}{bh}\left[1 - \left(\frac{2y}{h}\right)^2\right]; \qquad S_v\ (\text{max}) = \frac{3}{2}\frac{Q}{bh} = \frac{3}{2}\frac{Q}{A},\ \text{for } y = 0.$$

For a circular cross section (Fig. 10b),

$$S_v = \frac{4}{3}\frac{Q}{\pi r^2}\left[1 - \left(\frac{y}{r}\right)^2\right]; \qquad S_v\ (\text{max}) = \frac{4}{3}\frac{Q}{\pi r^2} = \frac{4}{3}\frac{Q}{A},\ \text{for } y = 0.$$

For a circular ring (thickness small in comparison with the major diameter), $S_v\ (\text{max}) = 2Q/A$, for $y = 0$.

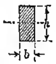


FIG. 10a.

FIG. 10b.

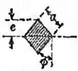


FIG. 10c.

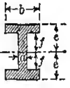


FIG. 10d.

For a square cross section (diagonal vertical, Fig. 10c),

$$S_v = \frac{Q\sqrt{2}}{a^2}\left[1 + \frac{y\sqrt{2}}{a} - 4\left(\frac{y}{a}\right)^2\right]. \qquad S_v\ (\text{max}) = 1.591\,\frac{Q}{A},\ \text{for } y = \frac{e}{4}.$$

obtainable thicknesses of wall. The apparent fiber stress under which the different tubes failed varied from about 7,000 lb per sq in. for the relatively thinnest to 35,000 lb per sq in. for the relatively thickest walls. Since the average yield point of the material was 37,000 and the tensile strength 58,000 lb per sq in., it would appear that the strength of a tube subjected to a collapsing fluid pressure is not dependent alone upon either the elastic limit or ultimate strength of the material constituting it. Prof. Stewart therefore suggests as a substitute for formula (a) the following:

$$P = 50{,}210{,}000(t/D)^3 \qquad (c)$$

The experiments by Stewart are considered to be the most reliable.

Carman (*Univ. Ill. Eng. Exp. Sta. Bull.* No. 5, June 1, 1906) states that the portion affected by collapse from hydraulic pressure is generally not greater in length than $12D$, that for greater lengths the collapsing pressure is independent of the length, and that the often-quoted law of Fairbairn type, in which the collapsing pressure varies inversely as the length, is only true for short tubes from $4D$ to $6D$ in length.

Carman's formulas are as follows:

For thin cold-drawn seamless steel tubes: $P = 50{,}200{,}000(t/D)^3$.

For thin brass tubes: $P = 25{,}150{,}000(t/D)^3$.

For thick cold-drawn seamless steel tubes: $P = 95{,}520(t/D) - 2090$.

For thick lap-welded steel tubes: $P = 83{,}270(t/D) - 1025$.

For thick brass tubes: $P = 93{,}365(t/D) - 2474$.

Limits of (t/D) are below 0.025 for thin tubes and above 0.03 for thick tubes.

When (t/D) is less than 0.06, **Carman's approximate formulas** are:

For cold-drawn seamless steel tubing, $P = 1{,}000{,}000(t/D)^2$.

For lap-welded steel, $P = 1{,}125{,}000(t/D)^2$.

These approximate formulas are stated to give satisfactory rough values for tubes of the most common commercial thicknesses.

A. E. H. Love gives the following **rational formulas**: $P = [2E/(1 - m^2)] \times (t/D)^3$ **for thin tubes** and $P = 2S_c[(t/D) - (t/D)^2]$ (special case of Lamé's general formula—see below) **for thick tubes**, where S_c is the ultimate compressive strength (yield point), lb per sq in. The average values of these constants for steel are: $E = 30{,}000{,}000$; $\mu = 0.295$; $S_c = 40{,}000$; and for brass: $E = 14{,}000{,}000$; $\mu = 0.357$; $S_c = 11{,}000$. Hence,

For thin steel tubes: $P = 65{,}720{,}000(t/D)^3$.

For thick steel tubes: $P = 80{,}000[(t/D) - (t/D)^2]$.

For thin brass tubes: $P = 32{,}090{,}000(t/D)^3$.

For thick brass tubes: $P = 22{,}000[(t/D) - (t/D)^2]$.

The **correction factor** for ellipticity and variation in thickness is $C = (D_{min}/D_{max})^3(t_{min}/t_{max})^3$. **Love's formula** becomes $P = [2CE/(1 - \mu^2)] (t/D)^3$ **for thin tubes**, in which $C = 0.69$ for Stewart's lap-welded steel flues, t = average thickness and D = maximum outside diameter, both in inches. **Lamé's formula** becomes $P = 2CS_c(t/D)[1 - C(t/D)^2]$ **for thick tubes**, in which $C = 0.8$.

Hollow sphere with normal pressure P. Pressure on meridian plane $= \pi r^2 P$. Tension or compression per lineal unit of shell $= \pi r^2 P/2\pi r = Pr/2$. Unit stress $S = Pr/2t$. The material carries an equal stress at right angles. True stress $T = S - (S/n) = Pr/3t$.

In thick cylinders (Fig. 18), **the stress is not uniformly distributed.** The hoop tension in the cylinder under internal pressure is greatest at the interior and diminishes toward the exterior. An interior hoop, from Fig. 18, is shown in Fig. 19. Cylinder unit pressure at bore $= p_1$. Radial internal

Table 1. Resilience per Unit of Volume, U_p

(S = longitudinal stress; S_s = shearing stress; E = tension modulus of elasticity; G = shearing modulus of elasticity.)

		TORSION	
Tension or compression....	$\frac{1}{2}S^2/E$	Solid, circular..........	$\frac{1}{4}S_s^2/G$
Shear....................	$\frac{1}{2}S_s^2/G$	Hollow, radii R_1 and R_2..	$\frac{(R_1^2 + R_2^2)}{R_1^2}\frac{1}{4}\frac{S_s^2}{G}$
BEAMS (free ends)		SPRINGS	
Rectangular section, bent in arc of circle; no shear.....	$\frac{1}{6}S^2/E$	Carriage..............	$\frac{1}{6}S^2/E$
Ditto, circular section.....	$\frac{1}{8}S^2/E$	Flat spiral, rect. section.	$\frac{1}{24}S^2/E$
Concentrated center load; rectangular cross section..	$\frac{1}{18}S^2/E$	Helical: axial load, circular wire...............	$\frac{1}{4}S_s^2/G$
Ditto, circular cross section.	$\frac{1}{24}S^2/E$	Helical: axial twist......	$\frac{1}{8}S^2/E$
Uniform load, rectangular cross-section............	$\frac{5}{90}S^2/E$	Helical: axial twist, rect. section...............	$\frac{1}{6}S^2/E$
I-beam section, concentrated center load.........	$\frac{3}{32}S^2/E$		

Unit rupture work U_R, sometimes called **ultimate resilience**, is measured by the area of the stress-deformation diagram to rupture.

$$U_R = \tfrac{1}{3}e_u(S_y + 2S_M), \text{ approx}$$

where e_u is the total deformation at rupture.

For structural steel, $U_R = \frac{1}{3} \times \frac{27}{100} \times [35{,}000 + (2 \times 60{,}000)] =$ 13,950 in.-lb per cu in.

Example 1. A load $P = 40{,}000$ lb compresses a wooden block of cross sectional area $A = 10$ sq in. and length = 10 in., an amount $e = \frac{4}{100}$ in. Stress $S = \frac{1}{10} \times 40{,}000 = 4{,}000$ lb per sq in. Unit elongation $s = \frac{4}{100} \div 10 = \frac{1}{250}$. Modulus of elasticity $E = 4{,}000 \div \frac{1}{250} = 1{,}000{,}000$ lb per sq in. Unit resilience $U_p = \frac{1}{2} \times 4{,}000 \times 4{,}000/1{,}000{,}000 = 8$ *in.-lb per cu in.*

Example 2. A weight $G = 5{,}000$ lb falls through a height $h = 2$ ft; V = number of cubic inches required to absorb the shock without exceeding a stress of 4,000 lb per sq in. Neglect compression of block. Work done by falling weight $= Gh = 5000 \times 2 \times 12$ in.-lb. Resilience of block $= V \times 8$ in.-lb $= 5000 \times 2 \times 12$. Therefore, $V = 15{,}000$ cu in.

ALLOWABLE UNIT STRESSES

The unit stresses used in the proportioning of machines and structures are called allowable unit stresses or **working stresses**. Unless the part is to be designed to fail under a single application of the load, the allowable stress is less than the ultimate stress. The ratio of the ultimate stress to the allowable stress is called the **factor of safety**. Since the stress at the yield point frequently limits the allowable unit stress, the factor of safety is sometimes defined as the ratio of the stress at the yield point to the allowable unit stress.

The magnitude of the **factor of safety** should be determined as a result of a consideration of the following factors: incorrect assumptions on which the computations are based; effects of temperature changes and initial stresses; possible increases of loads; effect of repetitions of stress (see p. 422); variations in the material and inaccuracies in workmanship; defects in the material; deterioration of the material; the effects of dynamic loads; effects of discontinuities due to notches, sharp fillets, and holes (see p. 421).

Numerous organizations have issued specifications which in their judgment provide for adequate designs. The A.S.M.E. has issued a code for the design of pressure vessels; the American Institute of Steel Construction specifications for steel buildings have been adopted in the building codes for many cities; the A.W.S. recommends allowable unit stresses to use in welded

Thick Cylinders with Shrinkage Rings. In these, it can be assumed that the stress under internal pressure is uniformly distributed over each ring, provided the ratio r/t is great.

Referring to Fig. 21, let t = sectional area (sq in.) of the cylinder for unit length in an axial direction; f = sectional area of the shrinkage rings per unit length of cylinder; S_1 = tensional hoop stress in hollow cylinder, lb per sq in.; S_2 = tensional hoop stress in shrinkage rings, lb per sq in.; p = interior pressure, lb per sq in.; and r = interior radius of cylinder, in. Then,

$$f = rp/[S_1(t/f) + S_2]$$

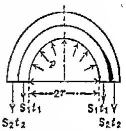


FIG. 21.

If the rings are shrunk on and R = outer radius of the inner cylinder; $R(1 - K)$ = inner radius of the rings before shrinking; $C_1 = 1/E_1$ for the inner cylinder; $C_2 = 1/E_2$ for the shrinkage rings; e_1 and e_2 = unit strains respectively in the inner cylinder and shrinkage rings when cold, then $K = e_1 + e_2 = C_2S_2 - C_1S_1$, and $e_1 = K/[1 + (C_2t/C_1f)]$.

The compressive stress in the cylinder before the pressure p is applied is $S' = e_1/C_1$; and the tensional stress in the shrinkage ring before the pressure p is applied is $S'' = e_2/C_2$. The pressure p causes a tensional stress of $S_1 + S'$ in the cylinder and of $S_2 - S''$ in the shrinkage rings.

The difference K found in this way is a minimum. This difference has to be increased, however, depending upon the condition of the surfaces of the cylinder and rings. If the rings are made of tough material, K may be increased considerably, as the effect is only to produce permanent expansion in the rings, while the inner cylinder will be compressed, thereby diminishing its working tensional stress.

The difference in temperature when the rings are forced on should be $d = K/c_w$, where c_w is the coefficient of expansion of the shrinkage ring.

Oval Hollow Cylinders. In Fig. 22, let a and b be the semiminor and semimajor axes. The bending moments at A and C will then be

$$M_0 = (pa^2/2) - (pI_x/2S) - (pI_y/2S)$$
$$M_1 = M_0 - p(a^2 - b^2)/2$$

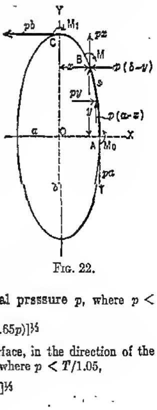


FIG. 22.

where I_x and I_y are the moments of inertia of the arc AC about the x and y axes, respectively. The bending moment at any point will be

$$M = M_0 - (pa^2/2) + (px^2/2) + (py^2/2).$$

Thick Hollow Spheres. With an internal prsssure p, where $p < T/0.65$,

$$r_2 = r_1[(T + 0.4p)/(T - 0.65p)]^{1/3}$$

The maximum tensile stress is on the inner surface, in the direction of the circumference. With an external pressure p, where $p < T/1.05$,

$$r_2 = r_1[T/(T - 1.05p)]^{1/3}$$

In both cases T is the true stress (see p. 444).

As on axial bars, **impact on beams** causes severe dynamic stresses. A weight W falling from height h on the center of a simple beam produces a maximum dynamic deflection f and a dynamic fiber stress S. Let f' and S' be the deflection and stress caused by W acting as a static load. By similar reasoning,

$$S = S' + S'[1 + (2h/f')]^{1/2} \tag{1}$$
$$f = f' + f'[1 + (2h/f')]^{1/2} \tag{2}$$

W must be large compared with the weight of the beam; otherwise, on account of the inertia of the beam, the energy will be used in local damage. Supports are rigid, and there is no friction.

The effect of inertia may be computed by the laws of collision of bodies. Assume impact **entirely inelastic.** nWh is the fractional part of the energy, producing elastic deformation in the beam or bar. W' = weight of beam or bar; W = weight of falling body; $m = W'/W$.

For longitudinal impact on a bar, $n = (1 + \frac{1}{3}m)/(1 + \frac{1}{2}m)^2$.

For center impact on a simple beam, $n = (1 + \frac{17}{35}m)/(1 + \frac{5}{8}m)^2$.

If the beam is fixed at the ends, use $\frac{13}{35}$ instead of $\frac{17}{35}$ and $\frac{1}{2}$ instead of $\frac{5}{8}$. For a cantilever struck at the end, use $\frac{33}{140}$ instead of $\frac{17}{35}$ and $\frac{3}{8}$ instead of $\frac{5}{8}$. **To compute stresses and deflections,** the value of h in (1) and (2) should be multiplied by n.

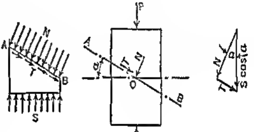


Fig. 11.

COMBINED STRESSES

The stresses treated here are *in one plane.* N is the unit normal stress, T the unit tangential stress on an internal inclined plane, S the unit stress on a plane normal to the geometric axis, P the external force, and R the resultant unit stress.

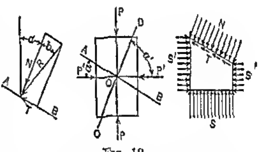


Fig. 12.

Stresses acting on different planes cannot be resolved or compounded like forces. The area upon which the stress acts must appear in the equation.

The resultant of several stresses acting together **on the same plane** is the geometric sum of their individual actions.

Case I. Force along One Axis (Fig. 11). The force P acts alone. The resultant stress on diagonal plane AB is resolved into a normal stress N and a tangential stress T.

$$N = S\cos^2 a \qquad T = S\sin a\cos a$$

The maximum value of T equals $\frac{1}{2}S$ when a equals 45 deg. The stresses on the diagonal plane CD (see Fig. 12) when $a + a' = \pi/2$, are $N' = S\sin^2 a$ and $T' = S\sin a\cos a$.

Case II. Forces along Two Right-angled Axes (Fig. 12). Here the forces P and P' act.

a distributed load may be replaced by its resultant acting at the center of gravity (cg) of the load area.

Simple Beams

The reactions at the support are computed. Thus, in Fig. 23, $\frac{1}{2}Wl = R_1l$, or $R_1 = \frac{1}{2}W$; $R_2 = \frac{1}{2}W$. In Fig. 24, $\frac{1}{3}Wl = R_1l$, or $R_1 = \frac{1}{3}W$; $R_2 = \frac{2}{3}W$. In general, the weight of the beam must be considered.

The **bending moment** (in ft-lb or in.-lb) at any section is the algebraic sum of the moments of the external forces on one side only of the section, or $M = \Sigma(Px)$. It is also equal to the moment of the internal forces or stresses at the section.

Examples: Uniform Load (Fig. 23). $M_A = \Sigma(Px)$ to left of point A at which moment is to be found. $M_A = \frac{1}{2}Wx - (wx \times \frac{1}{2}x)$. **Uniformly Varying Load** (Fig. 24). (Weight of beam neglected.) $M = \Sigma(Px) = \frac{1}{3}Wx - \frac{1}{2}kwx^2\frac{1}{3}x$, where k is a constant ($= h/l$) and w is the weight of the loading per unit of volume.

A **bending moment** that bends a beam convex downward is **positive** (+), and convex upward is **negative** (−).

The **shear** V (lb) is the algebraic sum of the external forces on one side only of the section and parallel to the section or $V = \Sigma P$. In Fig. 23, $V = \frac{1}{2}W - wx$. In Fig. 24, $V = \frac{1}{3}W - \frac{1}{2}kwx^2$

The moment and shear may be expressed by an ordinate drawn to scale at the section of the beam under consideration. A **bending moment diagram** is a series of such ordinates throughout the beam showing the variation of bending moment. A **shear diagram** is similarly constructed. Table 2 shows bending moment and shear diagrams for common cases of flexure, and gives values of reactions (R), moments (M), shears (Q), and deflections (f).

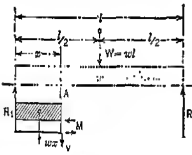


Fig. 23.

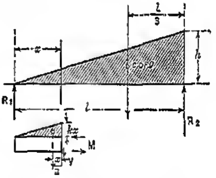


Fig. 24.

Maximum Safe Load on Steel Beams. To obtain maximum safe load (or maximum deflection under maximum safe load) for any of the conditions of loading given in Table 5, multiply the corresponding coefficient in that table by the greatest safe load (or deflection) for distributed load for the particular section under consideration as given in Table 4.

The following factors for reducing the load should be used when beams are long in comparison with their breadth:

Ratio of unsupported (lateral) length to flange width or breadth	20	30	40	50	60	70
Ratio of greatest safe load to calculated load	1	.9	.8	.7	.6	.5

shown in a testing machine, is required to cause failure. This method (2) is commonly used. According to (3), failure will begin when shearing stress $= \frac{1}{2}(S_x - S_y)$ is reached. Researches (1904–1908) by Hancock, Guest, and Coker point to truth of theory (3). This discussion is for elastic stresses and must not be extended to ultimate strength.

CYLINDERS AND SPHERES; TUBES

Thin Cylinders. In a thin envelope or ring (Fig. 17) subjected to an internal unit pressure p, the force of tension in ring P (hoop tension) in a unit length is constant and equal to pr. The uniform tensile stress S in the ring equals pr/t, or $t = pr/S$. An increase in t must be made to compensate for rivet holes in joints.

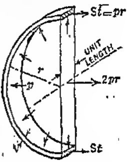


FIG. 17.

A portion of any circular curved ring under normal pressure carries a force $P = pr$. Internal pressure yields tension; external pressure yields compression.

If the **cylinder is closed at its ends,** a longitudinal stress S' acts. $S' = \frac{1}{2}rp/t = \frac{1}{2}S$. When $n = 3$ (see Poisson's ratio) the true stress $T = S - \frac{1}{3}S' = \frac{5}{6}S$.

Collapsing Pressure of Tubes. Tubes under external load may collapse. The following notation is used in the various formulas given for collapsing pressure:

P = collapsing pressure, lb per sq in. D = outside diam of tube, in.
E = modulus of elasticity. d = inside diam of tube, in.
μ = Poisson's ratio. l = length of tube, in.
t = thickness of tube, in.

For collapsing pressure, where l is not greater than $6D$, **Fairbairn's empirical formula** (1858) is: $P = 9{,}672{,}000(t^{2.19}/lD)$, or, more simply, $P = 9{,}675{,}600(t^2/lD)$.

For **large iron pipe flues,** diameters 30 to 50 in., **D. K. Clark's formula** is: $P = 200{,}000\ t^2/D^{1.75}$.

In 1906, two series of experiments were made by Prof. Reid T. Stewart on Bessemer steel lap-welded tubes (see *Trans. A.S.M.E.*, 28, p. 730). In one series, the tubes tested were $8\frac{5}{8}$ in. o. d. and of varying thickness and length, while in the other series they were 20 ft long and of varying thickness and diameter. The tests showed that all the old formulas were inapplicable to the wide range of conditions found in modern practice. Prof. Stewart found that the length of tube between transverse joints tending to hold it to a circular form has no practical influence upon the collapsing pressure of a commercial lap-welded steel tube, so long as this length is not less than about six diameters of tube.

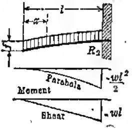
As based upon his researches, **Stewart's formulas for the collapsing pressure of modern lap-welded Bessemer steel tubes** are as follows:

$$P = 1000[1 - \sqrt{1 - 1600(t^2/D^2)}] \qquad (a)$$
$$P = 86{,}670(t/D) - 1386 \qquad (b)$$

Formula (a) is for values of P less than 581 lb per sq in., or for values of (t/D) less than 0.023, while formula (b) is for values greater than these.

These formulas are correct for tubes that are 20 ft in length between transverse joints tending to hold them to a circular form, and at the same time are substantially correct for all lengths greater than about six diameters. They have been tested for seven diameters, ranging from 3 to 10 in., in all

Table 2. Beams of Uniform Cross Section, Loaded Transversely— *(continued)*



$R_2 = W = wl$

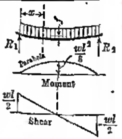
$M_x = -\frac{wx^2}{2}$

$M_{max} = -\frac{wl^2}{2}, \ (x=l)$

$Q_x = -wx$

$Q_{max} = -wl, \ (x=l)$

$f = \frac{W}{EI}\frac{l^3}{8}$ (max)



$R_1 = \frac{W}{2} = \frac{wl}{2}$

$R_2 = \frac{W}{2} = \frac{wl}{2}$

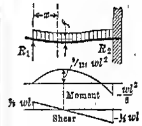
$M_x = \frac{wx}{2}(l-x)$

$M_{max} = \frac{wl^2}{8}, \ (x=\tfrac{1}{2}l)$

$Q_x = \frac{wl}{2} - wx$

$Q_{max} = \frac{wl}{2} \ (x=0)$

$f = \frac{W}{EI}\frac{5l^3}{384}$ (max)



$R_1 = \frac{3}{8}W = \frac{3}{8}wl,$

$R_2 = \frac{5}{8}W = \frac{5}{8}wl$

$M_x = \frac{wx}{2}\left(\frac{3}{4}l - x\right)$

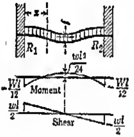
$M_{max} = \frac{9}{128}wl^2, \ \left(x = \frac{3}{8}l\right)$

$M_{max} = -\frac{wl^2}{8}, \ (x=l)$

$Q_x = \frac{3}{8}wl - wx,$

$Q_{max} = -\frac{5}{8}wl$

$f = \frac{W}{EI}\frac{l^3}{185}$ (max)



$R_1 = \frac{W}{2} = \frac{wl}{2}, \ R_2 = \frac{W}{2} = \frac{wl}{2}$

$M_x = -\frac{wl^2}{2}\left(\frac{1}{6} - \frac{x}{l} + \frac{x^2}{l^2}\right)$

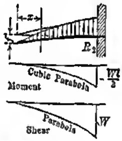
$M_{max} = -\frac{1}{12}wl^2,$

$(x=0, \text{ or } x=l)$

$Q_x = \frac{wl}{2} - wx$

$Q_{max} = \pm\frac{wl}{2}.$

$f = \frac{W}{EI}\frac{l^3}{384}$ (max)



$R_2 = W =$ total load

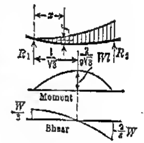
$M_x = -\frac{W}{3}\frac{x^3}{l^2}$

$M_{max} = -\frac{Wl}{3}$

$Q_x = -\frac{Wx^2}{l^2}$

$Q_{max} = -W$

$f = \frac{W}{EI}\frac{l^3}{15}$ (max)



$R_1 = \frac{1}{3}W, \quad R_2 = \frac{2}{3}W$

$M_x = \frac{Wx}{3}\left(1 - \frac{x^2}{l^2}\right)$

$M_{max} = \frac{2}{9\sqrt{3}}Wl,$

$\left(x = \frac{l}{\sqrt{3}}\right)$

$Q_x = W\left(\frac{1}{3} - \frac{x^2}{l^2}\right)$

$Q_{max} = -\frac{2}{3}W, \ (x=l)$

$f = 0.01304\frac{Wl^3}{EI}$ (max)

compressive unit stress $= p$. External unit pressure $= p_2$. Hoop unit tension $= S$. Longitudinal unit stress $= S_L$.

In **Barlow's formula**, it is assumed that the volume of the metal does not change during expansion of the cylinder. Therefore S varies inversely as the radius squared. $Sr^2 = S_1r_1^2$.

The total internal pressure $2r_1p_1 = 2\int Sdr$, or $p_1 = S_1t/(r_1 + t)$.

In **Lamé's formula**, the cylinder is assumed to stretch longitudinally so that cross sections remain plane surfaces; the longitudinal deformation e_L is therefore constant. e_L is so connected with S and p that $Ee_L = S_L - [(S - p)/n]$. Therefore

$$S - p = \text{constant} = K \qquad (a)$$

Considering equilibrium of the internal hoop (*Fig. 19*),

$$2rp - [(r + dr)2(p \pm dp)] = 2Sdr, \text{ or}$$
$$d(pr)/dr = S \qquad (b)$$

Algebraic development of (a) and (b) leads to Lamé's formula

$$S = [r_1^2p_1 - r_2^2p_2 + (p_1 - p_2)r_1^2r_2^2/r^2]/(r_2^2 - r_1^2)$$

When $r = r_1$ and $p_2 = 0$, $S = S_1$ the hoop tension at the bore, or

$$S_1 = p_1(r_2^2 + r_1^2)/(r_2^2 - r_1^2) \qquad (1)$$

or $$r_2 = r_1[(S_1 + p_1)/(S_1 - p_1)]^{1/2}$$

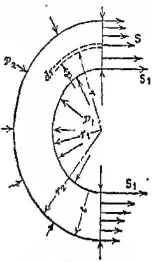


Fig. 18.

When $r = r_2$ and $p_2 = 0$, $S = S_2$ the hoop tension at exterior, or $S_2 = 2p_1r_1^2/(r_2^2 - r_1^2)$. If $r_2 = 2r_1$, $S_1 = 5p_1/3$ and $S_2 = \frac{2}{3}p_1$.

These stresses are apparent stresses. The true unit hoop stresses T are found by methods of p. 445.

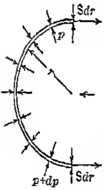


Fig. 19.

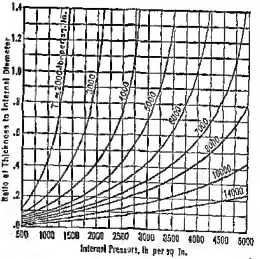


Fig. 20.

For **true stresses**, Hütte states that $r_2 = r_1[(T + 0.4p)/(T - 1.3p)]^{1/2}$ where T is the working tensile strength. Values of T from this equation are given in Fig. 20. When p is external this becomes $r_2 = r_1[T/(T - 1.7p)]^{1/2}$ when $p < T/1.7$ only.

Table 2. Beams of Uniform Cross Section, Loaded Transversely—(*continued*)

Concentrated Load W'
Uniformly Dist. Load $W = wl$; $c < c_1$

$$R_1 = W'\frac{c_1}{l} + \frac{W}{2}$$

$$R_2 = W'\frac{c}{l} + \frac{W}{2}$$

(a) $\frac{W'}{W} < \frac{c_1 - c}{2c}$

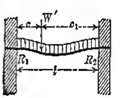
$$M_{\max} = R_1\frac{x_1}{2} = \frac{R_1^2 l}{2W}, \quad \left(x_1 = \frac{R_1 l}{W}\right)$$

(b) $\frac{W'}{W} > \frac{c_1 - c}{2c}$

$$M_{\max} = \left(W' + \frac{W}{2}\right)\frac{cc_1}{l}, \quad (x_1 = c_1)$$

Deflection of beam under W':

$$f = \left(W' + \frac{l^2 + cc_1}{8cc_1}W\right)\frac{c^2c_1^2}{3EIl}$$



$c < c_1$

$$R_1 = W'\frac{(3c + c_1)c_1^2}{l^3} + \frac{W}{2}$$

$$R_2 = W'\frac{(c + 3c_1)c^2}{l^3} + \frac{W}{2}$$

$$M_{\max} = M_1 = W'\frac{cc_1^2}{l^2} + \frac{Wl}{12}$$

Deflection under W'

$$f = \frac{1}{EI}\left(W'\frac{c^3c_1^3}{3l^3} + W\frac{c^2c_1^2}{24l}\right)$$

Table 3. Uniformly Distributed Loads on Rectangular Beams 1 In. Wide*

(Calculated for unit fiber stress of 1,000 lb per sq in.; nominal size)
TOTAL LOAD IN POUNDS, INCLUDING WEIGHT OF BEAM

Span, ft	Depth of beam in inches										
	6	7	8	9	10	11	12	13	14	15	16
5	800	1090	1420	1800	2220	2690	3200	3750	4350	5000	5690
6	670	910	1180	1500	1850	2240	2670	3130	3630	4170	4740
7	570	780	1010	1290	1590	1920	2280	2680	3110	3570	4060
8	500	680	890	1120	1390	1680	2000	2350	2720	3130	3560
9	440	600	790	1000	1230	1490	1780	2090	2420	2780	3160
10	400	540	710	900	1110	1340	1600	1880	2180	2500	2840
11	360	490	650	820	1010	1220	1450	1710	1980	2270	2590
12	330	450	590	750	930	1120	1330	1560	1810	2080	2370
13	310	420	550	690	850	1030	1230	1440	1680	1920	2190
14	290	390	510	640	790	960	1140	1340	1560	1790	2030
15	270	360	470	600	740	900	1070	1250	1450	1670	1900
16	250	340	440	560	690	840	1000	1170	1360	1560	1780
17	230	320	420	530	650	790	940	1100	1280	1470	1670
18	220	300	400	500	620	750	890	1040	1210	1390	1580
19	210	290	380	470	590	710	840	990	1150	1320	1500
20	200	270	360	450	560	670	800	940	1090	1250	1420
22	180	250	320	410	500	610	730	850	990	1140	1290
24	160	230	290	370	460	560	670	780	910	1040	1180
26	150	210	270	340	420	520	610	720	840	960	1090
28	140	190	250	320	390	480	570	670	780	890	1010
30	130	180	240	300	370	450	530	630	730	830	950

* This table is convenient for wooden beams. For any other fiber stress S', multiply the values in table by $S'/1000$.

PRESSURE BETWEEN BODIES WITH CURVED SURFACES

Two Spheres. The radius A of the compressed area is obtained from the formula $A^3 = 0.68P(c_1 + c_2)/[(1/r_1) + (1/r_2)]$, in which P is the compressing force, c_1 and c_2 ($= 1/E_1$ and $1/E_2$) are reciprocals of the respective moduli of elasticity, and r_1 and r_2 are the radii. (Reciprocal of Poisson's ratio assumed to be $n = 10/3$.) The greatest compressive stress (in.-lb units) in the middle of the compressed surface will be $S_{max} = 1.5(P/\pi A^2)$, and

$$S^3_{max} = 0.235P[(1/r_1) + (1/r_2)]^2/(c_1 + c_2)^2$$

The total deformation of the two spheres will be Y, which is obtained from

$$Y^3 = 0.46P^2(c_1 + c_2)^2[(1/r_1) + (1/r_2)]$$

For $c_1 = c_2 = 1/E$, *i.e.*, two spheres with the same modulus of elasticity, it follows that $A^3 = 1.36\ P/E[(1/r_1) + (1/r_2)]$, $S^3_{max} = 0.059PE^2[(1/r_1) + (1/r_2)]^2$, and $Y^3 = 1.84P^2[(1/r_1) + (1/r_2)]/E^2$. If the radii of these spheres are also equal, $A^3 = 0.68Pr/E = 0.34Pd/E$; $S^3_{max} = 0.235PE^2/r^2 = 0.94PE^2/d^2$; and $Y^3 = 3.68P^2/E^2r = 7.36P^2/E^2/d$.

Sphere and Flat Plate. In this case, $r_1 = r$ and $r_2 = \infty$, and the above formulas become $A^3 = 0.68Pr(c_1 + c_2) = 1.36Pr/E$, and

$$S^3_{max} = 0.235P/r^2(c_1 + c_2)^2 = 0.059PE^2/r^2$$

Also $$Y^3 = 0.46P^2(c_1 + c_2)^2/r = 1.84P^2/E^2r.$$

Two Cylinders. The width b of the rectangular pressure surface is obtained from $(b/4)^2 = 0.29P(c_1 + c_2)/l[(1/r_1) + (1/r_2)]$, where r_1 and r_2 are the radii, and l the length.

$$S^2_{max} = (4P/\pi bl)^2 = 0.35P[(1/r_1) + (1/r_2)]/l(c_1 + c_2)$$

For cylinders with the same moduli of elasticity, $c_1 = c_2 = 1/E$, and $(b/4)^2 = 0.58P/El[(1/r_1) + (1/r_2)]$; and $S^2_{max} = 0.175PE[(1/r_1) + (1/r_2)]/l$. When $r_1 = r_2 = r$, $(b/4)^2 = 0.29Pr/El$, and $S^2_{max} = 0.35PE/lr$.

Cylinder and Flat Plate. Here $r_1 = r$, $r_2 = \infty$, and the above formulas reduce to $(b/4)^2 = 0.29Pr(c_1 + c_2)/l = 0.58Pr/El$, and

$$S^2_{max} = 0.35P/lr(c_1 + c_2) = 0.175PE/lr$$

BEAMS

For Properties of Structural Steel, see steel manufacturers' handbooks.

Notation

R	= Reaction	S_v	= Vertical shearing unit stress
M	= Bending moment	Z	= Horizontal shearing unit stress
W	= Total distributed load	I	= Rectangular moment of inertia
w	= Distributed load per longitudinal unit	I_P	= Polar moment of inertia
		r	= Radius of curvature
P	= Concentrated load	i	= Slope
V or Q	= Total vertical shear	f	= Deflection
S	= Unit normal apparent stress	l	= Distance between supports

A **simple beam** is a bar resting on supports near its ends. A **cantilever beam** projects out beyond a support. To compute reactions and moments,

Theory of Flexure

A bent beam is shown in Fig. 25. The concave side is in compression and the convex side in tension. These are divided by the **neutral plane** of zero stress $A'B'BA$. The intersection of the neutral plane with the face of the beam is the **neutral line** or **elastic curve** AB. The intersection of the neutral plane with the cross-section is the **neutral axis** NN'.

It is assumed that a beam is prismatic, of a length at least 10 times its depth, and that the external forces are all at right angles to the axis of the beam and in a plane of symmetry, and that flexure is slight. Other assumptions are: (1) That the material is homogeneous, and obeys Hooke's law. (2) That stresses are within the elastic limit. (3) That every layer of material is free to expand and contract longitudinally and laterally under stress as if separate from other layers. (4) That the tensile and compressive moduli of elasticity are equal. (5) That the cross section remains a plane surface.

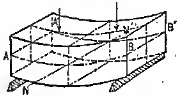


FIG. 25.

(The assumption of plane cross sections is only strictly true when the shear is constant or zero over the cross section, and when the shear is constant throughout the length of the beam.)

It follows then that: (1) The internal forces are in horizontal balance. (2) The **neutral axis contains the center of gravity** of the cross section, when there is no resultant axial stress. (3) The stress intensity varies directly with the distance from the neutral axis.

The moment of the elastic forces about the neutral axis, *i.e.*, the **stress moment** or **moment of resistance**, is $M = SI/c$, where S is an elastic unit stress at outer fiber whose distance from the neutral axis is c; and I is the rectangular moment of inertia about the neutral axis.

$$M = SI/c$$

This formula is for the **strength of beams**. For rectangular beams, $M = \frac{1}{6}Sbh^2$, where b = breadth, and h = depth. That is, the elastic **strength of beam sections** varies as follows:

(1) For equal width, as the square of the depth. (2) For equal depth, directly as the width. (3) For equal depth and width, directly as the strength of the material. (4) If span varies, then for equal depth, width and material, inversely as the span.

If a beam is cut in halves horizontally, the two halves laid side by side will carry only one-half as much as the original beam.

The term **section modulus** is given to the value of I/c, where c is the distance to the fiber carrying greatest stress. Moment of inertia of cross section = I.

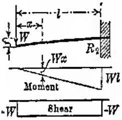
Tables 6, 7, and 8 give the properties of various beam cross sections.

For **horizontal** or **longitudinal shear** in beams, see p. 439.

Relation of Moment and Shear. The shear V is the first differential of the bending moment M with respect to x. $dM/dx = V$. When M is a maximum, V is zero. In Fig. 23, $dM/dx = \frac{1}{2}W - wx$, which equals V.

Oblique Loading. It should be noted that Table 6 includes certain cases for which the horizontal axis is not a neutral axis, assuming the common case of vertical loading. The rectangular section with the diagonal as a

Table 2. Beams of Uniform Cross-section, Loaded Transversely —



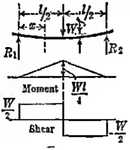
$R_2 = W$

$M_x = -Wx$

$M_{max} = -Wl,\ (x = l)$

$Q_x = -W$

$f = \frac{Wl^3}{3EI}$ (max)



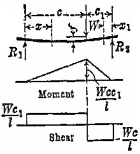
$R_1 = \frac{W}{2},\ R_2 = \frac{W}{2}$

$M_x = \frac{Wx}{2}$

$M_{max} = \frac{Wl}{4},\ \left(x = \frac{l}{2}\right)$

$Q_x = \pm\frac{W}{2}$

$f = \frac{W}{EI}\frac{l^3}{48}$ (max)



$R_1 = \frac{Wc_1}{l},\ R_2 = \frac{Wc}{l}$

$M_x = \frac{Wc_1x}{l},\ M_{x1} = \frac{Wcx_1}{l}$

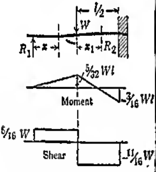
$M_{max} = \frac{Wcc_1}{l},$

$(x_1 = c_1$ or $x = c)$

$Q_x = \frac{Wc_1}{l},\ Q_{x1} = \frac{Wc}{l}$

$f = \frac{Wc_1}{3EIl}\left[\frac{c(l+c_1)}{3}\right]^{3/2}$ (max)

Max f occurs at $x = \sqrt{c(l+c_1)/3}$



$R_1 = \frac{5}{16}W,\ R_2 = \frac{11}{16}W$

$M_x = \frac{5}{16}Wx$

$M_{x1} = Wl\left(\frac{5}{32} - \frac{11}{16}\frac{x_1}{l}\right)$

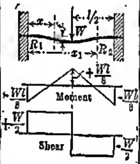
$M_{max} = -\frac{3}{16}Wl,\ \left(x_1 = \frac{l}{2}\right)$

$Q_x = +\frac{5}{16}W,\ Q_{x1} = -\frac{11}{16}W$

$Q_{max} = -\frac{11}{16}W,$

$\left(x = \frac{l}{2}\text{ to } x = l\right)$

$f = \frac{W}{EI}\frac{7l^3}{768}$



$R_1 = \frac{W}{2},\ R_2 = \frac{W}{2}$

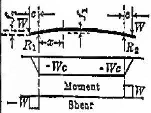
$M_x = \frac{Wl}{2}\left(\frac{x}{l} - \frac{1}{4}\right)$

$M_{x1} = \frac{Wl}{2}\left(\frac{x}{l} - \frac{3}{4}\right)$

$M_{max} = \frac{Wl}{8},\ \left(x = \frac{l}{2}\right)$

$Q_x = \frac{W}{2}\ \ Q_{x1} = -\frac{W}{2}$

$f = \frac{W}{EI}\frac{l^3}{192}$ (max)



$R_1 = W$

$R_2 = W$

$M_x = -Wc =$ Const

$Q_{W\text{ to }R_1} = -W$

$Q_{R_1\text{ to }2} = 0$

$Q_{2\text{ to }W} = +W$

$f_1 = \frac{W}{EI}\frac{l^2}{8}\frac{c}{l}$ (max)

$f_2 = \frac{W}{EI}\frac{c^2}{3}\left(c + \frac{3l}{2}\right)$ (max)

Table 6. Properties of Various Cross Sections—(*continued*)

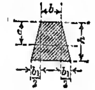
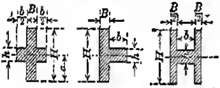
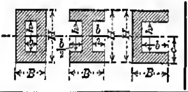
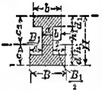
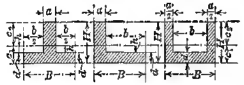
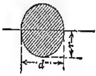
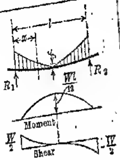
Section	Moment of inertia	Section modulus	Radius of gyration
Equilateral Polygon A = area, (see p. 39) R = rad circumscribed circle r = rad inscribed circle n = no. sides a = length of side Axis as in preceding section of octagon	$I=\frac{A}{24}(6R^2-a^2)$ $=\frac{A}{48}(12r^2+a^2)$ $=\frac{AR^2}{4}$ (approx)	$\frac{I}{c}=\frac{I}{r}$ $=\frac{I}{R\cos\frac{180^\circ}{n}}$ $=\frac{AR}{4}$ approx	$\sqrt{\frac{6R^2-a^2}{24}}=\frac{R}{2}$ $\sqrt{\frac{12r^2+a^2}{48}}$
	$I=\frac{6b^2+6bb_1+b_1^2}{36(2b+b_1)}h^3$ $c=\frac{1}{3}\,\frac{3b+2b_1}{2b+b_1}\,h$	$\frac{I}{c}=\frac{6b^2+6bb_1+b_1^2}{12(3b+2b_1)}h^2$	$\frac{h\sqrt{12b^2+12bb_1+2b_1^2}}{6(2b+b_1)}$
	$I=\frac{BH^3+bh^3}{12}$ $\frac{I}{c}=\frac{BH^3+bh^3}{6H}$		$\sqrt{\frac{BH^3+bh^3}{12(BH+bh)}}$
	$I=\frac{BH^3-bh^3}{12}$ $\frac{I}{c}=\frac{BH^3-bh^3}{6H}$		$\sqrt{\frac{BH^3-bh^3}{12(BH-bh)}}$
	$I=\frac{1}{3}(Bc_1^3-B_1h^3+bc_2^3-b_1h_1^3)$ $c_1=\frac{1}{2}\,\frac{aH^2+B_1d^2+b_1d_1(2H-d_1)}{aH+B_1d+b_1d_1}$		$\sqrt{\frac{I}{(Bd+b_1d_1)+a(h+h_1)}}$
			$I=\frac{1}{3}(Bc_1^3-bh^3+ac_2^3)$ $c_1=\frac{1}{2}\,\frac{aH^2+bd^2}{aH+bd}$ $c_2=H-c_1$ $r=\sqrt{\frac{I}{[Bd+a(H-d)]}}$
	$I=\frac{\pi d^4}{64}=\frac{\pi r^4}{4}=\frac{A}{4}r^2$ $=0.05d^4$ approx	$\frac{I}{c}=\frac{\pi d^3}{32}=\frac{\pi r^3}{4}=\frac{A}{4}r$ $=0.1d^3$ approx	$\frac{r}{2}=\frac{d}{4}$

Table 2. Beams of Uniform Cross Section, Loaded Transversely—(*continued*)



$$R_1=\frac{W}{2},\quad R_2=\frac{W}{2}$$

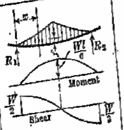
$$M_x= Wx\left(\frac{1}{2}-\frac{x}{l}+\frac{2x^2}{3l^2}\right)$$

$$M_{\max}=\frac{Wl}{12},\ \left(x=\frac{1}{2}l\ \right)$$

$$Q_x=W\left(\frac{1}{2}-\frac{2x}{l}+\frac{2x^2}{l^2}\right)$$

$$Q_{\max}=\pm\frac{W}{2},\ (x=0)$$

$$f=\frac{W}{EI}\frac{3l^3}{320}\ (\max)$$



$$R_1=\frac{W}{2},\ R_2=\frac{W}{2}$$

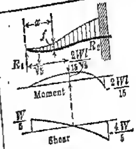
$$M_x= Wx\left(\frac{1}{2}-\frac{2}{3}\frac{x^2}{l^2}\right)$$

$$M_{\max}=\frac{Wl}{6},\ \left(x=\frac{1}{2}l\right)$$

$$Q_x=W\left(\frac{1}{2}-\frac{2x^2}{l^2}\right)$$

$$Q_{\max}=\pm\frac{W}{2},\ (x=0)$$

$$f=\frac{W}{EI}\frac{l^3}{60}\ (\max)$$



$$R_1=\frac{W}{5},\ R_2=\frac{4W}{5}$$

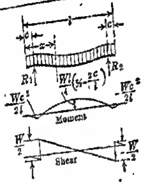
$$M_x=Wx\left(\frac{1}{5}-\frac{x^2}{3l^2}\right)$$

$$M_{\max}=-\frac{2}{15}Wl \text{ at support 2}$$

$$Q_x=W\left(\frac{1}{5}-\frac{x^2}{l^2}\right)$$

$$Q_{\max}=-\frac{4W}{5}$$

$$f=\frac{16Wl^3}{1500\sqrt{5}EI}=\frac{0.00477\,Wl^3}{EI}(\max)$$



$$R_1=\frac{W}{2}=\frac{wl}{2},\ R_2=\frac{W}{2}=\frac{wl}{2}$$

$$M_x=\frac{Wx}{2}\left(1-\frac{c}{x}-\frac{x}{l}\right),\ (x>c)$$

$$M_x=-\frac{Wx^2}{2l},\ (x\leqq c)$$

$$M_{\max}=\frac{Wl}{4}\left(\frac{1}{2}-\frac{2c}{l}\right),\ c\leqq\left(\frac{\sqrt{2}-1}{2}\right)l$$

$$Q_x=\frac{W}{2}-wx_1\ (x>c)$$

$$Q_x=-wx_1\ (x\leqq c)$$

Concentrated Load W'
Uniformly Dist. Load $W=wl$

$$R_1=W'\frac{c_1^2(3c+2c_1)}{2l^3}+\frac{3}{8}W$$

$$R_2=W'\frac{(2c^2+6cc_1+3c_1^2)c}{2l^3}+\frac{5}{8}W$$

$$M_2=W'\frac{cc_1(2c+c_1)}{2l^2}+W\frac{l}{8}$$

$$M_{W'}=W'\frac{cc_1^2(3c+2c_1)}{2l^3}+W\frac{(3c_1-c)c}{8l}$$

$$(a)\ \frac{W'}{W}<\frac{l^2}{4c_1^2}\ \frac{5c-3c_1}{3c+2c_1}$$

$$M_{x\max}=\frac{R_1^2}{2W}l,\quad\left(x=\frac{R_1l}{W}\right)$$

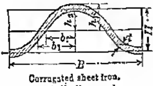
$$(b)\ \frac{W'}{W}<\frac{l^2(3c_1-5c)}{4c(2c^2+6cc_1+3c_1^2)}$$

$$M_{c_1\max}=W'c+\frac{(R_1-W')^2}{2W}l,\quad\left(x=\frac{R_1-W'}{W}l\right)$$

Deflection under W'

$$f=\frac{W'}{EI}\ \frac{c^3c_1^3(4c+3c_1)}{12l^3}+\frac{W}{EI}\ \frac{cc_1^3(3c+c_1)}{48l}$$

Table 6. Properties of Various Cross Sections—(*continued*)

Section	Moment of inertia and section modulus	Radius of gyration
 Corrugated sheet iron, parabolically curved	$I = \frac{64}{105}(b_1h_1^3 - b_2h_2^3)$, where $h_1 = ½(H+t)$ | $b_1 = ½(B+2.6t)$ $h_2 = ½(H-t)$ | $b_2 = ½(B-2.6t)$ $\frac{I}{c} = \frac{2I}{H+t}$	$r = \sqrt{\frac{3I}{t(2B+5.2H)}}$

(Approximate values of *least* radius of gyration r)

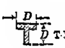
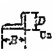
	 Phoenix Column	 Carnegie Z-Bar Column	 I-Beam	 Channel	 Deck Beam
$r =$	$0.3636D$	$0.295D$	$D/4.58$	$D/3.54$	$D/6$
	 T-Beam	 Angle Equal legs	 Angle Unequal legs	 Cross	
$r =$	$D/4.74$	$D/5$	$BD/2.6(B+D)$	$D/4.74$	

Table 7. Moments of Inertia of Rectangles
(Unit widths; height $= h$; axis at middle of depth)

h	I	h	I	h	I	h	I	h	I
1	0.08333	13	183.1	25	1302	37	4221	49	9,804
2	0.6667	14	228.7	26	1465	38	4573	50	10,420
3	2.250	15	281.3	27	1640	39	4943	51	11,050
4	5.333	16	341.3	28	1829	40	5333	52	11,720
5	10.42	17	409.4	29	2032	41	5743	53	12,410
6	18.00	18	486.0	30	2250	42	6174	54	13,120
7	28.58	19	571.6	31	2483	43	6626	55	13,860
8	42.67	20	666.7	32	2731	44	7099	56	14,630
9	60.75	21	771.8	33	2995	45	7594	57	15,430
10	83.33	22	887.3	34	3275	46	8111	58	16,260
11	110.9	23	1014.0	35	3573	47	8652	59	17,110
12	144.0	24	1152.0	36	3888	48	9216	60	18,000

principal axes are axes with respect to which the moment of inertia is, respectively, a maximum and a minimum, and for which the product of inertia is zero. For symmetrical sections, axes of symmetry are always principal axes. For unsymmetrical sections, like a rolled angle section (Fig. 26), the inclination of the principal axis with the X axis may be found from the formula $\tan 2\theta = 2I_{xy}/(I_y - I_x)$, in which θ = angle of inclination of the principal axis to the X axis, I_{xy} = the product of inertia of the section with respect to the X and Y axes, I_y = moment of inertia of the section with respect to the Y axis, I_x = moment of inertia of the section with respect to the X axis. When this principal axis has been found, the other principal axis is at right angles to it.

Table 4. Approximate Safe Loads in Pounds on Steel Beams

(Pencoyd Iron Works)

Allowable fiber stress for steel, 16,000 lb per sq in. (basis of table); for iron, reduce values given in table by one-eighth. Beams supported at both ends.

L = distance between supports, ft. a = interior area, sq in.
A = sectional area of beam, sq in. d = interior depth, in.
D = depth of beam, in. w = total working load, net tons.

Shape of section	Greatest safe load, pounds		Deflection, inches	
	Load in middle	Load distributed	Load in middle	Load distributed
Solid rectangle......	$\frac{890AD}{L}$	$\frac{1780AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Hollow rectangle...	$\frac{890(AD-ad)}{L}$	$\frac{1780(AD-ad)}{L}$	$\frac{wL^3}{32(AD^2-ad^2)}$	$\frac{wL^3}{52(AD^2-ad^2)}$
Solid cylinder.......	$\frac{667AD}{L}$	$\frac{1333AD}{L}$	$\frac{wL^3}{24AD^2}$	$\frac{wL^3}{38AD^2}$
Hollow cylinder.....	$\frac{667(AD-ad)}{L}$	$\frac{1333(AD-ad)}{L}$	$\frac{wL^3}{24(AD^2-ad^2)}$	$\frac{wL^3}{38(AD^2-ad^2)}$
Even-legged angle or tee	$\frac{885AD}{L}$	$\frac{1770AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Channel or Z bar...	$\frac{1525AD}{L}$	$\frac{3050AD}{L}$	$\frac{wL^3}{53AD^2}$	$\frac{wL^3}{85AD^2}$
Deck beam.........	$\frac{1380AD}{L}$	$\frac{2760AD}{L}$	$\frac{wL^3}{50AD^2}$	$\frac{wL^3}{80AD^2}$
I beam............	$\frac{1695AD}{L}$	$\frac{3390AD}{L}$	$\frac{wL^3}{58AD^2}$	$\frac{wL^3}{93AD^2}$

Table 5. Coefficients for Correcting Values in Table 4 for Various Methods of Support and of Loading

Conditions of loading	Maximum relative safe load	Maximum relative deflection under max relative safe load
BEAM SUPPORTED AT ENDS		
Load uniformly distributed over span..........	1.0	1.0
Load concentrated at center of span...........	½	0.80
Two equal loads symmetrically concentrated....	$l/4c$	.....
Load increasing uniformly to one end..........	0.974	0.976
Load increasing uniformly to center...........	¾	0.96
Load decreasing uniformly to center...........	3/2	1.08
BEAM FIXED AT ONE END, CANTILEVER		
Load uniformly distributed over span.........	¼	2.40
Load concentrated at end.....................	⅛	3.20
Load increasing uniformly to fixed end........	⅜	1.92
BEAM CONTINUOUS OVER TWO SUPPORTS EQUIDISTANT FROM ENDS		
Load uniformly distributed over span:		
1. If distance $a > 0.2071l$..................	$l^2/4a^2$	
2. If distance $a < 0.2071l$..................	$\frac{l}{l-4a}$	
3. If distance $a = 0.2071l$..................	5.83	
Two equal loads concentrated at ends.........	$l/4a$	

l = length of beam; c = distance from support to nearest concentrated load; a = distance from support to end of beam.

Internal Moment beyond the Elastic Limit

Ordinarily, the expression $M = SI/c$ is used for stresses above the elastic limit, in which case S becomes an experimental coefficient S_R, the **modulus of rupture**, and the formula is empirical. The true relation is obtained by applying to the cross section a stress-strain diagram from a tension and compression test, as in Fig. 28. Figure 28 shows the side of a beam of depth d under flexure beyond its elastic limit; line 1–1 shows the distorted cross section; line 3–3, the usual rectilinear relation of stress to strain; and line 2–2, an actual stress-strain diagram, applied to the cross section of the beam, compression above and tension below. The neutral axis is then below the gravity axis. The outer material may be expected to develop greater ultimate strength than in simple stress, on account of the reinforcing action of material nearer the neutral axis that is not yet overstrained. This leads to an **equalization of stress** over the cross section. S_R exceeds the ultimate strength S_M in tension as follows: for cast iron, $S_R = 2S_M$; for sandstone, $S_R = 3S_M$; for concrete, $S_R = 2.2S_M$; for wood (green), $S_R = 2.3S_M$.

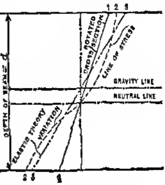


Fig. 28.

In the case of steel I beams, failure practically begins when the elastic limit in the compression flange is reached.

On account of support of adjoining material, the elastic limit in flexure S_p is also greater than in tension, depending upon the relation of breadth to depth of section. For the same breadth, the difference decreases with increase of height. No difference will occur in the case of an I beam, or with hard materials. Bauschinger quotes for soft steel plates, 1.27; Considère, 1.37; Hatt, 1.5 (*R. R. Gaz.*, 1899).

Wide plates will not expand and contract freely, and the value of E will be increased on account of side constraint. As a consequence of lateral contraction of the fibers of the tension side of a beam and lateral swelling of fibers at the compression side, the cross section becomes distorted to a trapezoidal shape, and the neutral axis is at the cg of the trapezoid. Strictly, this shape is one with a curved perimeter, the radius being r/m, where r is the radius of the neutral line of the beam, and m is Poisson's ratio.

Deflection of Beams

When a beam is subjected to bending, the fibers on one side elongate, while the fibers on the other side shorten (Fig. 29). These changes in length cause the beam to deflect. All points on the beam except those directly over the support fall below their original position, as shown by Fig. 25.

The **elastic curve** is the curve taken by the neutral axis. The radius of curvature at any point is

$$r = EI/M$$

A beam bent to a **circular curve** of constant radius has a constant bending moment.

Replacing r in the equation by its approximate geometrical value, $1/r = d^2y/(dx)^2$, the fundamental equation from which the elastic curve of a bent beam can be developed and the deflection of any beam can be obtained is,

$$M = EId^2y/(dx)^2 \text{ (approximate)}$$

This is a formula for **stiffness of beams**.

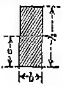
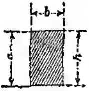
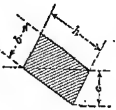
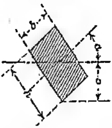
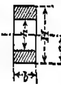
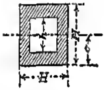
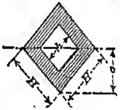
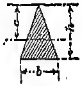
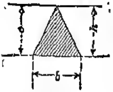
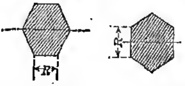
Substituting the value of M, in terms of x, and integrating once, gives the slope of the tangent to the elastic curve of the beam at point x, y; tan i =

horizontal axis (Table 9) is such a case. These cases must be handled by the principles of oblique loading.

Every section of a beam has two principal axes passing through the center of gravity, and these two axes are always at right angles to each other. The

Table 6. Properties of Various Cross Sections

(I = Moment of inertia; I/c = section modulus; $r = \sqrt{I/A}$ = radius of gyration)

			
$I = \frac{bh^3}{12}$	$\frac{bh^3}{3}$	$\frac{b^3h^3}{6(b^2+h^2)}$	$\frac{bh}{12}(h^2\cos^2 a + b^2\sin^2 a)$
$\frac{I}{c} = \frac{bh^2}{6}$	$\frac{bh^2}{3}$	$\frac{b^2h^2}{6\sqrt{b^2+h^2}}$	$\frac{bh}{6}\left(\frac{h^2\cos^2 a + b^2\sin^2 a}{h\cos a + b\sin a}\right)$
$r = \frac{h}{\sqrt{12}} = 0.289h$	$\frac{h}{\sqrt{3}} = 0.577h$	$\frac{bh}{\sqrt{6(b^2+h^2)}}$	$\sqrt{\frac{h^2\cos^2 a + b^2\sin^2 a}{12}}$
			
$I = \frac{b}{12}(H^3 - h^3)$	$\frac{H^4 - h^4}{12}$	$\frac{H^4 - h^4}{12}$	$\frac{bh^3}{36}$; $c = \frac{2}{3}h$
$\frac{I}{c} = \frac{b}{6}\frac{H^3 - h^3}{H}$	$\frac{1}{6}\frac{H^4 - h^4}{H}$	$\frac{\sqrt{2}}{12}\frac{H^4 - h^4}{H} = 0.1179\frac{H^4 - h^4}{H}$	$\frac{bh^2}{24}$
$r = \sqrt{\frac{H^3 - h^3}{12(H-h)}}$	$\sqrt{\frac{H^2 + h^2}{12}}$	$\sqrt{\frac{H^2 + h^2}{12}}$	$\frac{h}{\sqrt{18}} = 0.236h$
			
$I = \frac{bh^3}{12}$	$\left(\frac{5\sqrt{3}}{16}R^4 = 0.5413R^4\right)$		$\frac{1+2\sqrt{2}}{6}R^4 = 0.6381R^4$
$\frac{I}{c} = \frac{bh^2}{12}$	$\frac{5}{8}R^3$;	$0.5413R^3$	$0.6906R^3$
$r = \frac{h}{\sqrt{6}} = 0.408h$	$\left(\sqrt{\frac{5}{24}}R = 0.456R\right)$		$0.475R$

Square, axis same as first rectangle, side $= h$; $I = h^4/12$; $I/c = h^3/6$; $r = 0.289h$.
Square, diagonal taken as axis: $I = h^4/12$; $I/c = 0.1179h^3$; $r = 0.289h$.

where C'' is a new constant $= n/m$. Other factors remaining the same, the **deflection varies directly as the stress and inversely as E.** If the span is constant, a shallow beam will submit to greater deformations than a deeper beam without exceeding a safe stress. If depth is constant, a beam of double span will attain a given deflection with only one-quarter the stress. Values of n, m, and C'' are as follows (for other values, see Table 2):

Beam	Load	n	m	C''
Cantilever	Concentrated at end	1	3	1/3
Cantilever	Uniform	2	8	1/4
Simple	Concentrated at center	4	48	1/12
Simple	Uniform	8	384/5	5/48
Fixed ends	Concentrated at center	8	192	1/24
Fixed ends	Uniform	12	384	1/32
One end fixed, One end supported	Concentrated at center	16/3	768/7	7/144
One end fixed, One end supported	Uniform	128/9	185	1/13
Simple	Uniformly varying, maximum at center	6	60	1/10

Graphical Relations

Referring to Fig. 31, the shear V acting at any section is equal to the total load on the right of the section, or

$$V = \int w dx$$

Since wdx is the product of w, a loading intensity (which is expressed as a vertical height in the load diagram), by dx, an elementary length along the horizontal, evidently *wdx is the area of a small vertical strip of the load* **diagram.** Then $\int wdx$ is the summation of all such vertical strips between two indefinite points. Thus, to obtain the shear in any section mn, find the area of the load diagram up to that section, and draw a second diagram called the **shear diagram,** any ordinate of which is proportional to the shear, or to the area in the load diagram to the right of mn. Since $V = dM/dx$,

$$\int V dx = M$$

By similar reasoning, a **moment diagram** may be drawn, such that the ordinate at any point is proportional to the area of the shear diagram to the right of that point. Since $M = EId^2f/(dx)^2$,

$$\int M dx = EI[(df/dx) + C] = EI(i + C)$$

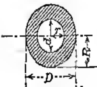
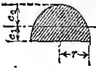
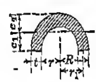
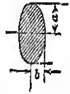
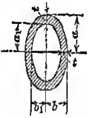
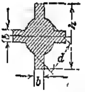
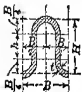
if I is constant. Here C is a constant of integration. Thus i, the slope or grade of the elastic curve at any point, is proportional to the area of the moment diagram $\int M dx$ up to that point; and a **slope diagram** may be derived from the moment diagram in the same manner as the moment diagram was derived from the shear diagram.

If I is not constant, draw a new curve whose ordinates are M/I and use these M/I ordinates just as the M ordinates were used in the case where I was constant; that is, $\int (M/I) dx = E(i + C)$. The ordinate at any point *of the slope curve is thus proportional to the area of the M/I curve to the* right of that point. Again, since $iE = Edf/dx$,

$$\int iE dx = \int E df = E(f + C')$$

and thus the ordinate f to the elastic curve at any point is proportional to the area of the slope diagram $\int i dx$ up to that point. The equilibrium polygon may be used in drawing the **deflection curve** directly from the M/I diagram.

Table 6. Properties of Various Cross Sections—*(continued)*

Section	Moment of inertia	Section modulus	Radius of gyration
 $d_m = \frac{1}{2}(D+d)$ $s = \frac{1}{2}(D-d)$	$I=\frac{\pi}{64}(D^4-d^4)$ $=\frac{\pi}{4}(R^4-r^4)$ $=\frac{1}{4}A(R^2+r^2)$ $=0.05(D^4-d^4)$ approx	$\frac{I}{c}=\frac{\pi}{32}\frac{D^4-d^4}{D}$ $=\frac{\pi}{4}\frac{R^4-r^4}{R}$ $=0.8d_m^2s$, approx, when $\frac{s}{d_m}$ is very small	$\frac{\sqrt{R^2+r^2}}{2}=\frac{\sqrt{D^2+d^2}}{4}$
	$I=r^4\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)$ $=0.1098r^4$	$\frac{I}{c_2}=0.1908r^3$ $\frac{I}{c_1}=0.2587r^3$ $c_1=0.4244r$	$\frac{\sqrt{9\pi^2-64}}{6\pi}r=0.264r$
	$I=0.1098(R^4-r^4)-\frac{0.283R^2r^2(R-r)}{R+r}$ $=0.3tr_1^3$, approx when $\frac{t}{r_1}$ is very small	$c_1=\frac{4}{3\pi}\frac{R^2+Rr+r^2}{R+r}$ $c_2=R-c_1$	$\sqrt{\frac{2}{\pi}\frac{I}{(R^2-r^2)}}=$ $0.31r_1$ (approx)
	$I=\frac{\pi a^3b}{4}=0.7854a^3b$	$\frac{I}{c}=\frac{\pi a^2b}{4}=0.7854a^2b$	$\frac{a}{2}$
	$I=\frac{\pi}{4}(a^3b-a_1^3b_1)$ $=\frac{\pi}{4}a^2(a+3b)t$ approx	$\frac{I}{c}=\frac{\pi}{4}a(a+3b)t$ approx	$\sqrt{\frac{I}{(\pi ab-a_1b_1)}}=$ $\frac{a}{2}\sqrt{\frac{a+3b}{a+b}}$ approx
	$I=\frac{1}{12}\left[\frac{3\pi}{16}d^4+b(h^3-d^3)+b^3(h-d)\right]$ $\frac{I}{c}=\frac{1}{6h}\left[\frac{3\pi}{16}d^4+b(h^3-d^3)+b^3(h-d)\right]$		$\sqrt{\frac{I}{\pi\frac{d^2}{4}+2b(h-d)}}$ approx
	$I=\frac{t}{4}\left(\frac{\pi B^3}{16}+B^2h+\frac{\pi Bh^2}{2}+\frac{2}{3}h^3\right)$ $h=H-\frac{1}{2}B$ $\frac{I}{c}=\frac{2I}{H+t}$		$\sqrt{\frac{I}{2\left(\frac{\pi B}{4}+h\right)t}}$

tion energy in the material of a beam in a length x is dU, and

$$U = \tfrac{1}{2}\int M^2dx/EI = \tfrac{1}{2}\int M di$$

where M is the moment at any point x, and di is the angle between the tangents to the elastic curve at the ends of dx. The values of resilience and deflection in special cases are easily developed from this equation.

Rolling Loads

Rolling or moving loads are those loads which may change their position on a beam. Figure 33 represents a beam with two equal concentrated moving loads, such as two wheels on a crane girder, or the wheels of a truck on a

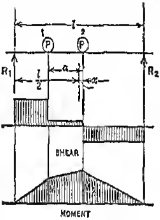


Fig. 33.

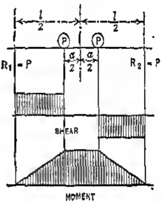


Fig. 34.

bridge. Since the maximum moment occurs where the shear is zero, it is evident from the shear diagram that the maximum moment will occur under a wheel. $x < a/2$:

$$R_1 = P\left(1 - \frac{2x}{l} + \frac{a}{l}\right)$$

$$M_2 = \frac{Pl}{2}\left(1 - \frac{a}{l} + \frac{2x}{l}\frac{a}{l} - \frac{4x^2}{l^2}\right)$$

$$R_2 = P\left(1 + \frac{2x}{l} - \frac{a}{l}\right)$$

$$M_1 = \frac{Pl}{2}\left(1 - \frac{a}{l} - \frac{2a^2}{l^2} + \frac{2x}{l}\frac{3a}{l} - \frac{4x^2}{l^2}\right)$$

M_2 max when $x = \tfrac{1}{4}a$,
M_1 max when $x = \tfrac{3}{4}a$.

$$M_{\max} = \frac{Pl}{2}\left(1 - \frac{a}{2l}\right)^2 = \frac{P}{2l}\left(l - \frac{a}{2}\right)^2$$

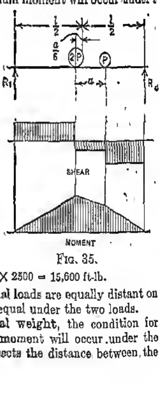


Fig. 35.

Example. Two wheel loads of 3,000 lb each, spaced on 5 ft centers, move on a span of l = 15 ft, x = 1.25 ft, and R_2 = 2,500 lb. ∴ Moment = 6.25 × 2500 = 15,600 ft-lb.

Figure 34 shows the condition when two equal loads are equally distant on opposite sides of the center. The moment is equal under the two loads.

If the **two moving loads are of unequal weight,** the condition for **maximum moment** is that the maximum moment will occur under the heavy wheel when the center of the beam bisects the distance between the

Calling the moments of inertia with respect to the principal axes, I_x' and I_y', the unit stress existing anywhere in the section at a point whose coordinates are x and y (Fig. 27) is $S = (My \cos \alpha / I_x') + (Mx \sin \alpha / I_y')$, in which M = bending moment with respect to the section in question, α = the angle which the plane of bending moment or the plane of the loads makes with the y axis, $M \cos \alpha$ = the component of bending moment causing bending about the principal axis which has been designated as the X axis, $M \sin \alpha$ = the component of bending moment causing bending about the principal axis which has been designated as the Y axis. The sign of the two terms for unit stress may be determined by inspection in the usual way, and the result will be tension or compression as determined by the algebraic sum of the two terms.

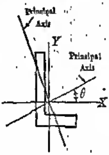


Fig. 26.

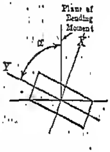


Fig. 27.

In general, it may be stated that when the plane of the bending moment coincides with one of the principal axes, then the other principal axis is the neutral axis. This is the ordinary case, in which the ordinary formula for unit stress may be applied. When the plane of the bending moment does not coincide with one of the principal axes, then the above formula for oblique loading may be applied.

Table 8. Moments of Inertia of Circular Cross Sections

I = moment of inertia about a diam $d = \pi d^4/64$
c = distance from outer fiber to neutral axis
I/c = section modulus $= \pi d^3/32$
(For polar moment of inertia, multiply values in table by 2.)

d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$	d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$	d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$	d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$
10	0.4909	0.09817	35	73.66	4.209	60	636.2	21.21	85	2,562	60.29
11	0.7187	0.1307	36	82.45	4.580	61	679.7	22.28	86	2,685	62.45
12	1.018	0.1696	37	92.00	4.973	62	725.3	23.40	87	2,812	64.65
13	1.402	0.2157	38	102.4	5.387	63	773.3	24.55	88	2,944	66.90
14	1.886	0.2694	39	113.6	5.824	64	823.6	25.74	89	3,080	69.21
15	2.485	0.3313	40	125.7	6.283	65	876.2	26.96	90	3,221	71.57
16	3.217	0.4021	41	138.7	6.766	66	931.4	28.23	91	3,366	73.98
17	4.100	0.4823	42	152.7	7.274	67	989.2	29.53	92	3,517	76.45
18	5.153	0.5726	43	167.8	7.806	68	1,050	30.87	93	3,672	78.97
19	6.397	0.6734	44	184.0	8.363	69	1,113	32.25	94	3,833	81.54
20	7.854	0.7854	45	201.3	8.946	70	1,179	33.67	95	3,998	84.17
21	9.547	0.9092	46	219.8	9.556	71	1,247	35.14	96	4,169	86.86
22	11.50	1.045	47	239.5	10.19	72	1,319	36.64	97	4,346	89.60
23	13.74	1.194	48	260.6	10.86	73	1,394	38.19	98	4,528	92.40
24	16.29	1.357	49	283.0	11.55	74	1,472	39.78	99	4,715	95.26
25	19.18	1.534	50	306.8	12.27	75	1,553	41.42	100	4,909	98.18
26	22.43	1.726	51	332.1	13.02	76	1,638	43.10	101	5,108	101.2
27	26.09	1.932	52	358.9	13.80	77	1,726	44.82	102	5,313	104.2
28	30.17	2.155	53	387.3	14.62	78	1,817	46.59	103	5,525	107.3
29	34.72	2.394	54	417.4	15.46	79	1,912	48.40	104	5,743	110.4
30	39.76	2.651	55	449.2	16.33	80	2,011	50.27	105	5,967	113.7
31	45.33	2.925	56	482.8	17.24	81	2,113	52.17	106	6,197	116.9
32	51.47	3.217	57	518.2	18.18	82	2,219	54.13	107	6,434	120.3
33	58.21	3.528	58	555.5	19.16	83	2,330	56.14	108	6,678	123.7
34	65.60	3.859	59	394.8	20.16	84	2,444	58.19	109	6,929	127.1

9 also gives the **maximum bending moment** which will occur between supports, and in addition the position of this moment and the points of inflection (see Fig. 40).

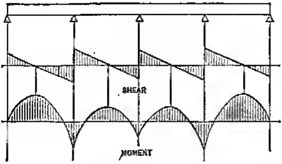


FIG. 39.

Figure 40 shows the values of the functions for a uniformly loaded continuous beam resting on three equal spans with four supports.

Table 9. Uniformly Loaded Continuous Beams over Equal Spans*
(Uniform load per unit length = w; length of each span = l)

Number of supports	Notation of support of span	Shear on each side of support. L=left, R=right. Reaction at any support is $L+R$		Moment over each support	Maximum moment in each span	Distance to point of maximum moment, measured to right from support	Distance to point of inflection, measured to right from support
		L	R				
2	1 or 2	0	½	0	0.125	0.500	None
3	1 2	0 ⅝	⅜ ⅝	0 ⅛	0.0703 0.0703	0.375 0.625	0.750 0.250
4	1 2	0 6/10	4/10 5/10	0 1/10	0.080 0.025	0.400 0.500	0.800 0.276, 0.724
5	1 2 3	0 17/28 13/28	11/28 15/28 13/28	0 3/28 2/28	0.0772 0.0364 0.0364	0.393 0.536 0.464	0.786 0.266, 0.806 0.194, 0.734
6	1 2 3	0 23/38 18/38	15/38 20/38 19/38	0 4/38 3/38	0.0779 0.0332 0.0461	0.395 0.526 0.500	0.789 0.268, 0.783 0.196, 0.804
7	1 2 3 4	0 63/104 49/104 53/104	41/104 55/104 51/104 53/104	0 11/104 8/104 9/104	0.0777 0.0340 0.0433 0.0433	0.394 0.533 0.490 0.510	0.788 0.268, 0.790 0.196, 0.785 0.215, 0.804
8	1 2 3 4	0 86/142 67/142 72/142	56/142 75/142 70/142 71/142	0 15/142 11/142 12/142	0.0778 0.0338 0.0440 0.0405	0.394 0.528 0.493 0.500	0.789 0.268, 0.788 0.196, 0.790 0.215, 0.785
Values apply to		wl	wl	wl^2	wl^2	l	l

* The numerical values given are coefficients of the expressions at the foot of each column.

$dy/dx = \int_0^x Mdx/EI$. Since i is usually small, $\tan i = i$, expressed in radians. A second integration gives the vertical deflection of any point of the elastic curve from its original position.

Example. In the cantilever beam shown in Fig. 29, the bending moment at any section $= -P(l - x) = EId^2y/(dx)^2$. Integrate and determine constant by the condition that when $x = 0$, $dy/dx = 0$. Then $EIdy/dx = -Plx + \frac{1}{2}Px^2$. Integrate again, and determine constant by the condition that when $x = 0$, $y = 0$. Then $EIy = -\frac{1}{2}Plx^2 + Px^3/6$. This is the equation of the elastic curve. When $x = l$, $y = f = -Pl^3/3EI$.

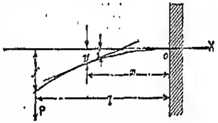


Fig. 29.

Deflection in general, f, may be expressed by the equation $f = Pl^3/mEI$, where m is a coefficient. See Tables 2 and 4 for values of f for beams of various sections and loadings. For coefficients of deflection of standard structural steel shapes, see steel manufacturers' handbooks.

Since I varies as the cube of the depth, the **stiffness,** or inverse deflection, of various **beams** varies, other factors remaining constant, inversely as the load, inversely as the cube of the span, and directly as the cube of the depth. This deflection is due to bending, but is increased by that due to vertical shear, especially in deep beams. For a **rectangular beam** with a center load P, the deflection due to vertical shear is $f' = Pl/4GA$, where G is the modulus of rigidity and A the area of cross section. The shear is assumed to be uniformly distributed. If the shear S_v is distributed according to the parabolic law, $f' = 3Pl/10GA$. If the **load is distributed,** the deflection due to vertical shear is one-third that due to same load concentrated at the center.

Usually, deflection due to shear is unimportant, but in case of **I-beam girders,** where the vertical shear in the web is large, f' may become important. Here the deflection due to shear $= Pl/4GA$, when the shear due to P is considered to act entirely on the web of area A.

Design of beams may be fixed either by consideration of strength or, if deformations must be low, by **stiffness.** Stiffness is controlled by design or by material (E).

When a load may pass by two paths to a support, the different paths take parts of the load in proportion to their stiffness.

Example (Fig. 30). Two wooden stringers—one (A) 8 × 16 in. in cross section and 20 ft in span, the other (B) 8 in. × 8 in. × 16 ft—carry the center load P_0 = 22,000 lb. Required, the load carried by each stringer. The deflections, f, of the two stringers must be equal. Load on $A = P_1$, and on $B = P_2$. $f = P_1l_1^3/48EI_1 = P_2l_2^3/48EI_2$. Then $P_1/P_2 = l_2^3I_1/l_1^3I_2 = 4$. $P_0 = P_1 + P_2 = 4P_2 + P_2$, whence $P_2 = 22{,}000/5 = 4{,}400$ lb and $P_1 = 4 \times 4400 = 17{,}600$ lb.

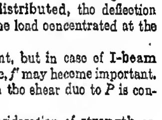


Fig. 30.

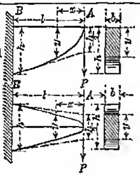
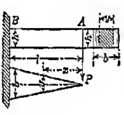
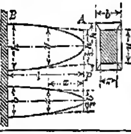
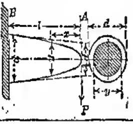
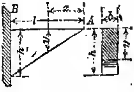
Relation between Deflection and Stress

Combine the formula $M = SI/c = Pl/n$, where n is a constant, P = load, and l = span, with formula $f = Pl^3/mEI$, where m is a constant. Then

$$f = C''Sl^2/Ec$$

Table 10. Beams of Uniform Strength (in Bending)

1. Fixed at One End, Load P Concentrated at Other End

Beam	Cross section	Elevation and plan	Formulas
1, 2 	Rectangle: width (b) constant, depth (y) variable	Elevation: 1, top, straight line; bottom, parabola. 2, complete parabola Plan: Rectangle	$y^2=\frac{6P}{bS_s}x$ $h=\sqrt{\frac{6Pl}{bS_s}}$ Deflection at A: $f=\frac{8P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Triangle	$y=\frac{6P}{h^2S_s}x$ $b=\frac{6Pl}{h^2S_s}$ Deflection at A: $f=\frac{6P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (z) variable, depth (y) variable: $\frac{z}{y}=k$ (const)	Elevation: Cubic parabola Plan: Cubic parabola	$y^3=\frac{6P}{kS_s}x$ $z=ky$ $h=\sqrt[3]{\frac{6Pl}{kS_s}}$ $b=kh$
	Circle: diam (y) variable	Elevation: Cubic parabola Plan: Cubic parabola	$y^3=\frac{32P}{\pi S_s}x$ $d=\sqrt[3]{\frac{32Pl}{\pi S_s}}$
2. Fixed at One End, Load P Uniformly Distributed over l			
	Rectangle: width (b) constant, depth (y) variable	Elevation: Triangle Plan: Rectangle	$y=x\sqrt{\frac{3P}{blS_s}}$ $h=\sqrt{\frac{3Pl}{bS_s}}$ $f=6\frac{P}{bE}\left(\frac{l}{h}\right)^3$

Thus, the five curves of load, shear, moment, slope, and deflection are so related that each curve is derived from the previous one by a process of graphical integration, and with proper regard to scales the deflection is thereby obtained.

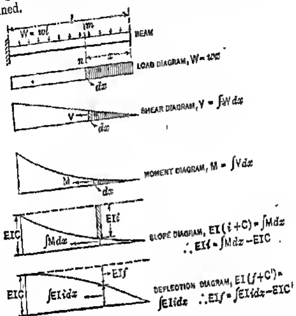


Fig. 31.

The vertical displacement of any point O_n (Fig. 32) in the elastic curve of a bent beam from the tangent line at any other point B, is Δf, and equals the area of the moment diagram between O'' and B'' times the distance X of the center of gravity of the area from O''. Or,

$$\Delta f = \text{Area } O''B''B'''O'''O'' \times X/EI$$

The angle d between the tangents at O_n and B equals the area of the moment diagram between O'' and B'' divided by EI. The intersection of the tangents at O_n and B is in the same vertical line as the center of gravity of the moment area between O'' and B''.

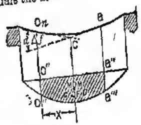


Fig. 32.

Resilience of Beams

The external work of a load gradually applied to a beam, and which increases from zero to P, is $\frac{1}{2}Pf$ and equals the **resilience** U. But, from formulas under "Relation between Deflection and Stress," p. 463, $P = nSI/cl$ and $f = nSl^2/mcE$, where n and m are constants that depend upon loading and supports, S = fiber stress, c = distance from neutral axis to outer fiber, and l = length of span. Substitute for P and f, and

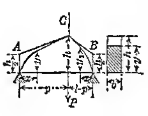
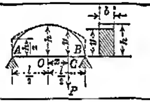
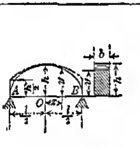
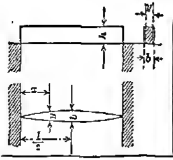
$$U = \frac{n^2}{m}\left(\frac{k}{c}\right)^2 \cdot \frac{S^2V}{2E}$$

where k is the radius of gyration, and V the volume of the beam. For values of U, see Table 1.

The resilience of beams of similar cross section at a given stress is proportional to their volumes. The **internal resilience**, or the elastic deforma-

Table 10. Beams of Uniform Strength (in Bending)—(*continued*)

3. SUPPORTED AT BOTH ENDS, LOAD P CONCENTRATED AT POINT C

Beam	Cross section	Elevation and plan	Formulas
	Rectangle: width (b) constant, depth (y or y_1) variable	Elevation: Two parabolas, vertices at points of support Plan: Rectangle	$y^2 = \frac{6P(l-p)}{blS_s}x$ $y_1^2 = \frac{6Pp}{blS_s}x_1$ $h = \sqrt{\frac{6P(l-p)p}{blS_s}}$
LOAD P MOVING ACROSS SPAN			
	Rectangle: width (b) constant, depth (y) variable	Elevation: Ellipse, Major axis $= l$ Minor axis $= 2h$ Plan: Rectangle	$\frac{x^2}{\left(\frac{l}{2}\right)^2} + \frac{y^2}{\frac{3Pl}{2bS_s}} = 1$ $h = \sqrt{\frac{3Pl}{2bS_s}}$
4. SUPPORTED AT BOTH ENDS, LOAD P UNIFORMLY DISTRIBUTED OVER l			
	Rectangle: width (b) constant, depth (y) variable	Elevation: Ellipse Plan: Rectangle	$\frac{x^2}{\left(\frac{l}{2}\right)^2} + \frac{y^2}{\frac{3Pl}{4bS_s}} = 1$ $h = \sqrt{\frac{3Pl}{4bS_s}}$ Deflection at O: $f = \frac{1}{64}\frac{Pl^3}{EI} = \frac{3}{16}\frac{P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Two parabolas with vertices at center of span	$y = \frac{3P}{S_sh^2}\left(x - \frac{x^2}{l}\right)$ $b = \frac{3Pl}{4S_sh^2}$

and depth d, $I/c = bd^2/6$; and $M = Sbd^2/6$. Thus, for a cantilever beam of rectangular cross section, under a load P, $Px = Sbd^2/6$. If b is constant, d^2 varies with x, and the profile of the shape of the beam will be a parabola, as Fig. 41. If d is constant, b will vary as x and the beam will be triangular in plan, as shown in Fig. 42.

Shear at the end of a beam necessitates a modification of the forms determined above. The area required to resist shear will be P/S_s in a can-

resultant of the loads and the heavy wheel. Figure 35 shows this position and the shear and moment diagrams.

When several wheel loads constituting a system occur, the several suspected wheels must be examined in turn to determine which will cause the greatest moment. The position for the greatest moment that can occur under a given wheel is, as stated, when the center of the span bisects the distance between the wheel in question and the resultant of all the loads then on the span. The position for maximum shear at the support will be when one wheel is passing off the span.

Constrained Beams

Constrained beams are those so held or "built in" at one or both ends that the tangent to the elastic curve remains fixed in direction. These beams are held at the ends in such a manner as to allow free horizontal motion, as illustrated by *Fig. 36. A constrained beam is stiffer* than a simple beam of the same material, on account of the modification of the moment by an end resisting moment. Figure 37 shows the two most common cases of constrained beams. See also Table 2.

Fig. 36.

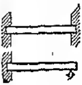


Fig. 37.

Continuous Beams

A continuous beam is one resting upon several supports which may or may not be in the same horizontal plane. The general discussion for beams holds for continuous beams. $S_vA = V$, $SI/c = M$, and $d^2f/dx^2 = M/EI$. The shear at any section is equal to the algebraic sum of the components parallel to the section of all external forces on either side of the section.

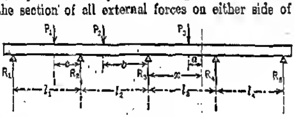


Fig. 38.

The bending moment at any section is equal to the moment of all external forces on either side of the section. The relations stated above between shear and moment diagrams hold true for continuous beams. The bending moment at any section is equal to the bending moment at any other section, plus the shear at that section times its arm, plus the product of all the intervening external forces times their respective arms. To illustrate (Fig. 38):

$$V_x = R_1 + R_2 + R_3 - P_1 - P_2 - P_3$$

$$M_x = R_1(l_1 + l_2 + x) + R_2(l_2 + x) + R_3x - P_1(l_2 + c + x) - P_2(b + x) - P_3a$$

$$M_x = M_3 + V_3x - P_3a$$

Table 16 gives the value of the moment at the various supports of a uniformly loaded continuous beam over equal spans, and it also gives the values of the shears on each side of the supports. Note that the shear is of opposite sign on either side of the supports and that the sum of the two shears is equal to the reaction.

Figure 39 shows the relation between the moment and shear diagrams for a uniformly loaded continuous beam of four equal spans (see Table 9). Table

thrust bearing plate might be considered a circular plate built in at the inner edge and loaded with a uniformly distributed load.

Let R = outside radius of plate. r = inside radius of plate. P = total load on plate, uniformly distributed along an edge, lb. p = load per sq in. of plate surface, lb. t = thickness of plate, in. E = modulus of elasticity, lb

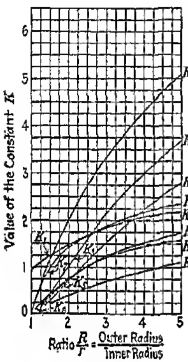


FIG. 48.—Values of K.

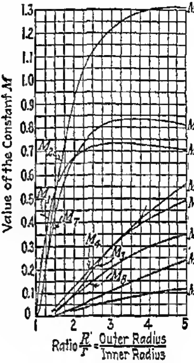


FIG. 49.—Values of M.

per sq in. S = maximum unit stress, lb per sq in., y = deflection of one edge of the plate with respect to the other, in.

For a plate loaded along the edge, $S = KP/t^2$; $y = MPR^2/Et^3$.

For a plate uniformly loaded $S = KpR^2/t^2$; $y = MpR^4/Et^3$.

The values of K and M depend on the ratio R/r and vary with the method of support of the two edges.

Values of K and M are given in Figs. 48 and 49. The constants are to be applied as follows:

Plate uniformly loaded

K_2, M_2, plate simply supported at inner edge, free at outer edge.

K_7, M_7, plate simply supported at outer edge, free at inner edge.

K_3, M_3, plate built in at inner edge, free at outer edge.

K_4, M_4, plate simply supported at outer edge, inner edge prevented from rotating.

K_5, M_5, plate built in at inner edge, outer edge prevented from rotating.

Plate loaded along edge

K_1, M_1, plate simply supported at outer edge, free at inner edge.

K_6, M_6, plate built in at inner edge, outer edge prevented from rotating.

K_8, M_8, plate built in at inner edge, outer edge free.

Continuous beams are stronger and much stiffer than simple beams. However, a small, unequal subsidence of piers will cause serious changes in sign and magnitude of the bending stresses, reactions, and shears.

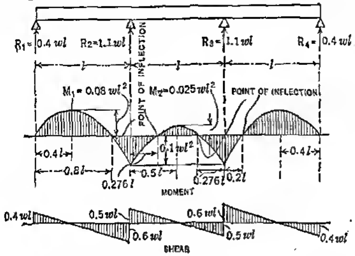


FIG. 40.

Maxwell's Theorem. When a number of loads rest upon a beam, the deflection at any point is equal to the sum of the deflections at this point due to each of the loads taken separately. Maxwell's theorem states that *if unit loads rest upon a beam at two points A and B, the deflection at A* due to the unit load at B equals the deflection at B due to the unit load at A.

Castigliano's theorem *states that the deflection of the point of application of an external force acting on a beam is equal to the partial derivative of the work of deformation with respect to this force.* Thus, if P be the force, f the deflection, and U the work of deformation, which equals the resilience,

$$dU/dP = f$$

According to the **principle of least work,** *the deformation of any structure takes place in such a manner that the work of deformation is a minimum.*

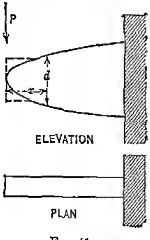


FIG. 41.

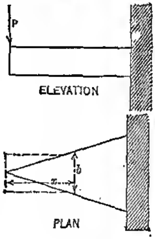


FIG. 42.

Beams of Uniform Strength

Beams of uniform strength so vary in section that the unit stress S remains constant, and I/c varies as M. **For rectangular beams,** of breadth b

9. Rectangular Plate Supported at the Periphery and Subjected to a Concentrated Load at the Center. (a and b = length and breadth, and $b/a = c$.) With the assumption made in (S),

$$S_{max} = 1.5K_1 \frac{c}{(1+c^2)} \frac{P}{t^2} \gtreqless S_s.$$

$K_1 = 1.75$ to 2.00 for cast iron. For $b = a$, $c = 1$ (square plate), and $S_{max} = 0.75K_1P/t^2 \gtreqless S_s$.

Deflection of Square Plates with Different Methods of Support. Deflection under uniformly distributed load $= f = 15b^4pK_2/\pi^4t^3E$. Values of K_2 are as follows:

Method of supporting plate	Center of plate K_2	Middle of unsupported edge K_2
2 edges supported, 2 rigidly fixed	0.134	.....
3 edges supported, 1 rigidly fixed	0.212	.....
4 edges supported, 0 rigidly fixed	0.310	.....
2 edges supported, 1 rigidly fixed, 1 not supported	0.430	0.825
3 edges supported, 1 not supported	0.602	0.915
2 edges supported, 2 not supported	0.994	1.110

Trapezoidal Plates. Calculations for these are made by assuming equivalent rectangular plates.

Flat Cylinder Heads with Flanged Edges (Fig. 51).

$$S_{max} = p\left\{K_1\frac{R}{t} + K_2\left[\frac{r - 0.5R\left(1 + \frac{R}{r}\right)}{t}\right]^2\right\}$$

in which R = radius of curvature of the flange and r = radius of head or attached cylinder, both in in.; p = internal pressure, in lb per sq in.

FIG. 51.

For steel heads riveted to the cylinder $K_1 = 0.5$, and $K_2 = 0.33$ to 0.38; for cast-iron heads integral with the cylinder, $K_1 = K_2 = 0.3$.

Curved Plates with Openings (Bach and Pfleiderer). With a single flue (Figs. 52 and 53) riveted into the plate, the greatest stress due to bending will be found at the flanges, and

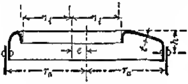


FIG. 52.

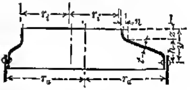


FIG. 53.

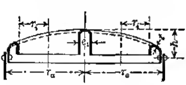


FIG. 54.

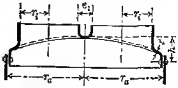
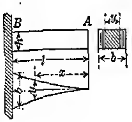
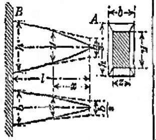
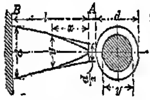


FIG. 55.

$$S_{max} = 0.45p\frac{r_a - r_i}{t^2}\left(r_a - r_i - 2e + \frac{5e^2}{h + 2e}\right).$$

Table 10. Beams of Uniform Strength (in Bending)—*(continued)*

2. Fixed at One End, Load P Uniformly Distributed over l

Beam	Cross section	Elevation and plan	Formulas
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Two parabolic curves with vertices at free end	$y = \frac{3P}{lS_s}\frac{x^2}{h^2}$ $b = \frac{3Pl}{S_s h^2}$ Deflection at A: $f = \frac{3P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (z) variable, depth (y) variable, $\frac{z}{y} = k$	Elevation: Semi-cubic parabola Plan: Semi-cubic parabola	$y^3 = \frac{3P}{kS_s l}x^2$ $z = ky$ $h = \sqrt[3]{\frac{3Pl}{kS_s}}$ $b = kh$
	Circle: diam (y) variable	Elevation: Semi-cubic parabola Plan: Semi-cubic parabola	$y^3 = \frac{16P}{\pi l S_s}x^2$ $d = \sqrt[3]{\frac{16Pl}{\pi S_s}}$

3. Supported at Both Ends, Load P Concentrated at Point C

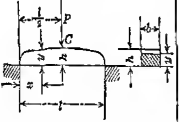
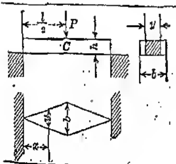
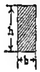
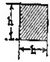
Beam	Cross section	Elevation and plan	Formulas
	Rectangle: width (b) constant, depth (y) variable	Elevation: Two parabolas, vertices at points of support Plan: Rectangle	$y = \sqrt{\frac{3P}{S_s b}x}$ $h = \sqrt{\frac{3Pl}{2bS_s}}$ $f = \frac{P}{2Eb}\left(\frac{l}{h}\right)^3$
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Two triangles, vertices at points of support	$y = \frac{3P}{S_s h^2}x$ $b = \frac{3Pl}{2S_s h^2}$ $f = \frac{3Pl^3}{8Ebh^3}$

Table 11. Torsion of Shafts of Various Cross Sections
(For strength and stiffness of shafts, see pp. 493 and 794)

Cross section	Torsional resisting moment M_t	Angular deflection, α_1 (length = 1 in., radius = 1 in.)		Work of torsion (V = volume)
		In terms of torsional moment	In terms of max shear	
	$\frac{\pi}{16} d^3 S_s$	$\frac{M_t}{GI_p} = \frac{32}{\pi d^4} \frac{M_t}{G}$	$2 \frac{S_s \max}{G} \frac{1}{d}$	$\frac{1}{4} \frac{S_s^2 \max}{G} V$ (Note 1)
	$\frac{\pi}{16} \frac{D^4 - d^4}{D} S_s$	$\frac{32}{\pi(D^4 - d^4)} \frac{M_t}{G}$	$2 \frac{S_s \max}{G} \frac{1}{D}$	$\frac{1}{4} \frac{S_s^2 \max}{G} \frac{D^2 + d^2}{D^2} V$ (Note 2)
	$\frac{\pi}{16} b^2 h S_s$ $(h > b)$	$\frac{16}{\pi} \frac{b^2 + h^2}{b^3 h^3} \frac{M_t}{G}$	$\frac{S_s \max}{G} \frac{b^2 + h^2}{b h^2}$	$\frac{1}{8} \frac{S_s^2 \max}{G} \frac{b^2 + h^2}{h^2} V$ (Note 3)
	$\frac{2}{9} b^2 h S_s$ $(h > b)$	$3.6 \frac{b^2 + h^2}{b^3 h^3} \frac{M_t}{G}$*	$0.8 \frac{S_s \max}{G} \frac{b^2 + h^2}{b h^2}$*	$\frac{4}{45} \frac{S_s^2 \max}{G} \frac{b^2 + h^2}{h^2} V$ (Note 4)
	$\frac{2}{9} h^3 S_s$	$7.2 \frac{1}{h^4} \frac{M_t}{G}$	$1.6 \frac{S_s \max}{G} \frac{1}{h}$	$\frac{8}{45} \frac{S_s^2 \max}{G} V$ (Note 5)
	$\frac{b^3}{20} S_s$	$46.2 \frac{1}{b^4} \frac{M_t}{G}$	$2.31 \frac{S_s \max}{G} \frac{1}{b}$	
	$\frac{b^3}{1.09} S_s$	$0.967 \frac{1}{b^4} \frac{M_t}{G}$	$0.9 \frac{S_s \max}{G} \frac{1}{b}$	

* When h/b =	1	2	4	8
Coefficient 3.6 becomes =	3.56	3.50	3.35	3.21 and
Coefficient 0.8 becomes =	0.79	0.78	0.74	0.71

NOTES.—1. $S_{s_{max}}$ at circumference. 2. $S_{s_{max}}$ at outer circumference. 3. $S_{s_{max}}$ at A; $S_{s_B} = 16M_t/\pi b h^2$. 4. $S_{s_{max}}$ at middle of side h; in middle of b, $S_s = 9M_t/2bh^2$. 5. $S_{s_{max}}$ at middle of side.

SPRINGS

It is assumed in the following formulas that the springs are in no case stressed beyond the elastic limit (*i.e.*, that they are perfectly elastic), and that they are subject to Hooke's law.

tilever and R/S_s in a simple beam. The dotted extensions in Figs. 41 and 42 show the changes necessary to enable these cantilevers to resist shear. The waste in material and extra cost in fabricating, however, make many of the forms impractical, except for cast iron.

Table 10 shows some of the simple **sections of uniform strength.** In none of these, however, is shear taken into account.

STRENGTH OF PLATES

Notation.

P = total load, lb.
p = load per unit area, lb per sq in.
t = thickness of plate, in.
S_{max} = maximum stress, lb per sq in.
S_s = safe stress, lb per sq in.
E = modulus of elasticity.
f = deflection, in.
K_1 and K_2 = coefficients.

The values of K_1 and K_2 are dependent upon the method of support and upon the initial force required to give a tight joint (to prevent leakage) before the load is applied. Cases 3 and 6 follow Grashof; the others, C. Bach. The formulas apply only within the elastic limit.

FIG. 43.

Flat Plates

1. Circular Plate Subjected to Uniformly Distributed Load.

FIG. 44.

$$S_{max} = K_1 r^2 p/t^2 \leqq S_s, \quad \text{and} \quad f = K_2 r^4 p/t^3 E.$$

For cast iron, K_1 = 0.8 (fixed edge) to 1.2 (freely supported); K_2 = 0.17 (fixed edge) to 0.60 (freely supported). For mild steel K_1 = 0.50 (not < 0.45) with fixed edge to 0.75 (not < 0.67) when freely supported.

In Fig. 43, S_{max} is at edge. In Fig. 44, S_{max} is at center. If the fixed ends deflect sufficiently to make the center and circumferential stresses equal, K_1 = 0.38, or not < 0.33.

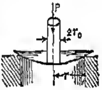


FIG. 45.

2. Circular Plate Loaded at the Center and Freely Supported at the Circumference (Fig. 45); P uniformly distributed over area πr_0^2.

$$S_{max} = \frac{3}{\pi}K_1\left(1 - \frac{2}{3}\frac{r_0}{r}\right)\frac{P}{t^2} \leqq S_s; \quad f = K_2\frac{r^2 P}{t^3 E}$$

For cast iron, K_1 = 1.5; K_2 = 0.4 to 0.5.

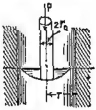


FIG. 46.

3. Circular Plate Loaded as in (2) but Fixed at the Circumference (Fig. 46).

$$S_{max} = \frac{1.365}{\pi}\frac{P}{t^2}\log_e\frac{r}{r_0} \leqq S_s;$$

$$f = \frac{0.6825}{\pi}\frac{r^2 P}{t^3 E} = 0.22\frac{r^2 P}{t^3 E}$$

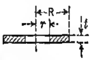
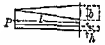
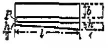
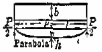
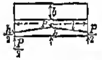


FIG. 47.

4. Circular Plates with Concentric Circular Holes (Fig. 47).

The following formulas are used by the Westinghouse Electric & Mfg. Co. for the calculation of the strength of flat circular plates with holes (Fig. 47), such as diaphragms, flanges, and Kingsbury thrust-bearing plates, where the thickness is not over 15 percent of the difference in inside and outside diameters and where the deflection is not over half the thickness. The Kingsbury

Table 12. Strength and Deflection of Single-leaf Flat Springs

Plans and elevations of springs	General				$S_s = 60{,}000$; $E = 30{,}000{,}000$		
	a	c	U	v	a	c	U
	Load P applied at end of spring						
	$\frac{S_s}{E}$	$\frac{6}{S_s}$	$\frac{S_s^2}{6E}$	$\frac{1}{2}$	$\frac{2}{1000}$	$\frac{1}{10{,}000}$	20.0
 Parabola	$\frac{4S_s}{3E}$	$\frac{6}{S_s}$	$\frac{S_s^2}{6E}$	$\frac{2}{3}$	$\frac{8}{3}\times\frac{1}{1000}$	$\frac{1}{10{,}000}$	20.0
	$\frac{2S_s}{3E}$	$\frac{6}{S_s}$	$\frac{0.33S_s^2}{6E}$	1	$\frac{4}{3}\times\frac{1}{1000}$	$\frac{1}{10{,}000}$	6.66
	$\frac{0.87S_s}{E}$	$\frac{6}{S_s}$	$\frac{0.70S_s^2}{6E}$	$\frac{5}{8}$	$\frac{1.75}{1000}$	$\frac{1}{10{,}000}$	14.0
	$\frac{1.09S_s}{E}$	$\frac{6}{S_s}$	$\frac{0.725S_s^2}{6E}$	$\frac{3}{4}$	$\frac{2.18}{1000}$	$\frac{1}{10{,}000}$	14.52
	Load P applied at center of spring						
	$\frac{S_s}{4E}$	$\frac{6}{4S_s}$	$\frac{S_s^2}{6E}$	$\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	20.0
	$\frac{0.87S_s}{4E}$	$\frac{6}{4S_s}$	$\frac{0.70S_s^2}{6E}$	$\frac{5}{8}$	$\frac{7}{16}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	14.0
 Parabola	$\frac{S_s}{3E}$	$\frac{0}{4S_s}$	$\frac{S_s^2}{6E}$	$\frac{2}{3}$	$\frac{2}{3}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	20.0
	$\frac{1.09S_s}{4E}$	$\frac{6}{4S_s}$	$\frac{0.725S_s^2}{6E}$	$\frac{3}{4}$	$\frac{0.54}{1000}$	$\frac{1}{40{,}000}$	14.52
	$\frac{S_s}{6E}$	$\frac{6}{4S_s}$	$\frac{0.33S_s^2}{6E}$	1	$\frac{1}{3}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	6.66

5. Laminated Triangular Plate Spring (Fig. 61). If the triangular plate spring shown at I be cut into an even number ($= 2n$) of strips of equal width (in this case 8 strips of width $b/2$), and these strips be combined, a laminated spring will be formed whose carrying capacity will equal that of the original uncut spring; or $P = nbh^2S_s/6l$; $n = 6Pl/bh^2S_s$.

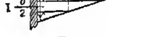


FIG. 61.

6. Laminated rectangular plate spring with leaf ends tapered in the form of a cubical parabola (Fig. 62); see (3) *ante*.

A plate which is "built in" is held rigidly and is subjected to both shearing force and bending moments. A plate which is "prevented from rotating" on an edge has zero slope in the radial direction at that edge but is permitted to move perpendicular to the plane of the plate; it is not subjected to shearing forces but only to bending moments. An example is the inner edges of a turbine diaphragm with a heavy hub and simply supported at the outer edge.

Example. A diaphragm having a clamped inner edge is loaded with a total load, P, 5.8 lb uniformly distributed along the outer edge.

$R = 1.12$ in.; $r = 0.5$ in.; $t = 0.017$ in.; $R/r = 2.25$; $E = 30 \times 10^6$ lb per sq in.
From Fig. 48, $K_1 = 0.9$; from Fig. 49, $M_1 = 0.12$.

The maximum stress, $S = K_1P/t^2 = 0.9 \times 5.8/(0.017)^2 = 18{,}000$ lb per sq in. The maximum deflection, $y = M_1PR^2/Et^3 = 0.12 \times 5.8 \times (1.12)^2/30 \times 10^6 \times (0.017)^3 = 0.006$ in.

5. Elliptical Plate Subjected to a Uniformly Distributed Load p (see Figs. 43 and 44).

Major axis $= 2a$; minor axis $= 2b$; ratio of axes $= b/a = c$.

$$S_{max} = K_1 \frac{b^2}{t^2} \frac{2p}{(1 + c^2)} \gtreqless S_t$$

For cast iron, $K_1 = 0.67$ (fixed) to 1.13 (freely supported).
For mild steel, $K_1 = 0.42$ (fixed) to 0.71 (freely supported).
For $c = 1$ (circle), $S_{max} = K_1b^2p/t^2$.

6. Elliptical Plate with Load P at the Center. Notation as in (5).

$$S_{max} = \frac{8}{5\pi} K_1 \frac{8 + 4c^2 + 3c^4}{3 + 2c^2 + 3c^4} c \frac{P}{t^2} \gtreqless S_t.$$

For cast iron, $K_1 = 1.50$ (fixed) to 1.67 (freely supported).
For $c = 1$ (circle), $S_{max} = 3K_1P/\pi t^2$.

7. Infinitely large plate subjected to a uniformly distributed load and supported at points distant a from each other (Fig. 50). For each single square,

$$S_{max} = 0.2275a^2p/t^2 \gtreqless S_t, \text{ and } f = 0.0284a^4p/t^3E.$$

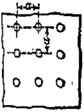


FIG. 50.

8. Rectangular Plate Supported at the Periphery and Subjected to a Uniformly Distributed Load. (a and b = length and breadth of plate; $b/a = c$.) For the calculation of stress along the diagonal, as in case (5) of an ellipse,

$$S_{max} = K_1 \frac{(b/2)^2}{t^2} \frac{2p}{(1 + c^2)} \gtreqless S_t$$

When $b = a$, $c = 1$ (square plate), and $S_{max} = 0.25K_1a^2p/t^2 \gtreqless S_t$.
For cast iron, $K_1 = 0.75$ (fixed) to 1.13 (freely supported).
For mild steel, $K_1 = 0.48$ (fixed) to 0.72 (freely supported).
If the plate is supported on two sides ($b = \infty$), the coefficient of a^2p/t^2 will be 0.50 (fixed) to 0.75 (freely supported).

Note. In cases (5) and (8), values of K_1 for mild steel are: for elliptical plates, 0.43 to 0.55 (max = 0.86); rectangular plates, 0.56 to 0.75 (max = 1.13). The maxima, which correspond to values for free support, are to be rarely used. The minimum value of K_1 corresponds to the case where the moment at the center equals the moment at the support, $M_{max} = pl^2/16$.

Tests on rectangular cast-iron plates at Case School of Applied Science (1896–1897), with load applied at center, give the following values for the breaking load W. For plates supported at the edges, $W = 276St^2/(l^2 + b^2)$; for plates with fixed edges, $W = 442St^2/(l^2 + b^2)$. The plates tested were 10 × 15 in., ½ to 1⅛ in. thick; modulus of rupture of the cast iron, 33,000 lb per sq in.

spring $= l$. See "Experiments on Helical Springs," by Benjamin and French, *Trans. A.S.M.E.*, **23**, p. 298.

For heavy closely-coiled **helical springs** the usual formulas are inaccurate and result in stresses greatly in excess of those assumed. See Wahl, "Stresses in Heavy Closely-coiled Helical Springs," *Trans. A.S.M.E.*, 1928. *In cases 10 to 12 and 15 to 18, the quantity k is unity for lighter springs and* has the stated values (supplied by Wahl) for heavy closely coiled springs.

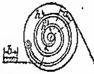


FIG. 65.

10. Spiral Coiled Spring of Rectangular Cross Section (Fig. 65).

$P = bh^2S_b/6rk$; $I = bh^3/12$; $U = Pf/2 = S_b^2V/6Ek^2$;
$f = ra = Plr^2/EI = 12Plr^2/Ebh^3 = 2rlS_b/hEk$.

For heavy closely-coiled springs $k = (3c - 1)/(3c - 3)$, where $c = 2R/h$ and R is the minimum radius of curvature at the center of the spiral.

11. Cylindrical Helical Spring of Circular Cross Section (Fig. 66).

$P = \pi d^3S_b/32rk$; $I = \pi d^4/64$; $U = Pf/2 = S_b^2V/8Ek^2$;
$f = ra = Plr^2/EI = 64Plr^2/\pi Ed^4 = 2rlS_b/dEk$.

For heavy closely-coiled springs, $k = (4c - 1)(4c - 4)$ where $c = 2r/d$.

12. Cylindrical Helical Spring of Rectangular Cross Section (Fig. 67).

$P = bh^2S_b/6rk$; $I = bh^3/12$; $U = Pf/2 = S_b^2V/6Ek^2$;
$f = ra = Plr^2/EI = 12Plr^2/Ebh^3 = 2rlS_b/hEk$.

For heavy closely-coiled springs, $k = (3c - 1)/3c - 3)$, where $c = 2r/h$.

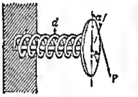


FIG. 66.

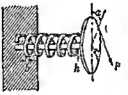


FIG. 67.

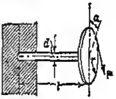


FIG. 68.

Springs Subjected to Torsion

The statements made concerning coiled springs subjected to bending apply also to (13) and (14).

13. Straight Bar Spring of Circular Cross Section (Fig. 68).

$P = \pi d^3S_v/16r = 0.1963d^3S_v/r$; $U = Pf/2 = S_v^2V/4G$;
$f = ra = 32r^2lP/\pi d^4G = 2rlS_v/dG$.

14. Straight Bar Spring of Rectangular Cross Section (Fig. 69).

$P = 2b^2hS_v/9r$; $K = b/h$; $U = Pf/2 = 4S_v^2V(K^2 + 1)/45G$ (maximum when $K = 1$);
$f = ra = 3.6r^2lP(b^2 + h^2)/b^3h^3G = 0.8rlS_v(b^2 + h^2)/bh^2G$.

Springs Loaded Axially Either in Tension or Compression

[Note. For springs 15 to 18, r = mean radius of coil; n = number of coils.]

15. Cylindrical Helical Spring of Circular Cross Section (Fig. 70).

$P = \pi d^3S_v/16rk = 0.1963d^3S_v/rk$; $U = Pf/2 = S_v^2V/4Gk^2$;
$f = 64nr^3P/d^4G = 4\pi nr^2S_v/dGk$.

With two flues (Figs. 54 and 55),

$$S_{max} = 22p\frac{r_a - 1.5r_i}{t^2}\left(r_a - r_i - 2e + \frac{5e^2}{h + 2e}\right).$$

In computing h (Fig. 53), assume $s = 1.5$ to $2t$.

TORSION

Under torsion, a bar (Fig. 56) is twisted by a couple of the value Pp. Elements of the surface become helices of angle d, and a radius rotates through an angle a in a length l, both d and a being expressed in radians. S_v = shearing unit stress at distance r from center; I_P = polar moment of inertia; G = shearing modulus of elasticity. It is assumed that the cross sections remain plane surfaces. The strain on the cross section is wholly tangential, and is zero at the center of the section. $ld = ra$.

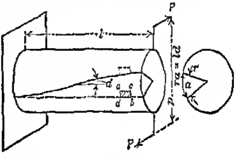


FIG. 56.

In the case of a **circular cross section**, the stress S_v increases directly as the distance of the strained element from the center.

The polar moment of inertia I_P for any section may be obtained from $I_P = I_1 + I_2$, where I_1 and I_2 are the rectangular moments of inertia of the section about any two lines at right angles to each other, through the center of gravity.

The **external twisting moment** M_t is balanced by the internal resisting moment.

For strength, $M_t = S_v I_P/r$.

For stiffness, $M_t = aGI_P/l$.

The torsional resilience $U = \frac{1}{2}Ppa = S_v^2 I_P l/2r^2G = a^2GI_P/2l$.

The state of stress on an element taken from the surface of the shaft, as in Fig. 57, is pure shear. Pure tension exists at right angles to one 45 deg helix and pure compression at right angles to the opposite helix.

Reduced formulas for shafts of various sections are given in Table 11.

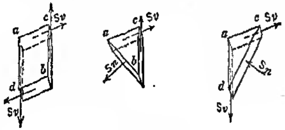


FIG. 57.

Failure under torsion in brittle materials is a tensile failure at right angles to a helical element of the surface. Plastic materials twist off squarely. Fibrous materials separate in long strips.

Torsion of Non-circular Sections. When a section is not circular, the unit stress no longer varies directly as the distance from the center. Cross-sections become warped, and the greatest unit stress usually occurs at a point on the perimeter of the cross-section *nearest* the axis of twist. There is no stress at the corners of square and rectangular sections, and the analyses become complex.

Table 13. Safe Working Loads and Deflections of Cylindrical Helical Steel Springs of Circular Cross Section

Allowable unit stress, lb per sq in.	Wire gage W.&M.	Diam, in.		Pitch diameter, D in.																			
			D	5/32	3/16	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/4
150,000	20	.035	F	16.2	13.4	10.0	8.10	6.66	5.75	4.96	4.05	3.39											
			f	.026	.037	.067	.105	.149	.200	.276	.420	.608											
	19	.041	F	26.2	21.6	16.2	13.0	10.8	9.27	8.10	6.52	5.35	4.57										
			f	.023	.032	.057	.089	.128	.175	.229	.362	.512	.697										
	18	.047	F	39.1	32.6	24.5	19.6	16.4	13.9	12.3	9.80	8.10	6.92	6.14									
			f	.019	.028	.049	.078	.112	.153	.200	.311	.449	.610	.800									
	17	.054	F	59.4	49.6	37.2	29.7	24.6	21.2	18.5	14.7	12.4	10.5	9.25	8.23								
			f	.016	.024	.043	.067	.098	.133	.174	.273	.390	.532	.695	.878								
	3/16"	.062	F	....	74.9	56.1	44.9	37.3	32.0	28.0	22.4	18.6	16.1	13.9	12.5	11.2							
			f	....	.021	.037	.058	.084	.115	.151	.235	.340	.460	.605	.760	.947							
	16	.063	F	....	78.2	58.7	46.9	39.2	33.9	29.4	23.5	19.6	16.8	14.7	13.2	11.9	10.7						
			f	....	.020	.037	.057	.083	.113	.148	.233	.335	.445	.591	.748	.930	1.12						
	15	.072	F	....	117.	80.7	70.0	58.7	50.2	43.6	35.2	29.0	25.0	21.9	19.5	17.5	16.0						
			f	....	.018	.032	.050	.077	.100	.130	.203	.294	.405	.521	.652	.802	.986						
	14	.080	F	....	....	121	96.6	80.5	69.1	60.4	48.2	40.1	34.6	30.1	26.7	24.2	22.1	20.2					
			f	....	....	.029	.045	.065	.090	.117	.183	.262	.359	.470	.593	.735	.886	1.105					
140,000	13	.092	F	....	....	171	136	113	97.6	85.5	68.9	57.3	48.8	42.6	37.8	34.5	31.3	28.6					
			f	....	....	.023	.037	.053	.072	.098	.148	.214	.291	.388	.481	.596	.720	.854					
	3/32"	.093	F	....	....	178	142	118	99.5	89.0	71.2	59.1	50.9	44.3	39.6	35.7	32.3	29.6	27.3				
			f	....	....	.023	.036	.052	.071	.093	.146	.211	.286	.376	.473	.585	.707	.841	.986				
	12	.105	F	....	....	....	204	170	147	127	102	85.4	73.0	63.4	56.6	51.1	46.3	42.6	38.9				
			f	....	....	....	.032	.047	.064	.083	.122	.188	.256	.336	.425	.512	.632	.755	.880				
	11	.120	F	....	....	....	303	253	217	190	152	126	108	95.2	84.2	76.2	69.2	63.5	58.5	54.3			
			f	....	....	....	.028	.041	.055	.073	.114	.174	.223	.296	.368	.449	.551	.657	.768	.893			
	1/8"	.125	F	....	....	....	....	286	245	214	171	143	121	107	95.5	85.2	78.0	71.5	65.8	60.8	57.2		
			f	....	....	....	....	.039	.053	.069	.109	.169	.213	.278	.353	.437	.528	.626	.731	.855	.981		
	10	.135	F	....	....	....	....	359	309	270	217	171	154	135	120	108	98.7	90.2	82.7	72.2	71.8	67.5	
			f	....	....	....	....	.036	.049	.064	.106	.145	.198	.260	.327	.399	.486	.581	.680	.791	.908	1.04	
	9	.148	F	....	....	....	....	....	408	356	285	237	207	178	158	142	130	118	109	102	95.0	89.0	
			f	....	....	....	....	....	.045	.059	.092	.132	.180	.235	.293	.370	.448	.530	.620	.723	.828	.945	
	5/32"	.156	F	....	....	....	....	....	480	418	330	270	234	208	185	167	152	139	128	119	111	104	92.7
			f	....	....	....	....	....	.042	.056	.087	.125	.171	.223	.282	.350	.422	.509	.588	.685	.785	.896	1.12
	8	.162	F	....	....	....	....	....	....	466	376	311	276	234	207	187	170	156	143	134	125	117	103
			f	....	....	....	....	....	....	.054	.084	.121	.165	.216	.273	.338	.409	.488	.566	.663	.757	.863	1.09

Notation:

P = Safe load, lb.
f = Deflection for a given load, P, in.
l = Length of spring, in.
V = Volume of spring, cu in.
S_s = Safe stress (due to bending), lb per sq in.
S_v = Safe shearing stress, lb per sq in.
U = Resilience, in.-lb.

For values of E, see p. 418. Benjamin and Hoffman (in "Machine Design") recommend the following values for S_v, G and E: For spring brass wire, S_v = 45,000 to 60,000; G = 6,000,000; E = 9,000,000. For spring steel, tempered, S_v = 75,000 to 115,000; G = 12,000,000 to 15,000,000: E = 30,000,000.

The work in inch-pounds performed in deflecting a spring from 0 to f (spring duty) is $U = Pf/2 = S_s^2V/CE$. This is based upon the assumption that the deflection is proportional to the load. C is a constant dependent upon the shape of the springs.

The time of vibration T (in seconds) of a spring (weight not considered) is equal to that of a simple circular pendulum whose length l_0 equals the deflection f (in ft) that is produced in the spring by the load P. $T = \pi\sqrt{l_0/g}$, where g = acceleration of gravity in ft per sec.[2]

Springs Subjected to Bending

(See first case in Table 2, p. 451)

1. Rectangular Plate Spring (Fig. 58).

$$P = bh^2S_s/6l; \quad I = bh^3/12; \quad U = Pf/2 = VS_s^2/18E;$$
$$f = Pl^3/3EI = 4Pl^3/bh^3E = 2l^2S_s/3hE.$$

2. Triangular Plate Spring (Fig. 59). The elastic curve is a circular arc.

$$P = bh^2S_s/6l; \quad I = bh^3/12; \quad U = Pf/2 = S_s^2V/6E;$$
$$f = Pl^3/2EI = 6Pl^3/bh^3E = l^2S_s/hE.$$

3. Rectangular plate spring with end tapered in the form of a cubical parabola (Fig. 60). The elastic curve is a circular arc. P, I, and f same as for triangular plate spring (Fig. 59). $U = Pf/2 = S_s^2V/9E$.

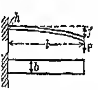


Fig. 58.

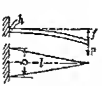


Fig. 59.

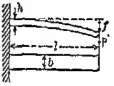


Fig. 60.

The strength and deflection of single-leaf flat springs of various forms are given (R. A. Bruce, *Am. Mach.*, July 19, 1900) by the formulas $h = al^2/f$ and $b = cPl/h^2$. The volume of the spring is given by $V = vlbh$. The values of the constants a and c and the resilience in inch-pounds per cubic inch are given in Table 12, both in terms of the safe stress S_s and for stated specific values of S_s and E. Values of v are given also.

4. Compound (Leaf or Laminated) Springs. If several springs of rectangular section are combined, the resulting compound spring should (1) form a beam of uniform strength that (2) does not open between the joints while bending (*i.e.*, elastic curve must be a circular arc). Only the type immediately following meets both requirements, the others meeting only the second requirement.

Table 13—(*continued*)

Allowable unit stress, lb per sq in.	Wire gage W.&M.	Diam, in.	Pitch diameter, D in.																	
			D	1¼	1⅜	1½	1⅝	1¾	1⅞	2	2¼	2½	2¾	3	3½	4	4½	5	5½	6
115,000	13/32″	.406	P	2400	2170	2000	1840	1710	1600	1500	1330	1200	1090	1000	855	750	666			
			f	.108	.134	.159	.186	.217	.248	.204	.353	.444	.525	.640	.867	1.13	1.43			
	00000	.430	P	2875	2610	2400	2210	2050	1918	1798	1598	1440	1308	1200	1028	900	800			
			f	.104	.126	.150	.175	.204	.234	.267	.338	.418	.503	.600	.815	1.06	1.35			
	7/16″	.437	P	3000	2730	2500	2310	2140	2000	1800	1665	1500	1365	1250	1074	940	835	750		
			f	.100	.124	.148	.173	.201	.231	.264	.327	.412	.490	.593	.810	1.05	1.33	1.64		
110,000	000000	.460	P	....	3065	2800	2580	2400	2230	2100	1865	1680	1530	1400	1200	1058	952	840		
			f	....	.112	.134	.157	.183	.209	.239	.303	.374	.447	.536	.729	.956	1.21	1.49		
	15/32″	.468	P	....	3265	2940	2725	2530	2375	2210	1970	1770	1610	1472	1265	1110	935	885		
			f	....	.111	.132	.154	.182	.206	.235	.295	.368	.444	.530	.720	.943	1.19	1.47		
	0000000	.490	P	....	3675	3270	3115	2890	2710	2535	2245	2025	1840	1690	1445	1268	1125	1015	920	
			f	....	.106	.126	.148	.172	.196	.225	.284	.351	.424	.506	.688	.900	1.13	1.40	1.70	
	½″	.500	P	....	....	3610	3320	3090	2890	2710	2410	2160	1970	1810	1550	1352	1205	1062	985	
			f	....	....	.123	.144	.168	.192	.220	.274	.347	.415	.495	.672	.880	1.11	1.37	1.65	
	9/16″	.562	P	....	....	....	4700	4390	4090	3830	3420	3080	2790	2565	2190	1913	1710	1535	1395	1280
			f	....	....	....	.128	.149	.175	.195	.248	.306	.372	.440	.596	.782	.990	1.22	1.47	1.75
	⅝″	.625	P	....	....	....	....	6100	5600	5260	4660	4210	3825	3505	3000	2630	2340	2110	1913	1750
			f	....	....	....	....	.134	.154	.176	.218	.275	.328	.397	.538	.705	.875	1.05	1.33	1.58
100,000	11/16″	.687	P	....	....	....	....	....	....	6325	5660	5090	4630	4250	3625	3195	2825	2560	2330	2125
			f	....	....	....	....	....	....	.145	.183	.228	.274	.327	.443	.580	.733	.908	1.00	1.30
	¾″	.750	P	....	....	....	....	....	....	....	7400	6640	6030	5540	4745	4150	3690	3325	3025	2770
			f	....	....	....	....	....	....	....	.178	.218	.252	.299	.402	.532	.671	.832	1.00	1.19
	13/16″	.812	P	....	....	....	....	....	....	....	....	8420	7660	7000	6000	5260	4675	4200	3825	3500
			f	....	....	....	....	....	....	....	....	.192	.232	.276	.376	.490	.620	.766	.880	1.10
	⅞″	.875	P	....	....	....	....	....	....	....	....	10830	9550	8700	7500	6560	5740	5250	4770	4730
			f	....	....	....	....	....	....	....	....	.179	.218	.257	.348	.456	.577	.712	.860	1.02
90,000	15/16″	.937	P	....	....	....	....	....	....	....	....	....	10600	9700	8400	7160	6470	5810	5290	4850
			f	....	....	....	....	....	....	....	....	....	.179	.217	.290	.383	.480	.591	.715	.855
	1″	1.000	P	....	....	....	....	....	....	....	....	....	....	11780	10100	8800	7850	7050	6330	5870
			f	....	....	....	....	....	....	....	....	....	....	.206	.276	.360	.454	.561	.680	.803
	1⅛″	1.125	P	....	....	....	....	....	....	....	....	....	....	....	14400	12600	11230	10100	9200	8400
			f	....	....	....	....	....	....	....	....	....	....	....	.244	.320	.405	.496	.600	.718
	1¼″	1.250	P	....	....	....	....	....	....	....	....	....	....	....	24700	18200	15300	13250	12540	11500
			f	....	....	....	....	....	....	....	....	....	....	....	.260	.287	.364	.442	.545	.648
80,000	1⅜″	1.375	P	....	....	....	....	....	....	....	....	....	....	....	....	20400	18100	16150	14850	13600
			f	....	....	....	....	....	....	....	....	....	....	....	....	.280	.294	.364	.440	.522

7. Laminated Trapezoidal Plate Spring with Leaf Ends Tapered. (Fig. 63.) The ends of the leaves are trapezoidal in form and are tapered according to the *formula*

$$z = h/1 + \sqrt[3]{\frac{b_1}{b}\left(\frac{a}{x} - 1\right)}$$

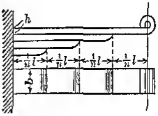


FIG. 62.

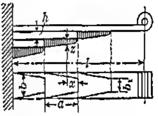


FIG. 63.

8. Semielliptic Springs (for locomotives, horse-drawn and automobile trucks, etc.). *Referring to Fig. 64, the load $2P$ (lb) acting on the spring center band produces a tensional stress $P/\cos a$ in each of the inclined shackle links.* This is resolved into the vertical force P and the horizontal force $P \tan a$, which together produce a bending moment $M = P(l + p \tan a)$. The *equations* given in (1), (2), and (3) apply to curved as well as straight springs. The bearing force $= 2P = (2nbh^2/6)[S_s/(l + p \tan a)]$, and the deflection $= (6l^2/nbh^3)P(l + p \tan a)/E = l^2S_s/hE$.

In addition to the bending moment, the leaves are subjected to the tensional force $P \tan a$ and the transverse force P, which produce in the upper leaf an additional stress $S = P \tan a/bh$, as well as a shearing stress to be calculated according to p. 465.

In determining the number of leaves n in a given spring, allowance should be made for an excess load on the spring caused by the vibration. This is usually made by decreasing the allowable stress about 15 percent.

The foregoing does not take account of initial stresses caused by the band. For more detailed information, see "The Design of Automobile Springs," by E. R. Morrison, *Machinery*, Jan., 1910.

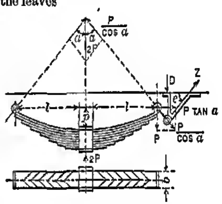


FIG. 64.

9. Elliptic Springs. Safe load P(lb) $= nbh^2S_s/6l$, where $l = \frac{1}{2}$ distance between bolt eyes (less $\frac{1}{2}$ length of center band, where used); deflection f (in.) $= 4l^2S_sK/hE$, where

$$K = \frac{1}{(1 - r)^3}\left[\frac{1 - r^2}{2} - 2r(1 - r) - r^2 \log_e r\right]$$

r being the number of full-length leaves $\div$ total number (n) of leaves in the spring. All dimensions in inches. For semielliptic springs, the deflection is only half as great. Safe load $= nbh^2S_s/3l$. (J. B. Peddle, *Am. Mach.*, Apr. 17, 1913.)

Coiled Springs. In these, the load is applied as a couple Pr which turns the spring while winding or holds it in place when wound up. If the spindle is not to be subjected to bending moment, P must be replaced by two equal and opposite forces ($P/2$) acting at the circumference of a circle of radius r. The formulas are the same in both cases. The springs are assumed to be fixed at one end and free at the other. The bending moment acting on the section of least resistance is always Pr. The length of the straightened

If, as in certain classes of masonry construction, the material cannot withstand tensile stress and thus no tension can occur, the center of moments (Fig. 75) is taken at the center of stress. For a rectangular section, P acts at distance k from the nearest edge. Length under compression $= 3k$, and $S_m = \frac{2}{3}P/hk$. For a circular section, $S_m = [0.372 + 0.056(k/r)]P/k\sqrt{rk}$, where r = radius and k = distance of P from circum-

Table 14. Values of the Ratio z/r (Fig. 76)

$\frac{e}{r}$ \ $\frac{r_1}{r}$	0.0	0.5	0.6	0.7	0.8	0.9	1.0	$\frac{e}{r}$
0.25	2.00	....	....	....	....	....	....	0.25
0.30	1.82	....	....	....	....	....	....	0.30
0.35	1.66	1.89	1.98	....	....	....	....	0.35
0.40	1.51	1.75	1.84	1.93	....	....	....	0.40
0.45	1.37	1.61	1.71	1.81	1.90	....	....	0.45
0.50	1.23	1.46	1.56	1.66	1.78	1.89	2.00	0.50
0.55	1.10	1.29	1.39	1.50	1.62	1.74	1.87	0.55
0.60	0.97	1.12	1.21	1.32	1.45	1.58	1.71	0.60
0.65	0.84	0.94	1.02	1.13	1.25	1.40	1.54	0.65
0.70	0.72	0.75	0.82	0.93	1.05	1.20	1.35	0.70
0.75	0.59	0.60	0.64	0.72	0.85	0.99	1.15	0.75
0.80	0.47	0.47	0.48	0.52	0.61	0.77	0.94	0.80
0.85	0.35	0.35	0.35	0.36	0.42	0.55	0.72	0.85
0.90	0.24	0.24	0.24	0.24	0.24	0.32	0.49	0.90
0.95	0.12	0.12	0.12	0.12	0.12	0.12	0.25	0.95

Table 15. Values of the Ratio S_{max}/S_{avg}

(NOTE. In determining S average, use load P divided by total area of cross section)

$\frac{e}{r}$ \ $\frac{r_1}{r}$	0.0	0.5	0.6	0.7	0.8	0.9	1.0	$\frac{e}{r}$
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
0.05	1.20	1.16	1.15	1.13	1.12	1.11	1.10	0.05
0.10	1.40	1.32	1.29	1.27	1.24	1.22	1.20	0.10
0.15	1.60	1.48	1.44	1.40	1.37	1.33	1.30	0.15
0.20	1.80	1.64	1.59	1.54	1.49	1.44	1.40	0.20
0.25	2.00	1.80	1.73	1.67	1.61	1.55	1.50	0.25
0.30	2.23	1.96	1.88	1.81	1.73	1.66	1.60	0.30
0.35	2.48	2.12	2.04	1.94	1.85	1.77	1.70	0.35
0.40	2.76	2.29	2.20	2.07	1.98	1.88	1.80	0.40
0.45	3.11	2.51	2.39	2.23	2.10	1.99	1.90	0.45
0.50	3.55	2.80	2.61	2.42	2.26	2.10	2.00	0.50
0.55	4.15	3.14	2.89	2.67	2.42	2.26	2.17	0.55
0.60	4.96	3.58	3.24	2.92	2.64	2.42	2.26	0.60
0.65	6.00	4.34	3.80	3.30	2.92	2.64	2.42	6.65
0.70	7.48	5.40	4.65	3.86	3.33	2.95	2.64	0.70
0.75	9.93	7.26	5.97	4.81	3.93	3.33	2.89	0.75
0.80	13.87	10.05	8.80	6.53	4.93	3.96	3.27	0.80
0.85	21.08	15.55	13.32	10.43	7.16	4.50	3.77	0.85
0.90	38.25	30.80	25.80	19.85	14.60	7.13	4.71	0.90
0.95	96.10	72.20	62.20	50.20	34.60	19.80	6.72	0.95
1.00	∞	∞	∞	∞	∞	∞	∞	1.00

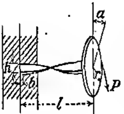
For heavy closely-coiled springs, $k = [(4c - 1)/(4c - 4)] + 0.615/c$, where $c = 2r/d$.

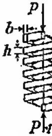
16. Cylindrical Helical Spring of Rectangular Cross Section (Fig. 71).

$P = 2b^2hS_v/9rk$; $K = b/h$; $U = Pf/2 = 4S_v^2V(K^2 + 1)/45GK^2$ (maximum when $K = 1$);

$f = 7.2\pi nr^3P(b^2 + h^2)/b^3h^3G = 1.6\pi nr^2S_v(b^2 + h^2)/bh^2Gk$.

For heavy closely-coiled springs, $k = [(4c - 1)/(4c - 4)] + 0.615/c$, where $c = 2r/b$.





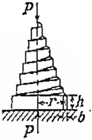


Fig. 69. Fig. 70. Fig. 71. Fig. 72. Fig. 73.

17. Conical Helical Spring of Circular Cross Section (Fig. 72).

l = length of developed spring; d = diameter of wire; r = maximum mean radius of coil.

$P = \pi d^3S_v/16rk = 0.1963d^3S_v/rk$; $U = Pf/2 = S_v^2V/8Gk^2$;

$f = 16r^2lP/\pi d^4G = 16nr^3P/d^4G = rlS_v/dGk = \pi nr^2S_v/dGk$.

For heavy closely-coiled springs, $k = [(4c - 1)/(4c - 4)] + 0.615/c$, where $c = 2r/d$.

18. Conical Helical Spring of Rectangular Cross Section (Fig. 73).

$K = b/h$; $P = 2b^2hS_v/9rk$; $U = Pf/2 = 2S_v^2V(K^2 + 1)/45GK^2$ (maximum when $K = 1$);

$f = 1.8r^2lP(b^2 + h^2)/b^3h^3G = 1.8\pi nr^3P(b^2 + h^2)/b^3h^3G$.

$= 0.4rlS_v(b^2 + h^2)/bh^2Gk = 0.4\pi nr^2S_v(b^2 + h^2)/bh^2Gk$.

For heavy closely-coiled springs, $k = [(4c - 1)/(4c - 4)] + 0.615/c$, where $c = 2r/b$.

19. Truncated Conical Springs (Types 17 and 18). The formulas under (17) and (18) apply for truncated springs. In calculating deflection f, however, it is necessary to substitute $(r_1^2 + r_2^2)$ for r^2, and $\pi n(r_1 + r_2)$ for πnr, r_1 and r_2 being respectively the greatest and least mean radii of the coils.

Note. The preceding formulas for various forms of coiled springs are sufficiently accurate when the cross section dimensions are small in comparison with the radius of the coil, and for small pitch. Springs (15) to (19) are for either tension or compression.

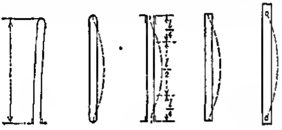
Safe working loads and deflections of cylindrical helical springs of round steel wire in tension or compression are given in Table 13. The table is based on the formulas given in (15) ante. d = diameter of steel, in.; D = pitch diameter (center to center of wire), in.; P = safe working load for given unit stress, lb; f = deflection of 1 coil for safe working load, in.

The table is based on the values of unit stress indicated, and G = 12,500,000. For any other value of unit stress, divide the tabular value by the unit stress used in the table and multiply by the unit stress to be used in the design. For any other value of G, multiply the value of f in the table by 12,500,000 and divide by the value of G chosen. **For square steel**, multiply values of P

critical load at which equilibrium no longer obtains. The ratio of length to radius of gyration, or **slendernsss ratio,** at which a column begins to fail by buckling, is between 100 and 120. Such columns are computed by Euler's formula. Few structural columns fail as long columns.

Short columns with values of l/r less than 100 begin to fail when the combination of direct stress and bending stress reaches the yield point of the material. The actual failure is dependent upon the homogeneity of the material, the straightness of the column, and the eccentricity of loading, all of which control bending stresses. Failure of built-up columns begins with a local crippling at some part of the column. Such elements are not susceptible of calculation, and short columns of this kind are to be computed by empirical formulas which are, however, modeled on rationally derived forms.

Failure of columns also depends upon the condition of their ends, as shown in Fig. 78: (*a*) **Free-ended** column, as a mast with a terminal load; (*b*) **round-ended,** as a spherical bearing; (*c*) fixed-ended, when rigidly con-



a b c d e

FIG. 78.

nected to a rigid support; (*d*) **square-ended,** when a square-ended column abuts upon a support; and (*e*) a combination of rounded and fixed ends as a pin. The theoretical curves of flexure are shown by dotted lines. Columns of types (*c*), (*d*), and (*e*) are common. Ideal conditions are only obtained practically in free-ended columns. Structural columns have various degrees of freedom. The elements controlling the strength of columns ars so indsterminate that refined analysis is misapplied.

Long Columns

Notation. A = Area of cross section; P = load; l = length; r = radius of gyration = $\sqrt{I/A}$; f = deflection; I = rectangular moment of inertia; S = unit axial stress.

Euler's Formula. A perfect or ideal column with a straight axis, of uniform material, and loaded in the direction of its axis, would fail in direct compression like a short block. On account of imperfections, a sudden lateral deflection will occur under some critical load on a long column, which will then be no longer in equilibrium. The **critical load,** acting with a continually increasing leverage, will continue to bend the column until it fails by buckling. This critical load, P, for rounded ends, is given by Euler's formula (published in 1759) as follows:

$$P = \pi^2EI/l^2$$

The assumptions made in deriving this formula are: (1) that the shortening of the column may be neglected; (2) that the bent length of arc is equal to the

Table 13—(continued)

Allowable unit stress, lb per sq in.	Wire gage W.&M.	Diam, in.	Pitch diameter, D in.																				
			D	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/4	2 1/2	2 3/4	3	3 1/2	4	4 1/2
140,000	7	.177	P f	608 .049	487 .077	406 .115	347 .151	305 .198	270 .251	243 .311	223 .375	205 .447	187 .522	174 .606	163 .695	152 .793	135 1.00	122 1.24					
125,000	3/16"	.187	P f	642 .041	522 .065	426 .093	367 .127	320 .166	284 .210	256 .260	233 .314	213 .373	197 .467	183 .510	170 .584	160 .665	142 .832	128 1.04					
	6	.192	P f	696 .040	556 .063	465 .091	396 .124	348 .160	309 .205	278 .252	254 .308	233 .366	214 .428	199 .499	186 .571	174 .652	154 .825	139 1.02	126 1.23				
	5	.207	P f	... ...	694 .059	579 .085	495 .115	432 .151	385 .191	346 .236	315 .286	288 .342	266 .396	247 .462	232 .570	216 .607	192 .757	173 .943	158 1.11	144 1.36			
	7/32"	.218	P f	... ...	812 .055	678 .080	580 .109	509 .142	452 .180	408 .223	360 .269	339 .321	310 .374	291 .437	270 .488	255 .570	225 .710	204 .891	185 1.08	169 1.28			
	4	.225	P f	... ...	895 .054	746 .078	640 .106	560 .138	498 .175	447 .213	407 .262	372 .312	345 .372	320 .425	299 .486	280 .565	248 .691	224 .866	203 1.05	187 1.24			
	3	.244	P f	... ...	1120 .049	950 .071	811 .098	711 .138	632 .161	570 .200	527 .240	475 .287	438 .336	406 .391	381 .449	356 .537	316 .646	284 .800	259 .965	237 1.14			
	1/4"	.250	P f	... ...	... ...	1027 .070	880 .095	760 .131	685 .157	617 .191	560 .236	513 .281	476 .328	440 .385	410 .439	385 .524	342 .624	308 .780	281 .946	266 1.12	222 1.53		
	2	.263	P f	... ...	... ...	1195 .066	1125 .089	895 .118	795 .149	717 .183	652 .224	598 .266	551 .312	501 .363	478 .416	448 .475	400 .592	359 .740	326 .896	298 1.06	256 1.44		
	9/32"	.281	P f	... ...	... ...	1450 .062	1240 .085	1087 .111	969 .140	863 .172	794 .209	724 .250	665 .292	620 .340	580 .390	543 .443	482 .562	437 .692	395 .840	362 1.02	310 1.36		
	1	.283	P f	... ...	... ...	... ...	1264 .084	1110 .111	985 .139	886 .169	805 .207	740 .246	682 .289	634 .338	592 .386	564 .440	492 .559	439 .690	402 .883	370 .990	317 1.35		
115,000	5/16"	.312	P f	... ...	... ...	... ...	1575 .070	1376 .092	1220 .116	1100 .144	1000 .174	915 .207	845 .242	775 .283	733 .322	687 .368	610 .467	550 .577	500 .697	460 .829	392 1.12	343 1.47	
	00	.331	P f	... ...	... ...	... ...	... ...	1636 .088	1455 .109	1316 .135	1187 .163	1090 .194	1000 .227	932 .264	870 .343	818 .346	725 .437	653 .541	594 .654	545 .770	468 1.05	410 1.30	
	11/32	.341	P f	... ...	... ...	... ...	... ...	1820 .082	1620 .105	1452 .127	1325 .156	1214 .186	1120 .218	1040 .256	970 .293	910 .330	808 .413	728 .522	661 .625	608 .745	520 1.02	454 1.32	
	000	.362	P f	... ...	... ...	... ...	... ...	2140 .079	1910 .100	1714 .123	1560 .149	1430 .177	1318 .207	1220 .243	1147 .273	1070 .317	950 .400	858 .495	778 .598	714 .713	612 .965	535 1.26	
	3/8"	.375	P f	... ...	... ...	... ...	... ...	... ...	2110 .079	1940 .117	1780 .144	1580 .172	1458 .201	1354 .234	1265 .268	1185 .308	1058 .382	950 .478	860 .579	790 .688	678 .938	592 1.22	528 1.54
	0000	.393	P f	... ...	... ...	... ...	... ...	... ...	2430 .092	2180 .114	1984 .137	1820 .164	1680 .195	1560 .223	1458 .256	1365 .292	1212 .369	1092 .457	990 .550	910 .657	780 .890	682 1.16	607 1.47

be the allowable unit stress for designing, and should be the ultimate compressive strength where ultimate load is required.

Table 17. Values of the Coefficient K in Rankine's Formula

Material	Both ends fixed	Fixed and rounded	Both ends rounded	Fixed and free
Timber	1/3000	1.95/3000	4/3000	16/3000
Cast iron	1/5000	1.95/5000	4/5000	16/5000
Wrought iron	1/36000	1.95/36000	4/36000	16/36000
Steel	1/25000	1.95/25000	4/25000	16/25000

Tetmajer's experiments on round-ended columns (*Z.V.d.I.*, 1896, p. 1404) show that K varies with the material of the column and with the value of l/r. For wrought iron, K varied from 0.000448 to 0.000136; for steel, from 0.000370 to 0.000130. Tetmajer's recommended values for K are as follows:

	l/r	K
Cast iron	20–150	0.00070
Wrought iron	20–250	0.00016
Soft steel	20–250	0.00014
Wood	20–200	0.00023

Tetmajer's formula for ultimate loads on round-ended columns is

$$S = S_c[1 - a(l/r) + b(l/r)^2]$$

Table 18. Values for Tetmajer's Formula for Short Columns

Materials	S_c	a	b	Limits of l/r	
				Min	Max
Cast iron	110,000	0.01546	0.00007	5.0	80
Wrought iron	43,000	0.00426	0.0	10.0	112
Hard steel	48,000	0.00185	0.0	....	90
Soft steel	44,000	0.00368	0.0	10.0	105
Wood	4,200	0.00662	0.0	1.8	100

Straight-line Formula for Columns. Plotted results of tests yield a straight-line relation between the load at failure and the slenderness ratio l/r. T. H. Johnson's formula is $P/A = S - C(l/r)$, where S is the unit strength of a very short column, and C is a constant obtained by drawing the straight line tangent to a curve representing Euler's formula at a point where $l/r = 150$. The values recommended for use in this formula are given in Table 19. For designing, S and C should be divided by a proper factor of safety.

Table 19. Constants for Johnson's Straight-line Column Formula

Kind of column		S lb per sq in.	C	Limit of l/r
Wrought iron:	Flat ends	42,000	128	218
	Hinged ends	42,000	157	178
	Round ends	42,000	203	138
Steel:	Flat ends	52,500	179	195
	Hinged ends	52,500	220	159
	Round ends	52,500	284	123
Cast iron:	Flat ends	80,000	438	122
	Hinged ends	80,000	537	99
	Round ends	80,000	693	77
Oak:	Flat ends	5,400	28	128

by 1.06, and values of f by 0.75. **For brass,** take S_s = 10,000 to 20,000, and multiply values of f by 2 (Howe).

When designing a spring for continuous work, as a car spring, a low unit stress should be chosen; for intermittent working a higher value may be used.

Examples of Use of Table 13. (1) Required the safe load (P) for a spring of ⅜ in. round steel with a pitch diameter (D) of 3½ in. In the line headed D, under 3½, is given the value of P, or 678 lb. This is for a unit stress of 115,000 lb per sq in. The load, P, for any other unit stress may be found by dividing the 678 by 115,000 and multiplying by the unit stress to be used in the design. To determine the number of coils, this spring would need to compress (say) 6 in. under a load of (say) 678 lb, take the value of f under 678, or 0.938, which is the deflection of one coil under the given load. Therefore 6/0.938 = 6.4, say 7, equals the number of coils required. The spring will therefore be 2⅝ in. long when closed (7 × ⅜), counting the working coils only, and must be 8⅝ in. long when unloaded. Whether there is an extra coil at one end which does not deflect will depend upon the details of the particular design. The deflection in the above example is for a unit stress of 115,000 lb per sq in. The rule is, divide the deflection by 115,000 and multiply by the unit stress to be used in the design.

(2) A 7/16 in. steel spring of 3½ in. outside diameter has its coils in close contact. How much can it be extended without exceeding the limit of safety? The maximum safe load for this spring is found to be 1074 lb and the deflection of one coil under this load is 0.810 in. This is for a unit stress of 115,000 lb per sq in. Therefore 0.810 is the greatest admissible opening between any two coils. In this way, it is possible to ascertain whether or not a spring is overloaded, without knowledge of the load carried.

ECCENTRIC LOADS

When **short blocks** are loaded **eccentrically** in compression or in tension, *i.e.*, not through the center of gravity (cg), a combination of axial and bending stress results. The maximum unit stress S_m is the algebraic sum of these two unit stresses.

In Fig. 74 a load P acts in a line of symmetry at the distance e from cg; r = radius of gyration. The unit stresses are (1) S_c, due to P, as if it acted through cg, and (2) S_b, due to the bending moment of P acting with a leverage of e about cg. Thus unit stress S at any point y is

$$S = S_c \pm S_b$$
$$= (P/A) \pm Pey/I$$
$$= S_c(1 \pm ey/r^2).$$

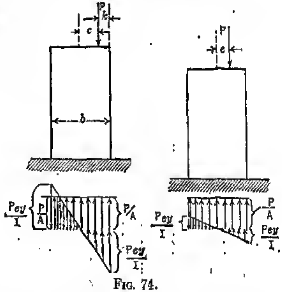


Fig. 74.

y is positive for points on the same side of cg as P, and negative on the opposite side. **For a rectangular cross section** of width b, the maximum stress $S_m = S_c(1 + 6e/b)$. When P is outside the middle third of width b and is a compressive load, tensile stresses occur.

For a **circular cross section** of diameter d, $S_m' = S_c(1 + 8e/d)$. The stress due to the weight of the solid will modify these relations.

Note. In these formulas e is measured from the gravity axis, and gives tension when e is greater than ⅙ the width (measured in the same direction as e), for rectangular sections; and when greater than ⅛ the diameter for solid circular sections.

M_t. $S = My/I = 8Pl/\pi d^3$, and $S_s = 16M_t/\pi d^3$ for a circular shaft. S and S_s combine (Fig. 81) to produce internal normal (S_n) and tangential (S_t) unit stresses on an interior plane (see p. 443). In Fig. 81, S_n, and S_t vary with the angle d. S_n is a maximum when $\cot 2d = -S/2S_s$, and S_t is a maximum when $\tan 2d = +S/2S_s$. The maximum apparent stresses are:

$$S_n = \tfrac{1}{2}S \pm \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \qquad S_t = \sqrt{S_s^2 + (\tfrac{1}{2}S)^2}$$

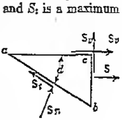


FIG. 81.

These formulas also apply to any case of normal stress combined with shear, as when a bolt is under tension and shear, or when the material of a beam is under tension (or compression) and shear. When S is tension, use (+) sign in order to find the maximum tensile stress S_n, and use (−) sign to find the maximum compressive stress S_n. For strength under combined stresses, see p. 443.

Combined Torsion and Longitudinal Force

A common case is the shaft of a vertical turbine. The longitudinal force is then compression. If W be the weight carried on the shaft in pounds, d the shaft diameter in inches, H the horsepower transmitted, and n the rpm,

$$S_n = \frac{2W}{\pi d^2} + \sqrt{\frac{321{,}000^2H^2}{n^2d^6} + \frac{4W^2}{\pi^2d^4}} \qquad S_t = \sqrt{\frac{321{,}000^2H^2}{n^2d^6} + \frac{4W^2}{\pi^2d^4}}$$

Combined Flexure and Longitudinal Force

Figure 82 shows a bar under flexure due to transverse and longitudinal loads. The maximum fiber stress S (Fig. 82) is made up of S_s, due to the direct action of load P, and S_b, due to the entire bending moment M. M is the algebraic sum of two bending moments, M_1 due to longitudinal load (+ for compression and − for tension), and M_2 due to transverse load. $M = M_2 \pm M_1$. Here $M_1 = Pf$ and $f = CS_bl^2/Ec$.

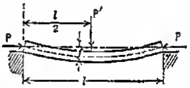


FIG. 82.

For the Case of Longitudinal Compression. $S_bI/c = M_2 + CPS_bl^2/Ec$, or $S_b = M_2c/(I - CPl^2/E)$. The maximum stress is $S = S_b + S_s$ compression. The constant C for the case of Fig. 82 is derived from the equations $Pl/4 = S_bI/c$ and $f = Pl^3/48EI$. Solving for f; $f = \frac{1}{12}S_bl^2/Ec$, or $C = \frac{1}{12}$. For a beam supported at the ends and uniformly loaded, $C = \frac{5}{48}$. Other cases can be similarly calculated.

For the Case of Longitudinal Tension. $M = M_2 - Pf$, and $S_b = M_2c/(I + CPl^2/E)$. The maximum stress is $S = S_b + S_s$, tension.

A common method for computing stresses on eccentrically loaded columns consists in adding to the uniform stress, P/A, the additional stresses due to column action, $(P/A)(Kl^2/r^2)$, as given by Rankine's formula, p. 491, and that due to eccentricity, $(P/A)(ey/r^2)$. Thus,

Total stress $= \dfrac{P}{A}\left(1 + \dfrac{ey}{r^2} + \dfrac{Kl^2}{r^2}\right)$ when $S_b = \dfrac{P}{A}\left(1 + \dfrac{ey}{r^2}\right)$ = stress due to flexure, and $S_c = \dfrac{P}{A}\left(1 + \dfrac{Kl^2}{r^2}\right)$ = stress due to longitudinal load.

ference. For a **circular ring,** S = average compressive stress on cross-section produced by P; e = eccentricity of P; z = length of diameter under compression (Fig. 76). Values of z/r and of the ratio of S_{max} to average S are given in Tables 14 and 15.

Chimney Problem. Weight of chimney = 563,000 lb; e = 1.56 ft; outside diam of chimney = 10 ft 8 in.; inside diam = 6 ft 6½ in. Overturning moment = Pe = 878,000 ft-lb, r_1/r = 0.6. e/r = 0.29. This gives $(z/r) > 2$. Therefore, the entire area of the base is under compression. Area under compression = 55.8 sq ft; I = 546; S = (563,000/55.8) ± (878,000 × 5.33)/546 = 18,700 (max) and 1,500 (min) lb compression per sq ft. From Table 15, by interpolation, S_{max}/S_{avg} = 1.85 ∴ S_{max} = (563,000/55.8) × 1.85 = 18,685 lb per sq ft.

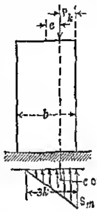


FIG. 75.

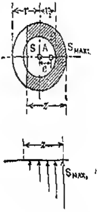


FIG. 76.

The kern is the area around the center of gravity of a cross section within which any load applied will produce stress of only one sign throughout the entire cross-section. Outside the kern, a load produces stresses of different sign. Figure 77 shows kerns (shaded) for various sections.

For a **circular ring,** the radius of the kern $r = D[1 + (d/D)^2]/8$.

For a **hollow square** (H and h = lengths of outer and inner sides), the kern is a square similar to Fig. 77(a), where

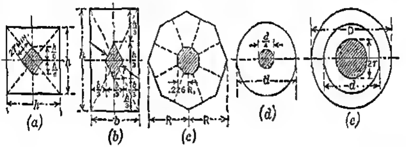
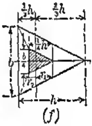


FIG. 77.

$$r_{min} = \frac{H}{6}\frac{1}{\sqrt{2}}\left[1 + \left(\frac{h}{H}\right)^2\right] = 0.1179H\left[1 + \left(\frac{h}{H}\right)^2\right]$$



For a **hollow octagon** [R_o and R_i = radii of circles circumscribing the outer and inner sides; thickness of wall = $0.9239(R_o - R_i)$], the kern is an octagon similar to Fig. 77(c), where $0.2256R$ becomes $0.2256R_o[1 + (R_i/R_o)^2]$.

COLUMNS

A column differs from a tension bar in that any non-uniform yielding in the cross section brings about further yielding. This non-uniform yielding is not serious in short blocks, but is serious in columns.

Columns are divided for analysis into **long columns** and **short columns,** in both of which initial inequalities introduce serious bending.

Long columns fail by buckling at a load less than the elastic limit of the material. **Buckling** is the sudden collapse of a long column at or above the

perpendicular to AB, giving F'. Then $E'F'$ is the modified width. Similarly, KH becomes $K'H'$. The new area A' is evaluated with a planimeter.

VALUES OF A'. Rectangle (height $= h$, width $= b$): $A' = bd[1 + (h/r)^2/12 + (h/r)^4/80 + \cdots]$. Circle (radius $= R$): $A' = 2\pi r[r - \sqrt{r^2 - R^2}]$. Any section: $A' = \Sigma\{[1 - (y/r) + (y^2/r^2) - (y^3/r^3) + (y^4/r^4) - \text{etc.}]\,dA\}$. Symmetrical sections: $A' = A + (I/r^2) + (1/r^4)\Sigma(y^4da) + \text{etc.}$, where I is the moment of inertia of the section about the axis from which y is measured.

Example—Stress in Hooks (Fig. 85). The central horizontal section AB is a trapezoid; area $=$ 9.855 sq in. Radius of curvature $r = 2.5 + 1.843 = 4.343$ in. Eccentricity of load $= 4.343$ in. $A' = 10.673$ sq in. $y' = 0.33$ in. $M = 20{,}000 \times 4.343$. Substituting in the above equations,

$$S_1 = \frac{20{,}000 \times 4.343}{4.343 \times 0.818}\left(\frac{4.343}{2.5} - \frac{10.673}{9.855}\right) + \frac{20{,}000}{9.855} = 17{,}700 \text{ lb per sq in. tension.}$$

$$S_1 = \frac{20{,}000 \times 4.343}{4.343 \times 0.818}\left(\frac{10.673}{9.855} - \frac{4.343}{7}\right) - \frac{20{,}000}{9.855} = 9120 \text{ lb per sq in. compression.}$$

By the ordinary theory (p. 460), tension at $B =$ 12,700 lb per sq in., and compression at $A =$ 13,380 lb per sq in.

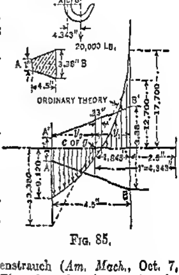


FIG. 85.

Crane Hooks. Experiments by Rautenstrauch (*Am. Mach.*, Oct. 7, 1909) yield the results given in Table 21. They show that the capacity of a hook calculated according to the ordinary theory is in excess of its real strength. See also p. 998.

Table 21. Elastic Limit of Crane Hooks

A = area, sq in. I = moment of inertia. l = distance from load line to gravity axis, in. y = distance from inner fiber to gravity axis, in.

Nominal capacity, tons	Material	Cross section dimensions				Load on hook at elastic limit, lb	Elastic limit determined on specimen cut from hook, lb per sq in.	Max unit stress by ordinary theory, lb per sq in.
		A	I	l	y			
30	C. steel	23.35	111.6	7.25	3.36	56,000	34,000	11,700
20	C. steel	14.48	....	5.90	2.75	30,000	28,500	10,100
15	C. steel	13.92	....	5.13	2.23	48,000	62,750	17,500
15	W. iron	8.40	11.9	5.00	1.87	16,000	30,000	14,500
10	C. steel	8.72	....	4.30	2.05	18,000	37,500	11,600
10	W. iron	6.08	6.5	4.00	1.50	16,000	30,000	17,600
5	C. steel	5.69	....	3.25	1.42	18,000	53,500	17,000
5	W. iron	4.80	3.8	3.47	1.35	14,000	30,000	20,400
3	C. steel	3.50	....	2.89	1.16	8,500	40,800	22,400
2	C. steel	2.03	....	2.03	0.83	4,700	45,900	16,200

length of the vertical projection; and (3) that shearing forces may be neglected. To take account of end conditions, Euler's formula includes a coefficient n.

$$P = n\pi^2EI/l^2$$

Table 16. Strength of Round-ended Columns According to Euler's Formula

P = allowable load, lb. l = length of column, in. b = smallest dimension of a rectangular section, in. d = diameter of a circular section, in. r = least radius of gyration of section.

Material	Cast iron	Wrought iron	Low-carbon steel	Medium-carbon steel
Ultimate compressive strength, lb per sq in.	107,000	53,400	62,600	89,000
Allowable compressive stress, lb per sq in. (maximum)	7,100	15,400	17,000	20,000
Modulus of elasticity	14,200,000	28,400,000	30,600,000	31,300,000
Factor of safety	8	5	5	5
Smallest I allowable at worst section, in.^{4}	$\frac{Pl^2}{17{,}500{,}000}$	$\frac{Pl^2}{56{,}000{,}000}$	$\frac{Pl^2}{60{,}300{,}000}$	$\frac{Pl^2}{61{,}700{,}000}$
Limit of ratio, $l/r >$	50.0	60.6	59.4	55.6
Rectangle ($r = b\sqrt{1/12}$), $l/b >$	14.4	17.5	17.2	16.0
Circle ($r = \frac{1}{4}d$), $l/d >$	12.5	15.2	14.9	13.9
Circular ring of small thickness ($r = d\sqrt{1/8}$), $l/d >$	17.6	21.4	21.1	19.7

Values of n: both ends rounded, $n = 1$; both ends fixed, $n = 4$; one rounded, one fixed, $n = 2$; one end fixed, one end free $n = \frac{1}{4}$. These values are theoretical. Experiments by Kirsch (*Z.V.d.I.*, 1905, p. 907) show that when $l/r = 100$ the relation of the first three coefficients, instead of being 1:4:2, is 1:1.13:1.05, and when $l/r = 200$, it is 1:3.0:1.78 in the case of wrought-iron bars 20 mm (0.79 in.) thick.

Euler's formula is proved to be reliable for long columns that fail by buckling within the elastic limit by Tetmajer's experiments (*Z.V.d.I.*, 1896, p. 1404). The value of the ratio (l/r) at which failure of round-ended columns by buckling occurred was as follows: wrought iron, 112; soft steel, 105; medium steel, 90; cast iron, 80; wood, 100. Table 16, for columns with rounded ends ($n = 1$), is based on the Euler formula.

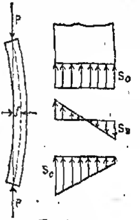


Fig. 79.

Short Columns

Rankine's formula applies to short columns, which fail under the stress S_c (Fig. 79), this stress being the sum of (1) the stress S_0, a uniform compression due to P acting as an axial load, and (2) S_B in the outer fiber due to moment M of P acting with a lever arm f around the gravity axis of the section. $S_0 = P/A$ and $S_B = Mc/I = Pfc/Ar^2$; $P = S_cA/[1 + (fc/r^2)]$, where r is the radius of gyration and c the distance from the outer fiber to the gravity axis. Here f is unknown and is replaced by Kl^2/c, giving Rankine's formula

$$P = S_cA/[1 + K(l^2/r^2)]$$

This is largely used in design. Values of K have been derived experimentally, and those recommended by Merriman are given in Table 17. Rankine's formula is to be used for values of $l/r = 20$ to 120. The value of S_c should

arch theory: $\Delta x = -K\int Myds/EI$, $\Delta y = K\int Mxds/EI$, and $\Delta\theta = K\int Mds/EI$ where the constant K is introduced to correct for the increased flexibility of a curved pipe, and where the integration is over the entire length of pipe between supports. In Table 1 are given equations derived by this method for moment and thrust at one reaction point for pipes in one plane that are fully fixed, hinged at both ends, hinged at one end and fixed at the other, or partly fixed. If the reactions at one end of the pipe are known, the moment distribution in the entire pipe then can be obtained by simple statics.

Since an initially curved pipe is more flexible than indicated by its moment of inertia, the constant K is introduced. Its value may be taken from Fig. 3, or computed from the equation given on page 499. $K = 1$ for all straight pipe sections since they act according to the simple flexure theory.

Table 1. General Equations for Pipe Lines in One Plane

Unsymmetrical	Symmetrical about y-axis
Both ends fully fixed:	
$M_0 = \dfrac{EI\Delta x(CF - AB) + EI\Delta y(BF - AG)}{2ABF + CGH - B^2H - A^2G - CF^2}$	$M_0 = \dfrac{EI\Delta xF}{GH - F^2}$
$F_x = \dfrac{EI\Delta x(CH - A^2) + EI\Delta y(BH - AF)}{2ABF + CGH - B^2H - A^2G - CF^2}$	$F_x = \dfrac{EI\Delta xH}{GH - F^2}$
$F_y = \dfrac{EI\Delta x(BH - AF) + EI\Delta y(GH - F^2)}{2ABF + CGH - B^2H - A^2G - CF^2}$	$F_y = 0$
$\Delta\theta = 0$	$\Delta\theta = 0$
Both ends hinged:	
$M_0 = 0$	$M_0 = 0$
$F_x = \dfrac{EI\Delta xC + EI\Delta yB}{CG - B^2}$	$F_x = \dfrac{EI\Delta x}{G}$
$F_y = \dfrac{EI\Delta xB + EI\Delta yG}{CG - B^2}$	$F_y = 0$
$\Delta\theta = \dfrac{\Delta x(AB - CF) + \Delta y(AG - BF)}{CG - B^2}$	$\Delta\theta = \dfrac{-\Delta xF}{G}$
Origin end only hinged, other end fully fixed	
$M_0 = 0$	
$F_x = \dfrac{EI\Delta xC + EI\Delta yB}{CG - B^2}$	
$F_y = \dfrac{EI\Delta xB + EI\Delta yG}{CG - B^2}$	
$\Delta\theta = \dfrac{\Delta x(AB - CF) + \Delta y(AG - BF)}{CG - B^2}$	
In general for any specific rotation $\Delta\theta$ and movement Δx and Δy:	
$M_0 = \dfrac{EI\Delta x(CF - AB) + EI\Delta y(BF - AG) + EI\Delta\theta(CG - B^2)}{2ABF + CGH - A^2G - CF^2 - B^2H}$	
$F_x = \dfrac{EI\Delta x(CH - A^2) + EI\Delta y(BH - AF) + EI\Delta\theta(CF - AB)}{2ABF + CGH - A^2G - CF^2 - B^2H}$	
$F_y = \dfrac{EI\Delta x(BH - AF) + EI\Delta y(GH - F^2) + EI\Delta\theta(BF - AG)}{2ABF + CGH - A^2G - CF^2 - B^2H}$	

In Fig. 3 are given the flexure constants K, α, β, and γ for initially curved pipes as functions of the quantity $\lambda = tR/r^2$. The flexure constants are derived from the equations

Table 20. Allowable Unit Stresses (Lb per Sq In.) on Steel Columns According to Various Formulas

$\frac{l}{r}$	American Bridge Co. $19{,}000-100\frac{l}{r}$ 13,000 max	A. R. E. Assn. Chicago and New York $16{,}000-70\frac{l}{r}$ 14,000 max	Gordon $\frac{12{,}500}{1+\frac{l^2}{36{,}000r^2}}$	Philadelphia $\frac{16{,}250}{1+\frac{l^2}{11{,}000r^2}}$	Boston $\frac{16{,}000}{1+\frac{l^2}{20{,}000r^2}}$
10	13,000	14,000	12,460	16,000	15,920
20	13,000	14,000	12,355	15,680	15,690
30	13,000	13,900	12,195	15,020	15,310
40	13,000	13,200	11,970	14,185	14,815
50	13,000	12,500	11,690	13,240	14,220
60	13,000	11,800	11,365	12,240	13,560
70	12,000	11,100	11,000	11,240	12,850
80	11,000	10,400	10,615	10,275	12,120
90	10,000	9,700	10,205	9,360	11,390
100	9,000	9,000	9,785	8,510	10,670
110	8,000	8,300	9,355	7,740	9,970
120	7,000	7,600	8,390	7,035	9,300

Table 20 gives the allowable unit stresses for **steel columns** as deduced from the American Bridge Co.'s formula and also as deduced from the American Railway Engineering Assn. formula (A.R.E. Assn.); the Gordon formula, and the formulas from the building codes of the cities of Chicago, New York, Philadelphia, and Boston.

For tables of structural-steel columns and steel-pipe columns, see steel manufacturers' publications or handbooks.

For columns carrying traveling machinery, cranes, etc., add 25 percent to the live load to allow for impact and vibration.

W. H. Burr's tests on **cast-iron columns** at Phoenixville (*Eng. News*, June 30, 1898) gave a wide range of results. The *average* **ultimate strength** (in pounds per square inch) for round columns is expressed by $P/A = 30{,}500 - 165(l/d)$, where d and l are expressed in the same units. A large factor of safety, 4, must be used with cast-iron columns. Maximum slenderness ratio is 70.

COMBINED STRESSES

Combined flexure and torsion arise when a twisted shaft is bent by belts or other forces. Elements of the shaft become helices of variable pitch. An

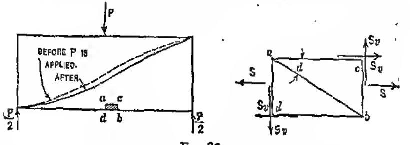
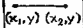
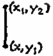
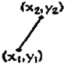


Fig. 80.

element of the surface (Fig. 80) is subjected to a flexural unit stress S due to bending moment M, and a shearing unit stress S_v, due to torsional moment

Table 2. Values of A, B, C, F, G, and H for Various Piping Elements

Section	$A = K\int x\,ds$	$B = K\int xy\,ds$	$C = K\int x^2\,ds$	$F = K\int y\,ds$	$G = K\int y^2\,ds$	$H = K\int ds$
	$\frac{s}{2}(x_1 + x_2)$	Ay	$s\left(\frac{s^2}{3} + x_1x_2\right)$	sy	Fy	s
	sx	$\frac{A}{2}(y_1 + y_2)$	Ax	$\frac{s}{2}(y_1 + y_2)$	$s\left(\frac{s^2}{3} + y_1y_2\right)$	s
	$\frac{s}{2}(x_1 + x_2)$	$\frac{A}{3}(y_1 + y_2) + \frac{s}{6}(x_1y_1 + x_2y_2)$	$\frac{s}{3}(x_1^2 + x_1x_2 + x_2^2)$	$\frac{s}{2}(y_1 + y_2)$	$\frac{s}{3}(y_1^2 + y_1y_2 + y_2^2)$	s
	πKRx	$A\left(y + \frac{2R}{\pi}\right)$	$A\left(x + \frac{R^2}{2x}\right)$	$(\pi y + 2R)KR$	$Fy + \left(2y + \frac{\pi}{2}R\right)KR^2$	πKR
		$A\left(y - \frac{2R}{\pi}\right)$		$(\pi y - 2R)KR$	$Fy - \left(2y - \frac{\pi}{2}R\right)KR^2$	
	$\left(\frac{\pi x}{2} - R\right)KR$	$Ay + \left(x - \frac{R}{2}\right)KR^2$	$Ax + \left(\frac{\pi R}{4} - x\right)KR^2$	$\left(\frac{\pi y}{2} + R\right)KR$	$Fy + \left(\frac{\pi R}{4} + y\right)KR^2$	$\frac{\pi KR}{2}$

BENDING OF CURVED BEAMS

(From Morley, "Strength of Materials," Longmans)

Simple beam equations apply when the radius of curvature of a curved beam is large (8 to 10 times), compared to dimensions of its cross section. This is not the case in hooks, links, and rings, when some modification of formulas is necessary. Plane cross sections, normal to the axis of curvature, are assumed. The deformation (e) is proportional to the distance from the gravity axis, *but the unit deformation* (s) *is not proportional*, since the lengths of the fibers vary. The elongation of any curved fiber is evaluated, and the stress determined. The neutral axis is not at the cg, but is near the center of curvature. The stress variation is not rectilinear (Fig. 83).

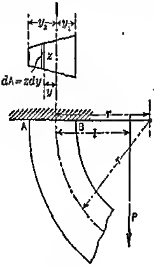


FIG. 83.

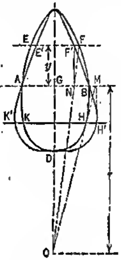


FIG. 84.

In Fig. 83, P = load; l = leverage of load about axis through cg of cross section; M = moment due to $P = Pl$; A = area of cross section; r = radius of curvature of undeformed beam from center of curvature to cg of cross section. Stress due to combination of direct load P (taken as acting at cg) and moment $M = S$, or the normal unit stress at distance y from axis through cg. The equations which follow are for pure bending, but apply with little error to ordinary bending, including shear.

$$S_1 \text{ (at } B) = \frac{M}{r(A' - A)}\left(\frac{r}{r - y_1} - \frac{A'}{A}\right) + \frac{P}{A} \text{ (tension)}$$

$$S_2 \text{ (at } A) = \frac{M}{r(A' - A)}\left(\frac{A'}{A} - \frac{r}{r + y_2}\right) - \frac{P}{A} \text{ (compression)}$$

where

$$A' = r\Sigma\left(\frac{dA}{y + r}\right)$$

The neutral axis is at a distance y' from cg, where $y' = r\left(\frac{A' - A}{A'}\right)$.

A' may be considered as a "modified" area, and is obtained graphically (Fig. 84) as follows: To find $A' = r\Sigma\left(\frac{dA}{y + r}\right)$, first find the cg of the original cross section. Lay off O so that $OG = r$. Change each width in the ratio of $r/(y + r)$. For example, draw OF intersecting AB in N. Draw NF'

Reactions and stresses are greatly influenced by **flattening** of the cross **section** of the curved portions of the pipe line.

It is recommended that **cold springing** allowances be discounted in stress calculations.

Application to Two- and Three-plane Pipe Lines. Pipe lines in more than one plane may be solved by the successive application of the preceding data, dividing the pipe line into two or more one-plane lines.

Example. The unsymmetrical pipe line of Fig. 4 has fully fixed ends From Table 2 use $K = 1$ for all sections, since only straight segments are involved,

Part of pipe	Values of integrals					
	A	B	C	F	G	H
0–1	$\frac{a^2}{2}$	0	$\frac{a^3}{3}$	0	0	a
1–2	ab	$\frac{ab^2}{2}$	a^2b	$\frac{b^2}{2}$	$\frac{b^3}{3}$	b
2–3	$\frac{c}{2}(2a + c)$	$\frac{bc}{2}(2a + c)$	$\frac{c^3}{3} + ac(a + c)$	bc	b^2c	c
Total 0–3	$\frac{a^2}{2} + ab + \frac{c}{2}(2a + c)$	$\frac{ab^2}{2} + \frac{bc}{2}(2a + c)$	$\frac{a^3}{3} + a^2b + \frac{c^3}{3} + ac(a + c)$	$\frac{b^2}{2} + bc$	$\frac{b^3}{3} + b^2c$	$a + b + c$

Upon introduction of $a = 120$ in., $b = 60$ in., and $c = 180$ in., into the preceding relations for A, B, C, F, G, H, the equations for the reactions at 0 from Table 1 become

$$M_0 = EI\Delta x(-7.1608 \times 10^{-5}) + EI\Delta y(-8.3681 \times 10^{-5})$$
$$F_x = EI\Delta x(+1.0993 \times 10^{-6}) + EI\Delta y(+3.1488 \times 10^{-7})$$
$$F_y = EI\Delta x(+3.1488 \times 10^{-7}) + EI\Delta y(+1.33717 \times 10^{-7})$$

Also it follows that

$$M_1 = M_0 + F_ya = EI\Delta x(+3.06248 \times 10^{-4}) + EI\Delta y(+7.6779 \times 10^{-5})$$
$$M_2 = M_1 - F_xb = EI\Delta x(+2.4029 \times 10^{-4}) + EI\Delta y(-1.1215 \times 10^{-4})$$
$$M_3 = M_2 + F_yc = EI\Delta x(+8.0707 \times 10^{-4}) + EI\Delta y(+1.2854 \times 10^{-4})$$

Thus the maximum moment M occurs at 3.

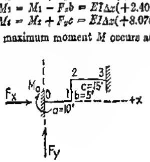


Fig. 4.

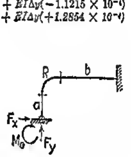


Fig. 5.—Right-angle Pipe Line.

The total maximum longitudinal fiber stress ($\alpha = .1$ for straight pipe)

$$= \frac{F_x}{2\pi rt} \pm \frac{Mr}{I}$$

There is no transverse flexure stress since all sections are straight. The maximum shearing stress is either (a) one-half of the maximum longitudinal fiber stress as given above, (b) one-half of the hoop-tension stress caused by any internal radial pressure that might exist in the pipe, or (c) one-half the difference of the maximum longitudinal

PIPE LINE FLEXURE STRESSES CAUSED BY EXPANSION OR MOVEMENT OF SUPPORTS

BY

H. V. HAWKINS AND W. H. SHIPMAN

References: Shipman, "Design of Steam Piping to Care for Expansion," *Trans. A.S.M.E.*, 1929. Wahl, "Stresses and Reactions in Expansion Pipe Bends," *Trans. A.S.M.E.*, 1927. Hovgaard, "The Elastic Deformation of Pipe Bends," *Jour. Math. Phys.*, Nov., 1926, and Oct., 1928.

Nomenclature.

M_0 = end moment at origin, in-lb.

M = max moment, in-lb.

F_x = end reaction at origin in x-direction, lb.

F_y = end reaction at origin in y-direction, lb.

S_l = $(Mr/I)\alpha$ = max unit longitudinal flexure stress, lb per sq in.

S_t = $(Mr/I)\beta$ = max unit transverse flexure stress, lb per sq in.

S_s = $(Mr/I)\gamma$ = max unit shearing stress, lb per sq in.

Δx = relative deflection of ends of pipe parallel to x-direction caused by either temperature change or support movement, or both, in.

Δy = same as Δx but parallel to y-direction, in. Note that Δx and Δy are positive if under the change in temperature the end opposite the origin tends to move in a positive x- or y-direction, respectively.

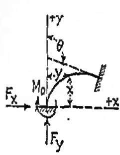


Fig. 1.

t = wall thickness of pipe, in.

r = mean radius of pipe cross section, in.

λ = constant $= tR/r^2$.

I = moment of inertia of pipe cross section about pipe center line, in[4].

E = modulus of elasticity of pipe at *actual working* temperature, lb per sq in.

K = flexibility index of pipe. $K = 1$ for all straight pipe sections, $K = (10 + 12\lambda^2)/(1 + 12\lambda^2)$ for all curved pipe sections where $\lambda > 0.335$ (see Fig. 3).

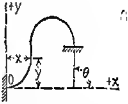


Fig. 2.

α, β, γ = ratios of actual max longitudinal flexure, transverse flexure, and shearing stresses to Mr/I for curved sections of pipe (see Fig. 3).

A, B, C, F, G, H = constants given by Table 2.

θ = angle of intersection between tangents to direction of pipe at reactions.

$\Delta\theta$ = change in θ caused by movements of supports, or by temperature change, or both, radians.

ds = an infinitesimal element of the length of the pipe.

R = radius of curvature of pipe center line.

General Discussion

Under the effect of changes in temperature of the pipe line, or of movement of support reactions (either translation or rotation), or both, the determination of stress distribution in a pipe becomes a statically indeterminate problem. In general the problem may be solved by a slight modification of the standard

fiber stress and hoop-tension stress, whichever of these three possibilities is numerically greatest.

The reactions for a right-angle pipe line (Fig. 5) may be determined from the following equations:

$$M_0 = C_1 EI\Delta x/R^2$$
$$F_x = C_2 EI\Delta x/R^3$$
$$F_y = C_3 EI\Delta x/R^3$$

In these equations, Δx is the x-component of the deflection between reaction points caused by *temperature change only*. The values of C_1, C_2, and C_3 are given in Fig. 6 for $K = 1$ and for $K = 2$. For other values of K, interpolation may be employed.

Example. With $a/R = 20$ and $b/R = 3$, the value of C_1 is 0.0192 for $K = 1$ and 0.0170 for $K = 2$. If $K = 1.75$, the interpolated values of C_1 is 0.0180. With the origin at the right-hand end (Fig. 5), C_1 is 0.248 for $K = 1$.

$$K = (10 + 12\lambda^2)/(1 + 12\lambda^2), \quad \text{when} \quad \lambda > 0.335$$
$$\alpha = \tfrac{2}{3}K\sqrt{(5 + 6\lambda^2)/18}$$
$$\beta = 18\lambda/(1 + 12\lambda^2)$$
$$\gamma = [8\lambda - 36\lambda^3 + (32\lambda^2 + \tfrac{20}{3})\sqrt{\tfrac{2}{3}\lambda^2 - \tfrac{5}{18}}]/(1 + 12\lambda^2), \quad \text{when} \quad \lambda < 0.58$$
$$= (12\lambda^2 + 18\lambda - 2)/(1 + 12\lambda^2), \quad \text{when} \quad \lambda > 0.58$$

The increased flexibility of the curved pipe is brought about by the tendency of its cross section to flatten. This flattening causes a transverse flexure stress whose maximum is S_t. Because the maximum longitudinal and maximum transverse stresses do not occur at the same point in the pipe's cross section, the resulting maximum shear is not one-half the difference of S_l and S_t; it is S_s. In the straight sections of the pipe, $\alpha = 1$, the transverse stress disappears, and $\gamma = \frac{1}{2}$. This discussion of S_s does not include the uniform transverse or longitudinal tension stresses induced by the internal pressure in the pipe; their effects should be added if appreciable.

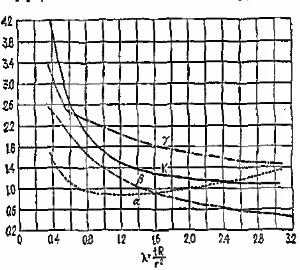


FIG. 3.—Flexure Constants for Initially Curved Pipes.

Table 2 gives values of *the constants A, B, C, F, G, and H for use* in equations listed in Table 1. The values may be used (1) for the solution of any pipe line, or (2) for the derivation of equations for standard shapes composed of straight sections and arcs of circles as in Fig. 5. Equations for shapes not given may be obtained by algebraic addition of those given. All measurements are from the left-hand end of the pipe line. Reactions and stresses are very greatly influenced by end conditions. Formulas are given to cover the extreme conditions. The following suggestions and comments should be considered when laying out a pipe line:

Avoid expansion bends, and design the entire pipe line to take care of its own expansion.

The movement of the equipment to which the ends of the pipe line are attached must be included in the Δx and Δy of the equations.

Maximum flexibility is obtained by placing **supports and anchors** so that they will not interfere with the natural movement of the pipe.

That shape is most efficient in which the **maximum length of pipe** is **working** at the maximum safe stress.

Excessive **bending moment at joints** is more likely to cause trouble than excessive stresses in pipe walls. Hence, keep pipe joints away from points of *high moment.*

The period of free vibration is

$$T = 2\pi/\omega_n = 2\pi\sqrt{W/kg} = 2\pi\sqrt{\delta_{st}/g} \tag{4}$$

It is seen that the period is the same as the period of a mathematical pendulum the length of which is equal to the static deflection of the spring under the action of the load W. This conclusion can always be used in calculating the period of free vibration of elastic systems when the static deflections are proportional to the load and the mass of the elastic portion of the system (in the above illustration the mass of the spring) can be neglected in comparison with the mass of the load. Taking, for instance, a beam with supported ends loaded at the middle, and neglecting the mass of the beam in comparison with the mass of the load W, the static deflection of the beam will be $\delta_{st} = Wl^3/48EI$ and the period of free vibration will be

$$T = 2\pi\sqrt{Wl^3/48EIg}$$

Vibrations with Damping. In practice, resisting forces are always present which cause a gradual damping of the original vibrations. These damping forces may arise from various sources such as air or fluid resistance, internal friction of the material of the vibrating body, or friction between sliding surfaces. When a body is vibrating in air or in a liquid and velocities are small, such as may exist in dashpots, the **resisting force is proportional to velocity** and the differential equation of vibration is

$$\frac{W}{g}\ddot{x} + c\dot{x} + kx = 0 \tag{5}$$

in which $c\dot{x}$ denotes the resisting force of the moving body. Using the notation

$$q^2 = \omega_n^2 - n^2, \tag{6}$$

where $n = cg/2W$, the general solution of Eq. (5) will be

$$x = e^{-nt}[(\dot{x}_0/q)\sin qt + x_0(\cos qt + (n/q)\sin qt)] \tag{7}$$

The damped vibratory motion has a period:

$$T = 2\pi/q = 2\pi/\sqrt{\omega_n^2 - n^2} \tag{8}$$

The quantity n is usually small in comparison with ω_n and the difference between q and ω_n is a small quantity of the second order. It can be assumed therefore that a small damping force does not affect the period of vibration.

The amplitude of the vibrations (7) gradually decreases with the time, due to the factor e^{-nt}, diminishing after every cycle in the ratio

$$e^{-nt}:1 \tag{9}$$

This ratio is used in experimental determination of the **coefficient of damping.**

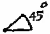
In the case of a **constant friction** such as exists between two dry surfaces of sliding bodies under constant pressure, the period of vibration is the same as in the case of free vibrations without resisting forces [Eq. (4)]. The amplitude of vibration diminishes after each half cycle by the quantity

$$a = 2R/k \tag{10}$$

where R denotes the resisting force and k is the spring constant.

Forced Vibrations. When a periodical disturbing force $Q \sin \omega t$ is acting on the vibrating body, the differential equation of vibration will be

$$\frac{W}{g}\ddot{x} + c\dot{x} + kx = Q\sin\omega t \tag{11}$$

	$\left(\frac{\pi x}{2} + R\right) KR$	$Ay + \left(x + \frac{R}{2}\right) KR^2$	$Ax + \left(\frac{\pi R}{4} + x\right) KR^2$	$\left(\frac{\pi y}{2} + R\right) KR$	$Fy + \left(\frac{\pi R}{4} + y\right) KR^2$	$\frac{\pi KR}{2}$
		$Ay - \left(x + \frac{R}{2}\right) KR^2$		$\left(\frac{\pi y}{2} - R\right) KR$	$Fy + \left(\frac{\pi R}{4} - y\right) KR^2$	
	$\left(\frac{\pi x}{2} - R\right) KR$	$Ay - \left(x - \frac{R}{2}\right) KR^2$	$Ax + \left(\frac{\pi R}{4} - x\right) KR^2$	$\left(\frac{\pi y}{2} - R\right) KR$	$Fy + \left(\frac{\pi R}{4} - y\right) KR^2$	
	$\left(\frac{\pi x}{4} - \frac{R}{\sqrt{2}}\right) KR$	$Ay + \left[\left(1 - \frac{\sqrt{2}}{2}\right)x - \frac{R}{4}\right] KR^2$	$Ax + \left[\left(\frac{\pi}{8} + \frac{1}{4}\right)R - \frac{\sqrt{2}x}{2}\right] KR^2$	$\left[\frac{\pi y}{4} + \left(1 - \frac{\sqrt{2}}{2}\right)R\right] KR$	$Fy + \left[\left(1 - \frac{\sqrt{2}}{2}\right)y + \left(\frac{\pi}{8} - \frac{1}{4}\right)R\right] KR^2$	$\frac{\pi KR}{4}$
		$Ay - \left[\left(1 - \frac{\sqrt{2}}{2}\right)x \frac{R}{4}\right] KR^2$		$\left[\frac{\pi y}{4} - \left(1 - \frac{\sqrt{2}}{2}\right)R\right] KR$	$Fy - \left[\left(1 - \frac{\sqrt{2}}{2}\right)y - \left(\frac{\pi}{8} - \frac{1}{4}\right)R\right] KR^2$	
	$\left(\frac{\pi x}{4} + \frac{R}{\sqrt{2}}\right) KR$	$Ay + \left[\left(1 - \frac{\sqrt{2}}{2}\right)x + \frac{R}{4}\right] KR^2$	$Ax + \left[\left(\frac{\pi}{8} + \frac{1}{4}\right)R + \frac{\sqrt{2}x}{2}\right] KR^2$	$\left[\frac{\pi y}{4} + \left(1 - \frac{\sqrt{2}}{2}\right)R\right] KR$	$Fy + \left[\left(1 - \frac{\sqrt{2}}{2}\right)y + \left(\frac{\pi}{8} - \frac{1}{4}\right)R\right] KR^2$	
		$Ay - \left[\left(1 - \frac{\sqrt{2}}{2}\right)x + \frac{R}{4}\right] KR^2$		$\left[\frac{\pi y}{4} - \left(1 - \frac{\sqrt{2}}{2}\right)R\right] KR$	$Fy - \left[\left(1 - \frac{\sqrt{2}}{2}\right)y - \left(\frac{\pi}{8} - \frac{1}{4}\right)R\right] KR^2$	
	$[x(\theta_2 - \theta_1) - R(\sin\theta_2 - \sin\theta_1)] KR$	$Ay\left[x(\cos\theta_2 - \cos\theta_1) + \frac{R}{2}(\sin^2\theta_2 - \sin^2\theta_1)\right] KR^2$	$Ax - \left[x(\sin\theta_2 - \sin\theta_1) - \frac{R}{4}(\sin 2\theta_2 - \sin 2\theta_1) - \frac{R}{2}(\theta_2 - \theta_1)\right] KR^2$	$[y(\theta_2 - \theta_1) - R(\cos\theta_2 - \cos\theta_1)] KR$	$Fy - \left[y(\cos\theta_2 - \cos\theta_1) + \frac{R}{4}(\sin 2\theta_2 - \sin 2\theta_1) - \frac{R}{2}(\theta_2 - \theta_1)\right] KR^2$	$(\theta_2 - \theta_1) KR$

also very useful for an approximate calculation of the frequency of more complicated systems. Considering the arrangement of Fig. 1 and neglecting at first the mass of the spring, the kinetic energy of the system during the vibration is

$$T = W(\dot{x})^2/2g \tag{15}$$

Measuring the displacement of x from the position of equilibrium and taking into consideration that the weight W is always in balance with the initial tensile force in the spring, the increase in potential energy during the displacement x will be

$$V = kx^2/2 \tag{16}$$

Then on the basis of the principle of conservation of energy, neglecting the damping, we have

$$(W\dot{x}^2/2g) + kx^2/2 = \text{const.} \tag{17}$$

This means that during vibration the sum of the kinetic and potential energy remains constant and equal to its initial value. Assuming, for instance, that at $t = 0$ the displacement is equal to x and the initial velocity is zero, Eq. (17) becomes

$$(W\dot{x}^2/2g) + kx^2/2 = kx_0^2/2 \tag{18}$$

When, during vibration, x becomes equal to x_0, the velocity of the weight W becomes equal to zero and the energy of the system consists of the potential energy only. When x becomes equal to zero, which occurs when the load during vibration passes through its middle position, the velocity has its maximum value and the corresponding magnitude of the kinetic energy, from Eq. (18), becomes

$$W(\dot{x})^2_{max}/2g = kx_0^2/2 \tag{19}$$

This means that at this moment the total energy of the system is kinetic and equal to the potential energy which has been stored in the system during the initial displacement x_0 from the position of equilibrium. Equation (19) can be used for calculating the frequency of vibration of the system. It has been shown (p. 505) that in this case we have a simple harmonic motion, *i.e.*, it can be taken

$$x = x_0 \cos \omega_n t \text{ and } (\dot{x})_{max} = x_0\omega_n$$

Substituting in (19) we obtain

$$\omega_n^2 = kg/W$$

This coincides with the result previously obtained (p. 509).

In the foregoing consideration, the mass of the spring was neglected in comparison with the mass of the weight W. In order to determine the effect of such a simplification on the frequency of vibration, the approximate method developed by Lord Rayleigh may be employed. Assuming that the mass of the spring is small in comparison with the mass of the load W, the type of vibration will not be substantially affected by the mass of the spring and it can be assumed with sufficient accuracy that during vibration the displacement of any cross section of the spring at a distance c from the fixed end (Fig. 1) is xc/l. Let w denote the weight of the spring per unit length, then the mass of an element of the spring of the length dc will be $w\,dc/g$ and the kinetic energy of the spring itself will be

$$\frac{w}{2g}\int_0^l \left(\frac{\dot{x}c}{l}\right)^2 dc = \frac{\dot{x}^2}{2g}\left(\frac{wl}{3}\right)$$

and the energy Eq. (18) becomes

$$(W\dot{x}^2_{max}/2g) + (wl/3)\dot{x}^2_{max}/2g = kx_0^2/2 \tag{20}$$

It can be concluded from this that in order to estimate the effect of the mass of the spring on the period of natural vibration it is only necessary to add one-third of the weight of the spring to the weight W. Applying the same manner of reasoning to the case shown in Fig. 3, it can be shown that in order to take into consideration the mass of the beam in calculating the frequency of natural vibrations it is necessary to add to the weight W of the load $^{17}\!/_{35}$ of the weight of the beam. In the case of a cantilever loaded at the end by load W, a satisfactory approximation in calculating frequency will be obtained by adding to the load W, $^{33}\!/_{140}$ of the weight of the cantilever.

Non-harmonic Vibrations. In the previous discussion, it was assumed that the spring constant of the system was independent of the displacement x.

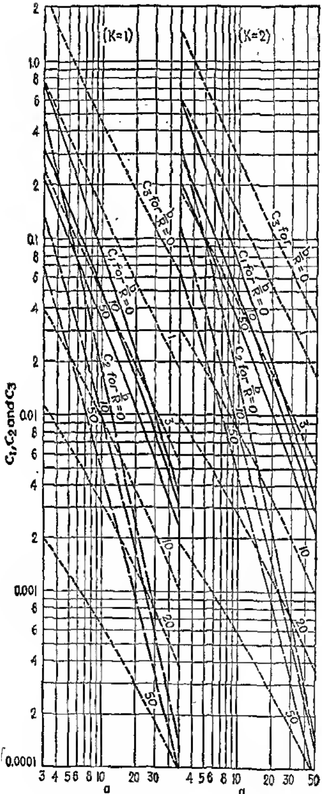


FIG. 6.—Reactions for Right-angle Pipe Lines.

distribution are always present. As a result of this, periodical disturbing forces occur which produce vibrations. In order to remove these vibrations in machines and establish quiet running conditions, balancing becomes necessary, especially in the case of large high-speed machines. Various conditions of unbalance of a rotor are shown in Fig. 6. Imagine the rotating body divided into two parts by any cross section mn. The three following typical cases of unbalance may arise:

1. The centers of gravity of both parts may be in the same axial plane and on the same side of the axis of rotation as shown in Fig. 6b. The center of gravity C of the whole body will consequently be in the same plane at a certain distance from the axis of rotation. This is called **static unbalance**, because it can be detected by a statical test. A statical balancing test consists of putting the rotor with the two ends of its shaft on absolutely-horizontal parallel rails. If the center of gravity of the whole rotor is in the axis, the rotor will be in static equilibrium in any position; if the center is off the axis, as in Fig. 6b, it will roll on the rails till the center of gravity reaches its lowest position.

2. The centers of gravity of both parts may be in the same axial plane but on opposite sides of the axis of rotation as shown in Fig. 6c and at such radial distances that the center of gravity C of the whole body will be exactly on the axis of rotation. In this case, the body will be in balance under static conditions, but during rotation a disturbing couple of centrifugal forces P will act on the rotor. This couple rotates with the body and produces vibrations in the foundation. Such a case is called **dynamic unbalance.**

3. In the most general case, the centers of gravity C_1 and C_2 may lie in different axial sections and during rotation a system of two forces formed by the centrifugal forces P and Q will act on the body, Fig. 6d. This system of forces can always be reduced to a couple acting in an axial section and a radial force, *i.e.*, static and dynamic unbalance will occur together.

In all cases, a complete balancing can be obtained by attaching to the rotor a weight in each of two cross-sectional planes arbitrarily chosen, provided the position and magnitude of the unbalance is known. For determining this unbalance, various types of **balancing machines** are used. A balancing machine represents usually an arrangement in which the effects of any unbalance in the rotor which is under test may be magnified by resonance. In the older types of balancing machines, the vibration was observed visually or recorded graphically. Sometimes the vibration was eliminated by adjustable compensating weights. In the more modern types, such as those made by the Tinius Olsen Co. of Philadelphia, electric recording is used. In the Gisholt-Westinghouse balancing machine, the entire operation has been made automatic by means of electric circuits.

Experience with large high-speed machines shows that while balancing carried out on the balancing machine in the shop may show good results, nevertheless this testing is usually done at comparatively low speed and in service, where the operating speeds are high, unbalance may still be apparent due to slight changes in mass distribution. It is therefore necessary also to check the balancing condition for normal operating speed. This test is carried out either in the shop where the rotor is placed for this purpose on rigid bearings or in the field after it is assembled in the machine. The magnitude and the location of the correction weight can then be found from measurements of the amplitudes of vibrations of the pedestal, which are recorded by a vibrometer.

VIBRATION

BY

S. TIMOSHENKO

Revised by J. P. Den Hartog

REFERENCES: Timoshenko, "Vibration Problems in Engineering," Van Nostrand. Hort, "Technische Schwingungslehre," Springer. Steuding, "Messung Mechanischer Schwingungen," V. D. I. Verlag. Den Hartog, "Mechanical Vibrations," McGraw-Hill.

Notation.

W = the weight of a vibrating body.
g = acceleration due to gravity.
I = moment of inertia of an oscillating disk about the axis of oscillation.
G = modulus of elasticity in shear.
E = modulus of elasticity in tension.
ν = Poisson's ratio.
k = the spring constant, *i.e.*, the force necessary to produce a deflection of a spring equal to unity or the couple necessary to produce an angle of twist of a shaft equal to one radian.
$\delta_{st} = W/k$ = the static deflection of the spring under the action of the load W.
T = the period, *i.e.*, the time of one oscillation.
$f = 1/T$ = the frequency, *i.e.*, the number of oscillations per second.
$\omega_n = \sqrt{gk/W}$ or $\omega_n = \sqrt{k/I}$ = the number of free oscillations per 2π sec.
A = the amplitude of the vibration.
α = the phase of the vibration.
c = the coefficient of damping.
$Q \sin \omega t$ = the periodical disturbing force.
$T_1 = 2\pi/\omega$ = the period of the disturbing force.
$f_1 = 1/T_1 = \omega/2\pi$ = the frequency of the disturbing force.

$\dot{x}$ and $\ddot{x}$ are the first and second derivatives of x with respect to time, *i.e.*, are velocity and acceleration, respectively.

Free Harmonic Vibration. The simplest case of a vibrating system with one degree of freedom is shown in Fig. 1. The weight W attached to the spring has freedom to vibrate in the vertical direction, and the mass of the spring is small in comparison with the mass of the load W.

If x denotes the displacement of the oscillating load from the position of equilibrium, the differential equation of free vibration will be

$$\ddot{x} + \omega_n^2 x = 0 \tag{1}$$

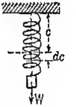


FIG. 1.

The general solution of this equation is

$$x = x_0 \cos \omega_n t + (\dot{x}_0/\omega_n) \sin \omega_n t \tag{2}$$

in which x_0 is the displacement of the body from the position of equilibrium at the initial moment ($t = 0$), and $\dot{x}_0$ is the initial velocity.

Substituting in Eq. (2),

$$x_0 = A \sin \alpha; \qquad \dot{x}_0/\omega_n = A \cos \alpha$$

we obtain

$$x = A \sin (\omega_n t + \alpha) \tag{3}$$

The unbalanced forces for various in-line engines with equal and equally spaced cylinders are shown in the table on page 511.

Engines of more cylinders are usually arranged in a V formation, with 6, 8, or 12 cylinders in each bank of the V. Since each bank is completely balanced in itself, the entire engine is also balanced for any angle between the two legs of the V. In the V-8 engine, consisting of two four-cylinder blocks of the 0-90-270-180 type crankshaft, each bank is unbalanced in primary moments. It is possible to balance this engine completely by so choosing the counterweight that the longitudinal and lateral unbalances of each individual cylinder are equal and opposite ($F_1 = -F_2$). By making the V angle 90 deg, the longitudinal forces of one block fall in line with the lateral forces of the other block, and vice versa, thus compensating each other, which ensures perfect balance.

Radial engines with master connecting rods as used in aircraft are not entirely symmetrical radially since the master rod is much heavier than the other rod. Therefore these engines labor under the same difficulty as single-cylinder engines, *i.e.*, they can be balanced in the direction of the master rod or across it, but not in both directions simultaneously. Larger radial engines with two-throw crankshafts and two stars of cylinders have the balance characteristics of two-cylinder-in-line engines.

Vibration Absorbers. The balancing of a machine can never be absolutely perfect, and there exists always a certain amount of unbalance which results in disturbing forces producing vibrations. In order to reduce the effect of disturbing forces on the foundation of a machine and to prevent the transmission of vibrations to the building, the introduction of special flexible elements between the machine and the foundation may be useful. Let W = the weight of the machine and $Q \sin \omega t$ = the impressed vertical force (Fig. 7), which is usually due to unbalance. If the machine is rigidly attached to the foundation, the entire disturbing force will be transmitted to the foundation. By introducing the flexible springs of a vibration absorber between the machine and the foundation, a substantial reduction in the force transmitted to the support can be obtained. It is only necessary to design the springs of the absorber in such a manner as to make the period T_n of natural vibration of the machine W on the springs large in comparison with the period $T = 2\pi/\omega$ of the machine. Vibration will then be reduced [see Eq. (13)] as compared with the displacements which would be produced by the disturbing force $Q \sin \omega t$ under static conditions. The forces which are transmitted to the foundation are proportional to the deflection of the springs, and therefore will be reduced in the same ratio also. Neglecting damping and taking, for instance, $T_n/T = 4$, *i.e.*, assuming that the operating speed is four times that corresponding to resonance, it will be found from Eq. (13) that the amplitude of the forced vibration is only $\frac{1}{15}$ of the statical deflection, or the force transmitted to the foundation will be reduced to $\frac{1}{15}$ of the impressed force.

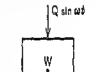


FIG. 7.

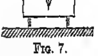


FIG. 8.

The actual design of an absorber frequently offers difficulties, principally on account of space limitation. It is difficult to have sufficient space for placing springs which will be strong enough to transmit the load to the foundation and at the same time flexible enough to make the resonance speed of the system small in comparison with the operating speed. In Fig. 8, a schematic view of a vibration absorber for a large single-phase generator is shown. In order to reduce the effect of the pulsating torque, flexible springs are introduced between the stator of the machine and the foundation. By the use of special guides, the possibility of lateral displacement of the stator is prevented and only a rotation about the 0 axis remains possible.

Let k = the torque necessary to produce a statical deformation of the springs such that the rotation of the stator about the 0 axis perpendicular to the plane of the figure is

Vibrations consist of two parts, free damped vibrations such as are represented by Eq. (5) and forced vibrations. The free vibrations will be damped out, and the forced vibrations alone are of practical importance. The general expression for forced vibrations is

$$x = A \sin (\omega t - \alpha) \tag{12}$$

where the amplitude of vibration and the phase are given by the following equations:

$$A = (Q/k)\left(1/\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + c^2\omega^2/k^2}\right) \tag{13}$$

$$\tan \alpha = c\omega/k(1 - \omega^2/\omega_n^2) \tag{14}$$

The first factor on the right side of Eq. (13) is the deflection calculated statically, *i.e.*, by division of the maximum magnitude of the disturbing force by the spring constant. The second factor is due to dynamical conditions and is called the **magnification factor.** In Fig. 2 the magnification factor is represented as a function of the ratio ω/ω_n, and for various values of the constant c, which is the coefficient of damping.

When the disturbing force has a very low frequency as compared with the frequency of the natural vibrations of the system, the ratio ω^2/ω_n^2 is small and the magnification factor approaches unity. Another extreme case occurs when the disturbing force has a very high frequency. In such a case, the ratio ω^2/ω_n^2 is a large number and the magnification factor approaches zero. When ω approaches ω_n, *i.e.*, the frequency of the disturbing force approaches the frequency of natural vibrations of the system and the damping forces are small, the magnification factor becomes large. This means that a small periodical disturbing force may produce very large forced vibrations provided it is in **resonance** with the natural vibrations of the system.

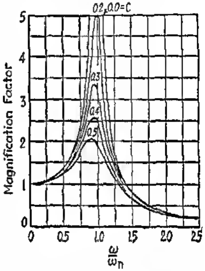


FIG. 2.—The Magnification Factor and the Coefficient of Damping.

From Eq. (12) it is seen that the forced vibration always lags behind the disturbing force. When c is small and ω is considerably less than ω_n, the difference of phase α is small. When ω approaches ω_n, a sharp variation in the phase of the forced vibrations takes place and at resonance α becomes equal to $\pi/2$. When ω is considerably larger than ω_n, the difference of phase approaches the value $\alpha = \pi$.

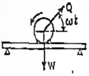


FIG. 3.

An example of forced vibrations is the vibrations produced by an unbalance of a rotating machine. Taking into consideration the vertical component of the centrifugal force Q only (Fig. 3), the disturbing force will be $Q \sin \omega t$. Heavy vibrations will be produced by this force when the number of revolutions of the machine per second approaches the frequency of the natural vibration of the system consisting of the mass of the machine and of the beams supporting the machine.

The Energy Method. The vibration problem of a system such as shown in Fig. 1 can be solved by a consideration of the energy of the system. This energy method is

in the bearings are such that free displacements of the cross sections $m - n$ and $m_1 - n_1$ during twist are possible, every crank can be replaced by an equivalent shaft of uniform cross section of an arbitrarily chosen torsional rigidity C. The length of the equivalent shaft will be found from the equation

$$l = C[(2b/C_1) + (a/C_2) + (2r/B)] \quad (28)$$

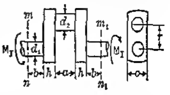


FIG. 11.

in which

$C_1 = \pi d_1^4 G/32$ = the torsional rigidity of the journal.

$C_2 = \pi d_2^4 G/32$ = the torsional rigidity of the crank pin.

$B = hc^3E/12$ = the flexural rigidity of the web.

b = half the length of the main journals and a, d_1, d_2, etc., are the dimensions shown in Fig. 11. If there are no clearences at the bearings, the length of the equivalent shaft will be found from the equation

$$l = C[(2b/C_1) + (a/C_2)(1 - r/s) + (2r/B)(1 - r/2s)] \quad (29)$$

in which

$$s = \left[\frac{r(a+h)^2}{4C_3} + \frac{ar^2}{2C_2} + \frac{a^3}{24B_1} + \frac{r^3}{3B} + \frac{1.2}{G}\left(\frac{a}{2F} + \frac{r}{F_1}\right)\right] \Big/ \left(\frac{ar}{2C_2} + \frac{r^2}{2B}\right)$$

$$C_3 = c^3h^3G/3.6(c^2 + h^2); \qquad B_1 = \pi d_2^4 E/64$$

In actual conditions, the length of the equivalent shaft will depend on the magnitude of clearances and will have an intermediate value between the two extreme values given by Eqs. (28) and (29).

In the case of a **shaft with three rotating masses** (*Fig. 12*), let I_1, I_2, I_3 = *the moments of inertia of the* rotating masses about the axis of the shaft and k_1, k_2 = spring constants for the two portions of the shaft. Then the frequency equation will be

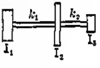
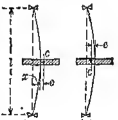


FIG. 12.

$$\frac{I_1I_2I_3}{k_1k_2}\omega_n^4 - \left(\frac{I_1I_2 + I_1I_3}{k_1} + \frac{I_2I_3 + I_1I_3}{k_2}\right)\omega_n^2 + (I_1 + I_2 + I_3) = 0 \quad (30)$$

from which the two frequencies of natural vibrations can be calculated. In the case of a shaft with many rotating masses as in the case of the Diesel engine, approximate numerical and graphical methods are usually applied in calculating frequencies of natural vibrations. (See Lewis, *Trans. Soc. Naval Architects Marine Eng.*, **33**, 1925, p. 109. See also Holzer, "Die Berechnung d. Drehschwingungen," Springer.) If a pulsating torque is acting on the shaft and the frequency of the torque approaches one of the frequencies of natural vibration of the shaft, the condition of resonance takes place and heavy forced torsional vibration will be produced.



(a) (b)

FIG. 13.

Critical Speed of a Rotating Shaft

It is known that rotating shafts at certain speeds, called critical speeds, become dynamically unstable and large vibrations are likely to develop. This phenomenon is due to resonance effects, and the critical speed for a shaft is that speed at which the number of revolutions per second of the shaft is equal to the frequency of its natural vibration.

The frequency of free vibration of such systems does not depend on the amplitude, *i.e.*, the vibrations are *isochronic*. There are cases where the spring constant of the vibrating system varies with the displacement and the restoring force is no longer proportional to the displacement. The natural frequency of systems involving such springs depends on the magnitude of the amplitude. By using such types of springs, the unfavorable effect of resonance can be diminished. If, due to the resonance, the amplitude of vibration begins to increase, the frequency of the vibration changes, *i.e.*, the resonance condition disappears. A simple example of such a spring is shown in Fig. 4. The flat spring, supporting the weight W, is built in at the end A. During vibration, the spring is partially in contact with one of two cylindrical surfaces AB or AC. Due to this fact, the free length of the cantilever varies with the amplitude so that the rigidity of the spring increases with increasing deflection, *i.e.*, the frequency of vibration increases with an increase in amplitude.

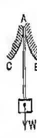


FIG. 4.

In Fig. 5, another example is given of a system in which the period of vibration depends on the amplitude. A body W performs vibrations between two springs by sliding without friction along the bar AB. This example illustrates the effect of clearance a on the frequency of vibration. The frequency will depend not only on the spring constant but also on the magnitude of the clearance a and on the initial conditions. Assume, for instance, that at the initial moment the body W is in its middle position and has an initial velocity v in the x-direction. Then the time necessary to cross the clearance a will be $t_1 = a/v$. After crossing the clearance, the body comes in contact with the spring and further motion in the x direction will be simple harmonic. The time during which the velocity is changing from v to 0 (quarter period of the simple harmonic motion) will be [see Eq. (4)] $t_2 = \pi\sqrt{W/kg}/2$. The period of a complete cycle of vibration is

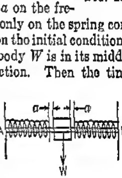


FIG. 5.

$$T = 4(t_1 + t_2) = (4a/v) + 2\pi\sqrt{W/kg} \tag{21}$$

It is seen that the period is very large for very small values of v and decreases with increase of v. If a periodic disturbing force, having a period larger than $2\pi\sqrt{W/kg}$ is acting, it will always be possible to give to the body W such an initial velocity that the corresponding period T will become equal to the period of the disturbing force and in such manner resonance conditions will be established. Some heavy vibrations in machines with clearances and in electric locomotives have been explained in this manner.

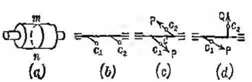


FIG. 6.

Balancing of Rotating Machines. One of the most important applications of the theory of vibrations is in the solution of balancing problems. It is known that a rotating body does not exert any variable disturbing action on the supports when the axis of rotation coincides with one of the principal axes of inertia of the body. It is difficult to satisfy this condition exactly in the process of manufacturing, because, due to errors in geometrical dimensions and non-homogeneity of the material, some irregularities in the mass

The value obtained by using in this formula the static loads W and the corresponding deflections y is generally within 1 percent of the correct value. By repeating the calculation, using W_1y_1, W_2y_2, W_3y_3, W_4y_4 . . . as the concentrated loads acting on the shaft and finding the corresponding deflections y_{1a}, y_{2a}, y_{3a}, y_{4a}, and then writing for the correct value N_c,

$$N_{c1} = N_c\sqrt{\frac{W_1y_1 + W_2y_2 + W_3y_3 + W_4y_4 + \cdots}{W_1y_{1a} + W_2y_{2a} + W_3y_{3a} + W_4y_{4a} + \cdots}} \tag{33}$$

a closer approximation to the true critical speed will be obtained, as the values y_{1a}, y_{2a} . . . will correspond more nearly to the elastic curve of the shaft *at the critical speed*. Similarly, by repeating the calculation again will W_1y_{1a}, W_2y_{2a} . . . as loads and finding corresponding deflections y_{1b}, y_{2b} . . . a still closer approximation to the true critical speed will be obtained. Unless, however, extreme care is taken to avoid cumulative errors of computation, the increased accuracy obtained for successive trials is likely to be to a large extent illusory, and of no practical value. If the shaft carries any distributed loads, they should be replaced by approximately equivalent concentrated loads before using Eq. (32). In case the loads W_1, W_2, W_3, W_4 . . . are not all on the same span (*e.g.*, on a three-bearing shaft), the loads on one span *must be assumed as acting vertically downward, and those on the other* vertically upward, so as to preclude the possibility of a zero deflection at any point except at the bearings. The same rule applies when the shaft has three or more spans.

In practice, the loads are usually shrunk on the shaft so that its stiffness (or its moment of inertia) at the loads is greatly increased; the loads are not exactly concentrated at the assumed points; and, finally, the gyroscopic action of the loads on the bent shaft, especially toward the bearings, tends to

Table 1. Critical Speeds of Shafts with a Single Concentrated Load
(Bearings R_1 and R_2; distance c. to c. = l)

Case	Bearings		Load W applied at distances from		Critical speed, N_c ($N_c = 187.7/\sqrt{y}$)	Static deflection y
	R_1	R_2	R_1	R_2		
1	Supported	Supported	a	b	$387{,}500\dfrac{d^2}{ab}\sqrt{\dfrac{l}{W}}$	$\dfrac{Wa^2b^2}{3EIl}$
2	Supported	Supported	$l/2$	$l/2$	$1{,}550{,}000\dfrac{d^2}{l\sqrt{Wl}}$	$\dfrac{Wl^3}{48EI}$
3	Fixed	Fixed	a	b	$387{,}500\dfrac{d^2l}{ab}\sqrt{\dfrac{l}{Wab}}$	$\dfrac{Wa^3b^3}{3EIl^3}$
4	Fixed	Fixed	$l/2$	$l/2$	$3{,}100{,}000\dfrac{d^2}{l\sqrt{Wl}}$	$\dfrac{Wl^3}{192EI}$
5	Fixed	Supported	a	b	$775{,}000\dfrac{d^2l}{ab}\sqrt{\dfrac{l}{Wa(3l+b)}}$	$\dfrac{Wa^3b^2}{12EIl^3}(3l+b)$
6	Fixed	Supported	$l/2$	$l/2$	$2{,}377{,}000\dfrac{d^2}{l\sqrt{Wl}}$	$\dfrac{7Wl^3}{768EI}$
7	Fixed	*	l	.....	$387{,}500\dfrac{d^2}{l\sqrt{Wl}}$	$\dfrac{Wl^3}{3EI}$

* One bearing, R_1; load W applied at free end of shaft, at distance l from bearing.

Balancing of Reciprocating Machines

For the calculation of the unbalanced forces in a single cylinder-crank mechanism, the moving parts are divided into a reciprocating weight and a rotating weight. The reciprocating weight W_{rec} consists of the piston and a fraction of the connecting rod. For this purpose, the connecting rod is laid horizontally on two scales, one under the crank-pin end and one under the piston-pin end. The fraction of the weight so found under the piston-pin end should be added to the weight of the piston to find the reciprocating weight. The rotating weight W_{rot} again consists of two parts. The first of these is *a weight so chosen that if it is concentrated at the crank-pin center* it causes a centrifugal force equal to that of the entire crank structure, including the counterbalance; the second is that fraction of the connecting rod found on the scale under the crank-pin end. The sum of these two contributions is the rotating weight W_{rot}. It is clear that W_{rec} is always positive, but that W_{rot} may be made zero or even negative by counterweighting the crankshaft.

The alternating unbalanced forces caused by these two weights are usually divided into *primary* forces, alternating once per revolution, and *secondary* forces, alternating twice per revolution. For a single crank, these forces are as follows:

$$\text{Primary}\begin{cases}\text{along cyl. center line, } (W_{rec} + W_{rot})\omega^2 r/g = F_1\\ \text{across cyl. center line, } W_{rot}\omega^2 r/g = F_2\end{cases}$$

$$\text{Secondary}\begin{cases}\text{along cyl. center line, } W_{rec}\omega^2 r^2/gl = F_3\\ \text{across cyl. center line, zero}\end{cases}$$

In these expressions, r is the crank radius, l the connecting rod length between its two bearing center, and ω the angular engine speed, in radians per second. By counterbalancing, one of the primary components can be reduced to zero, but the other primary as well as the secondary force cannot be eliminated. For a two-cylinder engine with 180 deg cranks, both primary forces are equal and opposite in direction for the two cylinders, so that a two-cylinder engine is in "primary force balance." However, the forces of the two cylinders are not in line, so that they form a couple; thence a two-cylinder engine is "unbalanced for primary moments."

No. of cylinders	Crank arrangement	Primary force		Secondary force longitudinal	Primary moment of		Secondary moments
		Longitudinal	Lateral		Long. force	Lat. force	
1	....	F_1	F_2	F_3			
2	0–180	0	0	$2F_3$	F_1l	F_2l	
3	0–120–240	0	0	0	$\sqrt{3}F_1l$	$\sqrt{3}F_2l$	$\sqrt{3}F_3l$
4	0–180–180–0	0	0	$4F_3$	0	0	
4	0–90–270–180	0	0	0	$\sqrt{10}F_1l$	$\sqrt{10}F_2l$	0
5	0–72–144–216–288	0	0	0	Unbalanced	Unbalanced	Unbalanced
6	0–120–240–240–120–0	0	0	0	0	0	0
7	Equal intervals of 360°/7	0	0	0	Unbalanced	Unbalanced	Unbalanced
8	0–180–90–270–270–90–180–0	0	0	0	0	0	0

In this table l is the distance between cylinder centerlines.

straighten out the shaft. For these reasons, the value of N_c as computed by Eq. (32) will usually be found to be somewhat low.

If a single load W_1 alone is acting on the (weightless) shaft, Eq. (32) becomes

$$N_c = 187.7/\sqrt{y} \tag{34}$$

and in this form it is occasionally applied to single-span shafts carrying more than one load by taking for y the *maximum* deflection of the shaft, no matter where it occurs.

A shaft may have as many critical speeds as the number of loads it carries. A shaft carrying a distributed load may therefore have an infinite number of critical speeds. For engineering purposes, only the first critical speed is usually of importance, while the second critical speed is only occasionally reached. For shafts of uniform or average diameter d (in.), carrying one or two concentrated loads or a uniformly distributed load (lb), formulas for the critical speed can be written down directly. Tables 1 and 2 are for steel shafts having a modulus of elasticity $E = 29{,}000{,}000$. Table 1 gives the single critical speed N_c. Table 2 gives directly or indirectly the two critical speeds N_{c1} and N_{c2}. In all cases, the weight of the shaft itself is either neglected, or a part of it (one-half to two-thirds) is added to the concentrated loads. Shafts with very short or self-aligning bearings are considered as *supported* at the bearings, and those with long rigid bearings as *fixed*.

As defined above, a shaft at its critical speed is in a state of maximum sensitiveness (or indifferent equilibrium), so that the smallest force may, if allowed sufficient time, deflect it to infinity and break it. The minutest deviation, however, from the mathematically exact critical speed is sufficient to restore to the shaft a considerable amount of its elastic resistance. In the neighborhood, therefore, of its critical speed a shaft merely undergoes more or less intense vibrations, which are generally transmitted also to the supporting frame. The deflecting force may be due to some external cause, but it is usually supplied by the almost unavoidable deviations of the centers of gravity of the various loads from the center line of the shaft. By reducing these deviations to a minimum by fine workmanship and very careful balancing, the vibration at the critical speed may be made scarcely noticeable.

Lateral Vibrations of Prismatical Bars

Let l = the length of the bar and $a = \sqrt{EIg/Ad}$, where EI = flexural rigidity of the bar, A = area of the cross section, d = weight per unit volume of the material of the bar; then the general equation for calculating frequencies of various types of natural vibration is

$$f_i = \alpha_i a/l^2 \tag{35}$$

The numerical values of the coefficient α_i depend on the conditions at the ends.

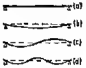


FIG. 14.

Bar with Simply Supported Ends. The consecutive types of natural vibrations shown in Fig. 14, have one, two, three . . . half-waves. The corresponding values of the factor α_i in Eq. (35) are

$$\alpha_1 = \pi/2; \quad \alpha_2 = 4\pi/2; \quad \alpha_3 = 9\pi/2; \quad \ldots \alpha_i = i^2\pi/2$$

The following tabulation gives values of α_i for three other cases of end support. Case I is for a bar with one end built in and the other free. Case II is for a bar with free ends or with built-in ends. Case III is for a bar with one end built in and the other end simply supported.

equal to one radian and I = moment of inertia of the stator about the same axis. Then the period of rotational vibration of the stator is [see Eq. (25) below]

$$T_n = 2\pi\sqrt{I/k} \tag{22}$$

Neglecting damping, the reduction of the torque transmitted to the foundation will be in the ratio [see Eq. (13)]

$$[(\omega^2/\omega_n^2) - 1]^{-1} = [(T_n^2/T^2) - 1]^{-1} \tag{23}$$

where T is the period of the pulsating torque acting on the stator.

In most cases, steel springs of various shapes may be used for mounting. Rubber springs are now available and are very convenient. The rubber usually is stressed in shear and is bonded to steel flanges which can be attached to the machine and to the floor with a common bolt connection. They are manufactured by the Goodrich Co., Akron, Ohio, under the trade name of "Vibro Insulators" and are available in several sizes. For the design of rubber supports in other cases, it is necessary to calculate the deflection characteristics of each design. Since these deflections are usually large in comparison with the original dimensions of the rubber, the ordinary formulas of strength of materials do not apply (see Smith, "Rubber Mountings," *Trans. A.S.M.E.*, A13, 1938).

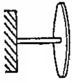


FIG. 9.

Torsional Vibrations of Shafts. The simplest case of torsional vibration is shown in Fig. 9 in which a circular disk is attached to the shaft. By twisting the shaft, torsional vibrations of the system can be produced.

Let

G = modulus of elasticity in shear.
I_p = polar moment of inertia of the circular cross section of the shaft.
$k = GI_p/l$ = the spring constant, *i.e.*, the torque necessary to produce an angle of twist equal to unity (one radian).
I = *moment of inertia of mass of the disk about the axis of the shaft.*
θ = the variable angle of twist of the shaft during vibration.

The differential equation of torsional vibration will be

$$I\ddot{\theta} + k\theta = 0 \tag{24}$$

and with the notation $\omega_n^2 = k/I$, the period of torsional vibration will be

$$T = 2\pi/\omega_n = 2\pi\sqrt{I/k} = 2\pi\sqrt{Il/GI_p} \tag{25}$$

In the case of a **system consisting of two disks** joined together by a shaft (Fig. 10), the disks will always rotate in opposite directions during vibration and there will be a nodal section mn which will remain stationary during oscillation. The distances l_1, l_2 of this section from the disks will be inversely proportional *to the moments of inertia of the disks.* Then $l_1 = lI_2/(I_1 + I_2)$ and $l_2 = lI_1/(I_1 + I_2)$. Substituting l_1 and I_1 instead of l and I in Eq. (25), the following equations for the system in Fig. 10 are obtained:

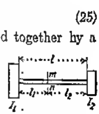


FIG. 10.

$$T = 2\pi\sqrt{I_1I_2l/(I_1 + I_2)GI_p}; \qquad f = \sqrt{(I_1 + I_2)GI_p/I_1I_2l}/2\pi \tag{26}$$

In the case of a shaft of variable cross section, such a shaft can always be replaced by an **equivalent shaft** of constant cross section noting only that a portion of a shaft of the length l and diameter d can be replaced, without changing the angle of twist of the shaft, by a portion of the length l_0 and diameter d_0 provided that

$$l_0 = ld_0^4/d^4 \tag{27}$$

In the case of **crankshafts** (Fig. 11) the problem of torsional rigidity depends on the conditions of constraint at the bearings. Assuming that the clearances

Bar of Variable Cross Section with Thicker End Built In and the Other End Supported. Assuming for I and A [Eqs. (37)] the approximate values of the fundamental and higher frequencies will be obtained by multiplying the corresponding frequencies of the prismatical bar with one end built in and another end supported (p. 518) with the factor (38) the constants β_i, β_i', γ_i, and γ_i' being taken from the table below:

i	β_i	γ_i	β_i'	γ_i'	i	β_i	γ_i	β'_i	γ_i'
1	0.431	0.569	0.626	0.857	4	0.494	0.506	0.628	0.662
2	0.480	0.520	0.612	0.724	5	0.496	0.504	0.631	0.654
3	0.490	0.510	0.623	0.680	6	0.497	0.503	0.633	0.649

Bar of Variable Cross Section with Free Ends. When the bar consists of two equal halves joined together at their thick ends, each half being generated by revolving the curve

$$y = ax^n \tag{39}$$

about the x-axis (x measured from the free ends) the frequency of the fundamental natural vibration will be given by Eq. (35). The constant α_i for various values of n in Eq. (39) is given below

$$\begin{array}{llllll} n &= 0 & \tfrac{1}{4} & \tfrac{1}{2} & \tfrac{3}{4} & 1 \\ \alpha_i &= 3.58 & 4.47 & 5.26 & 5.96 & 6.52 \end{array}$$

Vibration of Turbine Blades. Considering the blade as a cantilever bar of variable cross section built in at its thick end, the frequencies of natural vibrations in each of the two principal flexural planes of the blade will be obtained from the equation

$$f = \sqrt{f_1^2 + f_2^2} \tag{40}$$

in which f_1 denotes the frequency of the blade when the rotor is stationary, and f_2 represents the frequency of the blade when the elastic forces are neglected and only the restoring action due to centrifugal force is taken into consideration. Assuming that the variation of the cross-sectional area and of the moment of inertia along the blade are given by Eqs. (36), the frequency f_1 of the fundamental type of vibration can be calculated by using the table following Eq. (36), the frequency f_2 will be found from the equation

$$f_2 = \beta\omega/2\pi \tag{41}$$

in which ω = angular velocity of the turbine and β = the constant given in the table below for various values of the ratio a/l, where a = the radius of the rotor at the built-in end of the blade, l = the length of the blade, and c is defined by Eq. (36).

a/l	$c = 0$	$c = .2$	$c = .4$	$c = .6$	$c = .8$	$c = 1.0$
0	1.00	1.00	1.00	1.00	1.00	1.00
1	1.57	1.58	1.59	1.61	1.64	1.71
2	1.98	2.00	2.01	2.04	2.09	2.19
4	2.62	2.64	2.66	2.70	2.77	2.92
6	3.13	3.15	3.18	3.23	3.31	3.50
8	3.56	3.59	3.62	3.68	3.78	4.00
10	3.95	3.98	4.02	4.08	4.19	4.44

Vibrations of Membranes

Assumed that the membrane is a perfectly flexible and infinitely thin sheet of uniform material and thickness and that it is uniformly stretched in all

Shaft with One Disk. In order to exclude from our consideration the effect of the weight of the shaft and of the disk, a vertical shaft will be considered (Fig. 13). Let C be the center of gravity of the disk (Fig. 13a).

e = a small eccentricity.
ω = the angular velocity of the rotating shaft.
x = the deflection of the shaft at the disk.
W/g = the mass of the disk.

Then the centrifugal force acting on the shaft will be

$$W\omega^2(x + e)/g \tag{a}$$

The elastic reactive force with which the shaft is acting on the disk will be equal to

$$kx \tag{b}$$

where k is the spring constant of the shaft. In the case of a shaft of a uniform cross section and with the disk attached at the middle, $k = 48EI/l^3$.

Equating (a) and (b), we obtain $W\omega^2(x + e)/g = kx$ from which

$$x = e/[(\omega_n^2/\omega^2) - 1]$$

where $\omega_n^2 = kg/W$

It can be concluded that the deflection x tends to increase rapidly as ω approaches p, *i.e.*, when the number of revolutions per second of the shaft approaches the frequency of the lateral vibrations of the shaft and disk. The critical value of the speed will be

$$\omega_{cr} = \sqrt{gk/W} \tag{31}$$

At speeds higher than the critical, the center of gravity C will be situated as shown in Fig. 13b and the equation for determining x will be

$$W\omega^2(x - e)/g = kx$$

from which

$$x = e/[1 - (\omega_n^2/\omega^2)]$$

It is seen that for speeds above critical, with increasing ω the deflection x decreases and approaches the limit e, *i.e.*, at very high speeds the center of gravity of the disk approaches the line joining the supports and the deflected shaft rotates about the center of gravity C.

The critical speed of a rotating shaft is the speed at which its elastic forces are completely neutralized so that it is incapable of offering any resistance to a deflecting force. This speed is equal numerically to the frequency of vibration of the shaft with the masses mounted on it, if deflected by an external force while the shaft is at a standstill. Its value depends on the length of the shaft, its various diameters, the manner in which it is supported, and on the magnitude and distribution of the loads it carries, but it is independent of whether the shaft is horizontal or vertical.

Shafts of Uniform Diameter. If a shaft on two supports carrying loads $W_1, W_2, W_3, W_4 \ldots$ (lb) has such dimensions that the static deflections at those loads, if the shaft were horizontal (although actually it may be vertical), would be $y_1, y_2, y_3, y_4 \ldots$ (in.), respectively, the first or lowest critical speed N_c in rpm is given by the formula

$$N_c = 187.7\sqrt{\frac{W_1y_1 + W_2y_2 + W_3y_3 + W_4y_4 + \ldots}{W_1y_1^2 + W_2y_2^2 + W_3y_3^2 + W_4y_4^2 + \ldots}} \tag{32}$$

For the fundamental type of vibration of a square plate,

$$f = (\pi/a^2)\sqrt{gD/dh} \tag{46}$$

Consecutive frequencies of a **square plate** with free edges are given by the equation

$$f_i = (\alpha_i/2\pi a^2)\sqrt{gD/dh} \tag{47}$$

in which α_i is a constant depending on the mode of vibration. For the three lowest modes, the values of this constant are

$$\alpha_1 = 14.10; \quad \alpha_2 = 20.56; \quad \alpha_3 = 23.91$$

Circular Plate Clamped at the Boundary. Equation (47) can be used for calculating frequencies, a denoting in this case the radius of the boundary. Denoting with s the number of nodal circles and with n the number of nodal diameters, the magnitude of the factor α_i in Eq. (47) will be given by the following table. The table also gives values of α_i in Eq. (47) for a **circular plate with free boundary.**

s	Plate clamped at boundary			Plate with free boundary			
	$n = 0$	$n = 1$	$n = 2$	$n = 0$	$n = 1$	$n = 2$	$n = 3$
0	10.21	21.22	34.84	.....	.....	5.251	12.23
1	39.78	.....	.....	9.076	20.52	35.24	52.91
2	88.90	.....	.....	38.52	59.86		

Circular Plate Fixed at the Center. The constant α_i for consecutive modes of vibration having s nodal circles is as follows:

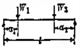
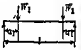
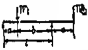
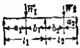
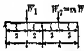
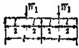
s = ...	0	1	2	3
α_i = ...	3.75	20.91	60.68	119.7

Vibration of Turbine Disks. It is well established that fractures which occur in turbine disks and which cannot be explained by defects in the material or by excessive stresses due to centrifugal forces may be attributed to flexural vibrations. Experiments show that such vibrations, at certain speeds of the turbine, become very pronounced and produce great additional bending stresses which may result in fatigue of the metal and in the gradual development of cracks, which usually start at the boundaries of the steam balance holes and other discontinuities in the web of the turbine disk where stress concentration is present. The most important cause producing these vibrations in turbine disks is that due to non-uniform steam pressure along the circumference. A localized pressure acting on the rim of a rotating disk is sufficient at certain speeds to maintain lateral vibrations in the disks. Experiments show that the failure should be attributed to those types of vibration having several diameters as nodal lines. These vibrations can be considered as combinations of two waves traveling circumferentially in the direction of rotation and in the opposite direction. The amplitudes of backward moving waves are usually larger than those of forward moving waves. Backward moving waves become especially pronounced under conditions of resonance when the backward speed of these waves in the disk coincides exactly with the forward angular velocity of the rotating disk so that the waves become stationary in space. This type of vibration is responsible in a majority of cases for disk failures. (See paper by

Table 2. Critical Speeds of Shafts Carrying Two Concentrated Loads
(The formulas are for shafts supported at the bearings)

General formula: N_{c1} and $N_{c2} = \sqrt{\frac{1}{2A}[U^2 + V^2 \mp \sqrt{(U^2+V^2)^2 - 4AU^2V^2}]}$

For N_{c1} use minus sign; for N_{c2} use plus sign. The values of A, U, and V are given in the table below, whenever N_{c1} and N_{c2} are not given directly. y_1 and y_2 = static deflections at W_1 and W_2 (shaft horizontal).

1		$A = 1 - \left(\frac{l^2 - a_1^2 - a_2^2}{2(l-a_1)(l-a_2)}\right)^2$ $U = 387{,}500\frac{d^2}{a_1(l-a_1)}\sqrt{\frac{l}{W_1}}$; $V = 387{,}500\frac{d^2}{a_2(l-a_2)}\sqrt{\frac{l}{W_2}}$
2		$N_{c2} = 548{,}000\frac{d^2}{a_1(l-2a_1)}\sqrt{\frac{l}{W_1}}$ $N_{c1} = 548{,}000\frac{d^2}{a_1}\sqrt{\frac{1}{W_1(3l-4a_1)}}$ $y_1 = \frac{W_1a_1^2}{6EI}(3l-4a_1)$
3		$A = 1 - \frac{(l+a)^2}{4l(l+c)}$ $U = 387{,}500d^2\sqrt{\frac{l}{W_1a^2b^2}}$; $V = 387{,}500\frac{d^2}{c\sqrt{W_2(l+c)}}$ $y_1 = \frac{W_1a^2b^2}{3EIl} - \frac{W_2ac}{6EIl}(l^2-a^2)$ $y_2 = \frac{W_2c^2}{3EI}(l+c) - \frac{W_1ac}{6EIl}(l^2-a^2)$
4		$A = 1 - \frac{9}{16}\ \frac{l}{l+c}$ $U = 1{,}550{,}000\frac{d^2}{l\sqrt{W_1l}}$; $V = 387{,}500\frac{d^2}{c\sqrt{W_2(l+c)}}$ $y_1 = \frac{W_1l^3}{48EI} - \frac{W_2cl^2}{16EI}$; $y_2 = \frac{W_2c^2(l+c)}{3EI} - \frac{W_1cl^2}{16EI}$
5		$C_1 = \frac{(l_1+a_1)^2}{4l_1(l_1+l_2)}$ $C_2 = \frac{(l_2+a_2)^2}{4l_2(l_1+l_2)}$ $A = 1 - \frac{C_1C_2}{(1-C_1)(1-C_2)}$ $U = 387{,}500\frac{d^2}{a_1b_1}\sqrt{\frac{l_1}{W_1(1-C_1)}}$; $V = 387{,}500\frac{d^2}{a_2b_2}\sqrt{\frac{l_2}{W_2(1-C_2)}}$
6		N_{c1} and $N_{c2} = 1{,}405{,}000\frac{d^2}{l\sqrt{W_1l}}\sqrt{1+m\mp\sqrt{1+m^2-1.388m}}$
7		$N_{c1} = 1{,}550{,}000\frac{d^2}{l\sqrt{W_1l}}$ $N_{c2} = 2{,}340{,}000\frac{d^2}{l\sqrt{W_1l}}$ $y_1 = \frac{7Wl^3}{768EI}$

lishes an air connection between the inside of the pipe and the ears. In use, the plunger is moved in the pipe very slowly till a pure singing note is heard. On the calibrated pipe, the corresponding frequency is read off. The instrument can be easily made up, the calibration being $f = 13{,}200/l$, where f = frequency, cycles per sec and l = length of pipe, inches.

Amplitude meters or vibrometers indicate amplitudes only and give no information regarding frequencies.

The vibrometer of the Vibration Specialty Co. (Philadelphia) is a seismic instrument with a natural frequency of 200 per min and very little damping, so that vibrations of frequencies above 500 per min can be measured. The relative motion of the weight and the frame is magnified 100 times. There is one indicator for horizontal and one for vertical vibrations on the same instrument which can be read simultaneously. Measurements can be made up to 2,200 cycles per min with the standard indicators and up to 4,000 cycles per min with special indicators, making the standard instrument fit for 1,800 rpm machinery. It is very rugged and intended for rough use in shops and power houses.

The vibrometer of Carl Schenk (Darmstadt, Germany) operates on the same principle as the previous ones. Its natural period is 40 per min, it has optical indicators, and can be used for frequencies from 200 to 5,000 per min. Both horizontal and vertical vibrations can be read on the same instrument. The magnification is 400 with the "precision" type or 75 with the "shop" type. The weights are critically damped, which is a great advantage.

The Davey Vibrometer (Vibroscope Inc., New York) is held still in space, and contact with the vibrating machine is established through a light pin. The relative motion sets a mirror vibrating which gives a band of light on a ground-glass scale. It can be bought with a magnification of 250 or 500. The low limit of frequency depends entirely on the kind of support. The upper frequency limit should be of the same order as the Schenk instrument on account of the optical magnification.

Vibrographs give a displacement-time record of the vibration.

The Geiger Vibrograph, Lehmann and Michels, Hamburg, Germany (See *Z. Ver. deut. Ing.* **60**, 1916, p. 811) can be used as a seismic instrument. Its magnification can be adjusted between 1:3 and 1:24. The record is traced on paper. A timing device tracing 1,500 cycle per min waves on the paper is built into the instrument. The seismic mass and spring can be easily replaced by others of different characteristics, so that it has a very wide frequency range. The standard instrument can be used from 500 up to 4,000 or 10,000 per min depending on the magnification. There is no damping in the instrument. Vibrations can be recorded in a vertical, horizontal, or any oblique direction. The seismic part can be removed and the instrument used as a direct vibration recorder with magnifications up to 72 times. In this case, the instrument itself should be held still in space by suspending it or holding it in the hands. The vibration is transmitted to the pen of the instrument by a light rod or needle. The instrument is capable of recording vibrations of small and light machinery, which is impossible with any seismic instrument. It can be changed to a torsiograph or stress recorder. Its disadvantage is the rather small magnification.

Tho Maihak Vibrograph (H. Maihak, Hamburg, Germany) is similar to Geiger's, but is a plain seismic instrument and cannot be transformed into a torsiograph or a direct recorder.

The Cambridge Vibrograph (Cambridge Instrument Co., Cambridge, England, see *Engineering*, 1925, p. 271) has a natural frequency of about 250 per min and no damping. To record vertical or horizontal vibrations two separate instruments are required, both on the seismic principle. The range of frequency is about 1,000 to possibly 10,000 per min. It records by means of a sharp needle on a band of celluloid, which is viewed through a hand microscope or can be microphotographed for reproduction. Its sensitivity is great; magnifications up to 2,000 give good pictures; amplitudes can be read to an accuracy of 0.01 mil. Since there is no provision for direct recording, the use of the instrument is restricted to heavy machines and structures.

Torsiographs. The Geiger **Torsiograph** (Lehmann and Michels, Hamburg) records non-uniformities of the angular rotation of shafts by means of the seismic principle. A very light aluminum pulley of 6 in. diameter carries inside itself a heavy steel flywheel, coupled to it by a flexible spiral spring. The pulley is driven, preferably by a steel

Case	α_1	α_2	α_3	α_4	α_5
I. Clamped-free	0.560	3.58	9.82	19.2	31.8
II. Free-free or clamped-clamped	3.58	9.82	19.2	31.8	47.5
III. Clamped-supported	2.45	7.96	16.6	28.4	43.3

Cantilever Bar of Variable Cross Section. If the variation of the cross-sectional area and of the principal moment of inertia of the cross section along the axis of the bar can be expressed in the form

$$A = a(1 - cx/l); \qquad I = b(1 - cx/l) \tag{36}$$

a, b, c being constants and x being measured from the built-in end, then the values of the coefficient α_i in Eq. (35) for the fundamental type of vibration are

$c =$	0	0.4	0.6	0.8	1.0
$\alpha_i =$	0.560	0.652	0.729	0.858	1.140

An approximate solution for a cantilever of variable cross section can be obtained by assuming that the variation of I and A along the axis of the bar (Fig. 15), measuring x from the built-in end, can be represented by the equations

$$I = I_0\left(1 - m\frac{x}{l} - m' \sin\frac{\pi x}{l}\right); \qquad A = A_0\left(1 - n\frac{x}{l} - n' \sin\frac{\pi x}{l}\right) \tag{37}$$

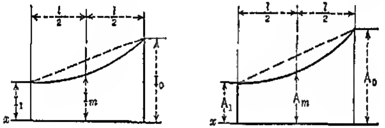


FIG. 15.

in which

$$m = (I_0 - I_1)/I_0; \qquad n = (A_0 - A_1)/A_0; \qquad m' = \frac{1}{I_0}\left(\frac{I_0 + I_1}{2} - I_m\right);$$

$$n' = \frac{1}{A_0}\left(\frac{A_0 + A_1}{2} - A_m\right)$$

Then the frequencies of the fundamental and higher modes of vibration will be obtained by multiplying the corresponding frequencies of the prismatical cantilever bar (see p. 518) with the factor

$$\sqrt{(1 - m\beta_i - m'\beta'_i)/(1 - n\gamma_i - n'\gamma'_i)} \tag{38}$$

The constants β_i, β_i', γ_i, and γ_i', for the consecutive modes of vibration are:

i	β_i	γ_i	β_i'	γ_i'	i	β_i	γ_i	β_i'	γ_i'
1	0.193	0.807	0.493	0.493	4	0.483	0.517	0.649	0.649
2	0.406	0.594	0.703	0.703	5	0.490	0.510	0.645	0.645
3	0.468	0.532	0.661	0.661	6	0.493	0.507	0.642	0.642

directions in its plane by a tension so large that the fluctuation in this tension due to the small deflections during vibrations can be neglected. Let

s = uniform tension per unit length of the boundary.
q = weight of the membrane per unit area.
A = the area of the membrane.

The frequency of the fundamental mode of vibration of the membrane is

$$f = (\alpha/2\pi)\sqrt{gs/Aq} \tag{42}$$

The constant α of this equation for various shapes of the boundary are as follows: circle, 4.261; square, 4.443; quadrant of a circle, 4.551; 60° sector of a circle, 4.616; rectangle, 3 × 2, 4.624; equilateral triangle, 4.774; semicircle, 4.803; rectangle, 2 × 1, 4.967; rectangle, 3 × 1, 5.736.

The frequencies of higher modes of vibrations of a rectangular membrane with the sides a and b, are given by the equation

$$f_{mn} = \frac{1}{2}\sqrt{\frac{gs}{q}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)},\ \text{where } m = 1, 2, 3 \cdots;\ n = 1, 2, 3 \cdots \tag{43}$$

The frequencies of higher modes of vibrations of a circular membrane is given by the equation

$$f_{ns} = (\alpha_{ns}/2\pi a)\sqrt{gs/q} \tag{44}$$

in which

a = the radius of the boundary of the membrane.
α_{ns} = the constant given in the table below and depending on the number n of nodal diameters and the number s of nodal circles (the boundary circle is included in this number).

s	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	s	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
1	2.40	3.83	5.13	6.38	7.59	8.78	5	14.9	16.5	18.0	19.4	20.8	22.2
2	5.52	7.02	8.42	9.76	11.06	12.3	6	18.1	19.6	21.1	22.6	24.0	25.4
3	8.65	10.17	11.6	13.02	14.4	15.7	7	21.2	22.8	24.3	25.7	27.2	28.6
4	11.8	13.3	14.8	16.2	17.6	19.0	8	24.4	25.9	27.4	28.9	30.4	31.8

Vibration of Plates

It is assumed that the plate consists of a perfectly elastic, homogeneous, isotropic material and that it has a uniform thickness, considered small in comparison with its other dimensions. The deflections are assumed to be small in comparison with the thickness of the plates.

Let:

h = the thickness of the plate.
$D = Eh^3/12(1 - \nu^2)$ = the flexural rigidity of the plate.
d = weight per unit volume of the material of the plate.

The frequencies of the consecutive modes of vibration of a rectangular plate with the sides a and b and simply supported along the edges are

$$f_{mn} = \frac{\pi}{2}\sqrt{\frac{gD}{dh}}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) \tag{45}$$

where $\quad m = 1, 2, 3 \cdots;\ n = 1, 2, 3 \cdots$

NON-FERROUS METALS

By CYRIL S. SMITH

CORROSION

By HENRY S. RAWDON

PAINTS AND PROTECTIVE COATINGS

By HENRY A. GARDNER

WOOD

By H. VON SCHRENK

By LIONEL S. MARKS

MISCELLANEOUS NON-METALLIC MATERIALS

By E. C. CROCKER AND ASSOCIATES

REFRACTORIES

By F. H. NORTON

LUBRICANTS

By C. M. LARSON

Wilfred Campbell, *Trans. A.S.M.E.*, **46**, 1924, p. 31. See also paper by Freundereich, *Engineering*, **119**, 1925, p. 2.)

VIBRATION INSTRUMENTS

By J. P. DEN HARTOG

REFERENCES: Steuding, "Messung Mechanischer Schwingungen," V. D. I. Verlag. Timoshenko, "Vibration Problems in Engineering," Van Nostrand.

Many vibration instruments consist of a weight, suspended on springs in a frame, which is made to follow the vibration to be measured. They are based on the **seismic principle**.

Let the point of suspension A of a mass-spring system be given a motion $a \sin \omega t$. The mass will swing with the same frequency but with a different amplitude. The amplitude of the relative motion between the mass and the frame, as a function of ω, is shown in Fig. 1. For very slow motion of the frame (ω nearly zero), the spring will not extend at all; for a motion in resonance with the natural frequency of the system ω_n, the relative motion will be large, while for a very rapid motion of A (ω large), the mass cannot follow and will stand still in space so that the relative motion is a, or equal to the vibration of the frame. An instrument recording **vibration amplitudes**, therefore, should have a natural frequency which is low with respect to the frequency of the vibration to be measured.

The acceleration belonging to the motion $a \sin \omega t$ is $-a\omega t^2 \sin \omega t$; again a harmonic motion with the amplitude $a\omega^2$. The equation of the fully drawn curve in Fig. 1 is

$$\text{Relative motion} = a\omega^2/(\omega_n^2 - \omega^2)$$

which for small values of ω is nearly equal to $a\omega^2$. Therefore, **accelerometers** should have a natural frequency which is high with respect to the frequency of the acceleration to be measured.

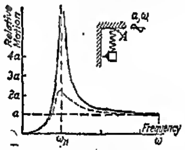


FIG. 1.—Vibration of Spring-suspended Weight.

A certain amount of **damping** is desirable in either class of instrument (dotted curve in Fig. 1). For the accelerometer, large damping is a fundamental necessity. An impure vibration contains, besides its fundamental frequency, many higher harmonic frequencies. An impure wave of 10 cycles per sec can cause resonance in systems with natural frequencies of 10, 20, 30, etc., cycles per sec, but not in a system of 5 cycles per sec. In an accelerometer, the natural frequency is higher than the measured frequency so that the danger of resonance always exists and can be avoided only by a great amount of damping. In a vibrograph, the natural frequency is lower than the measured one and no resonance can occur, so that such instruments can be built without much damping.

Frequency Meters. The **Fullarton Vibrometer** (Kelvin, Bottomley and Baird, Glasgow) is a single cantilever of which the length can be varied. The instrument is clamped to the structure under investigation, and at a certain length of the cantilever it responds. The corresponding frequency can be read off a scale.

The above instrument can be made for frequencies from 5 to about 100 cycles per sec. For frequencies higher than this, which may cause objectionable noise, the adjustable **organ pipe** is useful. It consists of a tube in which a plunger is inserted. The plunger has a small hole, which by means of rubber tubing and two stethoscope earpieces estab-

ejection from, the atom, thereby enabling it, because of its increased energy, to combine with other atoms to form molecules of either elementary substances or compounds. This convenient atomic model should be regarded as only a working hypothesis for coordinating a number of phenomena about which much yet remains to be known.

Chemical Elements[1]

Element	Symbol	Atomic weight*	Valence	Element	Symbol	Atomic weight*	Valence
Aluminum	Al	26.97	3	Molybdenum	Mo	95.95	3, 4, 5, 6, 8
Antimony	Sb	121.76	3, 5	Neodymium	Nd	144.27	3
Argon[2]	A	39.94	0	Neon[2]	Ne	20.183	0
Arsenic[3]	As	74.91	3, 5	Nickel	Ni	58.69	2, 3, 4
Barium	Ba	137.36	2	Nitrogen[5]	N	14.008	3, 5
Beryllium	Be	9.02	2	Osmium	Os	190.2	2, 3, 4, 6, 8
Bismuth	Bi	209.00	3, 5	Oxygen[5]	O	16.000	2
Boron[3]	B	10.82	3	Palladium	Pd	106.7	2, 4
Bromine[4]	Br	79.916	1, 3, 5	Phosphorus[3]	P	31.02	3, 5
Cadmium	Cd	112.41	2	Platinum	Pt	195.23	2, 4
Cesium	Cs	132.91	1	Polonium	Po	(210)	2, 4
Calcium	Ca	40.08	2	Potassium	K	39.096	1
Carbon[3]	C	12.010	2, 4	Praseodymium	Pr	140.92	3
Cerium	Ce	140.13	3, 4	Protactinium	Pa	231	5
Chlorine[5]	Cl	35.457	1, 3, 5, 7	Radium	Ra	226.05	2
Chromium	Cr	52.01	2, 3, 6	Radon[2] (radium emanation)	Rn	222	0
Cobalt	Co	58.94	2, 3	Rhenium	Re	186.31	1, 4, 7
Columbium (Niobium)	Cb	92.91	2, 3, 4, 5	Rhodium	Rh	102.91	3, 4
Copper	Cu	63.57	1, 2	Rubidium	Rb	85.48	1
Dysprosium	Ds	162.46	3	Ruthenium	Ru	101.7	3, 4, 6, 8
Erbium	Er	167.2	3	Samarium	Sa	150.43	3
Europium	Eu	152.0	2, 3	Scandium	Sc	45.10	3
Fluorine[6]	F	19.00	1	Selenium[3]	Se	78.96	2, 4, 6
Gadolinium	*Gd*	156.9	3	*Silicon*[3]	Si	28.06	4
Gallium	Ga	69.72	2, 3	Silver	Ag	107.880	1
Germanium	Ge	72.60	2, 4	Sodium	Na	22.997	1
Gold	Au	197.2	1, 3	Strontium	Sr	87.63	2
Hafnium	Hf	178.6	4	Sulphur[3]	S	32.06	2, 4, 6
Helium[2]	He	4.003	0	Tantalum	Ta	180.88	4, 5
Holmium	Ho	163.5	3	Tellurium[3]	Te	127.61	2, 4, 6
Hydrogen[7]	H	1.0081	1	Terbium	Tb	159.2	3
Indium	In	114.76	1, 2, 3	Thallium	Tl	204.39	1, 3
Iodine[4]	I	126.92	1, 3, 5, 7	Thorium	Th	232.12	4
Iridium	Ir	193.1	2, 3, 4, 6	Thulium	Tm	169.4	3
Iron	Fe	55.84	2, 3	Tin	Sn	118.70	2, 4
Krypton[2]	Kr	83.7	0	*Titanium*	Ti	47.9	3, 4
Lanthanum	La	138.92	3	Tungsten	W	183.92	3, 4, 5, 6
Lead	Pb	207.21	2, 4	Uranium	U	238.07	4, 6, 8
Lithium[8]	Li	6.940	1	Vanadium	V	50.95	1, 2, 3, 4, 5
Lutecium	Lu	175.0	3	Xenon[2]	Xe	131.3	0
Magnesium	Mg	24.32	2	Ytterbium	Yb	173.04	2, 3
Manganese	Mn	54.93	2, 3, 4, 6, 7	Yttrium	Yt	88.92	3
Mercury	Hg	200.61	1, 2	Zinc	Zn	65.38	2
				Zirconium	Zr	91.22	4

* The atomic weights are based upon oxygen having a weight of 16.000 by definition.
[1] All the elements are metals, except as otherwise indicated.
[2] Inert gas. [3] Metalloid. [4] Liquid. [5] Gas.
[6] Most active gas. [7] Lightest gas. [8] Lightest metal. [9] Not placed.

Calculation of the Percentage Composition of Substances. Add together the atomic weights of the elements in the compound to obtain its molecular weight. Multiply the atomic weight of the element to be calculated by the number of atoms present (indicated in the formula by a subscript number) and by 100, and divide by the molecular weight of the compound. *For example, hematite iron ore (Fe_2O_3) contains 69.94 per cent of iron by weight, determined as follows:* Molecular weight of Fe_2O_3 = (55.84×2)

belt, from the shaft under test, and thus follows the motion of this shaft exactly. The flywheel, loose on its shaft, is dragged along by the spring and rotates uniformly on account of its great inertia. The relative motion of the two is recorded on the paper band in the same manner as with the vibrograph. The instrument is of great usefulness for the investigation of torsional vibrations of gas and Diesel engine applications.

Accelerometers. The Cambridge accelerometer is a seismic instrument with a high natural frequency and critical electromagnetic damping and has the same celluloid recording device as the Cambridge vibrograph. A single instrument can record vertical and horizontal accelerations simultaneously. By inserting different springs, it can be adapted to various uses approximately as follows:

Spring	a	b	c	d
Natural frequency per sec....................	24	29	43	71
Record can be read as close as	$g/400$	$g/200$	$g/100$	$g/30$
Max acceleration measured..................	$g/2$	g	$2g$	$6g$

Velocity Meters. A reversed ratio loudspeaker may be used for transforming the motions of a vibration into an alternating current which may then be passed through an amplifier and to an oscillograph, where it is either observed visually or recorded photographically. In such an electromagnetic device, the electric current or voltage is proportional to the velocity of motion. The loudspeaker element must be mounted on a seismic mass but the pick-up instrument can be made quite small. The instrument is made by the Sperry Gyroscope Co. (See Draper and Bentley, "Measurement of Aircraft Vibration During Flight," *Jour. Aeronautical Sciences*, 3, 1936, p. 116; and Hull, "Modern Aids to Vibration Study," *Trans. A.S.M.E.*, 1937, A-151.)

Stress recorders measure variations in strain. They are clamped or screwed to two points on the member to be tested and the relative motion of these points is recorded.

The Cambridge Stress Recorder has a gage length of 10 in. and scribes its record on a film of celluloid 15 times magnified. By optical magnification of this record, the stress for steel can be read with an accuracy of about 200 lb per sq in. It can be used for frequencies up to 90 per sec.

The Peters telemeter (Southwark Foundry and Machine Co., Philadelphia) is made in gage lengths of 2 and 8 in. It operates on the change in electrical resistance due to compression of a stack of thin carbon disks. Two carbon piles are built in each instrument, connected in a Wheatstone bridge circuit and recorded in an oscillograph. With the 8 in. gage length and a sensitive oscillograph, stresses in steel can be read with an accuracy of 200 lb per sq in. It is possible to have several telemeters in different places record simultaneously on the same oscillograph film. The instrument can be used for frequencies up to about 300 per sec. It can also be placed on the rotating part of machines; in that case, three slip rings have to be mounted on the shaft. The pile principle has been used also for a recording pressure gage and for an accelerometer for a special purpose.

The Moullin stress recorder (Cambridge Instrument Co.) operates with alternating current, utilizing the variation in reluctance due to the displacement of an iron core. The oscillograph record is not a definite line as with the previous instrument, but rather a band of light of which the envelope is to be read. It was originally built for use on airships. The same recording principle is also used on a torsion meter for measuring torque variations in a shaft.

The smallest strain recorder consists of a piece of carbon glued to the test member with a thin piece of paper in between for electrical insulation (see p. 440). By inserting the carbon in one of the branches of a Wheatstone bridge, and by amplifying, these variations can be recorded on an oscillograph (Kearns and Guerke, "Vibration Stress Measurements in Strong Centrifugal Field," *Trans. A.S.M.E.*, 1937, A-156).

Approximate Specific Gravities and Densities

(Water at 39 F and normal atmospheric pressure taken as unity)

For more detailed data on any material, see the section dealing with the properties of that material. Data given are for usual room temperatures.

Substance	Specific gravity	Avg. density, lb per cu ft
Metals, Alloys, Ores		
Aluminum, cast-hammered	2.55–2.80	165
Aluminum, bronze	7.7	481
Brass, cast-rolled	8.4–8.7	534
Bronze, 7.9 to 14% Sn	7.4–8.9	509
Bronze, phosphor	8.88	554
Copper, cast-rolled	8.8–8.95	556
Copper ore, pyrites	4.1–4.3	262
German silver	8.58	536
Gold, cast-hammered	19.25–19.35	1205
Gold coin (U. S.)	17.18–17.2	1073
Iridium	21.78–22.42	1383
Iron, gray cast	7.03–7.13	442
Iron, cast, pig	7.2	450
Iron, wrought	7.6–7.9	485
Iron, spiegel-eisen	7.5	468
Iron, ferrosilicon	6.7–7.3	437
Iron ore, hematite	5.2	325
Iron ore, limonite	3.6–4.0	237
Iron ore, magnetite	4.9–5.2	315
Iron slag	2.5–3.0	172
Lead	11.34	710
Lead ore, galena	7.3–7.6	465
Manganese	7.42	475
Manganese ore, pyrolusite	3.7–4.6	259
Mercury	13.546	847
Monel metal, rolled	8.97	555
Nickel	8.9	537
Platinum, cast-hammered	21.5	1330
Silver, cast-hammered	10.4–10.6	656
Steel, cold-drawn	7.83	489
Steel, machine	7.80	487
Steel, tool	7.70–7.73	481
Tin, cast-hammered	7.2–7.5	459
Tin ore, cassiterite	6.4–7.0	418
Tungsten	19.22	1200
Zinc, cast-rolled	6.9–7.2	440
Zinc, ore, blende	3.9–4.2	253
Various Solids		
Cereals, oats, bulk	0.41	26
Cereals, barley, bulk	0.62	39
Cereals, corn, rye, bulk	0.73	45
Cereals, wheat, bulk	0.77	48
Cork	0.22–0.26	15
Cotton, flax, hemp	1.47–1.50	93
Fats	0.90–0.97	58
Flour, loose	0.40–0.50	28
Flour, pressed	0.70–0.80	47
Glass, common	2.40–2.88	162
Glass, plate or crown	2.45–2.72	161
Glass, crystal	2.90–3.00	184
Glass, flint	3.2–4.7	247
Hay and straw, bales	0.32	20
Leather	0.86–1.02	59
Paper	0.70–1.15	58
Potatoes, piled	0.67	44
Rubber, caoutchouc	0.92–0.96	59
Rubber goods	1.0–2.0	94
Salt, granulated, piled	0.77	48
Saltpeter	2.11	132
Starch	1.53	96
Sulphur	1.93–2.07	125
Wool	1.32	82
Timber, air-dry		
Apple	0.66–0.74	44
Ash, black	0.55	34
Ash, white	0.64–0.71	42
Birch, sweet, yellow	0.71–0.72	44
Cedar, white, red	0.35	22
Cherry, wild red	0.43	27
Chestnut	0.48	30
Cypress	0.45–0.48	29
Fir, Douglas	0.48–0.55	32
Fir, balsam	0.40	25
Elm, white	0.56	35
Hemlock	0.45–0.50	29
Hickory	0.74–0.80	48
Locust	0.67–0.77	45
Mahogany	0.56–0.85	44
Maple, sugar	0.68	43
Maple, white	0.53	33
Oak, chestnut	0.74	46
Oak, live	0.87	54
Oak, red, black	0.64–0.71	42
Oak, white	0.77	48
Pine, Oregon	0.51	32
Pine, red	0.48	30
Pine, white	0.43	27
Pine, Southern	0.61–0.67	38–42
Pine, Norway	0.55	34
Poplar	0.43	27
Redwood, California	0.42	26
Spruce, white, red	0.45	28
Teak, African	0.99	62
Teak, Indian	0.66–0.88	48
Walnut, black	0.59	37
Willow	0.42–0.50	28
Various Liquids		
Alcohol, ethyl (100%)	0.789	49
Alcohol, methyl (100%)	0.796	50
Acid, muriatic, 40%	1.20	75
Acid, nitric, 91%	1.50	94
Acid, sulphuric, 87%	1.80	112
Chloroform	1.500	95
Ether	0.736	46
Lye, soda, 66%	1.70	106
Oils, vegetable	0.91–0.94	58
Oils, mineral, lubricants	0.88–0.94	57
Turpentine	0.861–0.867	54

SECTION 6

MATERIALS OF ENGINEERING

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

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(Weights are *in vacuo*)

Temp, deg C	Specific gravity	Density, lb per cu ft	Temp, deg C	Specific gravity	Density, lb per cu ft	Temp, deg C	Specific gravity	Density, lb per cu ft	Temp, deg C	Specific gravity	Density, lb per cu ft
0	0.99987	62.4183	20	0.99823	62.3164	40	0.99224	61.9428	60	0.98324	61.380
2	0.99997	62.4246	22	0.99780	62.2894	42	0.99147	61.894	62	0.98220	61.315
4	1.00000	62.4266	24	0.99732	62.2598	44	0.99066	61.844	64	0.98113	61.249
6	0.99997	62.4246	26	0.99681	62.2278	46	0.98982	61.791	66	0.98005	61.181
8	0.99988	62.4189	28	0.99626	62.1934	48	0.98896	61.737	68	0.97894	61.112
10	0.99973	62.4096	30	0.99567	62.1568	50	0.98807	61.682	70	0.97781	61.041
12	0.99952	62.3969	32	0.99505	62.1179	52	0.98715	61.624	72	0.97666	60.970
14	0.99927	62.3811	34	0.99440	62.0770	54	0.98621	61.566	74	0.97548	60.896
16	0.99897	62.3623	36	0.99371	62.0341	56	0.98524	61.505	76	0.97428	60.821
18	0.99862	62.3407	38	0.99299	61.9893	58	0.98425	61.443	78	0.97307	60.745

Compressibility of Liquids

If v_1 and v_2 are the volumes of the liquids at pressures of p_1 and p_2 atm, respectively at any temperature, the coefficient of compressibility b is given by the equation

$$b = \frac{1}{v_1} \times \frac{v_1 - v_2}{p_2 - p_1}.$$

The value of $b \times 10^6$ for oils at low pressures at about 70 F varies from about 55 to 80; for mercury at 32 F it is 3.9; for chloroform at 32 F it is 100 and increases with the temperature to 200 at 140 F; for ethyl alcohol it increases from about 100 at 32 F and low pressures to 125 at 104 F; for glycerin it is about 24 at room temperature and low pressure.

Volume of Water as a Function of Pressure and Temperature
(From "International Critical Tables")

Temperature, deg F (C)	Pressure in atmospheres								
	0	500	1,000	2,000	3,000	4,000	5,000	6,500	8,000
32(0)	1.0000	0.9769	0.9566	0.9223	0.8954	0.8739	0.8565	0.8361	
68(20)	1.0016	0.9804	0.9619	0.9312	0.9065	0.8855	0.8675	0.8444	0.8244
122(50)	1.0128	0.9915	0.9732	0.9428	0.9183	0.8974	0.8792	0.8562	0.8369
176(80)	1.0287	1.0071	0.9884	0.9568	0.9315	0.9097	0.8913	0.8679	0.8481

Composition of Air

The percent volumetric composition of dry air at sea level is N_2, 78.03; O_2, 20.99; A, 0.94; CO_2, 0.03; H_2, 0.01; Ne, 0.00123; He, 0.0004; Kr, 0.00005; Xe, 0.000006. For ordinary purposes, this is taken as N_2, 79; O_2, 21. On this basis, the ratio of N_2 to O_2 is 3.76. On the basis of mass or weight, the percent composition is N_2, 75.5; O_2, 23.2; A, 1.33; CO_2, 0.045 or approximately N_2, 76.8; O_2, 23.2 and the nitrogen-oxygen weight ratio is 3.32 (data from "International Critical Tables," vol. 1, p. 393, McGraw-Hill).

GENERAL PROPERTIES OF MATERIALS

BY

H. W. BEARCE

(Originally prepared by Louis A. Fischer)

Chemistry

Every elementary substance is made up of exceedingly small particles called **atoms** which are all alike and which cannot be further subdivided or broken up by **chemical processes**. It will be noted that this statement is virtually a definition of the term elementary substance and a limitation of the term chemical process. There are as many different classes or families of atoms as there are **chemical elements**.

Two or more atoms, either of the same kind or of different kinds, are, in the case of most elements, capable of uniting with one another to form a higher order of distinct particles called **molecules**. If the molecules or atoms of which any given material is composed are all exactly alike, the material is a **pure substance**. If they are not all alike, the material is a **mixture**.

If the atoms which compose the molecules of any pure substance are all of the same kind, the substance is, as already stated, an **elementary substance**. If the atoms which compose the molecules of a pure chemical substance are not all of the same kind, the substance is a **compound substance**.

The atoms are to be considered as the smallest particles which occur separately in the structure of molecules of either compound or elementary substances, so far as can be determined by ordinary chemical analysis. The molecule of an element consists of a definite (usually small) number of its atoms. The molecule of a compound consists of one or more atoms of each of its several elements, the numbers of the various kinds of atoms and their arrangement being definite and fixed, and determining the character of the compound. This notion of molecules and their constituent atoms is useful for interpreting the observed fact that chemical reactions—*e.g.* the analysis of a compound into its elements, the synthesis of a compound from the elements, or the changing of one or more compounds into one or more different compounds—take place so that the masses of the various substances concerned in a given reaction stand in definite and fixed ratios.

It appears from recent researches that some substances which cannot by any available means be decomposed into simpler substances and which must therefore be defined as elements, are continually undergoing spontaneous changes or **radioactive transformation** into other substances which can be recognized as physically and chemically different from the original substance. Radium is an element by the definition given and may be considered as made up of atoms. But it is assumed that these atoms, so called because they resist all efforts to break them up and are therefore apparently indivisible, nevertheless split up spontaneously, at a rate which scientists have not been able to influence in any way, into other atoms, thus forming other elementary substances of totally different properties.

The view generally accepted at present is that the atoms of all the chemical elements, including those not yet known to be radioactive, consist of several kinds of still smaller particles, two of which are known as protons and electrons. The protons with other particles are bound together in the atomic nucleus and are positively charged. The electrons are negatively charged particles, all alike, external to the nucleus, and sufficient in number to neutralize the nuclear charge in an atom. The differences between the atoms of different chemical elements are due to the different numbers of these smaller particles composing them. According to the original Bohr theory, an ordinary atom is conceived as a stable system of such electrons revolving in closed orbits about the nucleus like the planets of the solar system around the sun. In a hydrogen atom, there is one proton and one electron; in a radium atom, there are 88 electrons surrounding a nucleus 226 times as massive as the hydrogen nucleus. Only a few, in general the outermost or valence electrons of such an atom, are subject to rearrangement within, or

Semisteel is a term that is now obsolete and is no guarantee of any specific or useful property in the metal.

Puddled iron (steel) is wrought iron (steel) made by the puddling process. Puddled steel is necessarily slag bearing.

Pig Iron

Iron ore is reduced in a blast furnace to form **pig iron**, which is the raw material for practically all iron and steel products. Nearly 90 percent of the iron ore mined in the United States comes from the Lake Superior region; this ore has the advantages of high quality and the cheapness with which it can be mined and transported by way of the Great Lakes. The ore consists mainly of hematite (Fe_2O_3) and contains 50 to 70 percent iron. Other important iron-ore districts in the United States are in Alabama, in the Adirondacks in New York, and in Pennsylvania.

The **blast furnace** consists of a vertical shaft over 20 ft diam and 100 ft high containing a descending column of iron ore, coke, and limestone and a large volume of ascending hot gas. The gas is produced by the burning of the coke in the hearth of the furnace and contains about 34 percent carbon monoxide. This gas reduces the iron ore to metallic iron which melts and picks up considerable quantities of carbon, manganese, phosphorus, sulphur, and silicon. The gangue (mostly silica) of the iron ore and the ash in the coke combine with the limestone to form the blast-furnace slag. The pig iron and slag are drawn off at intervals from the hearth through the iron notch and cinder notch, respectively. The modern blast furnace produces 100 to 1,200 tons of pig iron per day. To produce 1 ton of pig iron requires approximately 2 tons of iron ore, 1 ton of coke, half a ton of limestone, and 4 tons of air.

Wrought Iron

Manufacture. Pig iron is heated in a puddling furnace at a temperature somewhat above its melting point with the addition of iron oxides (ore, mill scale, etc.). The puddling furnace is a reverberatory furnace with an oxidizing flame which plays over the bath of molten metal. Air is allowed to enter the furnace and further promotes the oxidation. As the impurities are gradually burnt out of the iron, its melting point is raised so that the resulting purer metal forms in globules which are collected together by means of long iron rods manipulated by the puddler. This iron is not molten, but is taken from the furnace in a pasty condition in the form of balls and contains semi-molten slag (principally iron silicates) mechanically included. The ball is put through a squeezer or hammered with a steam hammer to remove a large portion of the slag and is now called a **bloom**. It passes through a rolling mill and is then known as **muckbar**. The bars are usually sheared, piled crosswise, and the pile is reheated and rerolled with further removal of slag, the purer iron product being called **refined bar iron**. This is the wrought iron of commerce. When refined bar iron is sheared, piled, and rerolled in a similar manner, the resulting material is called **double-refined iron**.

In a new method of manufacturing wrought iron, known as the **Aston process** or **New Byers process**, a very-low-carbon ferrous metal is prepared in a suitable furnace, usually a Bessemer converter. The relatively pure molten iron is poured into a ladle containing slag of the proper composition. An instantaneous and violent action occurs upon solidification of the metal, with profuse gas liberation, and the metal becomes a pasty mass of iron

+ (16 × 3) = 159.68. Percentage of iron in compound = (55.84 × 2) × 100/159.68 = 69.94.

Solubility of Inorganic Substances in Water

Number of grams of the anhydrous substance soluble in 1,000 grams of water. The common name of the substance is given in parenthesis.

	Composition	Temperature, deg F		
		32	122	212
Aluminum sulphate	$Al_2(SO_4)_3$	313	521	891
Aluminum potassium sulphate (potassium alum)	$Al_2K_2(SO_4)_4 \cdot 24H_2O$	30	170	1540
Ammonium bicarbonate	NH_4HCO_3	119		
Ammonium chloride (sal ammoniac)	NH_4Cl	297	504	760
Ammonium nitrate	NH_4NO_3	1183	3440	8710
Ammonium sulphate	$(NH_4)_2SO_4$	706	847	1033
Barium chloride	$BaCl_2 \cdot 2H_2O$	317	436	587
Barium nitrate	$Ba(NO_3)_2$	50	172	345
Calcium carbonate (calcite)	$CaCO_3$	0.018[1]	....	0.88
Calcium chloride	$CaCl_2$	594	....	1576
Calcium hydroxide (hydrated lime)	$Ca(OH)_2$	1.77	....	0.67
Calcium nitrate	$Ca(NO_3)_2 \cdot 4H_2O$	931	3561	3626
Calcium sulphate (gypsum)	$CaSO_4 \cdot 2H_2O$	1.76	2.06	1.69
Copper sulphate (blue vitriol)	$CuSO_4 \cdot 5H_2O$	140	334	753
Ferrous chloride	$FeCl_2 \cdot 4H_2O$	644[4]	820	1060
Ferrous hydroxide	$Fe(OH)_2$	0.0067[3]		
Ferrous sulphate (green vitriol or copperas)	$FeSO_4 \cdot 7H_2O$	156	482	
Ferric chloride	$FeCl_3$	730	3160	5369
Lead chloride	$PbCl_2$	6.73	16.7	33.3
Lead nitrate	$Pb(NO_3)_2$	403	....	1255
Lead sulphate	$PbSO_4$	0.042[2]		
Magnesium carbonate	$MgCO_3$	0.13[3]		
Magnesium chloride	$MgCl_2 \cdot 6H_2O$	524	....	723
Magnesium hydroxide (milk of magnesia)	$Mg(OH)_2$	0.009[2]		
Magnesium nitrate	$Mg(NO_3)_2 \cdot 6H_2O$	665	903	
Magnesium sulphate (Epsom salts)	$MgSO_4 \cdot 7H_2O$	269	500	710
Potassium carbonate (potash)	K_2CO_3	893	1216	1562
Potassium chloride	KCl	284	435	566
Potassium hydroxide (caustic potash)	KOH	971	1414	1773
Potassium nitrate (saltpeter or niter)	KNO_3	131	851	2477
Potassium sulphate	K_2SO_4	74	165	241
Sodium bicarbonate (baking soda)	$NaHCO_3$	69	145	
Sodium carbonate (sal soda or soda ash)	$NaCO_3 \cdot 10H_2O$	204	475	452
Sodium chloride (common salt)	$NaCl$	357	366	392
Sodium hydroxide (caustic soda)	$NaOH$	420	1448	3388
Sodium nitrate (chili saltpeter)	$NaNO_3$	733	1148	1755
Sodium sulphate (Glauber salts)	$Na_2SO_4 \cdot 10H_2O$	49	466	422
Zinc chloride	$ZnCl_2$	2044	4702	6147
Zinc nitrate	$Zn(NO_3)_2 \cdot 6H_2O$	947		
Zinc sulphate	$ZnSO_4 \cdot 7H_2O$	419	768	807

(1) 59 F. (2) 68 F. (3) In cold water. (4) 50 F.

Solubility of Gases in Water

(By volume, at atmospheric pressure)

t (deg F) =	32	68	212	t (deg F) =	32	68	212
Air	0.032	0.020	0.012	Hydrogen	0.023	0.020	0.018
Acetylene	1.89	1.12		Hydrogen sulphide	5.0	2.8	0.87
Ammonia	1250	700	......	Hydrochloric acid	560	480	
Carbon dioxide	1.87	0.96	0.26	Nitrogen	0.026	0.017	0.0105
Carbon monoxide	0.039	0.025	......	Oxygen	0.053	0.034	0.0185
Chlorine	5.0	2.5	0.00	Sulphuric acid	87	43	

Uses. Wrought iron can be obtained in the form of plates, sheets, forging billets, structural shapes, bars, pipes, and tubing. The principal use for wrought iron is in the form of pipe used for mildly corrosive conditions. It is also used extensively for general forging purposes. Wrought iron was formerly used for making crucible steel, and is still employed by some manufacturers for that purpose.

Ingot Iron

Ingot iron (Armco iron) is a commercially pure iron which is made in a basic open-hearth furnace in a manner similar to steel. The refining operation is carried on considerably further by the addition of a very pure grade of iron ore which oxidizes out the impurities to a low point. A high furnace temperature is required for this operation owing to the high melting point of pure iron. A typical analysis of ingot iron is given in Table 1.

Physical Constants. Sp gr, 7.866; melting point, 2795 F; specific heat at 77 F, 0.108; heat of fusion, 36 Btu per lb; thermal conductivity at 212 F., 465 Btu per hr per sq ft per in. per deg F; thermal coefficient of expansion at 212 F, 7 millionths per deg F; electrical resistivity at 32 F, 9.50 michohm-cm; temperature coefficient of electrical resistance between 32 and 212 F, 0.0031 per deg F. Many of these constants are affected considerably by small changes in composition, grain size, or mechanical treatment.

Table 3. *Mechanical Properties of Ingot Iron*
(R. L. Kenyon, A.S.M. Metals Handbook, 1939)

	Hot-rolled rods or plates	Dead soft	Cold-worked (approx max)	Forgings			
				Finished cold	Finished hot	Finished hot, annealed	Finished hot, quenched 1725 F in water
Tensile yield strength, kips	26–32	19	.....	26	19	18	30
Tensile strength, kips	42–48	38	100	43	42	41	47
Elongation, percent	22–28	43–48	.....	42	45	47	36
Gage length, in.	8	2	.....	2	2	2	2
Reduction of area, percent	65–78	70–77	65–70	76	77	71	70
Hardness, Brinell No.	82–100	67	220	101	90	82	110

Table 4. Effect of Temperature on Mechanical Properties of Ingot Iron
(A.S.M. Metals Handbook, 1939)

Temp F	300	350	400	450	500	550	600	650	700	750	800	850
Tensile strength, kips	59.8	62.7	65.5	65.0	64.3	62.9	60.8	57.3	51.7	45.8	42.0	36.0
Percent elongation	26.5	26.5	26.8	28.9	31.7	36.8	40.3	43.9	47.8	51	51	50
Percent reduction of area	60	57	54	54	54	56	58	60	63.8	67.5	67.5	65.3
Modulus $E \times 10^{-6}$	28.4	28.1	27.8	27.4	27.1	26.6	26.7	25.9	25.5	25.0	24.4	
Limiting creep stress,[a] kips	....	....	....	58	49	41	36	32	27	22	18	14

[a] 0.001 in. in 300 days.

Substance	Specific gravity	Avg density, lb per cu ft
Various Liquids		
Water, 4 C, max. density	1.0	62.428
Water, 100 C	0.9584	59.830
Water, ice	0.88–0.92	56
Water, snow, fresh fallen	.0.125	8
Water, sea water	1.02–1.03	64
Gases, see pp. 310 and 365		
Ashlar Masonry		
Granite, syenite, gneiss	2.4–2.7	159
Limestone	2.1–2.8	153
Marble	2.4–2.8	162
Sandstone	2.0–2.6	143
Bluestone	2.3–2.6	153
Rubble Masonry		
Granite, syenite, gneiss	2.3–2.6	153
Limestone	2.0–2.7	147
Sandstone	1.9–2.5	137
Bluestone	2.2–2.5	147
Marble	2.3–2.7	156
Dry Rubble Masonry		
Granite, syenite, gneiss	1.9–2.3	130
Limestone, marble	1.9–2.1	125
Sandstone, bluestone	1.8–1.9	110
Brick Masonry		
Hard brick	1.8–2.3	128
Medium brick	1.6–2.0	112
Soft brick	1.4–1.9	103
Sand-lime brick	1.4–2.2	112
Concrete Masonry		
Cement, stone, sand	2.2–2.4	144
Cement, slag, etc.	1.9–2.3	130
Cement, cinder, etc.	1.5–1.7	100
Various Building Mat'ls		
Ashes, cinders	0.64–0.72	40–45
Cement, Portland, loose	1.51	94
Portland cement	3.1–3.2	196
Lime, gypsum, loose	0.85–1.00	53–64
Mortar, lime, set	1.4–1.9	103
Mortar, Portland		94
cement	2.08–2.25	...135
Slags, bank slag	1.1–1.2	67–72
Slags, bank screenings	1.5–1.9	98–117
Slags, machine slag	1.5	96
Slags, slag sand	0.8–0.9	49–55
Earth, etc., Excavated		
Clay, dry	1.0	63
Clay, damp plastic	1.76	110
Clay and gravel, dry	1.6	100
Earth, dry, loose	1.2	76
Earth, dry, packed	1.5	95
Earth, moist, loose	1.3	78
Earth, moist, packed	1.6	96
Earth, mud, flowing	1.7	108
Earth, mud, packed	1.8	115
Riprap, limestone	1.3–1.4	80–85
Riprap, sandstone	1.4	90
Riprap, shale	1.7	105
Sand, gravel, dry, loose	1.4–1.7	90–105
Sand, gravel, dry, packed	1.6–1.9	100–120
Sand, gravel, wet	1.89–2.16	126
Excavations in Water		
Sand or gravel	0.96	60
Sand or gravel and clay	1.00	65
Clay	1.28	80
River mud	1.44	90
Soil	1.12	70
Stone riprap	1.00	65
Minerals		
Asbestos	2.1–2.8	153
Barytes	4.50	281
Basalt	2.7–3.2	184
Bauxite	2.55	159
Bluestone	2.5–2.6	159
Borax	1.7–1.8	109
Chalk	1.8–2.8	143
Clay, marl	1.8–2.6	137
Dolomite	2.9	181
Feldspar, orthoclase	2.5–2.7	162
Gneiss	2.7–2.9	175
Granite	2.6–2.7	165
Greenstone, trap	2.8–3.2	187
Gypsum, alabaster	2.3–2.8	159
Hornblende	3.0	187
Limestone	2.1–2.86	155
Marble	2.6–2.86	170
Magnesite	3.0	187
Phosphate rock, apatite	3.2	200
Porphyry	2.6–2.9	172
Pumice, natural	0.37–0.90	40
Quartz, flint	2.5–2.8	165
Sandstone	2.0–2.6	143
Serpentine	2.7–2.8	171
Shale, slate	2.6–2.9	172
Soapstone, talc	2.6–2.8	169
Syenite	2.6–2.7	165
Stone, Quarried, Piled		
Basalt, granite, gneiss	1.5	96
Limestone, marble, quartz	1.5	95
Sandstone	1.3	82
Shale	1.5	92
Greenstone, hornblende	1.7	107
Bituminous Substances		
Asphaltum	1.1–1.5	81
Coal, anthracite	1.4–1.8	97
Coal, bituminous	1.2–1.5	84
Coal, lignite	1.1–1.4	78
Coal, peat, turf, dry	0.65–0.85	47
Coal, charcoal, pine	0.28–0.44	23
Coal, charcoal, oak	0.47–0.57	33
Coal, coke	1.0–1.4	75
Graphite	1.64–2.7	135
Paraffin	0.87–0.91	56
Petroleum	0.87	54
Petroleum, refined (kerosene)	0.78–0.82	50
Petroleum, benzine	0.73–0.75	46
Petroleum, gasoline	0.70–0.75	45
Pitch	1.07–1.15	69
Tar, bituminous	1.20	75
Coal and Coke, Piled		
Coal, anthracite	0.75–0.93	47–58
Coal, bituminous, lignite	0.64–0.87	40–54
Coal, peat, turf	0.32–0.42	20–26
Coal, charcoal	0.16–0.23	10–14
Coal, coke	0.37–0.51	23–32

scrap, then melt the scrap with an oxidizing flame. When melting is well advanced, molten pig iron is added and a reaction takes place between the iron oxide and the impurities in the pig iron. The acid slag formed by the oxidation of silicon and manganese is run off, and with further heating the lime comes up through the bath causing a violent boil and forms a basic slag. This slag makes it possible to remove a large percentage of the phosphorus in the iron. When all the impurities are lowered to the desired extent, the metal is tapped through a hole in the rear of the furnace and poured into a ladle. Various additions are made to the steel either in the furnace before tapping, or in the ladle, to deoxidize the steel and to obtain the desired composition. The time required for the complete operation is 8 to 12 hr.

Acid-open-hearth steel is produced in an open-hearth furnace having an acid or siliceous lining. The process is very similar to the basic except that an acid slag is used so that no phosphorus can be removed from the steel. This requires a scrap and pig-iron charge of low phosphorus content.

Bessemer steel is made in a pear-shaped converter which is mounted on trunnions so as to be tilted easily for charging and pouring. Molten pig iron is poured into the converter, and air is blown through the liquid metal. Heat is liberated by the oxidation of the impurities—silicon, manganese, and carbon. When the carbon has been nearly eliminated, the metal is recarburized by the addition of alloys containing carbon, manganese, and silicon. From 8 to 20 tons of steel are made in one blow requiring only 10 or 15 min.

Acid Bessemer steel is made in a converter having a siliceous lining and is the only Bessemer steel made in the U. S., for our ores are not suitable for the production of pig iron for basic Bessemer steel. The latter is made in a converter having a magnesite lining, and limestone is added during the process to form a basic slag which removes a large proportion of the phosphorus in the pig. The process requires a pig iron very high in phosphorus, whereas the basic open-hearth process can use pig iron containing phosphorus in all but the highest amounts.

Electric Steel. Refining can be carried further in an electric furnace than in either the Bessemer converter or the open-hearth in consequence of better control of the composition of the slag and higher obtainable temperatures. The electric furnace commonly used in the production of electric steel is the **three-phase arc furnace** in which the electric arc heats the bath. The furnace has a basic lining and is usually charged with cold steel scrap. When the bath is molten, iron oxide is added to produce an oxidizing slag. The slag is then made reducing by the addition of carbon, some sulphur is removed, and the oxygen in the steel is greatly reduced. The desired composition is then obtained by the addition of the necessary alloys. The **induction furnace** is simply a melting furnace to which the various metals are added to make the desired alloy. When steel scrap is used as a charge, it will be a high-grade scrap the composition of which is well known.

Crucible steel is made in small graphite crucibles which are usually heated in regenerative gas furnaces. The process consists of melting iron together with various alloys, the purity of the product depending on the purity of the raw materials. This method of producing steel results in a high quality product but is almost obsolete.

Steel Ingots. After refining the steel by one of the preceding methods, the steel is tapped into a ladle and poured into iron ingot molds. Many

IRON AND STEEL

BY

B. R. QUENEAU

REFERENCES: Stoughton, "The Metallurgy of Iron and Steel," McGraw-Hill. Bullens, "Steel and Its Heat Treatment," Wiley. Bain, "The Alloying Elements in Steel," A.S.M. "National Metals Handbook," A.S.M. Alloys of Iron Monographs, Engineering Foundation, McGraw-Hill. Woldman and Dornblatt, "Engineering Alloys," A.S.M.

Classification of Iron and Steel

Iron (Fe) is not a high-purity metal commercially but contains other chemical elements which have a large effect on its physical and mechanical properties. The amount and distribution of these elements are dependent upon the method of manufacture. The most important commercial forms of iron are listed below. (Definitions are those given in the National Metals Handbook, 1939 ed.)

Pig iron is the product of the blast furnace and is made by the reduction of iron ore.

Cast iron is an alloy of iron containing so much carbon that, as cast, it is not appreciably malleable at any temperature.

White cast iron contains carbon in the combined form. The presence of cementite or iron carbide (Fe_3C) makes this metal hard and brittle, and the absence of graphite gives the fracture a white color.

Malleable cast iron is an alloy in which all the combined carbon in a special white cast iron has been changed to free or temper carbon by suitable heat-treatment.

Gray cast iron is a cast iron which, as cast, has combined or cementitic carbon not in excess of a eutectoid percentage—the balance of the carbon occurring as graphite flakes. The term "gray iron" is derived from the characteristic gray fracture of this metal.

Mottled cast iron is cast iron, the fracture of which is mottled, with white parts in which no graphite is seen and gray parts in which graphite is seen.

Ingot iron is an open-hearth iron very low in carbon, manganese, and other impurities.

Wrought iron is a ferrous material aggregated from a solidifying mass of pasty particles of highly refined metallic iron with which is incorporated, without subsequent fusion, a minutely and uniformly distributed quantity of slag.

Steel is a malleable alloy of iron and carbon, usually containing substantial quantities of manganese.

Carbon steel is steel that owes its distinctive properties chiefly to the carbon that it contains.

Alloy steel is steel that owes its distinctive properties chiefly to some element or elements other than carbon, or jointly to such other elements and carbon. Some of the alloy steels necessarily contain an important percentage of carbon, even as much as 1.25 percent. There is no agreement as to where the line between the alloy steels and the carbon steels shall be drawn.

Bessemer steel, open-hearth steel, crucible steel, and electric-furnace steel are steels made by the Bessemer, open-hearth, crucible, and electric-furnace processes, irrespective of carbon content.

Pipe, segregation, and inclusions are defects in the steel which cannot be remedied to any large extent. Surface defects such as seams, laps, and scabs can be removed by *chipping* the surface with air hammers, *scarfing* or *deseaming* with oxyacetylene torches, grinding, or, mechanically, by machining. To inspect the surface of steel products, the scale can be removed by *pickling* in acid, by the use of a torch in *flame scaling*, or by mechanical means.

Mechanical Treatment of Steel

At present, all but 2 or 3 percent of the total steel produced is cast in the form of ingots and then subjected to some form of mechanical treatment. The solid ingot is heated to between 2000 and 2600 F, depending on the composition of the steel, and then hot-worked by rolling, pressing, or hammering. **Hot work** consists of any mechanical treatment at temperatures above the thermal critical range of the steel. Considerable improvement in mechanical properties is obtained by hot work. It will increase to some extent the yield and tensile strength, but it is especially beneficial to the ductility of the steel since it breaks up the dendritic structure in the ingot and minimizes the effects of segregation and inclusions. The temperature at which hot work is completed is important and should be above, but as near as practicable to, the thermal critical range, the exact temperature depending upon the carbon and alloy content.

The effect of **rolling** is to elongate the inclusions in the direction of rolling, giving the steel excellent properties on samples taken parallel to this direction, although tests on samples taken in a transverse direction will not have as high mechanical properties. **Hammer forging** is more effective than rolling in that working can be done in more than one direction thus eliminating directional properties to the steel. The slow application of pressure in a **forging press** works the interior of a large forging more effectively than hammering; the press is used to a large extent for large high-quality forgings. Rolling, hammering, or pressing effectively breaks up the coarse crystallization of a cast ingot when a reduction in area of 3 or 4 to 1 has been obtained.

Shaping of steel by rolling is adopted whenever possible because of the rapidity of the operation and its relatively low cost. Rolling operations are carried out in mills which derive their name from the name of the product that they produce. Thus an ingot may be rolled into blooms in a blooming mill and slabs in a slabbing mill. In the blooming mill, the ingot is reduced by several passes to a **bloom**, having dimensions about 6 in. square or larger. The bloom may then be further reduced to a **billet** which is somewhere between 1½ and 6 in. square. The names blooms and billets still apply if the products are rectangular in form when the widths are less than twice the thickness. However, if the width far exceeds the thickness of the rectangular section, it is called a **slab**. Blooms are rolled on finishing mills into *structural shapes*, *rails*, *wheels*, *sheet bar* for further rolling into sheet, and **skelp** to be used in the manufacture of pipe. Blooms may also be used for forging purposes. Slabs are rolled into plates, and billets are rolled into rods, bars, bands, hoops, small shapes, and seamless tubes.

Cold work is work done on the metal below the thermal critical range and usually is done at atmospheric temperature. It greatly increases the yield strength and tensile strength, especially the former, and reduces the ductility. It includes all operations such as cold rolling, cold pressing, twisting, and wire drawing. Large tonnages of sheets are cold-rolled, which greatly

particles thoroughly mixed with slag. This pasty ball of iron is similar to the old puddled ball except that it is 6 or 7 times as large. The ball is taken to a squeezer and compacted into a 1,000 lb bloom which can be rolled directly into muckbar, slabs, rods, skelp, or any other desired form. In this process, no rabbling or agitation by hand or machine is required. The wrought iron produced has the metallographic characteristics and physical properties that are normal to a hand-puddled material, with an advantage of greater control of operation and of uniformity of product.

Composition. Typical analyses for hand-puddled and Byers wrought iron are given in Table 1. Wrought iron is distinguished from other irons in that it has a high phosphorus content and contains a large amount of slag inclusions.

Table 1. Typical Compositions of Wrought Iron, Ingot Iron, Electrolytic Iron, and Steel (Percent)

Material	C	Mn	P	S	Si	Slag by weight
Wrought iron (Hand puddled)	0.06	0.05	0.068	0.009	0.10	2.0
Wrought iron (Byers)	0.08	0.02	0.062	0.010	0.16	1.20
Ingot iron (Armco)	0.015	0.020	0.005	0.025	Trace	
Electrolytic iron	0.006		0.005	0.005	0.005	
Low-carbon steel	0.06	0.40	0.012	0.030	0.009	

Properties. Wrought iron can be readily worked and welded at temperatures close to its melting point. It is ductile when cold, has good forming qualities, and is said to be superior to mild steel in resistance to corrosion and fatigue failure. It has many practical applications owing to its ability to take on and hold protective metallic and paint coatings and its good machining and threading qualities. The mechanical properties of wrought iron are dependent upon the type of finished product. Table 2 gives the A.S.T.M. mechanical test requirements for single- and double-refined bars.

Table 2. Mechanical Properties of Wrought Iron
(A.S.T.M. Designation A189-39T)

	Single-refined bars			Double-refined bars		
	Bars under 1⅝ in. diam or thickness	Bars 1⅝ to 2½ in. excl. in diam or thickness	Bars over 2½ in. diam or thickness and all flat bars	Bars under 1⅝ in. diam or thickness	Bars 1⅝ to 2½ in. excl. in diam or thickness	Bars over 2½ in. diam or thickness and all flat bars
Tensile strength, min, lb per sq in.	48,000	47,000	46,000	48–54,000	47–54,000	46–54,000
Yield point, min, lb per sq in.	0.60 T.S.	0.55 T.S.	0.50 T.S.	0.60 T.S.	0.55 T.S.	0.50 T.S.
Elong in 8 in., min, percent	25	22	20	28	25	22
Reduction in area, min, percent	40	35	30	45	40	35

If austenite containing 0.80 percent carbon (eutectoid composition) is slowly cooled through the critical temperature, ferrite and cementite are rejected simultaneously, forming alternate plates or lamellae. This microstructure is called **pearlite** since when polished and etched it has a pearly luster. When examined under a microscope, however, the individual plates of cementite can easily be distinguished. If the austenite contains less than 0.80 percent carbon (hypoeutectoid composition), free ferrite will *first be rejected* on slow cooling through the critical until the composition of the remaining austenite reaches 0.80 percent carbon when the simultaneous rejection of both ferrite and carbide will again occur, producing pearlite. So a hypoeutectoid steel at room temperature will be composed of areas of free ferrite and areas of pearlite, the higher the carbon percentage, the greater the amount of pearlite present in the steel. When austenite containing more than 0.80 percent carbon (hypereutectoid composition) is slowly cooled, cementite is thrown out at the austenite grain boundaries, forming a cementite network until the austenite contains 0.80 percent carbon at which time pearlite is again formed. Thus, a hypereutectoid steel when slowly cooled will have areas of pearlite surrounded by a thin carbide network.

As the cooling rate is increased, the spacing between the pearlite lamellae becomes smaller; with the resulting greater dispersion of carbide preventing slip in the iron crystals, the steel becomes harder. Also with an increase in the rate of cooling, there is less time for the separation of excess ferrite or cementite and the equilibrium amount of these constituents will not be precipitated before the austenite transforms to pearlite. Thus with a fast rate of cooling, pearlite may contain more or less carbon than given by the eutectoid composition. When the cooling rate becomes very rapid (as obtained by quenching), the carbon does not have sufficient time to separate out in the form of carbide, and the austenite transforms to a highly stressed structure supersaturated with carbon called **martensite**. This structure is exceedingly hard but brittle and requires **tempering** to increase the ductility. Tempering consists of heating martensite to some temperature below the critical causing the carbide to precipitate in the form of small spheroids. The higher the tempering temperature, the larger the carbide particle size, the greater the ductility of the steel, and the lower the hardness.

In a carbon steel, it is possible to have a structure consisting either of parallel plates of carbide in a ferrite matrix, the distance between the plates depending upon the rate of cooling, or of carbide spheroids in a ferrite matrix, the size of the spheroids depending upon the temperature to which the hardened steel was heated. (Some spheroidization occurs when pearlite is heated, but only at high temperatures close to the critical temperature range.) At low tempering temperatures, the carbide particles are submicroscopic in size, therefore a hardened steel when tempered at a low temperature and examined under a microscope after etching will appear dark. This dark etching microstructure has been called **troostite**. When the carbide particles can be resolved under a microscope, the structure has been known as **sorbite**. Higher tempering temperatures cause the carbide spheroids to appear clearly, and the microstructure is usually referred to as a spheroidized structure. On account of the lack of a clear demarcation between troostite, sorbite, and a spheroidized structure, it is becoming customary simply to refer to the structure as **tempered martensite** and give the tempering temperature or the hardness of the product. The terms troostite and sorbite have for many years been used to denote very fine nodular pearlite and imperfectly developed pearlite formed in steels cooled just too slowly to be fully martensite, but this use of the terms is disappearing.

Heat-treating Operations

The following definitions of terms have been adopted by the A.S.T.M., S.A.E., and A.S.M. in substantially identical form.

Heat-treatment. An operation, or combination of operations, involving the heating and cooling of a metal or an alloy in the solid state, for the purpose of obtaining certain desirable conditions or properties.

Quenching. Rapid cooling by immersion in liquids or gases or by contact with metal.

Mechanical Properties. The average mechanical properties of ingot iron after various treatments are given in Table 3 (from A.S.M. Metals Handbook).

Young's modulus for ingot iron is 29,300,000 lb per sq in. in both tension and compression, and the shear modulus is 11,800,000 lb per sq in. Poisson's ratio is 0.28. The **effect of temperature** on the mechanical properties is shown in Table 4. The effect of **cold rolling** on the tensile strength, yield strength, elongation, and shape of the stress-strain curve is shown in Fig. 1. Armco iron welds evenly and easily; it holds paint well and has superior enameling properties; it has a high magnetic permeability at high inductions and low retentivity.

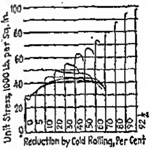

FIG. 1.—Effect of Cold Rolling on the Stress-strain Relations of Armco Ingot Iron. (Kenyon and Burns.)

Uses. In galvanized-sheet form for culverts, flumes, roofing, and siding; in plate form in oil tanks, water tanks, boilers, gas holders, and large pipe; in enameled form for ranges, refrigerators, tables and kitchen furniture, lighting fixtures, and similar articles. The purity of Armco iron makes it valuable as melting stock in making high-grade tool steel.

Electrolytic Iron

Electrolytic iron has been produced in commercial quantities since 1904 and has been useful in the production of metal with unusual properties. Many different processes have been tried out but, for economic reasons, most of them have been short-lived. The production of thin-wall large-diameter tubes has been successful since it is comparatively expensive to form them by rolling. Large revolving mandrels are used as cathodes, and a ferrous-chloride electrolyte is usually employed. It requires about 31 hr to plate 0.1 in. of iron in the form of a tube on the mandrel, but little labor or supervision is necessary, and a high percentage of satisfactory tubes is obtained. The tube is given a preliminary anneal to make it ductile and is then stripped off by special machines and reannealed to remove the hardness due to the cold-work of the stripping operation. Electrolytic iron is extremely brittle as deposited so that it can be readily pulverized to a fine powder, and this has been one way of producing a high-purity iron powder. Table 1 gives a typical analysis.

Mechanical Properties. Annealed electrolytic iron has tensile strength 35,000 to 40,000 lb per sq in.; yield strength, 10,000 to 20,000 lb per sq in.; elongation in 1.5 in., 40 to 60 percent; reduction of area, 70 to 90 percent. Large variations in mechanical properties will occur with change in amount of impurities, grain size, or previous mechanical or thermal treatment.

STEEL

Steel Manufacturing. Steel is produced from pig iron by the removal of impurities in either an open-hearth furnace, a Bessemer converter, or an electric furnace. In the United States, over 85 percent of steel is produced in the basic open-hearth furnace. This is a reverberatory furnace having a capacity up to 150 tons, although some special tilting furnaces have a capacity up to 400 tons. The hearth has a basic lining of magnesia and dolomite, and the roof is made of high-grade silica brick. A practice in wide use for making structural steels is to charge limestone, iron ore, and

"S curve." Various cooling rates are shown diagrammatically, and it will be seen that the faster the rate of cooling, the lower the temperature of transformation, and the harder the product formed. At around 1000 F, the austenite transforms rapidly to fine pearlite, and to form martensite it is necessary to cool very rapidly through this temperature range to prevent the formation of pearlite before the specimen reaches the temperature for the formation of martensite. The minimum rate of cooling that is required to form a fully martensitic structure is called the **critical cooling rate.** No matter at what rate the steel is cooled, the only products of transformation of this steel will be pearlite or martensite. However, if the steel is given an interrupted quench by quenching in a molten bath at some temperature between 400 and 1000 F, an acicular structure is obtained which is called **Bainite** and the heat-treatment is called **austempering.** Bainite has considerable toughness combining high strength with high ductility, and although austempering is not employed to any large extent today, it would appear to be a very promising method of heat-treating steels. Its limitation is that it can be applied only to articles of small cross section, for molten baths do not cool carbon steels sufficiently rapidly to prevent the formation of pearlite in samples more than ½ in. approx diam.

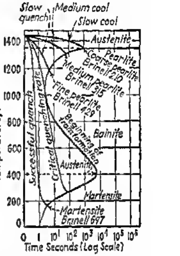

FIG. 3.—Influence of Cooling Rate on the Product of Transformation in a Eutectoid Carbon Steel.

The maximum hardness obtainable in a high-carbon steel with a fine pearlite structure is approximately 400 Brinell, although a martensitic structure would have a hardness of approximately 700 Brinell. Besides being able to obtain structures of greater hardness by forming martensite, a spheroidal structure will have considerably higher proof stress (*i.e.*, stress to cause a permanent deformation of 0.01 percent) and ductility than a lamellar structure of the same tensile strength and hardness. It is essential therefore to form martensite when optimum properties are desired in the steel. This can be done with a piece of steel having a small cross section by heating the steel above the critical and quenching in water; but when the cross section is large, the cooling rate at the center of the section will not be sufficiently rapid to prevent the formation of pearlite. The characteristic of steel that determines its capacity to harden throughout the section when quenched is called **hardenability.** This term should not be confused with the ability of a steel to attain a certain hardness. The intensity of hardening, *i.e.*, the maximum hardness of the martensite formed, is largely dependent upon the carbon content of the steel.

Three main factors affect the hardenability of steel: (1) austenite composition; (2) austenite grain size; and (3) amount, nature, and distribution of undissolved or insoluble particles in the austenite. The austenite composition will determine the position of the "nose" of the S curve, *i.e.*, the rate of decomposition, in the range of 1000 F. The slower the rate of decomposition, the larger the section that can be hardened throughout, and therefore the greater the hardenability of the steel. Everything else being equal, the higher the carbon content, the greater the hardenability. The question of austenitic grain size is of considerable importance in any steel that is to be heat-treated since it affects the properties of the steel to a considerable extent. When a steel is heated to just above the critical temperature, small polyhedral grains of austenite are formed. With increase in temperature, there is an increase in the size of grains, until at temperatures close to the melting point the grains are very large. The relation between the grain size developed and the heating temperature will vary considerably among steels

defects in rolled steel products may be introduced by incorrect ingot practice, and the production of high-grade steel requires great care at this stage.

High-quality machine and tool steels are deoxidized in the ladle with silicon and aluminum to such a degree that there is no gas evolution in the steel upon solidification, and they are called **killed** steels. When this steel cools in the mold, shrinkage of the steel on solidifying causes **piping**, the cavity or "pipe" being usually found in the upper portion of the ingot. To minimize this condition, a large-end-up mold is used together with a refractory "hot top" which supplies molten steel to the main body of the ingot while solidification proceeds. This minimizes the amount of metal that has to be discarded on account of pipe, but the top discard still amounts to 15 or 20 percent of the total weight of the ingot. In 1936, 15 percent of the total steel production was killed steel.

In order to reduce the cost of hot tops and large percentage of metal discard when making mild steel for structural purposes, the steel is not fully deoxidized. This results in blowholes in the steel on solidification; the presence of these blowholes minimizes piping by distributing throughout the ingot small voids instead of having one large one in the upper center of the ingot. If not exposed at the surface of the ingot, these blowholes weld together during rolling. Steel deoxidized in this manner is called **semikilled** steel. If the steel is deoxidized still less in the ladle, a reaction takes place during solidification in which the oxygen and the carbon in the steel form carbon monoxide which is freely evolved from the ingot. The intensity of this reaction affects the ingot structure greatly. If the reaction is allowed to go to completion, the product is called **rimmed** steel, but if the reaction is stopped after a short while by preventing, in a mechanical manner, further evolution of gas from the top of the ingot, the steel is called **capped** steel. The gas evolution results in an outer skin on the ingot which is clean and very low in carbon. In capped steel, the skin is thinner and there is less segregation or concentration of impurities than in rimmed steel. The presence of this nearly pure iron skin enables the production of an excellent surface finish on the rolled product, and therefore sheet and strip is made nearly exclusively from rimmed or capped steel.

Some defects besides the occurrence of pipe in rolled steel products are segregation, ingot cracks, seams, scabs, laps, and inclusions. **Segregation**, or the concentration of impurities, occurs in all steels upon solidification, but it can be minimized by proper mold design and low pouring temperatures. Rough mold surfaces or molds containing cracks or cavities interfere with the normal contraction of the ingot, and transverse cracks in the ingot skin may result. Cracks thus produced soon have their surfaces oxidized, and when the ingot is rolled out, these defects will be elongated in the direction of rolling and are called **seams**. Any oxidized crack or blowhole at the surface will roll out into a seam. Improper pouring conditions such as splashing of steel in the molds will form **scabs**. When rolling with grooved rolls which are not properly designed or set up, fins are liable to result from the flow of metal between the flat bodies of the rolls. If the fin is thin and wide, it will be folded over when the steel passes through the next set of rolls and will form a **lap**. **Non-metallic inclusions** consisting of sulphides, silicates, etc., are found to some extent in all steels and are introduced in the refining and deoxidation of the steel. Steels containing many inclusions are said to be "dirty," but the presence of these inclusions, unless they are large in size, is not necessarily detrimental to the physical properties.

than of plain carbon steel. The quenching operation does not have to be so drastic in the alloy steel as would be necessary to harden the plain carbon steel. Consequently, there is a smaller difference in temperature between the surface and center during quenching, and cracking and warping resulting from sharp temperature gradients in a steel during hardening can be avoided. The elements most effective in increasing the hardenability of steel are manganese, silicon, and chromium.

Such elements as molybdenum, tungsten, and vanadium are effective in increasing the hardenability when dissolved in the austenite, but they are usually present in the austenite in the form of carbides. The main advantage of these carbide-forming elements is that they prevent the agglomeration of carbides in tempered martensite. Tempering relieves the internal stresses in the hardened steel and causes spheroidization of the carbide particles with resultant loss in hardness and strength. The presence of these stable carbide-forming elements enables higher tempering temperatures to be used without sacrificing strength. This permits these alloy steels to have a greater ductility for a given strength, or, conversely, greater strength for a given ductility, than plain carbon steels.

The third factor which contributes to the strength of alloy steel is the presence of the alloying element in the ferrite. Any element in solid solution in a metal will increase the strength of the metal, so that these elements will materially contribute to the strength of hardened and tempered steels. The elements most effective in strengthening the ferrite are phosphorus, silicon, manganese, nickel, molybdenum, tungsten, and chromium.

A final important effect of alloying elements is their influence on the austenitic grain size. Martensite formed from a fine-grained austenite has considerably greater resistance to shock than when formed from a coarse-grained austenite. The oxides formed by the deoxidation of the steel by different elements apparently prevent grain growth above the critical temperature over a considerable temperature range. Aluminum is the most effective element to form grain-growth inhibitors, and most killed steels

Table 5. Trends of Influence of the Alloying Elements
(Bain, A.S.M. Metals Handbook, 1939)

Element	As dissolved in ferrite, strength	As dissolved in austenite, hardenability	As undissolved carbide in austenite, fine grain, toughness	As dispersed carbide in tempering, high temp strength, and toughness	As fine nonmetallic dispersion, fine grain, toughness
Al	Mild	Mild	None	None	Very strong
Cr	Mild	Moderate	Strong	Moderate	Slight
Co	*Strong*	*Negative*	None	None	None
Cb	?	?	Strong	Strong	None
Cu	Mild	Mild	None	None	None
Mn	Strong	Moderate	Mild	Mild	Slight
Mo	Moderate	Strong	Strong	Strong	None
Ni	Mild	Mild	None	None	None
P	Strong	Mild	None	None	None
Si	Moderate	Moderate	None	None	Moderate
Ta	Moderate ?	Strong ?	Strong	Strong	None
Ti	?	Strong ?	Very strong	Little ?	Moderate ?
W	Moderate	Strong	Strong	Strong	None
V	?	Very strong	Very strong	Very strong	Moderate ?
Zr	?	?	None ?	None	Strong ?

improves their surface finish as well as increases their strength (see p. 573 for cold-rolled sheet).

Constitution and Structure of Steel

As a result of the methods of production, the following elements are always present in steel: carbon, manganese, phosphorus, sulphur, silicon, and traces of oxygen and aluminum. Various alloying elements are frequently added, such as nickel, chromium, copper, molybdenum, and vanadium. The most important of the above elements in steel is carbon, and it is necessary to understand the effect of carbon on the internal structure of the steel to understand the heat-treatment of carbon and low-alloy steels.

In Fig. 2 is the iron-iron carbide equilibrium diagram which shows the phases that are present in steels of various carbon contents over a range of temperatures under equilibrium conditions. Pure iron when heated to 1670 F changes its internal crystalline structure from a body-centered cubic arrangement of atoms, **alpha iron**, to a face-centered cubic structure, **gamma iron**. At 2535 F, it changes back to the body-centered cubic structure, **delta iron**, and at 2795 F the iron melts. When carbon is added to iron, it is found that it has only slight solid solubility in alpha iron (only 0.007 percent at room temperature). On the other hand, gamma iron will hold up to 1.7 percent carbon in solution at 2066 F. The alpha iron containing carbon or any other element in solid solution is called **ferrite**, and the gamma iron containing elements in solid solution is called **austenite**. Usually when not in solution in the iron, the carbon forms a compound Fe_3C (iron carbide) which is extremely hard and brittle and is known as **cementite**.

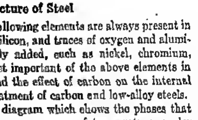

Fig. 2.—Iron-Iron Carbide Equilibrium Diagram.

The temperatures at which the phase changes occur are called **critical points** (or temperatures) and, in the diagram, represent equilibrium conditions. In practice there is a lag in the attainment of equilibrium, and the critical points are found at lower temperatures on cooling and at higher temperatures on heating than those given, the difference increasing with the rate of cooling or heating.

The various critical points have been designated by the letter *A*; when obtained on cooling, they are referred to as *Ar*, on heating as *Ac*. The various critical points are distinguished from each other by numbers after the letters, being numbered in the order in which they occur as the temperature increases. Ac_0 represents the magnetic change in cementite on heating; Ac_1, the beginning of transformation of ferrite to austenite on heating (line *PSK*); Ac_2, the magnetic change in ferrite on heating (line *MO*); Ac_3, the end of transformation of ferrite to austenite on heating (line *GOSK*); and Ac_4, the change from austenite to delta iron on heating (line *NJ*). On cooling, the critical points would be referred to as Ar_4, Ar_3, Ar_2, Ar_1, and Ar_0, respectively. It must be remembered that the diagram represents the pure iron-iron carbide system. The varying amounts of impurities in commercial steels affect to a considerable extent the position of the curves and especially the lateral position of point *S*.

Carbon steel in equilibrium at room temperature will have present both ferrite and cementite. The physical properties of the ferrite are approximately those of pure iron and are characteristic of the metal. The presence of cementite does not in itself cause steel to be hard, but rather it is the shape and distribution of the carbides in the iron that determines the hardness of the steel. The fact that the carbides can be dissolved in austenite is the basis of the heat-treatment of steel since the steel can be heated above the critical temperature (above line *GOSE* in the diagram) to dissolve all the carbides, and then suitable cooling through the cooling range will produce the desired size and distribution of carbides in the ferrite.

cooling rates (quenching) to form the hard martensitic structures. In selecting a **quenching medium** (see A.S.M. Metals Handbook), it is important to select the quenching medium for a particular job on the basis of size, shape, and allowable distortion before choosing the steel composition. It is convenient to classify steels in two groups on the basis of depth of hardening, *viz.*, shallow hardening and deep hardening. **Shallow-hardening steels** may be defined as those which, in the form of 1 in. diam rounds,

Table 7. Approximate Critical Temperatures for S.A.E. Steels
(Condensed from A.S.M. Metals Handbook, 1939)

S.A.E. Number	Ac_1 F	Ac_3 F
Carbon Steels		
1010	1350	1605
1015	1355	1585
1020	1355	1570
1025	1355	1545
1030	1350	1495
1035	1345	1475
1040	1340	1455
X1040	1340	1450
1045	1340	1450
X1045	1335	1420
1050	1340	1425
X1050	1335	1400
1055	1340	1425
X1055	1335	1400
1060	1340	1410
1065	1340	1385
X1065	1335	1380
1070	1345	1370
1075	1350	1365
1080	....	1360
1090	....	1360
1095	....	1360
10150[a]	....	1355
Free-cutting Steels		
1112	1355	1590
1120	1355	1550
X1315	1345	1520

S.A.E. Number	Ac_1 F	Ac_3 F
Manganese Steels		
T1330	1325	1480
T1335	1315	1460
T1340	1315	1435
T1345	1315	1410
T1350	1310	1400
T1360[a]	1305	1405
Nickel Steels		
2015	1375	1575
2115	1345	1525
2315	1300	1440
2320	1285	1420
2330	1275	1400
2335	1275	1375
2340	1280	1360
2345	1280	1350
2350	1280	1340
2515	1250	1420
2520[a]	1240	1390
Nickel-chromium Steels		
3115	1355	1500
3120	1350	1480
3125	1350	1465
3130	1345	1460
3135	1340	1445
3140	1355	1415
X3140	1350	1430

S.A.E. Number	Ac_1 F	Ac_3 F
Nickel-chromium Steels		
3145	1355	1395
3150	1335	1380
3215	1350	1465
3220	1350	1460
3230	1340	1435
3240	1335	1425
3245	1345	1400
3250	*1340*	*1375*
3312	1330	1435
3325	1335	1400
3330[a]	1320	1380
3335	1310	1360
3340	1290	1380
3415	1330	1425
3435	1290	1380
3450	1290	1360
Molybdenum Steels		
4130	1395	1485
X4130	1395	1480
4135	1395	1475
4140	1380	1460
4150	1365	1395
4340	1350	1425
4345	1345	1415
4615	1335	1485
4620	1335	1470
4640	1320	1430
4650[a]	1315	1410
4815	1300	1440
4820	1300	1440

S.A.E. Number	Ac_1 F	Ac_3 F
Chromium Steels		
5120	1410	1540
5140	1370	1440
5150	1330	1420
52100	1340	1415
Chromium-vanadium *Steels*		
6115	1420	1550
6120	1410	1545
6125	1400	1490
6130	1390	1485
6135	1390	1480
6140	1390	1455
6145	1390	1450
6150	1385	1450
6195	1370	1425
Tungsten Steels		
7260	1360	1430
Silicon-manganese Steels		
9255	1400	1500
9260	1400	1500

[a] This is not a standard S.A.E. steel.

have, after brine quenching, a completely martensitic shell not deeper than $\frac{1}{4}$ in. The shallow-hardening steels are those of low or no alloy content, whereas the deep-hardening steels have a substantial content of those alloying elements that increase penetration of hardening, notably chromium, manganese, and nickel. The high cooling rates required to harden shallow-hardening steel produce severe distortion in all but simple, symmetrical shapes having a low ratio of length to diameter or thickness. Plain carbon steels cannot be used for complicated shapes where distortion must be avoided. In this case, water quenching must be abandoned and a less active quench used which materially reduces the temperature gradient during quenching.

Hardening. Heating and quenching certain iron-base alloys from a temperature either within or above the critical range for the purpose of producing a hardness superior to that obtained when the alloy is not quenched. Usually restricted to the formation of martensite.

Annealing is a heating and cooling operation implying usually a relatively slow cooling. The purpose of such a heat-treatment may be (1) to remove stresses; (2) to induce softness; (3) to alter ductility, toughness, electrical, magnetic, or other physical properties; (4) to refine the crystalline structure; (5) to remove gases; or (6) to produce a definite microstructure. The temperature of the operation and the rate of cooling depend upon the material being heat-treated and the purpose of the treatment. Certain specific heat-treatments coming under the comprehensive term annealing are as follows:

1. Full Annealing. Heating iron-base alloys above the critical temperature range, holding above that range for a proper period of time, followed by slow cooling to below that range. The annealing temperature is generally about 100 F above the upper limit of the critical temperature range, and the time of holding is usually not less than 1 hr for each inch of section of the heaviest objects being treated. The objects being treated are ordinarily allowed to cool slowly in the furnace. They may, however, be removed from the furnace and cooled in some medium that will prolong the time of cooling as compared with unrestricted cooling in the air.

2. Process Annealing. Heating iron-base alloys to a temperature below or close to the lower limit of the critical temperature range followed by cooling as desired. This heat-treatment is commonly applied in the sheet and wire industries, and the temperatures generally used are from 1020 to 1200 F.

3. Normalizing. Heating iron-base alloys to approximately 100 F above the critical temperature range followed by cooling to below that range in still air at ordinary temperature.

4. Patenting. Heating iron-base alloys above the critical temperature range followed by cooling below that range in air, in molten lead, or a molten mixture of nitrates or nitrites maintained at a temperature usually between 800 to 1050 F, depending on the carbon content of the steel and the properties required of the finished product. This treatment is applied in the wire industry to medium- or high-carbon steel as a treatment to precede further wire drawing.

5. Spheroidizing. Any process of heating and cooling steel that produces a rounded or globular form of carbide. The following spheroidizing methods are used. (1) Prolonged heating at a temperature just below the lower critical temperature, usually followed by relatively slow cooling. (2) In the case of small objects of high-carbon steels, the spheroidizing result is achieved more rapidly by prolonged heating to temperatures alternately within and slightly below the critical temperature range. (3) Tool steel is generally spheroidized by heating to a temperature of 1380 to 1480 F for carbon steels and higher for many alloy tool steels, holding at heat from 1 to 4 hr and cooling slowly in the furnace.

6. Tempering (also termed **drawing**). Reheating hardened steel to some temperature below the lower critical temperature, followed by any desired rate of cooling. Although the terms "tempering" and "drawing" are practically synonymous as used in commercial practice, the term "tempering" is preferred.

Figure 3 summarizes the rates of decomposition of a eutectoid carbon steel over a range of temperatures, the resulting curve being frequently called an

quenching medium, and finally the condition of the surface of the steel before quenching. Steel that carries a heavy coating of scale will not cool so rapidly as a steel that is comparatively scale free, and soft spots may be produced; or, in extreme cases, complete lack of hardening may result. It is therefore essential to minimize scaling as much as possible. Decarburization can also produce undesirable results such as non-uniform hardening and thus lower the resistance of the material to alternating stresses.

Tempering of fully hardened steel is carried out for relief of quenching stresses and for the recovery of a limited degree of toughness and ductility. The operation will give almost any desired combination of properties by proper selection of the time-temperature conditions. Table 8 gives the Brinell hardness obtained in several carbon and alloy steels when tempered for 1 hr at various temperatures. It should be emphasized that steel should

Table 8. Brinell Hardness of Carbon and Alloy Steels

Composition of steel	Brinell hardness when hardened and tempered at						
	As quenched	200 F	400 F	600 F	800 F	1000 F	1200 F
0.35 carbon	550	534	495	415	352	269	217
0.80 carbon	697	712	653	555	461	363	269
1.2 carbon	728	755	682	601	477	375	285
S.A.E. 2340	668	614	534	461	375	307	237
S.A.E. T1340	627	614	539	469	401	311	241
S.A.E. 5140	601	589	545	477	408	331	262
S.A.E. 4140	614	614	545	485	429	363	302
S.A.E. 6140	601	601	555	485	415	363	302
0.70 C; 18 W; 4 Cr; 1 V	720	720	682	663	668	710	653

be tempered immediately after hardening to prevent cracking caused by internal stresses. When pyrometers are not available, the tempering temperatures may be controlled by the examination of the roughly polished surface of the metal. A thin film of oxide forms upon the steel, the color *of this film varying with the tempering temperature. Although it will vary* somewhat with different steels, the following temperatures are given with their corresponding colors: 400 F, faint straw; 440 F, straw; 475, deep straw; 520, bronze; 540, purple; 590, full blue.

The Relation of Design to Heat-treatment

Care must be taken in the design of a machine part to prevent cracking or distortion during heat treatment. With proper design the entire piece may be heated and cooled at approximately the same rate during the heat-treating operation. A light section should never be joined to a heavy section. Sharp reentrant angles should be avoided. Sharp corners and inadequate fillets produce serious stress concentration, causing the actual service stresses to build up to a point where they amount to 2 to 5 times the normal working stress calculated by the engineer in the original layout. The use of generous fillets is especially desirable with all high strength alloy steels.

It is well for the designer to remember that the modulus of elasticity of all commercial steels, either carbon or alloy, is the same so far as practical designing is concerned. The deflection under load of a given part is therefore entirely a function of the section of the part and is not affected by the composition or heat-treatment of the steel. Consequently if a part deflects excessively, a change in design is necessary; either a heavier section must be used or the points of support must be increased.

on account of variations in the deoxidation practice. By suitable deoxidation, steels can be made to be coarse grained at comparatively low temperatures, and others are fine grained over a considerable range of temperatures. Since the transformation of austenite usually starts at grain boundaries, a fine-grained steel will transform more rapidly than a coarse-grained steel because the latter has much less surface area bounding the grains than a steel with a fine grain size. Everything else being equal, coarse-grained austenite will have a higher hardenability than fine-grained. Small particles in the austenite will act as nuclei for the beginning of transformation in a manner similar to grain boundaries, and therefore the presence of a large number of small particles (sometimes submicroscopic in size) will result in low hardenability.

Determination of Austenitic Grain Size. The subject of austenite grain size is of considerable interest because of the fact that the grain size developed during heat-treatment has a large effect on the physical properties of the steel. In steels of similar chemical analysis, the steel developing the finer austenitic grain size will have a lower hardenability, but will, in general, have greater toughness, show less tendency to crack or warp on quenching, be less susceptible to grinding cracks, have lower internal stresses, and retain less austenite than coarse grained steel. It is for these reasons that most alloy steels are fine-grain steels. There are several methods of determining the grain-size characteristics of a steel, the one most commonly used in steel specifications being the **McQuaid-Ehn test.** In this test, a representative sample is carburized for 8 hr at 1700 F and cooled slowly. The high-carbon case on slow cooling will reject cementite at the austenite grain boundaries, and, by polishing and etching, the grains will be clearly seen under a microscope. There are several ways to report the grain size observed under the microscope, the one used most extensively being the A.S.T.M. index numbers. The numbers are based on the formula: number of grains per square inch at $100x = 2^{N-1}$, in which N is the grain-size index. The usual range in steels will be from 1 to 128 grains per sq in. at $100x$, and the corresponding A.S.T.M. numbers will be 1 to 8. Steels having an A.S.T.M. grain size of 1 to 4 are usually considered coarse-grained, and those from 5 to 8 are fine-grained steels. It should be noted that the McQuaid-Ehn test will give only the grain size developed in the steels when heated to one temperature for a given length of time. To determine fully the grain-size characteristics of a steel, tests should be made over a range of temperatures, but the McQuaid-Ehn test has proved of great help to both producers and consumers of steel since the test is inexpensive and reproducible. For further information on grain size, reference should be made to the A.S.M. Metals Handbook, 1939.

Effect of Alloying Elements on the Properties of Steel

When relatively large amounts of alloying elements are added to steel, the characteristic behavior of carbon steels is obliterated. Most alloy steel is medium- or high-carbon steel to which various elements have been added to modify to an appreciable extent its properties, but it still owes its distinctive characteristics to the carbon that it contains. It is not possible to give an average value of the percentage of alloy element required for a given purpose, since they vary in the intensity of their effect, but the range would be from a few hundredths of a percent to possibly as high as 5 percent.

When ready for service, these steels will usually contain only two constituents, ferrite and carbide. The only way that an alloying element can affect the properties of the steel is to change the dispersion of carbide in the ferrite, change the properties of the ferrite, or change the properties of the carbide. The effect on the distribution of carbide is the most important factor, since in sections amenable to close control of structure, carbon steel is only moderately inferior to alloy steel. However, in large sections where carbon steels will fail to harden throughout the section even on a water quench, the hardenability of the steel can be increased by the addition of any alloying element (except possibly cobalt). The effect of the alloying element when dissolved in the austenite is to displace the S curve (Fig. 3) to the right. This permits the hardening of a larger section of alloy steel

elements such as nickel, chromium, manganese, or molybdenum to give sufficient hardenability. In regard to the correct **heat-treatment**, it must be remembered that a carburized steel is a duplex material having a high-carbon case and a low-carbon core with a more or less gradual transition from one to the other. Two critical temperatures are thus involved in the hardening of a carburized part. The temperatures for several representative steels are given in Table 9.

Table 9. Critical Points of Carburized S.A.E. Steels, Deg F

S.A.E. No.	1015	1315	2315	2515	3115	3215	3312	3415	4115	4615	4815	5115	6115	4340
Ac_1 of case	1360	1345	1300	1250	1355	1350	1330	1330	1395	1335	1300	1410	1420	1350
Ac_3 of core	1605	1520	1440	1420	1500	1465	1435	1425	1500	1485	1440	1550	1550	1475

Annealing and Hardening of Carburized Parts. To minimize distortion, a part should be annealed at a temperature 50 F above the carburizing temperature and the rate of cooling from the annealing temperature should be regulated to provide the required structure for best machinability. For **hardening**, there are three treatments in general use. First, a direct quench from the carburizing temperature into a suitable quenching medium. This method was not used until recently except with parts of regular shape because of the danger of cracking and distortion. With the advent of steels *that maintain a fine grain size at carburizing temperatures, distortion is* found to be less by direct quenching than by any other method. A second treatment is to cool slowly from the carburizing temperature, reheat to above the critical of the case, and quench. This facilitates handling of large quantities of carburized parts delivered discontinuously from a batch-type furnace and minimizes the retention of austenite in the high-carbon case. The old "double-quench" method in which the part was slowly cooled from the carburizing temperature, reheated and quenched from above the core critical temperature, then reheated and quenched from above the critical of the case is no longer favored.

Cyaniding. Where it is necessary to have only a superficial hard wear-resisting surface, a rapid method of casehardening is to use a molten cyanide bath. The bath usually consists of sodium cyanide with sodium chloride and sodium carbonate to retard the decomposition of the cyanide. The cyaniding should be carried out at a temperature just above the critical of the core, and the steel should be quenched directly from the cyanide bath. A uniform case depth of around 0.010 in. is obtained in 1 hr in both carbon and low-alloy steels, and besides carbon the case will contain very hard iron nitrides which increase the wear resistance of the surface of the steel part. Cyanide baths are frequently used simply as a heating medium in connection with the hardening of steels to prevent surface decarburisation and produce work with a clean surface.

Nitriding. The introduction of nitrogen into the outer surface of steel parts in order to give an extremely hard, wear-resisting case is called nitriding. The treatment consists in heating steel to a temperature of 950 to 1000 F inside a chamber through which a controlled stream of ammonia gas is passed. The treatment usually lasts 50 to 90 hr depending upon the composition of the steel and the depth to which it is desired to effect nitrification. The nitriding temperature is below the thermal critical range and even below

will have some aluminum added during deoxidation. The presence of finely scattered carbides in the austenite appears to have a similar effect on the austenite grain size, so the elements forming stable carbides will also contribute to the formation of a fine-grained austenite.

In Table 5, a summary of the effects of various alloying elements is given. It must be remembered that this table indicates only the trends of the various elements, and the fact that one element has an important influence on one factor does not prevent it from having an equally strong influence on another one.

Principles of Heat-treatment of Iron and Steel

When heat-treating a steel for a given part, certain precautions have to be taken to develop optimum mechanical properties in the steel. Some of the major factors that have to be taken into consideration are outlined below. For detailed information, reference should be made to the A.S.M. Metals Handbook.

Heating. The first step in the heat-treatment of steel is the heating of the material to above the critical to make it fully austenitic. The **heating rate** should be sufficiently slow to avoid injury to the material through excessive thermal and transformational stresses. In general, hardened steel should be heated more slowly and uniformly than is necessary for soft stress-free materials. Large sections should not be placed in a hot furnace, the allowable size depending upon the carbon and alloy content. For high-carbon steels, care should be taken in heating sections as small as 2 in. diam, and in medium-carbon steels precautions are required for size over 6 in. diam. The **maximum temperature** selected will be determined by the chemical composition of the steel and its grain-size characteristics. In hypoeutectoid steel (below 0.80 percent carbon), a temperature just above the upper critical range is used, and in hypereutectoid steels (above 0.80 percent carbon), a temperature between the lower and the upper critical is generally used so as to avoid heating to high temperatures with consequent grain growth. See Table 6 for heat-treating temperatures for plain carbon steels and Table 7 for approximate critical temperatures of S.A.E. steels. The **time at maximum temperature** should be such that a uniform temperature is obtained throughout the cross section of the steel. Care should be taken to avoid undue length of time at temperature since this will result in undesirable grain growth, scaling, or decarburization of the surface. A practical figure often given for the total time in the hot furnace is ½ hr per in. of cross-sectional thickness. When the steel has attained a uniform temperature, the **cooling rate** must be such as to develop the desired structure: slow cooling rates (furnace or air cooling) to develop the softer pearlitic structures and high

Table 6. Temperatures for Heat-treatments for Carbon Steels
(A.S.M. Metals Handbook, 1939)

S.A.E. No.	Normalize, deg F	Anneal, deg F	Quench, deg F	S.A.E. No.	Normalize, deg F	Anneal, deg F	Quench, deg F
1010	1650–1800	1600–1700	1650–1700	1050	1550–1625	1550–1600	1450–1525
1020	1650–1750	1600–1700	1575–1675	1060	1525–1600	1500–1575	1425–1550
1030	1600–1675	1575–1650	1550–1625	1070	1500–1575	1475–1550	1425–1550
1035	1575–1650	1575–1625	1525–1600	1080	1500–1575	1475–1550	1400–1525
1040	1575–1650	1550–1600	1500–1575	1095	1500–1575	1475–1550	1400–1525
1045	1550–1650	1550–1600	1475–1550				

blocks surrounding the bearing. The extent of the heated zone can be so closely controlled that the fillets will remain soft while the bearing surface is hardened, thereby reducing the possibility of fatigue failure without sacrificing wear resistance at the bearing surface.

Commercial Steels

The variety of applications of steel for engineering purposes is due to the wide range of physical properties obtainable by changes in carbon content and heat-treatment. For the A.S.T.M. specifications, see p. 577. Applications of carbon steels are given in Table 11. Carbon steels can be subdivided

Table 11. Applications of Carbon Steels

Percent C	Uses
0.05–0.10	Sheet, strip, tubing, wire, nails
0.10–0.20	Rivets, screws, parts to be casehardened
0.20–0.35	Structural steel, plate, forgings such as camshafts, etc.
0.35–0.45	Machinery steel—shafts, axles, connecting rods, etc.
0.45–0.55	Large forgings—crankshafts, heavy-duty gears, etc.
0.60–0.70	Bolt-heading and drop-forging dies, rails, set screws
0.70–0.80	Shear blades, cold chisels, hammers, pickaxes, band saws
0.80–0.90	Cutting and blanking punches and dies, rockdrills, hand chisels
0.90–1.00	Springs, reamers, broaches, small punches and dies
1.00–1.10	Small springs, lathe, planer, shaper and slotter tools
1.10–1.20	Twist drills, small taps, threading dies, cutlery, small lathe tools
1.20–1.30	Files, ball races, mandrels, drawing dies, razors

roughly into three groups: (1) low-carbon steel, 0.05 to 0.25 percent carbon, where only moderate strength is required together with considerable plasticity; (2) machinery steels, 0.30 to 0.55 percent carbon, which can be heat-treated to develop high strength; and (3) tool steels containing from 0.60 to 1.30 percent carbon. This last range also includes rail and spring steels.

Low-carbon Steels. Of the many low-carbon-steel products, sheet and strip steels are becoming increasingly important. The consumption of steel in the sheet and tin-plate industry in 1937 accounted for approximately 40 percent of the total steel production in the United States, and recent trends would indicate even higher consumption in the future. This increased consumption has been made possible by the development of the continuous sheet and strip rolling mills. Applications in which large quantities of sheet are employed are tin plate for food containers, black, galvanized, and terne-coated sheets for building purposes, and high-quality sheets for automobiles, furniture, refrigerators, and countless other stamped, formed, and welded products. The difference between sheet and strip is based on width and is arbitrary. The difference between hot- and cold-rolled products is that in hot rolling the steel is heated before the final rolling and in cold rolling it is not. Cold working produces a better surface finish, increases the mechanical properties, and permits the rolling of thinner gage material than hot rolling. Approximate mechanical properties of cold-rolled strip steel are given in Table 12.

Sheets for deep-drawing applications must be dead soft so as to have a maximum amount of plasticity. They must also have a relatively fine grain size, since a large grain size will cause a rough finish, an "orange peel" effect, on the deep-drawn article. It is necessary to eliminate the sharp yield point that is characteristic of low-carbon steel so as to prevent sudden local elongations in the sheet during forming which result in strain markings

Certain oils are satisfactory but are incapable of hardening shallow-hardening steels of substantial size. A change in steel composition is required with a change from water to an oil quench. Quenching in oil does not entirely prevent distortion. When the degree of distortion produced by oil quenching is objectionable, recourse is taken to air hardening. The cooling rate in air is very much slower than in oil or water, so an exceptionally high alloy content is required. This means that a high price is paid for the advantage gained, both in terms of metal cost and loss in machinability, though it may be well justified when applied to expensive tools. In this case, danger of cracking is negligible.

Liquids for Quenching Shallow-hardening Steels. Shallow-hardening steels require extremely rapid surface cooling in the quench particularly in the temperature range around 1020 F. A submerged water spray will give the fastest and most reproducible quench practicable. Such a quench is limited in application to simple short objects which are not likely to warp. Because of difficulty in obtaining symmetrical flow of the water relative to the work, the spray quench is conducive to warping. The ideal practical quench is one that will give the required surface cooling without agitation of the bath. The addition of ordinary salt, sodium chloride, greatly improves the performance of water in this respect, the best concentration being around 10 percent. Most inorganic salts are effective in suppressing the formation of vapor at the surface of the steel and thus aid in cooling steel uniformly and eliminating the formation of soft spots. To minimize the formation of vapor, water-base quenching liquids must be kept cold, preferably under 70 F. The addition of some other soluble materials to water such as soap is extremely detrimental because of increased formation of vapor.

Liquids for Quenching Deep-hardening Steels. When oil quenching is required, use a steel of sufficient alloy content to produce a completely martensitic structure at the surface over the heaviest section of the work. To minimize the possibility of cracking, especially when hardening tool steels, keep the quenching oil warm, preferably between 100 and 150 F. If this expedient is insufficient to prevent cracking, the work may be removed just before the start of the hardening transformation and cooled in air. Whether or not transformation has started can be determined with a permanent magnet, the work being completely non-magnetic before transformation if completely hardened by the quench.

The cooling characteristics of quenching oils are difficult to evaluate and have not been satisfactorily correlated with the physical properties of the oils as determined by the usual tests. The standard tests are important with regard to secondary requirements of quenching oils. Low viscosity assures free draining of oil from the work and therefore low oil loss. A high flash and fire point assures a high boiling point and reduces the fire hazard which is increased by keeping the oil warm. A low carbon residue indicates stability of properties with continued use and little sludging. The steam-emulsion number should be low to assure low water content, water being objectionable because of its vapor film forming tendency and high cooling power. A low saponification number assures that the oil is of mineral base and not subject to organic deterioration of fatty oils which give rise to offensive odors. Viscosity index is a valuable property for maintenance of composition.

Effect of the Condition of Surface. The factors that affect the depth of hardening are the hardenability of the steel, the size of specimen, the

Table 13. Characteristics of Low-alloy High-strength Steels

Company	Trade name	Chemical composition, percent								Yield point, kips	Tensile strength, kips	Elong in 2 in., percent	Impact, ft-lb
		C	Mn	Si	Cu	Ni	Mo	P*	Cr				
Steel Corp..........	Cor-Ten	0.10 max	0.10–0.50	0.50–1.00	0.30–0.50	....	....	0.10–0.20	0.50–1.50	50 min	70 min	22.0 min	40 C
Steel Corp..........	Man-Ten	0.30 max	1.25–1.70	0.30 max	0.20 min	....	....	0.04 max	....	50 min	80 min	20.0 min	30 C
Steel Corp..........	Sil-Ten	0.40 max	0.60 min	0.20 min	0.20 opt	....	....	0.04 max	....	45 min	80–95	18.0 min	27 C
gstown Sheet & Tube.	Yoloy	0.05–0.25	0.30–0.90	0.10–0.25	0.85–1.10	1.50–2.00	....	....	....	53–69	70–95	21.0	45 C at 70 F 20 C at −100 F
blic Steel Corp.......	R.D.S.1	0.12 max	0.50–1.00	....	0.50–1.50	0.50–1.00	0.10 min	0.10 max	....	55 min	70 min	25.0	
blic Steel Corp	R.D.S. 1A	0.30 max	0.50–1.00	....	0.50–1.50	0.50–1.00	0.10 min	0.04 max	....	70 min	90 min	15.0	
l Steel Co...........	Hi-steel	0.12 max	0.50–0.70	0.30 max	0.90–1.25	0.45–0.65	....	0.10–0.15	....	55–60	75 min	20.0 min	65 I at 73 F 48 I at −40 F
Rolling Mill Co	H.T.-50	0.12 max	0.20 min	0.10 max.	0.35 min	0.50 min	0.05 min	0.05–0.15	....	50 min	65–75	25.0–28.0	130 I
& Laughlin Steel Corp	Jal-Ten	0.35 max	1.25–1.75	0.30 max	0.40 max	....	....	0.04 max	....	50 min	80 min	20.0 min	40 I
nental Steel Corp.....	Konik	....	....	....	0.10–0.30	0.30–0.50	....	....	0.07–0.30				
hem Steel Co........	Mayari-R	0.14 max	0.50–1.00	0.05–0.50	0.50–0.70	0.25–0.75	....	0.04–0.12	0.20–1.00	50 min	70 min		75 I
Wood Steel Co.......	A. W. Dyn-El	0.11–0.14	0.50–0.80	....	0.30–0.50	....	....	0.06–0.10	....	52–60	68–78	25.0 min	
Lakes Steel Corp....	Great Lakes	Not announced								49–67	78–88	32.0–50.0	59 C

* P and S average 0.035 to 0.055 unless otherwise stated.
I = Izod. C = Charpy.

Casehardening. The production of articles having a soft ductile interior and a very hard surface can be accomplished by carburizing a low-carbon steel at an elevated temperature and then quenching. The process of carburizing followed by hardening is known as casehardening.

Carburizing methods can be divided into three groups: pack carburizing where a solid carburizing agent is used, gas carburizing, and liquid carburizing. In **pack carburizing**, the usual compound contains 20 percent metallic carbonates (mostly barium carbonate) bound to a hardwood charcoal by the use of oil, tar, or molasses. Sometimes up to 20 percent coke is added in order to increase the rate of heat transfer. Before using again, used compound should have some new compound added to make up for loss of carbonates, powdering of the compound, and burning of the charcoal and coke. One part new compound is often added to 3 parts used compound, but with care in handling this can be cut down to 1 part new to 5 parts old.

Carburizing boxes are usually made from a high-nickel-chromium heat-resisting alloy. Cast-steel or welded-steel plate boxes may be substituted when the boxes are used infrequently or when the work being carburized is large and nonsymmetrical. Work should be packed with its longest dimension vertical, and the minimum amount of compound should be used so as to decrease the heating time. Normal **carburizing temperatures** are around 1700 F, but for nickel steels a temperature of 1650 F is used and for shallow-case depth, temperatures as low as 1550 F. It is difficult to prevent variations of 0.010 in. in case depth, and therefore it is standard practice to specify case depths of at least 0.025 in. when pack carburizing. By using a minimum of carburizing compound, the over-all carburizing time for a case of 0.040 to 0.050 in. will be about 9 hr. The pack method of carburizing is adaptable to both batch and continuous furnace operation; warping is held to a minimum. The main disadvantages are the time consumed in heating the charge and the high labor cost of packing and unpacking.

Gas carburizing is being developed and may eventually replace other methods. Low labor costs and automatic quenching reduce the operating costs to a low figure, but the high initial investment is justified only by large units operating at high output. The carburizing gases used are carbon monoxide, methane, ethane, and propane. The hydrocarbons will break down, if undiluted, liberating an excessive amount of carbon in the form of soot on all exposed surfaces. By diluting with gases such as nitrogen and hydrogen, the amount of free carbon deposit will be decreased. Cases of 0.040 to 0.050 in. can be obtained in 4 hr at 1700 F. By suitable adjustment of time, temperature, and gas composition, the surface carbon content and the carbon gradient can be varied to meet almost any requirement.

Liquid carburizing is done in activated baths of calcium cyanamid, sodium or potassium cyanide, and controlling chemicals which govern the decomposition of the cyanides. The baths are operated at between 1500 and 1650 F, and cases of 0.020 in. are obtained in 90 min. For case depths of more than about 0.030 in. it is more economical to use either pack or gas carburizing. The process is extremely flexible, easily controlled, and particularly well adapted to small units. With continuous operation and automatic quenching, labor costs are low.

The type of **steel for carburizing** will depend upon section and distortion limits of the finished part and the stresses to which it will be subjected. Plain carbon steels of between 0.15 and 0.25 percent carbon are used extensively and after heat-treatment will develop core strengths as high as 100,000 lb per sq in. To develop higher core strengths, it is necessary to have alloying

manganese sulphide inclusions causing the chips to break short on machining. Manganese and phosphorus harden and embrittle the steel which also contributes toward free machining. The familiar steels S.A.E. 1112 and X1112 known as **Bessemer screw stock** have excellent machinability. Steels having a small percentage of lead (about 0.25) have good machinability. These **leaded steels** increase the machinability of free-cutting steels more

Table 14. Chemical Compositions of S.A.E. Steels
(S.A.E. Handbook, 1939)
CARBON STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus max	Sulphur max
1010	0.05–0.15	0.30–0.60	0.045	0.055
1015	0.10–0.20	0.30–0.60	0.045	0.055
X1015	0.10–0.20	0.70–1.00	0.045	0.055
1020	0.15–0.25	0.30–0.60	0.045	0.055
X1020	0.15–0.25	0.70–1.00	0.045	0.055
1025	0.20–0.30	0.30–0.60	0.045	0.055
X1025	0.20–0.30	0.70–1.00	0.045	0.055
1030	0.25–0.35	0.60–0.90	0.045	0.055
1035	0.30–0.40	0.60–0.90	0.045	0.055
1040	0.35–0.45	0.60–0.90	0.045	0.055
X1040	0.35–0.45	0.40–0.70	0.045	0.055
1045	0.40–0.50	0.60–0.90	0.045	0.055
X1045	0.40–0.50	0.40–0.70	0.045	0.055
1050	0.45–0.55	0.60–0.90	0.045	0.055
X1050	0.45–0.55	0.40–0.70	0.045	0.055
1055	0.50–0.60	0.60–0.90	0.040	0.055
X1055	0.50–0.60	0.90–1.20	0.040	0.055
1060	0.55–0.70	0.60–0.90	0.040	0.055
1065	0.60–0.75	0.60–0.90	0.040	0.055
X1065	0.60–0.75	0.90–1.20	0.040	0.055
1070	0.65–0.80	0.60–0.90	0.040	0.055
1075	0.70–0.85	0.60–0.90	0.040	0.055
1080	0.75–0.90	0.60–0.90	0.040	0.055
1085	0.80–0.95	0.60–0.90	0.040	0.055
1090	0.85–1.00	0.60–0.90	0.040	0.055
1095	0.90–1.05	0.25–0.50	0.040	0.055

FREE-CUTTING STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus range	Sulphur range
1112	0.08–0.16	0.60–0.90	0.09–0.13	0.10 –0.20
X1112	0.08–0.16	0.60–0.90	0.09–0.13	0.20 –0.30
1115	0.10–0.20	0.70–1.00	0.045 max	0.075–0.15
1120	0.15–0.25	0.60–0.90	0.045 max	0.075–0.15
X1314	0.10–0.20	1.00–1.30	0.045 max	0.075–0.15
X1315	0.10–0.20	1.30–1.60	0.045 max	0.075–0.15
X1330	0.25–0.35	1.35–1.65	0.045 max	0.075–0.15
X1335	0.30–0.40	1.35–1.65	0.045 max	0.075–0.15
X1340	0.35–0.45	1.35–1.65	0.045 max	0.075–0.15

the usual tempering temperature so that the mechanical properties of the previously heat-treated core metal are not affected by nitriding. The big advantage of nitriding is that hardening is accomplished without a quenching operation, so that complicated shapes and articles of uneven cross section can be treated with safety. The steel used is usually heat-treated before machining, rough-machined, given a heat-treatment at the nitriding temperature without ammonia in order to produce whatever slight distortion is likely to occur, then the part is finish machined and nitrided. Besides wear resistance, nitriding aids in the retention of hardness at elevated temperatures and to some extent increases the resistance to corrosion. The disadvantages of nitriding include high cost, time of operation, and expert attention.

Stock for Nitriding. A series of steel alloys containing aluminum, known as **Nitralloy**, give the best results in the nitriding process. The compositions of the most commonly used nitriding steels are given in Table 10 (A.S.M. Metals Handbook, 1939).

Table 10. Compositions of Nitriding Steels (Nitralloy), Percent

Element	N 125	N 125N	N 135	N 135 modified	N 230
Carbon..............	0.20-0.30	0.20-0.27	0.30-0.40	0.38-0.45	0.25-0.35
Manganese...........	0.40-0.60	0.40-0.70	0.40-0.60	0.40-0.70	0.40-0.60
Silicon...............	0.20-0.30	....	0.20-0.30	....	0.20-0.30
Aluminum...........	0.90-1.40	1.10-1.40	0.90-1.40	0.95-1.35	1.00-1.50
Chromium...........	0.90-1.40	1.00-1.30	0.90-1.40	1.40-1.80	
Molybdenum.........	0.15-0.25	0.20-0.30	0.15-0.25	0.30-0.45	0.60-1.00
Nickel................	....	3.25-3.75			

Flame hardening is the local heating of steel above the critical temperature so that on subsequent quenching a hardened layer will be produced. The depth of the flame-hardened layer will vary from $\frac{1}{16}$ to about $\frac{1}{4}$ in. depending upon the service requirements. For local hardening or for hardening the surface of large steel parts, this method has been very useful, especially since distortion of the part is kept to a minimum. The chemical analysis of the steel has to be such as to respond readily to heat-treatment. For plain carbon steels, the carbon content should be between 0.35 to 0.70 percent, although steels with higher carbon content may be flame hardened if care is taken to prevent surface cracking. To obtain satisfactory results by flame hardening, the character of the flame, its distance from the surface of the work, its speed of travel, and the timing of the quench must all be under perfect control. After quenching, tempering is essential to relieve the stresses, a temperature of 400 F usually being sufficient. Flame hardening may be adapted to castings, forgings, or rolled sections irrespective of size. Typical applications are for the hardening of gear teeth, cams, wheel treads, rail ends, and many machine parts.

Local hardening can also be accomplished by local heating with electricity. Resistance heating is useful in hardening local sections of some forgings and castings, but in general its principal application is for heating parts having a uniform cross section. Induction heating is well adapted for the surface hardening of cylindrical parts. Crankshaft bearings are hardened by the patented "Tocco" process by applying a high-frequency current to the bearing section for a few seconds. When the steel is heated to the desired depth, water is sprayed on the heated surface through holes in the inductor

Table 14. Chemical Compositions of S.A.E. Steels.—(*Continued*)
(S.A.E. Handbook, 1939)

MANGANESE STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur, max
T1330	0.25-0.35	1.60-1.90	0.040	0.050
T1335	0.30-0.40	1.60-1.90	0.040	0.050
T1340	0.35-0.45	1.60-1.90	0.040	0.050
T1345	0.40-0.50	1.60-1.90	0.040	0.050
T1350	0.45-0.55	1.60-1.90	0.040	0.050

NICKEL STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur, max	Nickel range
2015	0.10-0.20	0.30-0.60	0.040	0.050	0.40-0.60
2115	0.10-0.20	0.30-0.60	0.040	0.050	1.25-1.75
2315	0.10-0.20	0.30-0.60	0.040	0.050	3.25-3.75
2320	0.15-0.25	0.30-0.60	0.040	0.050	3.25-3.75
2330	0.25-0.35	0.50-0.80	0.040	0.050	3.25-3.75
2335	0.30-0.40	0.50-0.80	0.040	0.050	3.25-3.75
2340	0.35-0.45	0.60-0.90	0.040	0.050	3.25-3.75
2345	0.40-0.50	0.60-0.90	0.040	0.050	3.25-3.75
2350	0.45-0.55	0.60-0.90	0.040	0.050	3.25-3.75
2515	0.10-0.20	0.30-0.60	0.040	0.050	4.75-5.25

NICKEL-CHROMIUM STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur, max	Nickel range	Chromium range
3115	0.10-0.20	0.30-0.60	0.040	0.050	1.00-1.50	0.45-0.75
3120	0.15-0.25	0.30-0.60	0.040	0.050	1.00-1.50	0.45-0.75
3125	0.20-0.30	0.50-0.80	0.040	0.050	1.00-1.50	0.45-0.75
3130	0.25-0.35	0.50-0.80	0.040	0.050	1.00-1.50	0.45-0.75
3135	0.30-0.40	0.50-0.80	0.040	0.050	1.00-1.50	0.45-0.75
3140	0.35-0.45	0.60-0.90	0.040	0.050	1.00-1.50	0.45-0.75
X3140	0.35-0.45	0.60-0.90	0.040	0.050	1.00-1.50	0.60-0.90
3145	0.40-0.50	0.60-0.90	0.040	0.050	1.00-1.50	0.45-0.75
3150	0.45-0.55	0.60-0.90	0.040	0.050	1.00-1.50	0.45-0.75
3215	0.10-0.20	0.30-0.60	0.040	0.050	1.50-2.00	0.90-1.25
3220	0.15-0.25	0.30-0.60	0.040	0.050	1.50-2.00	0.90-1.25
3230	0.25-0.35	0.30-0.60	0.040	0.050	1.50-2.00	0.90-1.25
3240	0.35-0.45	0.30-0.60	0.040	0.050	1.50-2.00	0.90-1.25
3245	0.40-0.50	0.30-0.60	0.040	0.050	1.50-2.00	0.90-1.25
3250	0.45-0.55	0.30-0.60	0.040	0.050	1.50-2.00	0.90-1.25
3312	max. 0.17	0.30-0.60	0.040	0.050	3.25-3.75	1.25-1.75
3325	0.20-0.30	0.30-0.60	0.040	0.050	3.25-3.75	1.25-1.75
3335	0.30-0.40	0.30-0.60	0.040	0.050	3.25-3.75	1.25-1.75
3340	0.35-0.45	0.30-0.60	0.040	0.050	3.25-3.75	1.25-1.75
3415	0.10-0.20	0.30-0.60	0.040	0.050	2.75-3.25	0.60-0.95
3435	0.30-0.40	0.30-0.60	0.040	0.050	2.75-3.25	0.60-0.95
3450	0.45-0.55	0.30-0.60	0.040	0.050	2.75-3.25	0.60-0.95

called **stretcher strains** or **Lüders' lines.** The sharp yield point can be eliminated by cold working (Fig. 1), a reduction of only 1 percent in thickness usually being sufficient. This cold reduction is usually done by cold rolling, known as **temper rolling,** followed by alternate bending and reverse bending in a roller leveler. Temper rolling must always precede roller leveling because soft-annealed sheets will "break" (yield locally) in the roller leveler. An important phenomenon in these temper-rolled low-carbon sheets is the return of the sharp yield point after a period of time. This is known as **aging** in steel. The return of the yield point is accompanied by an increase in hardness and a loss in ductility. Greater reductions by temper rolling decrease the rate of the aging process, and an increase in temperature greatly increases its rate. It is therefore necessary to fabricate the sheets soon after temper rolling in order to avoid stretcher strains. Recently, non-aging sheets have been developed by using killed steels, but these will probably be used only in special applications because of the added cost.

Table 12. Approximate Physical Properties for Various Tempers of Cold-rolled Strip Steel

(A.S.T.M. Specifications for Cold-rolled Strip: A109-38)

Grade or temper	Rockwell hardness B scale, 1/16 in. ball 100 kg load	Depth of cup for 0.050 in. thickness of strip, mm[a]	Tensile strength,[b] kips	Elong in 2 in. for 0.050 in., thickness of strip,[c] percent	Remarks
No. 1 hard	90 ± 6[d]	6-7	80 ± 12	3 ± 2	[e]
No. 2 half hard	80 ± 5	7-8	64 ± 8	9 ± 5	[f]
No. 3 quarter hard	69 ± 5	8-9	54 ± 6	20 ± 7	[g]
No. 4 soft or planished	58 ± 6	9-10½	48 ± 5	30 ± 6	[h]
No. 5 dead soft	45 ± 7	10-10½ 10-11½	44 ± 4	39 ± 6	[i]

[a] Cup depth varies with thickness of strip. For grade 5, dead soft temper, the depth is given approximately by the formula $D = 10.5 \text{ mm} + 6.4 \log t$ (t = thickness in mm). Other tempers vary in a similar way.

[b] Tensile properties are based on the standard tension test specimen for sheet metals, Fig. 7, A.S.T.M. E 8, 1939, Standards, Part I, p. 756.

[c] Elongation in 2 in. varies with the thickness of strip. For grade 5, dead soft temper, the percentage elongation $= 41 + 10 \log t$ (t = thickness in mm). Other tempers vary in a similar way.

[d] For cold strip 0.069 in. and thinner in thickness the Rockwell B hardness range is 96 ± 6, with corresponding increase in tensile strength and drop in depth of cup test.

[e] Intended for flat blanking only.

[f] Intended for easy bending up to 90 deg across the grain. (No bending along the grain.)

[g] For shallow drawing and stamping, where a very smooth surface is required. Bends 180 deg across the grain. Bends up to 90 deg along the grain.

[h] For fairly deep drawing where no sign of surface strain is permissible. Bends 180 deg both ways of the grain.

[i] For deep drawing where slight stretcher strains are permissible. Also for drifting. (Erroneously called "extrusion.") Bends 180 deg both ways of the grain.

Other important low-carbon steel products are plates, structural shapes, tubing, pipe, and wire. Boiler plate, rivets, and tubes are usually in the

Table 15. Mechanical Properties of Certain S.A.E. Steels with Various Heat-treatments
(Sections up to $1\frac{1}{2}$ in. diam. or thickness)

Draw temp, deg F	Tensile strength, kips	Yield point, kips	Reduction of area, percent	Elong in 2 in., percent	Brinell hardness	Tensile strength, kips	Yield point, kips	Reduction of area, percent	Elong in 2 in., percent	Brinell hardness
	S.A.E. 1035 quenched in water at 1525–1575 F					S.A.E. 1045 quenched in water at 1475–1525 F				
800	106	75	52	23	212	121	91	46.5	16	248
900	98	70	55	25	202	115	84	48.5	20	235
1000	95	67.5	56.5	27.5	192	111	77	51	22.5	229
1100	93	63	60	28.5	187	107.5	71	52.5	25	217
1200	89.5	57	64	30	183	103	66	55	27.5	212
1300	85	49	68	33	174	97.5	60	56	28	197
	S.A.E. T1330 normalized at 1650–1750 F, quenched in water at 1525–1575 F					S.A.E. 2340 normalized at 1625–1725 F, quenched in oil at 1425–1475 F				
800	161.5	137.5	42.5	13	331	165	148	46.5	14	331
900	140	117.5	48	16	285	148	127.5	50	16	302
1000	122	100	52	19	248	130	108	54	20	248
1100	108	86	57	21.5	217	115	90	58	25	229
1200	98.5	73	59.5	23.5	201	102.5	75	60	27	217
1300	92	68	61.5	25.5	187	93	63	61	28	183
	S.A.E. 3140 normalized at 1600–1700 F, quenched in oil at 1475–1525 F					S.A.E. 3240 normalized at 1625–1725 F quenched in oil at 1475–1525 F				
800	175	151	45	12	331	202	180	44	16	388
900	157	132.5	50	16	302	177.5	152.5	50	17.5	341
1000	148	110	55	18	269	155	132.5	56	19	321
1100	122.5	100	58.5	19	248	137.5	116	60	21	285
1200	111	85	62	20	223	122.5	100	62.5	22	255
1300	102.5	70	85	20	217	110	87.5	65	23	229
	S.A.E. 3335 normalized at 1600–1700 F, quenched in oil at 1425–1475 F					S.A.E. 3435 normalized at 1550–1650 F, quenched in oil at 1425–1475 F				
800	188	168.5	50	12.5	375	188	162.5	55	15	341
900	167	146	56	17.5	331	166	144	57.5	16.5	311
1000	148	127.5	59.5	20	302	157	127	60	18.5	285
1100	132	112	61.5	21	277	130	110	62.5	20	262
1200	118.5	100	62.5	22	241	119.5	97.5	65	22	229
	S.A.E. 4140 normalized at 1650–1750 F, quenched in oil at 1525–1625 F					S.A.E. 4340 normalized at 1625–1725 F, quenched in oil at 1475–1550 F				
800	180	156	36	10	363	210	190	40	11	415
900	170	146	40	11	352	195	175	42	11	401
1000	157.5	132	45	12	321	178	157.5	45	12	363
1100	142	118	51	14	293	160	138	48.5	15	331
1200	126.5	101	56	17.5	255	140	119	52.5	18	285
1300	110	86	59	21	223	120	95	60	20	241

range of 0.15 to 0.30 percent carbon. Structural steel has approximately the same carbon range, although some higher carbon steel shapes are sometimes used in bridges and heavy machinery. A special group of low-alloy high-strength steels has recently been developed for the transportation industries where a high ratio of strength to weight is required. Although these steels have a lower strength-weight ratio than high-strength stainless steels and light aluminum alloys, they are used to a large extent in the construction of passenger, freight, and tank cars, truck and trailer bodies, unloading buckets, and similar applications. In general, the low-alloy high-strength steels have a yield point of 50,000 and a tensile strength of 90,000 lb per sq in. and the ductility will be only slightly lower than that of mild steel. These steels can be formed either hot or cold, welded, flame cut, punched, reamed, and machined. In welding or flame cutting, precautions must usually be taken against hardening with resultant loss in ductility. These steels are definitely more difficult to machine than structural carbon steels, higher tool pressures being necessary so that tool speeds should be reduced and a cooling agent used. For detailed information on these steels, see *Metals and Alloys*, **9**, 1938, p. 243. Table 13, taken from this article, lists the trade names and manufacturing companies and gives approximate compositions and mechanical properties of the different steels.

Machinery Steels. The S.A.E. has specifications (S.A.E. Handbook) for steels that cover the requirements in the automotive industry, and these specifications have been widely adopted by industry in nearly all fields of mechanical manufacturing both in America and abroad. A numerical index, which in general consists of four digits, is used to identify the composition of the S.A.E. steels. The first digit indicates the alloy type, and, in the simple alloy steels, the second digit indicates the approximate percentage of the predominant alloying element. The last two digits indicate the average carbon content in "points," or hundredths of 1 percent. Thus 2340 indicates a nickel steel of approximately 3 percent nickel (3.25 to 3.75) and 0.40 percent carbon (0.35 to 0.45); and 71360 indicates a tungsten steel of about 13 percent tungsten (12 to 13) and 0.60 percent carbon (0.50 to 0.70). In some instances, it has been found necessary to depart from this system in order to avoid confusion. The prefix X is used in numerous instances to denote variations in the range of elements. The prefix T is used with the manganese steels (1300 series) to avoid confusion with steels of somewhat different manganese range that have been identified by the same numericals but without the prefix.

Specific applications of S.A.E. steels cannot be given, since the selection of a proper steel for a given part must depend upon an intimate knowledge of factors such as the availability and cost of the material, the detailed design of the part, and the severity of the service to be imposed, machinability, size, etc. To a large extent, the mechanical properties desired in the part to be heat-treated will determine the carbon and alloy content of the steel. Table 15 gives a résumé of mechanical properties of typical S.A.E. steels as given in the S.A.E. Handbook, 1939 (for sizes up to 1½ in. diam). Table 16 gives the effect of mass on the mechanical properties of heat-treated steels.

The low-carbon S.A.E. steels are used for carburized parts (see p. 553), cold-headed bolts and rivets, and for similar applications where high quality is required. An important series of steels is the S.A.E. low-carbon free-cutting steels for high-speed screw-machine stock and other machining purposes. These steels have high sulphur present in the steel in the form of

Table 16. Effect of Mass of Specimen on the Mechanical Properties of Some S.A.E. Steels

Diam of section, in.	Tensile strength, kips	Yield point, kips	Reduction of area, percent	Elong in 2 in., percent	Brinell hardness	Tensile strength, kips	Yield point, kips	Reduction of area, percent	Elong in 2 in., percent	Brinell hardness
S.A.E. 1045 quenched in water at 1500 F, drawn at 1000 F						S.A.E. 2340 normalized at 1625–1675 F, quenched in oil at 1450 F, drawn at 1000 F				
1	113	81	52	20	232	140	116	58	18	284
2	104	70	50	26	218	127	104	54	19	260
3	97	63	49	26	204	118	92	53	20	239
4	94	59.5	48.5	26	192	113	84	52.5	20.5	230
5	93	58	48	26	188	110	80	52	21	223
S.A.E. 3140 normalized at 1600–1700 F, quenched in oil at 1500 F, drawn at 1000 F						S.A.E. 3240 normalised at 1625–1675 F, quenched in oil at 1500 F, drawn at 1000 F				
1	145	122	60	15	277	161	142	57	18	332
2	135	109	53	17	266	145	122	54	19	302
3	128	98	49	18	263	133	107	53	19.5	274
4	123	90	48	18	260	125	95	52	20	250
5	122	88	48	18	257	120	87	52	20	239
S.A.E. 3340 normalized at 1600–1700 F, quenched in oil at 1450 F, drawn at 1000 F						S.A.E. 3435 normalized at 1600–1700 F, quenched in oil at 1460 F, drawn at 1000 F				
1	165	148	52	17	316	154	136	56	17	300
2	156	138	49	18	301	142	122	54	17	
3	152	131	48	18	294	137	115	53	17	
4	149	127	47	18	291	135	110	52.5	17	
5	148	128	46	18	287	133	105	52	17	

Table 17. Average Physical Properties of Cold-drawn Steel

(A.S.M. Metals Handbook, 1939)

Sizes ⅝ to 2 in. diam, test specimens 2 × 0.505 in.

S.A.E. steels	Tensile strength, kips	Yield point, kips	Elongation in 2 in., percent	Red of area, percent	Brinell hardness
1010	67	55.0	25.0	57.0	137
1015	71	60.3	22.0	55.0	149
1020	75	63.7	20.0	52.0	156
1025	80	68.0	18.5	50.0	163
1030	87	73.9	17.5	48.0	179
1035	92	78.2	17.0	45.0	187
1040	97	82.4	16.0	40.0	197
1045	102	86.7	15.0	35.0	207
1112	87	73.9	17.0	45.0	183
1120	78	66.3	19.5	49.0	159
X1314	80	68.0	19.0	51.0	163
X1315	82.5	70.1	18.5	50.0	167
X1330	98	83.3	18.0	40.0	201
X1335	105	89.2	16.0	35.0	217
X1340	112	95.2	14.0	30.0	223

Table 14. Chemical Compositions of S.A.E. Steels.—*(Continued)*
(S.A.E. Handbook, 1939)
MOLYBDENUM STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur max	Chromium range	Nickel range	Molybdenum range
4130	0.25-0.35	0.50-0.80	0.040	0.050	0.50-0.80	....	0.15-0.25
X4130	0.25-0.35	0.40-0.60	0.040	0.050	0.80-1.10	....	0.15-0.25
4135	0.30-0.40	0.60-0.90	0.040	0.050	0.80-1.10	....	0.15-0.25
4140	0.35-0.45	0.60-0.90	0.040	0.050	0.80-1.10	....	0.15-0.25
4150	0.45-0.55	0.60-0.90	0.040	0.050	0.80-1.10	....	0.15-0.25
4320	0.15-0.25	0.40-0.70	0.040	0.050	0.30-0.60	1.65-2.00	0.20-0.30
4340	0.35-0.45	0.50-0.80	0.040	0.050	0.50-0.80	1.50-2.00	0.30-0.40
X4340	0.35-0.45	0.50-0.80	0.040	0.050	0.60-0.90	1.50-2.00	0.20-0.30
4615	0.10-0.20	0.40-0.70	0.040	0.050	....	1.65-2.00	0.20-0.30
4620	0.15-0.25	0.40-0.70	0.040	0.050	....	1.65-2.00	0.20-0.30
4640	0.35-0.45	0.50-0.80	0.040	0.050	....	1.65-2.00	0.20-0.30
4815	0.10-0.20	0.40-0.60	0.040	0.050	....	3.25-3.75	0.20-0.30
4820	0.15-0.25	0.40-0.60	0.040	0.050	....	3.25-3.75	0.20-0.30

CHROMIUM STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur, max	Chromium range
5120	0.15-0.25	0.30-0.60	0.040	0.050	0.60-0.90
5140	0.35-0.45	0.60-0.90	0.040	0.050	0.80-1.10
5150	0.45-0.55	0.60-0.90	0.040	0.050	0.80-1.10
52100	0.95-1.10	0.20-0.50	0.030	0.035	1.20-1.50

CHROMIUM-VANADIUM STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur, max	Chromium range	Vanadium	
						Min	Desired
6115	0.10-0.20	0.30-0.60	0.040	0.050	0.80-1.10	0.15	0.18
6120	0.15-0.25	0.30-0.60	0.040	0.050	0.80-1.10	0.15	0.18
6125	0.20-0.30	0.60-0.90	0.040	0.050	0.80-1.10	0.15	0.18
6130	0.25-0.35	0.60-0.90	0.040	0.050	0.80-1.10	0.15	0.18
6135	0.30-0.40	0.60-0.90	0.040	0.050	0.80-1.10	0.15	0.18
6140	0.35-0.45	0.60-0.90	0.040	0.050	0.80-1.10	0.15	0.18
6145	0.40-0.50	0.60-0.90	0.040	0.050	0.80-1.10	0.15	0.18
6150	0.45-0.55	0.60-0.90	0.040	0.050	0.80-1.10	0.15	0.18
6195	0.90-1.05	0.20-0.45	0.030	0.035	0.80-1.10	0.15	0.18

Table 18. Classification of American Tool Steels.—(*Continued*)
(Adapted from Gill, "Tool Steels," *Metal Progress*, Oct., 1938)

TUNGSTEN FINISHING TOOL STEELS AND DRAWING DIES
(Brittle but intensely hard and keen edges for cutting hard materials)
Water hardening; medium wear resistance; low toughness; high warpage; no red hardness; deep hardening

C	W	Cr	Remarks	
1.20–1.40	5.00–6.00	....	Slightly better wear resistance than lower tungsten	High movement; best for drawing dies that must be rehardened after wear
1.20–1.40	3.00–5.00	....	Slightly tougher than higher tungsten	
1.20–1.40	4.00–6.00	0.40–0.80	Chromium improves heat treatability and reduces volume change	
1.20–1.40	4.00–6.00	1.00–1.50		

SILICON-MANGANESE PUNCH AND CHISEL STEELS
(A water hardening and inexpensive steel for cold cutting)
Medium wear resistance; medium toughness; medium warpage; medium hot hardness medium deep hardening

C	Si	Mn	Cr	Mo	V	Remarks
0.50–0.60	1.80–2.20	0.60–0.90	....	....	....	Spring steel analysis; all high-silicon steels liable to soft skin
0.60–0.75	1.70–2.25	0.70–0.90	....	....	....	More carbon gives higher hardness
0.50–0.60	1.75–2.25	0.70–0.90	0.20–0.35	....	0.15–0.30	Alloys increase hardenability and refine grain
0.50–0.60	1.75–2.25	0.70–0.90	....	0.40–0.60	....	Molybdenum greatly increases hardenability
0.50–0.60	0.75–1.25	0.35–0.60	0.20–0.40	0.40–0.60	....	Low silicon reduces brittleness and wear resistance

HIGH CARBON, HIGH CHROMIUM PUNCH AND DIE STEELS
(Durable rolls, mandrels, punches, dies, and shears for cold work)
High wear resistance; low toughness; low warpage; high hot hardness; deep hardening, difficultly machinable

C	Cr	V	Mo	Co	Ni	Remarks
		Oil Hardening Types ("Non-Deforming")				
2.25–2.45	12.00–14.00	....	....	....	....	
2.10–2.20	12.00–14.00	0.75–1.00	....	....	....	Somewhat tougher
2.15–2.25	12.00–14.00	....	....	....	0.50–0.75	Slightly air hardening; most difficult to machine
2.15–2.25	12.00–13.00	0.75–1.00	....	0.60–1.00	....	Slightly red hard
		Air Hardening Types (Tougher Than Above; and Deform Less in Hardening)				
1.40–1.60	12.00–13.00	....	0.50–0.60	3.00–4.00	....	Red hard properties. Good for cutting tools on non-ferrous materials
1.50–1.70	16.50–18.00	....	....	....	....	Liable to harden non-uniformly
1.50–1.60	12.00–13.00	....	0.80–1.00	....	....	
1.50–1.60	12.00–13.00	0.80–1.00	0.75–0.90	0.40–0.60	....	Vanadium imparts greater toughness
1.40–1.55	12.00–13.00	0.80–1.00	0.75–0.90	....	0.60–0.80	Most difficult to machine

CHROMIUM DIE STEELS FOR HOT WORK
(Gripper, bending, and heading dies for light work up to 600 F.)
Air or oil hardening, medium wear resistance and toughness; low warpage; medium hot hardness; deep hardening

C	Cr	Mo	Remarks
0.85–1.00	3.75–4.00	....	Usually quenched in light air blast
0.85–1.00	3.25–3.75	....	Lower chromium reduces cracks during oil quenching
0.65–0.75	3.75–4.25	....	Oil quenching (lower carbon) but not so rigid at 500 F
0.85–1.00	3.75–4.25	0.40–0.00	Best air hardener

Table 14. Chemical Compositions of S.A.E. Steel.—(*Continued*)
(S.A.E. Handbook, 1939)
TUNGSTEN STEELS

S.A.E. No.	Carbon range	Manganese, max	Phosphorus, max	Sulphur, max	Chromium range	Tungsten range
71360	0.50-0.70	0.30	0.035	0.040	3.00-4.00	12.00-15.00
71660	0.50-0.70	0.30	0.035	0.040	3.00-4.00	15.00-18.00
7260	0.50-0.70	0.30	0.035	0.040	0.50-1.00	1.50- 2.00

SILICON-MANGANESE STEELS

S.A.E. No.	Carbon range	Manganese range	Phosphorus, max	Sulphur, max	Silicon range
9255	0.50-0.60	0.60-0.90	0.040	0.050	1.80-2.20
9260	0.55-0.65	0.60-0.90	0.040	0.050	1.80-2.20

CORROSION- AND HEAT-RESISTING ALLOYS

S.A.E. No.	Carbon, max	Manganese, max	Silicon, max	Phosphorus, max	Sulphur, max	Chromium range	Nickel range
30905	0.08	0.20-0.70	0.75	0.030	0.030	17.00-20.00	8.00-10.00
30915	0.09-0.20	0.20-0.70	0.75	0.030	0.030	17.00-20.00	8.00-10.00
51210	0.12	0.60	0.50	0.030	0.030	11.50-13.00	
X51410	0.12	0.60	0.50	0.030	0.15-0.50	13.00-15.00	
51335	0.25-0.40	0.60	0.50	0.030	0.030	12.00-14.00	
51510	0.12	0.60	0.50	0.030	0.030	14.00-16.00	
51710	0.12	0.60	0.50	0.030	0.030	16.00-18.00	

Silicon range of all S.A.E. basic open-hearth alloy steels shall be 0.15 to 0.30. For electric and acid open-hearth alloy steels, the silicon content shall be 0.15 min.

than that of other grades. Cold drawing improves the machinability of free-cutting steels; they are usually supplied cold-drawn.

Cold-finished carbon-steel bars are used for parts such as bolts, nuts, typewriter and cash-register parts, motor and transmission power shafting, piston pins, bushings, oil pump shafts and gears, etc. Average mechanical properties of cold-drawn steel are given in Table 17. Besides increasing the mechanical properties, cold-finished steel has better machining properties than hot-rolled products. The surface finish and dimensional accuracy are also greatly improved by cold finishing.

Forging steels, between 0.30 and 0.40 percent carbon, are used for axles, bolts, pins, connecting rods, and similar applications. These steels are readily forged and, after heat-treatment, develop considerably higher mechanical properties than low-carbon steels. For heavy sections, where high strength is required, such as in crankshafts and heavy duty gears, the carbon is increased to 0.40 to 0.50 percent (S.A.E. 1045).

Tool Steel. Tool steels can be subdivided into four classifications: (1) plain carbon steels, (2) oil-hardening steels, (3) semi-high-speed steels, and (4) high-speed steels.

roughing operations. Molybdenum may be substituted for part of the tungsten to reduce cost and to give increased toughness.

Very high heating temperatures are required for the heat-treatment of high-speed steel. The steel should be heated slowly to 1600 F, held until a uniform temperature is reached, then heated rapidly to 2300 F by transferring to another furnace held at the latter temperature. It is held in the furnace only for the time necessary to bring it to temperature, then is quenched in warm oil or in an air blast. The high-speed steel should then be tempered at about 1100 F to increase the toughness; owing to a secondary hardening effect, the hardness of the tempered steel may be higher than as quenched. In general, cobalt high-speed steels require higher, and molybdeaum steels lower, quenching temperatures than the 18-4-1. A list of the more common American tool steels is given in Table 18.

Spring Steel. *For small springs, steel is often supplied to spring manufacturers in a form that requires no heat-treatment except perhaps a low-temperature anneal to relieve forming strains.* Types of previously treated steel wire for small helical springs are **music wire** which has been given a special heat-treatment called patenting and then cold-drawn to develop a high yield strength, **hard-drawn wire** which is of lower quality than music wire since it is usually made of lower grade material and is seldom patented, and **oil-tempered wire** which has been quenched and tempered. The wire usually has a Brinell hardness between 352 and 415, although this will depend on the application of the spring and the severity of the forming operation. Steel for small flat springs has either been cold-rolled or quenched and tempered to a similar hardness.

Steel for both helical and flat springs which is hardened and tempered after forming is usually supplied in an annealed condition. Plain carbon steel is satisfactory for small springs; for large springs it is necessary to use alloy steels such as chrome-vanadium or silicon-manganese steel in order to obtain a uniform structure throughout the cross section. Table 19 gives the chemical composition and heat-treatment of several spring steels (see p. 549 for further information on heat-treatment). It is especially important for springs that the surface of the steel be free from all defects.

Table 19. Chemical Composition and Heat-treatment of Spring Steels

Steels	Composition, percent					Heat-treatment temperatures, deg F		
	C	Mn	Si	Cr	V	Normalizing	Quenching	Tempering
Low carbon	0.70–0.80	0.50–0.80	0.15–0.30	......	....	1575	1450	700–850
Carbon	0.90–1.05	0.25–0.50	0.15–0.30	......	....	1575	1500	850–1050
Silicon carbon	0.90–1.05	0.25–0.50	0.25–0.50	......	....	1600	1525	850–1050
Silicon carbon-vanadium	0.88–0.98	0.45–0.55	0.50–0.75	......	0.15	1625	1600	750–1050
Chrome-vanadium	0.45–0.55	0.70–0.90	0.15–0.30	1.00–1.20	0.15	1600	1600	850–1050
Silicon manganese	0.50–0.60	0.60–0.80	1.80–2.20	......	....	1600	1575	850–1050

Table 15. Mechanical Properties of Certain S.A.E. Steels with Various Heat-treatments.—(*Continued*)

Draw temp, deg F	Tensile strength, kips	Yield point, kips	Reduction of area, percent	Elong in 2 in., percent	Brinell hardness	Tensile strength, kips	Yield point, kips	Reduction of area, percent	Elong in 2 in., percent	Brinell hardness
S.A.E. 4640 normalized at 1650–1700 F, quenched in oil at 1450–1500 F						S.A.E. 5140 normalized at 1625–1725 F, quenched in oil at 1500–1600 F				
800	178.5	147.5	47	10	363	180	160	42.5	13	363
900	158.5	130	49	11	321	160	140	47.5	14	331
1000	142	115	51	12	293	141	120	52	15	293
1100	138.5	102.5	55	15	262	126	102	56	17	255
1200	119	93	58	19	241	112	88	59	19	223
1300	115	88	61.5	22	229	100	78	62	21	202
S.A.E. 6140 normalized at 1650–1750 F, quenched in oil at 1550–1650 F										
800	180	160	42.5	13	363					
900	160	140	47.5	14	331					
1000	142	121	52	15	293					
1100	127.5	115	56	17.5	262					
1200	115	92.5	59	19	229					
1300	108	87	62	20.5	217					

Plain carbon tool steels, containing 0.60 to 1.40 percent carbon, are widely used on account of their low cost and excellent properties. For tools such as files, twist drills, threading dies, and small taps for which the primary requirement is a high hardness, a carbon content of 1.20 percent or higher is necessary. In tools subjected to shock, such as shear knives, chisels, hammers, and forging dies, the carbon content has to be kept below about 0.80 percent. For applications requiring both a high hardness and considerable toughness, the carbon content will be around 1 percent. Carbon steels do not have the ability to retain their hardness at the elevated temperatures developed in the tip of metal-cutting tools when operating under severe conditions. Carbon steels are liable to crack or warp on hardening whenever the tool has an intricate shape.

The **oil-hardening steels**, sometimes called non-deforming steels, have sufficient alloy content to be hardened on quenching in oil; distortion and cracking is therefore minimized. They usually contain up to 1.75 percent manganese with small percentages of chromium, molybdenum, vanadium, or tungsten and have the same general performance characteristics as plain carbon tool steels.

Semi-high-speed steels can operate at higher temperatures than carbon steels without losing their hardness in consequence of fairly high proportions of tungsten and chromium. The alloy carbides resist the tempering effect of high tool temperatures and also contribute to the abrasion resistance of the steel. On account of the high alloy content, the steels can be hardened in oil and thus are not liable to crack or warp on hardening.

High-speed steels retain considerable hardness at a red heat. A commonly used high-speed steel has 18 percent tungsten, 4 percent chromium, and 1 percent vanadium (known as 18-4-1 high-speed steel). Cobalt, sometimes added as an alloying element, improves the cutting qualities for

C. Have high creep strength up to 1200 F. Toughness impaired in relatively high carbon alloys by service at 800 to 1500 F.

Hot-working Qualities. A. Readily forged, pierced, or rolled at 2000 to 1700 F. Preheat and soak stock at 1600 F. Plain chromium alloys air harden on cooling.

B. May be forged, rolled, or pierced. Should be heated quickly. Forge from 2200 F down to 1750 F. On last heat continue cold working to 1400 F to refine grain. Alloys do not air harden.

C. May be forged, rolled, or pierced. Preheat and soak at 1600 F, heat quickly to 2200 F, forge down to 1850 F. Hot short range: 1800 to 1300 F. Alloys do not air harden.

Cold-working Qualities. A. Low carbon varieties can be easily cold drawn into wire, cold rolled, bent, formed, upset, coined, and deep drawn.

B. Can be cold drawn into wire, cold rolled, bent, formed, upset, coined, and deep drawn, especially when warm (300 to 500 F).

C. Can be cold drawn into wire, cold rolled, bent, formed, upset, coined, and deep drawn. Work harden twice as rapidly as Groups A and B.

Machinability. A. Machine satisfactorily with properly designed tools when heat treated to 200 to 250 Brinell. *Free-cutting grade contains zirconium sulphide.*

B. Machine satisfactorily with properly designed tools. Cold working and high sulphur improve machinability.

C. Most difficult of all. Use sharp tool having greater top rake than usual, and cut continually. Free-cutting grade contains selenium and phosphorus.

Riveting. A. Make excellent cold rivets. Not recommended for hot rivets driven above 1500 F, on account of air hardening.

B. Extra precautions required to avoid brittle rivets. Conical heads should be cold upset on ground bars; rivets driven at 1425 F into chamfered holes.

C. Excellent for either hot or cold rivets. Hot rivets may be driven at a high temperature (1900 F).

Welding Properties. A. Can be welded with gas, electric arc, or resistance. Weld air hardens. Little grain growth.

B. Can be welded. Anneal at 1450 F to reduce embrittlement alongside weld. Those metals subject to grain growth are brittle adjacent to the weld. Those metals not subject to grain growth yield satisfactory welds.

C. Can be welded with gas, electric arc, or resistance, if carburization is avoided. Weld does not air harden and is very tough. Only the relatively low carbon of "stabilized" metal should be welded if article must resist severe corrodents.

Corrosion Resistance. A. Very satisfactory for resisting weather, water, steam, and many organic and inorganic corrodents when chromium is 11.5 percent or more. If carbon is relatively high, metal must be hardened and tempered (below 1000 F).

B. Possess corrosion resisting properties superior to Group A. Especially good for nitric and other oxidizing acids.

C. Corrosion resistance depends largely upon total alloy content. This group will resist nearly all corrodents measurably better than Groups A and B. Especially good for organic acids. Severe pitting may occur in some stagnant solutions, under particles of foreign matter.

Scale Resistance. A. Useful for continuous temperatures up to 1200 F, and in some services up to 1500 F.

B. Superior to Group A, especially when chromium is above 25 percent; then resist reducing atmospheres up to 2100 F, oxidizing up to melting points, and sulphur gases up to 1800 F.

C. Excellent where combination of high temperature and corrosion is to be met. Low nickel alloys required to resist sulphurous gases.

Group A (Martensitic). The hardenable alloys were the first stainless steels to be developed and, because of their high hardness and stain resisting properties, are used extensively for table cutlery, pocket knives, surgical and dental instruments, springs for high-temperature operation, ball valves and seats, and similar applications. The highest grade of stainless cutlery steel has 0.70 percent C and 16 percent Cr. Forging and annealing treatments are given in Table 20. The proper hardening range depends on

Table 18. Classification of American Tool Steels

(Adapted from Gill, "Tool Steels," *Metal Progress*, Oct., 1938)

GENERAL PURPOSE CARBON TOOL STEELS

Water hardening; low wear resistance; high warpage; no red hardness; shallow hardening

C	Si	Mn	S	P	V	Common Uses
0.60–1.25 [a]	0.15–0.50	0.10–0.35	0.03 max.	0.03 max.	[b]	Almost universal;[c]

CHROMIUM-VANADIUM OR LOW CHROMIUM TOOL STEELS

(Substitutes for carbon tool steels)

Mostly water hardening; low wear resistance; high warpage; no red hardness; medium deep hardening

C	Si	Mn	Cr	V	Remarks
0.50–1.40	0.15–0.50	0.10–0.35	0.10–0.25	....	Chromium corrects tendency toward soft spots
0.50–1.40	0.15–0.50	0.10–0.35	0.25–0.50	....	More intense hardness
0.50–1.40	0.15–0.40	0.10–0.35	0.60–1.20	0.10–0.20	Water hardening } Very tough in low-carbon ranges
0.50–1.40	0.15–0.40	0.40–0.60	0.60–1.20	0.10–0.20	Oil hardening }

HIGH CARBON, LOW TUNGSTEN TOOL AND DIE STEELS

(Finishing tools for hard steels or non-ferrous alloys)

Oil hardening; medium wear resistance; medium toughness; low warpage; no red hardness; medium deep hardening

C	Si	Mn	Cr	W	V (Optional)	Remarks
0.90–1.10	0.20–0.40	0.15–0.30	....	1.00–1.50	....	} Somewhat erratic in heat treatment
1.15–1.25	0.20–0.40	0.15–0.30	....	1.75–2.50	0.10–0.25	}
0.90–1.10	0.20–0.40	0.15–0.30	0.35–0.75	1.50–2.50	0.10–0.25	Water hardening } more dependable
1.15–1.30	0.20–0.40	0.15–0.30	0.35–0.75	1.50–2.50	0.10–0.25	Oil hardening }

MANGANESE OIL-HARDENING DIE STEELS ("NON-DEFORMING")

(General purpose tools and especially dies, punches, and broaches)

Oil hardening; low wear resistance; medium toughness; low warpage; no red hardness; medium deep hardening

C	Si	Mn	Cr	W	Mo	V	Remarks
0.85–0.95	0.20–0.40	1.50–1.75	....	....	....	0.10–0.25	More subject to grain growth
0.85–1.00	0.20–0.40	1.15–1.45	0.30–0.60	0.30–0.60	....	0.10–0.25	Corrects above, but hardness is lower
0.85–1.00	0.20–0.40	1.35–1.65	....	....	0.20–0.35	0.10–0.25	Attains highest hardness
0.90–1.00	0.20–0.40	0.90–1.15	0.50–0.90	....	....	....	Least susceptible to hardening cracks

TUNGSTEN ALLOY CHISEL AND PUNCH STEELS

(Oil hardening steels; shears and battering tools for cold metal; heading dies)

Medium wear resistance; high toughness; low warpage; medium red hardness; medium deep hardening

C	W	Cr	V	Si	Remarks
0.45–0.60	0.75–1.25	0.75–1.25	....	1.00–1.50	Good wear resistance but somewhat brittle
0.45–0.60	1.50–2.00	0.75–1.25	....	1.00–1.50	Higher tungsten improves wear resistance
0.45–0.60	1.00–1.75	0.50–1.00	....	....	Low silicon increases toughness 25 percent
0.40–0.55	1.75–2.25	0.75–1.25	0.10–0.30	....	} Most popular analyses, tough and fine grained
0.55–0.65	1.75–2.25	0.75–1.25	0.10–0.30	....	}

[a] Usually subdivided by 0.10 percent steps; for instance, 0.65 to 0.75 percent, 0.75 to 0.85 percent, etc.

[b] Plain carbon steels have no vanadium; carbon-vanadium tool steels may have from 0.08 to 0.40 percent vanadium, depending on grade.

[c] In lower carbon ranges, the steels make shear blades, hammers, striking dies, rock drills. In medium ranges of carbon, the steels make chisels, smith's tools, dies, and cutters for machine tools. In the higher ranges of carbon are small cutters, woodworkers' tools, and cutlery.

composition and size, but, in general, the higher the quenching temperature, the harder the article. Oil quenching is preferable, but with thin and intricate shapes, which might warp on quenching, satisfactory hardening is obtained by cooling in air. Tempering around 900 F does not lower the tensile strength, and in this condition the steel shows remarkable resistance to weathering, to attack by fruit and vegetable acids, lye, ammonia, and other corrosive agents to which cutlery may be subjected.

Group B (Ferritic). This group is frequently called stainless iron because of its low carbon content. The alloys possess considerable ductility, ability to be worked hot or cold, excellent corrosion resistance, and are relatively inexpensive. Although these low-carbon chromium alloys cannot be hardened by heat-treatment, they can be hardened to a considerable extent by cold working. Alloys containing 16 to 18 percent Cr are probably the most useful of the straight chromium steels on account of their forming and medium-deep-drawing properties. They are used extensively for kitchen equipment, dairy machinery, interior decorative work, automobile trimmings, and for chemical equipment to resist nitric acid corrosion.

For resisting oxidizing conditions at high temperatures, the Cr content is increased to between 25 and 30 percent. These alloys are useful for all types of furnace parts not subjected to high stress. Since the oxidation resistance is independent of carbon content, soft forgeable alloys low in carbon can be rolled into plates, shapes, and sheets, and hard and wear-resistant castings can be made from higher carbon non-forgeable alloys. The mechanical properties of 12, 17, and 27 percent Cr low-carbon alloys are given in Table 20.

Group C (Austenitic). The addition of substantial quantities of Ni to the high-chromium alloys stabilizes the austenite to such an extent that they are austenitic at room temperature. They cannot be hardened except by cold working, although some excellent properties can be obtained in this manner. Table 21 gives mechanical properties of several austenitic stainless steels. These alloys are highly resistant to many acids including hot or cold nitric acid and at temperatures above 1200 F are stronger and scale less than any of the plain chromium alloys. They are useful for parts subjected to severe stress at elevated temperatures. The most common composition is 18 Cr and 8 Ni (known as 18–8), although many modifications have been developed for special purposes. Tungsten and molybdenum increase the strength at elevated temperatures, silicon and aluminum improve the resistance to scaling, and selenium and sulphur are sometimes added to improve the machinability.

The austenitic stainless steels are not highly resistant to hot sulphurous gases and are sometimes subject to embrittlement and intergranular corrosion. Normal corrosion resistance can be restored by heating the steel above 1700 F and cooling rapidly. When titanium or columbium is added to a low carbon 18–8, the steel is immune to this intergranular attack.

Alloy steels containing 5 percent Cr are used extensively as seamless tube in the oil and power industries because of their low cost and moderate resistance to corrosion. These steels usually contain about 0.5 percent Mo to aid in their resistance to creep at elevated temperatures. Table 20 gives their mechanical properties. Another chromium steel of commercial importance is **silcrome** (0.45 C, 3.25 Si, 9 Cr) used principally for gas-engine exhaust valves. It has a high transformation point, has considerable hardness at operating temperatures, and resists the corrosion and erosion of hot gases.

Table 18. Classification of American Tool Steels.—(*Continued*)
(Adapted from Gill, "Tool Steels," *Metal Progress*, Oct., 1938)

TUNGSTEN DIE STEELS FOR HOT WORK
(Blanking, forming, extrusion, and casting dies to work up to 1100 F.)
Air or oil hardening; medium wear resistance; medium toughness; low warpage; high red hardness; deep hardening

C	W	Cr	V	Remarks
0.25–0.35	8.00–10.00	2.50–3.50	0.30–0.60	In most general use; serviceable up to 1000 F
0.35–0.45	8.00–10.00	2.50–3.50	0.30–0.60	Higher carbon gives higher hardness
0.40–0.50	9.00–12.00	1.25–1.75	....	Chromium lowered to increase toughness
0.25–0.35	12.00–16.00	2.50–3.25	0.30–0.60	Increased tungsten raises serviceability to 1100 F
0.35–0.50	12.00–16.00	2.50–3.25	0.30–0.60	Hardenability and brittleness rise with carbon con-
0.50–0.60	12.00–16.00	2.50–3.25	0.30–0.60	tent
0.50–0.60	17.00–19.00	3.00–4.50	0.60–1.20	Low-carbon, high-speed steel

TUNGSTEN-CHROMIUM STEELS FOR HOT WORK AND DIE-CASTING DIES
Air or oil hardening; medium wear resistance; good toughness; low warpage; high red hardness; deep hardening

C	W	Cr	V	Mo	Si	Remarks
0.40–0.50	6.50–7.50	6.50–7.50	0.20–0.60	....	0.30–0.80	Maximum alloy for hottest services (1000 F)
0.35–0.45	5.50–6.50	5.00–6.00	....	....	0.30–0.80	Tougher
0.30–0.40	0.75–1.25	4.50–5.00	....	1.00–1.50	0.80–1.00	Less expensive substitute
0.35–0.40	....	4.50–5.00	....	1.00–1.50	0.80–1.00	Properties similar to steel above

HIGH SPEED STEELS
(Cutting tools of all types; tools for severe hot work)
Air or oil hardening; high wear resistance; low toughness; low warpage; high red hardness; deep hardening

C	W	Cr	V	Mo	Co	Remarks
			Conventional Types			
0.65–0.75	17.00–19.00	3.50–4.50	0.75–1.25	....	....	Most used; brittleness and cutting properties vary directly with carbon content
0.65–0.75	19.00–21.00	3.75–4.50	0.75–1.25	....	....	Better cutting ability but more brittle
0.75–0.85	17.00–19.00	3.50–4.50	1.75–2.25	0.40–0.90	....	Best cutting ability; excellent for finishing cuts
0.55–0.75	13.00–15.00	3.50–4.50	1.75–2.25	....	....	Roughing tools; somewhat erratic in hardening
			Molybdenum High Speed Steel			
0.60–0.85	1.00–2.50	3.50–4.50	0.75–1.25	6.00–8.00	....	Less expensive; "strategic" alloying element used
0.70–0.90	....	3.50–4.50	1.75–2.50	6.00–9.00	....	Improved by high vanadium content
			Cobalt High Speed Steel			
0.65–0.80	17.00–19.00	3.50–4.50	0.75–1.25	....	3.50–5.00	For cutting hard, gritty or tough materials
0.65–0.80	17.00–19.00	3.50–4.50	1.50–2.25	0.50–1.00	6.00–9.00	Cutting ability varies as total alloy content
0.65–0.80	18.00–21.00	3.50–4.50	1.75–2.25	0.50–1.00	10.00–13.00	Maximum alloy to be forgeable
0.65–0.80	12.00–15.00	3.50–4.50	1.75–2.25	....	5.00–8.00	Good service on special jobs

Silcrome is being replaced by austenitic steels containing silicon and tungsten in heavy-duty engines, since it does not retain its hardness at elevated temperatures or resist scaling as well as the austenitic steels.

Special-property Alloys

Iron-nickel alloys are used extensively in the electrical industry owing to their exceptional magnetic properties. Alloys containing 20 to 30 Ni are non-magnetic and are used to some extent for non-magnetic parts in electrical machinery. Alloys having a high permeability and low hysteresis loss have a composition between 45 and 80 Ni, two of the better known being **Permalloy** with 78.5 Ni and **Hipernik** with 50 Ni. **Perminvar** (45 Ni, 25 Co) has a constant permeability over a range of flux densities. The magnetic properties of various alloys are given in Table 22.

Table 22. Magnetic Properties of Various Alloys
(A.S.M. Metals Handbook, 1939)

Material*	Initial permeability	Maximum permeability	Hysteresis loss, ergs per cc per cycle[a]	Residual induction, gausses	Coercive force, oersteds[a]	Saturation value, gausses[b]	Resistivity, microhm cm
Armco iron	250	7,000	5,000	13,000	1.0	22,000	11
4 % Silicon-iron	600	6,000	3,500	12,000	0.5	20,000	50
78.5 Permalloy, quenched	10,000	105,000	200	6,000	0.05	10,700	16
45 Permalloy	2,700	23,000	1,200	8,000	0.3	16,000	45
3.8–78.5 Cr-Permalloy	12,000	62,000	200	4,500	0.05	8,000	65
3.8–78.5 Mo-Permalloy	20,000	75,000	200	5,000	0.05	8,500	55
45–25 Perminvar, baked	400	2,000	2,500	3,000	1.2	15,500	19
7–45–25 Mo-Perminvar, baked	550	3,700	2,600	4,300	0.65	10,300	80
70–7.5 Perminvar, annealed	750	3,500	.....	.....	0.8	12,000	16

* Single numbers preceding the word "Permalloy" signify the nickel content, and double numbers signify first the content of chromium or molybdenum, and second the nickel content, the balance being iron in each case. The two large numbers before "Perminvar" indicate the nickel and cobalt contents, respectively, and the small initial number indicates the molybdenum content.

[a] For saturation value of the flux density.

[b] Saturation value of the intrinsic induction.

Another important group of nickel-iron base alloys are those with low coefficients of expansion. **Invar**, containing 36 Ni, has an exceedingly low coefficient of linear expansion. Within limits of atmospheric temperature change, its expansion is proportional to the temperature and it is therefore used for secondary standards of length. **Elinvar** (32 Ni with small percentages of Cr, W, Mn, Si, and C) not only has a low coefficient of expansion but also has a constant modulus of elasticity over the temperature range of 0 to 100 F and is thus useful in hairsprings for watches and springs for other precision instruments. **Platinite**, a 46 Ni alloy, has the same thermal coefficient of expansion as platinum; and **Dumet wire**, a 42 Ni alloy covered with copper to prevent gassing at the seal, is used to replace platinum as the "seal-in" wire in incandescent lamps and vacuum tubes.

Electrical sheet steels are alloys of iron and silicon with C, Mn, P, and S *kept as low as possible. The silicon increases the electrical resistivity of iron* and greatly decreases the hysteresis loss; silicon-alloy sheets are used in almost all magnetic circuits where alternating current is used. For transformers, the silicon content is around 5 percent, but in structures subjected

Stainless Steels

Corrosion and Heat-resisting Steels. Certain alloys of iron and chromium are highly resistant to corrosion and oxidation at high temperatures and maintain considerable strength at these temperatures. These alloys sometimes contain nickel and small percentages of silicon, molybdenum, tungsten, or copper. This large and complex group of alloys is known as **stainless steels,** although none of them are truly stainless and many are not steels in the sense that they do not harden on quenching. See Thum, "The Book of Stainless Steels" (A.S.M., 1935) for details of the production, fabrication, and properties of these alloys.

Stainless steels may be classified according to their microstructure as follows: (A) **hardenable alloys** containing up to 16 percent chromium and 0.70 percent carbon which are martensitic when quenched; (B) **low-carbon, non-hardenable alloys** which are ferritic and contain more than 16 percent chromium; and (C) **chromium-nickel alloys** which are austenitic. Some of the properties of these three different groups of stainless steels are as follows.

Chemical Analysis. Group A. (Martensitic.) Chromium less than about 16 percent; carbon less than about 0.40 percent. May contain small percentages of tungsten, copper, nickel, silicon, molybdenum. Group is magnetic.

B. (Ferritic.) Chromium more than about 16 percent; carbon quite low, but can increase as chromium goes up. May contain small percentages of copper, nickel, silicon, molybdenum, tungsten. This group is magnetic.

C. (Austenitic.) Contain enough chromium and nickel to make steel austenitic and non-magnetic. Usually contain twice as much chromium as nickel or vice versa; total alloy content at least 26 percent. Carbon quite low.

Heat-treatment. A. Respond to hardening, tempering, and drawing. Resulting physical properties depend on chemical analysis (principally carbon content).

B. 18 percent chromium toughened by long anneal at more than 1400 F, and air cooling. Avoid decarburizing the skin. 25 percent chromium gets best strength and toughness by rapid cooling from 1650 F.

C. Do not respond to hardening by heat-treatment. Must be rapidly cooled from 1800 to 2150 F to have austenitic structure (Brinell 140 to 170).

Toughness. A. Are structurally dependable. After tempering are not brittle in notched sections or under impact.

B. Laminated structure, from coarse ferrite in ingot, causes low impact values, but proper rolling and heating gives adequate toughness in rods, bars, and sheets.

C. Extremely tough at all times, including liquid air temperatures. Dependable against shock except when corroded at grain boundaries (a preventable condition).

Grain Growth and Structural Changes at High Temperatures. A. Not subject to excessive grain growth. Thoroughly dependable for supporting any load or shock within their carrying capacity up to 1400 F.

B. The pure chromium irons low in carbon and those containing high silicon or aluminum (when cold worked) are subject to excessive grain growth, especially above 1900 F. Grain growth is reduced by carbon, manganese, copper, nickel, titanium, and vanadium. Some alloys containing them are not subject to grain growth.

Long service at 800 to 950 F makes them brittle when cold, although they are not brittle at working temperatures.

C. Tend to precipitate carbides at grain boundaries during service at 800 to 1500 F, losing some toughness and becoming susceptible to intergranular attack by strong corrodents. This is controlled by very low carbon, by titanium or columbium, or by prior "stabilization."

Strength at Elevated Temperatures. A. Much better than straight carbon steel for temperatures up to 1000 or 1200 F. Retain tensile properties up to 750 F.

B. Heat-resisting varieties quite tough at temperatures up to 1600 F. Superior in ductility to Group C but not in creep resistance.

Table 23. A.S.T.M. Specifications for Steels (1939)

A.S.T.M. Designation No	Grade	Tensile strength, kips	Mechanical properties: Yield Point	Yield Point: not under, kips	Minimum elong., percent: In 8 in.	Minimum elong., percent: In 2 in.	Bend test, see notes	Reduction of area, percent	Method of manufacture[a]
A7-39	....	60-72	0.5 T.S.	33	(49)	22	6, 18, 31, 38, 43	.....	B, OH, E
	Unannealed	67-82	0.5 T.S.	36	(49)	20	8, 21, 34, 40, 48	.....	B, OH, E
A10-39	....	35-65	0.5 T.S.	30	(49)	24	5, 17, 30, 37, 42	.....	OH, E
A78-39	A	45 min	0.5 T.S.	24	28(49)	...	1	.....	OH, E
A141-39	....	52-62	0.5 T.S.	28	(49)	...	1		
A8-39	....	90-115	0.5 T.S.	55	(50)	(61)	8, 25, 27	30(52)	OH, E
A94-39	....	80-95	....	45	(60)	(62)	7, 19, 29, 27, 42	30(62)	OH, E
A131-39	....	60-72	0.5 T.S.	33	(49)	22	7, 25, 28, 43	.....	OH, E
A70-39	....	55-65	0.5 T.S.	....	(64)	(65)	7, 18, 31, 37, 45, 47	.....	OH, E
A201-39	A	55-65	0.5 T.S.	....	(64)	(66)	6, 17, 30, 36, 46, 48	.....	OH, E
A212-39	A	65-77	0.5 T.S.	....	(66)	(67)	8, 19, 32, 37, 45, 47	.....	OH, E
A31-39	....	45-55	0.5 T.S.	....	(68)	...	1	.....	OH, E
A53-36	Welded Acid B	50 min	....	30	16	...	....	.....	B
	Welded OH	45 min	....	25	20	30	....	.....	OH
	Seamless Low C	48 min	....	26.5	...	35	....	.....	OH, E
	Seamless Med C	62 min	....	35	...	25	....	.....	OH, E
A106-39	Top welded	45 min	....	25	...	...	....	.....	OH
	Seamless A	48 min	....	30	35	(60)	....	.....	OH, E
A187-38	....	60 min	....	25 min	30	...	....	.....	E
A113-39	For cars	50-65	0.5 T.S.	(49)	22	...	5, 24, 26, 41	.....	OH, E
	For locomotives	55-65	0.5 T.S.	(49)	22	...	5, 24, 26, 41	.....	OH, E
	For pressing	48-58	0.5 T.S.	(49)	...	...	1	.....	OH, E
A26-39	Class B	115 min	....	....	10	...	....	12	OH, E
A15-39	Plain Bars { Structural	55-75	....	33	(58)	...	7, 12		
	Plain Bars { Intermediate	70-90	....	40	(59)	...	9, 13		
	Plain Bars { Hard	80 min	....	50	(60)	...	10, 14		

[a] OH. Open hearth. B. Bessemer. E. Electric furnace.

Table 20. Mechanical Properties of High-chromium Iron Alloys
(A.S.M., The Book of Stainless Steels, 2d ed.)

Mechanical properties at room temperature	5 % Cr		12 % Cr		Cutlery		17 % Cr		27 % Cr	
	Annealed	Quenched and drawn at 1100 F	Annealed	Quenched and drawn at 1100 F	Annealed	Oil quenched from 1850 F and drawn	Annealed	Cold-worked (wire)	Annealed	Cold-worked (wire)
Ultimate strength, kips	66	115	65	125	100	230–260	75	100–190	75–95	85–175
Yield point, kips	27	103	35	100	65	200–220	40		50–60	55–155
Elastic modulus, 1,000 kips	.....	.....	28	.....		.....	29	....		
Elongation in 2 in., percent	38	20	35	20	27	8–2	27		30–20	
Elongation in 10 in., percent	.....	.....	.....	.....	...	.....		25–2	......	25–2
Reduction in area, percent	76	66	65	60	59	20–2	55	40–20	60–50	55–25
Impact, ft-lb, Charpy	.....	.....	.....	75						
Izod	80	73	80	.....	...	....	8–25			
Brinell hardness number	136	250	140	230	170	480	175	185–270	160–190	150–250
Rockwell hardness number	B 75	C 24	B 76	C 22	...	C 56	B 85	B 90–105	B 80–90	C 0–25
Erichsen value, mm	.....	.....	.....	.....	...	....	7–9			
Stress, lb per sq in., causing 1 percent "creep" in 10,000 hr at	1000 F, 7000		1000 F, 13,000 1200 F, 2300 1350 F, 1400		. .		1000 F, 8500 1200 F, 2100 1350 F, 1200		1200 F, 1600 1350 F, 400	
Scaling temperature, F	1200		1300		1750		1550		2100	
Initial forging temperature	2100		2100		2000		2000		2200	
Finishing temperature, F	About 1400		Not over 1450		1700		Not over 1400		Not over 1400–1450	
Annealing treatment	Furnace cool from 1580 F		Prolonged heating at 1250–1350 F		1575–1625 F		[a]		1 hr or more at 1450 F and quench	

[a] Small cold reduction, followed by anneal at 1400 F and quench.

of 0.125 percent shall be made for each increase of $\frac{1}{32}$ in. in thickness above $\frac{3}{4}$ in., to a minimum of 18 percent for flange steel and 19 percent for firebox steel.

(57) Elongation in 2 in., min, percent = 1,750,000/T.S. For material over $2\frac{1}{2}$ in. in thickness, a deduction from the above elongation of 0.5 percent shall be made for each increase of $\frac{1}{2}$ in. in thickness above 2 in.

(58) Elongation in 8 in., min, percent = 1,500,000/T.S., but need not exceed 30 percent.

(59) *Elongation in 2 in., min, in transverse test = 30 percent; longitudinal test = 35 percent.*

(60) Elongation in 8 in. min, percent = 1,400,000/T.S. (1,300,000/T.S.) [1,200,000/T.S.], but not less than 20 (16) percent for structural (intermediate) [hard] grade.

Table 24. Specifications for Steel Forgings

(American Society for Testing Materials, Standards, 1939)
Serial Designation A18–39

Class	Treatment	Size	Tensile strength, min (except Class A), 1,000 lb per sq in. U	Yield point (or elastic limit, *E.L.*) 1,000 lb. per sq in.	Elongation in 2 in., min, percent		Reduction of area, min, percent	
					Inverse ratio	Not under	Inverse ratio	Not under
A	None	All sizes	47–60	$\frac{1}{2}U$	1500/U	......	2500/U	
B	None	(a), (b)	60	$\frac{1}{2}U$	1550/U	22	2400/U	35
		(c)	60	$\frac{1}{2}U$	1480/U	21	2220/U	32
C	Annealed	(a), (b)	60	$\frac{1}{2}U$	1700/U	25	2700/U	38
		(c)	60	$\frac{1}{2}U$	1600/U	24	2520/U	36
D	None	(a)	75	$\frac{1}{2}U$	1600/U	18	2200/U	24
		(b)	75	$\frac{1}{2}U$	1500/U	17	2000/U	22
		(c)	75	$\frac{1}{2}U$	1400/U	16	1800/U	20
E	Annealed	(a)	75	$\frac{1}{2}U$	1800/U	20	2800/U	33
		(b)	75	$\frac{1}{2}U$	1725/U	19	2640/U	31
		(c)	75	$\frac{1}{2}U$	1650/U	18	2400/U	29
F	Annealed	(a)	80	$\frac{1}{2}U$	1800/U	20	2800/U	32
		(b)	80	$\frac{1}{2}U$	1725/U	19	2640/U	30
		(c)	80	$\frac{1}{2}U$	1650/U	18	2400/U	28
G	Quenched and tempered	(d)	90	(*E.L.* 55)	2100/U	20.5	4000/U	39
		(e)	85	(*E.L.* 50)	2000/U	20.5	3800/U	39
		(f)	85	(*E.L.* 50)	1900/U	19.5	3600/U	37
		(g)	82.5	(*E.L.* 48)	1800/U	19	3400/U	36
H (nickel steel)	Annealed	(a), (b)	80	(*E.L.* 50)	2000/U	22	3600/U	40
		(c)	80	(*E.L.* 50)	1900/U	21	3400/U	38
J (nickel steel)	Quenched and tempered	(d)	100	(*E.L.* 70)	2200/U	20	4500/U	41
		(e)	100	(*E.L.* 65)	2100/U	20	4300/U	41
		(f)	90	(*E.L.* 60)	2000/U	20	4100/U	41
		(g)	85	(*E.L.* 55)	1900/U	20	3900/U	41
K (alloy steel)	Quenched and tempered	(h)	95–115	70	....	20	....	50
		(d), (e), (f)	90–110	65	....	20	....	50
		(g)	85–105	60	....	20	....	50
L (alloy steel)	Quenched and tempered	(h)	105–125	80	....	20	....	50
		(d), (e)	100–120	75	....	20	....	50
		(f)	100–120	75	....	18	....	45
		(g)	95–115	75	....	18	....	45
M (alloy steel)	Quenched and tempered	(h)	$\geq$125	$\geq$ 105	....	16	....	50
		(d)	$\geq$115	$\geq$ 95	....	16	....	45
		(e)	$\geq$110	$\geq$ 85	....	16	....	45
		(f)	$\geq$100	$\geq$ 75	....	18	....	45
		(g)	$\geq$100	$\geq$ 70	....	18	....	45

Notes to Table 24

The steel shall be made either by the open-hearth or electric furnace or by both processes.

Table 21. Mechanical Properties of High-chromium Nickel Iron Alloys
(A.S.M., The Book of Stainless Steels, 2d ed.)

Composition, Cr-Ni	18-8			18-12		25-12		15-35
Mechanical properties at room temperature	Annealed	Cold-worked (wire)	Stabilized	Annealed	Cold-worked (wire)	Annealed	Cold-worked (wire)	Castings
Ultimate strength, kips	80–90	105–300	85–95	80–90	105–275	90–110	110–270	50–70
Yield point, kips	40	60–250	40–45	40	......	40–60	65–230	40–50
Elastic modulus, 1,000 kips	29							
Elongation in 2 in., percent	60	......	55	60	......	50–35	......	7–3.5
Elongation in 10 in., percent	......	50–2	......	......	50–2	......	35–2	
Reduction in area, percent	70	65–30	55	65	65–30	60–45	55–20	8–4
Impact, ft-lb, Charpy	......	......	77					
Izod	75–110							
Fatigue endurance limit, kips	47							
Brinell hardness number	135–165	170–460	150–185	135–165	170–380	150–185	170–375	170–180
Rockwell hardness number	B 75–85	C 5–47	B 80–90	B 75–85	C 5–40	B 80–90	C 5–40	B 87–90
Erichsen value, mm	11–14							
Stress, lb per sq in., causing 1 percent "creep" in 10,000 hr at	1000 F. 17,000 1200 F. 7000 1350 F. 3000 1500 F. 850					1500 F. 15,000 (unconfirmed)		1400 F. 3500[a] 1600 F. 1800 1800 F. 1000
Scaling temperature, F	1650		1650	1650		2100		1900
Initial forging temperature, F	2200		......	As for 18-8		2200–2300		
Finishing temperature, F	Not under 1600–1700		......	As for 18-8		Not under 1600–1700		
Annealing treatment	Heat at 1900–2000 F and quench		[b]	As for 18-8		Heat at 2000–2150 F and quench		

[b] Final heat-treatment must consist of 2 to 4 hr soak at 1550 F. [a] Safe working stress.

WIRE, SHEETS, AND BARS

Wire and Sheet Metal Gages. Wire and sheet metal of the smaller thicknesses are made to various gages. Steel wire is usually made to the Washburn and Moen (W & M) or Roebling gage. The U. S. standard for sheet metal is based upon weight per square foot; the tabulated values are the corresponding thicknesses for wrought iron weighing 480, and for steel and open-hearth iron weighing 489.6 lb per cu ft. Stubs steel wire gage is also used for numbered twist drill sizes. The Birmingham wire gage is used in the U. S. for brass wire. The Brown and Sharpe gage is a uniform geometrical progression, each gage being equal to 0.89053 times the preceding gage. See p. 1690 for more data on the Brown and Sharpe gage.

Weights of Rolled Sheet Steel

Gage No.	Weight per sq ft, lb		Gage No.	Weight per sq ft, lb		Gage No.	Weight per sq ft, lb		Gage No.	Weight per sq ft, lb	
	B. W. G.	U. S. S. G.		B. W. G.	U. S. S. G.		B. W. G.	U. S. S. G.		B. W. G.	U. S. S. G.
7-0's	....	20 00	6	8.2824	8.125	18	1.9992	2	29	0.5304	0.5625
6-0's	....	18.75	7	7.344	7.5	19	1.7126	1.75	30	0.4896	0.5
5-0's	....	17.50	8	6.732	6.875	20	1.428	1.50	31	0.408	0.4375
0000	18.5232	16.25	9	6.0384	6.25	21	1.3056	1.375	32	0.3672	0.4063
000	17.34	15	10	5.4672	5.625	22	1.1424	1.25	33	0.3264	0.375
00	15.504	13.75	11	4.896	5	23	1.02	1.125	34	0.2856	0.3438
0	13.872	12.50	12	4.4972	4.375	24	0.8976	1	35	0.2040	0.3125
1	12.24	11.25	13	3.876	3.75	25	0.816	.875	36	0.1632	0.2813
2	11.5872	10.625	14	3.3864	3 125	26	0.7344	.75	37	....	0.2656
3	10.5672	10	15	2.9376	2.813	27	0.6528	.6875	38	....	0.25
4	9.7104	9.375	16	2 651	2.5	28	0.5712	.625			
5	8.976	8.75	17	2.3664	2.25						

Properties of Steel Wire

(Breaking stress = 100,000 lb per sq in.)

No., Roebling gage	Breaking load, lb	Weight, lb per 1,000 ft	No., Roebling gage	Breaking load, lb	Weight, lb per 1,000 ft	No., Roebling gage	Breaking load, lb	Weight, lb per 1,000 ft	No., Roebling gage	Breaking load, lb	Weight, lb per 1,000 ft
6-0's	16,619	558.4	6	2,895	97.3	17	229	7.70	27	23.0	0.763
5-0's	14,522	487.9	7	2,461	82.7	18	174	5.83	28	20 0	0.676
0000	12,130	407.6	8	2,061	69.3	19	132	4.44	29	18 0	0.594
000	10,292	345.8	9	1,720	57.8	28	96	3.23	30	15.0	0.517
00	8,605	289.1	10	1,431	48.1	21	80	2.70	31	14.0	0.481
0	7,402	248.7	11	1,131	38.0	22	62	2.07	32	13.0	0 446
1	6,290	211.4	12	866	29.1	23	49	1.65	33	9.5	0.319
2	5,433	182.5	13	665	22.3	24	42	1.40	34	7.9	0.264
3	4,676	157.1	14	503	16.9	25	31	1.06	35	7.1	0.238
4	3,976	133.6	15	407	13.7	26	25	0.855	36	6.4	0.214
5	3,365	113.1	16	312	10.5						

to vibration, such as motor armatures, the silicon is usually kept below 4 percent because of the brittleness of high silicon sheets.

Austenitic manganese steel (Hadfield's manganese steel) is a nonmagnetic alloy containing around 12 Mn and 1 C. It is relatively soft but work hardens on the surface when subjected to severe abrasion, so that it is extremely useful in crushing machinery, for railroad crossings and frogs, tractor shoes, etc. As cast, this alloy is partly martensitic and therefore hard and brittle. By quenching from a high temperature (1900 F), a homogeneous austenite is retained and the alloy has the high toughness, strength, and ductility characteristic of austenitic steels.

A.S.T.M. Specifications

The A.S.T.M. Standards, Part I, 1939, contain over two hundred specifications for ferrous materials and products. For detailed specifications and for chemical analyses reference should be made to these Standards. The specifications contain also details of the preparation of test specimens, the location from which they should be taken, and permissible variations in the dimensions of material ordered.

The physical properties specified for some of the more common ferrous products are contained in Tables 23 and 24. The products designated by the specification number in the first column of Table 23 are as follows:

STEEL FOR BRIDGES AND BUILDINGS

- A7–39 Plates, Sections, Bars, and Eyebar Flats
- A10–39 Mild Steel Plates
- A78–39 Plates for Forge Welding
- A141–39 Structural Rivet Steel
- A8–39 Structural Nickel Steel
- A94–39 Structural Silicon Steel
- A131–39 Structural Steel for Ships

STEEL FOR BOILERS

- A70–39 Carbon Steel Plates
- A201–39 Carbon-silicon Steel Plates
- A212–39 High Tensile Stress Carbon-silicon Steel Plates
- A31–39 Boiler Rivet Steels
- A53–36 Steel Pipe
- A106–39 Steel Pipe for High Temperatures
- A187–38 Seamless Cold-drawn Alloy Heat Exchanger Tubes

RAILROAD STEELS

- A113–39 Structural Steel for Locomotives and Cars
- A26–39 Steel Tires

BILLET-STEEL BARS FOR CONCRETE REINFORCEMENT

- A15–39

Weights of Square and Round Steel Bars

(For iron, subtract 2 percent)

Size, in.	Weight, lb per lin ft Square	Weight, lb per lin ft Round	Size, in.	Weight, lb per lin ft Square	Weight, lb per lin ft Round	Size, in.	Weight, lb per lin ft Square	Weight, lb per lin ft Round	Size, in.	Weight, lb per lin ft Square	Weight, lb per lin ft Round
0	......	......	3	30.60	24.03	6	122.4	96.1	9	275.4	216.3
1/16	0.013	0.010	1/16	31.89	25.05	1/16	125.8	98.2	1/16	279.2	219.3
1/8	0.053	0.042	1/8	33.20	26.08	1/8	127.6	100.2	1/8	283.1	222.4
3/16	0.120	0.094	3/16	34.54	27.13	3/16	130.2	102.2	3/16	287.0	225.4
1/4	0.213	0.167	1/4	35.91	28.21	1/4	132.8	104.3	1/4	290.9	228.5
5/16	0.332	0.261	5/16	37.31	29.30	5/16	135.5	106.4	5/16	294.9	231.6
3/8	0.478	0.376	3/8	38.73	30.42	3/8	138.2	108.5	3/8	298.8	234.7
7/16	0.651	0.511	7/16	40.18	31.55	7/16	140.9	110.7	7/16	302.8	237.8
1/2	0.850	0.668	1/2	41.63	32.71	1/2	143.7	112.8	1/2	306.9	241.0
9/16	1.076	0.845	9/16	43.15	33.89	9/16	146.4	115.0	9/16	310.9	244.2
5/8	1.328	1.043	5/8	44.68	35.09	5/8	149.2	117.2	5/8	315.0	247.4
11/16	1.607	1.262	11/16	46.23	36.31	11/16	152.1	119.4	11/16	319.1	250.6
3/4	1.913	1.502	3/4	47.81	37.55	3/4	154.9	121.7	3/4	323.2	253.9
13/16	2.245	1.763	13/16	49.42	38.81	13/16	157.8	123.9	13/16	327.4	257.1
7/8	2.603	2.044	7/8	51.05	40.10	7/8	160.7	126.2	7/8	331.6	260.4
15/16	2.988	2.347	15/16	52.71	41.40	15/16	163.6	128.5	15/16	335.8	263.7
1	3.400	2.670	4	54.40	42.73	7	166.6	130.9	10	340.0	267.0
1/16	3.838	3.015	1/16	56.11	44.07	1/16	169.6	133.2	1/16	344.3	270.4
1/8	4.303	3.380	1/8	57.85	45.44	1/8	172.6	135.6	1/8	348.6	273.8
3/16	4.795	3.766	3/16	59.62	46.83	3/16	175.6	137.9	3/16	352.9	277.1
1/4	5.313	4.172	1/4	61.41	48.23	1/4	178.7	140.4	1/4	357.2	280.6
5/16	5.857	4.600	5/16	63.23	49.66	5/16	181.0	142.8	5/16	361.6	284.0
3/8	6.428	5.049	3/8	65.08	51.11	3/8	184.9	145.2	3/8	366.0	287.4
7/16	7.026	5.518	7/16	66.95	52.58	7/16	188.1	147.7	7/16	370.4	290.9
1/2	7.650	6.008	1/2	68.85	54.07	1/2	191.3	150.2	1/2	374.9	294.4
9/16	8.301	6.519	9/16	70.78	55.59	9/16	194.5	152.7	9/16	379.3	297.9
5/8	8.978	7.051	5/8	72.73	57.12	5/8	197.7	155.3	5/8	383.8	301.5
11/16	9.682	7.604	11/16	74.71	58.67	11/16	200.9	157.8	11/16	388.4	305.0
3/4	10.413	8.178	3/4	76.71	60.25	3/4	204.2	160.4	3/4	392.9	308.6
13/16	11.170	8.773	13/16	78.74	61.85	13/16	207.5	163.0	13/16	397.5	312.2
7/8	11.953	9.388	7/8	80.80	63.46	7/8	210.9	165.6	7/8	402.1	315.8
15/16	12.763	10.024	15/16	82.89	65.10	15/16	214.2	168.2	15/16	406.7	319.5
2	13.600	10.681	5	85.00	66.76	8	217.6	170.9	11	411.4	323.1
1/16	14.463	11.359	1/16	87.14	68.44	1/16	221.0	173.6	1/16	416.1	326.8
1/8	15.353	12.058	1/8	89.30	70.14	1/8	224.5	176.3	1/8	420.8	330.5
3/16	16.270	12.778	3/16	91.49	71.86	3/16	227.9	179.0	3/16	425.5	334.2
1/4	17.213	13.519	1/4	93.71	73.60	1/4	231.4	181.8	1/4	430.3	338.0
5/16	18.182	14.280	5/16	95.96	75.36	5/16	234.9	184.5	5/16	435.1	341.7
3/8	19.178	15.062	3/8	98.23	77.15	3/8	238.5	187.3	3/8	439.9	345.5
7/16	20.201	15.866	7/16	100.53	78.95	7/16	242.1	190.1	7/16	444.8	349.3
1/2	21.250	16.690	1/2	102.85	80.78	1/2	245.7	192.9	1/2	449.7	353.2
9/16	22.326	17.534	9/16	105.20	82.62	9/16	249.3	195.8	9/16	454.6	357.0
5/8	23.428	18.400	5/8	107.58	84.49	5/8	252.9	198.7	5/8	459.5	360.9
11/16	24.557	19.287	11/16	109.98	86.38	11/16	256.6	201.5	11/16	464.4	364.8
3/4	25.713	20.195	3/4	112.41	88.29	3/4	260.3	204.5	3/4	469.4	368.7
13/16	26.895	21.123	13/16	114.87	90.22	13/16	264.0	207.4	13/16	474.4	372.6
7/8	28.103	22.072	7/8	117.35	92.17	7/8	267.8	210.3	7/8	479.5	376.6
15/16	29.338	23.042	15/16	119.86	94.14	15/16	271.6	213.3	15/16	484.5	380.5

Notes to Table 23
Cold Bend Tests

Test specimen to bend cold 180 deg unless otherwise noted, around pin of diameter d, *without cracking on outside of bend.*

Note	Material thickness t, in.	Diam of pin, d, in.	Note	Material thickness t, in.	Diam of pin, d/t
1	All thicknesses	0[a]	25	¾–1¼ incl	2
2	All thicknesses	1	26	1¼–2	2
3	All thicknesses	1½	27	1¼–2	2½
4	All thicknesses	2	28	1¼–2	3
		d/t	29	1–1½	2
5	Up to ¾, incl	0[a]	30	1–1½	1
6	Up to ¾, incl	½	31	1–1½	1½
7	Up to ¾, incl	1	32	1–1½	2
8	Up to ¾, incl	1½	33	1–1½	2½
9	Up to ¾, incl	2	34	1–1½	3
10	Up to ¾, incl	3	36	1½–3	1½
11	Up to ¾, incl	4	37	1½–2	2
12	¾ and over	1	38	1½–2	2½
13[b]	¾ and over	2	39	1½–2	3
14[b]	¾ and over	3	40	1½–2	4
15	¾ and over	4	41	Over 2	2
16	¾ to 1 incl	0[a]	42	Over 2	2½
17	¾ to 1 incl	½	43	Over 2	3
18	¾ to 1 incl	1	44	Over 2	4½
19	¾ to 1 incl	1½	45	2–3	2
20	¾ to 1 incl	2	46	3–4	2
21	¾ to 1 incl	2½	47	3–4	2½
22	¾ to 1 incl	3	48	4–6	2½
23	¾ to 1 incl	4			
24	¾ to 1¼ incl	1			

[a] Specimen to bend flat on itself without cracking on outside of the bend.
[b] To bend 90 deg without cracking.

Modifications in Elongation

[49] Elongation in 8 in., min., percent = 1,500,000/T.S. For materials over ¾ in. in thickness or diameter, a deduction from the above elongation of 0.25 percent shall be made for each increase of 1/32 in. of thickness or diameter above ¾ in., to a minimum of 18 percent, except for eyebar flats unannealed, which shall have a minimum of 14 percent.

For material under 5/16 in. in thickness or diameter, a deduction from the above elongation of 1.25 percent shall be made for each decrease of 1/32 in. of the specified thickness or diameter under 5/16 in.

[50] Elongation in 8 in., min, percent = 1,600,000/T.S. for structural nickel steel; 1,500,000/T.S. for structural silicon steel. For material over ¾ in. diam or thickness, a deduction from the above elongation of 0.25 percent shall be made for each increase of 1/16 in. diam or thickness over ¾ in., to a minimum of 14 percent. For material under 5/16 in. diam or thickness, *a deduction from the above elongation of 1.25 percent* shall be made for each decrease of 1/32 in. diam or thickness below 5/16 in.

[51] Elongation in 2 in., min, percent = 1,700,000/T.S.

[52] For material over ¾ in. diam or thickness, a deduction from the percentage reduction in area of 0.50 percent shall be made for each increase of 1/16 in. diam or thickness above ¾ in. (to a minimum of 24 percent in the case of silicon steel).

[53] Elongation in 2 in., min, percent = 1,600,000/T.S.

[54] Elongation in 8 in., min, percent = 1,500,000/T.S. for flange; 1,550,000/T.S., but not less than 25 percent, for firebox.

For material over ¾ in. in thickness a deduction from the above elongation of 0.125 percent shall be made for each increase in thickness of 1/32 in. over ¾ in. to a minimum of 21 percent for flange steel and 22 percent for firebox steel.

[55] Elongation in 2 in., min, percent = 1,700,000/T.S. *For material over 2½ in. in* thickness a deduction from the above elongation of 0.5 percent shall be made for each increase of ½ in. of thickness above 2½ in.

[56] Elongation in 8 in., min, percent = 1,550,000/T.S. for flange; 1,600,000/T.S. *for* firebox. For material over ¾ in. *in* thickness, a deduction from the above elongation

IRON AND STEEL CASTINGS

BY

CHARLES W. BRIGGS

REFERENCES: Bolton, "Gray Cast Iron," Penton. Hall, "The Steel Foundry," McGraw-Hill. Moldenke, "Principles of Iron Founding," McGraw-Hill. Sisco, "Alloys of Iron and Carbon," vol. II, McGraw-Hill. "Metals Handbook," A. S. M. "Cast Metals Handbook," American Foundrymen's Association. "Steel Castings Handbook," Steel Founders' Society.

Classification of Castings

The following classification of the regular ferrous foundry product is customary. See also p. 535.

Gray-iron Castings. The vast majority of gray irons fall within the range of composition: carbon, 2.50 to 3.70 percent and silicon, 0.50 to 3.00 percent. Three types of castings are made in the ordinary gray-iron jobbing foundry.

Soft gray-iron machinery castings are made from an iron that can be machined readily, is not specially strong, and is sound and reliable; they should be used for the ordinary run of machine construction work. Composition varies with the weight of the casting, light, medium, and heavy. Medium machinery mixtures (see Table 1) are used for ordinary castings satisfactorily, but it is not safe to make heavy castings from light machinery mixtures, as they will be very weak; very light castings from heavy machinery mixtures will be quite hard.

Strong gray-iron machinery castings are adapted for medium and heavy work where a greater strength than that of ordinary cast iron is desired, but not the special strength of the steel casting. They are made by adding 10 to 40 percent of steel scrap to the ordinary mixtures in which the required silicon has been provided to carry the steel burden safely. There is much variation in the strength of such castings; where extreme care must be exercised, it is safer to use steel castings.

Chilled-iron Castings. Such castings as crusher jaws, grinding plates, etc., requiring chilled-iron surfaces and soft bodies, are made in the jobbing foundry from special low-silicon irons. Chilled rolls, car wheels (chilled treads), and work of a similar nature are better obtained from roll or car-wheel foundries, as these work to service guarantees and are specially careful in the selection of their melting stock.

Special Gray-iron Castings. Special attention must be given to certain speciality castings. Thus, ornamental castings are made from high-phosphorous high-silicon iron, which is very fluid when molten, in order to fill the finest lines in the mold. See Table 1 for such special castings.

Malleable castings range in strength between gray iron and the steel castings; they are used where gray iron is too weak and steel too expensive. They are made ordinarily in sections less than ¾ in. thick, the pieces not much over 4 ft in length, and the weight not over 300 lb. The usual weights are a few ounces for light, 15 lb for medium, and 150 lb for heavy castings. The metal will bend, twist, and resist shock remarkably well—its special characteristic—better even than cast steel, is cheap, easily made, and specially adapted to repetition work, and therefore in general use for car castings, pipe fittings, agricultural work, hardware, etc. The minimum tensile strength should be 50,000 lb with the yield point at 30,000 lb per sq in. min and at least 10 percent elongation in 2 in.

USES OF STEELS LISTED

Class A, for forgings which may be welded or case-hardened.

Class B, for mild-steel forgings for structural purposes, for minor ship fittings, etc.

Class C, for mild-steel forgings for structural purposes, for ships, etc.

Classes D, E, F, G, H, and I, for various machinery forgings, choice depending upon design and upon the stresses and services to be imposed.

Classes K, L, and M, for various machinery forgings, choice depending upon design and upon the stresses and services to be imposed, and upon the character of machining operations to be done.

PERCENTAGE CHEMICAL COMPOSITION

Class A: Mn, 0.30–0.60, P, ≦0.05, both acid and basic; S, ≦0.05.

Classes B, C, D, E, F, and G: Mn, 0.40–0.80; P, ≦0.05, both acid and basic; S, ≦0.05.

Classes H and I: Mn, 0.40–0.80; P, ≦0.04, both acid and basic; S, ≦0.05; Ni, ≦3.00.

Classes K and L: P, ≦0.05 (acid), or ≦0.04 (basic); S, ≦0.05.

Class M: P, ≦0.04 (acid or basic); S, ≦0.05. The composition of alloy steel, other than P and S, to be agreed upon between manufacturer and purchaser.

SIZES

(a) Not over 8 in. in outside diameter or overall thickness.

(b) Over 8 to 12 in., inclusive.
(c) Over 12 to 20 in., inclusive.
(d) Up to 4 in., 2-in. max wall.
(e) Over 4 to 7 in., 3½-in. max wall.
(f) Over 7 to 10 in., 5-in. max wall.
(g) Over 10 to 20 in., 5- to 8-in. max wall.
(h) Up to 2 in., 1-in. max wall.

Specifications for Classes A, B, C, D, E, F, H are for forgings whose max outside diameter or overall thickness is not over 20 in. Specifications for Classes G, I, are for forgings whose max outside diameter or thickness is not over 10 in. when solid, and not over 20 in. when bored.

Steel castings are divided into two main classes, carbon and alloy. Each class is subdivided according to composition.

Carbon cast-steels are divided into two classes: (1) medium-carbon cast steels, containing 0.20 to 0.40 C, and (2) special-carbon cast steels containing (*a*) less than 0.20 C and (*b*) more than 0.40 C.

Alloy cast steels are divided into two classes: (1) low-alloy cast steels containing special alloying elements totaling less than 8 percent, and (2) high-alloy cast steels containing 8 percent or more special alloying elements. This class includes the heat- and corrosion-resistant steels and austenitic manganese steel.

The steel casting should be selected where strength, ductility, and reliability are essential.

CAST IRON

Effect of Element Additions. Cast iron includes a whole series of alloys of iron, carbon, and silicon, all of which contain free graphite flakes in the as-cast condition. Besides carbon, iron, and silicon, commercial cast-iron contains appreciable percentages of phosphorus, sulphur, and manganese.

Carbon. Beginning with the smallest percentages of carbon, there are the three commercial grades of steel castings. Just beyond a 2 percent content will be found the hard castings making tough (when good) malleables. Also, some of the special high-strength irons may have carbon contents well under 2.50 percent. From 2.50 percent on, good malleables are obtained, and from 2.75 percent upward the strong cast-iron varieties. The upper limit for ordinary cast irons is 3.75 percent. Low-silicon charcoal irons used for chilled-casting work run up to 4.25 percent.

The carbon in iron is of two general kinds: (1) **combined carbon** as carbides of iron; (2) **graphite**, or uncombined carbon, present as a mechanical admixture. The latter form of carbon is subdivided into two classes: (*a*) The crystalline graphite present in gray iron, produced in the "setting" of the molten metal and varying in size from the big-flaked "kish" thrown off as excess carbon from blast-furnace casts to the very finely crystallized graphite of the soft cast irons made to set very quickly by means of chills; (*b*) graphite found in malleable castings and resulting from the heat-treatment of white irons of the proper composition. Instead of separating out in the molten mass while setting, this graphite is formed from the solid, the amorphous particles reposing in the interstices of the crystalline iron structure which are opened up by the long-continued high-temperature application. It is amorphous in shape and is called **temper carbon.**

In cast-iron founding, where subsequent heat-treatment is used only for removing casting strains, as in softening iron pipe centrifugally-cast in metal molds, the total carbon present may be either all in the combined state, as one extreme, making white-fracture hard brittle and almost useless castings, or all graphitic, as the other extreme, making a black-fracture soft easily-machined and highly useful iron. This range is due largely to the percentage of the other elements present (notably the silicon), the degree of superheat attained in melting, the rate of cooling in the mold after pouring (a function of the thickness of section), and, in some degree, the pouring temperature.

Silicon has a powerful softening effect; its presence in cast iron reduces the ability of the iron to retain carbon in chemical combination. With silicon almost entirely absent, the iron will retain all its carbon in combination, making white iron. With about 3 percent silicon present, almost no carbon can be held in chemical combination, and gray iron results. Beyond 3

Wire and Sheet Metal Gages

(Diameters and thicknesses in decimal parts of an inch.)

Gage No.	American wire gage, or Brown & Sharpe (for non-ferrous sheet and wire)	Steel wire gage or Washburn & Moen or Roebling (for steel wire)	Birmingham wire gage (B.W.G.) or Stubs iron wire (for steel rods or sheets)	Stubs steel wire gage	British Imperial standard wire gage (S.W.G.)	U. S. standard gage for wrought iron sheet (480 lb per cu ft)	U. S. standard gage for steel and open-hearth iron sheet (489.6 lb per cu ft)	British standard for iron and steel sheets and hoops, 1914 (B.G.)
0000000	....	0.4900	....	......	0.500	0.500	0.4902	0.6666
000000	....	0.4615	....	......	0.464	0.469	0.4596	0.6250
00000	....	0.4305	....	......	0.432	0.438	0.4289	0.5883
0000	0.460	0.3938	0.454	......	0.400	0.406	0.3983	0.5416
000	0.410	0.3625	0.425	......	0.372	0.375	0.3676	0.5000
00	0.365	0.3310	0.380	......	0.348	0.344	0.3370	0.4452
0	0.325	0.3065	0.340	......	0.324	0.312	0.3064	0.3964
1	0.289	0.2830	0.300	0.227	0.300	0.281	0.2757	0.3532
2	0.258	0.2625	0.284	0.219	0.276	0.266	0.2604	0.3147
3	0.229	0.2437	0.259	0.212	0.252	0.250	0.2451	0.2804
4	0.204	0.2253	0.238	0.207	0.232	0.234	0.2298	0.2500
5	0.182	0.2070	0.220	0.204	0.212	0.219	0.2145	0.2225
6	0.162	0.1920	0.203	0.201	0.192	0.203	0.1991	0.1981
7	0.144	0.1770	0.180	0.199	0.176	0.188	0.1838	0.1764
8	0.128	0.1620	0.165	0.197	0.160	0.172	0.1685	0.1570
9	0.114	0.1483	0.148	0.194	0.144	0.156	0.1532	0.1398
10	0.102	0.1350	0.134	0.191	0.128	0.141	0.1379	0.1250
11	0.091	0.1205	0.120	0.188	0.116	0.125	0.1225	0.1113
12	0.081	0.1055	0.109	0.185	0.104	0.109	0.1072	0.0991
13	0.072	0.0915	0.095	0.182	0.092	0.094	0.0919	0.0882
14	0.064	0.0800	0.083	0.180	0.080	0.078	0.0766	0.0785
15	0.057	0.0720	0.072	0.178	0.072	0.070	0.0689	0.0699
16	0.051	0.0625	0.065	0.175	0.064	0.062	0.0613	0.0625
17	0.045	0.0540	0.058	0.172	0.056	0.056	0.0551	0.0556
18	0.040	0.0475	0.049	0.168	0.048	0.050	0.0490	0.0495
19	0.036	0.0410	0.042	0.164	0.040	0.0438	0.0429	0.0440
20	0.032	0.0348	0.035	0.161	0.036	0.0375	0.0368	0.0392
21	0.0285	0.0317	0.032	0.157	0.032	0.0344	0.0337	0.0349
22	0.0253	0.0286	0.028	0.155	0.028	0.0312	0.0306	0.0313
23	0.0226	0.0258	0.025	0.153	0.024	0.0281	0.0276	0.0278
24	0.0201	0.0230	0.022	0.151	0.022	0.0250	0.0245	0.0248
25	0.0179	0.0204	0.020	0.148	0.020	0.0219	0.0214	0.0220
26	0.0159	0.0181	0.010	0.146	0.018	0.0188	0.0184	0.0196
27	0.0142	0.0173	0.016	0.143	0.0164	0.0172	0.0169	0.0175
28	0.0126	0.0162	0.014	0.139	0.0148	0.0156	0.0153	0.0156
29	0.0113	0.0150	0.013	0.134	0.0136	0.0141	0.0138	0.0139
30	0.0100	0.0140	0.012	0.127	0.0124	0.0125	0.0123	0.0123
31	0.0089	0.0132	0.010	0.120	0.0116	0.0109	0.0107	0.0110
32	0.0080	0.0128	0.009	0.115	0.0108	0.0102	0.0100	0.0098
33	0.0071	0.0118	0.008	0.112	0.0100	0.0094	0.0092	0.0087
34	0.0063	0.0104	0.007	0.110	0.0092	0.0086	0.0084	0.0077
35	0.0056	0.0095	0.005	0.108	0.0084	0.0078	0.0077	0.0069
36	0.0050	0.0090	0.004	0.106	0.0076	0.0070	0.0069	0.0061
37	0.0045	0.0085	....	0.103	0.0068	0.0066	0.0065	0.0054
38	0.0040	0.0080	....	0.101	0.0060	0.0062	0.0061	0.0048
39	0.0035	0.0075	....	0.099	0.0052	0.0059	0.0057	0.0043
40	0.0031	0.0070	....	0.097	0.0048	0.0055	0.0054	0.0039
41	....	0.0066	....	0.095	0.0044	0.0053	0.0052	0.0034
42	....	0.0062	....	0.092	0.0040	0.0051	0.0050	0.0031
43	....	0.0060	....	0.088	0.0036	0.0049	0.0048	0.0027
44	....	0.0058	....	0.085	0.0032	0.0047	0.0046	0.0024
45	....	0.0055	....	0.081	0.0028	....	....	0.0022
46	....	0.0052	....	0.079	0.0024	....	....	0.0019
47	....	0.0050	....	0.077	0.0020	....	....	0.0017
48	....	0.0048	....	0.075	0.0016	....	....	0.0015
49	....	0.0046	....	0.072	0.0012	....	....	0.0014
50	....	0.0044	....	0.069	0.0010	....	....	0.0012

effect in carbide stabilization, the use of chromium in gray irons is restricted to percentages usually below 1 percent. High-chromium cast irons (20 to 35 percent chromium) show good corrosion and heat resistance and resistance to erosive wear.

Other Elements. Molybdenum, titanium, vanadium, aluminum, copper, etc., are seldom found in pig iron in sufficient quantity to be considered; they are, however, added to cast iron in small quantities in order to produce special effects.

Physical Characteristics

Shrinkage and Contraction. The term shrinkage is usually employed where contraction is really meant. Contraction in a casting is the reduction in dimensions due to the cooling from the temperature at the moment of solidification to the ordinary temperature of the atmosphere. Shrinkage, or liquid contraction, on the other hand, is the separation of the portions of metal from each other, due to an insufficient supply of liquid metal to fill up the spaces left by the reduction in volume as the iron solidifies. This forms spongy spots and actual cavities, the walls of which may be lined with pine-tree crystals of iron. Shrinkages greatly weaken the interior structure of the metal and are highly dangerous through their concealed position.

Irons that throw out graphite on solidifying will not contract nearly so much as those in which this does not occur. The ordinary contraction of gray iron is ⅛ in. per ft. That of white iron and steel is about ¼ in., but varies with differences in the rate of cooling, the composition of the metal, and the shape of the pattern. In the case of malleable castings, where the original hard-iron casting should contract ¼ in. (one-half of which is restored in the annealing process), some parts do not contract at all, although others show a contraction of $\frac{5}{16}$ in. per ft. This indicates that some portions are held tight in the mold and are actually stretched in the setting, and others readily pull away from the walls of the mold.

The most frequent **segregations** consist of hard spots of manganese and sulphur-iron compounds, which interfere with machining where the two elements are too high, and of iron phosphide, accompanied by increased hardness in the center of a heavy section made with too-high-phosphorus iron. There may be an artificial chilling of the metal in spots, through improper cooling, with consequent hardening effects.

Aging of Iron Castings. In order to reduce or, if possible, remove casting strains, it has been the custom, where permanent accuracy in finishing castings is desired, to store the rough casting for about 6 months before using them. In present-day production this is seldom possible, and the same object can be attained by a short heat-treatment. The castings, after cleaning, are brought to a temperature of between 500 and 600 F and left over night. Another way is to anneal at 1250 F for about 4 hr after the interior is fully heated and then cool gradually. The best result is obtained by annealing at the high temperature after the first cut has been taken. The metal will then be free from casting strains when the finishing cut is given.

Heat-treatment. Heat-treatments applied to cast irons are relatively new developments. They offer opportunities for improvements in quality and certain economies. Different treatments are used for different purposes. These may be summarized as follows:

1. Quick aging to remove internal stresses. Usual temperature 900 F.
2. Quenching and drawing to increase hardness and strength. Usual oil quench from 1500 to 1550 F followed by a draw to 700 F or above.

Weights of Flat Rolled Steel, Pounds per Linear Foot*

(The last line of the table gives weights per square feet)

(For iron, subtract 2 percent)

Width, in.	Thickness, in. $\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
$\frac{1}{4}$	0.053	0.106	0.159	0.213	0.27	0.32	0.37	0.43	0.48	0.53	0.58	0.64	0.69	0.74	0.80	0.85
$\frac{1}{2}$	0.106	0.213	0.319	0.425	0.53	0.64	0.74	0.85	0.96	1.06	1.17	1.28	1.38	1.49	1.59	1.70
$\frac{3}{4}$	0.159	0.319	0.478	0.638	0.80	0.96	1.12	1.28	1.43	1.59	1.75	1.91	2.07	2.23	2.39	2.55
1	0.213	0.425	0.638	0.850	1.06	1.23	1.49	1.70	1.91	2.13	2.34	2.55	2.76	2.98	3.19	3.40
2	0.425	0.850	1.275	1.700	2.13	2.55	2.98	3.40	3.83	4.25	4.68	5.10	5.53	5.95	6.38	6.80
3	0.638	1.275	1.913	2.550	3.19	3.83	4.46	5.10	5.74	6.38	7.01	7.65	8.29	8.93	9.56	10.20
4	0.850	1.700	2.550	3.400	4.25	5.10	5.95	6.80	7.65	8.50	9.35	10.20	11.05	11.90	12.75	13.60
5	1.063	2.125	3.188	4.250	5.31	6.38	7.44	8.50	9.56	10.63	11.69	12.75	13.81	14.88	15.94	17.00
6	1.275	2.550	3.825	5.100	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15.30	16.58	17.85	19.13	20.40
7	1.488	2.975	4.463	5.950	7.44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17.85	19.34	20.83	22.31	23.80
8	1.700	3.400	5.100	6.800	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20.40	22.10	23.80	25.50	27.20
9	1.913	3.825	5.738	7.650	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22.95	24.86	26.78	28.69	30.60
10	2.125	4.250	6.375	8.500	10.63	12.75	14.88	17.00	19.13	21.25	23.38	25.50	27.63	29.75	31.88	34.00
20	4.25	8.50	12.75	17.00	21.25	25.50	29.75	34.00	38.25	42.50	46.80	51.0	55.30	59.50	63.8	68.00
30	6.38	12.75	19.13	25.50	31.88	38.25	44.63	51.00	57.38	63.75	70.10	76.5	82.90	89.30	95.6	102.00
40	8.50	17.00	25.50	34.00	42.50	51.00	59.50	68.00	76.50	85.00	93.50	102.0	110.50	119.00	127.5	136.00
12	2.55	5.10	7.65	10.20	12.75	15.30	17.85	20.40	22.95	25.50	28.05	30.60	33.15	35.70	38.25	40.80

* For other widths the weights are obtainable by addition; for example, $54 \times \frac{3}{4}$ in. $= [(10 \times 5) + 4] \times \frac{3}{4}$ in., and weight $= (10 \times 12.75) + 10.20 = 137.7$ lb. Similarly, for greater thicknesses, the weights are obtainable by addition.

melting charges to cut down the total carbon, and at the same time superheat to higher temperature ranges, the **tensile strength** is raised, so that, in rather small sections, it is not uncommon to get a value of 45,000 lb per sq in. High-test cast iron can now be made by advanced foundrymen up to 55,000 lb and, with nickel added, even higher. These high values are, however, not yet readily obtainable. Where special strength is wanted, an iron not under 30,000 lb per sq in tensile strength should be specified. The tensile test for cast iron is not to be recommended unless the greatest care is taken with the preparation and testing of the specimen. American specifications make the tensile test optional and at the expense of the purchaser.

Table 3. Alloy Cast Iron

	Composition, percent													
Iron	Total carbon	Silicon	Graphitic carbon	Combined carbon by difference	Phosphorus	Manganese	Sulphur	Nickel	Chromium	Molybdenum	Steel	Pig-iron	Tapping temperature, deg F	Melting unit
G	3.49	2.08	2.78	0.71	0.19	0.55	0.08	1.17	0.65	.....	20	35	2894	Cupola
J	2.61	2.38	1.73	0.88	0.06	0.77	0.10	1.08	0.09	.....	85	..	2721	Cupola
Z	2.79	2.44	1.94	0.85	0.03	0.50	0.06	0.50	0.09	.....	68	32	2779	Arc
L	3.05	2.70	2.20	0.85	0.16	0.79	0.09	0.60	0.24	0.45	20	0	3000	Arc

Mechanical Properties

Iron	Tensile strength, psi	Modulus of elasticity in tension at ¼ load, psi × 10⁶	Modulus of rupture, psi	Deflection in 18 in. span, in.	Shear strength, psi	Endurance limit, psi	Compressive strength, psi	Brinell hardness 3,000 kg load	Izod impact unnotched, ft-lb
G	30,000	10.7	70,300	0.331	42,300	13,500	108,800	181	4.2
J	53,500	16.2	92,500	0.222	61,000	24,000	156,600	269	5.7
Z	56,700	16.4	111,700	0.490	57,100	25,000	135,700	236	8.2
L	39,200	14.3	81,800	0.301	44,200	18,200	131,200	228	5.1

The **transverse strength** of cast iron is the one most readily obtainable for specification purposes. The test bar of the A.S.T.M. is 1.20 in. diam, 21 in. long, and is broken transversely on supports 18 in. apart. A breaking strength of about 2,200 lb for the test bar, with a deflection of 0.25 in. or over, indicates a good iron for ordinary use.

The **elastic limit** of cast iron is close to its ultimate breaking strength. The **crushing strength** of cast iron is probably its most important characteristic, and gives it its value for structural use. It runs from 80,000 to 140,000 lb per sq in. The resistance to shock of cast iron is not high. On the other hand, the malleable casting has this as its prime characteristic. Hardness in ordinary cast irons is a condition that may be brought about artificially, if desired, by chilling the molten metal in the molds. Only the lower silicon irons are amenable. The mechanical properties of cast irons are given in

Table 1. Recommended Analyses for Various Classes of Castings
(All values in percent)

Castings	Light					Medium					Heavy				
	Si	Mn	S	P	C	Si	Mn	S	P	C	Si	Mn	S	P	C
Acid-resisting	2.00	0.75	0.05*	0.20*	3.25*	1.50	1.25	0.05*	0.20*	3.25*	1.00	1.25	0.05*	0.20*	3.25*
Agricultural	2.50	0.60	0.06	0.75	3.75	2.25	0.70	0.08	0.70	3.50	2.00	0.80	0.10	0.60	3.25
Air cylinders	1.90	0.70	0.08	0.50	3.40	1.50	0.80	0.09	0.40	3.25	1.00	0.90	0.10	0.30	3.00
Automobile cylinders	2.25	0.65	0.08*	0.40*	3.25*	2.00	0.75	0.08*	0.40*	3.25*					
Balls for grinding						0.75	0.50	0.15*	0.40*	3.75	0.50	0.50†	0.15*	0.40*	3.75
Bed plates	2.00	0.70	0.08	0.60	3.75	1.75	0.75	0.10	0.50	3.50	1.50	0.80	0.12	0.40	3.25
Boiler castings						2.00	0.80	0.06*	0.20*	3.50*					
Car wheels											0.65	0.50	0.08	0.35	3.50
Chilled castings												1.00	0.06*	0.20*	3.50*
Crusher jaws						1.00	1.00	0.04	0.20	3.50	0.80	1.25	0.06	0.20	3.25
Dynamo castings	2.50	0.50	0.05	0.75	3.75						2.15	0.50	0.06	0.50	3.25
Engine frames						2.00	0.60	0.08	0.50	3.50	1.75	1.00	0.10	0.40	3.00
Fire pots, grates	2.25	0.60	0.05	0.20	3.50	2.00	0.60	0.06	0.20	3.25					
Fly wheels	2.00	0.50	0.05	0.50	3.50	1.50	0.60	0.06	0.40	3.25	1.25	0.70	0.08	0.30	3.25
Furnace castings	2.40	0.60	0.05	0.60	3.75	2.15	0.80	0.06	0.50	3.50					
Gas engine cylinders	2.00	0.70	0.08	0.40	3.25	1.50	0.80	0.09	0.30	3.00	1.25	0.90	0.10	0.20	2.85
Gears	2.25	0.60	0.08	0.70	3.75	2.00	0.80	0.09	0.60	3.50	1.50	1.00	0.10	0.50	3.25
Grate bars						2.00	0.60	0.05	0.20	3.50					
Hardware	2.50	0.70	0.08	0.80	3.75										
Heat-resistant iron						2.00	0.80	0.06	0.20	3.25	1.50	1.00	0.06	0.20	3.00
High-test cast iron (best)	2.75	1.00	0.06	0.07	2.75	2.50	1.00	0.06	0.07	2.75	2.25	1.00	0.06	0.07	2.75
Hydraulic cylinders						1.50	0.80	0.05	0.40	3.25	1.00	1.00	0.08	0.20	2.85
Ingot molds											1.25	0.80	0.06	0.20	3.75
Machinery castings	2.50	0.60	0.08	0.70	3.75	2.00	0.80	0.09	0.60	3.50	1.50	1.00	0.10	0.50	3.25
Malleable castings (hard)	1.00	0.30*	0.08*	0.20*	3.50*	0.65	0.25*	0.08*	0.20*	3.50*					
Ornamental castings	2.75	0.60	0.06	0.90	3.75	2.25	0.70	0.08	0.80	3.50					
Pipe (water)	2.25	0.60	0.06	0.80	3.75	2.00	0.80	0.08	0.70	3.50	1.50	1.00	0.10	0.60	3.50
Piston rings	2.80	0.70	0.05	0.40	3.50	1.75	0.80	0.06	0.40	3.25					
Pulleys	2.40	0.50	0.05	0.70	3.75	2.15	0.60	0.07	0.60	3.50	1.90	0.70	0.09	0.50	3.25
Radiators	2.25	0.70	0.06	0.80	3.50										
Rolls (chilled)							0.80	0.06	0.40	3.25		1.00	0.08	0.30	3.00
Soft castings	2.60	0.50	0.06	0.60	3.75	2.40	0.60	0.08	0.50	3.50					
Soil pipe	2.25	0.60	0.08	0.80	3.75	2.00	0.80	0.10	0.60	3.50					
Steam cylinders	2.00	0.60	0.08	0.50	3.50	1.60	0.80	0.09	0.40	3.50	1.25	1.00	0.10	0.30	3.50
Stove plates	2.50	0.50	0.06	1.00	3.75	2.25	0.60	0.08	0.80	3.50					
Valves	2.25	0.60	0.07	0.50	3.25	1.75	0.80	0.08	0.40	3.00	1.25	1.00	0.09	0.30	2.85
White iron castings						0.75*	0.20†	0.25*	0.75*	2.50†					

* Below. † Above.

Other specifications for cast iron are as follows:

Automotive Gray-iron Castings, Spec. A159-40, A.S.T.M. Standards, 1940, Part 1, p. 143. "Gray-iron Castings for Valves, Flanges and Pipe Fittings," A126-40, A.S.T.M. Standards, 1940, Part 1, p. 140. "Lightweight and Thin-sectioned Gray-iron Castings," Specification A190-40, A.S.T.M. Standards, 1940, Part 1, p. 138. "Locomotive Cylinders, Cast Iron," A45-14, A.S.T.M. Standards, 1939, Part 1, p. 491.

Foundry Practice

Materials. **Pig irons** may be divided into: charcoal irons, coke irons, and electrically made pig irons. **Charcoal iron** was formerly used much for malleable castings, car wheels, and other work requiring chilling in whole or in part. Today it is used only for chilled rolls, crusher jaws, and castings that require chilled parts and great strength. Electrically made pig iron is now being made in Norway and imported into America. It is of very high quality and expensive. The bulk of foundry pig iron is made with coke as the fuel. Pig iron is now bought entirely on its composition.

The requirements of the foundry trade being sharply defined by the classes of castings made, the product of the blast furnace readily divides itself on the phosphorus content. Thus, high-phosphorus pig irons go to the stove trade, and very low-phosphorus irons, everything else being acceptable, go to the steel foundry. In a lesser degree, the sulphur plays a part, the very high-sulphur irons being unsuitable for the better grades of castings. High sulphur means a low selling price. Manganese is kept between 0.60 and 1.00 as a general rule. The silicon content is the determining factor in the purchase. The foundryman specifies the silicon and usually also the maximum sulphur.

In order that there may be uniformity in quotations, the following percentages and variations are used. The permissible variations are from the A.S.T.M. specification A43-24. These specifications do not advise that all five elements be specified in all contracts for pig iron, but do recommend that when these elements are specified the following percentages be used:

Recommended Chemical Specifications of Pig Iron

Silicon (0.25 allowed either way)	Sulphur (maximum)	Total carbon (minimum)	Manganese (0.20 either way)	Phosphorus (0.15 either way)
1.00	0.04	3.00	0.20	0.20
1.50	0.05	3.20	0.40	0.40
2.00	0.06	3.40	0.60	0.60
2.50	0.07	3.60	0.80	0.80
3.00	0.08	3.80	1.00	1.00
	0.09		1.25	1.25
	0.10		1.50	1.50

Percentages of any element may be specified one-half way between the above. In case of phosphorus and manganese, the percentages may be used as maximum or minimum figures, but unless so specified they are considered to include the variations above given.

Scrap is of two kinds, **domestic**, or that made in the foundry itself and which remains and is used in the daily heats; and **foreign**, or scrap which is bought in the open market and consists of several grades of heavy and light scrap machinery, pipe, stove plate, etc. Purchased scrap must be watched closely for undesirable elements.

Steel when used in foundry practice should be neither light nor heavy in section. Clippings from structural steel, rail ends, boiler plate, etc., are best. Steel should be charged directly on the fuel, pig iron over it, and then scrap cast iron. In this way, all portions of the mixture melt at about the same time and good mixing results.

percent, silicon begins to act as a hardener; the Scotch or silvery irons, the high-percentage-silicon pig irons, and finally the ferrosilicons up to metallic silicon are brittle substances.

In gray-iron work, the proper selection of the silicon content in the mixture is the most important factor. Within given limits of the other elements, every change desired can be brought out in the castings, so far as their strength, machining qualities, etc., are concerned, by variation in the silicon.

In malleable-casting work the silicon is even more important, as the range is quite small for a casting of given section.

Acidproof cast iron is now made in large quantities for the chemical industry. It is an ordinary cast iron with the silicon content raised to 14 percent, is white in fracture and very brittle. All surfaces are curved and fairly thick to reduce the casting strains. Castings of this description have trade names, such as **Duriron** and **Tantiron.**

Sulphur is an objectionable impurity in iron castings of practically all kinds, the iron-sulphur compounds being both brittle and weak. Sulphur ranges from about 0.08 to 0.15 percent in the majority of commercial irons, probably averaging about 0.10 percent. Its only redeeming feature is that high-sulphur castings cut freely under the tool, giving clean threads and surfaces.

Sulphur hardens iron by counteracting silicon in its power to form graphite. Hence, where high sulphur is present, proportionally more silicon must be charged.

Manganese promotes the retention of carbon in the combined form and hence counteracts silicon. This effect is not specially noticeable until the percentage of Mn passes 1.00. Manganese content usually ranges from 0.50 to 0.80 percent, although irons slightly above or below that range are not uncommon. In the case of malleable castings, the effect of over 0.40 Mn is noticeable in the annealing process, where it retards the opening up of the structure of the metal to allow the separating out of the "temper carbon" and for that reason is injurious. When sulphur is high, manganese will combine with it, and if, after tapping, with extremely hot iron, the metal is allowed to stand for a while, manganese sulphide rises and can be skimmed off with what slag has come up also, thus often reducing the sulphur of the metal by one-half.

Phosphorus. The great value of phosphorus is to make iron very fluid when molten. Hence its use for art work, stoves, radiators, and similar thin and not necessarily strong castings. Phosphorus is a hardener and requires the presence of the proper amount of silicon. The castings made with phosphorus irons being thin, there must be enough silicon present to prevent the retention of combined carbon, as thin sections cool at a rapid rate in the sand after casting. Up to 0.25 percent is considered low phosphorus, 0.30 to 0.70 percent medium, and above this to 1.50 percent is high.

Nickel is becoming an important factor in high-quality cast iron. As cast iron may be considered as essentially a matrix of indifferent steel, cut up and weakened by mechanically mixed graphite, anything to improve the matrix will be of value. Nickel up to 2 percent is best adapted for this purpose. While increasing the strength, it keeps the iron readily machinable by promoting the graphite formation. Higher percentages of nickel, from 4 to 20 percent, are used for castings of a specialized nature.

Chromium is employed in cast irons for two major purposes, *viz.*, to stabilize the carbides and increase corrosion resistance. Owing to its marked

A cupola 30 in. diam inside the lining is about as small as should be used in commercial practice.

The air furnace is a straight-draft reverberatory furnace. Metal in the hearth is melted by heat absorbed from the flame and hot gases passing over the hearth and by radiation from the roof and sides. Air furnaces regularly used in gray-iron melting usually are of large capacity, 20 tons or more of iron being melted in each batch. Pulverized coal is the common fuel, though oil is also used. The charge should be stacked in such a manner that the hot gases can pass through readily. In melting down, a short hot flame is desired, and the usual practice is to keep excessive slag scraped off. The bath should be well rabbled to mix the metal thoroughly. There is always some oxidation and silicon, manganese, and carbon are lowered. The advantage of the air furnace is that it gives a large amount of high-grade metal at one tap. The absence of direct contact of metal and fuel makes possible the very considerable reduction in total carbon by steel additions, without danger of picking up carbon from the fuel afterward. Heavy pieces of scrap, which could not go into a cupola, may be used.

The electric furnace is generally used for producing high-strength irons and highly superheated irons for light work. Metal composition is altered but little during the melting processes, and oxidation loss is avoided. The electric furnace is well adapted to melting special high-alloy cast irons.

An economical application of the electric furnace in the iron foundry is in connection with the cupola melting process. Molten metal from the cupola is transferred to the electric furnace and given 20 to 30 min treatment. The sulphur can readily be cut down to 0.04 in this time (in a basic lined furnace) and a high degree of superheat and deoxidation achieved.

MALLEABLE IRON CASTINGS

Malleable iron castings as manufactured in this country are used in large quantities in the construction of automobiles, at least half of the yearly production being used for this purpose. The agricultural-implement industry and the railroads are large users, and miscellaneous castings for diversified use account for the balance of the output. They are preferable to cast iron for any use where capacity to withstand impact, occasional overloads, or constant abuse in service is a requisite.

Malleable iron is strong, tough, and ductile; it consists of a matrix of silico-ferrite throughout which are uniformly distributed small nodules of temper carbon. The castings, from the air furnace, the electric, or open-hearth furnace, are hard or white-iron castings; they are made up wholly of pearlite and free cementite.

For the past 10 years, the average ultimate strength, yield point, and elongation in over 60 plants in different parts of the country are, respectively, 54,117 lb, 36,371 lb per sq in., and 18.49 percent in 2 in. The yield point is about 67 percent of the ultimate strength, which is very exceptional in the case of a soft and very ductile metal.

The following average data are taken from the A.S.T.M. and the Am. Foundrymen's Assoc., 1937 Joint Symposium on Malleable Iron Castings: Modulus of elasticity in tension, 25,000,000 lb per sq in.; Brinell hardness, 115; shearing strength, 48,000 lb per sq in.; modulus of rupture in torsion, 58,000 lb per sq in.; specific gravity, 7.40.

The average chemical composition of the hard-iron castings can be taken as: Si, 1.00; P, 0.16; S, 0.085; Mn, 0.280; combined C, 2.45.

3. Production of high-strength semi-malleable by anneal, quench, and draw.
4. Production of an extreme hard case by nitriding after suitable heat-treatment and machining operations.
5. Annealing to soften for increase in machinability. Usual range 1350 to 1500 F.

Many combinations of treatments are possible; only the more typical are mentioned.

Mechanical Properties

Strength of Cast Iron. A cast iron with a tensile strength of 25,000 lb per sq in. is considered a good metal. By the addition of steel scrap in the

Table 2. Gray Cast Iron

Iron	Composition, percent									Tapping temperature, deg F	Melting unit
	Total carbon	Silicon	Graphitic carbon	Combined carbon by difference	Phosphorus	Manganese	Sulphur	Mix: 100 percent minus cast-iron scrap: Steel	Mix: 100 percent minus cast-iron scrap: Pig-iron		
I	3.41	2.44	2.85	0.56	0.63	0.57	0.07	0	53	2649	Cupola
U	3.52	2.94	2.68	0.84	0.04	0.66	0.04	12	88	3005	Arc
F	3.60	2.06	2.93	0.68	0.75	0.52	0.06	10	75	2602	Cupola
O	3.43	2.35	2.75	0.68	0.20	0.73	0.09	13	41	2862	Cupola
D	3.25	2.08	2.66	0.59	1.99	0.53	0.05	13	67	2730	Cupola
E	3.12	2.18	2.44	0.68	0.63	0.44	0.10	4	56	2669	Cupola
R	3.24	1.63	2.41	0.83	0.42	0.56	0.08	0	50	2687	Arc
Q	2.88	1.99	2.25	0.63	0.43	0.51	0.10	23	0	2894	Arc
V	2.50	0.79	1.50	1.00	0.04	0.74	0.09	85	15	2653	Cupola

Mechanical Properties

Iron	Tensile strength, psi	Modulus of elasticity in tension at ½ load, psi × 10⁶	Modulus of rupture, psi	Deflection in 18 in. span, in.	Shear strength, psi	Endurance limit, psi	Compressive strength, psi	Brinell hardness, 3000 kg load	Izod impact, unnotched, ft-lb
I	20,000	8.0	48,800	0.274	27,700	9,400	87,200	158	3.4
U	19,400	6.0	48,300	0.370	29,600	10,000	72,800	146	3.6
F	22,400	8.7	49,200	0.251	33,000	11,400	91,000	163	3.6
O	25,000	9.7	58,700	0.341	35,500	11,800	95,000	163	4.9
D	21,300	10.5	33,300	0.141	27,500	12,300	90,900	179	2.2
E	32,500	13.6	73,200	0.301	44,600	16,500	120,800	192	4.2
R	35,100	13.3	77,000	0.326	47,600	17,400	120,800	196	4.4
Q	40,900	14.8	84,200	0.308	47,300	19,600	119,100	215	3.9
V	47,700	20.0	92,000	0.230	60,800	25,200	159,000	266	4.4

Irons I,U,F,O—Miscellaneous soft-iron castings.
Irons D,E,R—Strong gray-iron machinery castings.
Irons Q,V—High-test castings.

Standard Specifications for Malleable Castings

(A.S.T.M. Specification A47-33)

1. These specifications cover malleable castings for railroad, motor-vehicle, agricultural-implement, and general-machinery purposes.

2. The castings shall be produced by either the air-furnace, open-hearth, or electric-furnace process.

3. The tension-test specimens specified shall conform to the following minimum requirements as to tensile properties:

	Grade 32510	Grade 35018
Tensile strength, psi	50,000	53,000
Yield point, psi	32,500	35,000
Elongation in 2 in., percent	10.0	18.0

4. *a.* All castings, if of sufficient size, shall have cast thereon test lugs of a size proportional to the thickness of the casting, but not exceeding 5/8 by 3/4 in. in cross section. On castings which are 24 in. or over in length, a test lug shall be cast near each end. These test lugs shall be attached to the casting at such a point that they will not interfere with the assembling of the castings, and may be broken off by the inspector.

b. If the purchaser or his representative so desires, a casting may be tested to destruction. Such a casting shall show good tough malleable iron.

5. *a.* Tension test specimens shall be of the form and dimensions shown in Fig. 1. Specimens whose mean diameter at the smallest section is less than 19/32 in. will not be accepted for test.

b. A set of three tension test specimens shall be cast from each melt, without chills, using heavy risers of sufficient height to secure sound bars. The specimens shall be suitably marked for identification with the melt. Each set of specimens so cast shall be placed in some one oven containing castings to be annealed.

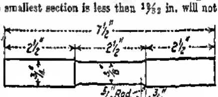

FIG. 1.—Tension Test Specimen for Malleable Iron.

6. *a.* After annealing, three tension test specimens shall be selected by the inspector as representing the castings in the oven from which these specimens are taken.

b. If the first specimen conforms to the specified requirements, or if, in the event of failure of the first specimen, the second and third specimens conform to the requirements, the castings in that oven shall be accepted, except that any casting may be rejected if its test lug shows that it has not been properly annealed. If either the second or third specimen fails to conform to the requirements, the entire contents of that oven shall be rejected.

7. Any castings rejected for insufficient annealing may be reannealed once. The reannealed castings shall be inspected, and if the remaining test lugs, or castings broken as specimens, show the castings to be thoroughly annealed, they shall be accepted; if not, they shall be finally rejected.

8. The castings shall conform substantially to the patterns or drawings furnished by the purchaser and also to gages which may be specified in individual cases. The castings shall be made in a workmanlike manner. A variation of 1/8 in. per ft will be permitted.

9. The castings shall be free from injurious defects.

STEEL CASTINGS

Effect of Element Additions. The elements always contained in steel and usually determined by analysis are carbon, manganese, silicon, phosphorus, and sulphur. Alloy additions consist of nickel, chromium, molybdenum, vanadium, copper, titanium, and aluminum.

Carbon. In general, tensile strength and yield point increase and ductility decreases with increasing carbon content.

Tables 2 and 3. The properties and composition of special alloy cast irons used for wear, heat, and oxygen resistance are given in Table 4.

Table 4. Special Alloy Cast Irons

Commercial name or type	Average composition, percent						Mechanical properties		Resistant to
	Total carbon	Ni	Cr	Si	Mo	Other elements	Brinell No.	Tensile strength, psi	
Ni-Hard	3.50	4.50	1.50	0.60	....	....	675	35,000	Wear
Nitrided cast iron	3.00	......	1.25	1.25	0.15	Al 1.00	...	......	Wear
Cr	3.70	......	0.75	1.10	....	Mn 0.50	250	35,000	Wear
High Cr	2.60	......	26.70	0.33	....	....	477	......	Wear
Oil quenched	3.00	1.20	0.40	1.50	....	....	480	......	Wear
Ni-Resist	3.00	13.50	3.20	1.60	....	Cu 6.48	140	24,000	Heat and corrosion
Ni-Mo	3.25	1.00	......	2.00	0.40	....	156	26,000	Heat
Cr	3.00	......	1.00	2.00	....	....	...	40,000	Heat
17 % Cr	2.00	......	17.00	1.75	....	....	420	70,000	Oxidation

Specifications for Castings

Standard Specifications for Gray-iron Castings

(Summary of A.S.T.M. Specifications A48-36)

1. These specifications apply to gray-iron castings where strength is a consideration. It is a purpose of these specifications to classify cast irons in respect to tensile strength.

2. Tensile Strength Classification. Gray-iron castings conforming to these specifications shall be known and listed by classes according to minimum tensile strengths of test bars, as follows:

Class	20	25	30	35	40	50	60
Min tensile strength, psi	20,000	25,000	30,000	35,000	40,000	50,000	60,000

3. Transverse tests shall be optional, and the following minimum breaking loads are specified:

Class		20	25	30	35	40	50	60
Min breaking load at center, lb	A	900	1,025	1,150	1,275	1,400	1,675	1,925
	B	1,800	2,000	2,200	2,400	2,600	3,000	3,400
	C	6,000	6,800	7,600	8,300	9,100	10,300	

4. The transverse breaking loads are based on actual averages of transverse test loads for each casting. It is not implied by these specifications that the ratio of tensile strength to transverse load is constant.

5. When an iron is specified by class and the transverse test bar fails to meet the load requirements, then the manufacturer shall have the right to have tested a tension-test specimen machined from a broken end of the transverse bar. In the event that this tension specimen meets the requirements of the specified class, the class requirement shall be considered as having been met, irrespective of the transverse breaking load.

6. Test bars shall be cast separately from the casting. Three standard sizes of transverse test bars are available under the specifications.

Transverse test bar A is 0.875 in. diam, 15 in. long, and 12 in. between supports. Transverse test bar B is 1.20 in. diam, 21 in. long, and 18 in. between supports. Transverse test bar C is 2.00 in. diam, 27 in. long, and 24 in. between supports.

7. The test bars shall be made under the same sand conditions as the castings and shall receive the same mechanical and thermal treatment as the castings.

to hinder contraction. Pattern makers in general use a "pattern-maker's" shrinkage of ¼ in. per ft for open construction and $\frac{3}{16}$ to ⅛ in. for hindered contraction. The solidifying contraction must be controlled by the use of risers or sink heads which are reservoirs of metal placed at points of final solidification within the casting in order to prevent shrinkage cavities and internal hot tears. Mold relieving methods must be provided for in order to prevent the external hot tears and cracking of members resulting from high stresses due to hindered contraction.

Steel castings are usually heat-treated to relieve casting strains and to produce a grain structure that gives higher ductility and impact resistance than that found in the as-cast material.

Heat-treatment. Many desired varieties of structure in cast steel of any grade may be developed by heat-treatment. In general, the normal run of carbon cast-steels receives the single normalizing or single annealing heat-treatment. To obtain the correct structural condition throughout the

Table 5. Approximate Chemical Analyses and Quenching and Tempering Temperatures for Alloy Castings

Steels 1–3 are vanadium; 4–6 chromium; 7–9 chrome-vanadium; 10–12 chrome-nickel; 13–15 chrome-molybdenum; 16–19 manganese.
(Max S, 0.060 percent. Max P, 0.050 percent.)

Steel No.	C	Mn	Si	Cr	Other constituents	Quenching temperatures, F	Maximum tempering temperature, F	Cool in: A = air F = furnace
1	.25–.35	.60–.90	.20–.50[1]	....	V, .15–.20	1550–1600[6]	1300	A or F
2	.35–.45	.60–.90	.20–.50[1]	....	V, .15–.20	1500–1550[6]	1300	A or F
3	.45–.55	.60–.90	.20–.50[1]	....	V, .15–.20	1475–1500[6]	1300	F
4	.25–.35	.60–.90	.20–.60	.80–1.10	....	1575–1650[6]	1275	A or F
5	.35–.45	.60–.90	.20–.60	.80–1.10	....	1525–1575[6]	1275	A or F
6	.45–.55	.60–.90	.20–.60	.80–1.10	....	1500–1525[6]	1275	F
7	.25–.35	.60–.90	.20–.60	.80–1.10	V, .15–.20	1575–1650[6]	1275	A or F
8	.35–.45	.60–.90	.20–.60	.80–1.10	V, .15–.20	1525–1575[6]	1275	A or F
9	.45–.55	.60–.90	.20–.60	.80–1.10	V, .15–.20	1500–1525[6]	1275	F
10	.25–.35	.60–.90	.20–.60	.50–.90	Ni, 1.25–1.75	1515–1550[5]	1250	A or F[2]
11	.35–.45	.60–.90	.20–.60	.50–.90	Ni, 1.25–1.75	1490–1515[5]	1250	A or F[3]
12	.45–.55	.60–.90	.20–.60	.50–.90	Ni, 1.25–1.75	1475–1490[5]	1250	F[2]
13	.25–.35	.60–.90	.20–.60	.70–1.00	Mo, .25–.35	1575–1650[4]	1300	A or F[3]
14	.35–.45	.60–.90	.20–.60	.70–1.00	Mo, .25–.35	1525–1575[5]	1300	A or F[3]
15	.45–.55	.60–.90	.20–.60	.70–1.00	Mo, .25–.35	1500–1525[5]	1300	F[3]
16	.25–.35	1.00–1.50	.20–.50	....	....	1600–1650[4]	1250	A or F
17	.25–.35	1.50–1.75	.20–.50	....	....	1600–1650[4]	1250	A or F
18	.35–.45	1.00–1.50	.20–.50	....	....	1550–1600[4]	1250	A or F
19	.35–.45	1.50–1.75	.20–.50	....	....	1550–1600[4]	1250	A or F

[1] 0.20 to 0.30 Si preferred.
[2] Shock resistance is sometimes improved by oil-quenching from the tempering temperature.
[3] A long soak is required at the tempering temperature.
[4] Water-quenched.
[5] Oil quenched.
[6] Either water- or oil-quenched.

Softeners are the higher silicon pig irons that are added to the mixture to increase the general silicon content. These irons run from 4 to 8 percent silicon and should be used with caution. It is far better to charge pig irons of nearly the right silicon than to use extremes.

Fluxes. As the pig iron and scrap charged into the cupola carry with them sand and rust, and as the ash of the fuel collects in the melting zone, it is essential for long heats that a flux be added to collect this material into a slag that can be drained off. Usually, slagging off a cupola is not resorted to for heats shorter than an hour. Limestone is the universal flux and should contain the maximum possible amount of carbonate of lime. Fluor spar is a flux that thins the slag very much. It should be used sparingly and in connection with limestone on account of its expense and powerful action on the cupola lining. The weight of the flux used is about 2 percent of the weight of the metal.

Melting Processes

The melting may be done in crucibles, in the cupola, in the air furnace, or the electric furnace.

The crucible process is used occasionally in making small amounts of cast iron. The absence of contact between metal and fuel makes this an ideal process, but the output is small and the fuel cost is high.

The cupola is the oldest unit for melting pig irons and scrap and today remains the most widely used. Initial investment is comparatively low, and upkeep expense is reasonable. Operating costs for fuel, refractories, labor, and power are less than for other iron-melting mediums. Anything from all pig iron to all steel charges can be melted. Melting is rapid and continuous. With careful supervision, temperatures of 2750 to 2850 F can be maintained consistently.

Uniformity of composition and physical properties can be held within fairly close limits. Once the metal is melted, readjustments in composition cannot be made. In this respect, the cupola is inferior to the other type of melting units, in which the carbon content can be adjusted subsequent to melting down. Cupola melting of uniform- and high-quality metal requires more knowledge and skill than are demanded by most other iron-melting processes.

The cupola is a shaft furnace, into which a fuel bed is introduced and upon which alternate layers of metal and fuel are charged. Air for combustion is supplied through tuyères near the bottom, placed just high enough to allow the space below to serve as a storage for the required quantity of molten iron. The cupola should be charged in even layers. The bed of fuel is first, with the pig iron spread uniformly upon this, and the scrap iron above. If steel is used, it is spread directly upon the coke below the pig iron. Melting should start about 6 min after air is allowed to enter through the tuyères. The bed of coke is previously heated.

It takes roughly 30,000 cu ft of air to melt 1 ton of iron, and the melting rate depends upon the rate of air supply. Definite sizes of cupolas are best adapted for given melting rates per hour. It is not wise to exceed greatly the rates of melting indicated below, or the quality of the metal produced will suffer.

Diam of cupola inside lining, in.	24	30	36	42	48	54	60	66	72	78	84	90
Melting rate, tons per hr.....	1.5	3	4.5	6	8	10	13	16	19	22	26	30

The heat treatments of alloy castings consist of normalizing, annealing, quenching, and tempering. The last two develop the maximum properties; intermediate properties may be secured by normalizing and tempering only. Full annealing produces only a slight increase in the physical properties not justified by the extra cost of the alloys, and in general should be employed only to increase the machinability of the steel, or for other special purposes.

The range of **mechanical properties** of cast steel is indicated in Tables 6 and 7.

Table 7. Alloy Compositions and Mechanical Properties Obtainable without Liquid Quenching

(From Metals Handbook, 1938.)

Type	C	Mn	Si	Cr	Ni	Va	Mo
C–V	0.22–0.40	0.58–0.96	0.31–0.40	....	....	0.18–0.26	
C–Cr	0.25–0.45	0.60–0.90	0.20–0.60	0.80–1.10			
C–Ni	0.20–0.30	0.80–1.00	0.25–0.45	....	2.00–2.25		
C–Mo	0.25–0.30	0.80–1.00	0.30–0.65	....	....	....	0.30–0.50
C–Mn	0.30–0.35	1.20–1.60	0.30–0.40				

	Heat-treatment		Properties			
Type	Normalize deg F	Tempered deg F	Yield point, 1,000 psi	Tensile strength, 1,000 psi	Percent elongation in 2 in.	Reduction of area, percent
C–V	1650	1200	46–64	79–94	23–30	30–50
C–Cr	1650–1550	700–1200	77–67	120–110	14–17	20–30
C–Ni	1650	1200	55–65	90–105	22–28	42–55
C–Mo	1700	1200	70–75	90–100	20–25	55–60
C–Mn	1650	1250	50–55	83–93	24–32	50–55

Mechanical properties are determined on a standard 0.505 in. diam test specimen. *Specimens are machined from test coupons about 1 in. thick and* $1\frac{1}{4}$ in. deep, usually attached to the casting. The influence of the mass of a casting on its mechanical properties is shown in Fig. 2.

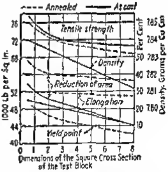

Fig. 2.—Influence of Size on the Mechanical Properties of the Center of a Block of Carbon Cast Steel (C, 0.26; Mn, 0.63; Si, 0.23).

Steel Foundry Practice

The processes used for melting steel for castings are the same as for ingots. The bulk of the tonnage of steel castings is produced *in the acid open-hearth furnace; next* comes the acid electric furnace. The basic electric furnace and the basic open-hearth are used when it is desirable to eliminate both phosphorus and sulphur from the metal during steelmaking. High-frequency induction furnaces are finding use in the production of high-alloy castings, especially those of low carbon content.

By **oxidation of the charge** during melting the carbon, silicon, and manganese are reduced. The carbon elimination is closely watched and is

After heat-treatment, the percentage of the various elements in castings of the foregoing composition will be practically the same, except in respect to the total carbon content, which instead of being in the combined form will be present not only in the form of temper carbon but considerably less in amount than was present in the hard-iron casting. In a ⅝ in thick section, the average loss will be around 0.55 points. Inasmuch as the matrix of malleable iron consists of silicoferrite, a phosphorus content even higher than 0.16 does not affect its toughness and ductility adversely, and if the manganese and sulphur are properly balanced, a sulphur content as high as 0.12 percent is not objectionable.

Annealing or Malleableizing. In the annealing of the original hard castings to produce the malleable-type structure, the castings are placed in metal boxes, usually surrounded with a packing material, supporting the castings in a manner that will prevent warpage. When a muffle furnace is used, the castings are annealed without being packed into boxes. The castings should be raised fairly rapidly to the annealing temperature, which ranges from 1560 to 1600 F. This temperature is maintained for about 60 hr. The temperature then is lowered at a rate of not more than 10 deg per hr until it reaches 1250 F, at which temperature the oven doors may be opened and the castings allowed to cool at a faster rate.

Owing to the high percentage of silicon and carbon, there is scant danger of oxides being present in the molten metal during melting or at the time the furnace is tapped, for which reason no ferroalloys or deoxidizing agents need be added to the bath. For this same reason, coupled with the fact that a relatively low pouring temperature is used, malleable iron is comparatively free from the presence of **non-metallic inclusions** and **blowholes**. The surfaces of the castings are very smooth, with corners sharp and letters or ornamental designs well defined. The examination of assemblies made up of malleable iron that have been exposed to the weather from 20 to 40 years shows that the **rust-resistant** properties of the castings are excellent.

Owing to the fact that the means used for converting the hard-iron castings into the finished product is by heat-treatment at a temperature not very much higher than the critical range, grain size is relatively small, and as the rate of cooling through and under the range is extremely slow, the castings are free from internal strains.

Although malleable castings are **readily machined**, they **cannot be** satisfactorily **welded**, for at a welding temperature the temper carbon will revert to the combined form with the result that most of the tensile properties will be adversely affected.

Malleable castings are extensively used for such purposes as bridge railings, bridge dams, lamp posts, drainage inlets, jack frames, stadium-seat brackets, curb armors, cable brackets, highway road supports, motor-truck parts, kevels, bitts, stair treads, bolts and nuts, awning parts and connecting rods. The castings may vary in weight from a few ounces to half a ton.

Cupola malleable, as distinguished from the regular product, has reference to the process in which the melting unit is the cupola. In this case, the mixture is melted in the presence of coke, instead of in a hearth out of contact with solid fuel, and consequently it is practically impossible to secure a low carbon content in the hard iron castings. The product is therefore inferior even to the lower grade of the A.S.T.M. specification.

Cupola malleable, however, is largely used in the making of fittings of all kinds.

The test coupons shall be cast separately or attached to one or more castings, of the kind ordered by the purchaser, in each melt used for manufacture of the purchased material.

Steel used for the castings shall conform to the following minimum requirements as to tensile properties:

Grade		Tensile strength, min psi	Yield point, min psi	Elongation in 2 in., min percent	Reduction of area, min percent
A-1	Unannealed	60,000	30,000	22	30
A-2	Normalized	60,000	30,000	26	38
A-3	Full annealed	60,000	30,000	24	35
B	Normalized	70,000	38,000	24	36
B-1	Full annealed	66,000	35,000	22	33
B-2	Full annealed	70,000	35,000	20	30
H	Normalized	60,000	43,000	17	25
H-1	Full annealed	80,000	40,000	17	25

7. The castings shall be free from injurious defects.

8. Notes regarding Selection and Characteristics of Grades. Grades A-1, A-2, and A-3 are produced with such low carbon contents as to make the material very soft and easily machinable. These grades are effectively adaptable for fusion fabrication. Grade B (normalized) is used in making a great many carbon-steel castings purchased for miscellaneous industrial application. Grades B-1 and B-2 (full annealed) are for castings which are to be full annealed either to obtain desired properties or to prevent internal stresses. Grade H (normalized) and Grade H-1 (full annealed) are used for

Table 8. Properties of Corrosion- and Heat-resistant Cast Steels
(Strauss)

Chemical composition, percent					Physical properties				Melting temp, deg F	Coeff of expansion $\times 10^6$, per deg F	Max temp for safe use, deg F
C	Cr	Ni	Mn	Si	Tensile strength, 1,000 psi	Yield point, 1,000 psi	Elong in 2 in., percent	Reduction of area, percent			
					Fe-Cr Alloys						
0.10	12.0	....	...	1.0	75–95	45–60	18–24	30–50	2730–2480	0.06	1400
0.12	19.0	....	...	1.0	75–100	45–70	10–20	15–35	2750–2500	0.06	1600
0.50	28.0	....	...	1.0	40–60	30–45	0–2	0–5	2640–2460	0.06	1800–2100
					Fe-Cr-Ni Alloys						
0.20	8.0	20.0	...	1.0	75–85	40–50	20–30	20–30	2710–2600	0.1	1400–1800
0.15	18.0	8.0	...	1.0	70–60	25–40	40–75	40–75	2670–2550	0.09	1400–1700
0.06	18.0	8.0	...	1.0	70–80	25–30	40–75	40–75	2670–2550	0.09	1400–1700
0.50	22.0	22.0	...	1.5	60–70	45–60	5–10	3–10	2580	0.09	2000
0.30	29.0	9.0	...	1.5	70–80	55–65	1–3	1–3	2730–2550	0.08	2000
0.30	26.0	16.0	...	1.5	......	.....	.....	.....	....	0.08	2100
0.50	18.0	36.0	...	2.0	60–70	40–55	1–8	2–10	2700–2550	0.08	1830–2000
					Nickel-rich Alloys						
0.60	13.0	62.0	1.0	1.5	60–75	35–45	1–5	1–5	2550–2300	0.07	1470–2100
0.40	20.0	64.0	1.5	1.5	50–70	40–50	1–3	1–3	2620–2280	0.07	2000–2300

Manganese acts as a deoxidizer on the liquid steel. It also combines with sulphur to form manganese sulphide and prevent "red-shortness." Besides increasing soundness, it increases toughness, tensile strength, and yield point. The normal manganese content of carbon steel castings is 0.50 to 1.00 percent. When cast steel contains over 1.00 percent manganese, it usually is considered to be an alloy cast steel.

Silicon acts as a deoxidizer and promotes soundness in steel. In normal amounts, 0.20 to 0.75 percent, it has little apparent effect on the tensile properties of steel at room temperature. In some grades of steel, silicon is added in larger than normal amounts as an alloying element, particularly in combination with other elements.

Phosphorus. In amounts up to a maximum of 0.05 percent, phosphorus has no practical effect on the structure or properties of steel.

The **sulphur** content should be kept as low as possible. When steel is deoxidized with critical amounts of aluminum, the sulphide inclusions are responsible for a falling off of the ductility.

When present in moderate amounts (0.50 to 5.00 percent), **nickel** produces a fine-grained structure and imparts strength, toughness, and, to a lesser degree, hardness, without decreasing ductility.

Although **chromium** is generally used in conjunction with an additional alloying element, simple chromium cast steels may be given properties far superior to those of carbon steels. With a given carbon content, chromium increases the tensile strength and yield point, but at the expense of some ductility. Castings subject to abrasion, corrosion, or high operating temperatures give much better service on the addition of chromium.

Because of their air-hardening qualities, **molybdenum** steels develop excellent properties in castings that must retain considerable strength and hardness, but which, because of large size or intricate design, cannot be liquid quenched. Representative compositions show molybdenum to be present from 0.20 to 0.50 percent. It finds application in castings requiring high tensile properties at moderate temperatures.

Vanadium is added to cast steel in amounts from 0.10 to 0.30 percent to produce grain refinement upon heat-treatment. Vanadium cast steels show high elastic properties and excellent resistance to impact.

The use of 0.50 to 2.00 percent **copper** in normalized copper-cast-steels when reheated to 850 to 1050 F, shows an increase in tensile strength, yield point, and hardness due to a precipitation hardening action.

Titanium and **aluminum** are excellent deoxidizers and scavengers. They are added as a final deoxidizer to the molten metal. Aluminum should be added at the rate of 2 lb or over per ton of metal in order to prevent low ductility.

Physical Characteristics

Steel that is made for steel castings should be thoroughly deoxidized, and care should be exercised in mold condition and metal transfer so that the casting will not have porosity.

Immediately upon the introduction of the molten steel into the mold cavity, two changes in volume of the metal take place. One occurs as the metal solidifies and amounts to about a 3 percent contraction and is known as the solidifying contraction. The other occurs during the cooling of the casting to room temperature and is referred to as the solid contraction. The amount of solid contraction that takes place depends on the shape and complexity of the casting, the amount of cored construction, etc., which acts

be fed directly by an outside reservoir, instead of through the legs, the section at S_2 will be sound and contain no cavity. As the location of feed heads depends largely on the foundryman, it is safer to design a casting so as to avoid the necessity for their use.

Joined Sections. There are five ways in which sections may be joined, L, T, V, X, and Y.

In L sections, if the outside corner at the junction is maintained, an increasing radius at the inner corner will bring about an increasing size of defect. If the section is uniform throughout, a defect will be found if the inner radius is small. If the inner radius is increased, but a uniform section is maintained, the defect will become smaller until it develops into centerline weakness, a condition likely to be found in uniform sections of any length. L sections should have from ½ to 1 in. inside radius; in a few cases, a radius up to 3 in. may be used. Making the section at the junction slightly smaller than that of the arms appears to give the best results.

To avoid a contraction cavity in a T section, it is necessary to core a hole at the center of the junction of the two members. Depressions in the arm of the T, although not eliminating the defect, reduce it markedly. A radius of ½ to 1 in. is recommended.

A uniform V section will not be free from contraction cavities; a slightly reduced section at the junction of the members is necessary. An inner radius of not less than 1 in. is recommended.

An X section cannot be designed free from a contraction cavity if the section is fed only through the arms. The defect can be made quite small by designing the joining section with a cored hole. By offsetting the arms of the X the hot spot would be reduced. A section having one arm completely offset allows the foundryman to use external chills to advantage.

In a Y section, the defect is about the same size regardless of the details of design. Arrangements should be made for feeding the junction.

A study of these sections leads to the following conclusions: In unfed joining sections in L or V shapes, all sharp corners at the junction should be replaced by radii so as to make this section slightly smaller than that of the arms. In X sections, two of the arms should be offset considerably. In all joining sections, sharp corners at the junctions should be replaced by radii. In the case of unfed T and X sections, these radii should not be large.

castings, they are usually held at temperature for approximately 1 hr for each inch of the thickness of the largest section of the castings.

In quality castings, a double heat-treatment is often given. An annealing treatment at a high temperature of 1600 to 2000 F is used for a homogenizing treatment which is followed by a second treatment at a temperature only slightly above the critical temperature range to refine the grain.

When exceptional mechanical properties, high degree of structural refinement, and resistance to wear are required, heating, quenching, and tempering will greatly assist in producing the desired results.

The following is the recommended practice for the heat-treatment of carbon-steel castings and alloy-steel castings of the A.S.M. (Metals Handbook, 1936).

Single normalizing or single annealing of carbon and alloy steel castings with a carbon range of 0.15 to 0.40 (0.40 to 0.50) should be carried out at a temperature of 1600 to 1700 (1550 to 1600) F. The cooling should be in still air for normalizing and in a furnace or other medium for slow cooling until the temperature is 800 F or below, for annealing.

Table 6. Properties Obtainable in Carbon Steel Castings
(From Metals Handbook, 1936.)

	As cast	Full annealed, 1650 F	Quenched, 1650 F, tempered, 500–1300 F
Analysis: C, 0.15–0.30; Mn, 0.50–0.90; Si, 0.20–0.60			
Tensile, psi	42,000–74,000	60,000–78,000	68,000–125,000
Yield, psi	15,000–37,000	34,000–44,000	39,000– 97,000
Elongation, 2 in., percent	25–18	33–24	33–10
Reduction of area, percent	40–21	55–40	65–20
Analysis: C, 0.30–0.40; Mn, 0.50–0.90; Si, 0.20–0.60			
Tensile, psi	74,000–86,000	86,000–97,000	92,000–140,000
Yield, psi	37,000–55,000	55,000–63,000	70,000–120,000
Elongation, 2 in., percent	24–16	25–22	19–5
Reduction of area, percent	31–21	41–31	28–19
Analysis: C, 0.40–0.60; Mn, 0.60–0.90; Si, 0.20–0.60			
Tensile, psi	79,000–87,000	89,000–100,000	[a] 80,000–120,000
Yield, psi	35,000–40,000	45,000– 50,000	[a] 50,000– 90,000
Elongation, 2 in., percent	7–3	16–7	[a] 25–8
Reduction of area, percent	5–3	20–8	[a] 45–15

[a] Quenched, 1575 F; tempered, 800–1300 F.

The quenching temperature of carbon-steel castings of 0.15 to 0.30 (0.30 to 0.40) [0.40 to 0.60] carbon range should be 1600 to 1650 (1550 to 1600) [1500 to 1550] F. The heating time should be ½ hr per in. diam or thickness, the quenching medium water at 70 (125) [125 or oil] F. Tempering should last at least 3 hr and should be at 800 to 1350 F.

Alloy castings should be cooled in the molds to a temperature below the critical range, and should be shaken out as soon as they become black and cleaned rapidly while still warm. They should go to the annealing furnace as hot as possible.

Table 1. Physical Constants of Principal Alloy-forming Elements

Element	Melting point, deg F	Boiling point 1 atm pressure, deg F	Specific heat, at about 70 F	Latent heat of fusion, Btu per lb	Latent heat of vaporization, Btu per lb	Linear coefficient of thermal expansion per deg F at about 70 F × 10⁶	Density, lb per cu ft	Specific gravity	Initial cubic compressibility per kg per sq mm × 10⁷	Modulus of elasticity, psi × 10⁻⁶	Electrical resistivity microhm-cm at 70 deg F	Atomic weight	Atomic number	Symbol
Aluminum	1220.4	3270	0.224	167.4	3506	12.9	168.6	2.70	13.43	10	2.655	26.97	13	Al
Antimony	1167	2624	0.049	69	670	5.5	413	6.62	16.48–5.26L	11	39	121.76	51	Sb
Arsenic	(1503)	1165S	0.082	.....	800	2.14	357	5.72	44	.....	46	74.91	33	As
Barium	1299	2980	0.068	.....	1130	10.5	225	3.6	101.9	.....	.....	137.36	56	Ba
Beryllium	2345	5040	0.400	500	.....	6.8	116	1.85	8.55	43	6.76	9.02	4	Be
Bismuth	520	2840	0.030	23	400	7.47	612	9.80	15.2–6.62L	4.6	120	209.00	83	Bi
Boron	417	4620	0.309	.....	.....	1.1	144	2.3	5.51	.....	10^{12}	10.82	5	B
Cadmium	610	1415	0.055	21	410	16.6	540	8.65	18.3–2.1L	.....	6.85	112.41	48	Cd
Calcium	1564	2710	0.15	.....	4280	12.2	97	1.55	56.97	.....	4.6	40.08	20	Ca
Carbon	.....	8700	0.165	.....	.....	0.67	141	2.25	1.8	0.7	1000	12.010	6	C
Cerium	1175	2550	0.05	.....	.....	.....	432	6.92	45.63	.....	78	140.13	58	Ce
Chromium	3350	4500	0.12	57	2650	3.4	446	7.14	5.19	.....	13.1	52.01	24	Cr
Cobalt	2715	5250	0.105	110	2770	7.00	550	8.80	5.39	.....	6.36	58.94	27	Co
Columbium (niobium)	3550	>6000	0.064	.....	.....	4.0	535	8.57	5.70	.....	20	92.91	41	Cb(Nb)
Copper	1981.4	4220	0.0918	91	3160	9.12	558	8.94	7.19	17	1.682	63.57	29	Cu
Gold	1945.4	5370	0.0310	29	803	8.0	1,206	19.32	5.77	11.3	2.42	197.2	79	Au
Hydrogen	−434	−423	3.141	27	.....	.....	53×10^{-4}	0.084×10^{-3}	.....	.....	.....	1.008	1	H
Indium	312	>2650	0.057	.....	.....	18.3	456	7.31	25.0	.....	8.4	114.76	49	In
Iridium	4450	8800	0.0309	.....	.....	3.56	1,399	22.4	2.68	74.7	6.08	193.1	77	Ir
Iron	2795	5430	0.122	117	2730	6.28	492	7.87	5.87	30.1	9.80	55.84	26	Fe
Lead	621.2	3000	0.0302	11.3	401	16.4	708	11.344	23.72	2.56	20.46	207.21	82	Pb
Lithium	367	2500	0.95	59	10,540	31	33.1	0.53	86.9	.....	8.5	6.940	3	Li

allowed to proceed until the desired low point is reached. The bath is then usually recarburized and finally is deoxidized, generally by additions of ferrosilicon and ferromanganese or of a silicon-manganese alloy. Titanium or aluminum are often used as final deoxidizers after the addition of the preceding deoxidizers.

In the **basic process**, the phosphorus is oxidized and becomes part of the slag, which is removed. Reducing conditions then are created by the reactions caused by a second slag, which reduces the sulphur to low amounts. In the **acid process**, sand and fluor spar are used for slag formation; in the basic process, lime and fluor spar are used.

Steel castings are made in sand molds almost exclusively, although steel has been cast centrifugally in chill molds for the manufacture of guns. Two general types of sand molds are used—dry sand and green sand. In the former case, the molds are baked in an oven prior to filling them with molten metal. Considerable care must be given to the construction of molds and cores and to the study of molding sands. The sands must be refractory, strong, permeable, and workable. Many sands are used, from the natural sands to those that are synthetically bonded. Coarse sands, fine sands, well-rounded sands, angular sands, and well-graded sands are all useful for particular purposes.

Specifications for Carbon-steel Castings for Miscellaneous Industrial Uses

A.S.T.M. A27-39 (In summary form)

1. These specifications cover carbon-steel castings to be used for miscellaneous industrial purposes, as distinguished from carbon-steel castings made for railroad and high-temperature applications. Ten grades of castings are covered, *viz.:*

Grade N-1. Castings not required to be physically tested or heat-treated.
Grade N-2. Castings not required to be physically tested but required to be annealed.
Grade A-1. Castings required to be physically tested but not required to be heat-treated.
Grade A-2, B, and H. Castings required to be annealed and physically tested.
Grade A-3, B-1, B-2, H-1. Castings required to be full annealed and physically tested.

2. The steel shall be made by one or more of the following processes: open-hearth, electric-furnace, converter, or crucible.

3. Unless otherwise specified, all castings may be annealed one or more times and may be given a supplementary heat-treatment by tempering or drawing. Castings in any grade except A-1, A-3, B-1, B-2, and H-1 may be given a supplementary heat-treatment that includes accelerated cooling by liquid quenching, liquid spraying, or air blasting.

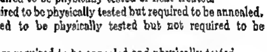

FIG. 3.—Tension Test Specimen for Cast Steel.

4. All castings that are to be heat-treated in any manner shall have been allowed to cool after pouring to a temperature below the critical range.

5. Chemical Composition. Steel used for the castings shall conform to the following maximum percentage requirements as to chemical composition: Mn, 1.00; P, 0.05; S, 0.06.

An analysis of each melt of steel shall be made by the manufacturer to determine the percentages of carbon, manganese, silicon, phosphorus, and sulphur. Check analysis may be made from the broken tension test specimen.

6. Tension Tests. Tension test specimens for castings of the grades that require physical testing shall conform to the dimensions shown in Fig. 3.

With increase of the annealing temperature beyond this point, the average grain size increases and the grains become larger and larger until the melting point is reached.

Starting from any annealed grain size the hardness, strength, and yield point can be increased progressively by cold working with concomitant decrease in ductility. The effects of cold working can be relieved by annealing at a suitable temperature. These are the only ways in which the properties of most non-ferrous alloys can be changed.

Precipitation Hardening. Many aluminum alloys and some alloys of copper and of nickel are hardened by a heat-treatment which is associated with precipitation from solution of a second constituent. These alloys, after heating, consist of homogeneous grains of solid solution, comparatively soft and indistinguishable microscopically from a pure metal. If cooled very slowly, the alloy will deposit crystals of a second constituent, the amount of which will increase as the temperature decreases. Rapid cooling from the solid solution zone will retain the alloy at room temperatures as a solid solution, but if the alloy is heated to a suitable temperature, particles of a new phase will form, and, in time, will grow to visible size. At some stage in this precipitation process, the hardness, strength, and (particularly) yield point of the alloy will be considerably improved. If the heat-treatment is carried out too long, the alloy becomes soft again. Hardening by precipitation involves (1) **solution heat-treatment** followed by rapid cooling and (2), precipitation heat-treatment or **aging treatment.** The temperature and time of the latter must be closely controlled to obtain the proper results. To some extent, precipitation hardening may be superimposed upon the hardness due to cold work. Precipitation-hardened alloys have an unusually high ratio of proportional limit to tensile strength, but the endurance limit is not raised nearly to the same extent.

Significance of Yield Point. The term "yield point" when applied to non-ferrous metals has a significance different from the yield point of mild steel. Few non-ferrous materials have a discontinuous break in the stress-strain curve such as produces the familiar drop of the beam in testing steel. Annealed alloys pass through a period of rather more rapid extension, but there is rarely the abrupt change that occurs with steels. Cold-worked materials commence imperceptibly to pass from the proportional state into the plastic range, and the whole stress-strain curve up to maximum stress is smooth. In testing material of this sort, two arbitrary figures are referred to as the yield strength. One is obtained by the **offset method** in which a stress-strain curve is plotted and the stress read at which the curve intersects a line parallel to the initial modulus line but distant from it by a specified amount of strain (usually 0.2 percent). The other method involves the determination of that stress which produces a specified (usually 0.5 percent) **total elongation** under load. This involves the simplest testing technique and is perfectly satisfactory for acceptance and comparative tests on a given class of material. A yield point on a non-ferrous metal without specification of offset or of strain under load is valueless.

Aluminum and Its Alloys[a]

Aluminum owes most of its applications to its light weight and the relatively high strength of its alloys, although other uses depend upon its comparatively

[a] Largely based on information supplied by Aluminum Company of America. See "The Aluminum Industry," by Edwards, Frary and Jeffries (New York, 1930) for a detailed treatment of the subject.

purposes that call for higher strength and less ductility than are typical of the other grades.

Other steel-casting specifications of the A.S.T.M. are as follows:

Alloy-steel Castings for Structural Purposes: A148-36.
Alloy-steel Castings for Valves, Flanges and Fittings for Temperatures from 750 to 1100 F: A157-39.
Austenitic Manganese Steel Castings: A128-33.
Carbon-steel and Alloy Steel Castings for Railroads: A87-36.
Carbon-steel Castings for Valves, Flanges and Fittings for High Temperature Service: A95-36.
20 Per Cent Chromium, 9 Per Cent Nickel Alloy Steel Castings (Tentative): A198-39.
Carbon-steel Castings suitable for Fusion Welding for Miscellaneous Industrial Uses: A215-40T.
Carbon-steel Castings for Fusion Welding for Service at Temperatures up to 850 F.
Alloy-steel Castings Suitable for Fusion Welding for Service at Temperatures from 750 to 1100 F.
Chromium Alloy Steel Castings: A221-39.
Chromium-nickel Alloy Steel Castings: A222-39.
Nickel-chromium Alloy Steel Castings: A223-39.

Design of Castings

Defects in castings result generally from poor design. **Hot tears** are due to large temperature differences in castings; the cracks occur at abrupt changes in section and at sharp angles. **Shrinkage cavities** are due to insufficient metal to care for metal contraction at the time of casting solidification. They are found in sections that must be fed through smaller sections.

The avoidance of hot tears requires the elimination of hot spots under stress. Hot spots occur in sections of extra mass and at positions of joining sections such as the hub of a cast wheel and the flanges or seat of a valve. All sections in a casting should, as far as possible, be made of a uniform thickness.

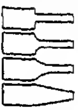

FIG. 4.—Designs Considered with Reference to Hot Tear Formation.

When the design cannot be changed to remove the hot spot, the stresses acting on the hot spots or other points of stress centralization should be diminished. Stresses usually concentrate at abrupt changes in section or at sharp corners. If liberal **fillets** replace sharp corner-junctions, the stresses will be reduced, but an increase in the radius of the fillet increases the size of the possible shrinkage cavity. Figure 4 shows improvements in design to reduce the probability of hot-tear formation.

FIG. 5.—Hot Spots.

Excessive stresses in a cast-steel structure can often be avoided by designing it in two or more parts and assembling these by **welding** or **bolting**.

When liquid steel solidifies, it contracts about 3 percent in volume, and since it solidifies progressively from the mold surface toward the center of the mold cavity, a **pipe** or **contraction cavity** will result unless the section is fed from a reservoir containing liquid steel. Castings with defects of this type may develop cracks extending from the cavity to the casting face. With an L section as in Fig. 5, a hot spot will be located somewhere near the center of the inscribed circle S_1 if the two arms of the L are of the same thickness. If, however, the L section is in a location where the junction can

Table 2. Aluminum Alloy Castings

A.S.T.M. designation	Aluminum Co. of America designation	Composition, percent[b] (aluminum remainder): Cu	Si	Mg	Fe	Other elements	Typical properties[a]: Treatment	Yield strength, kips	Tensile strength, kips	Elongation in 2 in., percent	Endurance limit, kips	Electrical conductivity, percent, I.A.C.S.
Aluminum-base Alloys for Sand Castings												
	12	8.0	....	....	....	....	SC	14	22	2.0	7.5	37
C	112	7.5	....	....	1.2	Zn 2.0	SC	14	23	2.0	8.5	30
CC	212	8.0	1.2	....	1.0	Zn 0.2	SC	14	22	2.0	7.5	
F	122	10.0	....	0.2	1.2	....	T2	20	25	1.0	9.5	41
							T61	30	36	1.0	...	33
G & GG	195	4.0	....	....	....	....	T4	16	31	8.5	6.0	
							T6	22	36	5.0	6.5	55
							T62	31	40	2.0	7.0	37
H	142	4.0	....	1.5	....	Ni 2.0	SC	24	28	1.0	8.0	36
							T61	32	37	0.5	8.0	37
							T571	28	32	0.5	8.0	
J & JJ	43	....	5.0	....	....	....	SC	9	19	6.0	6.5	37
H	47	....	12.5	....	....	....	SC	11	26	8.0	6.0	40
L	214	....	....	3.8	....	....	SC	12	25	9.0	5.5	55
M	356	....	7.0	0.3	....	....	T4	16	28	6.0	...	39
							T6	22	32	4.0	8.0	39
							T51	20	25	2.0	6.0	43
N	355	1.3	5.0	0.5	....	....	T4	20	30	5.0	...	35
							T6	25	35	3.5	...	36
							T51	23	28	1.5	6.5	43
	406	....	....	....	....	Mn 2.0	SC	9	18	5.0	5.5	
	145	2.5	....	....	1.25	Zn 10.0	SC	22	29	4.0		
	220	....	....	10.0	....	....	T4	25	45	14.0	7.0	
	(3L5)	2.5	....	....	....	Zn 13.5	SC	9	22	3.0		
Aluminum-base Alloys for Permanent Mold Castings												
1 & 1A	B195	4.5	3.0	....	....	....	T4	22	36	7.5	9.5	35
							T6	33	39	5.0	...	37
2	C113	7.5	4.0	....	1.2	Zn 2.0	CC	24	28	0.5		
3	A108	4.5	5.75	....	....	....	CC	16	28	2.0	...	31
4	122	10.0	....	0.25	1.2	....	T52	31	35	0.5		
							T65	36	48	0.0		
							T551	35	37	0.0	8.5	
							T552	32	34	0.0		
5	138	10.0	4.0	0.25	1.2	....	CC	24	28	0.5		

NON-FERROUS METALS

BY

CYRIL STANLEY SMITH

REFERENCES: Sachs and Van Horn, "Practical Metallurgy," A.S.M. "Metals Handbook," A.S.M. Carpenter and Robertson, "Metals," Oxford. Jeffries and Archer, "The Science of Metals," McGraw-Hill. Doan, "The Principles of Physical Metallurgy," McGraw-Hill. Van Wert, "Introduction to Physical Metallurgy," McGraw-Hill. Desch, "Metallography," Longmans. Rosenhain, "Introduction to the Study of Physical Metallurgy," Constable. Stoughton and Butts, "Engineering Metallurgy," McGraw-Hill. *Trans. A.I.M.E. Jour. Inst. Metals. Proc. A.S.T.M.* "Metallurgical Abstracts," Institute of Metals (London). Also, publications of the Copper Development Assoc. (London), the International Tin Research and Development Council (London), and other organizations. For a comprehensive list of trade names, compositions, properties, and manufacturers of commercially available nonferrous alloys see Woldman and Dornblatt, "Engineering Alloys," A.S.M., 1936.

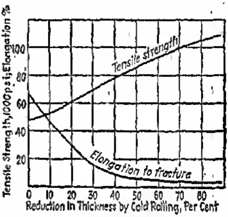

FIG. 1.—Effect of Rolling on Annealed Brass (Cu 72, Zn 28).

Cold Working and Heat-treatment

Heat-treatment. Most of the industrially important non-ferrous alloys are used either "as cast" or in the work-hardened condition. Heat-treatment for the purpose of hardening, as with steel, is not usual, but heat-treatment is nevertheless important. A cast metal consists of an aggregate of grains variously oriented and jutting one against the other. On deformation, the grains deform by a process involving the slip of blocks of atoms over each other along crystallographic planes. As this process of cold working proceeds, further deformation becomes more difficult and the metal is strengthened, hardened, and rendered less ductile. The effect of progressive cold rolling on brass is shown in Fig. 1 and may be considered as typical. With metals like copper, aluminum, and nickel, the added hardness resulting from cold work is stable at room temperature, but with lead, zinc, or tin it will disappear in time. The first effect of heating a cold-worked material is to relieve macro stresses in the object without loss of strength; indeed the strength is often increased slightly and the proportional limit raised considerably. Above a certain temperature, softening commences and proceeds rapidly with increase in temperature (Fig. 2). The old distorted grains break up into new smaller equiaxed unstrained grains. The temperature at which this occurs is lower and the resulting grain size is smaller the more extensive the working of the original piece.

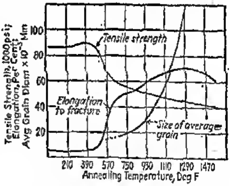

FIG. 2.—Effect of Annealing on Cold-rolled Brass (Cu 72, Zn 28).

Aluminum-copper-zinc Alloys. Alloy 3L5 is typical of a class of alloys used in Europe for general purposes. It is easy to handle, machines well, but, like all alloys containing zinc, is very weak at high temperatures. An American alloy, No. 145 of the Aluminum Co., is similar to this. This alloy is one of the best for producing sand castings with a combination of high strength and ductility without the necessity of special heat-treatment.

Aluminum-manganese alloys with about 2 per cent manganese (406) are used in places where corrosion resistance is the principal requirement, but since the casting qualities are poor, it is to a large extent being replaced by the aluminum-silicon alloys.

Permanent Mold-casting Alloys. Alloys for casting in permanent molds must be free from hot-shortness. The silicon alloys mentioned above are extensively used, also those containing copper, and alloys with both silicon and copper.

In the United States, the greater use of permanent mold castings is for internal-combustion-engine pistons for which light weight, low thermal expansion, and good properties at high temperatures are desirable. Most of these are cast in alloy 4. The more complex alloy 5 was developed primarily for this application and has a thermal expansion about 15 percent less than alloy 4, with approximately the same properties at ordinary and high temperatures and a higher thermal conductivity. Most of these alloys can be heat-treated to improve their properties or to render them less liable to dimensional change during use.

Die-casting Alloys. Aluminum alloys for pressure die-casting must possess considerable fluidity and be free from hot-shortness. The physical properties are usually of less importance than the casting qualities. Absorption of iron is difficult to avoid under operating conditions but should be kept low. There is still some use of the 8 and 12 percent copper alloys, although the trend is largely toward alloys containing silicon. A.S.T.M. specifications now include only those alloys given in Table 2.

Wrought Aluminum Alloys. Wrought aluminum alloys are divided into two classes, those hardened and strengthened by cold work alone and those which owe their improved properties to heat-treatment. Some of the latter alloys age-harden spontaneously at room temperature although others need to be heated moderately for precipitation hardening to occur. Table 3 shows the properties and compositions of wrought alloys of both types.

The most important work-hardening alloys are commercially pure aluminum (2S) or the 1.25 percent manganese alloy (3S). Both of these are available in a wide range of sheet, rod, tube, and wire sizes and extruded shapes. The latter alloy is used extensively for the manufacture of cooking utensils, conduit pipe, and other products where strength and hardness somewhat greater than those of pure aluminum are desired. Other alloys used in this class are the aluminum-manganese-magnesium alloy, 4S, and alloy 52S. These alloys can be hardened by cold work but are not heat-treated except when annealed to remove hardness resulting from cold work.

In England and Continental Europe, some other alloys are used. The most important of these are the alloys of aluminum with comparatively large amounts of magnesium. These are used in the work-hardened condition and possess properties little inferior to alloys of the duralumin type after heat-treatment.

Where a high strength alloy is necessary, it is usual to use an alloy of the **duralumin** type. The most common is 17S. This alloy is readily hot-

Magnesium	1204	2015	0.248	92	2700	14.4	108	1.739	9.84L	6.5	4.46	24.32	12	Mg
Manganese	2268	3905	0.107	116	1880	12.8	462	7.4	7.91	.....	28.	54.93	25	Mn
Mercury	— 38	674.4	0.033	5	127	.....	846	13.55	37.1	.....	95.8	200.61	80	Hg
Molybdenum	4750	8680	0.0601	.....	3180	2.6	637	10.2	3.60	50	5.11	95.95	42	Mo
Nickel	2650	6100	0.111	130	2790	7.2	556	8.90	5.29	30	6.84	58.69	28	Ni
Nitrogen	—346	—320	0.247	11	.....	.....	73×10^{-3}	1.165×10^{-3}	.....	.....	.....	14.008	7	N
Osmium	4890	9000	0.031	.....	.....	3.17	1,405	22.5	.....	.....	9.5	190.2	76	Os
Oxygen	—361	—297	0.218	6	.....	.....	83×10^{-3}	1.33×10^{-3}	.....	.....	.....	16.000	8	O
Palladium	2830	6850	0.0587	65	.....	6.41	750	12.0	5.28	17	10.00	106.7	46	Pd
Phosphorus	111	536	0.177	9	.....	69	114	1.82	.....	.....	.....	30.98	15	P
Platinum	3224	8100	0.0324	48	1140	4.90	1,340	21.45	3.60	2.4	9.83	195.23	78	Pt
Potassium	144	1425	0.134	26	925	46	54.7	0.86	274	.....	7.0	39.096	19	K
Rhodium	3570	7750	0.060	.....	.....	4.55	756	12.44	3.72	42.5	4.51	102.91	45	Rh
Selenium	428	1270	0.080	.....	168	20.6	300	4.81	11.8	.....	10^{11}	78.96	34	Se
Silicon	2580	4150	0.1762	.....	2600	3	145	2.32	3.1	16.4	80,000	28.06	14	Si
Silver	1761	3635	0.057	44	995	10.5	656	10.5	9.87	10.3	1.62	107.880	47	Ag
Sodium	207.5	1640	0.295	49	1960	39.5	61	0.97	142	.....	4.6	22.997	11	Na
Sulphur	235.4	832.3	0.175	17	120	37.5	129	2.07	.....	.....	10^{23}	32.06	16	S
Tantalum	5160	11,000	0.0332	.....	.....	3.6	1,036	16.6	4.79	27	14.6	180.88	73	Ta
Tellurium	846	1990	0.047	13	287	9.3	390	6.24	4.14–27.48L	.....	250,000	127.61	52	Te
Tin	449.4	4175	0.0541	26	690	11.9	450	7.30	6.72–6.02L	7.9	11.5	118.70	50	Sn
Titanium	3270	9210	0.142	.....	.....	3.96	281	4.5	7.97	.....	81	47.90	22	Ti
Tungsten	6120	10,700	0.035	79	2130	2.4	1,206	19.3	3.18	57	5.51	183.92	74	W
Vanadium	3110	5430	0.118	.....	.....	.....	350	5.6	6.09	.....	26	50.95	23	V
Zinc	787.2	1661	0.094	43	765	21.9	446	7.14	12.98–1.95L	6.0	5.92	65.38	30	Zn
Zirconium	3090	9125	0.066	.....	.....	3.5	400	6.4	10.97	.....	41.1	91.22	40	Zr

NOTE: Although most of the physical constants are fairly well established for the common metals, there is considerable uncertainty regarding the others. Purity has a very great effect on electrical resistivity and to a smaller degree on melting point. Specific heat, density, expansion, compressibility, and modulus are comparatively insensitive to purity and treatment.

L = linear compressibility: varies between limits shown in different crystallographic directions.

S = sublimes.

Table 3. Composition and Typical Properties of Wrought Aluminum Alloys.—(*Continued*)

A.S.T.M. specification	Alloy designation[a]	Nominal composition, percent (balance aluminum) Cu	Si	Mn	Mg	Other elements	Temper[b]	Density, lb per cu in.	Yield strength at 0.2% offset, kips	Tensile strength, kips	Elongation, percent in 2 in.	Brinell hardness (500 kg load)	Fatigue endurance limit, kips	Electrical conductivity, % I.A.C.S.
	25S	4.5	0.8	0.8	...	....	T	0.101	30	55	16	100	15	40
	51S	...	1.0	...	0.6	....	O	0.097	6	16	30	28	6.5	55
							W	.....	20	35	24	64	10.5	45
							T	.....	40	48	14	95	10.5	45
	A51S	...	1.0	...	0.6	Cr 0.25	W	0.097	34[c]	44[c]	14[c]	90[c]	10[c]	45
	53S	...	0.7	...	1.3	Cr 0.25	O	0.097	7	16	25	26	7.5	45
							W	.....	20	33	22	65	10	40
							T	.....	33	39	14	80	11	40
	(Y)	4.0	...	...	1.5	Ni 2.0	T	.....	31.4[c]	51.5[c]	15[c]			
	(RR56)	2.0	0.6	...	0.8	Ni 1.3 Fe 1.2	T	0.099	47[c]	60.5[c]	10[c]	129		
	(RR77)	2.2	...	1.0	3.0	Zn 5.0	T	0.101	60.5[c]	73.9[c]	8[c]	160	28	

These are typical values on 0.06 in. sheet. Elongation values on thicker material are higher.

Thermal conductivity (cgs. units) is approximately (% I.A.C.S. × 0.87) + 0.03 for all aluminum alloys not containing over 5 percent silicon.

[a] Follows nomenclature of Aluminum Co. of America, except those in ( ) which are English commercial designations.

[b] O Soft annealed. ½H & H Half hard and hard (about 40 and 80 percent reduction of area of all gages, respectively). T Solution heat-treated and spontaneous aging at room temperature. W Artificially aged. RT Cold-worked prior to aging.

[c] The properties given are specification minimum, not typical values.

worked. It hardens spontaneously at room temperature after solution heat-treatment. Extensive cold working must be done within a few hours after quenching. Cold working following age-hardening is less easy but gives the highest strength obtainable with the alloys; temper RT. For aircraft construction, 17S has been largely superseded by 24S because this alloy has a higher strength. 51S has better fabricating qualities in the quenched condition. Alloy 53S is used on account of its good combination of physical properties and corrosion resistance. It is available in various rolled structural shapes; most architectural extruded sections are made from it.

The last three entries in Table 3 are typical of the strong complex alloys containing magnesium, nickel, and copper used in England. They have not gained favor in the United States.

Most of the heat-treatable alloys are rather less resistant to corrosion than is pure aluminum or aluminum-manganese alloy; alloy 53S is an exception. The corrosion resistance of the duralumin-type alloys is greatest in the quenched condition and decreases with aging at room temperature. Oil quenching and artificial aging decrease the resistance still further. Alloys 17S and 24S are available in sheet form with an integral coating of high-

good corrosion resistance, good working properties, or electrical or thermal conductivity and reflectivity.

It is produced by the electrolysis of alumina dissolved in a bath of molten cryolite. The alumina is produced by chemical refinement of bauxite, an ore containing the hydrated oxides of aluminum. Aluminum reaches the market as wrought and cast products and in the form of ingots or notched bars for remelting. The impurities (principally iron and silicon with less copper) do not exceed 0.5, or 1.0 percent, respectively, in the grades usually specified. Material of high purity is available, and metal containing over 99.99 percent aluminum can now be obtained commercially.

Commercial aluminum is a soft and ductile metal and is used for many applications where high strength is not desired. It is available in extruded or rolled forms and can be hardened by cold working but not by heat-treatment. The alloys of aluminum possess better casting and machining characteristics and better mechanical properties and therefore are used more extensively than the pure metal.

Aluminum Alloys for Sand Castings. The compositions and typical properties of aluminum alloys used for foundry work are listed in Table 2. Most of these are based on either aluminum copper or aluminum silicon systems with additions to improve the casting or service characteristics. Among aluminum-copper alloys, the one containing 8 percent copper has been used longest as a general-purpose alloy, although the additions of silicon and iron to this, CC, improve the casting characteristics, particularly in rendering the alloy less hot-short. Additions of zinc to this alloy, C, are made to improve the machinability. Alloys G and GG with 4 percent copper are comparatively weak in the cast condition but may be heat-treated to give various degrees of toughness or strength. These alloys are more resistant to corrosion than the higher copper alloys, but the precipitation treatment decreases this somewhat. The 4 percent alloys do not have so good casting properties as those containing 8 percent copper.

Alloys containing 12 percent copper are slightly stronger than the 8 percent alloy but considerably less tough. They owe their employment to the fact that it is easy to produce castings free from leaks in this alloy, although they have now been largely replaced by aluminum silicon alloys.

Alloy F, which has a small magnesium content, retains its strength and hardness up to comparatively high temperatures. It was developed primarily for pistons but is used in other high-temperature applications. It age-hardens spontaneously after casting, and further hardening may be produced by a precipitation heat-treatment at a moderate temperature, particularly if this follows a solution heat-treatment.

Alloy H, known as Y alloy, is a complex alloy, rather difficult to cast, and is susceptible to heat-treatment with the development of high strength. Although extensively used in Europe, it has found little favor in the United States. Alloy F or 6 is used for similar applications in this country.

Aluminum-silicon alloys have come into considerable use because of their excellent casting qualities and resistance to corrosion. The alloys are not hot-short and are easy to cast sound in thin or thick sections. They are rather difficult to machine. The most commonly used aluminum-silicon alloy is that containing 5 percent silicon (J). Alloy K solidifies normally with a coarse hypereutectic structure, but this is "modified" before casting by the addition of a small amount of sodium to give a fine eutectic structure of greater strength and toughness. With all alloys containing substantial amounts of silicon, the iron content must be low to avoid brittleness.

steel. In general, such tools are similar to those used for working wood but should be harder. Cemented hard carbide tools are almost essential for aluminum-silicon alloys. The casting alloys containing copper and all the wrought heat-treated alloys possess good machinability. For products in which physical properties are subordinate to high machinability, as in automatic screw machine work, alloy 11S is used, for the additions of lead and bismuth render this free-cutting.

Riveting is the most commonly used method of joining aluminum alloys, especially in structures of the heat-treatable alloys that cannot be welded without loss of strength. In general, rivets of similar composition to the base metal are used. When heat-treatable rivets are driven cold, it is important that they be used in the freshly quenched condition prior to aging, but they may be kept for long times in cold storage. Large rivets can sometimes be driven hot from their solution treatment temperature, depending on contact with tools and surrounding metal to produce an effective quench.

Welding. (See p. 1848.) The wrought-aluminum alloys are readily welded by experienced operators by either the fusion or resistance method. Fusion welding of the strong alloys is not recommended unless subsequent heat-treatment is possible, but spot and seam welding can be done if automatically controlled. Most casting alloys may be welded, but experience is necessary to overcome the danger of strains and cracks resulting from thermal contraction. Welding should be done prior to heat-treatment. The rod used should generally be of the same composition as the alloy. Aluminum alloys can be soldered, but the resultant joints are rarely satisfactory and are not recommended for highly stressed parts or for joints that cannot be given adequate protection against corrosion action.

Corrosion Resistance. Although aluminum is chemically active, the presence of a firmly adherent self-healing oxide coat on the surface prevents action except under conditions that tend to remove this surface film. Concentrated nitric and acetic acids are handled in aluminum not only because of its resistance to attack but also because any resulting corrosion products are colorless. For the same reason, aluminum is employed in the preparation of foods and beverages. Hydrochloric acid and most alkalies dissolve the protective film at the surface and permit fairly rapid attack. Moderately alkaline soaps and the like can be used with aluminum if a small amount of sodium silicate is added. Aluminum is very resistant to sulphur and most of its gaseous compounds.

Ordinary atmospheric corrosion is resisted by aluminum and most of its alloys, and they may be used without any protective coating. The pure metal is most resistant to attack, and additions of alloying elements usually decrease resistance, particularly after heat-treatment. Under severe conditions of exposure such as may prevail on shipboard or where the metal is continually in contact with wood or other absorbent material in the presence of moisture, a protective coat of paint is desirable as an added precaution.

The resistance to corrosion of aluminum alloys may be augmented by coating the material with a surface layer of high-purity aluminum or in some cases an alloy, which is rolled as an integral part of the sheet. The corrosion resistance of any of the alloys may be improved by giving an **anodizing treatment**, which comprises making the parts to be treated the anode in an electrolytic bath (chromic, sulphuric, or oxalic acid). This produces a tough adherent coating of aluminum oxide. The film will be colorless on pure aluminum and tends to be gray or colored on alloys containing silicon, copper, or other constituents. This film is very adherent and cannot be readily detached by bending or ordinary fabricating processes. If a colored finish is desired, the electrolytically oxidized article may be treated with a dye solution. Chemical methods are available for producing a similar but thinner film without electrolytic action by mere dipping in hot alkaline oxidizing solution or applying a paste.

Table 2. Aluminum Alloy Castings.—*(Continued)*

A.S.T.M. designation	Aluminum Co. of America designation	Composition, percent[b] (aluminum remainder)					Typical properties[a]					
		Cu	Si	Mg	Fe	Other elements	Treatment	Yield strength, kips	Tensile strength, kips	Elongation in 2 in., percent	Endurance limit, kips	Electrical conductivity, percent I.A.C.S.
6	A132	0.8	12.0	1.0	0.8	Ni 2.5	T4 T551	30 28	38 36	1.5 0.5		
7	.....	1.5	12.25	0.55	....	Mn 0.75	T2? T6?	.. ..	32[c] 36[c]			
8	.....	....	11.0	....	....	....	CC	..	27[c]	3.0[c]		
9 & 9A	43	....	5.0	....	....	....	CC	9	24	6.0	...	37
10	.....	....	7.0	0.3	....	....	T2? T6?	.. ..	28[c] 30[c]	5.0[c] 3.0[c]		
11	142	4.0	.....	1.5	....	Ni 2.0	T61 T571	42 34	47 40	0.5 0.00	9.5	37

Aluminum Base Alloys for Die Casting

A.S.T.M. designation	Aluminum Co. of America designation	Composition, percent[b] (aluminum remainder)			Treatment	Yield strength, kips	Tensile strength, kips	Elongation in 2 in., percent	Endurance limit, kips	Charpy impact, ft-lb
		Cu	Si	Ni						
IV	43	...	5.0	....	DC	13	29	3.5	..	4.5
V	13	...	12.0	....	DC	18	33	1.5	15	2.0
VI	83	2.0	3.0	....	DC	14	30	3.5	14	5.0
VII	85	4.0	5.0	....	DC	19	35	2.0	17	2.5
VIII	..	1.5	1.0	2.25	DC	..	29	4.0	..	4.5
IX	93	4.0	1.75	4.0	DC	20	33	1.5	..	2.0
XI	..	2.0	8.0	....	DC	..	32	1.7	..	3.0
XII	..	7.0	2.0	....	DC	..	34	1.6	..	3.2
	81	7.0	3.0	....	DC	24	32	1.3	16	

Sand-casting alloys are covered by A.S.T.M. Specification B26-37T; Die-casting alloys by A.S.T.M. B85-39T and permanent mold alloys by A.S.T.M. B108-38T.

G & GG, J & JJ differ in impurity content but not in specified mechanical properties.

Treatments SC, CC, and DC refer to alloys in sand-cast, chill-cast, or die-cast conditions, respectively.

T4 is solution heat-treatment followed by rapid cooling and aging at room temperatures.

T6, 61, and 62 are solution heat-treatments followed by precipitation treatment.

T2, T5, T51, T571, and T59 are precipitation treatments only.

[a] Tensile properties are on ½ in. diam sand cast or ¼ in. diam chill or die-cast specimens, tested without machining surface. Yield strength corresponds to 0.2 percent offset. Endurance limit is based on 500,000,000 cycles on R. R. Moore type of machine and machined specimens. Charpy Impact is on unnotched ¼ in. square die casting.

[b] All these alloys may contain as impurities, iron in amounts not to exceed 0.4 up to 1.0 percent, silicon 0.2 to 1.2 percent, and manganese up to about 0.3 percent, according to the alloy. An intentional addition of titanium up to 0.2 percent is often made.

[c] Specification minimum values.

Table 6. Composition and Properties of Wrought Copper Base Alloys

Material	Form tested	Nominal composition, percent: Copper	Zinc	Lead	Tin	Other elements	Tensile strength, kips: Hard	Soft	Elongation, percent in 2 in.: Hard	Soft	Yield strength (0.5 % extension under load), kips: Hard	Soft	Rockwell B hardness: Hard	Soft	Melting point, deg F	Density, lb per cu ft	Coeff. of expansion (avg 77–572 deg F) per deg F × 10⁶	Elec. conductivity (annealed) percent I.A.C.S.	Thermal conductivity, Btu per hr per sq ft per ft per deg F
Copper and Brass																			
Copper, tough pitch	Sheet	99.90+	.....	.....	.....	0.04 O_2	46	33	5	35	40	10	51	F35	1979	556	9.85	100.0	223
	Wire	99.90+	.....	.....	.....	0.04 O_2	66	35	1[a]	35[a]									
	Rod	99.90+	.....	.....	.....	0.04 O_2	45	32	15	45	40	10	50	F35					
Copper, deoxidized	Tube	99.90	.....	.....	.....	0.02 P	45	35	10	45	40	10	50	F40	1980	558	9.85	80.0	187
Gilding	Sheet	95.0	5.0	.....	.....	.....	55	35	5	38	44	11	61	F45	1950	553	10.05	54.6	139
Commercial bronze, 90 %	Sheet	90.0	10.0	.....	.....	.....	62	37	6	40	47	12	70	1	1915	549	10.10	40.9	108
Red brass, 85 %	Sheet	85.0	15.0	.....	.....	.....	69	40	7	45	55	15	76	5	1870	546	10.49	37.0	92
	Tube	85.0	15.0	.....	.....	.....	69	40	10	50	55	15	76	5					
Red brass, 80 %	Sheet	80.0	20.0	.....	.....	.....	73	43	8	50	60	16	81	10	1830	541	10.60	32.5	81
	Wire	80.0	20.0	.....	.....	.....	100	47	1[a]	45[a]	..	..	..	....	....	....	....	32.5	
Spring brass	Sheet	72.0	28.0	.....	.....	.....	76	47	10	65	62	17	83	20	1770	534	11.00	28.60	71
Cartridge brass	Sheet	70.0	30.0	.....	.....	.....	76	47	10	65	62	17	83	20	1750	532	11.05	27.58	70
Drawing or spinning brass	Sheet	66.7	33.3	.....	.....	.....	73	45	10	60	60	17	80	15	1720	529	11.15	25.8	69
Common high (yellow) brass	Sheet	65.0	35.0	.....	.....	.....	73	45	10	60	60	17	80	15	1705	529	11.20	26.8	69
Rivet and pin wire brass	Rod	63.0	37.0	.....	.....	.....	65	46	20	60	50	17	75	20	1690	527	11.40	25.9	69
	Wire	63.0	37.0	.....	.....	.....	105	50	1[a]	50[a]									
Muntz metal	Sheet	60.0	40.0	.....	.....	.....	80	54	8	45	60	20	85	45	1660	524	11.55	28.6	73

Table 3. Composition and Typical Properties of Wrought Aluminum Alloys

A.S.T.M. specification	Alloy designation[a]	Nominal composition, percent (balance aluminum) Cu	Si	Mn	Mg	Other elements	Temper[b]	Density, lb per cu in.	Yield strength, 0.2% offset, kips	Tensile strength, kips	Elongation, percent in. 2 in.	Brinell hardness (500 kg load)	Fatigue endurance limit, kips	Electrical conductivity, % I.A.C.S.
						Work-hardened Alloys								
B25-38T	2S	...	...	...	...	....	O	0.098	5	13	35	23	5	59
							½H	.....	14	17	9	32	7	
							H	0.098	21	24	5	44	8.5	57
B79-38T	3S	...	...	1.2	...	....	O	0.099	6	16	30	28	7	50
							½H	0.099	10	21	8	40	9	41
							H	0.099	25	29	4	55	10	40
	4S	...	...	1.2	1.0	....	O	0.098	10	26	20	45	14	42
							½H	0.098	27	34	9	63	15	42
							H	0.098	34	40	5	77	16	42
B109-38T	52S	...	...	...	2.5	Cr 0.25	O	0.096	14	29	25	45	17	40
							½H	.....	29	37	10	67	19	
							H	0.096	36	41	7	85	20.5	40
	56S	...	...	0.1	5.2	Cr 0.1	O	.....	18	42	25			
							½H	.....	36	50	10			
							H	.....	45	57	7			
	(Birmabright)	...	...	0.5	3.5	....	O	.....	.....	31.4[c]	20[c]	58		
							H	.....	33.6[c]	44.8[c]	3[c]	80		
	(Mg 7)	...	...	0.5	7.0	....	O	.....	22.4[c]	44.8[c]	20[c]			
							H	.....	38.1[c]	56[c]	15[c]			
						Heat-treated Alloys								
B89-36T B78-36T	17S	4.0	...	0.5	0.5	....	O	0.101	10	26	20	45	11	45
							T	.....	37	60	20	100	15	30
							RT	.....	47	65	13	110		
	Alclad 17S	...	...	...	...	....	T	.....	33	56	18			
							RT	.....	40	57	11			
	11S	5.5	...	...	...	Pb 0.5 Bi 0.5	T3	0.102	42	49	..	95	12.5	40
	14S	4.4	0.8	0.8	0.4	....	T	0.101	50[c]	65[c]	10[c]	130[c]	15[c]	40
	18S	4.0	...	...	0.5	Ni 2.0	T	0.101	35[c]	55[c]	10[c]	100[c]	....	40
	24S	4.4	...	0.5	1.5	....	O	0.100	10	26	20	42	12	50
							T	.....	44	68	19	105	16	30
							RT	.....	55	70	13	116		
	Alclad 24S	...	...	...	...	....	T	.....	41	62	18			
							RT	.....	50	66	11			

Table 6. Composition and Properties of Wrought Copper Base Alloys.—(*Continued*)

Material	Form tested	Nominal composition, percent					Tensile strength, kips		Elongation, percent in 2 in.		Yield strength (0.5% extension under load), kips		Rockwell B hardness		Melting point, deg F	Density, lb per cu ft	Coeff. of expansion (avg 77–572 deg F) per deg F × 10⁶	Elec. conductivity (annealed) percent I.A.C.S.	Thermal conductivity, Btu per hr per sq ft per ft per deg F
		Copper	Zinc	Lead	Tin	Other elements	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft					
						TIN BRONZES													
Special bronze	Sheet	98.75	.....	....	1.25	......	65	40	6	48	50	14	75	F60	1970	555	.....	43.0	126
Signal bronze	Wire	98.25	.....	....	1.75	......	105	45	1[a]	40[a]	..	..	....	....	1960	554	.....	35.0	85
Phosphor bronze	Sheet	95.75	.....	....	4.0	0.25 P	80	48	8	50	65	20	86	28	1920	553	10.55	12.6	36
Phosphor bronze, Grade A	Sheet	94.9	.....	....	5.0	0.1 P	80	48	8	50	65	20	86	28	1920	553	9.90	18.4	47
Leaded phosphor bronze B	Rod	93.9	.....	1.0	5.0	0.1 P	65	..	25	..	55	..	75	....	1920	556	.....	18.4	48
Phosphor bronze, Grade C	Sheet	91.9	.....	....	8.0	0.1 P	93	60	10	65	68	24	94	50	1880	550	10.10	13.0	36
Phosphor bronze, Grade D	Sheet	89.4	.....	....	10.5	0.1 P	102	66	12	65	70	28	98	55	1830	548	10.15	10.6	29
Free Cut'g Phos. bronze	Rod	88.0	4.0	4.0	4.0	......	60	..	20	..	45	..	75	..	1830	553	.....	12.2	32
						COPPER-NICKEL ALLOYS													
30% Cupronickel	Tube	70.0	.....	....	.....	30.0 Ni	70	55	10	45	60	22	80	35	2240	558	9.00	4.7	17
20% Cupronickel	Sheet	80.0	.....	....	.....	20.0 Ni	74	48	5	40	67	20	79	....	2190	558	.....	6.5	21
15% Cupronickel	Sheet	85.0	.....	....	.....	15.0 Ni	65	45	5	35	55	18	....	....	2150	558	.....	8.2	27
Ambrac A	Sheet	75.0	5.0	....	.....	20.0 Ni	82	50	5	35	55	18	85	25	2100	553	9.10	6.2	22
	Rod	75.0	5.0	....	.....	20.0 Ni	70	50	20	40	60	18	80	25					
	Wire	75.0	5.0	....	.....	20.0 Ni	100	55	1[a]	35[a]									
30% Nickel silver	Wire	47.0	23.0	....	.....	30.0 Ni	150	75	1[a]	35[a]	..	..	....	....	2085	546	.....	3.6	13
25% Nickel silver	Sheet	55.0	20.0	....	.....	25.0 Ni	110	70	3	30		..	....	....	2075	545	.....	4.0	14
18% Nickel silver	Sheet	65.0	17.0	....	.....	18.0 Ni	85	56	4	40	70	..	85	40	2030	548	.....	5.9	19

purity aluminum on each side; the thickness of each layer is approximately 5 percent of the total. These products, known as **alclad alloys**, possess an excellent resistance to corrosion. Because of *electrolytic action*, exposed cut edges of the base metal are protected and ordinary bare rivets may be used. The strength is slightly less than bare sheet of similar gage.

Forgings may be made under a hammer or press, using dies on such work as propeller blades, pistons, and connecting rods. Alloy 25S is generally used, but for forgings requiring the highest strength 14S can be used. Complicated forgings where the highest mechanical properties are not needed are best made of A51S.

Heat-treatment. Intermediate annealing to relieve cold work is done at a temperature of about 650 F for pure aluminum and 52S or about 750 F for 3S and 4S. The rate of cooling is unimportant. The heat-treatable alloys are best cold-worked when in the quenched condition. They may be *fully annealed only by heating to 750 to 800 F* and cooling slowly to 500 F. A partial softening can be obtained by an ordinary anneal at 650 F.

The heat-treatable alloys must have a double heat-treatment; one at a high temperature to dissolve the alloy constituents later responsible for hardening and the other at a low temperature to permit them to commence to come out of solution in the critical state that causes hardening of the alloy. The second action may take place spontaneously at room temperatures on some alloys and is then known as *natural aging*, but on other alloys it has to be carried out at a somewhat elevated temperature referred to as *artificial aging* or precipitation treatment. The correct treatment for the various alloys is given in Table 4. The solution treatment is usually done in a nitrate bath or in a furnace with forced air circulation. The temperature must be controlled closely. The solution heat-treatment of duralumin should be followed by a rapid quench, preferably in cold water since slower quenches in hot water or oil, although they minimize distortion, render the alloy susceptible to intergranular corrosion. In the alloys that age spontaneously at room temperature, hardening starts immediately after quenching and is practically complete in four days. Severe cold-forming operations must be done within less than an hour *after* quenching. If it is desired to hold the alloy for later cold-working, aging may be retarded by storing the quenched material at low temperatures, for example, in ice, which will permit working up to 24 hr, or in "dry ice" (solid CO_2) which will retard aging almost indefinitely.

The precipitation heat-treatment, if necessary, is done in a furnace with forced air circulation and heated by steam coils or electrically.

Table 4. Conditions for Heat-treatment of Aluminum Alloys

Alloy	Solution heat-treatment[a]		Precipitation heat-treatment		
	Temperature, deg F	Temper designation	Temperature, deg F	Time of aging	Temper designation
17S	930–950	....	Room	4 days	17S-T
A17S	930–950	....	Room	4 days	A17S-T
24S	910–930	....	Room	4 days	24S-T
51S	960–980	51SW	315–325	18 hr	51S-T
53S	960–980	53SW	315–325	18 hr	53S-T

[a] In a molten nitrate bath, the time varies from 10 to 60 min depending upon the size of the load and the thickness of the material. In an air furnace, proper allowance must be made for a slower rate of bringing the load up to temperature. For heavy material, a longer time at temperature may be necessary. All quenching is performed in cold water.

Machining. There are many aluminum alloys which are easily machined without special technique. Pure aluminum and the aluminum-manganese alloys are hard to machine unless special tools are used with greater rake than is customary for

When painting or lacquering aluminum it is important that the surface be properly prepared, prior to the application of the paint. A thin anodic film makes an excellent paint base, or the metal may be chemically treated with a dilute phosphoric acid solution. Where corrosive conditions are to be met, zinc chromate may be used as the pigment in the primer coat and aluminum paint for the top.

Aluminum Conductors. For electrical uses (see p. 1689), aluminum is generally unalloyed. As commercially produced in the hard-drawn condition, aluminum has a conductivity of 61 percent IACS. For power transmission lines, the necessary strength for long spans is obtained by the use of steel reinforcing cores.

Copper and Its Alloys

Copper alloys are useful on account of their heat or electrical conductivity, good cold or hot working properties, machinability, or corrosion resistance. For high thermal or electrical conductivity, commercially pure copper should be used; if greater strength is required combined with high conductivity, alloys containing cadmium or other elements are used. The cheapest copper alloy is brass of high zinc content and is used unless high corrosion resistance under stress or the special mechanical properties of other alloys are required. When good cold-working properties are desired, as in deep drawing or forming operations, a brass with 30 to 35 percent zinc is used. Leaded brass is used when much machining must be done, particularly for automatic screw machine work. For high elastic strength, the tin bronzes are used. The alloys of copper with aluminum or silicon or nickel are good for corrosion resistance.

Several hundred different copper alloys are available but requirements can generally be met satisfactorily by one or more of the standard alloys listed in Table 6.

Commercial Copper. Much high-grade copper is still made by fire refining, but the larger proportion is refined electrolytically from anodes of crude blister copper. Sometimes the copper is leached directly from the ore and the solution electrolyzed to give a pure cathode copper directly. **Lake copper** originates in the Great Lakes district; it is not electrolytically refined but is of high purity and conductivity. Secondary copper, fully equal to primary copper, is reclaimed from copper-alloy scrap. A less pure grade of copper known as **casting copper** is produced for brass-foundry use in making alloys. The electrolytic refining operation produces cathodes that may measure about 3 ft square and weigh as much as 300 lb. Cathodes may be used directly for making alloys, but if copper is to be rolled to fabricated forms, it is melted and cast into wire bars, cakes, or billets. The remelting or so-called refining operation involves melting in a large fuel-fired reverberatory furnace, oxidation to eliminate sulphur and gas absorbed from the fuel, and "poling" to reduce the oxygen to about 0.04 percent. When correctly refined, castings solidify with an approximately level surface, the gas evolved during solidification balancing the shrinkage that would otherwise occur. This is known as **tough-pitch copper**. It has a density of 8.4 to 8.7 g per cc when cast, 8.89 to 8.92 when worked and annealed.

The presence of oxygen is desirable in making the copper slightly harder and, before electrolytic refining was adopted, was useful in neutralizing the effect of certain impurities. Oxygen is harmful if the copper is to be welded or otherwise heated in a reducing gas, as it causes an embrittlement of the

Table 5. Typical Tensile Properties of Aluminum Alloys at Elevated Temperatures

Alloy and temper	Property[a]	Temperature, deg F 75	300	400	500	700
Wrought						
2S-H	T.S.	24,000	17,500	6,000	3,500	1,500
	Y.P.	21,000	14,000	3,000	2,000	1,000
	El.	15	16	70	85	95
3S-H	T.S.	29,000	23,000	17,000	10,500	3,000
	Y.P.	25,000	16,000	8,000	5,000	2,000
	El.	10	12	15	25	60
4S-H	T.S.	40,000	32,000	23,000	11,500	4,000
	Y.P.	34,000	22,000	9,500	5,000	2,500
	El.	6	14	27	70	100
17S-T	T.S.	60,000	40,000	23,000	13,000	4,500
	Y.P.	37,000	33,500	20,000	9,500	3,000
	El.	22	16	23	35	100
24S-T	T.S.	68,000	46,000	27,000	14,000	5,000
	Y.P.	44,000	38,000	21,000	9,500	3,500
	El.	22	22	25	45	100
51S-W	T.S.	35,000	27,000	13,000	5,000	2,000
	Y.P.	20,000	23,000	10,000	3,500	1,500
	El.	30	16	30	75	100
51S-T	T.S.	48,000	28,000	14,000	5,000	2,000
	Y.P.	40,000	24,000	11,000	3,500	1,500
	El.	16	20	30	65	100
52S-¾H	T.S.	39,000	32,000	25,000	12,000	5,000
	Y.P.	34,000	27,000	11,000	8,000	2,500
	El.	10	16	35	80	120
53S-T	T.S.	39,000	25,000	13,000	6,000	2,500
	Y.P.	33,000	22,000	10,000	3,500	2,000
	El.	20	17	30	70	90

Alloy and temper	Temperature, deg F 75	300	400	500	600
Sand Castings					
122-T2	25,000	25,000	22,000	17,000	8,000
	20,000	17,000	14,000	11,000	4,500
	1.0	1.2	1.5	3.0	14.0
122-T61	36,000	35,000	22,000	10,000	8,000
	30,000	30,000	16,000	5,000	4,500
	1.0	1.2	2.0	6.0	14.0
142	28,000	28,000	22,000	12,000	7,500
	24,000	24,000	18,000	5,000	3,500
	1.0	1.0	1.0	9.0	10.0
195-T4	31,000	24,000	15,000	9,500	4,000
	16,000	13,000	9,000	6,000	3,000
	8.5	9.0	20.0	25.0	80.0
355-T4	30,000	30,000	13,000	8,000	6,000
	20,000	25,000	9,000	5,000	3,500
	5.0	3.0	12.0	22.0	30.0
356-T4	28,000	21,000	13,000	8,000	4,500
	16,0[illegible]	15,000	9,000	5,500	3,000
	6.[illegible]	7.0	8.0	20.0	45.0
Permanent Mold Castings					
122-T551	37,000	33,000	26,000	18,000	10,000
	35,000	29,000	20,000	12,500	6,000
	0	0	1	3	10
A132-T551	36,000	31,000	23,000	17,500	11,000
	28,000	22,000	13,000	9,500	5,000
	0.5	1	2	2	8
142-T571	40,000	37,000	28,000	15,000	9,000
	34,000	33,000	22,000	9,000	5,000
	0	1	2	10	30
B195-T4	36,000	30,000	15,000	7,500	4,000
	22,000	20,000	9,000	5,000	2,500
	7.5	8	12	25	65
355-T4	38,000	31,000	12,000	8,000	4,500
	23,000	25,000	9,000	6,000	3,000
	6	3	20	25	50

[a] T.S. Tensile strength lb per sq. in.; Y.P. Yield strength, 0.2 percent offset, lb per sq in.; El. Elongation in 2 in., percent.

Tensile tests made on A.S.T.M. standard test pieces, 0.5 in. diam, maintained at elevated temperatures until properties become constant.

Speed of straining 0.1 in. per min per in. of gage length.

For properties of aluminum at low temperatures see Table 9.

Table 7. A.S.T.M. Specification Properties of Copper Wire

Diam, in.	Hard-drawn wire: Tensile strength, psi, min	Hard-drawn wire: Elongation, percent in 60 in., min	Medium-drawn wire: Tensile strength, psi: Min	Medium-drawn wire: Tensile strength, psi: Max	Medium-drawn wire: Elongation, percent in 60 in., min	Soft-annealed wire: Tensile strength, psi, max	Soft-annealed wire: Elongation, percent in 10 in., min
0.460	49,000	3.75[a]	42,000	49,000	3.75[a]	36,000	35
0.325	54,500	2.40[a]	45,000	52,000	3.0[a]	36,000	35
0.229	59,000	1.79[a]	48,000	55,000	2.25[a]	36,000	35
0.162	62,100	1.14	49,000	56,000	1.15	37,000	30
0.114	64,300	1.02	50,000	57,000	1.06	37,000	30
0.081	65,700	0.95	51,000	58,000	1.00	38,500	25
0.057	66,400	0.89	52,000	59,000	0.94	38,500	25
0.040	67,000	0.85	53,000	60,000	0.88	38,500	25
Electrical resistivity, ohms (mile pound)	910.15		905.44[b]			891.58	
Microhms per cm cube......	1.7930		1.7837			1.7564	

[a] Elongation in 10 in. gage length.
[b] On wire below 0.324 in. diam, 896.15 for larger wires.

Table 8 summarizes the A.S.T.M. specification requirements for 65/35 brass of various rolled and annealed tempers. Practically all wrought copper alloys

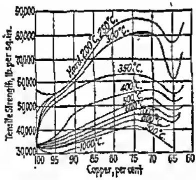

FIG. 3.—Tensile Strength of Copper-zinc Alloy.

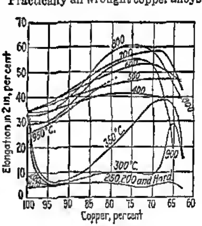

FIG. 4.—Percentage Elongation in 2 in. of Copper-zinc Alloys.

are used in the cold-worked condition to gain additional strength. Articles are often made from annealed stock and depend on the work of the forming operation to shape and harden them. When the work involved is too small to do this, brass rolled with more or less temper depending on requirements should be used.

Brass for spring purposes should be rolled as hard as consistent with the subsequent forming operations. For articles requiring sharp bends, or for deep-drawing operations, annealed brass must be used. In general, the lower grain sizes are to be preferred.

Leaded Brasses																			
Brass for pipe	Tube	67.5	32.0	0.5	....	....	73	45	10	55	60	17	80	15	1715	531	....	26.8	68
Hinge butt brass	Sheet	64.0	35.0	1.0	....	....	73	45	8	55	60	17	80	15	1695				
Leaded commercial bronze	Rod	88.5	10.0	1.5	....	....	54	37	15	40	45	12	58	1	1905	551	10.15	40.5	107
Leaded red brass	Rod	78.5	20.0	1.5	....	....	60	43	20	50	47	16	70	10	1815	542	10.65	28.9	73
Leaded brass	Rod	69.0	29.5	1.5	....	....	65	46	20	60	48	17	75	15	1740	534	11.10	27.5	70
Clock brass	Sheet	61.5	37.0	1.5	....	....	73	45	7	50	60	17	80	15	1665				
Hardware bronze	Rod	85.0	13.25	1.75	....	....	56	40	15	45	45	15	60	5	1875	548	....	35.6	93
Forging brass	Rod	60.0	38.0	2.0	....	....	...	54	..	45	..	20	..	45	1645	527	....	26.5	62
Extruded architectural bronze	Shapes	56.0	41.5	2.5	....	....	..	65	..	25	..	..	..	..	1625	527	....	25.0	62
Free cutting yellow brass	Rod	62.0	35.0	3.0	....	....	58	47	18	60	42	18	70	20	1625	530	11.35	25.0	62
Special Brasses																			
Aluminum brass	Tube	76.0	22.0	....	....	2.0 Al. 0.05 As	85	52	10	65	..	20	..	30	1780	520	10.65	22.6	58
Oreide	Sheet	87.25	11.5	....	1.25	....	72	43	8	47	60	16	80	15	1865				
Admiralty	Tube	70.0	29.0	....	1.0	....	85	52	10	65	..	20	..	30	1715	532	11.20	24.6	64
Trumpet-brass	Tube	81.0	18.0	....	1.0	....	80	48	10	60	..	..	..	25	1840				
Naval brass	Rod	60.0	39.25	....	0.75	....	63	56	30	40	35	22	65	50	1625	...	11.90		
Tobin bronze	Rod	60.0	39.25	....	0.75	....	63	56	35	45	35	22	65	50	1625	525	11.75	24.9	67
	Sheet	60.0	39.25	....	0.75	....	85	56	8	45	62	22	90	50					
Fourdrinier wire	Wire	61.0	18.75	....	0.25	....	..	49	..	43[a]	..	..	..	....	....	544	....	37.2	62

Table prepared by Technical Department of The American Brass Co. Values shown are typical and should not be used for specification purposes as they vary with shape and size tested and manufacturing variables. Tests on sheet 0.040 in. thick; rod 1 in. diam., wire 0.08 in. diam; tube 1 in. O.D. by 0.065 in. wall thickness. "Hard" sheet was reduced 4 B. & S. gage numbers in thickness by cold rolling. "Hard" rod, wire, or tube corresponds to commercial hard-drawn temper.

Higher values of yield and tensile strength and hardness are obtained by a greater amount of cold working, but this is limited to relatively narrow sheet and wire, rod, and tube of small diameter.

[a] Elongation on wire is on 10 in. gage length.

HT Precipitation heat-treatment on form shown.

F. Rockwell F hardness.

qualities, hardens them somewhat, and has given rise to the misleading name **phosphor bronze.** The bronzes are characterized by excellent elastic properties.

The **aluminum bronzes** with 5 and 8 percent aluminum find application because of their high strength and corrosion resistance, and sometimes because of their golden color. Those with 10 percent aluminum and other alloys of the type of "Tempaloy 917" listed in Table 6 with even higher amounts are very plastic when hot and have exceptionally high strength, particularly after heat-treatment.

The **copper-silicon** alloy "Everdur" listed in Table 6 may be considered as typical of its class. Other alloys (trade names, Duronze, Herculoy, Olympic Bronze, P M G Metal) are made in which zinc, iron, or tin is substituted for the manganese without a substantial change in physical properties. These alloys are as corrosion resistant as copper (slightly more so in some solutions) and possess excellent hot workability with high strength. Their outstanding characteristic is that of ready weldability by all methods. The alloys find extensive application for tanks, for chemical processing vessels and hot-water storage, fabricated by arc or acetylene welding.

The **cupronickels** and so-called **nickel silvers** are white in color and find application on this account, and because of their comparative freedom from tarnishing under atmospheric conditions. Nickel silver is the base for most silver-plated ware. The cupronickels are extremely malleable and may be worked extensively without annealing. Because of their excellent corrosion resistance, they are used for condenser tubes for most severe service. Alloys containing nickel have the best high-temperature properties of any copper alloy.

Many copper alloys capable of precipitation hardening have been proposed, but those containing **beryllium** and **chromium** are the only wrought alloys that find extensive application in the precipitation-hardened condition. The former is expensive, but its strength and elastic properties are so far superior to other materials that it is ideal for springs, diaphragms, and the like where high fatigue resistance under somewhat corrosive conditions is needed. The substitution of cobalt for part of the beryllium produces a cheaper alloy of only slightly inferior properties. Chromium copper is used either as castings or wrought shapes principally for electrodes in resistance welding machines. It has a high conductivity (about 80 percent of copper) and does not soften on prolonged heating at temperatures below about 850 F. This alloy is used in strip form for electrical switch parts because of its high softening temperature.

Young's modulus of all copper alloys in all conditions lies between 14 and 22×10^6 lb per sq in; for pure copper it is 17×10^6; for brass and bronze about 15×10^6. Nickel raises the modulus. The exact value for any alloy depends on residual stresses and to some degree on the extent and character of working that it has received. The proportional limit of any cold-worked copper alloy can be raised (sometimes very considerably) by a low temperature heat-treatment or relief anneal.

Fabrication. Practically any of the copper alloys listed in Table 6 can be obtained in sheet, rod, and wire form and many in the form of tubes. Most of them, in the annealed condition, will withstand extensive amount of cold work and may be shaped to the desired form by deep drawing, flanging, forming, bending, and similar operations. If extensive cold work is planned, the material should be purchased in the annealed condition. Very extensive operations need intermediate annealing either to avoid failure of the metal or to minimize the power consumption. This is done at 900 to 1300 F depending on the alloy, and is usually followed by air cooling. Because of the

18% Nickel silver	Sheet	55.0	27.0	...	...	18.0 Ni	99	60	4	45	75	22	93	45	1930	543	...	5.6	17
18% Nickel silver	Wire	56.0	26.0	...	...	18.0 Ni	130	65	1[a]	40[a]	..	..	...	...	1940	543	...	5.5	17
15% Nickel silver	Sheet	64.0	21.0	...	...	15.0 Ni	85	55	5	40	70	20	90	33	1970	543	...	6.3	20
15% Nickel silver	Sheet	57.0	28.0	...	...	15.0 Ni	90	55	5	40	..	..	...	...	1885	539			
Leaded nickel silver	Sheet	61.0	25.0	1.5	...	12.5 Ni	90	...	5	..	..	..	88	...					
10% Nickel silver	Sheet	65.0	25.0	...	...	10.0 Ni	88	55	7	42	70	20	87	30	1850	541	...	8.3	27
5% Nickel silver	Sheet	63.0	32.0	...	...	5.0 Ni	80	50	7	50	..	..	...	...	1760	...	...	12.0	34
ALUMINUM BRONZES																			
5% Aluminum bronze	Sheet	95.0	...	...	...	5.0 Al	92	55	7	65	65	22	92	35	1940	510	...	17.7	48
8% Aluminum bronze	Sheet	92.0	...	...	...	8.0 Al	105	65	7	60	65	25	96	50	1905	486	9.95	14.8	42
	Rod	92.0	...	...	...	8.0 Al	80	65	30	65	50	25							
Aluminum bronze	Rod	90.0	...	...	...	9.5 Al 0.5 Fe	95	80	16	22	55	40	...	...	1910	473	9.40	12.6	35
Tempaloy 917	Rod	81.9	...	...	...	(9.6 Al 2.5 Fe) (5.0 Ni 1.0 Mn)	105	...	12	..	60	..	105	...	1930	472	...	7.5	22
Nickel aluminum bronze	Sheet	92.0	...	...	...	4.0 Al 4.0 Ni	90	50	7	45	..	..	...	...	2000	522	9.45	15	39
SPECIAL ALLOYS																			
Manganese bronze	Rod	59.0	39.0	0.7	...	0.8 Fe 0.5 Mn	75	60	20	30	45	30	85	...	1645	522	...	24.6	58
Everdur 1010	Sheet	96.0	...	...	...	3.0 Si 1.0 Mn	95	58	7	60	60	22	92	35	1865	532	10.00	6.7	19
	Rod	96.0	...	...	...	3.0 Si 1.0 Mn	90	58	18	70	60	22	90	35					
	Wire	96.0	...	...	...	3.0 Si 1.0 Mn	145	60	1[a]	50[a]									
Everdur 1015	Tube	98.25	...	...	...	1.5 Si 0.3 Mn	65	40	8	50	50	15	75	F55	1930	546			
	Rod	98.25	...	...	...	1.5 Si 0.3 Mn	70	40	15	50	55	15	80	F55	...	...	...	12.0	31
	Sheet	98.25	...	...	...	1.5 Si 0.3 Mn	65	40	8	46	50	15	77	F55					
Beryllium copper	Sheet	97.4	...	...	...	2.15 Be 0.35 Ni	118	70	4	45	85	30	102	70	1750	515	...	17.0	46
Beryllium copper HT	Sheet	97.4	...	...	...	2.15 Be 0.35 Ni	190	175	2	5	97	95	114	112	...	...	...	18–25	47–64
CONDUCTOR ALLOYS																			
Calsun bronze	Wire	95.5	...	2.0	...	2.5 Al	135	52	1[a]	40[a]	..	..	...	...	1930	532	...	17.0	47
Cadmium copper	Wire	99.35	...	...	...	0.65 Cd	85	38	1[a]	40[a]	..	..	...	...	1975	555	...	85.0	
Cadmium copper	Sheet	99.0	...	...	...	1.0 Cd	55	37	6	50	48	12	65	F47	1970	555	...	80.0	199
	Wire	99.0	...	...	...	1.0 Cd	99	40	1[a]	40[a]									
Cadmium copper	Wire	98.6	...	0.6	...	0.8 Cd	95	42	1[a]	40[a]	..	..	...	...	1960	555	...	55.0	135
Chromium copper HT	Rod	99.1	...	...	...	0.85 Cr .05 Si	72	63	25	25	61	45	77	65	1975	...	...	75	177

Table 9. Tensile Properties of Non-ferrous Materials at Low Temperature

	Properties[a]	Temperature, deg F				Temperature, deg F		
		Room temp	−112	−292		Room temp	−112	−292
Copper	T.S.	31.4	38.5	58.0	Manganese bronze	72.4	75.5	94.8
99.985% Cu	Y.P.	8.6	10.1	11.5	56.45 Cu	24.0	27.0	28.8
	El	48	47	58	38.85 Zn	28	31	37
	R.A.	76	74	77	1.43 Mn	44	43	41
	Izod	43	44	50	1.25 Pb	20	22	21
					1.08 Fe			
					0.90 Sn			
Nickel	T.S.	65.5	76.4	97.9	Aluminum bronze	77.2	82.6	96.0
99.70 Ni	Y.P.	24.6	27.5	27.9	91.1 Cu	26.6	27.1	29.2
0.26 Mg	El	42	43	53	0.44 Mn	26	31	29
0.23 Si	R.A.	78	73	75	1.02 Zn	29	30	30
0.10 Fe	Izod	89	92	99	7.31 Al	24	24	21
Aluminum	T.S.	9.8	11.9	20.8	"Y" alloy	47.9	50.4	62.3
0.054 Si	Y.P.	4.4	4.3	4.5	3.46 Cu	25.4	25.9	27.5
0.07 Fe	El	36	38	44	0.30 Si	22	23	27
	R.A.	90	92	87	0.45 Fe	34	31	28
	Izod	19	20	27	1.86 Ni	7	8	8
					Bal. Al			
Cupronickel	T.S.	51.5	61.6	73.7	Solder[b]	8.1	.....	15.4
79.71 Cu	Y.P.	27.7	28.9	32.5	53.1 Sn	3.9	.....	8.7
20.58 Ni	El	26	29	36	47.4 Pb	28	.....	4
	R.A.	78	76	72		54	.....	4
	Izod	77	79	85		15	.....	2
Cupronickel	T.S.	60.0	72.2	89.7	Copper beryllium	76.2	86.7	111.6
54.30 Cu	Y.P.	19.6	22.1	26.3	2.56% Be	24.9	29.1	50
45.78 Ni	El	40	48	57	Q. 800 C	36	38	41
	R.A.	77	75	76		50	54	57
	Izod	81	81	86		41	40	40
Copper-nickel	T.S.	70.8	85.3	112.7	Copper beryllium	186.6	201.6	214.4
28.80 Cu	Y.P.	20.9	27.1	29.6	H.T. 300 C	125.4	147.4	155
69.58 Ni	El	41	40	51	2.56% Be	2.6	0.4	3.0
	R.A.	75	74	72		5.0	5	6
	Izod	90	90	97		2	3	3
Brass	T.S.	51.1	57.1	73.5	Everdur[c]	74	83	100.5
69.56 Cu	Y.P.	28.2	27.3	29.6	96.09 Cu			
30.50 Zn	El	49	60	75	0.15 Fe	40	32	36
	R.A.	77	79	73	2.75 Si	75	72	72
	Izod	66	69	79	0.97 Mn	73	69	
Nickel silver	T.S.	75.3	83.1	104.2				
55.15 Cu	Y.P.	27.9	27.5	28.4				
30.50 Ni	El	33	39	41				
14.30 Zn	R.A.	53	52	55				
	Izod	80	83	87				

Mainly from Colbeck and MacGillivray, *Trans. Inst. Chem. Eng*, 11, 1933, pp. 107–123. For copper alloys at low temperatures see *Proc. A.S.T.M.*, 39, 1939, pp. 642–648.

[a] T.S. Tensile strength, kips. Specimens 0.25 in. diam.
Y.P. Yield strength, kips, 0.1 percent offset.
El. Elongation in 2 in., percent.
R.A. Reduction of area in fracture, percent.
Izod. Impact resistance, ft lb, Izod specimens. Most specimens unfractured.

[b] Tensile tests on solder on 0.504 in. diam specimen. Yield point by drop of beam.

[c] Tensile tests on Everdur made at U. S. Bureau of Standards for American Brass Co. 0.505 in. diam specimens. Impact tests (Battelle Memorial Institute) on annealed … in. tensile strength. Charpy keyhole notch.

copper and renders it useless. **Deoxidized copper** is usually made by adding phosphorus, but this decreases the conductivity. **Oxygen-free copper** of high conductivity and density is now available, made by casting without contact with air. Like deoxidized copper, this is more ductile than tough-pitch copper and is immune from embrittlement by hot reducing gases unless it has been allowed to absorb oxygen during fabrication.

Copper may be rolled extensively at any temperature up to about 1900 F but is preferably hot worked at 1600 F. **Sheet copper** for roofing is sometimes used as it leaves the rolls, but for other purposes it is commonly employed after it has been cold-rolled to increase its hardness and strength. **Copper wire** above 0.04 in. diam is commonly made by drawing from a hot-rolled rod without annealing, but smaller sizes involve intermediate anneals. Copper shapes for switch parts are made by **extrusion**, brushes and commutator sections by rolling and **drawing**. Copper for electrical purposes must be very pure; the presence of even traces of certain impurities (particularly phosphorus, arsenic, and iron) decreases the conductivity very considerably.

Copper containing small amounts of silver or antimony retains the effect of cold working to a higher temperature than pure copper (about 610 F compared with about 400 F). This is useful where comparatively high temperatures are to be withstood, as in soldering or enameling operations.

Copper for **electrical purposes** should have a minimum copper content of 99.90 percent (silver being counted as copper) and a resistance, measured on a drawn and annealed wire, not to exceed 0.15436 ohms per m-g at 20 C (A.S.T.M. specification B5-27). Selected values of the strength of copper wire from A.S.T.M. specifications B1-40, B2-40, and B3-39 are shown in Table 7.

Brasses

The useful copper-zinc alloys contain up to 40 percent zinc. Those with 30 to 35 percent find the greatest application as they are cheap, very ductile, and readily worked. With decreasing zinc content, the alloys approach copper more and more in their properties and improve in corrosion resistance. **Season cracking** may occur with high-zinc brasses but rarely with 15 percent zinc or below; this is spontaneous cracking, occurring on exposure to atmospheric corrosion, in brass objects with high residual tensile stresses at the surface. It may be prevented by avoiding the production of internal macrostresses or by removing such stresses by relief annealing at 475 to 530 F without softening the work. It should be noted that alloys susceptible to spontaneous season cracking, even if they are free from internal strains, will crack when exposed to corrosive conditions under high service stresses.

The 5 to 20 percent zinc alloys find application because of freedom from season cracking, because of their red color, and because their high melting point is desirable in brazing operations. The properties of these alloys are included in Table 6. Figures 1 and 2 show the effect of progressive cold rolling and progressive annealing on a brass containing 30 percent zinc. Cold working increases the hardness and tensile strength and decreases the ductility as measured by elongation or reduction in area. Annealing below a certain temperature has practically no effect, but in the recrystallization range a rapid decrease in strength and increase in ductility occurs. At this point, the effect of cold working is almost entirely removed. Heating beyond this point results in the growth of the grains with comparatively little further increase in ductility. Figures 3 and 4 show the variation of properties of brass with composition after annealing at the temperatures indicated.

Corrosion Resistance. All copper alloys are highly resistant to atmospheric attack but for outdoor exposure those containing over 80 percent of copper (or copper itself) are preferred because of their relative immunity from season cracking (p. 663).

Water pipes are commonly made of deoxidized copper, red brass, and yellow brass, the last being least resistant to acid waters. Alloys with over 80 percent copper, such as red brass, the silicon-copper alloys, cupronickel, or aluminum bronze, are generally more resistant to corrosion than the alloys with low copper content. Deoxidized copper, arsenical copper, or Admiralty brass have proved satisfactory for condenser tubes operating with fresh water. In sea water, copper is less suitable because of its inability to form protective films. Admiralty brass was for years the standard alloy for sea water installations; as now made it often contains minute additions of arsenic, antimony, or phosphorus to retard dezincification, which is in effect selective corrosion of the alloy with redeposition of copper. Aluminum brass and 70/30 cupronickel are preferred for the most severe sea water service.

Copper and its alloys are not suitable for use in oxidizing acidic solutions or in the presence of moist free ammonia, or mercury.

Effect of Temperature. Copper and all its alloys increase in strength slightly and uniformly as the temperature decreases from room temperatures (Table 9). No low-temperature brittleness is encountered. Copper is useless for prolonged stressed service much above 400 F, but some of its alloys may be used up to 550 F. For service above this, the cupronickel and copper-aluminum alloys alone have satisfactory properties. Aluminum bronzes, particularly those containing 10 percent aluminum, find application for valve seats in internal-combustion engines.

Table 10a. Maximum Allowable Working Stresses for Non-ferrous Materials

(A.S.M.E., Unfired Pressure Vessel Code, Table U-4, condensed, 1940)

Material	Equivalent A.S.T.M. Spec.	Stresses, lb per sq in., for temperatures not exceeding deg F						
		Up to 150	250	350	400	450	500	550
Muntz metal condenser tubes and high brass pipe	B 43-33 B111-37T	5,000	4,000	2,500				
Red brass tubes	B 43-33	6,000	5,500	5,000	4,500			
Copper plates, tubes, and pipe	B 11-33 B 13-33 B 42-33	6,000	5,000	4,500	4,000			
Admiralty condenser tubes and pipe	B 43-33 B111-37T	7,000	6,500	6,000	5,500	4,500		
Copper-silicon alloy plates (types A and C)	B 96-36T B 98-36T	10,000	10,000	5,000				
Monel (annealed 70,000 psi tensile)		14,000	14,000	14,000	14,000	13,500	13,000	12,500
Cast steam bronze (88-6-1.5-4.5)	B 61-36	6,800	6,300	5,800	5,400	5,000	4,200	3,300
Cast steam bronze (Ounce metal, 85-5-5-5)	B 62-36	5,500	5,000	4,500	3,500			
Cupronickel (70/30 and 80/20)	B111-37T	10,000	10,000	9,000	8,400	7,500	6,300	5,000

Tensile properties of several copper alloys at various high temperatures will be found in the A.S.T.M.-A.S.M.E. "Symposium on the Effect of Temperature of the Properties of Metals," 1931 (see pp. 425 to 427). Because of the influence of the time factor, the tensile strength is a poor indication of

Table 8. Properties of Rolled Brass
(A.S.T.M. Specification B 36-40T)

Temper	Nominal grain size, mm	Annealing temp, F, (approx)[b]	Tensile strength, psi		Rockwell hardness
			Min	Max	
Annealed A	0.120	1300	......	......	F50-62
B	0.070	1180	......	......	54-67
C	0.050	1070	......	......	61-73
D	0.035	930	......	......	65-76
E	0.025	800	......	......	67-79
	0.015	700	......	......	72-85
	Nominal reduction, B. & S. gage No.[a]	Percentage reduction of thickness			
Quarter hard	1	10.9	49,000	59,000	B40-65
Half hard	2	20.7	55,000	65,000	57-74
Three-quarter hard	3	29.4	62,000	72,000	70-80
Hard	4	37.1	68,000	78,000	76-84
Extra hard	6	50.0	79,000	88,500	82-89
Spring	8	60.5	86,000	95,000	87-92
Extra spring	10	68.7	89,500	98,500	88-93

[a] Since the B. & S. scale is a uniform geometric progression (see p. 582) the actual reduction in thickness of 1 number will decrease as the piece is progressively cold rolled, but since the actual thickness also decreases the effect of cold work remains approximately the same.

[b] The grain size obtained after a given annealing treatment is dependent on the prior grain size and on the amount of final rolling.

The addition of **lead** to brass renders it free cutting and remarkably machinable. Additions of 0.75 to 1.25 percent of **tin** improve the corrosion resistance. **Aluminum** is added to brass to improve the corrosion resistance *particularly in condenser-tube applications.* ***Manganese bronze*** *is a complex* brass. It has hot-working properties, high strength, and abrasion resistance. The related alloys, Tobin bronze and Naval brass, are used as boat shafting.

All the brasses may be hot worked if they are free from lead, particularly those alloys containing 60 percent or less of copper, for these contain a constituent beta, which is extremely plastic at high temperatures even in the presence of lead. Alloys for extrusion and hot pressing are all in this range.

Extruded sections of many copper alloys are made in a wide variety of shapes. In addition to those used architecturally for moldings, etc., extrusion is important to the engineer since many objects such as pinions, hinges, brackets, and lock barrels can be made directly from extruded bars. Special extruded shapes used as stock for automatic screw machine operations frequently reduce scrap considerably.

Bronzes

The three most common *tin bronzes* contain about 4.5, 8, and 10 percent tin and are known as grades A, C, and D, respectively. They usually contain phosphorus, from a trace up to 0.4 percent, which improves the casting

Table 11. Composition and Properties of Copper-base Alloys for Sand Castings

A.S.T.M. Classification	A.S.T.M. No.	Commercial Designation	Nominal composition, percent								Tensile strength, kips	Yield strength,* kips	Elongation, percent in 2 in.
			Cu	Sn	Pb	Zn	Ni	Fe	Al	Mn			
Tin bronze	1A	88-10-2; "G" bronze	88	10	....	2	....	....	....	....	40	18	20
	1B	88-8-4; modified "G" bronze	88	8	....	4	....	....	....	....	40	18	20
Leaded tin bronze	2A	Steam or valve bronze, (Navy "M")	88	6	1.5	4	0.5	....	....	....	34	16	20
	2B	Commercial 88-10-2	86.5	10	1	2	0.5	....	....	....	36	16	15
High-leaded tin bronze	3A	80-10-10	80	10	10	....	....	....	....	....	25	12	8
	3B	83-7-7-3	83	7	7	3	....	....	....	....	30	14	12
	3C	85-5-9-1	85	5	9	1	....	....	....	....	25	12	8
	3D	78-7-15	78	7	15	....	....	....	....	....	25	14	10
	3E	70-5-25	70	5	25	....	....	....	....	....	21	11	7
Leaded red brass	4A	85-5-5-5, Ounce metal	85	5	5	5	....	....	....	....	30	14	20
	4B	Commercial red brass	83	4	6	7	....	....	....	....	29	12	15
Leaded semi-red brass	5A	Valve composition	81	3	7	9	....	....	....	....	28	12	15
	5B	Semi-red brass	76	3	6	15	....	....	....	....	25	12	15
Leaded yellow brass	6A	High-copper yellow brass	71	1	3	25	....	....	....	....	35	12	25
	6B	Commercial No. 1 yellow brass	66	1	3	30	....	....	....	....	30	11	20
	6C	60-40 yellow brass	60	1	1	37.85	....	....	0.15	....	40	14	15
Leaded high-strength yellow brass	7A	Leaded manganese bronze	61	0.75	0.75	bal.	....	1.0	0.75	0.25	60	..	20
High-strength yellow brass	8A	No. 1 manganese bronze	58	....	....	bal.	....	1.25	1.25	0.25	65	..	25
	8B	High-strength manganese bronze	62	....	....	bal.	....	3.00	5.00	3.50	100	..	10
Aluminum bronze	9A	Grade A	87.5	....	....	....	....	3.50	9.25	....	65	32	22
	9B	Grade B	89.0	....	....	....	....	1.00	10.0	....	65	30	20
	9B	Grade B—heat treated	89.0	....	....	....	....	1.00	10.0	....	80	50	9
Leaded nickel brass	10A	12% nickel silver	57	2	9	20	12	....	....	....	30	15	8
	10B	16% nickel silver	60.5	3	5	15.5	16	....	....	....	35	17	15
Leaded nickel bronze	11A	20% nickel silver	64	4	4	8	20	....	....	....	30	17	8
	11B	25% nickel silver	66.5	5	1.5	2	25	....	....	....	45	22	15
		Nickel tin bronze (cast)	88	5	2	....	5	....	....	....	48	24	40
		Nickel tin bronze (heat treated)	88	5	2	....	5	....	....	....	87	70	10
Silicon bronze		Everdur	94.9	....	....	....	....	Si 4	....	1.1	50	20	25
		Die Casting Alloys											
		Brass die-casting alloy	60.0	1.0	1.0	37.8	....	0.2	0.1	....	55	35	8
		Tombasil	81.5	....	....	14.1	....	4.3	....	....	85	50	8

* 0.5 percent extension under load.

ready workability of brass, it is often cheaper to use than steel. Brass may be drawn at higher speeds than ferrous metals and with less wear on the tools. In cupping operations, a take-in of 45 percent is usual and on some jobs, may be larger. Brass hardened by cold working is softened by annealing at about 1100 F.

Finish. Protection of the surface is usually unnecessary with copper alloys unless freedom from tarnishing is desired. In the latter case, electrodeposited nickel and chromium is perhaps the best finish. Brass makes the best base for chromium plating as the inevitable defects in the plating do not cause rust spots. Objects in which the decorative yellow or red color of the alloy is desired are finished by transparent lacquer. Copper is the standard base metal for high-quality vitreous enamel work.

Welding. Deoxidized copper will weld satisfactorily by the oxyacetylene method. Sufficient heat input to overcome its high heat conductivity must be maintained by the use of torches considerably more powerful than those customary for steel, and preferably by preheating the work in addition. The filler rod must be deoxidized. Tough-pitch copper will not give high-strength welds because of embrittlement due to the oxygen content. Copper may be arc welded, using either metallic or carbon-arc welding with experienced operators. Filler rods of phosphor bronze or silicon bronze will give strong welds more consistently and are used where the presence of a weld of different composition and corrosion-resistance characteristics is not harmful. Brass may be welded by the oxyacetylene process but not with the arc. A filler rod of about the same composition is used, although silicon is frequently added to prevent zinc fumes.

The copper-silicon alloys typified by Everdur are characterized by remarkable ease of welding by all methods. The conductivity is not too high, and the alloy is to a large extent self-fluxing. Metallic- and carbon-arc welding are particularly easy, and if the welds are peened the strength will be equal to that of the annealed sheet. The presence of silicon aids materially the resistance welding of copper alloys.

All copper alloys, except those containing large amounts of aluminum, can be readily soft soldered or silver soldered. **Brazing** is possible only with those alloys whose melting points are sufficiently greater than that of the solder used.

Machining. See "Machining of Copper," Copper Development Association, London, 1939, and Crampton and Croft, National Metals Handbook, 1939. Free-cutting brass rod has been for years the standard material for automatic screw machine work where the very highest machinability is necessary. The use of this material will often result in considerable savings over steel at a lower base price. Most copper alloys are readily machined by usual methods using standard tools designed for steel but at higher speeds. Consideration of the wide range of characteristics presented by various types of copper alloys and the adaptation of the machining practice to the particular material concerned will give greatly improved results. For purposes of machining, copper alloys can be divided into three groups depending on their structure and related characteristics.

Group A is composed of alloys of homogeneous structure; copper, the wrought bronzes up to 10 percent tin, brasses and nickel silvers up to 37 percent zinc, aluminum bronzes up to 8 percent aluminum, the silicon bronzes, and cupronickel. These alloys are all tough and ductile and form a long continuous chip. When they are severely cold-worked, they approach the second classification in their characteristics.

Group B includes lead-free alloys of duplex structure. Some cast bronzes and most of the high-strength copper alloys in the wrought condition belong in this group. They form a continuous but brittle chip by a process of intermittent shearing against the tool edge which causes chatter unless work and tool are rigid.

Group C. Many of the basic brasses and bronzes are rendered particularly fit for machining operations by the addition of 0.5 to 3.0 percent of lead. This exists in the structure as minute uniformly distributed droplets which apparently serve the function of breaking up the chip and lubricating the tool. Chips are fine, almost needlelike, and readily removed. Very little heat is evolved, but the tendency to chatter is greater than in Type A alloy. Lead additions may be made to most copper alloys.

For lathe turning, the tough alloys of Group A need a sharp top rake angle (20 to 30 deg for copper and cupronickel; 12 to 16 deg for the brasses, bronzes, and silicon bronzes with high-speed steel tools; 8 to 12 deg with carbide tools except for copper for which 16 is recommended). Type C (leaded) materials should have a much smaller

portable tools where weight is a factor. In the United States, magnesium is produced by the electrolysis of the molten chloride extracted from salt brines of Michigan, although electrothermic processes depending on the reduction of magnesite by carbon at high temperatures are being developed experimentally. Commercially pure metal contains a minimum of 99.8 percent magnesium, a typical analysis of impurities being 0.02 percent aluminum, 0.03 percent iron, 0.002 percent manganese, 0.01 percent silicon. The pure metal is used in the chemical and radio industries, as a minor constituent in non-ferrous alloys (particularly those containing nickel, where additions of less than 0.1 percent serve as deoxidizers and desulphurizers), and as a constituent of aluminum-rich alloys, some of which contain up to 10 percent magnesium. A large and increasing amount of magnesium is used in the form of magnesium-rich alloys.

Table 13. Magnesium-casting Alloys

A.S.T.M. No.	Properties and uses	Condition	Yield strength (0.2% offset) kips	Tensile strength, kips	Elongation in 2 in., percent	Brinell hardness No.	Fatigue endurance limit, kips
	Sand-casting Alloys						
2	General purpose alloy	Sand-casting	13	22	2	54	9
		Heat-treated	12	32	7	52	10
		H.T. and aged	19	34	2	69	9
3	Strong but brittle	Sand casting	16	21	1	59	9
		H.T. and aged	21	29	0.5	78	9
4	Good properties and corrosion resistance when heat-treated. Most widely used sand casting alloy	Sand casting	12	27	5	55	10
		Heat-treated	12	38	11	55	10
		H.T. and aged	19	38	5	69	10
14	Strongest casting, not subject to shock	H.T. and aged	22	36	1	77	
	Die-casting Alloys						
12	Die-casting Alloy	Die casting	22	30	1	62	
13	Standard die-casting alloy	Die casting	21	33	3	60	
11	Max resistance to salt-water corrosion	Die casting	9	28	9	35	

Most of the alloys are with aluminum, although all contain small amounts of manganese which improves the corrosion resistance. Metal that has to be extensively worked contains about 4 to 6, forgings up to 8.5, and castings as high as 12 percent aluminum. Alloys with over 6.5 percent aluminum may be heat-treated and aged to develop increased yield strengths. Zinc additions are made to many of the sand castings and forging alloys for improved strength and corrosion resistance; silicon is added to die-casting alloys.

Table 12 lists the composition of the important magnesium alloys produced by American manufacturers, most of which are covered by A.S.T.M. specifications. The mechanical properties of these alloys in various fabricated forms

rake to minimize chatter, a maximum of 8 deg with high-speed steels and 3 to 6 deg with carbide. Type B materials are intermediate, working best with 6 to 12 deg with high-speed tools, 3 to 8 deg with carbide, the higher angle being used with the tougher materials. Side clearance angle should be 5 to 7 deg except for tough "sticky" materials like copper and cupronickel, where a side rake of 10 to 15 deg will give better service.

Many copper alloys will drill satisfactorily with the standard helix angle of 30 deg. Straight fluted tools (helix angle 0 deg) are preferable for the leaded Group C alloys. An angle of 10 deg is to be preferred for Group B and 40 deg for copper and cupronickel. Feeds may generally be 2 to 3 times those used for steel. With Type B alloy, a fairly coarse feed helps breaking up the chip, and with Type A a fine feed and high speed gives best results provided that sufficient feed be used to prevent rubbing and work hardening.

Table 10. Creep Data on Wrought-copper Alloys

Designation	Chemical composition, percent					Treatment[a]	Temp, deg F	Stress to produce designated rate of creep, lb per sq in.	
	Cu	Zn	Sn	Mn	Other elements			0.01 % per 1000 hr	0.10 % per 1000 hr
85/15 Brass	85.00	14.92	....	....	....	Cold drawn	400 600	8,800 1,000	12,000 2,600
70/30 Brass	70.48	30.44	....	....	....	Cold drawn	400 600	12,700 290	18,000 850
60/40 Brass	60.21	39.72	0.16	....	....	Annealed (0.02 mm)	300 400	9,000 2,000	12,000 4,750
Admiralty brass	71.05	27.95	0.97	....	....	Annealed (0.03 mm)	400 600 800	13,000 1,000 54	19,000 1,950 160
Aluminum brass	77.26	21.61	1.18	....	....	Annealed (0.02 mm)	400 600	10,500 1,200	13,000 2,500
Tobin bronze	58.79	40.43	0.88	....	....	Annealed (0.02 mm)	300 400	12,000 3,500	15,000 5,700
Elect. copper	....	....	....	....	....	Full anneal	400	3,100	6,700
Deox. copper	99.97	....	....	....	P 0.015	Relief anneal	400	9,400	20,500
Everdur	95.57	0.20	....	1.25	Si 2.84	Anneal (842 F)	400 550	8,100 3,750	14,900 6,400
Everdur	....	....	....	....	....	Full Anneal	400	10,500	22,500
Ambrac	74.23	5.08	....	0.69	Ni 20.0	Anneal (1202 F)	600 750	13,800 8,400	27,800 13,500
Cupronickel	68.88	0.13	....	0.65	Ni 30.32	Anneal (1022 F)	750	9,100	18,800
Aluminum bronze	92.5	....	....	....	7.5 Al	Die cast	560 840	....	10,000 3,000

[a] Figures following anneal are resulting grain size (mm) or temperature of anneal.

References: "Compilation of Available High Temperature Creep Data of Metals and Alloys," A.S.T.M. and A.S.M.E., 1938. "Symposium on Effect of Temperature on Properties of Metals," A.S.T.M. and A.S.M.E., 1931. *Proc. A.S.T.M.*, 32 (Pt. III) 1932.

treatments are available which impart a high degree of resistance to salt water when subsequently painted. Alloys containing manganese show the greatest resistance to salt-water corrosion; this is accelerated by traces of copper and nickel.

Sand castings are made in dry sand, or in green sand to which fluorides or other substances have been added to reduce the action of water vapor on the hot metal. Shrinkage allowance is $^{5}/_{16}$ in. per ft for castings of moderate size, or $^{1}/_{8}$ in. per ft for large castings or where free shrinkage is prevented by cores or bosses. *Castings are frequently heat-treated* for severe service requirements. Die castings or permanent mold castings may be made when the initial mold cost is warranted.

Fabrication. A wide range of extruded structural sections, up to 8 in. I beams, is now obtainable. Forging and extruding and rolling operations are done at temperatures between 400 and 800 F. Sheet is often used in the cold-worked condition and will stand bends over a radius of 5 to 10 times the thickness of the sheet. Annealed sheet will bend over a radius 3 times its thickness. More severe forming operations must be carried out at a temperature of 500 to 650 F. All magnesium alloys machine with ease. High-speed steel tools are advisable or stellite or carbide tools in high production work. Top rake should be 10 to 20 deg, side rake 0 to 10 deg, and clearance 6 to 10 deg. Relatively high speed and heavy feed give best results. Tools must be kept sharp and smooth to avoid overheating of chips. Finely divided magnesium constitutes a fire hazard, and cleanliness in the machine shop is essential.

Many magnesium alloys can be welded by oxyacetylene, if the proper technique and fluxes are used. Electric spot or seam welds also can be made. Riveting is done with aluminum-alloy rivets.

Table 15. Composition of Nickel Base Alloys

Designation[a]	Composition, percent							Melting point, deg F	Density, lb per cu ft	Coefficient of expansion × 10⁻⁶ per deg F (80–212 F)	Electrical resistivity, microhm-cm (32 F)	Elastic modulus, lb per sq in. × 10⁻⁶
	Nickel	Copper	Iron	Manganese	Silicon	Carbon	Other elements					
Wrought Alloys												
Nickel	99.4	0.1	0.15	0.2	0.05	0.1	....	2640	555	7.2	10	30
Z Nickel	98	.....	.....	.....	.....	.....	....	2640	551	7.2	16	30
Monel	67.0	30.0	1.4	1.0	0.1	0.15	....	2460	551	7.8	43	26
R Monel	67.0	30.0	1.7	1.1	0.05	0.1	0.035 S	2460	551	7.8	..	26
K Monel	66 0	29.0	0.9	0.4	0.25	0.15	2.75 Al	2430	536	7.8	62	26
Inconel	79.5	0.2	6.5	0.25	0.25	0.08	13.0 Cr	2540	530	6.4	96	31
Casting Alloys												
Cast nickel	96.7	0.3	0.5	0.5	1.5	0.5	....	2650	551	7.4	22	30
Cast monel	67.0	29.0	1.5	0.9	1.25	0.3	....	2450	551	6.8	54	26
Cast H monel	65.0	29.5	1.5	0.9	3.0	0.1	....	2400	551	6.8	61	26
Cast S monel	63.0	30.0	2.0	0.9	4.0	0.1	....	2350	551	6.8	63	26

[a] Follows International Nickel Co.'s usage.

service characteristics, and creep tests are greatly to be preferred. Available creep data on copper alloys is summarized in Table 10. Table 10a gives values of allowable working stresses in non-ferrous materials according to the A.S.M.E. unfired pressure-vessel code.

Copper-base Alloys for Castings

(See "Cast Metals Handbook," Amer. Foundrymen's Assoc.)

Table 11 shows the composition of the important copper-base casting alloys (mainly from A.S.T.M. Specification B30-40T), together with typical properties to be expected from sand castings.

Test results obtained on standard test pieces (either attached to the casting or separately poured) indicate the quality of the metal used, but are not identical with the properties of the casting itself because of variation of thickness, soundness, and other factors. The ideal casting is one with a fairly uniform metal section with ample fillets and a gradual transition from thin to thick parts.

Tin bronze (alloys 1A, 1B) is an excellent steam and structural bronze and is used for expansion joints, pipe fittings, gears, bolts, nuts, valves, pump pistons, casings, bushings, bearings, and in general where good strength and resistance to salt-water corrosion are required. The **leaded tin bronzes** (alloys 2A, 2B) are more machinable and somewhat more pressure-tight; these are the most commonly used alloys for steam valves and the like. All four bronzes may be used at temperatures up to 500 F. The **high-lead bronzes** (3A-3E) are used principally for bearings and bushings. **Leaded red brass** (alloy 4A), also known as "composition brass" or "hydraulic bronze," is a standard composition for general service and is satisfactory for castings to withstand hydraulic pressures up to 350 lb per sq in. It has the attributes of fairly low cost, good physical properties, good casting properties, and easy machinability. The less costly **leaded semi-red brasses** and **leaded yellow brasses** (alloys 5A, 5B; 6A-6C) are used where corrosion resistance and other requirements are less severe.

High-strength yellow brass (manganese bronze, alloys 7A; 8A, 8B) may be used for castings requiring strength, resistance to abrasion, erosion, or sea-water corrosion; examples are marine propeller hubs and blades, gears, worm wheels, gun mount castings, etc. The **aluminum bronzes** (9A, 9B) have high strength and great resistance to corrosion, shock, and fatigue. They need careful foundry practice. They are used for such castings as gun slides and mountings, worm gears and wheels, valve seats, some types of bearing, and propellers. Alloy 9B may be heat-treated by quenching from 1550 F and reheating at 700 to 1100 F for improvement of its physical properties.

The **leaded nickel alloys** (10A, 10B; 11A, 11B) are used, principally on account of their white color, in plumbing fixtures, builders' hardware, and ornamental work. Alloy 11B is also used in sand castings needing corrosion resistance and for steam valves, etc., for service up to 550 F. The **nickel-tin bronze** is a strong wear-resisting alloy. The hardness and strength may be varied greatly by heat-treatment. The properties shown are for a casting subjected to quenching from 1400 F and reheating 5 hr at 550 F. Nickel in amounts up to about 1 percent is commonly added to many of the other foundry mixtures listed in Table 11 for its effect on strength, grain refinement, and lead distribution. **Silicon bronzes** (copper-silicon alloys) have a wide range of use as sand castings because of their high strength and resistance to the corrosive action of many chemical solutions.

It is impossible to cast pure copper in the foundry. Castings for applications needing high thermal or electrical conductivity are made from copper

nickel are required and where corrosion resistance is needed in parts that have to be worked extensively.

Commercial nickel may be forged or rolled at 1600 to 2300 F. It becomes increasingly harder below 1600 F but has no brittle range. The recrystallization temperature of cold-worked pure nickel is about 660 F, but commercial nickel recrystallizes at about 1100 F and is usually annealed at temperatures between 1100 and 1750 F.

The addition of certain elements to nickel renders it susceptible to a precipitation or aging treatment to increase its strength and hardness. In the unhardened or quenched state, the alloy Z nickel fabricates almost as easily as nickel and when finished may be hardened by heating for about 8 hr at 900 to 935 F. Intermediate anneals during fabrication are at 1350 to 1750 F, and the final anneal should be at 2000 F and be followed by quenching to render the material susceptible to aging. Treatment must be in an atmosphere that will prevent loss of carbon. The increase in strength due to aging is to a great extent superimposed on that due to cold work.

Nickel castings are made in dry sand molds, but need special technique because of the high temperatures involved. The addition of 1½ percent silicon and lesser amounts of carbon and manganese is necessary to obtain good casting properties.

Alloys of nickel with 2 to 5 percent manganese are used generally for spark-plug electrodes, an application that requires resistance to hot corrosive gases. They are slightly stronger than nickel but have, in general, the same properties.

Copper-nickel Alloys. Alloys containing less than 50 percent nickel are discussed under Copper Alloys, p. 628 An important nickel-rich alloy is known by the trade name **Monel metal** (see Tables 15 and 16). It combines high strength, high ductility, and excellent resistance to corrosion. It is a homogeneous solid solution alloy; hence its strength can be increased by cold working alone. In the annealed state, its tensile strength is about 70,000 lb per sq in., and this may be increased to 170,000 in the hardest drawn wires. It is available in practically all fabricated forms. Monel metal is hot worked in the range 1850 to 2150 F after rapid heating in a reducing but sulphur-free atmosphere. It must be cold-worked in the same manner as mild steel, but requires more power. Very heavily cold-worked Monel may commence to recrystallize at 800 F, but in practice no softening will occur below 1200 F. Annealing is done in boxes for 2 to 6 hr at about 1400 F or open for 2 to 5 min at about 1725 F. Non-scaling atmospheres are desirable and sulphur-free atmospheres essential.

Because of its toughness, Monel must be machined with slower cutting speed and lighter cuts than mild steel. High-speed tools are necessary. The special grade of Monel containing sulphur ("R" Monel) should be used where high cutting speeds must be maintained. This is slightly inferior to the sulphur-free alloy in mechanical properties and corrosion resistance, and it cannot be hot forged.

The short-time tensile strength at elevated temperatures and the creep properties of cold-drawn Monel are summarized in Tables 17 and 18. Monel does not suffer impairment of properties at low temperatures as do most ferrous materials, in fact the tensile strength and yield strengths actually increase appreciably down to liquid air temperatures, although the elongation remains the same.

The fatigue endurance limit of Monel is about 37,000 (52,000) lb per sq in. when annealed (hard drawn). The action of corrosion during fatigue is

with additions of deoxidizers such as silicon, manganese, phosphorus, or zinc, which decrease the conductivity 10 to 50 percent.

Brass die castings are made when greater accuracy of dimensions or a better surface finish is required than can be obtained with sand castings. While inferior in properties to hot-pressed parts, die castings are adaptable to a wider range of design, for they may be made with intricate coring and with considerable variation in section thickness.

Where resistance to corrosion or elevated temperatures is of importance, the manufacturer of the alloy should be consulted as to its suitability under the proposed conditions.

Table 12. Magnesium and Magnesium Alloys

A.S.T.M. No.	Dow Chemical Co. No.	American Magnesium Corp. No.	Composition, nominal, percent			Forms available	Density, lb per cu ft	Thermal conductivity 32–212 F, Btu per sq ft per ft per deg F	Electrical resistivity, microhm-cm at 68 F
			Al	Mn	Other elements				
			Commercially pure Mg, 99.8 +			ingot s	109	90.9	4.46
2	G	240	10.0	0.1		c, p	114	41	15.0 c
3	B	246	12.0	0.1			114	39	16.5 c
4	H	265	6.0	0.2	3.0 Zn	c	114	44[a]	11.5 c
6	F	244	4.0	0.3		c, e, s	111	56	9.5 c
7	E		6.0	0.3		s	112	48	13.0 e
8	J	57S	6.5	0.2	0.7 Zn	f, e	112	46[a]	12.5 e
9	O	58S	8.5	0.2	0.5 Zn	f, e	112	44[a]	14.5 e
10	..	61S	....	1.0	6.0 Sn	f	116		
11	M	403	....	1.5		d, f, e, s	111	73[a]	6.5 c
12	K	230	10.0	0.1	0.5 Si	d	114	41[a]	17.5 d
13	R	...	9.0	0.1	0.6 Zn	d	114	41	17.0 d
	L	...	2.5	0.3	3.5 Cd	f	114	..	8.5 e
14	P	...	10.0	0.1	1.0 Zn	c	114	39	12.5HTA
15	X	65S	3.0	0.2	3.0 Zn	f, e	112	51	10.0 e

[a] Approximate.
HTA Aged.
c Sand casting.
d Die casting.
e Extruded shapes.
f Forged.
p Permanent-mold casting.
s Plate, sheet, strip.

A.S.T.M. Tentative Specifications cover Sand Castings (B80-38T), Die Castings (B94-39T), Sheet (B90-38T), Forgings (B91-38T), Bars, Rods and Shapes (B107-38T).

Typical properties of many of the alloys are given in *Proc. A.S.T.M.*, 1934, 34 (ii), pp. 277–306.

Mean specific heat (68–662 F) 0.27 approx.

Of the alloys listed, those containing lead are very readily machined and no particular difficulty will be encountered with the others.

Castings of aluminum bronze and the alloys containing lead are not considered suitable for **welding**. Silicon bronze castings can be readily welded by the usual methods and brasses free from lead and aluminum can be gas-welded.

Magnesium and Magnesium Alloys*

Magnesium is the lightest metal of structural importance (density 108.6 lb per cu ft). Its principal applications are in transport construction and in

*Largely based on publication of Dow Chemical Co. See also publications of Am. Magnesium Corp.; A.S.M. Metals Handbook; Haughton and Prytherch, "Magnesium and Its Alloys," Chemical Publishing Co., N. Y.

Monel-metal castings are made from the alloys with silicon additions listed in Table 15. Casting temperatures are of the order of 2800 F, and the composition must be carefully controlled. Magnesium is added as a deoxidizer. The castings may be softened slightly by annealing if better machining properties are desired. Cast "S" Monel can be rehardened by heating to 1100 F and cooling slowly.

The **nickel-chromium alloys** possess excellent resistance to corrosion and oxidation. They are commercially available in various fabricated forms, usually in modifications containing additions of iron, copper, or molybdenum. The standard electrical resistance materials for high-temperature service are the 80-20 nickel-chromium and 60-15-25 nickel-chromium-iron alloys. The former can be operated at temperatures up to 2100 F and the latter to 1700 F. The resistivities at 77 F are 108 and 112 microhm-cm, respectively.

Magnetic Properties of Nickel Alloys. Nickel is slightly ferromagnetic but loses its magnetism at a temperature of 695 F when pure. In commercial nickel, this temperature is about 650 F. Monel is feebly magnetic and loses all ferromagnetism above 200 F. K Monel is non-magnetic down to at least −110 F. The degree of ferromagnetism and its temperature sensitivity are very susceptible to variations in composition and mechanical and thermal treatment.

Lead

Most lead is obtained from the sulphide ore, galena, by concentration, flotation, and reduction in blast furnaces. The crude lead is purified by drossing, the precious metals removed electrolytically or by washing with molten zinc (Parkes process), and the resulting lead cast into pigs. Three grades of pig lead are recognized by the A.S.T.M. specifications (Table 19).

Table 19. Composition Specifications for Lead, Percent
(A.S.T.M. Specification B29-35)

	Grade I corroding lead	Grade II chemical lead[a]	Grade III common lead	Grade IIIa common lead
Silver	0.0015 max	0.002-0.020	0.002 max	0.002 max
Copper	0.0015 max	0.040-0.080	0.0025 max	0.0025 max
Copper + silver	0.0025 max			
Arsenic	0.0015 max			
Arsenic + antimony + tin	....	0.002% max	0.015% max	0.015% max
Arsenic + antimony	0.0095 max			
Zinc	0.0015 max	0.001 max	0.002 max	0.002 max
Iron	0.002 max	0.0015 max	0.002 max	0.002 max
Bismuth	0.05 max	0.005 max	0.15 max	0.25 max
Lead (difference)	99.94 min	99.90 min	99.85 min	99.73 min

[a] Term applied to undesilverized lead from S. E. Missouri ores.

Corroding lead is the highest purity commercial lead and is used for making white lead. **Chemical lead** is extensively employed in chemical plants for withstanding corrosion, particularly of sulphuric acid. Its copper content confers added stiffness on it. **Common lead** is the usual grade for alloying.

Antimonial lead is used in places where greater strength is needed. For storage-battery plates, lead containing 6 to 7 percent antimony is used. In the cast condition, this has a tensile strength of about 7,000 lb per sq in.

are shown in Table 13. These are typical figures; specification values are lower.

Designs employing magnesium alloys must take into account the lower modulus value (6.5×10^6 lb sq in.), compared with other structural materials, and the higher thermal expansion coefficient. All magnesium alloys have an expansion coefficient per degree Fahrenheit of about 14×10^{-6} at 32 F and an average coefficient in the range 68 to 752 F of 16×10^{-6} per deg F.

Table 14. Wrought-magnesium Alloys

A.S.T.M. No.	Properties and uses	Condition	Yield strength (0.2% offset) kips	Tensile strength, kips	Elongation in 2 in., percent	Brinell hardness No.	Fatigue endurance, kips
	Forging Alloys						
8	General forging requirements	PF	24	40	10	57	17
11	Maximum resistance to salt-water corrosion	PF	19	33	6	43	
10	Good hot-working properties	HF	19	35	5	45	9
9	High-stressed parts of simple design	PF	30	45	8	78	18
		PFA	33	46	6	82	18
L (Dow)	Easily forged under hammer	HF	26	37	11	51	10.5
15	Moderate strength with corrosion resistance	PF	24	41	16	59	17
		PFA	28	42	14	62	17
	Extruding Alloys						
6	Extrudes in thin sections	E	29	40	16	47	14
8	Strong simple sections	E	30	43	17	54	17
9	Strong heavy sections	E	33	47	11	61	17
11	Resistance to salt water corrosion	E	27	42	6	42	6
X (Dow)	Common extrusion alloy for general purposes	E	30	42	19	51	18
		EA	34	44	13	54	17
	Sheet and Strip Alloys						
...	Comm. pure magnesium. Not used structurally	HR	27	37	9	50	9
		An	10	27	15	37	
7	Usual alloy for sheet. Good strength and formability	HR	34	45	9	70	
		An	20	39	15	57	
6	Largely replaced by #7	HR	30	43	8	60	
		An	18	35	13	49	
11	Best welding and forming qualities of magnesium alloy. Resistant to salt corrosion	HR	27	35	9	53	
		An	19	33	14	48	

Modulus of elasticity is 6,500,000 lb per sq in.
PF. Press forged. A. Aged. An. Annealed.
HF. Hammer. E. Extruded bars less than 1½ in. diam. HR. Hard rolled.

Magnesium alloys are not resistant to acid corrosion or continued exposure to salt water but are moderately resistant to atmospheric action, particularly if they are given a surface treatment in an acid dichromate pickle such as is usually applied to practically all fabricated forms supplied by the producers. The treated surface forms an excellent paint base. Several other chemical

the linotype machine, the metal must be fluid and capable of rapid solidification; hence metal of very nearly eutectic composition is used. It is very rarely used as the actual printing surface and therefore need not be as hard as stereo- and monotype metals. Foundry type is used for hand setting and needs greater hardness to withstand handling and repeated use.

Zinc and Zinc Alloys[a]

Zinc, one of the least expensive non-ferrous metals, is produced from sulphide, silicate, or carbonate ores by a process involving concentration and roasting followed either by reduction of the zinc ore by carbon and simultaneous distillation of the zinc in batch or continuous retorts or by leaching out the oxide with sulphuric acid and electrolizing the solution after purification. Distilled zinc contains impurities (principally Pb, Cd, and Fe) that may be eliminated by fractional redistillation to produce zinc of 99.99+ percent purity. Metal of equal purity can be produced by the electrolytic process. Zinc reaches the market in the form of slabs, 1 to 1½ in. thick, 8½ to 10 in. wide, 18 to 20 in. long. In this form, it is frequently called **spelter.**

The important grades of zinc available in the United States are covered by A.S.T.M. specification B6-37. The composition requirements are as follows:

Table 22. A.S.T.M. Specification B6-37 for Slab Zinc

Grade	Max percent			
	Lead	Iron	Cadmium	Sum of lead, iron, cadmium
Special high grade	0.007	0.005	0.005	0.010
High grade	0.07	0.02	0.07	0.10
Intermediate	0.20	0.03	0.50	0.50
Brass special	0.60	0.03	0.50	1.0
Selected	0.80	0.04	0.75	1.25
Prime western	1.60	0.08		

The special high-grade zinc is used in the manufacture of die castings, where impurities have a marked harmful effect on corrosion resistance and dimensional stability. Galvanizing (which consumes by far the largest proportion of zinc) utilizes principally Prime Western zinc. All grades are used for rolled-zinc products as the presence of impurities is often desirable for their strengthening effect. For brass manufacture and other alloys, there is an increasing tendency towards the use of high grades of zinc.

Rolled Zinc. Zinc rolled in the form of sheet, strip, or plate of various thicknesses is used extensively. Such zinc is unalloyed except for the presence of impurities resulting from incomplete refining. It is produced by hot rolling unless some stiffness and temper are required, in which case one or more of the finishing passes are done cold. The softer purer grades of zinc are used for deep drawing or forming operations, and the less pure metal is used for weather strip, roofing, and other applications where some stiffness is necessary, or where specific chemical properties are desired, as in photo-engravers' plates.

[a] Largely based on data supplied by The New Jersey Zinc Co.

Nickel and Nickel Alloys*

About 85 percent of the world's production of nickel comes from the sulphide deposits in Ontario. The smelting process involves the production of a matte containing both copper and nickel and repeatedly resmelting this with sodium sulphide to cause a separation into two layers of sulphide, one rich in nickel. The roasted sulphide may be reduced with carbon, cast to anodes, and electrolyzed to the pure metal, or reduced and vaporized as gaseous nickel carbonyl to be subsequently decomposed to form nickel pellets. If necessary, the nickel is remelted and cast as block or shot. If it is to be rolled, it is cast in ingot molds.

Monel, an important alloy of nickel, is made by reducing the copper-nickel matte without separating the metals from each other. Other alloys are made in the usual way by incorporation of the elements in the molten state.

The nominal composition and typical properties of nickel and various nickel-rich alloys are summarized in Tables 15 and 16.

Table 16. Mechanical Properties of Nickel and Its Alloys

Alloy	Form and temper tested	Yield strength, (0.2% offset) kips	Tensile strength, kips	Elongation, percent in 2 in.	Brinell hardness
Nickel	Rod, hot rolled	20–30	65–80	45–35	100–140
	Rod, cold drawn	60–90	80–115	35–15	140–200
	Rod, annealed	20–30	65–85	50–30	100–150
	Wire, spring temper	....	135–165	4–2	
Z Nickel	Rod, cold drawn	50–130	90–150	35–15	150–300
	Rod, drawn, heat-treated	120–150	160–190	20–7	300–380
	Wire, spring, heat-treated	....	200–240	10–5	
Monel	Rod, hot rolled	40–65	80–95	45–30	130–170
	Rod, drawn, annealed	30–40	70–85	50–35	120–160
	Strip, annealed	25–35	65–80	40–20	60–68[a]
	Strip, full-hard	90–115	110–130	15–2	>98[a]
	Wire, spring temper	....	140–170	10–2	
K Monel	Rod, hot rolled	40–60	90–110	45–35	140–180
	Rod, cold drawn	70–100	100–125	35–20	175–250
	Rod, drawn, heat-treated	100–130	140–170	30–15	260–320
Inconel	Rod, hot rolled	35–70	85–120	45–30	120–240
	Rod, cold drawn	25–40	80–95	55–35	130–170
	Wire, spring temper	....	165–185	10–2	
Cast nickel	As cast	20–30	55–70	30–15	100–130
Cast monel	As cast	32–40	65–80	45–25	125–150
Cast monel H	As cast	45–65	70–90	20–10	175–250
Cast monel S	As cast	70–90	90–115	3–1	275–350

[a] Rockwell B scale.

Commercially pure nickel is available as sheet, rod, wire, and other fabricated forms. It is used where the thermal or electrical properties of

* Largely based on information supplied by International Nickel Co.

Table 23. Properties of Zinc-base Die-casting Alloys

	A.S.T.M. No. XXIII	A.S.T.M. No. XXI	A.S.T.M. No. XXV
Nominal Composition, percent:			
Aluminum	4.1	4.1	4.1
Copper	0.0	2.7	1.0
Magnesium	0.04	0.03	0.03
Typical Properties:			
Tensile strength, lb per sq in.	40,300	47,900	45,400
Elongation, percent in 2 in.	5	5	3
Charpy impact, ft-lb	20	20	20
Electrical resistivity, 77 F, microhm-cm²	64	69	67
Electrical conductivity, 77 F $\times 10^2$ Mho cm³	155	144	151
Thermal conductivity, Btu per ft per sq ft per hr per deg F	66	61	62
Thermal expansion $\times 10^{-6}$ per deg F	15.2	15.4	15.2
Density, lb per cu ft	410	415	415
Melting point, deg F	718	715	717

The A.S.T.M. specification B86-38T calls for a maximum percent of 0.1 iron, 0.007 lead, 0.005 cadmium, 0.005 tin in alloys XXIII and XXI. The same impurity limits hold for alloy XXV with the exception of the tin limit, which is set at 0.002 maximum. The low limits of impurities are necessary to avoid disintegration of the castings by intergranular corrosion under moist atmospheric conditions. The presence of magnesium prevents this effect if the impurities are not higher than the specification values. The mechanical properties in Table 23 are average figures for die-cast tensile-test pieces of 0.25 in. diam or impact specimens 0.25 in. square. Specification values for these properties, if used, would naturally be lower than the typical ones quoted, and, in the case of the actual castings, considerable variations must be expected.

The alloys containing copper are stronger and possess improved corrosion resistance but are more subject to changes of dimension and properties (particularly impact resistance) on aging. Where extreme stability is important, the copper-free alloy should be used.

A measurement of the expansion of the die casting after exposure to water vapor at 203 F for 10 days is a suitable index of stability and freedom from susceptibility to intergranular corrosion and is frequently specified as an acceptance test. Analysis for impurity content is, however, more widely used.

Aging of Die Castings. Because of changes occurring in the structure of zinc die castings, they commence to shrink immediately after removal from the mold, the change being about two-thirds complete in 5 weeks. The maximum extent of this is about 0.001 in. per in. In the alloys containing copper, this is followed by an expansion. A.S.T.M. XXI alloy has a resultant expansion as much as 0.004 in. per in., whereas in alloy XXV the expansion amounts to only 0.001 in. per in. In alloy XXV, this change requires a period of many years at room temperatures. It is greatly accelerated at elevated temperatures. Coincident with expansion is some loss of impact resistance. Alloy XXI has a normal impact strength of 20 ft-lb on a 0.25 in. square casting, which drops to 3 ft-lb on dry aging at room temperature for 5 years and to about 1 ft-lb on accelerated aging. Alloy XXIII (copper-free) is unaffected by either treatment, and alloy XXV, with 1 percent copper, is

Table 17. Short-time High-temperature Properties of Hot-rolled Monel and Nickel

Temperature, deg F	70	300	600	700	800	1000	1200	1500
Monel								
Tensile strength, kips	31.8	29.5	27.6	28.3	29.4	23.1	17.5	9.0
Yield strength, 0.2 percent offset, kips	81.0	77.5	78.3	70.3	71.0	51.1	30.3	16.3
Elongation in 2 in., percent	46	48	51	53	52	29	34	58
Reduction of area, percent	61	64	66	68	72	31	39	64
Nickel								
Tensile strength, kips	72	75	79	82	77	40	33	22
Yield strength, 0.2 percent offset, kips	23	22	21	21	20	17	15	9
Elongation in 2 in., percent	51	48	47	47	51	51	51	63

Table 18. Stress to Produce Stated Rate of Creep in Cold-drawn Monel[a]

(In 1,000 psi)

Temperature, deg F	600	800	1000	1200
Creep rate, 0.01 percent per 1,000 hr	26	19	1.65	
Creep rate, 0.1 percent per 1,000 hr	36	23.5	4.3	
Creep rate, 1.0 percent per 1,000 hr	46	29	11.5	1.7

[a] Clark and White, "Symposium on Effect of Temperature on Metals," *A.S.T.M.-A.S.M.E.*, 1931, p. 383.

much less drastic on Monel than on steels of equal or higher endurance limit. McAdam found a limit of 30,000 lb per sq in. in brackish water.

Monel may be welded by the usual electric and gas methods but needs special fluxes. The gas flame should be slightly reducing and the work done rapidly without rewelding. Flux-coated electrodes should be used for arc welding. Spot and seam welding can be used on thin sheet. Soft soldering, brazing, and silver soldering are readily applied.

Monel is highly resistant to atmospheric action, sea water, steam, foodstuffs, and many industrial chemicals. It deteriorates rapidly in the presence of moist chlorine and ferric, stannic, or mercuric salts in acid solutions. It must not be exposed when hot to molten metals, sulphur, or gaseous products of combustion containing sulphur.

If about 2.75 percent aluminum is added to the base 70/30 nickel-copper alloy, it becomes susceptible to precipitation hardening. This alloy (trade name K Monel) is available in rod, strip, and wire form. It is sufficiently ductile in the annealed state to permit drawing, forming, bending, or other cold-working operations but work hardens rapidly and requires more power than mild steel. It is hot worked at 2150 to 1700 F and should be cooled and quenched from about 1450 F if the metal is to be further worked or to be hardened. Heat-treatment consists of quenching from 1450 F, cold working if desired, and reheating for 10 to 16 hr at 1080 to 1100 F. If no cold working is intended, the quench may be omitted on sections less than 2 in. thick and the alloy partially hardened during cooling from 1450 F. The properties of the heat-treated alloy remain quite stable, at least up to 1000 F for several months.

Other cobalt-chromium-tungsten alloys, usually with less tungsten than the above, find extensive application as **hard facing materials** which are applied by fusing with an oxyacetylene flame on to the edges and surfaces of parts that are subject to severe erosion or abrasion. Cast tungsten carbide and various nickel-chromium-boron compounds are also used for hard facing. These are applied in fragments or small pieces that are welded to the underlying metal with a suitable welding rod, or a composite rod is applied in a pasty condition.

Fusible Alloys

The more important fusible alloys are listed in Table 24 together with their melting points. These alloys are used as fusible links in sprinkler heads, electric cutouts, etc., for making castings in organic molds, for patterns in making match plates, for setting punches in multiple dies, and various other uses. They can be cast against wood, paper, and other organic materials without damaging them and against hard steel without drawing its temper. Many of the alloys may be used for making seals against glass. Those of lowest melting point can be melted in boiling water, a property made use of in bending thin wall tubing and forming objects from sheet which are filled with the alloy, to be subsequently melted out. Alloys containing about 50 percent bismuth undergo practically no change of volume on solidification.

For sprinkler heads with a rating of 160 F, the quaternary eutectic is used, for 200 F the bismuth-lead-tin eutectic, for 286 F the bismuth-tin, and for 360 F the lead-tin eutectic. It should be noted that intermediate temperatures between the various eutectics are not obtainable. Alloys of true eutectic composition will melt sharply at the temperatures shown and be completely solid just below it. Alloys not of the correct composition will be pasty over a range of temperatures.

Table 24. Fusible Alloys[a]

Designation	Melting point, deg F	Composition, percent				
		Bi	Pb	Sn	Cd	Other elements
Quinary eutectic Bi-Pb-Sn-Cd-In	117	40.9	22.1	10.7	8.2	In 18.1
Quaternary eutectic Bi-Pb-Sn-Cd (Wood's metal)[b]	158	49.4	27.7	12.9	10.0	
Ternary eutectic Bi-Pb-Cd	197	51.6	40.2	....	8.2	
Ternary eutectic Bi-Pb-Sn	203	52.0	32.0	16.0		
Binary eutectic Bi-Pb[b]	256	55.5	44.5			
Binary eutectic Bi-Sn	281	57.0	....	43.0		
Ternary eutectic Pb-Sn-Cd	288	....	30.6	51.2	18.2	
Binary eutectic Bi-Cd	291	60.0	....	....	40.0	
Binary eutectic Sn-Cd	351	....	....	67.8	32.2	
Binary eutectic Pb-Sn[b]	362	....	61.9	38.1		
Ternary eutectic Pb-Sn-Sb	462	....	84.0	4.0	....	Sb 12.0
Binary eutectic Pb-Sb	477	....	88.0	....	....	Sb 12.0
Matrix alloy[b]	217–440	48.0	28.5	14.5	....	Sb 9.0

[a] Selected from Booklet B-5 "Fusible Alloys Containing Tin" published by International Tin Research Council, 1937. Wood's metal is often made up roughly in the workshop from 4 parts Bi, 4 parts Pb, 1 part Sn, 1 part Cd, which is not far from the ideal composition. For other low melting alloys, see Type Metals and Solders.

[b] Most useful compositions.

with an elongation of about 22 percent, density 677 lb per cu ft. Lead for sheathing telephone and electric power cables is usually made of an alloy containing about 1 percent antimony. This alloy when extruded as cable sheath and aged 1 month at room temperature has a tensile strength of 2,750 to 3,050 lb per sq in. at a testing speed of 0.25 in. per min per in. of free length, elongation 30 to 40 percent. Endurance limit 800 lb per sq in. (50 million cycles at 700 per min).

Cast lead-antimony alloys containing 6 to 14 percent antimony have a tensile strength of 7,000 to 8,000 lb per sq in. with elongation decreasing from 24 to 10 percent. Allowable fiber stresses for indefinitely long service in extruded lead and lead alloy pipe according to the Lead Industries Association are given in Table 20. The lead-antimony alloys, particularly in the range 2 to 8 percent antimony, are susceptible to heat-treatment which considerably increases their strength. This treatment is rarely employed in practice. An alloy also used for cable sheathing is ordinary chemical lead with about 0.06 percent copper. Alloys with 0.1 percent tellurium or 0.01 to 0.10 percent calcium have been proposed for special purposes where higher creep and fatigue resistance are needed. Alloys with larger amounts of calcium (0.8 percent) and smaller quantities of alkali metals are used to limited extent as bearing metals.

Lead for coating steel (terne plate) and copper contains 5 to 25 percent of tin to aid adhesion to the base metal. Lead is an important constituent of alloys with tin and copper described elsewhere and of type metals.

Type Metals. The principal type metals are listed in Table 21. Considerable variations from these compositions are often made. Electrotype metal serves only as a backing to the shell and does not need to be hard. In

Table 20. Allowable Fiber Stress in Extruded Pipe, Lb per Sq In.

Temperature, deg F	"Chemical" lead	6% Antimony-lead	Temperature, deg F	"Chemical" lead	6% Antimony-lead
68	200	400	194	136	195
86	190	370	212	127	165
104	180	340	230	118	137
122	172	310	248	110	110
140	162	280	266	100	80
158	153	254	284	90	50
176	144	222	302	80	

Table 21. Composition and Properties of Type Metals[a]

Service	Composition, percent			Melting point, deg F	Brinell hardness
	Sn	Sb	Pb		
Electrotype	3	3	94	570	14
Linotype	4	12	84	475	22
Stereotype	6	14	80	500	24
Monotype	8	16	76	515	26
Foundry	14	24	62	605	32

[a] Gonser and Epstein, *Metals and Alloys*, 8, 1937, p. 59.

silver solders. The commercial range of standard compositions of silver and brazing solders as shown in Table 26 covers the range 1280 to 1600 F melting point. The special alloys listed in this table will be found desirable for purposes where the lower melting point justifies the higher cost. Alloys containing phosphorus should not be used for soldering steel or nickel-rich alloys on account of the formation of a brittle layer of phosphide.

Table 26. Silver Solders and Brazing Solders

Designation	Composition, percent				Melting range, deg F	
	Ag	Cu	Zn	Cd	First melting point	Point of complete liquefaction
A.S.T.M. 1	10	52	38	<0.5	1510	1600
A.S.T.M. 2	20	45	35	<0.5	1430	1500
A.S.T.M. 3	20	45	30	5	1430	1500
A.S.T.M. 4	45	30	25	....	1250	1370
A.S.T.M. 5	50	34	16	....	1280	1425
A.S.T.M. 6	65	20	15	....	1280	1325
A.S.T.M. 7	70	20	10	....	1335	1390
A.S.T.M. 8	80	16	4	....	1360	1460
A.S.T.M. 50–50	...	50	50	....	1595	1620
A.S.T.M. 52–48	...	52	48	....	1600	1620
Black Button	1Pb	27.3	64.7	7.5 Sn	1385	1440
Easy-Flo[a]	50	15.50	16.5	18 Cd	1160	1175
Sil-fos[a]	15	80	....	5 P	1300	1300
Phos Copper[a]	...	93	....	7 P	1317	1470

[a] Proprietary alloy.
See A.S.T.M. specifications B64-38T (Brazing Solders) and B73-29 (Silver Solders).

Bearing Metals

Typical compositions and properties of some copper-base alloys are shown in Table 27. For low-speed operation under high pressures such as bridge bearing and expansion plates, hard bronzes containing up to 20 percent tin are used. These can be used only with a mating surface of hardened steel and under conditions of proper alignment. For lower loads, the tin content may be decreased to 10 percent or less. Additions of lead are made to bronzes for use under conditions where alignment and lubrication are apt to be poor. The alloy 80/10/10, copper-tin-lead, is most commonly used for general machinery. Alloy 88/10/2, copper-tin-zinc, has good casting properties and is used for housings, etc., cast with integral bearings. Chill casting gives a finer structure and is preferable to sand casting for the best bearing properties.

Copper alloys containing high lead are finding increased application. The bronzes with high lead and tin contents need skilled foundry technique if the lead is to be properly distributed. The maximum amount of lead that can be retained with ordinary foundary practice is 25 percent, with 5 percent tin. This constitutes the so-called plastic bronze which is singularly proof against poor installation and abuse. Maintaining the copper constant at 70 percent and increasing the tin up to 16 percent at the expense of the lead produces a useful series of bearing alloys that will withstand progressively higher pressures. Straight copper-lead alloys with 25 percent lead, the balance copper with small amounts of nickel or tin, are used for internal-combustion engines for high loads. Because of the softness of the alloy, it is applied at a high temperature in a very thin layer (0.02 in.) on a steel backing strip which is subsequently formed to fit the journal or backing.

Strip is usually purchased on specifications (A.S.T.M. B69-39) involving a "dynamic ductility" test, which is an Erichsen cupping test performed at press speeds (about 90 rpm). This indicates its ability to withstand forming operations, although it is not exactly related to drawing properties. An arbitrary "temper" test is also used which measures spring back after a definite deformation at a prescribed speed. Creep tests at room temperature are necessary to estimate the behavior of the metal under service stresses. Zinc should not be used in applications where high continuous stresses are involved.

Alloys of zinc containing 0.65 to 1.25 percent copper are significantly stronger than unalloyed zinc and possess good ductility and working properties. They can be work hardened and may be employed for parts that must withstand loads somewhat higher than would be permissible in unalloyed zinc. The addition of about 0.01 percent magnesium (*Trans. A.I.M.E.*, 1930, p. 481) to this alloy increases the creep resistance considerably, and the alloy finds some application for roofing and the like with design stresses up to 10,000 lb per sq in. Magnesium additions, however, decrease the ductility and general fabricating characteristics.

The usual tensile test is practically meaningless with zinc because the creep that occurs even at quite small loads causes the breaking load to vary with testing speed: the results are unrelated to service conditions. The speed in tensile testing is usually controlled at 0.25 in. per min under which conditions the soft grades of rolled zinc will have a tensile strength of 16,000 to 19,000, hard-rolled impure zinc 19,000 to 26,000, and the zinc-copper-magnesium alloy in the cold-worked condition as much as 50,000 lb per sq in. The elongation will vary between 5 and 65 percent but bears no direct relation to formability because of the different speed of testing. The properties of zinc vary with the direction of testing, and the across-grain tensile values will be approximately 20 percent higher than those obtained with the direction of the grain, although the elongations are correspondingly lower.

Zinc strip or sheet can be fabricated by the usual methods, **cupping, forming**, etc., provided it is not at too low a temperature. A take-in of 42 percent on the first cupping operation is usual. Warm soapy water is widely used as a lubricant. The soft grades are self-annealing at room temperature, and only the harder alloys need intermediate annealing between operations as most other metals do. When necessary, the hard zincs are annealed at 212 F and the zinc-copper alloys at about 440 F. Welding is possible, and soldering is exceptionally easy. Simple extrusion of rods, molding, and tubing is possible but expensive because of the slow speeds necessary. The impact extrusion process, however, is being more and more widely used for producing battery cups and similar articles.

Zinc is resistant to atmospheric **corrosion** but is attacked by acids and alkalies. Soap tends to inhibit the action of water. Surface finishes for corrosion resistance or improving the appearance are readily applied. These include electroplating with copper, nickel, and chromium, lacquering, enameling, or chemically coating.

Zinc Die Castings. Zinc alloys are particularly suited for making die castings since the melting point is reasonably low, resulting in long die life even with ordinary steels, and a high accuracy and good surface finish is possible.

The alloys at present used for die castings in the United States are practically limited to the three which are covered by A.S.T.M. specification B86-38T. Nominal compositions and typical properties of die-cast test pieces are given in Table 23.

hardest babbitt and when well seated gives the best service. It should not be used with soft steel for wear will occur against the hard tin-copper constituent present. No. 4 with 10 percent lead is an economical bearing for general machinery purposes. Nos. 5 and 6 are intermediate alloys containing still higher lead content and are unsuited for operations at moderately elevated temperatures. They are not widely employed. Nos. 7 to 12 are all lead-base alloys and constitute the cheapest of all bearing materials. They should contain no copper, but all the tin should be combined with antimony. They may be used for light service under comparatively high speeds if properly fitted. In general, their bearing value increases with an increase of tin content. All lead-base metals may be mated with soft steel.

Cadmium-base bearing metals find application in fields similar to the copper-lead type for severe service in internal-combustion engines. They consist of cadmium alloyed with about 1.25 percent nickel or 1.25 percent silver and 0.25 percent copper and are applied as a thin coating on steel backing or strip. These metals owe their superiority to the fact that they retain their properties up to comparatively high temperatures.

Table 28. Composition and Properties of White Metal Bearing Alloys (Babbitt Metal)

Alloy No.	Composition, percent				Density, lb per cu in.	Compression yld pt. (0.125% reduction of length) psi		Compression strength (25% reduction of length) psi		Brinell hardness (500 kg 30 sec)		First melting pt, F	Temperature of complete liquefaction, F	Pouring temperature, F
	Cu	Sn	Sb	Pb		68 F	212 F	68 F	212 F	68 F	212 F			
1	4.5	91.0	4.5	....	0.265	4,400	2,650	12,850	6,950	17.0	8.0	433	...	825
2	3.5	89.0	7.5	....	0.267	6,100	3,000	14,900	8,700	24.5	12.0	466	669	795
3	8.3	83.3	8.3	....	0.270	6,600	3,150	17,600	9,900	27.0	14.5	464	792	915
4	3.0	75.0	12	10	0.272	5,550	2,150	16,150	6,900	24.5	12.0	363	583	710
5	2.0	65.0	15	18	0.280	5,050	2,150	15,050	6,750	22.5	10.0	358	565	690
6	1.5	20.0	15	63.5	0.337	3,800	2,050	14,550	8,050	21.0	10.5	358	531	655
7	...	10.0	15	75	0.332	3,550	1,600	15,650	6,150	22.5	10.5	464	514	640
8	...	5.0	15	80	0.363	3,400	1,750	15,600	6,150	20.0	9.5	459	522	645
9	...	5.0	10	85	0.370	3,400	1,550	14,700	5,850	19.0	8.5	459	493	620
10	...	2.0	15	83	0.364	3,350	1,850	15,450	5,750	17.5	9.0	468	507	630
11	...	....	15	85	0.371	3,050	1,400	12,800	5,100	15.0	7.0	471	504	630
12	...	....	10	90	0.386	2,800	1,250	12,900	5,100	14.5	6.5	473	498	625

The compression test specimens were cylinders 1.5 in. long and 0.5 in. diam machined from chill castings 2 in. long and 0.75 in. diam. Brinell tests (10 mm ball, 500 kg applied for 30 sec) made on bottom machined face of specimens cast in 2 in. diam × 5/8 in. deep steel mold at room temperature.

From A.S.T.M. Standards, 1939, p. 679.

Porous Bearings. For medium-duty application in small-size bearings, bushings are made by pressing mixtures of copper, tin, and (often) graphite powder together and sintering these in a reducing atmosphere without melting. By controlling the conditions under which they are made, porosity may be adjusted so that distributed voids of up to about 30 percent by volume may be left for lubricating oil. Once impregnated with oil, these bearings will operate for long periods without lubrication. They are best used for simple cylindrical or disk shapes in quantity production.

not greatly affected at room temperature. Alloy XXIII may be partly stabilized with respect to shrinkage by heating for 3 to 5 hr at 212 F. The castings should be at temperature for this period of time and may be cooled normally in air to room temperature.

Effect of Temperature on Zinc and Zinc Alloys. The properties of zinc and zinc alloys are very sensitive to temperature. Creep resistance decreases rapidly with increasing temperature, and this must be considered in designing articles to withstand continuous loads.

Ductility and general fabricating characteristics increase with temperature. Forming and drawing operations on strip or sheet zinc should not be attempted below 70 F, and the more severe operations can be performed more readily at somewhat higher temperatures (up to 125 F).

Zinc and zinc alloys become somewhat brittle below the range 0 to 32 F, depending on the particular composition, but recover their normal properties on reaching room temperature again. Even at low temperature, the die-casting alloys have residual impact strength superior to ordinary cast iron.

Tin

The principal deposits of tin ore are in the Federated Malay States, Dutch East Indies, and Bolivia, with China and other countries contributing smaller quantities. The principal smelters are in the Strait Settlements, England, Belgium, and China. The concentrated oxide ore is reduced with carbon in reverberatory or blast furnaces, purified by liquation for the ordinary grades of tin or electrolytically or by chemical processes for high-purity material. Common grades of tin usually exceed 99 percent tin content, the principal impurities being lead, antimony, arsenic, and copper. Secondary tin is reclaimed principally from miscellaneous tinned iron and soldered scrap and scrap copper alloys. The largest single use of metallic tin is for making tin plate, with solder and bearing metals the next largest consumers. In making **tin plate**, the clean and pickled steel sheets are immersed in a long bath of pure tin at a temperature of 575 to 650 F at the entering side and only 10 to 15 deg above the melting point of tin where the sheets leave the bath through a layer of palm oil. **Terne plate** is steel coated with lead containing 12 to 25 percent tin, for pure lead will not adhere satisfactorily to steel.

Pewter was originally an alloy of tin and lead containing not over 20 percent of the latter element in the best practice, but it is now frequently replaced by an alloy of tin and antimony more commonly called **britannia metal**. This is stable and less likely to tarnish. A composition now being produced in rolled form for working into decorative tableware and the like is 91 percent tin, 7 percent antimony, 2 percent copper. Tensile strength is about 8,500 lb per sq in. With the exception of these alloys, solders, and white bearing metals, there are no other important tin-rich alloys. Tin is an important minor constituent of fusible alloys, bronzes, lead Babbitts, and type metals.

Cobalt Alloys

Various alloys of cobalt with chromium and tungsten possess exceptional hardness at high temperatures and hence they find application as **cutting tools**. The important tool materials known as Stellite (grades J and 3) are in the composition range 12–17 percent tungsten, 30–35 chromium, 2.25–2.75 carbon, balance principally cobalt. Stellite J has a Brinell hardness of about 600 at room temperatures; this drops only to 320 at 1470 F, at which temperature high-speed steel suffers almost complete loss of hardness. Stellite cannot be machined or worked and must be cast and ground to shape.

temperature is above the melting point of one of the constituents or products of reaction and simple liquid surface tension consolidates the mass. In the copper-tin alloys, the tin melts first and then is absorbed by diffusion to form a solid alloy. Generally, the sintered product is the final one ready for use, but not infrequently the material is machined to exact size or submitted to cold-pressing operation to improve the surface finish or accuracy of dimensions. There is considerable shrinkage during the sintering operation, but this can be anticipated when preparing the dies and the dimensions of the final product can be held accurately.

Porous bearing metals are made by incorporating with the metallic powder, volatile materials which are driven off on heat-treatment, leaving interconnected pores that are subsequently filled with oil. Other types of bearing are made by the admixture of graphite powder which remains intimately associated with the bronze. Brushes for electric motors are similarly made from copper and graphite powders.

Powder metallurgy is employed in the manufacture of tungsten, molybdenum, tantalum, and some other refractory metals of which cast ingots (because of impurity content or grain size) cannot be worked. By the methods of powder metallurgy, a bar is obtained which by swaging or rolling at suitable temperatures and, if necessary, annealing in hydrogen or a vacuum becomes quite ductile and can then be worked into any form.

Hard Carbides. One of the most important contributions of powder metallurgy is the production of cemented hard carbides for tool materials. Although melted and cast dies of tungsten carbide have found use as extrusion dies, in general the material is too brittle to be used unless made by a powder metallurgical process. The tungsten carbide (with or without admixture of tantalum or titanium carbides) is produced in finely granular form and, after mixing with cobalt powder and pressing, sintered at a temperature where a cobalt-rich constituent melts and binds the carbide particles together. (For the design, welding and brazing, grinding, and use of cemented carbides in the machine shop, see *Am. Machinist*, **83**, 1939, pp. 127–134.

Tungsten carbide is the basis of most of the tool materials. Combinations with tantalum carbide have desirable properties for certain applications and the double carbide of titanium and tungsten is also used. The carbides are made by adding carbon to powdered metal or oxide and then heating in a reducing atmosphere to a temperature in the neighborhood of 3600 F. The time and temperature must be controlled to give the proper carbon absorption and appropriate particle size. The double carbide $WTiC_2$ is made by solidifying a molten alloy of nickel containing tungsten, titanium, and carbon and extracting the nickel chemically. The carbide powder is intimately mixed with the binder by prolonged ball milling and then pressed in hard steel dies to a plain slab or ingot of appropriate thickness for use in tool tips or to a special shape. The pressure varies in the range 15 to 30 tons per sq in. The resulting compacts are quite delicate, and to permit handling they are usually presintered at about 1550 F which gives them some strength and allows them to be turned or cut to shape with ordinary tools. Shrinkage of the sintered material must be allowed for. The formed objects are then heated at a temperature in the neighborhood of 2500 F to develop their full hardness and strength. Although this sintering temperature is below the melting point of the cobalt binder, there is a ternary constituent formed that is definitely liquid at this temperature and is largely responsible for the bonding strength.

The principal use of hard carbides is for tipping lathe and other cutting tools. The shaped tips are copper-welded, brazed, or silver soldered to a

Additions of about 10 percent mercury to the bismuth-lead-tin eutectic gives an alloy melting at 140 F which is used for anatomical castings.

Solders

Soft Solders. The composition and melting points of the standard soft solders (tin-lead alloys) are shown in Table 25. A.S.T.M. specification B32-21 allows a variation of ±1 percent on tin or lead contents from the normal composition. Solders should have not over 0.08 percent copper (Class A) or 0.15 percent copper (Class B) and a total of less than 0.10 percent other impurities (except bismuth) with no aluminum. The running properties will be impaired if these are exceeded. The eutectic composition (37 percent Pb) has the lowest melting point and in general the best wetting

Table 25. Composition and Melting Point of Soft Solders

Designation or use	Composition, percent			Melting range, F	
	Sn	Pb	Sb	First melting point	Point of complete liquefaction
Pure tin	100	....	....	449.6	449.6
A.S.T.M. 0A	63	37	0.12[a]	360	360
A.S.T.M. 1A	50	50	0.12[a]	360	415.4
A.S.T.M. 2A	45	55	0.12[a]	360	437
A.S.T.M. 3A	40	60	0.12[a]	360	459
A.S.T.M. 4A	37.5	62.5	0.12[a]	360	468
A.S.T.M. 5A	33	67	0.12[a]	360	486
Pure lead	.....	100	....	621	621
A.S.T.M. 1B	49.25	50	0.75[a]	365	397
A.S.T.M. 2B	43.5	55	1.5	370	428
A.S.T.M. 3B	38	60	2	370	442
A.S.T.M. 4B	35.5	62.5	2	370	412
A.S.T.M. 5B	31	67	2	370	455
For lead coating steel	25	75	....	360	514
For lead coating copper	5	95	....	(555)	595
Joints in copper tubing	95	....	5	450	465
Wiped joints in lead pipe	37.5	60	2.5	367	435
Wiped joints in lead pipe	23	68	9 Cd	294	455
"High" temp soft solder	......	94.5	5.5 Ag	580	695
"High" temp soft solder	......	95 Cd	5 Ag	640	740
"High" temp soft solder	......	82.5 Cd	17.5 Zn	508	508
"High" temp soft solder	......	50 Cd	50 Zn	508	619

[a] Maximum.

and running properties but is also the most expensive of the series. For wiped joints in lead pipe and cable, a wide solidification range is needed. The 95/5 tin-antimony alloy used for joining copper water pipe is said to have superior strength at hot-water temperatures. The high-melting-point soft solders are useful for the joining of objects that must be exposed to higher temperatures than are permissible with tin-lead solders. Those containing cadmium are more suitable for soldering zinc and galvanized iron than tin-lead solders. Alloys containing antimony should not be used on these materials.

Silver and Brazing Solders. There are no good alloys available for solders in the range from 600 to 1160 F which is the lowest melting point for

CORROSION

BY

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REFERENCES: Cushman and Gardner, "Corrosion and Preservation of Iron and Steel," McGraw-Hill. Whitney, "Corrosion of Iron," *Jour. Am. Chem. Soc.*, 25, 1903, p. 394. Walker, "Corrosion of Iron and Steel and Modern Methods of Prevention," *Jour. Iron Steel Inst.*, 90, 1909. Evans, "Metallic Corrosion, Passivity and Protection," Arnold, London. Speller, "Corrosion: Causes and Prevention," 2d ed., McGraw-Hill. Rawdon, "Protective Metallic Coatings," *Am. Chem. Soc. Monograph* 40, Reinhold. McKay and Worthington, "Corrosion of Metals and Alloys," Reinhold. Burns and Schuh, "Protective Coatings for Metals," *Am. Chem. Soc. Monograph* 79, Reinhold.

General Considerations

Corrosion, in its most familiar form, is a reaction between a metal and water. It may be represented by the equation, metal + water = hydroxide of metal + hydrogen. It is electrochemical in nature, the solution of the metal in water and the "plating" out of the hydrogen being accompanied by a flow of electricity. The reaction may be considered as being in two parts, the **anodic reaction,** the passing of the metal into solution, and the **cathodic reaction,** the plating out of the hydrogen. Although the anodic and cathodic areas are often so close together as to be indistinguishable, as in a uniform general corrosive attack, the anodic areas may be relatively widely separated from the cathodic areas as in localized corrosion. The principal factors that affect the reaction are as follows:

The Metal. Every metal has a certain tendency to dissolve in water, which is known as *solution pressure.* A measure of this tendency can be obtained from the electrical potential which must be applied in order to prevent any action when the metal is immersed in a solution of one of its salts of a given concentration. From such information, the metals may be arranged in a series in the order of their solution pressures. Complete agreement as to the order has not been reached. The order of the common metals with respect to decreasing solution pressure appears to be as follows: potassium, sodium, barium, strontium, calcium, magnesium, aluminum, manganese, zinc, cadmium, iron, cobalt, nickel, lead, tin, hydrogen, bismuth, copper, antimony, arsenic, mercury, silver, palladium, platinum, gold. Although this series indicates the relative tendencies of the metals toward corrosion (gold being minimum), it should be used with caution. Other factors may often greatly affect the rate of corrosion of a metal.

The Water. Water is always dissociated to some extent into its constituent ions, H^+ and OH^-. Any condition that promotes an increase in the concentration of the hydrogen ions favors the formation of hydrogen and *hence favors corrosion of iron immersed in the water.* For this reason, the solution of an acid or an acid gas in water promotes corrosion. On the other hand, an increase in the hydroxyl-ion concentration, for example, by the addition of an alkali or alkaline salt, retards or even prevents corrosion of iron. Limewater or dense cement concrete will inhibit the rusting of iron.

The hydroxide of the metal can influence directly the corrosion of the metal, while in solution, by opposing the anodic reaction, *i.e.*, the formation of more metal ions. Most of the hydroxides of the metals are relatively insoluble and have little direct bearing on corrosion. The tendency of the

Aluminum bronzes containing 8 to 11 percent aluminum with 3 to 7 percent iron are used for wearing surface for heavy duty. Unlike the leaded bronzes, they will not withstand conditions of poor lubrication. The most extensive uses of these is for bushings, guides, gears, screw-down nuts, and the like in machine-tool construction.

Table 27. Copper-base Bearing Metals[a]

A.S.T.M. designation	Nominal composition, percent					Compression deformation limit,[b] psi	Uses
	Cu	Sn	Pb	Zn	P		
B30-36 Grade 15	85	5	9	1	......	12,000	Small bearings machined by broaching
16	80	10	10	....	0.05+	12,500	Heavy pressures. Not broached
17	79.25	10	10	0.75	......	12,500	Harder bearings for use as No. 15
18	73	8	15	1.5	......	12,000	"ExB" metal of Penn. R. R. Locomotive and general bearings for moderate pressures
19	71.5	4.5	17	4	......	12,000	Car journal bearings and similar service
20	70.5	6	20	1	......	11,000	High-speed bearings under light and moderate pressure
B22-38T Class A	Rem.	19	..	....	<1.0	24,000	Bridge turntable plates. Low-speed high-pressure bearings (3,000 lb per sq in.). Must be mated with hard steel
B	Rem.	16	..	....	<1.0	10,000	As Class A. Trunnions, bearing, and expansion plates. Pressures up to 25,000 lb per sq in. with hard steel, 1,500 lb with medium hard steel
C	<82	10	10	....	0.7-1.0	......	Machinery bearings. Bridge expansion plates up to 1,000 lb per sq in.
D	<89	10	..	2.0	<0.5	......	Gears, worms, nuts, etc.
	70	....	30	....	......	......	Used as liner (0.02 to 0.06 in. thick) for internal-combustion engine bearings at high speeds with average loads over 1,200 lb per sq in. Needs special casting technique. See text

[a] Many of the cast bronze alloys in Table 11 may also be used for bearings.
[b] Deformation limit is compressive stress producing permanent set of 0.001 in. on machined sand cast specimens 1 in. sq area 1 in. high.

Babbitt metal is a general term used for soft lead and tin-base metals which are used cast as liners in bronze or steel backing. In general, they are used in preference to the bronzes for higher speeds and alternating loads but are less proof against abuse. The tin-base alloys are to be preferred except where initial price is the primary consideration. The lead-base alloys lose their hardness rapidly as the temperature increases and are generally employed only for comparatively light service with good lubrication.

The important **tin- and lead-base** bearing alloys (Babbitt metal) are listed in Table 28. No. 1 is used for internal-combustion engine bearings. It is plastic and unlikely to crack but must be used in a thin layer cast on to bronze or steel backing to give suitable support. No. 2 containing more antimony is somewhat harder and less likely to pound out. No. 3 is the

The same factors, although sometimes in other guise, apply to immersed corrosion and to underground corrosion.

Factors inhibiting corrosion are: the use of a self-sacrificing metal, anodic to the metal it is desired to protect, as zinc to prevent the corrosion of iron; protective coating; the rendering of metals "passive" (or so treating them that they are insoluble in acids and do not precipitate metals from solutions); neutralization of corrosive fumes or liquids; treatment of water to render it somewhat alkaline (high pH) or to induce formation of protective surface films (as of chromate); deactivation of water by elimination of oxygen; reversed polarity to counteract the effects of stray currents; and the application of an external emf so as to render the structure cathodic and concentrate corrosion on auxiliary anodic parts used for the purpose.

Corrosion due to electrolysis, which is not to be confused with the fact that corrosion is fundamentally an electrochemical phenomenon, is generally caused by the leakage of current from electric circuits, and may take place at a point far removed from where the leakage occurs. Undue emphasis is often given to this phase of the corrosion problem. Stray currents of extremely feeble intensity and voltage may serve to accelerate corrosion, even when they have not initiated it. Corrosion due to electrolysis may be minimized by providing thorough insulation; grounding all metallic conduits; avoiding combinations of dissimilar metals in a circuit; and by maintaining apparatus in an electronegative state with reference to possible sources of current, either by a "drainage" system, or by "tying up" with some sources of current of a higher potential.

Corrosion of Metals

Iron and Steel. Under similar conditions, iron and steel corrode at practically the same rate, but the distribution of the corrosion may be different for the two. The slag interspersed through wrought iron may result in a generally distributed attack under atmospheric corrosion rather than a severely localized (pitting) attack. In underground corrosion, and in continuous immersion in water, tests have not shown so much difference between the two materials.

Polished surfaces resist corrosion much better than rougher surfaces. Variations in surface finish may have a greater influence than ordinary variations in chemical composition other than pronounced segregation. The presence of mill scale on the surface favors localized or irregularly distributed corrosion. Frequently a polished surface will withstand exposure for a considerable length of time before showing signs of corrosion.

External conditions overbalance composition of the metal in determining rates of corrosion. **Oxygen** determines the commencement of corrosion of iron and steel under ordinary conditions; it not only acts as a depolarizer but also unites with ferrous iron at the corrosion anodes. In the general absence of oxygen, corrosion drops to a negligible rate. In local absence of oxygen, a differential oxygen cell may be set up which serves to accelerate corrosion in the oxygen-poor portion. In salt solutions, corrosion depends both upon the amount of oxygen present and upon the salt in solution.

Rust may accelerate corrosion and cause pitting. The probable explanation (Evans) is that surface accumulations of rust shield the underlying metal from free access to oxygen thus rendering such portions anodic (corrodible) with respect to unshielded areas to which oxygen has freer access (cathodic areas). Under certain conditions of exposure, especially atmospheric, rust may form so continuous and adherent a coating that it protects the underlying metal from further corrosive attack. This is especially true of copper-bearing steel and iron under atmospheric exposure. Rust adheres much better to cast iron than to rolled iron or steel; the superior corrosion resistance of cast iron is attributable in large measure to this fact.

Cold working of metals results in an increased rate of attack by acids; it also increases heterogeneity in metals which may lead to increased rate of corrosion. Local cold working, as in the punching of rivet holes, builds up internal stresses and greatly

Powder Metallurgy*

The production of objects from metal powders involves (1) the production of powdered metal of suitable size and characteristics, (2) the mechanical consolidation of the powder with or without previous admixture of other metallic or non-metallic powder constituents, and (3) the heating of the pressed mass to a point where the individual particles strongly cohere by a process involving grain growth or, sometimes, fusion of one of the constituents. Powder metallurgy is valuable in producing workable ingots of extremely refractory metals like tungsten, molybdenum, and tantalum which cannot be melted without harmful contamination; in making cemented carbide tools and parts of properties superior to those when cast; and in the manufacture of alloys of components (metallic or non-metallic) which do not mix in the liquid state, such as porous (oil-filled) or graphite-containing bronzes for bearings, copper-tungsten, or silver-tungsten alloys for electrical contacts and various magnetic alloys, some of which are made with each particle insulated from its neighbors. Mechanical reasons for the use of powder metallurgical processes involve the ability to fabricate securely united articles of varying composition and properties in different parts, and the almost entire elimination of scrap in the production of small parts which would otherwise involve casting, rolling, and blanking losses. The limitations of the process lie in the facts that, compared with ordinary press operations, the processes are not so readily adapted to high-speed production methods and only comparatively small and simple parts can be pressed, since extremely high pressures are needed. Moreover, the resulting strength and ductility does not usually compare favorably with similar alloys in the cast and rolled condition.

Metal powders are produced by various methods including principally grinding in stamp or ball mills or mechanical disintegrators, granulating, atomizing, condensing metal vapors, reducing oxide powders, and precipitation by chemical or electrolytic means. Practically all the common metals and many alloys are now available in powder form. The grain size of the powder, its shape and flowing characteristics must be closely controlled for they are very important both in relation to the production of the parts and their final properties. When alloys are required, either an alloy powder may be used or, more commonly, mixed powders of the constituent metals are employed. The powder is pressed in hard-steel dies and produces the final object directly under a pressure varying between 5 and 100 tons per sq in. The powder is fed into the die automatically in carefully measured quantities. Its bulk is usually from 3 to 8 times that of the pressed part; the height of the die must allow for this. The prevention of bridging and the elimination of air is important. The pressed objects will usually have a density in the neighborhood of 80 percent of that of the solid metal. Except in the case of low-melting-point metals, the pressed compact is mechanically weak and must be handled with care.

The next operation involves heating in a neutral or reducing gas or sometimes in vacuum to a point where the desired sintering occurs. In the case of pure metals and most alloys, no part is melted during this operation but the sintering involves the growing together of the various grains by a process akin to recrystallization and grain growth in cold-worked metals. In a number of cases (hard carbides, copper-tungsten alloys), the sintering

* See Jones, "Powder Metallurgy," Arnold. Symposium on Powder Metallurgy, M.I.T. 1940, *Trans. A.I.M.E.*, 128.

alkalies; as the iron is weak and brittle, its use is limited to shapes cast to size. Trade names for this type of alloy are **Duriron, Corrosiron,** and **Tantiron** (see p. 589). Nickel cast iron (13 to 15 percent Ni) has high corrosion resistance toward many chemicals and to dilute acids and has the advantage of superior strength and toughness (see Table 4, p. 593).

Metals in Brine. According to the work of M. B. Smith (*Ice and Refrig.*, 12), with neutral calcium chloride brine (sp gr 1.2) copper alloys are most corrosion resistant, wrought iron, steel, and cast iron constitute an intermediate group, and lead, solder, and zinc constitute a group having lower corrosion resistance. The durability of any metal in brine, however, varies with the condition of the brine. For general purposes, a pH of 8.5 (pH 7.0 being the neutral point) is recommended. Brine with a high pH is corrosive to aluminum and zinc. The corrosiveness of a brine is increased 2 or 3 times by saturating it with air.

Calcium chloride, calcium-magnesium chloride, calcium-magnesium-sodium chloride, or sodium chloride brines, which are neutral products or even alkaline in reaction, show no appreciable difference in their attack upon metals. Magnesium chloride is to be avoided, since it may produce an acid reaction (low pH). Ammonium chloride, sometimes formed by ammonia leaking into the brine, is very corrosive to steel.

Brines should be maintained in a slightly alkaline condition by the occasional addition of milk of lime, caustic soda, or other alkali. When in proper condition, they should give the characteristic pink color with the phenolphthalein indicator. An agent, such as $K_2Cr_2O_7$, may also be used to retard corrosion although its effectiveness is lowered in a chloride solution.

Corrosion of Pipes, Boilers, and Structural Work

Pipe Materials. Tests made for the purpose of showing the relative corrodibility of iron and steel lead to the conclusion that external conditions determine the corrosion rate more than differences of composition. This has been strikingly emphasized by the soil-corrosion investigation of the National Bureau of Standards.

The presence of mill scale on pipes favors the formation of centers of corrosion, and its removal in the process of manufacture is advisable.

Removal of Oxygen from Water. In any water system, the rate of corrosion is directly proportional to the oxygen concentration of the water. In a closed system, as hot-water heating systems in which the water is used repeatedly, no appreciable corrosion normally occurs after the oxygen initially present has been used up. Removal of the oxygen (air) from water is one of the best means of eliminating corrosion of pipe and other apparatus. There are, in general, two means for doing this. The **deactivation** method consists in removing the oxygen, carried by the water, by chemically combining it with some substance before it enters the system that is to be protected. The **deaeration** method consists in the mechanical removal of the air from the water.

The **deactivation** method is generally used for hot-water heating systems. Iron or steel, which is the material used for removing the oxygen, is employed in such a form as to expose as much surface as possible. Expanded metal may be arranged so as to secure approximately 70 sq ft of exposed surface to 1 cu ft of tank space. In ½ hr at 170 F, the oxygen in water may be reduced as low as 0.3 cc per liter when fixed by chemical union with iron. The water in boilers is often treated with a substance such as sodium sulphite which unites with free oxygen.

steel shank both for cheapness and for support of the brittle tip. In designing tips, it is important to avoid large changes of thickness and to design the backer so that stresses due to thermal contraction after brazing cannot crack the tip. In designing tools, it is important to use the lowest rake angle possible so the tool may have the maximum support. Other applications of hard carbides include lathe centers, gage tips, guides of various kinds, Brinell balls, and the like. The piece should be shaped as closely as possible to final size before sintering, but a limited amount of forming can be done by grinding with special silicon-carbide or diamond impregnated wheels.

The modulus of elasticity of tungsten carbide is about 79,000,000 lb per sq in.—higher than any other known material. The compressive strength of tungsten carbide varies between 800,000 and 500,000 lb per sq in. depending on cobalt content. Thermal expansion 3.3×10^{-6} per deg F. Thermal conductivity is about 500 Btu per hr per sq ft per in. per deg F for cemented tungsten carbide, decreasing to 185 for the double tungsten-titanium carbides. A low thermal conductivity seems to favor resistance to cratering in machining steels, but in machining cast iron and other materials with discontinuous chip the high thermal conductivity and superior hardness of tungsten carbide is desirable. The lower modulus of the carbides containing tantalum or titanium (about 58×10^6 lb per sq in.) permits greater deflection before fracture than the harder tungsten carbides.

Average New York Prices of the Principal Metals 1919–1939[a]

Year	Electrolytic copper, cents per lb, New York	Lead, cents per lb, New York	Zinc, cents per lb, St. Louis	Straits tin, cents per lb, New York	Antimony, cents per lb, New York	Quicksilver, dollars per flask of 76 lb, New York	Aluminum, cents per lb, New York	Silver, cents per troy oz, New York	Platinum, dollars per troy oz, New York
1919	18.691	5.759	6.988	63.328	8.190	92.15	32.14	111.122	114.61
1920	17.456	7.957	7.671	49.101	8.485	81.12	32.72	100.900	110.90
1921	12.502	4.545	4.655	29.916	4.957	45.46	21.11	62.654	75.03
1922	13.382	5.734	5.716	32.554	5.471	58.95	18.68	67.528	97.62
1923	14.421	7.267	6.607	42.664	7.897	66.50	25.41	64.873	116.54
1924	13.024	8.097	6.344	50.176	10.836	69.76	27.03	66.781	118.82
1925	14.042	9.020	7.622	57.893	17.494	83.13	27.19	69.065	119.09
1926	13.795	8.417	7.337	65.285	15.988	91.90	26.99	62.107	113.27
1927	12.920	6.755	6.242	64.353	12.393	118.16	25.40	56.370	84.64
1928	14.570	6.305	6.027	50.427	10.305	123.51	23.90	58.176	78.58
1929	18.107	6.833	6.512	45.155	8.956	122.15	23.90	52.993	67.66
1930	12.982	5.517	4.556	31.694	7.667	115.01	23.79	38.154	45.36
1931	8.116	4.243	3.640	24.467	6.720	87.35	23.30	28.700	35.67
1932	5.555	3.180	2.876	22.017	5.592	57.93	23.30	27.892	36.46
1933	7.025	3.869	4.029	39.110	6.528	59.23	23.30	34.727	30.99
1934	8.428	3.860	4.158	52.191	8.901	73.87	21.58	47.973	36.47
1935	8.649	4.065	4.328	50.420	13.616	71.99	20.50	64.273	34.15
1936	9.474	4.710	4.901	46.441	12.240	79.92	20.50	45.087	42.93
1937	13.167	6.009	6.519	54.337	15.355	90.18	20.08	44.883	51.77
1938	10.000	4.739	4.610	42.301	12.349	75.47	20.00	43.225	35.90
1939	10.965	5.053	5.110	50.323	12.359	103.94	20.00	39.082	36.75

[a] From 1939 Year Book of the American Bureau of Metal Statistics.

solution. Constituents that may form **hydrochloric acid** are chloride of magnesium plus steam, sulphate of magnesium and the alkaline chlorides, silica and alkaline chlorides, ferric chloride, ferrous chloride, carbonate of magnesium plus chlorides, and chloride of ammonium. Constituents that may form **sulphuric acid** are normal ferric sulphate, ferrous sulphate, sulphurous acid and, ferric chloride, sulphites, hydrogen sulphide, sulphate of calcium plus organic matter, sulphate of ammonia, and sulphate of copper. Constituents that may form **nitric acid** are normal ferric nitrate, alkaline nitrates and acid sulphates or sulphuric acid, and nitrate of ammonia.

Air sucked in by the feed pumps is a cause of serious corrosion. Air bubbles, forming below the water line, may serve to localize corrosion and result in pit formation. In marine practice, it has been found advantageous to pump the water from the hot well to a filter tank above the feed-pump suction valves. The bubbles are liberated from the surface of the tank, and a head is assured for the suction end of the pump. The corrosive action of air may also be reduced by introducing the feed water into the steam space above the water line.

Both general corrosion and pitting may be reduced or practically eliminated by using an open feed-water heater to expel the air, or better still, a feed-water heater connected to a vacuum pump.

Galvanic action takes place in certain instances. The remedy for it is usually the installation of zinc plates within the boiler, which must have intimate metallic contact with the metal of the boiler. The zinc plates are corroded instead of the boiler, and the latter is protected at the expense of the former. The necessary positive contact is difficult to maintain, and the efficiency of such plates is questionable except for a relatively short period following their installation. The practice of using 1 sq ft of zinc surface for 50 sq ft of surface has been followed.

Some of the **other causes of corrosion** in steam boilers are polluted feed water containing manufacturing wastes; rain water which, through contact with acid-containing soot on roofs, etc., becomes acid in nature; water containing organic matter; certain alkaline waters, the addition of barium chloride to which would probably be advisable; autoelectrolysis between the boiler metal and carbonaceous matter such as soot or ashes, especially around mud drums and blowoff pipes; decomposition of sulphate scale, especially calcium sulphate, with the formation of sulphuric acid (some feed waters, apparently neutral at ordinary temperatures, become acid when heated strongly); coal-pit water—generally acid; boiler metal containing much impurity, especially if segregated; and straining of the boiler metal, frequently caused by lack of proper staying, which leads to buckling of the plates.

Peculiar cracks sometimes develop in riveted joints of a boiler below the water line. Those cracks are intercrystalline in their character and are usually attributed to hydrogen embrittlement. The term **caustic embrittlement** is often applied. Caustic soda, formed from sodium carbonate in the water, acting upon the steel, while under stress as it normally is in service, is said to be responsible for this phenomenon (S. W. Parr). It is claimed that this does not occur with water containing sodium sulphate.

Corrosion has been reported as resulting from **copper** precipitated on the steel surfaces from the boiler water. Presumably the character of the water favored the solution of some copper from copper feed pipes or other brass or copper parts with which it came in contact. The dissolved copper later precipitates in metallic form on iron with which it comes in contact and in so doing induces corrosion.

The Cumberland process for protecting boilers, economizers, tanks, evaporators, etc. consists in impressing an emf so as to maintain the parts to be protected in a cathodic state. Properly distributed anodes of steel or iron are used with direct current at approximately 10 volts, and the application must be continuous. The success attained is dependent upon the current reaching all parts of the surface to be protected, and this is often difficult. A film of hydrogen over the cathodic surfaces is necessary at all times otherwise local corrosion may occur even within the sphere of influence of the applied emf. The method is not so simple as is often claimed.

hydroxides to form, under certain conditions, relatively impervious coatings or masses on the surface of the metal may often have a pronounced (indirect) influence upon the corrosion of the metal.

The Hydrogen. When a metal dissolves in water, an equivalent amount of hydrogen is set free (cathodic reaction). In an acid solution, the action is usually violent and appears as a gas; in most cases, however, the hydrogen "plates" out from the solution as an invisible film on the surface of the metal which usually retards and may prevent further action (polarization) until the hydrogen has been removed.

Oxygen of the Air. Oxygen and hydrogen unite to form water with extreme slowness under ordinary conditions, but when brought into contact with certain metallic surfaces they unite easily. Upon platinum, the reaction is very rapid, on iron much slower, and on zinc and aluminum practically negligible. It is in preventing the formation of the hydrogen film (depolarization) that the presence of oxygen dissolved in the water is so necessary to corrosion. If the cathodic reaction (the plating out of hydrogen) is facilitated, by the removal or the prevention of the formation of a film of hydrogen, the anodic reaction (the solution of the metal) is likewise facilitated. If air (oxygen) is removed from the water in a boiler or heating system, practically no corrosion, after the slight initial attack, will take place. The greater the oxygen pressure, or the more active the surface of the metal in accelerating the union of oxygen and hydrogen, the more rapid is the solution of the metal. Thus it happens that although zinc has a larger solution pressure than iron, it has a much higher overvoltage and is also more readily polarized by hydrogen than is iron; consequently, iron continues to corrode in a damp atmosphere, whereas zinc does not. When, however, zinc and iron are in contact, the hydrogen equivalent of the zinc which dissolves can and does plate out on the iron; hence, under these circumstances, the zinc continues to corrode and the iron does not.

The depolarizing action takes place with extreme ease on forge or mill scale—the magnetic oxide of iron that covers a piece of iron or steel after it has been highly heated. If this scale forms a close adherent coat, it is of value as a protecting surface; but if is ruptured at any point, corrosion at this point takes place with great rapidity, and a pit is the result. For this reason, mill scale should be removed as carefully as possible from ferrous structures exposed to corrosive conditions. An entirely secondary action of oxygen is the oxidation of the ferrous hydroxide formed by the action of the water, to produce the red ferric hydroxide which is familiar as rust.

Factors Stimulating Corrosion. Atmospheric corrosion is stimulated by a very damp atmosphere, since this maintains a film of water on the metal, an essential condition for corrosion. Other factors are: oxygen (air) dissolved in this water film; acids such as acid gases in the atmosphere or sulphur compounds from cinders, coke, coal dust, etc.; salts that dissociate in water producing an acid reaction; contact of dissimilar metals; and the presence on the metal of a depolarizing surface, such as mill scale on iron. Any condition of non-uniformity within the metal such as may arise from improper or non-uniform annealing or cold working favors corrosion. Similarly, non-uniformity in the concentration of the corrosive medium is favorable to corrosive attack. This is especially true of non-uniformity in the oxygen distribution as in a stack of sheets. Areas where the oxygen concentration is low are anodic with respect to those of higher oxygen concentration; hence, localized corrosion of such areas is stimulated.

(2) longitudinal, straight cracks of considerable length. Both are caused by high internal stress. Loss of ring in a 70:30 brass is a sign of incipient cracking. Prevention is accomplished by a stress-relief anneal, *e.g.*, 70:30 brass at 590 F; the stress may also be relieved by mechanical means, *i.e.*, by springing the tubes back and forth. By covering the surface with a thin impervious metal coat such as nickel electroplating, season cracking can be prevented but this has not proved commercially practicable.

Pumps *may be corroded by the solvent action* of the liquid passing through them. When dissimilar metals in contact are used, for example, a cast-iron body with brass glands, seats, etc., autoelectrolysis is almost certain to result when brines or other electrolytes are pumped. If it is not possible to confine the construction to one metal, the less resistant metal should be used only for easily renewable parts.

Corrosion may result from the leakage of current from the driving motor. This can be prevented by using an insulating coupling between the pump and motor shafts.

Air in the pump, either free or in solution in the liquid pumped, accelerates corrosion. Pumps should be vented so that they may be completely filled with the liquid they are handling.

Chemical Equipment. The development of corrosion-resistant steels (p. 571), of high-nickel alloys (p. 575), and of ferrosilicon alloys (p. 604) has extended greatly the uses of the ferrous materials in the chemical field. Enameled steel or iron ware is used widely, but most enamels are generally attacked by alkalies. Rubber-lined tanks and other chemical equipment together with hard rubber piping and connectors now serve a very useful purpose. The number of non-ferrous metals and alloys useful for chemical equipment is exceedingly large. Lead is widely used, and the development of lead-lined steel pipe and tanks has greatly extended its sphere of usefulness. Nickel, nickel-clad steel, and alloys high in nickel are widely used (see p. 575). Special aluminum bronzes that have been modified by the addition of some other element such as iron are now coming into use for handling acids. For certain uses, especially when natural flavors and odors must be retained, silver is used. It is also used widely in the chemical industry.

Underground Pipes. Pipes buried in the ground are sometimes corroded by the action of stray electric currents from trolley-car tracks, the corrosion manifesting itself at the points where the current leaves the pipes. They should be maintained in an electronegative relation to the tracks. Corrosion may be minimized by the use of insulating joints or by draining of the current through grounded copper conductors (see *B. of S. Bull.* 55, 72, 80).

Steel, wrought iron, cast iron, and lead often corrode severely in certain natural soils without the accelerating effect of stray electric currents. Soils containing organic or carbonaceous matter such as coke, coal, cinders, etc., or impregnated with acid wastes from manufacturing plants are highly corrosive in their action. Scofield and Stenger (*El. Ry. Jour.*, Nov. 14, 1914) found that black peaty soils and clays as electrolytes in contact with metals give rise to the following voltages: sheet iron, 0 to 0.5; steel pipe, 0 to 0.5; cast-iron pipe, 0 to 0.3; pitted cast iron, 0 to 0.7; clean cast iron, 0 to 0.15; the current leaving the metal at clean spots, returning to the impurities in the metal through the external circuit. Clean iron is therefore anodic toward impurities. *The autoelectrolysis of iron buried in soils is recurrent, since* oxygen is always present in sufficient amount to act as a depolarizer.

The N.B.S. investigation of soil corrosion [*Research Papers* 883 and 945 (1936); 982 and 1058 (1937)] has established the following facts:

Ferrous Metals. Serious corrosion underground occurs in the absence of stray currents although electrical currents, which did not originate in power plants, have been

increases corrosion rates of such parts; thorough annealing after cold working is the remedy.

Stress Corrosion. The effect of corrosion on a metal while it is under stress is often much more severe than under ordinary conditions. This is particularly true of metal members subjected simultaneously to fatigue stress and to corrosion. Under such conditions, the number of stress applications required for failure of the metal is very much less than under non-corrosive conditions. A condition of internal stress, such as may result from cold working, severe heat-treatment, local overheating as in welding, etc., may also serve to accentuate the destructive effect of what otherwise would be a mild corrosive attack. For example, hard-drawn brass tubing may crack under the action of relatively slight surface corrosion, or the heads of rivets may snap off. Tightly drawn-up bolts of certain non-ferrous metals will sometimes behave similarly.

Alloys of commercially pure iron with either cobalt, nickel, or copper in small amounts (0.25 to 0.30 percent) are more resistant to atmospheric corrosion than the original iron from which they were made; all such alloys tend to form protective rust coatings. **Manganese and copper** retard corrosion when alloyed with iron or steel. The results of extensive exposure tests of black steel sheets of different copper contents (*Trans. A.S.T.M.*, 1918–1940) have shown that for black sheets exposed to the atmosphere the presence of a small amount of copper, *e.g.*, 0.20 percent, is very advantageous. Although such materials rust readily when exposed to the atmosphere, the rusting is not progressive. Unlike the rust coating that forms on ordinary steel, the coating on copper-bearing steel is smoother, more adherent, and relatively impervious, and serves to protect the underlying metal. As with ordinary steel, the "life" of copper-bearing steel likewise varies with the conditions to which it is subjected. In A.S.T.M. tests, 16 gage sheet steel and iron have not yet failed after over 24 years continuous exposure in marine atmosphere at Annapolis, Md. In Pittsburgh, Pa., on the other hand, the life of comparison materials was not more than one-fourth as long.

According to other A.S.T.M. tests, the presence of copper in the same amounts as in the materials used in atmospheric exposure tests does not materially improve the life of steel immersed continuously in water, either in treated city water (Washington), brackish river water (Annapolis), or acid mine water (Pittsburgh). Data leading to similar conclusions concerning the life of underground structures were obtained in the soil-corrosion tests carried out by the National Bureau of Standards.

Certain low-alloy low-carbon steels with an alloy content below 5 percent (see Table 13, p. 558) have an improved corrosion resistance. These steels have a relatively high yield strength and tensile strength and permit thin sections which would not be admissible without the improved corrosion resistance.

Stainless steels (see p. 571) depend primarily on the presence of chromium for their characteristic properties. Plain chromium steels (4 to 6 percent Cr) are not of the stainless type. The original stainless cutlery steel contained approximately 13 percent and was used in the hardened form. The chromium nickel (18–8) steel, is the best known, but since it is austenitic it cannot be hardened by heat-treatment. Nitric acid does not attack it but hydrochloric and sulphuric acids do. The corrosion-resistance of 18–8 steel can be seriously impaired by improper heat-treatment, whereby the metal becomes susceptible to intercrystalline corrosion. This tendency can be overcome by proper alloying additions stabilizing the material. The 18–8 steel containing about 3 percent molybdenum is widely used in the sulphite paper industry. The 18–8 steel often does not remain bright and stainless on prolonged exposure in the atmosphere, and some care and attention are desirable for the material used for architectural purposes. In sea water, severe cases of pitting and perforation have been reported; these are usually associated with sluggish flow or stagnant water. Its remarkable corrosion resistance is attributed to a self-repairing oxide film which forms over the surface. Under conditions that favor the maintenance of this film or its re-formation, if injured, the resistance of the steel to corrosive attack is remarkable.

Cast Iron. Corrosion occurs in buried cast iron in certain types of soil. The product of this "graphitic corrosion" plugs the hole so that a pipe, if undisturbed, is still serviceable in low-pressure systems. Cast iron can be greatly improved in corrosion resistance by alloying elements, such as chromium and nickel. Silicon cast iron (13 to 14 percent Si) is resistant to most acids, and many other chemicals, though not to

Deposits of soot and fly ash assist very materially in this by adsorbing the corrosive gases in the products of combustion. These combine with atmospheric water vapor, and the corrosive attack is essentially that of a weak acid. The ordinary materials used for the construction of smokestacks do not differ very much among themselves in their resistance to the corrosive attack, although there is some indication that some of the newly developed low-alloy steels are superior.

Concrete Structures. The alkaline nature of cement assures great permanence to structural steel embedded in concrete. This has been repeatedly confirmed by the examination of the steel in reinforced-concrete buildings which have been demolished. The following information on deterioration by corrosion resulting from electrical current is from *B. of S. Tech. Paper* 18.

When current passes from an iron anode into concrete, oxides of iron form on the anode, occupying about 2.2 times the volume of the equivalent iron, and giving rise to mechanical pressure (sometimes as high as 4,700 lb per sq in.) which may result in cracking the concrete. At temperatures below 113 F, and with a potential gradient less than 60 volts per ft, this action is very slight, even in wet concrete.

Concrete near the cathode, or metal through which the current leaves the concrete, becomes softened and remains brittle and friable after drying, destroying the bond between the iron and concrete. This effect is noted at all voltages, high or low, and is due to the concentration of sodium or potassium near the cathode. The content of these elements in the cement should therefore be kept low.

Salt (NaCl), or calcium chloride, should never be added to concrete used in structures that will be subject to electrolytic action. Concrete in contact with salt water is very susceptible to electrolysis. In structures exposed to the action of salts, pickling solutions, etc., the potential gradient must be kept low. In the absence of metallic electrodes, the action of the current is similar to slow seepage, the water-soluble elements in the concrete migrating toward the cathode. Grounding of electric conductors in such a structure is equivalent to installing electrodes.

Waterproofing compounds when mixed with concrete have but little effect in preventing electrolysis. Waterproofing membranes, properly applied, are fairly efficacious in preventing the entry of earth currents. Painting or otherwise coating the reinforcing metal may minimize the danger from electrolysis, but it prevents proper bonding between the metal and the concrete.

All d-c circuits within a building should be kept free from grounds. All pipe lines entering a building should be installed with insulating joints outside the building; if passing through, they should have insulating joints on both sides of the building. If the potential drop around the isolated section is 8 volts or more, the isolated section should be shunted by a copper cable. Lead-covered cable should be kept out of contact with the building.

All metallic structures within a building should be interconnected, provided that all lines entering the building are installed with insulating joints, but they should not be grounded to any ground plate lying outside the insulating joints. Maintaining the reinforcing metal negative is worse than no protection at all.

Cinder concrete is frequently used in certain types of building construction. When the concrete is properly made and poured so as to eliminate voids adjacent to the reinforcing steel, very satisfactory service can be relied upon. If it is porous, pipes buried in it sometimes suffer severe corrosive attack by the acid-forming constituents that may be present. Proper precautions should be taken to neutralize any acidity in the cinder especially that which is in immediate contact with the pipes.

An important use of cement in preventing corrosion is the use of **cement linings for pipes** carrying water. They have proved very useful for corrosive waters such as mine drainage as well as for less severe service, as in water mains. The process is now being applied regularly to cast-iron pipe and also to steel pipe even as small as that used in ordinary water service. A special type coupling for cement-lined pipe is required. Lead-lined couplings are very useful.

Lead embedded in concrete is sometimes severely corroded as a result of free lime present. Lead shower-bath pans are a common example. Protection of the surface

Deaerators remove dissolved gases from water by mechanical means. In most deaerators, the water is agitated under suitable conditions of pressure and temperature. For deaeration at ordinary temperatures, a low-pressure or vacuum system is necessary; for higher temperatures, the system may be operated at atmospheric pressure. In many deaerators, the gas as it separates from the water is swept off by a current of steam. Water cannot be completely freed of its dissolved gas by mechanical means; 4 or 5 percent of the oxygen initially present may remain. For more complete removal, deaeration must be supplemented by deactivation. After deaeration, contact of the water with air must be prevented so far as possible; otherwise air is quickly absorbed by the water.

Boiler Tubes. Although results of tests, such as the extensive investigations by Ford (U. S. Navy) of 3 in. steel and iron boiler tubes in air-saturated distilled water at 68 to 77 F, have been interpreted as justifying a certain order of merit with respect to corrosion resistance of different boiler-tube materials, the differences are small, and would seem to have no very practical significance. Conditions external to the metal are most potent in determining the character and degree of corrosive attack.

The use of **nickel-steel tubes** (30 percent Ni) according to some authorities is economical because of their much longer life. Such tubes are not heat-treated. Tubes of **corrosion-resistant steel** (18 percent Cr, 8 percent Ni) are being used in superheaters.

The most frequent cases of tube failure are those resulting from imperfect heat transmission (mainly the result of scale). Physical imperfections in the metal, particularly such as result from non-uniform cold working (expanding, hammering, etc.), are responsible for numerous failures by corrosion.

Steam Boilers. (See p. 1018.) Corrosion in boilers may take place in three ways. **General corrosion** is the least dangerous, but the boiler must be watched closely lest it be gradually weakened to an unsafe degree. **Pitting** is readily detected as a rule and is frequently difficult to stop without a thorough search for the causes. **Grooving** is often very hard to locate before leaks take place, since such corrosion takes place at points where the metal has been bent or strained and is hidden from any but the most careful inspection. General corrosion of steam boilers may usually be traced to the water employed, but it is also associated with the action of certain **boiler feed-water compounds.**

A corrosive action resulting from the presence of acid in the water (or of oil containing fatty acids), which decomposes and causes pitting wherever the sludge can find a lodging place, may be overcome by the neutralization of the water by carbonate of soda. This should be carried to a point where the water will just turn red litmus paper blue. As a preventive, only the highest grades of hydrocarbon oils should be used.

Acidity may appear where salt water makes its way into a boiler, as may occur in marine practice from leaky condenser tubes, priming in the evaporators, etc. This acidity is caused by the dissociation and hydrolysis of magnesium chloride, hydrochloric acid and magnesium hydroxide being formed under high pressure. The acid in contact with the metal forms an iron salt, which, as soon as formed, is neutralized by the free magnesia in the water, thereby precipitating iron oxide and re-forming magnesium chloride. Where it is unavoidable that some salt water should make its way into a boiler, the water should be neutralized by milk of lime, which converts the magnesium chloride into magnesia and chloride of calcium, neither of which is corrosive, but both of which are scale-forming.

Decomposition of Boiler Waters. Under the action of heat and pressure, some waters may become corrosive by the decomposition of certain constituents carried in

The **electrolytic or cold process** consists in setting up the articles to be coated as cathodes in an electrolytic bath of soluble zinc salts, the anode being metallic zinc. Small articles are placed in metallic baskets in contact with the basket and with each other, the basket being attached to the cathode of the system. Both the acid sulphate and the cyanide bath are used, the latter particularly for small articles and, with suitable addition agents, for the recently developed "bright" coatings. The use of zinc plating on a large commercial scale has been advanced by its application to the coating of steel wire. The high ductility of the pure zinc coating obtained is the outstanding feature of such a coating. The ease of control of the uniformity and thickness is also advantageous. The coating of sheet steel by plating appears to be imminent.

In the **Schoop metal-spraying process**, a stream of very small hot metal particles (perhaps molten) formed by atomizing (by means of air or gas pressure) the molten end of a wire as it is fed into a blow-pipe flame is played over the surface to be coated. If the surface has been given a proper preliminary treatment (usually by sandblasting), a very adherent coating results, the thickness of which can be varied as desired. Very thick coatings are brittle. The imperviousness of metal-sprayed coatings can often be improved by using two or more metals in the same coating. Both metallic and nonmetallic materials can be coated by this means. Coatings of any metal available in wire form that can be melted in the flame used (oxygas for common metals, Pb, Zn, Sn, Cd, Cu, etc., oxyhydrogen or oxyacetylene for metals of high melting point, such as Ni) can be made. Metal-sprayed coatings are always porous to some extent, but they can be improved if the spraying is done by means of an inert gas instead of air. The electric-arc metal-spraying method is not as yet commercially available. Higher temperatures are available with the electric arc than with the gas method together with other advantages such as the ability to spray a coating formed of two different metals in one operation. The metal-spraying process has been used for coating with zinc of assembled structures such as bridges, radio and transmission line towers, and canal gates. Ship propellers, railway-car frames, and oil-storage tanks are other large structures which are metal sprayed. The coating of a structure, after complete assembly, is the outstanding advantage of the method. The spraying of furnace and stoker parts and superheater pipes with aluminum for protection at rather high temperature is also an important application of the method. The coating metal must usually be available in wire form, but in the **gravitas method**, metal is used in the form of a fine powder or dust. A cabinet for the semiautomatic coating of small objects is available commercially; but most articles must be handled individually.

Cadmium behaves similarly to zinc as a coating metal for iron in affording electrochemical protection from corrosion. Cadmium coatings are applied commercially only by electroplating and are preferred by many, the claim being made that a thin coating of cadmium gives the same degree of protection as a thicker one of zinc. This is true to some extent under marine conditions, but in an industrial atmosphere, where sulphur compounds are present, the useful life of cadmium coatings is shorter than that of comparable zinc coatings.

Coating with Tin, Lead, Nickel, Copper, or Chromium. Coatings of tin or of a tin-lead alloy are used principally on thin sheets of iron, being applied as a rule in a manner similar to that of the hot process of galvanizing. When so-called **terne plate** is made, a mixture of tin (25 percent) and lead

Condenser Tubes. (See Evans, *loc. cit.*) The conditions that obtain in condenser service are usually most severe in marine condensers and particularly in harbors and estuaries with contaminated water. Corrosion in land service may become a serious problem if the water is polluted by industrial wastes, mine water, etc.

Corrosion in condenser tubes may take place as a rather uniform surface attack, pits, or dezincification. The first is usually not objectionable and may be advantageous if a protective surface film is built up. Uniform thinning of condenser tubes results usually from a general dezincification or frequent harsh cleaning. Localized attack by pitting is serious and leads to perforations of the wall. Dezincification occurs in certain copper alloys usually having two microconstituents and results in conversion of the material into a weak brittle state which, in time, may cause holes. A small content of arsenic in the brass increases resistance to this form of deterioration. **Season cracking** (see p. 625 and below), the combined action of stress and corrosion, resulting in split tubes, is no longer a serious problem since brass condenser tubes are now furnished in a lightly annealed condition by manufacturers. Crosswise cracking may sometimes occur as a result of fatigue because of insufficient transverse support of the tubes. Corrosion may possibly contribute to such a failure.

There are several forms of localized attack: (1) pitting associated with surface deposits in the tube in slowly moving water; (2) impingement from air bubbles, in very rapidly moving water and occurring only near the inlet end; and (3) pitting along the length of the tube associated with erosion from suspended solids in the water. Long life of any condenser tube is dependent upon the building up, by initial corrosion, of a uniform protective surface film. A deposit on the wall surface favors electrolytic attack of the metal by shielding a portion from the action of the oxygen dissolved in the water. A condition of lower oxygen content thus obtains within the shielded areas, and such an area is anodic (corrodible) with respect to the surrounding areas to which oxygen has free access. Impingement of air bubbles at high velocity promotes rapid pitting by preventing the formation of a protective film and also in a mechanical way as shown by undercutting in the pits. The high water velocity of the modern high-speed condenser has increased the erosive effect of suspended solids in removing the protective film and thus promoting pitting. In high velocity water, an area on the downstream side of surface deposits may be severely eroded and early perforation result. Reduction of impingement and erosion can be effected by cutting down turbulence in the inlet water box. The impingement effect can also be reduced by rolling and flaring the inlet end of the tubes, the flared end being flush with the tube sheet. The insertion of a short sleeve of non-metallic material at the inlet end has proved effective in preventing impingement pitting.

The practice of coating condenser tubes with tin to prevent corrosion has been widely used. The coating is effective as long as it is intact, but when worn away in spots the attack is accelerated. Nonmetallic coatings have been used with some success, but they are not favored because of their low heat transmission.

Admiralty brass (Cu, 70; Zn, 29; Sn, 1) is extensively used for condenser tubes and for years has been the standard material for marine service. For severe service, it is being superseded by **aluminum brass** (Cu, 76; Al, 2; Zn, 22) and **cupronickel** (Cu, 70; Ni, 30) which are both resistant to impingement pitting. The cupronickel cannot suffer dezincification, and deterioration from this cause is negligible in the aluminum brass. The materials are widely used especially in foreign shipping. Arsenical copper tubing is coming into use for fresh-water or inland service where Muntz metal and Admiralty may be affected by dezincification. Red brass is also successfully used for inland service. Copper and red brass are not suitable for salt or brackish water, however. See Table 6, p. 636, for the properties of these alloys.

Season cracking of brass is associated with corrosion; it takes place in two ways: (1) by irregular cracks, commonly known as season cracking, and

few uses are soot-blowing apparatus, carbonizing boxes, furnace parts, and condenser and economizer tubes. Good results are regularly obtainable up to a temperature of 1700 F and in some cases to 1830 F. Calorizing is carried out by either a powder or a dip process. In the powder process, the parts to be treated are placed in a tight receptacle partly filled with a mixture of finely divided metallic aluminum and aluminum oxide. The air is replaced by hydrogen, and the receptacle is subjected to a high temperature for a time that depends on the depth of penetration of aluminum desired. In the dip process, the parts are fluxed, then immersed in molten aluminum, and then heated to promote alloying. This method gives a thinner coating of aluminum alloy but is much more expeditious. A calorized surface resists continued temperatures up to 1800 F but begins to burn at approximately 2000 F, whereas ordinary steel begins at about 930 F. It is not affected by ordinary oxidizing furnace conditions.

Calite, which may be considered as representative of a large class of heat-resistant alloys, is an alloy of iron, nickel, and aluminum and resists oxidization up to 2200 F for an indefinite time and, for short periods, up to 2370 F. The protective oxide formed does not snap off on quenching from extremely high temperatures. It is practically non-corrodible under ordinary conditions of exposure. Many other combinations, containing chromium and nickel as essential alloying constituents, are commercially available.

A process termed **chromizing**, similar in its operation to calorizing, uses powdered chromium or ferrochromium, the articles to be coated being heated while embedded in the powdered metal within a hydrogen or other non-oxidizing atmosphere. It has only a very limited commercial application.

Magnetic Oxide on Iron Surfaces. In the **Bower-Barff process**, the iron or steel articles to be coated are heated in a closed retort to a temperature of 1600 F, after which superheated steam is admitted. This results in the formation of red oxide (Fe_2O_3) and magnetic oxide (Fe_3O_4). Carbon monoxide is then admitted to the retort to reduce the red oxide to magnetic oxide, which is highly resistant to corrosion. Each operation takes about 20 min. The glossy black coating of magnetic oxide on **Russia iron** is produced by laying up sheets of iron with powdered charcoal between, the whole mass being then heated and hammered.

Iron and steel may also be **oxide coated** by electrolytic means, the object to be coated being made the anode (anodic oxidation) in an alkaline solution. Such coatings are primarily for appearance such as for cast-iron stove parts. Though experimentally successful, the commercial application of the process is limited. The **Chemag process**, a German development, is of this kind.

Phosphate Coatings for Rust Proofing Iron and Steel. (Eckelmann, *Chem. Met. Eng.*, Dec. 24, 1919.) In the **Coslett process**, iron or steel articles immersed for 3 or 4 hr in a boiling solution, made by mixing iron filings with concentrated H_3PO_4 (sufficient to form a paste) and then adding to weak phosphoric acid, become coated with a rust-resisting deposit of basic ferrous phosphate. This process was improved by the addition of an oxidizing agent, the **Parkerizing** process, and later by the addition of other accelerators (**Bonderizing**). The phosphate coating, in itself, affords only a very slight degree of protection against corrosion. Oiling the coating improves the corrosion-resistance greatly and imparts an attractive lustrous black appearance. Coatings of this kind are not suitable for severe out-of-doors service. Phosphating a steel surface is an excellent method of priming prior to subsequent painting or lacquering. The phosphate is applied by

detected on pipe lines. Soil conditions play a major role. Wrought iron and steel corroded at approximately the same rate, and the presence of copper in the steel did not improve its corrosion resistance. In certain soils, the corrosion rate of cast iron was greater than that of steel. The removal of mill and foundry scale from the surface did not greatly affect the rate of corrosion. Depth of pitting was directly proportional to duration of exposure in certain soils, but in others the pit depth increased very slowly after the soil conditions had become stabilized. Distribution of corrosion tended to become more uniform as the exposure period increased.

Non-ferrous Metals. No metal was found outstandingly superior under all conditions. The corrosion rate (loss of weight and depth of penetration) was, with but few exceptions, greater for ferrous than non-ferrous metals. In some soils, lead pitted severely within a few years, the purest lead being most resistant to corrosion. The presence of chlorides, bicarbonates, and sulphates in the soil retarded the rate of corrosion of lead, although in certain soils, as in tidal marshes, the rate of attack did not decrease with time. Copper and high-copper alloys corroded very slowly in most soils, the presence of sulphides being the principal accelerator of corrosion, the rate of attack being particularly high in cinders. High-zinc brass became weak and brittle by dezincification in many soils. Zinc corroded rapidly in a few soils, the rate being proportional to the duration of exposure. This limits the effective life of galvanized coatings on ferrous metals. The useful service is proportional to the thickness of the zinc coating; a coating of 2.8 oz per sq ft prevented the formation of pits for 10 years except in one very corrosive soil. Over a 10-year period, the loss of weight of galvanized steel was one-half, or less, that of the companion bare steel. **Aluminum** corroded rapidly under most of the soil conditions to which it was exposed.

Pipes buried in **mixtures of two dissimilar soils** or in two unlike soils in contact (not mixed) generally corrode more rapidly than in either of the two soils separately. Lead in a mixture or simple contact of dissimilar soils will often corrode markedly. Pipes buried in trenches fulfill these conditions of soils in contact or mixed. Metals corrode at the junction line of dissimilar soils. Cast iron in soils takes on a hard coat of rust and soil, pits being filled with carbon and black iron oxide. Lead shows both the gray and brown oxides when corroded in soils. Potentials up to 1 volt may readily be generated by placing unlike metals in a given soil or by using one metal in two dissimilar soils.

Bituminous coatings are widely used, especially natural asphalt and blown asphalt made from oil residuum and coal-tar pitch. Coatings of this last substance were somewhat inferior to others in the B. of S. investigation in that they were more susceptible to soil stresses, temperature changes, softening, and brittleness although they were more resistant to water absorption. The method of application of all such coatings is important. Dipping, brushing, and spraying methods are all used. For severe soil conditions, the coating should be reinforced by wrapping the coated pipe spirally with a strong fabric of some kind which has been impregnated with a waterproof bituminous mixture. For exceptionally severe corrosive soil conditions, pipes may be encased in concrete.

Pipes are often destroyed by the action of dissimilar metals to which they are connected, such as brass valves. Short connections, readily replaced, should be used on either side of the brass fittings.

Application of a negative potential booster to diminish potential differences in electrical rails has proved to be the most effective means of reducing electrolysis of underground iron and steel, such as gas and water mains.

Bridges, Roofs, Stacks, Etc. Iron and steel structures are corroded by the presence in the atmosphere of moisture and waste products from manufacturing and metallurgical plants such as carbon dioxide, sulphur dioxide, chlorine, ammonia, zinc and acid fumes, and soot. As a rule, they are not corroded in a dry atmosphere. To minimize the corrosive effects of gases, protective coatings (see p. 673) are used. Metal work exposed to the action of sulphur fumes should be covered with brick or with a paint highly resistant to sulphur dioxide. Bridges are now protected by means of so-called **drip floors** placed over the floor proper. The failure of **metal smokestacks** by corrosion occurs only during idle periods and particularly in the summer.

adhere firmly at all temperatures to which it will be exposed; coating shall be readily removable with cotton waste wet with kerosine; polished iron, steel, copper, or brass shall show no staining when exposed to weather at any temperature below 212 F for not less than 5 days; in the salt-spray test, no rust shall be formed in 24 hr, practically none in 5 days, and no appreciable rust in 60 days. Typical formulas are: (1) 20 g rosin of "H" grade + 100 g petrolatum (U.S.P.) + 10 cc kerosine. The rosin is melted and mixed with the hot petrolatum after which the kerosine is stirred in. Rosin greatly increases the adhesiveness of the petrolatum. Wax may be added to raise the melting point of the petrolatum if necessary. (2) 3 parts candellila wax, 6 parts "H" rosin, 50 parts petrolatum (U.S.P.). (3) 2 parts carnauba wax, 5 parts "H" rosin, 50 parts petrolatum (U.S.P.). In each case, melt the ingredients together at 255 F, stir and cool. Flow an excess over the metal surface (*B. of S. Tech. Paper*, 176 and *Circ.* 200 and 214). Lanolin is the best grease to use as a basis of slushing oils. A small amount of sodium chromate is desirable in slushing greases unless all traces of water have been eliminated.

of the lead by a bituminous coating is always recommended, and the use of alumina cement, as an added precaution, can also be recommended. Aluminum and aluminum alloys in intimate contact with plaster or cement may suffer corrosion for the same reasons. Similar precautionary measures are advisable.

Methods for Minimizing Corrosion

Corrosion may be minimized by (1) the use of a coating of protective metal such as zinc, tin, lead, nickel, or copper; (2) the production of oxide, phosphate, or similar coatings on iron and steel surfaces; (3) the application of protective paints (p. 673); and (4) rendering the surface of the metal passive.

Coating Metals with Zinc. Galvanizing. Zinc is applied to metal surfaces by the Sherardizing process, by dipping into a bath of molten zinc, by electrodeposition, or by metal spraying.

In the **Sherardizing process**, the articles, after being thoroughly cleaned by pickling and sandblasting, are placed in a metal drum together with zinc dust and heated to a temperature of from 500 to 600 F, depending on their size and shape, the drum being rotated so as to promote "rumbling" of the contents. The coating that results is not pure zinc, but an alloy of about 90 percent zinc and 10 percent iron (melting point, 1260 F approx.), and is highly resistant to corrosion. The process is especially suited for screws, bolts and nuts, chains, pipe fittings, nails, small castings, and such other articles as may conveniently be placed within the drum. The cost varies with the character of the articles coated. The term **electro-Sherardizing**, which is often used, merely connotes that the Sherardizing furnace is heated electrically. By a suitable annealing or heat-treatment, the zinc coating produced on sheet and wire by hot dipping can be converted into a very similar alloy coating. A cheap method used for nails, etc., consists in "rumbling" them in zinc dust. The coating is inferior in quality.

In the **hot process**, the articles after being thoroughly cleaned are dipped into a bath of molten zinc. The bath must be maintained at a temperature somewhat higher than the melting point of zinc, which necessitates a large fuel consumption and also results in a considerable loss of zinc (approximately 10 percent). A greater source of loss is the iron-zinc alloy (dross) which forms as a heavy sediment in the zinc bath. That portion of the zinc surface through which the material to be coated enters the zinc bath must be kept covered with a flux; ammonium chloride and zinc chloride are widely used for this. The process is used almost exclusively for sheet and pipe, and, until recently, for wire also. Exposed structural steel work, such as towers, is generally zinc coated by this means. Sheet and wire are coated by mechanical means, pipe and structural shapes by hand. Many irregular shapes (pots, vats, tubs, etc.) and small shapes (bolts, nuts, nails, screws) are coated by hand. For the latter, some means for removing excess zinc, such as centrifuges or shaking devices, is generally used. A very small percentage of aluminum renders the zinc bath very fluid and is favored by many in coating irregular shapes. One or two percent tin is often added in the coating of sheets in order to obtain a very uniform coating and to improve the surface appearance. A coating applied by hot dipping never consists of a simple layer of zinc. It is always of a composite nature, the layer adjacent to the basis metal consisting of zinc-iron alloys. This layer is relatively brittle and, thereby, imposes some limitations on hot-dipped galvanized sheet and wire for certain uses. By a recent process, formation of the alloy can be practically eliminated. A coating of 1 oz per sq ft of exposed surface is considered very suitable for most conditions of service.

Paint Oils. Pure linseed oil (see p. 721), raw or boiled, on account of rapid and hard drying, is preferable for general painting work. Refined raw oil is generally used in white paints for wooden surfaces. A mixture of raw and boiled oils is sometimes preferred in paints for metal surfaces. **Menhaden oil**, which is extracted from the menhaden fish, is occasionally used in special paints for sea exposure. This oil is apt to take dust and become darkened. **Soybean oil**, extracted from the soybean, a legume grown widely for forage, has been used as a partial substitute for linseed oil. In some cases, it has given fair results in combination with linseed oil and tung or oiticica oil in different types of paint. **Tung oil** and **oiticica oils** are (see p. 721) heat-treated with resins and used in technical and waterproofing paints. Their rapid drying to a clear, glossy film makes them valuable for such purposes. **Dehydrated castor oil** and **perilla oil** are now widely used as drying oils. **Rosin oil** and **petroleum oil** are dangerous to use in paints for the protection of wooden or steel surfaces, on account of their slow drying and tendency to check.

Paste Paints. **White-lead paste** contains approximately 9 lb of oil per 100 lb. **Iron-oxide paste** contains approximately 30 percent of oil. **Red lead** is sometimes bought in the dry form and mixed with oil but is usually purchased as a paste and then thinned. Red lead is one of the most widely used of all metal preservatives and gives good service under many conditions. Red lead containing some free litharge protects metal better than neutral red lead. The litharge forms with the linseed oil a hard water-resistant film. Combination pigment paints contain on an average 30 to 35 percent oil and 65 to 70 percent pigment. Instructions for reduction and application are given on the label.

Paint Thinners. Paints of a very heavy body or thick consistency are difficult to apply and often show brush marks after application. Such paints should be thinned with **turpentine** or other neutral thinner. **Petroleum spirits** of the same boiling point and gravity are being used in most paints with very satisfactory results.

Driers. Salts or oxides of lead, cobalt, and manganese are dissolved in oil or naphthenic acids to form driers. When these driers are added to paint, they act as catalytic agents and accelerate drying by attracting oxygen. **Litharge** (PbO) is most generally used. **Sugar of lead**, incorporated in oil, is used for light tints. **Sulphate of zinc** and **manganese dioxide** are used by the grinder for certain paints having zinc white as a base. Cobalt driers are used where very rapid drying is essential.

Paints for Structural Steel. The priming coat of paint should be made of linseed oil containing thoroughly inhibitive or inert pigments such as basic chromate of lead, red lead, basic sulphates of lead, zinc oxide, iron oxide, zinc chromate, etc. A mixture of the lead, zinc, and iron pigments is sometimes preferred to the use of any single pigment.

The surface area of a steel structure may be calculated by use of the following formulas in connection with the material bills giving the tonnage of the various shapes.

Let S = sq ft of surface per ton of metal, and w = weight of shape per running foot, lb. Then, for

I beams: $S = (428h + 1720)/w$, where h = depth of beam, in.

Channels: $S = (434b + 500)/w$, where b = width of channel, in.

Angles (equal-leg): $S = (660b - 21)/w$, where b = width of each leg, in.

Angles (unequal-leg): $S = 3220b/w$, where b = sum of widths of both legs, in.

Z bars: $S = (528h + 1090)/w$, where h = depth of web (out to out), in.

Plates (both sides): $S = 100/t$, where t = thickness, in.

(75 percent) is generally used. Such coatings, if free from pinholes, are highly resistant to corrosion. The excellent "paint-holding" properties of terne plate fit it for many uses in building construction. Its "lubricating properties," in drawing and stamping processes, assist greatly in the manufacture of containers and fuel tanks.

Lead coatings on steel are most efficient in a polluted atmosphere as in industrial centers. In rural atmosphere, pinhole corrosion is soon evident. A "bonding" agent, either as an alloy in the lead or as an undercoat, is necessary for the process commonly used in applying lead coating, *i.e.*, the hot-dip process.

In coating or electroplating with **nickel or copper**, the object to be coated is made the cathode, the anode consisting of a block of the metal to be deposited, and the electrolyte, a solution of the metal to be deposited. In nickel plating, a copper coating is generally applied before the nickel to render the latter more adherent and more corrosion-resistant by making the entire coating more impervious. **Chromium** coatings which are now widely used are also produced by electroplating from chromic acid solutions. They are nearly always applied as a very thin finish on a nickel coating and are almost perfect in their tarnish resistance. Coatings of these three metals protect the underlying, or basis, metal from corrosion only in so far as they exclude air and moisture. Hence, the imperviousness of such coatings as determined by the conditions of deposition is of very great importance. Chromium plating on account of its great hardness is also used to some extent as a protection against wear and abrasion (see Blum, "Chromium Plating," *Mech. Eng.*, Jan., 1927, and "Mechanical Applications of Chromium Plating," *Mech. Eng.*, Dec., 1928).

Copper is sometimes applied in coats integral with the basis metal. Copper is cast around a steel billet which is afterward worked down to the required size. Most of the steel coated in this way is used for wire and gives a combination of high electrical conductivity and high tensile properties. It is also used for purposes requiring the combination of high corrosion resistance and strength as in concrete revetment mats used in river control. Steel clad with nickel or stainless steel is also available commercially.

Aluminum coatings cannot be made by electrodeposition in the ordinary way, although aluminum has been deposited from complex organic liquids in the form of coatings on steel. Aluminum coatings are usually produced by mechanical means. Such a coating has proved very useful on duralumin (p. 630), but on iron or steel its usefulness is limited by the brittleness of the intermediate alloy layer which generally forms.

Steel coated by immersion in molten aluminum (see below) resists atmospheric corrosion admirably. Its rough unattractive appearance restricts its use.

Calorizing is a process by which a coating of aluminum and aluminum-iron alloys is produced on iron and steel (brass, copper, or nickel may also be calorized) which protects the metal against high temperatures because of the formation of aluminum oxide on the surface. It does not protect against ordinary corrosion in the atmosphere or in liquids. Sulphurous acid and carbon monoxide have no appreciable effect on calorized metal. Bending or working of calorized metal should be done at a bright-red heat; threading must be done before calorizing, followed by chasing of the threads to make them fit smoothly without breaking the coating; dimensions and weights are increased by calorizing. Thermal and electrical conductivities are not appreciably changed. The base metal is soft annealed by the process. A

surface that is easily rubbed or washed. High-grade varnishes should be judged comparatively by their body, working properties, flowing, clearness, gloss, and durability. Practical exposure tests are best suited to determine the latter point. Hardness of film, resistance to scraping, elasticity when rolled with a knife blade, and resistance to moisture are some of the quicker tests used. Setting in from 4 to 8 hr and hardening in 24 hr are the limits sometimes specified. (See *Specifications of Federal Specifications Board.*) Varnishes which dry in 4 hr are made from tung oil and synthetic resins; they are very durable if of the long oil type. Gilsonite, elaterite, and other asphalts are melted into oils to form black lustrous asphalt varnishes that are used for protection of water tanks, piping, etc., with great success. Hard, flexible, clear, and rapid-drying insulating varnishes are used extensively for dipping and brushing commutator rings, transformer coils, and other electrical equipment. Baking varnishes for armature coils must be resistant to heat without blistering. (See *A.S.T.M. Specifications.*) Shellac (see p. 760) and other spirit varnishes, chlorinated diphenyl, and phenolic resins are used extensively for the above purposes.

Synthetic Resins. Phenolic compounds condensed with formaldehyde, as well as glycerin phthalate resins, are now widely used in the manufacture of quick-drying water-resisting varnishes. These and numerous other synthetic resins are also applied in the manufacture of spirit varnishes, lacquers, and other protective coatings. For a full discussion of the physical properties of such resinous materials, see the "Resin Index" in "Physical and Chemical Examination of Paints, Varnishes, Lacquers and Colors."

Ceramic Coatings. Metal fittings that are to be permanently protected are often coated and baked with a mixture of feldspar, borax, tin and lead oxides, and other similar materials. The enamel coating produced by this method is very resistant to corrosive agencies. It is, however, subject to cracking from expansion or contraction of the metal upon which it is placed.

Cement and Stucco Coatings. Cement and stucco are rough and hold dirt and soot. They may present water absorptive surfaces and may become spotted during rain storms. They may show surface fissures after weathering, under some conditions. Painting cement and stucco produces a smooth surface which will not hold dirt and will prevent rain spotting. If the cement is freshly formed, the free lime present is usually neutralized by applying a water solution of 3 lb of zinc sulphate per gal of water. After thorough drying, oil paints may be applied. If the cement or stucco surface is old and well weathered, this preliminary wash is not necessary. The priming coat of paint should consist of China wood oil varnish containing pigments. This dries hard and binds the surface particles, and prevents saponification. The second and third coats may be prepared cement paints or the regular type of linseed oil house paints. Emulsified pigmented glycerine-phthalate coatings are now being developed for this purpose.

Factory White. The interior walls and ceilings of factories are usually coated with gloss or flat mill whites made upon a titanium pigment lithopone-zinc oxide base. These paints are washable and sanitary and have a high coefficient of reflection. They are also used on interior surfaces of dwellings and public buildings. (See *U. S. Federal Specifications for Gloss and Flat Interior Whites.*)

Paint-destroying Agencies. Atmospheres containing sulphurous acid or ammoniacal gases are very severe in their action upon paint coatings. Sea air is also very destructive in its effect upon paint. Saline drippings from underground tunnels and fatty acids from machinery are also to be avoided. In dry climates and in communities of little industrial activity, paint coatings last for several years without decay.

Application. For painting large surfaces, the spray system of painting is now used. A paint spray gun operated by two men will usually cover as much surface in 1 day as could be covered by two men using hand brushes in 6 days. (See Reports on Spray Painting in "Papers on Paint and Varnish.")

spraying. It finds extensive application for automobile bodies and is known commercially as **Bonderizing** and **Granodizing**. Some of the phosphate treatments are electrolytically applied. The phosphate treatment is also applicable to zinc surfaces. The advantages of this latter process over the Bower-Barff and similar process are in greater cheapness and simplicity and the use of low temperatures.

Treatments somewhat analogous to the Parkerizing treatment are widely used for the treatment of magnesium alloys prior to the application of other types of coatings, such as aluminum-pigmented spar varnish and pyroxalin coatings of the duco type.

Protection of Aluminum Alloys. Wrought-aluminum alloys, especially of the duralumin type, are now largely used for aircraft purposes. Alloys containing copper as an essential alloy constituent, in sheet form, are susceptible to intercrystalline corrosive attack which results in the material becoming very brittle with little or no surface evidence of the change. Aluminum alloys containing magnesium or magnesium and silicon as the essential alloying constituents are very stable under prolonged weathering conditions. The protective coatings used on aluminum alloys depend on the severity of the service. A preliminary anodizing treatment to produce a film of oxide on the surface (formed by making the article the anode of a cell with chromic or sulphuric acid as the electrolyte) is now common practice. Such a surface oxide coating forms an excellent basis for the application of other coatings. Aluminum-pigmented spar varnish is excellent for this. For very severe marine conditions, the only coatings that give permanent protection are those of aluminum. Such a coating may be applied by the Schoop metal-spraying process and is especially useful for heavy pieces. A sheet product known as **Alclad** is now available commercially. This consists of a sheet of duralumin or other high-strength aluminum alloy coated with aluminum, the coating forming an integral part of the sheet. The aluminum is applied to the alloy billet, and the whole is rolled into sheet form. This product is not yet (1941) commercially available in tube form.

The passivating of iron surfaces may be accomplished in several ways, the most common consisting of immersion of the metal in nitric acid (sp gr 1.4) after it has been highly polished. Other methods consist in immersing the metal in fuming sulphuric acid, potassium ferrocyanide, or potassium chromate solution or in chromic acid; coating with a manganese dioxide paint; cathodic pickling in a weak acid solution, the metal being made the cathode in a circuit of low voltage; treatment with arsenic, sodium nitrite, etc. This condition of passivity is temporary, and, thus far, passivating has been of doubtful value, except for stainless steel for which it is a regular practice. A chromate treatment now used commercially on zinc surfaces is somewhat analogous to some of the above treatments.

The treatment of water with a bichromate, sodium silicate, or similar chemical to form a protective surface film on iron (steel) with which it comes in contact is common practice for such purposes as air conditioning in which the same volume of water must be used repeatedly over and over.

Slushing oils are usually non-drying oils or greases which remain soft for prolonged periods, are strongly adhesive on metals, but can readily be removed when desired. The best protection to metals is afforded by acid-free semisolid oils applied in a melted condition. Types of commercial slushing oils are petroleum residues, mixtures of lithopone and iron oxide with heavy petroleum residues, petrolatumlike compounds emulsified with chromate solutions, blown vegetable oils, soft asphalts thinned, rosin-base materials. A specification proposed several years ago is as follows: Coating shall

quality into "dense southern pine" and "sound southern pine"; dense southern pine should show on either end an average of at least six annual rings per inch and at least one-third summerwood, or else the greater number of rings should show at least one-third summerwood, all as measured over the third, fourth, and fifth inches on a radial line from the pitch; wide-ringed material excluded by this rule is acceptable, provided the amount of summerwood, as above measured, is at least one-half; the contrast in the color between summerwood and springwood should be sharp, and the summerwood should be dark in color, except in pieces having considerably above the minimum requirement for summerwood; sound southern pine includes pieces of southern pine without any ring or summerwood requirement); **Northern white pine** (wood from tree of that name grown in Maine, Michigan, Wisconsin, Minnesota, and Canada); **Norway pine** (Norway or red pine grown in Michigan, Minnesota, Wisconsin, and Canada); **Idaho white pine** (a species of white pine grown in Western Montana, Northern Idaho, and Eastern Washington); **Ponderosa pine** (timber known as white pine grown in Arizona, California, New Mexico, Colorado, Oregon, and Washington; sometimes known as Western yellow or ponderosa pine, or California white pine or Western white pine); **sugar pine** (white pine from California); **poplar** (wood from the tulip tree, otherwise known as whitewood, yellow poplar, and canary wood as well as aspen, cottonwood, and balsam poplar); **redwood, Eastern spruce** (spruce timber from points east of and including Minnesota and Canada, covering white, red, and black spruce); **Engelmann spruce** (spruce timber from the Pacific Coast); **sycamore; tamarack** (tamarack or eastern tamarack, grown in states east of and including Minnesota); **tupelo** (tupelo gum and bay poplar); **walnut** (black walnut).

Physical Properties

Weight and Specific Gravity. The weight of wood will vary with the amount of water contained, and (within a given species) with the age, part of tree from which the wood is cut, geographical location, etc. In general, green wood will contain 50 to 75 percent, air-dry wood 10 to 20 percent water. In general practice, a wood weighing less than 30 lb per cu ft is called light; one between 30 and 40 lb, medium; and one more than 40 lb, heavy. The approximate air-dry weight of any wood is from 10 to 20 percent higher than its absolute dry weight. For the **specific gravity** of various woods, see p. 687.

Hardness. According to Schenck ("Forest Utilization"), dense woods are the harder. Wide rings in oak and narrow rings in pine indicate superior hardness. Heartwood may be harder than sapwood, while dry wood is generally harder than green wood of the same kind.

Relative Hardness of Woods

Hard: Hickory, dogwood, sugar maple, sycamore, locust, hornbeam, persimmon.

Medium: Ash, oak, elm, beech, cherry, mulberry, birch, sour gum, longleaf pine.

Soft: Chestnut, tulip tree, sweet gum, Douglas fir, Southern pine, larch, basswood, horse chestnut, hemlock, cottonwood, spruce.

Very soft: White pine, sugar pine, redwood, willow.

A scale of hardness frequently used is as follows:

Scale of Side Hardness of Wood

(Based on Wood Handbook)

Dogwood	1.50	Ash (white)	0.95	Chestnut	0.41
Maple (hard)	1.09	Walnut (black)	0.76	Butternut	0.37
Oak (white)	1.00	Elm (American)	0.75	Poplar (yellow)	0.34
Birch (yellow)	1.00	Cherry (black)	0.72	Pine (eastern white)	0.30
Beech	0.98	Birch (paper)	0.68	Cedar (western red)	0.26
Oak (red)	0.98	Pine (longleaf)	0.65	Cedar (northern white)	0.24

PAINTS AND PROTECTIVE COATINGS

BY

HENRY A. GARDNER

REFERENCES: Gardner, "Physical and Chemical Examination of Paints, Varnishes, Lacquers, and Colors," Inst. Paint & Varn. Research. Circulars Nos. 1 to 610, Scientific Sec., National Pt. Varn. & Lacquer Assn. Reports of Committee D-1 of the *A.S.T.M.*, 1903–1940.

Preparation of Surfaces for Painting. If **new wood** is to be painted, it should be sandpapered lightly in rough spots. All knots should be coated with aluminum varnish just before applying the paint. Before applying a second coat, all nail holes and crevices should be stopped with putty. If the surface needs repainting, all "alligatoring," scaling, or blistering appearing on the old paint should be leveled with sandpaper. When **new metal** is to be painted, it should be cleaned of rust spots with a wire brush, scraper, or sandblast. Small articles are generally pickled in a hot 15 percent solution of sulphuric acid to remove the oxide scale from the surface, and then thoroughly washed and dried before painting. **Old painted steel surfaces** should be sandblasted, if in bad condition. Three coats of paint should be used on structural materials if good results are to be obtained.

Spreading Rates. The average spreading rate for a paint of normal consistency is as follows:

Spreading Rate for Paint in Sq Ft Per Gal

Coat	Wooden surfaces	Metal surfaces	Cement and concrete
Priming coat	300 to 400	500 to 700	150 to 250
Second and third coats	400 to 600	700 to 800	300 to 400

Cost of Painting. The cost of labor for painting any structure is generally four times the cost of the paint. The best paints from the standpoint of efficiency are those described below.

Pigments and Prepared Paints for Wooden Surfaces. The opaque white pigments used in paints for exterior surfaces are corroded **white lead** (basic carbonate), sublimed white lead (basic sulphate), **zinc white** (zinc oxide), titanium oxide pigments, and **leaded zincs** (lead sulphate combined with zinc oxide). Lead pigments used alone are apt to chalk and "alligator," while zinc pigments often scale and check. By combining the two types of pigments, these defects are minimized. Equal parts of lead or titanox and zinc form a good paint. Mixtures of lead and zinc pigments or lead, zinc, and titanium pigments are often combined with about 10 percent of the extender pigments such as **barytes** (barium sulphate), **china clay** (aluminum silicate), **silex** (silicon dioxide), **asbestine** (magnesium silicate), etc., to increase the durability of the product. Such combinations when finely ground by machine with pure linseed oil form excellent paints. They are tinted with **colored pigments** such as chrome yellow (lead chromate), Prussian blue (ferro-ferricyanide of iron), sienna, ocher, umber, and lampblack and sold as prepared paints. In combination with zinc oxide or white lead and zinc oxide, titanium pigments will give remarkable service on wood, metal or cement. (See *U. S. Federal Specifications for Exterior White Paints.*)

less costly plywood products. Plywood glues are usually mixed mechanically and spread by revolving spreaders. The assembled plywood must go under a hydraulic press (at approximately 75 lb per sq in. pressure) before the initial set of the glue has begun, and remain under pressure or in clamps for not less than 4 hr. It is then dried, cut to size, and sanded.

Decay and Destruction of Timber

Decay in timber is caused by fungi growing in the wood fiber and is favored by the presence of oxygen, water, heat, and food supply (see Boyce, "Forest Pathology," McGraw-Hill). Timber suffering from the forms of decay usually distinguished as dry rot, moist rot, wet rot, brown rot, etc., has little practical value. Decayed wood is lighter than sound wood, and loses its strength rapidly as the decay progresses.

Absolutely dry wood will not decay, nor will wood decay when constantly submerged in water, nor when kept more than 4 ft under ground. Poles and posts decay chiefly at the ground line. Construction timbers in buildings, bridges, etc., decay most rapidly at points where they come in contact either with other timbers, with the ground, or with stone or concrete walls. Any conditions favoring the retention of water in the timbers, particularly where the temperature is from 60 to 85 F, will bring about decay. Sapwood of all timbers decays very rapidly, heartwood is usually more resistant. In structural timbers, decay frequently appears in the inner sapwood, so as not to *be observable from the outside, even by the most careful inspection.* This form of decay is usually termed "internal sap rot." Where strength is the principal requirement, the most careful examination of timbers with large amounts of sapwood should be made, particularly if there is evidence that such timbers have been cut from the tree more than 2 or 3 months after felling. This applies particularly to all forms of piling, stringers, posts, caps, etc.

Blue stain is a grayish-blue discoloration found in the sapwoods of pines and other coniferous woods, due to a minute fungus (*Ceratostomella pilifera*) growing in the wood fiber. It has no effect on the strength of the wood, and may be prevented by dipping the freshly sawed lumber in a 5 percent solution of sodium carbonate kept at about 140 F.

Decay of Living Trees. Where decay, or dote, is found in the heartwood of timbers, it is usually due to disease of the living trees. Different forms of such decay are distinguished in conifers as red heart, dote, rot, and in hardwoods as piped rot, brown rot, speckled rot, etc. All these forms of decay cease after trees are felled, and *cannot be communicated to other pieces of sound structural timber.*

Weathering is the wearing away of the surface of timbers caused by exposure to the elements, and is of importance only with comparatively soft woods. Different forms are distinguished according to the color, as white, gray, or brown weathering.

Life of Woods. The natural length of life of wood and its resistance to decay vary with the kind of wood and the conditions under which it is used. In general, woods may be classed as long-lived, medium-lived, and short-lived, as indicated below.

Long-lived: Cypress, redwood, red cedar, white cedar, osage orange, catalpa.

Medium-lived: White oak, slippery elm, black walnut, hickory, longleaf pine, tamarack, Douglas fir.

Short-lived: Red oak, red gum, beech, elm, spruce, shortleaf pine, hemlock.

Destruction of Wood by Marine Animals. Piling and other timbers exposed to salt water in warm climates are destroyed by various species of marine wood borers, of which the principal forms are the teredo (known as the shipworm) and the limnoria (sometimes, through resemblance, called a wood louse). The teredo thrives in waters with a saline density above 1.0054, and at temperatures of from 55 F to the highest

One gallon of pure red-lead paint (20.3 lb red lead in 5.62 lb of linseed oil without turpentine or other thinner) when applied to new steel surfaces will cover from 500 to 700 sq ft for the first coat, 650 to 850 sq ft for the second coat, and from 800 to 1,000 sq ft for the third coat; or, for two-coat work, 1 gal will cover from 300 to 400 sq ft and for three-coat work from 215 to 285 sq ft. (Cloyd M. Chapman, *Eng. Rec.*, Feb. 15, 1913.)

Aluminum paint contains 1¾ to 2 lb of aluminum powder per gallon. In highly-diluted varnish liquids it withstands a temperature of 900 F. As a primer, on wood, it has high moisture-proofing efficiency, when covered with light tints of regular linseed oil paint. Suspended in spar varnish, aluminum powder increases greatly the moisture resisting power of the varnish. The low radiation loss makes aluminum paint valuable for furnaces and the high heat reflecting power reduces evaporation losses from oil storage tanks.

Zinc dust is an excellent protective of iron and steel surfaces. A so-called **liquid galvanizing** is made of zinc dust in boiled linseed oil. Mixed with 20 percent zinc oxide it makes a good primer for iron and steel and an exceptional primer on galvanized iron. Zinc dust is also effective when mixed with iron oxide and zinc oxide, in proportions zinc dust 50, iron oxide 30, zinc oxide 20 percent. As zinc dust retards drying some additional drier may be used under adverse drying conditions. Zinc dust may be added to gray topside paints for vessels to increase resistance to corrosion.

Carbon Paints. Paints containing graphite, carbon black, or lampblack are unsafe to use as a priming coat for metal. Because of their electrical conductivity, they are apt to excite corrosion. Their use as second-coaters or top-coaters is to be recommended.

Ship-bottom Paints. Anticorrosive priming paints made of the rust-inhibitive pigments noted above and topped with quick-drying antifouling paints made of shellac dissolved in alcohol or resin-coal tar solutions and mixed with powdered zinc and the oxides of zinc, iron, and mercury are widely used in the Navy and merchant marine service for the protection of steel vessels. Some synthetic resin paints with light-colored pigments also give good results. Barnacles are phototropic and do not go to light-colored surfaces.

Paints for Water Carriers. Cast-iron and riveted-steel water carriers are generally heated and dipped in hot preparations of mineral asphalt, coal tar, and lime, or special baking enamels. This treatment is occasionally followed by baking at a high temperature to make the coating more resistant to abrasion. Recently certain types of synthetic resins have been found to be very effective.

Paints for Tinned and Galvanized Surfaces. After wiping off the greasy surface with benzine, a linseed-oil paint made of red lead, iron oxide, and chromate pigments, thinned with a little varnish, gives good results on tin. For galvanized iron use zinc-dust primers. Applying a 5 percent water solution of copper nitrate and sal ammoniac or a phosphoric acid compound to galvanized iron previous to painting is also recommended.

Rubber Paints. Paints made of caoutchouc, gutta percha, chlorinated rubber, etc., dissolved in coal-tar distillates form elastic films which give some protection to metal. Upon weathering, the films soon show pinholing and brittleness if ordinary rubber is used.

Metal Lacquers. Cellulose nitrate dissolved in butyl acetate and other solvents and thinners are widely used in coating automobiles and other manufactured metal objects. They are usually pigmented. They are generally applied by spraying. They dry with great rapidity and produce highly durable films. Phenolic and glycerine phthalate resins are usually present in such lacquers.

Varnishes. Gums melted into oils and thinned with turpentine form oleoresinous varnishes. For exterior purposes, a "long" oil varnish (one that contains a high percentage of oil) is superior. For interior use, a "short" oil varnish gives a highly lustrous

tive. In one run of a thousand ties treated with creosote, where all the ties were exactly the same size, the average absorption was 23 lb of creosote, but some of the pieces absorbed as low as 2 lb and some as high as 90 lb.

Whenever possible, only thoroughly air-seasoned timber should be treated, because wood containing appreciable amounts of water will not absorb creosote. Steaming of green timber reduces its strength (see Hatt, "On the Effect of Steaming on Wood," *Proc. A.R.E.A.*, 28, 1927, p. 1164). Whenever possible, all injuries to wood after treatment should be avoided. All framing, boring, and adzing should be done before treatment.

The principal preservatives used, mentioned in the order of quantities used in 1939, are: coal-tar creosote (creosote oil), coal-tar creosote solution, creosote-petroleum mixture, chromated zinc chloride, and zinc chloride. Small quantities of mercuric chloride were used. In addition to the above, petroleum oils and petroleum residues were used mixed with creosote. Of these preservatives, the order of efficiency is as follows: coal-tar creosote and creosote coal-tar solution, mercuric chloride, and zinc chloride. Water-gas tars have some antiseptic value but are probably not equal to coal-tar creosote. Petroleum has no antiseptic value, but is added to creosote to reduce checking. In addition to the above, there are numerous proprietary preservatives whose value is frequently not commensurate with the cost. A number of new preservatives have developed whose ultimate value has however not yet been demonstrated.

Description and Specifications for Preservatives

(For standard specifications see *Manual A.R.E.A.; Manual A.W.P.A.;* For methods of analysis see also *A.S.T.M. Standards*)

Creosote. Coal tar yields on distillation from 30 to 50 percent of creosote. It is not a simple substance but contains a large number of chemical constituents. Creosote specifications confine themselves to the requirement that the creosote must be a direct product of coal-tar distillation, free from adulteration, with limited material insoluble in benzol, and that certain definite percentages distil at standard temperatures.

At 65 F, creosote weighs about 8.7 lb per gal; at 100 F (the standard temperature) the specific gravity ranges from 1.03 to 1.12. It has high antiseptic properties and is practically insoluble in water. There are three grades of coal-tar creosote recognized in the trade which are distinguished chiefly by their specific gravities and percentages of low and high boiling compounds. The heavier oils are considered the more valuable. **Creosote coal-tar solution,** is a solution of coal-tar in creosote in varying proportions (20 to 40 percent coal tar). **Creosote-petroleum mixture** is a mixture of creosote and petroleum.

Water-gas tar distillate is obtained from water-gas tar by distillation. **Water-gas tar solution** is a solution of water-gas tar in water-gas tar distillate (not more than 40 percent water-gas tar).

Chromated zinc chloride is a mixture of approximately 80 percent zinc chloride and 20 percent sodium dichromate.

Zinc chloride is soluble in water and tends to leach out of timber in wet locations or in regions of heavy rainfall. Combinations of zinc chloride with creosote and of zinc chloride and water-gas tar and water-gas-tar solution are in use (see Card process, p. 683).

Processes of Treatment

(For detailed recommendations and Standard specifications, see *Proc. A.R.E.A.*, 1939; *Manual A.R.E.A.; Manual A.W.P.A.*)

Bethel or Full Cell Process. After a preliminary vacuum, creosote is injected under pressure varying from 40 to 200 lb per sq in., which is continued until the desired absorption has been obtained. The amount of creosote injected will vary from 10 to 25 lb per cu ft. This process is used largely for the treatment of piling, bridge materials, paving blocks, telegraph poles, and other timbers from which a long service is expected and where mechanical or destructive processes, aside from decay, are of minor importance. In general, bridge materials, piling, poles, and similar timbers are treated with about 16 lb of creosote per cu ft. For marine piling 22 lb or refusal treatment should be used.

WOOD

BY

HERMANN VON SCHRENK

(Revised by C. C. Forsaith)

References: Bulletins and Circulars of the U. S. Forest Service. "Wood Handbook," U. S. Dept. Agr. "Guide to the Grading of Structural Timbers and the Determination of Working Stresses," *U. S. Dept. Agr. Misc. Pub.* 18. Also *U. S. Dept. Agr. Tech. Bull.* 158 and 479. Weiss, "Preservation of Structural Timbers," McGraw-Hill. Record, "Identification of the Economic Woods of the United States," Wiley. Brown and Panshin, "Identification of the Commercial Timbers of the United States," McGraw-Hill. Koehler and Thelen, "Kiln Drying of Lumber," McGraw-Hill.

Definitions and Classification. Timbers are classed commercially as hardwoods and softwoods. Typical **hardwoods** are oak, ash, chestnut, hickory, maple, and poplar; typical **softwoods** are pines, hemlock, spruce, larch, fir, and cedar. The terms "hard" and "soft" do not necessarily refer to actual hardness (see table, p. 678). In all timbers, two forms are distinguished, heart and sapwood. **Heartwood** is the inner part of a tree, usually darker in color than **sapwood** (the outer part), frequently heavier, and more decay-resistant. Knots or branch inclusions (loose or solid) are defined as **pin knots** (not over ½ in. diam), **standard knots** (not over 1½ in.) **large knots** (more than 1½ in.), **sound knots** (solidly grown together with the surrounding wood), **loose knots** (not held firmly in place by growth or position), **pith knots** (sound knots with pith holes not more than ¼ in. in diam), **encased knots** (*i.e.*, entirely surrounded by bark or pith), **rotten** (decayed) **knots**, **spike knots** (knots sawed lengthwise). Recognized **defects** are **wane**, bark or the lack of wood from any cause on the edge; **shakes**, splits or checks in timber, which usually cause a separation of the wood between annual rings; **sap stain**, a discoloration of the sapwood; **pitch pockets**, openings between the grain of the wood, containing more or less pitch; **dote** and **red heart**, various forms of decay.

Timbers are sold under various names. The following list of **standard commercial names of timbers** has been adopted by the N.B.S. (Simplified Practice Recommendation, No. 16, July 1, 1926 and R16-39) and in part by the A.R.E.A. (Vol. 27, 1926, p. 833) and the A.S.T.M. (1927 Standards, Pt. II, p. 700). The species included in each name are given in parentheses.

Ash (white, black, blue, green, and red ash); **basswood** (linden, linn, lind or lime tree); **beech** (red and white beech); **birch** (paper, white, yellow, and black); **buckeye** (wood from the same genus as the horse-chestnut tree); **butternut** (butternut also known as white walnut); **cherry** (sweet, sour, red, wild, and especially the black cherry); **chestnut**; **cottonwood**; **cypress** [red, (coast type); yellow (inland type); white (inland type)]; **elm, soft** (American or white, red or slippery elm); **elm, rock** (rock or cork elm); **Douglas fir** [(coast type); red, (intermountain type); red (Rocky Mountain type)]; **gum** (red gum, black gum); **sweet gum or satin walnut**; **Eastern hemlock** (from all states east of and including Minnesota); **West Coast hemlock** (hemlock from the Pacific Coast); **hickory** (shell-bark, kingnut, mockernut, pignut, black, shagbark, and bitternut); **Western larch** (larch or tamarack from the Rocky Mountain and Pacific Coast regions); **maple, soft** (soft and white maple); **maple, hard** (hard, rock, and sugar maple); **white oak** (white, burr or mossy cup, rock, post or iron, overcup, swamp, post, live, chestnut or tan bark, yellow, and basket or cow oak); **red oak** (red, pin, black, water, willow, Spanish, scarlet, Turkey, black jack or barn, and shingle or laurel oak); **pecan**; **Southern pine** (all pines of the Southern states manufactured into lumber, including longleaf, shortleaf, loblolly, and Cuban pines. The lumber is divided according to

both sides, S-1-S or S-2-S, is $\frac{25}{32}$ in. thick. A standard **industrial board,** surfaced smooth on one or both sides, S-1-S or S-2-S, is $\frac{25}{32}$ in. thick. For details as to sizes, measurements, shipping provisions, tally, grade marking, etc., see above references.

Structural timbers are **graded,** by inspection, as follows: **Select** (S2) grade is especially adapted to heavy construction, such as railway, bridge, and mill work. **Standard** (S3) grade is primarily suitable for general building use and common mill construction. **Common** (S4) grade is especially recommended for small house construction where stiffness is a controlling factor and where strength requirements are not so critical. These three grades furnish the greater portion of the structural timbers used. **Extra Select** (S1) grade is an exceptional grade intended to meet the most exacting strength requirements for construction purposes. The grades are based on the density of the wood and on the defects present. Density is determined by observing the spacing of the annual rings and the percentage of summer wood. Permissible defects, such as knots, checks, shakes, and cross grain, vary with the proposed use of the structural member and with the location of the defects in that member (see also *Manual A.R.E.A.*)

Structural materials are also graded according to the use to which they are to be put, *i.e.*, joist and plank, beams and stringers, posts and timbers.

Shipping Weights. The weights of lumber are given either as green, shipping weight (air dry), or kiln dried. The average weight will vary materially, depending upon the item of lumber, such as sheathing, flooring, timbers, etc.

Table 1. Estimated Shipping Weights of Lumber
Lb per 1000 Ft B. M.
(Based on rough 1 in.)

Species	Green from saw	Shipping dry	Well seasoned	Kiln dried	Species	Green from saw	Shipping dry	Well seasoned	Kiln dried
HARDWOODS					HARDWOODS				
Ash, black	4600	......	3200	3000	Oak, white	5700	4500	4100	3600
Ash, white	4600	......	3800	3300	Poplar	3900	3000	2800	2400
Basswood	4200	2800	2500	2100	Sycamore	4750	......	3000	
Beech	5750	......	4000	......	Tupelo	4200	......	3000	
Birch	5500	......	4000	......	Walnut	4900	4000	3800	
Butternut	4000	......	2500	......	SOFTWOODS				
Chestnut	5000	......	2800	2450	Cedar, incense	......	......	2300	
Cherry	5000	......	......	......	Cedar, western red	3000	......	2700	
Cottonwood	4600	3100	2800	2400	Cypress	5000	......	3000	
Elm, rock	5400	4300	4000	3500	Fir, Douglas	3500	3300	3000	
Elm, soft	4750	3300	3100	2900	Hemlock, eastern	......	2500	......	
Gum, red	5400	3600	3300	3050	Hemlock, west coast	3500	3300	3000	
Gum, sap	5000	3300	3000	2750	Pine, southern	4500	3400	3200	
Hickory	6000	......	4500	......	Pine, northern white	4000	2900	2400	
Mahogany	4000	......	3000	......	Pine, Pondosa	......	......	2600	
Maple, hard	5400	4150	3900	3400	Pine, sugar	......	......	2250	
Maple, soft	5000	3650	3300	3000	Redwood	......	......	2400	
Oak, red	5500	4250	4000	3400	Spruce, Sitka	3500	......	2600	

Shingles. Pine shingles are usually 16 in. long, $2\frac{1}{2}$ to 14 in. wide, $\frac{1}{16}$ in. thick at the small end, and $\frac{5}{16}$ in. thick at the butt end. A standard shingle

Cleavability is inversely as the resistance to splitting in a lengthwise direction. According to Schenck ("Forest Utilization"), cleavability is affected by: (a) the straightness, length, and elasticity of the fiber; (b) the heaviness of the wood rays; (c) straightness of growth; (d) branchiness; (e) moisture (the higher the moisture the easier the wood can be split); (f) frost (reduces cleavability); (g) hardness.

Relative Cleavability of Woods

Hard to split: Black gum, elm, sycamore, dogwood, beech, holly, maple, birch, hornbeam.
Medium: Oak, ash, larch, cottonwood, linden, yellow poplar, hickory.
Easy to split: Chestnut, pines, spruce, fir, cedar.

Heat Value. The specific heat of practically all kinds of wood when oven-dry is 0.327 (Dunlap, *Forest Service Bull.* 110, 1912). The heat value of wood depends on its specific gravity (oven-dry), heavier woods giving more heat than light woods. According to Schenck, 1 cord of green wood may contain 250 gal of water (dependent upon the species, density of the wood, degree of dryness, etc.), and the heat required to evaporate this into steam is not available for other heating purposes. According to German experiments, wood with 45 percent moisture gives only 50 percent as much heat as oven-dry wood. Resin increases the heating power by about 12 percent. According to Roth, 100 lb of wood, as sold in wood yards, contains 25 lb of water, 74 lb of (oven-dry) wood, and 1 lb of ashes. Thus, 100 lb of green wood (50 percent moisture) furnish about 270,000 Btu, 100 lb of air-dry wood (10 percent moisture) about 580,000 Btu, and 100 lb of kiln-dry wood about 630,000 Btu.

Relative Values of Woods as Fuels

Best: Hickory, beech, hornbeam, locust, heart pine.
Good: Oak, ash, birch, maple.
Moderate: Spruce, fir, chestnut, hemlock, sap pine.
Poor: White pine, alder, linden, cottonwood.

Strength of Woods. See p. 687.

Plywood

Plywood is laminated or compound wood and consists of several sheets (plies or folds) of thin wood (or veneer) glued together, either in flat or curved shapes. Usually, the adjacent layers or sheets of wood have their respective fibers or grain at right angles to each other. Internal stresses and strains are balanced in such construction, resulting in permanence of dimension and shape.

The veneer used in assembling into plywood is manufactured in four ways: (1) Rotary cut from logs revolved in a veneer lathe; about 92 percent of all veneers are made in this manner. (2) Slice cut from the flat surface of logs or flitches moved angularly against a slicing knife. (3) Sawed on a segment saw; not economical on account of the saw-kerf waste. (4) Half-round cut from flitches mounted on a wide-swing or eccentric lathe.

The straightening out or flattening of rotary-cut veneer results in checks on the concave side. In veneer $\frac{1}{16}$ in. thick and less, these cutting checks are negligible, but in veneer $\frac{1}{8}$ and $\frac{3}{16}$ in. thick their presence affects the strength.

The glues used in plywood (see p. 697) include the following: Animal glue penetrates wood well, and is best applied in a hot room; it is soluble in water. Vegetable glue is economical in cost, principally used for plywood, of abundant strength, easy to use, but not waterproof. Casein glue is practically non-soluble in water and is usually specified for plywood for vessels, aircraft, and other exposed locations. Albumen glue when properly coagulated is practically waterproof, and its use is permitted under the most rigid waterproof specifications. Silicate glue is a low-priced glue used in box shooks and

Bark is usually sold and bought by the cord. The tanneries, however, apply the name to a weight of 2,240 lb, instead of to 128 cu ft. Twelve cords of bark fill one common (old) freight car. A stack of bark contains 30 to 40 percent solid bark. The specific gravity of fresh oak bark is 0.874; dried, it is 0.764. The bark of white oak varies from 55 percent of the wood in trees 20 years old to 21 percent in trees 140 years old. The average bark yield of chestnut oaks per tree (in cords) is as follows: trees 6 in. in diam, 0.013; 12 in. trees, 0.073; 18 in. trees, 0.195; 24 in. trees, 0.375.

Table 3. Number of Cubic Feet of Solid Wood per Cord, with Corresponding Diameters of the Average 4 ft Stick
(From U. S. Forest Service Bulletin 96)

Diam of standing timber breast-high, inches	Chestnut		Black oak		White oak	
	Diam of average stick, inches	Cubic feet per cord	Diam of average stick, inches	Cubic feet per cord	Diam of average stick, inches	Cubic feet per cord
1	0.9					
2	1.8	63	1.8	63	1.8	63
3	2.6	70	2.5	69	2.5	69
4	3.3	75	3.1	74	3.1	74
5	4.0	79	3.6	77	3.5	76
6	4.7	83	4.1	80	3.9	79
7	5.2	85	4.5	82	4.2	81
8	5.8	88	4.8	84	4.5	82
9	6.2	89	5.0	85	4.7	83
10	6.7	91	5.3	86	4.9	84
11	7.0	92	5.4	86	5.0	85
12	7.4	93	5.6	87	5.1	85
13	7.7	94	5.7	88	5.2	85
14	7.9	94	5.7	88	5.2	85
15	8.2	95	5.8	88	5.3	86
16	8.4	95	5.9	88	5.4	86
17	8.5	95	5.9	88		
18	8.7	95	6.0	89		

STRENGTH OF WOOD

By LIONEL S. MARKS

(For references, see p. 877)

The strength of wood-substance must be distinguished from the strength of large timbers containing defects such as knots and crooked grain. Wood-substance exhibits an elastic limit under usual rates of test loading; under continuous loads it is markedly plastic and the modulus of elasticity may be only half as great as listed in the tables. Under impact, the elastic limit is frequently twice as great as listed in the tables. Strength is also affected by temperature, and very greatly affected by moisture. Table 1 represents results of a series of timber tests of the Forest Service on representative trees collected from the forest. Inasmuch as the strength of species is affected very greatly by the age of the tree, site conditions, and the inherent variability of individuals, strength values of wood cannot be quoted with any exactness.

Relation of Physical Properties to Specific Gravity. As a rule, strength varies directly with the dry specific gravity. An approximate value S of various strength functions is given by the equation $S = mg^n$, where g is

found along our coasts. It lives in clear and turbid water but cannot live in waters with high sewage contamination, and seldom to a depth below 30 ft. It works most rapidly in warm water. Unprotected pine piles will be destroyed in approximately the following times at various points: Norfolk, Va., 1 to 5 years; Pensacola, Fla., 1 to 3 years; Galveston, Tex., 5 months to 1½ years; Colon, Panama, 9 months to 1 year; Puget Sound points, 1 year; Klawak, Alaska, 1½ to 3 years. The limnoria requires pure salt water and cannot live in dirty water. It occurs sparingly in Long Island Sound, is quite abundant along the coast of Massachusetts, and does great damage along the Gulf of Mexico and along the North Pacific Coast. All untreated woods grown in the United States used for piling are subject to the attack of marine borers. A number of tropical woods are immune. For preventive measures, see Creosoting, p. 682. For description, see *U. S. Forest Service Circular* 128, "Preservation of Piling against Marine Borers"; report on the San Francisco Bay Marine Piling Survey (*Proc. Am. Preservers, Assn.*, 1921, 1922, 1923, and 1927); report of National Research Council Marine Piling Committee, "Marine Structures, Their Deterioration and Preservation" by Atwood and Johnson, 1924, with bibliography.

Destruction of Wood by Insects. Termites, or white ants, destroy untreated wood. They live in the ground and enter buildings and structures only from the ground. The mature insects, or "flying ants," are harmless. The workers can dissolve lime mortar; hence only cement mortar should be used for foundations. Their entrance into buildings can best be prevented by absolutely sealing the foundation walls, both sides and top, with rich cement mixture and coal-tar pitch. Joints between concrete beams or floors should have a coal-tar pitch seal. All wood, except creosoted wood, should be kept away from ground connection. Creosoted wood is immune. In the tropics, several species of ants attack wood aboveground (such as telegraph and telephone poles, cross arms, etc.); the use of creosoted wood is recommended in such regions (see Snyder, *U. S. Dept. Agr., Bull.* 94, *Pt.* II, 1915).

Timber Preservation

Seasoning of Timber. Timber can be seasoned either by air drying or by artificial means. In **air drying**, excessive splitting may be prevented either by painting the exposed ends of logs, timbers, or planks with common paint or preferably with coal-tar creosote, or by driving in sharp-edged irons of wedge section, shaped in the form of the letter S, and usually called S irons. The base of the wedge section should be ⅛ in. thick. **Artificial drying** is usually done in kilns (see Tiemann, "The Kiln Drying of Lumber"). Coniferous woods can be dried more rapidly than hardwoods and at higher temperatures, without affecting their strength. The latter should be dried very slowly. Small pieces of wood can be dried so as to prevent checking by soaking them from 1 to 7 days in a concentrated salt solution. The soaked pieces should then be piled in the air to dry. Another plan for drying small pieces is to pile them in bone charcoal.

Timber Preservation. The prevention of decay due to fungi and the protection of timber against wood borers are accomplished by the injection of various chemicals. The principal requirements for successful chemical preservation are: as thorough impregnation of the wood as can be obtained; the injection of sufficiently large quantities of the preservative; the injection of efficient chemicals; and the use of thoroughly sound and seasoned wood.

The **penetration** obtained will vary with the preservative. Water solutions, such as zinc chloride, will usually penetrate clear through a stick. Coal-tar creosote will penetrate sapwood only in coniferous woods (the heartwood of pines is slightly penetrable in large pieces, wholly so in small pieces like paving blocks), and in many hardwoods such as gum, white oak, hickory, and ash. It will penetrate into the heartwood of red oak, elm, and sycamore, and into the heartwood of small pieces of pine, spruce, etc., such as paving blocks. Individual pieces of timber absorb different quantities of preserva-

the specific gravity of oven-dry wood and m and n have values which vary with the condition of the wood and the kind of stress to which it is subjected. Thus for small clear specimens, the modulus of rupture is obtained by putting $m = 25{,}530$, and $n = 1.20$; when the wood is green, $m = 19{,}140$ and $n = 1.27$; for green longleaf pine, $m = 20{,}800$ and $n = 1.5$. (Betts, International Engineering Congress, 1915.) See also Fig. 1. The value of n varies with the strength function under consideration and ranges from unity for compression parallel to the grain to 3 for ultimate rupture work. For shearing stress, n is approximately unity.

The density and strength of *summer wood* of the pines are about twice as great as in the *spring wood*. The proportion of summer wood gives an excellent indication of strength.

Moisture Content, Shrinkage, Specific Gravity. The percentage of moisture in wood is based upon the oven-dry weight of the wood. Thus wood which weighs 35 lb per cu ft when oven-dry and contains 50 percent moisture when green will then weigh 35 plus 17½, or 52½ lb per cu ft. In large beams, the cubic foot reckoned is not the shrunk cubic foot when oven-dry, but the cubic foot of volume occupied by the wood at the time of the test.

The **specific gravity** as based on oven-dry volume may be obtained from the specific gravity as based on green volume by allowing for shrinkage.

The **volumetric shrinkage** of wood from the green to the oven-dry state, expressed in percentage of volume, is approximately 26.5 times the specific gravity as based on green volume. Thus, for white oak, shrinkage $= 0.60 \times 26.5 = 15.9$ percent, and consists almost entirely of lateral contraction.

The specific gravity of **wood-substance**, of which 50 to 60 percent is **cellulose**, is very nearly **1.5**. A solid cubic foot of wood-substance would weigh nearly 93.6 lb.

Toughness and Strength. The order of technological values of various woods does not coincide with that indicated by the strength values. Toughness and strength combined indicate this value, which is shown by the work U_R required to break a beam. Values of U_R (in.-lb per cu in.) are given in Table 1; for hickories and rock elm U_R varies from 24 to 40; ashes and elms, 14 to 24; oaks, black spruce and maple, 8 to 14; white spruce, firs, and other pines, below 8. This factor U_R does not vary directly with specific gravity. For instance, eucalyptus, a strong and heavy wood, is comparatively brittle and ranks low in rupture work.

Tensile Strength. The tensile strength of wood parallel to the fibers (or grain) exceeds all other strength values. At 15 percent moisture, values of tensile strength (pounds per square inch) according to Johnson, are: elm, 29,000; hickory, 32,000; larch, 19,000; longleaf pine, 17,300. Bovey gives 11,612 for Douglas fir, and Isaacs, 15,900. Tensile strength parallel to fiber is not important, for other stresses, as shear, or tension perpendicular to fiber, bending, or bearing, govern design. **Tensile strength perpendicular to the fibers** may be taken at 900 lb per sq in. for hardwoods and 250 lb per sq in. for conifers.

Shearing Strength. The shearing strength parallel to the fibers of small, clear, green specimens may be taken as approximately 1,100 lb per sq in. for hard woods and 600 lb per sq in. for conifers except the following: cypress, Douglas fir, red pine, tamarack, 800; and longleaf pine, 1,000. The values are increased from 50 to 100 percent by careful seasoning. For **shear in flexure**, see Table 2.

The Lowry process aims to secure a good penetration with comparatively small quantities of creosote. Air-dry timber only is treated. Creosote oil is forced into the timber without a preliminary vacuum until a large quantity of creosote is absorbed, usually stated as "treatment to a refusal." A quick final vacuum is then applied and a considerable amount of the injected oil is withdrawn. In standard railway-tie treatment, about 2½ gal of creosote remain in a 6 in. × 8 in. × 8 ft tie and relatively more for larger sizes.

The Rüping process is intended to secure a good penetration with comparatively small quantities of oil. Compressed air is first forced into the wood up to a pressure of 75 lb per sq in., after which creosote oil is forced in at a higher pressure until treatment to a refusal is obtained. A final vacuum aids the compressed air in driving out a considerable quantity of the injected oil. In standard practice, 1.8 to 2 gal are left in a 6 in. × 8 in. × 8 ft railway tie.

The boiling process is used chiefly for Douglas fir. The green timber is placed in the creosoting cylinder, which is then filled with creosote and heated under vacuum to a point slightly above the boiling point of water. This heating is maintained until practically no water comes out of the condenser. Pressure is then applied and the preservative is forced into the timber to the requisite amount.

In the **Card process**, the preserving liquid is made up of 15 to 20 percent of creosote or water-gas distillate or water-gas-tar solution and the remainder of a 3 to 5 percent solution of zinc chloride. The creosote and zinc chloride are mixed in a centrifugal pump, and the emulsion thus produced is forced into the timber under pressure.

The open-tank treatment includes a number of methods for treating timber, without pressure, with either zinc chloride or creosote. The timber is put in an open tank and heated in hot liquid for several hours. It is then quickly immersed in cold liquid. During the cooling process, a partial vacuum is produced in the wood and the preservative enters under atmospheric pressure. This process is adapted particularly to the treatment of small quantities of timber in localities where larger treating plants are not available (for details, see *U. S. Forest Service Circulars* 101, 104, 111, and 117).

The length of life obtained from treatment will vary with the preservative used, the kind of material treated, the type of use and the region in which used. According to the A.R.E.A. reports (vol. 20, p. 150, 1919), cross tie streated with zinc chloride may be expected to give double the length of life of untreated timber, regardless of where used. Creosoted ties may be expected to give 25 years or more service depending upon the kind of wood, the amount of mechanical protection given, and the traffic. Piling when treated with sufficient creosote will give 15 years or more. Bridge timbers treated with sufficient creosote will give 25 years or more. In all cases, the higher the initial retention of creosote per cubic foot, the greater the probability of long life.

Standard Grades and Sizes of Lumber

By agreement of lumber manufacturers, users, and the U. S. Forest Service, and under the auspices of the U. S. Department of Commerce, all softwood lumber products have been standardized as to grades, sizes, and nomenclature. This agreement is known as the **American Lumber Standards.** (For details, see Revised Simplified Practice Recommendation, No. 16, July 1, 1926, U. S. Dept. of Commerce. See also *Manual A.R.E.A.*, and *A.S.T.M. Standards*, 1939, Pt. II, pp. 494–512.) Lumber is classified into: **yard lumber** (less than 5 in. in thickness) for general building purposes; **structural timbers** (over 5 in. in thickness) for structural purposes; and **factory or shop lumber** intended to be cut up for use in further manufacture.

Yard lumber is classified into grades, either select (good appearance and finishing), or common (containing defects but of general utility). Select grades may be grades A to D depending on defects and use for certain purposes. Common grades may be grades 1 to 5 depending on defects and use for specific purposes.

The unit of lumber measurement is the board foot, a piece 12 × 12 × 1 in. A **standard** commercially-dry **yard board,** surfaced smooth on one or

tension face of a beam seriously influence its strength. Knots greater than 1½ in. diam diminish compression strength from 15 to 20 percent. They do not influence stiffness or the elastic limit of beams. For beams the range of stress between the elastic limit and modulus of rupture is seriously influenced by knots, crooked grain, and other defects. Checks and shakes both in green and seasoned timber bring about failure of beams by splitting at the ends under horizontal shear. When failure by shear occurs, however, the modulus of rupture is approximately equal to that of beams failing otherwise.

Tests of **large sizes in compression** show the following general ranges of ultimate strength (Lanza): white pine and spruce, 2,000 to 3,000; yellow pine, 3,500 to 5,500; white oak, 3,000 to 5,000.

References: Talbot, *Eng. Exp. Sta., Univ. Ill. Bull.* 41; McFarland, *Bull. Am. Ry. Eng. Assn.*, vol. 14, No. 149, Sept., 1912. For complete list of full-sized tests, see *Trans. Am. Ry. Eng. Assn.*, vol. 10, Part I, 1909.

Strength as Affected by Moisture Content. A comparison of the results of tests on air-seasoned material with those on green material shows that, in general, all the **mechanical properties** are **improved by seasoning.** Increase in strength is especially marked on small pieces free from defects. Increase in strength of wood fiber due to drying is, in the case of large timbers, largely offset by a weakening of the timber due to the formation of checks. If the moisture content of a seasoned timber is increased, it loses strength rapidly, and if thoroughly soaked with water will become slightly weaker than when green. On this account, it is not safe in practice to depend upon an increase in strength in timbers due to seasoning. When, however, large beams are seasoned with ordinary care, it is safe to assume that they will not at any time be weaker than they were when green.

Table 3. Relation of Moisture Content to Compressive Strength

(From Circular 108, U. S. Forest Service Compression parallel to grain)

Percentage of moisture in the wood	Relative maximum crushing strength compared to that of wood containing 2 percent of moisture		
	Red spruce	Longleaf pine	Douglas fir
2	1.000	1.000	1.000
4	0.926	0.894	0.929
6	(c)0.841	0.790	0.850
8	0.756	0.702	0.774
10	0.661	0.623	0.714
12	0.617	9.552	0.643
14	(b)0.554	0.468	0.589
16	0.505	0.431	0.535
18	0.463	0.377	0.494
20	0.426	(a)0.328	0.458
22	0.394	0.278	0.428
24	0.362	....	(a)0.398
26	0.335		
28	0.314		
30	0.292		
32	0.271		
34	0.255		

(a) Green. (b) Air-dry.
(c) Kiln-dry (approx).

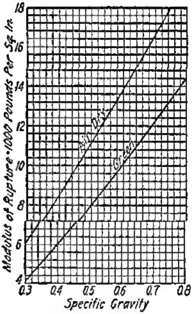

Fig. 1.—Modulus of Rupture of Wood.

is 4 in. wide. Cypress shingles are 16 in. long, and 5 butts measure 2 in. in thickness. A thousand shingles (4 to 5 bundles) will cover 100 to 150 sq ft of roof, according as laid from 4 to 6 in. to the weather. Shingles are usually laid to show 4 in. of their length, forming with 16 in. shingles a quadruple layer on a roof. The higher the grade of the shingles, the larger the weather face permissible. Cypress shingles are the most durable, followed by redwood, cedar, pine, and spruce (poor). Redwood shingles are the least inflammable. The weight of shingles (pounds per 1,000 shingles) is approximately as follows: cypress, 300; western white pine, 250; Washington cedar, 160 to 220; redwood (6 × 16 in., per bundle of 147), 42.5.

Cordwood. Firewood, pulpwood, and other small pieces of wood are usually bought and sold by the cord. A cord is equal to 128 cu ft of stacked wood. The standard cord is a stack of wood cut into 4 ft lengths, *i.e.*, 4 ft high and 8 ft long. A cord foot is one-eighth of a cord. The expression "surface feet" means the number of square feet measured on the side of the stack. In countries using the metric system, wood is piled in cubic meters (1 cu m = 0.274 cord). The actual volume of solid wood in a cord will depend, among other factors, on the form and size of the sticks, the amount of bark, whether split, and on the method of stacking (see Table 3). Ordinary conifers produce a greater amount of wood per cord than hardwoods, because of the smoother, straighter sticks. One cord of first-class split wood obtained from sound pieces 12 in. diam contains 102.4 cu ft of solid wood. Composed of inferior pieces having a diam of 6 in., a cord contains 97 cu ft of solid wood. While no absolute ratio can be given of the relation between cord measure and board measure, a fair average is 6 board-feet to each cubic foot.

Table 2. Contents in Feet (B. M.) of Joists, Scantlings, and Timbers

Size, inches	Length, feet									
	12	14	16	18	20	22	24	26	28	30
2 × 4	8	9	11	12	13	15	16	17	19	20
2 × 6	12	14	16	10	20	22	24	26	28	30
2 × 8	16	19	21	24	27	29	32	35	37	40
2 × 10	20	23	27	30	33	37	40	43	47	50
2 × 12	24	28	32	36	40	44	48	52	56	60
2 × 14	28	33	37	42	47	51	56	61	65	70
3 × 8	24	28	32	36	40	44	48	52	56	60
3 × 10	30	35	40	45	50	55	60	65	70	75
3 × 12	36	42	48	54	60	66	72	78	84	90
3 × 14	42	49	56	63	70	77	84	91	98	105
4 × 4	16	19	21	24	27	29	32	35	37	40
4 × 6	24	28	32	36	40	44	48	52	56	60
4 × 8	32	37	43	40	53	59	64	69	75	80
4 × 10	40	47	53	60	67	73	80	87	93	100
4 × 12	48	56	64	72	80	88	96	104	112	120
4 × 14	56	65	75	84	93	103	112	121	131	140
6 × 6	36	42	48	54	60	66	72	78	84	90
6 × 8	48	56	64	72	80	88	96	104	112	120
6 × 10	60	70	80	90	100	110	120	130	140	150
6 × 12	72	84	96	108	120	132	144	156	168	180
6 × 14	84	98	112	126	140	154	168	182	196	210
8 × 8	64	75	85	96	107	117	128	139	149	160
8 × 10	80	93	107	120	133	147	160	173	187	200
8 × 12	96	112	128	144	160	176	192	208	224	240
8 × 14	112	131	149	160	187	205	224	243	261	280
10 × 10	100	117	133	150	167	183	200	217	233	250
10 × 12	120	140	160	180	200	220	240	260	280	300
10 × 14	140	163	187	210	233	257	280	303	327	350
12 × 12	144	168	192	216	240	264	288	312	336	360
12 × 14	168	196	224	252	280	308	336	364	392	420
14 × 14	196	229	261	294	327	359	392	425	457	490

The relation between the specific gravity (dry) and the modulus of rupture of both air-dry and green wood as determined by the U. S. Forest Service is shown in Fig. 1. (Newlin, *Am. Lumberman*, Jan. 16, 1915.)

Determination of Working Stresses

(From *U. S. Dept. Agr. Misc. Pub.* 185)

Basic Stresses

The values listed in Table 4 afford a basis for the computation of design stresses. They are basic stresses for clear material used under such conditions of exposure that deterioration is not to be expected—*e.g.*, timbers that remain too dry to support the growth of decay. Except for stress in compression perpendicular to grain, they also apply to timbers that will remain too wet for decay to attack them. They are applicable to long-time loading and require modification (1) for the grade of timber used and (2) for the conditions of exposure to which it will be subjected in service. With certain exceptions, multiplication of the values in Table 4 by the strength ratio corresponding to the grade gives the design stress for material that will remain either dry or saturated.

Higher initial strength than would otherwise be required is desirable in timbers exposed to decay or other deterioration in order to offset the effects of deterioration. It may be provided by arbitrarily increasing the size of the timbers or by using lower design stresses. Table 5 includes values for three types of exposures.

Table 5. Average Ratios among Stresses for Different Exposures

Kind of stress	Type of exposure, percent		
	Continuously dry or continuously submerged	Occasionally wet but quickly dried	More or less continuously damp or wet
Stress in extreme fiber in bending	100	85	71
Stress in compression perpendicular to grain	100	70	58
Stress in compression parallel to grain	100	92	78
Stress in horizontal shear	100	100	100
Modulus of elasticity	100	100	100

The "occasionally wet but quickly dried" classification applies to timbers in outside locations but not in contact with the soil as in open-deck structures of railway trestles. In such instances, the material is subjected to wet or moist conditions at intervals but may be expected to dry quickly enough so that the decay hazard is only moderate. The "more or less continuously damp or wet" classification covers material, such as piling, sills, or dock timbers, where wet and drier conditions alternate, with the drying taking place slowly so that the average moisture condition is more favorable to decay.

No simple rule for modifying design stresses can cover all conditions of use. Factors to be considered include the extent to which the exposure favors decay, the expected life of the structure, the frequency and thoroughness of inspection, the original cost and cost of replacements, and the amount of sapwood permitted. If treated timbers are used, the efficiency of the treat-

Table 1. Strength of Small Specimens of Green and Air-seasoned Timber

(Average physical and mechanical properties, based on tests of small clear specimens 2 × 2 in. in cross section; bending span, 28 in. Mainly from Circular No. 213, U. S. Forest Service)

Species, common name	Moisture content, percent	Specific gravity based on volume when tested	Shrinkage in volume from green to oven-dry condition, percent	Static bending: Fiber stress at elastic limit, lb per sq in.	Static bending: Modulus of rupture, lb per sq in.	Static bending: Modulus of elasticity, 1,000 lb per sq in.	Static bending: Work to max load, in.-lb per cu in., U_R	Compression parallel to grain: Crushing strength, lb per sq in.	Compression perpendicular to grain: Stress at elastic limit, lb per sq in.	Hardness—load required to imbed a 0.444-in. ball to one-half its diam: End, lb	Hardness—load required to imbed a 0.444-in. ball to one-half its diam: Side, lb	Shearing strength, parallel to grain, lb per sq in.
Hardwoods												
Ash, black	11.6	.497	....	6310	11,620	1395	13.1	5590	893	1101	792	1660
Ash, white	10.8	.580	....	9860	15,665	1778	13.7	7600	1300	1870	1285	2023
Basswood	9.2	.345	....	5910	7310	1674	3.2	4770	557	445	364	1524
Beech	13.1	.620	....	8140	14,830	1830	16.4	6450	1185	1463	1217	1908
Birch, yellow	10.3	.624	....	13,360	19,400	2396	17.9	9560	1340	1542	1280	1428
Elm, rock	11.2	.630	....	8000	16,350	1755	20.4	7570	1603	1593	1257	2154
Elm, white	10.8	.469	....	6790	12,140	1504	13.4	5840	727	892	679	1447
Hickory, water	8.9	.652	....	11,750	20,200	2165	19.4	10,140	2208	....	....	1865
Hickory, shagbark	9.6	.730	....	12,400	22,300	2186	23.4	10,420	2385	....	....	2324
Maple, red	12.1	.548	....	8650	13,420	1761	12.4	6610	1291	1531	1024	1789
Maple, sugar	11.4	.612	....	9660	15,260	1878	13.1	7840	1367	1925	1344	2250
Oak, red	11.2	.661	....	8790	14,080	1915	12.9	7270	1100	1528	1359	1796
Oak, white	11.6	.688	....	8090	15,190	1779	13.7	7750	1345	1577	1425	2141
Sycamore	11.5	.484	....	5680	9350	1365	6.4	5340	885	957	814	1554
Tupelo	12.8	.510	....	6310	8970	1286	5.4	5850	865	1244	847	1577
Softwoods												
Arborvitae	55.0	.293	7.0	2600	4250	643	5.7	1990	288	321	225	617
Cedar, incense	80.0	.363	....	3950	6040	754	....	3030	518	....	....	613
Cypress, bald	79.0	.452	11.5	4430	7110	1378	5.1	3960	548	460	355	836
Fir: Douglas	32.0	.418	10.9	3570	6340	1242	6.6	2920	427	415	399	853
White	156.0	.350	10.2	3880	5970	1131	5.2	2800	445	381	322	742
Hemlock	129.0	.340	9.2	3410	5770	917	6.6	2750	420	463	354	790
Pine: Lodgepol	44.0	.370	11.3	3080	5130	1015	5.1	2530	364	288	307	672
Longleaf	63.0	.528	12.8	5090	8630	1662	8.1	4280	491	574	502	1060
Shortleaf	52.0	.477	....	4360	7710	1395	....	3570	400	....	....	746
Sugar	123.0	.360	8.4	3330	5270	966	5.0	2600	353	334	307	702
Western yellow	98.0	.353	9.2	2660	4760	879	4.9	2220	342	310	311	644
White	74.0	.363	7.8	341[illegible]	5310	1073	5.9	2720	314	304	294	649
Redwood	69.0	.366	....	5020	7460	1101	....	4160	618	....	....	....
Spruce: Engelmann	45.0	.325	10.5	2740	4550	866	4.8	2170	302	272	253	607
Red	31.0	.396	....	3440	5820	1143	6.0	2920	322	....	....	754
White	41.0	.318	....	3160	5200	968	6.6	1940	262	....	....	645
Tamarack	52.0	.491	13.6	4200	7170	1236	7.2	3480	480	401	380	883

NOTE: The hardwoods listed above were all dried. The softwoods are green.

MISCELLANEOUS NON-METALLIC MATERIALS

BY

E. C. CROCKER

(With the Assistance of Various Members of the Staff of Arthur D. Little Inc.)

ABRASIVES

REFERENCES: "Abrasives: Their History and Development," The Norton Co. Searle, "Manufacture and Use of Abrasive Materials," Pitman. Jacobs, "Abrasives Handbook," Penton. "Boron Carbide," The Norton Co. "Abrasive Materials," Annual review in "Minerals Yearbook," U. S. Bureau of Mines.

Natural Abrasives

Diamond of the **bort** variety, crushed and graded into usable sizes and bonded with synthetic resinous and metal powders, finds use in grinding tungsten- and tantalum-carbide cutting tools.

Corundum is a mineral composed essentially of crystallized alumina (93 to 97 percent Al_2O_3) with a density of 3.68 to 3.94, average coefficient of expansion of about 0.0000045 per deg F, and a hardness (Mohs scale) of 9. When of high purity, it is insoluble in acids and melts at close to 3700 F. The present commercial source for corundum is South Africa (Northern Transvaal). Although it has been largely replaced by manufactured abrasives, it is still used in the grinding of lenses and for certain types of rough grinding of steel.

Emery, a cheap and impure form of corundum which has been used for centuries as an abrasive, has been largely superseded by manufactured corundum for grinding. It is still used to some extent in the metal- and glass-polishing trades. The principal deposits of commercial emery are located in Asia Minor and on the island of Naxos.

Garnet. Certain deposits of garnet having a hardness between quartz and corundum are used in the manufacture of abrasive paper. **Quartz** is also largely used for this purpose. In the electrical trades, quartz colored to imitate garnet is sometimes substituted. Garnet costs about twice as much as quartz and generally lasts proportionately longer. Silicon carbide is superior to garnet for leather working.

Grindstones and pulpstones are quarried directly and are generally made from sandstone.

Buhrstones and millstones are generally made from cellular quartz. Chasers (or stones running on edge) are also made from the same mineral.

Oilstones. The great majority of oilstones are quarried in Arkansas and are known as "Arkansas" and "Washita" stones. The **Arkansas stone** is a true novaculite, homogeneous, gritty, and of a fine, siliceous composition. The hard variety is used for sharpening tools requiring a very fine edge, such as those of surgeons, engravers, and dentists. The soft variety is more porous and coarser and is used for less careful work.

Washita stone is less dense, more porous, and of the same general composition. The chief use is for whetstones and for coarser tools.

Pumice, of volcanic origin, is extensively used in leather, felt, and woolen industries and in the manufacture of polish for wood, metal, and stone. An artificial pumice is made from sand and clay in five grades of hardness, grain, and fineness.

Infusorial earth or **tripoli** resembles chalk or clay in physical properties. It can be distinguished by absence of effervescence with acid, is generally white or gray in color, but may be brown or even black. Owing to its porosity, it is very absorptive. It is used extensively in polishing powders, scouring soaps, etc., and, on account of its porous structure, in the manufacture of dynamite as a holder of nitroglycerin, also as a non-conductor for steam pipes and as a filtering medium. It is also known as **kieselguhr, fossil flour,** or **diatomaceous earth.**

The strength of large-sized pieces of structural timber containing knots and other defects is usually not more than two-thirds of the values in Table 1. Table 2 (abstracted from publications of Forest Service) gives results of tests. The strength of small specimens cut from the beams after test is included. The strength of timbers of the same apparent grade varies greatly. The average of the lowest 10 percent group of a series of values will be from 60 to 70 percent of the general average value for the group. The range of individual values, low to high, may be from 50 to 150 percent of the average. Bulletin 108, Forest Service, U. S. Department of Agriculture (Cline and Heim), is an authoritative discussion of tests of large sizes of timber and the effects of defects upon strength. See also Newlin and Johagan, "Tests of Large Timber Columns," *U. S. Dept. Agr. Bull.* 167.

Influence of Knots and Other Defects on Strength. Forest Service tests indicate the following relations: Knots which interrupt the grain in the

Table 2. Strength of Air-seasoned Timber, Structural Sizes
(U. S. Forest Service)

Species	Density of oven-dry wood, lb per cu ft	Number of rings per in.	Bending				Compression				Shear
							Parallel to fibers			Perp. to fibers	
							Crushing strength				
			Fiber stress at elastic limit, lb per sq in.	Modulus of rupture, lb per sq in.	Modulus of elasticity, 1000 lb per sq in.	Horizontal shear,* lb per sq in.	At elastic limit, lb per sq in.	At maximum load, lb per sq in.	Modulus of elasticity, 1000 lb per sq in.	Crushing strength at elastic limit, lb per sq in.	Shearing strength, lb per sq in.
Longleaf pine:											
Structural sizes	39	15.4	3,691	5,749	1,705	272	3,480	4,800	......	572	
Small specimens	....	....	6,750	11,520	1,740	......	......	......	......	......	934
Douglas fir:											
Structural sizes	28	13.1	4,563	6,372	1,549	221	3,271	4,258	1,038	639	
Small specimens	....	....	6,686	10,378	1,695	......	3,842	5,002	1,084	......	822
Shortleaf pine:											
Structural sizes	31	12.4	4,675	6,573	1,726	364	4,070	6,030	1,951	796	
Small specimens	....	....	7,780	12,120	1,792	......	......	6,380	......	926	1,135
Western larch:											
Structural sizes	29	22.7	3,503	5,856	1,487	340	......	5,746	......	597	
Small specimens	....	....	5,880	10,254	1,564	......	......	5,934	......	......	905
Loblolly pine:											
Structural sizes	32	6.3	3,504	6,118	1,487	434	3,011	4,292	1,206	655	
Small specimens	....	....	5,170	9,400	1,467	......	......	5,547	......	......	1,115
Tamarack:											
Structural sizes	31	12.7	3,730	5,498	1,341	299	3,349	4,320	1,351	......	
Small specimens	....	....	7,630	13,080	1,620	......	......	4,790	......	697	879
West. hemlock:											
Structural sizes	28	17.7	4,398	6,420	1,737	307	4,840	5,814	2,140	473	
Small specimens	....	....	6,333	10,369	1,666	......	4,560	5,403	1,923	......	924
Redwood:											
Structural sizes	22	20.1	3,442	3,891	890	......	......	4,276	......	525	
Small specimens	....	....	4,777	7,798	1,146	......	......	5,119	......	564	671
Norway pine:											
Structural sizes	27	10.0	4,069	6,054	1,418	278	3,047	4,228	1,367	......	
Small specimens	....	....	5,280	8,470	1,158	......	......	7,550	......	924	1,154

* Only those pieces which failed first by horizontal shear are included in this column.

is not possible to obtain as wide a range of grades as by the vitrified method, but the time required in manufacture is relatively short. The process is often used to facilitate quick delivery, and very large diameter wheels can be made with fewer manufacturing hazards. Silicate bonded wheels and shapes give best results on such work as wet tool grinding, knife sharpening, and surface grinding.

Organic Bonded Wheels. Organic bonds can be used for large wheels, and are equally well adapted to the manufacture of very thin wheels because of the flexibility of such wheels as compared with vitrified and silicate process wheels. Organic bonded wheels possess high resistance to centrifugal force and are operated at relatively high speeds. There are three distinct types in the group.

The **shellac process** consists of mixing abrasive grains with shellac, heating the mass until the shellac is viscous, stirring, cooling, crushing, forming in molds, and reheating sufficiently to permit the shellac to set firmly upon cooling. Wheels made by this process are popular for light grinding operations, such as saw gumming, roll grinding, ball race and cam grinding, and in the cutlery trade.

In the **rubber process** the bond is hard rubber. The initial mixture of grain, rubber, and sulphur (and such special ingredients as accelerators, fillers, and softeners) may be obtained by rolling or other methods. Having formed the wheel, the desired hardness may then be developed by vulcanization. Wheels can be made in a wide variety of grain combinations and grades and have a high factor of safety as regards resistance to breakage in service. Wheels made by the rubber process may be used for practically every class of grinding from gear teeth to the snagging of large castings.

The **synthetic resinoid process** is a relatively recent development which is finding an important application in grinding and is revolutionizing many of the older theories regarding grinding wheels and grinding practice. The general practice is to form the wheel by the cold-press process using an especially prepared resinoid. After heating, the resultant bond is an insoluble infusible product of notable strength and resiliency. The resinoid bonded wheel gives promise of largely supplanting the other organic bonded wheels for light grinding. It is also being adapted to the heavy snagging (or rough grinding) of castings and steel billets.

The ordinary abrasive wheel is operated at speeds of 4,000 to 16,000 surface fpm. Abrasive wheels are trued up by a diamond set in the end of a soft metal rod and held against the face of the wheel while revolving.

The **grain** of a wheel is determined by the size or combination of sizes of the abrasive grain used. The Grinding Wheel Manufacturers Assoc. of the United States and Canada has standardized sizes under the arbitrarily assigned numbers 6, 8, 10, 12, 14, 16, 20, 24, 30, 36, 46, 54, 60, 70, 80, 90, 100, 120, 150, 180, 200, 220, and 240. The finer grades known as **flour** are designated as 280, 320, 400, 500, 600, 800, 900, or as F, FF, FFF, and XF.

The **grade** is determined by the hardness of the wheel. The wheel from which the particles are easily broken, causing it to wear away rapidly, is called soft, and one that is able to retain its particles longer is called hard.

The following table gives the most generally used **grade designations** and letter for abrasive wheels:

Grade designation	Very soft	Soft	Medium soft	Medium
	A, B, C,	D, E, F, G	H, I, J, K	L, M, N, O
Grade designation		Medium hard	Hard	Very hard
		P, Q, R, S	T, U, V, W, X	Y, Z

Seasoned wood in small pieces is much stronger than green wood. The relation of strength to moisture content depends upon species, whether softwood or hardwood, and upon the kind of loading.

Table 3 shows the reduction in compressive strength of several woods due to increasing the moisture content from 2 percent to the amount indicated. Thus, red spruce with 16 percent of moisture has but half (0.505) the strength it has with 2 percent of moisture. The latter moisture content could not be maintained in use. The table also serves to permit calculations of change in strength for other changes in moisture content. See also Wilson, *U. S. Dept. Agr. Bull.* 282.

Table 4. Basic Stresses for Clear Timber

(All values are in pounds per square inch)

Species	Extreme fiber in bending	Compression perpendicular to grain	Compression parallel to grain $L/d = 10$	Maximum horizontal shear	Modulus of elasticity
1	2	3	4	5	6
Softwoods:					
Cedar, Alaska	1,466	250	1,066	120	1,200,000
Cedar, northern and southern white	1,000	175	733	93	800,000
Cedar, Port Orford	1,466	250	1,200	120	1,200,000
Cedar, western red	1,200	200	933	106	1,000,000
Cypress, southern	1,733	300	1,466	133	1,200,000
Douglas fir, coast region	2,000	325	1,466	120	1,600,000
Douglas fir, coast region, close-grained	2,133	345	1,565	120	1,600,000
Douglas fir, Rocky Mountain region	1,466	275	1,066	113	1,200,000
Douglas fir, dense, all regions	2,333	380	1,711	140	1,600,000
Fir, commercial white	1,466	300	933	93	1,100,000
Fir, balsam	1,200	150	933	93	1,000,000
Hemlock, eastern	1,466	300	933	93	1,100,000
Hemlock, western	1,733	300	1,200	100	1,400,000
Pine, western white, northern white, sugar, and ponderosa	1,200	250	1,000	113	1,000,000
Pine, Norway	1,466	300	1,066	113	1,200,000
Pine, southern yellow	2,000	325	1,466	146	1,600,000
Pine, southern yellow, dense	2,333	380	1,711	171	1,600,000
Redwood	1,600	250	1,333	93	1,200,000
Redwood, close-grained	1,707	267	1,422	93	1,200,000
Spruce, Engelmann	1,000	175	800	93	800,000
Spruce, red, white, and Sitka	1,466	250	1,066	113	1,200,000
Tamarack	1,600	300	1,333	126	1,300,000
Hardwoods:					
Ash, commercial white	1,866	500	1,466	167	1,500,000
Ash, black	1,333	300	866	120	1,100,000
Beech	2,000	500	1,600	167	1,600,000
Birch, sweet and yellow	2,000	500	1,600	167	1,600,000
Chestnut	1,266	300	1,066	120	1,000,000
Elm, rock	2,000	500	1,600	167	1,300,000
Elm, American and slippery	1,466	250	1,066	133	1,200,000
Gum, black and red	1,466	300	1,066	133	1,200,000
Hickory, true and pecan	2,533	600	2,000	187	1,800,000
Maple, sugar and black	2,000	500	1,600	167	1,600,000
Oak, commercial red and white	1,866	500	1,333	167	1,500,000
Tupelo	1,466	300	1,066	133	1,200,000

(Not for use in design but for determining working stresses according to the grade of timber and the condition of exposure)

Fish glue is prepared by leaching heads, bones, and other fish scrap. It is sold in liquid form in various concentrations for general gluing work. It is unusually hygroscopic and slow drying. Its greatest importance is in the engraving field.

Casein is obtained by precipitation from cow's milk. It is sold in powdered form and can be brought into solution with ammonia or fixed alkali by heating to not over 150 F. Recently it has become available in cold-water-soluble form. Its principal uses are in laying wood veneer, cork floors, setting brush bristles, etc., where strength and moisture-resistance are desirable and strong alkalinity is no objection. Casein has a tendency to deteriorate unless strongly preserved even after the film has dried out thoroughly. Formaldehyde should be avoided as a preservative because it tends to diminish the solubility of casein.

Vegetable glue is a general classification for all adhesives produced by treating starches or dextrins. The most important sources are wheat, corn, tapioca, sago, and potato starch. These starches lend themselves to various treatments such as cooking with alkali and other chemicals to produce stable liquid solutions. Another method of converting starch into soluble adhesives is by the use of enzymes. In certain cases, an acid treatment is resorted to. Starches can be treated by roasting to produce dextrins from which stable liquid vegetable glues can be produced.

Vegetable adhesives are increasing in use on account of their low cost and convenience, and because they are used cold. Principal uses are in the manufacture of paper bags, paper boxes, envelopes, shoes, and similar lines. The starches or dextrins can be prepared by the consumer. The principal sources of dextrins in the United States are corn, tapioca, and sago. Corn dextrins are not cold-water soluble whereas sago and tapioca are.

Rubber cements are available in two types, the latex or compounded-latex type and the solvent rubber-cement type. Owing to its water-resistance and excellent flexibility, latex is becoming more and more important as an adhesive. Ammonia is used as a stabilizer to prevent coagulation, but deammoniated types have appeared without the objectionable ammonia odor. Latex can be compounded with some glues and pastes to lend them greater adhesive powers.

Solvent rubber cements are produced by dissolving freshly-milled crepe rubber in naphtha with or without the addition of benzol or carbon tetrachloride. This produces a tacky very viscous adhesive which is unusually fast drying but not particularly strong. An objection to its industrial use is its inflammability.

Natural gums are of two types, some being soluble in water, others insoluble. The most important soluble gum is **gum arabic**, from which glue can be produced which is unusually light in color and clear. The insoluble gums, of which **gum copal** and **gum damar** are most generally used, are brought into solution with solvents such as alcohol to produce waterproof adhesives.

Silicate of soda, commonly known as **water glass**, is made by fusing quartz sand with potash or sodium hydrate. It is a heavy oil-like liquid containing 33 to 60 percent sodium silicate and must be protected from air to prevent drying out, or gellation. It is produced in various strengths and used as a cheap adhesive and also for fireproofing cloth and wood. It also finds a use as an ingredient in artificial stone cements.

Miscellaneous. Ordinary **putty** is made from whiting and pure raw linseed oil. Glazier's putty generally contains some varnish in addition. Linseed-oil varnishes mixed with litharge or red lead and ocher are extensively used as a hard-drying putty. **Mastic cement** is composed of powdered limestone, sand, and litharge, mixed with linseed-oil varnish. **Litharge and glycerin** produce a very hard-drying cement, resistant to alkalies, acids, and petroleum solvents. Water glass and chalk produce a hard waterproof mass, as do also water glass and Portland cement. **Magnesia cement** consists of calcined magnesia in a 30 percent solution of chloride of magnesium and hardens to a stonelike mass. **Marine glue** is made from pitch and oils.

ALCOHOL

(See also p. 296)

Grain alcohol (ethyl alcohol) is made by the distillation of fermented sugar solutions derived either from grain, molasses, or fruit sirups, or by the treatment of wood with sulphurous acid. It is sold in the United States

ment and the loss of strength should be taken into account.

The Grading of Structural Timbers

Because of variations in the quality of the wood, the number and size of knots, the degree of cross grain, and the extent of shakes and checks, one timber of a species may be several times as strong as another. Without grading of timbers, working stresses must be adapted to the strength of the poorer pieces of each species. With grading, each piece can be fully loaded and economy and dependability promoted.

In the *U. S. Dept. Agr. Misc. Publ.* 185, there are given proposed specification requirements for grading which take account of the quality of the wood, decay, slope of grain, size of knots in relation to timber dimension, location of knots, size of shakes, and other defects. Different specifications are suggested (1) for beams, girders, stringers, etc.; (2) for joists, rafters, planks, flooring, etc.; and (3) for posts, columns, struts, caps, sills, etc. The influence of these various defects on the strength of the timber is given in the form of **strength ratios** which give the ratio of strength to be anticipated as a percentage of the strength of clear green material.

The major **lumber associations** have graded the structural lumber with which they deal on the basis of the magnitude of the defects, using such terms as "prime," "select," "heart," "merchantable," "common," "No. 1," etc., to indicate the grades. They publish strength ratios for these various grades. A selection of their grades with corresponding strength-ratios is contained in the *Misc. Pub.* 185 already referred to. Further information can be obtained from the California Redwood Association, San Francisco, Calif.; the Northern Hemlock and Hardwood Manufacturers' Association, Oshkosh, Wis.; the Southern Pine Association, New Orleans, La.; the Southern Cypress Manufacturers' Association, Jacksonville, Fla.; the West Coast Lumbermen's Association, Seattle, Wash.; and other regional lumber associations. See also "Wood Structural Design Data," National Lumber Manufacturers Association, Washington, D. C.

The **working stresses** as determined from Tables 4 and 5 must be multiplied by the strength ratios of the lumber associations in order to find the permissible working stresses for lumber of any stated grade.

show no cracking when bent around a 1 in. circular mandrel, grain side outside. Under the piping test, a piece of belting bent 180 deg around a 3 in. mandrel with the grain surface in shall show no signs of coarse wrinkling or piping on the grain surface.

The United States Navy specifications require oak-tanned leather cut entirely from the center or back of the hide and free from any loading. Single belts must show an average tensile **strength** of 4,000 lb per sq in.; double belts a tensile strength of 3,600 lb. When subjected to a stress of 2,250 lb per sq in. for 1 hr, single belts should show not more than 13.5 percent elongation and double belts not more than 12.5 percent. The modulus of elasticity in tension varies from 17,800 lb per sq in. for new belts to 32,000 lb for old. The proportional elastic limit is about 2,300 lb per sq in. The strength of chrome leather varies from 8,500 to 12,900 lb per sq in.

Care of Leather Belting. A harsh dry belt should be dressed by first removing the surface dirt with a coarse cloth and then applying the dressing, a little on the pulley side and more on the outside. Only as much dressing as the leather will readily absorb should be used and preferably when the belt is idle. Neatsfoot oil and castor oil make very good dressings. Surface dressings such as rosin should be avoided, except as first aid in an emergency, for they seriously shorten the life of the belt.

Commercial sizes of leather belting as adopted by the Leather Belting Asso. range from ½ to 72 in. in width, the width increasing by ⅛ in. up to 1 in., by ¼ in. up to 4 in., then by ½ in. to 7 in., and above that by inches.

Rubber belts are made by saturating a strong woven duck belt with rubber and subsequently vulcanizing. Their advantages over leather belts are claimed to be uniformity in width and thickness, durability when exposed to steam and dampness, great tensile **strength**, and grip on pulley. With proper weight of duck, a three- or four-ply rubber belt is claimed to equal a single, and a five- or six-ply to a double leather belt. Rubber belts are cheaper than leather or other belts suitable for the same kind of work and are of particular value in damp locations or under unusual temperature conditions.

Rubber belts should never be treated with animal oils or greases, as these substances rapidly destroy their life. Good results are said to be obtained by painting with a mixture of equal parts of red lead, black lead, French yellow, and litharge mixed with boiled linseed oil and sufficient japan to cause quick drying. This produces a smooth polished surface and, if the belt should slip, the side next the pulley can be slightly moistened with boiled linseed oil. Commercial sizes run from 1 to 60 in., increasing by ⅛ in. up to 2 in., by ¼ in. to 4 in., and then by inches. Special seamless and endless belts can be obtained.

Cotton belts, when woven, have a tensile strength of about 5,000 lb per sq in.; when sewed, from 6,500 to 7,500 lb, per sq in.

BRICK

REFERENCES: "Brick Engineering," Vols. I, II and III, Brick Manufacturers' Assoc. of America; "Brick Structures," Brick Manufacturers' Assoc. of America; Stang, Parsons, and McBurney, "Compressive Strength of Clay Brick Walls," *B. of S. Research Paper* 108. A.S.T.M. Specifications Covering Paving Brick, Building Brick, Sewer Brick and Fire Brick.

Building brick are classified as either common brick or face brick depending on whether or not they have been artificially colored or textured.

Manufactured Abrasives

Manufactured abrasives, although more expensive than the natural, have largely superseded them commercially, owing to a much closer control of chemical composition and crystal structure, thus enabling the maintenance of uniform strength properties or "temper" in the manufactured abrasives.

Crystalline alumina consists essentially of the same mineral as corundum, but the physical properties such as crystal structure, size, and shape of grain are so controlled as to produce the most desirable abrasives for specific types of grinding. The method of cooling the pig of ore and the percentages of impurities present (TiO_2, Fe_2O_3, and SiO_2), although usually less than 5 percent total, greatly influence the properties of the product. Crystalline alumina has a concboidal fracture, and the grains, when crushed or broken, possess sharp cutting points and edges. Manufactured corundum, particularly the fines, is also an excellent refractory material for electric-furnace parts, laboratory ware, and many special industrial installations. As an abrasive it is best adapted to the grinding of high-tensile-strength materials such as steel and annealed malleable iron. Manufactured corundum is available under several trade names such as **Borolon** (Abrasive Co.), **Aloxite** (Carborundum Co.), **Lionite** (General Abrasive Co.), and **Alundum** (Norton Co.). A product made by fusing chemically purified alumina (Al_2O_3) is not so tough as the material of 95 percent purity but is better adapted to certain operations such as precision grinding of sensitive steels.

Silicon carbide (SiC) (see p. 730) corresponding to the mineral moissanite has been found in meteors. The hardness is between that of corundum and diamond; the specific gravity is 3.2. It is insoluble in acid and infusible. It decomposes above 4060 F. It is manufactured by fusing together coke and sand in an electric furnace of the resistance type. Sawdust is used also in the batch and burns away leaving passages for the carbon monoxide to escape. The grains are characterized by great brittleness. Abrasives of silicon carbide are best adapted to the grinding of low-tensile-strength materials such as cast iron, brass, bronze, marble, concrete, stone, and glass. It is available under several trade names such as **Carborundum** (Carborundum Co.), **Carbolon** (Exolon Co.), and **Crystolon** (Norton Co.).

Boron carbide (B_4C), a recent addition to commercial manufactured abrasives, has a scratch hardness greater than that of silicon carbide. It is available under the trade name of **Norbide** (Norton Co.) and is being used in loose grain form for the lapping of tungsten- and tantalum-carbide-type tools. In the form of molded shapes, it is used for sandblast nozzles, wire-drawing dies, bearing surfaces for gages, etc.

Grinding Wheels

Vitrified Process. In wheels, segments, and other abrasive shapes of this type, the abrasive grains are bonded with a glass obtained by mixing the grains with such materials as clays and feldspars in various proportions, forming the mass, drying, and subjecting to heat. It is possible to manufacture wheels as large as 60 in. diam by this process, and even larger wheels may be obtained by building up with segments. Most of the grinding wheels and shapes (segments, cylinders, bricks, etc.) now manufactured are of the vitrified type and are quite satisfactory for general grinding operations.

Silicate Process. Wheels and shapes of the silicate type are manufactured by mixing the grain with sodium silicate (water glass) and fillers that are more or less inert, forming the mass, and baking at a moderate temperature. It

The common requirements are as follows: size, $8\frac{1}{4} \times 2\frac{1}{4} \times 4$, $8\frac{1}{4} \times 3 \times 3\frac{1}{4}$, $8\frac{1}{4} \times 3 \times 4$, with permissible variations of $\frac{1}{8}$ in. in either transverse dimension, and $\frac{1}{4}$ in. in length; loss in rattler test, 22 to 26 percent of original weight.

Although brick have always been used in construction, their use has been limited, until recently, to resisting compressive-type loadings. By adding steel in the mortar joints to take care of tensile stresses, **reinforced brick masonry** extends the use of brick masonry to additional types of building construction such as floor slabs. Reinforced brick masonry behaves like a homogeneous material within its appropriate range of application.

Sand-lime brick are made from a mixture of sand and lime, molded under pressure and cured under steam at 200 F. They are usually a light gray in color and are used primarily for backing brick and for interior facing.

Cement brick are made from a mixture of cement and sand, manufactured in the same manner as sand-lime brick. In addition to their use as backing brick, they are also used below grade for manholes and catch basins where there is no danger of attack from acid or alkaline conditions.

Firebrick. (See p. 728.)

Measuring the amount of **brick required** is done in one of two ways, depending on the types of brick to be used. Where a common brick is to be used throughout the wall, the superficial area in square feet is determined and multiplied by 6(12)[18] for a 4(8)[12] in. wall; this is for masonry joints $\frac{1}{4}$ in. thick. For thinner joints, a slightly larger number of brick are required. Where a face brick is being used with common brick for the backing of a wall, the total number of brick required is determined as above and then the number of face brick is found and subtracted from the total to determine the number of backing brick. For common bond, using a row of headers for every sixth course, the number of face brick required per square foot of wall area is 7. To determine the face brick required for various bonds, increase the number per square foot of area over 6 as follows; for English bond, 50; for Flemish bond, $33\frac{1}{3}$; for Flemish cross bond, $16\frac{2}{3}$; for garden wall bond, 25; for running header bond, 100; for common bond, $16\frac{2}{3}$ percent.

CELLULOID

Celluloid (xylonite, pyralin) is a thermoplastic mixture of nitrocellulose (pyroxylin) and camphor. This material, with the addition of alcohol and pigments or dyes if desired, is pressed or bent into shape in a heated mold. Sp gr is approximately 1.35; easily deformable when warm, but stiff when cold, and about as strong as the softer woods. Modulus of elasticity varies from 200,000 to 500,000 lb per sq in. Chemically unstable at higher temperatures, but stabilized by various admixtures. Very inflammable unless fireproofing agents or other materials are added in large quantities.

CLEANSING MATERIALS

Soap is made by chemically combining fats with alkalies. Most soaps are made by the use of caustic soda; only soft soaps, liquid soaps, and shaving soaps are made by replacing caustic soda in whole or part by caustic potash. Cocoanut oil gives a hard but very soluble soap having a quick, abundant lather, and, therefore, soaps made from this material alone are advantageous for use with hard water. Soaps made from palm oil have a good lather and are readily soluble. Beef tallow is by far the most important material, and the usual soaps on the market contain more tallow than any other material. Olive oil produces a rather soft-bodied soap which becomes firm and hard on aging, of fair lathering properties, and rather slimy feel. Drying and semi-drying oils are usually used only in small amounts, since in neutral soaps

Abrasive Paper

Flint, garnet, emery, corundum, and the artificial abrasives are all used on paper and cloth fastened by glue or some special adhesive. The abrasive papers are used chiefly for soft materials, such as wood, and the abrasive cloths are used for metal work.

Crushed steel is made by heating high-grade crucible steel to nearly white heat and then quenching in a bath of cold water. The fragments thus produced are crushed to particles varying from fine powder up to $\frac{1}{16}$ in. diam and are classified as diamond crushed steel, diamond steel, emery, and steelite. Chief use in the stone, brick, glass, and metal trades.

Rouge and crocus are finely powdered oxide of iron used for buffing and polishing. Rouge is the red oxide, crocus is purple.

ADHESIVES

REFERENCES: J. A. Taggart, "The Glue Book," Inland Printer Co. Bogue, "Chemistry and Technology of Gelatine and Glue," McGraw-Hill. Sherer, "Casein," Van Nostrand. Dulac, "Industrial Cold Adhesives," Lippincott. Standish, "Cements, Pastes, Glues and Gums," Reinhold.

Glue is generally divided into animal glue, fish glue, casein, vegetable glue, latex and rubber cements, natural gums, silicate of soda, miscellaneous.

Animal glue is made by boiling out hides, skins, bones, and sinews of various animals, principally cattle and rabbits, and drying the resulting liquor. It is obtainable commercially in sheets, strips, flakes, ground, and powdered.

The modern method for **grading glue** has been set up and is controlled by The American Glue Manufacturers Assoc. The important properties of glues are viscosity and jelly strength. The viscosity is tested by timing the flow of a definite concentration of glue in water at a set temperature through a pipette. The jelly strength represents the tenacity of a solution of glue that has been allowed to set by cooling to form a jell; it is measured according to the **Bloom Jellometer** and expressed in millipoises. Practically all animal glue bought today is bought in specifications expressed in seconds of viscosity and millipoises of jelly strength. These two properties are most important in the use of the glues and bear a very definite relation to its quality. The higher the jelly and viscosity, the better the grade of glue. Bone glues are usually low in viscosity and jelly strength as compared with hide glues and gelatines.

Undissolved glues will retain their properties indefinitely and will not decay. When brought into solution with water they should be preserved with phenol or a similar powerful preservative. Formaldehyde must be avoided as it tends to reduce the solubility. Good animal glues are neutral or only very slightly acid. The better grades when made up in sheet form will bend without breaking and when broken will show a splintery edge. A clean fracture indicates brittle low-grade glue.

In dissolving, glue should always be soaked in cold water and then warmed. It is of great importance to employ no more heat than necessary to assure complete solution. It should never be heated above 150 F; jacket heat or double boilers should be used but never live steam. The "Glue Book," by Taggart, gives the following valuable suggestions regarding selection:

Wood joints: High-test hide glues. *Veneers:* Moderately high-test mixtures of bone and sinew or, better, bone and hide. *Sizing:* Glue free from grease and foam. *Paper boxes:* Quick setting glues for setting up, low-test glues for covering. *Belting and leather:* High-test glues preferred. *Bookbinding:* For covers, low-grade bone glue; for rounding and backing, high-grade hide glue. For *emery purposes:* Very high-grade and pure glue.

From animal glues are prepared (1) *flexible glue* consisting of a mixture of glue, glycerin, or other plasticizers and water and (2) *liquid animal glues* by treating the raw glue so as to prevent it from jelling. Flexible glue is used principally in the paper-box and bookbinding trade; liquid glues are used for general gluing work.

fittings such as wormline, ratline and spunyarn, also marline and roundline used in serving rigging.

In making cordage, the fiber is opened, run into a sliver (a continuous ribbon of fiber laying parallel and without twist), spun into a yarn, formed *into a strand*, and *laid into a rope*. *Spinning* is twisting the fibers into a continuous length of the size required and with the proper number of twists to the foot. *Forming* is twisting the number of yarns required to make the size of the strand, the twist in the strand opposite to the twist in the yarn.

Ropes are plain laid or cable laid. **A plain-laid rope** may be made of three, four, or six strands twisted together in the opposite direction to the twist in the strand. **Cable-laid** or water-laid rope consists of three plain-laid *three-strand ropes* twisted together in the opposite direction to the twist in the rope. They can be made four or six strand, but such rope is seldom required. Such ropes have approximately 65 percent of the strength of a plain-laid rope of corresponding size. They have greater elasticity and present much greater resistance to abrasive wear and water absorption than plain-laid ropes.

The term **lay** is used to designate the length of one turn in the rope measured parallel to the axis and is usually expressed as hard, medium hard, medium soft, and soft lay. Examples of special lays are yacht rope, bolt rope, sail-maker's lay, coal falls, and transmission lay. A yarn, strand, or rope has S (or left) twist if, when held in a vertical position, the spirals conform in slope to the central portion of the letter S, and Z (or right) twist if the spirals conform in slope to the central portion of the letter Z.

A common-laid rope will have the strength required by the U. S. Government Federal specifications (see below). A soft-laid rope will have less elongation, greater tensile strength, and less resistance to abrasive wear than a hard-laid rope.

Ropes made of selected yarns or bolt-rope stock may be required for severe work. Such rope will exceed the tensile strength required by Federal

Weight and Strength of Different Sizes of Manila Rope, Specification Values

(From U. S. Government Specification TR601A, dated Nov. 26, 1935, and formulated jointly by cordage manufacturers and Government representatives)

Approx diam, in.	Circumference, in.	Max net weight, lb per ft	Min breaking strength, lb	Approx diam, in.	Circumference, in.	Max net weight, lb per ft	Min breaking strength, lb	Approx diam, in.	Circumference, in.	Max net weight, lb per ft	Min breaking strength, lb
3/16	5/8	0.015	450	13/16	2½	0.195	6,500	1 13/16	5½	0.895	26,500
¼	¾	0.020	600	15/16	2¾	0.225	7,700	2	6	1.08	31,000
5/16	1	0.029	1,000	1	3	0.270	9,000	2¼	7	1.46	41,000
⅜	1⅛	0.041	1,350	1 1/16	3¼	0.313	10,500	2⅝	8	1.91	52,000
7/16	1¼	0.053	1,750	1⅛	3½	0.360	12,000	3	9	2.42	64,000
½	1½	0.075	2,650	1¼	3¾	0.418	13,500	3¼	10	2.99	77,000
9/16	1¾	0.104	3,450	1 5/16	4	0.480	15,000	3⅝	11	3.67	91,000
⅝	2	0.133	4,400	1½	4½	0.600	18,500	4	12	4.36	105,000
¾	2¼	0.167	5,400	1⅝	5	0.744	22,500				

The approximate length of coil is 1,200 ft for diam 7/16 in. and larger. For smaller sizes it is longer; up to 3,000 ft for 3/16 in. diam.

by "degree proof" or "proof," the figure representing twice the percentage of alcohol by volume; thus, 100 deg proof spirit at 60 F contains 50 percent of absolute alcohol. Common custom in the United States expresses the percentage of alcohol "by volume." Grain alcohol is miscible in all proportions with most liquids except fatty oils, which it dissolves very slightly.

Methanol (methyl alcohol), formerly known as **wood alcohol**, is now principally produced synthetically by passing highly compressed hydrogen and carbon dioxide over catalysts, and condensing the reaction product. The material produced in this way is of extreme purity and uniformity. It is waterwhite, and has only a feeble odor. Its specific gravity is 0.7965 at 60 F, and it boils at 152 F. It burns with a non-luminous flame, is miscible in all proportions with water, alcohol, ether, and glycerin, and is a good solvent for certain resinous materials including shellac.

Some methanol is still being made from wood, principally to supply the denaturing trade, which demands certain types of raw materials.

Denatured Alcohol. The object of denaturing alcohol is to permit the use of ethyl alcohol for purposes other than drinking free from the heavy excise tax. The common denatured alcohol of commerce is made under Government regulation by adding a small quantity of wood alcohol and benzine to grain alcohol. Such a mixture is unfit for consumption as a beverage, but for most industrial purposes it serves equally as well as the pure product. For special industries where such a denaturing would be objectionable, the Government has permitted the use of special denaturing materials, such as pyridine, nicotine, "aldehol," sodium salicylate, diethyl phthalate, and a number of other materials.

BELTING

Leather Belts. High-quality leather belting is made of leather cut from the part of the hide which is back of the shoulders and close to the backbone. This section varies from 24 to 32 in. in width and from 40 to 54 in. in length. Such leather is uniform and has minimum stretch and maximum elasticity and produces belts which will run straight and quiet on the pulleys. Single belting can regularly be obtained up to about $\frac{5}{16}$ in. in thickness, double (two-ply) belting up to about $\frac{5}{8}$ in. in thickness. Belting of over two plies is usually made to suit individual conditions. Very wide belting may be made of two or more strips laterally. Extra long sections may include some inferior shoulder stock which is characterized by the coarse neck wrinkles in contrast to firm center stock which is smooth and without wrinkles.

The quality of leather belting may be determined by the "piping" test which consists of taking a piece of belting and doubling it up grain side in. If the wrinkles or piping appearing on the grain side are large and coarse, the leather is not of high quality. Fine, delicate wrinkles indicate firm, sound stock.

The leather most generally used for belting is vegetable tanned, but chrome-tanned and combination-tanned leathers are also used. Leather belting may be regularly obtained which has been made up with waterproof cement, rendering the belt substantially proof against dampness, steam, and water.

The A.P.I. **Tentative Specifications for Leather Belting** (1926) require an elongation of not less than 11 percent nor more than 24 percent for a load of 700 (1,300) lb per in. of width on a single (double) belt. The ultimate strength shall be not less than 700 lb per in. of width for single, and 1,300 lb per in. of width for double belts. A sample of belting must

solvents; drying-oil varnishes based on cashew-nut-shell liquid, combined with resins (Harvel varnishes).

Varnishes have changed greatly in the last few years, since new oils have become available, and particularly since phenolic resins have been employed in their manufacture. The presence of plenty of linseed oil is no longer sufficient evidence of a good varnish, or the finding of rosin evidence of a poor one. Evaluation should be based on performance tests.

Impregnating Compounds. Bitumens and waxes are used to impregnate motor coils, the melted mix being forced into the coil in a vacuum tank, forming a solid insulation when cooled. Brittle compounds, which gradually pulverize due to vibration in service, and soft compounds, which melt and run out under service temperatures, should be avoided as far as possible. Impregnating materials for high-tension paper-insulated cables are usually a blend of petroleum products containing enough thickener to give the right degree of mechanical consistency. Chemical homogeneity of the materials is very important.

Oil. Refined grades of petroleum oils are extensively used for the insulation of transformers, switches, and lightning arresters. The following specification covers the essential points:

Specific gravity, 0.860; flash test, not less than 335 F; cold test, not more than 14 F; viscosity (Saybolt) at 100 F, not more than 120 sec.; loss on evaporation (8 hr at 200 F), not more than 0.5 percent; dielectric strength, not less than 35,000 volts; freedom from water, acids, alkalies, saponifiable matter, mineral matter, or free sulphur. Moisture is particularly dangerous in oil. Skinner claims that 0.06 percent H_2O will reduce the dielectric strength approximately 50 percent.

Paraffin is used to impregnate paper and cloth. Specific inductive capacity, about 2; dielectric strength, 8,000 volts per mm; specific resistance, 240,000,000 to 3,900,000,000 megohms per cu cm.

Impregnated Fabrics. Fabrics serve as a framework to hold a film of insulating material, and must therefore be of proper thickness, texture, and mechanical strength, and free from nap or acidity. Cambric, muslin, lonsdale, and batiste are commonly used (see *Ind. Eng. Chem.*, July, 1928, p. 698).

Empire cloth is cambric treated with linseed oil by a special process. Similar products are oiled linen, oiled canvas, varnished cambric.

Insulating tape, also known as **friction tape** or **binding** tape, consists of a flat woven braid or tape impregnated with an adhesive and insulating compound adhering firmly to the braid. ¾ in. tape should have a breaking strength of not less than 33 lb and run not less than 150 ft per lb; other widths in proportion.

Cumar, or paracumaron resin, is a synthetic resin made from low-boiling coal-tar distillates. It is made in different grades with melting points as low as 120 F and as high as 320 F. As an insulator, it has a dielectric constant somewhat lower than paraffin, but it possesses high surface resistivity particularly when exposed to moist air. It is a neutral resin and unaffected by water, acids, alkalies, and other reagents. It is used in molded insulation in combination with the usual fillers such as asbestos, fiber, pulp, etc. Also used in paint and varnish, rubber compounds, printing inks, and as a substitute for other resins.

Thermosetting substances of the phenol-aldehyde type (Bakelite, Durez, Durite), and of the urea-formaldehyde type (Beetle) first soften and then undergo a chemical reaction which converts them quickly to a strong infusible product. The best all-around properties are available in the phenolics where numerous special types have been developed for high heat resistance (Durez 34), low power factor (Durez 1601, Bakelite XM262).

Common brick are usually red in color due to the iron oxide in the clay. They are used below grade for sewers, manholes, catch basins, tunnels, footings, and foundations, and are used above grade for the facing and backing of exterior walls and for interior partitions.

Face brick may be red in color, but are also available in grays and buffs where fire clays are the basic materials used in their manufacture. By flashing with chemicals, various surface colors such as green, blue, black, yellow, and orange are obtained. Ceramic glazes on face brick are obtainable in almost any color. Various artificial textures lead to such names as "tapestry," "mattex," "vertical scored," "horizontal scored," "oak bark," and "rug." Face brick are used primarily for the facing of exterior walls.

Brick are manufactured by the dry-press, the stiff-mud, or the soft-mud process. The **dry-press brick** is made in molds under high pressure and from relatively dry clay mixes. Usually all six surfaces are smooth and even, with sharp arrises and geometrical uniformity. The **stiff-mud brick** are made from mixes of clay or shale with more moisture than in the dry-press process, but less moisture than used in the soft-mud process. The clay is extruded from an auger machine in a ribbon and cut by wires into the required lengths. These brick may be side cut, or end cut, depending on the cross section of the ribbon and the length of the section cut off. The two faces cut by wires are rough in texture, the other faces may be smooth or artificially textured. The **soft-mud process** uses a wet mix of clay which is placed in molds under slight pressure. Where sand is used to prevent adherence of the clay to the molds, the result is called a **sand-struck brick;** a **water-struck brick** is a soft-mud brick made in molds where water is used as the lubricant.

The various **methods of burning** commonly used are described as scove kilns, tunnel kilns, round or downdraft kilns, continuous kilns, and permanent kilns. The products of the scove kiln are: light hard brick, or tops, coming from the tops and sides of the kiln; body brick coming from the center of the kiln; and bench brick coming from the arches or benches of the kiln. In the round and the permanent kilns and in the continuous kilns, a more even temperature over the entire body of brick allows a greater percentage of body brick. The light hard brick from these kilns come from the sides where the heat is less intense. There are no arch brick or bench brick. In tunnel kilns, all brick are burned approximately the same. Placed on cars, the brick move slowly through the tunnel and into gradually increasing temperatures, until the maximum of approximately 2000 F is reached. From there on, they are gradually cooled.

Brick are highly resistant to freezing and thawing, to attacks of acids and alkalies, and to fire. They furnish good thermal insulation and good insulation against sound transference.

The use of brick masonry in building construction is due primarily to the **strength of brick in compression.** This strength varies according to the hardness to which the brick is burned and according to the clay from which it is made. Strengths of 1,500 to 24,000 lb per sq in. are available. The requirements for brick exposed to the weather are as follows: compressive strength, 4,500 lb per sq in. min; modulus of rupture, 600 lb per sq in. min; absorption, 5 to 12 percent; ratio of 24 hr absorption to porosity as determined by the 5 hr boil test, 80 percent max (see A.S.T.M. "Specification for Building Brick").

Paving brick are made of clay or shale usually by the stiff-mud or dry-press process. Brick for use as paving brick are burned to vitrification.

tensile strength, lb per sq in., 1,800; compressive strength lb per sq in., 15,000; dielectric strength, volts per mm, 16,350. J. E. Boyd (A.S.M.E., Dec., 1915) gives the compressive strength of porcelain and high-grade stoneware as 20,000 lb per sq in.; the tensile strength of porcelain > 3,000 lb per sq in., of stoneware 1,100 to 2,200 lb; the modulus of elasticity in tension and compression of porcelain, 10,000,000 lb per sq in., of stoneware, 6,000,000 to 9,000,000. Much research work has been done on the improvement of porcelain, particularly for spark plugs, resulting in superior material. The pure porcelain is called **mullite, or sillimanite.** Excellent types applied to electrical uses are **isolantite and alsimag.**

Glass finds many special applications in electrical insulation, particularly for low-tension, small-power uses. A new development of interest is glass fiber for the insulation of magnet wire for electrical machinery.

EXPLOSIVES

(Material Contributed by Hercules Powder Co.)

Black Powder is a black granular slow-acting explosive, which does not detonate but burns with explosive rapidity when ignited under confinement. **Gunpowder** and **"A" blasting powder** are made with potassium nitrate, sulphur, and charcoal. **"B" blasting powder** is made with sodium nitrate, sulphur, and charcoal. Black powder is a mechanical mixture of the substances mentioned above, the nitrates furnishing oxygen for the combustion of the inflammable charcoal and sulphur.

The B grade is ordinarily used for blasting. It is used for blasting in non-gassy and non-dusty coal mines and for blasting where a heaving action is desired.

High explosives contain certain chemical compounds which are themselves explosives; they decompose with extreme rapidity.

Nitroglycerin is formed by the action of concentrated nitric and sulphuric acids on glycerin. It is the base of all dynamites. Commercially it is used only for blasting oil wells.

Straight nitroglycerin dynamite consists of nitroglycerin absorbed in a mixture of carbonaceous materials and oxidizing salts. It is very sensitive and has a high rate of detonation in the higher strengths. It is used for demolition, mud capping, submarine blasting, and ditching by the propagated method. It is not recommended for other uses.

Extra (ammonia) dynamite has the same strength, grade for grade, as nitroglycerin dynamite. Ammonium nitrate is substituted in these grades for part of the nitroglycerin. Extra dynamite is less hazardous to handle and use than straight nitroglycerin dynamite and has the advantage of lower cost. Rate of detonation is lower, grade for grade, than the straight nitroglycerin dynamite. It is used for quarrying, stump and boulder blasting, and much general blasting.

High-ammonia dynamite contains ammonium nitrate as the principal explosive ingredient, nitroglycerin being present only in quantities sufficient to give the desired sensitiveness and other properties. These explosives are relatively insensitive to flame, shock, and friction. They are not recommended for wet work. High-ammonia dynamites are used for mining, quarrying, construction, agricultural blasting, and general blasting work.

Blasting gelatin consists principally of nitroglycerin which is gelatinized with nitrocotton. It is a dense rubberlike explosive that is highly water-resistant and has approximately the same strength as nitroglycerin.

larger amounts of these materials would affect adversely the keeping qualities of the soap.

For cleaning paint and varnish, use pure neutral soap and water; any free caustic or carbonated alkali will damage the paint. Unpainted floors are usually washed with soap powder containing large quantities of sodium carbonate. Cleansers for cleaning pots and pans, sinks, baths, etc., usually consist of finely pulverized pumice, quartz, volcanic ash, etc., with small percentages of soap and sodium carbonate.

Soda ash, which is practically pure sodium carbonate, is extensively used for cleaning purposes, and, in solution, as a lubricant for grinding steel dies and cutting steel and as a preventive of rust on steel tools. For the latter purpose, the strength of solution should be kept low. **Sal soda** is a crystalline form of sodium carbonate containing about 63 percent of water by weight. Although extensively used, it can generally be replaced to advantage by soda ash—which should be at least 97 percent pure. **Caustic soda** (sodium hydrate) is extensively used in textile mills for scouring, cleaning, etc., and for cleansing metals from oil and dirt. A good grade should contain not less than 94 percent of caustic soda. It is very hygroscopic.

Caustic potash, used for cleansing, should contain not less than 88 percent of caustic potash; a crude variety, however, containing 65 percent together with considerable caustic soda, is frequently employed. **Carbonate of potash** should contain not less than 75 percent of potassium carbonate.

Solvent naphtha and gasolines, the lighter distillates of coal tar, and petroleum are used to dissolve oil and grease in cleansing metal, fabrics, etc. **Benzol**, in combination with acetone or alcohol is the basis of most varnish removers. **Carbon tetrachloride** is also used as a solvent for oils and grease; it is non-inflammable. **Oxalic acid** is used in solution for the removal of ink and iron stains. **Sodium hydrosulphite**, a bleaching material, is used to remove dye spots from white goods by sprinkling a few dry crystals on the spot, and then directing steam from a small jet at the end of a glass tube onto the crystals. **Ammonia**, a mild alkali, is used to replace the stronger alkalies in cases where damage might result from their use. **Borax** is also used as a mild alkali for cleaning purposes.

CORDAGE

REFERENCES: Printed matter issued by Plymouth Cordage Co.

The term cordage is applied to products made by twisting together a number of vegetable fibers. **Sisal** and **Manila** (structural fibers) are classed as **hard** fibers. **Hemp**—American, Russian and Italian—(bast fibers) is classified as **soft** fiber. **New Zealand flax**, also known as New Zealand hemp, is generally included with the hard fibers (see p. 709). Manila (**abaca**) is the fiber best suited for marine purposes and general construction work.

A Manila rope constructed from good fiber has strength, elasticity, resistance to deterioration and abrasive wear. Mexican sisal has approximately 65 percent of the strength of Manila; it is used where a cheap fiber is required, also for harvest twine. Java and African sisal have approximately 80 percent of the strength of Manila and are commonly used in the manufacture of tying twines and cores for wire rope. Hemp rope is used where softness and pliability are required, such as hand ropes for elevators. Hemp is the best fiber for making tarred cordage as it will absorb tar readily and retain it; hence, its former use for standing rigging and its present use for all tarred

ramie; and vascular fibers. Cotton and ramie are nearly white; linen, grayish brown; jute and hemp have a decided brown color. Those containing large proportions of cellulose are flexible and elastic, while those which are highly lignified or woody are stiff and brittle. The tensile strength varies considerably, but the relative strength of the fibers is in the following order: Hemp, jute, manila, linen, cotton, ramie. Vegetable fibers are much less hygroscopic than wool or silk, the latter averaging 12 to 16 percent of moisture normally, while cotton and linen average but 6 to 8 percent.

Cotton fiber is obtained from the seed hair of the cotton plant, the seeds being separated from the fiber by a process known as "ginning." The small hairs left on the seed after the first ginning, known as "linters," are used in the manufacture of cotton batting, guncotton, etc. The best quality of cotton fiber is known as "Sea Island" cotton, as it is raised on islands off the coast of Carolina.

The natural spiral-like twist of the fiber makes it specially adapted to spinning. The spinning quality is also dependent upon the length and fineness of the staple. Sea Island cotton is spun into very fine yarns, up to 300, *i.e.*, 300 hanks of 840 yd each weigh 1 lb. The tensile strength of cotton is between the tensile strength of silk and wool.

Mercerized Cotton. When cotton is treated with strong caustic alkali and simultaneously subjected to mechanical tension to prevent contraction, the fiber is greatly changed, taking on a high luster and an increase in tensile strength of 30 to 50 percent. The mercerized fiber also has a greater power of absorption for dyestuffs.

Linen is obtained from the flax plant by "retting." The linen fiber consists of nearly pure cellulose. Natural linen varies from pale yellowish-white to grey. It may be bleached with chloride of lime, but the fiber suffers considerable deterioration. It is of pronounced luster, silky in appearance, stronger than cotton, and a better conductor of heat. Its hygroscopic moisture is about the same as cotton. It does not withstand the action of boiling alkalies, bleaching powders, and other oxidizing agents as well as cotton, nor does it dye so readily. Linen can be mercerized in the same manner as cotton.

Jute is obtained from the stalks of the jute plant by steeping in water. The fiber so obtained is free from woody fiber and generally has a length of 4 to 7 ft. It is of pale yellowish-brown color, the best qualities showing a yellowish-white or silver-gray, and having considerable luster and tensile strength. The short fibers are employed as a raw material for papermaking. The fiber is smooth and lustrous and has no jointed ridges or transverse markings, such as in linen or other bast fibers. It cannot be readily bleached owing to the disintegration of the fiber. Its valuable qualities are fineness, silklike luster, and adaptability for spinning. Its strength is less than that of most other bast fibers. Its chief defect is lack of durability, dampness causing rapid deterioration, and under ordinary conditions the fiber becomes brittle and weak. Bleached fiber is especially unstable; its principal uses are in the manufacture of coarse-woven fabrics such as gunny sacks, bagging, binding thread in carpets and rugs. As a substitute for hemp, it has been extensively used in the manufacture of twine and small ropes, and, owing to its cheapness, is largely used to adulterate better fibers.

Ramie and China grass are obtained from the bast of the stingless nettle. They are two distinct fibers, but the names are generally used interchangeably. The fiber is the strongest and most durable of all vegetable fibers and the least affected by moisture. Ramie can be separated into fibers of great fineness and possesses one-third the strength of hemp. It is white in color, resembling bleached cotton, and possesses a higher luster than linen. It lacks the elasticity of wool and silk and the flexibility of cotton, and consequently produces a harsher fabric in which the fine fibers do not adhere well to each other.

Hemp is a name applied to a number of bast fibers of similar appearance and properties, and is obtained from the plant by "retting." Its composition is a mixture of cellulose and lignocellulose. It contains more hygroscopic moisture than cotton or linen. It is principally used for the manufacture of twine and cordage on account of its great strength and durability. It is not readily affected by water as contrasted with jute. It is not much used

specifications 10 to 15 percent. **Coal falls,** generally used as a single whip, and rope used for the transmission of power are made from selected yarns, but because of the hard lay are not so strong as common-laid rope made of fiber of the same grade.

Drilling cables are a cable-laid product. They are generally left laid but can be made right laid and from selected yarns if required.

ELECTRICAL INSULATING MATERIALS

REFERENCES. Murphy and Morgan, "The Dielectric Properties of Insulating Materials," *Bell System Tech. Jour.*, 16, Oct., 1937, p. 493. Taylor, "Evaluation of Plastics," *India Rubber World*, 98, May, 1938, p. 41.

The insulating properties of any material are dependent upon: dielectric strength or the ability to withstand high voltages without breakdown; ohmic resistance or the ability to prevent leakage of small currents; and power loss or the absorption of electrical energy, which is transformed into heat. Power loss depends upon a number of influences, particularly the molecular symmetry of the insulation and frequency of the voltage, and is the basis of power factor, an important consideration whenever efficient handling of alternating currents is concerned, and a dominating consideration when high frequencies are used, as in radio circuits. Materials may have one of these qualities to a far greater extent than the other, for example, air has a very high specific resistance but very little dielectric strength and no power loss at any frequency; glass has great dielectric strength yet much lower resistance than air. The **ideal insulator** is one having the maximum dielectric strength and resistance and also mechanical strength and chemical stability. Moisture is by far the greatest enemy of insulation, consequently the absence of hygroscopic quality is desirable.

The common insulating materials are described below. For their electrical properties, see p. 1706.

Rubber. See Rubber, Gutta Percha, and Balata, p. 724

Mica and Mica Compounds. Mica is a natural mineral varying widely in color and composition, and occurs in sheets which can be subdivided down to a thickness of 0.00025 in. White mica is best for electrical purposes. The green shades are the softest varieties, and the white amber from Canada the most flexible. Mica has high insulating qualities, the best grades having a disruptive strength of 12,000 rms volts per tenth of a millimeter. Its lack of flexibility and non-uniformity and its surface leakage are disadvantages. To offset these, several mica products have been developed, in which small pieces of mica are built up into finished shapes by means of binders such as shellac, gum, and Bakelite. Mechanical strength: modulus of rupture (beam tests) 50,000 to 70,000; modulus of elasticity 25,000,000 to 30,000,000; shearing strength 18,000 to 38,000 lb per sq in.

Micanite consists of thin sheets of mica built into finished forms with insulating cement. It can be bent when hot and machined when cold, and is obtainable in thicknesses of 0.01 to 0.12 in. Flexible micanite plates, cloth, and paper are also obtainable in various thicknesses. **Mogohmit** is similar to micanite except that it is claimed not to contain adhesive matter. It can be obtained in plates, paper, linen, and finished shapes. **Megotalc,** built up from mica and shellac, is similar to the above-named products and obtainable in similar forms.

Insulating Varnishes. Three general types of insulating varnish are used: asphalt, bitumen, or wax, in petroleum solvent; drying-oil varnishes based on linseed, chinawood, perilla, or soybean oil, compounded with resins from natural or synthetic sources, thinned principally with petroleum

all over the receptacle. It also tends to corrode metals. For freezing temperatures, see p. 1879.

Calcium chloride ($CaCl_2$) is a white solid substance widely used for preventing freezing of solutions and (owing to its great hygroscopic power) for keeping sizing materials and other similar substances moist. It does not "creep" as in the case of salt. It does not rust metal but attacks solder.

Calcium chloride solutions are much less corrosive on metal if made alkaline by the use of a little lime, and also if a trace of sodium chromate is present. They are not suitable for use in automobile radiators, because of corrosive action while hot, and because of tendency of any spray therefrom to ruin the insulation of spark plugs and high-tension cables. For freezing temperatures see p. 1879.

Glycerin is a colorless, viscid liquor without odor and miscible with water in all proportions. It should have a specific gravity of approximately 1.25. It has no effect on metals but disintegrates rubber and loosens up iron rust.

Denatured alcohol is free from the disadvantages of calcium chloride, salt, and glycerin solutions, but is volatile from water mixtures which run hot. A solution containing 50 percent alcohol becomes inflammable, but it is rarely necessary to use more than 30 percent. Cost, approximately 50 cents per gal. See p. 693.

Methyl alcohol (Zerone), variously trade named, is sold widely for automobile antifreeze. It is a desirable and effective antifreeze, but care must be taken not to breathe its fumes, which are poisonous.

Ethylene glycol (Prestone, Zerex) is used as a freezing preventive and also permits the use of high jacket temperatures in aircraft and other engines. Sp gr 1.125 (1.098) at 32 (77) F; boiling point, 387 F; specific heat, 0.575 (0.675) at 68(212) F. Miscible with water in all proportions.

Non-freezing Percentages by Volume in Solution

Temp. deg F	20	10	0	−10
Methyl alcohol	13	20	25	30
Prestone	17	25	32	38
Alcohol	17	26	34	42
Glycerine	22	33	40	47

GLASS

REFERENCES: Journal of the Society of Glass Technology, Sheffield, England. Hodkin and Cousen, "A Textbook of Glass Technology," Van Nostrand. Hovestadt, "Jena Glass," Macmillan. Rosenhain, "Glass Manufacture," Van Nostrand. Long, "Propriétés physiques et fusion du verre," Dunod. Dralle-Keppeler, "Die Glasfabrikation," vols. 1 and 2, Oldenbourg. Eitel-Pirani-Scheel, "Glastechnische Tabellen," Springer. Jebsen-Marwedel, "Glastechnische Fabrikationsfehler," Springer.

Glass is obtained by melting together silica, alkali, and stabilizing ingredients, such as lime, alumina, lead, and barium. Bottle, plate, and window glass, usually contain SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, and Na_2O. Small amounts of the oxides of manganese and selenium are added to obtain colorless glass.

Special glasses, such as tableware, laboratory ware, thermometer glass, and optical glass, *require different manufacturing methods and different compositions.* The following oxides are either substituted in or added to the above base glass: B_2O_3, ZnO, K_2O, As_2O_3, PbO, MnO, etc., to secure the requisite properties. Colored glasses are obtained by adding the oxides of iron, manganese, copper, cadmium, cerium, titanium, etc.

Molten glass possesses the ability to be fabricated in a variety of ways and to be cooled down to room temperature rapidly enough to prevent crystallization of the con-

arc resistance (Bakelite XM1000), and other specialized properties. Materials are available for such specialized applications as the molding of telephone handles (Durez 1543), radio-tube bases (Durez 2260T), and automotive ignition parts (Durez 2491). The urea plastics are lacking in heat resistance but are suitable for general-purpose molding and have fair arc resistance.

Paper. The present tendency is to use paper only as a backing or framework for an insulating film or compound, owing to hygroscopic qualities. Manila and **kraft** papers possess the best dielectric and mechanical strength, and when coated with good insulating varnish are excellent insulators. **Japanese paper** is extremely thin and strong, and is used in tubular insulation and also as a backing for mica paper. Various types of paper, paraffined, are used in condensers.

Vulcanized fiber, also known as **hard fiber,** is prepared by treating paper pulp or a textile material with a saturated solution of zinc chloride and afterward consolidating it under heavy pressure. The soluble materials are then carefully leached out by long soaking in water and the fiber dried out. It has a consistency like horn, but is generally found, in commerce, treated with glycerin or glucose, which makes it very pliable. In the hard form, it can be planed, sawed, and worked like wood, and in the soft form can be readily molded. It is claimed to be an efficient non-conductor of heat and an excellent insulator. The soft varieties are used in making artificial leather and rubber products. Its **specific gravity** is about 1.3; **specific resistance,** 53 megohms per cu cm. Furnished in 42 × 66 in. sheets 0.010 to 1½ in. thick, and in tubes ³⁄₁₆ in. to 2½ in. diam by 22 to 36 in. long. It must be thoroughly seasoned, tough and flexible, and, when unbaked, must stand bending around a circle having a radius of 10 times its thickness without cracking or splitting. After 12 hours' baking at 250 F, it must bend around a circle of radius 50 times its thickness. **Tensile strength** of sheets, lb per sq in.: less than ¹⁄₁₆ in. in thickness and over ⅝ in., 5,000; ¹⁄₁₆ in. to ¼ in., 8,000; ¼ in. to ⅝ in., 7,000 lb per sq in. **Electrical strength:** about 1,200 volts per 0.01 in. of thickness.

Presspahn or **fuller board** is a specially prepared paper board which, when impregnated with oil, is an excellent insulator. Insulator presspahn is very hygroscopic. The material is mechanically strong but its dielectric strength is greatly injured by creasing. **Pergamyn** is cellulose beaten into fine fibrillae and consolidated under pressure without gelatinization or chemical treatment. Its uses are similar to those of vulcanized fiber. **Cellulith** is produced by grinding wood pulp to a homogeneous mass and then drying. It is very hard, can be worked like wood, and is claimed to be a substitute for horn or ebonite. **Isolit** is a form of papier maché impregnated and covered with special insulating compound. **Adit** is a variety of isolit claimed to have great tensile strength and toughness and to ignite with difficulty. **Litholite** is a paper product used for insulation. It has high disruptive strength; good insulation resistance, and is stated to be inflammable but tough. **Psychiloid** is a chemically treated paper pulp obtainable in sheets from ¹⁄₁₆ in. to 1½ in. in thickness. It is very strong and rigid, and stated to have high disruptive strength and insulation resistance.

Porcelain is made from the purest white clay agglutinized by some substance such as powdered feldspar which softens and fuses at the temperature at which the ware is fired, rendering the mass semitransparent. The material possesses great solidity, strength, and ability to resist sudden changes of temperature. It is used extensively for electrical purposes on account of its strength, durability, and electrical resistance. External glaze is of importance in protecting porcelain. A simple test for quality consists in chipping off the glaze and noting whether ink produces a flowing stain, which it will do on poor grades. **Properties:** The Locke Insulator Mfg. Co. gives the following average values: specific gravity, 2.427; specific inductive capacity, 4.4;

Luxfer prisms are white glass plates whose interior side is provided with parallel prismatic, transverse ridges. They are mounted vertically in front of windows or as protecting roofs more or less obliquely on the outside, and serve to diffuse the light passing through them into the darker portion of the apartment. They possess great strength and considerable resistance to fire.

Pyrex glass is a low-expansion borosilicate glass containing no metals of the magnesia-lime-zinc group and no heavy metals. Physical properties: modulus of elasticity 8,800,000 lb per sq in.; sp gr 2.25; refractive index 1.4754; dispersion 0.00738; linear thermal expansion coefficient (65–660 F) 0.0000032; specific heat 0.20; thermal conductivity 0.21 Btu per sq ft per in. per deg F per hr; can be used without softening up to 1200–1300 F. Principal uses are chemical ware, baking ware, high-tension insulators, sight glasses for chemical apparatus, boiler gage glasses, glass pipe lines for chemical plants, railroad lantern globes and battery jars. Owing to its low coefficient of expansion, Pyrex glass withstands sudden changes of temperature without breaking. Chemical ware withstands the test of plunging from molten paraffin at 390 F into ice water.

Fused quartz is made by melting rock crystal or purest quartz sand in the electric furnace. It is unaffected by changes of temperature, is fireproof and acid-resistant, does not conduct electricity, and has practically no expansion under heat. It is used considerably for high-temperature laboratory apparatus. (Paget, *Jour. Royal Society of Arts*, Apr. 4, 1924.)

GRAPHITE

REFERENCES: "Graphite, Its Properties, Occurrence, Refining and Uses," Department of Mines, Canada. "Mineral Resources of the United States," U. S. B. of M. Acheson, "Seventeen Years of Experimental Research and Development," Am. Acad. of Arts and Sciences, 1908. "Mineral Industry." Seton, "Graphite and Its Uses." *Jour. Soc. Chem. Ind.*, **43**, 1939. Vosburgh, "Electrodes—Carbon and Graphite," *Elec. Eng.*, **52**, 1933. Szymowitz, "Colloidal Graphite," *Jour. Chem. Educ.*, **16**, 1939.

Graphite is an amorphous form of carbon, other forms being the diamond and lampblack. Three commercial varieties are obtainable—flake, amorphous, and artificial. Flake and amorphous graphite exist in nature and are separated from their accompanying rock mechanically. The latter is much more abundant but of less value commercially. The best flake graphite comes from Ceylon, although certain valuable varieties occur in the United States, particularly those in New York State controlled by the Jos. Dixon Crucible Co. The **specific gravity** of commercial graphite ranges from 2.015 to 2.583; for refined graphite it is 1.802. Its **hardness** on Mohs scale is between 1 and 2. The **specific heat** of natural graphite is 0.2019; of furnace graphite, 0.1970. The **heat value** of natural graphite is 14,033 Btu (Favre and Siberman), and of artificial graphite, 14,222 Btu (Berthelot and Petit) per lb. According to Fizeau, the **coefficient of linear expansion** per deg F is 0.00000437 at 104 F, and the elongation of a unit length from 32 to 212 F is 0.000796. Graphite conducts heat better than diamond and is also a good conductor of electricity.

Specific electrical resistance: Natural, 14.20 ohms (Streintz), 12.20 ohms (Muraoka); graphitized electrode, 12 ohms; Ceylon, 2 to 8 ohms (Zellner); Acheson electrode, 12 ohms (Zellner).

Flake graphite is extensively used in the manufacture of crucibles and other refractories, owing to its resistance to heat and its ability to form a stiff mix with clay. It is also used in stove polish, foundry facings, paint and pencil manufacture, and as a lubricant.

Amorphous graphite is extensively used in paints owing to its resistance to acids and alkalies and ability to form an elastic film with oil; also for packings, lubrication, electrodes, and pencils. For lubrication, it must be entirely free from grit; owing to its unctuous nature, it is an excellent lubricant either alone or with oil.

Straight-gelatin dynamites are made in various strengths and are dense plastic water-resistant explosives with fumes that make them suitable for underground use. The nitroglycerin used in gelatin dynamites is gelatinized with a small percentage of nitrocotton, which is responsible for their consistency. These explosives have been almost entirely replaced by **extra gelatins** or **ammonia gelatins**, which contain ammonium nitrate.

Semigelatins contain a higher percentage of ammonium nitrate than extra gelatins. They are more economical and possess sufficient plasticity and water-resistance to make them suitable for much work formerly requiring gelatin. They are more economical than gelatins, and their fumes are such that they are suitable for most underground work.

The gelatins and semigelatins are used in mining, quarrying, construction, tunnel driving, shaft sinking, and for other uses where their water-resistance, fumes, or plasticity make them specially adapted.

Permissible explosives are explosives similar in all respects to samples which have passed certain tests by the U. S. Bureau of Mines and when used in accordance with the directions prescribed by that Bureau. Permissible explosives are used in coal mining and particularly recommended for use in gassy and dusty mines. There are two general classes; nongelatinous permissibles, which are used for most work; and gelatinous permissibles, which are especially adapted for wet conditions.

FIBERS

REFERENCES: Matthews, "The Textile Fibers," Wiley. Graff, "Color Atlas for Fiber Identification," Inst. of Paper Chem., 1940. Hünlich, "Textile Fibers and Materials," Skinner. Astbury, "Fundamentals of Fiber Structure," Oxford. Hess, "Textile Fibers and Their Use," Lippincott.

Fibers for industrial purposes are either mineral, animal, or vegetable. They may be further classified as natural and synthetic. The principal mineral fiber of importance is asbestos (see p. 717). Finely spun glass, slag wool, and metal threads are also used. The animal fibers of commercial importance are either animal hairs or the silk of the silkworm and the larvae of other moths. Sheep, goats, and camels supply the principal hair fibers. Cotton, linen, jute, and hemp are the principal vegetable fibers of natural origin, and rayon is the most important synthetic fiber.

Fibers may readily be distinguished under the microscope. Animal and vegetable fibers may be distinguished by their odor on burning, and by the fact that vegetable fibers burn off sharply while animal fibers fuse to a rounded beadlike end.

Animal Fibers

Silk fiber consists of a continuous thread spun by the mulberry silkworm. Inferior silks are obtained from other varieties of caterpillars and are known as "wild" silk.

Silk fiber possesses great tensile strength, the average breaking strength being approximately 64,000 lb per sq in., and the elongation 15 to 20 percent.

Wool. The wool of the sheep is that principally employed. The tensile strength and elasticity of wool fibers vary widely; the average strength being about half that of cotton. The length of the fiber varies between 1 and 8 in. and the diam between 0.0018 and 0.004 in. Wool is graded according to the length of staple into "tops" and "noils," the former being used for worsted yarns and the latter for making woolen and carded yarns.

Vegetable Fibers

All vegetable fibers consist mainly of cellulose and may be classified as follows: seed hairs, such as cotton; bast fibers, such as flax, hemp, jute, and

which has the most complete and most finely divided cellular structure. The obstruction to heat flow is offered by the surface resistances at the boundaries of the air space. Air itself is not a good insulator. The material should not absorb moisture and should be free from rot, mold, or offensive odors. The principal materials used for this purpose are cork, asbestos, and mixtures of asbestos and magnesia and mineral wool.

Cork is the outer bark of the cork oak. It is highly compressible, possesses great permanent elasticity, is an excellent nonconductor of heat, is waterproof, unaffected by moisture, and slow burning. The natural cork wood weighs 10 to 12 lb per cu ft. Cork is used for bottle stoppers, life preservers, cold-storage insulation, carburetor floats, polishing wheels, washers and gaskets, cork paving brick, isolating pads for machinery, and in the manufacture of linoleum and cork tile flooring.

For cold-storage insulation, pure corkboard is made in sheets 12 in. × 36 in. × thicknesses up to 6 in. No artificial binder is used, the natural gums and resins in the cork sufficing to hold the material together after proper baking. The **U. S. Navy Department** specification requires that such pure corkboard shall withstand boiling in water for 3 hr without disintegration or more than 2 percent expansion in any direction. Granulated cork is likewise used for cold storage insulation as a filling material between confining walls. Granulated cork ordinarily refers to new corkwood finely divided, while the term regranulated cork applies to the granulated material which has been baked in making corkboard and which is a by-product of this material. Tests made at the Bureau of Standards give the following values of heat conductivity for cork products in Btu per sq ft per in. thickness per deg F per 24 hr: corkboard, no artificial binder, density 6.9 (9.9) [11.3] lb per cu ft, 6.5 (7.3) [7.4]; corkboard with bituminous binder, density 16 lb per cu ft, 8.4; ground cork, less than $\frac{3}{16}$ in., density 9.4 lb per cu ft, 7.1; regranulated cork, about $\frac{3}{16}$ in., density 8.1 lb per cu ft, 7.5. The pure corkboard used commercially weighs approximately 10½ lb per cu ft (see also Tables 4 and 6, pp. 389, 390).

Cork tile, made by essentially the same process as pure corkboard, is a quiet, non-slippery floor largely used for bank working spaces and in public libraries.

Linoleum is a mixture of cork dust with linseed and other drying oils, reinforced on one side by burlap material. Machinery isolation is made from pure cork baked under pressure in thicknesses up to 6 in.

Balsa is the wood of a large tropical tree found in Latin America. It is the lightest wood known, commercial grades weighing 8 to 15 lb per cu ft. When properly prepared, over 90 percent of the volume of the wood consists of air spaces. U. S. Bureau of Standards tests show conductivity of 7.5 to 8.3 Btu per sq ft per in. thickness per deg F in 24 hr. Tests at the Massachusetts Institute of Technology show about half the strength of spruce; weight, kiln dry 7 to 11 lb per cu ft; modulus of rupture 2,357 lb per sq in; maximum crushing strength in compression parallel to grain 2,317 lb per sq in.; shearing strength parallel to grain 351 lb per sq in. Natural balsa is subject to decay. It can be made waterproof and non-absorbent to oil and gasoline. It can be sawed, planed, and worked like ordinary wood, and painted, stained, or varnished. Largest commercial size of balsa boards is 4 in. × 6 in. × 6 ft. Principal uses: refrigerator lining, life preservers, floats, and other purposes where lightness combined with strength is desired.

Diatomaceous silica is a mineral of low density consisting of the fossilized cellular skeletons of microscopic organisms called diatoms. Other names for

for woven textiles on account of its harshness and stiffness and lack of pliability and elasticity. The fiber is of dark brown color and cannot be successfully bleached without injury.

Manila hemp is obtained from the leaf stalks of the *Musa textilis*. The fiber is white and lustrous, light and stiff, easily separated, very strong, and of great durability. The coarser fibers are used for the manufacture of cordage on account of their great strength. Manila hemp ropes are stronger than cotton ropes.

Sisal hemp is obtained from the leaves of a plant found in Central America, and to some extent in the West Indies and Florida. It is of light-yellowish color, straight and smooth. Its principal use is in cordage manufacture, its strength being second only to that of Manila hemp. It is also used for papermaking.

Waste. The best **wool waste** consists of all-wool carpet yarn in threads not less than 3 in. long and comparatively free from moisture or dirt. **Cotton waste** should consist of equal parts of white and colored new cotton threads properly mixed and free from water and dirt. White cotton waste should be made up entirely of new white cotton threads not less than 3 in. long and free from dirt or water. It is customary to purchase waste by sample. Inferior quality is indicated by the presence of fibers other than wool or cotton, by sweepings or dirt, or by evidence that the waste has been previously used.

Synthetic Fibers

Rayon of several types (nitrocellulose, cuprammonium, viscose, cellulose acetate) is produced by the extrusion of a cellulose solution obtained from cotton linters or purified wood pulp, dissolved in one of several cellulose solvents, through the fine holes of a spinneret into air or a hardening bath, to form a continuous filament. Several such filaments are twisted to form a thread. A wide range of filament diameters is possible from very coarse to finer than silk, and by a modification of the extrusion orifices, **cellophane**, hollow tubes, and artificial bristles may be produced. Rayon thread is being used in tire fabric because of its excellent heat resistance. When wet, most rayon fibers lose strength.

Glass wool, made from molten glass by methods similar to those used in the manufacture of slag wool from molten rock or slag, can be produced in a wide variety of filament diameters. The coarser material is extensively used in filters for air conditioning and is widely used for thermal insulation. The finer fibers can be woven into cloth of unusual chemical inertness, strength, fireproofness, durability, and electrical resistance. Woven glass tape is used for electrical insulation, and glass cloth for filtration involving corrosive liquids.

Casein wool, or **prolon**, made from milk casein, **wood wool** made from cellulose but permanently crimped in manufacture, and **synthetic bristles** made by the interaction of an organic acid and an amine to produce an organic salt capable of being formed into filaments of varying thickness, are other synthetic fibers. **Exton** is coarse fiber for bristles and **nylon** is finer fiber either for bristles or for yarn. **Vinyon** is a vinyl acetate-vinyl chloride extruded fiber, used for yarn.

FREEZING PREVENTIVES

Common salt is sometimes used to prevent the freezing of water; it does not, however, lower the freezing point sufficiently to be of use in very cold weather, and in concentrated solution tends to "creep" and to crystallize

Mineral wool and **glass wool** are fibers made by the steam blowing of molten slag or glass. Bats of these substances should be of fine fibrous structure and as free as possible of lumps.

Hair felt is made from lime-washed cattle hair formed into felts from 1/4 to 2 in. thick. It is used for the insulation of refrigerator cars and for insulation of piping, fittings, and tanks at low temperatures. It is also furnished stitched between layers of waterproof paper for various low-temperature insulation purposes and between layers of asbestos paper for passenger-car insulation.

Vegetable fiber insulations, in the form of boards 7/16 to 1/2 in. thick, made from wood pulp or bagasse, are widely used in frame-building construction. In addition to having considerable insulating value, they possess sufficient structural strength to replace wood sheathing and plaster lath. They are also used in the construction of refrigerators, refrigerator cars, etc. Finely divided wood or other vegetable fibers secured between layers of paper are used for frame building insulation, etc.

For commercial insulating materials, see also pp. 389–391. For commercial pipe coverings, see also p. 953.

LACQUERS

From 1926 until about 1937 practically all automobiles were finished with duco or other pyroxylin lacquers. Some cars still are finished with pyroxylin, but the trend toward the use of baked enamels is very marked. Pyroxylin has the advantages of requiring about one-tenth the time necessary for paint and varnish and of better resistance to weather and usage.

The foundation or film-making material used in lacquers is nitrocellulose or pyroxylin, largely of so-called low-viscosity type. This is improved in flexibility and color dispersion by the use of plasticizers, and improved in body, density, waterproofness, adhesion, and gloss by the use of various resinous materials. All colors are obtained by pigments which must be of extreme fineness and well dispersed, and capable of withstanding indefinite exposure to sunlight and rain without fading.

The liquid portion of lacquers consists of solvents and diluents which for spray application boil mostly between 212 and 280 F, but small amounts boiling lower or higher are permissible or desirable for certain purposes. In general, pyroxylin lacquers dry by evaporation of the solvent, but several types depend upon the oxidation of some drying oils in addition to the first-set of the pyroxylin by evaporation.

Some use is made of lacquer for the inside finishing of houses, offices, and hotels, where speedy drying is essential. Lacquer is also being used on railroad cars, but, so far, not extensively for out-door large-scale painting. In general, pyroxylin lacquer has not body enough for satisfactory use on furniture or other woodwork, but is used on a few items, such as radio cabinets.

LEATHER

Belts. See Belting, p. 699.

Rawhide is prepared by rubbing oil or fats into a strip of hide, twisting and stretching it until the moisture is removed and the skin thoroughly filled with oil. It is very strong, tough, and waterproof, and is chiefly used for belt lacing and similar purposes.

Leather substitutes generally consist of refuse leather reduced to a pulp, molded, pressed, or wound into suitable shapes, and frequently waterproofed by a final treatment. The earlier of these preparations were composed of glue, starch, waxes, and adhesives and also included water-repellent ingredients. Of the leather-free artificial leathers, of which there are many types, the best known are **leatherine,** made by coating calico with rubber and rubber substitute; **leather board,** prepared by pulping fibrous materials

stituents. It is a rigid material at ordinary temperatures, but may be remelted and molded any number of times by the application of heat. Ordinary glass is melted at about 2600 F and will soften enough to lose its shape at about 1100 F.

Properties of Glass. *The following data are for ordinary glass: specific* gravity, 2.3 to 2.6; tensile strength, 10,000 lb per sq in.; compressive strength, 50,000 lb per sq in.; modulus of elasticity, 10,000,000 lb per sq in.; coefficient *of expansion,* 40 to 60 $\times 10^{-7}$ per deg F; thermal conductivity, 0.4 to 0.6 Btu per sq ft per in. per deg F per hr; specific heat, 0.16 to 0.20.

Glass always breaks under a tensile stress which arises in the system, even though it fails under an applied compressive force. The value obtained for crushing strength depends on the material of the plates used and upon the precision of the testing apparatus. Glass resists atmospheric corrosion and is one of the best materials for resisting the attack of water, bases, and acids, with the exception of hydrofluoric acid.

Window glass is a soda-lime-silica glass, fabricated in continuous sheets up to a width of 6 ft. The sheets are made in two thicknesses, SS and DS, which are, respectively, $\frac{3}{32}$ and $\frac{1}{8}$ in. Both thicknesses are made in A, B, C, and D grades.

Plate glass is similar in composition to window glass. It is fabricated in continuous sheets up to a width of 15 ft, and is polished on both sides. *It may be obtained in* various thicknesses and grades.

Skylight Glass. The weight of fluted or roof plate glass required for one square of roof (no allowance being made for lap) is approximately 350 lb for $\frac{1}{4}$ in. glass; in proportion for other thicknesses. A square of roof is 10 ft square.

Safety glass consists of two layers of plate glass firmly held together by an intermediate layer of organic material, such as pyroxylin plastic or cellulose acetate. The *safety glass, which is sealed to prevent deterioration of the plastic layer, is ordinarily* $\frac{1}{4}$ in. thick, but can be obtained in various thicknesses. This plate is shatterproof and is used for windshields, bank cashier's windows, etc.

Toughened glass is made from sheet glass in thicknesses up to 1 in. It possesses great mechanical strength, which is obtained by rapidly chilling the surfaces while the glass is still hot. This process sets up a high compression on the glass surfaces, which have the capacity of withstanding very high tensile forces before failure occurs.

Wire glass is a glass having an iron wire screen thoroughly embedded in it. It offers about $1\frac{1}{2}$ times the resistance to bending that plain glass does; very thin sheets may be walked on. If properly made, it does not fall apart when cracked by shocks or heat, and is consequently fireproof. It is used for flooring, skylights, fireproof doors, fire walls, etc. It must be ordered exactly to measure, as it cannot be readily cut. Less light passes through it than through plain glass.

Pressed glass is made by forming heat-softened glass in molds under pressure. Such articles as cheap tableware, insulators, and glass blocks are made by this process.

Glass insulators are pressed and are made in a variety of shapes and sizes for both low and high voltages. Glass is a good conductor in the molten condition, but is a good insulator at ordinary temperatures. The dielectric constant and specific resistance of glass can be varied within wide limits, making it an ideal material for many electrical purposes.

Glass blocks find wide application for building purposes, and are made $3\frac{7}{8}$ in. thick in the following sizes: 5 $\times$ 8, 6 $\times$ 6, 8 $\times$ 8, 12 $\times$ 12 in. The thermal conductivity of a glass block panel, for a thickness of $3\frac{7}{8}$ in., is 0.49 Btu per sq ft hr per deg F. The solar heat transmission can be varied within wide limits by using different colored glasses and by changing the reflection by means of surface configurations. The light transmission can be varied from 25 to 80 percent by changing the surface configurations of the blocks.

Fiber glass is a term used to designate articles that consist of a multitude of tiny glass filaments ranging in size from 0.0001 to 0.01 in. in diam. The larger fibers are used in air filters; those 0.0005 in. in diam, for thermal insulation; and the 0.0001 to 0.0002 in. diam fibers, for glass fabrics, which are stronger than ordinary textiles of the same size. Insulating tapes made from glass fabric have found wide application in electrical equipment, such as motors and generators.

Vitrolite structural glass is used for many structural purposes, such as store fronts and table tops, and is available in thicknesses of $\frac{5}{16}$ and $\frac{7}{16}$ in., and 128 $\times$ 177 in.

Properties of Common Minerals*

Mineral	Composition	Hardness, Mohs scale	Specific gravity
Amphibole	$Ca_5(ClF)(PO_4)_3$	5.0	3.2
Andalusite	Al_2SiO_5	7.5	3.1 to 3.2
Anhydride	$CaSO_4$	3.0 to 3.5	2.9 to 3.0
Apatite	$Ca_5(ClF)(PO_4)_3$	5.0	3.2
Aragonite	$CaCO_3$	3.5 to 4.0	2.9
Barite	$BaSO_4$	2.5 to 3.5	4.3 to 4.6
Bauxite	$Al_2O(OH)_4$	1.0 to 3.0	2.4 to 2.5
Biotite	$(HK)_2(MgFe)_2Al_2(SiO_4)_3$	2.5 to 3.0	2.7 to 3.1
Borax	$Na_2B_4O_7.10H_2O$	2.0 to 2.5	1.7
Calcite	$CaCO_3$	3.0	2.7
Chrysolite	$(MgFe)_2SiO_4$	6.5 to 7.0	3.3 to 3.6
Corundum	Al_2O_3	8.5 to 9.0	3.95to 4.1
Cryolite	$AlNa_3F_6$	2.5	2.9 to 3.0
Cuprite	Cu_2O	3.5 to 4.0	5.8 to 6.1
Dolomite	$CaMg(CO_3)_2$	3.5 to 4.0	2.8 to 2.9
Epidote	$Ca_2(AlFe)_2(AlOH)(SiO_4)_3$	6.0 to 7.0	3.2 to 3.5
Garnet	$(SiO_4)_3$	6.5 to 7.5	3.1 to 4.3
Graphite	C	1.0 to 2.0	2.1 to 2.2
Gypsum	$CaSO_4.2H_2O$	1.5 to 2.0	2.3
Hematite	Fe_2O_3	5.5 to 6.5	4.9 to 5.3
Hypersthene	$(MgFe)SiO_3$	5.0 to 6.0	3.4 to 3.5
Kaolinite	$H_4Al_2Si_2O_9$	2.0 to 2.5	2.6
Lepidolite	$(KLi)_3Al(SiO_3)_3$	2.0 to 2.5	2.0 to 3.2
Leucite	$KAl(SiO_3)_2$	5.5 to 6.0	2.4 to 2.5
Magnesite	$MgCO_3$	3.5 to 4.5	3.0 to 3.12
Magnetite	Fe_3O_4	5.5 to 6.5	4.9 to 5.2
Malachite	$Cu_2(OH)_2CO_3$	3.5 to 4.0	3.9 to 4.0
Microcline	$KAlSi_3O_8$	6.0 to 6.5	2.5 to 2.6
Nephelite (Elaeolite)	$(NaK)_8Al_8Si_9O_{34}$	5.5 to 6.0	2.5 to 2.6
Orthoclase	$KAlSi_3O_8$	6.0 to 6.5	2.4 to 2.6
Plagioclase	$nNaAlSi_3O_8 + mCaAl_2Si_2O_8$	5.0 to 7.0	2.6 to 2.7
Pyrite	FeS_2	6.0 to 6.5	4.9 to 5.2
Pyrolusite	MnO_2	1.0 to 2.5	4.7 to 4.86
Pyroxene	$RSiO_3$	5.0 to 6.0	3.2 to 3.6
Pyrrhotite	FeS	3.5 to 4.5	4.5 to 4.6
Quartz	SiO_2	7.0	2.6
Serpentine	$H_4Mg_3Si_2O_9$	2.5 to 4.0	2.5 to 2.65
Siderite	$FeCO_3$	3.5 to 4.0	3.8 to 3.9
Talc	$H_2Mg_3(SiO_3)_4$	1.0 to 1.5	2.5 to 2.9

* From "A Guide to the Sight Recognition of 120 Common or Important Minerals," by A. J. Moses in Farrell, "Practical Field Geology," McGraw-Hill.

Clay schist is composed of hardened clay and quartz, often with little lamellae of mica. It cleaves readily and is therefore suitable for roofing. See also Slate, p. 724.

Limestone (carbonate of lime), if capable of taking high polish, is called **marble.** The color varies with amounts of iron oxide, copper oxide, etc. By ignition, it produces **caustic lime. Calcite** is transparent and used as sculptor's marble. **Chalk,** another form of limestone, is white and earthy.

Marl is a dense, earthy or slaty rock, consisting of a mass of carbonate of lime mixed with clay and siliceous sand. Certain species are well suited for the manufacture of cement.

Dolomite, a magnesian carbonate of lime, is whitish-yellow or gray to brown in color, crystallized and dense. Certain deposits produce a hard durable building stone, well suited for road building.

Gypsum, hydrated sulphate of lime, is generally white, but often yellowish to reddish, or gray to blackish; it is not durable when exposed to weather.

Sandstone consists of sand grains made up into a more or less compact rock by a binder consisting generally of silicon, lime, or clay. It often contains calcite, mica, iron ore, and admixtures of red and green clay. Iron pyrites has a disintegrating effect; an abundance of mica is also undesirable.

Artificial graphite is made by the International Acheson Graphite Co., of Niagara Falls. Amorphous carbon is converted to graphite by subjecting carbonaceous material to a temperature in the electric furnace above the volatilization point of impurities such as iron and silicon. It is possible to produce absolutely pure graphite in this manner. Its principal use at present is in the manufacture of electrodes. Large amounts are sold as a lubricant after a special treatment called "deflocculation," which makes possible its colloidal suspension in water or oil. **Aquadag** and **oildag** are products of this kind containing deflocculated graphite in water and oil, respectively.

Graphite for foundry use is often called **plumbago**, as it was originally supposed that graphite was a form of black lead. The U. S. Navy specifies *that plumbago shall be dry, free from coal dust or grit,* and shall *contain* not less than 55 percent of graphite carbon.

Graphite refractory brick are used for furnace linings on account of their great heat-resisting qualities. The product known as **kryptol**—composed of graphite, carborundum, and clay—is used for electric resistance furnaces as its resistance is sufficient to localize a high degree of heat without the material itself being destroyed.

HEAT INSULATORS

REFERENCES: "Asbestos, Its Origin and Production," *Chemical World*, Jan., 1913. Printed matter issued by the H. W. Johns-Manville Co., Armstrong Cork Co., Celite Products Co., and American Balsa Co. Symposium on Thermal Insulating Materials, A.S.T.M., 1939. Thomas, "Cork Insulation," Nickerson & Collins. Dalzell and McKinney, "Air-conditioning Insulation," Am. Tech. Soc., 1937.

Chemical composition is no index of insulating value. For example, the conductivity of solid graphite is more than two hundred times as great as that of the same material in finely powdered form. The composition of the material should be suited to the service required of it, but the insulating value is determined by the **physical structure** which should be such that the optimum number of surface resistances are interposed in the path of heat flow.

Powdered or Fibrous Insulating Materials. Mineral wool, powdered gypsum, graphite, charcoal, shavings, sawdust, diatomaceous earth, basic *magnesium carbonate, asbestos, cork-dust, wool fiber.*

Molded Insulators. Powdered cork may be molded into corkboard by heat and pressure. Some powdered and fibrous insulating materials may be molded with water and afterward dried. In other cases, a binder must be used, such as waterglass, clay, or asphalt. The presence of even a small amount of binder decreases the insulating quality of the material very considerably.

Flexible insulations are usually made by stitching fibrous material between layers of kraft paper. Common fillers are eel grass, wood fiber, flax tow, kapok, jute, and hair.

Cast Insulations. Insulex and **pyrocel** are gypsum materials, cast in place, leaving a cellular structure. Porous concrete has also been made, using crushed ice or cinders instead of gravel.

Sprayed Insulation. Spray-o-flake is paper scrap mixed with water glass and asphalt. Thickness can be built up by spraying to 2 in.

Air. Commercial insulators owe their insulating value to the spaces or voids in their structure and not to the air itself. The best insulation is the one

Definitions of Paper," Lockwood. "Dictionary of Paper," Am. Paper & Pulp Assn., 1940. Clapperton and Henderson, "Modern Papermaking," Blackwell. Jahans, "Paper Testing and Chemistry for Printers," Pitman. Sutermeister, "Chemistry of Pulp and Papermaking," Wiley.

Paper is a matted or felted structure of fibrous material formed into a relatively thin sheet through the medium of a dilute suspension of pulp and water, and composed essentially of cellulose fibers obtained from vegetable growths in a more or less pure state.

Pulp for papermaking is prepared by grinding wood mechanically, by cooking chemically according to either of the three standard cooking processes, *i.e.*, sulphite, kraft, or soda, and also by chemically treating cotton, linen, and hemp rags, waste, esparto, straw, etc. So-called semichemical pulp processes, involving only partial disintegration of the wood structure, have been introduced lately. By these methods, wood wastes, such as sawmill refuse, which are usually burned as fuel, can be worked into cheap boards. The use of rags is at present confined to only the most particular grades of papers, as, for example, the highest grades of bond and writing, condenser papers, etc.

The lower grades of paper, such as **newspaper**, are made largely from groundwood mixed with a certain percentage of chemically prepared pulp. **Book paper** is made from a mixture of sulphite and soda pulp, with the addition of varying amounts of fillers such as clay and talc. These are added to improve the surface and printing qualities so necessary in this grade. **Insulating paper** for use in cable winding, while formerly made from all high-grade manila stock, is at present being made almost entirely from kraft. **Kraft** is now used to make practically all twisting and wrapping papers.

Box boards, container boards, Bristols, etc. are made from ground wood and chemically prepared pulp, such as sulphite or kraft, or mixtures of these, and waste papers. Thick, lightweight, insulating boards for building construction are made from waste materials such as bagasse and newspapers, licorice root, groundwood, and sulphite screenings.

In testing papers, **tensile strength** (500 to 2,500 lb per sq in.), stretch, and bursting are of importance; tests should be made on special machines. Comparison of different samples of the same grade of paper can be roughly made by tearing, crumpling and by visual examination for freedom of dirt and other indications of careless manufacture.

ROOFING MATERIALS

(Material Furnished by Bird & Son, East Walpole, Mass.)

REFERENCES: Abraham, "Asphalts and Allied Substances," Van Nostrand. A.S.T.M. Tentative Standards. Grondal, "Certigrade Handbook of Red Cedar Shingles," Red Cedar Shingle Bureau, Seattle, Wash.

Asphalt. Asphalts are bitumens, and the one most commonly seen in roofing and paving is obtained from petroleum residuals. Lake asphalts such as Trinidad and Bermudez, are used primarily in paving. Natural asphalts such as **Gilsonite** and **Elaterite** are too hard and brittle for use in roofing or paving, but are used extensively in asphalt coating, paints, and varnishes.

Petroleum residuals are obtained by the refining of petroleum. The qualities of asphalt are affected by the nature of the crude and the process of refining. Mid-Continent, Mexican, Venezuelan, Texan; and certain Californian asphalts are very satisfactory. When the flux asphalts obtained from the oil refineries are treated by blowing air through them while the asphalt is maintained at a high temperature, a material is produced which is very stable and has good weathering properties.

this type of material are **kieselguhr, diatomaceous earth, infusorial earth, Sil-O-Cel,** and **celite.** Material from the better deposits is almost pure silica which fact accounts for its high heat resistance. Where the material has been contaminated with clay, lime, sand, and other foreign matter, the density is increased and the insulating value and heat resistance are decreased. Insulating bricks are sawed directly from the material in the better deposits, for use at temperatures up to 1600 F.

Uses. Calcined bricks are made for temperatures as high as 2500 F. The material is also used in powdered form as an insulating fill and is mixed with asbestos fiber and bonding material for the production of moulded blocks and pipes insulation for use at temperatures up to 1600 F. Calcined granular material is used as an insulating fill at higher temperatures and is mixed with Portland cement to provide semirefractory insulating concrete for furnace doors, furnace foundations, the fireproofing of steel members, etc.

Other uses of diatomaceous silica are in the manufacture of a clarifying agent for use in filtering, as an admixture to improve the workability of concrete, stucco, and mortar, as a lightweight mineral filler for use in compounds such as polishes, paints, varnishes, enamels, lacquers, insecticides, matches, linoleum, catalysts, dynamite, etc.

Asbestos is a heat-resisting fibrous mineral, the most important deposits of which are in Canada. The **chrysotile variety,** because of its fineness of fiber, tensile strength, elasticity, and flexibility, is commercially the most important and references to asbestos without other designation presumably indicate this variety. Other useful varieties are Amosite or Rhodesian asbestos and blue or Cape asbestos, both from South Africa. The amphibole or hornblende variety is of little commercial value because of its brittleness and lack of fiber strength.

Uses. High-quality asbestos because of its incombustibility, its low heat and electrical conductivity, and its resistance to the action of most chemical agents has a wide variety of uses among which are heat and electrical insulations, packings and gaskets for high-temperature service, brake linings and friction materials, cloth for theater curtains, and heat-protective clothing, roofings, and building materials. It is spun and woven into yarn, rope, and cloth, is formed into asbestos felt, paper, roll board, and mill board for insulation, is mixed with Portland cement to make fireproof shingles and asbestos wood, and it constitutes either the principal ingredient or the essential reinforcing material for most insulations in the range of temperatures from 150 to 1600 F. Among the insulations consisting principally of asbestos are laminated-felt types made up of successive layers of thin felts, the felted fiber types (asbestos fiber and bonding material) and the air cell types made up of successive layers of plain and corrugated asbestos paper.

It has an approximate **dielectric strength** of 4,000 volts per mm and a specific resistance of 16×10^4. Good asbestos should not burn or show any flame under the blowpipe nor should it be affected by concentrated hydrochloric acid. Asbestos wood is obtainable in standard sheets 36 × 48 in., and in thicknesses from ⅛ in. by ⅛ in. to 1 in., and then by ¼ in. increases to 2 in.

The **U. S. Navy specifications** call for **asbestos millboard** in standard sheets 40 × 40 in., the various thicknesses to have the following weights: ½ in., 25 to 29 lb; ⅜ in. 21 to 24 lb; ¼ in., 14 to 16 lb; 3/16 in., 11 to 12 lb; ⅛ in., 7 to 8 lb; 1/16 in., 3½ to 4½ lb. Asbestos millboard is generally made hard, but, if desired, can be made medium or soft. The material should stand a dry heat of 400 F without injury and should not be affected by acids.

The U. S. Navy specifies for **asbestos plaster for pipe covering** a mixture of 5 percent of long asbestos fiber, 65 percent of infusorial earth (see above) and 30 percent of fireproof binding material. The material must mix with water to proper consistency, and, after drying, should not burn or show any flame. It should not bake hard like a brick but be capable of compression. One bag, 140 lb net weight, should cover 40 sq ft of surface 1 in. thick.

Gulf States is preferable. Redwood shingles come 5½ butts to 2 in.; lesser thicknesses are more liable to crack and have shorter life. Shingles 8 in. wide or over should be split before laying. Dimension shingles of uniform width are obtainable. Dipping in oil or creosote adds to life. Creosote stains are both preservative and decorative. Shingles should be dipped before laying for best results. Wood shingles make a lasting roof, but materially increase the fire hazard.

Slate should be hard and tough and have a well-defined vein that is not too coarse. If too soft, they will absorb water; if too brittle, they are easily broken on cutting and installing. The surface when freshly split should have a bright metallic luster, be free from loose flakes or dull surfaces, and be straight and true. Slate should give a clear metallic ring when struck. Color is not an indication of quality. Black ribbon slate is cheaper than all-black slate and equally good if appearance is unimportant. The color of slate varies from dark blue, bluish black, and purple to gray and green. Stock sizes range from 7 × 9 in. to 14 × 24 in.; thickness, from ⅛ to ⅜ in., $^{3}/_{16}$ in. being the usual thickness. Approximate weight varies from 650 to 850 lb per square. Slate roofs rank well in regard to fire hazard but are not so good as tile on exposure to adjacent conflagrations.

Metallic roofings are laid in large sheets without sheathing (often strengthened by corrugating), sometimes cut into small sizes and laid as shingles, bent into interlocking shapes like tile, or soldered into a single structure as in tin roofing. Metal tile and metal shingles are usually made of copper, copper-bearing galvanized steel, tin plate, or zinc. The lightest weight metal shingle is the one made from copper which weighs from 72 to 118 lb per square. The metal radiates the heat resulting in lower temperatures beneath these roofs than with most other types of uninsulated roofs.

Tile. Hard-burned clay tiles with overlapping or interlocking edges cost about the same as slate. They should have a durable glaze and be well made. Unvitrified tiles with *slip* glaze are satisfactory in warm climates, but vitrified tiles only should be used in the North. Tile roofs weigh from 750 to 1,200 lb per square. Properly made tile does not deteriorate, is a poor conductor of heat and cold, and is not so brittle as slate.

Asbestos shingles and sidings are composed of Portland cement reinforced with asbestos fiber and formed under great pressure. They resist the destructive effect of time, weather, and fire. Asbestos shingles—American method—weigh about 500 lb per square and carry Underwriter's Class A label. Dutch lap and hexagonal or French method asbestos shingles weigh 250 to 300 lb per square and carry the Underwriter's B label. Asbestos shingles are made in a variety of colors and shapes. The asbestos roofing shingles have a smooth surface or a textured surface which represents wood graining.

Asbestos siding is commonly made in 12 × 24 in. strips and applied with a 10½ × 24 in. exposure. Such siding weighs about 200 lb per square and is produced in white, gray, and colors.

RUBBER, GUTTA PERCHA, AND BALATA

REFERENCES: Bedford and Winkelman, "Survey of Rubber Chemistry," Reinhold. Davis and Blake, "Chemistry and Technology of Rubber," Reinhold. Geer, "Rubber Compounding 1918–1938," Chemical Industries, vol. 42, p. 640, 1938. Hauser, "Latex," Reinhold. Schidrowitz, "Rubber," Van Nostrand. Whitby, "Plantation Rubber and the Testing of Rubber," Longmans.

These three substances, which are frequently confused, are all produced by the coagulation of the latex from certain shrubs and trees and are similar

of various sorts rendered absorptive by the addition of powdered chalk or whiting and after being molded into form covered with a glazing solution composed of starch, gelatin and turpentine; and **leatheroid**, a combination of chemically treated paper with rubber and sandarac, often used for trunks and suitcases.

NATURAL STONES

REFERENCE: Baker, "Masonry Construction," Wiley.

The important **qualities** in stone for building construction are cheapness, durability, and strength. Stones are seriously affected by weather, chemicals, gases, and temperature. Resistance to weathering depends upon both the hardness and the absorbent properties of the stone. Porosity is an objectionable element. If the constituents differ greatly in characteristics, weathering is apt to be unequal, consequently starting disintegration.

In general, crystalline structure is more durable than amorphous. Examination of the condition of the stone in the quarry affords some idea as to its durability. Further information can be obtained by artificial tests of weight, crushing strength, absorption, and resistance to freezing, to acids, and to heat.

The **structure** of the stone is of prime importance where heavy loads are to be carried. All stones are classified on the basis of the mineral forming the chief constituent, *viz.*, **siliceous stones**, such as granite, syenite, gneiss, trap, and quartz; **argillaceous or clayey stones**, in which alumina is the predominating mineral, such as slate; **calcareous stones** in which carbonate of lime predominates, such as limestone, marble, and dolomite. From a practical standpoint, stones are also divided into two classes—**stratified** and **unstratified**—the former being represented by such stones as marble, slate, etc., whereas the **unstratified** stones consist of an aggregate of crystalline grains such as granite, trap, and basalt lava.

The table on p. 720 gives the composition and some of the properties of the more common minerals.

Granite, composed of feldspar, quartz, and mica, is the strongest and most durable of stones in common use and is quarried into shape with facility. It is extremely hard and tough and can be wrought into elaborate shapes only with great difficulty. The feldspar content determines the coloring, while the quartz determines essentially the hardness.

Trap is a very strong and durable stone, but quarried and wrought with great difficulty. It is exceedingly tough and is widely used for roads and railroad ballast.

Syenite, composed of feldspar and hornblende, is crystalline, granular, and speckled black and white; it is hard, and takes a good polish. **Diorite** and **diabase** are similar stones and are frequently known as **greenstone**.

Serpentine is generally green in color and disintegrates in the open air. Soapstones and asbestos are related to it.

Trachytes are dense, frequently porous, feldspathic rocks with admixtures of crystals of hornblende, biotite, and magnetite. They are gray in color. Pumice stone is a variety.

Augitites. Basalt, a solidified product of volcanic emission, is a common variety, and the most durable and pressure-resisting structural stone; it is valuable for road-building and for bank and supporting walls. **Basalt lava** is very porous rock varying in color and hardness, used for stairs, paving blocks, and millstones.

Gneiss, a schistose structure of granite, is more subject to decomposition than granite, especially if rich in feldspar and mica.

Quartzite (pure or almost pure quartz) is characterized by its vitreous crystallization. When porous, it is used for millstones; it is also used in the manufacture of glass, and as broken stones or as gravel in road building, for preparing concrete, etc.

Mica schist consists of gray quartz between layers of mica. It is used for roofing plates.

To soften (for purposes of manipulation or for final properties): bituminous bodies such as the so-called "mineral rubbers," coal tar, wood tar, and their products, various vegetable and mineral oils, paraffin and petrolatum, sulphurized and sulphur chlorinated oils.

Vulcanization accessories, dispersion and wetting mediums, etc.: magnesium oxide zinc oxide, litharge, lime, stearic and other organic acids, degras, pine tars.

Coloring pigments: iron oxides, especially the red grades, zinc oxide, lithopone, titanium compounds, chromium oxide, ultramarine blues, carbon and lampblacks, antimony sulphides, etc., and organic pigments of various shades.

Specifications should state the percentage and kind of rubber desired, the specific gravity of compound, and suitable physical tests. Tensile strength and stretch tests are of importance, but vary widely with different compounds. A good 30 percent para compound should have a tensile strength of at least 1,000 lb per sq in. and should stretch to at least 5 times its length before breaking. A piece stretched to 3 times its length should show a permanent elongation of not more than 20 percent. High-grade sheet rubber shows a tensile strength of 2,700 lb per sq in., an ultimate elongation of over 600 percent, and a set of about 10 percent measured after 300 percent elongation for 1 min with 1 min rest. Lower grades show lower strength and elongation but greater set. The treads of the better grades of tires have a tensile strength of about 4,000 lb per sq in. Rubber compounds with a strength of even 5,000 lb per sq in. have been produced. Chemical analysis is of value in detecting rubber substitutes, reclaimed rubber, and foreign substances, but, generally speaking, does not serve to identify the grade of rubber. Rubber for permanent use should be slightly undervulcanized rather than overvulcanized. Rubber compounds are extensively used for electrical insulation, hose, steam and water packing, buffers, tiling, and numerous other purposes where electrical strength, resilience, and waterproofing qualities are desired.

Latex. The use of preserved latex has extended the use of rubber, improving old products and permitting the production of valuable new products. A striking new product is **vulcanized latex,** the invention of Schidrowitz. This latex retains its creamy fluidity and in spite of the fact that the particles of rubber are vulcanized, the liquid dries, by evaporation of the water, to give a strong, tough, and coherent film. The use of this latex permits the application of rubber to materials which are too sensitive to withstand the physical and chemical manipulation of the usual rubber factory.

Hard rubber, also known as **ebonite** and vulcanite, is made by vulcanizing rubber with a large quantity of sulphur and for a considerable time. It can be softened by heat and then be shaped or molded as desired, is capable of being highly polished when free from mineral admixtures, does not oxidize readily, and is not affected by air and sunlight, is very impervious to electricity but becomes charged with static electricity when rubbed. The substance is brown to black and resembles horn in appearance. The solvents that dissolve raw rubber and partly dissolve vulcanized rubber have no influence on hard rubber. It offers great resistance to all acids. If exposed for a long time to temperatures above 400 F, it does not melt but carbonizes. It is extensively used for switch handles, for covering tools for electrical purposes, insulating tubes for electrical conduits, fountain-pen handles, linings for acid vats, and numerous other purposes. Its use is being somewhat superseded by synthetic resinous plastics.

A **semihard rubber** is also made by varying the degree of vulcanization. This is more flexible and elastic than hard rubber and more durable than ordinary vulcanized rubber.

Reclaimed rubber is chiefly used as a cheap source of rubber hydrocarbon. A better product can be made at a low price by the use of reclaimed rubber than by the overloading of crude rubber with mineral fillers. In addition, reclaimed rubber is employed for its own specific properties. It is insoluble in the solvents of crude rubber, and its use is thus indicated where rubber comes into contact with oils, gasoline, etc. It may act as a dispersing medium, accelerates vulcanization, and often imparts manipu-

Conglomerates consist of rounded pebble stones formed by the aggregation of various rocks, held together by a binder such as clay or marl.

Gravel and **sand** consist of small fragments of quartzy minerals, often mixed with lime, marl, and clay, from which they can be liberated by washing. Pit sand is ordinarily gritty but frequently more impure than river sand.

Clay is a hydrous silicate of aluminum; as it occurs in nature it is frequently a mixture of clay with sand, limestone, and oxide of iron produced by the erosion of rocks containing feldspar minerals. The purest variety, **kaolin**, is used for the production of chinaware, and as a papermaking material. Plastic varieties are used for pipe clays, fireproof clays, and potter's clays. Clay shrinks in drying and subsequently in burning without losing its form as a whole, and becomes extremely hard. Certain varieties known as **fuller's earth** are very absorbent of oils or dyes. See also Brick, p. 700.

For strength and other physical properties of natural stones, see Table 3, p. 419.

PAINT OIL

(See also p. 674)

China wood oil (tung oil), expressed from the tung nut, is being increasingly used as a substitute for linseed oil in paints and varnishes. It has a peculiar rancid odor and yellow color, usually darker than that of linseed oil. It has the unique property of apparently drying throughout at a uniform rate when spread in a film instead of forming a skin by surface drying as does linseed oil. It dries "flat" instead of glossy, with an opaque, white film. When heated to about 400 F, it is converted into a jellylike mass insoluble in ordinary solvents. Specific gravity, from 0.936 to 0.944; its other constants are very similar to those of linseed oil. It can be mixed with lead and manganese driers. Its peculiar drying properties and resistance to water when dry have been successfully utilized in varnish making. It costs more than linseed oil.

Linseed oil is a brownish-yellow vegetable oil with a specific gravity between 0.931 and 0.937. It possesses the property of absorbing oxygen and consequently drying to a greater extent than most other oils; hence largely used for paints and varnishes. It is frequently adulterated with cottonseed, rosin, fish, mineral, and soybean oils. **Adulteration** can sometimes be detected by odor, color, or gravity, but chemical analysis is usually necessary. Mixtures of China wood, soybean, and linseed oils can be made which are extremely difficult to detect. Adulteration with mineral or rosin oil can frequently be detected by a green or blue fluorescence which appears when the oil is viewed on a black background with the back to the light.

As raw oil dries very slowly, it is frequently used as **"boiled oil."** The name was derived from the original practice of boiling the oil in open kettles at a temperature of 300 to 500 F and incorporating oxides of lead and manganese, this process greatly increasing the drying properties. Little if any boiled oil is now made in this way, as the original process was expensive and darkened the oil greatly. So-called "boiled oil" is now made by heating a small quantity of oil with lead and manganese oxides and subsequently adding this drier to the raw oil. A third class, known as **bunghole oil**, is made by adding driers dissolved in benzine or turpentine to the raw oil. If properly made the quicker processes produce satisfactory oil. Boiled oil should have a similar odor and taste to raw oil; specific gravity, between 0.931 and 0.950.

Perilla oil; treated **fish oil, soybean oil,** and **oiticica oil** are prominent among the newer oils used in paints and varnishes, generally in conjunction with some synthetic resins.

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REFRACTORIES

BY

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Types of Refractories

Fire-clay Refractories. Fire-clay brick are made from fire clays, which comprise all refractory clays that are not white burning. Fire clays can be divided into plastic clays and hard flint clays; they may also be classified as to alumina content.

Firebricks are usually made of a blended mixture of flint clays and plastic clays which is then formed, after mixing with water, to the required shape. Some or all of the flint clay may be replaced by highly burned or calcined clay, called **grog**. A large proportion of the modern bricks is molded by the dry-press or power-press process where the forming is carried out under high pressure and with a low water content. Extruded and hand-molded brick are still made in large quantities.

The dried bricks are burned either in periodic or tunnel kilns at temperatures varying between 2200 and 2600 F. Tunnel kilns give continuous production and a uniform temperature of burning.

Fire-clay bricks are divided into grades according to the A.S.T.M. designation: *Low-heat-duty brick*, fusion point above 2770 F (cone 19). *Moderate-heat-duty brick*, fusion point above 2905 F (cone 26). *Intermediate-heat-duty brick*, fusion point above 2940 F (cone 28). *High-heat-duty brick*, fusion point above 3055 F (cone 31). *Superduty fire-clay brick*, fusion point above 3171 F (cone 33).

Fire-clay bricks are used for boiler settings, kilns, malleable-iron furnaces, incinerators, and many portions of steel and non-ferrous metal furnaces. They are resistant to spalling and stand up well under many slag conditions, but are not generally suitable for use with high-lime slags, fluid-coal-ash slags, or under severe load conditions.

High-alumina brick are manufactured from raw materials rich in alumina, such as diaspore. They are graded into groups with 50, 60, 70, 80, and 90 percent alumina content. When well fired, these brick contain a large amount of mullite and less of the glassy phase than is present in the firebricks. Corundum is also present in many of these bricks. High-alumina brick are generally used for unusually severe temperature or load conditions. They are employed extensively in limekilns and rotary cement kilns, the ports and regenerators of glass tanks and for slag resistance in some metallurgical furnaces; their price is higher than firebrick.

Silica brick are manufactured from crushed ganister rock containing about 97 to 98 percent silica. A bond consisting of 2 percent lime is used, and the bricks are fired in periodic kilns at temperatures of 2700 to 2800 F for several days until a stable volume is obtained. They are especially valuable where a good strength is required at high temperatures.

Silica brick are used extensively in coke ovens, the roofs and walls of open-hearth furnaces, in the roofs and side walls of glass tanks, and as linings of

Coal Tar. Coal tar is more susceptible to temperature change than asphalt, and therefore for roofing purposes its use is usually confined to flat decks. Coal-tar built-up roofs are used extensively.

Asphalt prepared roofing is manufactured by impregnating a dry roofing felt with a hot asphaltic saturant. A coating consisting of a harder asphalt compounded with a fine mineral filler is applied to the weather side of the saturated felt. Into this coating is embedded mineral surfacing such as mineral granules, powdered talc, mica, or soapstone. The reverse side of the roofing has a very thin coating of the same asphalt which is usually covered with powdered talc or mica to prevent the roofing from sticking in the package. The surfacing used on **smooth-surfaced roll roofing** is usually powdered talc or mica. The surfacing used on **mineral or slate-surfaced roll roofing** is roofing granules in either natural colors prepared from slate or artificial colors usually made by applying a coating to a rock granule base. **Asphalt shingles** usually have a granular surfacing. They are made in strips and as individual shingles. The different shapes and sizes of these shingles provide single, double, and triple coverage of the roof deck.

Materials Used in Asphalt Prepared Roofing. The felt is usually composed of a continuous sheet of felted vegetable and animal fibers. High-grade roofing felts contain usually about 50 to 70 percent cotton, 10 to 20 percent wool, and 5 to 20 percent jute, manila, and wood fibers. The constituents can be varied to give felts with varying absorptive capacities.

The most satisfactory roofing **asphalts** are obtained by air blowing a steam- or vacuum-refined petroleum residual. Saturating asphalts must possess a low viscosity in order for the felt to become thoroughly impregnated. Coating asphalts must have good weather-resisting qualities and possess a high fusion temperature in order that there will be no flowing of the asphalt after the application to the roof. The coating asphalt is blown for a much longer time than is the saturating asphalt. It has been found that the addition of 30 to 40 percent of a finely ground mineral filler to the coating asphalt greatly improves its weathering qualities.

The fine **mineral surfacings** used on smooth-surfaced roll roofings are mica, talc, or soapstone. The coarse grade of mineral surfacing which is used on slate-surfaced roll roofing and asphalt shingles is either a crushed slate or rock. The coarse mineral granules are often colored artificially by coating them with pigmented silicate, cement, or a ceramic glaze and firing the granules to such a temperature that the coatings are permanent.

Asphalt built-up roofing usually consists of several layers of asphalt saturated felt with a continuous layer of hot mopped asphalt between the layers of felt. The top layer of such a roof may consist of a hot mopping of asphalt with slag or gravel embedded therein or a slate-surfaced cap sheet. Asphalt built-up roofs are never applied to an absolutely flat or dead level deck; the pitch must be from ¼ to 6 in. per ft.

Coal-tar built-up roofs are similar in construction to the asphalt built-up roofs. Several layers of tar saturated felt have a continuous layer of hot mopped pitch between each layer of felt. The tar built-up roofs usually have a top surfacing of slag or gravel. As coal tar is more susceptible to temperature change than asphalt, it is used only when the pitch is less than 2 in. per ft. Asphalt or asphalt-saturated felts should not be applied on the same deck with pitch or tar-saturated felts.

Wood Shingles. Wood shingles are usually manufactured in three different lengths: 16, 18, and 24 in. There are three grades in each length, the No. 1 grade being the best and the No. 3 grade being intended for purposes where the presence of defects will not be objectionable. See p. 684.

Clear white-pine shingles are the best. Red-cedar shingles of good quality are obtainable from the Pacific Coast. In the South, red cypress from the

of severe temperature and heavy load, or severe spalling conditions, as in the case of high-temperature oil-fired boiler settings, or piers under enameling furnaces. This type of brick has also been found of considerable value in the upper structure of glass tanks and checkers. Another brick for the same uses is a high-fired brick of Missouri aluminous clay.

There are a number of bricks on the market made from electrically fused materials, such as fused **mullite**, fused alumina, and fused **magnesite**. These bricks although high in cost are particularly suitable for certain severe conditions.

Bricks of **silicon carbide** (see p. 695), either recrystallized or clay bonded, have a high thermal conductivity and find use in muffle walls and as a slag-resisting material.

Other types of refractory that find certain limited use are **forsterite, zirconia,** and **zircon.** Acid-resisting brick consisting of a dense body like stoneware are used for lining tanks and conduits in the chemical industry.

Natural stones such as sandstone or mica schist are used as refractories in metallurgical furnaces such as the linings of Bessemer converters. Soapstone is used in soda and sulphate recovery furnaces, but is being replaced to some extent by chrome.

The **chemical composition** of some of the refractories is given in Table 1. The **physical properties** are given in Table 2. Reference should be made to A.S.T.M. standards for details of standard tests.

Table 1. Chemical Composition of Typical Refractories[a]

No.	Refractory type	SiO_2	Al_2O_3	Fe_2O_3	TiO_2	CaO	MgO	Cr_2O_3	SiC	Alkalies	Resistance to: Siliceous steel-slag	High-lime steel-slag	Fused mill-scale	Coal-ash slag
1	Alumina (fused)	8-10	85-90	1-1.5	1.5-2.7					0.8-1.3[c]	E	G	F	G
2	Chrome	6	23	15[b]							G	E	E	G
3	Chrome (unburned)	5	18	12[b]	...	...	...	...	...	...	G	E	E	G
4	Fire clay (high-heat duty)	50-57	36-42	1.5-2.5	1.5-2.5	...	...	...	...	1-3.5[c]	F	P	P	F
5	Fire clay (super-duty)	52	43	1	2	...	...	...	...	2[c]	F	P	F	F
6	Forsterite	34.6	0.9	7.0	...	1.3	55.4							
7	High-alumina	22-26	68-72	1-1.5	3.5	...	...	...	...	1-1.5[c]	G	F	F	F
8	Kaolin	52	45.4	0.6	1.7	0.1	0.2	...	...	...	P	P	G[d]	F
9	Magnesite	3	2	6	...	3	86		...	...	P	E	E	E
10	Magnesite (unburned)	5	7.5	8.5	...	2	64	10	...	...	P	E	E	E
11	Magnesite (fused)	...	...	...	...	...	...	...	...	...	F	E	E	E
12	Refractory porcelain.	25-70	25-60	...	...	...	...	...	...	1-5	G	F	F	F
13	Silica	96	1	1	...	2	...	...	...	...	E	P	F	P
14	Silicon carbide (clay bonded)	7-9	2-4	0.3-1	1	...	...	...	85-90	...	E	G	F	E
15	Sillimanite (mullite)	36	62	0.5	1.5	...	...	...	...	0.5[c]	G	F	F	F
16	Insulating fire-brick (2600 F)	67.7	36.0	2.4	1.5	0.6	0.5	...	...	...	P	P	G[e]	P

[a] Much of this data has been taken from a table prepared by L. J. Trostel, *Chem. Met. Eng.*, Nov., 1938.

[b] As FeO.

[c] Includes lime and magnesia.

[d] Excellent if left above 1200 F.

[e] Oxidizing atmosphere.

E. Excellent. G. Good. F. Fair. P. Poor.

in many respects. The essential difference between gutta percha and rubber is that the former becomes markedly soft and plastic on immersion in hot water, retaining any shape then given to it on cooling, whereupon it becomes hard but not brittle like other gums. Rubber or caoutchouc, on the other hand, does not soften materially in hot water and retains its original elasticity and strength almost unimpaired. Balata behaves on heating very much the same as gutta percha but possesses very little elasticity. It can be vulcanized, but without improved properties, whereas rubber and gutta percha are readily vulcanized and possess different properties after vulcanization.

Rubber is obtained principally from the far East—from the cultivated plantations of Ceylon, the Malay Peninsula, the Straits Settlement, Java, Sumatra, etc. In addition, about 5 percent of the world's supply is obtained as wild rubber, the best of which is the well-known fine para from Brazil. The tropical sections of South America and Africa produce various grades of rubber which are of declining importance. The cultivated Guayule of Mexico and New Mexico furnishes excellent rubber, which may, in time, compete to some extent with Hevea rubber.

High-grade rubber will stretch to approximately 10 times its length and at this point will bear a load of 10 tons per sq in. It can be compressed to one-third its thickness thousands of times without injury. When vulcanized for elasticity, it behaves in some respects like other engineering materials up to a load of 30 percent of the breaking load and at an extension of one-half the maximum. From this point on, however, the resistance increases in greater proportion than the extension, and finally assumes a comparatively high value. Even when stretched almost to the point of rupture, it restores itself very nearly to its original dimensions on being released, and gradually recovers a part of the residual loss of form.

The dielectric constant at 1,000 cycles is 2.37 for purified rubber, 2.68 for soft vulcanized, and 2.82 for hard rubber. The specific gravities are 0.906, 0.923, and 1.173, respectively. The color of vulcanized rubber containing only rubber and sulphur is yellow to brown; mineral and other additions may vary the color nearly to black.

Freshly cut or torn raw rubber possesses the power of self-adhesion which practically disappears in vulcanized rubber. Cold water indefinitely preserves rubber, but if exposed to the air, and particularly to the sun, all rubber goods tend to become hard and brittle. Dry heat up to 120 F should have little deteriorating effect, but at temperatures of 360 to 400 F rubber begins to melt, loses its elasticity, and becomes sticky; at higher temperatures, it becomes entirely carbonized. Unvulcanized rubber is soluble in carbon bisulphide, benzine, petroleum ether, and turpentine, but only readily so directly after milling.

Rubber is seldom used alone industrially except for surgical tape, rubber cements, and other minor purposes. Most rubber is vulcanized, *i.e.*, made to combine with sulphur, or in the case of some thin goods, sulphur chloride. Vulcanization if properly carried out improves the tensile properties, eliminates tackiness, renders the rubber less susceptible to temperature changes, and makes it insoluble in all known solvents. It is impossible to dissolve vulcanized rubber unless it is first decomposed. Other ingredients are added for general effects as follows:

To increase tensile strength and resistance to abrasion: carbon black, zinc oxide, magnesium carbonate, glue, certain clays, as well as most of the organic vulcanization accelerators.

To cheapen and stiffen: whiting, barytes, soapstone, silica and silicates, stale flour, infusorial earth, clays, fibrous materials.

The variation of the specific heat of refractories with temperature is indicated in Table 3 which gives data from Bradshaw and Emery (*Trans. Ceram. Soc.* (England), 19, 84–92, 1919–'20.)

Table 3. Specific Heat of Refractories

Temperature, deg F	Mean specific heat, 77 to t deg F			
	Silica 1	Silica 2	Fireclay	Zircon
1112	0.225	0.223	0.225	0.137
1832	0.255	0.252	0.255	0.157
2192	0.252	0.255	0.254	0.167
2552	0.293	0.295	0.297	0.175

Heyn (*Mitt. konig. Materialprufungsamt*, Jahrg. 32, 1914 p. 185) gives the following data on the specific heat of magnesite bricks:

Temp, t deg F	200	400	600	800	1000	1500	2000	2500
Mean specific heat between 77 and t deg F	0.219	0.233	0.244	0.252	0.257	0.270	0.282	0.294

Wilson, Holdcroft, and Mellor (*Trans. Ceram. Soc.* (England), 12, 1912–1913, p. 279) give the following formula for the specific heat of fire-clay bricks:

Mean specific heat $= 0.192 + 0.000033t$, where t is in deg F.

Standard and Special Shapes

There are a large number of **standard refractory** shapes carried in stock by most manufacturers. The manufacturers' catalogues should be consulted in selecting these shapes, but the more common ones are shown in Table 4. These shapes have been standardized by the American Refractories Institute and by the Bureau of Simplification of the U.S. Department of Commerce.

Special shapes are more expensive than the standard refractories, and, as they are usually hand molded, will not be so dense or uniform in structure as the regular brick. When special shapes are necessary, they should be laid out as simply as possible and the maximum size should be kept down below 30 in. if possible. It is also desirable to make all special shapes with the vertical dimension as an even multiple of 2½ in. plus one joint so that they will bond in with the rest of the brickwork.

Mortars, Coatings, Plastics, Castables, and Ramming Mixtures

Practically all brickwork is laid up with some type of **mortar** to give a more stable structure and to seal the joints. This mortar may be ground fire clay or a specially prepared mortar containing grog to reduce the shrinkage. The bonding mortars may be divided into three general classes. The first are **air-setting mortars** which often contain silicate of soda to give a strong bond when dried or fired at comparatively low temperatures. Many of the air-setting mortars should not be used at extremely high temperatures because the fluxing action of the air-setting ingredient reduces the fusion point. The second class is called **heat-setting mortar** and requires temperatures of over 2000 F to produce a good bond. These mortars vary in vitrifying point, some producing a strong bond in the lower temperature ranges, and the others requiring very high temperatures to give good strength. The third classification comprises **special-base mortars** such as silica, magnesite,

lative properties which are difficult to obtain when crude rubber is the sole source of the rubber hydrocarbon.

Rubber Substitutes. This name has been applied to sulphurized or sulphurchlorinated oils which are compounding ingredients and in no way true substitutes for rubber.

Synthetic rubbers are now available of several chemical types. **Neoprene** is a *polymerized chloroprene* and differs from rubber structurally in that chlorine replaces the methyl group in the molecule. **Buna S** and **Buna N** are butadiene rubbers, combined with Styrene and acrylonitrile, respectively. **Butyl** rubber is a synthetic product (1940) made principally from the olefines from the cracking of petroleum, but containing a small proportion of diolefines. **Thiokol** is a rubberlike polymerized ethylene sulfide. *Each of these synthetic types offers one or more advantages over* natural rubber, such as resistance to oil, to oxidation, or to heat or light, and some of them are of great strength or resistance to wear.

Gutta Percha. Pure gutta percha is colorless, and, when cut thin, transparent. It is tasteless, inodorous, softens at 99 F, and can be molded readily at 195 F. If heated above 265 F, it melts to a colorless oil; it is not affected by cold and is very resistant to water. When exposed to air and sunlight, it rapidly deteriorates. It possesses excellent heat resistance, and the greatest electrical resistance of any plastic material.

Gutta percha is insoluble in most reagents, is slightly soluble in pure alcohol and partly dissolved by turpentine, olive oil, and benzine. The best solvents are carbon bisulphide and chloroform. It is very little affected by acids or alkalies. On account of its great electrical resistance, waterproof qualities, and permanency under water, it was largely used in the first transatlantic cables. *It is little used today except for the manufacture* of golf balls and minor uses.

Balata. *Raw balata is gray, brown, or whitish red and possesses little elasticity.* It is softer than gutta percha under ordinary temperatures, does not become so firm when cooled, is soluble in turpentine, benzene, and carbon bisulphide, but resistant to acid and alkalies. It can be readily molded at 125 F. On account of its great toughness and resistance to moisture and air, it was formerly used for the manufacture of rubber belts. Its present chief uses are for the covers of golf balls and as a heat-plastic adhesive for combining fabrics and mending tissue.

SHELLAC

The most commonly used shellac for pattern and varnish work is the T. N. grade, *which usually contains about 3 percent of rosin. A grade called* Garnet lac sells at about the same price as T. N. and is free from rosin or lac-wax.

Shellac is soluble in alcohol, hot aqueous solutions of sodium and potassium carbonates, caustic alkalies, borax, and acetic acid. Its specific resistance is approximately 9×10^9 megohms per cu cm; **specific inductive capacity**, about 2.74. It is slightly hygroscopic but not sufficiently so to impair its value as an electrical insulator. Shellac softens at 181 F, flows readily at 230 to 248 F, hardens again at 356 to 384 F and begins to carbonize at 446 F.

Shellac is used extensively as a sticker and insulator in the manufacture of electrical apparatus, for bonding abrasive wheels, and as a bond for a variety of molded goods. The most common **adulterant** is rosin, which makes the shellac brittle. A common test is to heat a piece of shellac with a lighted match until one corner is melted, then blow out the flame and attach the match to the piece of shellac and let the latter drop to the floor. If the shellac is of good quality and ordinarily free from adulteration, it will string out in a thin flexible thread from the height of a man's hand held as high as possible from the floor. It has been found that the addition of 5 percent of rosin increases the adhesiveness. **Bleached shellac** is used very largely in lacquers and has less adhesive strength than **orange shellac**. It is not completely soluble in alcohol and nearly always contains some chlorine. The ordinary spirit varnish is made by dissolving from 4½ to 9 lb of shellac in a gallon of alcohol. 4½ lb per gal makes 1½ gal of varnish and 9 lb per gal makes 2 gal of varnish.

Table 5. Additional Arch, Wedge, and Tier Bricks

Name of shape	a	b	b'	c	c'	Name of shape	a	b	c
Large 9 in. No. 1 wedge	9	6¾	.....	2½	1⅞	13½ in. No. 1 wedge	13½	6	2¾‡
Large 9 in. No. 2 wedge	9	6¾	.....	2½	1½	13½ in. No. 2 wedge	13½	6	2½‡
No. 1 flat-back arch	9	6	.....	3½	2¾	13½ in. No. 3 wedge	13½	6	2‡
No. 2 flat-back arch	9	6	.....	3½	2	No. 102 angle bung	12¾*	4½	‡
9 × 6 in. No. 1 key	9	6	5⅜	2½†	.....	No. 103 arch bung	13	4½	2⅝‡
9 × 6 in. No. 2 key	9	6	4 13/16	2½†	.....	No. 104 arch angle bung	12¾*	4½	2⅝‡
13½-in. No. 1 key	13½	6	5	2½†	.....	No. 105 arch bung	13	4½	2⅛‡
13½ in. No. 2 key	13½	6	4⅜	2½†					

* a' = 11⅝. † = 2½ or 3. ‡ = 3.

Table 6. Circle Bricks and Blocks

The dimensions ($a \times b \times c$) of circle bricks are 9 × 4½ × 2½; of rotary kiln blocks 9 × 9 × 4; of 6 in. cupola block, 9 × 6 × 4; of 9 in. cupola block 9 × 4½ × 9 in.

Name of brick	Diameter of circle: Outside	Diameter of circle: Inside	No. of bricks to a circle	Name of brick	Diameter of circle: Outside	Diameter of circle: Inside	No. of bricks to a circle	Name of brick	Diameter of circle: Outside	Diameter of circle: Inside	No. of bricks to a circle
24 in. Circle brick	33	24	12	9-90 Rotary-kiln block	108	90	38	No. 90 6 in. Cupola block	102	90	36
36 in. Circle brick	45	36	16	9-96 Rotary kiln block	114	96	40	No. 96 6 in. Cupola block	108	96	38
48 in. Circle brick	57	48	20	9-102 6 in. kiln block	120	102	42	No. 102 6 in. Cupola block	114	102	40
60 in. Circle brick	69	60	24	No. 30 6 in. Cupola block	42	30	15	No. 108 6 in. Cupola block	120	108	42
72 in. Circle brick	81	72	28	No. 36 6 in. Cupola block	48	36	17	No. A 9 in. Cupola block	25	16	9
84 in. Circle brick	93	84	32	No. 42 6 in. Cupola block	54	42	19	No. B 9 in. Cupola block	30	21	11
9-48 Rotary kiln block	66	48	23	No. 48 6 in. Cupola block	60	48	21	No. C 9 in. Cupola block	36	27	13
9-54 Rotary kiln block	72	54	25	No. 54 6 in. Cupola block	66	54	23	No. D 9 in. Cupola block	39	30	14
9-60 Rotary kiln block	78	60	27	No. 60 6 in. Cupola block	72	60	25	No. E 9 in. Cupola block	49	40	17
9-66 Rotary kiln block	84	66	29	No. 66 6 in. Cupola block	78	66	27	No. F 9 in. Cupola block	60	51	21
9-72 Rotary kiln block	90	72	31	No. 72 6 in. Cupola block	84	72	29	No. G 9 in. Cupola block	69	60	24
9-78 Rotary kiln block	96	78	33	No. 78 6 in. Cupola block	90	78	31	No. H 9 in. Cupola block	82	73	29
9-84 Rotary kiln block	102	84	36	No. 84 6 in. Cupola block	96	84	33				

acid electric steel furnaces. Although silica brick is readily spalled (cracked by a temperature change) below red heat, it is very stable if the temperature is kept above this range, and for this reason stands up well in regenerative furnaces. Any structure of silica brick should be heated up slowly to the working temperature; a large structure often requires two weeks or more to bring up.

Magnesite brick are made from crushed magnesium oxide which is produced by calcining raw magnesite rock to high temperatures. A rock containing several percent of iron oxide is preferable as this permits the rock to be fired at a lower temperature than if pure materials were used. Magnesite brick are generally fired at a comparatively high temperature in periodic or tunnel kilns, though large tonnages of unburned brick are now produced. The latter are made with special grain sizing and hydraulically pressed.

Magnesite brick are basic and are used wherever it is necessary to resist high-lime slags such as in the basic open-hearth furnace. They also find use in furnaces for the lead and copper refining industry. The hydraulically-pressed unburned brick find extensive use as linings for cement kilns. Magnesite brick are not so resistant to spalling as fire-clay brick.

Chrome brick are manufactured in much the same way as magnesite brick, but are made from natural chromite ore. Commercial ores always contain magnesia and alumina. Unburned hydraulically-pressed chrome brick are also made.

Chrome bricks are very resistant to all types of slag. They are used as separators between acid and basic refractories, also in soaking pits and floors of forging furnaces. The unburned hydraulically-pressed brick now find extensive use in the walls of the open-hearth furnace. Chrome bricks are used in sulphite-recovery furnaces and to some extent in the refining of nonferrous metals.

The insulating refractory is a class of recently developed bricks which consist of a highly porous fire clay or kaolin. They are light in weight (about ½ to ¼ that of fireclay), low in thermal conductivity, and yet sufficiently resistant to temperature to be used successfully on the hot side of the furnace wall, thus permitting thin walls of low thermal conductivity and low heat content. The low heat content is particularly valuable in saving fuel and time on heating up, allows rapid changes in temperature to be made, and permits rapid cooling. These bricks are made in a variety of ways, such as mixing organic matter with the clay and later burning it out to form pores; or a bubble structure can be incorporated in the clay-water mixture which is later preserved in the fired brick. The insulating refractories are classified into several groups according to the maximum temperature that they can withstand on the hot face; the ranges are, up to 1550, 1950, 2250, 2550, and above 2750 F for 2 percent linear shrinkage.

Insulating refractories are used mainly in the heat-treating industry for furnaces of the periodic type; the low heat content permits noteworthy fuel savings as compared with firebrick. They are also used extensively in stress-relieving furnaces, chemical-process furnaces, oil stills or heaters, and in the combustion chambers of domestic-oil-burner furnaces; they usually have a life equal to the heavy brick that they replace. They are particularly suitable for constructing experimental or laboratory furnaces because they can be cut or machined readily to any shape. They are not resistant to fluid slag.

There are a number of types of **special brick**, obtainable from individual manufacturers. High-burned kaolin refractories are particularly valuable under conditions

Solid walls built with standard 9 in. brick are made up in various ways; 9 in. walls with alternate header and stretcher courses; 13½ in. walls with alternate header and stretcher courses, or with four stretcher courses to one header course on the hot face, or with four header courses to one stretcher course on the hot face; 18 in. walls with alternate header and stretcher courses.

Table 7. Transmitted Heat Losses and Heat-storage Capacities of Wall Structures under Equilibrium Conditions

(Based on still air at 80 F)

Condensed from "B & W Insulating Firebrick" Bulletin of the Babcock and Wilcox Co.

Thickness, in.		Hot face temperature, deg F									
Wall	Of insulating refractory and firebrick	1200		1600		2000		2400		2800	
		HL	HS	HL	HS	HL	HS	HL	HS	HL	HS
	4½ 20	355	1,600	537	2,300	755	2,900				
4½	4½ 28	441	2,200	658	3,100	932	4,000	1,241	4,900	1,589	5,900
	4½ FB	1,180	8,480	1,870	11,700	2,660	14,800	3,600	18,100	4,640	21,600
7	4½ 28 + 2½ 20	265	3,500	408	4,900	567	6,500	751	8,100	970	9,600
	4½ FB	423	12,500	660	17,700	917	23,000	1,248	28,200		
9	4½ 28 + 4½ 20	203	4,100	311	5,900	432	7,900	573	9,900	738	12,200
	4½ FB + 4½ 20	285	13,700	437	19,200	615	24,800				
	9 20	181	3,100	280	4,300	395	5,500				
	9 28	233	4,100	349	5,800	480	7,500	642	9,300	818	11,100
	9 FB	658	15,800	1,015	21,600	1,430	27,600	1,900	34,000	2,480	40,300
11½	9 28 + 2½ 20	169	5,700	260	8,000	364	10,500	484	13,100	623	15,800
	9 FB + 2½ 20	335	22,300	514	31,400	718	40,600	962	50,400	1,233	60,300
	9 28 + 4½ 20	143	6,500	217	9,300	305	12,300	404	15,300	514	18,700
	9 FB + 4½ 20	241	24,100	367	34,500	516	44,800	690	55,100		
13½	9 20 + 4½ FB	165	5,300	255	7,300	348	9,900				
	9 28 + 4½ FB	200	6,900	302	9,700	415	12,600	556	15,700	710	19,100
	13½ FB	452	22,300	700	31,000	980	39,900	1,310	49,100	1,683	58,300
16	13½ FB + 2½ 20	275	31,200	423	43,300	588	56,300	780	70,000	994	84,200
18	9 20 + 9 FB	147	8,500	225	11,900	319	15,700				
	9 28 + 9 FB	175	10,700	266	15,100	375	19,700	493	24,600	635	29,800
	13½ FB + 4½ 20	210	34,100	318	48,400	440	62,600	587	77,500	753	92,600
	18 FB	355	28,800	532	40,300	745	52,200	1,000	64,200	1,283	76,500
20½	18 FB + 2½ 20	234	39,000	356	55,400	500	72,000	665	89,200	847	107,000
22½	18 FB + 4½ 20	182	43,200	281	61,000	392	79,200	519	97,700	667	117,600
	22½ FB	287	36,000	435	49,500	612	64,100	814	78,800	1,040	93,400

HL Heat loss in Btu per sq ft per hr. HS Heat storage capacity in Btu per sq ft. 20 = 2000 F insulating refractory. 28 = 2800 F insulating refractory. FB Fire-clay brick.

Many modern furnaces are constructed with air-cooled walls, with refractory blocks held in place against a casing by alloy steel holders. Sectional walls made up with steel panels having lightweight insulating refractories attached to the inner surface are also used and are especially valuable for use in the upper parts of large boiler furnaces, oil stills, and similar types of construction. The sections can be made up at the plant and shipped as a

Table 2. Physical Properties of Typical Refractories[a]
(Refractory numbers refer to *Table 1*)

Refractory No.	Fusion point, Deg F	Fusion point, Pyrometric cone	Deformation under load, percent at deg F and lb per sq in.	Spalling resistance	Reheat shrinkage after 5 hr percent at (deg F)	Wt. of straight 9 in. brick, lb
1	3390+	39+	1 at 2730 and 50	Good	+0.5 (2910)	9-10.6
2	3580+	41+	Shears 2740 and 28	Poor	−0.5 to 1.0 (3000)	11.0
3	3580+	41+	Shears 2955 and 28	Fair	−0.5 to 1.0 (3000)	11.3
4	3060-3170	31-33	2.5–10 at 2460 and 25	Good	±0 to 1.5 (2550)	7.5
5	3170-3200	33-34	2–4 at 2640 and 25	Excellent	±0 to 1.5 (2910)	8.5
6	3430	40	10 at 2950	Fair	....	9.0
7	3290	36	1–4 at 2640 and 25	Excellent	−2 to 4 (2910)	7.5
8	3200	34	0.5 at 2640 and 25	Excellent	−0.7 to 1.0 (2910)	7.7
9	3580+	41+	Shears 2765 and 28	Poor	−1 to 2 (3000)	10.0
10	3580+	41+	Shears 2040 and 28	Fair	−0.5 to 1.5 (3000)	10.7
11	3580+	41+		Fair	....	10.5
12	2640-3000	16-30		Good		
13	3060-3090	31-32	Shears 2900 and 25	Poor[b]	+0.5 to 0.8 (2640)	6.5
14	3390	39	0–1 at 2730 and 50	Excellent	+2[c] (2910)	8-9.3
15	3310-3340	37-38	0–0.5 at 2640 and 25	Excellent	−0 to 0.8 (2910)	8.5
16	2980-3000	29-30	0.3 at 2200 and 10	Good	−0.2 (2600)	2.25

Refractory No.	Porosity	Specific heat 60–1200 F	Mean coefficient of thermal expansion from 60 F to shrinkage point × 10[6]	Mean thermal conductivity, Btu per sq ft per hr per deg F per in. thickness; Mean temperatures between the hot and cold face, deg F: 200	400	800	1200	1600	2000	2400
1	20–26	0.20	0.43	..	20	22	24	27	30	32
2	20–26	0.20	0.56	..	8	9	10	11	12	12
3	10–12	0.21								
4	15–25	0.23	0.25–0.30	5	6	7	8	10	11	12
5	12–15	0.23	0.25–0.30	6	7	8	9	10	12	13
6	23–26	0.25								
7	28–36	0.23	0.24	6	7	8	9	10	12	13
8	18	0.22	0.23	..	....	11	12	13	13	14
9	20–26	0.27	0.56–0.83	...	40	35	30	27	26	25
10	10–12	0.26								
11	20–30	0.27	0.56–0.80							
12	.....	0.23	0.30	..	14	15	17	18	19	20
13	20–30	0.23	0.46[d]	..	8	10	12	13	14	15
14	13–28	0.20	0.24	....	....	100	80	65	55	50
15	20–25	0.23	0.30	..	10	11	12	13	14	15
16	75	0.22	0.25	..	1.6	2.0	2.6	3.2	3.8	

[a] Much of this data has been taken from a table prepared by L. J. Trostel, *Chem. Met. Eng.*, Nov., 1938.
[b] Excellent if left above 1200 F.
[c] Oxidizing atmosphere.
[d] Up to .56 at red heat.

the shut-down period of the furnace. Where slag or abrasion is severe, brick with a dense structure is desirable. If spalling conditions are important, a brick with a more flexible structure is better, although there are cases where a very dense structure gives better spalling resistance than a more open one.

High-lime slag can be taken care of with magnesite, chrome, or high alumina brick; but if severe temperature fluctuations are encountered also, no brick will give long life. For coal-ash slag, dense fire-clay bricks give fairly good service if the temperature is not high. At the higher temperatures, a chrome-plastic or silicon-carbide refractory often proves successful. When the conditions are unusually severe, air- or water-cooled walls must be resorted to; the water-cooled stud-tube wall has been very successful in boiler furnaces.

With a general freedom from slag, it is often most economical to use an insulating refractory. Although this brick may cost more per unit, it allows thinner walls, so that the total construction cost may be no greater than the regular brick. The substitution of insulating refractory for heavy brick in periodic furnaces has sometimes halved the fuel consumption.

The stability of a refractory installation depends largely on the bricklaying. The total cost, in addition to the bricks, of laying brick varies with the type of construction, locality, and refractory; but a figure of $100 per thousand is a fair allowance on small jobs; on a large structure, such as an open-hearth furnace, where approximately 1,000,000 brick would be laid, the total cost in addition to the bricks might be somewhat less than $50 per thousand.

Table 8. Refractory Prices

(Per thousand, carload lots at destination)

	Approximate costs in 1939
Fire-clay brick	$48.00 F.W.
Superduty fire clay brick	$61.00 F.W.
High-alumina brick	(50%, $85) (60%; $105) (70%, $130) F.W.
Kaolin brick	$240 (with mortar) F.W.
Mullite brick	$500 F.W.
Magnesite brick	$335 F.W.
Chrome brick	$270 F.W.
Clay-bonded silicon-carbide brick	$1,050 F.F.
Fused-magnesite brick	$1,500 F.F.
Fused-alumina brick	$1,140 F.F.
Insulating fire brick	(1960 F, $78) (2250 F, $102) (2550 F, $124) D.P.
Finely-ground fire clay for laying high-heat duty fire clay brick, in sacks	$11.35 N.T.
Superduty fire clay, in sacks	$22.30 N.T.
High-alumina fire clay, in sacks	$47.50 N.T.
Silica fire clay (for laying silica brick), in sacks	$12.80 N.T.
Silica-base high-temperature bonding-mortar, carload lots (in drums)	$65.00 N.T.
Chrome-base high-temperature bonding-mortar, carload lots (in drums)	$75.00 N.T.

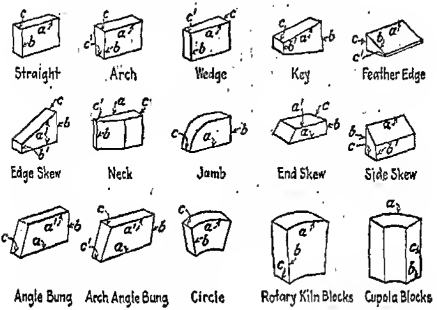
F.W. = F.O.B. works. F.F. = F.O.B. factory. D.P. = delivered in Pittsburgh. N.T. = net ton.

or chrome, which are specially blended for use with their respective bricks. The chrome-base mortar may be satisfactorily used with fire-clay bricks in many cases.

The refractory bonding mortars should preferably be selected on the advice of the manufacturer of the refractory to obtain good service, although there

Table 4. Sizes and Shapes of Firebrick

(All dimensions in in.)



Name of bricks	Straight bricks Length a	Width b	Thickness c	Name of bricks	Straight bricks Length a	Width b	Thickness c
9 in. straight.....	9	4½	2½	9 × 4½ × 3 in. straight........	9	4½	3
Small 9 in.......	9	3½	2½	9 × 6 × 3 in. straight.........	9	6	3
Soap............	9	2¼	2½	13½ in. straight..	13½	6	2½
Checker.........	9	2¾	2¾	Bridge block......	13½	6	3
Split brick.......	9	4½	1¼	Stock-hole tile....	18	9	4½
2 in. brick.......	9	4½	2	Square-edge tile ..	12	12	3
Large 9 in.......	9	6¾	2½	No. 101 square bung	13	4½	3
Flat back straight	9	6	2½	Open-hearth checker........	10½	4½	4½

Regenerator tile sizes, $a \times b \times c$ are: 18 × 6 or 9 × 3; 18 × 9 or 12 × 4; 22½ × 6 or 9 × 3; 22½ × 9 or 12 × 4; 27 × 9 × 3; 27 × 9 or 12 × 4; 31½ × 12 × 4; 36 × 12 × 4.

The following **arch, wedge, and key bricks** have maximum dimensions, $a \times b \times c$ of 9 × 4½ × 2½ in. The minimum dimensions a', b', c' are as noted: No. 1 arch, $c' = 2\frac{1}{8}$; No. 2 arch, $c' = 1\frac{3}{4}$; No. 3 arch, $c' = 1$; No. 1 wedge, $c' = 1\frac{7}{8}$; No. 2 wedge, $c' = 1\frac{1}{2}$; No. 3 wedge, $c' = 2$; No. 1 key, $b' = 4$; No. 2 key, $b' = 3\frac{1}{2}$; No. 3 key, $b' = 3$; No. 4 key, $b' = 2\frac{1}{4}$; Edge skew, $b' = 1\frac{1}{2}$; Feather edge, $c' = \frac{1}{8}$; No. 1 neck, $a' = 3\frac{1}{2}$, $c' = \frac{5}{8}$; No. 2 neck, $a' = 2\frac{1}{2}$, $c = \frac{5}{8}$; No. 3 neck, $a' = 0$, $c' = \frac{5}{8}$; End skew, $a' = 6\frac{3}{4}$; Side skew, $b' = 2\frac{1}{4}$; Jamb brick, 9 × 2½; Bung arch, $c' = 2\frac{3}{8}$.

S.A.E. viscosity number	Saybolt viscosity range, sec at 130 F	S.A.E. viscosity number	Saybolt viscosity range, sec at 210 F
10	90–120	40	<80
20	120–185	50	80–105
30	185–255	60	105–125
40	>255	70	125–150

In the case of prediluted oils, S.A.E. Viscosity Numbers by which the oils are classified shall be determined by the viscosity of the undiluted oils.

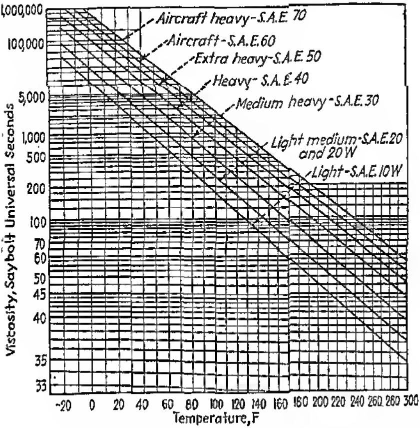


FIG. 1.—Variation of Viscosity with Temperature (A.S.T.M. D341-39 Viscosity Chart).

The S.A.E. viscosity numbers for transmission and rear axle lubricants constitute a classification in terms of viscosity and also of consistency at low temperatures. They are as follows:

S.A.E. viscosity number	Viscosity range, Saybolt Universal	Consistency. Must not channel in service at deg F
80	100,000 sec at 0 F, max.	Minus 20
90	800 to 1500 sec at 100 F	Zero
140	120 to 200 sec at 210 F	Plus 35
250	200 sec at 210 F, min.	....

are a considerable number of independent manufacturers of mortars who supply an excellent product. From 300 to 400 lb of dry mortar per thousand brick is required for thin joints, which are desirable in most furnace construction. For thicker trowel joints, up to 500 lb per thousand brick is required. In the case of chrome-base mortars, 600 lb per thousand brick should be allowed, and for magnesite cement 800 lb.

The working properties of the bonding mortar are important. Mortars for insulating refractories should be carefully selected as many of the commercial products do not retain their water sufficiently long to enable a good joint to be made. There are special mortars for this purpose which are entirely satisfactory.

Coatings are used to protect the hot surface of the refractories, especially when they are exposed to dust-laden gases or slags. These coatings usually consist of ground grog and fire clay of a somewhat coarser texture than the mortar. There are also chrome-base coatings which are quite resistant to slags, and in a few cases natural clays containing silica and feldspar are satisfactory.

The coatings can be applied to the surface of the brickwork with a brush in thin layers about $\frac{1}{16}$ in. thick, or they may be sprayed on with a cement gun, the latter method generally giving the best results. Some types of coating can be put on in much thicker layers, but care should be taken to assure that the coating selected will fit the particular brick used, otherwise it is apt to peel off in service. The coating seals the pores and openings in the brickwork and presents a more continuous and impervious service to the action of the furnace gases and slag; it is not a cure-all for refractory troubles.

Plastics and ramming mixtures are generally a mixture of fire clay and coarse grog of somewhat the same composition as the original fire-clay brick. They are used in repairing furnace walls which have been damaged by spalling or slag erosion, and also for making complete furnace walls in certain installations such as small boiler furnaces. They are also used to form special or irregular shapes, in temporary wooden forms, in the actual furnace construction.

Some of the plastics and ramming mixtures contain silicate of soda and are air setting, so that a strong structure is produced as soon as the material is dry. Others have as a base chrome ore or silicon carbide, which make a mixture having a high thermal conductivity and a good resistance to slag erosion. These mixtures are often used in the water walls of large boiler furnaces; they are rammed around the tubes and held in place by small studs welded to the tube walls. The chrome plastic has been used with good success for heating-furnace floors and subhearths of open-hearth furnaces.

Castable mixes are a refractory concrete usually containing high-alumina cement to give the setting properties. These find considerable use in forming intricate furnace parts in wooden molds; large structures have been satisfactorily cast by this method. This type of mixture is much used for baffles in boilers where it can be cast in place around the tubes. Lightweight castables with good insulating properties are used to line furnace doors.

Furnace Walls

The modern tendency in furnace construction is to make a comparatively thin wall, well supported by ironwork. The wall may be made of heavy refractories backed up with insulating material, or of insulating refractory. Table 7 gives heat losses and heat contents of a number of wall combinations and may enable the designer to pick out a wall section to suit his purpose.

Average values of VI for other crudes and their derivatives are about as follows: Mid-Continent, 70; East Texan, 60; Colombian, 40; Peruvian, 20.

By the use of addition compounds, such as paratone and acrylic ester, lubricating oils are raised to a higher viscosity index than normally obtained. Solvent refining also raises the viscosity index but has disadvantages that the oils are corrosive and are lower in lubricating value.

When lubricating oils are subjected to high pressure, there is a marked increase in viscosity. Paraffinic oils of high viscosity index vary less with pressure than do naphthenic oils, and fixed oils vary least.

Lubricants with good temperature-viscosity curves (high viscosity index) are desirable. In cold starting, the flatter the temperature-viscosity curve, the less the energy required and better the fluidity. In normal operation and at high temperatures and at high pressures, the flatter temperature-viscosity curve oils have less friction and higher load-carrying capacity.

Cloud and Pour Points. Petroleum oils, when cooled, may become plastic solids as a result either of partial separation of wax or of congealing of the hydrocarbons composing the oil. With some oils, the separation of wax becomes visible at temperatures slightly above the solidification point, and when that temperature is reached under prescribed conditions, it is known as the **cloud point** (A.S.T.M. D97-39). With oils in which wax does not separate prior to solidification, or in which the separation is invisible, the cloud point cannot be determined. That temperature at which the oil will just flow under prescribed conditions is known as the **pour point** (A.S.T.M. D97-39).

The pour point indicates the lowest temperature at which an oil will flow to the pump, bearings, or cylinder walls. It is particularly important for immediate oil circulation in connection with cold starting of engines, or with gravity lubricating systems, as the fluidity is a factor of pour point and viscosity of the cold oil. Pour-point depressants may be added to wax-containing oils to lower the pour points instead of dewaxing the oils.

Gravity. Lubricating-oil gravities are expressed either in specific gravity or A.P.I. gravity (see p. 85). Low-viscosity oils have higher A.P.I. gravities than the higher viscosity oils of the same crude-oil series. Paraffinic oils are the lightest or highest A.P.I. gravities, naphthenic are intermediate, and animal and vegetable oils are the heaviest or low A.P.I. gravity. The value of the sp gr s_1, at a temperature t_1 F, of a petroleum oil which has the sp gr s at 60 F is given approximately by the equation

$$s_1 = s - k(t_1 - 60)$$

where k has the following approximate values

$s =$	1	0.9	0.85	0.8	0.75	0.7
$k =$	0.00035	0.00035	0.00037	0.00040	0.00043	0.00048

The gravity of lubricating oils is of no value in predicting quality although it gives a clue to the source of the crude-oil base.

Flash and Fire Points. The flash point of an oil is the temperature to which an oil has to be heated until sufficient inflammable vapor is driven off to flash when brought into contact with a flame. The fire point is the higher temperature at which the oil vapors will continue to burn when ignited. The A.S.T.M. D92-33 standard method for flash and fire points by means of open cup tester is used for lubricating oils. In general, the open flash point is 30 F higher than the closed flash, and the fire point is some 50 to 70 F above this flash point.

unit. They have the advantage of low cost because of the light ironwork required to support them.

Many failures in furnace construction result from improper expansion joints. Expansion joints should usually be installed at least every 10 ft, although in some low-temperature structures the spacing may be greater. For high-temperature construction, the expansion joint allowance per foot in inches should be as follows: fire clay, $\frac{1}{16}$ to $\frac{3}{32}$; high alumina, $\frac{3}{32}$ to $\frac{1}{8}$; silica, $\frac{1}{8}$ to $\frac{3}{16}$; magnesite, $\frac{1}{4}$; chrome, $\frac{5}{32}$; forsterite, $\frac{1}{4}$. Figure 1 shows some typical examples of expansion joints. Corrugated cardboard is often used in the joints.

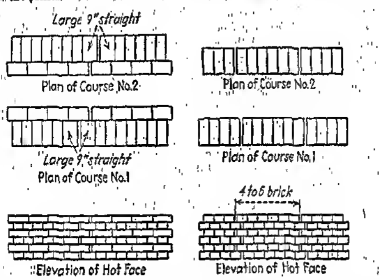


FIG. 1.—Expansion Joints.

The **roof** of the furnace is usually either a sprung arch or a suspended arch. A **sprung arch** is generally made of standard shapes using an inside radius equal to the total span. In most cases, it is necessary to build a form on which the arch is sprung. The arch dimensions can be calculated when the span and rise are known by the use of the table on p. 84. The number of brick per course is then easily determinable.

In the case of arches with a considerable rise, it has been found that an inverted catenary shape is better than a circular shape for stability, and it is possible to run the side walls of the furnace right down to the floor in one continuous arch with almost complete elimination of the ironwork. The catenary can be readily laid out by hanging a flexible chain from two points on a vertical wall.

The **suspended arch** is used when it is desirable to have a flat roof (curved suspended arches are also made); it presents certain advantages in construction and repair but is more difficult to insulate than the sprung arch. Special suspended arch shapes are commercially available. The insulating refractory is suited to this type of construction because the steel supports are light and the heat loss is low.

Selection of Refractories

The selection of the most suitable refractory for a given purpose demands experience in furnace construction. A brick that costs twice as much as another brand and gives twice the life is preferable since the total cost includes also the laying cost. Furthermore, a brick that gives longer service reduces

with 90 ml of petroleum naphtha as specified and centrifuged under prescribed conditions. The volume of sediment at the bottom of the centrifuge tube is reported as the A.S.T.M. precipitation number.

Corrosion and Neutralization Number. The usual method of checking lubricating oils for acidity is by determining the A.S.T.M. (D188-27T) neutralization number. The tests are designed to indicate, in petroleum lubricants and compounded products, the presence of organic constituents having acid characteristics and the contamination by alkalies and mineral acids. The neutralization number is the weight in milligrams of potassium hydroxide required to neutralize one gram of oil.

The neutralization number of used oil in no way indicates the corrosive action of the used oil in service. In some cases, a neutralization number of 1.0 may not attack the bearing metals; yet in other service tests of a different oil, 0.2 number will show high corrosive tendencies in short periods of operation.

Crude oils contain a natural inhibitor which is removed by the more drastic types of refining such as solvent extraction. Certain phosphite and sulphite additives have been used to check corrosion in lubricating oils. The use of oiliness additives in some instances may result in bearing corrosion especially where cadmium and certain lead-base bearings are used.

The Underwood bearing-corrosion test machine has been useful in the study of the corrosive resistance of lubricating oils. The bearing metals met with in service are used in the test so that their relationship to the oil in question is checked. In the Underwood tests, 0.01 percent of Fe_2O_3 is added, in the form of iron naphthenate, to accelerate the corrosion.

Products, such as slushing oils and greases, which are used to protect metal surfaces from corrosion are generally required to pass special tests. The tests measure the degree of protection afforded by the product under specified conditions such as salt spray or immersion in water.

Oiliness. The property of lubrication known as oiliness is of considerable importance. It is a phenomenon that becomes strongly evident only when the oil film separating rubbing surfaces is exceedingly thin. In films of molecular dimensions, viscosity effects are negligible although oiliness has a marked bearing. Oiliness depends on both the lubricant and the surface to which it is attached. Oiliness is the property that causes a difference in the friction when lubricants of the same viscosity at the same temperature and pressure of the film are used with different bearings.

Extreme-pressure Lubricants. The high tooth pressures and high rubbing velocities often encountered in hypoid and spur-type gearing have developed a class of lubricants called extreme-pressure or hypoid-gear lubricants. When mineral oils alone are used, metal to metal contact occurs which results in scoring, galling, and local seizure of the gear teeth. Most extreme-pressure lubricants are mineral oils containing loosely held sulphur or chlorine or some highly reactive material. The important characteristics of extreme-pressure lubricants are: ability to prevent galling or scoring; tendency to corrode; chemical stability; ability to reduce wear; ability to give low friction; and freedom from abrasive materials.

The S.A.E. film-strength testing machine is used for checking extreme-pressure and hypoid lubricants. This machine consists of a regular Timken bearing cup which rotates in contact with a second bearing cup at various speeds and rates of slippage. The hypoid-gear lubricants have higher load-carrying capacity at speeds above 750 rpm on the S.A.E. machine.

Properties of Various Lubricating Fats and Fatty Oils. Animal and vegetable fats and fatty oils are distinguished from mineral oils by being saponifiable with caustic alkalies. These organic oils oxidize, becoming rancid and setting free fatty acids. Oxidation also causes gumming particularly with cottonseed and corn oil. Subjected to high temperatures they tend to decompose to corrosive acids.

LUBRICANTS

BY

C. M. LARSON

REFERENCES: A.S.T.M., Committee D-2, "Standards on Petroleum Products and Lubricants," "Symposium on Motor Lubricants," "Symposium on Lubricants." Archbutt and Deeley, "Lubrication and Lubricants," Griffin. Bacon and Hamor, "The American Petroleum Industry," McGraw-Hill. Battle, "Industrial Oil Engineering," Lippincott. Burwell, "Oiliness," Alox Chemical Corp. Corse, "Bearing Metals and Bearings," Reinhold. Dunstan, "The Science of Petroleum," Oxford. Gill, "Short Handbook on Oil Analysis," Lippincott. Hersey, "Theory of Lubrication," Wiley. "Lubrication and Lubricants," A.S.M.E. Kalichevsky, "Modern Methods of Refining Lubricating Oil," Reinhold. Klemgard, "Lubricating Greases, Their Manufacture and Use," Reinhold. Lewkowitsch, "Analysis of Fats, Oils and Waxes," Macmillan. Nash and Bowen, "Principles and Practice of Lubrication," Chapman & Hall. Thomsen, "Practice of Lubrication," McGraw-Hill.

Type and Properties. The correct type and properties of a lubricant are controlled by the application. The lubricant is manufactured to meet the service requirements of minimum coefficient of friction; maximum adhesion to the surfaces to be lubricated; maximum film strength; physical stability with regard to temperatures and pressure; chemical stability against oxidation; freedom from corrosive acids; resistance to emulsion; non-volatility; proper fluidity at low temperatures; minimum consistency; and purity control of abrasives, fillers, soaps, or addition agents.

Liquid-petroleum lubricants are generally used because of their suitability to modern engineering design. Greases, which are a mixture of mineral oil and soap, are recommended only where leakage is too high to retain liquid lubricants.

Petroleum Mineral Oils

The petroleum mineral oils are manufactured by fractionation of crude oils and are refined by acids or solvents and clay. Lubricating crude oils are mainly from the Pennsylvania, Mid-Continent, Gulf Coast, or California fields.

Physical tests of lubricants are widely used since their utility depends to a large extent upon their physical characteristics. The usual tests are viscosity, pour, gravity, flash and fire, demulsibility, and color; chemical tests comprise carbon residue, oxidation, corrosion, acidity, oiliness, extreme pressures, sulphur, and ash.

Viscosity. (See p. 244.) The Saybolt Universal viscometer (p. 244) is the standard instrument for testing petroleum products and lubricants; 100, 130, and 210 F are the temperatures specified (A.S.T.M. designation D88-38).

The variation of viscosity with temperature of petroleum oils may be determined with considerable accuracy when the viscosities are known at any two temperatures. The A.S.T.M. publishes charts for that purpose covering both Saybolt Universal viscosity and kinematic viscosity. If the two known points are plotted on the chart and are joined by a straight line, viscosities at other temperatures can be read off from that line. Figure 1 shows such a chart. The lines drawn on it are typical lines for the grades indicated.

S.A.E. Viscosity Number. The S.A.E. viscosity numbers for crankcase oils constitute a classification in terms of viscosity only. They are as follows:

Table 1. Properties of Animal and Vegetable Oils

Oil	Specific gravity at 60 F	Saponification number	Cold test, deg F	Flash test, deg F	Viscosity Saybolt, sec at 100 F	Price, cents per gal
Blown rapeseed	0.968	198–215	40	460	800–900	65
Castor oil	0.963	176–187	14	505	1485	85
Colza or rape	0.916	170–179	30	455	215–270	51
Corn	0.922	189–192	14	480	180–190	60
Cottonseed	0.922	181–197	40	580	180–195	70
Degras, commercial	0.951	110–210	65	520	165–180[a]	60
Lard, prime	0.915	195–198	35	565	210–215	70
Lard No. 1	0.915	193–198	60	550	205–220	55
Neatsfoot	*0.915*	*193–204*	*40*	*440*	*220–225*	*65*
Porpoise jaw	0.925	269–273	0	415	100–105	$50
Rosin	0.981	91–95	25	257	220–230	35
Sperm	0.880	120–140	45	455	115–120	60
Tallow, acidless	0.927	193–198	100	575	220–230	60
Whale (blubber)	0.927	185–193	60	515	75–80	40
25 deg paraffin	0.900	0.20	15	410	180	26

[a] At 210 F; is semisolid at 100 F.

soap stock. When the soap base has been prepared, mineral oil is added in small quantities and the mixing continued, heat having been supplied all the while by means of a steam coil. As the percentage of mineral oil is increased, the grease becomes softer. Soda greases are dehydrated (less than ½ percent moisture), whereas lime soap greases contain 1 to 2 percent moisture.

The A.S.T.M. **dropping point** of grease is the temperature at which it changes from a semisolid to a liquid state when the determination is made according to the prescribed A.S.T.M. D566-40T. Calcium- or lime-soap greases have melting points below 200 F; for dropping-points of 300 F or higher soda-soap greases are used.

The A.S.T.M. Method (D217-38T) is used in measuring the worked or the unworked **consistency** of lubricating greases which have a worked consistency less than 400. In this test, a standardized double-pitch cone is allowed to drop in the product at a definite temperature. The depth of penetration is measured. The unworked (original) consistency of lubricating greases is affected by the soap content, the kind of fat used, the method of manufacture, the final water content, the rate of cooling, and the basic metallic constituent of the soap. It is impractical to control the consistency of a grease to narrow limits. Any working or remelting of a grease after it is in the container will change the consistency. Although many tests are based on the unworked consistency, this property bears no definite relationship to worked values. Final tests are usually based on worked consistency (A.S.T.M.) where possible although hard railroad greases and soda-soap greases are tested for unworked consistency.

The texture of a grease refers to its structure such as smooth, fibrous, spongy, or rubbery. Calcium-base greases are smooth, soda-soap greases are fibrous or spongy, and aluminum-soap greases are stringy or rubbery.

The A.S.T.M. methods (D128-40) of **analysis** permit determination of the constituents of grease likely to be covered by specifications. Such constituents are soap base and content, fat, water, fillers, ash, excess alkali or acid, insaponifiable matter, and lubricating-oil content. Two greases showing the same analysis may show marked differences in lubricating performance and storage-stability properties. The soda-soap greases are more stable over much longer periods of service than lime-base greases.

Miscellaneous Oils and Lubricants

Air-compressor oils are refined straight mineral oils of S.A.E. 20 or 30 viscosity numbers. **Automobile and gasoline-engine oils** are well-

In the case of fluid grease, made by adding soap or other thickening ingredients to oil, the viscosity number by which the lubricant is classified is determined by the viscosity of the oil before the addition of the soap or other thickening ingredients. The S.A.E. viscosity numbers listed above are in general use for the description of lubricants for all uses.

The recommended minimum viscosities at operating temperatures for straight mineral oils are as follows:

	Viscosity, Saybolt sec
Well-designed high-speed bearings (turbines)	40
Automobile bearings (circulated oil)	45
Heavy-duty automotive engines (circulated oil)	70
Aircraft engines (circulated oil)	110
Reduction gears and speed reducers	200– 500
Transmissions and rear-axle gears	400– 1,000
Exposed gears	10,000–30,000

Viscosity index (VI) is an arbitrary method of stating the rate of change of viscosity of an oil with change of temperature. Pennsylvania crude oils, refined by conventional methods, suffer comparatively little change in viscosity with temperature; Gulf Coastal crudes change considerably. These two crudes and their fractions have been assigned viscosity index numbers of 100 and 0, respectively. The average viscosity characteristics of these two standardized oils have been obtained by Dean and Davis (*Chem. Met. Eng.*, 36, 1929, p. 618). The procedure in finding the VI of an oil is to determine its viscosity (Saybolt Universal) at 210 and at 100 F. The viscosities at 100 F of oils that have the same viscosity at 210 F, but are of VI 100 and 0, are then found from A.S.T.M. D567-40T. The VI of the oil under consideration is then found, by simple ratio, by comparison of its increase in viscosity from 210 to 100 F with the corresponding increases in the two standardized oils.

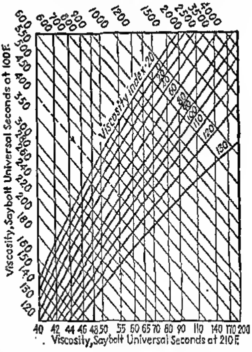


Fig. 2.—Viscosity-index Chart (Dean and Davis Scale).

Since the viscosity at 210 F is taken at the same value for all three oils, the VI of the oil under investigation can be written

$$\mathrm{VI} = \frac{L - U}{L - H} \times 100$$

where U, H, and L are the viscosities at 100 F of the oil under test, of the VI = 100 oil, and of the VI = 0 oil, respectively. Values of the VI may be obtained from Fig. 2. As shown by the broken lines the VI of an oil with SV of 55 at 210 F and 400 at 100 F is 80. It should be noted that values of VI may be negative or may exceed 100.

Flash and fire points depend upon the nature of the original crude oil, the viscosity, and the method of refining. For the same viscosities and degree of refinement, the paraffinic oils are higher than the naphthenic flash and fire points.

Steam Emulsion Test. A.S.T.M. D157-36 Steam Emulsion Test is used for lubricating oils and is commonly used for turbine oils. This method is suitable for use on all oils when emulsion, demulsibility, or emulsification tests are required. The Steam Emulsion number (S.E. No.) is the number of seconds required for an oil to separate when emulsified and separated under prescribed conditions.

The demulsification of pure well-refined mineral oils is very good below 300 S.E. No. Next to viscosity, the demulsibility is the most important test for steam-turbine oils. There is some doubt as to the necessity of high demulsification of lubricants for other service applications.

Color. The color of a lubricating oil is obtained by reference to transmitted light. The color by reflected light, called the bloom, is a distinctive feature capable of giving information on the origin and refining of the oil.

The color of an oil may vary with regard to intensity of light transmitted. Color measurement (A.S.T.M. D155-39T) is intended for the determination of lubricating oils through the use of the A.S.T.M. Union Colorimeter. The A.S.T.M. color numbers range from 1 (lily white) to 8 (darker than claret red). Oils darker than 8 color are diluted with kerosine (15 percent lubricating oil and 85 percent by volume of kerosine) and are then observed in the same way as lighter colored oils.

Carbon Residue. The Conradson carbon-residue test (A.S.T.M. D189-39) is a means of determining the amount of carbon residue left on evaporating an oil under specified conditions and is intended to throw some light on the relative carbon-forming properties of an oil. The carbon-residue test was originally developed for comparison of the carbon-forming properties of lubricating oils for internal-combustion engines. The deposits vary with the type and mechanical condition of the engine, the service conditions, the time of continuous operation, the viscosity of the oil, and the carburation of the fuel. Modern refining methods tend to make this test less useful for predicting carbon-forming tendency of motor oils.

The Ramsbottom coking test (A.S.T.M. D524-40T) is widely used in Great Britain and yields more reproducible results than the Conradson test. The A.S.T.M. has recommended that the Conradson test be replaced by the Ramsbottom test. The carbon residues from the two methods are not comparable.

Ash. The ash determination of lubricating oil is conducted on the carbon remaining after the carbon-residue test. The nature of the ash is used to determine iron (rate of wear), sand or grit (from atmosphere), or lead (from leaded gasoline) in the used lubricating oil.

Oxidation Testing Methods. Lubricating oils may be subjected to relatively high temperatures in the presence of air and catalytically active metals or metallic compounds. The resultant oxidation of the oil develops increased viscosity, acids, carbon residue, sludge, and asphaltenes. There are several oxidation tests. The Navy Work Factor (*U. S. Navy, Bur. Ships*, N.B.S. 431—1940) and the U. S. Army Oxidation Test (No. 29 LA, Mar. 1, 1938, F5) are met in aircraft lubricating-oil specifications.

When the carbon residue of motor oils is lowered below certain limits, the oxidation products are soluble in hot oil and lacquer deposits form on the metallic surfaces such as on the pistons, in the ring grooves, and on the inlet valve stems where there are low rates of oil flow in internal-combustion engines. It appears that the unsaturated and asphaltic constituents yield a sludge of carbonaceous matter and the pure paraffins of high solvent refining form corrosive acids.

Phosphites are being tried as antioxidant addition agents in gasoline-engine lubricating oils. The tin salts also have been tried. In the Diesel engine field, calcium-base addition agents have been used successfully to prevent sludge and ring sticking.

Precipitation Number. Steam cylinder oils and black oils are checked by the (A.S.T.M. D91-40) precipitation number. This method is also applied to used crankcase oils of internal-combustion engines to check the oxidized products, carbonaceous matter, and asphaltenes formed through use. The A.S.T.M. precipitation number is the number of milliliters of precipitate formed when 10 ml of lubricating oil are mixed

MECHANISM

BY

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REFERENCES: Reuleaux, "Kinematics of Machinery," Macmillan. Unwin, "Elements of Machine Design," Longmans. Schwamb and Merrill, "Elements of Mechanism," Wiley.

Definition. A mechanism is that part of a machine which contains two or more pieces so arranged that the motion of one compels the motion of the others according to a definite law depending upon the nature of the combination.

Linkages

Links may be of any form so long as they do not interfere with the desired motion. The simplest form is that of four bars, *A*, *B*, *C*, *D*, fastened together at their ends by cylindrical pins, and which are all movable in parallel planes. If the links are of different lengths and each is fixed in turn, there will be four possible combinations; but as two of these are similar there will be produced three mechanisms having distinctly different motions. Thus, in Fig. 1, if

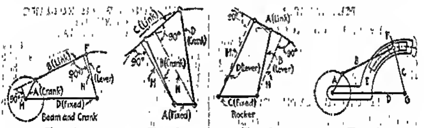

FIG. 1. FIG. 2. FIG. 3. FIG. 4.

D is fixed *A* can rotate and *C* oscillate, giving the **beam-and-crank** mechanism, as used on side-wheel steamers. If *B* is fixed the same motion will result; but if *A* is fixed (Fig. 2) links *B* and *D* can rotate, giving the **drag-link** mechanism used to feather the floats on paddle wheels. Fixing link *C* (Fig. 3), *D* and *B* can only oscillate, and a **rocker** mechanism sometimes used in straight-line motions is produced. It is customary to call a rotating link a **crank**; an oscillating link a **lever**, or beam; and the connecting link a **connecting rod**. The fixed link is often enlarged and used as the supporting frame.

If in the linkage (Fig. 1) the pin joint *F* is replaced by a slotted piece *E* (Fig. 4) no change will be produced in the resulting motion, and if the length of links *C* and *D* is made infinite the slotted piece *E* will become straight and the motion of the slide will be that of pure translation, thus obtaining the engine, or **sliding-block, linkage** (Fig. 5).

If in the sliding-block linkage (Fig. 5) the long link *B* is fixed (Fig. 6), *A* will rotate and *E* will oscillate and the infinite links *C* and *D* may be indicated as shown. This gives the **swinging-block linkage**. When used as a quick-return motion the slotted piece and slide are usually interchanged (Fig. 7) which in no way changes the resulting motion. If the short link *A* is fixed (Fig. 8) *B* and *E* can both rotate, and the mechanism known as the **turning-block linkage** is obtained. This is better known under the name of the

Saponification Number. The saponification number (A.S.T.M. D94-39T) of animal and vegetable oils and of compounded lubricants is the number of milligrams of potassium hydroxide required to saponify 1 g of oil. It is a measure of both free and combined fatty acids. When the sample contains appreciable amounts of sulphur and sulphurized oils, the saponification number obtained will be greater for the amount of fatty matter actually present. The percentage of fatty oil (or fat) in a compounded petroleum product can be calculated from the saponification number when the fatty oil is known (see Table 1). Unless the saponification number is known, commercial practice uses a value 195 for calculation.

The saponification number is the best obtainable index of the percentage of fat or fatty oil in a given lubricant.

Animal Oils. Animal oils are commonly obtained by rendering the fatty parts of the different animals, usually with high-pressure steam. Extraction with solvents is seldom practiced. As a result of the rendering operation, a more or less solid fat is obtained, as beef tallow, mutton tallow, or lard. When this is chilled and pressed, the corresponding oil and stearin result. **Tallow** should be "acidless," *i.e.*, free from sulphuric acid and contain only the smallest possible quantity of free fatty acid. It is hard and does not turn rancid easily. It forms a constituent of belt dressings, cylinder oils, and lubricating greases. **Tallow oil** is obtained by chilling and pressing tallow; it is a light-yellow bland oil used for mixing with steam cylinder oils. **Lard oil** is prepared similarly to tallow oil and appears in the market in five or six different varieties including Prime, Pure, No. 1, and No. 2, which are graded according to color, the first being very light straw-yellow and the last dark brown and ill-smelling: it is used in thread-cutting oils. **Neatsfoot oil** is obtained by boiling the bones of the feet of horned cattle with water and running off the oil. It is often adulterated with sheep and horse-foot and hide oil, as well as rape, cottonseed, and mineral oils. It is light yellow and bland, with little tendency to turn rancid. It is used similarly to lard oil and particularly for softening leather. **Whale oil** is made by rendering the blubber of various species of whales, and consequently is of very variable composition. It is used as a leather dressing and to mix with other oils as a lubricant. **Sperm oil** comes from the great cavity in the head of the sperm whale and also from its blubber. It is a limpid, pale yellow oil adapted for fine machinery. **Degras** is the grease obtained in scouring wool. It usually contains sulphuric acid, except when made by the naphtha-extraction process. It possesses the property of emulsifying with water to a marked degree and finds use as a leather dressing and in cylinder oils.

Vegetable Oils. Vegetable oils are usually prepared by pressing the crushed cooked seed or by extraction with benzine or carbon bisulphide. The chief vegetable oils are castor, corn, cottonseed, and rape. **Castor oil,** obtained from the castor bean, is a very viscous oil which gums on standing and does not naturally mix well with mineral oils. To render this possible, it is heated to 212 F and a current of air forced through, whereby it becomes more viscous and is known as "blown oil." **Cottonseed oil** is a pale or deep yellow oil of slight drying properties, chiefly used as an adulterant for other oils and, as well as rapeseed and castor, for the manufacture of blown oils. **Corn** (or maize) **oil** is a somewhat viscous oil with some drying properties. It is used with cottonseed oil for the preparation of soft soaps for lubricating greases. **Rapeseed oil** should be clear, *i.e.*, free from gelatinous matter. Blown rapeseed is compounded with mineral oils for marine steam-engine oils.

Greases. Grease is a mineral oil that has been thickened by compounding with soap. An alkali, usually lime or soda, is mixed with a fat to form the

representing the velocities of any number of points on the link will be on a straight line (Fig. 12): abc = a straight line. To find the velocity of any point when the velocity and direction of any two other points are known, condition (2) may be used, or a combination of (1) and (3). The **linear velocity ratio** of any two points on a linkage may be found by determining the distances e and f to the instant center (Fig. 13); then $V_c/V_b = e/f$. This may often be simplified by noting that a line drawn parallel to e and cutting B produced, forms two similar triangles efB and sAy, which gives $V_c/V_b = e/f = s/A$.

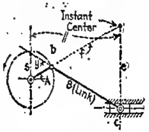

FIG. 13.

The angular velocity ratio for any position of two oscillating or rotating links A and C (Fig. 1), connected by a movable link B, may be determined by scaling the length of the perpendiculars M and N from the axes of rotation to the center line of the movable link. The angular velocity ratio is inversely proportional to these perpendiculars, or $O_C/O_A = M/N$. This method may be applied directly to a linkage having a sliding pair if the two infinite links are redrawn perpendicular to the sliding pair, as indicated in Fig. 14. M and N are also shown in Figs. 1, 2, 3, 5, 6, 8. In Fig. 5 one of the axes is at infinity, therefore N is infinite, or the slide has pure translation.

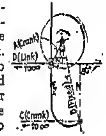

FIG. 14.

Forces. A mechanism must deliver as much work as it receives, neglecting friction, therefore the force at any point F multiplied by the velocity V_F in the direction of the force at that point must equal the force at some other point P multiplied by the velocity V_P at that point; or the forces are inversely as their velocities and $F/P = V_P/V_F$. It is at times more convenient to equate the moments of the forces acting around each axis of rotation (sometimes using the instant center) to determine the force acting at some other point. Applying this to Fig. 15, $(F \times a \times c)/(b \times d) = P$.

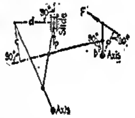

FIG. 15.

CAMS

Cam Diagrams. A cam is usually a plate or cylinder which communicates motion to a follower by means of its edge or a groove cut in its surface. In the practical design of cams the angular velocity ratio is not directly involved, but the follower must assume a definite series of positions while the driver occupies a corresponding series of positions. The relationship of driver to follower may be represented by a diagram in which the ordinates represent the rise or fall of the follower and the abscissas the angular motion of the cam.

FIG. 16.

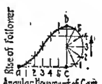

FIG. 17.

The three most common forms of motion used are uniform motion (Fig. 16), harmonic motion (Fig. 17), and uniformly accelerated and retarded motion

refined straight mineral *oils with low carbon residue, low pour points*, and free from corrosive acids. **Belt dressings** are (1) mixtures of solid fats, waxes, degras, or tallow with castor or fatty oils; (2) vulcanized corn or cottonseed oil thinned with naphtha; (3) preparations containing rosin which is undesirable (see also p. 700): (4) neats foot oil. **Black oils** (summer and winter) or **well oils** are reduced *crude oil from which the* naphtha, illuminating, and gas oils have been removed by distillation. The summer oils have higher viscosity and the winter oils zero pour points. **Car or journal-box oils** (summer and winter) are usually made from a light subzero pour oil blended to suitable viscosities with heavy cylinder or residuum oils to meet climatic conditions. *The minimum of winter grade is* 40 sec Saybolt at 210 F. The summer grade is about 60 sec at 210 F. Car or journal-box oils are more highly refined than black or well oils. **Chassis grease or pressure-gun grease** are lime or lime soda greases, using at least an S.A.E. 30 grade oil; *they have the proper consistency* to feed through pressure guns and yet do *not drip from the shackles or bearings*. **Crankcase oils** *are filtered straight* mineral oils and are usually light turbine oils which do not emulsify with water. **Cylinder or valve oils** are mixtures of S.A.E. 60 or 70 mineral oils and 3 to 10 percent animal oils, the percentage of compound decreasing with increasing dryness of steam. When straight mineral, they are prefixed *mineral*. **Superheat valve oils** *have high flash and fire and are 180 sec* or higher Saybolt viscosity at 210 F. **Cutting oils** are S.A.E. 10 or 20 petroleum oils, usually engine oils, compounded with 10 to 30 percent lard oil or S.A.E. 10 or 50 oils blended with sulphurized-lard or sulphur-chloride-lard base. **Diesel-engine oils** are highly filtered straight mineral oils, usually *of S.A.E. 10, 20, 30, or 40 viscosity numbers. The four-cycle engines work* well on Gulf Coast oils whereas for the two-cycle, Pennsylvania oils are recommended. **Dynamo or electric-motor oils** are well-refined straight mineral oils, S.A.E. 10 or 20 grades. Where low temperatures are encountered, zero pour or lower is required. **Engine and machine oils** are straight *mineral oils of S.A.E. 10 or 20 grades, either red or pale color.* The degree of refinement depends on whether moisture comes in contact and if oil is reclaimed. **Graphite or plumbago** (see p. 714) should be finely ground and low in ash or abrasives. When mixed with grease in proportions of 2 to 18 percent, the result is graphite grease. **Oildag, aquadag, and gredag** are *colloidal suspensions of artificial* deflocculated *graphite in oil,* water and grease, respectively (see p. 715). **Marine-engine oils** are petroleum oils of S.A.E. 20 or 40 grades, compounded with lard oil or blown rapeseed. They should emulsify with water. **Non-fluid oils** are light pale petroleum oils with small percentages of lime soap or aluminum oleate. **Pinion or gear greases** *are either straight petroleum products of 500 to 5,000 sec* Saybolt viscosity at 210 F or heavy mineral cylinder oils with rosin, talc, graphite, or asbestos. **Soluble oils** are pale mineral oils, light S.A.E. 10 grade with soap or water-saponifiable oil. When added to water, they saponify immediately into a milky oily cutting or grinding solution. **Spindle oils** (textile) are highly *refined pale or lily-white oils of light S.A.E. 10 body.* For a stainless spindle oil, the lily-white-color oil is compounded with prime lard oil. **Transformer oils** are highly refined filtered light-petroleum oils, water free, so as to have high dielectric strength. **Turbine oils** are highly refined filtered pale oils with high demulsibility. They are S.A.E. 10 grade except for geared turbines *where S.A.E. 20 or 30 body is* used or S.A.E. 30 or 70 viscosity number for ring-oiled turbine grades. **Watch oil** is obtained from the porpoise, dolphin, or blackfish where it exists in the cavities in the jaw and in the brain or "melon" of the fish.

CYLINDRICAL CAM (Fig. 22). **In this type of cam more than one complet turn may be obtained, provided that in all cases the follower returns to it starting point.** Draw rectangle *wxyz* (Fig. 22) representing the developmen of cylindrical surface of the cam. Subdivide the desired motion of the fol lower *bc* horizontally in the manner indicated in Figs. 16, 17, and 18, and plo the corresponding angular displacements 1′, 2′, 3′, etc., of the cam vertically then through the intersection of lines from these points draw a smooth curve This may best be shown by an example, assuming the following data for th diagram in Fig. 22: Total motion of follower = *bc*; circumference of cam = $2\pi r$. Follower to move with harmonic motion 4 units to the right in 0.6 turn then rest (or "dwell") 0.4 turn, and finish with uniform motion 6 units to th right and 10 units to the left in two turns.

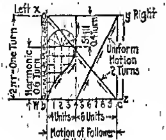

FIG. 22.

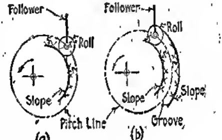

FIG. 23.

Cam Design. In the practical design of cams the following point must be noted. If only a small force is to be transmitted, sliding con tact may be used, otherwise **rolling contact.** For the latter the pitch lin must be corrected in order to get the true slope of the cam. An approximat construction (Fig. 23) may be employed by using the pitch line as the cente of a series of arcs the radii of which are equal to that of the follower roll to b used; then a smooth curve drawn tangent to the arcs will give the slop desired for a roll working on the periphery of the cam (Fig. 23*a*) or in a groov (Fig. 23*b*). For plate cams the roll should be a small cylinder, as in Fig 24*a*. In cylindrical cams it is usually sufficiently accurate to make th

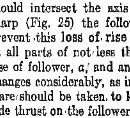

FIG. 24.

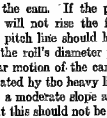

FIG. 25.

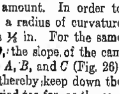

FIG. 26.

roll conical, as in Fig. 24*b*, in which case the taper of the roll produce should intersect the axis of the cam. If the pitch line *abc* is made to sharp (Fig. 25) the follower will not rise the full amount. In order t prevent this loss of rise the pitch line should have a radius of curvatur at all parts of not less than the roll's diameter plus ⅛ in. For the sam rise of follower, *a*, and angular motion of the cam, *O*, the slope of the cam changes considerably, as indicated by the heavy lines *A*, *B*, and *C* (Fig. 26) Care should be taken to keep a moderate slope and thereby keep down th side thrust on the follower, but this should not be carried too far, as the cam would become too large and the friction increase.

SECTION 7

MACHINE ELEMENTS

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From "Mechanical Engineers' Handbook," edited by Lionel S. Marks

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PACKINGS

By F. C. Thorn

PIPE AND PIPE FITTINGS

By C. W. Ham

WIRE ROPE, NAILS, ETC.*

* Staff Contribution.

and D will have rotated the amount indicated in the tabulation. Then the algebraic sum will give the relative turns of each gear. That is, in Fig. 31, for one turn of the arm, *B does not move and C turns in the same direction $3\frac{1}{2}$ rev, and D in the opposite* direction $\frac{1}{4}$ rev; whereas in Fig. 32, for one turn of the arm, B does not turn, but C and D turn in the same direction as the arm respectively $2\frac{1}{2}$ and $1\frac{3}{7}$ rev. (Note: The arm in the above case was turned $+1$ for convenience, but any other value might be used.)

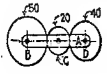

Fig. 31.

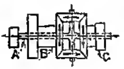

Fig. 33.

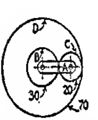

Fig. 32.

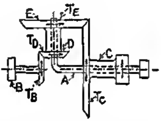

Fig. 34.

Fig. 35.

Bevel epicyclic trains are epicyclic trains containing bevel gears and may be calculated by the preceding method, but it is usually simpler to use the general formula which applies to all cases of epicyclic trains:

$$\frac{\text{Turns of } C \text{ relative to arm}}{\text{Turns of } B \text{ relative to arm}} = \frac{\text{Absolute turns of } C - \text{turns of arm}}{\text{Absolute turns of } B - \text{turns of arm}}$$

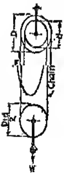

Fig. 36.

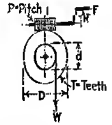

Fig. 37.

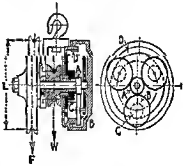

Fig. 38.

The left-hand term gives the value of the train and can always be expressed in terms of the number of teeth (T) on the gears; care must be used, however, to express it as either plus (+) or minus (−), depending upon whether the gears turn in the same or opposite directions.

$$\frac{\text{Relative turns of } C}{\text{Relative turns of } B} = \frac{C - A}{B - A} = -1, \text{ in Fig. 33; } = +\frac{T_B}{T_C} \times \frac{T_E}{T_D} \text{ in Fig. 34.}$$

Whitworth quick-return motion, and is generally constructed as in Fig. 9. The **ratio of time of advance to time of return** H/K of the two quick-return motions (Figs. 7 and 9) may be found by locating, in the case of the swinging block (Fig. 7), the two tangent points (t) and measuring the angles H and K made by the two positions of the crank A. If H and K are

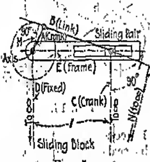

FIG. 5.

FIG. 6.

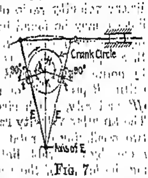

FIG. 7.

known the axis of E may be located by laying off the angles H and K on the crank circle and drawing the tangents E, their intersection giving the desired point. For the turning-block linkage (Fig. 9), determine the angles H and K made by the crank B when E is in the horizontal position; or, if the angles are known, the axis of E may be determined by drawing a

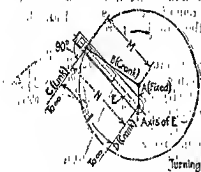

FIG. 8.

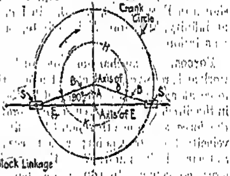

FIG. 9.

horizontal line through the two crank-pin positions (S) for the given angle, and the point where a line through the axis of B cuts E perpendicularly will be the axis of E.

Velocities of any two or more points on a link must fulfill the following conditions (see p. 195): (1) Components along the link must be equal and

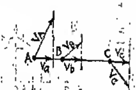

FIG. 10.

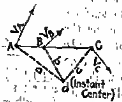

FIG. 11.

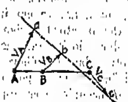

FIG. 12.

in the same direction (Fig. 10): $V_a = V_b = V_c$. (2) Perpendiculars to V_A, V_B, V_C from the points A, B, C must intersect at a common point d, the **instant center** (or **instantaneous axis**). (3) The velocities of points A, B and C are directly proportional to their distances from this center (Fig. 11): $V_A/a = V_B/b = V_C/c$. For a straight link the tips of the vectors

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(Originally prepared by W. Rautenstrauch)

REFERENCES: Spooner, "Machine Design, Construction and Drawing," Longmans. Unwin, "Elements of Machine Design," Longmans. Kimball and Barr, "Elements of Machine Design," Wiley. Smith and Marx, "Machine Design," Wiley. Hess, "*Machine Design*," *Lippincott*. *Benjamin and Hoffman*, "*Machine Design*," *Holt*. Reuleaux, "The Constructor," Bach, "Die Maschinen Elemente," Kroner. Leutwiler, "Elements of Machine Design," McGraw-Hill. Berard and Waters, "Elements of Machine Design," Van Nostrand. French, "Engineering Drawing," McGraw-Hill.

SCREW FASTENINGS

Machine Screw Threads: Forms and Proportions

The American (National) Standard thread (formerly called the Sellers or U. S. Standard thread) for machine screws, shown in Fig. 1, is proportioned by the formulas p = pitch = 1/No. of threads per inch, d = depth = $0.6495p$, f = flat = $p/8$. See Table 1.

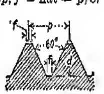
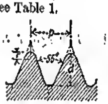


FIG. 1.—American (National).



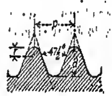


FIG. 2.—Whitworth. FIG. 3.—British Association. Standard Screw Threads.

The Whitworth Standard thread (Fig. 2) is used in Great Britain and is based on the formulas $d = 0.6403p$, r = radius = $0.1373p$. See Table 2.

The British Association screw thread is shown in Fig. 3, and dimensions of bolts with such threads are given in Table 3.

The French (Metric) screw thread (Fig. 4) is based on the formulae p = pitch in mm., $d = 0.6495p$, $f = p/8$. See Table 4.

Table 1. American (National) Standard Screw Threads

(Formerly called the Sellers or U. S. Standard Threads. See Fig. 1)

Diam of screw, in.	Threads per in.	Diam at root of thread, in.	Double depth of thread, in.	Width of flat, in.	Area of section at root of thread, sq in.
1/4	20	0.1850	0.0650	0.0063	0.027
5/16	18	0.2403	0.0717	0.0069	0.045
3/8	16	0.2936	0.0814	0.0078	0.068
7/16	14	0.3447	0.0923	0.0089	0.093
1/2	13	0.4001	0.0999	0.0096	0.126
9/16	12	0.4542	0.1078	0.0104	0.162
5/8	11	0.5069	0.1181	0.0114	0.202
3/4	10	0.6201	0.1299	0.0125	0.302
7/8	9	0.7307	0.1443	0.0139	0.420
1	8	0.8376	0.1624	0.0156	0.550
1 1/8	7	0.9394	0.1856	0.0179	0.694
1 1/4	7	1.0644	0.1856	0.0179	0.893
1 3/8	6	1.1585	0.2165	0.0208	1.057
1 1/2	6	1.2835	0.2165	0.0208	1.295
1 5/8	5 1/2	1.3888	0.2362	0.0227	1.515
1 3/4	5	1.4902	0.2598	0.0250	1.746
1 7/8	5	1.6152	0.2598	0.0250	2.051

Diam of screw, in.	Threads per in.	Diam at root of thread, in.	Double depth of thread, in.	Width of flat, in.	Area of section at root of thread, sq in.
2	4 1/2	1.7113	0.2887	0.0278	2.302
2 1/4	4 1/2	1.9613	0.2887	0.0278	3.023
2 1/2	4	2.1752	0.3248	0.0313	3.719
2 3/4	4	2.4252	0.3248	0.0313	4.620
3	3 1/2	2.6288	0.3712	0.0357	5.428
3 1/4	3 1/2	2.8788	0.3712	0.0357	6.510
3 1/2	3 1/4	3.1003	0.3997	0.0385	7.548
3 3/4	3	3.3170	0.4330	0.0417	8.641
4	3	3.5670	0.4330	0.0417	9.963
4 1/4	2 7/8	3.7982	0.4518	0.0435	11.329
4 1/2	2 3/4	4.0276	0.4724	0.0455	12.753
4 3/4	2 5/8	4.2551	0.4949	0.0476	14.226
5	2 1/2	4.4804	0.5196	0.0500	15.763
5 1/4	2 1/2	4.7304	0.5196	0.0500	17.572
5 1/2	2 3/8	4.9530	0.5470	0.0526	19.267
5 3/4	2 3/8	5.2030	0.5470	0.0526	21.262
6	2 1/4	5.4226	0.5774	0.0556	23.098

(Fig. 18). In plotting the diagrams (Fig. 18) for this last motion, divide ac into an even number of equal parts and bc into the same number of parts with lengths increasing by a constant increment to a maximum and then decreasing by the same decrement, as, for example, 1, 3, 5, 5, 3, 1, or 1, 3, 5, 7, 9, 9, 7, 5, 3, 1. In order to prevent shock when the direction of motion changes, as at a and b in the uniform motion, the harmonic motion may be used; if the cam is to be operated at high speed, the uniformly accelerated and retarded motion should preferably be employed; in either case there is a very gradual change of velocity.

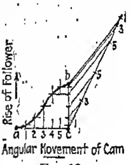


FIG. 18.

Pitch Line. The actual pitch line of a cam varies with the type of motion and with the position of the follower relative to the cam's axis. Most cams as ordinarily constructed are covered by the following four cases.

FOLLOWER ON LINE OF AXIS (Fig. 19). To draw the pitch line, subdivide the motion bc of the follower in the manner indicated in Figs. 10, 17, 18. Draw a circle with a radius equal to the smallest radius of the cam $a0$ and subdivide it into angles $0a1'$; $0a2'$, $0a3'$, etc., corresponding with angular displacements of the cam for positions 1, 2, 3, etc., of the follower. With a as a center and radii $a1$, $a2$, $a3$, etc., strike arcs cutting radial lines at d, e, f, etc. Draw smooth curve through points d, e, f, etc.

OFFSET FOLLOWER (Fig. 20). Divide bc as indicated in Figs. 16, 17, and 18. Draw a circle of radius ac (highest point of rise of follower) and one tangent to cb produced. Divide the outer circle into parts 1′, 2′, 3′, etc., corresponding with the angular displacement of the cam for positions 1, 2, 3, etc., of the follower, and draw tangents from points 1′, 2′, 3′, etc., to the small circle. With a as a center and radii $a1$, $a2$, $a3$, etc., strike arcs cutting tangents at d, e, f, etc. Draw a smooth curve through d, e, f, etc.

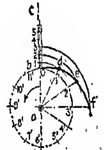


FIG. 19.

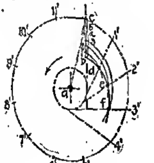


FIG. 20.

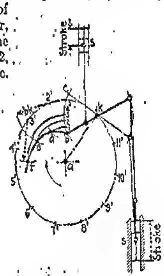


FIG. 21.

ROCKER FOLLOWER (Fig. 21). Divide the stroke of the slide S in the manner indicated in Figs. 16, 17, and 18, and transfer these points to the arc bc as points 1, 2, 3, etc. Draw a circle of radius ak and divide it into parts 1′, 2′, 3′, etc., corresponding with angular displacements of the cam for positions 1, 2, 3, etc., of the follower. With k, 1′, 2′, 3′ etc., as centers and radius bk, strike arcs kb, $1'd$, $2'e$, $3'f$, etc., cutting at $bdef$ arcs struck with a as a center and radii ab, $a1$, $a2$, $a3$, etc. Draw a smooth curve through b, d, e, f, etc.

Standard Screw Threads for Bolts, Machine Screws, Nuts, and Commercially Tapped Holes

The National Screw Thread Commission appointed by Congress in 1918 to standardize and unify screw threads presented a revised report in 1928 and a further revision in 1933. This was issued as *N. B.S. Misc. Pub.* 141 and reference to it for all matters relating to screw threads for various uses should be made.

The report includes certain additions to the coarse-thread series and to the fine-thread series but eliminates all sizes above 1¾ in. in the latter series. It also makes certain additions to hose-coupling threads and revises the tolerances for pipe-thread gages. Specification for Acme threads are also revised.

The thread profile is that previously known as the Sellers or U. S. Standard (see p. 758) and is to be known hereafter as the **American (National) form of thread.** Two thread series are adopted, a coarse-thread and a fine-thread series with basic diameters and threads per inch substantially as given in Table 10, p. 764. The coarse-thread series is recommended for general engineering work and where rapid and easy assembly is desired; the fine-thread series is recommended for automotive and airplane use and wherever minimum weight is a desideratum. Four classes of fits are specified: (1) **Loose fits** are recommended as a commercial standard for tapped holes in the numbered (small) sizes only and for rough work when ease of assembly is desirable and snugness of fit is unnecessary. (2) **Free fits** include the great bulk of screw-thread work of ordinary quality of finished and semifinished bolts and nuts, etc. (3) **Medium fits** include the better grades of interchangeable screw-thread work such as automobile bolts and nuts. (4) **Close fits** include work requiring a fine snug fit such as is necessary in airplane work. In this class of fits, selective assembly of parts may be necessary. This is not practicable as a commercial standard for tapped holes of the numbered (small) sizes. The same quality of fit as given by medium-fit screws and threaded holes can be obtained by using screws to free-fit tolerances in holes made to close-fit tolerances, or vice versa.

The following specifications apply to all classes of fit, except loose-fit.

Uniform Minimum Nut. The pitch diameter of the minimum threaded hole or nut corresponds to the basic size, variations being permitted above the basic size. The major and minor diameters of the minimum nut are the same for all classes.

Uniform Tap Drill Sizes. The maximum and minimum minor diameters are the same for all nuts of a given size for all classes of fit. This permits uniform tap-drill sizes for all classes of fit.

Uniform Major and Minor Diameter of Screws. The maximum and minimum major and minor diameters are the same for all screws of a given size for all classes of fit.

Length of Engagement. The tolerances are based on a length of engagement not to exceed the nominal diameter of the thread. Where greater lengths of engagement are required, a corresponding increase in accuracy is necessary.

Limiting dimensions and tolerances for free, medium, and close fits are given in Table 5, which also includes tolerances for nuts of loose fit in the numbered sizes only. If larger tolerances are required than those given for free fit, the loose-fit dimensions of Table 6 may be used. With loose fit, the minimum nut diameter is basic as with the other fits but the maximum screw diameter is below basic.

Additional dimensions to those given in Table 5 are as follows:

Screws. Maximum major diameter = basic major diameter (Col. 3). Minimum major diameter = Col. 3—Col 5. Maximum pitch diameter = basic pitch diameter (Col. 4). Minimum pitch diameter = Col. 4—Col. 9, 10, 11, or 12.

Nuts. Minimum major diameter = basic major diameter (Col. 3). Minimum pitch diameter = basic pitch diameter (Col. 4). Maximum pitch diameter = Col. 4 + Col. 9, 10, 11, or 12. Minimum minor diameter = Col. 7—Col. 8.

Rolling Surfaces

In order to connect two shafts so that they shall have a definite angular velocity ratio, rolling surfaces are often used; and in order to have no slipping between the surfaces they must fulfill the following two conditions; the line of centers must pass through the point of contact, and the arcs of contact must be of equal length. The angular velocities, expressed usually in rpm, will be inversely proportional to the radii: $N/n = r/R$. The two surfaces most commonly used in practice, and the only ones having a constant angular velocity ratio, are cylinders where the shafts are parallel, and cones where the shafts (produced) intersect at an angle. In either case there are two possible directions of rotation, depending upon whether the surfaces roll in opposite directions (external contact) or in the same direction (internal contact). In Fig. 27, $R = nc/(N + n)$ and $r = Nc/(N + n)$; in Fig. 28, $R = nc/(N - n)$ and $r = Nc/(N - n)$. In Fig. 29, $\tan B = \sin A/[(n/N) + \cos A]$, and $\tan C = \sin A/[(N/n) + \cos A]$; in Fig. 30, $\tan B = \sin A/[(n/N) - \cos A]$, and $\tan C = \sin A/[(N/n) - \cos A]$. With the above values for the angles B and C, and the length d or e of one of the cones, R and r may be calculated.

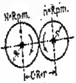


FIG. 27.

FIG. 28.

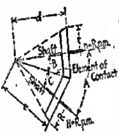


FIG. 29.

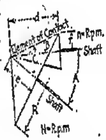


FIG. 30.

Epicyclic Trains

Epicyclic trains are combinations of gears in which some or all of the gears have a motion compounded of rotation about an axis and a translation or revolution of that axis. The gears are usually connected by a link called an arm, which often rotates about the axis of the first gear. Such trains may be calculated by first considering all gears locked and the arm turned; then the arm locked and the gears rotated. The algebraic sum of the separate motions will give the desired result. The following examples and method of tabulation will illustrate this. The figures on each gear refer to the number of teeth for that gear.

	A	B	C	D
Gear locked, Fig. 31	+1	+1	+1	+1
Arm locked, Fig. 31	0	−1	$+1 \times {}^{50}/_{20}$	$-1 \times {}^{50}/_{20} \times {}^{20}/_{40}$
Addition, Fig. 31	+1	0	$+3\frac{1}{2}$	$-\frac{1}{4}$
Gears locked, Fig. 32	+1	+1	+1	+1
Arm locked, Fig. 32	0	−1	$+1 \times {}^{30}/_{20}$	$+1 \times {}^{30}/_{20} \times {}^{20}/_{40}$
Addition, Fig. 32	+1	0	$+2\frac{1}{2}$	$+1\frac{3}{4}$

For common epicyclic trains, see also p. 817.

In Figs. 31 and 32 lock the gears and turn the arm A right-handed through 1 revolution (+1), then lock the arm and turn the gear B back to where it started (−1); gears C

Table 6. Coarse-thread Series, Loose Fit

Size	Major screw diameter, in.		Screw pitch diameter, in.		Max minor screw diameter	Max nut pitch diameter	Min nut minor diameter, in.
	Max	Min	Max	Min			
1	0.0723	0.0671	0.0622	0.0596	0.0531	0.0655	0.0561
2	0.0852	0.0796	0.0736	0.0708	0.0633	0.0772	0.0667
3	0.0981	0.0990	0.0846	0.0815	0.0725	0.0886	0.0764
4	0.1110	0.1042	0.0948	0.0914	0.0803	0.0992	0.0849
5	0.1240	0.1172	0.1070	0.1044	0.0933	0.1122	0.0979
6	0.1369	0.1293	0.1166	0.1128	0.0986	0.1215	0.1042
8	0.1629	0.1553	0.1426	0.1388	0.1246	0.1475	0.1302
10	0.1887	0.1795	0.1616	0.1570	0.1376	0.1675	0.1449
12	0.2147	0.2055	0.1876	0.1830	0.1636	0.1935	0.1709
1/4"	0.2485	0.2383	0.2160	0.2109	0.1872	0.2226	0.1959
5/16"	0.3109	0.2995	0.2748	0.2691	0.2427	0.2821	0.2524
3/8"	0.3732	0.3606	0.3326	0.3263	0.2965	0.3407	0.3073
7/16"	0.4354	0.4214	0.3890	0.3820	0.3478	0.3981	0.3602
1/2"	0.4978	0.4830	0.4478	0.4404	0.4034	0.4574	0.4167
9/16"	0.5601	0.5443	0.5060	0.4981	0.4579	0.5163	0.4723
5/8"	0.6224	0.6054	0.5634	0.5549	0.5109	0.5745	0.5266
3/4"	0.7472	0.7288	0.6822	0.6730	0.6245	0.6942	0.6417
7/8"	0.8719	0.8519	0.7997	0.7897	0.7356	0.8128	0.7547
1"	0.9966	0.9744	0.9154	0.9043	0.8432	0.9299	0.8647
1 1/8"	1.1211	1.0963	1.0283	1.0159	0.9458	1.0446	0.9704
1 1/4"	1.2461	1.2213	1.1533	1.1409	1.0708	1.1696	1.0954
1 1/2"	1.4956	1.4666	1.3873	1.3728	1.2911	1.4062	1.3196
1 3/4"	1.7448	1.7110	1.6149	1.5980	1.4994	1.6370	1.5335
2"	1.9943	1.9575	1.0500	1.8316	1.7217	1.8741	1.7594
2 1/4"	2.2443	2.2075	2.1000	2.0816	1.9717	2.1241	2.0094
2 1/2"	2.4936	2.4528	2.3312	2.3108	2.1896	2.3580	2.2294
2 3/4"	2.7436	2.7028	2.5812	2.5608	2.4369	2.6080	2.4794
3"	2.9936	2.9528	2.8312	2.8108	2.6869	2.8580	2.7294
3"	2.9927	2.9469	2.8071	2.7842	2.6421	2.8373	2.7294
Fine-thread Series, Loose Fit							
0	0.0593	0.0545	0.0512	0.0480	0.0440	0.0543	0.0465
1	0.0723	0.0673	0.0633	0.0608	0.0553	0.0665	0.0508
2	0.0853	0.0801	0.0752	0.0726	0.0661	0.0785	0.0691
3	0.0982	0.0926	0.0866	0.0830	0.0763	0.0902	0.0797
4	0.1111	0.1049	0.0976	0.0945	0.0855	0.1016	0.0894
5	0.1241	0.1177	0.1093	0.1061	0.0962	0.1134	0.1004
6	0.1370	0.1302	0.1208	0.1174	0.1063	0.1252	0.1109
8	0.1629	0.1557	0.1449	0.1413	0.1288	0.1496	0.1339
10	0.1889	0.1813	0.1686	0.1648	0.1506	0.1735	0.1562
12	0.2148	0.2062	0.1916	0.1873	0.1710	0.1971	0.1773
1/4"	0.2488	0.2402	0.2256	0.2213	0.2050	0.2311	0.2113
5/16"	0.3112	0.3020	0.2841	0.2795	0.2601	0.2900	0.2674
3/8"	0.3737	0.3645	0.3466	0.3420	0.3226	0.3525	0.3299
7/16"	0.4360	0.4258	0.4035	0.3984	0.3747	0.4101	0.3834
1/2"	0.4985	0.4883	0.4660	0.4609	0.4372	0.4726	0.4459
9/16"	0.5609	0.5495	0.5248	0.5191	0.4927	0.5321	0.5024
5/8"	0.6234	0.6120	0.5873	0.5816	0.5552	0.5946	0.5649
3/4"	0.7482	0.7356	0.7076	0.7013	0.6715	0.7157	0.6823
7/8"	0.8729	0.8589	0.8265	0.8195	0.7853	0.8356	0.7977
1"	0.9979	0.9839	0.9515	0.9445	0.9103	0.9606	0.9227
1 1/8"	1.1226	1.1068	1.0685	1.0606	1.0204	1.0788	1.0348
1 1/4"	1.2476	1.2310	1.1935	1.1856	1.4545	1.2038	1.1598
1 1/2"	1.4976	1.4818	1.4435	1.4356	1.3954	1.4538	1.4098
1 3/4"	1.7472	1.7288	1.6822	1.6690	1.6245	1.6982	1.6417
2"	1.9972	1.9788	1.9322	1.9190	1.0745	1.9482	1.8917
2 1/4"	2.2466	2.2244	2.1654	2.1509	2.0932	2.1833	2.1147
2 1/2"	2.4966	2.4744	2.4154	2.4009	2.3432	2.4333	2.3647
2 3/4"	2.7466	2.7244	2.6654	2.6509	2.5932	2.6833	2.6147
3"	2.9966	2.9744	2.9154	2.9009	2.8432	2.9333	2.8647

Other nut dimensions as follows: Minimum major diameter = basic major diameter, Col. 3, Table 5. Minimum pitch diameter = basic pitch diameter, Col. 4, Table 5. Maximum minor diameter = **Col. 7, Table 5.**

Hoisting Mechanisms

Pulley Block (Fig. 35). Given the weight W to be raised, the force F necessary is $F = V_W \times W/V_F = W/n$ = load/number of ropes; V_W and V_F being the respective velocities of W and F.

Differential Chain Block (Fig. 36). $F = V_W \times W/V_F = W(D - d)/2D$.

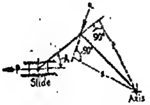


FIG. 39.

Worm and Wheel (Fig. 37). $F = \pi d(n/T)W/2\pi R = WP(d/D)/2\pi R$, where n = number of threads, single, double, triple, etc.

The triplex chain block (Fig. 38) is a geared hoist making use of the epicyclic train. $F = W \times M/L\{1 + [(T_D/T_C) \times (T_B/T_A)]\}$, where T = number of teeth on gears.

Toggle Joint (Fig. 39). $P = F \times s \cos A/t$.

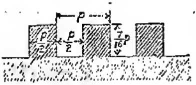


Fig. 7.—Sellers Square Thread.

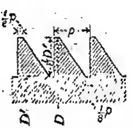


Fig. 8.—Buttress Thread.

Table 9. Sellers Standard Square Threads

(See Fig. 7)

Diam of bolt, in.	Threads per in.	Diam of root of thread, in.	Diam of bolt, in.	Threads per in.	Diam of root of thread, in.	Diam of bolt, in.	Threads per in.	Diam of root of thread, in.	Diam of bolt, in.	Threads per in.	Diam of root of thread, in.
1/4	10	0.1625	3/4	5	0.575	1 1/2	3	1.2084	3	1 3/4	2.5
5/16	9	0.2153	13/16	4 1/2	0.6181	1 5/8	2 7/8	1.307	3 1/4	1 3/4	2.75
3/8	8	0.2658	7/8	4 1/2	0.6806	1 3/4	2 3/4	1.4	3 1/2	1 5/8	2.962
7/16	7	0.3125	15/16	4	0.7163	1 7/8	2 1/2	1.525	3 3/4	1 1/2	3.168
1/2	6 1/2	0.3656	1	4	0.7813	2	2 1/2	1.612	4	1 1/2	3.418
9/16	6	0.4167	1 1/8	3 1/2	0.875	2 1/4	2 1/4	1.862			
5/8	5 1/2	0.4666	1 1/4	3 1/2	1.00	2 1/2	2	2.0626			
11/16	5	0.512	1 3/8	3	1.0834	2 3/4	2	2.3126			

The Buttress thread (Fig. 8) is a strong form, adapted for taking load in one direction only.

The Dardelet thread (Fig. 9) made by the Dardelet Threadlock Corp., New York, is a thread on which the nut is self-locking. It is similar to the Acme thread; the sides are inclined at 14 1/2 deg.

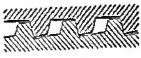


Fig. 9.—Dardelet Thread.

The S.A.E. Standard Screw-thread Series (Aug., 1922) includes a coarse-thread series sometimes called the **American** or **National Coarse Screw-thread Series,** which in the numbered sizes below 1/4 in. correspond to the Special Standard of the A.S.M.E. (1907) and from 1/4 in. up are the same as the

Table 10. S. A. E. Standard Screw-thread Series (1922)

Screw No.	Diam, in.	Threads per in.		Diam, in.	Threads per in.			Diam, in.	Threads per in.		
		Coarse	Fine		Coarse	Fine	Extra fine		Coarse	Fine	Extra fine
0	0.060	......	80	1/4	20	28	36	1 1/8	7	12	18
1	0.073	64	72	5/16	18	24	32	1 1/4	7	12	18
2	0.086	56	64	3/8	16	24	32	1 1/2	6	12	18
3	0.099	48	56	7/16	14	20	28	1 3/4	5	12	16
4	0.112	40	48	1/2	13	20	28	2	4 1/2	12	16
5	0.125	40	44	9/16	12	18	24	2 1/4	4 1/2	12	16
6	0.138	32	40	5/8	11	18	24	2 1/2	4	12	16
8	0.164	32	36	3/4	10	16	20	2 3/4	4	12	16
10	0.190	24	32	7/8	9	14	20	3	4	10	16
12	0.215	24	28	1	8	14	20	Over 3 to 6	......	10	16
								Over 6	......	8	16

Table 2. Whitworth Standard Screw Threads (See Fig. 2)

Diam of screw, in.	Threads per in.	Depth of thread, in.	Diam at root of thread, in.	Diam of screw, in.	Threads per in.	Depth of thread, in.	Diam at root of thread, in.
¼	20	0.0320	0.1860	1⅝	5	0.1281	1.3689
5/16	18	0.0356	0.2414	1¾	5	0.1281	1.4939
⅜	16	0.0400	0.2950	2	4½	0.1423	1.7154
7/16	14	0.0457	0.3460	2¼	4	0.1601	1.9298
½	12	0.0534	0.3933	2½	4	0.1601	2.1798
9/16	12	0.0534	0.4558	2¾	3½	0.1830	2.3841
⅝	11	0.0582	0.5086	3	3½	0.1830	2.6341
11/16	11	0.0582	0.5711	3¼	3¼	0.1970	2.8560
¾	10	0.0640	0.6219	3½	3¼	0.1970	3.1060
13/16	10	0.0640	0.6844	3¾	3	0.2134	3.3231
⅞	9	0.0711	0.7327	4	3	0.2134	3.5731
1	8	0.0800	0.8399	4½	2⅞	0.2227	4.0546
1⅛	7	0.0915	0.9420	5	2¾	0.2328	4.5343
1¼	7	0.0915	1.0670	5½	2⅝	0.2439	5.0121
1⅜	6	0.1067	1.1616	6	2½	0.2561	5.4877
1½	6	0.1067	1.2866				

Table 3. British Association Screw Threads (See Fig. 3)

Number	Diam of screw, mm	Approx diam, in.	Pitch, mm	Approx pitch, in.	Diam at root of thread, mm	Number	Diam of screw, mm	Approx. diam, in.	Pitch, mm	Approx pitch, in.	Diam at root of thread, mm
0	6.0	0.236	1.00	0.0394	4.8	13	1.20	0.047	0.25	0.0098	0.90
1	5.3	0.209	0.90	0.0354	4.22	14	1.00	0.039	0.23	0.0091	0.72
2	4.7	0.185	0.81	0.0319	3.73	15	0.90	0.035	0.21	0.0083	0.65
3	4.1	0.161	0.73	0.0287	3.22	16	0.79	0.031	0.19	0.0075	0.56
4	3.6	0.142	0.66	0.0260	2.81	17	0.70	0.028	0.17	0.0067	0.50
5	3.2	0.126	0.59	0.0232	2.49	18	0.62	0.024	0.15	0.0059	0.44
6	2.8	0.110	0.53	0.0209	2.16	19	0.54	0.021	0.14	0.0055	0.37
7	2.5	0.098	0.48	0.0189	1.92	20	0.48	0.019	0.12	0.0047	0.34
8	2.2	0.087	0.43	0.0169	1.68	21	0.42	0.017	0.11	0.0043	0.29
9	1.9	0.075	0.39	0.0154	1.43	22	0.37	0.015	0.10	0.0039	0.25
10	1.7	0.067	0.35	0.0138	1.28	23	0.33	0.013	0.09	0.0035	0.22
11	1.5	0.059	0.31	0.0122	1.13	24	0.29	0.011	0.08	0.0031	0.19
12	1.3	0.051	0.28	0.0110	0.96	25	0.25	0.010	0.07	0.0028	0.17

Table 4. French (Metric) Standard Screw Threads (See Fig. 4)

Diam of screw, mm	Pitch, mm	Diam at root of thread, mm	Width of flat, mm	Diam of screw, mm	Pitch, mm	Diam at root of thread, mm	Width of flat, mm	Diam of screw, mm	Pitch, mm	Diam at root of thread, mm	Width of flat, mm
3	0.5	2.35	0.06	18	2.5	14.75	0.31	40	4.0	34.80	0.50
4	0.75	3.03	0:09	20	2.5	16.75	0.31	42	4.5	36.15	0.56
5	0.75	4.03	0.09	22	2.5	18.75	0.31	44	4.5	38.15	0.56
6	1.0	4.70	0.13	22	3.0	18.10	0.38	45	4.5	39.15	0:56
7	1.0	5.70	0.13	24	3.0	20.10	0.38	46	4.5	40.15	0.56
8	1.0	6.70	0.13	26	3.0	22.10	0.38	48	5.0	41.51	0.63
8	1.25	6.38	0.16	27	3.0	23.10	0.38	50	5.0	43.51	0.63
9	1.0	7.70	0.13	28	3.0	24.10	0.38	52	5.0	45.51	0.63
9	1.25	7.38	0.16	30	3.5	25.45	0.44	56	5.5	48.86	0.69
10	1.5	8.05	0.19	32	3:5	27.45	0.44	60	5.5	52.86	0.69
11	1.5	9.05	0.19	33	3.5	28.45	0.44	64	6.0	56.21	0.75
12	1.5	10.05	0.19	34	3.5	29.45	0.44	68	6.0	60.21	0.75
12	1.75	9.73	0.22	36	4.0	30.80	0.5	72	6.5	63.56	0.81
14	2.0	11.40	0.25	38	4.0	32.80	0.5	76	6.5	67.56	0.81
16	2.0	13.40	0.25	39	4.0	33:80	0.5	80	7.0	70.91	0.88

The Whitworth pipe thread is the standard used in Great Britain. It is cut either straight or with a taper of ¾ in. per ft as in the Briggs standard.

Table 12. Whitworth Pipe Threads

Pipe size, in.	Diam, in.	Diam at bottom of thread, in.	Pipe size, in.	Diam, in.	Diam at bottom of thread, in.	Pipe size, in.	Diam, in.	Diam at bottom of thread, in.	Pipe size, in.	Diam, in.	Diam at bottom of thread, in.
⅛	0.3825	0.3367	⅞	1.189	1.0975	1⅝	2.021	1.905	2¾	3.247	3.1305
¼	0.518	0.4506	1	1.309	1.1925	1¾	2.116	1.9996	3	3.485	3.3685
⅜	0.6563	0.5889	1⅛	1.492	1.3755	1⅞	2.245	2.1285	3¼	3.6985	3.582
½	0.8257	0.7342	1¼	1.65	1.5335	2	2.347	2.2305	3½	3.912	3.7955
⅝	0.9022	0.8107	1⅜	1.745	1.6285	2¼	2.5875	2.471	3¾	4.1255	4.009
¾	1.041	0.9495	1½	1.8825	1.765	2½	3.0013	2.8848	4	4.339	4.223

Number of threads per inch: ⅛ in., 28; ¼ and ⅜ in., 19; ½, ⅝, ¾ and ⅞ in., 14; 1 in. and larger, 11.

Bolt Machine Screw Heads and Nuts: Forms and Proportions

The **A.S.M.E. Standard machine screws** have threads of the same form and proportions as the American (National) Standard (Fig. 1). The following diameters and pitches are standardized, the relation being: threads per in. $= 6.5/(\text{diameter} + 0.02)$. Certain special pitches are also recommended in addition to the standards.

Table 13. A.S.M.E. Standard Machine Screws (1907)

Screw No.	Basic diam, in.	Standard number of threads	Maximum screw diameters		Minimum screw diameters			Special number of threads
			Pitch diam, in.	Root diam, in.	External diam, in.	Pitch diam, in.	Root diam, in.	
0	0.060	80	0.0519	0.0438	0.0572	0.0505	0.0411	
1	0.073	72	0.064	0.0550	0.070	0.0625	0.052	64
2	0.086	64	0.0759	0.0657	0.0820	0.0743	0.0624	56
3	0.099	56	0.0874	0.0758	0.0955	0.0857	0.0721	48
4	0.112	48	0.0985	0.0839	0.1082	0.0966	0.0808	40, 36
5	0.125	44	0.1102	0.0955	0.1210	0.1082	0.0910	40, 36
6	0.138	40	0.1218	0.1055	0.1330	0.1197	0.1007	36, 32
7	0.151	36	0.1330	0.1149	0.1466	0.1308	0.1097	32, 30
8	0.164	36	0.146	0.1279	0.1596	0.1438	0.1227	32, 30
9	0.177	32	0.1567	0.1364	0.1722	0.1544	[illegible]	30, 24
10	0.190	30	0.1684	0.1467	0.1852	0.166	0.1407	32, 24
12	0.216	28	0.1928	0.1696	0.2111	0.1903	0.1633	24
14	0.242	24	0.2149	0.1879	0.2368	0.2123	0.1807	20
16	0.268	22	0.2385	0.209	0.2626	0.2358	0.2013	20
18	0.294	20	0.2615	0.229	0.2884	0.2587	0.2208	18
20	0.320	20	0.2875	0.255	0.3144	0.2847	0.2468	18
22	0.346	18	0.3099	0.2730	0.3402	0.3070	0.2649	16
24	0.372	16	0.3314	0.2908	0.366	0.3284	0.281	18
26	0.398	16	0.3574	0.3168	0.392	0.3544	0.307	14
28	0.424	14	0.3776	0.3312	0.4178	0.3745	0.3204	16
30	0.450	14	0.4036	0.3572	0.4438	0.4005	0.3464	16

Table 5. Coarse-thread Series, Free, Medium, and Close Fits

1	2	3	4	5	6	7	8	9	10	11	12
Sizes	Threads per in.	Major diam, in	Pitch diam, in.	Major screw diam tolerance, in.	Max minor screw diam, in.	Max minor nut diam, in.	Minor nut tolerance, diam, in.	Pitch diam tolerances, in.[a]			
								Loose fit[b]	Free fit	Medium fit	Close fit
1	64	0.0730	0.0629	0.0038	0.0538	0.0578	0.0043	0.0026	0.0019	0.0014	0.0007
2	56	0.0860	0.0744	0.0040	0.0641	0.0686	0.0048	0.0028	0.0020	0.0015	0.0007
3	48	0.0990	0.0855	0.0044	0.0754	0.0787	0.0056	0.0031	0.0022	0.0016	0.0008
4	40	0.1120	0.0958	0.0048	0.0813	0.0876	0.0064	0.0034	0.0024	0.0017	0.0009
5	40	0.1250	0.1088	0.0048	0.0943	0.1006	0.0064	0.0034	0.0024	0.0017	0.0010
6	32	0.1380	0.1177	0.0054	0.0997	0.1076	0.0076	0.0038	0.0027	0.0019	0.0010
8	32	0.1640	0.1437	0.0054	0.1257	0.1336	0.0076	0.0038	0.0027	0.0019	0.0011
10	24	0.1900	0.1629	0.0066	0.1389	0.1494	0.0092	0.0046	0.0033	0.0024	0.0012
12	24	0.2160	0.1889	0.0066	0.1649	0.1754	0.0092	0.0046	0.0033	0.0024	0.0012
¼"	20	0.2500	0.2175	0.0072	0.1887	0.2013	0.0101	......	0.0036	0.0026	0.0013
5/16"	18	0.3125	0.2764	0.0082	0.2443	0.2584	0.0106	......	0.0041	0.0030	0.0015
⅜"	16	0.3750	0.3344	0.0090	0.2983	0.3141	0.0111	......	0.0045	0.0032	0.0016
7/16"	14	0.4375	0.3911	0.0098	0.3499	0.3679	0.0119	......	0.0049	0.0036	0.0018
½"	13	0.5000	0.4500	0.0104	0.4056	0.4251	0.0123	......	0.0052	0.0037	0.0019
9/16"	12	0.5625	0.5084	0.0112	0.4603	0.4813	0.0127	......	0.0056	0.0040	0.0020
⅝"	11	0.6250	0.5660	0.0118	0.5135	0.5364	0.0131	......	0.0059	0.0042	0.0021
¾"	10	0.7500	0.6850	0.0128	0.6273	0.6526	0.0136	......	0.0064	0.0045	0.0023
⅞"	9	0.8750	0.8028	0.0140	0.7387	0.7667	0.0142	......	0.0070	0.0049	0.0024
1"	8	1.0000	0.9188	0.0152	0.8466	0.8782	0.0148	......	0.0076	0.0054	0.0027
1⅛"	7	1.1250	1.0322	0.0170	0.9497	0.9858	0.0154	......	0.0085	0.0059	0.0030
1¼"	7	1.2500	1.1572	0.0170	1.0747	1.1108	0.0154	......	0.0085	0.0059	0.0030
1⅜	6	1.3750	1.2667	0.0202	1.1705	......	......	......	0.0101	0.0071	
1½"	6	1.5000	1.3917	0.0202	1.2955	1.3376	0.0180	......	0.0101	0.0071	0.0036
1¾"	5	1.7500	1.6201	0.0232	1.5046	1.5551	0.0216	......	0.0116	0.008[illegible]	0.0041
2"	4½	2.0000	1.8557	0.0254	1.7274	1.7835	0.0241	......	0.0127	0.0089	0.0044
2¼"	4½	2.2500	2.1057	0.0254	1.9774	2.0335	0.0241	......	0.0127	0.0089	0.0044
2½"	4	2.5000	2.3376	0.0280	2.1933	2.2564	0.0270	......	0.0140	0.0097	0.0048
2¾"	4	2.7500	2.5876	0.0280	2.4433	2.5064	0.0270	......	0.0140	0.0097	0.0048
3"	4	3.0000	2.8376	0.0280	2.6933	2.7564	0.0270	......	0.0140	0.0097	0.0048
3"	3½	3.000	2.8144	0.0314	2.6494	2.7216	0.0309	......	0.0157	0.0107	0.0053
3¼	4	3.2500	3.0876	0.280	2.9533	......	......	......	0.0140	0.0097	
3½	4	3.5000	3.3376	0.280	3.1933	......	......	......	0.0140	0.0097	
3¾	4	3.7500	3.5876	0.280	3.4433	......	......	......	0.0140	0.0097	
4	4	4.0000	3.8376	0.280	3.693[illegible]	......	......	......	0.0140	0.0097	
Fine-thread Series, Free, Medium, and Close Fits											
0	80	0.0600	0.0519	0.0034	0.0447	0.0478	0.0027	0.0024	0.0017	0.0013	0.0006
1	72	0.0730	0.0640	0.0036	0.0560	0.0595	0.0030	0.0025	0.0018	0.0013	0.0007
2	64	0.0860	0.0759	0.0038	0.0668	0.0708	0.0033	0.0026	0.0019	0.0014	0.0007
3	56	0.0990	0.0874	0.0040	0.0771	0.0816	0.0037	0.0028	0.0020	0.0015	0.0007
4	48	0.1120	0.0985	0.0044	0.0864	0.0917	0.0043	0.0031	0.0022	0.0016	0.0008
5	44	0.1250	0.1102	0.0046	0.0971	0.1029	0.0045	0.0032	0.0023	0.0016	0.0009
6	40	0.1380	0.1218	0.0048	0.1073	0.1136	0.0049	0.0034	0.0024	0.0017	0.0009
8	36	0.1640	0.1460	0.0050	0.1299	0.1369	0.0052	0.0036	0.0025	0.0018	0.0009
10	32	0.1900	0.1697	0.0054	0.1517	0.1596	0.0056	0.0038	0.0027	0.0019	0.0010
12	28	0.2160	0.1928	0.0062	0.1722	0.1812	0.0060	0.0043	0.0031	0.0022	0.0011
¼"	28	0.2500	0.2268	0.0062	0.2062	0.2152	0.0060	......	0.0031	0.0022	0.0011
5/16"	24	0.3125	0.2854	0.0066	0.2614	0.2719	0.0065	......	0.0033	0.0024	0.0012
⅜"	24	0.3750	0.3479	0.0066	0.3239	0.3344	0.0065	......	0.0033	0.0024	0.0012
7/16"	20	0.4375	0.4050	0.0072	0.3762	0.3888	0.0072	......	0.0036	0.0026	0.0013
½"	20	0.5000	0.4675	0.0072	0.4387	0.4513	0.0072	......	0.0036	0.0026	0.0013
9/16"	18	0.5625	0.5265	0.0082	0.4943	0.5084	0.0076	......	0.0041	0.0030	0.0015
⅝"	18	0.6250	0.5889	0.0082	0.5568	0.5709	0.0076	......	0.0041	0.0030	0.0015
¾"	16	0.7500	0.7094	0.0090	0.6733	0.6891	0.0080	......	0.0045	0.0032	0.0016
⅞"	14	0.8750	0.8286	0.0098	0.7874	0.8054	0.0085	......	0.0049	0.0036	0.0018
1"	14	1.0000	0.9536	0.0098	0.9124	0.9304	0.0085	......	0.0049	0.0036	0.0018
1⅛"	12	1.1250	1.0709	0.0112	1.0228	1.0348	0.0090	......	0.0056	0.0040	0.0020
1¼"	12	1.2500	1.1959	0.0112	1.1478	1.1688	0.[illegible]	......	0.0056	0.0040	0.0020
1⅜	12	1.3750	1.3209	0.0112	1.2728	......	......	......	0.0056	0.0040	
1½"	12	1.5000	1.4459	0.0112	1.3978	1.4188	0.0090	......	0.0056	0.0040	0.0020

[a] These tolerances are cumulative and include all errors of lead and angle. For other dimensions, see p. 760.

[b] This fit is recommended for nuts only and only in the numbered series.

Weight of 100 Bolts and Nuts, Lb

Length under head, in.	Bolts with square heads and nuts									Bolts with hexagon heads and nuts				
	Diameter of bolt, in.									Diameter of bolt, in.				
	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1/2	5/8	3/4	7/8	1
1	4	7	11	15	22	37	56	84	122	19	33	52	76	110
1 1/4	4	7	11	16	23	39	59	88	128	20	34	54	80	116
1 1/2	5	8	12	17	24	41	62	93	133	22	36	57	85	121
1 3/4	5	8	13	18	26	43	64	97	139	23	38	60	89	127
2	5	9	14	19	27	45	67	101	144	24	40	63	93	132
Lb per in. additional	1.4	2.2	3.1	4.3	5.6	8.7	12.5	17.0	22.3	5.6	8.7	12.5	17.0	22.3

Weights of Nuts, Bolt Heads, and Shanks

(For calculating the weight of large bolts)

Diameter of bolt, in.	1 1/4	1 1/2	1 3/4	2	2 1/2	3
Wt of 1 hex head and 1 hex nut, lb....	1.7	2.9	4.6	6.8	13.0	22.0
Wt of 1 sq head and 1 sq nut, lb......	2.0	3.5	5.5	8.1	15.5	26.2
Wt of shank per inch, lb............	0.35	0.5	0.68	0.89	1.40	2.00

Drills for Pipe Taps. The sizes of twist drills used in boring holes to be reamed with pipe reamers and threaded with pipe taps, for both the Whitworth and American (Briggs) systems, are given in Table 16.

Table 16. Twist Drills for Use with Pipe Taps

Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.	Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.	Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.	Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.
1/8	21/64	5/16	3/4	29/32	29/32	1 3/4	......	1 15/16	3	3 3/16	3 9/32
1/4	27/64	27/64	7/8	......	1 1/16	2	2 3/16	2 5/32	3 1/4	......	3 1/2
3/8	9/16	9/16	1	1 1/8	1 1/8	2 1/4	......	2 13/32	3 1/2	3 11/16	3 3/4
1/2	11/16	11/16	1 1/4	1 15/32	1 15/32	2 1/2	2 9/16	2 25/32	3 3/4	......	4
5/8	......	25/32	1 1/2	1 23/32	1 23/32	2 3/4	......	3 1/32	4	4 5/16	4 1/4

Table 17. Oval Fillister Head Machine Screws. A.S.M.E. Standard

A = diam of body. $B = 1.64A - 0.009$ = diam of head and radius for oval. $C = 0.66A - 0.002$ = height of side. $D = 0.173A + 0.015$. $E = \frac{1}{2}F$ = depth of slot. $F = 0.134B + C$ = height of head.

(Letters refer to Fig. 11. All dimensions in inches)

A	B	C	D	E	F	A	B	C	D	E	F
0.060	0.0894	0.0376	0.025	0.025	0.0496	0.216	0.3452	0.1405	0.052	0.093	0.1868
0.073	0.1107	0.0461	0.028	0.030	0.0609	0.242	0.3879	0.1577	0.057	0.105	0.2097
0.086	0.132	0.0548	0.030	0.036	0.0725	0.268	0.4305	0.1748	0.061	0.116	0.2325
0.099	0.153	0.0633	0.032	0.042	0.0830	0.294	0.4731	0.192	0.066	0.128	0.2554
0.112	0.1747	0.0719	0.034	0.048	0.0953	0.320	0.5158	0.2092	0.070	0.140	0.2783
0.125	0.196	0.0805	0.037	0.053	0.1068	0.346	0.5584	0.2263	0.075	0.150	0.3011
0.138	0.217	0.089	0.039	0.059	0.1180	0.372	0.601	0.2435	0.079	0.162	0.3240
0.151	0.2386	0.0976	0.041	0.065	0.1296	0.398	0.6437	0.2606	0.084	0.173	0.3469
0.164	0.2599	0.1062	0.043	0.071	0.1410	0.424	0.6863	0.2778	0.088	0.185	0.3698
0.177	0.2813	0.1148	0.046	0.076	0.1524	0.450	0.727	0.295	0.093	0.201	0.4024
0.190	0.3026	0.1234	0.048	0.082	0.1639						

The International Standard screw thread, as adopted by the "Congrès International pour l'Unification des Filetages," in Zurich, Oct. 24, 1898, is shown in Fig. 5, in which $d = 0.7036p$ and $t = 0.866p$. See Table 7.

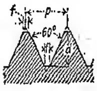


FIG. 4.—Metric.

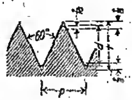


FIG. 5.—International.

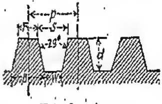


FIG. 6.—Acme.

Standard Screw Threads.

Table 7. International Standard Metric Screw Threads
(See Fig. 5)

Diam of screw, mm	Pitch, mm	Diam of screw, mm	Pitch, mm	Diam of screw, mm	Pitch, mm	Diam of screw, mm	Pitch, mm	Diam of screw, mm	Pitch, mm	Diam of screw, mm	Pitch, mm
6	1.00	12	1.75	24	3.00	42	4.50	64	6.00	96	8.00
7	1.00	14	2.00	27	3.00	45	4.50	68	6.00	116	9.00
8	1.25	16	2.00	30	3.50	48	5.00	72	6.50	136	10.00
9	1.25	18	2.50	33	3.50	52	5.00	76	6.50	......	......
10	1.50	20	2.50	36	4.00	56	5.50	80	7.00	......	......
11	1.50	22	2.50	39	4.00	60	5.50	88	7.50	......	......

Power Transmission Screw Threads: Forms and Proportions

The Acme 29 deg screw thread is shown in Fig. 6. The proportions of screws with Acme threads, given in Table 8, are obtained from the formulas $d = 0.5p + 0.01$ in.; flat (top) $= 0.3707p$; flat (bottom) $= 0.3707p - 0.0052$ in.

Table 8. Acme 29 Deg Screw Threads
(Letters refer to Fig. 6)

N	p	d	F	W	S	B
Number of threads per in.	Pitch of single thread, in.	Depth of thread, in.	Width of top of thread, in.	Width of space at bottom of thread, in.	Width of space at top of thread, in.	Thickness at root of thread, in.
1	1.0	0.5100	0.3707	0.3655	0.6293	0.6345
1⅓	0.750	0.3850	0.2780	0.2728	0.4720	0.4772
2	0.500	0.2600	0.1853	0.1801	0.3147	0.3199
3	0.3333	0.1767	0.1235	0.1183	0.2098	0.2150
4	0.250	0.1350	0.0927	0.0875	0.1573	0.1625
5	0.200	0.1100	0.0741	0.0689	0.1259	0.1311
6	0.1667	0.0933	0.0618	0.0566	0.1049	0.1101
7	0.1428	0.0814	0.0530	0.0478	0.0899	0.0951
8	0.125	0.0725	0.0463	0.0411	0.0787	0.0039
9	0.1111	0.0655	0.0413	0.0361	0.0699	0.0751
10	0.10	0.0600	0.0371	0.0319	0.0629	0.0681

The square thread (Fig. 7) has the proportions given in Table 9 for the range of sizes tabulated.

Table 21. Machine Screw Standards
(American Screw Co., Providence, R. I.)

Trade number	Diam of body, in.	Threads per in.	No. of drill	Size of drill, in.	Length, in.
2	0.0842	48, 56, 64	49	0.0730	1/8 – 3/8
3	0.0973	48, 56	45	0.0820	1/8 – 7/8
4	0.1105	32, 36, 40	42	0.0935	1/8 – 2
5	0.1236	32, 36, 40	38	0.1015	1/8 – 2 1/4
6	0.1368	30, 32, 36	35	0.1100	1/8 – 2 1/4
7	0.1500	30, 32	30	0.1285	1/8 – 3
8	0.1631	30, 32, 36	29	0.1360	1/8 – 3
9	0.1763	24, 30, 32	27	0.1440	3/16 – 3 1/2
10	0.1894	24, 30, 32	25	0.1495	3/16 – 3 1/2
12	0.2158	20, 24	17	0.1730	1/4 – 3 1/2
14	0.2421	18, 20, 24	13	0.1850	1/4 – 4
16	0.2684	16, 18, 20	6	0.2040	5/16 – 4
18	0.2947	16, 18, 20	1	0.2280	3/8 – 4
20	0.3210	16, 18	*D*	0.246	3/8 – 4
22	0.3474	16, 18	*J*	0.277	5/8 – 4
24	0.3737	14, 16, 18	*N*	0.302	5/8 – 4
26	0.4000	14, 16	*P*	0.323	3/4 – 4
28	0.4263	14, 16	*R*	0.339	3/4 – 4
30	0.4526	14, 16	*U*	0.368	3/4 – 4

Standard lengths vary by the following increments: 1/8 in. to 1 in. by 1/16 in.; 1 in. to 2 in. by 1/8 in.; 2 in. to 4 in. by 1/4 in.

Thread-cutting screws, made by the Shakeproof Lock Washer Co., Chicago, are adaptable to the American National Coarse- and Fine-thread Series and have been standardized from No. 4 screw to 3/8 in. diameter and for metal thicknesses up to 1/2 in. These are supplied in all types of head.

Eyebolts used by the Union Iron Works are made to the proportions given in Fig. 15 and Table 22.

Table 22. Proportions of Eyebolts
(See Fig. 15)

Diam of bolt, in.	Diam of stock in eye, in.	A, in.	B, in.	C, in.	D, in.	E, in.	F, in.	Capacity, lb
3/8	3/8	1 1/2	2 1/4	1 3/8	5/8	1	2	767
1/2	3/8	1 1/2	2 1/4	1 3/8	5/8	1	2	1,104
5/8	1/2	2	3	1 3/4	1	1 1/4	2 3/4	1,963
3/4	1/2	2	3	1 3/4	1	1 1/4	2 3/4	2,485
7/8	5/8	2 1/4	3 1/2	2 1/8	1 1/4	1 3/4	3 3/8	3,712
1	5/8	2 1/4	3 1/2	2 1/8	1 1/4	1 3/4	3 3/8	5,185
1 1/8	3/4	2 1/2	4	2 1/2	1 1/2	2 1/4	4	6,903
1 1/4	3/4	2 1/2	4	2 1/2	1 1/2	2 1/4	4	7,854
1 3/8	7/8	2 3/4	4 1/2	2 3/4	2	2 1/2	4 3/4	9,940
1 1/2	7/8	2 3/4	4 1/2	2 3/4	2	2 1/2	4 3/4	12,270
1 5/8	1 1/8	3 1/4	5 1/4	3 1/4	2 1/2	3	6	13,520
1 3/4	1 1/8	3 1/2	5 1/4	3 1/4	2 1/2	3	6	16,210
1 7/8	1 1/4	4	6 1/2	4	3	3 1/2	7	19,150
2	1 1/4	4	6 1/2	4	3	3 1/2	7	22,340

Strength of Eyebolts. The following formula by C. M. Sames (*Ind. Eng.*, Sept., 1911) closely represents the practice of the General Electric Co.: Load in lb at which deformation begins $= 33{,}000\ d^3/(D_1 - 1.2D_2)$, in which d = diam of stock in eye, D_1 = mean diam of eye (= internal diam + d), and D_2 = diam of bolt shank at root of thread—all in inches. In designing, for eyebolts forged from double refined iron, take safe load equal to 1/2 that given by formula.

American Standard, Table 1; a fine-thread series, sometimes called the **American** or **National Fine Series,** which in the numbered sizes corresponds to the A.S.M.E. Standard (1907) and from ¼ in. up with the S.A.E. Standard, Table 23; and an **extra-fine series** (S.A.E., 1915).

Screw Threads for Pipes

The American National taper pipe thread was formerly known as the Briggs pipe thread. Its form is shown in Fig. 10, and it is made to the following specifications: Taper of pipe end = ¾ in. per ft = 1 in 16. Depth of thread, $h = 0.8 \times$ pitch (p). Pitch diameter at end of pipe, $E_0 = D - (0.05D + 1.1)p$. Length of effective thread, $L_2 = p(0.8D + 6.8)$. D = outside diameter of pipe (see p. 907). See Table 11. Further details in A.S.A., B2.

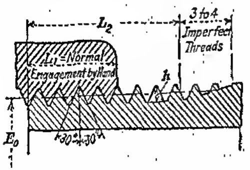


FIG. 10.—American National Taper Pipe Thread.

Table 11. American National Taper Pipe Threads
(Letters refer to Fig. 10)

Nominal inside diam of pipe, in.	No. of threads per in.	Length of normal engagement, in. L_1	Length of effective thread, in. L_2	Pitch diameter at end of pipe, in. E_0	Nominal inside diam of pipe, in.	No. of threads per in.	Length of normal engagement, in. L_1	Length of effective thread, in. L_2	Pitch diameter at end of pipe, in. E_0
⅛	27	0.180	0.26385	0.36351	3½	8	0.821	1.25000	3.83750
¼	18	0.200	.40178	0.47739	4	8	0.844	1.30000	4.33438
⅜	18	0.240	.40778	0.61201	4½	8	0.875	1.35000	4.83125
½	14	0.320	.53371	0.75843	5	8	0.937	1.40630	5.39073
¾	14	0.339	.54571	0.96768	6	8	0.958	1.51250	6.44609
1	11½	0.400	.68278	1.21363	7	8	1.000	1.61250	7.43984
1¼	11½	0.420	.70678	1.55713	8	8	1.063	1.71250	8.43359
1½	11½	0.420	.72348	1.79609	9	8	1.130	1.81250	9.42734
2	11½	0.436	.75652	2.26902	10	8	1.210	1.92500	10.54531
2½	8	0.682	1.13750	2.71953	11	8	1.285	2.02500	11.53906
3	8	0.766	1.2000	3.34062	12	8	1.360	2.12500	12.53281

For O.D. pipe 14 to 30 in. diam, 8 threads per in.

American National straight pipe threads have a pitch of $D - (0.05D + 1.1)p + \frac{1}{16}L_1$. See Table 11 for values of L_1. *Couplings with straight internal threads* may be used for ordinary pressures; taper threads should be used for high pressures.

Dimensions of Shakeproof Lock Washers
(All dimensions in inches)

Size of screw	External teeth		Internal teeth		Extra-heavy, internal teeth		Size of screw, in.	External teeth		Internal teeth		Extra-heavy, internal teeth	
	Outside diam	Thickness	Outside diam	Thickness	Outside diam	Thickness		Outside diam	Thickness	Outside diam	Thickness	Outside diam	Thickness
No. 2	....	....	3/16	0.010	....	....	7/16	25/32	0.035	25/32	0.035	27/32	0.062
3			[illegible]	0.012			1/2	7/8	0.040	7/8	0.040	59/64	0.062
4							9/16	31/32	0.040	31/32	0.040	1 1/32	0.062
6(1/8")							5/8	1 1/16	0.045	1 1/16	0.045	1 1/8	0.062
8(5/32")							11/16	....	....	1 5/32	0.045	..	
10(3/16")							3/4	1 1/4	0.050	1 1/4	0.050	1 1/4	0.072
12							7/8	1 7/16	0.062	1 3/8	0.050	1 7/16	0.078
1/4"							1	1 5/8	0.062	1 5/8	0.062		
5/16"							1 1/8	....	....	1 13/16	0.062		
3/8"							1 1/4	....	....	1 15/16	0.062		

Foundation bolts may be made to the forms shown in Figs. 20 to 22. The difficulty in replacing such bolts as may become broken or injured in service causes the form shown in Fig. 20 to be favored whenever it can be used. $l \geq 5d$.

The S.A.E. Standard for Bolts and Nuts is shown in Fig. 23, proportions of bolts made to this standard being given in Table 23.

Table 23. S.A.E. Screw Standard
(See Fig. 23)

D	$1/p$	A	A_1	B	C	E	H	I	K	d
1/4	28	9/32	7/32	7/16	3/32	5/64	3/16	3/32	1/16	1/16
5/16	24	21/64	17/64	1/2	3/32	5/64	15/64	7/64	1/16	1/16
3/8	24	13/32	21/64	9/16	1/8	1/8	9/32	1/8	3/32	3/32
7/16	20	29/64	3/8	5/8	5/8	1/8	21/64	1/8	3/32	3/32
1/2	20	9/16	7/16	3/4	9/16	1/8	3/8	1/8	3/32	3/32
9/16	18	39/64	31/64	7/8	3/16	5/32	27/64	1/8	3/32	1/8
5/8	18	23/32	35/64	15/16	1/4	5/32	15/32	1/8	3/32	1/8
11/16	16	[illegible]	[illegible]	1	1/4	[illegible]	[illegible]	1/8	3/32	1/8
3/4	16	13/16	21/32	1 1/16	1/4	5/32	9/16	1/8	3/32	1/8
7/8	14	29/32	49/64	1 1/4	1/4	5/32	21/32	1/8	3/32	1/8
1	14	1	7/8	1 7/16	1/4	5/32	3/4	1/8	3/32	1/8
1 1/8	12	1 5/32	63/64	1 5/8	5/16	7/32	27/32	7/32	5/32	11/64
1 1/4	12	1 1/4	1 3/32	1 13/16	5/16	7/32	15/16	7/32	5/32	11/64
1 3/8	12	1 13/32	1 13/64	2	3/8	1/4	1 1/32	1/4	3/16	13/64
1 1/2	12	1 1/2	1 5/16	2 3/16	3/8	1/4	1 1/8	1/4	3/16	13/64

All dimensions in inches. **$1/p$ = number of threads per inch.** All heads and nuts to be semifinish. Material—steel **of not less than 100,000 lb** tensile strength and not less than 60,000 lb elastic limit. **These threads not to be used in** cast iron, brass, or aluminum.

Coach and lag screws as shown in Fig. 24 are made to the proportions given in Table 24.

Wood screws are made in lengths from 1/4 to 5 in. for steel and from 1/4 to 3 1/2 in. for brass screws, increasing by 1/8 in. up to 1 in., by 1/4 in. up to 3 in., and by 1/2 in. up to 5 in. **The American National Standard sizes** are given in Table 25. Screws are made with flat, round, or oval heads.

Wrench head bolts and nuts and wrench openings have been standardized by the A.S.A. The **wrench openings** selected are given in Table 14.

Table 14. Open-end Wrench Openings
(All dimensions in inches)

Basic width across flats, bolt heads and nuts	Wrench openings Maximum	Wrench openings Minimum	Basic width across flats, bolt heads and nuts	Wrench openings Maximum	Wrench openings Minimum	Basic width across flats, bolt heads and nuts	Wrench openings Maximum	Wrench openings Minimum	Basic width across flats, bolt heads and nuts	Wrench openings Maximum	Wrench openings Minimum
5/32	0.163	0.158	1/2	0.510	0.503	1	1.015	1.006	2 1/4	2.277	2.262
3/16	0.194	0.189	9/16	0.573	0.566	1 1/8	1.142	1.132	2 5/8	2.656	2.639
1/4	0.257	0.252	5/8	0.636	0.629	1 1/4	1.267	1.257	3	3.035	3.016
5/16	0.321	0.314	3/4	0.763	0.755	1 5/16	1.331	1.320	3 3/8	3.414	3.393
11/32	0.353	0.346	13/16	0.826	0.818	1 1/2	1.521	1.509	3 3/4	3.793	3.770
3/8	0.384	0.377	7/8	0.888	0.880	1 11/16	0.710	1.697	4 1/8	4.171	4.147
7/16	0.447	0.440	15/16	0.952	0.943	1 7/8	1.898	1.885	4 1/2	4.549	4.523

Bolt heads and nuts have been standardized so as to fit the wrench sizes. See Table 15.

Table 15. Square and Hexagonal Regular Bolt Heads
(All dimensions in inches)

Bolt diameter	Rough and semifinished					Finished				
	Width across flats		Min width across corners		Height	Width across flats		Min width across corners		Height
	Max	Min	Hex	Square		Max	Min	Hex	Square	
1/4	3/8	0.363	0.414	0.498	11/64	7/16	0.428	0.488	0.588	3/16
5/16	1/2	0.484	0.552	0.665	13/64	9/16	0.552	0.629	0.758	15/64
3/8	9/16	0.544	0.620	0.747	1/4	5/8	0.613	0.699	0.842	9/32
7/16	5/8	0.603	0.687	0.828	19/64	3/4	0.737	0.840	1.012	21/64
1/2	3/4	0.725	0.827	0.995	21/64	13/16	0.799	0.911	1.097	3/8
9/16	7/8	0.847	0.966	1.163	3/8	7/8	0.861	0.982	1.182	27/64
5/8	15/16	0.906	1.033	1.244	27/64	15/16	0.922	1.051	1.266	15/32
3/4	1 1/8	1.088	1.240	1.494	1/2	1 1/8	1.108	1.263	1.521	9/16
7/8	1 5/16	1.269	1.447	1.742	19/32	1 5/16	1.293	1.474	1.775	21/32
1	1 1/2	1.450	1.653	1.991	27/32	1 1/2	1.479	1.686	2.031	3/4
1 1/8	1 11/16	1.631	1.859	2.239	3/4	1 11/16	1.065	1.898	2.286	27/32
1 1/4	1 7/8	1.813	2.067	2.489	27/32	1 7/8	1.850	2.109	2.540	15/16
1 1/2	2 1/4	2.175	2.480	2.986	1	2 1/4	2.222	2.533	3.051	1 1/8
1 3/4	2 5/8	2.538	2.893	3.485	1 5/32	2 5/8	2.593	2.956	3.560	1 5/16
2	3	2.900	3.306	3.982	1 11/32	3	2.964	3.379	4.070	1 1/2
2 1/4	3 3/8	3.263	3.720	4.480	1 1/2	3 3/8	3.335	3.802	4.579	1 11/16
2 1/2	3 3/4	3.625	4.133	4.977	1 21/32	3 3/4	3.707	4.226	5.090	1 7/8
2 3/4	4 1/8	3.988	4.546	5.476	1 53/64	4 1/8	4.078	4.649	5.599	2 1/16
3	4 1/2	4.350	4.959	5.973	2	4 1/2	4.449	5.072	6.108	2 1/4

Regular nuts (rough, semifinished, and finished) have a maximum width across flats of $1\frac{1}{2}D$ except for $D = \frac{1}{4}$ to $\frac{5}{16}$ when the width $= 1\frac{1}{2}D + \frac{1}{16}$. D is bolt diameter. Tolerance for width is $-0.050D$. Thickness is $\frac{7}{8}D$.

Table 26. Dimensions of Round and Square Washers

Size of bolt, in.	Size of hole, in.	U. S. Standard, round: Outside diam, in.	U. S. Standard, round: Thickness, in.	U. S. Standard, round: Approx No. in 100 lb	Narrow gage, round: Outside diam, in.	Narrow gage, round: Thickness, in.	Standard sizes, square: Width, in.	Standard sizes, square: Thickness, in.
3/16	1/4	9/16	3/64	44,300				
1/4	5/16	3/4	1/16	18,100	5/8	3/16		
5/16	3/8	7/8	1/16	13,600	3/4	1/16		
3/8	7/16	1	5/64	7,700	7/8	1/16	1 1/2	1/8
7/16	1/2	1 1/4	5/64	4,500	1 1/8	5/64	1 3/4	1/8
1/2	9/16	1 3/8	3/32	3,400	1 1/4	3/32	2	3/16
9/16	5/8	1 1/2	3/32	2,700	1 3/8	3/32		
5/8	11/16*	1 3/4	1/8	1,400	1 1/2	1/8	2 1/4	1/4
3/4	13/16*	2	1/8	1,200	1 3/4	1/8	2 1/2	1/4
7/8	15/16*	2 1/4	5/32	760	2	5/32	3	1/4
1	1 1/16*	2 1/2	5/32	570	2 1/4	5/32	3 1/2	3/8
1 1/8	1 1/4	2 3/4	5/32	490	2 1/2	5/32	4	3/8
1 1/4	1 3/8	3	5/32	415	2 5/8	5/32	4 1/2	3/8
1 3/8	1 1/2	3 1/4	11/64	325	3	11/64	5	3/8
1 1/2	1 5/8	3 1/2	11/64	275	3 1/4	11/64	6	3/8
1 5/8	1 3/4	3 3/4	11/64	245				
1 3/4	1 7/8	4	11/64	200	......	......	6	3/8
1 7/8	2	4 1/4	11/64	185				
2	2 1/8	4 1/2	11/64	170	......	......	6	3/8
2 1/4	2 3/8	4 3/4	3/16	140				
2 1/2	2 5/8	5	7/32	115				

* Holes in square washers 1/32 in. larger for these four sizes.

Carriage bolts (Fig. 26) may be obtained in the sizes given in Table 27 and to any lengths desired. Length of thread, 2 to 4 times thickness of nut, depending on length of bolt.

FIG. 26.—Carriage Bolts.

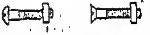


FIG. 27.—Stove Bolts.

Table 27. Dimensions of Carriage Bolts

(See Fig. 26)

Diameter of bolt, in.	0.19	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4
Diameter of head, in.	7/16	9/16	11/16	13/16	15/16	1 1/16	1 3/16	1 5/16	1 9/16
Thickness of head, in.	0.094	1/8	5/32	3/16	7/32	1/4	9/32	5/16	3/8

Stove bolts (Fig. 27) are made in the sizes given in Table 28.

Table 28. Dimensions of Stove Bolts

(See Fig. 27)

Diam of bolt, in.	1/8	5/32	3/16	7/32	1/4	5/16	3/8
No. of threads per inch	32	28	24	22	18	18	16

Materials, Strength, and Service Adaptability of Machine Bolts and Screws

The materials to be used for bolts, screws, and nuts depend on service conditions and relative costs. The stresses to be allowed in determining the proportions in any case depend on the nature of the loading and the material.

As a guide to the selection of **bolts and nuts for fastenings** in stationary machinery, the specifications of the Bureau of Steam Engineering of the U. S. Navy are given in Table 29.

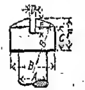


FIG. 11. Oval Fillister.

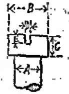


FIG. 12. Flat Fillister.

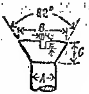


FIG. 13. Flat.

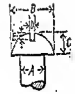


FIG. 14. Round.

Table 18. Flat Fillister Head Machine Screws. A.S.M.E. Standard

A = diam of body. $B = 1.64A - 0.009$ = diam of head. $C = 0.66A - 0.002$ = height of head. $D = 0.173A + 0.015$ = width of slot. $E = \frac{1}{2}C$ = depth of slot. (Letters refer to Fig. 12. All dimensions in inches)

A	B	C	D	E	A	B	C	D	E
0.060	0.0894	0.0376	0.025	0.019	0.216	0.3452	0.1405	0.052	0.070
0.073	0.1107	0.0461	0.028	0.023	0.242	0.3879	0.1577	0.057	0.079
0.086	0.132	0.0548	0.030	0.027	0.268	0.4305	0.1748	0.061	0.087
0.099	0.153	0.0633	0.032	0.032	0.294	0.4731	0.1920	0.066	0.096
0.112	0.1747	0.0719	0.034	0.036	0.320	0.5158	0.2092	0.070	0.104
0.125	0.196	0.0805	0.037	0.040	0.346	0.5584	0.2263	0.075	0.113
0.138	0.217	0.0890	0.039	0.044	0.372	0.601	0.2435	0.079	0.122
0.151	0.2386	0.0976	0.041	0.049	0.398	0.6437	0.2606	0.084	0.130
0.164	0.2599	0.1062	0.043	0.053	0.424	0.6863	0.2778	0.088	0.139
0.177	0.2813	0.1148	0.046	0.057	0.450	0.727	0.295	0.093	0.147
0.190	0.3026	0.1234	0.048	0.062					

Table 19. Flat-head Machine Screws. A.S.M.E. Standard

A = diam of body. $B = 2A - 0.008$ = diam of head. $C = (A - 0.008)/1.730$ = depth of head. $D = 0.173A + 0.015$ = width of slot. $E = \frac{1}{3}C$ = depth of slot. (Letters refer to Fig. 13. All dimensions in inches)

A	B	C	D	E	A	B	C	D	E
0.060	0.112	0.029	0.025	0.010	0.216	0.424	0.120	0.052	0.040
0.073	0.138	0.037	0.028	0.012	0.242	0.472	0.135	0.057	0.045
0.086	0.164	0.045	0.030	0.015	0.268	0.528	0.150	0.061	0.050
0.099	0.190	0.052	0.032	0.017	0.294	0.580	0.164	0.066	0.055
0.112	0.216	0.060	0.034	0.020	0.320	0.632	0.179	0.070	0.060
0.125	0.242	0.067	0.037	0.022	0.346	0.682	0.194	0.075	0.065
0.138	0.262	0.075	0.039	0.025	0.372	0.732	0.209	0.079	0.070
0.151	0.294	0.082	0.041	0.027	0.398	0.788	0.224	0.084	0.075
0.164	0.320	0.090	0.043	0.030	0.424	0.840	0.239	0.088	0.080
0.177	0.346	0.097	0.046	0.032	0.450	0.892	0.254	0.093	0.085
0.190	0.372	0.105	0.048	0.035					

Table 20. Round-head Machine Screws. A.S.M.E. Standard

A = diam of body. $B = 1.85A - 0.005$ = diam of head. $C = 0.7A$ = height of head. $D = 0.173A + 0.015$ = width of slot. $E = \frac{1}{2}C + 0.01$ = depth of slot. (Letters refer to Fig. 14. All dimensions in inches)

A	B	C	D	E	A	B	C	D	E
0.060	0.106	0.042	0.025	0.031	0.216	0.394	0.151	0.052	0.085
0.073	0.130	0.051	0.028	0.035	0.242	0.443	0.169	0.057	0.094
0.086	0.154	0.060	0.030	0.040	0.268	0.491	0.187	0.061	0.103
0.099	0.178	0.069	0.032	0.044	0.294	0.539	0.205	0.066	0.112
0.112	0.202	0.078	0.034	0.049	0.320	0.587	0.224	0.070	0.122
0.125	0.226	0.087	0.037	0.053	0.346	0.635	0.242	0.075	0.131
0.138	0.250	0.096	0.039	0.058	0.372	0.683	0.260	0.079	0.140
0.151	0.274	0.105	0.041	0.062	0.398	0.731	0.278	0.084	0.149
0.164	0.298	0.114	0.043	0.067	0.424	0.779	0.296	0.088	0.158
0.177	0.322	0.123	0.046	0.071	0.450	0.827	0.315	0.093	0.167
0.190	0.346	0.133	0.048	0.076					

The American Screw Co's machine-screw standards are in Table 21.

bolt depends on its relative tightness, the tighter bolts carrying the greater loads. When the conditions of attendance are such as to occasion a great difference in tightness, lower working stresses must be used in designing the bolts than otherwise are necessary. On the other hand, it may be desirable to have the bolts the weakest part of the machine, since their breakage from overload in the machine will result in a minimum cost of replacement. In such cases, the breaking load of the bolts may well be equal to the load which causes the weakest member of the machine connected to be stressed up to the elastic limit.

Table 31. Safe Loads for U. S. Standard Bolts

Nominal diam., in.	No. of threads per in.	Ultimate strength, lb per sq in.						
		20,000	40,000	50,000	60,000	65,000	80,000	95,000
		Alloy Cu, 88%, Sn, 10%, Zn, 2%	Phosphor-bronze	Wrought iron and best rolled bronze	Class B bolt material	Class A bolt material	Class A Nos. 1 and 2 machinery forgings	High-grade machinery forgings
1/4	20	57	115	143	172	186	229	272
5/16	18	99	190	247	297	322	396	470
3/8	16	150	301	376	451	488	601	714
7/16	14	207	415	519	623	675	830	986
1/2	13	282	564	704	845	915	1,125	1,340
9/16	12	365	730	912	1,095	1,186	1,460	1,730
5/8	11	456	913	1,140	1,370	1,480	1,020	2,170
3/4	10	690	1,380	1,725	2,070	2,240	2,760	3,280
7/8	9	964	1,930	2,410	2,900	3,140	3,860	4,580
1	8	1,265	2,530	3,170	3,800	4,120	5,060	6,010
1 1/8	7	1,595	3,190	3,990	4,790	5,180	6,380	7,570
1 1/4	7	2,070	4,140	5,180	6,210	6,730	0,280	9,830
1 3/8	6	2,440	4,890	6,110	7,330	7,940	9,780	11,600
1 1/2	6	3,020	6,040	7,540	9,060	9,800	12,050	14,300
1 5/8	5 1/2	3,530	7,060	8,820	10,600	11,500	14,100	16,730
1 3/4	5	4,060	8,120	10,150	12,200	13,200	16,200	19,250
1 7/8	5	4,800	9,600	12,000	14,400	15,600	19,200	22,800
2	4 1/2	5,360	10,750	13,400	16,100	17,400	21,500	25,500
2 1/4	4 1/2	7,120	14,200	17,000	21,400	23,100	28,500	33,800
2 1/2	4	8,750	17,500	21,900	26,300	28,400	35,000	41,500
2 3/4	4	11,000	22,000	27,500	33,000	35,700	44,000	52,200
3	4	13,400	26,800	33,500	40,200	43,600	53,600	63,600
3 1/4	4	16,100	32,200	40,200	48,400	52,400	64,400	76,400
3 1/2	4	19,000	38,100	47,600	57,200	61,900	76,200	90,400
3 3/4	4	22,200	44,500	55,600	66,700	72,300	89,000	105,500
4	4	25,700	51,400	64,200	77,000	83,400	102,800	122,000
4 1/4	4	29,350	58,700	73,400	88,100	95,400	117,400	139,300
4 1/2	4	33,300	66,600	83,200	100,000	108,000	133,000	158,000
4 3/4	4	37,400	75,000	93,700	112,000	122,000	150,000	178,000
5	4	41,900	83,800	105,000	126,000	136,000	167,500	199,000
5 1/4	4	46,600	93,200	116,500	140,000	151,000	186,000	221,000
5 1/2	4	51,500	103,000	129,000	154,500	167,000	206,000	244,500
5 3/4	4	56,700	113,500	142,000	170,000	184,000	227,000	269,000
6	4	62,000	124,000	155,000	186,000	202,000	248,000	295,000

Bolts screwed up tight have an initial stress due to the tightening before any external load is applied to the machine member. According to Prof. Barr, the initial tensile load due to screwing up for a tight joint varies about as the diameter of the bolt, and may be estimated at 16,000 lb per in. of diameter. There is consequently danger of excessive stresses for bolts of less than

Setscrews for fastening collars and the like to shafting may have forms of heads and points as shown in Figs. 16, 17, and 18.

The round point will be found to give better service, since it will not work loose under repeated load application.

The **safe holding power** P of cup or flat-point setscrews (B. H. D. Pinckney, *Am. Mach.*, Oct. 15, 1914) may be calculated from P (lb) $= 63{,}025N/nr$, where N = hp transmitted, n = rpm, and r = radius of shaft, in. Values of P for various diameters d of set screws are as follows:

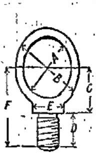


FIG. 15. Eyebolt.

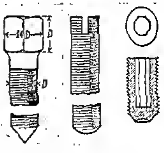


FIG. 16. FIG. 17. FIG. 18. Set Screws.

d (in.) =	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1	1 1/8	1 1/4
P (lb) =	100	168	256	368	500	658	840	1,280	1,830	2,500	3,288	4,198

Thus, for a pulley transmitting 6 hp at 300 rpm on a 2 in. shaft, $P = 63{,}025 \times 6/(300 \times 1) = 1{,}260$ lb, indicating the use of either one 3/4 in. screw or two 9/16 in. screws.

Lock nuts and special devices to prevent bolts and nuts from coming loose as may result when vibrations are encountered may be made as shown in Fig. 19. In the type shown in Fig. 19*f*, the thicker nut is shown at the top; since practically the whole load is taken by the upper nut it is considered

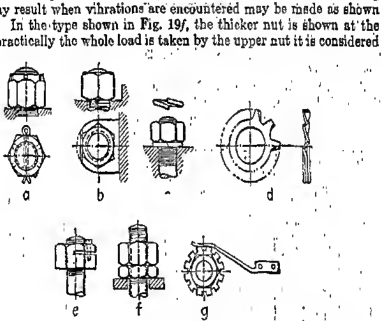


FIG. 19.—Lock Nuts.

good practice to put the thinner nut next to the work and the thicker one on top. Lock washers for bolts (Fig. 19*c*) have been standardized by the S.A.E. (see 1939 Handbook) as follows:

Bolt diameter, in.	1/4	3/8	1/2	5/8	3/4
Washer, width × thickness	5/32 × 1/16	1/8 × 3/32	11/64 × 1/8	13/64 × 5/32	1/4 × 3/16
Bolt diameter, in.		7/8	1	1 1/4	1 1/2
Washer, width × thickness		17/64 × 3/16	5/16 × 1/4	7/16 × 1/4	1/2 × 5/16

An efficient type of lock washer (Fig. 19*d*) made by the Shakeproof Lock Washer Co., forces teeth to dig into the work and nut, resisting thereby any tendency to unscrew because of vibration. The dimensions are as follows:

Table 34 gives tap drill sizes for taps with American (National) Standard, "V," and Whitworth threads.

Table 34. Tap Drill Sizes

Size of tap, in.	No. of threads per in.	Size of drill, in. American (National)	"V" thread	Whitworth	Size of tap, in.	No. of threads per in.	Size of drill, in. American (National)	"V" thread	Whitworth
3/16	24	0.128	0.111	0.129	15/16	9	0.808	0.790	0.810
1/4	20	0.191	0.184	0.192	1	8	0.854	0.832	0.856
5/16	18	0.248	0.239	0.249	1 1/16	8	0.917	0.894	0.919
3/8	16	0.302	0.293	0.303	1 1/8	7	0.957	0.932	0.960
7/16	14	0.354	0.345	0.355	1 1/4	7	1.082	1.057	1.085
1/2	13	0.409	0.399	0.416	1 3/8	6	1.179	1.144	1.182
1/2	12	0.402	0.391	0.403	1 1/2	6	1.304	1.269	1.307
9/16	12	0.465	0.453	0.466	1 5/8	5 1/2	1.412	1.372	1.416
5/8	11	0.518	0.506	0.520	1 5/8	5	1.390	1.347	1.394
11/16	11	0.581	0.568	0.583	1 3/4	5	1.515	1.472	1.519
3/4	10	0.632	0.618	0.634	1 7/8	5	1.640	1.597	1.644
13/16	10	0.695	0.680	0.697	1 7/8	4 1/2	1.614	1.566	1.619
7/8	9	0.745	0.728	0.747	2	4 1/2	1.739	1.691	1.744

RIVET FASTENINGS

Forms and Proportion of Rivets. The forms of rivet heads for structural and boiler work, as well as proportions which represent good practice, are shown in Fig. 28.

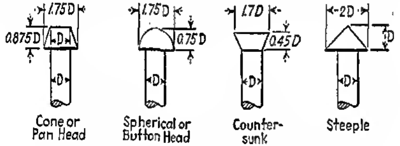


FIG. 28.—Rivet Heads.

Weights of 100 Structural Rivets with Button Heads, Lb

Length under head, in.	Diameter of rivet, in. 3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
1 1/2	7	13	23	35	50	68	91	130
1 5/8	7	14	24	36	52	71	95	134
1 3/4	8	15	25	37	54	74	98	139
1 7/8	8	15	26	39	56	77	102	143
2	9	16	27	41	58	80	105	148
2 1/8	9	17	28	43	60	82	109	152
2 1/4	9	18	29	44	62	85	112	156
2 3/8	10	18	30	46	64	88	116	161
Lb per inch additional	3.0	5.6	8.7	12.5	17.0	22.2	28.2	34.7
	Weights of 100 button heads, lb							
On rivets	2.4	5.0	9.7	16.0	24.0	35.0	49.0	78.0
As driven	1.9	4.0	7.5	12.5	18.5	27.0	37.5	51.0

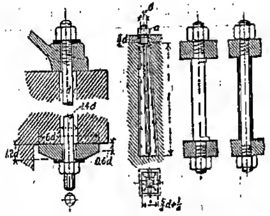


FIG. 20. FIG. 21. FIG. 22.
Foundation Bolts.

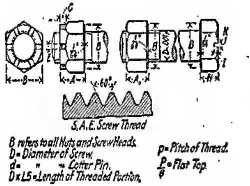


FIG. 23.—S.A.E. Standard for Bolts and Nuts.

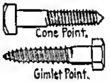


FIG. 24.—Coach and Lag Screws.

FIG. 25.—Wood Screw.

Table 24. Coach and Lag Screws

Diam of screw, in.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
No. of threads per in.	10	9½	7	7	6	5	5	4½	4½	3
Across flats of hexagon and square heads, in.	3/8	15/32	9/16	21/32	3/4	27/32	15/16	1 1/8	1 5/16	1 1/2
Thickness of hexagon and square heads, in.	3/16	1/4	5/16	3/8	7/16	1/2	17/32	5/8	3/4	7/8

Length of Threads for Screws of all Diameters

Length of screw, in.	1½	2	2½	3	3½	4	4½
Length of thread, in.	To head	1½	2	2¼	2½	3	3½
Length of screw, in.	5	5½	6	7	8	9	10–12
Length of thread, in.	4	4	4½	5	6	6	7

Table 25. American National Wood Screws

(Included angle of flat head = 82 deg, see Fig. 25)

Number	0	1	2	3	4	5	6	7	8
Threads per in.	32	28	26	24	22	20	18	16	15
Diameter, in.	0.060	.073	.086	.099	.112	.125	.138	.151	.164
Number	9	10	11	12	14	16	18	20	24
Threads per in.	14	13	12	11	10	9	8	8	7
Diameter, in.	.177	.190	.203	.216	.242	.268	.294	.320	.372

Washers for bolts and lag screws, either round or square, are made to the dimensions given in Table 26.

Table 35. Dimensions of Riveted Lap Joints

(All dimensions in inches. Letters refer to Fig. 29. E = percent efficiency of joint. Dimensions based on a tensile strength of 55,000 lb per sq in. for the plate and a shearing strength of 44,000 lb per sq in. for the rivets.)

Thickness of plate	Diam of rivet holes	Single riveted				Double riveted			
		E	P	G	Method of failure	E	P	G	S
1/4	11/16	60.7	1 3/4	1 1/16	T.P.	69.5	2 1/4	1 1/16	1 3/4
9/32	11/16	60.3	1 3/4	1 1/16	S.R.	69.5	2 1/4	1 1/16	1 3/4
5/16	13/16	59.4	2	1 1/4	T.P.	69.1	2 5/8	1 1/4	1 7/8
11/32	13/16	59.4	2	1 1/4	T.P.	69.1	2 5/8	1 1/4	1 7/8
3/8	15/16	58.3	2 1/4	1 7/16	T.P.	68.9	3	1 7/16	2
13/32	15/16	58.3	2 1/4	1 7/16	T.P.	68.9	3	1 7/16	2
7/16	1 1/16	57.5	2 1/2	1 5/8	T.P.	68.5	3 3/8	1 5/8	2 1/8
15/32	1 1/16	57.5	2 1/2	1 5/8	T.P.	68.5	3 3/8	1 5/8	2 1/8
1/2	1 1/16	56.7	2 1/2	1 5/8	S.R.	68.5	3 3/8	1 5/8	2 1/8

should be 3 times the rivet diameter. Frequently, angle connections such as shown in Fig. 38 are used. In such construction, it is good practice to

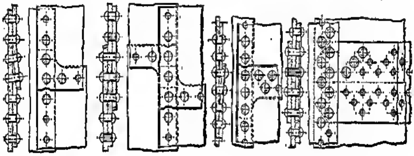


FIG. 33. FIG. 34. FIG. 35. FIG. 36.
Forms of Riveted Joint at Junction of Three or Four Plates.

have the rivet diameter $d = 2 \times$ plate thickness t and the dimension $a = \frac{1}{2}(w - t_1)$, in which w = width of angle leg and t_1 = thickness of leg = $t + \frac{1}{16}$ in. The other dimensions in the figure are in terms of t.

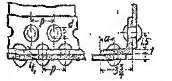


FIG. 37. FIG. 38.
Joining Plates at Right Angles.

Straining Actions in Riveted Joints. The straining actions to which the several elements of a riveted joint are subjected are easily determined as to kind, but are most difficult to evaluate. *More particularly is this true when* the joint is one of several in a structure such as a boiler and receives its load according to the elasticity of the parts through which these loads are transmitted. Without regard to the friction between the plate and assuming all rivets in the joint to be equally well fitted and consequently sharing equally

Table 29. Materials for Bolts and Nuts

Class	Material	Minimum tensile strength, lb per sq in.	Minimum elastic limit, lb per sq in.	Minimum elongation in 8 in. percent	Maximum percentage of	
					Phosphorus	Sulphur
A	Open-hearth nickel or carbon steel	75,000	40,000	23	0.04	0.03
B	Open-hearth carbon steel	58,000	30,000	28	0.04	0.03

Bending Tests. Class A material must bend cold after quenching about an inner diameter equal to the thickness of the test piece in each case without cracking; quenching temperature, 80 to 90 F. For Class B, the same test applies, except that inner diameter equals one-half thickness of test piece.

Hammer Test. Class A material must flatten out cold to one-half the original diameter without showing cracks. Class B must flatten out while heated to a cherry red in daylight to a thickness equal to one-third the original diameter without showing cracks.

Bolts requiring unusual strength are made from material specified under forgings in Table 30. Connecting-rod bolts, for example, are made from "high-grade forgings." For the S.A.E. specifications, see p. 559.

Setscrews may be made from the following material recommended by the Carnegie Steel Co.: carbon, 0.15 to 0.20 percent; manganese, 0.6 to 0.8 percent; phosphorus, not over 0.06 percent; sulphur, 0.06 percent.

Safe loads in tension for U. S. Standard bolts, as determined by Harvey D. Williams, of the Bureau of Steam Engineering, U. S. Navy, are given in Table 31. Colvin and Stanley ("American Machinists' Handbook") compute the tensile and shearing strengths as given in Table 32.

Table 30. Steel Forgings for Bolts

Class	Material	Treatment	Minimum tensile strength, lb per sq in.	Minimum elastic limit, lb per sq in.	Minimum elongation in 2 in., percent	Maximum percentage of		Cold bend about an inner diam of
						phosphorus	sulphur	
High grade..	Open-hearth nickel steel	Anneal and oil-temper	95,000	65,000	21	0.06	0.04	1 in. through 180 deg
A......	Open-hearth nickel or carbon steel	Anneal; oil tempering optional	80,000	50,000	25	0.06	0.04	1 in. through 180 deg
B......	Open-hearth carbon steel	Anneal	60,000	30,000	30	0.06	0.04	½ in. through 180 deg

General Notes on the Design of Bolts

Bolts subjected to shock and sudden change in load are found to be more serviceable when the body of the bolt is turned down or drilled to the area of the root of the thread. The drilled bolt is stronger in torsion.

When a **number of bolts are employed in fastening** together two parts of a machine, such as a cylinder and cylinder head, the load carried by each

Table 38. Dimensions of Quadruple-riveted Butt Joints (Steel Rivets in Steel Plates)

(All dimensions in inches. Letters refer to Fig. 32. Dimensions based on the tensile and shearing strengths given for Table 36.)

Thickness of plate	Thickness of straps	Diam of rivet hole	Efficiency, percent	Long pitch	Middle pitch	Short pitch	F	E	A	B	C	D
1/4	1/4	13/16	93.8	11	5 1/2	2 3/4	16 1/2	7 3/4	1 3/16	1 3/4	2 1/8	2 1/4
9/32	1/4	13/16	93.8	11	5 1/2	2 3/4	16 1/2	7 3/4	1 3/16	1 3/4	2 1/8	2 1/4
5/16	9/32	13/16	93.8	13	6 1/2	3 1/4	18 7/8	8 3/4	1 1/4	1 7/8	2 7/16	2 5/8
11/32	9/32	13/16	93.8	13	6 1/2	3 1/4	18 7/8	8 3/4	1 1/4	1 7/8	2 7/16	2 5/8
3/8	5/16	13/16	94.2	14	7	3 1/2	19 3/8	8 3/4	1 1/4	1 7/8	2 7/16	2 3/4
13/32	5/16	13/16	94.2	14	7	3 1/2	19 1/2	8 3/4	1 1/4	1 7/8	2 7/16	2 3/4
7/16	3/8	15/16	94.0	15 1/2	7 3/4	3 7/8	21 3/8	9 3/4	1 7/16	2	2 3/4	3 3/16
15/32	3/8	15/16	94.0	15 1/2	7 3/4	3 7/8	21 3/8	9 3/4	1 7/16	2	2 3/4	3 3/16
1/2	7/16	15/16	94.1	16	8	4	21 1/2	9 3/4	1 7/16	2	2 3/4	3 1/8
17/32	7/16	15/16	94.1	16	8	4	21 1/2	9 3/4	1 7/16	2	2 3/4	3 1/8
9/16	7/16	15/16	94.1	16	8	4	21 1/2	9 3/4	1 7/16	2	2 3/4	3 1/8
9/16	7/16	1 1/16	93.4	16	8	4	23 5/8	11	1 5/8	2 1/4	3	3 5/16
19/32	1/2	1 1/16	93.4	16	8	4	23 5/8	11	1 5/8	2 1/4	3	3 5/16
5/8	1/2	1 1/16	93.4	16	8	4	23 5/8	11	1 5/8	2 1/4	3	3 5/16
21/32	1/2	1 1/16	93.4	16	8	4	23 5/8	11	1 5/8	2 1/4	3	3 5/16
11/16	1/2	1 3/16	92.8	16 1/2	8 1/4	4 1/8	25 5/8	12	1 13/16	2 5/8	3 1/4	3 9/16
23/32	1/2	1 3/16	92.8	16 1/2	8 1/4	4 1/8	25 5/8	12	1 13/16	2 5/8	3 1/4	3 9/16
3/4	1/2	1 3/16	92.7	16 1/2	8 1/4	4 1/8	25 5/8	12	1 13/16	2 5/8	3 1/4	3 9/16
25/32	9/16	1 5/16	92.3	17	8 1/2	4 1/4	27 7/8	13 1/4	2	2 5/8	3 1/2	3 13/16
13/16	9/16	1 5/16	92.3	17	8 1/2	4 1/4	27 7/8	13 1/4	2	2 5/8	3 1/2	3 13/16
27/32	9/16	1 5/16	91.8	17	8 1/2	4 1/4	27 7/8	13 1/4	2	2 5/8	3 1/2	3 13/16
7/8	5/8	1 5/16	91.2	17 1/2	8 3/4	4 3/8	28	13 1/4	2	2 5/8	3 1/2	3 7/8
29/32	5/8	1 5/16	90.5	17 1/2	8 3/4	4 3/8	28	13 1/4	2	2 5/8	3 1/2	3 7/8
15/16	11/16	1 5/16	90.1	18	9	4 1/2	28 1/8	13 1/4	2	2 5/8	3 1/2	3 15/16
31/32	11/16	1 5/16	89.5	18	9	4 1/2	28 1/8	13 1/4	2	2 5/8	3 1/2	3 15/16
1	3/4	1 7/16	90.2	19	9 1/2	4 3/4	30 1/2	14 1/2	2 3/16	2 7/8	3 3/4	4 1/4
1 1/32	3/4	1 7/16	89.6	19	9 1/2	4 3/4	30 1/2	14 1/2	2 3/16	2 7/8	3 3/4	4 1/4
1 1/16	3/4	1 7/16	89.0	19	9 1/2	4 3/4	30 1/2	14 1/2	2 3/16	2 7/8	3 3/4	4 1/4
1 3/32	3/4	1 7/16	88.5	19	9 1/2	4 3/4	30 1/2	14 1/2	2 3/16	2 7/8	3 3/4	4 1/4
1 1/8	3/4	1 7/16	88.0	19	9 1/2	4 3/4	30 1/2	14 1/2	2 3/16	2 7/8	3 3/4	4 1/4
1 5/32	3/4	1 7/16	87.5	19	9 1/2	4 3/4	30 1/2	14 1/2	2 3/16	2 7/8	3 3/4	4 1/4
1 3/16	13/16	1 7/16	87 7	20	10	5	30 5/8	14 1/2	2 3/16	2 7/8	3 3/4	4 5/16
1 7/32	13/16	1 7/16	87.2	20	10	5	30 5/8	14 1/2	2 3/16	2 7/8	3 3/4	4 5/16
1 1/4	7/8	1 7/16	86.8	20	10	5	30 5/8	14 1/2	2 3/16	2 7/8	3 3/4	4 5/16

of tank, in. S = capacity of joint per pitch width to resist shearing with a given intensity of shearing stress, lb. C = capacity of joint per pitch width to resist crushing with a given intensity of crushing stress, lb. T = capacity of joint per pitch width to resist tearing of plate between the last or outside row of rivets with a given intensity of tensile stress, lb. U = capacity of unpunched plate per pitch width to resist tearing with a given intensity of tensile stress, lb.

Then $S = \pi d^2(nf_s + 2mf'_s)/4$; $C = dt(nf_c + mf'_c)$; $T = (p - d)tf_t$;

$U = p \times t \times f_t$;

and E_s = efficiency in shearing $= S/U = \pi d^2(nf_s + 2mf'_s)/4ptf_t$

E_c = efficiency in crushing $= C/U = d(nf_c + mf'_c)/pf_t$

E_t = efficiency in tearing $= T/U = (p - d)/p$

Table 32. Strength of U. S. Standard Bolts from 1/4 to 3 In. in Diam

Bolt		Areas		Tensile strength, lb			Shearing strength, lb			
							Full bolt		Bottom of thread	
Diam of bolt, in.	No. of threads per in.	Full bolt, sq in.	Bottom of thread, sq in.	At 10,000 lb per sq in.	At 12,500 lb per sq in.	At 17,500 lb per sq in.	At 7,500 lb per sq in.	At 10,000 lb per sq in.	At 7,500 lb per sq in.	At 10,000 lb per sq in.
1/4	20	0.049	0.027	270	340	470	380	490	200	270
5/16	18	0.077	0.045	450	570	790	580	770	340	450
3/8	16	0.110	0.068	680	850	1,190	830	1,100	510	680
7/16	14	0.150	0.093	930	1,170	1,630	1,130	1,500	700	930
1/2	13	0.196	0.126	1,260	1,570	2,200	1,470	1,960	940	1,260
9/16	12	0.248	0.162	1,620	2,030	2,840	1,860	2,480	1,220	1,620
5/8	11	0.307	0.202	2,020	2,520	3,530	2,300	3,070	1,510	2,020
3/4	10	0.442	0.302	3,020	3,770	5,290	3,310	4,420	2,270	3,020
7/8	9	0.601	0.419	4,190	5,240	7,340	4,510	6,010	3,150	4,190
1	8	0.785	0.551	5,510	6,890	9,640	5,890	7,850	4,130	5,510
1 1/8	7	0.994	0.693	6,990	8,660	12,130	7,450	9,940	5,200	6,930
1 1/4	7	1.227	0.890	8,890	11,120	15,570	9,200	12,270	6,670	8,900
1 3/8	6	1.485	1.054	10,540	13,180	18,450	11,140	14,850	7,910	10,540
1 1/2	6	1.767	1.294	12,940	16,170	22,640	13,250	17,670	9,700	12,940
1 5/8	5 1/2	2.074	1.515	15,150	18,940	26,510	15,550	20,740	11,360	15,150
1 3/4	5	2.405	1.745	17,450	21,800	30,520	18,040	24,050	13,080	17,440
1 7/8	5	2.761	2.049	20,490	25,610	35,860	20,710	27,610	15,370	20,490
2	4 1/2	3.142	2.300	23,000	28,750	40,250	23,560	31,420	17,250	23,000
2 1/4	4 1/2	3.976	3.021	30,210	37,770	52,870	29,820	39,760	22,660	30,210
2 1/2	4	4.909	3.716	37,160	46,450	65,040	36,820	49,090	27,870	37,160
2 3/4	4	5.940	4.620	46,200	57,750	80,840	44,580	59,400	34,650	46,200
3	3 1/2	7.069	5.428	54,280	67,850	94,990	53,020	70,690	40,710	54,280

1 in. diam. If the bolt is manifestly more yielding than the connected members, it should be designed simply to resist the initial tension or the external load, whichever is the greater. If the probable yielding of the bolt is 50 to 100 percent of that of the connected members, take the resultant stress as the initial tension plus one-half the external load. If the yielding of the connected members is probably 4 to 5 times that of the bolt (as when certain packings are used), take the resultant stress as the initial tension plus three-fourths the external load.

In any given machine, the bolts and screws used should be of as few sizes as possible in order to reduce the number of spanners required to tighten them and the number of drill and tap sizes needed in manufacture.

In drilling and tapping cast iron for studs, it is necessary to tap to a depth equal to 1 1/2 times the stud diameter in order that the strength of the cast-iron threads in shear may equal the tensile strength of the bolt. Drill sizes and depths of hole and thread are given in Table 33.

Table 33. Depths to Drill and Tap Cast Iron for Studs

Diam of stud, in......	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
Diam of drill, in......	13/64	17/64	5/16	3/8	27/64	31/64	17/32	41/64	3/4	55/64
Depth of thread, in...	3/8	15/32	9/16	21/32	3/4	27/32	15/16	1 1/8	1 5/16	1 1/2
Depth to drill, in....	7/16	17/32	5/8	23/32	27/32	15/16	1 1/32	1 1/4	1 7/16	1 5/8

It is not good practice to drill holes to be tapped through the metal into pressure spaces, as the leakage resulting, even though the bolt fits tightly, will occasion much trouble.

Drill Sizes for American (National) or A.S.M.E. Standard Thread Taps. Diameter of drill = outside diameter of stud *minus* pitch of thread.

cheaper, is objectionable. Further, the holes in different plates cannot be spaced with sufficient accuracy to register perfectly on being assembled. If the hole is punched out say $\frac{1}{16}$ in. smaller than is required and then reamed to size, the metal injured by cold flow during punching will be removed. Annealing after punching also largely obviates the injury. Drilling, while more expensive, permits of greater accuracy and does not injure the metal. The Massachusetts Boiler Inspection Law states: "Rivet holes, excepting for attaching stays or angle bars to heads, shall be drilled full size with plates, butt straps, and heads bolted up in position; or they may be punched not to exceed ¼ in. less than full size for plates over $\frac{5}{16}$ in. in thickness, and ⅛ in. less than

Table 39. Lengths of Rivets for Various Grips for Boilers

Grip, in. (see Fig. 40)	ROUND-HEAD RIVETS Diam, in.					COUNTERSUNK-HEAD RIVETS Diam, in.				
	½	⅝	¾	⅞	1	½	⅝	¾	⅞	1
	Length, in.					Length, in.				
1	2	2¼	2⅜	2½	2⅝	1⅝	1¾	1¾	1⅞	1⅞
1¼	2¼	2½	2⅝	2¾	2⅞	1⅞	2	2	2⅛	2⅛
1½	2⅝	2⅞	3	3⅛	3¼	2½	2¼	2⅜	2⅜	2½
1¾	2⅞	3⅛	3¼	3⅜	3½	2⅜	2½	2⅝	2⅝	2¾
2	3⅛	3⅜	3½	3⅝	3¾	2⅝	2¾	2⅞	2⅞	3
2¼	3⅜	3⅝	3¾	3⅞	4	2⅞	3	3⅛	3⅛	3¼
2½	3⅝	3⅞	4	4⅛	4¼	3⅛	3¼	3⅜	3⅜	3½
2¾	3⅞	4⅛	4¼	4⅜	4½	3⅜	3½	3⅝	3⅝	3¾
3	4¼	4½	4⅝	4¾	4⅞	3¾	3⅞	3⅞	4	4⅛
3¼	4½	4¾	4⅞	5	5⅛	4	4⅛	4⅛	4¼	4⅜
3½	4¾	5	5⅛	5¼	5⅜	4¼	4⅜	4⅜	4½	4⅝
3¾	5	5¼	5⅜	5½	5⅝	4½	4⅝	4⅝	4¾	4⅞
4	5¼	5½	5⅝	5¾	5⅞	4¾	4⅞	4⅞	5	5⅛
4½	5¾	6	6¼	6⅜	6½	5¼	5⅜	5½	5½	5⅝
5	6⅜	6⅝	6¾	6⅞	7	5⅜	6	6	6	6⅛

full size for plates not exceeding $\frac{5}{16}$ in. in thickness, and then drilled or reamed to full size with plates, butt straps and heads bolted up in position." The U. S. Navy Department specifies that all holes in boiler plates must be drilled with the plates in place. In structural work, the holes are generally punched for shop riveting, while for field riveting it is usual to drill them to a template, or, if punched, to ream them with the parts to be connected bolted in place.

The tensile strength of the metal between holes drilled in a plate has been found to be greater than that of an undrilled plate. For the spacings usual in riveted work, the increase in strength as shown by experiment is 10 to 12 percent. Punching, on the other hand, results in a loss of tensile strength, experiments showing the strength of the metal between the holes to be 5 to 20 percent under that of unpunched plates in the case of iron, and 8 to 35 percent in the case of steel plates. With the latter, the loss increases with the thickness of the plate.

Hand-riveted joints are of practically the same strength as those machine-riveted, but the load at which visible slip occurs is much greater with machine riveting. In hot riveting, pressures up to 150,000 lb per sq in. and even higher are used, and in cold riveting the pressure required is about 300,000 lb per sq in. of rivet section. Where the pressure exceeds 225,000 to 270,000 lb,

Forms and Proportions of Riveted Joints for Boilers and Tanks. Riveted joints for steam boilers and tanks may have the forms and proportions recommended by the Hartford Steam Boiler Inspection and Insurance Co., as given in Figs. 29 to 32, and Tables 35 to 38.

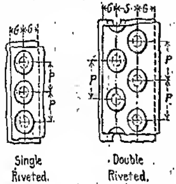


FIG. 29.—Riveted Lap Joints.

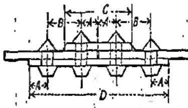


FIG. 30.

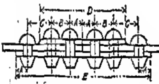


FIG. 31.

FIG. 32.

FIGS. 29–32.—Forms of Riveted Butt Joints.

Lap-riveted Joints for Girth Seams of Cylindrical Vessels with Unstayed Heads

In horizontal tubular boilers, tanks, and similar vessels, it is customary for the sake of convenience to use the same size of rivets in the girth seams as in the longitudinal seams. Where the heads of such vessels are not stayed by tubes or through braces, the strength of the circumferential joints should be at least 50 percent of that required for the longitudinal joints of the vessel. The joints in the table below are designed to meet the above requirements, and it should be understood that a higher efficiency could be obtained in some instances by using a different size of rivets. It is assumed in each case that the efficiency of the corresponding longitudinal joint is not greater than that of the quadruple-riveted butt joints of Table 38.

SINGLE-RIVETED LAP JOINTS

Thickness of plate, in.	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	9/16
Diam of rivet holes, in.	11/16	11/16	13/16	13/16	13/16	13/16	15/16	15/16	15/16	1 1/16
Pitch, in.	1 3/4	1 3/4	1 7/8	1 7/8	1 7/8	1 7/8	2 1/8	2 1/8	2 1/8	2 3/8
Efficiency, percent	60.7	60.3	55.7	56.7	56.7	54.5	55.9	55.4	52.0	53.3

DOUBLE-RIVETED LAP JOINTS

Thickness of plate, in.	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/16	1 1/8	1 3/16	1 1/4
Diam of rivet hole, in.	1 1/16	1 1/16	1 3/16	1 3/16	1 5/16	1 5/16	1 5/16	1 7/16	1 7/16	1 7/16	1 7/16	1 7/16
Pitch, in	3 1/4	3 1/4	3 3/4	3 3/4	4	4	4	4 1/2	4 1/2	4 1/2	4 1/2	4 1/2
Efficiency, percent	67.3	67.3	68.3	63.0	66.6	61.9	67.7	57.7	54.3	51.3	48.6	46.2

The form of the **joint at the junction of three or four plates** may be as shown in Figs. 33 to 36.

When **plates are to be joined at right angles** and riveted, the joint may have the form shown in Fig. 37. With such joints, the outer radius of curvature of the plate should be at least 4 times the plate thickness. The overlap

Woodruff keys (Fig. 41) are made by the Whitney Mfg. Co., Hartford, Conn., in the sizes given in Table 40, and for use with shafts not larger than 2½ in. diam. *The S.A.E. Standards should also be consulted. Cutters for* milling out the key seats, as well as special machines for using the cutters, are to be had from the manufacturer. Where the hub of the gear or pulley is relatively long, two keys should be used. Slightly rounding the corners or ends of these keys will obviate any difficulty met with in removing pulleys from shafts. They are not to be used as sliding keys or feathers.

Gib-head Keys (Fig. 42). This form of taper key is necessary when the smaller end is inaccessible for drifting out and the larger end is accessible. It can be used with care with all sizes of shafts. Its use is forbidden in certain jobs and places for safety reasons. Proportions are given in Table 41.

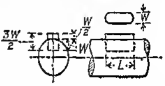


Fig. 43.—Sunk Key.

Sunk keys are made by the Pratt & Whitney Co., of Hartford, Conn., to the form and dimensions given in Fig. 43 and Table 43. These keys are particularly adapted to the case of hubs fitting adjacent parts such that neither end of the key is accessible. The difficulties attending the sinking of keyways for these keys have militated against their frequent use. The Pratt & Whitney Co., however, has devised a successful spline miller which greatly facilitates this ordinarily troublesome operation.

Feather keys are used to prevent parts from turning on a shaft while allowing them to move in a lengthwise direction. They may be of the forms shown in Fig. 44, with dimensions as given in Table 43.

Table 41. Gib-head Taper Stock Keys

(Letters refer to Fig. 42. All dimensions in inches.)
(Approved by A.S.A., 1934.)

Diameters of shafts (inclusive)	Square type					Flat type					Tolerances on keys	
	Key		Gib head			Key		Gib head				
	W	H[1]	C	D	E	W	H[1]	C	D	E	Width (minus)	Height (plus)
½ – 9/16	⅛	⅛	¼	7/32	5/32	⅛	3/32	3/16	⅛	⅛	0.0020	0.0020
⅝ – ⅞	3/16	3/16	5/16	9/32	7/32	3/16	⅛	¼	3/16	5/32	0.0020	0.0020
15/16–1¼	¼	¼	7/16	11/32	11/32	¼	3/16	5/16	¼	3/16	0.0020	0.0020
1 5/16 –1¾	⅜	⅜	11/16	15/32	15/32	⅜	¼	7/16	⅜	5/16	0.0020	0.0020
1 13/16–2¼	½	½	⅞	19/32	⅝	½	⅜	⅝	½	7/16	0.0025	0.0025
2 5/16 –2¾	⅝	⅝	1 1/16	23/32	¾	⅝	7/16	¾	⅝	½	0.0025	0.0025
2⅞ –3¼	¾	¾	1¼	⅞	⅞	¾	½	¾	¾	⅝	0.0025	0.0025
3⅜ –3¾	⅞	⅞	1½	1	1	⅞	⅝	1 1/16	⅞	¾	0.0030	0.0030
3⅞ –4½	1	1	1¾	1 3/16	1 3/16	1	¾	1¼	1	13/16	0.0030	0.0030
4¾ –5½	1¼	1¼	2	1 7/16	1 7/16	1¼	⅞	1½	1¼	1	0.0030	0.0030
5¾ –6	1½	1½	2½	1¾	1¾	1½	1	1¾	1½	1¼	0.0030	0.0030

[1] This height of the key is measured at the distance W from the gib head.

The minimum stock length of keys is equal to 4 times the key width, and the maximum stock length of keys is equal to 6 times the key width. The increments of increase of length are equal to 2 times the width.

Table 36. Dimensions of Double-riveted Butt Joints (Steel Rivets in Steel Plates)

(All dimensions in inches. Letters refer to Fig. 30. Dimensions based on a tensile strength of 55,000 lb per sq in. for plates and straps and a shearing strength of 44,000 lb per sq in. for rivets in single shear and 88,000 lb for rivets in double shear.)

Thickness of plate	Thickness of straps	Diam of rivet hole	Efficiency, percent	Long pitch	Short pitch	*D*	*C*	*A*	*B*
1/4	1/4	11/16	82.8	4	2	8 1/2	4 1/4	1 1/16	2 1/8
9/32	1/4	11/16	82.8	4	2	8 1/2	4 1/4	1 1/16	2 1/8
5/16	9/32	13/16	81.9	4 1/2	2 1/4	9 7/8	5	1 1/4	2 7/16
11/32	9/32	13/16	81.9	4 1/2	2 1/4	9 7/8	5	1 1/4	2 7/16
3/8	5/16	13/16	81.9	4 1/2	2 1/4	9 7/8	5	1 1/4	2 7/16
13/32	5/16	13/16	81.9	4 1/2	2 1/4	9 7/8	5	1 1/4	2 7/16
7/16	3/8	15/16	81.3	5	2 1/2	11 3/4	5 3/4	1 7/16	2 3/4
15/32	3/8	15/16	81.3	5	2 1/2	11 3/4	5 3/4	1 7/16	2 3/4
1/2	7/16	15/16	81.3	5	2 1/2	11 3/4	5 3/4	1 7/16	2 3/4

Table 37. Dimensions of Triple-riveted Butt Joints (Steel Rivets in Steel Plates)

(All dimensions in inches. Letters refer to Fig. 31. Dimensions based on the tensile and shearing strengths given for Table 36.)

Thickness of plate	Thickness of straps	Diam of rivet hole	Efficiency, percent	Long pitch	Short pitch	*E*	*D*	*A*	*B*	*C*
1/4	1/4	11/16	87.5	5 1/2	2 3/4	12	7 3/4	1 1/16	1 3/4	2 1/8
9/32	1/4	11/16	87.5	5 1/2	2 3/4	12	7 3/4	1 1/16	1 3/4	2 1/8
5/16	9/32	13/16	87.5	6 1/2	3 1/4	13 5/8	8 3/4	1 1/4	1 7/8	2 7/16
11/32	9/32	13/16	87.5	6 1/2	3 1/4	13 5/8	8 3/4	1 1/4	1 7/8	2 7/16
3/8	5/16	13/16	88.4	7	3 1/2	13 5/8	8 3/4	1 1/4	1 7/8	2 7/16
13/32	5/16	13/16	88.4	7	3 1/2	13 5/8	8 3/4	1 1/4	1 7/8	2 7/16
7/16	3/8	15/16	87.9	7 3/4	3 7/8	15 1/4	9 3/4	1 7/16	2	2 3/4
15/32	3/8	15/16	87.9	7 3/4	3 7/8	15 1/4	9 3/4	1 7/16	2	2 3/4
1/2	7/16	15/16	88.3	8	4	15 1/4	9 3/4	1 7/16	2	2 3/4
17/32	7/16	15/16	88.3	8	4	15 1/4	9 3/4	1 7/16	2	2 3/4
9/16	7/16	1 1/16	86.7	8	4	17	11	1 5/8	2 1/4	3
19/32	1/2	1 1/16	86.7	8	4	17	11	1 5/8	2 1/4	3
5/8	1/2	1 1/16	86.7	8	4	17	11	1 5/8	2 1/4	3
21/32	1/2	1 1/16	86.7	8	4	17	11	1 5/8	2 1/4	3
11/16	1/2	1 3/16	85.6	8 1/4	4 1/8	18 1/2	12	1 13/16	2 3/8	3 1/4
23/32	1/2	1 3/16	85.6	8 1/4	4 1/8	18 1/2	12	1 13/16	2 3/8	3 1/4
3/4	1/2	1 3/16	85.5	8 1/4	4 1/8	18 1/2	12	1 13/16	2 3/8	3 1/4
25/32	9/16	1 5/16	84.6	8 1/2	4 1/4	20 1/4	13 1/4	2	2 5/8	3 1/2
13/16	9/16	1 5/16	84.6	8 1/2	4 1/4	20 1/4	13 1/4	2	2 5/8	3 1/2
27/32	9/16	1 5/16	84.2	8 1/2	4 1/4	20 1/4	13 1/4	2	2 5/8	3 1/2
7/8	5/8	1 5/16	84.1	8 3/4	4 3/8	20 1/4	13 1/4	2	2 5/8	3 1/2
29/32	5/8	1 5/16	83.6	8 3/4	4 3/8	20 1/4	13 1/4	2	2 5/8	3 1/2
15/16	11/16	1 5/16	83.7	9	4 1/2	20 1/4	13 1/4	2	2 5/8	3 1/2
31/32	11/16	1 5/16	83.2	9	4 1/2	20 1/4	13 1/4	2	2 5/8	3 1/2
1	3/4	1 7/16	83.4	9 1/2	4 3/4	22	14 1/2	2 3/16	2 7/8	3 3/4

in taking up load, the accepted method of analyzing the stresses in the joint is as follows:

Let t = thickness of plate, in. d = diameter of rivet, in. p = pitch of rivets, in. n = number of rivets per pitch width of plate, in single shear. m = same, in double shear. f_t = intensity of stress in tension, lb per sq in. f_c = intensity of stress in crushing, lb per sq in., single shear. f_s = intensity of stress in shearing, lb per sq in., single shear (f'_c and f'_s for double shear). P = pressure existing in the tank or boiler, lb per sq in. D = internal diam

the proportions (see Fig. 48) $w = 0.241D$; $h = 0.075D$; $d = 0.850D$: for sliding when not under load, $w = 0.241D$; $h = 0.125D$; $d = 0.750D$. For **six-spline fittings**, $w = 0.25D$ and for permanent fit $h = 0.05D$; $d = 0.90D$: for sliding when not under load, $h = 0.075D$; $d = 0.85D$: for sliding when under load, $h = 0.10D$; $d = 0.80D$. **For ten-spline fittings**, $w = 0.156D$ and for permanent fit $h = 0.045D$; $d = 0.91D$: for sliding when not under load,

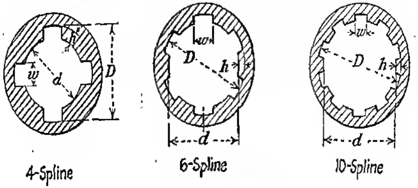


Fig. 48.—Splined Shaft.

$h = 0.07D$; $d = 0.86D$: for sliding when under load $h = 0.095D$; $d = 0.81D$. **For sixteen-spline fittings**, $w = 0.098D$ and for permanent fit $h = 0.045D$; $d = 0.91D$: for sliding when not under load, $h = 0.07D$; $d = 0.86D$: for sliding when under load, $h = 0.095D$; $d = 0.81D$.

Table 44. Torque Capacity of Splined Shafts

(Torque capacity in in.-lb per in. of bearing length at 1,000 lb pressure per sq in. on the sides of the splines.)

D, in.	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3
4-spline, P.F.	78	107	139	175	217	262	311	367	424	555	703	865	1249
4-spline, S.N.L.	123	167	219	277	341	414	491	577	670	875	1106	1365	1969
6-spline, P.F.	80	109	143	180	223	269	321	376	436	570	721	891	1283
6-spline, S.N.L.	117	159	208	263	325	393	468	550	637	833	1052	1300	1873
6-spline, S.L.	152	207	270	342	421	510	608	713	827	1080	1367	1688	2430
10-spline, P.F.	120	165	215	271	336	406	483	566	658	860	1088	1343	1934
10-spline, S.N.L.	183	248	326	412	508	614	732	860	997	1302	1647	2034	2929
10-spline, S.L.	241	329	430	545	672	813	967	1135	1316	1720	2176	2688	3869

***Taper pins** are sometimes used in transmitting very small torques,* and may be placed in either of the ways shown in Fig. 47. They should be fitted so that the parts are drawn together, when the pin is driven home, to prevent their working loose.

Table 45. Morse Standard Taper Pins

(Taper, $\frac{1}{4}$ in. per ft. Lengths increase by $\frac{1}{4}$ in. Dimensions in inches.)

Size number	0	1	2	3	4	5	6	7	8	9	10
Diam at large end	0.156	0.172	0.193	0.219	0.250	0.289	0.341	0.409	0.492	0.591	0.706
Length	$\frac{3}{4}$–$1\frac{1}{4}$	$\frac{3}{4}$–2	$\frac{3}{4}$–$2\frac{1}{4}$	$\frac{3}{4}$–3	$\frac{3}{4}$–3	$\frac{3}{4}$–3	$\frac{3}{4}$–4	1–4	$1\frac{1}{4}$–$4\frac{1}{4}$	$1\frac{1}{2}$–$5\frac{1}{4}$	$1\frac{1}{2}$–6

The Groov-Pin Corp., New Jersey, have developed a special **grooved pin** which may be used instead of taper pins in certain cases.

Cottered joints may be employed for fastening rods to other rods, rods to pistons and cross heads, yokes to rods (as in the case of connecting rods), and for services of similar kinds. Some forms of such joints and proportions recommended are shown in Figs. 49–52.

The preceding equations may be used to determine the efficiency of any form of riveted joint of known dimensions. When designing, the most economic relations between the pitch and diameter of rivet and the thickness of plate obtain when the three efficiencies are equal. If then these be equated and solved for p, d, and t, it will be found that, for the **longitudinal seams** of boilers and tanks,

$$t = PD(nf_c + mf'_c + f_t)/2(nf_c + mf'_c)f_t$$
$$d = 4(nf_c + mf'_c)t/\pi(nf_s + 2mf'_s)$$
$$p = [\pi d^2(nf_s + 2mf'_s)/4tf_t] + d$$

A factor of safety of 5 or 4½ is used in determining the value of t.

For iron plates and rivets, $f_s = 38{,}000$, $f_c = 65{,}000$, $f'_s = 35{,}500$, $f'_c = 80{,}000$; for steel plates and rivets, $f_s = 44{,}000$, $f_c = 90{,}000$, $f'_s = 45{,}000$, $f'_c = 110{,}000$; $f_t = 40{,}000$ for iron and 55,000 for steel.

For the **girth seam** of single- or double-riveted lap joints, since the thickness of plate and diameter of rivets are established (all holes being of the same diameter for economy in fabrication), the pitch is determined such that a sufficient number of rivets may be placed to take up the total load on the joint with the proper limits of stress in shear and compression. The total load on the rivets in a girth seam is equal to $\pi D^2P/4$. The equations for t, d, and p will be found to give proportions ensuring steamtightness up to pressures of 200 lb per sq in. when joints are properly calked.

The following **general practice rules** will also be found to be serviceable:

1. Butt joints are to be preferred to lap joints, since the latter occasion severe bending stresses in the rivet, which is a most frequent cause for failure.
2. The distance from the center line of the row of rivet holes nearest the edge of plate to edge of plate should be 1½ to 2 times rivet diameter.
3. The rivet diameter should be $d = 1.2\sqrt{t}$ to $1.4\sqrt{t}$.

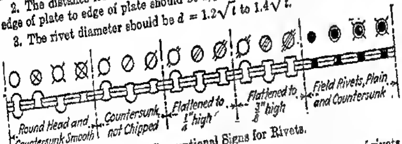


FIG. 39.—Conventional Signs for Rivets.

4. In multiple-riveted joints, the minimum distance between rows of rivets is 1.7d or 0.6 to 0.8 pitch for staggered riveting. For chain riveting, this should be at least 2d and preferably 2½d.

Materials Specifications for Rivets and Plates. See pp. 577–581. A factor of safety of 4½ to 5 is usually employed for boiler joints.

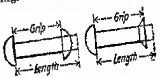


FIG. 40.

Conventional signs to indicate the form of the head to be used and whether the rivet is to be driven in the shop or the field at the time of erection are given in Fig. 39. The lengths of rivets for various grips are given in Table 39 (see also Fig. 40).

Punched vs. Drilled Plates. Holes in plates forming parts of riveted structures are punched, punched and reamed, or drilled. Punching, while

space between mating parts. **Tolerance** is defined as the amount of variation permitted in the size of a part.

Loose Fit (Class 1), Large Allowance. This fit provides for considerable freedom and embraces certain fits where accuracy is not essential, such as machined fits of agricultural and mining machinery; controlling apparatus for marine work; textile, rubber, candy, and bread machinery; general machinery of a similar grade; some ordnance material.

Free Fit (Class 2), Liberal Allowance. For running fits with speeds of 600 rpm or over and journal pressures of 600 lb per sq in. or over. For dynamos, engines, many machine-tool parts, and some automotive parts.

Medium Fit (Class 3), Medium Allowance. For running fits under 600 rpm and with journal pressures less than 600 lb per sq in.; also for sliding fits and the more accurate machine-tool and automotive parts.

Snug Fit (Class 4), Zero Allowance. This is the closest fit which can be assembled by hand and necessitates work of considerable precision. It should be used where no perceptible shake is permissible and where moving parts are not intended to move freely under a load.

Wringing Fit (Class 5), Zero to Negative Allowance. This is also known as a **tunking fit** and is practically metal-to-metal. Assembly is usually selective and not interchangeable.

Tight Fit (Class 6), Slight Negative Allowance. Light pressure is required to assemble these fits, and the parts are more or less permanently assembled, such as the fixed ends of studs for gears, pulleys, and rocker arms. These fits are used for drive fits in thin sections or extremely long fits on very light sections. Used in automotive, ordnance, and general machine manufacturing.

Medium Force Fit (Class 7), Negative Allowance. Considerable pressure is required to assemble these fits, and the parts are considered permanently assembled. These fits are used in fastening locomotive wheels, car wheels, armatures of dynamos and motors, and crank disks to their axles or shafts. They are also used for shrink fits on medium sections or long fits. These fits are the tightest which are recommended for cast-iron holes or external members as they stress cast iron to its elastic limit.

Heavy Force and Shrink Fit (Class 8), Considerable Negative Allowance. These fits are used for steel holes where the metal can be highly *stressed without exceeding its elastic limit. They cause excessive stress for* cast-iron holes. Shrink fits are used where heavy force fits are impractical, as on locomotive wheel tires and heavy crank disks of large engines.

Allowances and Tolerances for Various Fits

The formulas for allowances and tolerances for the various fits enumerated above, as recommended tentatively by the A.S.A., are given in Table 46. These are based on good practice or, where this varies, a compromise is used. The tolerances on holes apply only to reamed holes or those having a finish equivalent to reaming. It should be noted that, in practice, to cut down cost, as large a tolerance should be allowed as possible, all factors being considered. In classes (5) to (8), selective assembly is usually required, *i.e.*, a large shaft must be used with a large hole, and vice versa.

In Table 47 is given a summary of the allowances, allowances plus tolerance, and average interferences for the various classes of fits, as recommended tentatively by the A.S.A. **Interference** here denotes negative allowance.

however, there is danger that the lateral pressure of the rivet may crack the plate.

Frictional Resistance of Riveted Joints. *Rivets in cooling contract* longitudinally and draw the plates together with considerable force. They also contract laterally and therefore do not completely fill their holes when cold. Before shearing can take place, it is consequently necessary that the plates shall slip on each other, such slipping, however, being resisted by the friction of the surfaces in contact. According to C. Bach, this frictional resistance when slipping begins ranges from 14,000 to 30,000 lb per sq in. of rivet section at each pair of surfaces in contact. As any appreciable slip of a boiler joint will result in leakage, it is the practice of European engineers to design such joints according to rules based by Bach on the resistance to slipping. The proportions specified in these rules, however, do not differ greatly from those based on a consideration of shearing strength.

KEYS, PINS, AND COTTERS

Square and flat plain taper keys have the same dimensions as gib-head keys (Table 41) up to the dotted line of Fig. 42.

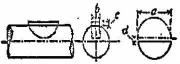


FIG. 41.—Woodruff Key.

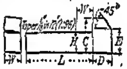


FIG. 42.—Gib-head Key.

Table 40. Dimensions of Standard Woodruff Keys

(*All dimensions in inches. Letters refer to Fig. 41. d = distance from center of stock from which key is made to top of key. Approved by A.S.A., 1930.*)

No. of key	Diam of key a	Thickness of key b	Depth of keyway c	d	No. of key	Diam of key a	Thickness of key b	Depth of keyway c	d
1	1/2	1/16	1/32	3/64	19	1 1/4	3/16	3/32	5/64
2	1/2	3/32	3/64	3/64	20	1 1/4	7/32	7/64	5/64
3	1/2	1/8	1/16	3/64	21	1 1/4	1/4	1/8	5/64
4	5/8	3/32	3/64	1/16	D	1 1/4	5/16	5/32	5/64
5	5/8	1/8	1/16	1/16	E	1 1/4	3/8	3/16	5/64
6	5/8	5/32	5/64	1/16	22	1 3/8	1/4	1/8	3/32
7	3/4	1/8	1/16	1/16	23	1 3/8	5/16	5/32	3/32
8	3/4	5/32	5/64	1/16	F	1 3/8	3/8	3/16	3/32
9	3/4	3/16	3/32	1/16	24	1 1/2	1/4	1/8	7/64
10	7/8	5/32	5/64	1/16	25	1 1/2	5/16	5/32	7/64
11	7/8	3/16	3/32	1/16	G	1 1/2	3/8	3/16	7/64
12	7/8	7/32	7/64	1/16	26	2 1/8	3/16	3/32	17/32
A	7/8	1/4	1/8	1/16	27	2 1/8	1/4	1/8	17/32
13	1	3/16	3/32	1/16	28	2 1/8	5/16	5/32	17/32
14	1	7/32	7/64	1/16	29	2 1/8	3/8	3/16	17/32
15	1	1/4	1/8	1/16	30	3 1/2	3/8	3/16	13/16
B	1	5/16	5/32	1/16	31	3 1/2	7/16	7/32	13/16
16	1 1/8	3/16	3/32	5/64	32	3 1/2	1/2	1/4	13/16
17	1 1/8	7/32	7/64	5/64	33	3 1/2	9/16	9/32	13/16
18	1 1/8	1/4	1/8	5/64	34	3 1/2	5/8	5/16	13/16
C	1 1/8	5/16	5/32	5/64					

STANDARD WOODRUFF KEYS TO USE WITH VARIOUS DIAMETERS OF SHAFTS

Diam of shaft	Numbers of keys	Diam of shaft	Numbers of keys	Diam of shaft	Numbers of keys
5/16–3/8	1	7/8–15/16	6, 8, 10	1 3/8 –1 7/16	14, 17, 20
7/16–1/2	2, 4	1	9, 11, 13	1 1/2 –1 5/8	15, 18, 21, 24
9/16–5/8	3, 5	1 1/16–1 1/8	9, 11, 13, 16	1 11/16–1 3/4	18, 21, 24
11/16–3/4	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1 1/4–1 5/16	12, 14, 17, 20	2 1/16–2 1/2	25

missible tolerance is the difference between the minimum and maximum allowances.

Allowances for Various Types of Fit

(All dimensions in inches.)

Diam, in.	Loose fit, min −	Loose fit, max, and running fit, min −	Running fit, max, and sliding fit, min −	Sliding fit, max, and push fit, min −	Push fit, max, and driving fit, min +	Driving fit, max, and press fit, min +	Press fit, max, and force fit, min +	Force fit, max +
3/64– 1/8	0.00083	0.00043	0.00020	0.00008	0.00012	0.00024	0.00039	0.00059
9/64– 1/4	0.00122	0.00063	0.00031	0.00012	0.00020	0.00035	0.00059	0.00098
17/64– 27/64	0.00165	0.00087	0.00043	0.00016	0.00028	0.00047	0.00083	0.00145
7/16– 23/32	0.00216	0.00119	0.00059	0.00020	0.00032	0.00059	0.00110	0.00197
47/64–1 3/16	0.00274	0.00157	0.00078	0.00025	0.00032	0.00071	0.00141	0.00253
1 13/64–1 57/64	0.00334	0.00196	0.00098	0.00031	0.00031	0.00086	0.00176	0.00319
1 29/32–2 63/64	0.00401	0.00236	0.00118	0.00039	0.00027	0.00102	0.00214	0.00394
2 31/32–4 17/32	0.00472	0.00275	0.00137	0.00047	0.00023	0.00118	0.00256	0.00480
4 33/64–6 57/64	0.00551	0.00315	0.00157	0.00056	0.00021	0.00137	0.00303	0.00579
6 29/32–10 7/16	0.00630	0.00355	0.00177	0.00066	0.00020	0.00157	0.00355	0.00689
10 29/64–15 3/4	0.00709	0.00394	0.00197	0.00075	0.00020	0.00177	0.00413	0.00807

Stresses Produced by Shrink or Press Fit. *Steel Hub on Steel Shaft.* The maximum equivalent stress, pounds per square inch, set up by a given press-fit allowance (in inches per inch of shaft diameter) is equal to $3x \times 10^7$,

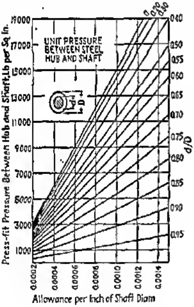


FIG. 56.—Press-fit Pressures between Steel Hub and Shaft.

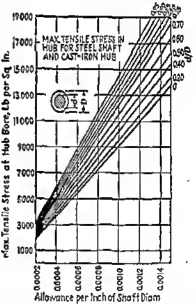


FIG. 57.—Variation in Tensile Stress in Cast-iron Hub with Press-fit Allowance.

where x is the allowance per inch of shaft diameter (Baugher, *Trans. A.S.M.E.*, 1931, p. 85). The press-fit pressures set up between a steel hub and shaft, for various ratios d/D between shaft and hub outside diameters, are given in Fig. 56. These curves are accurate to 5 percent even if the shaft is hollow, provided the inside shaft diameter is not over 25 percent of the outside. The

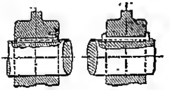


Fig. 44.—Feather Keys.

Saddle Key. Flat Key.

Fig. 45.

Table 42. Keyway Dimensions, Inches

Diam of shaft	Width of key seat	Depth of key seat	Diam of shaft	Width of key seat	Depth of key seat	Diam of shaft	Width of key seat	Depth of key seat
3/4 –1 1/4	1/4	1/8	3 13/16–4 1/4	1	1/2	7 5/16– 8 1/4	2	3/4
1 5/16–1 3/4	3/8	3/16	4 5/16–4 3/4	1 1/8	9/16	8 5/16– 9 1/4	2 1/4	7/8
1 13/16–2 1/4	1/2	1/4	4 13/16–5 1/4	1 1/4	5/8	9 5/16–10 1/4	2 1/2	7/8
2 5/16–2 3/4	5/8	5/16	5 5/16–5 3/4	1 3/8	11/16	10 5/16–11 1/4	2 3/4	7/8
2 13/16–3 1/4	3/4	3/8	5 13/16–6 1/4	1 1/2	3/4	11 5/16–12 1/4	3	7/8
3 5/16–3 3/4	7/8	7/16	6 5/16 7 1/4	1 3/4	3/4			

Table 43. Dimensions of Sunk Keys

(All dimensions in inches. Letters refer to Fig. 43.)

Key No.	*L*	*W*	Key No.	*L*	*W*	Key No.	*L*	*W*	Key No.	*L*	*W*
1	1/2	1/16	13	1	3/16	22	1 3/8	1/4	54	2 1/4	1/4
2	1/2	3/32	14	1	7/32	23	1 3/8	5/16	55	2 1/4	5/16
3	1/2	1/8	15	1	1/4	F	1 3/8	3/8	56	2 1/4	3/8
4	5/8	3/32	B	1	5/16	24	1 1/2	1/4	57	2 1/4	7/16
5	5/8	1/8	16	1 1/8	3/16	25	1 1/2	5/16	58	2 1/2	5/16
6	5/8	5/32	17	1 1/8	7/32	G	1 1/2	3/8	59	2 1/2	3/8
7	3/4	1/8	18	1 1/8	1/4	51	1 3/4	1/4	60	2 1/2	7/16
8	3/4	5/32	C	1 1/8	5/16	52	1 3/4	5/16	61	2 1/2	1/2
9	3/4	3/16	19	1 1/4	3/16	53	1 3/4	3/8	30	3	3/8
10	7/8	5/32	20	1 1/4	7/32	26	2	3/16	31	3	7/16
11	7/8	3/16	21	1 1/4	1/4	27	2	1/4	32	3	1/2
12	7/8	7/32	D	1 1/4	5/16	28	2	5/16	33	3	9/16
A	7/8	1/4	E	1 1/4	3/8	29	2	3/8	34	3	5/8

Saddle keys and flat keys (Fig. 45) are used only for the transmission of small torques or turning efforts that are never liable to sudden changes in magnitude. Under excessive stresses, they turn around the shaft and damage it.

In transmitting large torques, it is customary to use two or more keys, as shown in Fig. 46. The arrangement shown at (*a*) permits more ready cutting of the keyway. If but one key is used with the arrangement shown at (*b*), torque can be taken only in one direction.

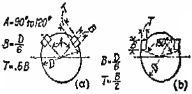


Fig. 46.—Double-keying of Shafts.

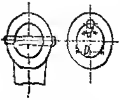


Fig. 47.—Taper Pins.

Splined shafts have been standardized by the S.A.E. (*S.A.E. Handbook*) for 4-, 6-, and 10-spline fittings. **Four-spline fittings** for permanent fit have

Cast-iron Hub on Steel Shaft. Where the shaft is solid, or hollow with an inside diameter not over 25 percent of the outside diameter, Fig. 57 may be used to determine maximum tensile stresses in the cast-iron hub, resulting from the press-fit allowance; for various ratios d/D, Fig. 58 gives the press-fit pressures. These curves are based on a modulus of elasticity of 30×10^6 lb per sq in. for steel and 15×10^6 for cast iron. For a hollow shaft with an inside diameter more than about ¼ the outside, the Lamé formulas may be used.

Pressure Required in Making Press Fits. The force required to press a hub on the shaft is given by $\pi fpdl$ where l is length of fit, p the unit press-fit pressure between shaft and hub, f the coefficient of friction, and d the shaft diameter. Values of f varying from 0.03 to 0.33 have been reported, the lower values being due to yielding of the hub as a consequence of too high a fit allowance; the average is around 0.10 to 0.15. (Horger and Nelson, *Jour. of Applied Mechanics*, March, 1938.)

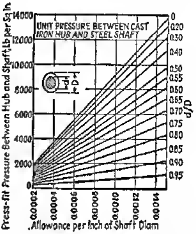


Fig. 58.—Press-fit Pressures between Cast-iron Hub and Shaft.

Torsional Holding Ability. The torque required to cause complete slippage of a press fit is given by $T = \frac{1}{2}\pi fpld^2$. Local slippage will usually occur near the end of the fit at much lower torques. If the torque is alternating, stress concentration and rubbing corrosion will occur at the hub face so that, eventually, fatigue failure may occur, at considerably lower torques. Only in cases of static torque application is it justifiable to use ultimate torque as a basis for design.

SHAFTS, AXLES, CRANKS

(For critical speeds of shafts see pp. 514–518, and for torsion of shafts of various cross sections, see Table 11, p. 478.

Strength of Shafting. Shafts may be subjected to torsion, to bending, to axial tension or compression, or to a combination of any or all of these actions. To determine the stress set up in shafts when subjected to these actions, let d = diameter of shaft, in.; T = torque moment, in.-lb; M = bending moment, in.-lb; S = normal stress in shaft, lb per sq in.; S_s = shearing stress in shaft, lb per sq in.; hp = horsepower transmitted; N = rpm of shaft. Then for solid circular shafts under pure torsion,

$$d = \sqrt[3]{5.1T/S_s} = 68.5\sqrt[3]{\text{hp}/NS_s}$$

Strength of Shafts under Combined Torque and Bending. Where shafts are subjected to combined bending and twist, the maximum resultant shearing stress is given by

$$S_s = 5.1\sqrt{M^2 + T^2}/d_1{}^3 \qquad \text{or} \qquad d = \sqrt[3]{5.1\sqrt{M^2 + T^2}/S_s}$$

Example. A 2½ in. round shaft carries a 30 in. pulley weighing 500 lb. The total pull of the belt is 2,000 lb, and the unbalanced pull is 1,500 lb horizontally. Pulley is 6 in. from hanger.

Resultant force acting on shaft $= \sqrt{500^2 + 2000^2} = 2{,}060$ lb. $M = 2060 \times 6 =$ 12,360 in.-lb at hanger, $T = 1500 \times 15 = 22{,}500$ in.-lb. From the preceding formula, $S_s = (5.1/15.625)\sqrt{12{,}360^2 + 20{,}500^2} = 8{,}400$ lb per sq in.

An analysis of the straining action in cotter joints which will serve as a guide in the determination of proportions for such joints in general may be made with reference to the type of joint shown in Fig. 53. The joint may fail in any one of the following ways, and the relation between load and stress is as specified. Let F = load on rod, lb;

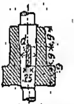


FIG. 49.

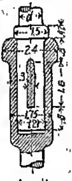


FIG. 50.

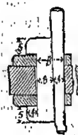


FIG. 51.

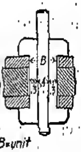


FIG. 52.

Cottered Joints.

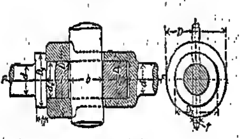


FIG. 53.—Cottered Joint.

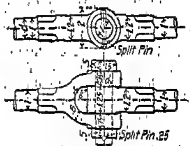


FIG. 54.—Knuckle Joint.

d = diam of rod, in.; f_t, f_s, and f_c the tensile, shearing, and crushing stresses, lb per sq in. Then (for wrought-iron rods),

1. Failure of rod by tension: $F = (\pi d^2/4) f_t$ (1)
2. Tearing of rod end across the cotter hole: $F = [(\pi d_1^2/4) - d_1 t] f_t$. (2)
   Equating (1) and (2) and letting $t = d_1/4$, then $d_1 = 1.21d$ and $t = 0.3d$.
3. Tearing of socket across the hole: $F = [\pi/4(D_2^2 - d_1^2) - (D_2 - d_1)t] f_t$, whence (allowing for clearance), $D_2 = 1.75d$.
4. Double shearing of cotter: $F = 2btf_s$, whence
   $$b = d_1(f_t/f_s) = 1.6d \text{ (approx) for iron and steel}$$
5. Crushing the cotter bearing surface in rod end: $F = d_1 t f_c$.
6. Crushing the cotter bearing surface in socket: $F = t(D - d_1) f_c$.
7. Shearing socket end: $F = 2[l(D - d_1) f_s]$. $(D = 2d_1)$
8. Shearing rod end: $F = 2l_1 d_1 f_s$. (Let $f_s = \frac{1}{2} f_s$ because of grain.)
   Letting $l = l_1$, then $l = 0.65d$, say, $l = 0.75d$.
9. Crushing collar on rod: $F = (\pi/4)(D_3^2 - d_1^2) f_c$, whence $D_3 = 1.4d$, say 1.5d.
10. Shearing collar off rod: $F = \pi d_1 t_1 f_s$, whence $t_1 = 0.42d$, say, 0.5d.

When two rods are to be joined so as to permit movement at the joint, a round pin is used in place of a cotter. In such cases, the proportions may be as shown in Fig. 54.

SHRINK, PRESS, DRIVE, AND RUNNING FITS

Classification of Fits. The A.S.A. tentatively recommends the following classification of fits. **Allowance** is defined as the minimum clearance

it is recommended that the above stresses be reduced 25 percent and where failure of the shaft would be a serious matter an additional 25 percent reduction in stress is proposed.

Charts based on the above formulas and given in the A.S.A. tentative codes for determining the necessary diameters of shafting subjected to combined torsion and bending or to simple torsion or simple bending are given in Figs. 59 and 60. These charts have as ordinates K_mM and as abscissas K_tT at the bottom or K_tP at the top where P is the horsepower

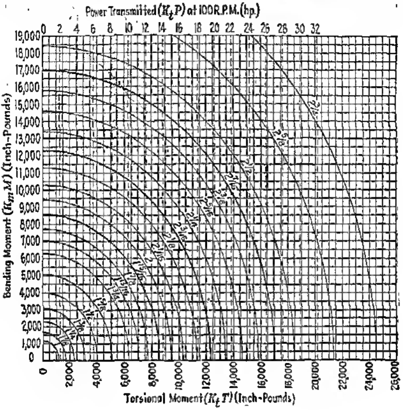


Fig. 59.

transmitted at 100 rpm corresponding to the torque T. To find the shaft diameter, it is necessary to find the intersection of the ordinate representing the bending moment K_mM with that representing the torque K_tT or the horsepower K_tP transmitted at 100 rpm. These values of shaft diameter are based on a stress of 6,000 lb per sq in.; to find the value at any other stress S_s, it is necessary to multiply the diameter obtained from the chart by $\sqrt[3]{6{,}000/S_s}$. The light arcs in the chart indicate standard sizes of machinery shafting; the heavy arcs indicate standard sizes of transmission shafting.

Example. A shaft is subjected to a maximum torsional moment of 6,900 in.-lb combined with a maximum bending moment of 11,500 in.-lb. The loads are steady, stress concentration is negligible, and the material has an elastic limit in tension of 27,000 lb per sq in. Taking 30 percent of this value, we obtain 8,000 lb per sq in. as the working stress. Since the loads are steady from Table 49, $K_m = 1.5$ and $K_t = 1$. Using the chart of Fig. 59 for $K_tT = 6{,}900$ and $K_mM = 17{,}200$, we get $d = 2.5$ in. This value given by the chart is for a stress of 6,000 lb per sq in. For a stress of 8,000

These fits are not intended to cover all work, since individual conditions may make changes necessary from the standard. However, it is believed they will apply to the majority of cases arising in practice.

Table 46. Formulas for Recommended Allowances and Tolerances

Class of fit	Method of assembly	Allowance	Selected average interference of metal	Hole tolerance	Shaft tolerance
(1) Loose	Strictly interchangeable	$0.0025\sqrt[3]{d^2}$	....	$0.0025\sqrt[3]{d}$	$0.0025\sqrt[3]{d}$
(2) Free	Strictly interchangeable	$0.0014\sqrt[3]{d^2}$	....	$0.0013\sqrt[3]{d}$	$0.0013\sqrt[3]{d}$
(3) Medium	Strictly interchangeable	$0.0009\sqrt[3]{d^2}$	....	$0.0008\sqrt[3]{d}$	$0.0008\sqrt[3]{d}$
(4) Snug	Strictly interchangeable	0.0000	....	$0.0006\sqrt[3]{d}$	$0.0004\sqrt[3]{d}$
(5) Wringing	Selective assembly	....	0.0000	$0.0006\sqrt[3]{d}$	$0.0004\sqrt[3]{d}$
(6) Tight	Selective assembly	....	$0.00025d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$
(7) Medium force	Selective assembly	....	$0.0005d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$
(8) Heavy force or shrink	Selective assembly	....	$0.001d$	$0.0006\sqrt[3]{d}$	$0.0006\sqrt[3]{d}$

Comparison of Shrink and Force Fits. Shrink fits are used in places where a force fit would be difficult to assemble, as for example, locomotive wheel tires. On the other hand, the convenience of assembling force fits makes their use advantageous in cases where a hydraulic press of sufficient capacity is available for this purpose. Force fits are sometimes made with the mating parts slightly tapered. This taper, while making the fit easier to assemble, has the disadvantage of making the fit less reliable since, if looseness develops, a slight axial movement serves to allow complete looseness between the parts. This disadvantage may be overcome if the tapered portion is allowed to project through the fit, as shown in Fig. 55, so that the part of the shaft in contact with the hub is cylindrical. The tapered end thus serves to make assembling easier.

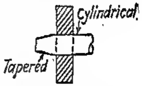


FIG. 55.

C. E. Johansson, Inc., have prepared the following table of tolerances for accurate work. The quantities given are permissible variations above or below the desired dimensions. For less accurate work, these tolerances may be doubled.

Tolerances (+ and −) for Accurate Work

Diam of hole in...	3/64–1/8	9/64–1/4	17/64–27/64	7/16–23/32	47/64–1 3/16	1 13/64–1 57/64
Tolerance, in.....	0.00008	0.00012	0.00017	0.00022	0.00028	0.00034

Diam of hole, in...	1 29/32–2 41/64	2 31/32–4 17/32	4 35/64–6 57/64	6 29/32–10 7/16	10 29/64–15 3/4
Tolerance, in.....	0.00042	0.00050	0.00059	0.00069	0.00079

The allowances which should be made for different kinds of fits are tabulated on p. 792. They are based on the use of the hole as the nominal size and give the amounts by which the shaft should be less than (−) or greater than (+) the nominal hole size. If the shaft is selected as the base, the allowances for the hole will be of the same magnitude but of opposite sign. The per-

limits, thus indicating that the mechanism of fatigue failure is related to the maximum shear theory, a more rational procedure is to design the shaft on the basis of the maximum shear theory.

CASE 1. Assume that a *pure torque* T consisting of a constant torque T_c on which is superimposed a variable torque T_v is acting on the shaft. Using the equations for working stress given by Soderberg (*Jour. Applied Mechanics*, Sept., 1935), the required diameter is given by

$$d = \sqrt[3]{\frac{32F_s}{\pi}\left\{\frac{KT_v}{S_e} + \frac{T_c}{S_y}\right\}}$$

where F_s = factor of safety (frequently taken as between 2 and 3); K = factor of stress concentration in torsion (see pp. 420–425 for values of K); S_e = endurance limit of material in reversed bending; S_y = yield stress in tension.

CASE 2. Assume the shaft is subject to *pure bending* only, consisting of a constant component of bending moment M_c on which is superimposed a variable component M_v. *The equation for required diameter is likewise*

$$d = \sqrt[3]{\frac{32F_s}{\pi}\left\{\frac{M_c}{S_y} + \frac{KM_v}{S_e}\right\}}$$

In this case, the factor K is the stress concentration factor in bending (see pp. 420–425). The stress concentration effect may be due to fillets, keyways, bearing fits, etc.

CASE 3. For the usual case of *constant torque T and reversed bending moment M*, the shaft diameter is obtained from

$$d = \sqrt[3]{\frac{32F_s}{\pi}\sqrt{\left(\frac{KM}{S_e}\right)^2 + \frac{T^2}{S_y^2}}}$$

In this case, K is the stress concentration factor in bending.

Examples. A shaft is subjected to an alternating bending moment M of 12,000 in.-lb and a constant torque T of 6,000 in.-lb. Assume that at the point of maximum bending moment there is a press fit for which the stress concentration factor is 2.5. The yield stress S_y = 36,000, and the endurance limit S_e in reversed bending is 30,000 lb per sq in. The factor of safety F_s is to be taken as 2. Since these are the conditions of Case 3, substitution in the formula for this case gives

$$d = \sqrt[3]{\frac{32 \times 2}{\pi}\sqrt{\left(\frac{2.5 \times 12{,}000}{30{,}000}\right)^2 + \left(\frac{6000}{36{,}000}\right)^2}} = 2\tfrac{3}{4} \text{ in.}$$

Stiffness of Shafting. Stiffness of shafting may become important where critical speeds, vibration, etc., may occur. Moreover, through lack of proper stiffness, much trouble may arise in upkeep of bearings. Therefore, in addition to determining the strength of shafting by the above methods, it is also necessary to check up on the stiffness. In practice, the angular twist of shafting is usually limited to 1 deg per ft of length. For calculating the diameter d required for a given total angular twist a in degrees of a shaft of length l subjected to a torque T, or horsepower hp at N rpm, the following formula may be used (G being the modulus of rigidity):

$$d = 4.9\sqrt[4]{Tl/aG} = 77.6\sqrt[4]{\text{hp} \times l/NaG}$$

It should be noted that alloy-steel shafting is no stiffer than ordinary carbon-steel shafting, the stiffness being proportional to the modulus of elasticity which does not change much for different steels.

equivalent stress given above is based on the maximum shear theory and is numerically equal to the radial fit-pressure added to the tangential tension in the hub. Where the shaft is hollow, with an inside diameter equal to more than about 25 percent of the outside diameter, the allowance in inches per

Table 47. Standard Fits, Summary of Tightest, Loosest, and Selected Condition for All Classes of Fit

(Allowances, tolerances, and interferences are given in ten-thousandths of an inch.)

Size, in.	Loose fit (Class 1)		Free fit (Class 2)		Medium fit (Class 3)		Snug fit (Class 4)		Wringing fit (Class 5)		Tight fit (Class 6)	Medium force fit (Class 7)	Heavy force and shrink fit (Class 8)
	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Tightest fit	Loosest fit	Selected fit	Selected fit	Selected fit
	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Allowance	Allowance + tolerances	Average interference	Average interference	Average interference
	+[1]	+[1]	+[1]	+[1]	+[1]	+[1]		+[1]	−[1]	+[1]	−[2]	−[2]	−[2]
0 – 3/16	10	30	4	18	2	10	0	5	2	3	0	1	1
3/16 – 5/16	10	50	6	22	4	14	0	7	3	4	1	1	3
5/16 – 7/16	10	50	7	25	5	17	0	7	3	4	1	2	4
7/16 – 9/16	20	60	9	29	6	18	0	8	3	5	1	3	5
9/16 – 11/16	20	60	10	32	7	21	0	8	3	5	2	3	6
11/16 – 13/16	20	60	12	36	7	21	0	9	4	5	2	4	8
13/16 – 15/16	20	60	13	37	8	24	0	10	4	6	2	4	9
15/16 – 1 1/16	30	90	14	40	9	25	0	10	4	6	3	5	10
1 1/16 – 1 3/16	30	90	15	43	10	26	0	10	4	6	3	6	11
1 3/16 – 1 3/8	30	90	16	44	10	28	0	10	4	6	3	6	13
1 3/8 – 1 5/8	30	90	18	48	12	30	0	12	5	7	4	8	15
1 5/8 – 1 7/8	40	100	20	52	13	33	0	12	5	7	4	9	18
1 7/8 – 2 1/8	40	100	22	54	14	34	0	13	5	8	5	10	20
2 1/8 – 2 3/8	40	100	24	58	15	35	0	13	5	8	6	11	23
2 3/8 – 2 3/4	50	110	26	62	17	39	0	13	5	8	6	13	25
2 3/4 – 3 1/4	50	130	29	67	19	43	0	15	6	9	8	15	30
3 1/4 – 3 3/4	60	140	32	72	21	45	0	15	6	9	9	18	35
3 3/4 – 4 1/4	60	140	35	77	23	49	0	16	6	10	10	20	40
4 1/4 – 4 3/4	70	150	38	80	25	51	0	17	7	10	11	23	45
4 3/4 – 5 1/2	70	150	41	85	26	54	0	17	7	10	13	25	50
5 1/2 – 6 1/2	80	180	46	94	30	60	0	18	7	11	15	30	60
6 1/2 – 7 1/2	90	190	51	101	33	63	0	19	8	11	18	35	70
7 1/2 – 8 1/2	100	200	56	108	36	68	8	20	8	12	20	40	80

[1] (+) denotes clearance or amount of looseness.
[2] Obtained by fitting or selection.

It is not necessary that both shaft and hole member be made to the same class of fit. For example, shaft members of Class 2 may be used with hole members of Classes 1 and 3, or vice versa.

inch to obtain an equivalent hub stress of 30,000 lb per sq in. may be determined by using Lamé's thick cylinder formulas (*Jour. Applied Mechanics*, Dec., 1937, p. A-185). It should be noted that these curves hold only when the maximum stress is below the yield point; above the yield point, plastic flow occurs and the stresses are less than calculated.

Standard stock lengths for cold-finished shafting are 16, 20, and 24 ft.

Cranks and Crankshafts. Crankshafts may be designed for strength by determining the bending and twisting moments at the weakest sections and combining these in accordance with the methods described above. As an example of this type of calculation, take a crank of the overhung type having the proportions given in Fig. 61 and made of wrought iron or steel. The following formulas apply (neglecting stress concentration):

Bending at section $a - b$: $Fl = tw^2S/6$; $t = 6Fl/w^2S$; $w = \sqrt{6Fl/t^2S}$ where S is the bending stress.

Bending at section $c - e$: $Fx = wt^2S/6$; $t = \sqrt{6Fx/w_1S}$; $w_1 = 6Fx/t^2S$.

Table 50. Horsepower Transmitted by Turned Steel Line Shafting

(Shafting well supported and with pulleys near to the bearings.)

For cold-rolled line shafting (up to 5 in.), add 30 percent. For head shafts of turned steel, *subtract 30 percent. For head shafts of cold-rolled steel, subtract 10 percent.* For transmission shafts (without pulleys) of turned steel, add 80 percent. For transmission shafts (without pulleys) of cold-rolled steel, add 125 percent.

Diam of shaft, in.	Number of revolutions per minute						Diam of shaft in.	Number of revolutions per minute				
	100	200	300	400	500	600		100	200	300	400	500
1½	3.7	7	11	15	18	22	4	71	142	213	284	356
1⅝	4.8	9	14	19	24	28	4¼	85	170	256	341	426
1¾	5.9	11	17	24	30	36	4½	102	203	305	405	507
1⅞	7.3	14	22	29	37	44	4¾	119	238	357	476	595
2	8.9	17	27	35	44	53	5	139	278	417	557	695
2⅛	10.6	21	32	43	53	64	5¼	161	322	483	644	805
2¼	12.6	25	38	51	63	76	5½	184	369	553	738	922
2⅜	14.9	30	45	60	74	89	5¾	211	422	633	844	1055
2½	17	35	52	69	87	104	6	240	480	720	960	1200
2⅝	20	40	60	80	100	120	6¼	271	542	813	1084	1355
2¾	23	46	69	92	115	138	6½	305	611	917	1222	1528
2⅞	26	53	79	105	132	158	6¾	341	682	1023	1364	1705
3	30	60	90	120	150	180	7	381	762	1143	1524	1905
3⅛	34	68	102	136	170	203	7¼	423	847	1270	1693	2116
3¼	38	76	114	153	191	229	7½	468	938	1406	1875	2344
3⅜	43	85	128	171	213	256	7¾	516	1033	1550	2066	2583
3½	48	95	143	190	238	286	8	568	1138	1707	2275	2844
3⅝	53	106	159	211	265	317	8½	681	1364	2047	2728	3411
3¾	59	117	176	234	293	351	9	809	1620	2430	3240	
3⅞	65	129	194	258	322	387	9½	951	1904	2858		
	.....	.....	.....	.....	.....	.....	10	1111	2222	3333		

In addition, the stress due to the combined bending and twisting moments at the main journal may be checked up by the use of the charts of Figs. 59 and 60. In these formulas, F = load on crank pin, lb; l and x are lever arms, in. (see Fig 61); w = width of crank web near shaft boss, in.; w_1 = width of crank web near pin boss, in.; t = web thickness, in.; S = allowable stress, lb per sq in. The value of the stress S is commonly taken as 4,500 to 6,000 for wrought iron and steel; this value is low enough to allow for stress concentration effects.

Fundamental methods for calculating the stresses and deformations for **single and multiple-throw crankshafts** are given by Timoshenko and Lessells, "Applied Elasticity," p. 183. The resulting formulas for multiple-throw crankshafts are, however, quite complicated.

For determining **torsional critical speeds** of internal-combustion engines, (see p. 513) it is important to know the stiffness of the crankshafts in torsion.

For hollow shafts, having a given ratio of inside to outside diameter $n = d_1/d_2$, the required diameter may be found by multiplying the required diameter of a solid shaft, as found above, by a factor $\sqrt[3]{1/(1-n^4)}$.

Thus, for a hollow shaft, the required outside diameter is given by

$$d_2 = \sqrt[3]{5.1\sqrt{M^2+T^2}/S_s(1-n^4)}$$

The relative torsional strength of solid and hollow shafts may be determined by means of Table 48.

Examples. What is the diameter of a solid shaft equal in strength to a 10 in. shaft having a 4 in. hole? $d_1/d_2 = 4/10 = 0.40$; for this ratio, $D/d_2 = D/10 = 0.991$, from which $D = 9.91$ in.

Required the outside diameter of a hollow shaft with hole $= 0.6 \times$ outside diameter equal in strength to an 8 in. solid shaft. d_2/D(for $d_1/d_2 = 0.60$) $= 1.047 = d_2/8$, whence $d_2 = 8.38$ in.

Required the dimensions of a hollow shaft but ⅔ the weight of a 12 in. solid shaft of the same strength. Here w = ⅔ = 66.7 percent, for which (from table) $d_1/d_2 = 0.64$ and $d_2/D = 1.063$; hence $d_2 = 12 \times 1.063 = 12.76$ in., and d_1 (from $d_1/12.76 = 0.64$) = 8.16 in.

Table 48. Relative Strengths of Solid and Hollow Shafts

D = diam of solid shaft, d_2 = outside diam of hollow shaft of equal torsional strength, d_1 = inside diam of hollow shaft of equal torsional strength, w = weight of hollow shaft in percentage of the weight of a solid shaft of equal torsional strength,

$$D = \sqrt[3]{(d_2^4 - d_1^4)/d_2}.$$

$\frac{d_1}{d_2}$	$\frac{d_2}{D}$	$\frac{D}{d_2}$	w	$\frac{d_1}{d_2}$	$\frac{d_2}{D}$	$\frac{D}{d_2}$	w	$\frac{d_1}{d_2}$	$\frac{d_2}{D}$	$\frac{D}{d_2}$	w
0.40	1.009	0.991	85.4	0.58	1.041	0.961	72.0	0.76	1.145	0.873	55.8
0.42	1.011	0.990	84.0	0.60	1.047	0.955	70.0	0.78	1.167	0.857	53.7
0.44	1.013	0.988	82.6	0.62	1.055	0.948	68.6	0.80	1.192	0.838	51.5
0.46	1.015	0.985	81.4	0.64	1.063	0.941	66.7	0.82	1.221	0.819	49.0
0.48	1.018	0.982	79.7	0.66	1.073	0.932	65.0	0.84	1.255	0.797	46.7
0.50	1.022	0.979	78.3	0.68	1.084	0.923	63.4	0.86	1.298	0.771	44.3
0.52	1.026	0.975	76.6	0.70	1.096	0.913	61.3	0.88	1.350	0.741	41.6
0.54	1.030	0.971	75.1	0.72	1.110	0.901	59.6	0.90	1.427	0.701	38.7
0.56	1.035	0.966	73.5	0.74	1.126	0.887	57.7				

The tentative **Code for Design of Transmission Shafting** of the A.S.A. recommends that the values of M and T in the preceding formula be multiplied by certain shock and fatigue factors K_m and K_t depending on the service conditions. The formula then becomes

$$d = \sqrt[3]{5.1\sqrt{(K_m M)^2 + (K_t T)^2}/S_s}$$

K_m and K_t are to be taken from Table 49.

Table 49. Values of the Factors K_m and K_t

	K_m	K_t
STATIONARY SHAFTS:		
Gradually applied load	1.0	1.0
Suddenly applied load	1.5–2.0	1.5–2.0
ROTATING SHAFTS:		
Gradually applied or steady loads	1.5	1.0
Suddenly applied loads, minor shocks	1.5–2.0	1.0–1.5
Suddenly applied loads, heavy shocks	2.0–3.0	1.5–3.0

The A.S.A. tentative code recommends that the working stress S_s in the foregoing formula be taken as 30 percent of the elastic limit in tension and not more than 18 percent of the ultimate tensile strength. For commercial steel shafting, this gives 8,000 lb per sq in. without allowance for keyways. Where keyways or fillets are present so as to produce stress concentration,

For marine-engine shafts, see p. 1105. For propulsion shafting, see p. 1432. **Diesel-engine crankshafts** (see p. 1438) should be designed not only for strength but for avoidance of critical speed. (See *Trans. A.S.M.E.*, Applied Mechanics, Vol. 50, No. 8, for methods of calculating critical speeds of Diesel engines.)

Crank disks of cast iron (Fig. 62) may be designed with the following proportions:

$$D_3 = 0.8D \qquad B = \tfrac{3}{4}D \qquad A = \tfrac{5}{8}D \qquad C = \tfrac{1}{4}D$$
$$E = \tfrac{1}{3}D \qquad d_1 = 2d \qquad D_1 = 1\tfrac{3}{4}D \qquad d_2 = 0.8d$$

The arc dimension a varies with the amount of counterweight to be provided.

COUPLINGS AND CLUTCHES

Solid Couplings

Solid couplings do not allow any relative movement between the coupled shafts.

Flange couplings (Fig. 63) are made of a plain face or of male and female type. The proportions are in terms of $k = (d + \tfrac{1}{2})$ in., where d is the shaft

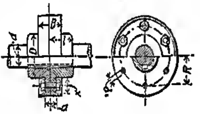


Fig. 63.—Flange Coupling.

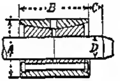


Fig. 64.—Double-cone Compression Coupling.

diameter in inches. $D = d + 0.8k$; $a = 0.55k$; d_1 = bolt diam $= 0.6d/\sqrt{N}$, where N is the number of bolts; $B = 0.55k + 3d_1$; $l = 1.4k$; $R = 1.5d$; $X = 2.5d_1$.

Flange couplings may be purchased for shafts 1 to 10 in. in diameter.

Double-cone compression couplings (Fig. 64) are convenient for connecting two shafts of considerable difference in diameters. Made by P. B. Wood's Sons Co., Chambersburg, Pa., in the sizes of Table 52.

Table 52. Dimensions of Double-Cone Compression Couplings
(Letters refer to Fig. 64. Dimensions in inches)

Shaft sizes	*A*	*B*	*C*	Shaft sizes	*A*	*B*	*C*	Shaft sizes	*A*	*B*	*C*
1 15/16	5 1/2	7 7/8	3	3 11/16	9 5/8	14 1/4	5 5/8	6 15/16	16 1/4	25	10 1/2
2 3/16	6	8 5/8	3 3/8	3 15/16	10 5/8	15	6	7 7/16	17 1/2	27	11 1/4
2 7/16	6 5/8	9 1/2	3 3/4	4 7/16	11 1/4	16 1/2	6 3/4	7 15/16	18 1/2	29	12
2 11/16	7	10 1/4	4 1/8	4 15/16	12 1/4	18	7 1/2	8 7/16	20 1/2	32	13 1/2
2 15/16	7 7/8	11 1/2	4 1/2	5 7/16	13 1/4	19 3/4	8 1/4	8 15/16	20 1/2	32	13 1/2
3 3/16	8 1/4	12 1/4	4 7/8	5 15/16	14	21 1/2	9				
3 7/16	9	13	5 1/4	6 7/16	15 1/4	23 1/4	9 3/4				

Clamp couplings (Fig. 65) are bored with halves separated during boring, to allow for clamping the shafts. Usual dimensions are shown in Table 53.

Ring compression couplings are used for outdoor service where danger of rusting exists. The sleeve is split in two. Proportions are shown in Fig. 66. These couplings are used with shafts up to 6 in. diam.

lb per sq in., $d \times \sqrt[3]{6000/8000} = 2.28$ in., the nearest standard machinery shaft being $2\frac{1}{4}$ in. If the shaft were hollow, this value would have to be multiplied by $\sqrt[3]{1/(1 - n^4)}$ where n is the ratio of outside to inside diameter.

Determination of Maximum Bending Moment. Frequently, the forces acting on the shaft do not act in the same plane. In such cases, the bending moments due to each force must be combined vectorially to get the resultant maximum bending moment M. This value is then used in the formulas or charts given above in combination with the torque moment.

If shafts are subjected to axial tensile or compressive loads, the stresses due to these loads must be superimposed on those due to the bending

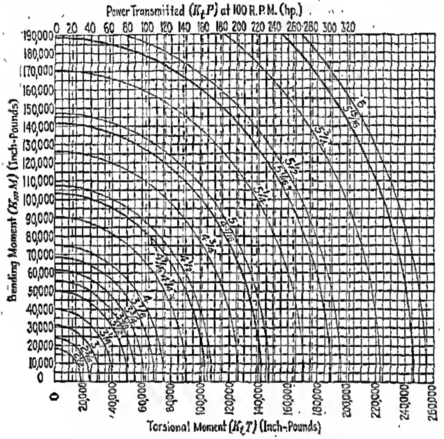


FIG. 60.

moment. These stresses may be thought of as due to an additional bending moment which may be added to the bending moment M used in the above formulas.

Rational Method for Designing Shafting. The preceding method of designing shafting has the disadvantage of basing the design on the properties of the materials under static loading without reference to its fatigue properties or the amount of stress concentration actually present in the shaft. The presence of the empirical factor K_m and K_t makes for further uncertainty. Inasmuch as most failures of shafting are fatigue failures, it is more logical to base the working stress on the endurance limit for the particular range of stress considered. Since test data show that endurance limits in reversed torsion are about half the corresponding bending endurance

made by the De Laval Co. are **given in Table 55.** The normal space between flanges is ⅛ in.

Table 54. Dimensions of Double Slider Couplings
(Letters refer to Fig. 67)

Maximum bore, in.	Dimensions, in. A	B	C	D	Hp at 100 rpm	Maximum rpm	Maximum bore, in.	Dimensions, in. A	B	C	D	Hp at 100 rpm	Maximum rpm
1¼	3¾	1⅞	1⅞	5⅝	2.8	1500	3½	10	5¼	4¾	15¼	55.0	1000
1½	4½	2¼	2¼	6¾	4.5	1500	4	12	6	5½	17½	90.0	750
1¾	5	2⅝	2⅝	7⅞	7.0	1500	4½	14	6¾	6¼	19¾	125.0	750
2	5½	3	3	9	10.5	1200	5	16	7½	7	22	172.0	500
2¼	6	3⅜	3⅜	10⅛	15.0	1200	5½	18	8¼	7¾	24¼	230.0	500
2½	7	3¾	3¾	11¼	20.0	1200	6	20	9	8⅝	26⅝	300.0	300
3	8	4½	4⅛	13⅛	36.0	1000							

Table 55. Dimensions of Rubber Bushing Couplings
(Letters refer to Fig. 68. Dimensions in inches)

Max bore, G	Out diam, E	Diam hub, F	Length shaft fit, H	No. of bolts	Max rpm	Hp per 100 rpm	Max bore, G	Out diam, E	Diam hub, F	Length shaft fit, H	No. of bolts	Max rpm	Hp per 100 rpm
1⅛	5	2	1½	6	4500	1.75	4	12¼	7	4	12	2000	45.0
1¼	5	2	1½	3	4500	0.875	5	12¼	7	4	6	2000	22.5
1¾	6½	3	2	12	3600	5.0	4⅝	13¾	8	4½	14	1800	65.0
2	6½	3	2	6	3600	2.5	5¾	13¾	8	4½	7	1800	32.5
2¼	8¼	4	2½	8	3000	10.0	5¾	16½	10	5	12	1500	116.0
2¾	8¼	4	2½	4	3000	5.0	7¼	16½	10	5	6	1500	58.0
2⅞	9¼	5	3	12	2600	20.0	7	18½	12	5½	16	1300	180.0
3½	9¼	5	3	6	2600	10.0	8¾	18½	12	5½	8	1300	90.0
3½	11¼	6	3½	8	2200	30.0	9	22½	16	6	20	1100	285.0
4¼	11¼	6	3½	4	2200	15.0	12	22½	16	6	10	1100	142.5

Flexible disk couplings (Fig. 69) consist of two flanges, driving and driven. Both have bolts engaging an intermediate flexible disk transmitting the power. The disk is made of leather, or rubber fabric for smaller sizes; steel laminations are used on some makes (Thomas Flexible Coupling Co., Warren, Pa.). The dimensions and capacities

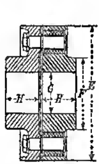


Fig. 68.—Rubber Bushing Coupling.

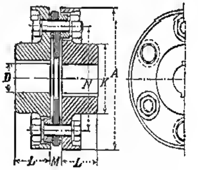


Fig. 69.—Flexible Disk Coupling.

of flexible disk couplings, as made by Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa., are given in Table 56.

The **Grundy flexible coupling** consists of two cast-iron flanges (Fig. 70) and a leather disk. The leather disk has lugs fitting into apertures cast in the flanges. Dimensions and capacities are given in Table 57. The capacities are shown for steady load.

The deflection of shafting is usually limited to 0.01 in. per ft of length. For slow or moderate speed shafts, if the loads acting are known, the deflections may be computed using the ordinary beam formulas. For higher speeds, the effect of centrifugal action enters, so that this method is no longer sufficient. In such cases, the rule for spacing hangers given by Pinkney may be used (see below).

Practical Rules for Proportioning Mill Shafting. In many cases, it is impossible to calculate accurately the loads causing bending and twisting in shafting. Because of this and because of the possibility that additional pulleys may be installed or changes in the arrangement of pulleys due to shifting of machines may take place, it may be necessary to depend on empirical formulas for the design of shafting although the more fundamental method outlined above is to be preferred.

The following rules have been suggested for determining suitable diameters of mill shafting:

For shafts subjected to torque alone, *i.e.*, for power transmission over considerable distances between pulleys, and for short countershafts having bearings not more than 8 ft apart and pulleys close to bearings,

$d = \sqrt[3]{50\ \mathrm{hp}/N}$ for turned shafting $= \sqrt[3]{40\ \mathrm{hp}/N}$ for cold-rolled shafting, where hp = horsepower to be transmitted, N = rpm, and d = shaft diam, in.

When the shaft is a first receiving shaft, *i.e.*, a line shaft having bearings, 8 ft apart,

$d = \sqrt[3]{90\ \mathrm{hp}/N}$ for turned shafting $= \sqrt[3]{70\ \mathrm{hp}/N}$ for cold-rolled shafting.

When the shaft is a head shaft carrying the main driving pulley and distributing power to receiving or line shafts and is well supported by bearings,

$d = \sqrt[3]{125\ \mathrm{hp}/N}$ for turned shafting $= \sqrt[3]{100\ \mathrm{hp}/N}$ for cold-rolled shafting.

Table 50 is based on the formula $d = \sqrt[3]{90 \times \mathrm{hp}/N}$ and affords a rapid means of determining the shaft diameter for any desired transmission.

In line with the common practice of limiting the deflection of shafting to 0.01 in. per ft, the following rules for spacing hangers have been recommended. Let L = max distance in feet between bearings for continuous shafting: Then $L = \sqrt[3]{870d^2}$ for bare shafting or $\sqrt[3]{175d^2}$ for shafts carrying a fair proportion of pulleys placed near the bearings. It should be noted that these empirical rules may be considerably in error and that where speeds are moderate and loads are known the exact deflection of the shaft should be computed using the ordinary beam formulas as given on p. 462. For higher speeds, the following formula (Pinkney, *Machinery*, May, 1915) takes into account the effect of shaft speed in producing a centrifugal whirl:

$$L = K[1500/(N + 1500)]d^{2/3}.$$

For head shafts or so-called prime movers, also those shafts subjected to shocks and the bending action of pulleys, gears, etc., and which may be reversed under full load, $K = 5$. For the common type of lineshafts (except that part where the power is applied) and also for countershafts, with ample allowance for the bending action of pulleys, gears, etc., which are not reversed under full load, $K = 6.6$. For shafts and countershafts which are merely to transmit power without any bending action except that due to the weight of the bare shaft and with all pulleys close to the bearings, $K = 9.5$.

Standard Shaft Diameters and Lengths. In Table 51 are given standard diameters and lengths of shafting as approved tentatively by the A.S.A.

Table 58. Cushioned Plate Couplings

Shaft diam, in.		Hp per 100 rpm	Max rpm	Dimensions, in.						Slots	
Min	Max			*A*	*B*	*C*	*D*	*F*	*H*	No.	Size
1	1 7/16	1.87	7300	4 3/16	4	2 9/16	2 1/8	1/16	5/8	5	3/4 × 3/4
1 1/4	1 13/16	3.64	5150	5 15/16	5 3/4	3 3/16	2 3/4	1/16	13/16	6	3/4 × 3/4
1 5/16	2 1/4	7.10	4575	6 11/16	6 1/2	4 1/16	3 1/2	1/16	1	6	3/4 × 1
2	2 7/8	14.95	3850	7 13/16	7 3/4	5 1/8	4 3/8	1/8	1 1/4	8	1 1/16 × 1 1/2
2 1/2	3 5/8	29.20	3350	9 3/16	9	6 3/8	5 1/2	1/8	1 9/16	8	1 1/8 × 1 1/2
3 1/8	4 1/2	57.30	2750	11 3/16	11	7 5/8	6 3/4	1/8	1 7/8	8	1 1/4 × 1 3/4
4	5 3/4	119.80	2200	14	13 3/4	10 1/8	8 5/8	1/8	2 5/16	12	1 1/2 × 2
5	7 1/4	234.0	1825	16 3/4	16 1/2	12 11/16	10 7/8	3/16	2 13/16	14	1 5/8 × 2
6 1/4	9	456.0	1475	20 3/4	20 1/2	15 11/16	13 1/2	3/16	3 1/4	16	1 3/4 × 2 1/2
8	11 1/2	957.0	1200	25 3/8	25	20 1/4	17 1/4	1/4	3 7/8	16	2 1/4 × 3
10	14 1/2	1870.0	925	33 3/8	33	25 1/4	21 3/4	1/4	4	22	2 1/2 × 3

Table 59. Belt-type Coupling
(Letters refer to Fig. 72; dimensions in inches)

Hp at 100 rpm	Max bore	*Dimensions*				Hp at 100 rpm	Max bore	*Dimensions*			
		A	*B*	*C*	*E*			*A*	*B*	*C*	*E*
1 1/4	1 1/8	6 1/4	6 3/8	2 3/8	1 5/8	35	4 3/4	14 7/8	11 5/8	4 1/2	2 3/8
2 1/2	1 1/2	7 1/4	6 5/8	2 1/2	1 5/8	55	6 1/2	17 1/4	11 7/8	4 3/4	2 5/8
5	1 3/4	8 5/8	7 3/8	2 7/8	1 5/8	80	7 1/2	19 3/4	13 5/8	5 3/8	2 7/8
7	2	9 1/2	7 5/8	3	1 5/8	115	9	22 3/8	15 1/8	6 1/8	2 7/8
10	2 1/2	10	8 1/8	3 1/4	1 5/8	160	11	25 5/8	17 5/8	7 1/8	3 3/8
14	2 3/4	11	9 1/8	3 5/8	1 7/8	208	13	29 1/2	20 1/4	8 1/8	4
20	3 1/4	12	9 5/8	3 7/8	1 7/8	270	15	33	22 3/4	9 1/8	4 1/2
27	4	13 3/4	10 7/8	4 1/4	2 5/8						

shafts and provides also a certain amount of torsional flexibility. Figure 73 shows the cantilever coupling made by Brown Engineering Co., Reading, Pa. and Table 60 gives

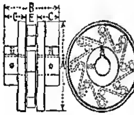


FIG. 72.—Belt-type Coupling.

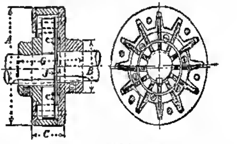


FIG. 73.—Flexible Steel Member Coupling.

the dimensions and capacities. The space *J* between flanges is 1/4 in. for shafts of 1 in. diam and over.

The **grid spring coupling** (Fig. 74) is a flexible steel member coupling manufactured by the Falk Corp., Milwaukee, Wis. The two hubs of the coupling have axial grooves through which the grid spring is laced back and forth. The resilient characteristics of the coupling are obtained through the design of the grooves. A cover shields the groove assembly and serves as a grease reservoir. The dimensions and capacities of the cou-

The angular twist, in degrees of a single throw, is given by

$$\theta = 57.3T[(2b + 0.9h)/C_1 + (a + 0.9h)/C_2 + 2r/B]$$

where T = applied torque. For the other symbols see p. 514.

Table 51. Tentative Standard Diameters and Lengths of Shafting As Approved by the A.S.A.

Diameters and tolerances

Transmission shafting sizes, in.	Machinery shafting sizes, in.	Tolerances[1] on diameters (−), in.	Transmission shafting sizes, in.	Machinery shafting sizes, in.	Tolerances[1] on diameters (−), in.	Transmission shafting sizes, in.	Machinery shafting sizes, in.	Tolerances[1] on diameters (−), in.
	1/2	0.002	1 11/16	1 11/16	0.003	....	3 1/4	0.004
	9/16	0.002	....	1 3/4	0.003	....	3 3/8	0.004
	5/8	0.002	....	1 13/16	0.003	3 7/16	3 1/2	0.004
	11/16	0.002	....	1 7/8	0.003	....	3 5/8	0.004
	3/4	0.002	1 15/16	1 15/16	0.003	....	3 3/4	0.004
	13/16	0.002	....	2	0.003	....	3 7/8	0.004
	7/8	0.002	....	2 1/16	0.004	3 15/16	4	0.004
15/16	15/16	0.002	....	2 1/8	0.004	....	4 1/4	0.005
	1	0.002	2 3/16	2 3/16	0.004	4 7/16	4 1/2	0.005
	1 1/16	0.003	....	2 1/4	0.004	....	4 3/4	0.005
	1 1/8	0.003	....	2 5/16	0.004	4 15/16	5	0.005
1 3/16	1 3/16	0.003	....	2 3/8	0.004	....	5 1/4	0.005
	1 1/4	0.003	2 7/16	2 7/16	0.004	5 7/16	5 1/2	0.005
	1 5/16	0.003	....	2 1/2	0.004	....	5 3/4	0.005
	1 3/8	0.003	....	2 5/8	0.004	5 15/16	6	0.005
1 7/16	1 7/16	0.003	....	2 3/4	0.004			
	1 1/2	0.003	....	2 7/8	0.004			
	1 9/16	0.003	2 15/16	3	0.004			
	1 5/8	0.003	....	3 1/8	0.004			

[1] These tolerances are negative and represent the maximum allowable variation below the exact nominal size. For instance, the maximum diameter of the $1\frac{15}{16}$ in. shaft is 1.938 in. and its minimum allowable diameter 1.935 in.

Engine Shafts. Engine shafts may be designed for strength by the more fundamental methods outlined on pp. 797, 798. Often, however, these shafts

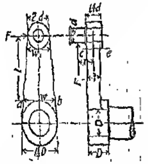


Fig. 61.—Overhung Crank.

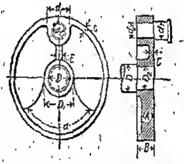


Fig. 52.—Crank Disk.

are designed from data representing good practice. Stationary steam-engine shafts were investigated by Trocien, who found the average proportions to be as follows:

For slow-speed and Corliss engines, $d = 7.2\sqrt[3]{hp/N} - 0.3$ in.

For high-speed center-crank engines, $d = 6.6\sqrt[3]{hp/N}$.

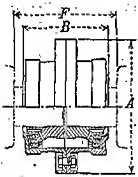


Fig. 75.—Fast Gear Coupling.

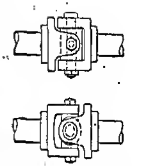


Fig. 76.—Hooke's Universal Joint.

Table 62. Dimensions of Fast Gear Couplings

(Letters refer to Fig. 75)

Maximum bore, in.	Hp per 100 rpm	Maximum rpm	Dimensions, in.			Maximum bore, in.	Hp per 100 rpm	Maximum rpm	Dimensions, in.		
			A	*B*	*F*				*A*	*B*	*F*
1½	9.5	12,000	6	4⅜	5¼	9	2,040	2,545	25¾	24	28½
2	22.5	9,300	7	5½	6¾	10	2,800	2,310	28⅞	26⅜	31
2½	43.8	7,900	8⅜	6¾	8¼	11	3,730	2,125	31	29	34
3	75.5	6,800	9 7/16	8⅛	9⅞	12	4,840	1,975	33⅝	31½	36½
3½	120	6,000	11	9⅜	11½	13	4,090	1,250	33	28¼	32¾
4	180	5,260	12⅜	10¾	13⅛	14	5,120	1,125	35¾	30	35
4½	250	4,770	13⅝	12	14¾	15	6,320	1,000	38	31¾	36¾
5	350	4,300	15 5/16	13½	16½	16	7,700	875	40½	33¾	38¾
6	605	3,690	17⅝	16	19½	17½	9,180	750	43	35¾	40¾
7	960	3,220	20¼	18½	22½	20	12,000	500	47¼	37	42
8	1,435	2,845	23⅝	21¼	25¾						

Universal Joints

Universal joints (Fig. 76) are couplings which are adapted to connecting *shafts fixed at an angle which is either constant or variable. The angular* velocity of the driving shaft being ω_1, the angular velocity ω of the driven shaft is varying during each revolution from a value $\omega_{max} = \omega_1/\cos\alpha$ to a value $\omega_{min} = \omega_1 \cos\alpha$, where α is the angle between the shafts. A double universal joint is often used to transmit motion between parallel offset shafts. If the pivots on the connecting jackshaft are set parallel to each other, the speed of the driven shaft is the same as that of the driving shaft.

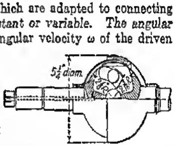


Fig. 77.—Weiss Universal Joint.

The **Weiss universal joint** (Fig. 77) has been used in automotive applications. This joint is designed to maintain equal speeds of the driving and driven shafts for any angle between the shafts. A number of steel balls *placed in intersecting races, cut in the two joint members, transmit* the motion. This joint permits axial movement of the shafts.

Clutches

Clutches are couplings providing for convenient disengaging of the coupled shafts during rotation.

Solid sleeve couplings are used for small shafts up to 2½ in. with moderate power and low speed. They consist of a cylindrical sleeve fitting closely on the coupled shafts which are held in place by set screws. The outside diameter is $3d$, the length is $4d$, where d is the diameter of the shafts.

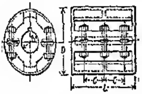


Fig. 65.—Clamp Coupling.

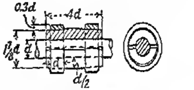


Fig. 66.—Ring Compression Coupling.

Table 53. Dimensions of Clamp Couplings
(Dimensions in inches)

Shaft sizes	Diameter	Length	Bolts each coupling	Shaft sizes	Diameter	Length	Bolts each coupling	Shaft sizes	Diameter	Length	Bolts each coupling
1 3/16	4 1/8	5 3/8	6	2 11/16	7 1/2	10 5/8	6	4 15/16	13 5/8	18	8
1 7/16	4 3/4	6 1/8	6	2 15/16	8 3/4	11 5/8	8	5 7/16	14	19	8
1 11/16	5	6 3/4	6	3 3/16	9 1/4	12 3/4	8	5 15/16	15 1/2	20	8
1 15/16	6	8	6	3 7/16	10	13 3/8	8	6 7/16	16 5/8	21	8
2 3/16	6 3/4	8 3/4	6	3 15/16	10 5/8	14 3/8	8	6 15/16	18 1/2	22	8
2 7/16	7 1/4	9 5/8	6	4 7/16	12	16 1/2	8				

Flexible Couplings

Flexible couplings allow for a slight occasional misalignment of the coupled shafts, both linear and angular. Following are shown typical flexible couplings. The ratings in horsepower per 100 rpm are given for smooth and steady loads. For uneven loads, the ratings must be ⅔, for shock loads ½ of the listed values.

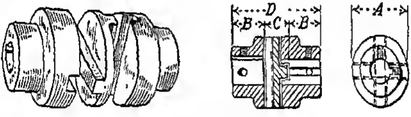


Fig. 67.—Double Slider Coupling.

Double slider couplings (Fig. 67) are used for heavy torques and moderate speed. The floating intermediate plate compensates the misalignment by a sliding movement. Dimensions, as made by Jones Foundry and Machine Co., Chicago, are given in Table 54.

For speeds greater than 100 rpm, the horsepower will be increased by the factor k.

Rpm	300	600	750	900	1200
k	2.5	3.5	3.75	4	4.25

Rubber bushing couplings (Fig. 68) consist of two steel disks, the driving disk having a number of rigid studs carrying a metal-lined rubber bushing. This projects into corresponding holes in the driven half. The dimensions and capacities of couplings

shaft axis and element of cone surface. f = coefficient of friction between the conical surfaces. N = rpm of shaft. W = width of cone surface, in. p = pressure normal to cone surface, lb per sq in.

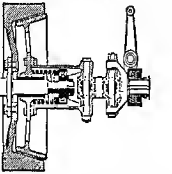


Fig. 80.—Cone Friction Clutch.

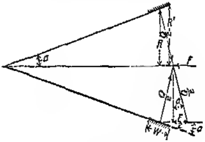


Fig. 81.

Then $T = QfR$; $F = Q(f\cos\alpha + \sin\alpha) = T(f\cos\alpha + \sin\alpha)/fR$.
Also $63{,}024 \times \text{h.p.}/N_1 = T = FfR/(f\cos\alpha + \sin\alpha)$
$\therefore F = 63{,}024 \times \text{hp}\,(f\cos\alpha + \sin\alpha)/NfR$
and $Q = 63{,}024 \times \text{hp}/NfR$. Also $Q = W \times 2\pi R \times p$.

Values of p are 7 to 8 lb per sq in. for leather faces and 40 to 50 lb for metal faces. In order that the clutch may not disengage of its own accord, tan α must be less than the coefficient of friction f. Accordingly, for leather or greasy metal, α would be 12 deg or less; or tan 12 deg = 0.213, which is less than f = 0.23. For free disengagement, wood or metal surfaces, $\alpha \gtreqless 20$ deg; leather-faced cone clutches with α = 18 to 20 deg need an operating device for disengagement (C. W. Hunt, *Trans. A.S.M.E.*, 30).

Ring friction clutches of a great variety of designs may be purchased on the market. The type manufactured by The Hill Clutch Co., Cleveland, Ohio, is illustrated in Fig. 82, dimensions and ratings being given in Table 65.

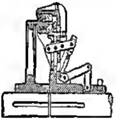


Fig. 82.—Hill Friction Clutch.

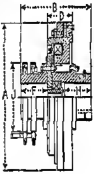


Fig. 83.—Cutler-Hammer Magnetic Clutch.

Table 65. Dimensions of Hill Clutch Couplings

No. of arms	Equivalent shaft diam, in.	Hp at 100 rpm	Max bore, in.	Total length (disengaged), in.	Dia open, in.	No. of arms	Equivalent shaft diam, in.	Hp at 100 rpm	Max bore, in.	Total length (disengaged), in.	Dia open, in.	No. of arms	Equivalent shaft diam, in.	Hp at 100 rpm	Max bore, in.	Total length (disengaged), in.	Dia open, in.
3	1 3/16	9	2¾	12⅛	15	4	3 7/16	60	4½	22⅜	29½	4	5½	230	6½	32½	57
3	1 15/16	12	3	13½	17½	4	3 11/16	75	5	23⅛	33	6	6	350	7	32½	57
3	2 3/16	15	3¼	14¼	20	4	3 15/16	90	5	24⅝	36½	6	7	480	7¾	37¼	64
3	2 7/16	20	3½	20⅜	22½	4	4 3/16	110	5½	25¾	41	6	7½	625	9	43½	70
3	2 11/16	27	4	18⅞	24¾	4	4 11/16	140	6	28⅛	46	6	8½	875	10½	47	80
3	2 15/16	35	4	20⅝	27½	4	4 15/16	175	6	30½	50	6	9½	1100	12	53	90
3	3 3/16	45	4½	22	29½												

Table 56. Dimensions of Flexible Disk Couplings
(Letters refer to Fig. 69. All dimensions in inches)

Maximum bore, D	Hp per 100 rpm	Maximum rpm	Maximum torque, ft-lb	Dimensions, in. A	K	L	M	N	Bolts Diam	Bolts Length	Bolts No.	Disk
1⅜	.5	3,400	90	5⅜	2½	1 49/64	19/32	4	⅜	2	6	Fabric
1⅝	1.0	2,800	125	6¾	3	1 57/64	21/32	4⅜	½	2½	6	
2	2.0	2,500	225	7⅝	3¾	2⅜	¾	5⅜	⅝	2¾	6	
2½	3.5	2,100	425	9¼	4¾	3 1/16	¾	7	¾	3½	6	
3	7.0	1,800	700	10¾	5¼	3 5/16	⅞	8	⅞	3¾	8	Leather
3½	10.0	1,550	1125	12¼	6¼	3½	1	9¼	1	4½	8	
4	16	1,350	1750	14	7	3 15/16	1⅛	10½	1¼	5	8	

Table 57. Grundy Couplings
(Dimensions in inches)

Outside diameter	Over-all length, both hubs	Hub diam	Max bore	Hp at 100 rpm	Outside diameter	Over-all length, both hubs	Hub diam	Max bore	Hp at 100 rpm
3	3	1 5/16	⅝	⅜	12	12½	6¼	3	55
4	4 5/16	1 13/16	1	1½	14	14 9/16	8	4	70
5	5 5/16	2⅜	1¼	3	16	15 9/16	9¼	5	95
6	5 13/16	3	1⅝	5	18	16⅜	10¾	6	125
7	6 9/16	3¼	1¾	8	21	18 11/16	13	7	180
8	6⅞	3¾	2⅛	13	24	18¾	14½	8	270
9	7⅜	4⅛	2⅜	16	30	20¾	19	11	500
10½	8 7/16	5⅛	2¾	35					

Cushioned plate couplings (Fig. 71) consist of two cast-steel flanged halves. Several rectangular slots are machined radially around their periphery to receive the cushioned torque transmitting steel plates. The plates are cushioned by four rectangular cushions, two in each flange, made of hydraulic packing. The plates and cushions are held in place by snap rings; each half is provided with a smooth steel cover, for safety. The dimensions and capacities for coupling of this type made by Westinghouse are given in Table 58.

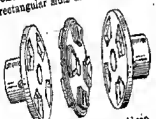


Fig. 70.—Grundy Flexible Coupling.

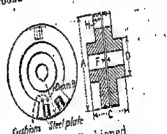


Fig. 71.—Cushioned Plate Coupling.

Flexible-band (belt-type) couplings (Fig. 72) have stud pins projecting from the driving and driven flanges. A tanned-leather belt is wound about the studs and acts as the intermediate driving connection. This coupling may be run in either direction and is very flexible. Dimensions and capacities of these couplings, as built by the Wood's Sons Co., Chambersburg, Pa., are given in Table 59.

Flexible steel member couplings transmit the torque from the driving to the driven element through a steel spring which allows for misalignment of the coupled

In Fig. 85, let C = leverage distance from fulcrum pin to line of action of P. Then, for clockwise rotation, $F(A + B) = RB - fRC$. For counterclockwise rotation, $F(A + B) = RB + fRC$.

In the case of clockwise rotation, it will be noted that C/B must be less than $1/f$, or the brake will be self-acting, *i.e.*, will bind.

In the arrangement shown in Fig. 86, for clockwise rotation, $F(A + B) = RB + fRC$ and for counterclockwise rotation, $F(A + B) = RB - fRC$.

In the latter case (for counterclockwise rotation), C/B must be less than $1/f$, or the brake will be self-acting, i.e., will bind.

Double blocks are used to eliminate bending of the shaft. Figure 88 illustrates one way in which such brakes may be rigged. The force relations are as follows:

Let F = load applied at end of lever, lb; R = reaction between wheel and *each* block, lb; P = tangential frictional resistance on each block surface, lb; f = coefficient of friction of materials in contact for a given condition of surface; r = drum radius, in.; T = torque on drum shaft, in-lb; other notations as in the figure. Then,

$$R = FA(c + d)/\cos(x/2)2Bc; \qquad P = Rf; \; T = 2Rfr.$$

The point o should be a floating pivot to permit adjustment as the blocks wear. Values of f are given in Table 64.

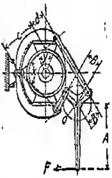


FIG. 88.

Should the face of the brake wheel and blocks be grooved, as shown in Fig. 87, $f/(\sin y + f \cos y)$ must be substituted for f in the foregoing equations, y being equal to half the angle included by the faces of the grooves and not less than 23 deg, to prevent binding; y may have any value up to 30 deg.

Band brakes are shown diagrammatically in Figs. 89, 90, and 91. The bands are usually of an asbestos fabric, sometimes reinforced with copper

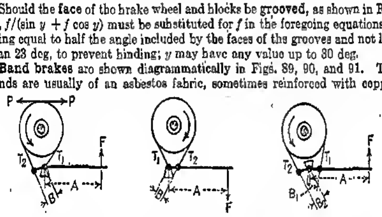


FIG. 89. FIG. 90. FIG. 91.

FIGS. 89–91.—Band Brakes.

wire and impregnated with asphalt. The force relations obtaining in their operations are as follows:

In Fig. 89, let F = force at end of brake handle; P = tangential force at rim of wheel; f = coefficient of friction of materials in contact; a = angle of wrap of band, deg; T_1 = total tension in band on tight side; T_2 = total tension in band on slack side. Then $T_1 - T_2 = P$ and $T_1/T_2 = 10^{0.0076fa} = 10^b$ where $b = 0.0076fa$. Also, $T_2 = P/(10^b - 1)$ and $T_1 = P \times 10^b/(10^b - 1)$. The values of $10^{0.0076fa}$ are given in Fig. 143 and Table 4, p. 242 (for a in radians).

For the arrangement shown in Fig. 89,

$$FA = T_2B = PB/(10^b - 1)$$

and

$$F = PB/A(10^b - 1)$$

For the construction illustrated in Fig. 90,

$$F = (PB/A)[10^b/(10^b - 1)]$$

For the differential brake shown in Fig. 91,

$$F = (P/A)[(B_2 - 10^bB_1)/(10^b - 1)]$$

Table 60. Dimensions of Flexible Steel Member Couplings
(Letters refer to Fig. 73. All dimensions in inches)

Max bore	Hp per 100 rpm	Max rpm	A	B	C	G	Max bore	Hp per 100 rpm	Max rpm	A	B	C	G
1/2	1/6	9500	1 15/16	7/8	1 5/32	1 27/32	3 3/4	48	1500	12	6 5/8	4 3/4	9 1/4
1 1/16	1/2	5500	3	1 3/8	1 3/8	2 23/32	4 1/4	68	1400	12 7/8	7 1/4	4 13/16	10 1/4
1	1	4200	4 3/4	1 3/4	1 15/16	3 7/16	4 3/4	96	1250	14 1/2	8 1/8	5 11/16	11 1/2
1 1/4	2	4000	5 3/8	2 1/4	1 31/32	3 15/16	5 1/4	120	1100	16 3/8	8 7/8	5 11/16	12 1/2
1 7/16	3	3700	6	2 1/2	2	4 1/8	6	150	1000	17 1/4	10 1/4	6 1/8	14 1/4
1 5/8	3 3/4	3500	6 5/8	2 3/4	2 1/16	4 1/8	7	240	950	18	11 3/4	6 1/8	15 1/4
1 7/8	6	2500	7 3/8	3 3/8	3 1/4	5	7 3/4	320	900	19	12 1/4	6 1/8	16 3/4
2 1/4	8	2100	8 5/16	3 15/16	3 1/4	5 3/4	8 1/2	510	850	22 3/4	14 1/2	6 1/2	19
2 1/2	10	2000	9 3/8	4 1/4	3 3/4	6 1/2	8 3/4	638	850	22 3/4	14 1/2	6 1/2	19
2 7/8	24	1900	10	4 7/8	4 3/16	7 1/4	10	1000	800	25 1/2	17	6 5/8	21 1/2
3 1/4	32	1650	11 1/2	5 1/2	4 9/16	8 1/4	11	1500	540	32	18	9 1/2	24 1/4

plings are shown in Table 61. With a fluctuating load, the capacities are 2/3 to 1/2; with shock load, they are 1/3 of those given in the table.

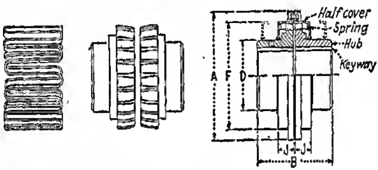


Fig. 74.—Grid-spring Coupling (Falk).

Table 61. Grid-spring Coupling
(Dimensions refer to Fig. 74; Dimensions in inches)

Basic rating, hp per 100 rpm	Max bores, in. Type FA	Max rpm	Dimensions, in. A	B	F	J	Gap, in.	Basic rating, hp per 100 rpm	Max bores, in. Type FA	Max rpm	Dimensions, in. A	B	F	J	Gap, in.
0.4	1	8,000	3 3/4	4 1/8	2 9/16	27/32	1/8	23	3 1/4	4,500	8 29/32	9 11/16	7 3/32	1 7/8	3/16
0.9	1 1/4	8,000	4 1/16	4 7/8	2 7/8	1 3/32	1/8	35	3 1/2	4,000	9 11/16	10 3/16	7 13/16	1 57/64	3/16
1.5	1 7/16	8,000	4 1/2	5 1/8	3 5/16	1 3/32	1/8	46	3	3,600	10 15/16	10 3/16	9 3/16	1 57/64	3/16
2	1 5/8	8,000	5	5 5/8	3 11/16	1 3/32	1/8	70	4 3/8	3,000	11 7/8	12	9 13/16	2 25/64	1/4
4	1 7/8	8,000	5 5/8	5 5/8	4 7/16	1 3/32	1/8	100	4 5/8	3,000	13 3/4	12 1/2	10 3/16	2 1/16	1/4
8	2 3/8	7,000	7 1/8	7 1/2	5 1/4	1 13/64	1/8	140	5 1/8	3,000	15 1/4	12 1/2	11 11/16	2 1/16	1/4
12	2 5/8	6,000	7 5/8	8 1/8	5 3/4	1 13/64	1/8	180	6	2,700	16 3/4	12 1/2	13 3/16	2 1/16	1/4
16	2 7/8	5,000	8 1/4	9 11/16	6 3/8	1 55/64	3/16	230	7	2,700	18 3/4	13 1/4	14 11/16	2 1/16	1/4

Fast gear couplings (Fig. 75, Bartlett Hayward Co., Baltimore) have two spur gears on flanges mounted on each shaft and continually in mesh with corresponding internal gears of a floating sleeve. The clearance in the gear teeth allows for a slight linear and angular misalignment. Table 62 gives the dimensions and capacities of these couplings.

Let R = mean radius of cone surface, in.; Q = force normal to cone surface, lb; P = tangential force at cone surface, acting with the leverage R; F = axial force, lb; f = coefficient of friction between cone surfaces; a = angle between shaft axis and element of cone surface, deg; b = angle of pitch of worm on worm wheel, deg; f_1 = coefficient of friction of worm and wheel teeth; r = pitch radius of worm, in.; r_1 = pitch radius of ratchet wheel, in.; L = load on ratchet teeth, lb. Let $x = f \cos a + \sin a$, and $y = (\tan b - f_1)$; then Lr_1 = torque on worm shaft (in-lb) due to F, where F is the axial force due to load on drum = Fry approximately.

The resisting moment of the clutch is $PR = QfR = RFf/x$. To hold the load and prevent its lowering, the actuating torque Fry must be less than the reacting torque RFf/x. Accordingly, $Rf/x \geq ry$, or $R \geq ryx/f$. The angles a and b may be assumed 22 deg, f at 0.08, and f_1 at 0.10, depending on the lubrication, whence $x = 0.45$ and $y = 0.30$. Then for the above assumptions $R \geq 1.7r$. If $a = 30$ deg, $b = 35$ deg, then, for f and f_1 as before, $x = 0.57$, $y = 0.60$, and $R \geq 4.3r$.

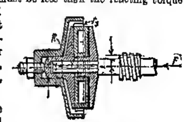


FIG. 94.—Disk Brake.

Disk brakes of the same general type of construction but using a flat face instead of a cone, are as shown in Fig. 94. There are two faces to be moved against friction.

The force relations are $Lr_1 \leq 2fFR = 2PR$, where $fF = P$. Also $R \geq ry/2f$.

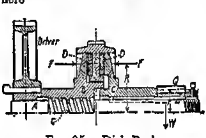


FIG. 95.—Disk Brake.

Frequently disk brakes are made as shown in Fig. 95. The pinion Q engages the gear in the drum (not shown). When the load is to be raised, power is applied through the gear and the connection between B and C is accomplished by the advancing of B along A and the clamping of the friction disks D and D and the ratchet wheel E. The reversal of the motor disconnects B and C. In lowering the load, only as much reversal of rotation of the gear is given as is needed to reduce the force in the friction disks so that the load may be lowered under control.

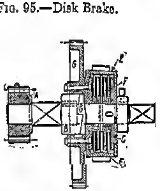


FIG. 96.—Multidisk Brake.

The force relations are as follows:

Let R = mean radius of friction plates, in.; f = coefficient of friction between plates; F = axial force along the screw = force in friction plates, lb; P = tangential force on friction plates at mean radius R, lb, $= fF$; W = load on pinion teeth, lb; r = radius of pinion pitch circle, in.; r_1 = radius of pitch circle of screw, in.; a = angle of screw thread, deg; f_1 = coefficient of friction in thread; $z = (\tan a + f_1)$.

Then the load in lowering causes a moment $Wr = fFR + Fr_1z$, approx. To sustain the load, $Wr \leq 2fFR$ and $fR \geq r_1z$.

Acceptable values of the several factors are: $a = 10$ deg; $f = 0.08$; $f_1 = 0.10$; $R = 9$ in. Substituting these in the last equation, $r_1 \leq 2.5$ in.+. Any radius of screw less than $2\frac{1}{2}$ in. consistent with strength will be satisfactory for the above conditions.

A multidisk brake is shown in Fig. 96. This type of construction results in an increase in the number of friction faces. The drum shaft is geared to the pinion A, while the motive power for driving comes through the gear G. In raising the load, direct connection is had between G, B, and A. In lower-

Square jaw clutches (Fig. 78) are used with shafts up to 4 in. diam for moderate speeds and power. Suitable proportions are: d = shaft diam in. and $k = d + 1$ in.; then $L = 2.65k$; $D = 2.1k$; $D_2 = 1.6k$; $D_3 = 1.5k$; $l = 1.25k$; $l_2 = 0.6k$; $x = y = 0.3k$.

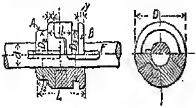


FIG. 78.—Jaw Coupling.

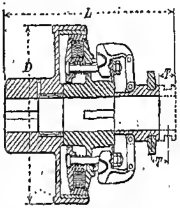


FIG. 79.—Plate Friction Clutch.

Plate Friction Clutches. A single disk clutch of this type is shown in Fig. 79 (Wood's Sons Co., Chambersburg, Pa.). Dimensions and capacities are given in Table 63. With two friction disks, the capacity of the coupling is doubled.

The force relation in plate friction clutches is as follows: Let R = mean radius of rubbing surface of rings, in.; T = torque transmitted, in.-lb; F = axial force, lb; f = coefficient of friction; n = number of rubbing surfaces;

Table 63. Dimensions of Plate Friction Clutches
(Letters refer to Fig. 79)

Size of shaft equal to capacity of coupling	Hp at 100 rpm	Dimensions, in.				Size of shaft equal to capacity of coupling	Hp at 100 rpm	Dimensions, in.			
		Max bore	D	L	T			Max bore	D	L	T
15/16	1¾	1½	6½	10⅝	1	2 15/16	45	6	24	23⅜	2½
1 3/16	3	1¾	7¾	12	1⅛	3 3/16	55	6	26	24⅜	2½
1 7/16	5½	2¼	10	13⅞	1¼	3 7/16	65	7½	29	26¾	2¾
1 11/16	10	3	12¼	15⅝	1⅜	3 15/16	85	8	34	30⅛	3½
1 15/16	15	4	14½	17	1½	4 7/16	120	8½	38	33⅝	4
2 3/16	20	4½	16½	18⅜	1¾	4 15/16	150	9	43	37⅞	4
2 7/16	25	5	19	20⅞	2	5 7/16	180	9½	49	39⅞	4
2 11/16	34	5½	21½	22	2¼	5 15/16	240	10	56	44¾	5

N = rpm of the shaft; Q = tangential force acting on ring surfaces at mean radius R, lb; hp = horsepower transmitted at N rpm; Then $Q = fFn$; $T = QR = nfFR = 63{,}024 \times \text{hp}/N$; $\text{hp} = NfnRF/63{,}024$.

Table 64. Friction Coefficients for Clutches

Cast iron on cast iron (dry)	0.15 to 0.20	Cork on metal	0.35
Cast iron on wood (dry)	0.20 to 0.25	Leather on metal (greasy)	0.23
Cast iron on brass (dry)	0.21	Cork on metal (greasy)	0.32
Leather on metal (dry)	0.56	Leather on metal (oily)	0.15

Cone friction clutches (Fig. 80) are adaptable to connecting shafts whose loads are frequently thrown on and off. The force relations obtaining in the operation of this clutch are shown diagrammatically in Fig. 81 and may be formulated as follows:

Let R = mean radius of clutch cone surface, in. T = torque transmitted through the clutch, in.-lb. Q = total force normal to conical surface, lb. P = tangential force at cone surface acting with leverage R, lb. F = axial force required to engage clutch (lb) = force of the spring. a = angle between

counteracted by the face of a solenoid or a centrifugal thruster. Interruption of current automatically applies the spring-activated brake shoes. Figures 99 and 100 show electric brakes built by the General Electric Co.; the capacities and dimensions of the brakes are given in Table 67.

Table 67. Electric Brakes
(Letters refer to Figs. 99 and 100)

Figure	Torque rating, lb-ft		Wheel		Dimensions, in.						
	Continuous	60 min inter.	Diam in in.	Face width, in in.	*N*	*B*	*C*	*D*	*M*	*O*	*F*
99	3	3	2¾	1½	6¼	9¼	11 3/32	2½	4 11/32	6¼	4¾
99	10	15	3 7/32	1¾	7 15/16	9⅛	10	3⅛	4¼	7 5/16	4⅝
99	25	35	4 15/32	3	9 11/16	12 13/16	14¼	4¼	6 7/16	10⅛	6 9/16
99	50	75	5¾	3	12⅜	16½	17½	5¼	6 15/16	12 5/16	9¼
100	125	160	8	3½	......	19	22⅝	6⅛	9¼	14 13/16	10
100	325	400	11	5	19 15/16	28	31¼	8¼	11¾	18½	18¾
100	600	800	14	6½	25⅝	33⅞	36⅜	10½	13	21⅜	22 3/16
100	1,200	1,600	19	8	29⅜	43	44¾	12½	16	26¾	26 23/32
100	2,400	3,600	24	10	37 3/16	53¾	.....	15½	20¼	36 11/16	29

GEARING

Spur Gears

Definitions. The parts of spur gears are shown in Fig. 101. The **diametral pitch** P is the ratio of the number of teeth in the gear to the diameter of the pitch circle D_p measured in inches. The **circular pitch**

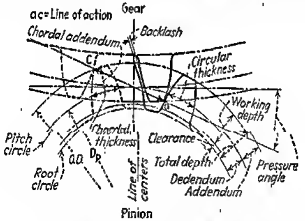


FIG. 101.—Gear Tooth Nomenclature.

p is the distance in inches on the circumference of the pitch circle between corresponding points of adjacent teeth. $P \cdot p = \pi$.

The **normal pitch** p_n is the distance along the line of action, between successive and corresponding involute tooth surfaces. $p_n = p \cos \phi$, where ϕ = pressure angle.

The **base circle diameter** D_b is the diameter of the base circle from which the involute is generated. $D_b = D_p \cos \phi$.

Magnetic clutches are being applied to the connecting of shafts even of large size. A clutch of this type manufactured by the Cutler-Hammer Mfg. Co. is shown in Fig. 83. Dimensions of C-H magnetic clutches and their capacities are given in Table 66.

Table 66. Cutler-Hammer Magnetic Clutches
(Letters refer to Fig. 83)

Hp per 100 rpm	Lining area, sq in.	Max safe rpm	D-c current at 230 V, amp	Max bore, in.	Dimensions, in.					
					A	B	D	F	H	J
0.3	13	3,540	0.2	2.00	7	7¼	4⅛	5	2	4⅞
1.7	28	2,840	0.5	2.00	10	8 7/16	4¼	4 9/16	3⅞	4⅞
3.0	42	2,380	0.5	2.00	12	8⅝	4⅛	4¼	4	4⅞
7.0	51	2,040	0.7	2.87	14	12 3/16	5 3/16	5	3 9/16	9½
15.0	83	1,790	0.8		16					
17.5	179	1,420	0.8	2.87	20	13⅝	5⅝	5⅝	3 13/16	9½
20.0	320	1,190	0.8		24					
30.0	113	1,420	0.9		20					
33.0	252	1,190	0.9	4.31	24	15 5/16	6	5½	5½	13½
35.0	415	1,030	0.9		28					
50.0	169	1,190	1.2		24					
53.5	332	1,030	1.2	5.50	28	16 9/16	6½	6¼	6¼	13½
58.0	521	900	1.2		32					
90.0	236	1,030	1.4		28					
97.0	425	900	1.4	5.50	32	16¾	6 11/16	6¼	6¼	13½
105.0	635	800	1.4		36					
130.0	274	900	1.8		32					
141.0	484	800	1.8	6.63	36	18½	6 11/16	7	7	13½
153.0	724	710	1.8		40					
190.0	354	800	2.1		36					
205.0	594	710	2.1	6.63	40	18⅞	7 1/16	7	7	13½
217.0	859	650	2.1		44					

BRAKES

Block brakes are shown diagrammatically in Figs. 84, 85, 86, and 87. They consist of a block or shoe of wood or cast-iron bearing upon an iron or steel

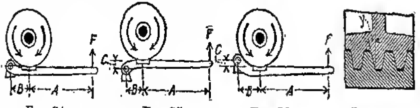


FIG. 84. FIG. 85. FIG. 86. FIG. 87.
FIGS. 84–87.—Block Brakes.

wheel. The force relations obtaining in the operation of these brakes may be formulated as follows:

In Fig. 84, let F = load applied at end of lever arm; A = distance from point of application of F to block center; B = distance from block center to center of fulcrum pin; R = reaction between wheel and block; f = coefficient of friction; P = tangential frictional resistance. Then, for rotation in either direction,

$$F(A + B) = RB, \quad R = P/f, \quad \text{and} \quad F = PB/f(A + B)$$

The shading in Figs. 102 to 107 shows the gear whose shaft has no motion.

Bevel Gears

Bevel gears may be used to connect two intersecting shafts in any given speed ratio. Occasionally bevel gears are used for non-intersecting shafts, in which case they are called skew bevel gears. A special type of skew bevel gearing developed by the Gleason Works and used widely in the automotive field is known as **hypoid gearing** (see *Jour. S.A.E.*, Vol. 18, No. 6). The involute tooth form is almost universally used for bevel gears.

Referring to Fig. 108, it is evident that the pitch surfaces of bevel gears must be frustrums of a pair of cones whose vertices are at the point of intersection of the axes.

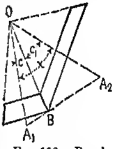


Fig. 108.—Bevel Gears.

There are three general types of bevel gears:

1. Right angle bevel gears (Fig. 109) in which the angle between intersecting shafts is 90 deg.
2. Acute angle bevel gears (Fig. 111) in which the angle between intersecting shafts is less than 90 deg.
3. Obtuse angle bevel gears (Fig. 112) in which the angle between intersecting shafts is greater than 90 deg.

Bevel gears having center angles, c and c_1, equal to 45 deg (Fig. 110) are called **miter gears**. The speeds of the shafts of bevel gears are determined by the following relation: $n/n_1 = \sin c_1/\sin c$, where $n(n_1)$ = rpm of pinion (gear) and $c(c_1)$ = center angle of pinion (gear) Fig. 109.

At the large end of the gear (Fig. 108), the tooth outlines will be approximately those generated on a pitch circle of radius BA_2. The formative number of teeth is the number of teeth which would be contained in a complete spur gear of pitch radius BA_2 (for the gear) and BA_1 (for the pinion). $T = N/\cos c$; $T_1 = N_1/\cos c_1$, where $T(T_1)$ = formative number of teeth in pinion (gear); $N(N_1)$ = actual number of teeth in pinion (gear). The formative number of teeth is used in selecting the proper cutter and also for obtaining the value of the Lewis factor when calculating the strength of the bevel gear.

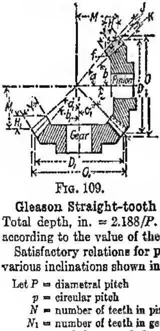


Fig. 109.

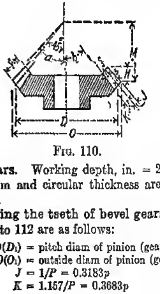


Fig. 110.

Gleason Straight-tooth Bevel Gears. Working depth, in. $= 2.000/P$. Total depth, in. $= 2.188/P$. Addendum and circular thickness are varied according to the value of the gear ratio.

Satisfactory relations for **proportioning** the teeth of bevel gears of the various inclinations shown in Figs. 109 to 112 are as follows:

Let P = diametral pitch
p = circular pitch
N = number of teeth in pinion
N_1 = number of teeth in gear
x = angle between shafts

$D(D_1)$ = pitch diam of pinion (gear)
$O(O_1)$ = outside diam of pinion (gear)
$J = 1/P = 0.3183p$
$K = 1.157/P = 0.3683p$

In this arrangement, the quantity $10^b \times B_2$ must always be less than B_1, or the band will grip the wheel and the brake, or part of the mechanism to which it is attached, will be ruptured.

It is usual in practice to have the leverage ratio A/B for block brakes about 5:1, and for hand brakes about 10:1. The bands are faced with maple blocks.

If f for wood on iron be taken at 0.3 and the angle of wrap for the band be 270 deg, *i.e.*, subtends $\frac{3}{4}$ of the circumference, then, $10^b = 4$ approx, and C/B be taken equal to $1/0.5 = 2$, the loads required for a given torque will be as follows for the cases just considered and for the leverage ratios stated above:

Block brake, Fig. 84	$F = 0.55P$
Block brake, Fig. 85 (clockwise rotation)	$F = 0.22P$
Block brake, Fig. 85 (counterclockwise rotation)	$F = 0.90P$
Block brake, Fig. 86 (clockwise rotation)	$F = 0.90P$
Block brake, Fig. 86 (counterclockwise rotation)	$F = 0.22P$
Band brake, Fig. 89	$F = 0.033P$
Band brake, Fig. 90	$F = 0.133P$
Band brake, Fig. 91	$F = 0.016P$

In the case of Fig. 91, the dimension B_2 must be greater than $B_1 \times 10^b$. Accordingly, B_1 is taken at $\frac{1}{4}$, A at 10, and, since $10^b = 4$, B_2 is taken at $1\frac{1}{2}$.

The principal function of a brake is to absorb energy. This energy appears at the surface of the brake as heat, which must be carried away at a sufficiently rapid rate to prevent burning of the wooden blocks. Suitable proportions may be arrived at as follows:

Let p = unit pressure on brake surface, lb per sq in. = R (reaction against block) /area of block; v = velocity of brake rim surface, fps = $2\pi rn/60$, where n = rpm of brake wheel. Then pvf = work absorbed per sq in. of brake surface per sec, and $pv \leqq 1{,}000$ for intermittent applications of load with comparatively long periods of rest and poor means for carrying away heat (wooden blocks); $pv \leqq 500$ for continuous application of load and poor means for carrying away heat (wooden blocks); $pv \leqq 1{,}400$ for continuous application of load with effective means for carrying away heat (oil bath).

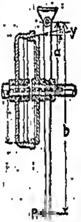


Fig. 92.—Cone Brake.

Cone brakes may be made to the form shown in Fig. 92. The force relations are as follows:

Let F = load applied at end of lever, lb; Q = normal pressure on cone surface, lb; P = tangential force on the rim of the brake, lb; r = mean radius of cone surface, in., and y = half angle of cone. Then $Q = F(b/a)/2(\sin y + f \cos y)$ and $P = fF(b/a)/(\sin y + f \cos y)$; $F = P(a/b)(\sin y + f \cos y)/f$. For $a = 1$, $b = 10$, $y = 15$ deg, and $f = 0.2$ for cast iron on cast iron, $F = 0.23P$, approx.

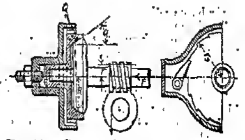


Fig. 93.—Cone Brake for Lowering Loads.

Such brakes are frequently used for lowering loads by the means shown in Fig. 93. The drum shaft is driven through the worm shaft by means of worm and wheel. In raising the load, the ratchet runs free. As the load tends to lower, the ratchet engages and the worm thrusts the cone surfaces together. Lowering of the load must be accomplished by the application of torque to the worm shaft. In the case of worms of small pitch, no brake is required, for the wheel cannot turn the worm. This brake is therefore adapted to worms of large pitch, and the reason for employing a large pitch is that a more efficient drive is obtained (see p. 827). The force relations are approximately as follows:

a 30-tooth worm wheel meshing with a single-threaded worm will have a velocity ratio of 1 to 30, *i.e.*, the worm must make 30 revolutions in order to revolve the worm wheel once. For a double-threaded worm, there will be 15 revolutions of the worm to one of the worm wheel, etc. High velocity ratios are thus obtained with relatively small wheels.

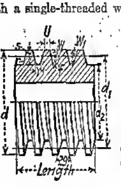


FIG. 114.

FIG. 115.—Hindley Worm Gear.

The following rules for **proportioning worm gears** are quoted from the "American Machinist Gear Book," by C. H. Logue, and refer to Figs. 113 and 114:

Let

- N = number of teeth in worm wheel
- N_1 = number of threads in worm
- p = circular pitch (dist c to c of teeth)
- L = lead (advance of worm in 1 rev)
- D_1 = pitch diam of worm wheel
- T = throat diam of worm wheel
- D = outside diam of worm wheel
- F = face of worm wheel
- a = distance from center line to point of tooth
- b = length of side
- d_1 = pitch diam of worm
- d = outside diam of worm
- d_2 = bottom diam of worm
- e = radius at throat of worm wheel
- c = angle of sides of face, deg
- B = center distance
- R = revs of worm per rev of wheel
- x = angle of teeth in wheel with axis (used in gashing)
- W = working depth
- W_1 = whole depth
- f = clearance
- t = thickness of tooth at pitch line
- t_n = normal thickness of tooth at pitch line
- p_n = normal circular pitch
- s = addendum
- U = width of worm thread at top
- Y = width of worm thread at bottom
- P = diametral pitch

Then

$$N = \pi D_1/p$$
$$T = 0.3183p(N + 2)$$
$$F = 2(0.5d + 0.17p)\sin c$$
$$= 0.5d + 0.17p \text{ approx}$$
$$a = F/2 - b\sin c$$
$$b = W_1 + 0.12p$$
$$d = d_1 + 2s$$
$$d_2 = d - 2W_1$$
$$e = \tfrac{1}{2}d_1 - s$$
$$c = 30 \text{ to } 35 \text{ deg}$$
$$\sin c = F/(d + 0.34p)$$
$$B = (D_1 + d_1)/2$$
$$D_1 = 0.3183Np$$
$$D = T + 2(s - e\cos c)$$
$$p = D_1/0.3183N$$
$$L = pN_1$$
$$N_1 = N/R$$
$$\tan x = L/\pi d_1$$
$$t_n = t\cos x, \text{ or}$$
$$t = t_n/\cos x$$
$$U = 0.335p = 1.0536/P$$
$$Y = 0.31p = 0.9744/P$$
$$p_n = p\cos x, \text{ or } p = p_n/\cos x$$

The formulas for the tooth parts of spur gears apply also to worm gears. N, D, and p are calculated the same as for spur gears.

T has the same value as the outside diameter of a spur gear of the same pitch and number of teeth.

F will usually be sufficiently accurate when calculated from the second formula.

d_1. The efficiency of a worm drive depends greatly on the helix angle x of the worm. When the lead is fixed, the angle is determined from $\tan x = L/\pi d_1$, and, in order to make it of a value that would ensure high efficiency, the diameter should be made small enough to provide a helix angle of about 30 deg. Little efficiency is gained in exceeding this angle (see Fig. 121). The preceding discussion assumes that the drive is to be speed reducing or that the worm drives. In the event of a speed increasing drive, where the gear drives, the designor should try to secure a helix angle between 45 deg and 60 deg.

ing, B moves relatively to G and forces the friction plates together, those plates fast to E being held stationary by the pawl on E. In the figure, there are three plates fast to E, one fast to G, and one fast to C.

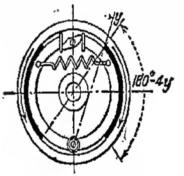


Fig. 97.—Internal Brake.

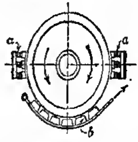


Fig. 98.—Eddy Current Brake.

In addition to the notation given in the previous case, let n = number of faces in sliding contact when the part C is moved relatively to part E which carries the ratchet. This condition obtains upon the load beginning to lower, and n_1 = number of *faces* in sliding contact when the parts G and C both move relatively to E. This obtains when full gripping of the plates takes place. The load in beginning to lower occasions the following force relation: $Wr = n_1fFR + Fr_2$. To sustain the load, it is necessary that $Wr \leqq n_1fFR$. Hence, to prevent the load from dropping, $r_2 \leqq fR(n_1 - n)$.

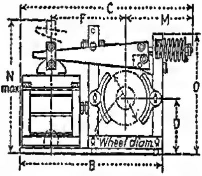


Fig. 99.—Solenoid-type Electric Brake.

Internal brakes are used extensively on most motor vehicles. The usual construction is shown in Fig. 97. The blocks are lined by a suitable friction material, usually asbestos woven with copper wire. In order to eliminate the possibility of binding, the angle y must be such that $\tan y \geqq f$. The angle a subtended by the brake is $a \leqq 180° - 4y$; $a \leqq 90°$ approx at $f = 0.4$ and $a \leqq 120°$ approx at $f = 0.2$.

Eddy current (brakes Fig. 98) are used with flywheels where quick braking is essential, and where large kinetic energy of the rotating masses precludes the use of block brakes due to excessive heating, as in reversible rolling mills. A number of poles a are electrically excited (north and south in turn) and create a magnetic flux which permeates the gap and the iron of the rim, causing eddy currents. The flywheel energy is converted through these currents into heat. The hand brake b may be used for quicker stopping when the speed of the wheel is considerably decreased, *i.e.* the eddy current brake is inefficient. Two brakes are provided to avoid bending forces on the shaft. (Siemens Schuckert Review, 1927, No. 5.)

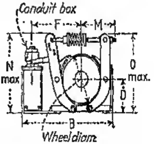


Fig. 100.—Thrustor-type Electric Brake.

Electric brakes are often used in cranes, bridges, turntables, and machine tools, where an automatic application of the brake is important as soon as power is cut off. The brake face is supplied by an adjustable spring which is

in common use are shown in Fig. 117. Correctly cut herringbone gears run much more smoothly than spur gears, and work silently and without vibration. They can also be run at higher speeds and higher velocity ratios. In the **Wuest system,** involute teeth with a 20 deg angle of obliquity are

Table 69. Formulas for Helical Gear Calculations

Driver		Follower		Remarks
To find	Formula	To find	Formula	
b	$\tan b = d_1r_1/d_2r_2$	a	$90 \text{ deg} - b$	Axes at right angles only
b	$\tan b = p_1/p_2$	a	$90 \text{ deg} - b$	
b	$\cos b = p_n/p_1$	a	$c - b$	
b	$\tan b = \pi d_1/L_1$	a	$c - b$	
p_n	$\pi d_1 \cos b/N_1$	p_n	$\pi d_2 \cos a/N_2$	Same in both gears
p_1	$\pi d_1/N_1$	p_2	$\pi d_2/N_2$	
L_1	$p_2N_1 = \pi d_1 \tan a$	L_2	$\pi d_2 \tan b = p_1N_2$	Axes at right angles only
N_1	$d_1P \cos b$	N_2	$d_2P \cos a$	
d_1	$2C\Big/\left(\frac{r_1}{r_2}\tan a + 1\right)$	d_2	$2C\Big/\left(\frac{r_1}{r_2}\tan b + 1\right)$	Axes at right angles only
d_1	$2C\Big/\left(\frac{r_1 \cos b}{r_2 \cos a} + 1\right)$	d_2	$2C - d_1$	
d_1	$0.3183N_1p_1$	d_2	$0.3183N_2p_2$	
D_1	$d_1 + 2S_n$	D_2	$d_2 + 2S_n$	
D_1	$d_1 + (2/P)$	D_2	$d_2 + (2/P)$	14½ deg standard only
Cutter*	$N_1/\cos^3 b$	Cutter*	$N_2/\cos^3 a$	
Center distance $C = (N_1/2P \cos b) + (N_2/2P \cos a)$				

* Spur-gear cutter to be used which is correct for the number of teeth given by the formulas.

employed, the pitch angle of the teeth being 23 deg. The face width is made equal to 6 × circular pitch p for gears with pinions of not less than 25 teeth, and $6p$ to $12p$ for high ratios with small pinions. For convenience in manufacture, the gears are grooved at the center of the face for a width equal to p and the two sets of teeth are stepped half the pitch apart. Pitch diameter $= N/P$ (for 20 teeth and over) $= (0.95N + 1)/P$ (for less than 20 teeth), where N = number of teeth and P = diametral pitch. Addendum $= 0.8/P$; dedendum $= 1/P$; working depth $= 1.6/P$.

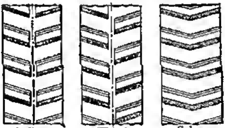


Fig. 117.—Herringbone Gears.

The gear teeth of De Laval herringbone gears for relatively high speeds are cut in steel forgings. For gears of relatively small diameters, the gear blanks and shaft are sometimes forged integral. On gears of relatively large size, a seamless rolled steel band is shrunk on a cast-iron center and the teeth are cut in this band. For pitch speeds above 3,000 fpm, a helical angle of approximately 45 deg is used in order that the maximum number of teeth may be engaged at the point of contact. The angle of obliquity is 14½ deg. *High speed helical gears have not less than 31 teeth in a pinion. Tooth* pressures in pounds per lineal inch of working face measured parallel to the shaft are limited to 60 times the diameter of the pinion in inches. The unsup-

The **gear ratio** m is the ratio of the number of teeth in the gear N_g to the number of teeth in the pinion N_p. $m = N_g/N_p$. The pinion is the wheel having the lesser number of teeth.

The **face** of a gear is the width of the tooth surface, measured parallel to the axis of the gear. The **center distance** is the distance between axes of mating gears.

Outlines and Proportions of Gear Teeth. The outlines of the teeth of gears in practice are almost always involute curves. The use of cycloidal and other curves is very rare. An involute curve (see p. 153) is generated by a point in a taut cord unwrapped from a base circle. One advantage of involute gearing is that uniform angular motion is transmitted, even though the center distance be changed slightly.

Tooth proportions for various systems adopted by the A.S.A. are given in Table 68.

Table 68. Tooth Proportions in Various Systems
(Dimensions in inches.)

	Full Depth or Composite 14½ deg.	Full Depth Involute 20 deg.	Stub Involute 20 deg.
Addendum	$1/P$	$1/P$	$0.8/P$
Minimum dedendum (incl. clearance)	$1.157/P$	$1.157/P$	$1/P$
Minimum clearance	$0.157/P$	$0.157/P$	$0.2/P$
Minimum total depth	$2.157/P$	$2.157/P$	$1.8/P$

Total depth is the radial distance from tip of tooth to root circle.

The speed ratios of spur gears are inversely proportional to their pitch diameters. The pitch diameters necessary for any given speed ratio and distance of shaft centers may be obtained in the following manner:

Let C = distance between shaft centers, in.; m = ratio of radius of larger to smaller gear. Then, diameter of smaller gear (D_s), in. $= 2C/(1 + m)$.

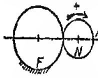


Fig. 102.

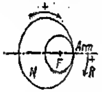


Fig. 103.

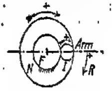


Fig. 104.

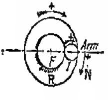


Fig. 105.

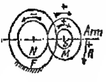


Fig. 106.

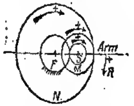


Fig. 107.

Figs. 102–107.—Epicyclic Gear Trains.

Diameter of larger gear (D_l), in. $= 2C/(1/m + 1)$. Accordingly, if $C = 24$ in. and $m = 5$, $D_l = 48/(6/5) = 40$ in., and $D_s = 48/6 = 8$ in.

For **epicyclic gear trains**, see p. 755.

The speed ratios (rpm of N ÷ rpm of R) for the more common gear trains are as follows:

Fig. 102. $1 + D_F/D_N$. Fig. 103. $1 - D_F/D_N$. Fig. 104. $1 + D_F/D_N$.
Fig. 105 (with internal gear driving). $D_R/(D_R + D_F)$.
Fig. 106. $1 - (D_F \times D_M)/(D_S \times D_N)$. Fig. 107. $1 + (D_F \times D_M)/(D_S \times D_N)$.

Table 70. Values of Form Factor (y) in Lewis Formula for Spur Gears

No. of teeth	*Full depth, composite 14½ deg	Full depth involute 20 deg	†Stub involute 20 deg	No. of teeth	*Full depth, composite 14½ deg	Full depth involute 20 deg	†Stub involute 20 deg
12	0.210	0.245	0.311	30	0.318	0.358	0.437
14	0.236	0.276	0.339	45	0.340	0.399	0.464
16	0.255	0.295	0.360	60	0.355	0.421	0.484
18	0.270	0.308	0.377	100	0.368	0.446	0.506
20	0.283	0.320	0.393	150	0.375	0.459	0.518
25	0.305	0.340	0.416	Rack	0.390	0.484	0.550

* Use these values also for Brown and Sharpe full depth teeth.
† Use these values also for Nuttall Stub, A.G.M.A. stub, and Fellows Stub, 4 or 8 pitch. For Fellows Stub, 5, 6, and 7 pitch, values are slightly greater; for 9, 10, and 12 pitch, slightly less.

Table 71. Lewis Form Factors for Gleason System

No. teeth in pinion \ Ratios	1.00 to 1.25	1.25 to 1.50	1.50 to 1.75	1.75 to 2.00	2.00 to 2.25	2.25 to 2.50	2.50 to 2.75	2.75 to 3.00	3.00 to 3.25	3.25 to 3.50	3.50 to 3.75	4.00 to 4.50	5.00 to ∞
Straight bevel gears													
10	0.231	0.260	0.280	0.294	0.305	0.315	0.324	0.332	0.340	0.347	0.353	0.365	0.377
12	0.248	0.265	0.281	0.295	0.308	0.318	0.328	0.335	0.341	0.345	0.348	0.353	0.356
16	0.252	0.261	0.269	0.277	0.285	0.292	0.298	0.304	0.308	0.312	0.314	0.319	0.323
19 to 21	0.265	0.272	0.279	0.286	0.294	0.300	0.307	0.312	0.317	0.320	0.324	0.328	0.332
26 to 30	0.284	0.291	0.297	0.304	0.310	0.317	0.322	0.327	0.332	0.336	0.339	0.344	0.347
Spiral bevel gears													
10	0.315	0.338	0.353	0.363	0.371	0.345	0.326	0.342	0.351	0.357	0.363	0.371	0.377
12	0.298	0.318	0.333	0.343	0.351	0.357	0.363	0.368	0.372	0.377	0.379	0.384	0.388
16	0.322	0.335	0.347	0.358	0.367	0.374	0.381	0.386	0.390	0.394	0.397	0.402	0.406
19 to 21	0.339	0.351	0.362	0.373	0.382	0.389	0.396	0.401	0.405	0.407	0.410	0.412	0.415
26 to 30	0.364	0.374	0.384	0.393	0.399	0.404	0.407	0.410	0.412	0.414	0.415	0.417	0.419

Herringbone Gears. Safe working pressure (lb) on the teeth of Wuest gears = $W = 0.4pFk$, where P = circular pitch, F = width of face (both in in.), and k has the following values determined by experiment:

	Velocity of pitch circle, fpm							
	200	400	800	1200	1600	2000	2400	2800
High-carbon-steel forgings	....	....	1550	1440	1320	1220	1120	1050
Steel castings	1300	1200	1100	1100	920	820	770	700
Phosphor bronze	1100	1040	920	830	750	690	630	590
Gun metal	960	880	750	650	580	530	500	460
Cast iron	760	690	590	500	450	410	390	370
Brass	600	520	440	400	360	330	310	300

Also W = hp × 33,000/V, where V = velocity of pitch circle, fpm.

Worm Gears. The allowable load (lb) on worm-gear teeth = $W = cbp$, where b = width of tooth and p = circular pitch—both in inches, and c = constant = 285 to 425 for cast-iron cut teeth = 455 to 711 for phosphor-bronze wheel and hardened steel worm such as are used for high speeds. These values are for the strength of the teeth only. For continuous running where heating and wear must be taken into consideration, c should have about the following values in the case of a phosphor-bronze wheel and a hardened and polished worm running in an oil bath:

NOTE: Table 70 from Marks, "Mechanical Engineers' Handbook," with modifications.

Then

$$N = DP = \pi D/p \qquad N_1 = D_1P = \pi D_1/p$$
$$D = N/P = 0.3183pN, \qquad D_1 = N_1/P = 0.3183pN_1$$
$$O = D + (2 \cos c/P) = D + 0.6366p \cos c\ [= D + (1.4142/P) = D + 0.45p \text{ for the miter gear in Fig. 110}]$$
$$O_1 = D_1 + (2 \cos c_1/P) = D_1 + 0.6366p \cos c_1$$
$$= D_1 + (2 \sin c/P) = D_1 + 0.6366p \sin c \text{ for Fig. 109}$$
$$\tan c = \cos(x - 90°)/[(N_1/N) - \sin(x - 90°)] \text{ for Fig. 112}$$
$$= \sin x/[(N_1/N) + \cos x] \text{ for Fig. 111; } = N/N_1 \text{ for Fig. 109}$$
$$c_1 = x - c$$
$$\tan s = 2 \sin c/N (= 1.4142/N \text{ for Fig. 110})$$
$$\tan f = 2.314 \sin c/N (= 1.6362/N \text{ for Fig. 110})$$
$$a = c + s (= 45° + s \text{ for Fig. 110}); \ a_1 = c_1 + s$$
$$b = c - f (= 45° - f \text{ for Fig. 110}); \ b_1 = c_1 - f$$
$$M = \tfrac{1}{2}O \cot a; \ M_1 = \tfrac{1}{2}O_1 \cot a_1$$
$$H = Y \cos a; \ H_1 = Y \cos a_1.$$

The number of teeth for which to select the cutter $= N/\cos c$ for pinions, and $N_1/\cos c_1$ for gears. For Fig. 110, this equals $1.414N$.

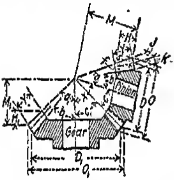


FIG. 111.

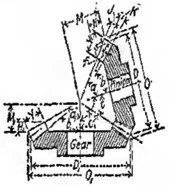


FIG. 112.

The pressure angles are as follows: for ratios having 14 or more teeth in pinion, 14½ deg; 13–13 to 13–24, 17½ deg; 13–25 and higher, 14½ deg; 12–12 and higher, 17½ deg; 11–11 to 11–14, 20 deg; 11–15 and higher, 17½ deg; 10–10 and higher, 20 deg.

Gleason Spiral Bevel Gears. Working depth, in. $= 1.700/P$. Total depth, in. $= 1.888/P$. The pressure angles are as follows: for ratios having 12 or more teeth in pinion, 14½ deg; 11–11 to 11–19, 17½ deg; 11–20 and higher, 14½ deg; 10–10 to 10–24, 17½ deg; 10–25 and higher, 14½ deg.

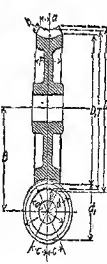


FIG. 113.

Worm Gears

Worm gearing is used for obtaining large speed reductions between non-intersecting shafts making an angle of 90 deg with each other. There are two classes of worm gearing in common use: straight worm gearing (Fig. 114) and **Hindley worm gearing** (Fig. 115). The latter gearing has a worm which is curved across the tops of the teeth and has a greater load-carrying capacity but lacks interchangeability and requires more careful machining and alignment.

With worm-and-wheel gearing, the **velocity ratio** is the ratio between the number of teeth on the gear and the number of threads on the worm. Thus,

In 1934, Schmitter published a design method based on this formula, with factors to take account of number of teeth in contact, face width, velocity effect, material, etc. This method has been developed with particular reference to the rating of helical and herringbone gear speed reducers (see "Manual of Gear Design," Section 3, Industrial Press, New York).

For application, the shear stress is multiplied by a service factor (Table 72).

Proportions of Gear Wheel Parts

The following represents Westinghouse-Nuttall practice in the design of industrial gearing.

The bores of gears and pinions are determined by the formula $b = \sqrt[3]{80h/n}$, where b = bore, in.; h = horsepower; n = rpm.

The hub diameter is made equal to $1.8b$, and the hub width equal to $1.25b$. For a gear with a face greater than $1.25b$, the width is made equal to the face.

The face of spur gears is made 3 or 4 times the circular pitch.

The minimum amount of metal permissible above the keyway of a pinion is determined by the following formula derived from application experience, $m = \sqrt{(t_p/5)}/P$, where m = minimum thickness of metal over keyway, in.; t_p = number of teeth in pinion; P = diametral pitch, in.

A six-arm gear is generally used whenever possible in the split and solid-type up to approximately 120 in. pitch diameter, and above this an eight-arm design is recommended. The four-arm design is generally used for split gears under 40 in. pitch diameter, with a narrow face and short hub to avoid special bolting. In calculating the arm sizes, the arm is considered as a cantilever beam with the load equally divided among the arms. First, the stalling load (or load at zero speed) is calculated according to the Lewis formula (p. 955). Then the section modulus is computed according to the flexure formula, $Z = Wr_p/Ns_o$, where Z = section modulus, in.^{3}; W = stalling load on teeth, lb; r_p = radius of pitch circle, in.; N = number of arms; s_o = working stress, lb per sq in.

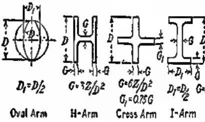


FIG. 120.—Gear Arm Sections.

The width of the arm in the plane of rotation is found by considering an elliptical cross section with a major axis of twice the length of the minor axis, $D = 2\sqrt[3]{Z/0.3927}$, where D = arm width at hub, in plane of rotation, in.; Z = section modulus, in.^{3}.

The width D found by the above method is used for arms having other cross sections (Fig. 120), the remaining dimensions of such cross sections being computed by the formulas given under the cross sections in Fig. 120.

The arms are made tapered ¼ in. per ft, with dimension D applying to the large end at the hub.

The rim thickness is proportioned according to the following formula: $J = \sqrt{(0.5t_g/N)}/P$, where J = thickness of rim inside of root diameter, in.; t_g = no. teeth in gear; N = number of arms; P = diametral pitch, in.

Phenolic Laminated Pinions. Certain special precautions should be taken in design of phenolic laminated pinions. The root diameter of the pinion should be such that the minimum distance from the edge of the keyway

c is generally from 30 deg to 35 deg. When the helix angle is 20 deg or over, c should not exceed 30 deg. When it is less than 20 deg, c may be taken as 35 deg.

L = circular pitch × number of threads in the worm; *e.g.*, a double-threaded worm of 1½ in. circular pitch has a lead of 3 in.

N_1. The number of threads in a worm of given velocity ratio = number of teeth in the wheel divided by the ratio; *e.g.*, for a wheel with 60 teeth and a velocity ratio of 30 : 1, number of threads = $N_1 = 60/30 = 2$.

W_1 should be proportioned from the normal pitch (p_n) rather than from the linear pitch (p), as is usually the case. It is wrong to take the depth from linear pitch even for single or double threads, and for quadruple and sextuple threads the error becomes so pronounced that it is likely to affect seriously the working of the gear.

t_n = thickness of tooth parallel with the axis of worm (= 0.5 × linear pitch) divided by the cosine of the helix angle x.

The length of the worm need be no greater than 3 times the circular pitch, as seldom are more than two teeth in contact at once. If the worm be made longer, however, it can be shifted lengthwise when worn, as it always wears away more rapidly than the wheel. It is common practice to make the length of worm = $6p$.

For strength of worm gears, see p. 824; for efficiency, pp. 240 and 827.

Helical Gears

Helical gears (commonly misnamed "spiral" gears) may be used to connect (1) parallel shafts, (2) shafts at right angles and not intersecting, and (3) shafts inclined at any angle to each other and not intersecting. Worm gears form a special case of (2). Figure 116 represents two helical gears connecting non-intersecting shafts inclined to each other in plan at an angle of c degrees. Gear A, of radius R, has teeth of a pitch angle equal to x deg, while the pitch angle of the teeth of gear B is equal to x_1 deg; the radius of gear B is R_1. Let n = rpm of A and n_1 = rpm of B; then the linear velocity of the pitch surface of $A = 2\pi Rn$, and the component of the velocity in the direction of the normal to the tooth profiles is $2\pi Rn \cos x$. Similarly, for gear B, the component of the velocity in the direction of the normal to the tooth profiles is $2\pi R_1 n_1 \cos x_1$. When helical gears are in mesh, however, the normals to the tooth profiles at the point of contact are coincident and the velocities in the direction of the normals are identical in value; hence $2\pi Rn \cos x = 2\pi R_1 n_1 \cos x_1$, or $n/n_1 = R_1 \cos x_1 / R \cos x$.

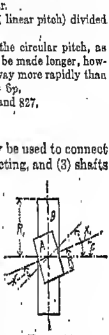


FIG. 116.

The problem of connecting a pair of shafts by helical gears for any velocity ratio admits of a number of solutions, since both the radii of the pitch surfaces and the angles of the teeth contribute to establishing the velocity ratio. The formulas given in Table 69 will be found of assistance in the computation of helical gears. The notation used in this table is as follows:

$N_1(N_2)$ = number of teeth in driver (follower)
$d_1(d_2)$ = pitch diam of driver (follower)
$p_1(p_2)$ = circular pitch of driver (follower)
p_n = normal circular pitch for both gears
P = normal diametral pitch for both gears
$D_1(D_2)$ = outside diam of driver (follower)
S_n = addendum of normal pitch
a = tooth pitch angle of follower
b = tooth pitch angle of driver
$L_1(L_2)$ = lead of driver (follower)
= lead of tooth helix
$r_1(r_2)$ = rpm of driver (follower)
c = angle between shafts in plan
C = center distance.

When helical gears are used to connect parallel shafts, the normal component of the tangential pressure on the teeth produces end thrust of the shafts. To remove this objection, such gears are made with right-handed helical teeth on one side of the face and left-handed on the other, and are then known as **herringbone gears**. The classes of herringbone gears

For **alloy steels for automobile transmission gears,** see S.A.E. specifications, p. 539.

The major non-metallic gear materials are phenolic laminated (Formica, Micarta, Textolite, etc.) and rawhide. Fiber and other materials are also used, but to a smaller extent. Non-metallic materials are used where quietness is of prime importance; usually one member of the mating pair is non-

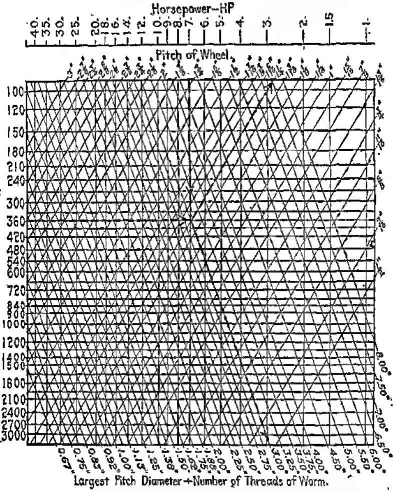


FIG. 122.—Diagram for Designing Worm Gearing of Maximum Efficiency.

metallic and the other metallic. Rawhide when made into a gear requires metallic side plates to provide adequate support; the phenolic laminated materials are self-supporting. Rawhide is softer and may under some conditions be applied for extreme quietness. The phenolic laminated materials are moistureproof and are resistant to chemical attack.

Table 78. Composition of Cast Iron for Gears

	Si	S	P	Mn	Total carbon
Large gears	1.00–1.50	0.08–0.10	0.30–0.50	0.80–1.00	Low
Medium gears	1.50–2.00	Under 0.09	0.40–0.60	0.70–0.90	
Small gears	2.00–2.50	Under 0.06	0.50–0.70	0.60–0.80	

ported width of the working face of a pinion measured from the inner ends of the supporting bearings is kept less than 2½ times the diameter of the pinion. Gear-case bearing supports are made rigid without any adjustment of the bearings; this construction is absolutely essential for the satisfactory operation of high-speed helical gears. If properly cut and supported, the life of a high-speed helical gear is found to be 25 to 30 years provided the tooth pressures are kept within proper limits and the gear is properly lubricated. An involute tooth is used with an addendum 0.8×3.1416 times the normal pitch.

For large herringbone gear sets, the Westinghouse Co. makes use of a **floating frame** which carries the pinion and allows adjustment due to various load distortions.

Beam Strength of Gear Teeth

Spur Gears. The Lewis formula, published in 1893 and still widely used, considers a gear tooth as a cantilever beam with a concentrated load applied at the tip along the line of action (Fig. 118). It assumes that the total load is applied to a single tooth. The formula is

$$W = FS_0yc/P$$

Fig. 118.—Load on Gear Tooth (Lewis Formula).

where W is the total working load, lb; F the active face width, in.; P the diametral pitch, in.; y the form factor which combines effective beam length and section modulus (see Table 70); and c a velocity factor. For ordinary metal gearing, $c = 600/(600 + V)$; for carefully cut or ground gearing $c = 1200/(1200 + V)$; and for laminated phenolic or rawhide gearing $c = 150/(200 + V) + 0.25$; where V = pitch line velocity, fpm. The working stress S_0 lb per sq in. should be as follows: for non-metallic laminated phenolic materials or rawhide, 6,000; for cast iron, 8,000; for bronze, 12,000; for cast-steel, untreated, 18,000; for cast-steel, heat-treated, 28,000; for forged steel, untreated, 20,000; for forged steel, heat-treated, 25,000 to 35,000.

Allowable torque and horsepower, as functions of the above working load, are expressed approximately by

$$T = Wr_p \qquad \text{hp} = WV/33{,}000$$

where r_p = pitch radius, in.;

Bevel Gears. The Lewis formula becomes

$$W_1 = FS_0ycQ/P$$

where W_1 = equivalent working load, lb (tangential component of total load acting at tip of large end of tooth, assuming one tooth carries total load); F = active face width, in., measured along pitch element; P = diametral pitch at large end; S_0 = working stress, lb per sq in.; y = form factor (Table 70 for standard bevel gears, using *formative* number of teeth; Table 71 for the Gleason system using the *actual* number of teeth); c = speed factor, same as for spur gears; and Q = taper factor $= 1 - f/l_1 + f^2/(3l_1^2)$ where l_1 = slant height of pitch cone, in. (OB, Fig. 108). Allowable torque and horsepower are found by

$$T = W_1r_1 \qquad \text{hp} = W_1V_1/33{,}000$$

where r_1 = pitch radius at large end, in., and V_1 = pitch line velocity at large end, fpm.

largest ones. Variations from the standard sizes may usually be obtained but at extra cost.

Table 75. Dimensions of Standard Cast-iron Pulleys (Single Arm)

Diameter, in.	Maximum bore, in.	Face widths, in.	Diameter, in.	Maximum bore, in.	Face widths, in.	Diameter, in.	Maximum bore, in.	Face widths, in.
3 to 5	$1\frac{7}{16}$	2 to 10	11	$2\frac{15}{16}$	2 to 12	18 to 24	$3\frac{15}{16}$	2 to 20
6	$2\frac{7}{16}$	2 to 10	12 to 14	$2\frac{15}{16}$	2 to 14	25 to 26	$4\frac{7}{16}$	2 to 20
7	$2\frac{7}{16}$	2 to 10	15	$2\frac{15}{16}$	2 to 16	27 to 48	$4\frac{7}{16}$	2 to 24
8	$2\frac{7}{16}$	2 to 12	16	$2\frac{15}{16}$	2 to 18	49 to 60	$4\frac{15}{16}$	6 to 24
9	$2\frac{7}{16}$	2 to 12	17	$3\frac{15}{16}$	2 to 18	61 to 72	$5\frac{15}{16}$	6 to 24
10	$2\frac{15}{16}$	2 to 12						

It should be noted that because of indeterminate shrinkage strains the calculation of stresses in the arms of cast-iron pulleys is, at best, very rough. *In general, such pulleys are not recommended for speeds over 5,000 fpm.* Where severe operating conditions, shocks, etc., are present, the use of an all-steel pulley is advisable. All-steel pulleys either of the solid or split type, consisting of wrought-steel arms cast into a semisteel hub, the arms being welded to a heavy plate steel rim, are made by Medart Co. The outer ends of the arms are bent to form rim lugs. By this means, shrinkage stresses are reduced and a much stronger pulley is obtained. Such pulleys may be purchased in standard sizes with diameters ranging from 24 to 192 in. and face widths from 6 to 50 in.

Wood split pulleys may be obtained from the manufacturer in diameters from 4 to 120 in. and face widths from 3 to 24 in. in standard sizes, with larger ones available on special order. Such pulleys, although much cheaper than the metal pulleys, should not be used in damp places, for high speeds, or where shock loads occur.

Stresses in Pulleys, Sheaves, and Flywheels—Proportions

Arms of pulleys, sheaves, and flywheels are subjected to stresses due to condition of founding, to details of construction (such as split or solid), and to conditions of service, which do not readily admit of analysis. For this reason, no accurate stress relations can be established, and the following formulas must be understood to be only approximately correct. It has been established experimentally by Benjamin (*Am. Mach.*, Sept. 22, 1898) that thin-rim pulleys do not distribute equal loads to the several pulleys arms. For these, it will be safe to assume the tangential force on the pulley rim as acting on half the number of arms. Pulleys with comparatively thick rims, such as engine band wheels, have all the arms taking the load. Furthermore, while the stress action in the arms is similar to that in a beam fixed at both ends, the amount of restraint at the rim depending on the rim's elasticity, it may, nevertheless, be assumed for purposes of design that cantilever action is predominant. The bending moment at the hub in arms of thin-rim pulleys will be $M = PL/\frac{1}{2}N$, where M = bending moment, in.-lb; P = tangential load on the rim, lb; L = length of the arm, in.; and N = number of arms. For thick-rim pulleys and flywheels, $M = PL/N$.

For arms of elliptical section having a width of 2 times the thickness, where E = **width of arm section** at the rim, in., and s_t = intensity of tensile stress, lb per sq in.,

Peripheral velocity of worm, fps... =	3.3	8.2	13.1	18	23
c =	426–570	356–426	285–340	213–256	142–170

Helical Gears. Use Lewis formula for spur gears, p. 823. Any error introduced by this method will be on the safe side.

Surface Strength of Gear Teeth

The general solution, by Hertz, of the stress distribution due to two elastic bodies in contact has been applied by Jandasek to gear design. Since contact stresses are quite localized, the special case of two cylinders in contact is applicable to the contact region of gear teeth. The basic Hertz equation for maximum compressive stress is

$$S_c = 0.583\sqrt{W\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\Big/\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}$$

where S_c is the maximum compressive stress; W the load per inch of face; E_1 and E_2 the moduli of elasticity; and R_1 and R_2 the radii of curvature at the contact point (Fig. 119).

The width of the contact area is

$$b = 2.185\sqrt{W\left(\frac{1}{E_1}+\frac{1}{E_2}\right)\Big/\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

Buckingham has developed a formula for spur gears which is also based on the Hertz maximum compressive stress (see "Manual of Gear Design," Section 2, Industrial Press, New York).

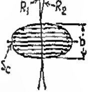


FIG. 119.—Pressure Distribution at Contact of Teeth.

In 1926, Timoshenko suggested that a more logical criterion would be the maximum shear stress.

$$S_s = 0.175\sqrt{W\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\Big/\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}$$

The maximum shear stress is approximately $3/10$ of the maximum compressive stress, so that qualitatively the methods are not essentially different. The maximum shear stress occurs at a depth equal to $0.393b$. For steel, the preceding equation becomes

$$S_s = 676\sqrt{W\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

Table 72. Service Factors for Shear Stress in Gear Teeth
(*Elec. Jour.*, Sept., 1937)

Character of load on driven machine	Turbine or electric motor drive			Multicylinder gas or steam engine			Single-cylinder gas engine		
	Intermittent, 3 hr per day	8–10 hr per day	24 hr per day	Intermittent, 3 hr per day	8–10 hr per day	24 hr per day	Intermittent, 3 hr per day	8–10 hr per day	24 hr per day
Uniform	0.8	1.0	1.25	1.0	1.25	1.5	1.25	1.5	1.75
Moderate shock	1.0	1.25	1.5	1.25	1.5	1.75	1.5	1.75	2.0
Heavy shock	1.5	1.75	2.0	1.75	2.0	2.25	2.0	2.25	2.5

joining arm. Then $s_t = 0.00034n^2R(w_1L_1 + wL/2)/Z$, where n = rpm of wheel; R = wheel radius, ft; and Z = modulus of rim section. The above equation gives the value of s_t for bending when the bolts are loose, which is the worst possible condition that may arise. On this basis of analysis, s_t should not be greater than 8,000. The stress due to bending in addition to the stress due to rim expansion as analyzed previously will be the probable maximum intensity of stress for which the rim should be checked for strength. The flange bolts, because of their position, do not materially relieve the bending action. In case a tie rod leads from the flange to the hub, it will be safe to consider it as an additional factor of safety. When the tie rod is kept tight, it very materially strengthens the rim.

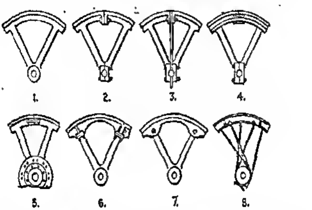


FIG. 128.—Types of Flywheel Construction (see Table 76).

A more accurate method for calculating maximum stresses due to centrifugal force in flywheels with arms cast integral with the rim is given by Timoshenko "Strength of Materials," Part II, 1930, p. 451. More exact equations for calculating stresses in the arms of flywheels and pulleys due to a combination of belt pull, centrifugal force, and changes in velocity are given by Heusinger, *Forschung Ing. Wes.*, 1938, p. 197. In both treatments, shrinkage stresses in the arms due to casting are neglected.

The relative strengths of different types of wheel construction are shown by the results of Benjamin's experiments on the bursting of flywheels (*Trans. A.S.M.E.*, 1899, 1902). The types of wheels experimented with and the speeds at which they burst are shown in Fig. 128 and Table 76.

Table 76. Test Data on the Flywheels of Fig. 128

Number.*	1	2	3	4	5	6	7	8
Number of arms	6	6	6	6	8	6	6	24
Rim speed at failure, fps = v	395	194	225	305	256	223	393	424
Comparative rim speeds at failure	100	49	57	77	65	56.5	100	107
Apparent rim tension at failure lb per sq in. = $v^2/10$	15,625	3,764	5,062	9,302	6,502	4,973	15,445	17,978
Efficiency† of construction	0.85	0.19	0.265	0.49	0.34	0.26	0.84	0.94

* Construction of flywheels: No. 1, solid wheel; No. 2, in halves, with flange joints; No. 3, in halves, with reinforced joints; No. 4, in halves, with link joints; No. 5, segmental, with link joints; No. 6, in halves, with pad joints; No. 7, solid rim, separate spider; No. 8, solid rim, with tangent spokes.

† Efficiency assuming tensile strength at 19,000 lb per sq in.

to the root circle shall be at least equal to the depth of tooth. The keyway compressive stress should not exceed 3,000 lb per sq in., which forms the basis of the formula:

$$LA = 84H/ND$$

where L is the length of keyway, in.; A the depth of key, in. (keyway in gear is approx ½ this value); H the horsepower; N the rpm; and D the diameter of shaft, in.

Taper keys which can be driven in place should ordinarily not be used for phenolic laminated pinions.

Efficiency of Gearing in Power Transmission

(See p. 240)

Spur and Bevel Gears. The efficiency of power transmission by the use of spur and bevel gears is very high. The loss of power for each set of spur gears in a train, where an oil bath is provided and the gears are generated by modern machinery, does not ordinarily exceed one-half of one percent. Where generated bevel gears are mounted under the same conditions and provided with antifriction thrust bearings, the efficiency per pair of gears should easily be 99 percent. The use of bevel gearing with plain thrust collars for power transmission is practically obsolete.

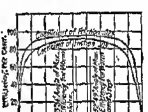


Fig. 121.—Efficiency of Worm Gearing.

The efficiency of **worm gearing,** when the friction of the thrust bearing is neglected, is $E = \tan \alpha\ (1 - f \tan \alpha)/(\tan \alpha + f)$ approx, where E = efficiency, α = angle of thread, and f = coefficient of friction. With values of $f = 0.025$ and 0.05, the variation of efficiency with thread angle is as shown in Fig. 121.

Self-locking worms are obtained when $f = \tan \alpha$. The efficiency resulting with such angles can never be as much as 50 percent. In practice, $\tan \alpha$ can be made greater than f and the gear will be self-locking because of the friction in other parts of the drive.

To find the pitch of worm-wheel teeth and the largest pitch diameter of worm **to transmit a given horsepower** at a given speed at maximum efficiency, the diagram shown in Fig. 122 may be employed (*Machinery*, May, 1912). To transmit 7 hp with a single thread worm at 360 rpm there is required, as shown by the diagram, a worm pitch diameter of 3 in. and a circular pitch of worm-wheel of 1¼ in. For cast-iron worm wheels and worms with unfinished teeth, the pitch should be 1⅓ times that obtained from the diagram.

The above remarks on efficiency of worm gearing apply to running conditions only. At the moment a worm drive is started from rest, the losses are from 2 to 3 times the above indicated running losses. Thus a single-thread worm drive showing by test a running efficiency of 70 percent showed a loss of over 50 percent at the moment of starting.

Materials for Gears

(See also p. 620 for phosphor bronze)

For cast-iron gears, the compositions given in Table 73 are used. For phosphor-bronze worm wheel with steel worm, a satisfactory composition is: Cu, 89.5 percent, Sn, 10 percent, P, 0.5 percent.

Table 78. Dimensions of Sheaves for Manila Rope Drives
(All dimensions in inches.)

Rope diameter	English system grooves A	B	C	E	American system grooves A	B	C V groove	C U groove	E
3/4	7/8	1 3/16	5/8	9/16	3/4	1 1/8	3/8	1/2	9/16
7/8	1	1 1/2	7/8	3/4	15/16	1 3/8	1/2	5/8	11/16
1	1 1/8	1 5/8	7/8	7/8	1	1 3/8	1/2	5/8	3/4
1 1/8	1 1/4	2	1 1/16	15/16	1 3/16	1 11/16	3/4	13/16	13/16
1 1/4	1 3/8	2 1/8	1 3/16	1 1/16	1 3/16	1 13/16	5/8	3/4	15/16
1 3/8	1 1/2	2 1/8	1 5/16	1 1/8	1 1/4	1 7/8	11/16	7/8	1 1/16
1 1/2	1 5/8	2 1/4	1 3/8	1 3/8	1 3/8	2 1/16	3/4	7/8	1 1/8
1 3/4	1 3/4	2 1/2	1 7/16	1 7/16	1 5/8	2 7/16	7/8	1 1/4	1 5/16
2	2	2 3/4	1 9/16	1 9/16	1 7/8	2 3/4	1	1 1/8	1 1/2

Width of face of sheave = (number of grooves − 1) × B + $2A$

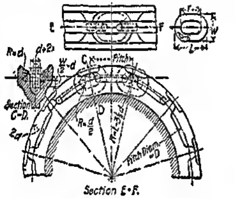


FIG. 132.—Sheave for Link Chain.

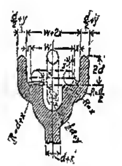


FIG. 133.

where a = 90 deg divided by the number of teeth in the wheel. Values of sin a and cos a are given in Table 79.

Table 79. Sprocket Wheels for Ordinary Link Chains
Values of a, sin a and cos a for use in the formula:
Pitch diam $D = \sqrt{(r/\sin a)^2 + (d/\cos a)^2}$. (See Figs. 132–133.)

No. of teeth	a	Sin a	Cos a	No. of teeth	a	Sin a	Cos a	No. of teeth	a	Sin a	Cos a
5	18° 0′	.30902	.95106	14	6° 25.7′	.11179	.99372	23	3° 54.78′	.06825	.99768
6	15° 0′	.25882	.96593	15	6° 0′	.10453	.99452	24	3° 45′	.06540	.99786
7	12° 51.4′	.22252	.97493	16	5° 37.5′	.09801	.99519	25	3° 36′	.06279	.99803
8	11° 15′	.19509	.98079	17	5° 17.64′	.09226	.99573	26	3° 27.69′	.06038	.99819
9	10° 0′	.17365	.98481	18	5° 0′	.08716	.99619	27	3° 20′	.05814	.99831
10	9° 0′	.15643	.98769	19	4° 44.22′	.08258	.99657	28	3° 12.85′	.05607	.99844
11	8° 10 9′	.14231	.98986	20	4° 30′	.07846	.99692	29	3° 6.18′	.05413	.98854
12	7° 30′	.13053	.99144	21	4° 17.14′	.07473	.99721	30	3° 0′	.05234	.99893
13	6° 55.4′	.12054	.99271	22	4° 5.45′	.07136	.99745				

The value of r, the internal length of the link, is given by $r = L - 2d$, where L is the outside length of the link and d is the diameter of chain stock. These last quantities and the width of the link W are given in Table 80. In

Friction Gearing

Friction gearing of either the spur or bevel gear type may be used for the transmission of power. In either case, it is good practice for the driver to be of paper, leather, or pulp composition, and the driven wheel of metal, which may be iron, type metal, aluminum, etc. Goss (*Trans.* A.S.M.E., 29, 1907), reports as follows on the characteristics of friction drives:

The **safe working pressures** in pounds per square inch are as follows: straw fiber, 150, leather fiber, 240; tarred fiber, 240; sulphite fiber, 140; leather, 150. They are taken as 20 percent of the ultimate resistance of the material to crushing.

The safe working values of the **coefficient of friction** may be 60 percent of the values found from laboratory experiments. The frictional and power coefficients in Table 74 are calculated upon this basis.

Table 74. Frictional and Power Coefficients for Friction Gearing

Hp transmitted = $kwd\,N$

where w = width of face of driver, in.; d = diam of driver, in.; N = rpm

Driver	Driven wheel	Coefficient of friction	k	Driver	Driven wheel	Coefficient of friction	k
Straw fiber.....	Iron........	0.25	0.00030	Tarred fiber....	Type metal.	0.16	0.00031
Straw fiber.....	Aluminum..	0.27	0.00033	Sulphite fiber...	Iron........	0.33	0.00037
Straw fiber.....	Type metal.	0.18	0.00022	Sulphite fiber...	Aluminum..	0.32	0.00035
Leather fiber...	Iron........	0.31	0.00059	Sulphite fiber...	Type metal..	0.31	0.00034
Leather fiber...	Aluminum..	0.30	0.00057	Leather........	Iron........	0.13	0.00016
Leather fiber...	Type metal.	0.18	0.00035	Leather........	Aluminum..	0.21	0.00026
Tarred fiber....	Iron........	0.15	0.00029	Leather........	Type metal.	0.25	0.00029
Tarred fiber....	Aluminum..	0.18	0.00035				

In the case of **bevel friction gears,** the horsepower may be had from the above values by taking for N a value which will correspond to the average velocity and radius of the pitch surface. In the case of **wedge-surface friction gears,** as shown in Figs. 123 and 124, the tangential force P for a given normal force Q pressing the wheels together is $P \leqq Qf/(\sin\alpha + f\cos\alpha)$, where f is the coefficient of friction. Generally, α = 10 to 15 deg. The radial depth of contact of the surfaces e (Fig. 123) should not exceed ½ in. The efficiency of friction drives, according to Ernst, may range from 88 to 90 percent. In **disk drives,** the distance between the face of the driver and the center of the driven wheel should not be less than 12 times the face width of the driver, otherwise the power capacity will be decreased due to slip.

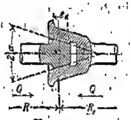


Fig. 123.

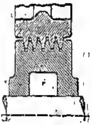


Fig. 124.

Wedge-surface Friction Gears.

PULLEYS, FLYWHEELS, SHEAVES, DRUMS

Cast-Iron pulleys are made in standard sizes, either solid, or with split hubs and rims, the single arm types being made in the standard sizes shown in Table 75 (data from Medart Co.). For multiple-arm pulleys, the standard diameters vary from 12 to 60 in., the face widths from 13 to 66 in., and the maximum bore sizes from $3\frac{7}{16}$ in. for the smallest pulleys to $6\frac{15}{16}$ in. for the

with fewer than 25 teeth, running at speeds above 500 or 600 rpm, should be heat-treated to give a tough wear-resistant surface testing between 35 and 45 on the Rockwell C hardness scale.

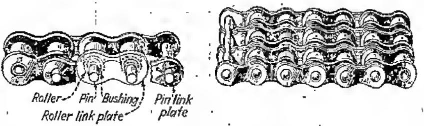
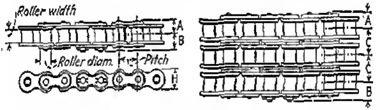


FIG. 137.—Roller Chain Construction.

If the speed ratio requires the larger sprocket to have as many as 128 teeth, or more than 8 times the number on the smaller sprocket, it is usually better, with few exceptions, to make the desired reduction in two or more steps. The American Standard tooth form allows roller chain to adjust itself to a larger pitch circle as the pitch of the chain elongates owing to natural wear in the pin-bushing joints. The greater the number of teeth, the sooner the chain will ride out too near the ends of the teeth.

Table 82. Roller-chain Data and Dimensions



Chain number			Roller			Roller link plate		Dimensions, in.				Recommended max rpm		
												Teeth		
A.S.A.	Diamond	Pitch, in.	Width, in.	Diam, in.	Pin diam, in.	Thickness, in.	Height H, in.	A	B	C	Tensile strength per strand, lb	12	18	24
35	82	3/8	3/16	0.200	0.141	0.050	0.344	0.224	0.290	0.400	2,100	2,380	3,780	4,200
41	65[a]	1/2	1/4	0.306	0.141	0.050	0.383	0.256	0.315	....	2,000	1,750	2,725	2,850
40	66	1/2	5/16	0.312	0.156	0.060	0.452	0.313	0.358	0.563	3,700	1,800	2,830	3,000
50	449[a]	5/8	3/8	0.400	0.200	0.080	0.594	0.384	0.462	....	6,100	1,300	2,030	2,200
50	148	5/8	3/8	0.400	0.200	0.080	0.545	0.384	0.462	0.707	6,600	1,300	2,030	2,200
60	433	3/4	1/2	0.469	0.234	0.094	0.679	0.493	0.567	0.892	8,500	1,025	1,615	1,700
80	434	1	5/8	0.625	0.312	0.125	0.903	0.643	0.762	1.160	14,500	650	1,015	1,100
100	470	1 1/4	3/4	0.750	0.375	0.156	1.126	0.780	0.910	1.411	24,000	450	730	850
120	472	1 1/2	1	0.875	0.437	0.187	1.354	0.977	1.123	1.796	34,000	350	565	650
140	474	1 3/4	1	1.000	0.500	0.220	1.647	1.054	1.219	1.929	46,000	260	415	500
160	478	2	1 1/4	1.125	0.562	0.250	1.900	1.250	1.433	2.301	58,000	225	360	420
200	480	2 1/2	1 1/2	1.562	0.781	0.312	2.275	1.533	1.850	2.800	95,000	170	260	300

[a] Not made in multiple strands.

$$E = \sqrt[3]{40PL/s_tN} \text{ (thin rim)} = \sqrt[3]{20PL/s_tN} \text{ (thick rim)}$$

For single-thickness belts, P may be taken as 50 B lb and for double-thickness belts $P = 75\ B$ lb, where B is the width of pulley face, in. Then $E = k \times \sqrt[3]{BL/s_tN}$, where k has the following values: for thin rim, single belt, 13; thin rim, double belt, 15; thick rim, single belt, 10; thick rim, double belt, 12. For cast iron of good quality, s_t due to bending may be taken at 1,500 to 2,000. The arm section at the rim may be made from ⅔ to ¾ the dimensions at the hub.

For high-speed pulleys and flywheels, it becomes necessary to check the arm for tension due to rim expansion. It will be safe to assume that each arm is in tension due to one-half the centrifugal force of that portion of the rim which it supports. That is, $T = As_t = Wv^2/2NgR$ (lb) where T = tension in arm, lb; N = number of arms; v = speed of rim, fps; R = radius of pulley, ft; A = area of arm section, sq in.; W = weight of pulley rim in lb and s_t = intensity of tensile stress in arm section, lb per sq in. Whence $s_t = WRn^2/5800NA$, where n = rpm of pulley.

Arms of flywheels having heavy rims may be subjected to severe stress action due to the inertia of the rim at sudden load changes. There being no means of predicting the probable maximum to which the inertia may rise, it will be safe to make the arms equal in strength to ¾ of the shaft strength in torsion. Accordingly, for elliptical arm sections,

$$N \times 0.5E^3s_t = ¾ \times 2s_sd^3, \qquad \text{or} \qquad E = 1.4d\sqrt[3]{s_s/s_tN}$$

For steel shafts with s_s = 8,000 and cast-iron arms with s_t = 1,500,

$$E = 2.4d/\sqrt[3]{N} = 1.3d \text{ (for 6 arms)} = 1.2d \text{ (for 8 arms)}$$

where $2E$ = width of elliptical arm section at hub, in. (thickness $= E$) and d = shaft diameter, in.

Rims of belted pulleys cast whole may have the following proportions (see Fig. 125):

$$t_2 = ¾h + 0.005D; \qquad t_1 = 2t_2 + C; \qquad w = ⅛B \text{ to } ¼B;$$

where h = belt thickness, $C = \frac{1}{24}\,w$, and B = belt width, all in inches.

Engine hand wheels, flywheels, and pulleys run at high speeds are subjected to the following stress actions in the rim:

FIG. 125. FIG. 126.
Rims for Belted Pulleys.

Considering the rim as a free ring (*i.e.*, without arm restraint) and made of cast iron or steel, $s_t = v^2/10$ (approx), where s_t = intensity of tensile stress, lb per sq in. and v = rim speed, fps. For beam action between the arms of a solid rim, $M = Pl/12$ (approx), where M = bending moment in rim, in.-lb; P = centrifugal force of that portion of rim between arms, lb, and l = length of rim between arms, in; from which $s_t = WR^2n^2/450N^2Z$, where W = weight of entire rim, lb; R = radius of wheel, ft; n = rpm of wheel; and Z = modulus of rim section, in.³. In case the rim section is of the forms shown in Fig. 126, care must be taken that the flanges do not reduce the section modulus from that of the rectangular section. **For split rims** fastened with bolts as shown in Fig. 127, the stress analysis is as follows:

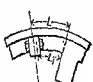


FIG. 127.

Let w = weight of rim portion L in. in length, lb; w_l = weight of lug, lb; L_l = lever arm of lug, in.; and s_t = intensity of tensile stress in rim section

Table 83. Horsepower Ratings for Single-strand, Diamond Roller-chain Drives.—(*Continued*)

Teeth	A.S.A. No. 100					1¼ pitch						Diamond No. 470
	Rpm of sprocket											
	25	50	100	200	300	400	500	650	700	750	800	870
12	1.72	3.19	5.8	9.9	13.0	15.6	17.2					
15	2.19	4.10	7.5	13.1	17.5	21.3	24.0	27.2	28.1			
18	2.55	4.97	9.1	16.0	21.6	26.6	30.2	34.5	35.7	36.8		
21	3.08	5.80	10.7	18.9	25.5	31.4	35.7	40.9	42.3	43.5	44.6	
24	3.52	6.62	12.2	21.5	29.2	35.4	40.5	46.5	48.1	49.5	50.6	52.0

Teeth	A.S.A. No. 120					1½ in. pitch						Diamond No. 472
	Rpm of sprocket											
	25	50	75	100	150	200	250	300	350	400	500	600
12	2.90	5.4	7.6	9.6	13.2	16.2	18.7	21.0	22.8	24.3		
15	3.71	6.9	9.8	12.5	17.3	21.6	25.3	28.6	31.4	33.9	38.0	
18	4.74	8.4	12.0	15.3	21.3	26.6	31.3	35.4	39.2	42.4	47.9	
21	5.24	9.9	14.0	17.9	24.9	31.2	36.8	41.7	46.2	50.0	56.7	61.7
24	5.99	11.3	16.0	20.4	28.5	35.7	41.9	47.6	52.6	57.1	64.6	70.3

Teeth	A.S.A. No. 140					1¾ in. pitch						Diamond No. 474
	Rpm of sprocket											
	20	30	50	100	150	200	250	300	350	400	450	475
12	3.72	5.4	8.4	14.8	20.1	24.5	28.1	31.0				
15	4.73	6.9	10.8	19.3	26.6	32.8	38.2	42.8	46.7			
18	5.73	8.3	13.1	23.7	32.7	40.5	47.3	53.2	58.4	62.9		
21	6.70	9.7	15.3	27.7	38.4	47.6	55.7	62.8	69.0	74.5	79.0	
24	7.65	11.1	17.5	31.7	43.7	54.3	63.6	71.6	78.7	84.8	89.9	92.4

Teeth	A.S.A. No. 160					2 in. pitch						Diamond No. 478
	Rpm of sprocket											
	10	20	40	80	120	160	200	240	280	320	360	400
12	2.9	5.5	10.1	18.0	24.6	30.1	34.8	38.6				
15	3.7	7.0	13.0	23.5	32.4	40.2	47.0	52.9	58.0	62.4		
18	4.4	8.5	15.8	28.7	39.7	49.5	58.1	65.7	72.4	78.3		
21	5.2	9.9	18.5	33.6	46.7	58.1	68.3	77.5	85.5	92.5	99	
24	5.9	11.3	21.1	38.4	53.5	66.5	78.0	88.3	97.4	105.4	112	118

Teeth	A.S.A. No. 200					2½ in. pitch						Diamond No. 480
	Rpm of sprocket											
	10	20	40	60	80	100	120	160	200	240	260	280
12	5.6	10.5	19.1	26.8	33.6	39.6	45.1	54.4				
15	7.1	13.4	24.7	34.8	43.9	52.2	59.8	73.4	85			
18	8.6	16.2	30.0	42.4	53.7	64.1	73.7	90.7	105	118		
21	10.0	18.9	35.1	49.7	63.1	75.3	86.6	106.9	124	139	146	
24	11.4	21.6	40.2	56.8	71.9	86.0	98.8	121.8	142	159	166	173

Flywheels of the shrunk-link-joint type may be constructed as shown in Fig. 129 and to the dimensions given in Table 77. With cast-iron flywheels the bearing pressure on the link may be 20,000 lb per sq in. and the maximum shear on the head of the link 5,600 lb per sq in. The length of the link may be made $0.999D$, which gives an initial tension in the link of 30,000 lb per sq in.

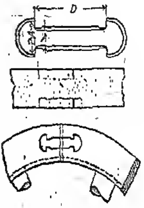


Fig. 129.—Shrunk-link Joint for Flywheels.

Large flywheels for high rim speeds and severe working conditions (as for rolling mill service) have recently been made from flat-rolled steel plates with holes bored for the shaft. A group of such plates may be welded together by circumferential welds to form a large flywheel. By this means, the welds do not carry direct centrifugal load, but serve merely to hold the parts in position. Flywheels up to 15 ft diam for rolling mill service have been constructed in this way.

Sheaves for Manila Rope Drives. The dimensions in inches of standard rope sheaves for manila rope drives (both for English and American systems) as used by Medart Co. are given in Table 78 where the letters refer to Figs. 130 and 131. For working sheaves or where arc of contact is under 90 deg, an angle of 45 deg is recommended; for tension and idler sheaves, 60 deg V sheaves are recommended.

Table 77. Data on Flywheels with Shrunk Link Joints

Flywheel effect in lb at 1 ft radius	Max safe speed, rpm	Mean rim speed, ft per sec at max safe rpm	Dimensions					No. of arms	Weight in lb	
			Rim			Hub				
Weight of rim × (rad gyr²) or moment of inertia of rim			Diam, ft	Face, in.	Thickness, in.	Std bore, in.	Length, in.		Rim	Total
4,000,000	121	104	18	19	20	24	30	8	59,730	86,954
6,060,000	59	67	23	13½	18	20	28	10	52,466	94,725
7,200,000	74	84	23	16	18	20	28	10	62,466	104,008
9,850,000	85	71	19	27½	36	24½	30	8	154,000	172,500
11,900,000	140	193	26	18½	14	30	30	10	65,100	118,100
12,300,000	110	118	23	16½	27½	29½	34	10	116,000	144,760
15,000,000	83	84	22	24	30	33	38	10	143,300	190,168
17,600,000	54	64	25	22	28	32	38	10	137,000	180,790
20,900,000	67	79	25	25	30	37	40	10	165,000	242,387
33,400,000	105	140	28	26	29	39	40	10	205,000	299,450
24,200,000	100	133	28	19½	29	37	40	10	148,500	264,610

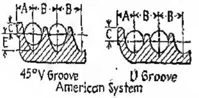


Fig. 130.

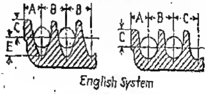


Fig. 131.

Figs. 130 and 131.—Dimensions of Grooves for Rope Drives.

Sheaves for ordinary link chains may be proportioned as shown in Figs. 132 and 133. The pitch diameter D is given by the equation

$$D^2 = (r/\sin a)^2 + (d/\cos a)^2$$

There should be at least 120 deg of wrap in the arc of contact on a power sprocket. For ratios of 3:1 or less, the wrap will be 120 deg or more for any center distance or number of teeth. To secure a wrap of 120 deg or more, for ratios greater than 3:1, the center distance must be not less than the difference between the pitch diameters of the two sprockets.

Sprocket Diameters. N = number of teeth; P = pitch of chain, in.; D = diameter of roller, in. The pitch of a standard roller chain is measured from the center of a pin to the center of an adjacent pin.

$$\text{Pitch diameter} = P/\sin\frac{180}{N}$$

$$\text{Bottom diameter} = \text{pitch diam} - D.$$

$$\text{Outside diameter} = P\left(0.6 + \cot\frac{180}{N}\right).$$

$$\text{Caliper diameter} = \left(\text{Pitch diam} \times \cos\frac{90}{N}\right) - D.$$

The exact bottom diameter cannot be measured for an odd number of teeth, but it can be checked by measuring the distance (caliper diameter) between bottoms of two tooth spaces nearest opposite to each other. Bottom and caliper diameters must not be oversize—all tolerance must be negative. A.S.A. negative tolerance $= 0.003 + (0.001 \times P\sqrt{N})$ in.

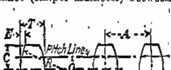


FIG. 139.—Sprocket Tooth Sections.

Design of Sprocket Teeth for Roller Chains. The section profile for the teeth of roller chain sprockets, recommended by the A.S.A., has the proportions shown in Fig. 139. Let P = chain pitch; W = chain width (length of roller); n = number of strands of multiple chain; M = over-all width of tooth profile section; H = nominal thickness of link plate, all in inches. Referring to Fig. 152, $T = 0.93W - 0.006$, for single strand chain; $= 0.90W - 0.006$, for double- and triple-strand chains; $= 0.88W - 0.006$, for quadruple or quintuple strand chains; and $= 0.86W - 0.006$, for sextuple-strand chain and over. $C = 0.5P$. $E = \frac{1}{8}P$. $R(\text{min}) = 1.063P$. $Q = 0.5P$. $A = W + 4.15H + 0.003$. $M = A(n - 1) + T$. Tolerance on sprocket thickness $= \pm(0.02W + 0.002)$.

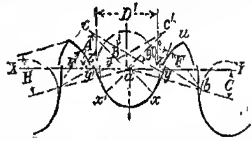


FIG. 140.—Laying Out a Standard Sprocket Tooth.

One of the most important requirements of a sprocket for roller chain is that the tooth space, or roller seat, should not be undersize. The size and shape of new straddle cutters, space cutters, or hobs should be checked carefully by cutting and testing a sample sprocket.

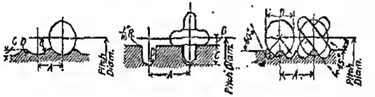
The method of laying out a standard sprocket tooth for roller chains is illustrated by Fig. 140. This form of tooth is recommended by the A.S.A. and is designed for maximum efficiency throughout the life of the drive. Because of the large pressure angle, the tendency of the teeth to wear hook-

Table 80. Dimensions of Sprocket Wheel Chains

(Letters refer to Fig. 132. d = diam of stock in link, in.; L and W = outside length and width of link, in.)

d	L	W	d	L	W	d	L	W	d	L	W
3/16	1 3/8	13/16	1/2	2 1/2	1 3/4	13/16	4	2 13/16	1 1/4	6	4 1/4
1/4	1 1/2	1	9/16	2 3/4	1 15/16	7/8	4 1/4	3	1 3/8	6 1/2	4 3/4
5/16	1 3/4	1 1/16	5/8	3 1/4	2 1/8	15/16	4 1/2	3 1/4	1 1/2	7 1/4	5 1/4
3/8	2	1 1/8	11/16	3 1/2	2 5/16	1	4 3/4	3 1/2	1 5/8	7 7/8	5 3/4
7/16	2 1/4	1 3/16	3/4	3 3/4	2 1/2	1 1/8	5 1/2	3 7/8			

Figs. 132 and 133, $x = 3/32$ in. for $d = 1/4$ to $1/2$ in.; $= 1/8$ in. for $d = 1/2$ to $1 1/8$ in.; $= 5/32$ in. for $d = 1 1/4$ to $1 5/8$ in. y is $3/32$ in. for $d = 3/16$ to $7/16$ in., and $3/16$ in. for $d = 1/2$ to $15/16$ in.



Figs. 134–136.—Drum Scores for Cables and Chains.

Drum scores for cables and chains may be made as illustrated in Figs. 134 to 136, with the dimensions given in Table 81.

Table 81. Dimensions in Inches of Standard Drum Scores

See Fig. 134	Rope	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
	A	5/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/16
	B	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32
	C	3/32	7/64	1/8	9/64	5/32	11/64	3/16	13/64	7/32	15/64	1/4
	D	3/32	3/32	3/32	1/8	1/8	1/8	5/32	5/32	5/32	3/16	3/16
See Fig. 135	Chain	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
	A	1 1/2	1 13/16	1 7/8	2 1/16	2 5/16	2 3/8	2 11/16	2 7/8	3 3/8	3 9/16	3 1/2
	B	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2
	C	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/16	1 1/8	1 3/16
	D	3/16	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32	9/16
See Fig. 136	Chain	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
	A	1 1/4	1 7/16	1 9/16	1 3/4	1 7/8	2 1/16	2 3/16	2 3/8	2 1/2	2 11/16	2 13/16
	B	11/32	3/8	7/16	15/32	17/32	9/16	5/8	21/32	23/32	3/4	27/32
	C	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2
	D	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4

CHAIN DRIVES

Roller Chain Drives. The advantages of finished steel roller chains are high efficiency (around 98 to 90 percent), no slippage, no initial tension required, chains may travel in either direction. The basic construction of roller chain is shown in Fig. 137.

The shorter the pitch, the higher the permissible operating speed of roller chain. Horsepower capacity in excess of that provided by a single chain may be had by the use of multiple chains which are essentially parallel single chains assembled on pins common to all strands. Because of its lightness in relation to tensile strength, the effect of centrifugal pull does not need to be considered; even at the unusual speed of 6,000 fpm, this pull is only 3 percent of the ultimate tensile strength.

Sprocket wheels with fewer than 16 teeth may be used for relatively slow speeds, but 18 to 24 teeth are desirable for high speed service. Sprockets

Having determined the proper pitch, the sprockets are next selected. On small motor drives, 17 to 21 teeth on the driver sprocket is good practice. For large speed ratios and a minimum number of teeth in the pinion, it is desirable to avoid extremely large driven sprockets; 127 teeth is a good limit, and less than 100 is preferred. Where the speed ratio is small, it is desirable to get at least 33 teeth in the driven wheel. It is usually well to select sprockets that will give a chain speed of 1,200 to 1,600 fpm. If in the design this figure is exceeded, this speed may be lowered by the use of smaller sprockets and smaller pitch. An odd number of teeth in both sprockets is desirable unless an exact ratio requires an even number. Although higher speeds than indicated in the table may often be used, it is best that such designs be submitted to the manufacturer for approval.

Table 85. Nominal Weights and Dimensions of Crane and Proof Coil Chain

Nominal size of chain bar, in.	Actual size of material, in.	Nom length of 100 links, in.		Nom weight per 100 ft, lb		Nominal dimensions of links, in.							
						Crane chain				Proof coil			
						Outside		Inside		Outside		Inside	
		Crane chain	Proof coil	Crane chain	Proof coil	Length	Width	Length	Width	Length	Width	Length	Width
1/4	9/32	86	100	78	70	1 27/64	1	55/64	7/16	1 3/16	1 1/16	1	1/2
5/16	11/32	100	111	115	105	1 11/16	1 3/16	1	1/2	1 51/64	1 3/16	1 7/64	1/2
3/8	13/32	109	123	166	158	1 29/32	1 7/16	1 3/32	5/8	2 3/64	1 7/16	1 15/64	5/8
7/16	15/32	122	138	220	210	2 5/32	1 5/8	1 7/32	1 1/16	2 5/16	1 11/16	1 3/8	3/4
1/2	17/32	134	150	275	265	2 13/32	1 13/16	1 11/32	3/4	2 9/16	1 7/8	1 1/2	13/16
9/16	19/32	156	175	350	335	2 3/4	1 15/16	1 9/16	3/4	2 15/16	2 1/16	1 3/4	7/8
5/8	21/32	169	188	430	410	3	2 3/16	1 11/16	7/8	3 3/16	2 5/16	1 7/8	1
3/4	25/32	188	213	615	580	3 7/16	2 9/16	1 7/8	1	3 11/16	2 11/16	2 1/8	1 1/8
7/8	29/32	225	250	820	780	4 1/16	3 1/16	2 1/4	1 1/4	4 5/16	3 1/16	2 1/2	1 3/8
1	1 1/32	256	275	1,045	1,000	4 5/8	3 7/16	2 9/16	1 3/8	4 15/16	3 9/16	2 3/4	1 1/2
1 1/8	1 5/32	288	...	1,310	.....	5 3/16	3 13/16	2 7/8	1 5/8				
1 1/4	1 9/32	306	...	1,600	.....	5 5/8	4 5/16	3 1/16	1 3/4				
1 3/8	1 13/32	363	...	1,930	.....	6 7/16	4 11/16	3 3/8	1 7/8				
1 1/2	1 17/32	387	...	2,335	.....	6 15/16	5 1/16	3 7/8	2				
1 5/8	1 21/32	425	...	2,740	.....	7 9/16	5 7/16	4 1/4	2 1/8				
1 3/4	1 25/32	475	...	3,180	.....	8 3/16	5 13/16	4 3/4	2 1/4				
1 7/8	1 29/32	525	...	3,6[illegible]	.....	9 1/16	6 5/16	5 1/4	2 1/2				
2	2 1/32	575	...	4,100	.....	9 13/16	6 11/16	5 3/4	2 5/8				

To find the required chain width, the total chain pull F is divided by the normal tension per inch as given in Table 84. Where the load is not uniform, the horsepower assumed in figuring the drive should be increased by multiplying the rated horsepower of the motor or prime mover used, by a factor as follows:

For electric motor (oil or gas engine) driver the factor with uniform load as in a line shaft, centrifugal pump exciter or most machine tools is 1 (1.25); with uneven or pulsating load as in a duplex pump, compressor with balance wheel, beaters, stokers, etc., 1.25 (1.5); with severe shock or reversing load as in a ball or tube mill, heavy rolling mills, drill rigs, crushers, etc., 1.5 (2). Where the drive is used 24 hr a day, an additional 25 percent should be added to the horsepower rating used in the computations. These ratings are based on chains in protecting cases with adequate lubrication.

Table 83. Horsepower Ratings for Single-strand Diamond Roller-chain Drives

Teeth	A.S.A. No. 35					⅜ in. pitch						Diamond No. 82
	Rpm of sprocket											
	200	400	8000	1,200	1,600	2,000	2,400	2,800	3,200	3,600	4,000	4,500
12	0.34	0.60	1.01	1.31	1.53	1.66	1.72	1.73				
15	0.43	0.78	1.35	1.78	2.12	2.37	2.54	2.65	2.70	2.69		
18	0.52	0.96	1.65	2.21	2.65	2.98	3.24	3.43	3.52	3.57	3.55	
21	0.61	1.12	1.95	2.61	3.14	3.53	3.86	4.08	4.22	4.28	4.28	
24	0.70	1.28	2.22	2.98	3.57	4.04	4.38	4.65	4.81	4.86	4.87	4.75

Teeth	A.S.A. No. 40					½ in. pitch						Diamond No. 66
	Rpm of sprocket											
	200	400	600	800	1,000	1,200	1,600	1,800	2,000	2,400	2,800	3,200
12	0.77	1.34	1.81	2.16	2.46	2.71	2.99	3.07	3.10			
15	0.99	1.76	2.40	2.93	3.38	3.77	4.32	4.52	4.67	4.81		
18	1.20	2.15	2.94	3.63	4.21	4.71	5.48	5.76	5.97	6.27	6.35	
21	1.41	2.52	3.47	4.27	4.97	5.57	6.30	6.86	7.13	7.50	7.63	
24	1.60	2.88	3.96	4.87	5.67	6.35	7.40	7.80	8.12	8.51	8.68	8.57

Teeth	A.S.A. No. 50 ⅝ in. pitch: Diamond No. 449 (single)—No. 148 (multiple)											
	Rpm of sprocket											
	100	200	300	400	600	800	1,000	1,200	1,400	1,600	1,800	2,200
12	0.80	1.44	1.99	2.48	3.26	3.86	4.3	4.6	4.8			
15	1.02	1.87	2.61	3.27	4.39	5.31	6.0	6.8	7.0	7.3	7.5	
18	1.23	2.27	3.19	4.01	5.41	6.58	7.5	8.3	8.9	9.4	9.7	
21	1.45	2.66	3.75	4.70	6.38	7.77	8.9	9.8	10.6	11.1	11.6	11.9
24	1.65	3.05	4.27	5.37	7.28	8.85	10.2	11.2	12.1	12.6	12.1	13.6

Teeth	A.S.A. No. 60					¾ in. pitch						Diamond No. 433
	Rpm of sprocket											
	50	100	200	300	400	600	800	1,000	1,200	1,400	1,600	1,800
12	0.73	1.34	2.41	3.30	4.05	5.2	6.1	6.6	6.9			
15	0.92	1.72	3.14	4.34	5.39	7.1	8.5	9.5	10.2	10.6		
18	1.12	2.10	3.82	5.31	6.63	8.9	10.6	12.0	13.0	13.7	14.1	
21	1.31	2.46	4.49	6.24	7.80	10.4	12.5	14.1	15.4	16.3	16.9	
24	1.50	2.80	5.11	7.12	8.90	11.9	14.3	16.1	17.6	18.6	19.2	19.5

Teeth	A.S.A. No. 80					1 in. pitch						Diamond No. 434
	Rpm of sprocket											
	50	100	150	200	300	400	500	600	700	800	1,000	1,160
12	1.68	3.07	4.28	5.3	7.2	8.7	9.8	10.7	11.4			
15	2.14	3.95	5.57	7.0	9.6	11.8	13.6	15.1	16.3	17.3		
18	2.59	4.81	6.79	8.6	11.8	14.5	16.9	18.9	20.5	21.9	24.0	
21	3.03	5.62	7.96	10.1	13.9	17.1	19.9	22.3	24.3	26.0	28.5	
24	3.46	6.43	9.10	11.5	15.8	19.5	22.7	25.4	27.7	29.6	32.5	33.9

Table 87. Safe Load for Sling Chains
(As used by Westinghouse Electric and Mfg. Co.)

Diam of chain, in.	Used straight, lb	Used at 60 deg angle, lb	Used at 45 deg angle, lb	Used at 30 deg angle, lb
3/8	1,800	1,550	1,250	900
1/2	3,000	2,600	2,100	1,500
5/8	4,600	4,000	3,250	2,300
3/4	6,750	5,850	4,800	3,400
7/8	9,350	8,100	6,600	4,670
1	12,400	10,700	8,750	6,200
1 1/8	15,600	13,500	11,000	7,800
1 1/4	19,200	16,600	13,600	9,600
1 3/8	23,000	19 900	16,300	11,500
1 1/2	27 200	23 600	19,200	13,600
1 3/4	35,000	30,300	24,700	17,500
2	44,400	38,500	31,400	22,200
2 1/2	69,600	60,300	49,200	34,800

The safe loads specified in the table are for each single chain; when used in any other multiple, the load may be increased proportionately. Stub hooks should be used for lifting apparatus which is provided with holes for that purpose.

Annealing and Care of Chains. In service, because of overloading or the peening or hammering action of the links on each other, chains tend to become brittle so that sudden failure may occur. To remove this condition, annealing of the chain at intervals is necessary. The following notes on annealing are published by the National Safety Council:

In cases where failure of a chain may result in loss of life or property, it is recommended that, where possible, the repair and annealing of such chains be entrusted to a company making a business of this work and having the necessary experience, equipment, and technical supervision to do it correctly. Such chains should be annealed as often as *careful inspection (to determine if a work-hardened condition exists, or if the chain has been overloaded)* shows this to be necessary. The annealing of wrought-iron chains should be carried out in a furnace provided with accurate temperature measurement and control, at 1350 to 1375 F for the following lengths of time: ¼ to ½ in. chain, ½ hr; ⅝ to 1 in. chain, ¾ hr; 1 to 1⅜ in. chain, 1 hr; over 1⅜ in., 2 hr. Cooling in air may follow the anneal. For steel chain, the corresponding annealing temperatures are 1600 to 1650 F. It is important that uniform temperature be obtained during annealing in all parts of the chain.

Table 88. Dimensions of Circular Rings Used at Ends of Sling Chains

Size chain, in.	Single-sling chain	Double-sling chain	Size chain, in.	Single-sling chain	Double-sling chain
	Size of ring,[a] in.	Size of ring,[a] in.		Size of ring,[a] in.	Size of ring,[a] in.
1/4	1/2 × 2 1/4	9/16 × 2 3/4	7/8	1 1/2 × 6	1 7/8 × 8
5/16	9/16 × 2 3/8	5/8 × 2 1/2	1	1 3/4 × 7	2 1/8 × 9
3/8	3/4 × 4	7/8 × 4	1 1/8	1 7/8 × 8	2 3/8 × 10
7/16	3/4 × 4	1 × 4	1 1/4	2 1/8 × 9	2 3/4 × 12
1/2	7/8 × 4	1 1/8 × 5	1 3/8	2 3/8 × 10	3 × 12
5/8	1 1/8 × 5	1 3/8 × 6	1 1/2	2 3/4 × 12	3 1/4 × 12
3/4	1 3/8 × 6	1 3/4 × 7			

[a] First figu[illegible]r, second figure to inside ring diameter.

Idler sprockets may be used on either side of the standard roller chain, to take up slack, to guide the chain around obstructions, to change the direction of rotation of a driven shaft, or to provide more wrap on another sprocket. Idlers should not run faster than the speeds recommended as maximum for other sprockets with the same number of teeth. It is desirable that idlers have at least two teeth in mesh with the chain, and it is advisable, though not necessary, to have an idler contact the idle span of chain.

Horsepower ratings are based upon the number of teeth and the rotative speed of the smaller sprocket, either driver or follower. The pin-bushing bearing area, as it affects the allowable working load, is the important factor for medium and higher speeds. For extremely slow speeds, the chain selection may be based upon the ultimate tensile strength of the chain. For chain speeds of 25 fpm and less, the chain pull should be not more than 1/5 of the ultimate tensile strength; for 50 fpm, 1/6; for 100 fpm, 1/7; for 150 fpm, 1/8; for 200 fpm, 1/9; and for 250 fpm, 1/10 of the ultimate tensile strength.

Ratings for **multiple strand chains** are proportional to the number of strands. The recommended numbers of strands for multiple chains are 2, 3, 4, 6, 8, 10, 12, 16, 20, and 24, with the maximum over-all width in any case limited to 24 in.

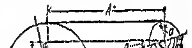


FIG. 138

Chain Length Calculations. Referring to Fig. 138, L = length of chain, in.; P = pitch of chain, in.; R and r = pitch radii of large and small sprockets, respectively, in.; D = center distance, in.; A = tangent length, in.; a = angle between tangent and center line; N and n = number of teeth on larger and smaller sprockets, respectively; $(180 + 2a)$ and $(180 - 2a)$ = angles of contact on larger and smaller sprockets, respectively, deg.

$a = \sin^{-1}[(R - r)/D]$. $A = D \cos a$.

$$L = NP(180 + 2a)/360 + nP(180 - 2a)/360 + 2D \cos a$$

If L_p = length of chain in pitches, and D_p = center distance in pitches,

$$L_p = (N + n)/2 + a(N - n)/180 + 2D_p \cos a$$

Avoiding the use of trigonometrical tables,

$$L_p = 2C + (N + n)/2 + K(N - n)^2/C$$

where C is the center distance in pitches and K is a variable depending upon the value of $(N - n)/C$. Values of K are as follows:

$(N - n)/C$	0.1	1.0	2.0	3.0	4.0	5.0	6.0
K	0.02533	0.02538	0.02555	0.02584	0.02631	0.02704	0.02828

Formulas for chain length on multisprocket drives are cumbersome except when all sprockets are the same size and on the same side of the chain. For this condition, the chain length in pitches is equal to the sum of the consecutive center distances in pitches plus the number of teeth on one sprocket.

Actual chain lengths should be in even numbers of pitches. When necessary, an odd number of pitches may be secured by the use of an offset link, but such links should be avoided if possible. An offset link is one pitch; half roller link at one end and half pin link at the other end. If center distances are to be non-adjustable, they should be selected to give an initial snug fit for an even number of pitches of chain. For the average application, a center distance equal to 40 ± 10 pitches of chain represents good practice.

Steam-engine cylinders may have walls of the thickness, t(in.) = $(PD/2000) + 0.3$ in. The thickness thus obtained allows for reboring. For jacketed cylinders, the liner, when of cast iron, may be of the thickness $t_0 = [PD/(5125 + 10P)] + 0.4$ in.

Cylinders for hydraulic presses are usually designed with high working stresses used to reduce the weight of the cylinders:

Steel or cast-steel cylinders	15,000 to 30,000 lb per sq in.
Phosphor bronze	7,000 to 12,500 lb per sq in.
Cast iron, bronze	4,500 to 11,000 lb per sq in.

Cylinder flanges should not be thicker than 1.2t to 1.25t. Heavier flanges cause severe cooling strains which materially weaken the casting.

Cylinder heads when flat may be proportioned as flat plates supported at the edges. $t_1 = 0.45D\sqrt{5P/6f}$, where t_1 = thickness of plate, in., and f = allowable stress, lb per sq in. = 3,000 for cast iron and 8,000 for cast steel. When heads are **dished and ribbed**, the same formula may be used for determining the maximum limit of dimension and the actual thickness made 20 to 40 percent less, depending on the ribbing.

Flat cover plates may be designed as rectangular plates supported at the edges (see p. 475). The thickness of the plate is $t_2 = ka\sqrt{P/f}$, where a = the shorter side of the plate, in.; b = the longer side of the plate, in.; f = 3,000 for cast iron, 9,000 for cast steel; k depends on the ratio b/a and is as follows:

b/a =	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0
k =	0.536	0.613	0.672	0.719	0.754	0.780	0.844	0.860

Large covers are usually dished and ribbed for added strength; the same values of k may be used for determining the maximum value of t_2 and the actual thickness made less, depending on the ribbing.

Stud bolts for holding down cylinder heads and steam-chest covers should never be less than ⅝ in. and preferably ¾ in. diam to prevent their being twisted off in tightening. The intensity of stress based on steam-pressure load should be limited to 6,000 lb per sq in., and the spacing should not be greater than 6½ in. or less than 3½ times the diameter of the bolt. If higher stress is necessary, special steel must be used. Stud holes should be tapped to a depth equal to 1½ times the diameter of stud, but never allowed to enter a steam space. The strength of the bolts should be much less than the strength of the frame or cylinder with load application along the center line of the engine. This will ensure the safety of the more expensive castings in case of unusual loading such as may be caused by water in the cylinder.

Pistons and Piston Rods

Small pistons may be made to the form and dimensions given in Fig. 143 and Table 90.

Larger pistons of this type are often reinforced by radial internal ribs. Proper holes, adequately bossed, must be provided for holding the cores in place and for removing them after casting. Ribs are not advisable for very high temperatures, superheated steam, or gas engines, causing excessive thermal stresses.

Disk pistons (Fig. 144) are often used. The thickness h of the disk, in., is $h = \sqrt{(k_1D^2 - k_2D_1^2)P/f}$; P = internal gage pressure lb per sq in.; f = the allowable stress lb per sq in.; k_1 and k_2 depend on the ratio d_1/D_1 and are as follows:

shaped is reduced, while the chain rides higher on the teeth as it elongates, thus accommodating itself to a larger pitch circle.

In Fig. 153, let P = pitch; D = nominal roller diameter, in.; N = number of teeth; $D' = 1.005D + 0.003$; $A = 35 + 60/N$ deg; $B = 18 - 56/N$, deg; $C = 180/N$ deg.

In laying out the tooth, first draw line XY. Locate point a, and with that as a center and radius ax equal to $1/2D'$, draw a circular arc for the "seating curve" xx'.

Draw line xac making angle A with line XY, and locate point c so that $ac = 0.8D$. Draw line cy making angle B with line cx. With center at c and radius cx, draw arc xy for the "working curve."

Draw line yz perpendicular to line cy. Draw line ab making angle C with line XY, and locate point b so that $ab = 1.24D$. Draw line bz parallel to line yc. With b as center and radius bz, draw the "topping curve," arc zu tangent to line zy.

A similar construction for the other half will complete the tooth outline.

Outside diameter of sprocket when tooth is pointed $= P \cot (180/N) + 2H$.

The recommended value for H is $0.3P$; and when this value is chosen, the outside diameter of the sprocket will be $P[0.6 + \cot (180/N)]$.

Silent Chains. Silent chain drives are widely used in various applications including front-end drives in automobile engines and in connecting high-speed electric motors to their loads. The advantages of silent chains in addition to their quietness in operation are high efficiency, suitability for high speeds, positive drive without slippage, ability to run on short or long centers, and the elasticity of their connection to the load.

FIG. 141.—Morse Silent Chain.

Morse Silent Chain. *The principal feature of* construction of the Morse silent chain is the rocker joint as shown in Fig. 141.

The procedure in the design of Morse silent chains is as follows: The pitch is selected first, the normal rpm of the high-speed shaft for each pitch being indicated in Table 84. In most drives, the largest pitch consistent with the maximum rpm as indicated in the table will be the most economical.

Table 84. Data on Design of Morse Silent-chain Drives

Pitch, in.	No. of pitches per ft	Normal rpm	Normal tension, 1 in. wide, lb	Min no. of teeth, driver	Desirable no. of teeth, driver	Min no. of teeth, driven
3/16	64	5,000	30	13	13-17	21
3/8	32	3,600	75	13	15-17	21
1/2	24	2,400	100	13	15-17	21
5/8	19.2	1,800	125	13	17-21	21
3/4	16	1,400	150	13	17-21	21
0.9	13.33	1,100	185	15	17-23	25
1	12	1,000	205	15	17-23	25
1.2	10	800	250	15	17-23	25
1¼	9.6	750	265	15	17-23	25
1½	8	600	335	17	17-27	25
2	6	400	600	17	17-31	27
3	4	250	935	17	19-31	27

When extreme quietness is desirable, it is recommended to use two pitches smaller than that indicated for the rpm given in the table.

the pressure p against the wall is $D = 2.4fA^2/EB$ or $2.4fA^2/EF$, where E is the modulus of elasticity of the ring material. The ring is preferably in segments for large engines, each segment being pressed against the cylinder walls by a spring.

Piston rods are subjected mainly to compressive stresses and may be designed as round free-ended columns, for which Euler's formula reduces to $P = 0.485Ed^4/sl^2$, in which P = total load, lb; E = modulus of elasticity ($= 30{,}000{,}000$ for mild steel); d = diam of rod, in.; l = length of rod, in. [For hollow piston rods, substitute $(d_2^4 - d_1^4)$ for d^4 in formula, d_2 and d_1 being, respectively, the outer and inner diameters.] The factor of safety s has the following values: For vertical engines $s = 8$ to 11 for load fluctuating between P and 0; $s = 15$ to 22 for load fluctuating between $+P$ and $-P$. In horizontal engines, the rods are also subjected to bending stress due to the combined weight of the piston and rod. In small engines with light pistons, the formula for vertical engines may be used with $s = 11$ to 22. In large engines with heavy pistons and rods running through glands in both heads (as in tandem steam and gas engines), the rod must be of such stiffness that it will not deflect at mid-length between its points of support more than 0.08 in. In this case, the piston diameter must be 0.12 to 0.16 in. less than that of the bore. Deflection in inches $= l^3(W_1 + 0.625W_2)/48EI$, where l = length between supported points, in.; W_1 and W_2 = weights of piston and rod, respectively, lb; I = moment of inertia of rod section $= \pi d^4/64$. Most reliable values of d may be found by the use of Ritter's long-column formula. This reduces to the following form for mild-steel rods:

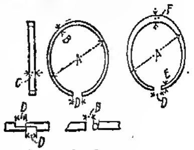


FIG. 146. FIG. 147.
Piston Rings.

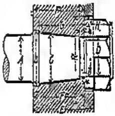


FIG. 148.—Piston-rod Ends.

$$d^2 = P(1 + \sqrt{1 + fl^2/190P})/1.57f$$

where f = allowable working stress, usually 5,000 lb per sq in., l = rod length, in., P = load on rod, lb.

Table 92. Typical Dimensions for Ends of Piston Rods
(All dimensions in inches. Letters refer to Fig. 148)

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*R*
2¾	3¼	3	2½	4½	3¾	3⅛	2⅜	4¾
3	3½	3¼	2¾	4½	3¾	3⅛	2⅜	5¼
3¼	3¾	3½	3	5½	4¼	4⅛	2⅜	5¾
3½	4	3¾	3¼	5½	4½	3⅞	2⅞	6
3¾	4¼	4	3½	6½	5½	4⅞	2⅞	6½
4	4½	4¼	3¾	6½	5½	4⅞	2⅞	6⅞
4¼	4¾	4½	4	7	6	5⅜	3⅜	7¼
4½	5	4¾	4¼	7	6	5⅜	3⅜	7¾

To check for stress in an existing design, the formula is more conveniently written as: $f = P[1 + (16SL^2/\pi^2Ed^2)]/A$, where A = area of rod section, sq

For calculating silent chain lengths, the procedure for roller chain drives (p. 992) may be used.

CRANE CHAINS AND HOOKS

Crane Chains. For applications where failure is dangerous to life or property, chains made from a high grade of wrought iron are widely used. Because of its extreme ductility, wrought iron has a large shock-absorbing capacity, and chains made of this material will stretch considerably under an overload, unless they have been overstrained or the links embrittled. Wrought iron has the additional advantage that it is easily welded. Welded chains made of nickel steel (S.A.E. 2320) are also coming into use where exacting requirements must be met.

Table 86. Proof and Break Test Loads of Crane and Proof Coil Chain

Nominal size of chain bar, in.	Proof test load, lb		Break test load, lb		Safe working load, lb	
	Crane chain, wrought iron	Proof coil, iron or steel	Crane chain, wrought iron	Proof coil, iron or steel	Crane chain, wrought iron	Proof coil, iron or steel
1/4	1,767	1,700	3,535	3,400	1,060	850
5/16	2,760	2,650	5,520	5,300	1,655	1,325
3/8	3,975	3,850	7,950	7,700	2,385	1,925
7/16	5,415	5,250	10,830	10,500	3,250	2,625
1/2	7,072	6,850	14,145	13,700	4,240	3,425
9/16	8,947	8,650	17,895	17,300	5,370	4,325
5/8	11,047	10,700	22,095	21,400	6,630	5,350
3/4	15,900	15,350	31,800	30,700	9,540	7,675
7/8	21,622	20,900	43,245	41,800	12,960	10,450
1	28,275	27,350	56,550	54,700	16,950	13,675
1 1/8	33,400	......	66,800	......	20,040	
1 1/4	41,250	......	82,500	......	24,750	
1 3/8	49,900	......	99,800	......	29 910	
1 1/2	59,350	......	118,700	......	35,600	
1 5/8	69,750	......	139,500	......	41,800	
1 3/4	80,800	......	161,600	......	48,450	
1 7/8	92,750	......	185,500	......	55,300	
2	105,500	......	211,100	......	63,300	

Nominal weights and dimensions for crane chains of wrought iron or steel as given in A.S.T.M. specification A-56-30 are given in Table 85; proof, break test, and safe working loads taken from the same specification are given in Table 86. The proof coil chain should be used only where life or property is not endangered by a chain failure and where corrosion is not a factor. **Proof coil** is made of steel—generally open-hearth—and is used for railroad cars and construction and forestry work.

The safe loads as recommended by Hantman (*Maintenance Engineering*, Oct., 1932) and used by Westinghouse Electric & Mfg. Co. for wrought-iron sling chains, used at various angles, are given in Table 87. These loads are somewhat lower than those indicated in Table 86.

The American Chain and Cable Co. recommends that chains used on the sheaves to operate cranes have straight-sided links (so as to operate properly in the sheaves), and chains used on slings should have oval-shaped links. Under overload, the oval-shaped links will tend to straighten out, thus giving visual warning that the chain is too light for the load.

Pin bushings are proportioned to have a maximum bearing pressure 1,000 to 1,200 lb per sq in. for steam engines, up to 2,500 lb per sq in. in oil or gas engines. In small connecting rods, this pressure is 300 to 700 lb per sq in., depending on the required rate of wear. The ratio of length to diameter of the bushing is 1.2 to 1.5.

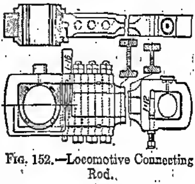


FIG. 152.—Locomotive Connecting Rod.

Shanks for connecting rods may be designed as long columns with round ends, using Ritter's formula (see p. 848), $d^2 = CP[1 + \sqrt{1 + (l^2kf/KCP)}]/kf$, where P = load on rod, lb; f = maximum allowable stress, lb per sq in.; l = length of rod, in. For rods of **circular cross section**, d = rod diam, in., C = 1.00, k = 1.57, and K = 300. For **rods of rectangular cross section** having height of section equal to 2.7 times the thickness, d = height of rod section, in.; C = 1.35, k = 1, and K = 420. These constants are for rods of mild steel having an elastic limit of 30,000 lb per sq in. and a modulus of elasticity in tension of 30,000,000 lb per sq in. In high-speed steam-engine practice, f may be 4,000. In slow-speed steam engines, gas engines and pumps, f may be 5,000 to 5,500. In general, the **working stress** for connecting rods should be ⅙ to ⅛ the stress at the elastic limit of the material, when designing for long-column action. Rods for high-speed engines having important inertia stresses should be designed with a factor of ⅙. The sum of the stresses due to long-column action and to inertia (whipping action) should not exceed ½ the stress at the elastic limit of the material.

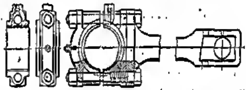


FIG. 153.—Connecting Rod for Small Engines.

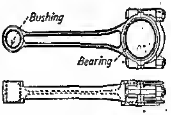


FIG. 154.—Connecting Rod of Automobile Engine.

The **maximum inertia stress** in a connecting-rod shank may be closely approximated by use of the following formulas:

For round rods, $f' = l^2wv^2/(0.098d^3 \times 16gR)$, where f' = stress due to inertia, lb per sq in.; w = weight of 1 in. length of rod, lb; v = velocity of crank pin fps; g = 32.2; R = crank radius, ft; d = mean diam of rod, in. For rectangular rods, $f' = l^2wv^2/(0.167bh^2 \times 16gR)$, where b = thickness of rod section, in., and h = height of rod section, in.

Connecting-rod ends as in Fig. 166 should have their bolts designed such that each may take ½ the maximum load on the cap with a stress of 5,000 lb per sq in. for wrought iron and 6,000 lb per sq in. for steel. In double-acting engines, the maximum load producing tension in the bolts may be occasioned by the internal pressure in the cylinder or the inertia of the reciprocating masses. In single-acting gas engines, the load on the bolts is caused by the inertia forces. **Strap ends** as shown at the crank-pin end of the rod in Fig. 152 should have the side straps designed to carry ½ the load, with a limiting

Hantman (*loc. cit.*) recommends that new chains be marked with prick-punch marks so that the distance between marks is 3 ft when the chain hangs freely. Annealing should be carried out where the stretch is 1 in. in 3 ft or if the chain shows a work-hardened condition. If the stretch is greater than this amount and the links bind, that portion of the chain should be scrapped. After repair or anneal, chains should be proof-tested. Chains that show gouge marks, cracks, bent links, excessive wear, or a tendency of the welds to open should be replaced. The safe load should be reduced for worn chains to compensate for wear.

For proportions of chain sheaves, see p. 833.

Dimensions of circular rings used at the ends of sling chains, (American Chain Co.) are as indicated in Table 88.

Crane Hooks. Since crane hooks act essentially as curved bars in bending, stresses in these hooks may be calculated by curved bar theory (Timoshenko, "Strength of Materials," Part 2, p. 423). For irregular cross sections as used in crane hooks, the graphical method described in this reference may be employed.

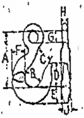


FIG. 142. Sling Hook.

Dimensions of standard sling hooks as made by American Chain and Cable Co. for the various chain sizes are given in Table 109 (dimensions refer to Fig. 142).

Table 89. Dimensions of Sling Hooks[a]
(American Chain and Cable Co.)

No.	Size	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*
55	1/8	2 1/2	1 3/16	15/16	1	3/4	13/16	7/16	5/16	3/8
56	3/16	2 3/4	1 5/16	1 1/32	1 1/16	7/8	15/16	1/2	3/8	1/2
57	1/4	3 1/4	1 9/16	1 9/32	1 1/4	31/32	1 1/16	5/8	3/8	5/8
58	5/16	3 3/4	1 11/16	1 1/16	1 7/16	1 1/8	1 1/4	3/4	7/16	3/4
59	3/8	4 1/8	2	1 1/2	1 5/8	1 1/4	1 3/8	7/8	1/2	7/8
60	7/16	5	2 7/16	1 7/8	1 13/16	1 1/2	1 11/16	1	5/8	1
61	1/2	5 3/4	2 13/16	2 1/8	2 1/4	1 3/4	2	1 1/8	3/4	1 1/8
62	5/8	6 5/8	2 3/4	2 1/16	3 5/16	2	2	1 1/4	1	1 1/2
63	3/4	8 1/2	3 1/4	2 11/16	3 3/8	2 1/2	2 5/8	1 3/4	1 1/8	1 5/8
64	7/8 and 1	10 1/8	4	3	4 3/4	2 15/16	3 1/8	2	1 1/4	1 5/8
65	1 1/8 1 1/4	12	5	3 3/4	6 1/16	3 7/16	3 3/4	2	1 5/8	2 5/8
68	1 3/8	15 3/8	6	4 1/2	8 3/4	3 3/4	4	2 3/8	1 7/8	3 1/8
69	1 1/2	16 3/4	6 1/2	5	9 3/4	4 1/4	4 1/2	2 1/2	2	3 3/4

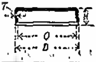
[a] All dimensions in inches.

ENGINE DETAILS

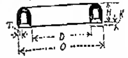
Cylinders

The wall thickness, inches, of a cylinder subjected to internal pressure, P, pounds per square inch, is $t = PD/2f$, where D = diam of bore, in.; f = allowable stress in the cylinder wall; $f = 6{,}000$ for cast iron, 15,000 for steel. When P is higher than $f/5$ (in steel hydraulic-press cylinders for $P > 3{,}000$), the cylinder walls may be proportioned for strength by Lamé's formula (see p. 446). With *low pressures*, the *minimum* thickness of the wall for cast-iron cylinders is: $t = D/50 + 0.4$ in., when cast vertically; $t = D/50 + 0.5$ in., when cast horizontally.

Table 93. Proportions for Oak-tanned Leather Hydraulic Packing



(All dimensions in inches.)
(K = clearance in cavity)



	End U packing				U-wall or neck packing				Cup packing		Stock	
Diam of ram	D	O	H	T	D	O	H	K	D	H	Diam	Thickness
4	2½	4	¾	3/16	4	5½	1	1/16	4	1	6	3/16
6	4½	6	1	3/16	6	7½	1 1/16	1/16	6	1¼	8½	3/16
8	6½	8	1¼	3/16	8	9½	1¼	1/16	8	1½	11	3/16
10	8½	10	1¼	¼	10	11½	1⅜	⅛	10	1½	13	¼
12	10½	12	1½	¼	12	13½	1½	⅛	12	1½	15	¼
13	11½	13	1½	¼	13	14½	1½	⅛	13	1¾	16½	¼
14	12½	14	1½	¼	14	15½	1¾	⅛	14	1¾	17½	¼
18	16½	18	1⅞	¼	18	19½	1⅞	3/16	18	2	22	¼
21	19½	21	1⅞	¼	21	22½	1⅞	3/16	21	2	25	¼
23	21½	23	1⅞	¼	23	24½	1⅞	3/16	23	2	27	¼
24	22½	24	1⅞	¼	24	25½	1⅞	3/16	24	2	28	¼

through a succession of very small clearances c formed by rings fitting alternately in grooves in the stator and rotor. These rings are usually composed

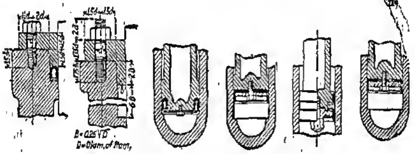


FIG. 157. FIG. 158. FIG. 159. FIG. 160. FIG. 161. FIG. 162.
Packings for Hydraulic Cylinders.

of short brass strips with clearance between their ends in order to preve
them from being forced out by expansion when heated. The clearance edg
of the rings are made 0.01 to 0.015 in. thick. The number of rings n require
to limit the leakage of steam to a given weight per second may be determine
from the formula

$$n = [40p - 2{,}600(w/a)]/[540(w/a) - p]$$

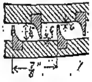


FIG. 163.

where p = absolute pressure of steam, lb per sq in., w = permissible leakage of steam, lb per sec, and a = cross-sectional area of the steam passage at the clearance rings, sq in. This formula is derived from a chart given in Morrow's "Steam Turbine Design," p. 293.

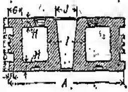
Example. Let dummy piston circumference = 70 in.; pressure before passin
through labyrinth = 180 lb per sq in. abs; pressure after passing labyrinth = 8
lb abs; clearance c = 0.01 in.; leakage w not to exceed 0.7 lb per sec. Here a = 7
× 0.01 = 0.7 and w/a = 0.7/0.7 = 1. Substituting in formula, number of rings fo
180 lb = 13; also number of rings for 85 lb = 2; whence, number of rings for leakage
of 0.7 lb per sec between the two pressures = 13 − 2 = 11.

ϑ_1 = ... 0.1 0.2 0.3 0.4 0.5
= 1.38 0.89 0.62 0.44 0.32
= 0.20 0.18 0.17 0.16 0.14

Conical pistons for use in vertical engines may be proportioned for ngth according to the formulas given by Kraft (*Proc. I.C.E.*, 127, p. 259):

$$T = p(R^2 - x^2)/2fx \sin a; \quad t = px'/f \sin a$$

ere p = steam pressure, lb per sq in.; f = maximum allowable stress, lb sq in. (= 3,000 for cast iron and 9,000 for steel); T and t = minimum



IG. 143.—Small Piston.

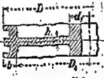


FIG. 144.—Disc Piston.

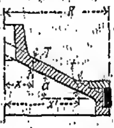


FIG. 145.—Conical Piston.

ckness of metal at sections near boss and rim, respectively, in.; x, x', a, and ire as in Fig. 145.

Table 90. Dimensions of Small Pistons
(All dimensions in inches. Letters refer to Fig. 156)

B	C	D	E	F	G	H	I	J	A	B	C	D	E	F	G	H	I	J
2½	⅜	½	¾	⅜	....	....	....	1 3/16	11	4½	¾	⅝	1¾	⅝	1¼	⅞	3½	1⅝
2½	⅜	½	¾	⅜	....	....	....	1 3/16	12	4¾	¾	⅝	1⅞	⅝	1¼	1	3¾	1⅞
3	½	½	1	½	....	....	....	1⅝	13	5¼	¾	⅝	2⅛	¾	1¼	1	3¾	2
3½	½	½	1⅛	½	1	⅝	2¼	1⅝	14	5½	¾	⅝	2⅜	¾	1⅜	1	4	2
3½	½	½	1⅛	½	1	⅝	2⅜	1¼	16	5½	¾	¾	2½	⅞	1¼	1	4⅛	2⅜
4	⅝	⅝	1¼	⅝	1⅛	⅞	3	1⅜	18	6	⅞	¾	2¾	1	1⅜	1⅛	4½	2⅝
4	⅝	⅝	1½	⅝	1⅛	⅞	3¼	1⅜	20	6½	⅞	¾	3¼	1	1½	1¼	5	3

Solid pistons.

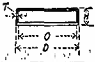
ble 91. Dimensions of Equal-section and Eccentric Piston Rings
(All dimensions in inches. Letters refer to Figs. 146 and 147)

t-rs	Diameters of cylinders, in.														
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
[illegible]	6 5/32	7 5/32	8 3/16	9 3/16	10 7/32	11¼	12¼	13 9/32	14 9/32	15 5/16	16 11/32	17 11/32	18⅜	19 13/32	20 13/32
[illegible]	¼	9/32	5/16	⅜	13/32	7/16	15/32	½	9/16	19/32	⅝	11/16	¾	¾	13/16
[illegible]	½	½	½	⅝	⅝	⅝	⅝	⅝	⅝	¾	¾	¾	¾	¾	¾
[illegible]	35/64	39/64	21/32	23/32	25/32	27/32	⅞	15/16	1	1 1/16	1⅛	1 3/16	1¼	1 9/32	1 11/32
[illegible]	3/16	7/32	¼	9/32	9/32	5/16	11/32	⅜	13/32	7/16	15/32	15/32	½	17/32	9/16
[illegible]	9/32	5/16	11/32	⅜	13/32	7/16	15/32	½	9/16	⅝	11/16	11/16	¾	13/16	⅞

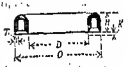
Piston rings up to 20 in. diam may be made to the forms and propor-ons given in Figs. 146 and 147 and Table 91. The eccentric rings give a ore uniform pressure against the cylinder walls. This pressure should 3 to 5.5 lb per sq in.; in high-pressure compression up to 7 lb per sq in.

The strength of the piston ring may be estimated from, $f = 3pA^2/B^2$ or A^2/F^2, where f is the stress in pounds per square inch in the section opposite · The length D, in., through which the ring must be sprung to produce

Table 93. Proportions for Oak-tanned Leather Hydraulic Packing



(All dimensions in inches.)
(K = clearance in cavity)



Diam of ram	End U packing D	End U packing O	End U packing H	End U packing T	U-wall or neck packing D	U-wall or neck packing O	U-wall or neck packing H	U-wall or neck packing K	Cup packing D	Cup packing H	Stock Diam	Stock Thickness
4	2½	4	¾	3/16	4	5½	1	1/16	4	1	6	3/16
6	4½	6	1	3/16	6	7½	1 1/16	1/16	6	1¼	8½	3/16
8	6½	8	1⅛	3/16	8	9½	1¼	1/16	8	1½	11	3/16
10	8½	10	1¼	¼	10	11½	1⅜	⅛	10	1½	13	¼
12	10½	12	1½	¼	12	13½	1½	⅛	12	1½	15	¼
13	11½	13	1½	¼	13	14½	1½	⅛	13	1¾	16½	¼
14	12½	14	1½	¼	14	15½	1¾	⅛	14	1¾	17½	¼
18	16½	18	1⅞	¼	18	19½	1⅞	3/16	18	2	22	¼
21	19½	21	1⅞	¼	21	22½	1⅞	3/16	21	2	25	¼
23	21½	23	1⅞	¼	23	24½	1⅞	3/16	23	2	27	¼
24	22½	24	1⅞	¼	24	25½	1⅞	3/16	24	2	28	¼

through a succession of very small clearances c formed by rings fitting alternately in grooves in the stator and rotor. These rings are usually composed

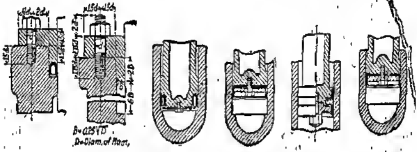


FIG. 157. FIG. 158. FIG. 159. FIG. 160. FIG. 161. FIG. 162.
Packings for Hydraulic Cylinders.

of short brass strips with clearance between their ends in order to prevent them from being forced out by expansion when heated. The clearance edges of the rings are made 0.01 to 0.015 in. thick. The number of rings n required to limit the leakage of steam to a given weight per second may be determined from the formula

$$n = [40p - 2{,}600(w/a)]/[540(w/a) - p]$$

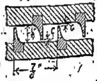


FIG. 163.

where p = absolute pressure of steam, lb per sq in., w = permissible leakage of steam, lb per sec, and a = cross-sectional area of the steam passage at the clearance rings, sq in. This formula is derived from a chart given in Morrow's "Steam Turbine Design," p. 293.

Example. Let dummy piston circumference = 70 in.; pressure before passing through labyrinth = 180 lb per sq in. abs; pressure after passing labyrinth = 85 lb abs; clearance c = 0.01 in.; leakage w not to exceed 0.7 lb per sec. Here a = 70 × 0.01 = 0.7 and w/a = 0.7/0.7 = 1. Substituting in formula, number of rings for 180 lb = 13; also number of rings for 85 lb = 2; whence, number of rings for leakage of 0.7 lb per sec between the two pressures = 13 − 2 = 11.

in.; E = modulus of elasticity in tension = 30,000,000 for mild steel; S = elastic limit of material = 30,000 lb per sq in. for mild steel.

A suitable design of a **piston-rod end** is shown in Fig. 148 and Table 92.

Crossheads

Engine crossheads of the **wing type** may have the features of construction shown in Fig. 149. The **box type** of crosshead may be designed as shown in Fig. 150. The substitution of pin shoulders on the crosshead and straight bores in the box for the taper construction reduces the cost of machining. The shoulders should bear on the face of the side opposite to which the nut

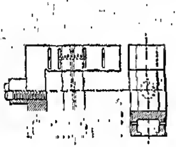


Fig. 149.—Wing Type of Crosshead.

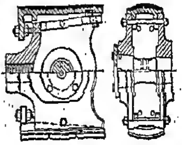


Fig. 150.—Box Type of Crosshead.

holding the pin is seated in order that screwing up the nut will not tend to bind the connecting-rod end in the box and thus cause heating. It is well to flatten the top and bottom portions of the pin where wear is a minimum to prevent the pin wearing to an oval shape. It is sometimes the practice to give the pin a quarter turn occasionally to promote more uniform wear. In locomotive and marine service, the type of crosshead shown in Fig. 151 is frequently used.

Crosshead Pins. In steam-engine practice, the crosshead pin is designed for a bearing pressure of 1,000 to 1,500 lb per sq in. According to Barr, American practice in proportioning crosshead pins is as follows: $dl = CA$, $l = Kd$; where d = pin diam in.; l = bearing length of pin, in.; A = piston area, sq in., C = 0.08 (0.07) for high (low) speeds, and K = 1.25 (1.3) for high (low) speeds.

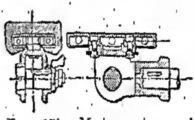


Fig. 151.—Marine-engine and Locomotive Type of Crosshead.

Shoes. The area of the shoes for stationary steam-engine crossheads should be such as to limit the maximum pressure to from 25 to 40 lb per sq in. The total maximum pressure of the crosshead shoes on the guide is $P \tan a$, where P = maximum steam-pressure load behind the piston, lb, and a = maximum angle of connecting rod with engine center line.

In **marine practice**, the area of the shoes, according to Bauer, is based in the following limiting pressures: 55 to 65 lb. per sq in. for cargo and slow-running passenger vessels; 65 to 80 lb per sq in. for mail steamers.

In **locomotive practice**, the limiting pressure on the crosshead shoe is about 85 lb per sq in.

Connecting Rods

Connecting rods for **locomotive** service may be made to the form shown in Fig. 152. For **small stationary engines**, rods of the form shown in Fig. 153 are used.

Table 95. Crank Angles and Piston Positions for Connecting Rods of Different Lengths

Calculated from $\cos^{-1}\left(\frac{1-x_1}{1-2x_1/\left(1+\frac{l}{r}\right)}-x_1\right)$ where $x_1 = \frac{x}{2r}$.

Fraction of stroke from commencement, $\frac{x}{2r}$	Ratio of length of connecting rod to length of crank										
	2.5	3	3.5	4	4.5	5	5.5	6	7	8	∞
	Angle through which crank has advanced from dead center, deg										
0.005	6.86	7.02	7.15	7.25	7.34	7.40	7.46	7.51	7.59	7.65	8.11
0.01	9.70	9.94	10.13	10.27	10.39	10.48	10.56	10.63	10.74	10.83	11.48
0.02	13.75	14.09	14.35	14.56	14.72	14.86	14.97	15.06	15.22	15.34	16.26
0.03	16.88	17.30	17.62	17.87	18.07	18.23	18.37	18.49	18.68	18.82	19.95
0.04	19.53	20.02	20.38	20.67	20.91	21.10	21.26	21.39	21.61	21.78	23.07
0.05	21.88	22.43	22.84	23.16	23.42	23.64	23.82	23.97	24.21	24.40	25.84
0.06	24.02	24.62	25.08	25.43	25.71	25.95	26.15	26.31	26.58	26.79	28.36
0.07	26.01	26.65	27.14	27.53	27.84	28.09	28.30	28.48	28.77	28.99	30.68
0.08	27.86	28.56	29.08	29.49	29.82	30.09	30.32	30.51	30.82	31.06	32.86
0.09	29.62	30.36	30.92	31.35	31.70	31.99	32.23	32.43	32.76	33.01	34.92
0.10	31.29	32.07	32.66	33.12	33.49	33.79	34.05	34.26	34.61	54.87	36.87
0.15	38.77	39.74	40.47	41.04	41.49	41.86	42.17	42.43	42.65	43.17	45.57
0.20	45.31	46.46	47.31	47.97	48.49	48.92	49.27	49.57	50.05	50.42	53.13
0.25	51.32	52.62	53.58	54.31	54.90	55.38	55.77	56.10	56.63	57.04	60.00
0.30	56.99	58.43	59.49	60.30	60.94	61.46	61.89	62.25	62.82	63.26	66.42
0.35	62.45	64.03	65.18	66.06	66.75	67.30	67.76	[illegible]	68.76	69.22	72.54
0.40	67.80	69.51	70.75	71.68	72.41	73.00	73.49	73.89	74.53	75.02	78.46
0.45	73.12	74.95	76.26	77.25	78.02	78.63	79.14	79.56	80.23	80.73	84.26
0.50	78.46	80.41	81.79	82.82	83.62	[illegible]	84.78	85.22	85.90	86.42	90.00
0.55	83.90	85.95	87.39	88.46	89.28	89.94	90.48	90.92	91.62	92.14	95.74
0.60	89.50	91.64	93.13	94.23	95.07	95.74	96.28	96.73	97.44	97.96	101.54
0.65	95.35	97.56	99.08	100.20	101.05	101.72	102.27	102.72	103.42	103.94	107.46
0.70	101.54	103.80	105.34	106.46	107.31	107.98	108.52	108.97	109.66	110.17	113.58
0.75	108.21	118.49	112.02	113.13	113.97	114.62	115.15	115.58	116.25	116.74	120.00
0.80	115.57	117.82	119.32	120.39	121.19	121.82	122.32	122.73	123.37	123.83	126.87
0.85	123.94	126.10	127.51	128.52	129.26	129.84	130.31	130.68	131.26	131.69	134.43
0.90	133.96	135.90	137.17	138.05	138.71	139.21	139.61	139.94	140.44	140.81	143.13
0.91	136.26	138.15	139.37	140.22	140.85	141.34	141.72	142.03	142.51	142.86	145.05
0.92	138.71	140.52	141.69	142.51	143.11	143.57	143.94	144.24	144.70	145.03	147.14
0.93	141.32	143.05	144.16	144.94	145.51	145.95	146.30	146.58	147.01	147.33	149.32
0.94	[illegible]	145.77	146.82	147.55	148.09	148.50	148.82	149.09	149.49	149.79	151.64
0.95	147.21	148.73	149.71	150.30	150.88	151.26	151.56	151.81	152.18	152.45	154.16
0.96	150.62	152.02	152.90	153.52	153.97	154.31	154.59	154.81	155.14	155.39	156.93
0.97	154.51	155.75	156.53	157.07	157.47	157.77	158.01	158.20	158.50	158.71	160.05
0.98	159.15	160.18	160.83	[illegible]	161.61	161.86	162.06	162.22	162.46	162.64	163.74
0.99	165.23	165.98	166.44	166.77	167.00	167.18	167.32	167.44	167.61	167.74	168.52
0.995	169.55	170.08	170.41	170.64	170.81	170.94	171.04	171.12	171.24	171.33	171.89
1.00	180.00	180.00	180.00	[illegible]	180.00	180.00	180.00	180.00	180.00	180.00	180.00

Fig. 164, the dead centers are unsymmetrical and the stroke is greater than $2r$, having the value

$$s = \sqrt{(l+r)^2 - h^2} - \sqrt{(l-r)^2 - h^2}$$

Determination of the Velocity of the Reciprocating Parts. The *velocity of the reciprocating parts is*

$$c = v\left(\sin\alpha + r\sin 2\alpha/2l\sqrt{1-\frac{r^2}{l^2}\sin^2\alpha}\right) = v(\sin\alpha + r\sin 2\alpha/2l) \text{ approx.}$$

when the center lines of the piston rod and crankshaft lie in the same plane. The ratio c/v is the tangential factor; Table 96 gives the values of velocities corresponding to various crank angles. When, however, the crank rotates about a center off the center line, as in Fig. 164 (offset cylinder), the above formula does not hold, and the graphical solution of Fig. 165 may be resorted

stress of 5,000 to 6,000 lb per sq in. The ends are made 1 to 1.6 times the thickness of the sides for both milled and strap ends.

Stuffing Boxes.

(For packings see p. 896)

The axial flow q of a liquid or gas through the clearance between a shaft and bushing can be estimated in cubic inches per minute.

$$q = 0.00108(1000c)^3dp/\mu l,$$

where c is the radial clearance in inches (half the diametral difference); p is the pressure drop across the bushing, lb per sq in.; l and d are the length and diameter of the bushing, in.; μ is the absolute viscosity of the liquid or gas, poises.

Stuffing boxes of the ordinary **bolted flange type** suitable for steam engines may be proportioned as shown in Fig. 155, where d = rod diam, in.; $d_1 = 1.22d + 0.6$ in.; $l_1 = 0.4d + 1$ in.; $l_2 = d + 1$ in.; $l_3 = 0.75l_2$;

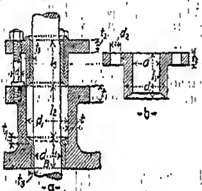


Fig. 155.—Bolted Flange Type.

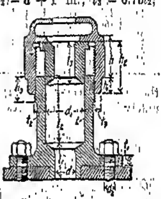


Fig. 156.—Screw Type.

Stuffing Boxes.

d_2 = bolt diam = $0.12d + \frac{1}{2}$ in. when 2 bolts are used, = $1.6(0.12d + \frac{1}{2}$ in.$)/\sqrt{n}$ where the number of bolts n is greater than 2; $t = 0.1d + 0.6$ in.; $t_1 = 1.4t$; $t_2 = t$; $t_3 = 0.04d + 0.2$ in. (not to exceed $\frac{1}{2}$ in.); $t_4 = 0.1d + 0.13$ in. (not to exceed 1 in.). All glands are to be of brass.

The screw type of stuffing box is not recommended but may be made as shown in Fig. 156, with

$d_1 = 1.3d + 0.6$ in.	$l_3 = 0.6d + 1$ in.	$h = 0.6d + 1$ in.
$d_2 = 0.15d + 0.5$ in.	$t = 0.1d + 0.3$ in.	$h_1 = 0.14d + 0.4$ in.
$l_1 = 0.4d + 1$ in.	$t_1 = 0.15d + 0.5$ in.	$h_2 = 0.67d + 1.2$ in.
$l_2 = d + 1.5$ in.	$t_2 = 0.13d + 0.4$ in.	$h_3 = 0.3d + 0.7$ in.

The gland is of brass.

Packings for hydraulic cylinders may be proportioned as shown in Figs. 157 and 158. The construction shown in Fig. 158 is adapted to pressures of 2,500 to 3,000 lb per sq in., and **hemp packing** is used.

Sometimes the ram of the press cylinder is **packed with leather**, as shown in Figs. 159 to 162. The proportions may be as given in Table 93. For high-pressure and superheated steam, stuffing boxes are usually provided with metallic packing.

Labyrinth Packing. Leakage past dummy or balance pistons of steam turbines of the Parsons type may be reduced to a small amount by a labyrinth packing (Fig. 163), where the steam pressure is throttled down by passing

this force is obtained by subtracting from the pressure on one face the corresponding pressure on the other face of the piston. In Fig. 167 are shown the indicator cards *ABCD* and *A'B'C'D'* for the forward and return strokes of a single-cylinder double-acting steam engine; the total pressures are given by the curves *abc* and *a'b'c'*. In Fig. 168 the indicator card *ABCDEF* of a single-cylinder, single-acting, four-cycle Diesel engine is shown.

Table 96. Tangential Factors and Piston Velocities

(Tangential pressure on crank = resultant horizontal pressure times tabular quantity. Forward stroke is toward crankshaft. Wrist-pin velocity = crank-pin velocity times tabular quantity.) Calculated from $\sin a + \dfrac{r \sin 2a}{2l\sqrt{1 - \frac{r^2}{l^2}\sin^2 a}}$

Crank angles forward, deg a	Ratio of connecting-rod length to crank length										
	$\frac{l}{r}$ = 2.5	3	3.5	4	4.5	5	5.5	6	7	8	
5	0.1219	0.1161	0.1120	0.1089	0.1065	0.1045	0.1029	0.1016	0.0996	0.0980	0.0872
10	0.2442	0.2307	0.2226	0.2164	0.2117	0.2079	0.2048	0.2022	0.1981	0.1950	0.1736
15	0.3594	0.3425	0.3304	0.3215	0.3145	0.3089	0.3043	0.3005	0.2946	0.2901	0.2588
20	0.4718	0.4499	0.4343	0.4227	0.4136	0.4064	0.4006	0.3957	0.3880	0.3822	0.3420
25	0.5781	0.5516	0.5329	0.5189	0.5081	0.4995	0.4925	0.4866	0.4774	0.4706	0.4226
30	0.6768	0.6464	0.6250	0.6091	0.5968	0.5870	0.5791	0.5724	0.5620	0.5542	0.5000
35	0.7667	0.7331	0.7097	0.6923	0.6788	0.6682	0.6595	0.6522	0.6409	0.6325	0.5736
40	0.8466	0.0108	0.7859	0.7675	0.7533	0.742[illegible]	[illegible].7329	0.7253	0.7134	0.7045	0.6428
45	0.9156	0.8786	0.8530	0.8341	0.8196	0.808[illegible]	0.7988	0.7910	0.7789	0.7699	0.7071
50	0.9730	0.9358	0.9102	0.8915	0.8771	0.8657	0.8565	0.8488	0.8368	0.8279	0.7660
55	1.0181	0.9820	0.9572	0.9392	0.9253	0.9144	0.9055	0.8982	0.8867	0.8782	0.8192
60	1.0507	1.0168	0.9937	0.9769	0.9641	0.9540	0.9457	0.9390	0.9284	0.9205	0.8660
65	1.0707	1.0402	1.0196	1.0046	0.9932	0.9842	0.9769	0.9709	0.9615	0.9545	0.9063
70	1.0784	1.0525	1.0350	1.0224	1.0127	1.0051	0.9990	0.9939	0.9860	0.9801	0.9397
75	1.0743	1.0539	1.0402	1.0303	1.0228	1.0169	1.0121	1.0081	1.0020	0.9974	0.9659
80	1.0592	1.0452	1.0357	1.0289	1.0238	1.0197	1.0164	1.0137	1.0095	1.0063	0.9848
85	1.0341	1.0269	1.0221	1.0186	1.0160	1.0139	1.0122	1.0109	1.0087	1.0071	0.9962
90	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
95	0.9583	0.9655	0.9703	0.9738	0.9764	0.9785	0.9801	0.9815	0.9837	0.9853	0.9962
100	0.9104	0.9245	0.9339	0.9407	0.9459	0.9499	0.9532	0.9559	0.9601	0.9633	0.9848
105	0.8575	0.8779	0.8916	0.9015	0.9090	0.9150	0.9198	0.9237	0.9299	0.9344	0.9659
110	0.8010	0.8269	0.8444	0.8570	0.8667	0.87[illegible]	0.8804	0.8855	0.8934	0.899[illegible]	0.9397
115	0.7419	0.7724	0.7930	0.8080	0.8194	0.8284	0.8357	0.8417	0.8511	0.8581	0.9063
120	0.6814	0.7153	0.7383	0.7551	0.7680	0.778[illegible]	0.7863	0.7931	0.8037	0.8116	0.8660
125	0.6202	0.6564	0.6811	0.6991	0.7130	0.7239	0.7328	0.7401	0.7516	0.7601	0.8192
130	0.5591	0.5963	0.6219	0.6406	0.6550	0.6664	0.6756	0.6833	0.6953	0.7042	0.7660
135	0.4986	0.5356	0.5612	0.5801	0.5946	0.6061	0.6154	0.6232	0.6353	0.6444	0.7071
140	0.4390	0.4748	0.4997	0.5181	0.5322	0.5435	0.5526	0.5602	0.5721	0.5810	0.6428
145	0.3805	0.4140	0.4375	0.4549	0.4683	0.4790	0.4877	0.4949	0.5062	0.5147	0.5736
150	0.3232	0.3536	0.3750	0.3909	0.4032	0.4130	0.4209	0.4276	0.4380	0.4458	0.5000
155	0.2672	0.2937	0.3124	0.3263	0.3371	0.3457	0.3528	0.3586	0.3678	0.3747	0.4226
160	0.2122	0.2342	0.2498	0.2614	0.2704	0.2776	0.2835	0.2884	0.2961	0.3018	0.3420
165	0.1583	0.1752	0.1872	0.1962	0.2033	0.2088	0.2133	0.2171	0.2231	0.2276	0.2588
170	0.1051	0.1165	0.1247	0.1309	0.1356	0.1394	0.1425	0.1451	0.1492	0.1523	0.1736
175	0.0524	0.0582	0.0623	0.0654	0.0679	0.0698	0.0714	0.0727	0.0748	0.0763	0.0872
180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The inertia force of the reciprocating parts per square inch of piston area is obtained by multiplying the mass of the parts per square inch of piston area (piston, piston-rod, crosshead, and 0.35 to 0.45 of the connecting rod) by the acceleration of the piston (see Table 97). It is represented by the ordinates of the curves *xyz* and *x'yz'* for the forward and return strokes, respectively. The *ordinates between the total steam or gas pressure and the inertia curves* (p_1, p_2, p', and p'') give the resultant pressures acting on the piston along the center line of the engine.

Crank-Gearing

Analysis of the Motion of the Reciprocating Parts. Let r = crank radius, in.; R = crank radius, ft; $s = 2r$ = stroke, in.; l = length of connecting rod, in.; x = displacement of piston from head end dead center, in.; a = crank angle corresponding to x; b = connecting-rod angle corresponding to x; v = crank-pin velocity, fps; c = piston velocity, fps; N = rpm of the crank. Then, when the center lines of the piston rod and crankshaft lie in the same plane,

$$x = r(1 - \cos a) + l(1 - \cos b) = r\left(1 - \cos a + \frac{r}{2l}\sin^2 a\right), \text{ approx.}$$

Table 94. Piston Positions for Various Crank Angles

(From beginning of stroke toward crankshaft. To find distance of piston from beginning of stroke, multiply tabular value by length of stroke.)

Calculated from $\frac{x}{2r} = \frac{1}{2}(1 - \cos a) + \frac{l}{2r}\left(1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2 a}\right)$

Crank angles, deg, a	Ratio of length of connecting rod to length of crank										
	$\frac{l}{r}$ = 2.5	3	3.5	4	4.5	5	5.5	6	7	8	∞
5	0.0027	0.0025	0.0024	0.0024	0.0023	0.0023	0.0022	0.0022	0.0022	0.0021	0.0019
10	0.0106	0.0101	0.0098	0.0095	0.0093	0.0091	0.0090	0.0089	0.0087	0.0085	0.0076
15	0.0238	0.0226	0.0218	0.0212	0.0208	0.0204	0.0201	0.0198	0.0194	0.0191	0.0170
20	0.0419	0.0399	0.0385	0.0375	0.0367	0.0360	0.0355	0.0350	0.0343	0.0338	0.0302
25	0.0648	0.0618	0.0597	0.0580	0.0568	0.0558	0.0550	0.054[illegible]	0.0532	0.0524	0.0468
30	0.0922	0.0880	0.0849	0.0827	0.0809	0.0795	0.0784	0.0774	0.0759	0.0748	0.0670
35	0.1238	0.1181	0.1141	0.1111	0.1088	0.1069	0.1054	0.1042	0.1022	0.1007	0.0904
40	0.1590	0.1518	0.1467	0.1430	0.1401	0.1377	0.1358	0.1342	0.1318	0.1299	0.1170
45	0.1975	0.1887	0.1825	0.1779	0.1744	0.1716	0.1693	0.167[illegible]	0.1643	0.1621	0.1464
50	0.2387	0.2283	0.2210	0.2156	0.2114	0.2081	0.2054	0.2032	0.1996	0.1970	0.1786
55	0.2822	0.2702	0.2618	0.2556	0.2508	0.2470	0.2439	0.2413	0.2373	0.2342	0.2132
60	0.3274	0.3139	0.3044	0.2974	0.2921	0.2878	0.2843	0.2814	0.2769	0.2735	0.2500
65	0.3737	0.3586	0.3484	0.3407	0.3348	0.3301	0.3263	0.3231	0.3182	0.3144	0.2887
70	0.4207	0.4045	0.3932	0.3850	0.3786	0.3735	0.3694	0.3660	0.3607	0.3567	0.3290
75	0.4677	0.4505	0.4386	0.4298	0.4230	0.4177	0.4133	0.4097	0.4041	0.3999	0.3706
80	0.5142	0.4963	0.4839	0.4747	0.4677	0.4621	0.4576	0.4539	0.4480	0.4436	0.4132
85	0.5599	0.5415	0.5288	0.5194	0.5122	0.5065	0.5019	0.4981	0.4920	0.4876	0.4564
90	0.6044	0.5858	0.5729	0.5635	0.5563	0.5505	0.5458	0.5420	0.5359	0.5314	0.5000
95	0.6471	0.6287	0.6160	0.6066	0.5994	0.5937	0.5891	0.5852	0.5792	0.5747	0.5436
100	0.6879	0.6699	0.6575	0.6484	0.6414	0.6358	0.6313	0.6275	0.621[illegible]	0.6172	0.5868
105	0.7265	0.7093	0.6974	0.6886	0.6819	0.6765	0.6722	0.6685	0.6629	0.6587	0.6294
110	0.762[illegible]	0.7465	0.7353	0.7270	0.7206	0.7156	0.7114	0.7080	0.7027	0.6987	0.6710
115	0.7963	0.7814	0.7710	0.7633	0.7574	0.7527	0.7489	0.7457	0.7408	0.7371	0.7113
120	0.8274	0.8139	0.8044	0.7974	0.7921	0.7878	0.7843	0.7814	0.7769	0.7735	0.7500
125	0.8558	0.8438	0.8354	0.8292	0.8244	0.8206	0.8175	0.8149	0.8108	0.8078	0.7868
130	0.8815	0.8711	0.8638	0.8584	0.8542	0.8509	0.8482	0.8459	0.8424	0.8398	0.8214
135	0.9046	0.8958	0.8896	0.8851	0.8815	0.8787	0.8764	0.8745	0.8715	0.8692	0.8536
140	0.9250	0.9179	0.9128	0.9090	0.9061	0.9038	0.9019	0.9003	0.8978	0.8960	0.8830
145	0.9429	0.9372	0.9332	0.9302	0.9279	0.9261	0.9246	0.9233	0.9213	0.9199	0.9096
150	0.9583	0.9540	0.9510	0.9487	0.9469	0.9455	0.9444	0.9434	0.9420	0.9408	0.9330
155	0.9711	0.9681	0.9660	0.9643	0.9631	0.9621	0.9613	0.9606	0.9595	0.9587	0.9532
160	0.9816	0.9796	0.9782	0.9772	0.9764	0.9757	0.9752	0.9747	0.9740	0.9735	0.9698
165	0.9897	0.9886	0.9878	0.9872	0.9867	0.9863	0.9860	0.9858	0.9854	0.9851	0.9830
170	0.9954	0.9949	0.9946	0.9943	0.9941	0.9939	0.9938	0.9937	0.9935	0.9933	0.9924
175	0.9989	0.9987	0.9986	0.9986	0.9985	0.9985	0.9984	0.9984	0.9984	0.9983	0.9981
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 94 gives correct piston positions corresponding to various crank angles. Table 95 gives also crank angles corresponding to various piston positions.

When, however, the center line of motion of the reciprocating masses is located a distance h from the center of rotation of the crank, as shown in

moving masses; and the areas *DEF* and *HAB* represent the energy per square inch of piston area liberated in the retardation of the moving masses for the production of useful work in the driven unit.

In the case of multicylinder engines, the tangential effort curves for each cylinder are superimposed, properly shifted with respect to each other, corresponding to the angular location of their respective cranks.

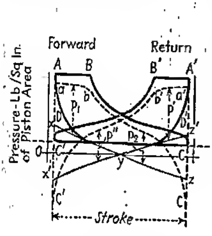


Fig. 167.—Indicator Cards of Double-acting Steam Engine.

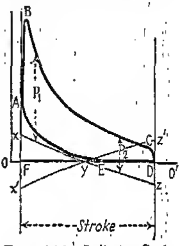


Fig. 168.—Indicator Card of Single-acting Diesel Engine.

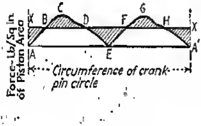


Fig. 169.—Tangential Force at Crankpin for Steam Engine of Fig. 167.

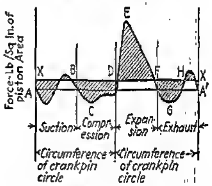


Fig. 170.—Tangential Force at Crankpin for Diesel Engine of Fig. 168.

Determination of Flywheel Weight. Let M = mass of wheel rim; W = weight of wheel rim, lb; v_1 = maximum linear rim velocity, fps; v_2 = minimum rim velocity, fps; $v = \frac{1}{2}(v_1 + v_2)$ = average rim velocity, fps; $k = (v_1 - v_2)/v$ = coefficient of velocity fluctuation; D = mean diam of rim, ft; N = rpm; A = piston area, sq in; E_m = energy represented by area *BCD* or *FGH* or *DEF* (Fig. 169) = maximum energy in ft-lb per sq in. of piston area which must be absorbed or liberated by the wheel rim with velocity variation between v_1 and v_2. Then

$$E_m \times A = M\frac{v_1^2}{2} - M\frac{v_2^2}{2} = \frac{W}{g}kv^2 = \frac{kWD^2N^2}{11,744}; \text{ or } W = \frac{11,744E_mA}{kD^2N^2}$$

About $\frac{9}{10}$ W may be placed in the rim to account for the flywheel effect

Crank Gearing

Analysis of the Motion of the Reciprocating Parts. Let r = crank radius, in.; R = crank radius, ft; $s = 2r$ = stroke, in.; l = length of connecting rod, in.; x = displacement of piston from head end dead center, in.; a = crank angle corresponding to x; b = connecting-rod angle corresponding to x; v = crank-pin velocity, fps; c = piston velocity, fps; N = rpm of the crank. Then, when the center lines of the piston rod and crankshaft lie in the same plane,

$$x = r(1 - \cos a) + l(1 - \cos b) = r\left(1 - \cos a + \frac{r}{2l}\sin^2 a\right), \text{ approx.}$$

Table 94. Piston Positions for Various Crank Angles

(From beginning of stroke toward crankshaft. To find distance of piston from beginning of stroke, multiply tabular value by length of stroke.)

Calculated from $\frac{x}{2r} = \frac{1}{2}(1 - \cos a) + \frac{l}{2r}\left(1 - \sqrt{1 - \frac{r^2}{l^2}\sin^2 a}\right)$

Crank angles, deg, a	Ratio of length of connecting rod to length of crank										
	$\frac{l}{r}$ = 2.5	3	3.5	4	4.5	5	5.5	6	7	8	∞
5	0.0027	0.0025	0.0024	0.0024	0.0023	0.0023	0.0022	0.0022	0.0022	0.0021	0.0019
10	0.0106	0.0101	0.0098	0.0095	0.0093	0.0091	0.0090	0.0089	0.0087	0.0085	0.0076
15	0.0238	0.0226	0.0218	0.0212	0.0208	0.0204	0.0201	0.0198	0.0194	0.0191	0.0170
20	0.0419	0.0399	0.0385	0.0375	0.0367	0.0360	0.0355	0.0350	0.0343	0.0338	0.0302
25	0.0648	0.0618	0.0597	0.0580	0.0568	0.0558	0.0550	0.0543	0.0532	0.0524	0.0468
30	0.0922	0.0880	0.0849	0.0827	0.0809	0.0795	0.0784	0.0774	0.0759	0.0748	0.0670
35	0.1238	0.1181	0.1141	0.1111	0.1088	0.1069	0.1054	0.1042	0.1022	0.1007	0.0904
40	0.1590	0.1518	0.1467	0.1430	0.1401	0.1377	0.1358	0.1342	0.1318	0.1299	0.1170
45	0.1975	0.1887	0.1825	0.1779	0.1744	0.1716	0.1693	0.1674	0.1643	0.1621	0.1464
50	0.2387	0.2283	0.2210	0.2156	0.2114	0.2081	0.2054	0.2032	0.1996	0.1970	0.1786
55	0.2822	0.2702	0.2618	0.2556	0.2508	0.2470	0.2439	0.2413	0.2373	0.234[illegible]	0.2132
60	0.3274	0.3139	0.3044	0.2974	0.2921	0.2878	0.2843	0.2814	0.2769	0.2735	0.2500
65	0.3737	0.3588	0.3484	0.3407	0.3348	0.3301	0.3263	0.3231	0.3182	0.3144	0.2887
70	0.4207	0.4045	0.3932	0.3850	0.3786	0.3735	0.3694	0.3660	0.3607	0.3567	0.3290
75	0.4677	0.4505	0.4386	0.4298	0.4230	0.4177	0.4133	0.4097	0.4041	0.3999	0.3706
80	0.5142	0.4963	0.4839	0.4747	0.4677	0.4621	0.4576	0.4539	0.4480	0.4436	0.4132
85	0.5599	0.5415	0.5288	0.519[illegible]	0.5122	0.5065	0.5019	0.4981	0.4920	0.4876	0.4564
90	0.6044	0.5858	0.5729	0.5635	0.5563	0.5505	0.5458	0.5420	0.5359	0.5314	0.5000
95	0.6471	0.6287	0.6160	0.6066	0.5994	0.5937	0.5891	0.5852	0.5792	0.5747	0.5436
100	0.6879	0.6699	0.6575	0.6484	0.6414	0.6358	0.6313	0.6275	0.6216	0.6172	0.5868
105	0.7265	0.7093	0.6974	0.6886	0.6819	0.6765	0.6722	0.6685	0.6629	0.6587	0.6294
110	0.7627	0.7465	0.7353	0.7270	0.7206	0.7156	0.7114	0.7080	0.7027	0.6987	0.6710
115	0.7963	0.7814	0.7710	0.7633	0.7574	0.7527	0.7489	0.7457	0.7408	0.7371	0.7113
120	0.8274	0.8139	0.8044	0.7974	0.7921	0.7878	0.7843	0.7814	0.7769	0.7735	0.7500
125	0.8558	0.8438	0.8354	0.8292	0.8244	0.8206	0.8175	0.8149	0.8108	0.8078	0.7868
130	0.8815	0.8711	0.8638	0.8584	0.8542	0.8509	0.8482	0.8459	0.8424	0.8398	0.8214
135	0.9046	0.8958	0.8896	0.8851	0.8815	0.8787	0.8764	0.8745	0.8715	0.8692	0.8536
140	0.9250	0.9179	0.9128	0.9090	0.9061	0.9038	0.9019	0.9003	0.8978	0.8960	0.8830
145	0.9429	0.9372	0.9332	0.9302	0.9279	0.9261	0.9246	0.9233	0.9213	0.9199	0.9096
150	0.9583	0.9540	0.9510	0.9487	0.9469	0.9455	0.9444	0.9434	0.9420	0.9408	0.9330
155	0.9711	0.9681	0.9660	0.9643	0.9631	0.9621	0.9613	0.9606	0.9595	0.9587	0.9532
160	0.9816	0.9796	0.9782	0.9772	0.9764	0.9757	0.9752	0.9747	0.9740	0.9735	0.9698
165	0.9897	0.9886	0.9878	0.9872	0.9867	0.9863	0.9860	0.9858	0.9854	0.9851	0.9830
170	0.9954	0.994[illegible]	0.9946	0.9943	0.9941	0.9939	0.9938	0.9937	0.9935	0.9933	0.9924
175	0.9989	0.9987	0.9986	0.9986	0.9985	0.9985	0.9984	0.9984	0.9984	0.9983	0.9981
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 94 gives correct piston positions corresponding to various crank angles. Table 95 gives also crank angles corresponding to various piston positions.

When, however, the center line of motion of the reciprocating masses is located a distance h from the center of rotation of the crank, as shown in

Nearly all governors depend for their action upon centrifugal force, and consist of a pair of masses rotating about a spindle driven by the prime mover and kept from flying outward by a controlling force, generally applied by springs. With an increase in speed, this controlling force is overcome and the masses move outward and this motion is transmitted to valves supplying the prime mover with its working fluid or fuel. In small machines, the governor acts directly upon the regulating valve or valves, while in large machines the governor moves small controlling valves admitting fluid under pressure into operating cylinders actuating the main governing valves (see also p. 218).

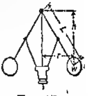


FIG. 172.

Conical Pendulum Governor (Fig. 172). In this governor, invented by Watt, the revolving masses are balls attached to a vertical spindle by link arms, and the controlling force consists of the weights of the balls themselves. There is a definite ball position for any particular speed, and the governor is said to be "static." For any position of the balls, the height of the cone of revolution of the balls and their arms is h (in.) $= 35{,}200/n^2$, where n = rpm. The sensitiveness of this governor, or the sleeve movement for a given change in speed, rapidly falls off as n increases.

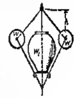


FIG. 173.

Loaded Governor. The **Porter governor** (Fig. 173) is a Watt governor with the addition of a heavy movable weight surrounding the spindle. For a given variation of speed, the height h (in.) is greater than with the Watt governor, being, when the four arms are of equal length, $h = [(w + w_1)/w] \times (35{,}200/n^2)$, where w = weight of one ball and w_1 = weight surrounding spindle, both in pounds. This type of governor can be run at much higher speeds than the Watt, and it is much more powerful with the same weights of balls.

Crossed-arm Governor. This is a conical pendulum governor in which each arm is suspended from a pin placed at the opposite side of the spindle to that of its ball. By proper proportioning, the balls may be made to move along an approximately parabolic path, and the height h made practically constant for the whole range of the governor, resulting in extreme sensitiveness.

Spring-loaded governors are those in which the controlling force is wholly or partly produced by springs. This permits the use of a horizontal or inclined axis of rotation.

Shaft governors consist essentially of a wheel keyed to the main shaft of the engine and carrying weights pivoted near its circumference, which are free to move outward under the influence of centrifugal force, this tendency being resisted by springs. In steam engines, the movement of the weights is communicated by links to an eccentric, thus varying the travel of the valve.

In **inertia shaft governors,** a weight arm is so pivoted and has its weight so distributed that its inertia assists in making the eccentric adjustment, and the more sudden the change in load the greater the assistance it renders. Thus, when the engine speeds up the weight tends to rotate at its old speed by reason of its inertia, and hence lags behind the wheel. Also, if the load is suddenly increased, the engine will slow down, but the weight arm will continue at its old speed, gaining on the wheel and again assisting the centrifugal force in altering the position of the eccentric with respect to the crank. In the Rites inertia governor, the weight arm is a long bar with heavy ends which is pivoted off its center to a pin some distance from the

to. AH is the center line of the piston rod produced, AP is the connecting rod, OP the crank with center at O. Draw OQ at right angles to AH. For any position AP of the connecting rod cutting OQ in E, the length OE represents the velocity of the reciprocating parts to the same scale to which OP

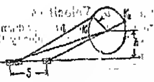


Fig. 164.

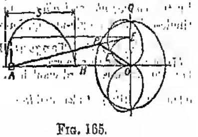


Fig. 165.

represents the crank-pin velocity. The graphical construction of Fig. 165 shows how to construct velocity ellipses for the whole revolution.

Determination of the Acceleration of the Reciprocating Parts. The acceleration p of the reciprocating parts is given by the equation

$$p = v^2(\cos a + r \cos 2a/l)/R, \text{ approx}$$

when the centerlines of the pistons and crankshaft lie in the same plane.

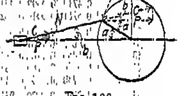


Fig. 166

Table 97 gives the values of the acceleration for different crank angles.

If the cylinder is offset, as in Fig. 164, the acceleration of the reciprocating parts may be found by the approximate formula

$$p = (v^2/R)[\cos a + (r/l) \cos 2a + h/l \sin a]$$

The inertia force in pounds per square inch of piston is $f = Mp/A = Wv^2(\cos a + r \cos 2a/l)/AgR = 0.00034WN^2R(\cos a + r \cos 2a/l)/A$, where W = total weight of reciprocating masses, lb; A = piston area, sq in. The value of W for any given engine is to be taken as the weight of the piston, piston rod, cross head, and a portion of the connecting rod. It is usual to include ½ to ⅔ the connecting-rod weight as part of the reciprocating masses. Values of $\cos a + (r/l) \cos 2a$ are given in Table 97.

Trooien shows that the weight of reciprocating parts for steam engines may be expressed by the formula $W = KD^2/SN^2$, where K = 1,370,000 to 3,400,000, with average value of 2,000,000; W = weight of parts, lb; D = cylinder diam, in.; S = stroke, in., and N = rpm.

Relation between the Force in the Line of Piston Travel and the Tangential Reaction at the Crank Pin. In Fig. 166, if P is the force in the line of piston travel, the force acting along the connecting rod is $C = P/\cos b$, and that on the guides $N = P \tan b$. The force tangent to the crank-pin circle at the crank pin is $T = C \sin (a + b) = P \sec b \sin (b + a) = P \sin a \cdot (1 + r \cos a/\sqrt{l^2 - r^2 \sin^2 a})$. Table 96 gives values of $\sec b \sin (b + a)$ for various values of l/r. They are equal to the tangential factors of piston velocities for different crank angles.

Determination of Flywheel Weight

Tangential Effort Curves. The total force on the piston at each instant is obtained from the indicator card. In the case of a double-acting engine,

between the limits of no steam supply and full steam supply. If this range is but a small percentage of n, the governor is said to be sensitive.

The **configuration of a governor** at any speed within its range may be determined, as in Fig. 174, a numerical example of a Watt governor being taken, in which $w = 10$ lb. Assuming the upper end of ball arm (Fig. 172) to be pivoted at the axis of spindle, $r^2 + h^2 = L^2 =$ constant, or, for any position, $r = \sqrt{L^2 - h^2}$. Now, $h = 35{,}200/n^2 = 9.78$ in. for 60 rpm, 7.19 in. for 70 rpm, and 5.5 in. for 80 rpm. Assuming radius $r = 10$ in. at 60 rpm, $L^2 =$ constant $= 195.6$, whence $r = 12$ in. at 70 rpm (for $h = 7.19$ in.) $= 12.8$ in. at 80 rpm. In Fig. 187, at any radius greater than 12.8 in., say 16 in., erect an ordinate AD. Calculate the values of $F = 0.00034rn^2w$ taking $r = 16$ in., $w = 10$ lb, and $n = 60$, 70, and 80, respectively, and lay these values off to scale, as AB, AC, AD. Draw OB, OC, and OD. Take $r_1 = 10$ in. by scale, and erect an ordinate cutting OB or the 60 rpm line at a; locate points b and c similarly, using radii r_2 and r_3 of 12 in. and 12.8 in., respectively, and connect a, b, and c by a smooth curve. This is the **curve of controlling force** of the governor, and the ball path radius for any intermediate speed may be obtained by suitably scaling AD, drawing a line from the desired speed value on this scale to O and dropping an ordinate to the radius scale from its intersection with the curve abc. When a tangent to any point makes a greater angle with OA than does a line drawn from O through the same point, the governor is stable throughout its range. The influence of friction can be readily seen by plotting $F + f$ and $F - f$ curves.

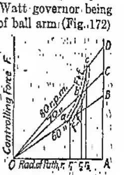


FIG. 174.

A governor is said to be **isochronous** (or astatic) when the speed of rotation is the same for all positions within its limits of movement. On account of the frictional resistances of the joints, however, the speed must always increase slightly before any additional centrifugal force is available, and this will move the weights to their extreme outward position; conversely, when the speed falls sufficiently for gravity or spring force to overcome the frictional resistance, the weights will move to their extreme inward position. In this way, the engine will fall into a state of speed oscillation or **hunting**, which is obviously objectionable and may be avoided by giving the governor a fair degree of stability, by reducing the frictional resistances as much as possible, and by the use of dashpots which offer great resistance to sudden movements but allow comparatively slow changes of position to take place freely.

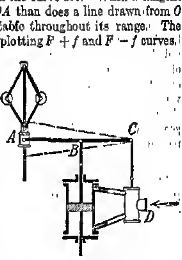


FIG. 175.—Governor Restoring Mechanism.

A **restoring mechanism** (Fig. 175) must be included between the governor sleeve and the pilot valve of a hydraulic relay to avoid "hunting." When the governor sleeve A rises, B forms at first a fixed fulcrum; pilot valve D is lowered and admits pressure fluid to the under side of the piston, which, in traveling upward, restores the pilot valve to its mid-position by using A as a fulcrum. This ensures for every speed of the turbine, in the steady state, a corresponding definite position of the valve, hence a definite output.

Table 97. Inertia Factors and Piston Accelerations

(Values of $\cos\alpha + r\cos 2\alpha/ly + r^2 \sin^2 2\alpha/4l^3y^3$, where $y = \sqrt{1 - \frac{r^2}{l^2}\sin^2\alpha}$. Algebraic signs relate to forward stroke; use opposite signs for return stroke.)

Crank angles forward, deg, α	Ratio of connecting-rod length to crank length										
	$\frac{l}{r}$ = 2.5	3	3.5	4	4.5	5	5.5	6	7	8	∞
0	1.4000	1.3333	1.2857	1.2500	1.2222	1.2000	1.1818	1.1667	1.1429	1.1250	1.0000
5	1.3908	1.3249	1.2778	1.2426	1.2152	1.1932	1.1753	1.1604	1.1369	1.1193	.9962
10	1.3635	1.2997	1.2543	1.2204	1.1941	1.1731	1.1559	1.1416	1.1192	1.1024	.9848
15	1.3183	1.2580	1.2155	1.1839	1.1594	1.1399	1.1239	1.1107	1.0899	1.0744	.9659
20	1.2558	1.2006	1.1621	1.1335	1.1116	1.0941	1.0799	1.0681	1.0496	1.0357	.9397
25	1.177[illegible]	1.1283	1.0948	1.0702	1.0514	1.0365	1.0244	1.0144	.9987	.9871	.9063
30	1.0829	1.0423	1.0149	.9950	.9799	.9681	.9585	.9505	.9382	.9290	.8660
35	.9750	.9439	.9236	.9091	.8983	.8898	.8830	.8775	.8688	.8624	.8192
40	.8551	.8349	.8225	.8140	.8078	.8031	.7993	.7963	.7917	.7883	.7660
45	.7252	.7172	.7133	.7112	.7100	.7092	.7086	.7083	.7078	.7076	.7071
50	.5878	.5929	.5980	.6026	.6064	.6097	.6124	.6148	.6186	.6215	.6428
55	.4455	.4643	.4787	.4899	.4988	.5061	.5121	.5171	.5250	.5310	.5736
60	.3013	.3338	.3574	.3751	.3889	.4000	.4091	.4167	.4286	.4375	.5000
65	.1583	.2041	.2363	.2601	.2785	.2931	.3050	.3149	.3305	.3420	.4226
70	.0197	.0776	.1175	.1468	.1692	.1869	.2013	.2132	.2319	.2458	.3420
75	−.1117	−.0434	.0030	.0368	.0625	.0828	.0993	.1129	.1341	.1499	.2588
80	−.2329	−.1567	−.1054	−.0682	−.0400	−.0178	.0002	.0150	.0381	.0553	.1736
85	−.3417	−.2605	−.2062	−.1669	−.1372	−.1139	−.0949	−.0793	−.0550	−.0369	.0872
90	−.4364	−.3536	−.2981	−.2582	−.2279	−.2041	−.1849	−.1690	−.1443	−.1260	0
95	−.5160	−.4348	−.3805	−.3412	−.3115	−.2881	−.2692	−.2536	−.2293	−.2112	−.0872
100	−.5802	−.5040	−.4527	−.4155	−.3873	−.3651	−.3471	−.3323	−.3092	−.2920	−.1736
105	−.6293	−.5610	−.5146	−.4809	−.4551	−.4348	−.4184	−.4048	−.3835	−.3677	−.2588
110	−.664[illegible]	−.6054	−.5665	−.5373	−.5140	−.4971	−.4827	−.4708	−.4521	−.4382	−.3420
115	−.6869	−.6411	−.6090	−.5851	−.5667	−.5521	−.5402	−.5303	−.5148	−.5032	−.4226
120	−.6987	−.6662	−.6426	−.6249	−.6111	−.6000	−.5909	−.5833	−.5714	−.5625	−.5000
125	−.7016	−.6829	−.6685	−.6573	−.6483	−.6411	−.6351	−.6301	−.6221	−.6161	−.5736
130	−.697[illegible]	−.6927	−.6875	−.6830	−.6792	−.6759	−.6732	−.6708	−.6670	−.6641	−.6428
135	−.6890	−.6970	−.7009	−.7030	−.7043	−.7050	−.7056	−.7059	−.7064	−.7066	−.7071
140	−.6770	−.6971	−.7096	−.7181	−.7243	−.7290	−.7328	−.7358	−.7404	−.7438	−.7660
145	−.6633	−.6944	−.7147	−.7292	−.7400	−.7485	−.7553	−.7609	−.7695	−.7759	−.8192
150	−.6491	−.6898	−.7172	−.7370	−.7521	−.7640	−.7736	−.7815	−.7939	−.8030	−.8660
155	−.6356	−.6843	−.7178	−.7424	−.7612	−.7761	−.788[illegible]	−.7982	−.8139	−.8256	−.9063
160	−.6236	−.6788	−.7173	−.7458	−.7678	−.7853	−.7995	−.8113	−.8298	−.8436	−.9397
165	−.6136	−.6738	−.7163	−.7480	−.7725	−.7920	−.8079	−.8212	−.8419	−.8575	−.9659
170	−.6061	−.6700	−.7153	−.7492	−.7755	−.7965	−.8137	−.8280	−.8504	−.8673	−.9848
175	−.6015	−.6675	−.7146	−.7498	−.7772	−.7991	−.8171	−.8320	−.8555	−.8731	−.9962
180	−.6000	−.6667	−.7143	−.7500	−.7778	−.8000	−.8182	−.8333	−.8571	−.8750	−1.0000

The tangential reactions at the crankpin can be obtained (p. 855) by multiplying the resultant pressure on the piston by the tangential factors for the various crank angles (see Table 96). In Fig. 169, curve *ACEGA'* shows the tangential reaction of the steam engine of Fig. 167 plotted against the travel of the crank, and Fig. 170 shows the same for the Diesel engine of Fig. 168.

In engine design, it is usual to construct the tangential effort curve from the cards obtained at normal or rated engine load and to assume that the engine is working against a constant and uniform torque. The uniform resisting torque is represented in Figs. 169 and 170 by *XX*, which is so located that the shaded areas above and below it are equal to one another. The areas *BCD* and *FGH* represent in foot-pounds per square inch of piston area the amount of energy which is effective in increasing the kinetic energy of the

$P = W/ld$ = unit pressure, lb per sq in.
N = rpm of a shaft.
μ = viscosity, in (lb)(sec)/(in.^{2}) units.
Z = viscosity, centipoises.
β = angle between load and entering edge of oil film.
c = coefficient for side leakage of oil.
h_0 = minimum film thickness.
$e = h_0/s$ = ratio of minimum film thickness to radial clearance.
f = coefficient of friction.
F = friction force, lb.
K = operating characteristic of a plain cylindrical bearing (see below).
K_f = coefficient for friction of a plain cylindrical bearing (see p. 867).
t_w = temp of bearing wall, deg F.
t_a = temp of air deg F.
t_1 = temp of oil film, deg F.

In bearings designed to work under conditions of greasy or semifluid *lubrication, the allowable mean intensity of pressure $P = W/ld$ is as* follows:

Hardened alloy steel on hardened steel	2,100
Hardened alloy steel on bronze	1,300
Alloy steel on bronze	850
Mild steel, smooth finish on bronze	550
Mild steel, ordinary finish, on bronze	400
Cast iron on bronze	400
Mild steel on cast iron	350
Mild steel on lignum-vitae with water lubrication	350

Fluid lubrication in plain cylindrical bearings depends on the viscosity of the lubricant and on its adhesion to the surfaces of the journal and the bearing. The radial clearance provided in the bearing forms, automatically, a wedge-shaped film between the journal and the bearing. The oil is entrained by the journal into the film. A hydrostatic pressure is created in the film, sufficient to float the journal and carry the load applied to it.

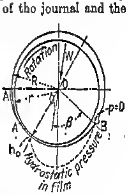


FIG. 1.—Journal Bearing with Perfect Lubrication.

The **minimum film thickness** h_0 determines the closest approach of the journal and bearing surfaces with complete lubrication (Fig. 1). The allowable closest approach depends on the degree of finish of these surfaces and on the rigidity of the journal and bearing structures. In practice, $h_0 = 0.00075$ in. is common in electric motors and generators of medium speed, with steel shafts in babbitted bearings; h_0 = 0.003 to 0.005 in. for large steel shafts running at high speed in babbitted bearings, (turbogenerators, fans) with pressure oil-supply for lubrication; h_0 = 0.0001 to 0.0002 in. in automotive and aviation engines, with very fine finish of the surfaces.

Figure 2 gives the ratio, e, of the minimum film thickness to the radial clearance for a plain cylindrical journal. The operating characteristic of the bearing is

$$K = \frac{P}{\mu N}\left(\frac{R-r}{r}\right)^2 \frac{1}{c}.$$

In Fig. 1, β is the angle between the direction of the load W and the entering edge of the load-carrying oil film, in degrees. The entering edge is at the

of the arms and other rotating masses. Acceptable values of k are as follows: pumps, $1/20$ to $1/30$; machine shops, $1/35$ to $1/40$; looms and paper mills, $1/40$; spinning mills, $1/60$; d-c generator (lighting), $1/150$; a-c generators, $1/300$.

Wittenbauer's Analysis for Flywheel Performance. The method does not involve more computation work than the one described above, but it is more accurate where the reciprocating parts are comparatively heavy. Wittenbauer's method avoids the inaccuracy resulting from the evaluation of the inertia forces on the reciprocating parts on the basis of the uniform nominal speed of rotation for the engine.

Let the crank-pin velocity be represented by v_r and the velocity of any moving masses (m_1, m_2, m_3, etc.) at any instant or phase be represented, respectively, by v_1, v_2, v_3, etc. The kinetic energy of the entire engine system of moving masses may then be expressed as

$$E = \tfrac{1}{2}(m_1v_1^2 + m_2v_2^2 + m_3v_3^2 \ldots) = \tfrac{1}{2}M_rv_r^2$$

where the single reduced mass M_r at the crank pin which possesses the equivalent kinetic energy is

$$M_r = [m_1(v_1/v_r)^2 + m_2(v_2/v_r)^2 + m_3(v_3/v_r)^2 + \ldots]$$

In an engine mechanism, sufficiently accurate values of M_r can be obtained if the weight of the connecting rod is divided between the crank pin and the wrist pin so as to retain the center of gravity of the rod in its true position; usually 0.55 to 0.65 of the weight of the connecting rod should be placed on the crank pin, and 0.45 to 0.35 of the weight on the wrist pin. M_r is a variable in engine mechanisms on account of the reciprocating parts, and should be found for a number of crank positions. It should include all moving masses except the flywheel.

The total energy E used in accelerating reciprocating parts from the beginning of the forward stroke up to any crank position can be obtained by finding from the indicator cards the total work done in the cylinder (on both sides of the piston) up to that time and subtracting from it the work done in overcoming the resisting torque, which may usually be assumed constant. The mean energy of the moving masses is $E_0 = \tfrac{1}{2}M_rv_r^2$.

In Fig. 171, the reduced weights of the moving masses $G_F + G_{rs}$ are plotted on the X axis corresponding to different crank positions. $G_F = gM_F$ is the reduced flywheel weight and $G_{rs} = gM_{rs}$ is the sum of the other reduced weights. Against each of these abscissas is plotted the energy E available for acceleration measured from the beginning of the forward stroke. The curve 0123456 is the locus of these plotted points.

This diagram possesses the following property: Any straight line drawn from the origin O to any point in the curve is a measure of the velocity of the moving masses; tangents bounding the diagram measure the limits of velocity between which the crank pin will operate. The maximum linear velocity of the crank pin in feet per second is $v_2 = \sqrt{2g \tan \alpha_2}$, and the minimum velocity is $v_1 = \sqrt{2g \tan \alpha_1}$. Any desired change in v_1 and v_2 may be accomplished by changing the value of G_F, which means a change in the flywheel weight or a change in the flywheel weight reduced to the crank pin. As G_F is very large compared with G_r and the point 0 cannot be usually located on the diagram unless a very large drawing is made, the tangents are best formed by direct calculation:

$$\tan \alpha_2 = \frac{v_r^2}{2g}(1 + k); \qquad \tan \alpha_1 = \frac{v_r^2}{2g}(1 - k)$$

where k is the coefficient of velocity fluctuation. The two tangents ss and tt to the curve 0123456, thus drawn, cut on the ordinate E_0 a distance ΔE. The reduced flywheel weight is then found to be

$$G_F(\Delta E)g/v_r^2k$$

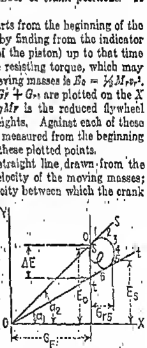


FIG. 171.—Analysis of Flywheel Performance.

Governors

Governors are mechanisms designed to maintain the speeds of prime movers (rpm) within reasonably constant limits, whatever the load may be.

be used. For crankpin bearings in punches and shears, mean pressures up to 3,000 lb per sq in. may be used.

Table 1. Current Practice in Mean Bearing Pressures

Type of bearing	Permissible pressure, lb per sq in. of projected area	*Type of bearing*	Permissible pressure, lb per sq in. of projected area
Diesel engines, main bearings	800–1,000	Gas engines, main bearings	500– 600
Crankpin	1,000–1,200	Crankpin	1,500–1,800
Wrist pin	1,800–2,000	Air compressors, main bearings	120– 240
		Crankpin	240– 400
Marine Diesel engines, main bearings	400– 500	Crosshead pin	400– 800
Crankpin	1,000–1,400	Aircraft engine crankpin	700–1,900
Marine line shaft bearings	25– 35	Centrifugal pumps	80– 100
Steam engines, main bearings	150– 500	Generators, low or medium speed	90– 140
Crankpin	800–1,500	Roll-neck bearings	1,500–2,000
Crosshead pin	1,000–1,800	Locomotive crankpins	1,500–1,700
Flywheel bearings	200– 250	Railway-car axle bearings	300– 350
Marine steam engine, main bearings	275– 500	Miscellaneous ordinary bearings	80– 150
Crankpin	400– 600	Light line shaft	15– 25
Steam turbines and reduction gears	70– 220	Heavy line shaft	100– 150

Length-diameter ratios are usually chosen between $l/d = 1$ and $l/d = 2$. In shorter bearings, the carrying capacity of the oil film is greatly impaired by the effect of side leakage. Longer bearings are used to restrain the shaft from vibration, as in line shafts, or to position the shaft accurately, as in

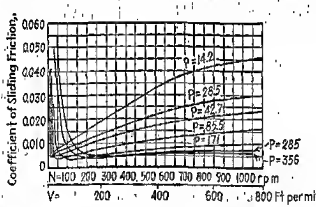


FIG. 4.—Variation of the Coefficient of Friction with Velocity and Pressure (in a 2¼ × 9¾-in. Bearing).

machine tools. In power machines, the tendency is toward shorter bearings. Typical values are as follows:

Turbogenerators, 0.8–1.5; stationary engines—for main bearings, 1.5–2.0, for crankpin bearings, 1; generators and motors 1.5–2.0; ordinary shafting—heavy, with fixed bearings, 2–3, light, with self-aligning bearings, 3–4; machine-tool bearings, 2–4.

For the **clearance between journal and bearing**, see Fits, p. 789. Medium fits may be used for journals running at speeds under 600 rpm, and

center of the shaft. In the Armstrong governor, a weight attached to the end of a laminated spring fixed to the hub of the governor wheel is acted on both by centrifugal force and inertia.

Helical **springs** for shaft governors may be calculated as follows: Let x = distance (ft) from center of gravity of weight to center of pin around which the weight turns; x_1 = distance (ft) from center of pin to point of attachment of spring. Then, pull P_1 on spring due to centrifugal force of weight W (lb) $= 0.00034WR_1^2n_1^2\,x/x_1$, where R_1 = distance (ft) from center of shaft to center of gravity of weight when running at n_1 rpm. Also, at n_2 rpm, $P_2 = 0.00034WR_2^2n_2^2x/x_1$. Then (see p. 486) P_2 = maximum tension (or compression) on spring, lb $= f\pi d^3/8D$, where d = diam of wire in spring, in.; D = mean diam of coil, in.; and f (safe) = 50,000. After obtaining d from the equation for a given diameter D and pull P_2, the number of active coils c required to give the necessary extension or compression S (in.) of the spring may be obtained from the formula $z = Cd^4S/S(P_2 - P_1)D^3$, where P_1 is the initial tension or compression on spring, lb and C = 12,000,000 for round steel wire. If with springs thus calculated the governing is too fine, reduce the number of active coils in order to stiffen spring and bring about a greater fluctuation in speed. If the governing is too coarse, a larger number of active coils is required. The **powerfulness** K of the two springs of a shaft governor depends upon the energy stored in the springs by extending them the amount S (from tension P_1 to tension P_2).

Hydraulic governors consist of an impeller acting as a centrifugal pump with oil as fluid; the oil pressure set up is applied to a piston subjected to a controlling force. With increase of speed, the piston moves until the controlling force is increased to balance the new oil pressure. Impellers with straight radial vanes are best, having a pressure characteristic practically independent of the volume of oil delivered by the impeller. The oil discharge in the operating mechanism does not therefore influence the governing pressure, which is nearly proportional to the square of the speed. For the same speed, the pressure varies as the square of the impeller diameter. The pressures generally used in oil governors vary from 10 lb in small machines to 60 lb per sq in. in large units. Lower pressures decrease friction losses in the impeller.

Stability, Isochronism, Hunting. The controlling force in a governor (whether gravity or that of springs) may be considered as a force F acting on each weight in the direction of the radius toward the axis of revolution. If w/g be the mass of the weight and r the radius of its center of gravity the governor will revolve in equilibrium when $F = 4\pi^2r(n/60)^2w/g = 0.00034rn^2w$. In order to ensure stability, F must increase more rapidly than r as the balls move outward. The frictional resistance to the action of a governor may be considered as a force f acting radially on each ball so that the whole controlling force is $F + f$ when the balls are moving out and $F - f$ when they are moving in. The corresponding speeds are given by the equation $F + f = 4\pi^2r[(n + \Delta n)/60]^2w/g$ and $F - f = 4\pi^2r[(n - \Delta' n)/60]^2w/g$. As a result of friction, the speed may change from Δn above to $\Delta' n$ below the normal speed n while the position of the balls is unchanged. It is apparent that a stable governor cannot maintain an absolutely constant speed in the engine it controls. If a change takes place in the load on a steam engine, it is necessary for the weights or balls to be displaced a certain amount in order to alter the steam supply, and the weights can retain their new position only at a changed speed. The maximum range of speed depends on that amount of change of n which is required to alter the configuration of the governor

shaft or flywheel journals in bronze bearing up to 84,000; in Babbitt bearing up to 224,000 and more for water-cooled journals. The value of WN/l can be determined for **high-speed bearings** by calculating H as follows (Lasche): For P from 15 to 215 lb per sq in., t from 68 to 212 F, V from 500 to 5,000 fpm; $H = 51.2V/(t - 32)$.

Representative **maximum values of H** for three types of medium- to high-speed bearing are given in the following table (after Lasche):

Heat Dissipation in Bearings
(Ft lb per min per sq in. of projected surface)

Temperature rise of bearing, deg F	60	90	120	150	180
Well-ventilated bearing	200	420	705	1470	[illegible]
Standard bearing, unventilated	110	220	360	560	830
Thin shell and housing, unventilated	40	60	80	105	140

If the calculated values of fPV exceed the radiating capacity at the maximum operating temperature, it is necessary to resort to artificial cooling. The highest bearing temperature permissible with normal lubricants is about 210 F. Bradford and Eaton gave the heat-dissipating capacity of bearings as $H = (T + 33)^2/K_h$, where $T = t_w - t_a$ and K_h is an experimental constant depending on the type of bearing and the air conditions surrounding it. For bearings of light construction in still air, $K_h = 55$. For bearings of heavy construction, well ventilated, $K_h = 31$.

Elements of Journal Bearings

Typical dimensions of solid and split **bronze bushings** are given in Table 2.

Table 2. Wall Thickness of Bronze Bushings, Inches

Diam of journal, in.	<1/4	1/4–1/2	1/2–1	1–1 1/2	1 1/2–2 1/2	2 1/2–4	4–5 1/2
Solid bushing, normal	1/16	3/32	1/8	3/16	1/4	3/8	1/2
Split bushing, normal	3/32	1/8	5/32	7/32	5/16	15/32	5/8
Solid bushing, thin	1/16	3/32	3/32	1/8	3/16	1/4	3/8
Split bushing, thin	1/16	3/32	1/8	3/16	1/4	3/8	1/2

Bronze bushings made from hard-drawn bronze sheets and rolled into cylindrical shape are made by the Cleveland Graphite Bronze Co. with a wall thickness of only 1/32 in. up to 1/2 in. diam and with a wall thickness of 1/16 in. for bearings from 1 in. diam up. The wall thickness of these bearings depends chiefly upon the strength of the material which supports them. Bushings of this type are pressed into place, and the bearing surface is finished by burnishing with a slightly tapered bar to a mirror finish. The allowable bearing pressures may exceed those of cast bronze shown in Table 1 by 10 to 20 percent.

Babbitt linings in larger bearings are generally employed in thickness of 1/8 in. or over and must be provided with sufficient anchorage in the supporting shell. The anchors take the form of dovetailed grooves or holes drilled in the shell and counterbored from the outside.

Improved conditions are obtained by sweating or bonding the Babbitt to the shell by tinning the latter, using potassium chlorate as flux. Tin-base Babbitts sweated to a bronze shell are sometimes used in thickness of only 1/32 in. and even less.

Figure 6 shows the principal types of bonded Babbitt linings. Figure 6*a* is for normal operating conditions. Figure 6*b* is for severer operating conditions.

PLAIN BEARINGS

BY

G. B. KARELITZ

REFERENCES: Archbutt and Deeley, "Lubrication and Lubricants," Griffin. Clower, "Lubricants and Lubrication," McGraw-Hill. Nash and Bowen, "Principles and Practice of Lubrication," Chapman and Hall. Lasche, "Construction and Material in the Building of Steam Turbines," Oliver and Boyd. Thomsen, "The Practice of Lubrication," McGraw-Hill. Hersey, "Theory of Lubrication," Wiley. Boswall, "Theory of Film Lubrication," Longmans. "General Conference on Lubrication and Lubricants," A.S.M.E.

Plain bearings, according to their function, may be

(1) **Journal bearings**, cylindrical in shape, carrying a rotating shaft.

(2) **Thrust bearings**, the function of which is to prevent lengthwise motion of a rotating shaft.

(3) **Guide bearings**, to guide a machine element in its lengthwise motion, usually without rotation of the element.

Friction in bearings depends on the type of lubrication employed (see p. 1899).

In exceptional cases of design, or with a complete failure of lubrication, a bearing may run dry. The coefficient of friction is then between 0.25 and 0.40, depending on the materials of the rubbing surfaces. With the **bearing barely greasy**, or when the bearing is well lubricated but the speed of rotation is very slow, boundary lubrication takes place. The coefficient of friction varies from 0.08 to 0.14. This condition occurs also in any bearing when the shaft is starting from rest.

Semifluid lubrication exists between the journal and bearing when the conditions are not such as to form a load-carrying fluid film and thus separate the surfaces. Semifluid lubrication takes place at comparatively low speed, intermittent or oscillating motion, heavy load, insufficient oil supply to the bearing (wick or waste lubrication, drop-feed lubrication). Semifluid lubrication exists also in thrust bearings with fixed parallel thrust collars, in guide bearings of machine tools, in bearings with copious lubrication where the shaft is bent or the bearing is misaligned, or where the bearing surface is interrupted by improperly arranged oil grooves. The coefficient of friction in such bearings is 0.02 to 0.08.

Fluid or **complete lubrication**, when the rubbing surfaces are completely separated by a fluid film, provides the lowest friction losses and prevents wear. A certain amount of oil must be fed to the oil film in order to compensate for end leakage and maintain its carrying capacity. Pressure lubrication, from a pump or gravity tank, is used, or automatic lubricating devices are provided in self-contained bearings (oil rings or oil disks), or the bearing is submerged in an oil bath (thrust bearings for vertical shafts).

Plain Cylindrical Journal Bearings

Notation.

R = radius of bearing, in.

r = radius of journal, in.

$s = R - r$ = radial clearance, in.

W = load on a bearing, lb.

l = length of bearing, in.

$d = 2r$ = diameter of journal, in.

In Fig. 8, W is the resultant force or load (lb) on the bearing or journal. The radial ordinates P_1, to the dotted curve, show the pressures (lb per sq in.) of the journal on the oil film due to the load when there is no axial groove, while the ordinates P_2, to the solid curve, show the pressures with an incorrectly located groove. Since there is no oil pressure near the groove, the permissible load W must be reduced or the film will be ruptured.

Groove dimensions (Fig. 9) are given by the following relations: $a = \frac{1}{3}$ wall thickness; $W_s = 2.5a$; $W_d = 3a$; $c = 0.5W_d$; $f = \frac{1}{16}$ in. to $0.5W_d$.

In order to maintain the oil film, the axial distributing groove should be placed in the unloaded sector of the bearing. The location of grooves in a variety of cases is shown in Figs. 10 to 26.

Horizontal Bearings, Rotational Motion

Direction of load known and constant.

Load downward or inside the lower 60 deg segment as in the case of ring-oiling bearings (Fig. 10).

Load at an angle more than 45 deg to the vertical center-line (Fig. 11).

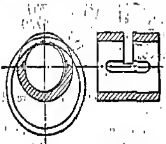


Fig. 10.

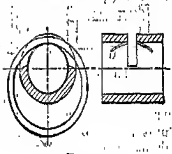


Fig. 11.

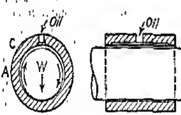


Fig. 12.

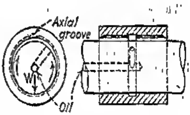


Fig. 13.

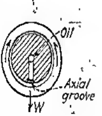


Fig. 14.

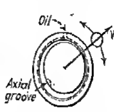


Fig. 15.

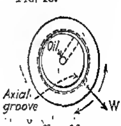


Fig. 16.

Force- or drop-feed oiling. The oil inlet may be anywhere within the no-load sector (Fig. 12).

Oil introduced through the center of the revolving shaft (Fig. 13).

Stationary journal and revolving bearing (Fig. 14).

place where the hydrostatic pressure is equal or nearly equal to the atmospheric pressure and may be at the location of the oil distributing groove B, or at the end of a machined recess pocket as at AA. For complete bearings *i.e.*, when the inner surface of the bearing is not interrupted by grooves—β may be taken as 60 deg (Fig. 2).

The coefficient c corrects for side leakage. There is a loss of load-carrying capacity caused by the drop in the hydrostatic gage-pressure p in the oil film from the mid-section of the bearing toward its ends; $p = 0$ at the ends. The value of c depends mainly on the length to diameter ratio l/d. Approximate values of c are as follows:

$l/d =$	3.0	2.0	1.5	1.0	¾	½	⅓
$c =$	0.80	0.70	0.60	0.50	0.35	0.20	0.10

The minimum film thickness $h_0 = e \times (R - r)$. Values of e are given in Fig. 2 for given values of β.

Example. A generator bearing, 6 in. diam $\times$ 9 in. long, carries a vertical downward load of 8,650 lb; $N = 720$ rpm. The diametral clearance of the bearing is $2s = 0.012$ in.; the bearing is split on its horizontal diameter, and the lower half is relieved 40 deg down on each side, for oil distribution along journal; the bearing arc is therefore 100 deg; with the load vertical, $\beta = 50$ deg. The absolute viscosity of the oil in the film is 23 centipoises (medium turbine oil).

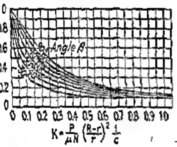


FIG. 2.—Minimum Film Thickness Ratio for a Plain Cylindrical Journal (for $K < 1$).

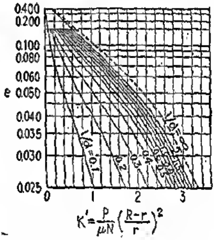


FIG. 3.—Minimum Film Thickness Ratio for a Plain Cylindrical Journal (for $K' > 1$).

$P = W/ld = 160$ lb per sq in.; $\mu = 23 \times 1.45 \times 10^{-7} = 33.4 \times 10^{-7}$ in (lb) (sec)/(in.^{2}) units. With $l/d = 1.5$, $c = 0.60$, and $K = 0.44$. From Fig. 2, with $\beta = 50$ deg, $e = 0.20$. The minimum film thickness $h_0 = e \times s = 0.66 \times 0.006 = 0.004$ in.

When the characteristic K is larger than 1.0 (heavy load, slow speed, low velocity), Fig. 3 should be used. In Fig. 3, the abscissas are K' which does not contain the coefficient c; the curves are drawn for various values of l/d, instead of values of β as in Fig. 2.

Allowable mean bearing pressures in bearings with fluid film lubrication are given in Table 1. If the load maintains the same magnitude and directions when the journal is at rest (heavily loaded shafts, heavy gears), the mean bearing pressure should be somewhat less than when bearings are loaded only when running. For the bearings of rope and chain sheaves, which turn intermittently, the mean bearing pressure is 350 to 850 lb per sq in.; where wear is not important, these values may be doubled. With crank (crosshead) pins of alloy steel running in bronze bushings, in ordinary steam engines, mean pressures from 700 to 1,000 (1,050 to 1,150) lb per sq in. may

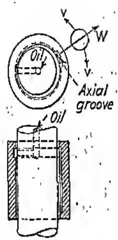


Fig. 21.

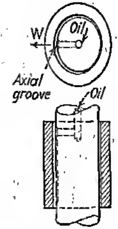


Fig. 22.

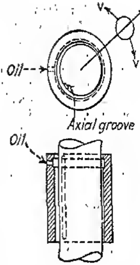


Fig. 23.

Bearings with Oscillatory Motion

Direction of load constant.

No oil film can be built up due to the small sliding velocity and unstable lubrication will exist. Axial grooves in the loaded sector distribute the lubricant to all parts of the bearing and avoid dry spots (Fig. 24).

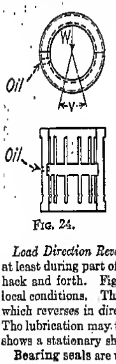


Fig. 24.

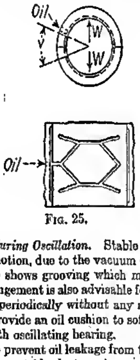


Fig. 25.

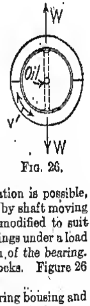


Fig. 26.

Load Direction Reversed during Oscillation. Stable lubrication is possible, at least during part of the motion, due to the vacuum caused by shaft moving back and forth. Figure 25 shows grooving which may be modified to suit local conditions. This arrangement is also advisable for bearings under a load which reverses in direction periodically without any rotation of the bearing. The lubrication may then provide an oil cushion to soften shocks. Figure 26 shows a stationary shaft with oscillating bearing.

Bearing seals are used to prevent oil leakage from the bearing housing and to protect the bearing from outside dust, water, vapors, etc. A drainage groove at the end of the bearing is effective to divert the oil passing through the bearing back into the oil well (Fig. 27*a*). The drain holes at the bottom

free fits for speeds over 600 rpm. Kingsbury suggests for these journals a diametral clearance $2s = 0.002 + 0.001d$ in. In journals running at high speed, $2s = 0.002d$ should be used in order to lower the friction losses in the bearing.

Figure 4 shows the dependence of the **coefficient of friction** f on the velocity and pressure for a journal bearing with oil-ring lubrication, as found by Stribeck for the stated unit pressures P. The descending branches of the curves give the friction in the semifluid range, at low speed. The minimum values of f are at the transition from semifluid to perfect lubrication (complete separation between bearing and journal surfaces). This transition takes place at lower speeds in bearings with a better finish of the bearing and journal surfaces. Figure 5 shows the influence of the bearing temperature on the value of f, observed by Stribeck for the bearing of Fig. 4, when the journal was run at 1,100 rpm.

The friction force F in plain journal bearings is $f \times W$. Also $F = K_f \mu N r l / \left(\frac{R - r}{r}\right)$ where μ is in (lb)(sec)/(in.^{2}) units. The value of K_f depends on the value of e (Figs. 2 and 3):

e =	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
K_f =	0.66	0.67	0.69	0.73	0.79	0.89	1.01	1.19	1.50	2.16

The **mechanical loss** in the bearing is $FV/33{,}000$ hp where V = the peripheral velocity of the journal, fpm. For high-speed bearings (turbines, generators), with a clearance ratio $(R - r)/r$ of .002, according to Soderberg, the loss is $0.00016 Zd^2 l (N/1000)^2$ hp, where Z is in centipoises.

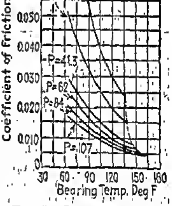


FIG. 5.—Influence of Bearing Temperature on the Coefficient of Journal Friction.

The **heat dissipation** L from the outside bearing wall in foot-pounds per minute, according to Karelitz, in quiet air, is $L = 0.184\, S(t_w - t_0)$, where S is the area of the bearing wall, sq in.; and t_w and t_0 are the respective temperatures of the wall and the ambient air, deg F. In an air draft, of velocity 500 fpm, the heat dissipation is 3 times as great. $S = (10 \text{ to } 15)dl$ for pillow blocks carrying bushings, and $S = (18 \text{ to } 25)dl$ for larger bearing-pedestals carrying cast-iron or steel bearing shells. In self-contained bearings (electric motors, line shafts, etc.) without artificial oil or water cooling, the dissipated heat is equal to the bearing losses; $L = FV$. The temperature rise of the wall can thus be determined.

For pedestals with steel bearing-shells with oil-ring lubrication operating without artificial cooling, the temperature of the oil film t_l is considerably higher than the temperature of the wall:

Temperature rise of the pedestal wall, $(t_w - t_0)$, deg F	20	30	40	50
Temperature of oil film above temperature of wall, $(t_l - t_w)$, deg F	20	24	27	30

The maximum **heat dissipation** H in foot-pounds per square inch of projected area of a journal per minute is given by $H = fWV/ld = fWN/3.82l$, where V = surface speed of journal, fpm. The values for WN/l may be taken as follows: *crankpin* in bronze bearing up to 210,000; in Babbitt bearing with low bearing pressures and short journals up to 494,000. *Main crank-*

Hardwood bearings containing lubricants are of lignum vitae which has a natural gum or hard maple which is impregnated with oil, grease, or wax. Lignum-vitae propeller bearings are used for ships, hard-maple bearings are used for loose pulleys, in textile machinery, wringer blocks.

Porous-metal bearings compressed from metal powders and sintered contain up to 35 percent of liquid lubricant. The porous metal generally consists of a 90–10 copper-tin bronze with 1½ percent graphite. These bearings do not require oil grooves since capillarity distributes the oil and maintains an oil film. If additional lubrication from an oil well should be provided, oil will be absorbed through the porous wall as required. For high temperatures where oil will carburize, a higher percentage of graphite (6 to 15 percent) is used.

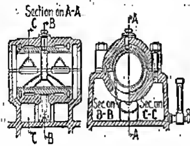


FIG. 31.—Split Bearing with One Chain. Main Crankshaft Bearing. Vertical Oil Engine.

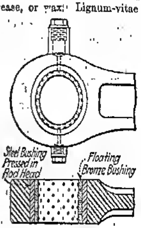


FIG. 32.—Floating Bushing. Big End Locomotive Main Rod.

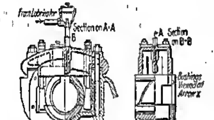


FIG. 33.—Crankshaft Main Bearing. Horizontal Steam Engine. Drop Feed Lubrication.

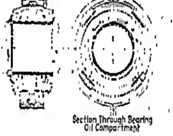


FIG. 34.—Large Turbine Bearing (Westinghouse). Self-aligning, with Additional Path for Cooling Oil in the Shell.

Porous-metal bearings are used where plain metal bearings are impracticable, due to lack of space or inaccessibility for lubrication, as in automotive generators and motors and vacuum-cleaner motors. Porous-metal bearings are manufactured by Bound Brook Oil-less Bearing Co.; Chrysler Corporation; Moraine Products Co., Dayton, Ohio; U. S. Graphite Corp., Saginaw, Mich.

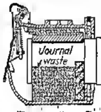


FIG. 35.—Car-wheel Journal Box with Oil-saturated Waste Packing in Lower Half.

Thrust Bearings

At low speeds, shaft shoulders or collars bear against flat bearing rings. The lubrication is semifluid, and the friction is comparatively high.

For die-cast automotive bushings, Heldt recommends the following proportions: $D = 1.2d$ for main crank bearings; $D = 1.2d$ for connecting-rod bearings; $t = 0.1d$; $e = 0.25d$; where d is the journal diameter, D the outside diameter of the bushing, t the thickness of the flange, and e the diameter of the anchorage lug.

General practice for the **thickness of Babbitt lining and shells** is as follows: FIG. 7, $b = \frac{1}{32}d + \frac{1}{8}$ in.; $S = 0.18$ in. for bronze or steel $= 0.2d$ for cast iron. FIG. 7a, $t = \frac{b}{2} + \frac{1}{16}$ in.; $W = 1.8t$; $W_1 = 2.2t$.

Solid bronze or steel bushings, when pressed into the bearing housing, must be finished after pressing in. Light press fits and securing by setscrews or keys are preferable to heavy press fits and no keying, since heavy pressure, especially in thin-walled bushings, will set up stresses which will

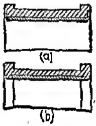


FIG. 6.

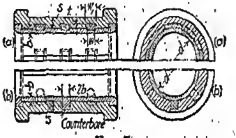


FIG. 7.

release themselves if bearings should run hot in service and will result in closing in on journal and scoring when cooling.

Uniform Load Distribution. Misalignment between journal and bearing should never be so great as to cause metallic contact. The maximum allowable inclination α of the shaft to the bearing is given by $\tan \alpha = 2s/l$.

Whenever the deflection angle of the bearing installation is greater than α, either the bearing length should be reduced or, if that is not feasible, the bearing should be mounted on a spherical seat to permit self-alignment.

As a general rule, fixed bearings may be used for l/d ratios up to 2 and self-aligning seats for l/d ratios from 3 to 4.

Oil grooves are of two kinds, axial and circumferential: the former distribute the oil lengthwise in the bearing; the latter distribute it around the

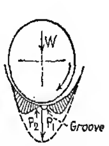


FIG. 8.

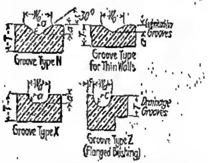


FIG. 9.—Lubrication and Drainage Grooves.

shaft at the oilhole, and also collect and return oil which would otherwise be forced out at the ends of the bearing. Grooves have often been put into bearings indiscriminatingly, with the result that they scrape off the oil and interrupt the film.

friction in these bearings varies from $f = 0.03$ at $P = 45$ to $f = 0.06$ at $P = 15$.

The performance of the bearing thrust rings is much improved by the introduction of grooves with tapered lands as shown in Fig. 37.[1] The lands extend on either side of the groove. The taper angle of the lands is very slight, so that a pressure-oil film is formed between the bearing ring and the collar of the shaft.

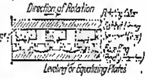


FIG. 41.—Kingsbury Thrust Bearings. (Developed Cylindrical Sections.)

For high speeds or where low friction losses and a low wear rate are essential, **pivoted segmental thrust bearings** are used (Kingsbury thrust bearing, or Michell bearing in Europe). The bearing members in this type are tiltable shoes which rest on hard steel buttons mounted on the bearing housing. The shoes are free to form automatically a wedge-shaped oil film between the shoe surface and the collar of the shaft (Figs. 38, 39, 40).

The minimum oil film thickness h_0 in., between the shoe and the collar, at the trailing edge of the shoe, is approximately $h_0 = 0.12\sqrt{\mu V l/P}$, where V is the velocity of the collar, on the mean diameter, fpm and $P = W/lb$. As indicated in Fig. 38, $b = l$ approx. The standard thrust bearings have six shoes. Load-carrying capacities of Kingsbury thrust bearings are given in Table 3.

The coefficient of friction in Kingsbury thrust bearings, referred to the mean diameter of the shoes, is $f = 2.7\sqrt{\mu V/lP}$. Figures 39 and 40 show typical pivoted segmental thrust bearings. They usually embody a system of rocking levers which are used for alignment and equalization of load on the several shoes (Fig. 41).

Sliding Bearings

All sliding bearings (Fig. 42) to wear true must have the sliding parts of nearly equal lengths. Bearings made in this way will be found not to wear out of true. Oiling is accomplished in several ways, an acceptable method being that shown in Fig. 43. Short slides *in many machine tools are lubricated by oil pads or direct oil application.* The weight of the table and work and thrust of the tool cause wear on the

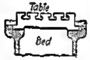


FIG. 42.

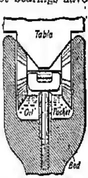


FIG. 43.

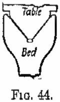


FIG. 44.

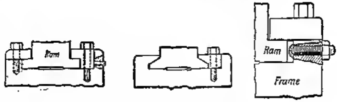


FIG. 45.

bottom and sides of the guides. To compensate for the wear in both directions, bearings are sometimes made V-shaped, as shown in Fig. 44.

Simpler sliding bearings in machine tools are made with provision for adjustment, as shown in Fig. 45, of which there are many modifications.

Load variable inside a given angle.

Figures 10, 11, 12, 13, and 14 are suitable, provided there is about 45 deg between the axial groove and the direction of the load W.

Rotating load.

For rotating shafts, a circumferential groove at the middle of the bearing and an axial groove on the no-load side (Fig. 15).

For stationary shafts and rotating bearings, a circumferential groove in the bearing and an axial groove on the no-load side. The oilhole is in the shaft at the mid-length of the bearing (Fig. 16).

Load direction uncertain.

Oil-ring bearings (Figs. 10 and 12) may be used although they have defects under certain load directions. With forced or drop feed, the oilhole enters a circumferential groove at the middle of the bearing and the axial groove is omitted (Fig. 17). Arrangements for introducing oil through the rotating shaft can be made.

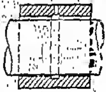


FIG. 17.

Vertical Bearings for Continuous Rotation

Direction of load known and constant.

Drop feed (Fig. 18). For force feed, put the hole nearer the middle of the bearing to avoid top leakage.

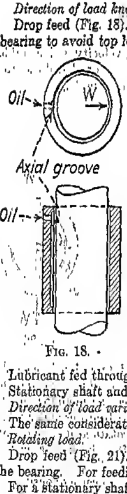


FIG. 18.

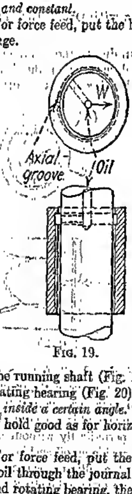


FIG. 19.

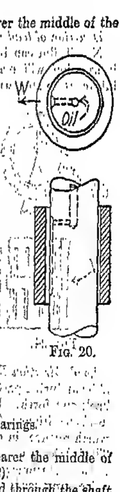


FIG. 20.

Lubricant fed through the running shaft (Fig. 19).

Stationary shaft and rotating bearing (Fig. 20).

Direction of load variable inside a certain angle.

The same considerations hold good as for horizontal bearings.

Rotating load.

Drop feed (Fig. 21). For force feed, put the hole nearer the middle of the bearing. For feeding oil through the journal (Fig. 22).

For a stationary shaft and rotating bearing, the oil is fed through the shaft and the axial groove is cut on the no-load side of the journal (Fig. 23).

Guide bearings. Direction of load generally unknown. Similar to Fig. 26 but omit the axial groove.

BEARINGS WITH ROLLING CONTACT

BY

G. A. UNGAR

REFERENCES: Ahrens, "Das Kugellager," Springer. Clinedienst, *Trans. A.S.M.E.*, June 2, 1938. Goodman, *Engineering*, Aug. 3 and 24, 1923; June 19, 1925. Herrman, *Trans. S.A.E.*, Aug. 2, 1932. Hertz, "Gesammelte Werke," Barth, Vol. I, pp. 155 *et seq.* Juergensmeyer, "Die Waelzlager," Springer. Metals Handbook, 1939. Murzoli, "Fatigue Resistance of Hard Steels," Stabilimento Tipograf. Nat. Trieste. Palmgren, *Z.V.d.I.*, 1924, p. 339; also *Trans. S.A.E.*, Oct. 1920. Timoshenko, "Theory of Elasticity," McGraw-Hill. Wawrziniok, *Auto. Tech. Z.*, 1928, Nos. 8 and 12.

THE MECHANICS OF ROLLING CONTACT BEARINGS

Rolling contact between journal and bearing is produced by inserting rolling members between concentric surfaces of the stationary and moving elements.

If free rolling in two directions is required, the rolling elements must be balls; ... ction, the rolling elements may ... , provided always that all the c... tersect in one point of the main axis of rotation and that the (geometrical) sum of all the individual angular velocities is equal to the angular velocity of rotation around the main axis. These theoretical requirements for pure rolling cannot always be fulfilled.

In Fig. 1, the location of the intersecting point on the main axis of rotation determines the direction of the load which the bearing can support. If the

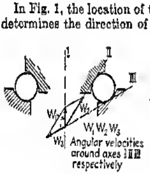


FIG. 1.

FIG. 2.

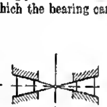


FIG. 3.

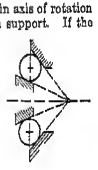


FIG. 4.

intersection is at infinity (Fig. 2), the load may be radial or annular. With intersection at the center of the bearing (Fig. 3), the loading is thrust or axial. With intersection in an intermediate position, (Fig. 4), the load is angular.

Notation.

B_r = Brinell hardness number.

D_i = inner race diameter, in.

d_1, d_2 = curvature diameters of two rolling bodies at their point of contact.

d_2', d_2'' = principal curvature diameters of a curved surface (at right angles to each other), in.

δ_r = curvature diameter of a substitute rolling body in contact with a flat surface, in.

ξd_r = diameter of the contact surface for balls or the width of the contact rectangle for rollers of length L, in.

E = modulus of elasticity for the material, lb per sq in.

of the groove must be ample for passage of the oil flow. An oil thrower mounted on the shaft is shown in Fig. 27*b*. The bearing housing may be provided with a single (Fig. 27*c*) or double collecting groove, or with brass or aluminum strip scrapers (Fig. 27*d*), to collect the oil creeping along the shaft.

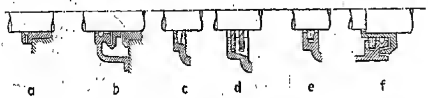


Fig. 27.—Sealing End Grooves.

For protection from dust, etc., felt packing rings are often used (Fig. 27*e*). The felt ring is soaked in oil to prevent charring by friction heat. In severe cases, additional protection by a labyrinth runner is very effective (Fig. 27*f*).

Line-shaft bearings are purchasable in standard sizes by steps of ¼ in. for the smaller diameters, and by steps of ½ in. for the larger sizes. **Hangers** for line-shaft bearings may be purchased in types permitting ceiling, floor, or wall suspension. Figure 28 shows bearings for mounting in line-shaft hangers.

Fig. 28.—Bearing for Drop Hanger with Oil Ring Lubrication.

Main Shaft Machine Bearings. Average practice in proportioning the diameters and lengths of headstock bearings in lathes is as follows:

Front bearings: $d = s/5$; $l = s/4$. Back bearings: $d = s/8$; $l = s/6$; where d(diam), l(length) and s(swing) are all in inches.

Types of bearings are shown in Figs. 29 to 35. They include the principal methods of lubrication and types of construction.

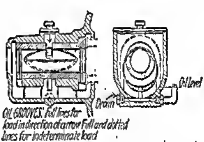


Fig. 29.—Ring-oiled Bearing Solid Bushing.

Oilless bearings (information by C. Claus, Bound Brook Oil-less Bearing Co.) is the accepted term for self-lubricating bearings containing lubricants in solid or liquid form in their material. Graphite is used as a solid lubricant in one group, and another group consists of porous structures (wood, metal), containing oil, grease, or wax.

Graphite-lubricated bearings (bridge bearings, sheaves, trolley wheels, high-temperature applications) consist generally of cast bearing bronze as a supporting structure containing various overlapping designs of grooves which are filled with graphite. The graphite is mixed with a binder, and the plastic mass is pressed into the cavities and baked to the hardness of a lead pencil; 45 percent of the bearing area is graphite. Drilled holes are also used for filling the bronze bearings with graphite in plastic form or with preformed graphite plugs. Strip bronze with indentations or grooves filled with graphite is used in sheet form or rolled to a butted cylindrical bushing. These bearings are manufactured by Bound Brook Oil-less Bearing Co.; Cleveland Graphite Bronze Co.; Merriman Bros., Boston.

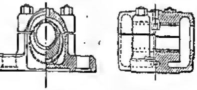


Fig. 30.—Rigid Ring-oiling Pillow Block (Link-Belt Co.).

The maximum compression stress p_{max} in a roller with diameter d_r and length L (for $L > d_r$) and in the flat surface with which it is in contact can be written according to Hertz

$$p_{max} = 4P_o/\pi\xi d_rL$$
$$(\xi d_r/2)^2 = 4P_o d_r(1 - 1/m^2)/EL$$
$$p^2_{max} = P_oE/(\pi Ld_r)(1 - 1/m^2) = 0.35P_oE/Ld_r$$
$$\xi = 4(1 - 1/m^2)p_{max}/E \text{ (with } 1/m = 0.3)$$

By introducing the mean stress p_m in the pressure area,

$$p_m = 2P_o/\pi\xi d_rL = \pi p_{max}/4$$
$$\xi = 4.64p_m/E \text{ (with } 1/m = 0.3)$$

If k_o is the specific load, lb per sq in., produced by P_o upon a rectangle with the area f surrounding the rolling element,

$$k_o = P_o/f = P_o/d_r^2 = [p_m(\xi d_r)^2\pi/4]/d_r^2, \ (= 14.3p_m^3/E^2, \text{ for balls})$$
$$k_o = P_o/f = P_o/d_rL = p_m\xi d_rL/d_rL, \ (= 4.64p_m^2/E, \text{ for rollers})$$

For materials following Hooke's law and for stresses within the elastic limit, the specific load capacity of balls does not vary exactly with d^2. Smaller balls have a relatively higher load capacity as indicated by the relation $P_o = k_od^2/1 + .002d)$. In rollers the relative load capacity appears to be unaffected by the diameter. For values of p_m and k_o, see under "Static Load Capacity."

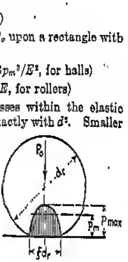


FIG. 5.

Stresses in Axial or Thrust Bearings. If each rolling element (ball or roller) carries an equal share of the axial load P_a, then $P_a = fk_on$.

The coefficient of utilization η_a of the axial bearing is given by $\eta_aF = nf$, where F is the ring area of the bearing $= \pi/4(D_o^2 - d_i^2)$. η_a is less than 1 in bearings with retainers and equals 1 in full-type bearings. The previous equation can then be written $P_a = \eta_aFk_o$ and the specific axial bearing load $p_a = P_a/F = \eta_ak_o$.

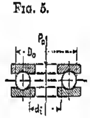


FIG. 6.

In order to ensure equal load distribution, the rolling elements must have uniform size and at least one of the supporting outside faces of the races should be spherical to permit of self-alignment.

The **influence of speed** on the load-carrying capacity of thrust bearings is particularly pronounced in ball thrust bearings. This is due to sliding and skewing of the balls driven by centrifugal force against the curved raceways in addition to the normally reduced dynamic load capacity.

If v is the mean velocity of the balls, in fps, the reduced thrust capacity P_a' lb, may be written as follows: $P_a' = 3.3P_a/(v + 3.3)$ where P_a is the static load capacity, lb. For instance, at 10 fps, $P_a' = 0.248P_a$.

Stresses in Radial or Annular Bearings. If the radial load P_r could be assumed to be supported by a flexibly yielding outer race, the load would be evenly distributed over all the rolling elements in the loaded half of the bearing and each element would support the load P_o. The specific radial bearing load p_r could therefore be written $p_r = P_r/D_iL = \eta_rk_o$.

Because of the comparative rigidity of the outer race, if the bearing is assembled with clearance and not under initial stresses, the rolling elements

For hardened-steel collars on bronze rings, with interrupted service, pressures up to 2,000 lb per sq in. are permissible; for continuous low-speed operation, 1,500 lb per sq in.; for steel collars on babbitted rings, 200 lb per sq in. In multicollar thrust bearings, the values are reduced considerably because of the difficulty in distributing the load evenly between the several collars.

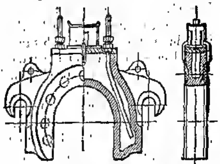


Fig. 36.—Horseshoe Bearing Ring for Marine Thrust Bearing.

Thrust bearings for marine service are sometimes made with adjustable horseshoe bearing rings as shown in Fig. 36 and with provision for water-cooling. The thrust collars are formed by grooving the shaft and have an outside diameter $D = 1.6d$ to $1.9d$, where d is the normal shaft diameter. The thickness of each collar $= w = 0.13d$ to $0.16d$, and the distance between collars $= s = 2w$ to $3w$. The number of collars required $= n = P/\pi d_1 bp$, where P = total thrust on bearing, lb; d_1 = mean diam of collar, in.; b = radial width of collar, in., and p = allowable bearing pressure, lb per sq in., as follows: for cargo steamers, $p = 40$ to 55; for passenger steamers, $p = 55$ to 80. According to Howarth, the coefficient of

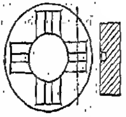


Fig. 37.—Thrust Collar with Grooves Fitted with Tapered Lands.

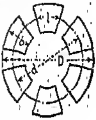


Fig. 38.—Kingsbury Thrust Bearing with Six Shoes.

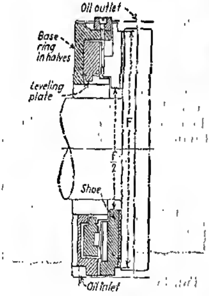


Fig. 39.—Left Half of Six-shoe Self-aligning Equalizing Horizontal Thrust Bearing for Load in Either Axial Direction.

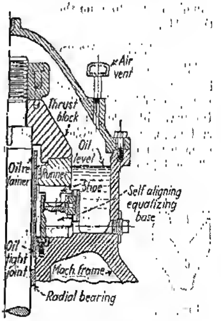


Fig. 40.—Half Section of Mounting for Vertical Thrust Bearing.

at rest, local permanent deformation interferes with subsequent smooth bearing operation.

With very slow uniform rotation, the non-brinelling static load may be exceeded by up to 30 percent because the permanent deformation results in cold-working of the material and produces a uniformly distributed change in the shape of the race surfaces.

The effect of race hardness upon the specific load factor k_o for short and long cylindrical rollers is shown in Table 1.

Table 1. Multipliers of Load Capacity with Varying Race Hardness

Brinell hardness No., kg per sq mm	Rockwell C hardness	Short solid rollers (Hyatt)	Long solid rollers, Bantam, needles
627	60	1.0	1.0
601	58	0.935	0.969
578	57	0.902	0.940
555	55	0.84	0.875
534	53	0.78	0.769
514	52	0.75	0.700
495	50	0.69	0.500

Although the dynamic load capacity changes comparatively little in the range above a Rockwell C hardness of 60, the static capacity depends greatly not only upon the surface hardness but also upon the hardness through a substantial depth of rolling elements and races.

The permissible mean stress p_m varies substantially in direct proportion with the Brinell hardness number B_r of races and rolling elements. In modern bearings, $p_m = 1422XB_r$ lb per sq in.

In ball bearings: $X = 0.85$ to 0.95.

In roller bearings with $L \lesseqgtr 2d$: $X = 0.6$ to 0.65.

In needle bearings with $L \gtreqless 4d$: $X = 0.375$–0.45 down to 514 Brinell hardness and drops gradually, reaching 0.325–0.375 at 495 Brinell.

The effect of high operating temperatures which reduce the hardness must be considered in selecting bearings of sufficient load capacities. The value of k_o of stainless-steel ball bearings even with a Rockwell C hardness of 60 (as compared with 63 to 64 for standard ball bearings) is only about 70 percent of k_o for standard ball bearings.

Examples. *Ball bearing.* A No. 307 Norma-Hoffman deep-groove radial bearing containing eight ½ in. balls, with races having 62–64 and balls having 63–65 Rockwell C hardness, has a static non-brinelling load capacity P_o per ball of 6,250 lb. The substitute radius d_r from formula (2) with a groove diameter of 0.510 in. and an inner race diameter of 1.75 in. is 0.765 in. The static $k_o = 6250/0.765^2 = 10{,}684$ lb per sq in. With $E = 30 \times 10^6$, $p_m^3 = 10{,}684E^2/14.3$ and the static $p_m = 876{,}000$ lb per sq in. This is equivalent to $0.92B_r \times 1422$ with a Brinell number $B_r = 667$ corresponding to an average race hardness of 63 Rockwell C.

The dynamic load capacity of the balls is substantially lower. For an average life of 2,500 hr according to the manufacturers' load ratings, P_o becomes 2,540 lb for 100 rpm and 1,240 lb at 1,000 rpm with k_o and p_m reduced accordingly.

Short Roller Bearings. A No. R-345-L (No. 309 S.A.E.) Norma-Hoffman cylindrical roller bearing with twelve ½ in. rollers of 0.425 in. effective length, with 62–64 Rockwell C races and 63–65 Rockwell C rollers, has a static non-brinelling capacity of 9,375 lb per roller. The substitute radius d_r from formula (1), with an inner-race diameter of 2.375 in., is 0.414 in. The static load $k_o = 9375/0.425 \times 0.414 = 54{,}300$ lb per sq in. With $E = 30 \times 10^6$, $p_m^2 = 54{,}300E/4.64$ and static $p_m = 593{,}000$ lb per sq. in. This is

Table 3. Capacities of 6-shoe Standard-duty Horizontal and Vertical Thrust Bearings

(Based on viscosity of 150 sec. Saybolt at operating temperatures. Capacities given may be increased from 10% to 25% if viscosity is increased in same proportion.)

Bearing size	Area, sq in.	Rpm 100	Rpm 200	Rpm 400	Rpm 800	Rpm 1,800	Rpm 3,600	Bearing size	Area, sq in.	Rpm 100	Rpm 150	Rpm 200	Rpm 300	Rpm 500	Rpm 700
		Safe load, thousands of pounds								Safe load, thousands of pounds					
5	12.5	1.44	1.7	2.0	2.4	2.9	3.5	19	180	40.00	44.0	48.0	53.0	60.0	65.0
6	18.0	2.30	2.7	3.2	3.8	4.6	5.5	21	220	51.00	57.0	61.0	68.0	77.0	84.0
7	24.5	3.30	3.9	4.7	5.6	6.8	8.0	23	264	65.00	72.0	77.0	85.0	97.0	105.0
8	32.0	4.60	5.5	6.6	7.8	9.6	11.4	25	312	80.00	88.0	95.0	105.0	119.0	123.0
9	40.5	6.20	7.4	8.8	10.4	13.0	15.0	27	364	97.00	107.0	115.0	127.0	144.0	146.0
10½	55.1	9.20	10.8	13.0	15.4	19.0	22.0	29	420	116.00	128.0	137.0	152.0	168.0	168.0
12	72.0	12.80	15.2	18.0	21.0	26.0	29.0	31	480	137.00	151.0	162.0	180.0	192.0	192.0
13½	91.1	17.20	20.0	24.0	29.0	35.0	36.0	33	544	160.00	177.0	189.0	210.0	220.0	220.0
15	112.5	22.00	26.0	32.0	37.0	45.0	....	37	684	215.00	235.0	250.0	275.0	275.0	
17	144.5	30.00	36.0	43.0	51.0	58.0	....	41	840	275.00	305.0	325.0	335.0	335.0	
								45	1012	345.00	385.0	405.0	405.0		

Figure 9 illustrates the difference in catalogue ratings of three manufacturers for the same ball-bearing size with the same size and number of balls, but with different life expectancies.

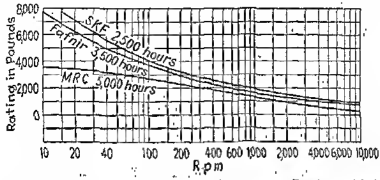


FIG. 9.—Ratings for No. 210 Single-row Deep-groove Bearings with Ten ½ in. Balls.

Figure 10 shows multipliers for catalogue ratings of five ball-bearing manufacturers to obtain bearing lives at loads other than those on which the catalog ratings are based [see Eq. (4)].

If both the load and speed of a bearing vary, an equivalent average load P_{av} at a speed n_s can be obtained from the equation

$$P_{av} = \sqrt[x]{r_1 n_1/n_s P_1^x + r_2 n_2/n_s P_2^x + \cdots r_n n_n/n_s P_n^x}$$

where r_1, r_2, etc., represent the time fractions; P_1, P_2, etc., the loads prevailing during each time fraction; and n_1, n_2, etc. are the rpm at those times. This value of P_{av} may be used in connection with Eq. (5).

Annular bearings with revolving inner races have a smaller number of stress reversals for the same rpm than when the outer races revolve. Therefore the life expectancy with outer race revolving is lower than with inner race revolving.

The type and size of the bearing determine this difference, and the manufacturers supply suitable correction factors for the ratings or speeds, which are always given for revolving inner races.

SKF gives, for outer races revolving, 90 percent of the carrying capacities of self-aligning ball bearings and 75 percent of the carrying capacities of all other bearing types. The multipliers for Bantam needle bearings range from 1.2 for small bearings up to 1.06 for large bearings.

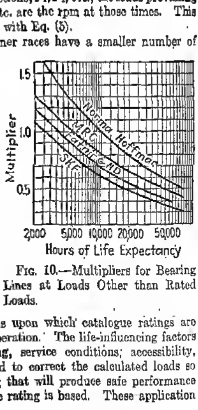


FIG. 10.—Multipliers for Bearing Lines at Loads Other than Rated Loads.

The ideal load and speed conditions upon which catalogue ratings are based do not prevail in normal field operation. The life-influencing factors of shock, temperature, pre-loading, service conditions, accessibility, and lubrication are therefore employed to correct the calculated loads so as to find from the catalogue a rating that will produce safe performance during the average life upon which the rating is based. These application

$1/m$ = Poisson's ratio
η_a and η_r = coefficients of utilization of bearings
k_0 = the specific load produced by P_o upon a rectangle of area which is d_r^2 for balls and d_rL for rollers, lb per sq in.
L = length of rolling element, in.
n = number of rolling elements.
P_a = static axial load capacity for a bearing, lb.
P_o = the load on a rolling element, lb.
P_r = static radial load capacity for a bearing, lb.
p_a = specific axial bearing load, lb per sq in.
p_r = specific radial bearing load, lb per sq in.
p_m = mean compressive stress for the contact area, lb per sq in.
p_{max} = maximum compressive stress for the contact area, lb per sq in.

To obtain the highest load-carrying capacity, the surfaces of races and rolling elements must be accurately finished with a high surface polish.

Individual Load Capacity of the Rolling Elements

The relative curvature of a rolling element in contact with the race determines the stresses and deformations of the contact surfaces when pressed together.

According to Hertz, the curvature diameter d_r of a substitute rolling body in contact with a flat surface can be used in place of the curvature diameters d_1 and d_2 of two rolling bodies in contact with each other.

$$1/d_r = 1/d_1 + 1/d_2 \tag{1}$$

If a ball of the diameter d_1 is in contact with a race having two principal curvature diameters (at right angles to each other) d_2' and d_2'', the diameter d_r of a substitute ball in contact with a flat surface may be written approximately as follows:

$$\frac{2}{d_r} = \frac{2}{d_1} + \frac{1}{d_2'} + \frac{1}{d_2''} \tag{2}$$

For concave surfaces, the signs in (1) and (2) are negative.

In order to keep d_r as large as possible in ball bearings, so as to increase the carrying capacity, the radii of the raceway grooves are kept as nearly equal to the ball radius as possible.

Under load, the race and rolling element flatten out at the contact surface. The area of this contact surface equals $(\xi d_r)^2\pi/4$ for balls and $\xi d_r L$ for cylindrical rollers, where ξd_r is the diameter of the contact surface for balls or the width of the contact rectangles for rollers of the length L and where d_r is the diameter of the substitute rolling element in contact with a flat surface.

The mean compression stress p_m, lb per sq in., for this contact area is

$$p_m = P_o/(\xi d_r)^2\pi/4 \text{ (for balls)} = P_o/\xi d_r L \text{ (for rollers)}$$

According to Hertz, the maximum compression stress p_{max} in a ball with diameter d_r and in a flat surface with which it is in contact is $1.5p_m$.

$$(\xi d_r/2)^3 = 1.5(1 - 1/m^2)P_o d_r/2E$$

Therefore

$$p^3_{max} = 1.5P_oE^2/\pi^3(d_r/2)^2(1 - 1/m^2)^2$$

For metallic bodies the value of $1/m$ may be taken as 0.3 and with $p_m = p_{max}/1.5$

$$\xi = 4.28p_m/E \text{ for balls}$$

The great majority of retainers are made from steel stampings. For high-speed-precision and ultra-precision bearings, carefully machined retainers are used, usually designed so that the ... the outside of the inner race. These high-speed ... made from fabric-reinforced phenol plastics.

Normal and Extreme Bearing Speeds. The speeds listed by the manufacturer apply usually only to standard bearing types. With precision and ultra-precision bearings, these speeds can be substantially exceeded (see Figs. 11 and 12). The limit speeds indicated apply to rotating inner races. For stationary inner races, the rpm shown must be multiplied by 0.6.

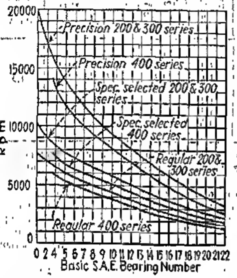


FIG. 11.—Limiting Speeds of Standard Ball Bearings (Single-row Deep-groove Angular and Duplex).

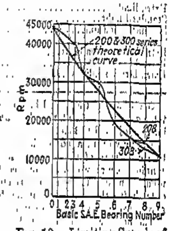


FIG. 12.—Limiting Speeds of Super-precision Bearings.

Friction of Rolling Contact Bearings

The total friction is composed of rolling friction, due to elastic deformation caused by the bearing loads, and of *sliding friction* due to speed variations (acceleration or oscillation), manufacturing inaccuracies, the rubbing of rolling elements against guiding race surfaces and cages, the displacement of the lubricant, and the rubbing of bearing seals.

According to Goodman's tests: the coefficient of friction of cylindrical roller bearings dropped from 0.0019 with increasing loads to 0.0013; in deep-groove ball bearings, the coefficient of friction first dropped from 0.0025 to 0.0014 with increasing load but upon further load increase it increased to 0.0020. Self-aligning ball bearings showed, with increasing load, first a drop from 0.0015 to 0.0011 and then an increase to 0.0014.

Muzzoli's tests indicate a steady decrease of the coefficient of friction with increasing load in cylindrical roller bearings without showing the reversal noticeable with ball bearing; the coefficient of friction is lower than that of ball bearings. They also show that deep-groove bearings with angular load have a greater friction increase with increasing angular loads than with radial loads alone and that angular-contact bearings have higher friction for

in the bearing axis perpendicular to the direction of load P_r are completely unloaded (see Fig. 7) and, in the axis coinciding with the load, are loaded approximately 50 percent higher than the individual uniform bearing load P_o of a bearing with a freely yielding outer race. Consequently, the specific radial load p_r is only about two-thirds of the specific axial load p_a or $p_r = P_r/D_iL \cong 0.65\eta_r k_o$.

In single-row ball bearings, D_iL becomes $D_i d_r$ and $\eta_r = nd_r/D_i\pi$ or $p_r \cong 0.65k_o nd_r/D_i\pi$. The radial load $P_r \cong fk_o n/5$ where f is the rectangle surrounding a rolling element $(= d_rL)$. This load is only one-fifth of the axial bearing load with the same number of balls.

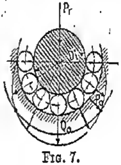


FIG. 7.

Stresses in Angular Bearings or Combination Radial-thrust Bearings. When radial bearings are used to support both axial and radial loads, an equivalent radial bearing load P may be used in place of P_r in the preceding formulas. The maximum ball load Q_o is given by the equation $Q_o = 5P_r/\eta \cos \alpha$. The value of P is given by

$$P = P_r + yP_a \qquad (3)$$

where y is a conversion factor and α the race contact pressure angle (Fig. 8).

The average y values for SKF bearings, according to the manufacturers, are as follows: for barrel-type roller-bearings, 2 to 3; for angular-contact ball-bearings, 0.33; for deep-groove bearings, 1 to 1.5; and for self-aligning ball-bearings, 1.5 to 4.5.

For single-row taper roller bearings, Timken gives the formula $P = 0.66P_r + kP_a$ where k averages 1.5 for standard bearings and 0.75 for steep-angle bearings intended for heavy thrust. If the calculated P is less than P_r, the thrust load P_a should be neglected and only the radial load P_r used.

Norma-Hoffman gives the following values for the equivalent bearing load P. With single-row deep-groove ball bearings when $P_a > \frac{1}{2}P_r$; $(P_a < \frac{1}{2}P_r)$; $[P_r = 0]$, the corresponding values of P are $0.5P_r + 1.5P_a$; (P_r); $[1.5P_a]$. With the single-row filling-notch type when $P_a > \frac{1}{3}P_r$; $(P_a < \frac{1}{3}P_r)$, the values of P are $\frac{1}{3}P_r + 2P_a$; (P_r). For double-row ball bearings, $P = P_r + 1.25P_a$. For angular-contact ball bearings, the equivalent thrust load is $2P_r + P_a$.

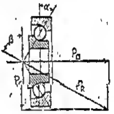


FIG. 8.

If a radial load P_r is imposed on an angular-contact bearing, it causes a thrust load P_a which necessitates an additional bearing on the same shaft. If α is the pressure angle (Fig. 8), $P_a = 1.22\,P_r \tan \alpha$ for ball bearings, $= 1.26P_r \tan \alpha$ for roller bearings (Juergensmeyer, *loc. cit.*).

Static Load Capacity

It is necessary to distinguish between static and dynamic load capacity. For the latter, see Relation of Load, Speed, and Bearing Life, p. 1036. The permissible static load upon a bearing is the non-brinelling static load, just before permanent surface deformation takes place. Exact experimental determination of this point is difficult; the first sign of brinelling can be noticed after reducing the bearing load and revolving it with an indicator against the races, a slight jump occurring when the rolling element travels over the brinelling mark. If the non-brinelling static load is exceeded with the bearing

Table 3. S.A.E. Standards for Radial and Angular Ball and Roller Bearings

B = Normal bore of inner race. D = Outside diameter,
W = Normal width of individual rings.

B, mm	Separable (open) type Fig. 13		Light series Figs. 14a, 14b, 15a, 15b, 16, 17			Medium series			Heavy series		
				W			W			W	
	D, mm	W, mm	D, mm	Single row, mm	Wide, in.	D, mm	Single row, mm	Wide, in.	D, mm	Single row, mm	Wide, in.
4	..	..	16	5							
5	16	5	19	6							
6	21	7	19	6							
7	22	7	22	7							
8	24	7	22	7							
9	28	8	26	8							
10	28	8	30	9	0.551	35	11	3/4			
12	32	7	32	10	5/8	37	12	3/4			
15	35	8	35	11	5/8	42	13	3/4			
17	44	11	40	12	11/16	47	14	7/8	62	17	1 3/16
20	47	14	47	14	13/16	52	15	7/8	72	19	1 3/8
25	52	15	52	15	13/16	62	17	1	80	21	1 3/8
30	..	..	62	16	15/16	72	19	1 3/16	90	23	1 9/16
35	..	..	72	17	1 1/16	80	21	1 3/8	100	25	1 3/4
40	..	..	80	18	1 3/16	90	23	1 7/16	110	27	1 15/16
45	..	..	85	19	1 3/16	100	25	1 9/16	120	29	2 1/8
50	..	..	90	20	1 3/16	110	27	1 3/4	130	31	2 5/16
55	..	..	100	21	1 5/16	120	29	1 15/16	140	33	2 1/2
60	..	..	110	22	1 7/16	130	31	2 1/8	150	35	2 5/8
65	..	..	120	23	1 1/2	140	33	2 5/16	160	37	2 13/16
70	..	..	125	24	1 9/16	150	35	2 1/2	180	42	3 1/8
75	..	..	130	25	1 5/8	160	37	2 11/16	190	45	3 1/4
80	..	..	140	26	1 3/4	170	39	2 11/16	200	48	3 7/16
85	..	..	150	28	1 15/16	180	41	2 7/8	210	52	3 5/8
90	..	..	160	30	2 1/16	190	43	2 7/8	225	54	3 7/8
95	..	..	170	32	2 3/16	200	45	3 1/16			
100	..	..	180	34	2 3/8	215	47	3 1/4			
105	..	..	190	36	2 9/16	225	49	3 7/16			
110	..	..	200	38	2 3/4	240	50	3 5/8			
115											
120											
125											
140											

Tolerances. For all dimensions +0.000

		Up to 19 mm	20–30 mm	35–50 mm	55–80 mm	85–110 mm
For B	Tolerance, in.	−0.0003	−0.0004	−0.0005	−0.0006	−0.0008

For D Open type, up to 28 mm, −0.0004 in.; 32–52 mm, −0.0005 in.

Other types up to 30 mm	32–47 mm	52–80 mm	90–120 mm	130–180 mm	190–240 mm
−0.0004	−0.0005	−0.0006	−0.0008	−0.0010	−0.0012

For W Open type ±0.002 in.; other types −0.005 in.

equivalent to $0.625B_r \times 1422$ with a Brinell number $B_r = 667$ corresponding to an average race hardness of 63 Rockwell C.

The dynamic load capacity of the rollers is again substantially lower. For an average life of 2,500 hr, P_o the load rating becomes 4,580 lb at 100 rpm and 2,140 lb at 1,000 rpm, with k_o and p_m reduced accordingly.

Long Roller Bearing. A No. 1-7194 R.B.C. needle-roller-bearing with 29 rollers of 0.1205 in. diam and 0.700 in effective length, with 60–63 Rockwell C outer race and roller hardness and a shaft hardness of 60 Rockwell C, has a static non-brinelling load capacity of 1,625 lb per roller. The substitute radius d_r from formula (2) with an inner race (shaft) diameter of 1 in. is 0.1075 in. The static load factor $k_o = 1625/0.1075 \times 0.7 = 21{,}620$ lb per sq in. With $E = 30 \times 10^6$, $p_m^2 = 21{,}620 \times E/4.64$ and static $p_m = 374{,}000$ lb per sq in. This is equivalent to $0.42B_r \times 1422$ with a Brinell number B_r 627 corresponding to a shaft hardness of 60 Rockwell C.

The dynamic load capacity of the rollers is substantially lower, and according to the load ratings of the manufacturer for a 3,000 hr life, P_o becomes 905 lb at 100 rpm and 552 lb at 1,000 rpm.

Relation of Load, Speed, and Bearing Life

In correctly made, properly installed and lubricated bearings, the life is determined by fatigue of the material of races and rolling elements in contact with each other, and the time when fatigue failure occurs is dependent upon both the magnitude of the stresses and the actual number of stress applications.

Tests with many bearings conducted under carefully controlled conditions have provided the basis for the load ratings of the bearing manufacturers which are always based upon a certain life expectancy percentage.

No uniform practice has so far been established for the minimum life-expectancy percentages. SKF, Norma-Hoffman, Timken, Tyson, and Bantam, for instance, base their ratings on a minimum life expectancy of 90 percent of a large number of bearings tested under identical conditions with regular lubricant renewal.

The Hyatt ratings are based on a 50 percent average life expectancy of large numbers of bearings tested without any lubricant renewal.

The average life expectancy of a large group of bearings is 3½ to 5 times their minimum life expectancy. Some of the manufacturers ratings are based on average and some on minimum life expectancy.

According to the tests of the bearing manufacturers, the life of a radial or angular bearing may be expressed by

$$H = C/NP^x \tag{4}$$

where H is the number of hours of life, N the rpm, P the load (actual radial or equivalent radial load from formula (3)), C a constant depending upon the type and size of the bearing, and x an exponent which is given as 3 for *SKF*, 3.3 for Martin-Rockwell, and 3.33 for Tyson bearings.

If H_c is the average life of a bearing as rated by a manufacturer, and P_{av} the average total bearing load, lb, the expected actual bearing life H can be written as

$$H = H_cF^x \tag{5}$$

where F is the ratio of the catalogue load rating to P_{av}.

H_c is selected differently by the various manufacturers. SKF uses 2,500 hr. Bantam 3,000 hr, Fafnir and New Departure 3,500 hr, Martin Rockwell and Hyatt 5,000 hr, Norma-Hoffman 10,000 hr, average bearing life. Timken and Tyson ratings are based on 3,000 hr of minimum bearing life.

For a true comparison of bearing ratings, it is therefore essential to convert the catalogue ratings to the same average or minimum life expectancy.

PRINCIPAL STANDARD BEARING TYPES AND COMPARATIVE CHARACTERISTICS

Radial Ball Bearings

Fig. 13. Separable or magneto type, with ball separator.

Fig. 14*a*. Deep groove with uninterrupted raceways. Conrad type, built as single- and two-row bearings. With ball separator.

Fig. 14*b*. Deep groove with filling slots. More balls than Conrad type. Built as single- and two-row bearings with ball separator.

Angular Ball Bearings

Fig. 15*a*. Single-row type. With ball separator. Used also in pairs as duplex bearing.

Fig. 15*b*. Two-row type with filling slots for assembling. With ball separator.

Fig. 16. Two-row self-aligning type. With ball separator.

Radial Roller Bearings

Fig. 17. Roller bearings with short solid cylindrical rollers. Built with two lips on either raceway, three lips and four lips, also as full-type bearing without retainer but with retaining snap rings with and without inner race.

Fig. 18. Roller bearings with long solid cylindrical rollers of small diameter (needle roller bearings). No retainer. With and without inner race. Require ample diametral and very little circumferential play for satisfactory operation. Originally used for slow speed and oscillating parts, especially in limited spaces. Now also used successfully for certain higher speed applications such as Diesel engine connecting rod big end bearings, textile spindles, etc. Because of simultaneous rolling and sliding of the needles, ample lubricant circulation and, frequently, cooling are necessary for other than intermittent short runs.

Fig. 19. Roller bearings with short or long wound cylindrical rollers (Hyatt type). With retainers and solid or split outer races. With and without inner race.

Bearings with Tapered Rollers

Fig. 20*a*. With retainer. Race cones intersect in one point of bearing axis. Lip at large diameter of inner race takes up roller thrust and keeps rolls aligned. Lip intact either on spherical surface or in two points. Also built as two-row bearing with one inner race and two outer races or one outer and two inner races.

Fig. 20*b*. With snap ring. Full type. Four-point contact between extended inner race lip and roller end face.

Spherical Race Bearings

Fig. 21. Self-aligning bearing with two rows of short barrel rollers (SKF type). Spherical outer race in one piece or in two parts for preloading. Roller thrust taken on spherically shaped surfaces of center flange on inner race. Two-piece retainer.

Fig. 22. Self-aligning bearing with hourglass rollers (Shafer type). Spherical inner race. Retainer. Also built as two-row bearing with one piece inner race.

Axial or Thrust Ball Bearings

Fig. 23. Multirow flat race ball bearings with retainer. Also built as single-row bearing.

Fig. 24. Grooved-race single-direction ball bearings with retainer. Also built as self-contained type with and without ball retainer. Two direction bearings with three races and two rows of balls.

Fig. 25. Four-point contact single-direction bearings (Auburn type). Contact points of each race located in cone intersecting with similar cone of companion race in one point of bearing axis. Usually built as self-contained bearing without ball retainer. Two-direction bearings with three races and two rows of balls.

Roller Thrust Bearings

Fig. 26. Single direction. Bearings with cylindrical rollers. Short rollers stacked radially to reduce sliding. Thrust taken on retaining ring of separator. Two-direction bearings with three races and two sets of rollers.

Fig. 27. Bearings with tapered rollers. Race cones intersect in one joint of bearing axis. Roller thrust supported on lips of races and by separator. Also built as self-contained bearing without separator.

factors are based upon the service experience of the individual manufacturers of bearings who should be consulted regarding them.

Table 2. Corrections for Carrying Capacity of Bearings with Revolving Outer Races

Bearing series	Multipliers for rpm				
	Marlin Rockwell	Norma-*Hoffman*	Fafnir	N.D.	Hyatt solid roller
Extra light		1.2			
Standard light	1.4	1.5	1.5	1.46	1.3
Standard medium	1.6	1.6	1.7	1.61	1.3
Standard heavy	1.8	1.7	1.7	1.74	
Extra large, extra light	1.3				
Extra large, light	1.4				
Extra large, medium	1.5				

Clearance is the initial looseness of a self-contained bearing. Play is the possible total movement (axial, radial, or angular) of one race against the other when the bearing is subjected to load.

The play of installed bearings depends on bearing clearance, fit of the races, temperature difference of the races, elastic deformation of the races under load, the normal running accuracy of the mounted bearing, and the special characteristics of the type used. The first three factors are closely interrelated. Inner races expand when pressed onto shafts, thereby reducing the clearance. With extreme press fits, the race expansion may cause destructive loads upon races and rolling elements.

The bearing manufacturers have established practices for proper housing and shaft fits which should be closely adhered to, and where tight press fits are necessary it is possible to obtain bearings of appropriately greater clearance. For installations where play must be avoided, preloaded angular-contact bearings in pairs are available. To avoid excessive loads from race expansion, correct shaft and housing fits are particularly essential. Bearings with greater manufacturing tolerances or variations require more play than precision bearings so as to avoid mounting damage. Bearings without self-aligning features with sufficient play permit of slight misalignment or shaft deflection but the maximum stresses upon the rolling elements are increased.

Bearing Materials. Modern rolling-contact bearing races and rolling elements of better quality are generally made from high-carbon chrome-alloy steel, hardened throughout or surface hardened. S.A.E. steel 52100, heat-treated to a Rockwell C hardness varying from over 60 up 64, is very extensively used. These bearing steels corrode readily, and therefore only neutral lubricants should be used. For the same reason, the bearings must be protected against the intrusion of water, acids, alkalies, moisture, and active gases. Where liquids of this nature cannot be kept out, bearings can be supplied in a number of standard sizes from stainless steel. Such bearings are widely used in gasoline pumps and meters.

According to Norma-Hoffman, a heat-treated stainless steel of an approximate analysis of 0.95 to 1.05 carbon and 17.0–18.0 chromium can be hardened to as high as 60 Rockwell C hardness and over.

Separators for Rolling Elements. The purpose of separators or cages is to hold the rolling elements in proper relation to one race; to avoid contact between the rolling elements; to hold the rolling elements in place when one or both races are removed; to guide the rolling elements if there are no other means provided by the race tracks; and to dampen noise.

Table 5. Criteria for Comparing Rolling Contact Bearings

Bearing type	L.C. Radial		L.C. X	L.C. Axial	Sh. C.		Sp. C—In multiples of 100 rpm	½″	1″	2″	4″	Fr.C.	A.C.	M.C.	A.L.C.	Note
A	1050		1 to 1.2	......	US MS	1.0 0.33	Pr UPr	250 500	150 320	.. ..	.. ..	L	S	NM	2	
B	L M H	1075 975 875	0.8 to 1.5	......	US MS ES	1.0 0.5 0.33	R Pr UPr	75 200 450	60 150 300	45 95 ..	20 45 ..	ML L L	FS FS S	SM	1	
C	L M H	1050 950 825	1.1	......	US MS ES	1.0 0.5 0.33	R Pr	65 150	50 100	30 80	15 35	ML ML	FS FS	NM	1	1
D	L M H	1500 1250 1150	2	......	US MS ES	1.0 0.5 0.33	R Pr	75 200	60 150	45 95	20 45	ML L	FS FS	SM	1	
E	....		0.5 to 0.33	L 3200 M 3000 H 2000	US MS ES	1.0 0.5 0.33	R Pr	75 200	60 150	45 95	20 45	ML ML	FS FS	NM	2	
F	L M H	1475 1275 1075	0.833 to 1.2	......	US MS ES	1.0 0.5 0.33	R Pr	55 140	45 95	35 70	15 30	M ML	FS FS	NM	1	1
G	L M H	975 825 750	2.5 to 4.5	......	US MS ES	1.0 0.5 0.33	Pr	200	150	95	45	L to ML	FS	EM	1	2
H	L M	825 725	1.5 to 3	......	US MS ES	1.0 0.5 0.33	Pr	200	150	95	45	ML	FS	EM	1	2
I	IR SH	1650 2225	......	0	US MS	1.0 0.77	R Pr	... ...	50 150	30 95	18 45	ML L	FS S	NM	ATB	3
J	IR SH	1725 2325	......	0	ES	0.5	R	...	45	28	16	M	FS	NM	ATB	4
K	1500		......	See note	US MS ES	1.0 0.77 0.5	Pr	...	75	45	25	ML	FS	NM	1 or 2	5
L	2000		......	0	US MS ES	1.0 0.77 0.5	Pr	...	20	15	8	MH to H	N	NM	ATB	6
M	IR SH	525 675	......	0	US MS ES	1.0 0.7 0.35	R	...	20	20	15	MH to H	N	NM	ATB	
N	IR SH	750 925	......	0	US MS ES	1.0 0.7 0.35	R	...	20	20	15	H	N	NM	ATB	
O *O*	IR SH IR SH	2500 3900 4300 6900	......	0	US MS ES	1.0 0.68 0.34	Pr	...	50	40	18	MH to H	FS to N	NM	ATB	7A 7B
P	RA SA	1500	1.43 0.7	 1450	US MS ES	1.0 0.77 0.33	R	...	50	40	18	MH to H	FS to N	NM	2	8
Q	RA SA	1775	1.39 0.725	 1700	US MS ES	1.0 0.77 0.33	R	...	50	40	18	MH to H	FS to N	NM	2	9

light axial load than deep-groove bearings but that the values approach each other with heavier loads.

Ball and cylindrical-short-roller bearings have low sliding friction because of small contact areas and high manufacturing accuracy. With [illegible] variations increase and shaft deflection and [illegible] to the increase in sliding friction.

Muzzoli's tests with needle bearings of 60 mm bore, 90 mm O.D., and 28 mm width with 79 needles of 3 mm diam and 19.8 mm long showed coefficients of friction ranging from 0.0083 under 440 lb load to 0.0044 under 46,400 lb load.

When sufficient space is available, oil-bath lubrication is preferable provided proper seals prevent oil leakage. For limited lubricant space, grease is preferable.

If the effect of seals is disregarded, which in properly designed installations can be entirely avoided, the friction of rolling contact bearing is not only very low, but also almost entirely independent of speed, load, and temperature; the starting friction is only very little higher than the running friction.

Under normal operating conditions, the temperature rise is slight, but at high speeds and with large bearings considerable heat is generated and oil circulation is necessary to carry off the heat.

BEARING STANDARDS

Various types of rolling contact bearings are now standardized as regards dimensions, in sizes ranging from fractions of an inch to 12 in. bore. The International and the major part of the S.A.E. standards for ball and roller bearings are based on metric dimensions. The S.A.E. standards for taper roller bearings are based on inch dimensions.

The regular radial and angular ball and roller bearing S.A.E. standard series are divided into light (200), medium (300), and heavy (400) series. The last one or two digits of the bearing number, beginning with digit 04, represent one-fifth of the bore in millimeters, and digits 00, 01, 02, and 03 represent 10, 12, 15, and 17 mm bore, respectively.

The wide metric ball and roller bearing S.A.E. series are also divided into light (5,200), medium (5,300), and heavy (5,400) series with the bore indicated in the same manner as in the regular series.

The S.A.E. standards covering one and two directions ball thrust bearings have not been generally adopted.

Ball and roller bearings of the same S.A.E. dimensions should not be used interchangeably in the same applications, because of different requirements for shaft and housing fits and, frequently, of different methods for holding the races in position. Taper roller bearings with standard bore and outside diameter require additional clearance space for projecting retainers.

In addition to the International and S.A.E. standard sizes, the bearing manufacturers have a great variety of other standardized sizes and types.

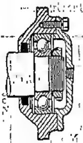


Fig. 32.

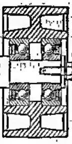


Fig. 33.

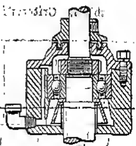
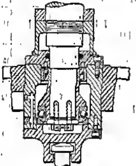


Fig. 34.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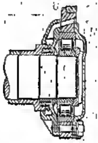


Fig. 36.

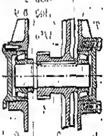


Fig. 37.

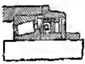


Fig. 38.

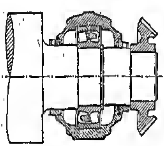


Fig. 39.

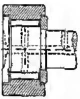


Fig. 40.

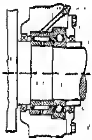


Fig. 41.

Figs. 32–41.—Bearing Installations.

Typical Bearing Installations

Fig. 32. Standard horizontal mounting for single- and double-row ball bearings. Felt seal. Grease lubrication.

Fig. 33. Loose pulley. Radial bearings with built-in felt seals.

Fig. 34. Vertical high-speed spindle mouting. Oil-circulation by flinger. External labyrinth dirt protection.

Fig. 35. Self-aligning vertical-shaft mounting with radial and thrust bearings.

Fig. 36. Railway-motor bearings. Grease lubrication with labyrinth seals.

Fig. 37. Taper roller bearings with shim and screw-clearance adjustment for outer races.

Fig. 38. Leather shaft seal with inner-race adjustment for taper roller bearing. Outside flinger added.

Fig. 39. Spherical roll bed bearings mounted in split pillow blocks. Floating bearing held on shaft with snap ring. External labyrinth flingers. Grease lubrication.

Fig. 40. Cam follower with needle rollers.

Fig. 41. Printing-press roll with needle roller bearings and radial ball bearing for axial stabilization. Grease lubrication. Felt or leather seals are used.

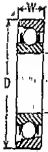


FIG. 13.


FIG. 14a.


FIG. 14b.


FIG. 15a.

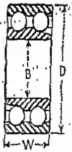


FIG. 15b.

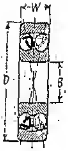


FIG. 16.

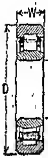


FIG. 17.


FIG. 18.

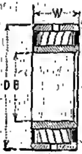


FIG. 19.

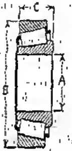


FIG. 20a.

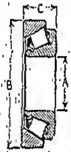


FIG. 20b.


FIG. 23.


FIG. 24.


FIG. 21.

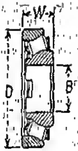


FIG. 22.


FIG. 25.


FIG. 26.


FIG. 27.

FIGS. 13–27.—Standard Bearing Types.

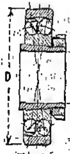


FIG. 28.


FIG. 29.


FIG. 30.


FIG. 31.

FIGS. 28–31.—Special-purpose Bearings.

PACKINGS

BY

F. C. THORN

Packings include gaskets, valve disks and seats, sliding contact packings, and diaphragms.

Gaskets are installed between stationary surfaces, which consist of parallel flanges, or concentric cylinders. Flat gaskets for flange joints may extend beyond the bolts, as in (Fig. 1), but, since the portion beyond the bolt is of no value, the ring gasket (Fig. 2) is more economical. The materials used for flat flange gaskets, their compression characteristics, and service for which they are recommended are indicated in Table 1.

For greater compressibility than can be obtained with a flat gasket, a thick gasket, installed in a groove, may be employed. Unless the material is very hard, it is not good practice to permit the gasket to protrude from the groove (Fig. 3) since it is liable to "mushroom." Better practice is to employ a tongue-and-groove construction (Fig. 4).

In flange joints, the following rules should be observed:

1. Employ bolts of sufficient size and number so that, *after* deducting the relieving effect of internal pressure, the residual unit gasket load shall be 5 to 10 times the internal pressure.

Table 1

Type	Service principally	Common thickness, in.	Percent reduction in thickness at pressure[b]			Permenent set at 10,000 lb., percent
			500 lb	2000 lb	10,000 lb	
Sheet rubber	Water (low pressure)	1/16	8	14	37	10
Cloth-inserted sheet rubber	Water (low pressure)	1/16	4	7	33	16[c]
Corkboard	Oil (low pressure)	1/16	57	69	78	58
Gasket paper	Oil (low pressure)	1/16	13	22	31	14
Rubberized asbestos cloth	Hot water (boiler manholes, etc.)	1/4	11	22	34	31
Compressed asbestos sheet[a]	All services up to 600 F	1/16	1	10	16	6
Corrugated sheet-metal, with filling (Fig. 5)	Steam, oil at high temperatures	1/8	8	11	16	2
Metal jacket over asbestos center (Fig. 6)	Steam, oil at high temperatures, boilers, automobile cylinders, etc.	1/16	2	4	8	4
Spirally wound steel strip with intervening asbestos layers (Fig. 7)	Steam, oil at high temperatures, boilers	3/16	3	4	11	9

Solid metal, in flat, round, oval, or profile (Fig. 8) cross sections, is used with steam, oil, and gases at very high pressures and temperatures.

[a] Rubber compound containing about 70 percent asbestos fiber.

[b] Determined by cold compression of disks, 1 sq in. area by thickness indicated, except in case of semimetallic gaskets (Figs. 5, 6, 7) in which a $3\frac{1}{2} \times 4\frac{1}{2}$ in. gasket was tested (zero point in all cases taken at 7 lb per sq in. load).

[c] Fabric threads ruptured.

Compression is largely controlled by friction between platen and gasket. Smooth or lubricated platens, thicker or narrower gaskets, and, in most cases, higher temperatures give higher compressions at all loads. Set is increased by higher temperature and longer time under load.

Special-purpose Bearings

Fig. 28. Bearings with split tapered adapter sleeves. For use on straight shafts in any desired location. Also applied to deep-groove ball-bearing, and cylindrical short-roller bearings, usually with outer races having spherical seats. Extended solid inner races with setscrews or other locking devices are used for the same purpose.

Fig. 29. Snap ring bearings. Eliminate the necessity for machining housing shoulders to locate bearings. Also applied to needle bearings.

Fig. 30. Shielded bearings. Stamped disks attached to outer races assist in dirt protection and grease retention. Made with single- and two-row ball bearings. Used extensively as prelubricated types for inaccessible locations with lubricant sufficient for estimated bearing life.

Ball Bearings with Unground Races

Fig. 31. Grooved solid inner race with two-piece outer race held together by spun shell. Built in many modifications for radial, angular, and thrust loads of medium intensity and at moderate speeds.

Table 4. S.A.E. Standards for Tapered Roller Bearings
(Figs. 20*a* and 20*b*)
A, bore; *B*, outside diameter; *C*, width (shoulder to shoulder)
Tolerances: −0.000; bore, 2½ in. or smaller, +0.0005; Over 2½ in., +0.001; O.D., +0.001 in.; width, +0.008 in.

A, in.	Light series		Medium series		Heavy series	
	B, in.	*C*, in.	*B*, in.	*C*, in.	*B*, in.	*C*, in.
0.375	1.2595	0.394				
0.500	1.3775	0.433				
0.625	1.625	0.5625				
0.750	1.8504	0.5662	1.938	0.7813		
0.875	2.047	0.591	2.240	0.7625		
1.000	2.3125	0.750	2.375	0.7813	2.615	0.9375
1.125	2.500	0.8125	2.615	0.9375	2.8593	1.1875
1.250	2.717	0.7813	2.750	0.9375	3.125	1.1563
1.375	2.844	0.8125	3.000	0.9375	3.3125	1.1563
1.500	3.000	0.8125	3.125	1.000	3.4375	1.1875
1.625	3.1496	0.8268	3.375	1.1875	3.750	1.250
1.750	3.3464	0.8125	3.4375	1.1875	4.000	1.375
1.875	3.500	0.8125	3.6718	1.1875	4.250	1.4375
2.000	3.6718	0.8125	3.875	1.1875	4.375	1.500
2.225	3.937	0.8268	4.125	1.1875	4.875	1.500
2.500	4.3307	0.8661	4.4375	1.1875	5.375	1.625
2.750	4.6875	1.000	4.875	1.1875	5.909	1.750
3.000	5.000	1.0625	5.375	1.1875	6.375	1.875
3.250	5.375	1.1875	5.5115	1.4375	6.625	1.875
3.500	5.625	1.1875	6.000	1.5625	7.125	1.875
3.750	5.875	1.250	6.375	1.5625	7.500	2.250
4.000	......	......	6.625	1.625	7.875	2.4375
4.250	......	......	......	......	8.375	2.625

2. Select a gasket wide enough to support, without crushing, the original bolt load *before* deducting for the internal pressure. Approximate values for crushing strength of gaskets at various thicknesses and temperatures can be obtained from the gasket manufacturer.

Most leaky joints are due to insufficient bolt pressure. If, however, the joint is designed to be self-tightening under pressure, as in the boiler manhole cover (Fig. 9), a single bolt in the center is sufficient.

Instances of gaskets packing concentric surfaces are the mechanical joint for cast-iron pipe (Fig. 10), the condenser tube-sheet ferrule (Fig. 11), and the ammonia condenser fitting. There are also gaskets shaped like cups and designed to be self-tightening under pressure, such as the gasket (Fig. 12) used with "boltless" autoclave doors.

Valve disks are specialized gaskets designed for joints that are frequently broken and restored. Disks for globe valves (Fig. 13) are usually encased in a disk holder with a swivel mounting, so that they do not rotate while closing. They are made of hard rubber or Bakelite. Hydrant valves (Fig. 14) are generally bevel-seated; they are made of medium hard rubber or leather. Pump valves (Fig. 15) are described on p. 1621. **Valve seats** of rubber are used with metal valve disks on some pumps, such as the *rotary drilling* pump (Fig. 16).

Sliding contact packings include all packings that operate against moving surfaces. A feature common to all of them is that they leak while in motion, although under favorable conditions the leak may be scarcely measurable and is frequently invisible if the fluid is a gas or becomes a gas on escaping. Leakage is increased by high pressures and low viscosity; it is minimized by deep packings and tight adjustment, but as both of these give rise to friction, it is not always possible to employ them, especially at high speeds. The only sure way to eliminate leakage is to inject into the packing another liquid at higher pressure, permitting it to leak both ways from the point of admission. This is the basis for the use of the **water seal** in centrifugal pumps (Fig. 27) and for the force-feed lubricating of metal packing sets (Fig. 24) when operating against steam or gases.

Sliding contact packings may be classified on the basis of the shape of the opposing surfaces—cylindrical, conical, spherical, or flat. Examples of the last three are, respectively, the plug-cock lining (Fig. 29), the ball-joint (Fig. 30), and the "rotary seal" when used in its restricted sense of a rotating collar operating against the flat end face of a stationary collar. Two types are shown: in the automobile water pump seal (Fig. 31), the rotating Bakelite sealing member is supported flexibly on a rubber cup; in the domestic refrigerator seal (Fig. 32), the stationary sealing member (bronze) is suspended by a diaphragm. Cylindrical packings may in turn be classified according to whether packing is accomplished on the outside perimeter, as in the piston packings (Figs. 17 to 20), or on the inside perimeter, as in the rod and shaft packings (Figs. 21 to 28). A second basis of classification is according to the type of motion—rotary, reciprocating, or helical (as in a rising stem valve packing). A third basis of classification is into automatic (*i.e.*, self-tightening under pressure) or non-automatic (*i.e.*, tightened by external means, generally a gland drawn into the opening of a stuffing box).

The simplest **piston packing** is the split ring (Fig. 17) constructed generally of cast iron and employed on pistons of gas, oil, and steam engines and compressors (see p. 848). Large pistons frequently employ segmental rings similar to metal rod packings (Fig. 24) but facing outward. These types are fully automatic, being driven against the cylinder wall by fluid pressure from within. Split rings made from layers of cotton fabric, bonded with rubber

Table 5. Criteria for Comparing Rolling Contact Bearings.—(*Continued*)

Bearing type	L.C. Radial		L.C. X	L.C. Axial	Sh. C.		Sp. C.—In multiples of 100 rpm	½"	1"	2"	4"	Fr.C.	A.C.	M.C.	A.L.C.	Note
R	A B	1200 1750	2.75	......	US MS ES	1.0 0.77 0.33	R	...	50	40	18	MH to H	FS to N	NM	1	10
S	A B	2150 1850	3 2	......	US MS ES	1.0 0.77 0.33	Pr	A B	... ...	.. 75	30 30	M to MH	FS	EM	1	11
T	A B	1100 1750	0.9 1	......	US MS ES	1.0 0.77 0.33	R	...	50	40	18	MH to H	FS to N	EM	2	12
U	1900		2	......	US MS ES	1.0 0.77 0.33	R	...	...	40	18	MH to H	FS to N	EM	1	13
V	A B	0 0	0 0	1800 2200	US MS ES	1.0 0.5 0.33	Pr	50	35	12	5	MH to H	N	NM	2	14
W	0		0	3000	US MS ES	1.0 0.77 0.15	Pr	...	...	15	10	H	N	NM	2	15
X	0		0	3500	US MS ES	1.0 0.77 0.5	R	...	...	..	6	H	N	NM	2	16

A, B.B. open type; *B*, B.B.-S.W. deep groove; *C*, B.B.-D.W. deep groove; *D*, B.B.-S.W. filling slots; *E*, B.B.-S.W. angular; *F*, B.B.-D.W. angular; *G*, B.B.-S.W. spherical; *H*, B.B.-D.W. spherical; *I*, R.B.-S.W. cyl. rollers; *J*, R.B.-D.W. cyl. rollers; *K*, R.B.-S.W. cyl. rollers; *L*, R.B.-S.W. cyl. rollers; *M*, R.B.-D.W. cyl. wound rollers; *N*, R.B.-E.W. cyl. wound rollers; *O*[7A], R.B. needle; *O*[7B], R.B. needle; *P*, R.B.-taper rollers; *Q*, R.B.-taper rollers; *R*, R.B.-taper rollers double bearing; *S*, R.B.-barrel rollers—two row spherical; *T*, R.B.-S.W. hourglass rollers spherical; *U*, R.B.-D.W. hourglass rollers spherical; *V*, B.B.-axial groove type; *W*, R.B. axial cyl. rollers; *X*, R.B. axial conical rollers.

[1] Two row; [2] self-aligning two row; [3] cyl. inner race; [4] cyl. inner race light ser. only; [5] races with lips for axial location only; [6] full type no retainer; [7A] ¾"–1½" bore inner race; [7B] 1⅝–4" bore inner race; [8] retainer type; [9] full type no retainer; [10] retainer type, A up to 3 in., B over 3 in. to 4 in.; [11] A light series 3 to 5 in. bore, B medium series 1½ to 5 in. bore; [12] A ⅝ to 1⅜ in. bore, B 1⅕ to 4 in. bore; [13] two-row 1⅛ to 4 in. bore; [14] single dir. flat, A ½ to 1½ in. bore, B 1¾ to 4 in. bore; [15] single dir. flat 2 to 5 in. bore; [16] single dir. flat 3 to 5 in. bore.

ABBREVIATIONS

B.B., ball bearing; R.B., roller bearing; L, light series S.A.E.; M, medium series S.A.E.; H, heavy series S.A.E.; S.W., single-row width; D.W. double width; E.W., extreme width; IR, bearing complete with inner race; SH, rollers operate direct on shaft; R, regular bearing finish; Pr, precision finish; UPr, ultra precision finish; ATB, additional thrust bearing required; RA, regular angle; SA, steep angle.

E AND PIPE FITTINGS

BY

C. W. HAM

Piping Standards

Standardization in the piping industry has made rapid progress in recent years. In 1908, the A.W.W.A. adopted a set of specifications for cast-iron water pipe and fittings, largely replacing tho specifications adopted by the New England Water Works Association in 1902. Specifications for cast-iron gas pipe and fittings adopted by the American Gas Institute in 1913 were later revised and adopted by the American Gas Association. The A.S.A. has approved numerous standards for screwed and flanged fittings. In the pages that follow, the significant portions of many of these standards are given.

In June, 1935, the A.S.A., under the sponsorship of the A.S.M.E., issued The American Tentative Standard *Code for Pressure Piping* (A.S.A. B31.1-1935) covering steam, gas and air, oil, and district heating piping systems and fabrication details of pipe supports, bends, joints, valves, fittings, etc. The code recommends, or presents as mandatory, comprehensive applications of materials and products to the piping systems indicated above. It is intended to serve as a guide in the drafting of regulations and as a standard of reference for minimum safety requirements for those concerned with pressure piping. References to the code and brief abstracts will be given in the pages that follow. The code may be obtained from the A.S.M.E.

Cast-iron Pipe

Cast-iron pipe is extensively used for water, gas, sewage, culverts, drains, etc., in a wide range of sizes and for varying pressures, and is particularly **adapted to underground and submerged service** because of its comparatively high corrosion-resisting qualities. It is more durable than wooden-stave, wrought-iron, or steel pipe, but its first cost is greater for the ordinary sizes required in a pressure pipe line or in a distributing system. The tensile strength of commercial cast-iron pipe is uncertain, and, because of its low elasticity, it is not suitable for lines subject to the strains of expansion, contraction, and vibration, unless it is of very heavy weight. This pipe may be had in various thicknesses and weights with either **flanged ends** or **bell-and-spigot ends.** The latter are generally used for underground work, making a tight joint when properly put together, calked, and leaded. For exposed piping, flanged ends are used, the joints being made up with gaskets. Flanged pipe has superior strength and tightness of the joint and is used where pipe lines can be well supported. The bell-and-spigot joint possesses greater flexibility, provides for expansion and contraction, and is therefore especially suitable for water pipe and almost exclusively used for that purpose. Figure 1 shows the standard form of the joint for ordinary pressures. Other forms sometimes used in the United States are tho bell-and-plain-spigot and the turned-and-bored joints. Plain-end pipe with couplings for high-pressure gas mains and threaded pipe for corrosive liquids are also manufactured. Cast-iron pipe, fittings, and valves have not been found suitable for superheated steam service. The Code for Pressure Piping (A.S.A. B31.1-1935)

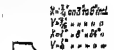


Fig. 1.

Comparative Features of the Principal Bearing Types

The criteria of Table 5 may be used for comparing the important characteristics of rolling contact bearings for the purpose of their appropriate selection.

Load Criterion (L.C.). Expressed in pound per square inch projected bearing area. The loads have been chosen arbitrarily, but with a view to a fair comparison of the load capacity of the different types. The principal manufacturers' load ratings at 300 rpm of bearings of the same inner race bore (or shaft diameter) have been converted into capacities for the same approximate minimum life expectancy of 3,000 hr and then divided by the projected bearing area (outside diameter minus bore times width). Averages from the calculated figures for bearings ½ to approx. 4 in. bore are used, unless otherwise stated. Either the radial L.C. or the axial L.C. are given and also the conversion factor X. (L.C. Radial) = X (L.C. Axial).

Shock Criterion (Sh. C.). The reciprocals of the recommended safety factors for shock as compared with a safety factor of 1 for steady load and speed conditions are used as criteria, bearings that require the largest shock safety factors therefore having the lowest relative shock resistance or criterion. The shock criteria are listed for US, uniform, steady load; MS, moderate shock; and ES, extreme and indeterminate shock.

Speed Criterion (Sp. C.). The average maximum rpm with rotating inner races of bearings having ½, 1, 2, and 4 in. bore of inner race (or shaft diameter) are used as criteria. For medium and heavy series S.A.E. dimension bearings, the bores of corresponding light series bearings with the same outside diameter apply.

Friction Criterion (Fr. C.). The relative magnitude are expressed as L (low), ML (medium low), M (medium), MH (medium high), H (high).

Acoustic Criterion (A.C.). The relative bearing noise is expressed by S (silent), FS (fairly silent), and N (noisy) at speeds from approximately ⅛ to ½ the maximum bearing rpm up to maximum bearing rpm.

Misalignment Criterion (M.C.). Expressed by internal ability to compensate for misalignment and shaft deflection. EM is extreme misalignment (±5 percent); SM, slight misalignment (±1 percent); and NM, no misalignment.

Axial Location Criterion (A.L.C.). Expressed by the number of bearings (1 or 2) needed to stabilize a shaft assembly axially.

Mounting and Maintenance of Rolling Contact Bearings

It is essential to protect rolling contact bearings from the effects of dirt, grit, and splash by suitable seals, which should also prevent lubricant leakage. Grease is used with limited axial space, in poorly accessible locations, and in prelubricated bearings. The principal seals used are felt, leather, and cork (usually for liquid-splashed or submerged bearing installations); and labyrinth seals, alone or in combination, with other seals. For high speed applications labyrinth seals only should be used. Where oil bath lubrication is employed, there should be ample axial space for the displaced lubricant in rotation without churning.

The most important advantage of rolling contact bearings over plain bearings is their availability as complete units, standardized over a wide range of sizes, and with definitely known load characteristics. In addition, they are largely free from leakage of lubricant, have low starting friction, can be operated at great speeds, require little repair, and cover any desired life expectancy. Lubrication is necessary only at long intervals.

The disadvantages are higher first cost, the impossibility of quick or temporary repairs, the more complicated shaft and housing design, greater noise (especially at high speeds), lower resistance to shock, general nonavailability as completely split types, substantially larger bearing diameters except with certain types of needle bearings, and the serious effects of dirt and grit.

states that cast iron shall not be used for any service over 450 F and not for oil over 300 F.

The **specifications** of the A.W.W.A. for bell-and-spigot pipe are now used generally as the manufacturers' standard throughout the United States. Standard dimensions are given in Tables 1 and 2. Brackett's formula for the thickness of cast-iron pipe is $T = 0.25 + (P + P')r/3,300$, in which T is the thickness, in.; P the maximum static pressure, lb per sq in., for which the pipe is designed; P' the allowance made for water ram; and r the radius, in. This gives a large factor of safety to cover inequalities of the castings, strains brought on the pipe from other causes than the water pressure, and to give sufficient thickness to ensure the pipe against breakage in shipping and laying. For ordinary waterworks conditions for pipes, 42 to 60 in. diam, 70 lb per sq in. is enough for P', but for smaller pipe Brackett allows the following values:

Diam of pipe, in.	= 36	30	24	20	16	12	10 to 3
P', lb per sq in.	= 75	80	85	90	100	110	120

Table 3. Weight of Lead Required for Cast-iron Bell-and-spigot Pipe

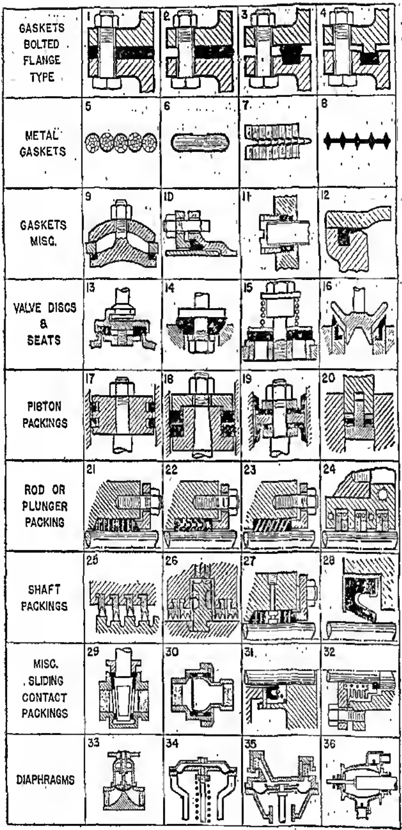
Depth of joint, in.	Diameter of pipe, in.															
	4	6	8	10	12	14	16	18	20	24	30	36	42	48	54	60
	Approximate weight of lead per joint, lb															
2	8	10	13	16	19	22	30	34	37	44	54	65	75	86	98	108
2½	9	12	16	19	23	26	36	40	44	53	65	77	86	102	117	130
Solid	13	18	23	31	37	39	65	72	80	95	118	140	155	202	239	256

For city work where great damage would be caused by breakage, and for single lines with no reserve, where an interruption of the supply would be a very serious matter, the pipes may be made thicker than computed by this formula. See the A.S.A. proposed new method of determining thickness.

The employment of **cast-iron pipe for** gas supply and distribution is second in importance only to its use for carrying water. In this country the bell-and-spigot type of joint, similar to that for water (Fig. 1) has generally been found to meet all requirements for gas. Abroad, however, practice varies and the turned and bored joint is preferred by many prominent users. Standard bell-and-spigot gas pipe has dimensions and weights approximately the same as water pipe of Class A, Table 1. Standard heavy gas pipe has dimensions and weights similarly approximating Class B, Table 1. Standard flanged gas pipe has dimensions and weights approximately the same as flanged water pipe of Class A, Table 5.

The **specifications** of the American Gas Association are the generally accepted standards for cast-iron gas pipe and fittings. These specifications are based largely on those of the A.W.W.A. (see above). While the flange diameters, bolt circles, etc., of the American Standards for steam (see p. 922) have been generally adopted by the manufacturers of flanged pipe for water, the standards of the American Gas Association are used for flanged gas pipe.

Pipe Joints. The most common method of making the bell-and-spigot pipe joint is by pouring and calking the lead into the bell. In the built-up sections of cities, where streets are crowded underground with other structures, and surface traffic is heavy, the tendency seems to be to increase the thickness of metal, and to use exclusively (even for gas) pipe having bells calked with lead, as securing the most flexible joint. Such pipe with lead joints are also preferable where conditions of subsoil, as in newly made ground, indicate a possible subsidence. One advantage of a lead joint is that if it leaks, simple recalking will usually correct the trouble, whereas a cement joint must



FIGS. 1-36.—Packings.

iron joint. By making the tapers of slightly different pitch, the joint provides *for flexibility while remaining tight.* Two bolts to the joint are sufficient, except for pressures above 175 lb per sq in. "Universal" cast-iron pipe is largely used for carrying gas and water and is suitable for all pressures and services. The pipe is tested with hydrostatic pressure of 300 to 500 lb per sq in. All universal pipe and special castings of a given diameter and of any class are interchangeable with those of a different class. Standard laying lengths, 6 ft. Thicknesses and weights are given in Table 7.

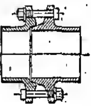


FIG. 4.—"Universal" C. I. Pipe.

Fittings for Cast-iron Water Pipe. Flanged fittings *of the dimensions of the American Standard for steam are* not often used with cast-iron water pipe. The longer fittings of the A.W.W.A. are generally preferred because of lower friction loss. The dimensions of the flanged fittings of this class conform very closely to the dimensions of the bell-and-spigot fittings of the *A.W.W.A. The flange thicknesses conform to those of the American Standard* (see pp. 923 and 924) and they are drilled to American Standard templates. These fittings, both flange and bell-and-spigot type, are made in a great variety of forms known as "Standard Special Fittings." For dimensions and weights, see manufacturers' catalogues, or standard specifications of the A.W.W.A.

Table 5. Standard Cast-iron Flanged Pipe for Water*

Inside diam of pipe, in.	Class A, Head, 100 ft; 43 lb pressure: Thickness of pipe, in.	Weights per Foot, lb	Weights per 12 ft length with flanges, lb	Class B, Head, 200 ft; 86 lb pressure: Thickness of pipe, in.	Weights per Foot, lb	Weights per 12 ft length with flanges, lb	Class C, Head, 300 ft; 130 lb pressure: Thickness of pipe, in.	Weights per Foot, lb	Weights per 12 ft length with flanges, lb	Class D, Head, 400 ft; 173 lb pressure: Thickness of pipe, in.	Weights per Foot, lb	Weights per 12 ft length with flanges, lb
3	0.39	13.0	168	0.42	14.6	188	0.45	15.5	199	0.48	16.4	211
4	0.42	18.0	235	0.45	20.1	254	0.48	21.3	272	0.52	22.8	297
6	0.44	27.9	357	0.48	31.1	391	0.51	32.9	417	0.55	35.3	451
8	0.46	38.7	492	0.51	42.7	547	0.56	48.0	603	0.60	56.2	648
10	0.50	51.9	666	0.57	58.0	762	0.62	65.5	831	0.68	71.4	914
12	0.54	67.0	869	0.62	76.4	1,000	0.68	85.4	1,100	0.75	93.7	1,220
14	0.57	82.3	1,060	0.66	94.7	1,230	0.74	108.1	1,390	0.82	119.2	1,540
16	0.60	98.8	1,280	0.70	114.6	1,500	0.80	133.3	1,710	0.89	147.5	1,910
18	0.64	118.3	1,520	0.75	137.8	1,780	0.87	162.4	2,070	0.96	178.4	2,290
20	0.67	137.4	1,770	0.80	163.1	2,120	0.92	190.6	2,440	1.03	212.3	2,730
24	0.76	186.5	2,410	0.89	217.3	2,820	1.04	257.6	3,300	1.16	266.0	3,690
30	0.88	266.1	3,490	1.03	312.6	4,050	1.20	366.9	4,760	1.37	421.2	5,440
36	0.99	358.7	4,740	1.15	410.7	5,500	1.36	497.7	6,500	1.58	581.9	7,560
40	1.06	427.2	5,684	1.23	497.0	6,590	1.48	601.6	7,921	1.72	703.4	9,200
42	1.10	464.6	6,180	1.28	542.2	7,200	1.54	657.4	8,640	1.78	754.1	10,000
48	1.26	608.0	8,120	1.42	687.2	9,130	1.71	832.7	11,000	1.96	960.8	12,600

* Flange dimensions and drilling conform to "American 125 lb Standard" (see Table 27, page 923). Thicknesses conform to those of the A.W.W.A. (see Table 1).

Cast-iron soil pipe and fittings are of the hub and spigot form, similar in design to the cast-iron water pipe shown in Fig. 1. Tapped openings and pipe plugs are threaded in accordance with the American Standard taper pipe thread, A.S.A.-B2.

or Bakelite, are employed for water pumps, gasoline pumps, etc., and are sometimes arranged so that they can be tightened by a gland as in (Fig. 18). Cups are fully automatic and are very tight; they are used principally for slow-speed applications, such as the air hoist (Fig. 19). See also p. 852. The Bridgman piston (Fig. 20) is a modification of the conventional piston designed to be self-tightening under pressure; it has been used for high-pressure gas experiments and employs a rubber ring.

The end rings used in the square **rod or plunger packing** (Fig. 21) are generally molded to an angle of 30 deg to the vertical, either in manufacture or by contact with the gland and bottom of the box. Many materials are employed, such as leather or braided flax saturated with wax for cold water; laminated rubberized-cotton fabric for hot water, low-pressure steam, ammonia, etc.; rolled rubberized-asbestos fabric for steam and air; and rolled or twisted metal foil for high temperature and high pressure conditions generally. The nested V and nested conical packings (Figs. 22 and 23) are semiautomatic and generally superior in performance to the square rings—they are made from the same materials and used for the same services. The floating metal rings (Fig. 24) are made of numerous radial or tangential segments to enable them to contract on the shaft; they are assembled generally in pairs to break joints, and with garter springs to hold them together. They are used for steam, gas, or air under the most severe conditions of temperature and pressure, and are supplied with lubricating oil. For general stuffing box design see p. 853; for pipe expansion joints, p. 940.

Among the rotary **shaft packings**, the **labyrinth** packing (Fig. 25) differs from all the preceding in that no attempt is made to hold to close clearances; leakage being controlled by forcing the escaping fluid to travel in a circuitous path (see p. 852 and 1198). The liquid seal (Fig. 26) likewise does not operate on close clearances; it depends on the principle of the U tube, the natural weight of the liquid in the example shown being magnified by centrifugal force. Both these are used for steam turbines (see p. 1200). Square packings as well as the nested-V and conical types are also largely employed for rotary shafts. Where it is desired to effect complete sealing, as on centrifugal pumps, it is common practice to inject water or other fluid into the packing by way of the H-shaped **lantern ring** shown in Fig. 27. Square packings for rotary shafts are made, in general, of the same materials as the corresponding reciprocating packings, braided asbestos yarn with a binder of grease and graphite being perhaps the most popular type, for both centrifugal pump and valve-stem applications. **Plastic packings** consisting of loose mixtures of asbestos fiber, graphite, metal, and a binder are being used to an increasing extent.

Cups with an inside lip, sometimes known as **hat packings**, are widely used as oil-retaining rings on rotary shafts. They are of leather or rubber composition (Fig. 28) and are provided generally with an internal spring and a metal case which is pressed into a corresponding cavity surrounding the shaft.

Diaphragms are used for the same purposes as sliding contact packings and are entirely leakless, although somewhat limited as to travel. The use of the cylindrical metal diaphragm (bellows) as a rotary seal suspension is shown in (Fig. 32). In the diaphragm valve (Fig. 33) the diaphragm replaces both the conventional stem packing and valve disk. Diaphragms of rubberized-cotton fabric are used in pumps (Fig. 35) and in motors (Fig. 34) to operate valves and switches. When correctly designed, they are made with slack to permit a natural rolling motion. The rubber diaphragm in the Abercrombie pump (Fig. 36) is unusual in that the diaphragm is under fluid pressure on both sides and is therefore unstressed.

sure gas mains. It is also concentric in section and of uniform thickness. For list of sizes, types of joints used, etc., refer to general catalogue of U. S. Cast Iron Pipe and Foundry Co.

Table 8. Cast-iron Soil Pipe and Fittings
(A.S.A. A40.1—1935)
Approximate weights, in pounds[b]

Fittings	Size of fitting, in.														
	2	3	4	5	6	3×2	4×2	4×3	5×2	5×3	5×4	6×2	6×3	6×4	6×5
Pipe, per ft.	5	9	12	15	19										
¼ bends, regular	5	10	15	19	24										
¼ bends, short sweep	6	13	18	23	28										
¼ bends, long sweep	8	16	22	28	34										
⅕ bends	5	10	14	18	22										
⅙ bends	5	9	13	17	20										
⅛ bends	4	8	12	15	18										
1/16 bends	4	8	11	13	16										
Return bends	7	14	21	27	34										
T branches	8	15	21	26	32	13	16	19	19	22	24	22	25	27	30
Tapped T branches[a]	7	..	..	..	..	12	15	..	18	..	..	20			
Sanitary T branch	8	16	22	28	34	14	17	20	20	23	26	23	26	29	32
Tapped sanitary T branch[a]	8	..	..	..	..	12	15	..	18	..	..	21			
Y branch	8	17	24	32	40	14	17	20	20	24	27	23	27	31	35
Inverted Y branch	9	18	25	33	41	15	18	22	22	25	29	25	29	33	37
Combination Y and ⅛ bend	10	20	29	38	50	15	18	24	21	27	33	24	30	36	42
Upright Y branch	10	20	28	37	47	16	19	23	22	27	32	25	30	35	40
Vent branch	9	18	25	32	41	14	18	21	21	24	28	22	27	31	36
Double hubs	5	8	11	13	15										
Reducers	..	..	..	..	..	6	7	9	8	10	11	9	11	12	13
Increasers	..	..	..	..	..	9	10	12	12	14	15	13	15	16	18
Tapped increasers	..	..	..	..	..	9	11	..	12	..	..	14			

[a] Tapped up to 2 in.
[b] Weights of pipe include the hub. Laying lengths of pipe are 5 ft. From the data given for staple fittings, weights of other fittings may be estimated. For data on pipe *sizes 8, 10, and 15 in. and other data, see A.S.A. A40.1-1935.*

Steel Pipe and Tubing

Designation of Pipe Size. Commercial sizes of wrought-iron and steel *pipes and tubes are known by their nominal inside diameter from ⅛ to 12 in.* Above 12 in. diam, pipes and tubes are usually known by their outside diameters (O.D.). The outside diameter of all classes of pipe and tubing heavier than standard is the same outside diameter as standard, the extra thickness always being on the inside.

In ordering pipe 12 in. and smaller designate weight desired, and in ordering tubes designate thickness desired.

Standard pipe (Table 9) is furnished in sizes ⅛ to 12 in., in random lengths, threaded both ends, with coupling on one end. The length measurement is from end to end including coupling. **Extra strong pipe** (Table 10) is heavy pipe used for high steam, gas, and hydraulic pressures. Sizes are ⅛ to 12 in., and random lengths 12 to 22 ft. **Double extra strong pipe** (Table 11) is suitable for very high pressures. Sizes are ½ to 12 in., and random lengths 12 to 22 ft. **Outside diameter pipe** runs in sizes from 14 in. up. Extra strong, double extra strong, and outside diameter pipe are furnished with plain ends and in random lengths unless otherwise specified.

Table 1. Standard Weights and Thicknesses of Cast-iron Bell-and-spigot Pipe for Water (Fig. 1)*

(American Water Works Assoc.)

Nominal inside diam, in.	Approximate laying length, ft	Class A 100 ft head 43 lb pressure			Class B 200 ft head 86 lb pressure			Class C 300 ft head 130 lb pressure			Class D 400 ft head 173 lb pressure			Approximate lb lead per joint 2 in. thick	Approximate lb hemp per joint
		Thickness, in.	Outside diam, in.	Wt per ft, lb	Thickness, in.	Outside diam, in.	Wt per ft, lb	Thickness, in.	Outside diam, in.	Wt per ft, lb	Thickness, in.	Outside diam, in.	Wt per ft, lb		
4	12	0.42	4.80	20.0	0.45	5.00	21.7	0.48	5.00	23.3	0.52	5.00	25.0	7.5	0.21
6	12	0.44	6.90	30.8	0.48	7.10	33.3	0.51	7.10	35.8	0.55	7.10	38.3	10.3	0.31
8	12	0.46	9.05	42.9	0.51	9.05	47.5	0.56	9.30	52.1	0.60	9.30	55.8	13.3	0.44
10	12	0.50	11.10	57.1	0.57	11.10	63.6	0.62	11.40	70.8	0.68	11.40	76.7	16.0	0.53
12	12	0.54	13.20	72.5	0.62	13.20	82.1	0.68	13.50	91.7	0.75	13.50	100.0	19.0	0.61
14	12	0.57	15.30	89.6	0.66	15.30	102.5	0.74	15.70	117.0	0.82	15.70	129.0	22.0	0.81
16	12	0.60	17.40	108.0	0.70	17.40	125.0	0.80	17.80	144.0	0.89	17.80	158.0	30.0	0.94
18	12	0.64	19.50	129.0	0.75	19.50	150.0	0.87	19.90	175.0	0.96	19.90	192.0	33.8	1.00
20	12	0.67	21.60	150.0	0.80	21.60	175.0	0.92	22.10	208.0	1.03	22.10	229.0	37.0	1.25
24	12	0.76	25.80	204.0	0.89	25.80	233.0	1.04	26.30	279.0	1.16	26.30	307.0	44.0	1.50
30	12	0.88	31.70	292.0	1.03	32.00	333.0	1.20	32.40	400.0	1.37	32.70	450.0	54.3	2.06
36	12	0.99	38.00	392.0	1.15	38.30	454.0	1.36	38.70	546.0	1.58	39.20	625.0	64.8	3.00
42	12	1.10	44.20	513.0	1.28	44.50	592.0	1.54	45.10	717.0	1.78	45.60	825.0	75.3	3.62
48	12	1.26	50.50	667.0	1.42	50.80	750.0	1.71	51.40	908.0	1.96	52.00	1050.0	85.5	4.37
54	12	1.35	56.70	800.0	1.55	57.10	933.0	1.90	57.80	1140.0	2.23	58.40	1340.0	97.6	6.25
60	12	1.39	62.80	917.0	1.67	63.40	1100.0	2.00	64.20	1340.0	2.38	64.80	1580.0	108.0	8.25
72	12	1.62	75.30	1280.0	1.9[illegible]	76.00	1550.0	2.39	76.90	1900.0	......	......	......	131.3	12.50
84	12	1.72	87.50	1630.0	2.22	88.50	2100.0	......	......	......	......	......	......	152.0	15.00

* All weights are approximate and include allowance for bell; proportionate allowance to be made for any variation.

Table 2. Cast-iron Bell-and-spigot Pipe for Fire Lines and Other High-pressure Service

Standard Weights and Thicknesses

(American Water Works Assoc.)

Nominal inside diam, in.	Class E 500 ft head 217 lb pressure		Class F 600 ft head 260 lb pressure		Class G 700 ft head 304 lb pressure		Class H 800 ft head 347 lb pressure	
	Thickness, in.	Lb per ft	Thickness, in.	Lb per ft	Thickness, in.	Lb per ft	Thickness, in.	Lb per ft
6	0.58	42.5	0.61	44.3	0.65	48.1	0.69	50.5
8	0.66	63.9	0.71	66.8	0.75	72.3	0.80	76.1
10	0.74	86.9	0.80	92.8	0.86	101.4	0.92	107.3
12	0.82	114.6	0.89	122.8	0.97	136.2	1.04	144.4
14	0.90	145.6	0.99	158.8	1.07	175.1	1.16	187.5
16	0.98	180.7	1.08	196.5	1.18	218.0	1.27	233.8
18	1.07	221.8	1.17	239.3	1.28	268.2	1.39	287.8
20	1.15	265.8	1.27	287.3	1.39	321.8	1.51	345.8
24	1.31	359.1	1.45	392.3				
30	1.55	530.9	1.73	588.8				
36	1.80	738.1	2.02	821.0				

Laying lengths, 12 ft; all weights are approximate and include allowance for bell; proportionate allowance to be made for any variation.

Table 11. Double Extra Strong Pipe

Nominal pipe size, in.	Outside diameter, in.	Nominal wall thickness, in. Wrought iron	Nominal wall thickness, in. Steel	Wt per ft, lb, plain ends	Nominal pipe size, in.	Outside diameter, in.	Nominal wall thickness, in. Wrought iron	Nominal wall thickness, in. Steel	Wt per ft, lb, plain ends
½	0.840	0.307	0.294	1.714	2½	2.875	0.565	0.552	13.695
¾	1.050	0.318	0.308	2.440	3	3.500	0.615	0.600	18.583
1	1.315	0.369	0.358	3.659	4	4.500	0.690	0.674	27.541
1¼	1.660	0.393	0.382	5.214	5	5.563	0.768	0.750	38.552
1½	1.900	0.411	0.400	6.408	6	6.625	0.884	0.864	53.160
2	2.375	0.447	0.436	9.029	8	8.625	0.895	0.875	72.424

American Standard Wrought-iron and Wrought-steel Pipe

(A.S.A. B36.10-1939)

The dimensional standarde for pipe are based on products such as are covered by the A.S.T.M. and A.S.A. specifications of Table 12. Nominal wall thicknesses are given in Tables 13 and 14 for steel and wrought-iron pipe, respectively. *Actual wall thicknesses should not be more than 12½ percent below the nominal wall thicknesses of the tables.* Screw threads of threaded pipe and couplings should conform to the American Standard A.S.A. B2 (Table 11) or, for threaded line pipe as used in the petroleum and gas industries, to A.P.I. 5-L.

Recommended values of S (see "Thickness of Pipe," below) are given in Table 10. The user should compute the exact value of wall thickness for his conditions as described below and then select a thickness from Tables 13 and 14.

Line Pipe Specifications (A.P.I. 5-L), tensile strength, min, lb per sq in. **Furnace welded:** Bessemer, 50,000; O.H.; Class I, 45,000; Class II, 48,000: wrought iron, 42,000. **Seamless or electric welded:** O.H.; Grade A, 48,000; Grade B, 60,000; Grade C, 75,000. **Seamless or furnace welded:** O.H., iron, 42,000. **Seamless carbon-molybdenum alloy steel pipe** (A.S.T.M. A206-38T): 55,000 lb per sq in., for temperatures 750 to 1000 F. **Seamless alloy-steel pipe** (A.S.T.M. A158-38T): 60,000 to 75,000 lb per sq in., for temperatures 750 to 1100 F. Scope: Pipe for line pipe purposes, to convey gas, water, or oil. Sizes covered, ⅛ in. nominal size to 24 in. outside diam. *Couplings for threaded line pipe are of special design.* Threads on pipe and in couplings are subject to official gage limits.

Thickness of Pipe. *The following notes, covering power piping systems,* have been abstracted from Section I of the Code for Pressure Piping (A.S.A. B31.1).

For inspection purposes, the minimum thickness of pipe wall to be used for piping at different pressures and for temperatures not exceeding those for the various materials given in Table 16 shall be determined by the formula:

$$t_m = (PD/2S) + C$$

where t_m = minimum pipe wall thickness, in. allowable on inspection; P = maximum internal service pressure, lb per sq in. gage (plus water hammer allowance in case of cast-iron pipe conveying liquids); D = outside

be entirely cut out and remade, and the dirt and old cement adhering to the pipe are likely to interfere with the perfect bonding of the new joint. Cement joints for gas pipe are fairly common practice, and, to a smaller extent, for water pipe.

Cast-iron pipe should be made of a soft and tough quality of iron and should be subjected to a test pressure of at least twice the working pressure. A good grade of cast iron shows the following properties: tensile strength, 20,000 lb per sq in.; a test bar 1 × 2 in. in section should support a central load of 2,000 lb when placed flatwise on supports 24 in. apart; if loaded to breaking, it should show a deflection of not less than 0.30 in. before fracture. Cast-iron pipe is coated by dipping in hot coal-tar pitch varnish before being laid in the ground.

Flexible Joint Pipe. The necessity for crossing streams and other waterways and of laying pipe lines into them has developed various forms of flexible joint pipe adapted to laying under water, which, when calked with lead, are capable of motion through several degrees without leakage. Figures 2 and 3 show two styles of such joints which have an adjustment of 10 to 12 deg in standard sizes. The lead flexible joint (Fig. 2) is the one generally used. Figure 3 shows a more expensive joint intended for use in swift water where there is danger of pulling apart; it can be calked before shipment.

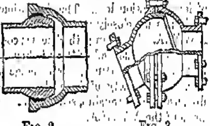


Fig. 2. Fig. 3.
Flexible Pipe Joints.

In selecting the thickness of a pipe for a submerged line, the internal service pressure is seldom the determining factor, as ample allowance should be made to minimize the risk of breakage in laying and to withstand external shocks from floating ice or other objects. The thicknesses and weights given in Table 4 are in accord with good practice.

Table 4. Thicknesses and Weights of Flexible Joint Pipe*

Nominal diam, in	Class	Thickness, in	Weight per 12-ft length, lb	Lead per joint, lb	Nominal diam, in	Class	Thickness, in	Weight per 12-ft length, lb	Lead per joint, lb
6	B	0.48	500	9	16	D	0.89	2250	60
6	D	0.55	550	9	18	B	0.75	2300	73
8	B	0.51	670	14	18	D	0.96	2760	73
8	D	0.60	780	14	20	B	0.80	2625	92
10	B	0.57	950	22	20	D	1.03	3220	92
10	D	0.68	1080	22	24	B	0.89	3530	112
12	B	0.62	1210	39	24	D	1.16	4300	112
12	D	0.75	1400	39	30	B	1.03	5100	146
14	B	0.66	1450	51	30	D	1.37	6400	146
14	D	0.82	1750	51	36	B	1.15	7000	177
16	B	0.70	1860	60	36	D	1.58	7900	177

* Joints furnished with screwed, flanged, or bell-and-spigot ends, as specified. Class B for 200 ft head. Class D for 400 ft head. Thicknesses conform to those of the A.W.W.A. (see Table 1).

"Universal" pipe (Fig. 4) is cast-iron pipe with hub-and-spigot ends, the contact surfaces of which are machined on a taper, giving an iron-to-

Table 13. Dimensions of Welded and Seamless Steel Pipe
(For temperatures not exceeding 750 F)

Nominal pipe size, in.	Outside diam, in.	Nominal wall thickness, in., for stated schedule numbers									
		10	20	30	40	60	80	100	120	140	160
1/8	0.405	.....	.....	.....	**0.068**	.....	**0.095**	.....	.....	.....	
1/4	0.540	.....	.....	.....	**0.088**	.....	**0.119**				
3/8	0.675	.....	.....	.....	**0.091**	.....	**0.126**				
1/2	0.840	.....	.....	.....	**0.109**	.....	**0.147**	.....	.....	.....	0.187
3/4	1.050	.....	.....	.....	**0.113**	.....	**0.154**	.....	.....	.....	0.218
1	1.315	.....	.....	.....	**0.133**	.....	**0.179**	.....	.....	.....	0.250
1 1/4	1.660	.....	.....	.....	**0.140**	.....	**0.191**	.....	.....	.....	0.250
1 1/2	1.900	.....	.....	.....	**0.145**	.....	**0.200**	.....	.....	.....	0.281
2	2.375	.....	.....	.....	**0.154**	.....	**0.218**	.....	.....	.....	0.343
2 1/2	2.875	.....	.....	.....	**0.203**	.....	**0.276**	.....	.....	.....	0.375
3	3.5	.....	.....	.....	**0.216**	.....	**0.300**	.....	.....	.....	0.437
3 1/2	4.0	.....	.....	.....	**0.226**	.....	**0.318**				
4	4.5	.....	.....	.....	**0.237**	.....	**0.337**	.....	0.437	.....	0.531
5	5.563	.....	.....	.....	**0.258**	.....	**0.375**	.....	0.500	.....	0.625
6	6.625	.....	.....	.....	**0.280**	.....	**0.432**	.....	0.562	.....	0.718
8	8.625	.....	0.250	**0.277**	**0.322**	**0.406**	**0.500**	0.593	0.718	0.812	0.906
10	10.75	.....	0.250	**0.307**	**0.365**	**0.500**	**0.593**	0.718	0.843	1.000	1.125
12	12.75	.....	0.250	**0.330**	**0.406**	**0.562**	**0.687**	0.843	1.000	1.125	1.312
14 O.D.	14.0	0.250	0.312	0.375	0.437	0.593	0.750	0.937	1.062	1.250	1.406
16 O.D.	16.0	0.250	0.312	0.375	0.500	0.656	0.843	1.031	1.218	1.437	1.562
18 O.D.	18.0	0.250	0.312	0.437	0.562	0.718	0.937	0.156	1.343	1.562	1.750
20 O.D.	20.0	0.250	0.375	0.500	0.593	0.812	1.031	1.250	1.500	1.750	1.937
24 O.D.	24.0	0.250	0.375	0.562	0.687	0.937	1.218	1.500	1.750	2.062	2.312
30 O.D.	30.0	0.312	0.500	0.625							

The schedule numbers are approximate values of 1000P/S (see p. 908 for the symbols). Thicknesses include a mill tolerance of 12.5 percent. Thicknesses in black type in schedules 30 and 40 agree with those of standard weight pipe, Table 9; those in schedules 60 and 80 with extra strong pipe, Table 10.

Table 14. Dimensions of Welded Wrought-iron Pipe
(For temperatures not exceeding 750 F)

Nominal pipe size, in.	Outside diam, in.	Nominal wall thickness, in., for stated schedule numbers					
		10	20	30	40	60	80
1/8	0.405	.....	.....	.....	0.070	.....	0.098
1/4	0.540	.....	.....	.....	0.090	.....	0.122
3/8	0.675	.....	.....	.....	0.093	.....	0.129
1/2	0.840	.....	.....	.....	0.111	.....	0.151
3/4	1.050	.....	.....	.....	0.115	.....	0.157
1	1.315	.....	.....	.....	0.136	.....	0.183
1 1/4	1.660	.....	.....	.....	0.143	.....	0.195
1 1/2	1.900	.....	.....	.....	0.148	.....	0.204
2	2.375	.....	.....	.....	0.158	.....	0.223
2 1/2	2.875	.....	.....	.....	0.208	.....	0.282
3	3.5	.....	.....	.....	0.221	.....	0.306
3 1/2	4.0	.....	.....	.....	0.231	.....	0.325
4	4.5	.....	.....	.....	0.242	.....	0.344
5	5.563	.....	.....	.....	0.263	.....	0.383
6	6.625	.....	.....	.....	0.286	.....	0.441
8	8.625	.....	.....	0.283	0.329	.....	0.510
10	10.75	.....	.....	0.313	0.372	0.510	0.606
12	12.75	.....	.....	0.336	0.414	0.574	0.702
14 O.D.	14.0	0.250	0.312	0.375	0.437	0.625	0.750
16 O.D.	16.0	0.250	0.312	0.375	0.500	0.687	
18 O.D.	18.0	0.250	0.312	0.437	0.562	0.750	
20 O.D.	20.0	.....	0.375	0.500	0.562		

The footnotes to Table 13 apply to this table also.

Table 6. Cast-iron Flanged Pipe for High-pressure Service

STANDARD WEIGHTS AND THICKNESSES*

Nominal inside diam, in.	Class E 500 ft head, 217 lb pressure		Class F 600 ft head, 260 lb pressure		Class G 700 ft head, 304 lb pressure		Class H 800 ft head, 347 lb pressure	
	Thickness, in.	Lb per ft	Thickness, in.	Lb per ft	Thickness, in.	Lb per ft	Thickness, in.	Lb per ft
6	0.58	37.7	0.61	39.5	0.65	42.9	0.69	45.2
8	0.66	54.7	0.71	60.6	0.75	65.1	0.80	68.8
10	0.74	78.8	0.80	84.7	0.86	92.5	0.92	98.5
12	0.82	104.2	0.89	112.4	0.97	124.6	1.04	132.9
14	0.90	133.1	0.99	146.2	1.07	160.2	1.16	172.6
16	0.98	165.0	1.08	180.8	1.18	199.2	1.27	215.0
18	1.07	202.3	1.17	219.8	1.28	244.6	1.39	264.1
20	1.15	241.1	1.27	262.5	1.39	294.4	1.51	318.3
24	1.31	328.5	1.45	361.6	1.75	446.2	1.88	476.9
30	1.55	484.7	1.73	538.0	......	......	......	......
36	1.80	674.2	2.02	748.7	......	......	......	......

* Weights are approximate and do not include flanges. For each flanged joint, add weight of 1 ft of pipe; laying lengths, 12 ft. Flange dimensions and drilling conform to "American 250 lb standard" (see Table 27, p. 924). Thicknesses conform to those of the A.W.W.A. (see Table 2).

Table 7. Standard Weights and Thicknesses of Universal Cast-iron Pipe

(Central Foundry Co.)

Nominal inside diam, in.	Class No. 100, 100 lb pressure			Class No. 130, 130 lb pressure			Class No. 175, 175 lb pressure			Class No. 250, 250 lb pressure		
	Approx thickness, in.	Estimated weight, lb per		Approx thickness, in.	Estimated weight, lb per		Approx thickness, in.	Estimated weight, lb per		Approx thickness, in.	Estimated weight, lb per	
		Ft	6 ft length		Ft	6 ft length		Ft	6 ft length		Ft	6 ft length
2	......	......	......	......	......	......	0.35	8½	51	0.39	9½	57
3	......	......	......	......	......	......	0.37	13	78	0.42	14½	87
4	0.37	18	108	0.40	18¾	112½	0.43	20¼	121½	0.45	21¼	127½
6	0.43	30	180	0.45	31	186	0.47	32	192	0.51	35½	213
8	0.47	44¼	265½	0.49	46	276	0.525	49¼	295½	0.58	53¼	319½
10	0.50	60½	363	0.53	63½	381	0.58	67¾	406½	0.64	74	444
12	0.53	75½	453	0.57	80½	483	0.62	87	522	0.70	97½	585
14	0.565	94½	567	0.60	99½	597	0.66	107½	645	0.76	124	744
16	0.60	115½	693	0.65	123	738	0.72	134	804	0.83	156	936

Inside diam, in.	2	3	4	6	8	10
Bolt sizes, in.	½ × 4	½ × 4¼	⅝ × 5	¾ × 6	⅞ × 6¾	1 × 7½

Inside diam, in.	12	14	16	20	24	30
Bolt sizes, in.	1 × 8	1⅛ × 9	1½ × 9½	1⅝ × 11½	1⅝ × 12	2 × 12½

DeLavaud Centrifugally Cast Cast-iron Pipe. The utilization of centrifugal force in the manufacture of cast-iron pipe has developed a method that produces pipe superior in many respects to that cast in sand molds. A metal mold is rotated horizontally as the molten metal is fed into the mold so that the rotary force throws the metal against the mold, forming the pipe. This process forces from the metal the impurities and gives it a very homogeneous structure. The outstanding feature of this pipe is the great increase in strength resulting from the process. Being free from blow holes and impurities and of closer grain than sand-cast pipe, it is of particular value for high-pres-

Table 16. Allowable S Values for Pipe in Power Piping Systems

Material[a]	Specification	Values of S, lb per sq in., for temperatures F not to exceed[b]					
		400	450	700	750	800	850
Seamless steel,							
Grade A	A.S.T.M. A106	.....	....	9,600	9,000	7,020	5,800
Grade B	A.S.T.M. A106	.....	....	12,400	11,500	9,160	7,520
Grade C	A.S.T.M. A106	.....	....	15,000	14,000	11,200	9,300
Seamless steel,							
Low carbon	A.S.T.M. A53	.....	....	9,600	9,000		
Medium carbon	A.S.T.M. A53	.....	....	12,400	11,500		
Seamless steel	A.S.T.M. A120	.....	9,600				
Electric-fusion-welded steel (high pressure high temperature service),							
Grade A	A.S.T.M. A155	.....	....	8,100	7,400	5,900	4,900
Grade B	A.S.T.M. A155	.....	....	9,000	8,200	6,600	5,450
Grade C	A.S.T.M. A155	.....	....	9,900	9,000	7,200	6,070
Electric-fusion-welded steel,							
Grade A[c]	A.S.T.M. A139	.....	....	7,700	7,200		
Grade B[c]	A.S.T.M. A139	.....	....	9,600	9,000		
Electric-fusion-welded steel, large size	A.S.T.M. A134	.....	....	0.16 × TS	0.15 × TS		
Electric-resistance-welded steel,							
Grade A	A.S.T.M. A135	.....	....	8,200	7,600		
Grade B	A.S.T.M. A135	.....	....	10,200	9,500		
Lap-welded steel	A.S.T.M. A106	.....	....	7,600	7,000		
Lap-welded steel	A.S.T.M. A53	.....	....	7,600	7,000		
Lap-welded steel	A.S.T.M. A120	.....	7,600				
Forge-welded steel,							
Grade A	A.S.T.M. A136	.....	....	7,200	6,700		
Grade B	A.S.T.M. A136	.....	....	8,000	7,500		
Butt-welded steel	A.S.T.M. A53	.....	....	6,000	5,600		
Butt-welded steel	A.S.T.M. A120	.....	6,000				
Steel or wrought iron, riveted joint	A.S.T.M. A138		(TS×E)/5				
Lap-welded wrought iron	A.S.T.M. A72	.....	....	5,700	5,300		
Butt-welded wrought iron	A.S.T.M. A72	.....	....	4,900	4,500		
Seamless brass pipe	A.S.T.M. B43	4,500					
Seamless copper pipe	A.S.T.M. B42	4,000					
Seamless copper tubing	A.S.T.M. B75	4,000					
Seamless copper tubing	A.S.T.M. B88	4,000					
Cast-iron pipe, centrifugally cast[d]	F.S.B. WW-P-421	.....	6,000				
Cast-iron pipe, pit-cast[d]	A.W.W.A	.....	4,000				

To the minimum pipe wall thickness calculated from any of the above S values, the manufacturing tolerance, demanded for the pipe considered, must be added to obtain the nominal wall thickness. (See A.S.A.-B36.)

[a] Pipe in accordance with A.P.I. specification 5L may be used.

[b] The several types and grades of pipe tabulated above shall not be used at temperatures in excess of the maximum temperatures for which the S values are indicated. If it is desirable to use an adjusted value of S for carbon-steel pipe at temperatures below 700 F., the curve of Fig. 5 may be utilized.

[c] If plate material having physical properties other than stated in Section 6 of A.S.T.M. specification A139 is used in the manufacture of ordinary electric-fusion-welded steel pipe, the allowable stress shall be taken as 0.15 times the tensile strength for temperatures above 700 F but not over 750 F, and 0.16 times the tensile strength for temperatures of 700 F and below.

[d] Cast-iron pipe shall not be used for lubricating oil lines for mechinery and in any case not for oil having a temperature above 300 F.

TS = ultimate tensile strength of the material.

E = efficiency of the joint.

Table 9. Standard Pipe and Line Pipe

Diam			Nominal thickness, in.	Circumference		Transverse areas			Length of pipe per sq ft of		Length of pipe containing 1 cu ft, ft	Nominal weight per ft, lb	No. of threads per in. of screw
Nominal internal, in.	Actual external, in.	Approx internal diam, in.		External, in.	Internal, in.	External, sq in.	Internal, sq in.	Metal, sq in.	External surface, ft	Internal surface, ft			
⅛	0.405	0.27	0.068	1.27	0.85	0.13	0.06	0.07	9.44	14.15	2513.00	0.24	27
¼	0.540	0.36	0.088	1.70	1.14	0.23	0.10	0.12	7.08	10.49	1383.30	0.42	18
⅜	0.675	0.49	0.091	2.12	1.55	0.36	0.19	0.17	5.66	7.76	751.20	0.57	18
½	0.840	0.62	0.109	2.63	1.95	0.55	0.30	0.25	4.55	6.15	472.40	0.85	14
¾	1.050	0.82	0.113	3.30	2.59	0.87	0.53	0.33	3.64	4.64	270.00	1.13	14
1	1.315	1.05	0.134	4.13	3.29	1.36	0.86	0.50	2.90	3.65	166.90	1.68	11½
1¼	1.660	1.38	0.140	5.22	4.34	2.16	1.50	0.67	2.30	2.77	96.25	2.27	11½
1½	1.900	1.61	0.145	5.97	5.06	2.84	2.04	0.80	2.01	2.37	70.66	2.72	11½
2	2.375	2.07	0.154	7.46	6.49	4.43	3.36	1.07	1.61	1.85	42.91	3.65	11½
2½	2.875	2.47	0.204	9.03	7.75	6.49	4.78	1.71	1.33	1.55	30.10	5.79	0
3	3.500	3.07	0.217	11.00	9.63	9.62	7.39	2.24	1.09	1.25	19.50	7.57	8
3½	4.000	3.55	0.226	12.57	11.15	12.57	9.89	2.68	0.96	1.08	14.57	9.11	0
4	4.500	4.03	0.237	14.14	12.65	15.90	12.73	3.18	0.85	0.95	11.31	10.79	8
5	5.563	5.05	0.259	17.48	15.85	24.31	19.99	4.32	0.69	0.76	7.20	14.62	8
6	6.625	6.07	0.280	20.81	19.05	34.47	28.89	5.59	0.58	0.63	4.98	18.97	8
8	8.625	8.07	0.276	27.10	25.35	58.43	51.15	7.28	0.44	0.47	2.82	24.69	0
8	8.625	7.98	0.322	27.10	25.07	58.43	50.02	8.41	0.44	0.48	2.88	28.55	8
9	9.625	8.94	0.344	30.24	28.08	72.76	62.72	10.04	0.40	0.43	2.29	33.91	8
10	10.750	10.19	0.278	33.77	32.01	90.76	81.55	9.21	0.36	0.37	1.76	31.20	0
10	10.750	10.14	0.306	33.77	31.86	90.76	80.75	10.01	0.36	0.38	1.78	34.24	8
10	10.750	10.02	0.366	33.77	31.47	90.76	78.82	11.94	0.36	0.38	1.82	40.48	8
12	12.750	12.09	0.328	40.06	37.98	127.68	114.80	12.88	0.30	0.32	1.25	43.77	0
12	12.750	12.00	0.375	40.06	37.70	127.68	113.10	14.59	0.30	0.32	1.27	49.56	8

Table 10. Extra Strong Pipe

Diam			Nominal thickness, in.	Circumference		Transverse areas			Length of pipe per sq ft of		Nominal weight per ft, lb
Nominal internal, in.	Actual external, in.	Approx internal diam, in.		External, in.	Internal, in.	External, sq in.	Internal, sq in.	Metal, sq in.	External surface, ft	Internal surface, ft	
⅛	0.405	0.21	0.100	1.27	0.64	0.13	0.03	0.10	9.43	18.63	0.31
¼	0.540	0.29	0.123	1.70	0.92	0.23	0.07	0.16	7.08	12.99	0.54
⅜	0.675	0.42	0.127	2.12	1.32	0.36	0.14	0.22	5.66	9.07	0.74
½	0.840	0.54	0.149	2.64	1.70	0.55	0.23	0.32	4.55	7.05	1.09
¾	1.050	0.74	0.157	3.30	2.31	0.87	0.43	0.44	3.64	5.11	1.47
1	1.315	0.95	0.182	4.13	2.99	1.36	0.71	0.65	2.90	4.02	2.17
1¼	1.660	1.27	0.194	5.22	4.00	2.16	1.27	0.89	2.30	3.00	2.99
1½	1.900	1.49	0.203	5.97	4.69	2.84	1.75	1.08	2.01	2.56	3.63
2	2.375	1.93	0.221	7.46	6.07	4.43	2.94	1.50	1.61	1.98	5.02
2½	2.875	2.32	0.280	9.03	7.27	6.49	4.21	2.28	1.33	1.65	7.66
3	3.500	2.89	0.304	11.00	9.09	9.62	6.57	3.05	1.09	1.33	10.25
3½	4.000	3.36	0.321	12.57	10.55	12.57	8.86	3.71	0.96	1.14	12.50
4	4.500	3.82	0.341	14.14	12.00	15.90	11.45	4.46	0.85	1.00	14.98
5	5.563	4.81	0.375	17.48	15.12	24.31	18.19	6.11	0.69	0.79	20.78
6	6.625	5.75	0.437	20.81	18.07	34.47	25.98	8.50	0.58	0.66	28.57
8	8.625	7.63	0.500	27.10	23.96	58.43	45.66	12.76	0.44	0.50	43.34
10	10.750	9.75	0.500	33.77	30.63	90.76	74.66	16.10	0.36	0.40	54.73
12	12.750	11.75	0.500	40.06	36.91	127.65	108.43	19.25	0.30	0.33	65.41

low-carbon seamless steel (A53) or wrought iron (A72) are suitable. Pipe permissible for this service may be used for temperatures above 450 F if the proper S is used in calculating the pipe-wall thickness.

Valves below 3 in. may have inside stem screws. Stop valves 8 in. and over must be by-passed. Bodies, bonnets, and yokes shall be of cast iron, malleable iron, steel, bronze, brass, or monel. Flanged-steel fittings must conform to the 300 lb American Standard B16e; if of cast iron, to the 250 lb American Standard B16b or for screwed fittings, B16d. Malleable-iron screwed fittings shall conform to the 300 lb M.S.S. SP-31, except that the 150 lb American Standard B16c may be used for pressures not greater than 150 lb. Welded fittings may be used.

Steam Pressures from 25 to 125 Lb, Temperature Not above 450 F. Pipe may be of steel, wrought iron, cast iron, copper or brass; valve bodies of cast iron, malleable iron, steel or brass. Fittings shall be of 125 lb American Standard with screwed (B16d), flanged (B16a), or malleable-iron screwed fittings (B16c).

Steam Pressures 25 Lb and less, Temperatures up to 450 F. Pipe may be of steel, wrought iron, spiral riveted steel, brass, copper, or cast iron. Flanged fittings shall conform to the 25 lb American Standard B16b2. Screwed fittings shall be of the 125 lb American Standard B16d or of the 150 lb American Standard B16c for cast iron or malleable iron, respectively, or the M.S.S. SP-10 for bronze. Welded fittings may be used.

Table 17. Capacity of Pipes and Cylindrical Tanks of Various Diameters in Gallons per Foot of Length

(For capacities in cubic feet, use table of areas, p. 33)

Feet	Inches											
	0	1	2	3	4	5	6	7	8	9	10	11
0	......	.0408	0.1632	0.3672	0.6528	1.020	1.469	1.999	2.611	3.305	4.080	4.937
1	5.875	6.895	8.00	9.18	10.44	11.79	13.22	14.73	16.32	17.99	19.75	21.58
2	23.50	25.50	27.58	29.74	31.99	34.31	36.72	39.21	41.78	44.43	47.16	49.98
3	52.88	55.86	58.92	62.06	65.28	68.58	71.97	75.44	78.99	82.62	86.33	90.13
4	94.00	97.96	102.0	106.1	110.3	114.6	119.0	123.4	128.0	132.6	137.3	142.0
5	146.9	151.8	156.8	161.9	167.1	172.4	177.7	183.2	188.7	194.3	199.9	205.7
6	211.5	217.6	223.4	229.5	235.7	242.0	248.2	254.7	261.1	267.7	274.3	281.1

Feet	Inches				Feet	Inches				Feet	Inches			
	0	3	6	9		0	3	6	9		0	3	6	9
7	287.9	308.8	330.5	352.9	16	1504	1551	1600	1648	25	3672	3746	3820	3896
8	376.0	399.9	424.5	449.8	17	1698	1748	1799	1851	26	3972	4048	4126	4204
9	475.9	502.7	530.2	558.5	18	1904	1957	2011	2066	27	4283	4363	4443	4524
10	587.5	617.7	647.7	679.0	19	2121	2177	2234	2292	28	4606	4689	4772	4856
11	710.9	743.6	777.0	811.1	20	2350	2409	2469	2530	29	4941	5027	5113	5200
12	846.0	881.7	918.0	955.1	21	2591	2653	2716	2779	30	5288	5376	5465	5555
13	992.9	1032	1071	1111	22	2844	2909	2974	3041	31	5646	5738	5830	5923
14	1152	1193	1235	1278	23	3108	3176	3245	3314	32	6016	6111	6206	6302
15	1322	1366	1412	1457	24	3384	3455	3527	3599	33	6398	6496	6594	6692

Pipe Bends. For theory and design, see p. 497. Pipe bends of any size and shape can be obtained to suit almost any condition arising in practice. Various shapes of pipe bends are shown in Fig. 6, some limiting dimensions of which are given in Table 18. Their uses may be broadly classed as follows: (1) For steam lines to provide flexibility and compensate for expansion and contraction; (2) to reduce the number of joints in a pipe line; (3) to avoid obstructions such as columns, pipe foundations, which otherwise would require several pieces of pipe and fittings; (4) to reduce friction in piping; (5) for coils in heaters, refrigerating systems, etc.

diam of pipe, in.; S = allowable stress in material, lb per sq in.; C = allowance for threading, mechanical strength, and corrosion, in.

Values of S for various materials and temperatures contemplated shall not exceed those given in Table 16. (Continued on p. 911.)

Table 12. A.S.A. Specifications for Tensile Strength of Pipe

A.S.A. designation	A.S.T.M. designation	Style of pipe	Tensile strength, min, lb per sq in.	Scope
B36.1	A53	Welded and seamless steel	Welded: Bessemer, 50,000; O.H. 45,000. Seamless: low carbon, 48,000; medium 62,000	(a)
B36.2	A72	Welded wrought iron	40,000	(b)
B36.3	A106	Lap-welded and seamless steel for high temperatures	Welded: O.H. 45,000. Seamless: Grade A, 48,000; Grade B, 62,000; Grade C, 75,000	(c)
B36.4	A134	Electric-fusion-welded steel, sizes 30 in. and over	See A.S.T.M. Standard A134	(d)
B36.5	A135	Electric-resistance-welded steel	Grade A, 48,000. Grade B, 60,000	(e)
B36.6	A136	Forge-welded steel	Grade A, 45,000. Grade B, 50,000	(f)
B36.7	A137	Lock-bar steel	Plates: 55,000 to 65,000. Lock-bars; 40,000 to 50,000	(g)
B36.8	A138	Riveted steel and wrought iron	Plates: Steel, 55,000 to 65,000; wrought iron, Class A, 48,000; Class B, 47,000 Rivet bar stock; steel, 45,000 to 55,000; wrought iron, 47,000	(h)
B36.9	A139	*Electric-fusion-welded* steel, 8 to 30 in.	*Grade A, 48,000. Grade B, 60,000* or other material	(i)
B36.11	A155-36	Electric-fusion-welded steel pipe for high-temperature high-pressure service	Grade A, 45,000; Grade B, 50,000; Grade C, 55,000	(j)
G8.7	A120	Black and hot-dipped galvanized welded and seamless steel	Same as B36.1	(k)

(a) Commercial steel pipe for general uses, also for coiling, bending, flanging, and similar forming operations when so specified.

(b) Commercial wrought-iron pipe for general uses, also for coiling, bending, flanging, and other special purposes.

(c) Lap-welded and seamless steel pipe for high-temperature service. Suitable for bending, flanging, and similar forming operations.

(d) Covers pipe 30 in. diam and over in wall thicknesses up to ¾ in., fabricated from steel plates by electric-fusion welding.

(e) Pipe up to 30 in. intended for conveying liquids, gas, or vapor at temperatures below 450 F. Adapted for flanging, bending, and similar forming operations in Grade A class.

(f) Covers sizes 14 to 96 in., wall thicknesses ¼ to 1¼ in., forge welded from steel plates and intended for various uses.

(g) Covers sizes 20 to 72 in., wall thicknesses $\frac{3}{16}$ to ½ in., fabricated from plates with H-shaped lock bars for the longitudinal seams. Suitable for conveying liquids or gases.

(h) Shop-fabricated pipe suitable for conveying liquids or gases; made from steel or wrought-iron plates with riveted seams.

(i) Covers sizes *8 to <30 in.* in wall thicknesses not over ⅝ in., fabricated from steel plates by electric-fusion welding. Intended for conveying liquids, gas, or vapor at temperatures below 450 F. Adapted for flanging and bending.

(j) Electric-fusion-welded steel pipe having an outside diameter of 18 in. and over for high-temperature and high-pressure service. Suitable for bending, flanging, corrugating, and similar forming operations. Welding in accordance with Par. U-68 of the A.S.M.E. code for unfired pressure vessels.

(k) Commercial steel pipe for ordinary uses such as low-pressure steam, liquid, or gas lines. Not intended for coiling or close bending, nor for high-temperature service.

The radii of pipe bends and lengths of tangents should preferably equal or exceed the minimum values given in Table 18. Bends with plain or beveled ends, for welding, can be furnished without tangents, but for convenience in erection it is recommended that they have tangents at least equal to the minimum lengths given in Table 18 for threaded ends. Bends with slip-on flanges can be furnished with a minimum length of tangent equal to the length through the hub of the flange. Those fitted with welding neck flanges can be furnished without tangents. When possible, however, the use of a tangent is recommended for convenience in erection.

Table 18. Limiting Dimensions of Pipe Bends
(Letters and dimensions refer to Fig. 6. All dimensions are in inches)
(From Crane Co. catalogue)

Size of pipe, in.[a]	Minimum recommended radius[b] R	Shortest radius, in., to which pipe can be bent								Minimum length of tangent, in.[b] X	
		Steel pipe				Wrought-iron pipe					
		Threaded ends, screwed, or welded flanges		Lap joints		Threaded ends, screwed, or welded flanges		Lap joints			
		Standard	Extra strong	Standard	Extra strong	Standard	Extra strong	Standard	Extra strong	Screwed flanges	Lap joints
¼	1¼	1	⅝	...	...	1½	1	.....	...	1	
⅜	1⅞	1¼	¾	...	...	2	1¼	.....	...	1¼	
½	2½	1½	1	2	2	2½	2	2½	2	1½	2
¾	3¾	1¾	1¼	3	3	3	2½	3	3	1¾	2
1	5	2	1½	4	4	4	3	4	4	2	2
1¼	6¼	2¼	1¾	4	4	5	4	5	4	2	2½
1½	7½	2½	2	5	5	6	5	6	5	2½	3
2	10	3	2½	6	6	8	5	8	6	3	4
2½	12½	5	4¼	10	10	10	8	10	10	4	5
3	15	8	6	12	12	12	10	12	12	4	6
3½	17½	10	8	12	12	14	12	14	12	5	6
4	20	12	10	14	14	16	12	16	14	5	6
5	25	18	14	18	14	20	15	20	15	6	7
6	30	22	16	22	16	26	16	26	16	7	7
8	40	30	23	30	23	30	23	30	23	9	8
10	50	36	30	36	30	36	32	36	32	12	10
12	60	46	36	46	36	46	42	46	42	14	10
14 O.D.	70	60	48	60	48	60	54	60	54	16	14
16 O.D.	95	80	60	80	60	80	70	80	70	18	16
18 O.D.	108	90	66	90	66	90	80	90	80	18	18
20 O.D.	120	100	72	100	72	100	90	100	90	18	18
24 O.D.	144	144	108	144	108	144	122	144	122	18	20

[a] For sizes 14 in. O.D. and larger, radii under standard pipe are based on a wall thickness of $\frac{3}{16}$ in. or less. The radii under extra-strong pipe are based on a wall thickness of ½ in. or greater.
[b] For steel or genuine wrought-iron pipe, in "standard" or "extra-strong" weight.

Pipe coils are made from any of the commercial sizes of iron, steel, brass, and copper pipe and tubing. Limiting center to center dimensions to which pipe coils can be fabricated in sizes ¼ to 2 in. according to the recommenda-

Table 15. A.P.I. Line Pipe
(All weights and dimensions are nominal)

Size, in.	Weight, lb per ft: Threads and couplings	Weight, lb per ft: Plain ends	Thickness, in.	External diam, in.	Internal diam, in.	Threads per in.	Couplings: Length, in.	Couplings: Ext diam, in.	Test pressure, lb per sq in.: Butt-weld	Test pressure: Lap weld and grade A seamless	Test pressure: Grade B seamless	Test pressure: Grade C seamless
⅛	0.25	0.24	0.068	0.405	0.269	27	1⅛	0.582	700	1,000	1,000	1,000
¼	0.43	0.42	0.088	0.540	0.364	18	1⅜	0.724	700	1,000	1,000	1,000
⅜	0.57	0.56	0.091	0.675	0.493	18	1⅝	0.898	700	1,000	1,000	1,000
½	0.86	0.85	0.109	0.840	0.622	14	1¾	1.085	700	1,000	1,000	1,000
¾	1.14	1.13	0.113	1.050	0.824	14	2⅛	1.316	700	1,000	1,000	1,000
1	1.70	1.67	0.133	1.315	1.049	11½	2⅜	1.575	700	1,000	1,000	1,000
1¼	2.30	2.27	0.140	1.660	1.380	11½	2⅝	2.054	1,200	2,500	2,500	2,500
1½	2.75	2.71	0.145	1.900	1.610	11½	2⅞	2.294	1,200	2,300	2,500	2,500
2	3.75	3.65	0.154	2.375	2.067	11½	3⅝	2.870	1,200	1,900	2,200	2,500
2½	5.90	5.79	0.203	2.875	2.469	8	4⅛	3.389	1,200	2,100	2,400	2,500
3	7.70	7.57	0.216	3.500	3.068	8	4⅛	4.014	1,200	1,900	2,100	2,300
3½	9.25	9.10	0.226	4.000	3.548	8	4⅝	4.628	.....	1,700	1,900	2,100
4	11.00	10.79	0.237	4.500	4.026	8	4⅝	5.216	.....	1,600	1,800	2,000
5	15.00	14.61	0.258	5.563	5.047	8	5⅛	6.420	.....	1,400	1,600	1,800
6	19.45	18.97	0.280	6.625	6.065	8	5⅝	7.482	.....	1,300	1,400	1,600
8	25.55	24.69	0.277	8.625	8.071	8	6⅛	9.593	.....	950	1,100	1,200
8	29.35	28.55	0.322	8.625	7.981	8	6⅛	9.593	.....	1,100	1,300	1,400
10	32.75	31.20	0.279	10.750	10.192	8	6⅝	11.958	.....	800	900	1,000
10	35.75	34.24	0.307	10.750	10.136	8	6⅝	11.958	.....	850	950	1,100
10	41.85	40.48	0.365	10.750	10.020	8	6⅝	11.958	.....	1,000	1,200	1,300
12	45.45	43.77	0.330	12.750	12.090	8	6⅝	13.958	.....	800	900	1,000
12	51.15	49.56	0.375	12.750	12.000	8	6⅝	13.958	.....	900	1,000	1,100
14 O.D.	57.00	54.56	0.375	14.000	13.250	8	7⅛	15.446	.....	800	900	1,000
15 O.D.	61.15	58.57	0.375	15.000	14.250	8	7⅛	16.446	.....	750	850	950
16 O.D.	65.30	62.57	0.375	16.000	15.250	8	7⅛	17.446	.....	700	800	900
17 O.D.	73.20	69.70	0.393	17.000	16.214	8	7⅛	18.683	.....	700	800	900
18 O.D.	81.20	76.84	0.409	18.000	17.182	8	7⅛	19.921	.....	700	750	850
20 O.D.	90.00	85.57	0.409	20.000	19.182	8	7⅝	21.706	.....	600	700	800

The permissible variation *in* weight *for any length of pipe is 10* percent above and 3½ percent below; but the carload weight shall not be more than 1¾ percent under the nominal weight.

Furnished with threads and couplings and in random lengths, unless otherwise ordered.

The weight per ft of pipe with threads and couplings is based on a length of 20 ft, including the coupling.

Lap-weld not furnished in sizes below 1½ in.

The value of C shall not be less than the following:

Type of pipe	Value of C, in.
Cast-iron pipe, centrifugally cast or cast horizontally in green sand molds	0.14
Cast-iron pipe, pit cast	0.18
Threaded steel, *wrought-iron or non-ferrous pipe* (n = number of threads per in.)	$0.8/n$
Grooved steel, wrought-iron or non-ferrous pipe	Depth of groove
Plain end steel, wrought-iron or non-ferrous pipe or tube for 1 in. size and smaller	0.05
Plain end steel, wrought-iron or non-ferrous pipe or tube for sizes above 1 in.	0.065

(Continued on p. 913.)

tion of Crane Co. are given in Table 19. Steel tubing cannot be bent to the absolute limits of brass or copper.

Tubular Products. The following material from the catalogue of the National Tube Co. covers tubular products such as boiler tubes; steel tubes; oil-well pipe, casing, and tubing; signal pipe; trolley poles; cylinders and shipping containers; seamless mechanical tubing; aircraft tubing; tubular forgings; etc.

These products are made by either of two basic processes, welded or seamless. Welded products are made in sizes $\frac{1}{8}$ to 96 in. diam and, in certain sizes, in lengths up to 40 ft. Seamless products are made in sizes from $\frac{1}{8}$ to 28 in. and, in certain sizes, in lengths up to 45 ft. Three processes are used in making welded products: butt weld in sizes $\frac{1}{8}$ to 3 in.; lap weld in sizes 2 to 24 in.; and electric weld in sizes 30 to 96 in. Seamless products are also made by three processes: piercing in sizes up to 14 in.; rotary rolling in sizes 14 to 28 in.; and cupping in sizes $5\frac{1}{2}$ to 30 in.

Table 21. Standard Dimensions of Lap-welded Steel or Charcoal-iron Boiler Tubes

External diam, in.	Internal diam, in.	Nominal thickness, in.	Nearest Birm. wire gage No.	Circumference: External, in.	Circumference: Internal, in.	Transverse areas: External, sq in.	Transverse areas: Internal, sq in.	Transverse areas: Metal, sq in.	Length of tube per sq ft of: External surface, ft	Length of tube per sq ft of: Internal surface, ft	Nominal weight per ft, lb
$1\frac{3}{4}$	1.560	0.095	13	5.498	4.901	2.405	1.911	0.494	2.183	2.448	1.68
2	1.810	0.095	13	6.283	5.686	3.142	2.573	0.569	1.909	2.110	1.93
$2\frac{1}{4}$	2.060	0.095	13	7.069	6.472	3.976	3.333	0.643	1.698	1.854	2.19
$2\frac{1}{2}$	2.282	0.109	12	7.854	7.169	4.909	4.090	0.819	1.528	1.674	2.78
$2\frac{3}{4}$	2.532	0.109	12	8.639	7.954	5.940	5.035	0.905	1.389	1.509	3.07
3	2.782	0.109	12	9.425	8.740	7.069	6.079	0.990	1.273	1.373	3.37
$3\frac{1}{4}$	3.010	0.120	11	10.210	9.456	8.296	7.116	1.160	1.175	1.269	4.01
$3\frac{1}{2}$	3.260	0.120	11	10.996	10.242	9.621	8.347	1.274	1.091	1.172	4.33
$3\frac{3}{4}$	3.510	0.120	11	11.781	11.027	11.045	9.676	1.369	1.018	1.088	4.65
4	3.732	0.134	10	12.566	11.724	12.566	10.939	1.627	0.955	1.024	5.53
5	4.704	0.148	9	15.708	14.778	19.635	17.379	2.256	0.764	0.812	7.67
6	5.670	0.165	8	18.850	17.813	28.274	25.249	3.025	0.637	0.673	10.28
8	7.670	0.165	8	25.133	24.096	50.266	46.204	4.062	0.477	0.498	13.81
10	9.594	0.203	6	31.416	30.140	78.540	72.292	6.248	0.382	0.398	21.24
12	11.542	0.229	$4\frac{1}{2}$	37.699	36.260	113.098	104.629	8.469	0.319	0.330	28.79

NOTE. In estimating effective steam-heating or evaporating surface of tubes, the surface in contact with air or gases of combustion, according to manner of application, as whether internal or external, is to be thus taken. For heating liquids by steam, superheating steam, or transferring heat from one liquid or one gas to another, mean surface of tubes to be computed.

The steels used for tubular products conform to standard specifications. For example, the A.P.I. specification 5A gives tensile strengths as follows: seamless steel, Grades *A*, *B*, *C*, and *D*, 48,000, 70,000, 75,000, and 95,000 lb per sq. in., respectively; open-hearth welded steel, classes I and II, 45,000 and 48,000 lb per sq in., respectively; Bessemer welded steel, 50,000 lb per sq in.

Seamless mechanical tubing is available in a wide range of diameters and wall thicknesses. Diameters of round tubing range from $\frac{1}{8}$ to 20 in. outside diameter and wall thicknesses from 20 gage (B.W.G.) to 2 in. Oval, square, and rectangular shapes are also available. Round tubing is listed in a range

Plain end pipe includes pipe joined by flared compression couplings, lapped joints, and by welding, *i.e.*, by any method that does not reduce the wall thickness of the pipe at the joint.

Water-hammer allowance for cast-iron pipe to be added to P in the formula is as follows:

Pipe size, in.	4–10	12–18	20	24–30	36–48	54–84
Water-hammer allowance, per sq in.	120	110	100	95	90	85

Having computed the exact nominal thickness, as outlined above, a schedule number may be selected from Tables 13 or 14 the thickness of which is not less than the computed value.

For pressure-temperature ratings of carbon-steel pipe at temperatures below 700 F, Fig. 5 may be used. For higher temperatures, the S values in Table 16 apply.

Physical and Chemical Properties of Pipes, Tubes, Etc. The design of piping for operation above 850 F presents many problems not encountered at lower temperatures. For the properties of steel applicable to high-temperature service (as well as to ordinary service) for pipes, tubes, fittings, bolting material, etc., see p. 571. For a discussion of creep properties, see p. 428. See also recent publications of the National Tube Co.

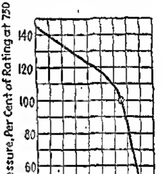


FIG. 5.—Pressure-temperature Rating Chart for Carbon-steel Pipe.

A.S.A. Code for Steam Piping

(A.S.A. B31.1-1935)

Quantities in parentheses are A.S.T.M. Specification numbers.

Steam Pressures from 250 to 1,500 Lb, Temperatures 450 to 750 F. For pressures in excess of 400 lb per sq in., the pipe may be seamless steel (A106) or, for high-pressure, high-temperature service, electric-fusion-welded steel (A155). For pressures between 250 and 400 lb per sq in., the pipe may be lapwelded or seamless steel (A106), or, for high-pressure, high-temperature service, electric-fusion-welded steel (A155), electric-resistance-welded steel (A135), or seamless steel (A53). For pressures not greater than 250 lb per sq in., the pipe may be electric-fusion-welded steel (A134 or A139), forge-welded steel (A136), welded steel (A53), or wrought iron (A72). For close coiling or cold bending, the pipe may be low-carbon or Grade A seamless steel (A155 and A156), wrought iron (A72), or Grade A electric-welded pipe (A135 or A139). When the temperature is higher than 750 F, unless otherwise prohibited, the proper S value may be obtained from Fig. 5.

Valves and fittings must have flange openings or welded ends and valves must have external stem threads. Valves must be of cast or forged steel or of forged or cast non-ferrous material. Malleable iron may be used up to 300 lb pressure and 500 F. For 3 (2) [1½] in. pipe and pressure from 250 to 400 (400 to 600) [600 to 1500] lb, forged and cast-steel screwed valves and fittings may be substituted. Malleable-iron screwed fittings (300 lb M.S.S. SP-31) may be used for pressures not greater than 300 lb and temperatures not over 500 F. Valves 8 in. and larger should have the by-pass of at least ¾ in., commercial size. Welded fittings may be used of the same material and thickness as the pipe with which they are to be used.

Steam Pressures from 125 to 250 Lb, Temperature Not Above 450 F. Pipe may be electric-fusion-welded steel (A134 or A139), forge-welded steel (A136), welded steel (A53), or wrought iron (A72). Copper and brass may be used if the temperature does not exceed 406 F. Cast iron may also be used. For close coiling or cold bending,

The sleeve-type coupling illustrated in Fig. 7 is particularly suitable for plain end pipe and is widely used. A gasket is used to make a tight joint. Advantages of this coupling are low cost, the use of unskilled labor in making the connections, and the fact that small changes in alignment and grade can be made with regular, straight lengths of pipe by a movement in the coupling. This type of coupling is used extensively in long oil lines, on high-pressure natural gas lines, and on water lines.

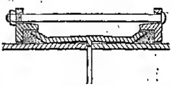


Fig. 7.—Sleeve-type Plain-end Coupling.

Pipes and Tubes of Copper, Brass, Lead, Tin, and Aluminum

Brass and copper tubes are listed by the manufacturers in even outside diameters and in wall thicknesses conforming to Stubs gage. **Brass and copper pipe** are available in "standard" and "extra-heavy" iron pipe sizes (Table 23). To obtain the approximate weight per foot of brass and copper pipe or tubes, use the following densities: yellow brass, 0.307 lb per cu in.; copper, 0.323 lb per cu in.

Table 23. Weight of Yellow Brass and Copper Pipe

Size, in.	Standard Iron Pipe Sizes				Extra-heavy Iron Pipe Sizes			
	Dimensions		Approx weights		Dimensions		Approx weights	
	Inside diam, in.	Outside diam, in.	Brass, Per lin ft, lb	Copper, Per lin ft, lb	Inside diam, in.	Outside diam, in.	Brass, Per lin ft, lb	Copper, Per lin ft, lb
⅛	0.281	0.405	0.246	0.259	0.205	0.405	0.353	0.371
¼	0.375	0.540	0.437	0.459	0.294	0.540	0.593	0.624
⅜	0.494	0.675	0.612	0.644	0.421	0.675	0.805	0.847
½	0.625	0.840	0.911	0.958	0.542	0.840	1.191	1.253
¾	0.822	1.050	1.235	1.298	0.736	1.050	1.622	1.706
1	1.062	1.315	1.740	1.829	0.951	1.315	2.386	2.509
1¼	1.368	1.660	2.557	2.669	1.272	1.660	3.291	3.460
1½	1.600	1.900	3.037	3.193	1.494	1.900	3.986	4.191
2	2.062	2.375	4.017	4.224	1.933	2.375	5.508	5.791
2½	2.500	2.875	5.830	6.130	2.315	2.875	8.407	8.839
3	3.062	3.500	0.314	8.741	2.892	3.500	11.24	11.82
3½	3.500	4.000	10.85	11.41	3.358	4.000	13.66	14.37
4	4.000	4.500	12.29	12.93	3.818	4.500	16.41	17.25
5	5.062	5.563	15.40	16.19	4.813	5.563	22.51	23.67
6	6.125	6.625	18.44	19.39	5.750	6.625	31.32	32.93
8	8.000	8.625	30.05	31.60	7.625	8.625	47.00	49.42
10	10.019	10.750	43.91	46.17	9.750	10.750	59.32	62.40

Plumber's Sizes

Size, in.	Diam, in.		Lb per ft		Size, in.	Diam, in.		Lb per ft	
	Outside	Inside	Brass	Copper		Outside	Inside	Brass	Copper
⅝	0.654	0.521	0.452	0.475	1¼	1.245	1.060	1.233	1.297
¾	0.768	0.631	0.554	0.583	1½	1.508	1.311	1.606	1.689
⅞	0.875	0.728	0.682	0.717	1¾	1.756	1.564	1.844	1.939
1	1.000	0.836	0.871	0.916	2	2.007	1.815	2.123	2.232

From tests on pipe bends made by Crane Co. (*Valve World*, Oct., 1915), the following facts were established:

1. Extra-strong pipe bends have practically the same expansion value as the corresponding shape full-weight pipe bends.

2. The tangents ordinarily furnished on pipe bends do not add materially to the expansion value.

3. Bends made to a shorter radius than 5 or 6 diameters of the pipe have practically no expansion value, as they will buckle when bending.

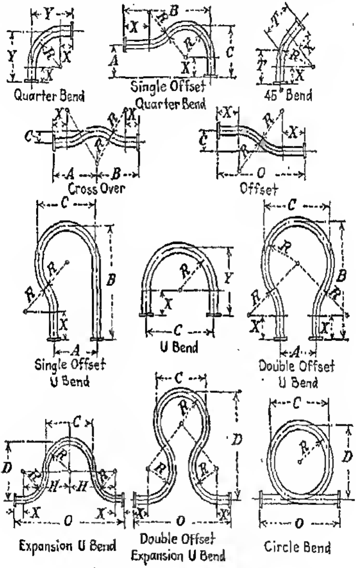


Fig. 6.—Various Types of Pipe Bends.

4. Bends are superior to the ordinary pipe-and-fitting structure because they are more flexible and do not throw all the load on the joints. Comparative tests showed that the built-up structure invariably leaked at the flanged joints, and in many cases before the fitting had been stressed near to the maximum allowable fiber stress.

5. A U bend has twice the expansion value of a 90 deg or quarter bend of the same size and radius, and an expansion U bend 4 times the expansion value of a quarter bend or twice that of a U bend. A double offset expansion U bend has 2½ times the expansion value of a U bend, and 1¼ times that of an expansion U bend.

Commercial sizes of **aluminum tubing** are listed by the manufacturers in even outside diameters and in wall thicknesses conforming to Stubs gage. **Aluminum pipe** is available in "standard" and "extra-heavy" iron pipe sizes. To obtain the approximate weight per foot of aluminum pipe or tubing, a weight of 0.098 lb per cu in. may be used.

Lead pipe is supplied in straight lengths, in coils, or in reels. The sizes and weights in Tables 24 to 26 are from the catalogue of the National Lead Co. The data in Table 25 conform to the standards advocated by the Lead Industries Assoc.

Table 26. Sizes and Weights of Block Tin Pipe

Inside diam, in.	Outside diam, in.	Weight per ft, oz	Inside diam, in.	Outside diam, in.	Weight per ft, oz	Inside diam, in.	Outside diam, in.	Weight per ft, oz
1/16	1/8	1/2	3/8	9/16 scant	6 1/2	5/8	3/4	7
1/8	3/16	1	3/8	9/16 full	7 1/2	5/8	25/32	9
3/16	1/4	1 1/2	3/8	19/32	8 1/2	5/8	13/16	10 1/2
3/16	5/16	2 1/2	3/8	5/8 scant	9 1/2	5/8	7/8	15
1/4	3/8	3	3/8	5/8 full	10 1/2	3/4	7/8 full	8
1/4	13/32	4	3/8	21/32	12	3/4	29/32	10
1/4	7/16	5	7/16	9/16 scant	4 1/2	3/4	29/32 full	11
1/4	15/32	6	7/16	19/32	6 1/2	3/4	15/16	12 1/2
1/4	1/2 scant	7	7/16	5/8	8	3/4	1 scant	17
1/4	1/2 full	8	1/2	19/32 scant	4	3/4	1 1/32	20
5/16	7/16 full	4	1/2	5/8	5	3/4	1 1/16	22 1/2
5/16	1/2 scant	5 1/2	1/2	5/8 full	5 1/2	1	1 1/8	11
5/16	17/32	7 1/2	1/2	21/32	7	1	1 3/16	16
5/16	9/16	8 1/2	1/2	21/32 full	7 1/2	1	1 3/16 full	17
3/8	1/2 scant	4	1/2	11/16	9	1	1 7/32	19 1/2
3/8	1/2 full	5	1/2	23/32	10 1/2	1	1 1/4	22
3/8	17/32	5 1/2	1/2	3/4	12 1/2	1	1 5/16	28

FITTINGS FOR WROUGHT-IRON AND STEEL PIPE

American Standard Cast-iron Pipe Flanges and Flanged Fittings

For Maximum Working Saturated Steam Pressure of 25 Lb, 125 Lb, and 250 Lb per Sq In. (Gage)

Sizes. The sizes of the fittings in the following tables are nominal pipe sizes. In the 25 lb standard, the nominal pipe size is the same as the port diameter of the fittings for *all* sizes. In the 125 and 250 lb standards the nominal pipe size is the same as the port diameter of fittings for pipe having inside diameters of 12 in. and smaller. For pipe 14 in. and larger, the corresponding outside diameter of the pipe is given, and consequently the fittings will have a smaller port diameter.

Pressure Rating. In the 25 lb standard, the sizes 36 in. and smaller may also be used for maximum non-shock working hydraulic pressures of 43 lb per sq. in. or a maximum gas pressure of 25 lb per sq in. at or near the ordinary range of air temperatures. In the 125 lb standard, the sizes 12 in. and smaller may also be used for maximum non-shock working hydraulic pressure of 175 lb per sq in. gage, at or near the ordinary range of air temperatures. In the 250 lb standard, the sizes 10 in. and smaller may also be used for maximum non-shock working hydraulic pressures of 325 lb per sq in. at a temperature of 250 F, and for maximum non-shock working hydraulic pressures of 400 lb per sq in. at or near the ordinary range of air temperatures.

The **physical requirements** of these flanges and fittings shall be as follows. Minimum tensile strength for light (medium) [heavy] castings 20,000 (21,000) [24,000] lb per sq in.; light castings are defined as those having any section less than 1/2 in. thick, heavy castings have no section less than 2 in. thick, and medium castings are in between. Sulphur content not to exceed 0.12 percent.

Facing. All 25 lb and 125 lb cast-iron flanges and flanged fittings shall be plain faced; *i.e.*, without projection or raised face. All 250 lb cast-iron flanges and flanged

Table 19. Center-to-center Dimensions of Pipe Coils

Size of pipe, in.	Recommended and advisable minimum, in.		Shortest possible, in.	
	Standard	Extra strong	Standard	Extra strong
¾	3½	2½	3½	2½
1	4	3	4	3
1¼	5	4	4½	3½
1½	6	5	5	4
2	8	6	6	5

Table 20. Round Seamless Steel Tubing

Approx Weight in Lb per Ft*

Thickness†	Outside diam, in.													
	½	¾	1	1¼	1½	2	2½	2¾	3	3½	4	4½	5	5½
20	0.17	0.27	0.36	0.45	0.55									
18	0.24	0.37	0.50	0.63	0.76									
16	0.30	0.47	0.65	8.82	1.00	1.34	1.69	1.86						
14	0.37	0.59	0.81	1.03	1.25	1.70	2.14	2.36						
13	0.41	0.66	0.92	1.17	1.42	1.93	2.44	2.69	2.95	3.45				
12	0.45	0.75	1.04	1.33	1.62	2.20	2.78	3.07	3.37	3.94				
11	0.49	0.81	1.13	1.45	1.77	2.41	3.05	3.37	3.69	4.33	4.97			
10	....	0.88	1.24	1.60	1.95	2.67	3.39	3.74	4.10	4.82	5.53	6.25	6.96	7.67
5/32	....	0.99	1.41	1.82	2.24	3.87	3.91	4.32	4.74	5.57	6.41	7.24	8.07	8.91
3/16	....	1.13	1.63	2.13	2.63	3.63	4.63	5.13	5.63	6.63	7.63	8.63	9.63	10.6
7/32	....	....	1.82	2.41	2.99	4.16	5.32	5.91	6.49	7.66	8.82	10.0	11.2	12.3
¼	....	....	2.00	2.67	3.33	4.67	6.00	6.67	7.33	8.67	10.0	11.3	12.7	14.0
5/16	....	....	....	3.13	3.96	5.63	7.29	8.13	8.96	10.6	12.3	14.0	15.6	17.3
⅜	....	....	....	3.50	4.50	6.50	8.50	9.51	10.5	12.5	14.5	16.5	18.5	20.5
½	....	....	....	....	5.33	8.00	10.7	12.0	13.3	16.0	18.7	21.3	24.0	26.7
⅝	....	....	....	....	....	9.17	12.5	14.2	15.8	19.2	22.5	25.8	29.2	32.5
¾	....	....	....	....	....	....	....	....	18.0	22.0	26.0	30.0	34.0	38.8
⅞	....	....	....	....	....	....	....	....	19.8	24.5	29.2	33.8	38.5	43.2
1	....	....	....	....	....	....	....	....	21.3	26.7	32.0	37.3	42.7	48.0

Approx Weight in Lb per Ft‡

Thickness, in.	Outside diam, in.										
	6	7	8	9	10	11	12	14	16	18	20
¼	15.4	18.0	20.7	23.4	26.0	28.7	31.7	36.7	42.1	47.4	52.7
5/16	19.0	22.3	25.7	29.0	32.3	35.7	39.0	45.7	52.4	59.0	65.7
⅜	22.5	26.5	30.5	34.5	38.6	42.6	46.6	54.6	62.6	70.6	78.6
½	29.4	34.7	40.1	45.4	50.7	56.1	61.4	72.1	82.8	93.5	104.0
⅝	35.9	42.6	49.2	55.9	62.6	69.3	75.9	89.3	103.0	116.0	129.0
¾	42.0	50.1	58.1	66.1	74.1	82.1	90.1	106.0	122.0	138.0	154.0
⅞	47.9	57.2	66.6	75.9	85.3	94.6	104.0	123.0	141.0	160.0	179.0
1	53.4	64.1	74.8	85.4	96.1	107.0	117.0	139.0	160.0	182.0	203.0
1⅛	58.6	70.6	82.6	94.6	107.0	117.0	131.0	155.0	179.0	203.0	227.0
1¼	63.4	76.8	90.1	103.0	117.0	130.0	144.0	170.0	197.0	224.0	250.0
1⅜	67.9	82.6	97.3	112.0	127.0	141.0	156.0	185.0	215.0	244.0	274.0
1½	72.1	88.1	104.0	120.0	136.0	152.0	168.0	200.0	232.0	264.0	296.0

* For weights of intermediate sizes (⅝, ⅞, 1⅛, 1⅜, 1¾, 2¼, 3¼, 3¾, 4¼, 4¾, and 5¾ in. diam), take one-half of sum of weights of next larger and smaller tabulated sizes.

† Nos. 20 to 10, B.W.G. numbers; remaining sizes in fractions of an inch.

‡ For weights of intermediate sizes (6½, 7½, 8½, 9½, 10½, 11½, 13, 15, 17, and 19 in. diam), take one-half of sum of weights of next larger and smaller tabulated sizes.

Table 27. Templates for Drilling Cast-iron Pipe Flanges, Flanged Valves, and Fittings.—(*Continued*)

Nominal pipe size, in.	Diameter of flange, in.	Thickness of flange (minimum), in.	Diameter of raised face, in.	Diameter of bolt circle, in.	Number of bolts	Diameter of bolts, in.	Diameter of drilled bolt holes, in.	Length of bolts, in.	Length of bolt stud with two nuts, in.	Total effective area bolt metal, sq in.	Stress in bolt metal, lb per sq in.[a]	Size of ring gasket, in.
250 Lb Standard												
1	4 7/8	11/16	2 11/16	3 1/2	4	5/8	3/4	2 1/4	......	0.808	970	1 × 2 7/8
1 1/4	5 1/4	3/4	3 1/16	3 7/8	4	5/8	3/4	2 1/2	......	0.808	1520	1 1/4 × 3 1/4
1 1/2	6 1/8	13/16	3 9/16	4 1/2	4	3/4	7/8	2 1/2	......	1.208	1345	1 1/2 × 3 3/4
2	6 1/2	7/8	4 3/16	5	8	5/8	3/4	2 1/2	......	1.616	1595	2 × 4 3/8
2 1/2	7 1/2	1	4 15/16	5 7/8	8	3/4	7/8	3	......	2.416	2090	2 1/2 × 5 1/8
3	8 1/4	1 1/8	5 11/16	6 5/8	8	3/4	7/8	3 1/4	......	2.416	2030	3 × 5 7/8
3 1/2	9	1 3/16	6 5/16	7 1/4	8	3/4	7/8	3 1/4	......	2.416	2460	3 1/2 × 6 1/2
4	10	1 1/4	6 15/16	7 7/8	8	3/4	7/8	3 1/2	......	2.416	3120	4 × 7 1/8
5	11	1 3/8	8 5/16	9 1/4	8	3/4	7/8	3 3/4	......	2.416	4385	5 × 8 1/2
6	12 1/2	1 7/16	9 11/16	10 5/8	12	3/4	7/8	3 3/4	......	3.624	3915	6 × 9 7/8
8	15	1 5/8	11 15/16	13	12	7/8	1	4 1/4	......	5.04	4400	8 × 12 1/8
10	17 1/2	1 7/8	14 1/16	15 1/4	16	1	1 1/8	5	......	8.80	3625	10 × 14 1/4
12	20 1/2	2	16 7/16	17 3/4	16	1 1/8	1 1/4	5 1/2	......	11.10	3975	12 × 16 5/8
14O.D.	23	2 1/8	18 15/16	20 1/4	20	1 1/8	1 1/4	5 3/4	......	13.88	3735	13 1/4 × 19 1/8
16O.D.	25 1/2	2 1/4	21 1/16	22 1/2	20	1 1/4	1 3/8	6	......	17.86	2255	15 1/4 × 21 1/4
18O.D.	28	2 3/8	23 5/16	24 3/4	24	1 1/4	1 3/8	6 1/4	......	21.43	4505	17 × 23 1/2
20O.D.	30 1/2	2 1/2	25 9/16	27	24	1 1/4	1 3/8	6 1/2	......	21.43	4845	19 × 25 3/4
24O.D.	36	2 3/4	30 5/16	32	24	1 1/2	1 5/8	7 1/2	9 1/2	31.06	4500	23 × 30 1/2
30O.D.	43	3	37 3/16	39 1/4	28	1 3/4	2	8 3/4	10 1/2	48.89	5590	29 × 37 1/2
36O.D.	50	3 3/8	43 11/16	46	32	2	2 1/4	9 1/2	11 1/2	73.70	5355	34 1/2 × 44
42O.D.	57	3 11/16	50 7/16	52 3/4	36	2	2 1/4	9 3/4	12	82.90	5945	40 1/4 × 50 3/4
48O.D.	65	4	58 7/16	60 3/4	40	2	2 1/4	10 1/2	13	92.08	7315	46 × 58 3/4

See "Introductory Notes," p. 922.
[a] The stress shown is that of internal pressure only assumed to act on a circular area equal in diameter to the outside diameter (1) of the ring gasket covering the flange to the inside of the bolts for the 25 lb standard, and (2) of the raised face for the 250 lb standard.

fittings shall have a raised face 1/16 in. high, of the diameters given in Table 27. The raised face is included in the minimum flange thickness and center-to-face dimensions.

An inspection limit of ±1/32 in. shall be allowed on all center to contact surface dimensions for sizes up to and including 10 in. and ±1/16 in. on sizes larger than 10 in. An inspection limit of ±1/16 in. shall be allowed on all contact-surface to contact-surface dimensions for sizes up to and including 10 in. and ±1/8 in. on sizes larger than 10 in.

Dimensions. In the 25 lb standard, the flange diameters, bolt circles, and number of bolts are the same as in the 125 lb American Standard (A.S.A. B16a-1939), with a reduction in the thickness of flanges and bolt diameters, thereby maintaining interchangeability between the two standards.

The center-to-face and face-to-face dimensions of 25 lb standard fittings are the same as for the 125 lb standard.

Bolting. Drilling templates are in multiples of four, so that fittings may be made to face in any quarter. *Bolt holes shall straddle the center line. For bolts smaller than* 1 3/4 in., the bolt holes shall be drilled 1/8 in. larger in diameter than the nominal size of the bolt. Holes for bolts 1 3/4 in. and larger shall be drilled 1/4 in. larger than nominal diameter of holts. Bolts shall be of steel with standard "rough square heads" and the nuts shall be of steel with standard "rough hexagonal" dimensions; all as given

of thicknesses for outside diameters as follows: cold drawn; ¼ to 1½ in. by sixteenths, 1½ in. to 5½ in. by eighths, and 5¾ to 10¾ in. by quarters. Hot drawn; 6½ to 12 in. by halves, and 12 to 20 in. by inches. Some of these sizes, with their thicknesses and weights, are given in Table 20. Dimensions and weights of boiler tubes are given in Tables 21 and 22.

Table 22. Seamless-steel Boiler Tubes

Outside diam, in.	Thickness B.W.G.	Thickness In.	Mfg. wt lb per ft	Outside diam, in.	Thickness B.W.G.	Thickness In.	Mfg. wt lb per ft	Outside diam, in.	Thickness B.W.G.	Thickness In.	Mfg. wt lb per ft
1	13	0.095	1.037	2½	12	0.109	3.171	4½	10	0.134	7.103
	12	0.109	1.168		11	0.120	3.457		9	0.148	7.817
	11	0.120	1.263		10	0.134	3.835		8	0.165	8.702
	10	0.134	1.384		9	0.148	4.207		7	0.180	9.447
1¼	13	0.095	1.328	2¾	12	0.109	3.504	5	9	0.148	8.720
	12	0.109	1.502		11	0.120	3.823		8	0.165	9.711
	11	0.120	1.628		10	0.134	4.244		7	0.180	10.550
	10	0.134	1.793		9	0.148	4.658		6	0.203	11.810
1½	13	0.095	1.619	3	12	0.109	3.838	5½	9	0.148	9.622
	12	0.109	1.836		11	0.120	4.189		8	0.165	10.720
	11	0.120	1.994		10	0.134	4.652		7	0.180	11.650
	10	0.134	2.201		9	0.148	5.110		6	0.203	13.050
1¾	13	0.095	1.910	3¼	11	0.120	4.555	6	7	0.180	12.750
	12	0.109	2.169		10	0.134	5.061		6	0.203	14.290
	11	0.120	2.360		9	0.148	5.561		5	0.220	15.410
	10	0.134	2.610		8	0.165	6.179		4	0.238	16.640
2	13	0.095	2.201	3½	11	0.120	4.921				
	12	0.109	2.503		10	0.134	5.469				
	11	0.120	2.726		9	0.148	6.012				
	10	0.134	3.018		8	0.165	6.683				
2¼	13	0.095	2.492	4	10	0.134	6.286				
	12	0.109	2.837		9	0.148	6.915				
	11	0.120	3.092		8	0.165	7.693				
	10	0.134	3.427		7	0.180	8.347				

Spiral Pipe. Spiral pipe is strong lightweight steel pipe with a single continuous helical seam (riveted or welded) from end to end stiffening it throughout. The **spiral-riveted pipe** is listed in sizes 3 to 42 in. inside diameter, in various thicknesses, and in lengths up to 40 ft. It is used for high- and low-pressure water lines, vacuum lines, exhaust-steam lines, low-pressure air lines, sand and gravel conveying, and similar services. It is stronger than straight-riveted pipe of equal size. The **spiral-welded pipe** is listed in sizes 3 to 30 in. inside diameter, in various thicknesses, and in lengths up to 40 ft. In addition to the uses given above for spiral-riveted pipe, it is used extensively by the petroleum industry, for oil and gas lines, for low-pressure steam lines, etc.

Spiral pipe may be asphalt coated or galvanized. The pipe is designed for special joints, flanges, and lightweight fittings, but the American Standard flanges and fittings can be furnished, if desired. For further details, refer to catalogue of Taylor Forge and Pipe Works, Chicago.

Table 28. General Dimensions of Flanged Fittings—Straight Sizes.—(*Continued*)

Size S	Center to face, ells, tees, crosses, and true Y *A*	Center to face, long-radius ells *B*	Center to face, 45 deg ells *C*	Face to face, laterals *D*	Center to face, laterals *E*	Center to face, true Y and laterals *F*	Face to face, reducers *G*	Diam of flanges	Minimum thickness of flanges	Minimum metal thickness of body
					250 Lb Standard					
1	4	5	2	8½	6½	2	....	4⅞	$\frac{11}{16}$	½
1¼	4¼	5½	2½	9½	7¼	2¼	....	5¼	¾	½
1½	4½	6	2¾	11	8½	2½	....	6⅛	$\frac{13}{16}$	½
2	5	6½	3	11½	9	2½	5	6½	⅞	½
2½	5½	7	3½	13	10½	2½	5½	7½	1	$\frac{9}{16}$
3	6	7¾	3½	14	11	3	6	8¼	1⅛	$\frac{9}{16}$
3½	6½	8½	4	15½	12½	3	6½	9	$1\frac{3}{16}$	$\frac{9}{16}$
4	7	9	4½	16½	13½	3	7	10	1¼	⅝
5	8	10¼	5	18½	15	3½	8	11	1⅜	$\frac{11}{16}$
6	8½	11½	5½	21½	17½	4	9	12½	$1\frac{7}{16}$	¾
8	10	14	6	25½	20½	5	11	15	1⅝	$\frac{13}{16}$
10	11½	16½	7	29½	24	5½	12	17½	1⅞	$\frac{15}{16}$
12	13	19	8	33½	27½	6	14	20½	2	1
14 O.D.	15	21½	8½	37½	31	6½	16	23	2⅛	1⅛
16 O.D.	16½	24	9½	42	34½	7½	18	25½	2¼	1¼
18 O.D.	18	26½	10	45½	37½	8	19	28	2⅜	1⅜
20 O.D.	19½	29	10½	49	40½	8½	20	30½	2½	1½
24 O.D.	22½	34	12	57½	47½	10	24	36	2¾	1⅝
30 O.D.	27½	41½	15	....	....	....	30	43	3	2

(For weights, see Table 30.)

design of heads. Hexagonal nuts for pipe sizes 48 to 96 in. in the 125 lb standard and 18 to 48 in. in the 250 lb standard can be conveniently pulled up with box wrenches.

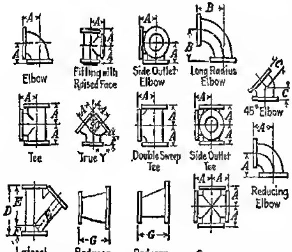


FIG. 8.—American Standard 25 Lb, 125 Lb, and 250 Lb Cast-iron Flanged Fittings. (25 Lb and 125 Lb Have Plain Face: 250 Lb Fittings Have $1/16$ In. Raised Face.) See Tables 28 to 34.

Thin brass tubing commonly used for ornamental work, brass hand railings, etc., is not of the proper size to take the standard pipe thread and is too thin for pressure work. Brass pipe is not liable to corrosion. The average ultimate strength is about 18,000 lb per sq in.

Copper pipe deteriorates rapidly under high temperatures and repeated stresses. At a temperature of 360 F, its strength is reduced 15 percent, and on this account it should never be used for high steam pressures and temperatures.

Table 24. Weights and Dimensions of Lead Tubing[a]

Inside diam, in.	Outside diam, in.	Weight per ft, oz	Inside diam, in.	Outside diam, in.	Weight per ft, oz	Inside diam, in.	Outside diam, in.	Weight per ft, oz
1/16	3/8	0.75	3/16	5/16	3.00	1/4	1/2	12.00
1/8	13/64	1.00	1/4	11/32	4.00	5/16	3/8	2.00
1/8	3/16	1.50	1/4	3/8	5.00	3/8	7/16	2.00
1/8	1/4	2.00	1/4	13/32	6.00	3/8	1/2	7.00
3/16	1/4	2.00	1/4	7/16	8.00	7/16	1/2	4.00

[a] Furnished in coils of approx 25 lb, or on reels carrying approx 50 lb or 100 lb.

Table 25. Weights and Dimensions of Lead Pipe

Size, inside diam, in.	Classification East	Classification West	Outside diam, in.	Weight per ft, lb	Size inside diam, in.	Classification East	Classification West	Outside diam, in.	Weight per ft, lb
3/8	E	AQ	0.520	0.50	1	E	AQ	1.192	1.63
	D	XL	0.549	0.63		D	XL	1.232	2.00
	C	L	0.577	0.75		C	L	1.284	2.50
	B	M	0.631	1.00		B	M	1.356	3.25
	A	S	0.725	1.50		A	S	1.428	4.00
	AA	XS	0.811	2.00		AA	XS	1.492	4.75
	AAA	XXS	0.888	2.50		AAA	XXS	1.596	6.00
1/2	E	AQ	0.625	0.56	1 1/4	E	AQ	1.442	2.00
	D	XL	0.666	0.75		D	XL	1.486	2.50
	C	L	0.712	1.00		C	L	1.528	3.00
	B	M	0.756	1.25		B	M	1.592	3.75
	A	S	0.798	1.50		A	S	1.670	4.75
	AA	XS	0.876	2.00		AA	XS	1.765	6.00
	AAA	XXS	1.012	3.00		AAA	XXS	1.889	7.75
5/8	E	AQ	0.765	0.75	1 1/2	E	AQ	1.740	3.00
	D	XL	0.803	1.00		D	XL	1.776	3.50
	C	L	0.881	1.50		C	L	1.830	4.25
	B	M	0.953	2.00		B	M	1.882	5.00
	A	S	1.019	2.50		A	S	1.984	6.50
	AA	XS	1.082	3.00		AA	XS	2.076	8.00
	AAA	XXS	1.137	3.50		AAA	XXS	2.272	11.25
3/4	E	AQ	0.906	1.00	1 3/4				
	D	XL	0.940	1.25		D	XL	2.024	4.00
	C	L	1.006	1.75		C	L	2.086	5.00
	B	M	1.068	2.25		B	M	2.146	6.00
	A	S	1.156	3.00		A	S	2.193	6.75
	AA	XS	1.212	3.50		AA	XS	2.404	10.50
	AAA	XXS	1.336	4.75		AAA	XXS	2.624	14.75

Additional standard sizes are 2, 2 1/2, 3, 4, 5, and 6 in.

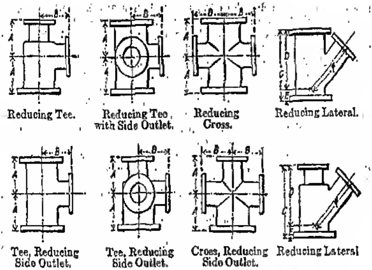


FIG. 9.—Reducing Tees, Crosses, and Laterals.

Additional dimensions for larger sizes of the 25 lb standard are as follow

Size	42	48	54	60	72
Size of outlets and smaller	24	30	36	40	48
Center to face, run	23	26	29	33	40
Center to face, outlet	30	34	37	41	48

Table 30. Weights of American Standard Cast-iron Flanged Fittings, (Straight Sizes), Lb

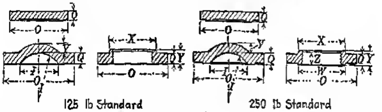
Nominal pipe size, in.	90 deg elbow	45 deg elbow	90 deg long radius elbow	Side-outlet elbow	Tees	Cross and side-outlet tees	Laterals (not ribbed)
			25 LB STANDARD				
4	35	30	45	....	50	65	
5							
6	55	50	65	....	85	100	
8	80	65	105	....	120	150	
10	135	100	160	....	185	225	
12	185	160	245	....	270	325	
14	250	195	330	....	370	450	
16	340	240	425	....	450	550	
18	385	285	530	....	550	670	
20	465	350	665	....	660	780	
24	695	490	950	....	960	1,130	
30	1050	840	1,550	....	1,500	1,750	
36	1620	1350	2,480	....	2,275	2,600	
42	2325	2000	3,620	....	3,200	3,675	
48	3205	2850	5,300	....	4,300	4,880	
54	4565	3970	7,500	....	6,250	6,880	
60	6000	5140	9,675	....	8,000	10,250	
72	9320	7525	14,175	....	12,150	13,450	

Table 27. Templates for Drilling Cast-iron Pipe Flanges, Flanged Valves, and Fittings

American Standard

Nominal pipe size, in.	Diameter of flange, in.	Thickness of flange (minimum), in.	Diameter of raised face, in.	Diameter of bolt circle, in.	Number of bolts	Diameter of bolts, in.	Diameter of drilled bolt holes, in.	Length of bolts, in.	Length of bolt stud with two nuts, in.	Total effective area bolt metal, sq in.	Stress in bolt metal, lb per sq in.*	Size of ring gasket, in.
25 Lb Standard												
4	9	3/4	......	7 1/2	8	5/8	3/4	2 1/4	......	1.616	570	4 × 6 7/8
5	10	3/4	......	8 1/2	8	5/8	3/4	2 1/4	......	1.616	750	5 × 7 7/8
6	11	3/4	......	9 1/2	8	5/8	3/4	2 1/4	......	1.616	930	6 × 8 3/4
8	13 1/2	3/4	......	11 3/4	8	5/8	3/4	2 1/4	......	1.616	1470	8 × 11
10	16	7/8	......	14 1/4	12	5/8	3/4	2 1/2	......	2.424	1440	10 × 13 3/8
12	19	1	......	17	12	5/8	3/4	2 3/4	......	2.424	2195	12 × 16 1/8
14	21	1 1/8	......	18 3/4	12	3/4	7/8	3 1/4	......	3.620	1750	14 × 18
16	23 1/2	1 1/8	......	21 1/4	16	3/4	7/8	3 1/4	......	4.830	1710	16 × 20 1/2
18	25	1 1/4	......	22 3/4	16	3/4	7/8	3 1/2	......	4.830	1965	18 × 22
20	27 1/2	1 1/4	......	25	20	3/4	7/8	3 1/2	......	6.040	1920	20 × 24 1/2
24	32	1 3/8	......	29 1/2	20	3/4	7/8	3 3/4	......	6.040	2690	24 × 28 3/4
30	38 3/4	1 1/2	......	36	28	7/8	1	4 1/4	......	11.760	2030	30 × 35 1/8
36	46	1 5/8	......	42 3/4	32	7/8	1	5	......	13.440	2610	36 × 41 7/8
42	53	1 3/4	......	49 1/2	36	1	1 1/8	5 1/4	......	19.800	2315	42 × 48 1/2
48	59 1/2	2	......	56	44	1	1 1/8	5 1/2	......	24.200	2475	48 × 55
54	66 1/4	2 1/4	......	62 3/4	44	1	1 1/8	5 3/4	......	24.200	3195	54 × 61 3/4
60	73	2 1/4	......	69 1/4	52	1 1/8	1 1/4	6	......	36.020	2515	60 × 68 1/8
72	86 1/2	2 1/2	......	82 1/2	60	1 1/8	1 1/4	6 3/4	......	41.570	3120	72 × 81 5/8
84	99 3/4	2 3/4	......	95 1/2	64	1 1/4	1 3/8	7 1/4	......	57.140	3005	84 × 94 1/4
96	113 1/4	3	......	108 1/2	68	1 1/4	1 3/8	7 3/4	......	60.570	3705	96 × 107 1/4
125 Lb Standard												
1	4 1/4	7/16	......	3 1/8	4	1/2	5/8	1 3/4	......	......	......	1 × 2 5/8
1 1/4	4 5/8	1/2	......	3 1/2	4	1/2	5/8	2	......	......	......	1 1/4 × 3
1 1/2	5	9/16	......	3 7/8	4	1/2	5/8	2	......	......	......	1 1/2 × 3 3/8
2	6	5/8	......	4 3/4	4	5/8	3/4	2 1/4	......	......	......	2 × 4 1/8
2 1/2	7	11/16	......	5 1/2	4	5/8	3/4	2 1/2	......	......	......	2 1/2 × 4 7/8
3	7 1/2	3/4	......	6	4	5/8	3/4	2 1/2	......	......	......	3 × 5 3/8
3 1/2	8 1/2	13/16	......	7	8	5/8	3/4	2 3/4	......	......	......	3 1/2 × 6 3/8
4	9	15/16	......	7 1/2	8	5/8	3/4	3	......	......	......	4 × 6 7/8
5	10	15/16	......	8 1/2	8	3/4	7/8	3	......	......	......	5 × 7 3/4
6	11	1	......	9 1/2	8	3/4	7/8	3 1/4	......	......	......	6 × 8 3/4
8	13 1/2	1 1/8	......	11 3/4	8	3/4	7/8	3 1/2	......	......	......	8 × 11
10	16	1 3/16	......	14 1/4	12	7/8	1	3 3/4	......	......	......	10 × 13 3/8
12	19	1 1/4	......	17	12	7/8	1	3 3/4	......	......	......	12 × 16 1/8
14 O.D.	21	1 3/8	......	18 3/4	12	1	1 1/8	4 1/4	......	......	......	14 × 17 3/4
16 O.D.	23 1/2	1 7/16	......	21 1/4	16	1	1 1/8	4 1/2	......	......	......	16 × 20 1/4
18 O.D.	25	1 9/16	......	22 3/4	16	1 1/8	1 1/4	4 3/4	......	......	......	18 × 21 5/8
20 O.D.	27 1/2	1 11/16	......	25	20	1 1/8	1 1/4	5	......	......	......	20 × 23 7/8
24 O.D.	32	1 7/8	......	29 1/2	20	1 1/4	1 3/8	5 1/2	......	......	......	24 × 28 1/4
30 O.D.	38 3/4	2 1/8	......	36	28	1 1/4	1 3/8	6 1/4	......	......	......	30 × 34 5/8
36 O.D.	46	2 3/8	......	42 3/4	32	1 1/2	1 5/8	7	......	......	......	36 × 41 1/4
42 O.D.	53	2 5/8	......	49 1/2	36	1 1/2	1 5/8	7 1/2	......	......	......	42 × 48
48 O.D.	59 1/2	2 3/4	......	56	44	1 1/2	1 5/8	7 3/4	......	......	......	48 × 54 1/2
54 O.D.	66 1/4	3	......	62 3/4	44	1 3/4	2	8 1/2	10 1/2	......	......	54 × 61
60 O.D.	73	3 1/8	......	69 1/4	52	1 3/4	2	8 3/4	11	......	......	60 × 67 1/2
72 O.D.	86 1/2	3 1/2	......	82 1/2	60	1 3/4	2	9 1/2	12	......	......	72 × 80 3/4
84 O.D.	99 3/4	3 7/8	......	95 1/2	64	2	2 1/4	10 1/4	13	......	......	84 × 93 1/2
96 O.D.	113 1/4	4 1/4	......	108 1/2	68	2 1/4	2 1/2	11 1/2	14 1/2	......	......	96 × 106 1/4

Table 31. Dimensions and Weights of American Standard Cast-iron Screwed Companion and Blind Flanges*



(All dimensions in inches)

Nominal pipe size	Diameter of port I	Diameter of flange O	Thickness of flange (min) Q	Metal thickness (min) V	Diameter of hub (min) X	Length through hub (min) Y	Length of threads (min) Z	Diameter of raised face W	Weights, lb: Companion flanges	Weights, lb: Blind flanges
125 Lb Standard										
1	......	4 1/4	7/16	......	1 13/16	11/16	......	......	2	2
1 1/4	......	4 5/8	1/2	......	2 5/16	13/16	......	......	2	3
1 1/2	......	5	9/16	......	2 9/16	7/8	......	......	3	3
2	......	6	5/8	......	3 1/16	1	......	......	5	5
2 1/2	......	7	11/16	......	3 9/16	1 1/8	......	......	7	7
3	......	7 1/2	3/4	......	4 1/4	1 3/16	......	......	8	9
3 1/2	......	8 1/2	13/16	......	4 13/16	1 1/4	......	......	11	12
4	......	9	15/16	......	5 5/16	1 5/16	......	......	14	16
5	......	10	15/16	......	6 7/16	1 7/16	......	......	17	20
6	......	11	1	......	7 9/16	1 9/16	......	......	22	25
8	......	13 1/2	1 1/8	......	9 11/16	1 3/4	......	......	31	42
10	......	16	1 3/16	......	11 15/16	1 15/16	......	......	45	63
12	......	19	1 1/4	13/16	14 1/16	2 3/16	......	......	63	88
14 O.D.	......	21	1 3/8	7/8	15 3/8	2 1/4	......	......	82	115
16 O.D.	......	23 1/2	1 7/16	1	17 1/2	2 1/2	......	......	105	160
18 O.D.	......	25	1 9/16	1 1/16	19 5/8	2 11/16	......	......	120	190
20 O.D.	......	27 1/2	1 11/16	1 1/8	21 3/4	2 7/8	......	......	150	250
24 O.D.	......	32	1 7/8	1 1/4	26	3 1/4	......	......	220	370
30 O.D.	......	38 3/4	2 1/8	1 7/16	......	......	......	......	......	620
36 O.D.	......	46	2 3/8	1 5/8	......	......	......	......	......	990
42 O.D.	......	53	2 5/8	1 13/16	......	......	......	......	......	1470
48 O.D.	......	59 1/2	2 3/4	2	......	......	......	......	......	2000
250 Lb Standard										
1	1	4 7/8	11/16	......	2 1/16	7/8	0.68	2 11/16	3	3
1 1/4	1 1/4	5 1/4	3/4	......	2 1/2	1	0.76	3 1/16	4	4
1 1/2	1 1/2	6 1/8	13/16	......	2 3/4	1 1/8	0.87	3 9/16	6	6
2	2	6 1/2	7/8	......	3 5/16	1 1/4	1.00	4 3/16	7	8
2 1/2	2 1/2	7 1/2	1	......	3 15/16	1 7/16	1.14	4 15/16	11	12
3	3	8 1/4	1 1/8	......	4 5/8	1 9/16	1.20	5 11/16	14	16
3 1/2	3 1/2	9	1 3/16	......	5 1/4	1 5/8	1.25	6 5/16	18	20
4	4	10	1 1/4	......	5 3/4	1 3/4	1.30	6 15/16	23	26
5	5	11	1 3/8	......	7	1 7/8	1.41	8 5/16	29	34
6	6	12 1/2	1 7/16	......	8 1/8	1 15/16	1.51	9 11/16	37	46
8	8	15	1 5/8	......	10 1/4	2 3/16	1.71	11 15/16	56	75
10	10	17 1/2	1 7/8	15/16	12 5/8	2 3/8	1.92	14 1/16	81	120
12	12	20 1/2	2	1	14 3/4	2 9/16	2.12	16 7/16	115	155
14 O.D.	13 1/4	23	2 1/8	1 1/8	16 3/4	2 11/16	2.25	18 15/16	155	210
16 O.D.	15 1/4	25 1/2	2 1/4	1 1/4	18 7/8	2 7/8	2.45	21 1/16	195	270
18 O.D.	17	28	2 3/8	1 3/8	20 5/8	3 1/8	2.65	23 5/16	240	350
20 O.D.	19	30 1/2	2 1/2	1 1/2	22 3/4	3 1/4	2.85	25 9/16	300	440
24 O.D.	23	36	2 3/4	1 5/8	27 5/8	3 11/16	3.25	30 5/16	450	670
30 O.D.	29	43	3	2	......	......	......	37 3/16	......	1070

See notes at bottom of p. 931.

in the American Standard on Wrench Head Bolts and Nuts and Wrench Openings of the National Screw Thread Commission (see p. 767)." For bolts, 1¾ in. diam and larger, bolt-studs with a nut on each end are recommended.

Hexagonal nuts for pipe sizes 1 to 48 in. in the 125 lb standard and 1 to 16 in. in the *250 lb standard can be conveniently pulled up with open wrenches of minimum*

Table 28. General Dimensions of Flanged Fittings—Straight Sizes

American Standard

(All dimensions in inches. Letters refer to Fig. 8)

Size S	Center to face, ells, tees, crosses, and true Y A	Center to face, long-radius ells B	Center to face, 45 deg ells C	Face to face, laterals D	Center to face, laterals E	Center to face, true Y and laterals F	Face to face, reducers G	Diam of flanges	Minimum thickness of flanges	Minimum metal thickness of body
					25 Lb Standard					
4	6½	9	4	....	....	....	....	9	¾	0.42
5	7½	10¼	4½	....	....	....	....	10	¾	0.44
6	8	11½	5	....	....	....	....	11	¾	0.44
8	9	14	5½	....	....	....	....	13½	¾	0.46
10	11	16½	6½	....	....	....	....	16	⅞	0.50
12	12	19	7½	....	....	....	....	19	1	0.54
14	14	21½	7½	....	....	....	....	21	1⅛	0.57
16	15	24	8	....	....	....	....	23½	1⅛	0.60
18	16½	26½	8½	....	....	....	....	25	1¼	0.64
20	18	29	9½	....	....	....	....	27½	1¼	0.67
24	22	34	11	....	....	....	....	32	1⅜	0.76
30	25	41½	15	....	....	....	....	38¾	1½	0.88
36	28	49	18	....	....	....	....	46	1⅝	0.99
42	31	56½	21	....	....	....	....	53	1¾	1.10
48	34	64	24	....	....	....	....	59½	2	1.26
54	39	71½	27	....	....	....	....	66¼	2¼	1.35
60	44	79	30	....	....	....	....	73	2¼	1.39
72	53	94	36	....	....	....	....	86½	2½	1.62
					125 Lb Standard					
1	3½	5	1¾	7½	5¾	1¾	....	4¼	7/16	5/16
1¼	3¾	5½	2	8	6¼	1¾	....	4⅝	½	5/16
1½	4	6	2¼	9	7	2	....	5	9/16	5/16
2	4½	6½	2½	10½	8	2½	5	6	⅝	5/16
2½	5	7	3	12	9½	2½	5½	7	11/16	5/16
3	5½	7¾	3	13	10	3	6	7½	¾	⅜
3½	6	8½	3½	14½	11½	3	6½	8½	13/16	7/16
4	6½	9	4	15	12	3	7	9	15/16	½
5	7½	10¼	4½	17	13½	3½	8	10	15/16	½
6	8	11½	5	18	14½	3½	9	11	1	9/16
8	9	14	5½	22	17½	4½	11	13½	1⅛	⅝
10	11	16½	6½	25½	20½	5	12	16	1 3/16	¾
12	12	19	7½	30	24½	5½	14	19	1¼	13/16
14 O.D.	14	21½	7½	33	27	6	16	21	1⅜	⅞
16 O.D.	15	24	8	36½	30	6½	18	23½	1 7/16	1
18 O.D.	16½	26½	8½	39	32	7	19	25	1 9/16	1 1/16
20 O.D.	18	29	9½	43	35	8	20	27½	1 11/16	1⅛
24 O.D.	22	34	11	49½	40½	9	24	32	1⅞	1¼
30 O.D.	25	41½	15	59	49	10	30	38¾	2⅛	1 7/16
36 O.D.	28	49	18	....	....	....	36	46	2⅜	1⅝
42 O.D.	31	56½	21	....	....	....	42	53	2⅝	1 13/16
48 O.D.	34	64	24	....	....	....	48	59½	2¾	2

Bolting material including nuts and washers are based on a high-grade product equal to that given in A.S.T.M. Standard Specifications for Alloy-steel Bolting Material for High Temperature Service No. A96-33 and with physical and chemical requirements in accordance with the tables given under A.S.A. B16e-1939. Commercial steel bolts should not be used at steam pressures over 250 lb per sq in. and temperatures over 450 F. Nuts should be of carbon or alloy steel. Washers when used under nuts should be of forged or rolled carbon steel.

Table 32.* Steam, Water, and Oil Pressure Temperature Ratings for Carbon Steel Flanged Fittings and Companion Flanges
(Other-than ring joints)

Steam and water pressure rating (primary)	150	300	400	600	900	1500
Hydrostatic shell test at 125 F	350	750	1000	1500	2000	3500
Service temperature, F	Max steam, water, and oil pressures, lb per sq in. (non-shock)					
100	230	500	670	1000	1500	2500
150	220	480	640	960	1440	2400
200	210	465	620	930	1395	2325
250	200	450	600	900	1350	2250
300	190	435	580	870	1305	2175
350	180	420	560	840	1260	2100
400	170	405	540	810	1215	2025
450	160	390	520	780	1170	1950
500	150[a]	375	500	750	1125	1875
550	140	360	480	720	1080	1800
600	130	345	460	690	1035	1725
650	120	330	440	660	990	1650
700	110	315	420	630	945	1575
750	100	300[a]	400[a]	600[a]	900[a]	1500[a]
Maximum steam and water pressures						
800	85	250	335	500	750	1250
850	70	300	270	400	600	1000
Maximum oil pressures						
800	92	275	370	550	830	1380
850	82	245	330	490	740	1230
900	70	210	280	420	630	1050
950	55	165	220	330	495	825
1000	40	120	160	240	360	600

* For temperature ratings for 2,500 lb pressure, see A.S.A. B16e-1939.
All pressures are in pounds per square inch (gage).
[a] Primary service pressure rating.

Bolting. Drilling templates are in multiples of four, so that fittings may be made to face in any quarter. Bolt holes straddle the center lines. Bolt holes are drilled 1/8 in. larger in diameter than the nominal size of bolt. Bolts or bolt studs threaded at both ends may be used and shall be equipped with cold-punched or cold-pressed semifinished nuts of American Standard rough dimensions, chamfered and trimmed.

All bolts and bolt studs having diameters from 1/2 in. to and including 1 in. and the corresponding nuts shall be threaded with the American (National) Standard Screw

Spot Facing. The bolt holes of 25 lb, 125 lb and 250 lb cast-iron flanges and flanged fittings shall not be spot faced for ordinary service. When required, the flanges and fittings in sizes 36 in. and larger may be spot faced or back faced to the minimum thickness of flange with a plus tolerance of ⅛ in.

Reducing Fittings. Reducing elbows and side-outlet elbows carry same dimensions center to face as straight-size elbows corresponding to the size of the larger opening.

Tees, side-outlet tees, crosses, and laterals sizes 16 in. and smaller, reducing on the outlet or branch, have the same dimension center to face and face to face as straight-size fittings corresponding to the size of the larger opening. Sizes 18 in. and larger, reducing on the outlet or branch, are made in two lengths depending on the size of the outlet as given in the tables of dimensions.

Tees, crosses, and laterals, reducing on the run only, have the same dimensions center to face and face to face as straight-size fittings corresponding to the size of the larger opening.

Reducers and eccentric reducers for all reductions have the same face-to-face dimensions for the larger opening as given in Table 28.

Special double branch elbows whether straight or reducing have the same dimension center to face as straight size elbows corresponding to the size of the larger opening.

Side-outlet elbows and side-outlet tees shall have all openings on intersecting center lines.

Elbows. Special degree elbows ranging from 1 to 45 deg have the same center to face dimension given for 45 deg elbows, and those over 45 deg and up to 90 deg shall have the same center-to-face dimensions given for 90 deg elbows. The angle designation of an elbow is its deflection from straight line flow and is the angle between the flange faces.

Screwed Companion Flanges. Screwed companion flanges in the 25 lb standard shall not be thinner than those in the 125 lb standard on sizes 24 in. and smaller. Other types of flanges may have thicknesses as given in Table 27.

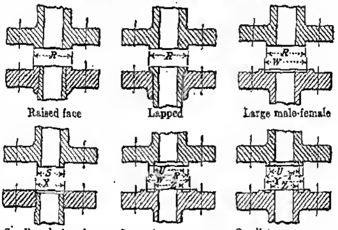
Table 29. General Dimensions Cast-iron Reducing Tees, Crosses, and Laterals

25 Lb, 125 Lb, and 250 Lb American Standard
(All dimensions in inches. Letters refer to Fig. 9)

Size	Tees and crosses					Laterals								
	Size of outlets and smaller*	Center to face, run, A		Center to face, outlet, B		Size of branches and smaller*	Face to face, run, C		Center to face, run, D		Center to face, run, E		Center to face, branch, F	
		25 lb and 125 lb	250 lb	25 lb and 125 lb	250 lb		125 lb	250 lb	125 lb	250 lb	125 lb	250 lb	125 lb	250 lb
1 to 16	All reducing fittings from 1 to 16 in. inclusive have the same center-to-face dimensions as straight-size fittings of the larger size													
18	12	13	14	15½	17	8	26	34	25	31	1	3	27½	32½
20	14	14	15½	17	18½	10	28	37	27	34	1	3	29½	36
24	16	15	17	19	21½	12	32	44	31½	41	½	3	34½	43
30	20	18	20½	23	25½	14	39		39		0		42	
36	24	20	......	26		.								

* Short-body patterns are used for sizes 18 in. and larger. Long-body patterns are used when outlets are larger than given above, and therefore have the same dimensions as straight-size fittings.

Table 34. Facing Dimensions for the American 150, 300, 400, 600, 900, 1500, and 2500 Lb Steel Flanges



Raised face | Lapped | Large male-female | Small male-female | Large tongue-groove | Small tongue-groove

(All dimensions given in inches)

Nominal pipe size	Outside diameter: Raised face, lapped, large male, and large tongue, R	Outside diameter: Small male, S	Outside diameter: Small tongue, T	I.D. of large and small tongue, U	Outside diameter: Large female and large groove, W	Outside diameter: Small female, X	Outside diameter: Small groove, Y	I.D. of large and small groove, Z	Height: Raised face 150 and 300 lb st'd*	Height: Raised face, large and small male and tongue 400, 600, 900, 1500 and 2500 lb st'ds	Depth of groove or female companion flanges
1/2	1 3/8	23/32	1 3/8	1	1 7/16	25/32	1 7/16	15/16	1/16	1/4	3/16
3/4	1 11/16	15/16	1 11/16	1 5/16	1 3/4	1	1 3/4	1 1/4	1/16	1/4	3/16
1	2	1 3/16	1 7/8	1 1/2	2 1/16	1 1/4	1 15/16	1 7/16	1/16	1/4	3/16
1 1/4	2 1/2	1 1/2	2 1/4	1 7/8	2 9/16	1 9/16	2 5/16	1 13/16	1/16	1/4	3/16
1 1/2	2 7/8	1 3/4	2 1/2	2 1/8	2 15/16	1 13/16	2 9/16	2 1/16	1/16	1/4	3/16
2	3 5/8	2 1/4	3 1/4	2 7/8	3 11/16	2 5/16	3 5/16	2 13/16	1/16	1/4	3/16
2 1/2	4 1/8	2 11/16	3 3/4	3 3/8	4 3/16	2 3/4	3 13/16	3 5/16	1/16	1/4	3/16
3	5	3 5/16	4 5/8	4 1/4	5 1/16	3 3/8	4 11/16	4 3/16	1/16	1/4	3/16
3 1/2	5 1/2	3 13/16	5 1/8	4 3/4	5 9/16	3 7/8	5 3/16	4 11/16	1/16	1/4	3/16
4	6 3/16	4 5/16	5 11/16	5 3/16	6 1/4	4 3/8	5 3/4	5 1/8	1/16	1/4	3/16
5	7 5/16	5 3/8	6 13/16	6 5/16	7 3/8	5 7/16	6 7/8	6 1/4	1/16	1/4	3/16
6	8 1/2	6 3/8	8	7 1/2	8 9/16	6 7/16	8 1/16	7 7/16	1/16	1/4	3/16
8	10 5/8	8 3/8	10	9 3/8	10 11/16	8 7/16	10 1/16	9 5/16	1/16	1/4	3/16
10	12 3/4	10 1/2	12	11 1/4	12 13/16	10 9/16	12 1/16	11 3/16	1/16	1/4	3/16
12	15	12 1/2	14 1/4	13 1/2	15 1/16	12 9/16	14 5/16	13 7/16	1/16	1/4	3/16
14 o.d.	16 1/4	13 3/4	15 1/2	14 3/4	16 5/16	13 13/16	15 9/16	14 11/16	1/16	1/4	3/16
16 o.d.	18 1/2	15 3/4	17 5/8	16 3/4	18 9/16	15 13/16	17 11/16	16 11/16	1/16	1/4	3/16
18 o.d.	21	17 3/4	20 1/8	19 1/4	21 1/16	17 13/16	20 3/16	19 3/16	1/16	1/4	3/16
20 o.d.	23	19 3/4	22	21	23 1/16	19 13/16	22 1/16	20 15/16	1/16	1/4	3/16
24 o.d.	27 1/4	23 3/4	26 1/4	25 1/4	27 5/16	23 13/16	26 5/16	25 3/16	1/16	1/4	3/16

See notes at bottom of p. 933.

Table 30. Weights of American Standard Cast-iron Flanged Fittings, (Straight Sizes), Lb—*(Continued)*

Nominal pipe size, in.	90 deg elbow	45 deg elbow	90 deg long radius elbow	Side-outlet elbow	Tees	Cross and side-outlet tees	Laterals (not ribbed)
125 Lb Standard							
1	5	4	7	8	9	11	10
1¼	7	6	9	10	11	15	13
1½	9	8	11	13	15	19	17
2	14	12	16	20	21	28	25
2½	19	17	23	28	30	39	36
3	24	20	28	34	37	48	44
3½	31	27	37	46	49	63	59
4	41	36	48	59	64	82	75
5	52	45	62	74	81	105	96
6	68	60	85	96	105	135	125
8	110	94	145	150	165	210	210
10	175	145	230	240	270	330	340
12	250	220	350	340	380	470	520
14 O.D.	350	270	470	470	530	650	680
16 O.D.	470	360	670	620	700	850	950
18 O.D.	580	420	840	760	860	1040	1150
20 O.D.	740	540	1080	970	1100	1330	1480
24 O.D.	1160	800	1640	1510	1730	2080	2080
30 O.D.	1850	1430	2800	2350	2710	3210	3680
36 O.D.	2800	2280	4450	3500	4050	4750	
42 O.D.	4010	3380	6610	4930	5790	6710	
48 O.D.	5400	4680	9250	6520	7620	8740	
250 Lb Standard							
1	9	7	10	13	14	18	15
1¼	11	10	13	17	18	23	20
1½	16	15	18	24	25	32	30
2	20	18	23	30	32	41	37
2½	30	28	34	43	46	58	57
3	40	35	44	55	58	74	73
3½	49	44	55	71	76	94	91
4	65	58	72	94	99	130	120
5	87	76	98	125	135	170	165
6	115	105	135	170	180	230	230
8	185	155	220	260	280	350	360
10	290	240	350	400	430	540	570
12	410	340	510	560	620	770	820
14 O.D.	560	440	710	790	870	1090	1180
16 O.D.	750	620	960	1040	1150	1430	1610
18 O.D.	970	780	1260	1330	1490	1840	2100
20 O.D.	1220	960	1630	1670	1880	2320	2670
24 O.D.	1840	1430	2470	2490	2800	3450	4020
30 O.D.	3120	2230	4290	4150	4740	5760	

All weights listed are for fittings faced and drilled, based on minimum thickness and dimensions given in preceding tables, without allowances for variation. Cast iron is assumed to weigh 0.26 lb per cu in.

Table 35. Templates for Drilling, American Standard Steel Pipe Flanges and Flanged Fittings*

(All dimensions in inches)

Nominal pipe size	400 lb Standard					600 lb Standard					900 lb Standard					1,500 lb Standard				
	Outside diam of flange	Thickness of flange minimum	Diam of bolt circle	Number of bolts	Size of bolts	Outside diam of flange	Thickness of flange minimum	Diam of bolt circle	Number of bolts	Size of bolts	Outside diam of flange	Thickness of flange minimum	Diam of bolt circle	Number of bolts	Size of bolts	Outside diam of flange	Thickness of flange minimum	Diam of bolt circle	Number of bolts	Size of bolts
1/2	For sizes below 4 in. use dimensions of 600 lb fittings					3 3/4	9/16	2 5/8	4	1/2	For sizes below 3 in. use the dimensions of 1,500 lb fittings					4 3/4	7/8	3 1/4	4	3/4
3/4						4 5/8	5/8	3 1/4	4	5/8						5 1/8	1	3 1/2	4	3/4
1						4 7/8	11/16	3 1/2	4	5/8						5 7/8	1 1/8	4	4	7/8
1 1/4	......	......	......	......	......	5 1/4	13/16	3 7/8	4	5/8	......	......	......	......	......	6 1/4	1 1/8	4 3/8	4	7/8
1 1/2	......	......	......	......	......	6 1/8	7/8	4 1/2	4	3/4	......	......	......	......	......	7	1 1/4	4 7/8	4	1
2	......	......	......	......	......	6 1/2	1	5	8	5/8	......	......	......	......	......	8 1/2	1 1/2	6 1/2	8	7/8
2 1/2	......	......	......	......	......	7 1/2	1 1/8	5 7/8	8	3/4	......	......	......	......	......	9 5/8	1 5/8	7 1/2	8	1
3	......	......	......	......	......	8 1/4	1 1/4	6 5/8	8	3/4	9 1/2	1 1/2	7 1/2	8	7/8	10 1/2	1 7/8	8	8	1 1/8
3 1/2	......	......	......	......	......	9	1 3/8	7 1/4	8	7/8										
4	10	1 3/8	7 7/8	8	7/8	10 3/4	1 1/2	8 1/2	8	7/8	11 1/2	1 3/4	9 1/4	8	1 1/8	12 1/4	2 1/8	9 1/2	8	1 1/4
5	11	1 1/2	9 1/4	8	7/8	13	1 3/4	10 1/2	8	1	13 3/4	2	11	8	1 1/4	14 3/4	2 7/8	11 1/2	8	1 1/2
6	12 1/2	1 5/8	10 5/8	12	7/8	14	1 7/8	11 1/2	12	1	15	2 3/16	12 1/2	12	1 1/8	15 1/2	3 1/4	12 1/2	12	1 3/8
8	15	1 7/8	13	12	1	16 1/2	2 3/16	13 3/4	12	1 1/8	18 1/2	2 1/2	15 1/2	12	1 3/8	19	3 5/8	15 1/2	12	1 5/8
10	17 1/2	2 1/8	15 1/4	16	1 1/8	20	2 1/2	17	16	1 1/4	21 1/2	2 3/4	18 1/2	16	1 3/8	23	4 1/4	19	12	1 7/8
12	20 1/2	2 1/4	17 3/4	16	1 1/4	22	2 5/8	19 1/4	20	1 1/4	24	3 1/8	21	20	1 3/8	26 1/2	4 7/8	22 1/2	16	2
14 O.D.	23	2 3/8	20 1/4	20	1 1/4	23 3/4	2 3/4	20 3/4	20	1 3/8	25 1/4	3 3/8	22	20	1 1/2	29 1/2	5 1/4	25	16	2 1/4
16 O.D.	25 1/2	2 1/2	22 1/2	20	1 3/8	27	3	23 3/4	20	1 1/2	27 3/4	3 1/2	24 1/4	20	1 5/8	32 1/2	5 3/4	27 3/4	16	2 1/2
18 O.D.	28	2 5/8	24 3/4	24	1 3/8	29 1/4	3 1/4	25 3/4	20	1 5/8	31	4	27	20	1 7/8	36	6 3/8	30 1/2	16	2 3/4
20 O.D.	30 1/2	2 3/4	27	24	1 1/2	32	3 1/2	28 1/2	24	1 5/8	33 3/4	4 1/4	29 1/2	20	2	38 3/4	7	32 3/4	16	3
24 O.D.	36	3	32	24	1 3/4	37	4	33	24	1 7/8	41	5 1/2	35 1/2	20	2 1/2	46	8	39	16	3 1/2

* See introductory notes, p. 931.

True Y's. The dimensions of true Y's, straight sizes, are given in Table 28. Other forms are considered special and should be made to suit conditions.

Laterals. Laterals (Y branches) both straight and reducing sizes 8 in. and larger shall be reinforced to compensate for the inherent weakness in the casting design.

The American Standard covers also dimensions (not included in the Tables) of base elbows and base tees and anchorage bases for straight tees and reducing tees.

American standard cast-iron pipe flanges and flanged fittings (A.S.A. B16b1-1931) are available for maximum non-shock working hydraulic pressure of 800 lb per sq in. (gage) at ordinary air temperatures.

Sizes of Wrenches for Bolting Up Flanges. Tests made by Crane Co. (*Valve World*, Sept., 1913) show that a compression of about 12,000 lb can be obtained by one man using a 16 in. wrench on ¾ to 1 in. bolts, a 36 in. wrench on 1⅛ to 1⅜ in. bolts, a 60-in. wrench on 1½ to 1¾ in. bolts, and a 72 in. wrench on 1⅞ and 2 in. bolts. Square nuts give 20 to 25 percent less compression than hexagon nuts with a given effort on the wrench. Rough-cut or uneven threads reduce compression 10 to 15 percent. Lubrication of threads and bearing surfaces increases compression about 50 percent over that obtained with dry threads and surfaces. It is not necessary to overstrain bolts specified in the American Standard if proper wrenches are used, and almost impossible to overstrain bolts larger than 1 in. diam. The following over-all lengths of box wrenches are recommended (interpolate for intermediate sizes):

Size of bolt, in.	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2¼
Length of wrench, in.	7¾	9⅞	11½	13¼	15	19	23	27	30⅛	34

American Standard Steel Pipe Flanges and Flanged Fittings

For Maximum Working Steam Pressure of 150, 300, 400, 600, 900, 1,500 and 2,500 Lb per Sq In. (Gage) at 750 F

Introductory Notes

Pressure Ratings and Tests. These standards shall be known as the "American 150, 300, 400, 600, 900, 1,500, and 2,500 lb Steel Flange Standards (A.S.A. B16e-1939)," said pressure designation being the recommended rating at a temperature such as given in Table 33. This table shows recommended ratings for various temperatures, together with hydrostatic shell test pressures for one set of conditions. For similar tables for other conditions, refer to A.S.A. B16e-1939.

Sizes. The size of the fittings and companion flanges in the Tables is identified by the corresponding nominal pipe size. For pipe 14 in. and larger, the corresponding outside diameter of the pipe is given.

Materials. The flanged fittings and flanges should be either steel castings or steel forgings of the grade complying with the A.S.T.M. specifications recommended under these standards for the various pressure-temperature ratings for which these standards are designed. A few of these characteristics selected from A.S.A. B16e-1939 are given in Table 33.

Notes to Table 31

[a] The weights of screwed companion flanges in the 25 lb standard, on sizes 24 in. and smaller may be assumed to be the same as in the 125 lb standard (see "Introductory Notes," p. 927).

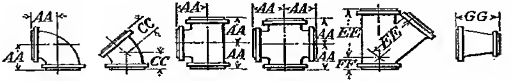
[b] Also length of threads in 125 lb standard.

All 250 lb cast-iron standard flanges have a $\frac{1}{16}$ in. raised face. This raised face is included in the minimum thickness of flange dimensions. Sizes 14 in. and larger are to be used with outside diameter pipe of the same sizes.

All blind flanges for sizes 12 in. (19 in. O.D.) for 125 lb standard and 10 in. (17½ in. O.D.) for 250 lb standard and larger must be dished, with inside radius equal to the port diameter.

All weights listed are for flanges faced only, based upon minimum thicknesses given in the table above without allowances for variation.

Table 37. Center to Contact Surface of American Standard Steel Flanged Fittings



Elbow 45°Elbow Tee Cross 45°Lateral Reducer

(All dimensions in inches)

Nom pipe size	400 lb Standard					600 lb Standard					900 lb Standard					1,500 lb Standard				
	AA	CC	EE	FF	GG	AA	CC	EE	FF	GG	AA	CC	EE	FF	GG	AA	CC	EE	FF	GG
½	For sizes below 4 in., use the dimensions of 600 lb fittings					3¼	2	5¾	1¾	5	For sizes below 3 in., use the dimensions of 1,500 lb fittings					4¼	3			
¾						3¾	2¼	6¾	2	5						4½	3¼			
1						4¼	2¼	7¼	2¼	5						5	3½	9	2½	5
1¼	...	...	...	...	...	4½	2¾	8	2½	5	...	...	...	...	...	5½	4	10	3	5¾
1½	...	...	...	...	...	4¾	3	9	2¾	5	...	...	...	...	...	6	4¼	11	3½	6¼
2	...	...	...	...	...	5¾	4¼	10¼	3½	6	...	...	...	...	...	7¼	4¾	13¼	4	7¼
2½	...	...	...	...	...	6½	4½	11½	3½	6¾	...	...	...	...	...	8¼	5¼	15¼	4½	8¼
3	...	...	...	...	...	7	5	12¾	4	7¼	7½	5½	14½	4½	7¾	9¼	5¾	17¼	5	9¼
3½	...	...	...	...	...	7½	5½	14	4½	7¾										
4	8	5½	16	4½	8¼	8½	6	16½	4½	8¾	9	6½	17½	5½	9¼	10¾	7¼	19¼	6	10¾
5	9	6	16¾	5	9¼	10	7	19½	6	10¼	11	7½	21	6½	11¼	13¼	8¾	23¼	7½	13¾
6	9¾	6¼	18¾	5¼	10	11	7½	21	6½	11¼	12	8	22½	6½	12¼	13⅞	9⅜	24⅞	8⅛	14½
8	11¾	6¾	22¼	5¾	12	13	8½	24½	7	13¼	14½	9	27½	7½	14¾	16⅝	10⅞	29⅞	9⅛	17
10	13¼	7¾	25¾	6¼	13¼	15½	9½	29½	8	15¾	16½	10	31½	8½	16¾	19	12	36	10¼	20¼
12	15	8¾	29¾	6½	15¼	16½	10	31½	8½	16¾	19	11	34½	9	17¾	22¼	13¼	40¾	12	23
14 O.D.	16¼	9¼	32¾	7	16½	17½	10¾	34½	9	17¾	20¼	11½	36½	9½	19	24¾	14¼	44	12½	25¾
16 O.D.	17¾	10¼	36¼	8	18½	19½	11¾	38½	10	19¾	22¼	12½	40¾	10½	21	27¼	16¼	48¼	14¾	28¼
18 O.D.	19¼	10¾	39¼	8½	19½	21½	12¼	42	10½	21¾	24	13¼	45½	12	24½	30¼	17¾	53¼	16½	31½
20 O.D.	20¾	11¼	42¾	9	21	23½	13	45½	11	23¾	26	14½	50¼	13	26½	32¾	18¾	57¾	17¾	34
24 O.D.	24¼	12¾	50¼	10½	24½	27½	14¾	53	13	27¾	30½	18	60	15½	30½	38¼	20¾	67¼	20½	39¾

Thread, Coarse Thread Series, Medium Fit, Class 3 (see p. 761). Bolts and bolt studs whose diameters are $1\frac{1}{8}$ in. and larger shall have special threads of the American (National) form whose pitch is $\frac{1}{8}$ in. (8 threads per in.). It is recommended that these special threads be allowed a pitch diameter tolerance of −0.006 in. and a lead tolerance of ±0.002 in.

Bolt studs with a nut at each end are recommended for high-temperature service.

Table 33. Physical Characteristics of Steels for Flanges and Fittings

	Steel castings[a]	Alloy-steel bolt material[b]			Forged-steel flanges[c]	
		Class A	Class B	Class C	Class I	Class II
Tensile strength, lb per sq in.	70,000	95,000	105,000	125,000	60,000	70,000
Yield point, min, lb per sq in.	36,000	70,000	80,000	105,000	30,000	36,000
Elongation in 2 in., min, percent	22	20	20	16	22	18
Reduction of area, min, percent	30	50	50	50	35	24

[a] Carbon steel (A.S.T.M. A95-36).
[b] (A.S.T.M. A96-33).
[c] (A.S.T.M. A181-37).
Phosphorus max., 0.05 percent; sulphur max. 0.05 percent.

The allowable working fiber stress, considering internal allowable working pressure only, in bolting material for valve bonnet flanges, cleanout flanges, etc., shall not exceed 9,000 lb per sq in. assuming the pressure to act upon an area circumscribed by the periphery of the outside of the contact surface.

All flanges shall be spot faced or back faced parallel to the flange face. Metal removed in spot facing or back facing shall not reduce the thickness of the flange below the minimum given in the tables. Spot facing does not apply to forged steel flanges if the back of the flange is parallel to the flange face.

Metal Thickness. Minimum metal thicknesses specified in the tables are based on an allowable fiber stress of 7,000 lb per sq in. using the modified Barlow formula of the A.S.M.E. Boiler Construction Code for cylindrical sections and adding 50 percent to the thickness thus determined to compensate for the shape of the fittings. The minimum commercial casting thickness is considered to be $\frac{1}{4}$ in., therefore the standards do not show thicknesses less than this. The minimum thickness in these standards means the minimum thickness in any part of the finished casting.

The modified Barlow formula is as follows: For pipes having nominal diameters of $\frac{1}{4}$ to 5 in., $P = 2S(t - 0.065)/D - 125$. For pipes of nominal diameters over 5 in., $P = 2S(t - 0.1)/D$, where P is the working pressure, lb per sq in.; t is the thickness of wall of pipe, in.; D is the actual outside diameter of pipe, in.; and S is 7,000 lb per sq in.

Ring Joints. The dimensions used for ring and groove joint facings were developed by a committee of the A.P.I. The corresponding dimensions and ring numbers incorporated in this standard (B16e-1939) are identical with those given in A.P.I. Std. No. 5-G-3-1937. The dimension for the depth of groove is added to the basic flange thickness which makes it necessary to include separate tables of dimensions for fittings having the ring joint facing.

Notes to Table 34

* Included in the minimum flange thickness dimensions. A $\frac{1}{16}$ in. raised face is also permitted on the 400, 600, 900, 1,500, and 2,500 lb flange standards, but it must be added to the minimum flange thicknesses. Regular facing for 400, 600, 900, 1,500, and 2,500 lb flange standards is a $\frac{1}{4}$ in. raised face not included in minimum flange thickness dimensions.

A tolerance of $\frac{1}{64}$ in. is allowed on the inside and outside diameters of all facings.

Gaskets for male-female and tongue-groove joints shall cover the bottom of the recess with minimum clearances taking into account the tolerances stated above.

union of the gasket type, at the center a female screw union having a brass to iron seat that is non-corrosive and a ground joint that eliminates the need for a gasket, and at the right a flange union of the gasket type. As in the case of other pipe fittings, unions and union fittings are available in the various pipe sizes and in materials and designs suitable for any service conditions.

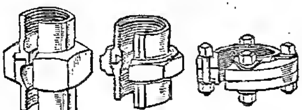


FIG. 11.—Types of Pipe Unions.

Very large flange unions can be made by bolting together two screwed companion flanges.

Expansion Joints for Steam Pipe Lines. The linear expansion of a long pipe must be taken care of by the use of some form of expansion joint or bend. The coefficients of expansion a of three metals used in pipe manufacture, within the range of temperatures common to boiler practice (32 to 392 F), as determined at the Bureau of Standards, Washington, D. C., are as follows: charcoal iron, 0.0000068S61; Bessemer steel, 0.0000069S0; seamless open-hearth steel (hot-finished), 0.0000068S3. The length of a tube at t F $= L_t = L\ (1 + at)$ in which L is the length of the tube at 0 F. See also p. 497. The theoretical expansion in 100 ft of iron or steel pipe, carrying steam of various pressures and temperatures, based on an outside temperature of 32 F, is as follows:

FIG. 12. FIG. 13. FIG. 14.
FIGS. 12-14.—Copper Expansion Joints for Steam Lines.

Gage pressure, lb per sq in.	30	100	125	150	175	200	Superheated steam				
Temperature of steam, F	274	338	353	366	377	388	420	500	550	600	650
Expansion of 100 ft of pipe, in.	2.03	2.72	2.89	3.05	3.19	3.31	3.73	4.77	5.48	6.23	7.03

Table 38 gives the linear expansion in 100 ft of cast-iron, wrought-iron, steel, brass, or copper pipe according to the A.S.A. Code for Pressure Piping.

Table 38. Thermal Expansion of Pipes
(Total expansion in inches per 100 ft above 32 F)

Temp, deg F	100	150	200	250	300	350	400	450	500	600	700
Steel and wrought iron	0.5	0.9	1.3	1.7	2.2	2.6	3.0	3.5	4.0	5.0	6.0
Cast iron	0.5	0.8	1.2	1.6	1.9	2.3	2.7	3.1	3.5		
Copper	0.8	1.4	2.0	2.5	3.1	3.7	4.3	4.9	5.6		
Brass and bronze	0.8	1.4	2.1	2.7	3.4	4.1	4.8	5.5	6.2		

Fitting Dimensions. An inspection limit of ±1/32 in. shall be allowed on all center-to-contact surface dimensions for sizes up to and including 10 in., and ±1/16 in. on sizes larger than 10 in. An inspection limit of ±1/16 in. shall be allowed on all contact-surface to contact-surface dimensions for sizes up to and including 10 in., and ±1/8 in., on sizes larger than 10 in.

When elbows having longer radii than specified in the standards are required, the use of pipe bends is recommended.

Laterals. The 45 deg laterals of the larger sizes may require additional reinforcement to compensate for the inherent weakness in this shape of casting.

Valve Dimensions. The center to contact surface and contact-surface to contact-surface dimensions of valves for the various pressures shall be in accordance with the proposed American Standard dimensions for ferrous flanged valves.

Reducing Fittings. Reducing fittings shall have the same center to flange edge dimensions as those of straight-size fittings of the largest opening.

Side Outlet Fittings. All side-outlet fittings shall have all openings on the intersecting center lines.

Welding Neck Flanges. The materials, facings, spot facings, etc., conform to the requirements given for other flanges, with the additional provision that the carbon content of the steel shall not exceed 0.35 percent.

Templates for drilling and center to contact surface dimensions of the American Standard 150 lb Steel Flanges and Flanged Fittings are the same as for the American Standard 125 lb Cast Iron Flanged Fitting Standard (Tables 27 to 29).

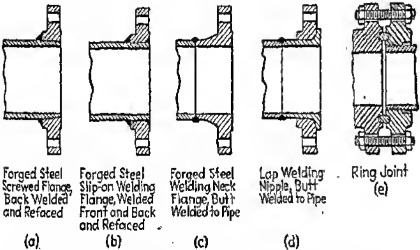


FIG. 10.—Welded Flange Joints and Ring Joint.

Templates for drilling and center to contact surface dimensions of the American Standard 300 lb Steel Flanges and Flanged Fittings are the same as for the American 600 lb Steel Flanged Fitting Standard for sizes 1/2 to 1 1/2 in. (Table 35); and the same as for the American 250 lb Cast Iron Flanged Fitting standard for sizes 2 to 24 in. (Tables 27 and 28).

Flanged Pipe Joints

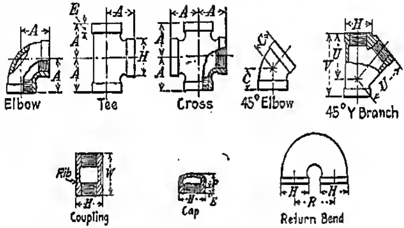
The usual form of pipe joint is that made up by bolting together flanges cast or forged integral with the pipe or fitting, threaded flanges, loose flanges on pipes with lapped ends, and flanges arranged for welding. These forms are illustrated above Table 34 and in Fig. 10. The threaded joint is satisfactory for low and medium steam pressures. The lapped joint is permitted in the same sizes and service ratings as for joints with integral flanges. It is extensively used in high-class work. With the ring joint a higher pressure

malleable-iron screwed fittings of the American Standard are given in Tables 41 and 42.

The normal **amount of thread engagement** necessary to make a tight joint for American Standard Pipe Thread joints as recommended by Crane Co. is as follows:

Size of pipe, in.	1/8	1/4	3/8	1/2	3/4	1	1¼	1½	2
Length of thread, in.	1/4	3/8	3/8	1/2	9/16	11/16	11/16	11/16	3/4
Size of pipe, in.	2½	3	3½	4	5	6	8	10	12
Length of thread, in.	15/16	1	1 1/16	1 1/8	1¼	1 5/16	1 7/16	1⅝	1¾

Table 41. Dimensions of American 150 Lb Standard Malleable-iron Screwed Fittings (Straight Sizes)*



(All dimensions in inches)

Size	A	H	E	C	V	U	W	P	R Close	R Medium	R Open
1/8	0.69	0.693	0.200	......	......	....	0.96				
1/4	0.81	0.844	0.215	0.73	......	....	1.06				
3/8	0.95	1.015	0.230	0.80	1.93	1.43	1.16				
1/2	1.12	1.197	0.249	0.88	2.32	1.71	1.34	0.87	1.000	1.25	1.50
3/4	1.31	1.458	0.273	0.98	2.77	2.05	1.52	0.97	1.250	1.50	2.00
1	1.50	1.771	0.302	1.12	3.28	2.43	1.67	1.16	1.500	1.875	2.50
1¼	1.75	2.153	0.341	1.29	3.94	2.92	1.93	1.28	1.750	2.25	3.00
1½	1.94	2.427	0.368	1.43	4.38	3.28	2.15	1.33	2.188	2.50	3.50
2	2.25	2.963	0.422	1.68	5.17	3.93	2.53	1.45	2.625	3.00	4.00
2½	2.70	3.589	0.478	1.95	6.25	4.73	2.88	1.70	....	....	4.50
3	3.08	4.285	0.548	2.17	7.26	5.55	3.18	1.80	....	....	5.00
3½	3.42	4.843	0.604	2.39	......	....	3.43	1.90			
4	3.79	5.401	0.661	2.61	8.98	6.97	3.69	2.08			
5	4.50	6.583	0.780	3.05	......	....	....	2.32			
6	5.13	7.767	0.900	3.46	......	....	....	2.55			

* The complete standard (A.S.A. B16c-1939) covers also reducing couplings, elbows, tees, crosses and service or street elbows and tees.

The Manufacturers' Standardization Society of Valve and Fitting Industry (M.S.S.) has standardized **malleable-iron and bronze** screw fittings for several pressures.

Brass screwed fittings are made in both the 125 and 250 lb standards. They are used for feed-water pipes where bad water makes steel pipe undesirable. Brass fittings

Table 36. Dimensions of American Standard Companion Flanges*

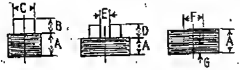
(All dimensions in inches)

Nom pipe size	150 lb			300 lb			400 lb			600 lb			900 lb			1,500 lb		
	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
½	1 3/16	5/8	5/8	1 1/2	7/8	7/8	For sizes below 4 in., use dimensions of the 600 lb flanges			1 3/8	7/8	7/8	For sizes below 3 in., use dimensions of 1,500 lb flanges			1 1/2	1 1/4	1 1/4
¾	1 1/2	5/8	5/8	1 7/8	1	1				1 7/8	1	1				1 3/4	1 3/8	1 3/8
1	1 15/16	11/16	11/16	2 1/8	1 1/16	1 1/16				2 1/8	1 1/16	1 1/16				2 1/16	1 5/8	1 5/8
1¼	2 5/16	13/16	13/16	2 1/2	1 1/16	1 1/16	....	....	....	2 1/2	1 1/8	1 1/8	....	....	....	2 1/2	1 5/8	1 5/8
1½	2 9/16	7/8	7/8	2 3/4	1 3/16	1 3/16	....	....	....	2 3/4	1 1/4	1 1/4	....	....	....	2 3/4	1 3/4	1 3/4
2	3 1/16	1	1	3 5/16	1 5/16	1 5/16	....	....	....	3 5/16	1 7/16	1 7/16	....	....	....	4 1/8	2 1/4	2 1/4
2½	3 9/16	1 1/8	1 1/8	3 15/16	1 1/2	1 1/2	....	....	....	3 15/16	1 5/8	1 5/8	....	....	....	4 7/8	2 1/2	2 1/2
3	4 1/4	1 3/16	1 3/16	4 5/8	1 11/16	1 11/16	....	....	....	4 5/8	1 13/16	1 13/16	5	2 1/8	2 1/8	5 1/4	2 7/8	2 7/8
3½	4 13/16	1 1/4	1 1/4	5 1/4	1 3/4	1 3/4	....	....	....	5 1/4	1 15/16	1 15/16						
4	5 5/16	1 5/16	1 5/16	5 3/4	1 7/8	1 7/8	5 3/4	2	2	6	2 1/8	2 1/8	6 1/4	2 3/4	2 3/4	6 3/8	3 9/16	3 9/16
5	6 7/16	1 7/16	1 7/16	7	2	2	7	2 1/8	2 1/8	7 7/16	2 3/8	2 3/8	7 1/2	3 1/8	3 1/8	7 3/4	4 1/8	4 1/8
6	7 9/16	1 9/16	1 9/16	8 1/8	2 1/16	2 1/16	8 1/8	2 1/4	2 1/4	8 3/4	2 5/8	2 5/8	9 1/4	3 3/8	3 3/8	9	4 11/16	4 11/16
8	9 11/16	1 3/4	1 3/4	10 1/4	2 7/16	2 7/16	10 1/4	2 11/16	2 11/16	10 3/4	3	3	11 3/4	4	4 1/2	11 1/2	5 5/8	5 5/8
10	12	1 15/16	1 15/16	12 5/8	2 5/8	3 3/4	12 5/8	2 7/8	4	13 1/2	3 3/8	4 3/8	14 1/2	4 1/4	5	14 1/2	6 1/4	7
12	14 3/8	2 3/16	2 3/16	14 3/4	2 7/8	4	14 3/4	3 1/8	4 1/4	15 3/4	3 5/8	4 5/8	16 1/2	4 5/8	5 5/8	17 3/4	7 1/8	8 5/8
14 O.D.	15 3/4	2 1/4	3 1/8	16 3/4	3	4 3/8	16 3/4	3 5/16	4 5/8	17	3 11/16	5	17 3/4	5 1/8	6 1/8	19 1/2	.....	9 1/2
16 O.D.	18	2 1/2	3 7/16	19	3 1/4	4 3/4	19	3 11/16	5	19 1/2	4 3/16	5 1/2	20	5 1/4	6 1/2	21 3/4	.....	10 1/4
18 O.D.	19 7/8	2 11/16	3 13/16	21	3 1/2	5 1/8	21	3 7/8	5 3/8	21 1/2	4 5/8	6	22 1/4	6	7 1/2	23 1/2	.....	10 7/8
20 O.D.	22	2 7/8	4 1/16	23 1/8	3 3/4	5 1/2	23 1/8	4	5 3/4	24	5	6 1/4	24 1/2	6 1/2	8 1/4	25 1/4	.....	11 1/2
24 O.D.	26 1/8	3 1/4	4 3/8	27 5/8	4 3/16	6	27 5/8	4 1/2	6 1/4	28 1/4	5 1/2	7 1/4	29 1/2	8	10 1/2	30	.....	13

* Other dimensions are given in Tables 34 and 35. Finished bore on lapped to be such as method of attachment of pipe requires.

is such that when the pipe is threaded to the American Standard dimensions, the end of the pipe will practically touch the shoulder when screwed in. They are especially adapted to plumbing work and vacuum-cleaning pipe installations.

Table 43. Dimensions of American Standards 125, 150, and 250 Lb Pipe Plugs[a]



(All dimensions in inches)

Nominal pipe size	Square-head pattern			Slotted pattern			Countersunk pattern		
	A	*B*	*C*	*A*	*D*	*E*	*A*	*F*	*G*
1/8	0.37	0.24	9/32						
1/4	0.44	0.38	3/8						
3/8	0.48	0.31	7/16						
1/2	0.56	0.38	9/16	....	....	....	0.56	3/8	0.16
3/4	0.63	0.44	5/8	....	....	....	0.63	1/2	0.18
1	0.75	0.50	13/16	....	....	....	0.75	1/2	0.20
1 1/4	0.80	0.56	15/16	....	....	....	0.80	3/4	0.22
1 1/2	0.83	0.62	1 1/8	....	....	....	0.83	3/4	0.24
2	0.88	0.68	1 5/16	....	....	....	0.88	7/8	0.26
2 1/2	1.07	0.74	1 1/2	....	....	....	1.07	1 1/8	0.29
3	1.13	0.80	1 11/16	....	....	....	1.13	1 3/8	0.31
3 1/2	1.18	0.86	1 7/8	....	....	....	1.18	1 1/2	0.34
4	....	....	.....	1.22	1.00	0.88	1.22	2	0.37
5	....	....	.....	1.31	1.00	0.88	1.31	2 1/4	0.46
6	....	....	.....	1.40	1.25	1.25	1.40	2 1/2	0.52
8	....	....	.....	1.57	1.38	1.50			
10	....	....	.....	1.76	1.75	1.75			
12	....	....	.....	1.94	1.75	1.75			

[a] *The material of this standard (A.S.A. B16e2-1936) to be cast iron, malleable iron,* cast steel or forged steel, for use in connection with fittings covered by the American Standard 125 and 250 lb cast-iron screwed fittings (A.S.A. B16d) and the American Standard 150 lb malleable-iron screwed fittings (A.S.A. B16c).

Ammonia valves and fittings must provide a high margin of safety against accidents. Flanged valves and fittings have tongue and groove faces to assure tightness at the joints and against blowing out gaskets. Gaskets are of lead or compressed asbestos sheet. Screwed valves and fittings have long threads and are recessed so that the joints may be soldered. These valves and fittings are made of malleable iron, ferrosteel, or forged steel, depending on the size and style. Valves are all iron, with steel stems, and have special lead disk faces or steel disks. No copper or brass must be used in their construction. Flanged valves are generally interchangeable with flanged fittings. All valves and fittings for ammonia are tested to 300 lb air pressure under water. For dimensions of valves, fittings, and specialties for ammonia, refer to catalogue of Crane Co.

VALVES

Tables 45 to 48 give the dimensions and working pressures of representative valves suitable for steam lines. These types represent but a very small

can be maintained with the same **total bolt stress** than is possible with the flat gasket type of joint. The **welded joint** eliminates possibility of leakage between flange and pipe. It is very successful in lines subject to high temperatures and pressures and heavy expansion strains. The welding-neck flange is available in the various pipe sizes. Specific requirements covering the application of all the types of joints in common use are outlined in the Code for Pressure Piping (A.S.A. B31.1).

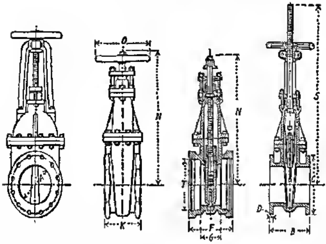
Facing of Flanges. Various styles of finish are used on the faces of flanges, having for their purpose the retention of the gasket used to make a tight joint. Those in general use are as follows: (see Table 34) plain straight face, plain face corrugated or scored, male and female, tongue and groove, and raised face. See pp. 897, 934.

The **plain straight face** has the entire face of the flange faced straight across and may be used with either a full face or ring gasket. The **plain face, serrated or V grooved,** is a plain face upon which concentric grooves have been cut with either a round-nose or V-shaped tool. This finish is sometimes of advantage when the service demands an exceptionally thick, loosely woven fibrous or soft metallic gasket, because the roughening of the faces of the flanges tends to keep the gasket from blowing out. The **male-and-female** facing consists of a recess in one flange and a corresponding raised face or projection on the other, extending from the inside of the pipe nearly to the inside of the bolt holes. In the **tongue-and-groove** facing, the tongue or raised face and the groove or recess are narrow rings located between the bolt holes and the port. The male-and-female and the tongue-and-groove facing have been extensively used, particularly on hydraulic lines, and to a more limited extent on high-pressure steam lines. Both of these types, however, have in common several objectionable features from the standpoint of manufacture, erection, and maintenance. These objections are removed by the use of the **raised-face** facing, which consists of a high narrow raised ring on each of the mating flanges, whose inside diameter is the same as that of the pipe or port. It is particularly recommended for high-pressure steam and hydraulic lines. Gaskets used in this type of joint are either soft fibrous material or soft metal and extend from the inside of the pipe to the bolt holes, and only the small portion in contact with the narrow raised face is subjected to the compressive effect of the bolts. The following advantages are claimed in the use of the raised-face type of facing: all mating of flanges eliminated; any valve or fitting may be removed from the line without springing the line apart; the gasket is automatically centered by its outer edge coming in contact with the bolts; the outside edges of the flanges are far enough apart to make it possible to determine whether the joint has been properly made.

The Bursting Pressure of Flanged Fittings. Results of hydraulic-pressure tests on flanged tees and ells made by Crane Co. (*Valve World*, Sept., 1913) are approximately expressed by the formula: bursting point, lb per sq in. $= tS/D$, where t = thickness of metal in the body of the fitting in in.; S = 65 percent of the tensile strength (T.S.) of the metal for fittings up to 12 in. diam (60 percent for larger diameters), and D = inside diam of fitting, in. A factor of safety of 4 to 8 should be used, depending on the size of the fitting. The formula is based on average of tests of 52 fittings made from cast iron (T.S. = 22,000) and "ferrosteel" (T.S. = 33,000).

Unions may be classified as **screw unions** and **flange unions.** Typical designs are shown in Fig. 11, where at the left is represented a female screw

Table 45. Dimensions of Standard Gate Valves
(Crane Co.)



(Iron body with brass trimmings: sizes 16 in. and smaller for steam working pressures up to 125 lb; for steam lines larger than 18 in., 150 lb cast-steel gate valves are recommended. All dimensions in inches. Letters refer to dimension sketches above. For drilling templates, see Table 27.)

Size, *A*	*B*	*C*	*D*	*K*	*N*	*O*	*S*	*F*	*T*	*G*	No. of turns to open
2	7	6	5/8	5 7/16	11 1/4	8	15	8 1/2	3 1/4	3	6
2 1/2	7 1/2	7	11/16	5 7/8	12 3/4	8	16 1/2	....	....	...	7 1/4
3	8	7 1/2	3/4	6 1/8	14 3/4	9	19 1/2	9	4 3/4	3 1/2	9 1/2
3 1/2	8 1/2	8 1/2	13/16	6 1/2	15 1/4	9	21 1/2	....	....	...	11
4	9	9	15/16	7 7/8	16 1/4	10	24 3/4	10 1/4	5 5/8	4 1/4	8 3/4
5	10	10	15/16	7 3/8	19	12	29 1/2	10 1/4	6 3/4	4 1/4	11
6	10 1/2	11	1	7 3/4	21 1/4	12	32 1/2	10 3/4	7 7/8	4 3/4	13 1/4
8	11 1/2	13 1/2	1 1/8	8 3/4	26	14	41	12	10	5	16 1/2
10	13	16	1 3/16	9 7/8	31	16	50	12 3/4	12 1/4	5 3/4	21
12	14	19	1 1/4	11 5/8	36	18	57 1/4	13 1/2	14 5/8	6 1/2	25

service in locations where the projecting thread is not likely to be injured; inside screws should be used for underground work (as in water or gas pipes) or wherever the projecting spindle is liable to injury. **Brass bodies** are softer and will stand less abuse than iron or steel bodies. They should not ordinarily be used for sizes larger than 2 in. to 3 in. **Iron bodies** with brass mountings are recommended for larger sizes for water lines, steam and hot-water heating systems, and for saturated steam up to 250 lb pressure. For the higher steam pressures and temperatures, cast-steel valves are generally used. All-iron valves are recommended for oil or gas or for fluids that corrode brass.

Cocks. The ordinary plug cock operated by a handle or wrench is a form of valve in comparatively small sizes suitable for ordinary service only. The Code for Pressure Piping requires that where cocks are used for high-temperature service they shall be so designed as to prevent galling, either by making the plugs of different material from the body of the cock or by treating the plugs to ensure different physical properties. By means of special design features that eliminate the tendency to leak and stick, the plug-cock type of valve has become available in large sizes and for severe service conditions.

Figure 12 is a **corrugated copper expansion joint** for low pressures. Figure 13 shows a **globe-type joint** and Fig. 14 a **non-collapsible joint**, both of copper, and for low-pressure work. Face-to-face dimensions of some of these joints, compiled from manufacturers' catalogues, are given in Table 39.

Table 39. Face-to-face Dimensions of Copper Expansion Joints

Size, in.	4	5	6	8	10	12	14	16	18	20	24	30	36	42	48
L, in.	8	9	9	10	11	11	12	12	13	13	14	16	17	19	20
G, in.	8	9	9	10	11	11	12	12	13	13	14	16	17	19	20
S, in.	5½	5½	6	6	6	6	6	6	6½	6½	7	8½	8½	9	9

Cast-iron standard and double expansion joints are shown in Figs. 15 and 16, and the dimensions of various sizes are given in Table 40. Cast-iron

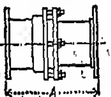


FIG. 15.

FIG. 16.

Cast-iron Expansion Joints for Steam Lines.

double expansion joints are of the same general proportions as standard expansion joints and are more particularly designed for underground work.

Table 40. Dimensions of Expansion Joints for 125 and 250 Lb Steam Working Pressure

(Walworth Company)

(All dimensions in inches. Letters refer to Figs. 15 and 16)

Size, in.	2	2½	3	3½	4	5	6
A, 125 lb	14½	14⅝	15½	16⅝	18³⁄₁₆	20¼	20¹³⁄₁₆
A, 250 lb	15¼	16	17⅝	18¹¹⁄₁₆	19½	22⅝	23¾
B, 125 lb	28	28½	29½	30¾	31¾	32½	34

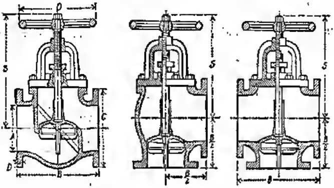
Size, in.	8	10	12	14	16	18
A, 125 lb	21¹⁵⁄₁₆	23³⁄₁₆	24⁷⁄₁₆	25⁵⁄₁₆	27	
A, 250 lb	25½	27⅝	29¾	31	33	34⅞
B, 125 lb	37¾	39¼	43	45	48½	

The **rubber expansion joint** has become an established part of the pipeline equipment of the modern power plant. Its special field of application is on low-pressure and vacuum lines in condenser applications, etc., and is recommended for water pressure up to 25 lb, where the maximum temperature does not exceed 180 F. Sizes may be obtained up to 72 in. (see manufacturers' catalogues).

Screwed Fittings

Screwed fittings are made in **cast iron, malleable iron**, cast steel, forged steel, and brass. Plain standard **fittings** are generally used for low-pressure gas and water, as in house plumbing and railing work, while the beaded fitting is the standard steam, air, gas, or **oil fitting**. Screwed fittings are supplied with a large factor of safety. The **questions of strength** involve much more than the pressure from within the pipe which induces a comparatively low stress in the material. The greater **strains** come from expansion, contraction, weight of piping, settling, water hammer, etc. Dimensions of cast-iron and

Table 47. Dimensions of Standard Globe, Angle, and Cross Valves
(Crane Co.)



(Iron-body valves with yokes and brass trimmings, for steam working pressures up to 125 lb. All dimensions in inches. Letters refer to sketches above. For drilling templates, see Table 27.)

Size, A	B	C	D	S*	O	Size, A	B	C	D	S*	O
2	8	6	5/8	10¾	8	5	13	10	15/16	17¼	10
2½	8½	7	11/16	11¾	8	6	14	11	1	19	12
3	9½	7½	¾	12¾	9	8	19½	13½	1⅛	23¾	16
3½	10½	8½	13/16	13	9	18	24½	16	1 3/16	28	18
4	11½	9	15/16	15¼	10	12	27½	19	1¼	34	20

* Approximate values.

with flange radiating area, gives lower first cost of piping under certain conditions, permits quicker delivery of pipe with welded outlets over that obtainable with special fittings, and reduces the cost of pipe covering, particularly in low-temperature work where cork insulation is used. The present tendency toward higher steam pressures and temperatures is leading to the elimination of as many gaskets as possible and to the sealing of steel flanges by welding.

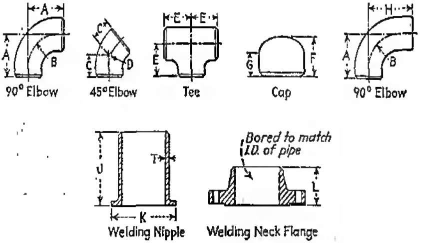
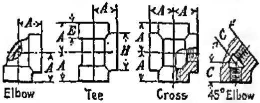


FIG. 17.—Fittings for Welding.

may be had in iron pipe sizes. Forged-steel screwed fittings are made for cold water or oil-working pressures up to 6,000 lb per sq in. hydrostatic. The A.S.A. has approved a standard (A.S.A. *A40.2-1936) for brass fittings for flared copper tubes for maximum* cold-water service pressure of 175 lb per sq in.

Table 42. Dimensions of American 125 and 150 Lb Standard Cast-iron Screwed Fittings (Straight Sizes)



(All dimensions in inches)

	125 lb				250 lb			
Size	*A*	*H*	*E*	*C*	*A*	*H*	*E*	*C*
¼	0.81	0.93	0.38	0.73	0.94	1.17	0.49	0.81
⅜	0.95	1.12	0.44	0.80	1.06	1.36	0.55	0.88
½	1.12	1.34	0.50	0.88	1.25	1.59	0.60	1.00
¾	1.31	1.63	0.56	0.98	1.44	1.88	0.68	1.13
1	1.50	1.95	0.62	1.12	1.63	2.24	0.76	1.31
1¼	1.75	2.39	0.69	1.29	1.94	2.73	0.88	1.50
1½	1.94	2.68	0.75	1.43	2.13	3.07	0.97	1.69
2	2.25	3.28	0.84	1.68	2.50	3.74	1.12	2.00
2½	2.70	3.86	0.94	1.95	2.94	4.60	1.30	2.25
3	3.08	4.62	1.00	2.17	3.38	5.36	1.40	2.50
3½	3.42	5.20	1.06	2.39	3.75	5.98	1.49	2.63
4	3.79	5.79	1.12	2.61	4.13	6.61	1.57	2.81
5	4.50	7.05	1.18	3.05	4.88	7.92	1.74	3.19
6	5.13	8.28	1.28	3.46	5.63	9.24	1.91	3.50
8	6.56	10.63	1.47	4.28	7.00	11.73	2.24	4.31
10	8.08	13.12	1.68	5.16	8.63	14.37	2.58	5.19
12	9.50	15.47	1.88	5.97	10.00	16.84	2.91	6.00
14 O.D.	10.40	16.94	2.00	....	11.00	18.40	3.10	
16 O.D.	11.82	19.30	2.20	....	12.50	20.88	3.45	

The 125 lb standard (A.S.A. B16d-1927) covers also reducing elbows and tees. The 250 lb standard (A.S.A. B16d-1927) covers only the straight sizes.

Railing Fittings. Fittings of special construction and of lighter weight than standard steam, gas, and water pipe fittings are widely used for hand railings around areaways, on stairs, for office enclosures with gates, and for permanent ladders. Railing fittings are made in various styles, generally globe-shaped in body, with ends reduced to take thread and recessed to cover all threads. They are furnished in malleable iron, black and galvanized, and in brass.

Special railing-fitting joints are available, such as the slip-and-screwed joint, where the post connection is screwed and the rim of the fitting is so made that the rail will slip into the fitting and allow for an angular variation of several degrees, being fastened by pins which are riveted over and filed smooth. The flush-joint stair-rail fitting is another special style of fitting which provides a hand rail with even surfaces at the joints.

Drainage fittings, as shown in the figures accompanying Table 44, have no pockets for the lodgment of solids, and the length of the thread chamber

Typical welded connections are shown in Fig. 10. In the back-welded screwed flange at (a), the threads retain the function of holding the flange securely in the pipe, hence, there is no shearing action on the weld. The fillet weld back of the flange forms a seal, assuring that no leakage will occur through the threads. The slip-on welding flange joint at (b) is limited by the Code to 300 lb pressure and a temperature of 700 F. The welding neck flange at (c) is machined with a beveled end and bored to match the thickness of the pipe. The welding nipple at (d) can be applied to piping cut to any desired length. Both the nipple and the pipe are machine beveled.

Table 49. Dimensions of Seamless-steel Welding Fittings
(Crane Co.)
(Letters refer to Fig. 17. All dimensions are in inches.)

Size of pipe	A	B	C	D	E	F	G	H	J	K	L	
											150 lb	300 lb
3/4	1 1/8	1										
1	1 1/2	1 5/16	1 1/8	1 5/16	1 3/8	.....	...	....	6	2	2 3/16	2 7/16
1 1/4	1 7/8	1 5/8	1 3/8	1 5/8	2	.....	...	....	6	2 1/2	2 1/4	2 9/16
1 1/2	2 1/4	1 7/8	1 1/2	1 7/8	2 1/4	.....	...	....	6	2 7/8	2 7/16	2 11/16
2	3	2 3/8	2	2 3/8	2 3/4	.....	...	4 1/4	6	3 3/8	2 1/2	2 3/4
2 1/2	3 3/4	2 7/8	2 1/4	2 7/8	3	.....	...	5	6	4 1/2	2 3/4	3
3	4 1/2	3 1/2	2 1/2	3 1/2	...	2 11/16	3/4	5 3/4	6	5	2 3/4	3 1/2
3 1/2	5 1/4	4 1/4	2 3/4	4	...	2 3/16	7/8	6 3/4	6	5 1/2	2 13/16	3 3/16
4	6	4 7/8	3	4 1/2	...	2 9/16	1	7 1/2	6	6 3/16	3	3 3/8
5	7 1/4	6 3/8	3 1/2	5 9/16	...	3 3/16	1 1/4	9	8	7 5/16	3 1/8	3 5/8
6	9	7 5/8	4	6 5/8	...	3 9/16	1 1/2	10 3/4	8	8 1/2	3 1/2	3 7/8
8	12	10 5/8	5	8 5/8	...	4 9/16	2	13 3/4	8	10 5/8	4	4 3/8
10	15	13 1/2	6	10 3/4	...	5 5/8	2 1/2	17	10	12 3/4	4	4 5/8
12	18	16 1/2	7	12 3/4	...	6 1/4	2 1/2	20 1/2	10	15	4 1/2	5 1/8
14 O.D.	16	14	7 1/2	14	...	6 3/4	2 1/2	....	12	16 1/4	5	5 5/8
16 O.D.	18	16	8 1/2	16	...	7 1/2	2 1/2	....	12	18 1/2	5	5 3/4
18 O.D.	20	18	9 1/2	18	...	7 3/4	2 1/2	....	12	21	5 1/2	6 1/4
20 O.D.	....	....	....	.....	...	8	2 1/2	....	12	23	5 11/16	6 3/8
24 O.D.	....	....	....	.....	...	8 3/4	2 1/2	....	12	27 1/4	6	6 5/8

"Standard" and "extra-strong" fittings have the same outside dimensions. Welding nipples are furnished in the regular pipe sizes and wall thicknesses. The facing, drilling, thickness, etc., of welding neck flanges conform to the American Standards. For other welding fittings and other sizes, working pressure ratings at various temperatures, etc., refer to manufacturers' catalogues.

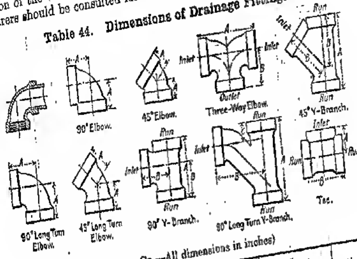
Pipe Supports

The code for Pressure Piping (A.S.A. B31.1-1935) includes many types of supports and gives directions for their application. A proper pipe support must have a strong rigid base properly supported, and an adjustable roll construction which will maintain the alignment in any direction. It is important to avoid friction caused by the movement of the pipe in the support and to have all parts of sufficient strength to maintain alignment at all times. Wire hangers, band iron hangers, wooden hangers, hangers made from small pipe and hangers having one vertical pipe support do not maintain alignment.

The direction of expansion in a pipe run can be predetermined by anchoring one end, both ends, or the middle. **Anchors** must be firmly fastened to a rigid and heavy part of the power plant structure, and must also be securely fastened to the pipe, otherwise, the equipment for absorbing expansion is useless, and severe stresses may be thrown on parts of the piping system. Some methods of support are shown in Figs. 18 to 20.

portion of the valves that are commercially available. Catalogues of manufacturers should be consulted for more detailed information.

Table 44. Dimensions of Drainage Fittings*



(Crane Co.—All dimensions in inches)

Size, in.†	90 deg elbows	45 deg elbows	90 deg long-turn elbows	45 deg long-turn elbows	Three-way elbows		Tees		90 deg Y branches		90 deg long-turn Y branches			45 deg Y branches	
	A	A	A	A	A	B	A	B	A	B	A	B	C	A	B
1¼	1¾	1⅜	2¼	1⅜	4½	2¼	1¾	3½	3¾	2¼	4¾	3⅝	1 3/16	5	3¼
1½	2 3/16	1 7/16	2½	1⅞	5¼	2⅝	2 3/16	4⅜	4¼	2⅝	5⅞	4⅜	1¾	5½	3⅝
2	2⅜	1¾	3 1/16	2¼	6¼	3⅛	2 5/16	4⅝	5 5/16	3 1/16	7 1/16	5 7/16	1⅝	6 7/16	4 5/16
2½	2 11/16	2 1/16	3 11/16	2⅝	7⅜	3 13/16	2 13/16	5⅝	6 5/16	3 15/16	8¼	6¼	2	7⅝	5⅝
3	3 3/16	2⅜	4¼	2⅞	8⅝	4 5/16	3 3/16	6⅜	7¼	4¼	9 13/16	7½	2 5/16	9	6 3/16
4	3 13/16	2¾	5 3/16	3 7/16	10⅜	5 3/16	4	8	8¾	5 3/16	13¾	9⅝	2⅞	10⅞	7 11/16
5	4½	3 3/16	6⅛	4⅜	12¼	6⅝	4⅝	9¼	10 5/16	6⅛	15¾	12¼	3¼	12 13/16	9 9/16
6	5 3/16	3½	7⅛	4⅞	14¼	7⅛	5 5/16	11⅞	11 15/16	7⅛	18 11/16	14 5/16	4⅜	14¾	10¾
8	6½	4 3/16	9	6	....	....	6½	13	15 1/16	9	24 9/16	19 9/16	5¼	18 13/16	13 9/16

* Other fittings which are available are as follows: 5⅝ deg, 11¼ deg, 22½ deg and 60 deg elbows; basin crosses; double 90 deg Y branches; double 90 deg long-turn Y branches; 45 deg double Y branches; S traps; half S traps; running traps; offsets.

† Other commercial sizes are 1, 10, 12, and 14 in.

According to the Code for Pressure Piping, valves for steam lines with pressures above 250 lb per sq in. but not over 1,500 lb and temperatures above 450 but not above 750 F should be of cast steel or forged steel. For pressures not over 250 lb and temperatures not over 450 F, they may be of cast iron, malleable iron, steel, or non-ferrous metal.

Several standards have been compiled and issued covering center-to-face dimensions of ferrous valves. The Manufacturers Standardization Society of the Valve and Fittings Industry issued SP-32 in 1937. The A.P.I. Standard No. 600A in 193[illegible] and No. 5-G-1 appeared in 1936 and the A.P.I. Standard No. 600A in 193[illegible] Dimension B in Tables 45 to 48 is in close agreement with the America[illegible] Standard "Face to Face Dimensions of Ferrous Flanged and Welding E[illegible] Valves" B16.10-1939, issued in 1939.

Selection of Valves. Gate valves reduce the frictional loss to a minimum [illegible] allow perfect drainage of the system. Outside screw valves are preferable for ind[illegible]

The distance between supports will vary with the kind of piping and number of valves and fittings. Supports should be provided near changes in direction, branch lines, and particularly near valves. The weight of piping should not be carried through valve bodies if they are to be kept tight.

In establishing the location of pipe supports, the designer should be guided by two requirements. (1) The horizontal span must not be so long that the sag in the pipe will impose an excessive stress in the pipe wall. (2) The pipe line must be pitched downward so that the outlet of each span is lower than the maximum sag in the span.

Tests conducted by Crane Co. to determine the deflection of horizontal standard pipe lines filled with water, in pipe sizes ¾ to 4 in., have indicated the following results: For pipes larger than 2 in. and with distance between supports greater than 10 ft, the resultant deflection is less than that determined by use of the formula for a uniformly loaded pipe fixed at both ends. For pipe sizes 2 in. and smaller, the test deflection was in excess of that determined by the formula for pipe having fixed ends and approached, for the shorter spans, the deflection as determined by the use of the formula for pipe having unrestrained ends.

Pipe Covering

(See p. 384 for heat-transmission data, and p. 715 for properties of insulating materials.)

The value of a steam-pipe covering is measured by the percentage reduction in condensation over that which occurs in bare pipe. This ranges from 65 to 85 percent, and even higher. The **standard covering** consists of 85 percent of carbonate of magnesia mixed with 15 percent of asbestos. The thickness of insulation necessary for most economical results under average conditions is shown in the following table.

Steam temperature, F		212–267	267–338	338–388	388–500	500–600
Thickness of insulation, in.	Pipes 4 in. and more	1	1½	2	2½	3
	Pipes 2 in. to 4 in.	1	1½	1½	2	2½
	Pipes ½ in. to 1½ in.	1	1	1	1½	2

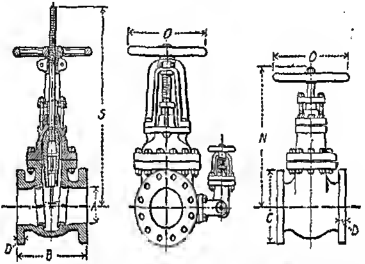
The covering is usually applied in molded sections (about 3 ft long) the seams of which are staggered and filled with magnesia plaster. Where the thickness of insulation is greater than 1½ in., it is desirable to apply it in two or more layers with joints broken or staggered. Fittings, valves, and flanges may be insulated with blocks and plastic material (plastic only on sizes smaller than 4 in.) to the same thickness as that of the adjacent pipe insulation. The sections are bound to the pipe by means of galvanized-iron wire or netting, over which is wrapped a coat of rosin-sized paper, followed by 8 oz canvas securely sewed on. **Asbestos air-cell covering** consists of several layers of corrugated asbestos sheets, making a laminated covering with air spaces between the corrugations.

A **wood casing** is sometimes used for underground steam or hot-water pipes, for exposed steam lines in mine shafts, etc., and for cold-storage and brine pipes. This covering is lined with tin for high-pressure steam lines, and is waterproofed for underground service.

L. B. McMillan (*Trans. A.S.M.E.*, 1915) finds, for coverings with white canvas surfaces, that the total heat flow in Btu per square foot of pipe surface per hour through a covering $= q_1 = Ut_d$, where U is taken from Table 52, and t_d = temperature difference, F, between pipe surface and air. The total Btu loss per hour per square foot of outside surface of covering $= q_2 = q_1r_1/(r_1 + s)$, where r_1 = outside radius of pipe, in., and s = thickness of covering, in. The temperature difference between the outer covering

Sizes are listed as high as 30 in. and are gear-operated in the larger sizes. For further details, refer to catalogue of the Merco-Nordstrom Valve Co., Pittsburgh, Pa.

Table 46. Dimensions of 175 Lb and 250 Lb Gate Valves
(Crane Co.)



(175 lb valves: ferrosteel body with brass trimmings, for steam working pressures, sizes 16 in. and smaller, up to 175 lb; for steam lines 18 in. and larger, 150 or 300 lb cast-steel gate valves are recommended. 250 lb valves: ferrosteel body with brass trimmings and hard metal seats; for steam working pressure up to 250 lb for steam lines 18 in. and larger, 300 lb cast-steel gate valves are recommended. All dimensions in inches. Letters refer to dimension sketches above. *For drilling templates, see Table 27.)*

Size, A	175 lb								250 lb							
	B^a	B^*	C	D	N	S	T†	W‡	B^a	B^*	C	D	N	S	O	W‡
1¼	....	....	....	....	....	....	....	......	6½	5½	5¼	¾	9¼	11½	7	12
1½	....	....	....	....	....	....	....	......	7½	6¼	6¼	13/16	9¾	13¼	8	11
2	7½	5½	6½	⅞	11½	15¼	....	6⅝	8½	7	6½	⅞	11	14¼	8	14
2½	8	6	7½	1	12¾	16¾	....	7⅞	9½	8	7½	1	13¼	17½	9	15
3	9½	7¼	8¼	1⅛	14¼	19¼	....	9¾	11⅛	9	8¼	1⅛	14¾	20½	10	14
3½	10	7½	9	1 3/16	15	21½	....	11¼	11⅞	10	9	1 3/16	15½	22	10	16
4	10½	7¾	10	1¼	17	24½	....	9	12	11	10	1¼	17½	24¾	12	28
5	11½	8½	11	1⅜	19¾	29¼	....	11	15	13½	11	1⅜	20¼	29¾	14	23
6	12	8¾	12½	1 7/16	21½	33	1¼	25	15⅞	15⅞	12½	1 7/16	23	34¼	16	28
8	13½	10	15	1⅝	25½	41¾	1½	34	16½	16½	15	1⅝	28¾	42¾	20	34
10	15	11½	17½	1⅞	30½	50¾	1½	42	18	18	17½	1⅞	33¾	52¾	22	39
12	16	12½	20½	2	34¼	57¾	2	50	19¾	....	20½	2	37¼	60	24	46

ª Includes $\frac{1}{16}$ in raised face (not shown).
* End to end, screwed, in.
† T = size of by-pass, in.
‡ W = number of turns to open.

Welding in Power-plant Piping

Welding is used as an economical or expedient method in making alterations and extensions to existing piping systems. It results in compactness, reduces the number of joints, minimizes radiation losses by doing away

Class	Color
F—Fire-protection equipment	Red
D—Dangerous materials	Yellow (or orange)
S—Safe materials	Green (or the achromatic colors, white, black, gray, or aluminum)
P—Protective materials	Bright blue
V—Extra valuable materials	Deep purple

Method of Identification. At conspicuous places throughout a piping system, color bands shall be painted on the pipes to designate to which one of the five main classes it belongs. If desired, the entire length of the piping system may be painted the main classification color.

Further, the actual contents of a piping system may be indicated by, preferably, a stenciled legend of standard size giving the name of the contents in full or abbreviated form. These legends shall be placed on the color bands. The identification scheme may be extended by the use of colored stripes placed at the edges of the colored bands.

The bands, legends, and stripes shall be placed at intervals throughout the piping system, preferably adjacent to valves and fittings to insure ready recognition during operation, repairs, and at times of emergency.

A recommended classification, under this color scheme of materials carried in pipes, includes as dangerous, combustible gases and oils, hot water and steam above atmospheric pressure; as safe, compressed air, cold water, and steam under vacuum.

Pressure Hose

Hose with durable rubber lining may be obtained to withstand any needed pressure. If the rubber compound is properly made, the life of a hose will be 7 to 10 years, while a cheaper hose, lined with inferior material, will probably not last more than 3 or 4 years. For fire hose, see p. 273.

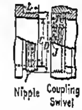


Fig. 21.—Typical Form of Standard Coupling.

American National Fire-hose Coupling Screw Thread. (Approved by the A.S.A., May, 1925, and indorsed and approved by the important engineering societies, and manufacturers and users of fire-hose couplings and hydrant outlets.) This standard is intended to cover the threaded part of fire-hose couplings, hydrant outlets, stand-pipe connections, and all other special fittings on fire lines, where fittings of the nominal diameters given in the following table are used. It also includes the limiting dimensions of the field inspection gages. The American National form of thread must be used.

Table 52. Dimensions of Standard Fire-hose Couplings
(All dimensions in inches. Letters refer to Fig. 23)

Inside diam C	Diam of thread D	No. of threads per in.	L	I	H	J	T
2½	3 1/16	7½	1	¼	15/16	3/16	1 1/16
3	3⅝	6	1⅛	5/16	1 1/16	¼	1 3/16
3½	4¼	6	1⅛	5/16	1 1/16	¼	1 3/16
4½	5¾	4	1¼	7/16	1 3/16	⅜	1 5/16

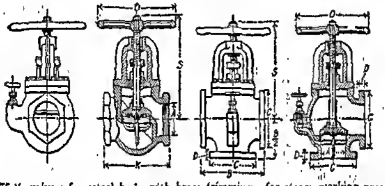
American Standard Hose Coupling Screw Threads. (A.S.A. B33.1-1935) (approved by A.S.A., July, 1935). These standards apply to the threaded parts of hose couplings, valves, nozzles, and all other fittings used in direct connection with hose intended for fire protection or for domestic,

The use of welded branches, line welds, and swedges have made possible almost the entire elimination of fittings from steam headers, with a consequent decrease in cost. The line weld permits pipe to be joined up to the limit of shipping dimensions, which for single cars is two full lengths.

Cut pipe, bends, flanges, fittings, and valves with beveled edges can be obtained in the various sizes, suitable for welding assemblies. On welded headers, center-to-face dimensions of flanged nozzles always correspond to center to face of flanged tees of like series. Dimensions of welding neck flanges and welding fittings may be added without clearance at welded joints to obtain over-all dimensions. Typical welding fittings are shown in Fig. 17, and their dimensions are given in Table 49. See also A.S.A. B16.9.

Table 48. Dimensions of Medium and Extra-heavy Globe, Angle, and Cross Valves

(Crane Co.)



(175 lb valves: ferrosteel body with brass trimmings, for steam working pressures up to 175 lb. 250 lb valves: ferrosteel body with brass trimmings, for steam working pressures up to 250 lb. All dimensions in inches. Letters refer to dimension sketches above. For drilling templates, see Table 27.)

Size, A	175 lb						250 lb					
	B[a]	C	D	K	S[b] open	O	B	C	D	K	S[b] open	O
2	9	6½	⅞	7¾	11¾	8	10½	6½	⅞	9½	13	9
2½	10	7½	1	8	12⅝	9	11½	7½	1	10¾	13¾	10
3	11	8¼	1⅛	8¼	14¼	10	12½	8¼	1⅛	11¾	16	10
3½	12	9	1³⁄₁₆	9½	15⅝	10	13¼	9	1³⁄₁₆	12¼	17	10
4	13	10	1¼	10½	16⅝	10						
5	14½	11	1⅜	12¼	18¼	12						
6	16	12½	1⁷⁄₁₆	14	20¼	14						
8	20	15	1⅝	18½	24⅛	16						
10	22½	17½	1⅞	....	28½	20						
12	25½	20½	2	....	31	20						

[a] Includes ¹⁄₁₆ in. raised face (not shown).
[b] Approximate values.

The Code for Pressure Piping (A.S.A. B31.1-1935) outlines specific requirements concerning the welding of pipe joints by the fusion process and relates to such joints as are formed by pipe end to end, pipe to flanges, fittings and valves, and pipe, valve, or fitting to other equipment. It covers materials, types of welds, welding procedure, stress relief, qualification of welders, the testing of welded joints, etc.

WIRE ROPE, NAILS, ETC.*

Wire Rope

Wire ropes are built up of **strands** made of wires twisted together, the numbers of wires commonly used being 4, 7, 12, 19, and 37. Ordinarily the wires are twisted into strands in the opposite direction to the twist of the strands into rope. When wires and strands are twisted in the same direction, the rope is known as **lang lay rope.** **Standard wire rope** is made of six wire strands and a hemp core. Wire strands are twisted around the core, either to the right or left, and the resulting rope is thereby designated as right lay or left lay. The twist may be long or short, the shorter twist forming the more flexible rope. The **core** of a wire rope is, as a rule, hemp saturated with a lubricant. It provides little additional strength, but acts as a cushion to preserve the shape of the rope and helps to lubricate the wires. A wire strand or wire rope core adds 7 to 10 percent to the strength of the rope, but will wear from the friction between it and the outer strands as rapidly as the outside of the rope. This does not apply to stationary ropes.

For great flexibility, the strands of a wire rope sometimes consist of wire ropes, which in turn are made of strands composed of wires, as in tiller rope. Running ropes and one construction of ship's hawsers are made with strands composed of 12 or 18 wires each, twisted about a hemp center. Ropes so made are very pliable and present good resistance to outside friction. Individual strands of wires are employed as smokestack guys, span wires for trolley roads, and wherever only moderate flexibility is needed.

Strength and Working Loads. The test strength of wire ropes seldom exceeds 90 percent of the aggregate strength of all of the wires, the average being about 82.5 percent.

The working load should never exceed 1/5 of the breaking strength and for many conditions should not be greater than 1/8 to 1/6. The proper factor of safety for a wire rope demands consideration of all loads; acceleration; deceleration; rope speed; rope attachments; the number, size, and arrangements of sheaves and drums; conditions producing corrosion and abrasion; length of rope; etc. The desirable factor of safety for given conditions can best be obtained by consulting the manufacturers of the wire rope.

Sizes of Drums or Sheaves. (See also page 833.) Tread diameters for 6 × 7 (6 × 19) [6 × 37] {8 × 19} rope should be approximately 72 (45) [27] {31} times the rope diameters, for average conditions. For economical service, these should often be increased. For example, on larger hoisting installations with the 6 × 7 (6 × 19) rope, the diameters may be 96 (90) times the rope diameter. In certain cases, the tread diameters may be less but should not be below 42 (30) [18] {21} times the rope diameters. Larger tread diameters give increased rope life and more economical service.

The size and condition of the sheave grooves is most important in determining rope life. The clearances for new or remachined grooves and the minimum clearances before sheave replacement or remachining grooves, should be as follows:

Normal rope diam, in.	1/4–5/16	3/8–1 1/4	1 3/8–1 5/8	1 3/4–2 1/4
New clearances, in.	1/32	1/16	1/8	1/8
Min clearances, in.	3/64	1/32	1/32	3/64

If a wire rope is operated over grooves that are too small, the rope will be abraded rapidly; in addition, a tight or corrugated sheave groove may

* Staff contribution.

Table 50 gives dimensions that may be used for cast-iron rolls and stands for pipe supports. Figure 19 shows a heavy wall bracket adapted for any size of pipe 5 to 20 in. diam. Dimensions: *A*, 15 to 19 in.; *B*, 8 in.; *C*, 18¾ in.; *D*, 27 in. The universal adjustable bracket shown in Fig. 20 is made for pipe 5 to 14 in. diam. The inside dimension *A* ranges in six sizes from 6 in. to 21 in.; the range of adjustment outward in each size is 3 in., and any adjustment *A* between 6 and 24 in. can be obtained from some one of the sizes made.

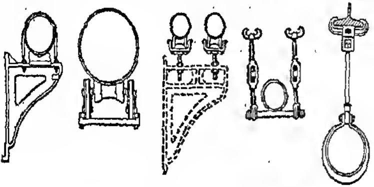


Fig. 18.—Methods of Supporting Pipes.

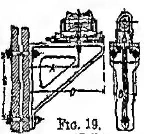


Fig. 19.

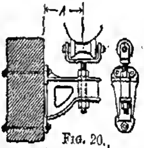
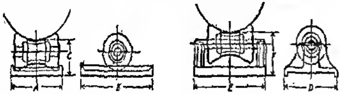


Fig. 20.

Wall Brackets for Supporting Pipes.

These rolls may also be adjustably mounted in various types of stands and hangers. Spring supports are recommended when vibration is present or when automatic adjustment for slight movement is necessary, in order to dampen the vibration and avoid line strains.

Table 50. Dimensions of Cast-iron Rolls with Plates and Chairs



(Letters refer to figures above. All dimensions in inches)

Size	A	B	C	D	E	F	Size	A	B	C	D	E	F
4	4½	8	2⅝	7⅛	6½	3¼	12	8½	10	4½	7⅛	11½	5$\frac{5}{16}$
5	4½	8	2⅝	7⅛	6½	3¾	14 O.D.	8½	10	4½	7⅛	11½	5$\frac{5}{16}$
6	4½	8	2⅝	7⅛	6½	3¾	16 O.D.	9½	11	5	7⅛	13	5¾
8	6½	9	3¾	7⅛	9¼	4⅜	18 O.D.	9½	11	5	7⅛	13	5¾
10	6½	9	3¾	7⅛	9¼	4⅜	20 O.D.	9½	11	5	7⅛	13	5¾
							24 O.D.	10½	12	5¼	7⅛	14½	6⅛

the abrasion which accompanies rapid hoisting. The wire in cast-steel rope has nearly twice the strength of that in Swedish iron rope and also has a much higher elastic limit. **Extra strong cast-steel rope** occupies an intermediate place between cast-steel and plow-steel rope. It is used in place of cast steel when it is desirable to increase the factor of safety for a given diameter. It may be used to advantage for general hoisting. **Plow-steel rope** is made of wire of great strength and toughness, capable of resisting severe abrasion. It can be used in place of cast-steel rope where it is desirable to reduce the dead weight of the rope itself, or where, by reason of increased loads, it is necessary to use a much stronger rope without increasing its diameter. Plow-steel rope is recommended especially for logging lines, scraper, dredge, and wrecking ropes, heavy cranes, ballast unloader ropes, and for all rough uses requiring maximum strength and toughness. **Improved plow-steel rope** is a special rope for extreme conditions. It has a maximum strength, toughness, and uniformity.

Standard hoisting rope (Fig. 1) is made of 6 strands, each of 19 wires, the strands being twisted around a hemp center.

Table 2. Extra Pliable Hoisting Rope
(Six 37-wire strands and hemp center)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons			
		"Blue Center" steel	Plow steel	Extra strong cast steel	Cast steel
3½	19.00	451.0	392.0	357.0	323.0
3¼	16.37	392.0	341.0	311.0	281.0
3	13.95	337.0	293.0	267.0	241.0
2¾	11.72	285.0	248.0	226.0	204.0
2½	9.69	237.0	206.0	188.0	170.0
2¼	7.85	194.0	168.0	153.0	139.0
2⅛	7.00	174.0	151.0	138.0	125.0
2	6.20	155.0	135.0	123.0	111.0
1⅞	5.45	137.0	119.0	108.5	98.0
1¾	4.75	119.5	104.0	95.0	86.0
1⅝	4.09	103.3	89.8	82.0	74.3
1½	3.49	88.2	76.7	70.0	63.5
1⅜	2.93	74.3	64.6	59.0	53.5
1¼	2.42	61.5	53.5	48.9	44.3
1⅛	1.96	49.9	43.5	39.7	36.0
1	1.55	39.5	34.4	31.5	228.6
⅞	1.19	30.5	26.5	24.2	22.0
¾	0.87	22.8	19.8	18.1	16.4
⅝	0.61	16.1	14.0	12.8	11.6
9/16	0.49	13.2	11.5	10.5	9.5
½	0.39	10.6	9.2	8.4	7.7
7/16	0.30	8.3	7.2	6.6	6.0
⅜	0.22	6.1	5.3	4.9	4.4
5/16	0.16	4.4	3.8	3.4	3.1
¼	0.10	2.8	2.4	2.2	2.0

Table 3. Extra Pliable Hoisting Rope
(Eight 19-wire strands and hemp center)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons			
		"Blue Center" steel	Plow steel	Extra strong cast steel	Cast steel
1½	3.26	79.50	69.00	63.30	57.50
1⅜	2.74	67.00	58.30	53.40	48.60
1¼	2.27	55.70	48.40	44.40	40.40
1⅛	1.84	45.20	39.40	36.00	32.80
1	1.45	35.80	31.20	28.60	26.00
⅞	1.11	27.60	24.00	22.00	20.00
¾	0.82	20.50	17.80	16.30	14.80
⅝	0.57	14.40	12.50	11.40	10.40
9/16	0.46	11.80	10.30	9.40	8.50
½	0.36	9.50	8.20	7.50	6.80
7/16	0.28	7.30	6.30	5.80	5.30
⅜	0.20	5.45	4.70	4.33	3.94
5/16	0.14	3.85	3.34	3.07	2.79
¼	0.09	2.47	2.15	1.79	1.79

ace and the air corresponding to the loss q_2 is $T_d = (328q_2 - 220)/(q_2 +$, and the coefficient of heat transfer for a given value of T_d is $k = \log_e r_1 - \log_e r_2)/T_d$, where $r_1 = r_1 + S$. Coefficients of heat transfer for 300 F are given in Table 51.

Table 51. Data on Commercial Pipe Coverings

Kind of covering (thickness of single covering, in.)	k (for temp. diff. of 300 F.)	Btu loss per sq ft per deg temp diff. per hr for single-thickness covered pipes (= U)					
		Temperature difference (pipe and room), F.					
		50	100	200	300	400	500
85 percent Magnesia (1.08)	0.507	....	0.381	0.397	0.413	0.429	0.445
Indented (1.12)	0.686	0.472	0.483	0.509	0.549	0.603	0.666
Vitribestos (0.96)	1.087	0.626	0.654	0.715	0.781	0.856	0.967
Eureka (1.04)	0.549	0.440	0.451	0.464	0.478	0.487	
Molded Asbestos (1.25)	0.778	0.517	0.522	0.539	0.561	0.596	
Wool Felt (1.10)	0.521	0.386	0.408	0.421	0.442	0.453*	
Mo Expanded Asbestos (1.07)	0.598	0.409	0.427	0.464	0.503	0.541	0.581
y Carocel (0.99)	0.540	0.358	0.378	0.421	0.466	0.510	0.562
y Serrated (1.00)	0.682	0.454	0.468	0.506	0.546	0.587	0.634
y Duplex (0.93)	0.636	0.423	0.447	0.498	0.548	0.574*	
y 85 percent Magnesia (1.10)	0.546	0.413	0.418	0.424	0.436	0.454	0.472
Mo Wool Felt (1.01)	0.510	0.395	0.410	0.433	0.459		
nreil High Pressure (1.16)	0.543	0.399	0.402	0.412	0.426	0.444	0.465
Asbestos Fire Felt (0.99)	1.093	0.694	0.711	0.749	0.795	0.845	0.901
Asbestos Sponge Felted (1.16)	0.468	0.336	0.347	0.369	0.391	0.414	0.439
Asbestocel (1.10)	0.596	0.418	0.429	0.454	0.493	0.544	0.609
Air Cell (1.00)	0.718	0.459	0.475	0.515	0.571	0.643	0.733
Plastic 85 percent Magnesia 51–3.24)	0.587	.....	0.470	0.488	0.505	0.522	0.539
Mo Air cell (0.95)	0.802	.....	0.539	0.603	0.681	0.771	0.871

For a temperature difference of 350 F.

he heat loss through any thickness of any material of which k is vn for $t_d \leqq 500$ F, is $q_2 = k(t_1 - t_2 - T_d)/[r_2(\log_e r_2 - \log_e r_1)]$, where t_1 t_2 are the temperatures of the pipe surface and room, respectively, F. he heat loss from bare pipe per degree temperature difference per hour square foot is as follows:

p. diff., F.	50	100	200	300	400	500
in Btu (= b)	1.95	2.152	2.665	3.26	4.035	5.18

he Btu saving due to covering per degree temperature difference per re foot per hour = $b - U$ (values of U in table), and the efficiency of covering in percent = $100(b - U)/b$. Thus, for the first covering in the ole, the saving per year per square foot of covering for a temperature ference of 300 F (for continuous operation, or 8,760 hr) = (3.26 − 0.413) 300 × 8760 = 7,482,000 Btu, and the efficiency = 100(3.26 − 0.413)/ 26 = 87.3 percent.

Identification of Piping

The A.S.A. has approved (1928) a **Scheme for the Identification of Piping** stems (A.S.A. A13-1928). This scheme is limited to the identification of piping stems in industrial plants, not including pipes buried in the ground, and electric conits. Fittings, valves, and pipe coverings are included, but not supports, brackets, or ner accessories.

Classification by Color. All piping systems are classified by the nature of the aterial carried. Each piping system is placed, by the nature of its contents, in the lowing classifications:

rope and requires larger sheaves. On account of the smaller number of wires, this rope should also be used with a higher factor of safety, as the breaking of one or two wires materially reduces the strength of the rope. The wires used are considerably larger in diameter than in hoisting rope, and consequently will stand greater wear. **Iron rope** of this construction is recommended for power transmissions equipped with large sheaves. **Cast-steel and extra strong cast-steel rope** are recommended for mine haulages, tramways, sand lines, and similar service where conditions tend to severe abrasion. **Plow-steel and improved plow-steel ropes** are recommended in place of cast steel when it is desirable to reduce the dead weight of the rope itself, or where, by reason of increased loads, it is necessary to use a stronger rope without increasing its diameter. This rope is particularly adapted for very long mine haulages.

Flat rope is composed of a number of wire ropes called "flat rope strands," of alternate right and left lay, placed side by side, then secured or sewed

Table 5. Flat Rope

Width and thickness, in.	Approx wt per ft, lb	Breaking strength, tons		Width and thickness, in.	Approx wt per ft, lb	Breaking strength, tons	
		Plow steel	Cast steel			Plow steel	Cast steel
7/8 × 8	10.69	275.3	224.3	1/2 × 3 1/2	2.82	75.2	61.0
7/8 × 7	9.63	247.8	201.9	1/2 × 3	2.47	65.8	53.4
7/8 × 6	8.56	220.2	179.4	1/2 × 2 1/2	2.13	56.4	45.8
7/8 × 5	7.50	192.7	157.0	3/8 × 6	3.63	93.7	76.5
3/4 × 8	9.70	228.3	184.5	3/8 × 5 1/2	3.42	88.5	72.2
3/4 × 7	8.13	207.5	167.7	3/8 × 5	3.03	78.1	63.8
3/4 × 6	7.31	186.8	150.9	3/8 × 4 1/2	2.83	72.8	59.5
3/4 × 5	6.50	166.0	134.2	3/8 × 4	2.44	62.4	51.0
5/8 × 8	8.32	216.0	174.6	3/8 × 3 1/2	2.23	57.2	46.7
5/8 × 7	7.23	187.0	151.3	3/8 × 3	1.84	46.8	38.3
5/8 × 6	6.14	158.0	128.1	3/8 × 2 1/2	1.64	41.6	34.0
5/8 × 5 1/2	5.59	143.8	116.4	3/8 × 2	1.25	31.2	25.5
5/8 × 5	5.04	129.4	104.8	5/16 × 4	2.29	58.5	48.0
5/8 × 4 1/2	4.50	115.0	93.1	5/16 × 3 1/2	2.03	51.2	42.0
5/8 × 4	3.95	100.6	81.5	5/16 × 3	1.75	43.8	36.0
5/8 × 3 1/2	3.40	86.3	69.9	5/16 × 2 1/2	1.47	36.5	30.0
1/2 × 7	5.85	150.4	122.0	5/16 × 2	1.17	29.2	24.0
1/2 × 6	4.85	131.6	106.7	5/16 × 1 1/2	.90	21.9	18.0
1/2 × 5 1/2	4.50	122.2	99.1	1/4 × 3	1.34	31.3	26.0
1/2 × 5	4.16	112.8	91.5	1/4 × 2 1/2	1.15	26.5	22.0
1/2 × 4 1/2	3.82	94.0	76.2	1/4 × 2	.88	21.7	18.0
1/2 × 4	3.16	84.6	68.6	1/4 × 1 1/2	.69	16.9	14.0

together with soft Swedish iron or steel wire (Fig. 5). The flat-rope strands consist of four 7-wire strands laid together with a center or core. The sewing or filling wire is much softer than the steel wires composing the strands of the rope, acts as a cushion or soft bed for the strands, and wears out much faster than the harder wires composing the latter. When the sewing wires are worn out, the flat rope can be resewed with new wire, and if any of the rope strands are also worn or damaged, these can be replaced by new por-

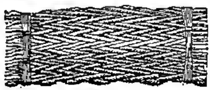


Fig. 5.—Flat Wire Hoisting Rope.

industrial, or general service in nominal sizes given in Table 53. The American National Standard thread form is used. This coupling is similar in design to the fire-hose couplings illustrated in Fig. 21.

Table 53. Dimensions of Standard Hose Couplings
(All dimensions in inches. Letters refer to Fig. 21)

Service and nominal size	Inside diam, C	Diam of thread, D	No of threads per in.	*L*	*I*	*H*	*T*
Garden: 1/2, 5/8, 3/4	25/32	1 1/16	11 1/2	9/16	1/8	17/32	3/8
Chemical: 3/4, 1	1 5/32	1 3/8	8	5/8	5/32	19/32	15/32
Fire: 1 1/2	1 17/32	2	9	5/8	5/32	19/32	15/32
Other connections: 1/2	17/32	13/16	14	1/2	1/8	15/32	5/16
3/4	25/32	1 1/32	14	9/16	1/8	17/32	3/8
1	1 1/32	1 9/32	11 1/2	9/16	5/32	17/32	3/8
1 1/4	1 9/32	1 5/8	11 1/2	5/8	5/32	19/32	15/32
1 1/2	1 17/32	1 7/8	11 1/2	5/8	5/32	19/32	15/32
2	2 1/32	2 11/32	11 1/2	3/4	3/16	23/32	19/32

Soldered-joint Fittings. The proposed American standard for these fittings (A.S.A. A40-) covers certain dimensions of soldered-joint wrought metal and cast brass fittings for copper tubing including (1) detailed dimensions of the bore, (2) minimum specifications for materials, (3) minimum inside diameter of the fitting, (4) metal thickness for both wrought metal and cast brass fittings, and (5) general dimensions for cast brass fittings including center-to-shoulder dimensions for both straight and reducing cast fittings. Pressure and temperature ratings also are given. Sizes of the fittings are identified by the nominal tubing size as covered by the American standard specifications for copper water tube, A.S.A. H23.1-1934 (A.S.T.M. B88-33).

Flattened-strand rope has little tendency to kink, and, owing to its smooth wearing surface, saves wear on pulleys, sheaves, and drums. It is made in the forms shown in Fig. 6. They are used for elevators, hoisting, dredging, etc. *For Type A, the minimum sheave size generally* recommended is 24 times the rope diameter, and the average, 36 times. For Type B, the corresponding values are 30 diameters minimum and 45 diameters average.

Non-spinning hoisting rope (Fig. 7) is constructed of 6 strands of 7 wires each, lang lay (wires in the strands and strands themselves twisted to the left) laid around a hemp core and covered with an outer layer composed of 12 strands, 7 wires, regular lay (wires in the strands twisted to the left and strands themselves twisted to the right). The object of this combination of lays is to prevent a free load suspended on the end of a single line from rotating. This type of rope is recommended for "back-haul" or single-line derricks; also for shaft sinking and mine hoisting where the bucket or cage swings free without guides. It works best where it does not overwind on the drum.

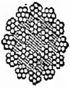


Fig. 7.

Non-rotating Hoisting Rope
(Composed of 18 strands and a hemp center, 7 wires to the strand)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons			
		"Blue center" steel	Plow steel	Extra-strong cast steel	Cast steel
1¾	5.30	115.0	100.0	91.0	81.0
1⅝	4.57	100.0	87.0	79.0	70.0
1½	3.89	85.5	74.5	67.0	60.0
1⅜	3.27	72.5	63.0	57.0	51.0
1¼	2.70	60.0	52.3	47.2	42.5
1⅛	2.19	49.0	42.5	38.4	34.2
1	1.73	38.8	33.8	30.5	27.3
⅞	1.32	29.8	25.9	23.5	21.1
¾	0.97	21.9	19.0	17.3	15.6
⅝	0.68	15.3	13.3	12.1	10.9
9/16	0.55	12.5	10.8	9.8	8.9
½	0.43	10.0	8.7	7.9	7.1
7/16	0.33	7.8	6.7	6.1	5.5
⅜	0.24	5.8	5.1	4.6	4.1

Either a closed or an open socket makes the best fastening on the end of non-spinning rope. These may be fastened in the same manner as any rope socket, but great care must be taken in attaching the socket to the rope to see that the strands do not untwist or allow any slack to work back into the rope. It is best to seize the end of the rope tightly for a distance of 4 or 5 in. just outside of the socket until the socketing is completed, when it may be taken off.

Steel-clad ropes (Fig. 8) are made in three constructions for the purpose of securing different degrees of flexibility: the 6 × 19, 6 × 37, and 6 × 61 types, respectively. Flat strips of steel wound spirally around each of the six strands composing the rope give additional wearing surface without sacrificing flexibility. When the outer flat-steel winding is worn through, a complete hoisting rope remains, with unimpaired strength. These ropes are designed to meet very severe conditions of service. The increased

disturb the strand relationship of the rope and necessitate its premature removal. The pressure of wire rope against a sheave groove is calculated by dividing the rope tension in pounds by the product of the radius to the bottom of the sheave groove and the diameter of the rope, both in inches. This quantity should not have a value greater than 450 lb per sq in. for cast iron, or 850 lb for cast steel. For greater pressures, a material of greater wear resistance, such as manganese steel, should be used.

Handling. Wire rope must not be coiled or uncoiled like hemp rope. When it is received upon a reel, the latter should be mounted upon a spindle or turntable and the rope then run off. When shipped in a coil, it should be rolled along the ground like a wheel. All untwisting and kinking must be avoided. When a wire rope is to be cut, soft iron wire should be served on each side of the place where the division is to be made to keep the rope from untwisting.

Materials. Rope made from **iron** wire is soft, tough, and pliable and has but little tendency to wear grooves in sheaves. This is of considerable advantage when wire rope is run over multiple-grooved sheaves. It is especially adapted for passenger elevators and similar service where the tendency to abrasion is comparatively slight, the speed is high, loads moderate. **Cast-steel rope** is recommended for general hoisting, such as mine hoists, derricks, coal hoists, cableways, conveyors, freight elevators, etc. It is tough and pliable, will bend over comparatively small sheaves and resist

Table 1. Standard Hoisting Rope

Composed of 6 strands and a hemp center, 19 wires to the strand
(John A. Roebling's Sons Co.)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons				
		"Blue Center" steel	Plow steel	Extra-strong cast steel	Cast steel	Iron
2¾	12.10	294.0	256.0	234.0	212.0	95.00
2½	10.0	246.0	214.0	195.0	176.0	79.10
2¼	8.10	202.0	176.0	160.0	144.0	64.80
2⅛	7.23	181.0	157.0	143.0	128.0	
2	6.40	161.0	140.0	127.0	114.0	51.80
1⅞	5.63	142.0	123.0	112.0	100.0	45.80
1¾	4.90	124.0	108.0	98.0	88.0	40.10
1⅝	4.23	108.0	94.0	85.0	76.0	34.80
1½	3.60	92.5	80.5	72.5	65.0	29.70
1⅜	3.03	78.5	68.0	61.5	55.0	25.20
1¼	2.50	65.0	56.5	51.0	46.0	21.00
1⅛	2.03	53.0	46.0	41.5	37.0	17.20
1	1.60	42.0	36.5	33.0	29.5	13.70
⅞	1.23	32.2	28.0	25.4	22.8	10.60
¾	0.90	23.7	20.6	18.7	16.8	7.86
⅝	0.63	16.6	14.4	13.1	11.8	5.52
9/16	0.51	13.5	11.7	10.6	9.6	4.49
½	0.40	10.8	9.4	8.5	7.7	3.57
7/16	0.31	8.4	7.3	6.6	6.0	2.76
⅜	0.23	6.3	5.5	5.0	4.5	2.05
5/16	0.16	4.5	3.9	3.5	3.2	1.43
¼	0.10	2.9	2.5	2.3	2.1	0.97

Wire rope slings are made up in numerous styles, some of which are manufactured endless in grommet construction and develop the full strength of the rope.

Tensile Strength of Galvanized Steel Wire Strands, Lb

(7-wire strand)

Diam, in. Wt of 1,000 ft, lb.	5/8 813	9/16 671	1/2 517	7/16 399	3/8 273	5/16 205	9/32 164	1/4 121	7/32 98.3	3/16 72.9	5/32 51.3	1/8 31.8
Siemens-Martin	19,100	15,700	12,100	9,350	6,950	5,350	4,250	3,150	2,560	1,900	1,470	910
High strength	29,600	24,500	18,800	14,500	10,800	8,000	6,400	4,750	3,850	2,850	2,140	1,330
Extra-high strength	42,400	35,000	26,900	20,800	15,400	11,200	8,950	6,650	5,400	3,990	2,940	1,830

Minimum elongation, percent, in 24 in.: S.M., 8; H.S., 5; E.H.S., 4.

Properties of Special Grades of Extra Galvanized Special Strands

Approx diam, in.	No. of wires in strand	Approx breaking strength, lb: Extra high strength	High strength	Siemens-Martin	Common soft strand	Approx wt per 1,000 ft, lb	Approx diam, in.	No. of wires in strand	Approx breaking strength, lb: Extra high strength	High strength	Siemens-Martin	Common soft strand	Approx wt per 1,000 ft, lb
1	19	104,500	73,200	47,000	28,700	2073	1¼	37	162,200	113,600	73,000	44,600	3,248
7/8	19	79,700	55,800	35,900	21,900	1581	1⅛	37	130,800	91,600	58,900	36,000	2,691
3/4	19	58,300	40,800	26,200	16,000	1155	1	37	102,700	71,900	46,200	28,300	2,057
5/8	19	40,200	28,100	18,100	11,000	796	1¾	61	315,000	220,600	141,800	86,600	6,346
9/16	19	33,700	24,100	16,100	9,640	637	1⅝	61	271,300	189,900	122,100	74,600	5,497
1/2	19	26,700	19,100	12,700	7,620	504	1½	61	229,600	160,700	103,300	63,100	4,665
							1⅜	61	192,000	134,400	86,400	52,800	3,901

Minimum elongation, percent, in 24 in.: E.H.S., 4; H.S., 5; S.M., 8.

Galvanized mast-arm rope is used for arc lights, mast arms, or other purposes where exposed to moisture. This rope is more durable than manila rope and does not shrink.

Extra Pliable Hoisting Rope. Made of 6 strands of 37 wires each and a hemp center (Fig. 2). The wires in this rope are much finer than those used in the standard hoisting rope and consequently not as suitable to withstand abrasion. These ropes are used on electric cranes, dredges, and for similar service requiring a strong tough rope that will operate successfully over small sheaves.

Extra pliable hoisting rope of 8 strands of 19 wires and a hemp center (Fig. 3) is much more pliable than the standard construction of 6 strands of 19 wires. The metallic area of an eight-strand rope is not so great as that of a six-strand rope, and the wires are smaller, but under severe bending stresses the decrease in strength is largely offset by the great pliability. It can be used over comparatively small sheaves and drums such as are frequently found on derricks. It is not good practice to use it except for

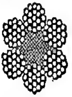


Fig. 1.

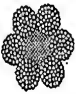


Fig. 2.

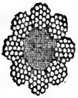


Fig. 3.

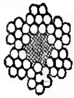


Fig. 4.

comparatively light loads or where there is much overwinding, because it would flatten or lose shape more quickly than 6×19 rope. Moreover it stretches more than a 6×19 rope.

Galvanized extra-pliable cast-steel hoisting rope is much more flexible than the six-strand hoisting rope, and is often used in preference to galvanized cast-steel running rope.

Standard coarse-laid rope (Fig. 4) is made of six strands and a hemp center, seven wires to the strand. It is much stiffer than standard hoisting

Table 4. Standard Coarse-laid Rope for Haulages and Transmissions
(Composed of six 7-wire strands and a hemp center)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons				
		"Blue center" steel	Plow steel	Extra strong cast steel	Cast steel	Iron
1½	3.38	86.5	75.00	68.70	62.50	29.70
1⅜	2.84	73.5	63.50	58.20	53.00	25.20
1¼	2.34	61.0	53.00	48.70	44.50	21.00
1⅛	1.90	50.0	43.60	40.00	36.40	17.20
1	1.50	40.0	34.80	31.90	29.00	13.70
⅞	1.15	30.8	26.80	24.60	22.40	10.50
¾	0.84	22.8	19.80	18.10	16.50	7.86
⅝	0.59	16.0	13.80	12.60	11.50	5.52
9/16	0.48	13.0	11.30	10.30	9.40	4.49
½	0.38	10.3	9.00	8.20	7.50	3.57
7/16	0.29	7.9	6.90	6.30	5.80	2.76
⅜	0.21	5.9	5.15	4.70	4.30	2.05
5/16	0.15	4.2	3.65	3.35	3.10	1.43
9/32	0.12	3.4	2.95	2.72	2.52	1.17
¼	0.094	2.7	2.35	2.15	2.00	

Galvanized Iron and Steel Running Rope

(Fig. 11, composed of 6 strands and a hemp center, each strand consisting of 12 wires and a hemp core)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons			Diam, in.	Approx wt per ft, lb	Breaking strength, tons		
		Plow steel	Cast steel	Iron			Plow steel	Cast steel	Iron
1 1/16	1.19	22.60	18.30	9.00	5/8	0.41	7.90	6.46	3.35
1	1.05	20.00	16.30	8.10	9/16	0.33	6.43	5.26	2.75
7/8	0.80	15.40	12.50	6.30	1/2	0.26	5.10	4.23	2.18
13/16	0.69	13.30	10.80	5.46	7/16	0.20	4.01	3.29	1.69
3/4	0.59	11.30	9.26	4.70	3/8	0.15	3.01	2.47	1.26
					5/16	0.10	2.13	1.76	0.88

Galvanized Cast-steel Rigging and Guy Rope

(Composed of 6 strands and a hemp center, either 7 or 19 wires to the strand)

Diam, in.	Approx wt per ft, lb		Breaking strength, tons	Diam, in.	Approx wt per ft, lb		Breaking strength, tons
	6×19	6×7			6×19	6×7	
1 1/4	2.50	2.34	40.0	3/4	0.90	0.84	14.8
1 3/16	2.26	2.12	36.4	5/8	0.63	0.59	10.4
1 1/8	2.03	1.90	32.8	9/16	0.51	0.48	8.5
1 1/16	1.81	1.70	29.4	1/2	0.40	0.38	6.8
1	1.60	1.50	26.1	15/32	0.35	0.33	6.0
7/8	1.23	1.15	20.0	7/16	0.31	0.29	5.3
13/16	1.06	0.99	17.3	3/8	0.23	0.21	3.9
				5/16	0.16	0.15	2.8

Galvanized Ships' Rigging and Guy Ropes

(Composed of 6 strands and a hemp center, 7 or 12 wires to the strand; 9/32 in. and finer of 5 strands)

Diam, in.	Approx weight per ft, lb	Approx strength, tons of 2,000 lb	Diam, in.	Approx weight per ft, lb	Approx strength, tons of 2,000 lb	Diam, in.	Approx weight per ft, lb	Approx strength, tons of 2,000 lb
1 3/4	4.85	42	1 1/8	2.00	18.0	1/2	0.39	3.39
1 11/16	4.42	38	1 1/16	1.77	16.1	7/16	0.30	2.35
1 5/8	4.15	35	1	1.58	14.1	3/8	0.22	1.95
1 1/2	3.55	30	7/8	1.20	11.1	5/16	0.15	1.42
1 7/16	3.24	28	13/16	1.03	9.4	9/32	0.125	1.20
1 3/8	3.00	26	3/4	0.89	7.8	1/4	0.09	0.99
1 1/4	2.45	23	5/8	0.62	5.7	7/32	0.063	0.79
1 3/16	2.21	19	9/16	0.50	4.46	3/16	0.04	0.61

tions. Flat rope is used principally for hoisting purposes. When large and long rope is used in hoisting heavy loads out of deep shafts, round rope requires large and heavy drums on which to wind, while flat rope, winding on itself, needs a reel but little wider than the width of the rope. Furthermore, flat rope does not spin or twist in the shaft. Flat rope is obtainable from 1¼ to 8 in. in width, and from ¼ to ⅞ in. in thickness, the length varying from 20 to 3,000 ft. Flat rope is particularly applicable to the operating of spouts on coal and ore docks, also for raising and lowering emergency gates on canals and similar machinery. It combines flexibility and great strength, thus making possible the use of simple and compact hoisting machinery.

The diameters of **drums and sheaves** for flat rope should be as large as possible, particularly for mine hoisting work. For average continuous duty of 4 × 7 flat rope, *i.e.*, 4 strands with 7 wires to the strand, sheaves (drums) should be at least 96 (80) times the nominal thickness of the rope for intermittent duty such as flood gate hoists 60 (48): for shaft hoists with long lengths of rope, best economy requires about 200 (120).

Table 6. Flattened-strand Hoisting Rope, Type A
(Five 28-wire strands and a hemp core)

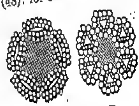
Diam. in.	Approx wt per ft, lb	Breaking strength, tons			
		Improved plow steel	Plow steel	Extra-strong cast steel	Cast steel
⅜	0.22	5.6	4.8	4.4	4
½	0.39	9.5	8.2	7.5	6.8
9/16	0.49	11.9	10.3	9.4	8.5
⅝	0.61	14.6	12.7	11.6	10.4
¾	0.87	20.9	18.2	16.5	14.8
⅞	1.19	28.5	24.8	22.4	20.1
1	1.55	37.	32.	29.	26.
1⅛	1.96	47.	41.	37.	33.
1¼	2.42	57.	50.	45.	41.

Table 7. Flattened-strand Hoisting Rope, Type B
(Six 25-wire strands and a hemp core)

Diam. in.	Approx wt per ft, lb	Breaking strength, tons			
		Improved plow steel	Plow steel	Extra-strong cast steel	Cast steel
⅜	0.25	6.9	6.	5.5	4.9
½	0.45	11.8	10.3	9.3	8.4
9/16	0.57	14.8	12.8	11.6	10.5
⅝	0.70	18.2	15.8	14.4	12.9
¾	1.01	26.	22.6	20.5	18.4
⅞	1.39	35.4	30.	27.9	25.
1	1.80	46.	40.	36.	32.
1⅛	2.28	58.	50.5	45.5	40.
1¼	2.81	71.5	62.	56.	50.5
1⅜	3.40	86.	74.	67.	60.
1½	4.05	101.	88.	79.	71.
1⅝	4.75	118.	103.	93.	83.
1¾	5.51	136.	118.	107.	96.
2	7.2	177.	154.	139.	125.
2¼	9.1	222.	193.	176.	158.
2½	11.2	270.	.....	214.	193.
2¾	13.6	323.	.....	257.	233.

Sheaves should be slightly crowned in the center and have deep flanges to guide the rope.

Flattened-strand wire rope is designed to give increased contact or wearing surface. The wear is consequently lessened upon any one individual wire and the necessity of the use of heavier wire diminished, which results in greater flexibility. The wearing surface is approximately 150 percent greater than that of a round-strand rope. Another feature of this type of rope is that the interstices between the strands being lessened, a greater number of wires are used for the same diameter. It is always made lang lay.



A. B.

Fig. 6.

Nails and Spikes

Nails are either **wire nails** of circular cross section and constant diameter, or **cut nails** of rectangular cross section with taper from head to point. The larger sizes are called **spikes**. The length of the nail is expressed in the "penny" *system*, the equivalents in inches being given in the following tables; the letter "d" is the accepted symbol for penny. A keg of nails weighs 100 lb.

Wire Nails for Special Purposes
(Washburn and Moen Gage)

Length, in.	Gage No.	No. to lb	Length, in.	Gage No.	No. to lb	Length, in.	Gage No.	No. to lb
Barrel nails			Barbed roofing nails			Slating nails		
5/8	15 1/2	1615	3/4	13	714	1	12	411
3/4	15 1/2	1346	7/8	12	469	1 1/4	10 1/2	225
7/8	14 1/2	906	1	12	411	1 1/2	10 1/2	187
1	14 1/2	775	1 1/8	12	365	1 3/4	10	142
1 1/8	14 1/2	700	1 1/4	11	251	2	9	103
1 1/4	14	568	1 3/8	11	230	Fine nails		
1 3/8	13	400	1 1/2	10	176	1	16 1/2	1351
1 1/2	13	367	1 3/4	10	151	1 1/8	15	718
			2	9	103	1	17	1560
Barbed dowel nails			Clout nails			1 1/8	16	1015
5/8	8	290				American felt roofing nails, 3/8 in. heads		
3/4	8	250	3/4	15	1160	7/8	12	195
7/8	8	210	7/8	14	808	1	12	180
1	8	190	1	14	705			
1 1/8	8	165	1 1/8	14	628			
1 1/4	8	150	1 1/4	13	423			
1 3/8	8	130	1 3/8	13	390			
1 1/2	8	120	1 1/2	13	350			

Approximate Number of Boat Spikes to a Keg of 200 Lb

Size of spike, in. sq	Length, in.									
	4	5	6	7	8	9	10	11	12	14
5/8	.....	.....	.....	.....	214	190	176	...	144	122
1/2	.....	.....	.....	.....	324	286	258	244	220	192
7/16	.....	.....	.....	480	438	...	378			
3/8	1,114	930	816	690	622	532	492	...	434	
5/16	1,776	1,342	1,124	978	858	776	706		...	...
1/4	2,576	2,134	1,778	1,488	1,382					

Approximate Number of Railroad Spikes to a Keg of 200 Lb

Size of spike, in. sq	Length, in.						
	2 1/2	3	3 1/2	4	4 1/2	5	5 1/2
3/8	1,600	1,408	1,242	1,111	105	917	
7/16	1,171	1,114	922	833	760	699	
1/2	918	800	712	608	556	526	
9/16	.....	.....	.....	.....	...	400	360

life obtained by the use of steel-clad rope is, in places where conditions are suitable, from 50 to 100 percent. It is recommended particularly for such service as dredging. The breaking strength of these ropes is less than that of a round strand rope of the same diameter and grade.

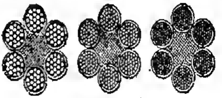


Fig. 8.—Steel-clad Ropes.

Galvanized wire rope has almost entirely superseded manila rope for shrouds and stays aboard ship. It is cheaper in first cost, is not affected by the weather, and does not stretch and contract with changes in atmospheric conditions, and thus saves a great deal of labor in setting up; it is as elastic as manila rope. There is great reduction in bulk and weight by its use, as it is only one-fifth or one-sixth as large as a manila rope of equal strength. Consequently, it offers only half as much surface to the wind. It is much less liable to accidents by being cut or chafed, and does not rot and give way suddenly without warning. Galvanized rope is better suited for guys for derricks than hemp rope or rods linked together.

Galvanized steel wire strand (Fig. 9) is used chiefly for guying poles and smokestacks, for supporting trolley wire, and for operating railroad signals. For overhead catenary construction of suspending trolley wire, the special grades of strand are preferable because they possess greater strength and toughness. The smallest sizes (sometimes called "galvanized seizing strand") are used for seizing or binding the ends of wire rope and thimble splices, and for tying rope into coils.

Fig. 9.

Wire Rope Fittings. When wire rope has to be cut to make attachment to fittings or for splicing, it is important that it should be properly seized: the wires and strands are twisted under uniform tension and this tension should be maintained. Three sets of seizings should be placed on each side of the cut with about 10 wraps of wire per seizing. Annealed iron wire should be used of the following diameters:

Rope diam, in.	3/8 and 1/2	5/8	3/4	7/8 and 1	1 1/8–1 7/8	2 and over
Seizing wire diam, in.	0.047	0.054	0.063	0.080	0.105	0.135

Attachment to a socket is made by separating and straightening the wires, cutting out the hemp center, cleansing with kerosine, dipping in one-half muriatic acid and one-half water, distributing the wires uniformly in the socket, and pouring in molten zinc after plugging up the lower end of the socket with fire clay; such an attachment will develop the full strength of the rope. Spliced eye connections when properly made will not pull out and will develop from 60 percent of the full strength of a 2 1/2 in. diameter rope up to 95 percent for a 1/2 in. rope. Clip and clamp connections are not desirable as permanent fastenings on hoisting ropes and will develop 75 to 85 percent of the strength of the rope.

Galvanized Common Steel Wire Strand

(Composed of 7 wires twisted together)

Diam, in.	Approx weight per 1,000 ft, lb	Approx strength, lb	Diam, in.	Approx weight per 1,000 ft, lb	Approx strength, lb	Diam, in.	Approx weight per 1,000 ft, lb	Approx strength, lb
5/8	813	11,600	3/8	273	4,250	7/32	98.3	1,540
9/16	671	9,600	5/16	205	3,200	3/16	72.9	1,150
1/2	517	7,400	9/32	164	2,570	5/32	51.3	870
[illegible]	399	5,700	1/4	121	1,900	1/8	31.8	540

Cut Steel Nails and Spikes

(Sizes, lengths, and approximate number per lb)

Sizes	Length, inches	Common	Clinch	Finishing	Casing and box	Fencing	Spikes	Barrel	Slating	Tobacco	Brads	Shingle
2d	1	740	400	1,100	......	......	......	450	340			
3d	1¼	460	260	880	......	......	......	280	280			
4d	1½	280	180	530	420	......	......	190	220			
5d	1¾	210	125	350	300	100	......	......	180	130		
6d	2	160	100	300	210	80	......	......	......	97	120	
7d	2¼	120	80	210	180	60	......	......	......	85	94	
8d	2½	88	68	168	130	52	......	......	......	68	74	90
9d	2¾	73	52	130	107	38	......	......	......	58	62	72
10d	3	60	48	104	88	26	......	......	......	48	50	60
12d	3¼	46	40	96	70	20	......	......	......	......	40	
16d	3½	33	34	86	52	18	17	......	......	......	27	
20d	4	23	24	76	38	16	14					
25d	4¼	20	......	......	......	......	......					
30d	4½	16½	......	......	30	......	11					
40d	5	12	......	......	26	......	9					
50d	5½	10	......	......	20	......	7½					
60d	6	8	......	......	16	......	6					
......	6½	......	......	......	......	......	5½					
......	7	......	......	......	......	......	5					

Sizes of American Wire Tacks

Oz	Length, in.	Size of wire, steel wire gage			Oz	Length, in.	Size of wire, steel wire gage		
		Upholsterers	Carpet	Bill postors			Upholsterers	Carpet	Bill posters
1	3/16	18	18	....	10	5/8	14½	15	12
1½	7/32	18	18	....	12	11/16	14½	15	12
2	¼	17	17	15	14	¾	14	14½	11½
2½	5/16	17	17	15	16	13/16	14	14½	11½
3	3/8	16	16	14	18	7/8	13½	13½	11
4	7/16	16	16	13½	20	15/16	13½	13½	11
6	½	15	15	13	22	1	13½	13½	10½
8	9/16	15	15	12½	24	1⅛	13	13	10½

Knots, Hitches, and Bends

No two parts of a knot which would move in the same direction if the rope were to slip should lie alongside of and touching each other. The knots shown in Fig. 13 are known by the following names:

A, bight of a rope; B, simple or overhand knot; C, figure 8 knot; D, double knot; E, boat knot; F, bowline, first step; G, bowline, second step; H, bowline, completed; I, square or reef knot; J, sheet bend or weaver's knot; K, sheet bend with a toggle; L, carrick bend; M, "stevedore" knot completed; N, "stevedore" knot commenced; O, slip knot; P, Flemish loop; Q, chain knot with toggle; R, half hitch; S, timber hitch; T, clove hitch; U, rolling hitch; V, timber hitch and half hitch; W, blackwall hitch; X, fisherman's bend; Y, round turn and half hitch; Z, wall knot commenced; AA, wall knot completed; BB, wall knot crown commenced; CC, wall knot crown completed.

The bowline G, one of the most useful knots, will not slip, and after being strained is easily untied. Knots H, K, and M are easily untied after being under strain. The knot M is useful when the rope passes through an eye

Galvanized Mast-arm Rope

(Composed of nine four-wire strands and a cotton center)

Diam, in.	3/8	5/16	1/4
Weight, lb per ft	0.158	0.107	0.070
Approx breaking stress, lb	2200	1530	1100

Diam, in.	Galvanized steel hawsers[a]		Galvanized steel mooring lines and hawsers[b]			Galvanized steel hawsers and warps[c]		
	Approx wt per ft, lb	Breaking strength, tons	Approx wt per ft, lb	Breaking strength, tons		Approx wt per ft, lb	Breaking strength, tons	
				Plow *steel*	Cast *steel*		Plow *steel*	Cast *steel*
2 3/8	8.74	173.3						
2 5/16	8.29	164.6						
2 1/4	7.85	156.2						
2 1/8	7.00	140.2						
2 1/16	6.59	132.6	5.87	118.00	98.00	4.47	81.3	66.6
2	6.20	125.0	5.52	112.00	92.00	4.20	76.9	62.7
1 15/16	5.82	117 8	5.18	105.00	86.60	3.94	72.2	59.0
1 13/16	5.09	103.4	4.53	92.30	76.20	3.45	63.4	51.7
1 3/4	4.75	96.5	4.23	86.20	71.20	3.22	59.3	48.3
1 11/16	4.41	89.8	3.93	80.20	66.30	2.99	55.3	45.0
1 5/8	4.09	63.4	3.64	74.50	61.60	2.77	51.5	41.8
1 1/2	3.49	71.2	3.11	63.60	52.60	2.36	44.3	35.8
1 7/16	3.20	65.5	2.85	58.50	48.40	2.17	40.7	33.0
1 3/8	2.93	60.0	2.61	53.60	44.40	1.99	37.4	30.2
1 1/4	2.42	49.7	2.16	44.40	36.70	1.64	31.0	25.1
1 3/16	2.19	44.9	1.95	40.10	33.20	1.48	28.1	22.8
1 1/8	1.96	40.3	1.75	36.00	29.90	1.33	25.3	20.5
1 1/16	1.75	36.0	1.56	32.20	26.70			
1	1.55	31.9	1.38	28.50	23.70			
7/8	1.19	24.6	1.06	22.00	18.30			
13/16	1.02	21.3	0.91	19.10	15.60			
3/4	0.87	18.3	0.78	16.40	13.60			
5/8	....	.....	0.54	11.60	9.59			
1/2	....	.....	0.35	7.63	6.37			
3/8	....	.....	0.194	4.40	3.67			

[a] Composed of 6 strands and a hemp center, 37 wires to the strand.

[b] Composed of 6 strands and a hemp center, each strand composed of 24 wires and a hemp core (Fig. 10).

[c] Composed of 6 strands and a hemp center, each strand consisting of 12 wires and a hemp core (Fig. 11).

with the full strength of (dry) rope (= 100), based on Miller's experiments (*Machinery*, p. 198, 1900), is as follows: Eye splice over iron thimble, 90; short splice in rope, 80; S and Y (see Fig. 13), 65; H, O and T, 60; I and J, 50; B and P, 45.

Recommended Standard Cross Sections

The A.S.A. Tentative Standard, Z.14.1-1935, for representing section lining and outside views is shown in Figs. 14 and 15. Subdivisions of any of the materials may be made by taking one of these standard cross sections as a basis and making minor changes or by writing on the standard cross section the name of the material.

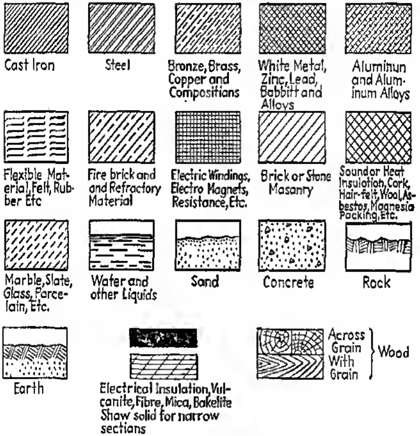


Fig. 14.—Standard Cross Sections.

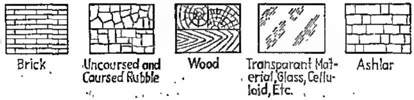


Fig. 15.—Standard Outside Views.

Galvanized Steel Cables for Suspension Bridges

(Fig. 12.—Composed of 6 strands with wire center)

Diam, in.	Approx weight per ft, lb	Approx strength, tons of 2,000 lb	Diam, in.	Approx weight per ft, lb	Approx strength, tons of 2,000 lb	Diam, in.	Approx weight per ft, lb	Approx strength, tons of 2,000 lb
2 3/4	12.7	310	2 1/8	7.60	185	1 5/8	4.34	106
2 5/8	11.6	283	2	6.73	164	1 1/2	3.70	90
2 1/2	10.5	256	1 7/8	5.90	144	1 3/8	3.10	75
2 3/8	9.50	232	1 3/4	5.10	124	1 1/4	2.57	62
2 1/4	8.52	208						

Tramway strand (see below) has usually 19, 32, 37, 61, or 91 wires, according to the diameter of the strand and the conditions of its use. It is recommended for the track or trolley cable on an aerial tramway on account of its compact construction, and is found, by reason of its comparatively smooth surface, to reduce greatly the friction on the carriage wheels. In long spans where the weight of the rope itself is a disadvantage, plow-steel quality is generally used.

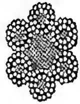


Fig. 10.

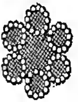


Fig. 11.

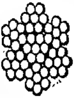


Fig. 12.

Tramway Strand

Diam, in.	1 1/2	1 3/8	1 1/4	1 1/8	1	7/8	3/4	5/8
Wt per 100 ft, lb	504	416	356	277	218	169	120	86
Approx strength, tons:								
Plow steel	127.5	105.0	84.6	70.7	58.0	44.4	32.5	22.8
Cast steel	108.4	88.8	71.8	60.0	49.2	37.6	27.6	19.2

Copper, Iron, Tinned, and Galvanized Sash Cords

(Six 7-wire strands and cotton core)

Diam, in.		1/4	7/32	3/16	5/32	1/8	3/32	1/16
Wt per ft, lb	Copper	0.108	0.083	0.061	0.044	0.026	0.015	0.007
	Iron	0.094	0.072	0.053	0.038	0.023	0.013	0.006
Breaking strength, lb	*Bright copper*	*1,225*	940	*688*	478	*306*	*172*	77
	Bright iron	2,040	1,570	1,150	840	560	315	140
	Ann'ld iron	1,225	940	688	478	306	172	77

Tiller Rope or Hand Rope

(6 strands of 42 wires each, 252 wires in all, 7 hemp cores)

Diam, in.	Approx wt per ft, lb	Breaking strength, tons		
		Plow steel	Cast steel	Iron
1	1.10	20.10	16.70	8.600
7/8	0.84	15.60	13.00	6.600
3/4	0.62	11.70	9.70	5.100
5/8	0.43	8.28	6.90	3.500
9/16	0.35	6.73	5.60	3.020
1/2	0.28	5.33	4.46	2.390
7/16	0.21	4.13	3.44	1.910
3/8	0.16	3.05	2.54	1.400
5/16	0.11	2.13	1.77	0.977
1/4	0.07	1.36	1.14	0.625

Heavy hinge nails or **track nails with countersunk heads** have chisel points unless diamond points are specified. **Plaster-board nails** are smooth with diamond points and ½ in. flat heads for lengths, 1, 1¼, and 1½ in.; 5/16 in. head for sizes Nos. 9, 10, and 11. **American felt roofing nails** have an extra large head and thin shank. **Spikes** are made either with flat heads and diamond points or with oval heads and chisel points.

Wire Nails and Spikes
(Washburn and Moen Gage)

Size of nail	Length, inches	Casing nails, smooth and barbed box: Gage	Casing nails, smooth and barbed box: No. to lb	Finishing nails: Gage	Finishing nails: No. to lb	Clinch nails: Gage	Clinch nails: No. to lb	Fence nails: Gage	Fence nails: No. to lb	Shingle nails: Gage	Shingle nails: No. to lb
2d	1	15½	1010	16½	1351	14	710				
3d	1¼	14½	635	15½	807	13	429	......	......	13	429
4d	1½	14	473	15	584	12	274	......	......	12	274
5d	1¾	14	406	15	500	12	235	10	142	12	235
6d	2	12½	236	13	309	11	157	10	124	12	204
7d	2¼	12½	210	13	238	11	139	9	92		
8d	2½	11½	145	12½	189	10	99	9	82		
9d	2¾	11½	132	12½	172	10	90	8	62		
10d	3	10½	94	11½	121	9	69	7	50		
12d	3¼	10½	87	11½	113	9	62	6	40		
16d	3½	10	71	11	90	8	49	5	30		
20d	4	9	52	10	62	7	37	4	23		
30d	4½	9	46								
40d	5	8	35								

Size of nail	Length, inches	Boat nails, Heavy: In.	Boat nails, Heavy: No. to lb	Boat nails, Light: In.	Boat nails, Light: No. to lb	Hinge nails, Heavy: In.	Hinge nails, Heavy: No. to lb	Hinge nails, Light: In.	Hinge nails, Light: No. to lb	Flooring nails: Gage	Flooring nails: No. to lb
4d	1½	¼	44	3/16	82	¼	50	3/16	82		
6d	2	¼	32	3/16	62	¼	38	3/16	62	11	157
8d	2½	¼	26	3/16	50	¼	30	3/16	50	10	99
10d	3	⅜	14	¼	22	⅜	12	¼	25	9	69
12d	3¼	⅜	13	¼	20	⅜	11	¼	23	8	54
16d	3½	⅜	12	¼	18	⅜	10	¼	22	7	43
20d	4	⅜	10	¼	16	⅜	9	¼	19	6	31

Size of nail	Length, inches	Common wire nails and brads: Gage	Common wire nails and brads: No. to lb	Barbed car nails, Heavy: Gage	Barbed car nails, Heavy: No. to lb	Barbed car nails, Light: Gage	Barbed car nails, Light: No. to lb	Spikes: Size	Spikes: Length in.	Spikes: Gage	Spikes: Approx no. to lb
2d	1	15	876	......	......	......	......	10d	3	6	41
3d	1¼	14	568	......	......	......	......	12d	3¼	6	38
4d	1½	12½	316	10	165	12	274	16d	3½	5	30
5d	1¾	12½	271	9	118	10	142	20d	4	4	23
6d	2	11½	174	9	103	10	124	30d	4½	3	17
7d	2¼	11½	161	8	76	9	92	40d	5	2	13
8d	2½	10¼	106	8	69	9	82	50d	5½	1	10
9d	2¾	10¼	96	7	54	8	62	60d	6	1	9
10d	3	9	69	7	50	8	57	......	7	5/16 in.	6
12d	3¼	9	63	6	42	7	50	......	8	⅜	4
16d	3½	8	49	6	35	7	43	......	9	⅜	3½
20d	4	6	31	5	26	6	31	......	10	⅜	3
30d	4½	5	24	5	24	6	28	......	12	⅜	2½
40d	5	4	18	4	18	5	21				
50d	5½	3	14	3	15	4	17				
60d	6	2	11	3	13	4	15				

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INTERNAL-COMBUSTION ENGINES

By LOUIS R. FORD

and is held by the knot, as it will not slip, and is easily untied after being strained. The wall knot is made as follows: Form a bight with strand 1 and pass the strand 2 around the end of it, and the strand 3 around the end of 2, and then through the bight of 1, as shown at Z in the figure. Haul the ends taut when the appearance is as shown at AA. The end of the strand 1 is now laid over the center of the knot, strand 2 laid over 1, and 3 over 2, when

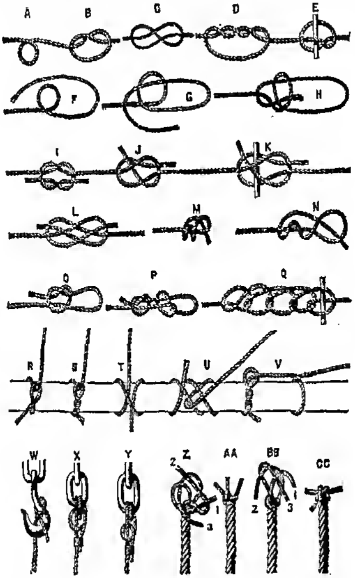

Fig. 13.—Rope Knots, Hitches, and Bends.
(From C. W. Hunt Co.'s Catalog)

the end of 3 is passed through the bight of 1, as shown at BB. Haul all the strands taut, as shown at CC. The "stevedore" knot (M, N) is used to hold the end of a rope from passing through a hole. When the rope is strained, the knot draws up tight, but it can be easily untied when the strain is removed. If a knot or hitch of any kind is tied in a rope, its failure under stress is sure to occur at that place. The shorter the bend in the standing rope, the weaker is the knot. The approximate strength of knots compared

PREFERRED NUMBERS

By C. H. BERRY

REFERENCES: Hirshfeld and Berry, "Size Standardization by Preferred Numbers," *Mech. Eng.*, Dec. 1922; Schlink, "A New Tool for Standardizers," *Am. Mach.*, July 12, 1923; Tornebohm, "The Development and Importance of Preferred Number Series," *Mech. Eng.*, Oct., 1923; Schlink, "Use of Preferred Numbers," *Jour. S.A.E.*, Feb., 1925; "Table of Preferred Numbers," *A.S.A. Standard* Z17.1-1936; Steczynski, "Preferred Numbers for American Practice," *Mech. Eng.*, Nov., 1928; von Dobbeler, "Preferred Numbers," *Mech. Eng.*, March, 1929.

Many manufactured articles are made in several sizes which may be designated by some dimension, speed, capacity, or other feature. Each such series of products may be paralleled by a series of numbers.

It is generally agreed that such number series should be **geometric progressions**, *i.e.*, each term should be a fixed percentage larger than the preceding. A geometric series provides small steps for small numbers, large steps for large numbers, and this best meets most requirements. The small steps in the diameter of the numbered twist drills would be absurd in drills of 1 in. diameter and larger.

In the case of sized objects that are used principally as raw material, *e.g.*, steel rod, an arithmetic progression may be preferable because it tends to reduce the cost of machining. It is desirable to be able to buy raw material a fixed amount (rather than a fixed percentage) larger than the finished article.

Preferred numbers is the name given to various series proposed for general use. These are either geometric progressions or approximations thereto. A geometric series is defined by one term and the ratio of each term to the preceding. On the choice of these elements for a preferred number series, there is as yet no general agreement. The same value would hardly be satisfactory for all cases. The idea of preferred numbers is to provide a master series from which terms can be chosen to suit any needs. This would ultimately lead to a comprehensive plan in all fields of manufacture, so that, for example, the sizes of shafting would be in accord with the sizes of bearings, and indeed with all manner of cylindrical machine elements.

An advantage of a geometric series is that if linear dimensions are chosen in the series, areas and volumes and other functions of powers of dimensions are also members of the same series.

In one of the most carefully considered systems of preferred numbers the base term is 1, and the ratio is $\sqrt[80]{10}$. In this series, the 81st term is 10, and accordingly the series from 10 to 100 or from 0.01 to 0.1, or, in general, from 10^n to 10^{n+1} is identical with the series from 1 to 10 with the decimal point shifted. This series will rarely be used in full; some will choose alternate terms, some every fourth, fifth, tenth, or twentieth term. The index of the root, 80, has as factors 2^4 and 5, so that the series readily yields sub-series having as ratios the roots of 10 with indices 2, 4, 8, 16, 5, 10, 20, 40, thus giving a wide range of choice.

The strict logic of this series has been somewhat impaired by the adoption of rounded values that are slightly different in the 1 to 10 and 10 to 100 intervals. For the United States, the A.S.A. has adopted a Table of Preferred Numbers (A.S.A. Z17.1-1936) which differs slightly from the system described in the preceding paragraph.

Another type of series is the **semigeometric** series (Steczynski, *loc. cit.*) consisting of a basic geometric series with 1 as the base term, and a ratio of 2, giving a series . . . $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4 . . . . Between consecutive terms are inserted arithmetic series of 2, 4, 8, or 16 terms, in general using different numbers of terms in different intervals.

SECTION 8

POWER GENERATION

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